

RMS Values of Commonly Observed Converter Waveforms

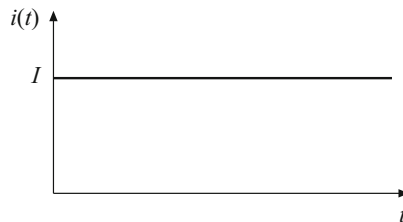
The waveforms encountered in power electronics converters can be quite complex, containing modulation at the switching frequency and often also at the ac line frequency. During converter design, it is often necessary to compute the rms values of such waveforms. In this appendix, several useful formulas and tables are developed which allow these rms values to be quickly determined.

RMS values of the doubly modulated waveforms encountered in PWM rectifier circuits are discussed in Sect. 21.5.

A.1 Some Common Waveforms

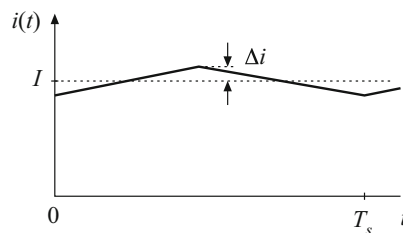
DC:

$$rms = I \quad (A.1)$$



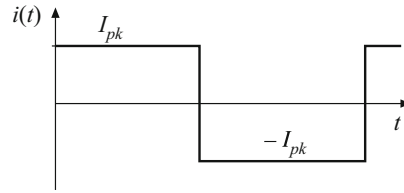
DC plus linear ripple:

$$rms = I \sqrt{1 + \frac{1}{3} \left(\frac{\Delta i}{I} \right)^2} \quad (A.2)$$



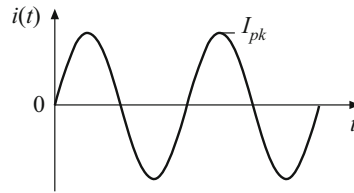
Square wave:

$$rms = I_{pk} \quad (A.3)$$



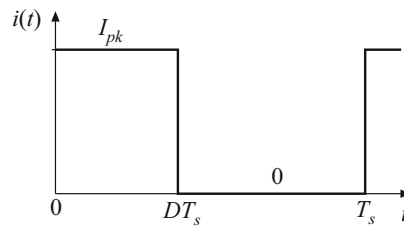
Sine wave:

$$rms = \frac{I_{pk}}{\sqrt{2}} \quad (A.4)$$



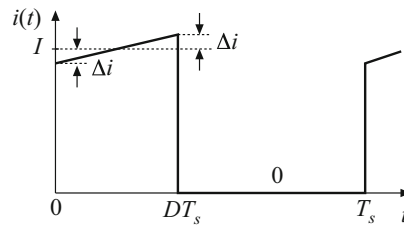
Pulsating waveform:

$$rms = I_{pk} \sqrt{D} \quad (A.5)$$



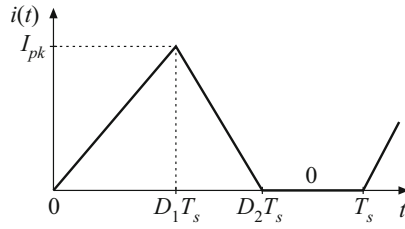
Pulsating waveform with linear ripple:

$$rms = I \sqrt{D} \sqrt{1 + \frac{1}{3} \left(\frac{\Delta i}{I} \right)^2} \quad (A.6)$$



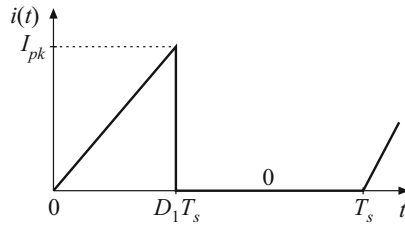
Triangular waveform:

$$rms = I_{pk} \sqrt{\frac{D_1 + D_2}{3}} \quad (A.7)$$



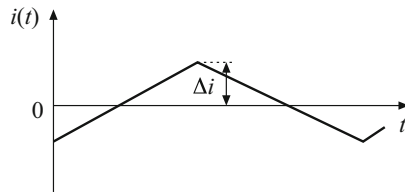
Triangular waveform:

$$rms = I_{pk} \sqrt{\frac{D_1}{3}} \quad (A.8)$$



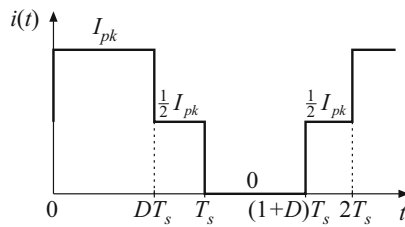
Triangular waveform, no dc component:

$$rms = \frac{\Delta i}{\sqrt{3}} \quad (A.9)$$



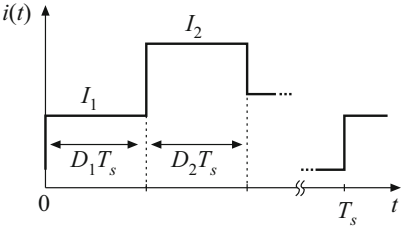
Center-tapped bridge winding waveform:

$$rms = \frac{1}{2} I_{pk} \sqrt{1 + D} \quad (A.10)$$

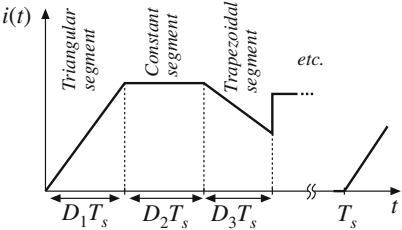


General stepped waveform:

$$rms = \sqrt{D_1 I_1^2 + D_2 I_2^2 + \dots} \tag{A.11}$$



A.2 General Piecewise Waveform



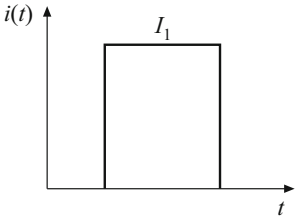
For a periodic waveform composed of n piecewise segments as shown above, the rms value is

$$rms = \sqrt{\sum_{k=1}^n D_k u_k} \tag{A.12}$$

where D_k is the duty cycle of segment k , and u_k is the contribution of segment k . The u_k s depend on the shape of the segments—several common segment shapes are listed below.

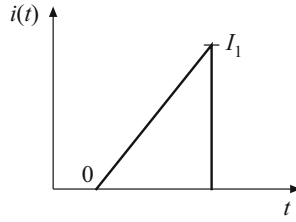
Constant segment:

$$u_k = I_1^2 \tag{A.13}$$



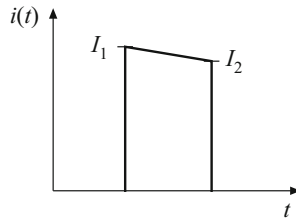
Triangular segment:

$$u_k = \frac{1}{3} I_1^2 \quad (\text{A.14})$$



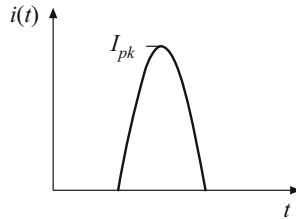
Trapezoidal segment:

$$u_k = \frac{1}{3} (I_1^2 + I_1 I_2 + I_2^2) \quad (\text{A.15})$$



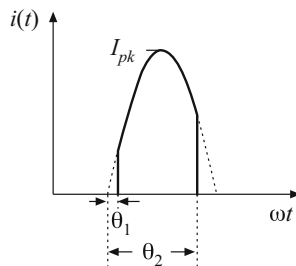
Sinusoidal segment, half or full period:

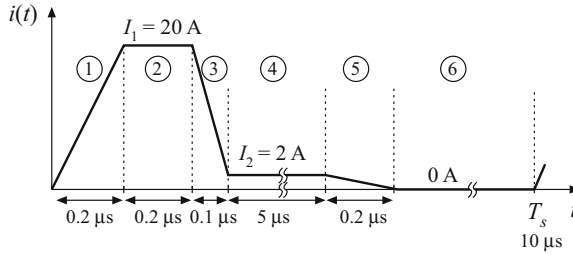
$$u_k = \frac{1}{2} I_{pk}^2 \quad (\text{A.16})$$



Sinusoidal segment, partial period: a sinusoidal segment of less than one half-period, which begins at angle θ_1 and ends at angle θ_2 . The angles θ_1 and θ_2 are expressed in radians:

$$u_k = \frac{1}{2} I_{pk}^2 \left(1 - \frac{\sin(\theta_2 - \theta_1) \cos(\theta_2 + \theta_1)}{(\theta_2 - \theta_1)} \right) \quad (\text{A.17})$$



Example

A transistor current waveform contains a current spike due to the stored charge of a free-wheeling diode. The observed waveform can be approximated as shown above. Estimate the rms current.

The waveform can be divided into six approximately linear segments, as shown. The D_k and u_k for each segment are

1. Triangular segment:

$$D_1 = (0.2\mu s)/(10\mu s) = 0.02$$

$$u_1 = I_1^2/3 = (20A)^2/3 = 133A^2$$

2. Constant segment:

$$D_2 = (0.2\mu s)/(10\mu s) = 0.02$$

$$u_2 = I_1^2 = (20A)^2 = 400A^2$$

3. Trapezoidal segment:

$$D_3 = (0.1\mu s)/(10\mu s) = 0.01$$

$$u_3 = (I_1^2 + I_2^2 + I_3^2)/3 = 148A^2$$

4. Constant segment:

$$D_4 = (5\mu s)/(10\mu s) = 0.5$$

$$u_4 = I_2^2 = (2A)^2 = 4A^2$$

5. Triangular segment:

$$D_5 = (0.2\mu s)/(10\mu s) = 0.02$$

$$u_5 = I_2^2/3 = (2A)^2/3 = 1.3A^2$$

6. Zero segment:

$$u_6 = 0$$

The rms value is

$$rms = \sqrt{\sum_{k=1}^6 D_k u_k} = 3.76A \quad (A.18)$$

Even though its duration is very short, the current spike has a significant impact on the rms value of the current—without the current spike, the rms current is approximately 2.0 A.

Magnetics Design Tables

Geometrical data for several standard ferrite core shapes are listed here. The geometrical constant K_g is a measure of core size, useful for designing inductors and transformers that attain a given copper loss [99]. The K_g method for inductor design is described in Chap. 11. K_g is defined as

$$K_g = \frac{A_c^2 W_A}{MLT} \quad (\text{B.1})$$

where A_c is the core cross-sectional area, W_A is the window area, and MLT is the winding mean-length-per-turn. The geometrical constant K_{gfe} is a similar measure of core size, which is useful for designing ac inductors and transformers when the total copper plus core loss is constrained. The K_{gfe} method for magnetics design is described in Chap. 12. K_{gfe} is defined as

$$K_{gfe} = \frac{W_A A_c^{2(1-1/\beta)}}{MLT \ell_m^{2/\beta}} u(\beta) \quad (\text{B.2})$$

where ℓ_m is the core mean magnetic path length, and β is the core loss exponent:

$$P_{fe} = K_{fe} B_{max}^\beta \quad (\text{B.3})$$

For modern ferrite materials, β typically lies in the range 2.6 to 2.8. The quantity $u(\beta)$ is defined as

$$u(\beta) = \left[\left(\frac{\beta}{2} \right)^{-\left(\frac{\beta}{\beta+2} \right)} + \left(\frac{\beta}{2} \right)^{\left(\frac{2}{\beta+2} \right)} \right]^{-\left(\frac{\beta+2}{\beta} \right)} \quad (\text{B.4})$$

$u(\beta)$ is equal to 0.305 for $\beta = 2.7$. This quantity varies by roughly 5% over the range $2.6 \leq \beta \leq 2.8$. Values of K_{gfe} are tabulated for $\beta = 2.7$; variation of K_{gfe} over the range $2.6 \leq \beta \leq 2.8$ is typically quite small.

Thermal resistances are listed in those cases where published manufacturer's data are available. The thermal resistances listed are the approximate temperature rise from the center leg of the core to ambient, per watt of total power loss. Different temperature rises may be observed under conditions of forced air cooling, unusual power loss distributions, etc. Listed window areas are the winding areas for conventional single-section bobbins.

An American Wire Gauge table is included at the end of this appendix.

B.1 Pot Core Data

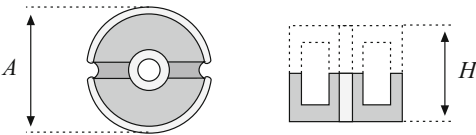


Fig. B.1 Pot core

Core type	Geometrical constant	Geometrical constant	Cross-sectional area	Bobbin winding area	Mean length per turn	Magnetic path length	Thermal resistance	Core weight
(AH) (mm)	K_g cm^5	K_{gfe} cm^x	A_c (cm^2)	W_A (cm^2)	MLT (cm)	ℓ_m (cm)	R_{th} ($^{\circ}\text{C}/\text{W}$)	(g)
704	$0.738 \cdot 10^{-6}$	$1.61 \cdot 10^{-6}$	0.070	$0.22 \cdot 10^{-3}$	1.46	1.0		0.5
905	$0.183 \cdot 10^{-3}$	$256 \cdot 10^{-6}$	0.101	0.034	1.90	1.26		1.0
1107	$0.667 \cdot 10^{-3}$	$554 \cdot 10^{-6}$	0.167	0.055	2.30	1.55		1.8
1408	$2.107 \cdot 10^{-3}$	$1.1 \cdot 10^{-3}$	0.251	0.097	2.90	2.00	100	3.2
1811	$9.45 \cdot 10^{-3}$	$2.6 \cdot 10^{-3}$	0.433	0.187	3.71	2.60	60	7.3
2213	$27.1 \cdot 10^{-3}$	$4.9 \cdot 10^{-3}$	0.635	0.297	4.42	3.15	38	13
2616	$69.1 \cdot 10^{-3}$	$8.2 \cdot 10^{-3}$	0.948	0.406	5.28	3.75	30	20
3019	0.180	$14.2 \cdot 10^{-3}$	1.38	0.587	6.20	4.50	23	34
3622	0.411	$21.7 \cdot 10^{-3}$	2.02	0.748	7.42	5.30	19	57
4229	1.15	$41.1 \cdot 10^{-3}$	2.66	1.40	8.60	6.81	13.5	104

B.2 EE Core Data

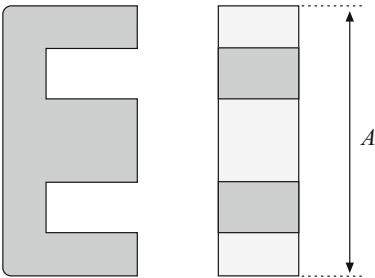


Fig. B.2 EE core

Core type	Geometrical constant	Geometrical constant	Cross-sectional area	Bobbin winding area	Mean length per turn	Magnetic path length	Core weight
(A) (mm)	K_g (cm ⁵)	K_{gfe} (cm ^x)	A_c (cm ²)	W_A (cm ²)	MLT (cm)	ℓ_m (cm)	(g)
EE12	$0.731 \cdot 10^{-3}$	$0.458 \cdot 10^{-3}$	0.14	0.085	2.28	2.7	2.34
EE16	$2.02 \cdot 10^{-3}$	$0.842 \cdot 10^{-3}$	0.19	0.190	3.40	3.45	3.29
EE19	$4.07 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	0.23	0.284	3.69	3.94	4.83
EE22	$8.26 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$	0.41	0.196	3.99	3.96	8.81
EE30	$85.7 \cdot 10^{-3}$	$6.7 \cdot 10^{-3}$	1.09	0.476	6.60	5.77	32.4
EE40	0.209	$11.8 \cdot 10^{-3}$	1.27	1.10	8.50	7.70	50.3
EE50	0.909	$28.4 \cdot 10^{-3}$	2.26	1.78	10.0	9.58	116
EE60	1.38	$36.4 \cdot 10^{-3}$	2.47	2.89	12.8	11.0	135
EE70/68/19	5.06	$75.9 \cdot 10^{-3}$	3.24	6.75	14.0	18.0	280

B.3 EC Core Data

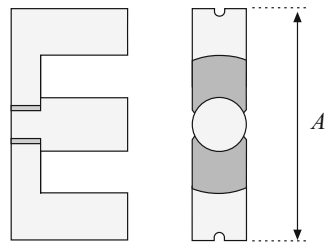


Fig. B.3 EC core

Core type	Geometrical constant	Geometrical constant	Cross-sectional area	Bobbin winding area	Mean length per turn	Magnetic path length	Thermal resistance	Core weight
(A) (mm)	K_g (cm ⁵)	K_{gfe} (cm ^x)	A_c (cm ²)	W_A (cm ²)	MLT (cm)	ℓ_m (cm)	R_{th} (°C/W)	(g)
EC35	0.131	$9.9 \cdot 10^{-3}$	0.843	0.975	5.30	7.74	18.5	35.5
EC41	0.374	$19.5 \cdot 10^{-3}$	1.21	1.35	5.30	8.93	16.5	57.0
EC52	0.914	$31.7 \cdot 10^{-3}$	1.80	2.12	7.50	10.5	11.0	111
EC70	2.84	$56.2 \cdot 10^{-3}$	2.79	4.71	12.9	14.4	7.5	256

B.4 ETD Core Data

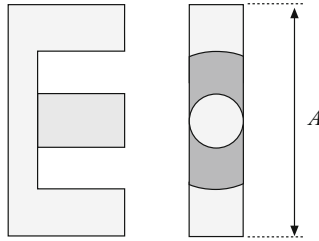


Fig. B.4 ETD core

Core type	Geometrical constant	Geometrical constant	Cross-sectional area	Bobbin winding area	Mean length per turn	Magnetic path length	Thermal resistance	Core weight
(A) (mm)	K_g (cm ⁵)	K_{gfe} (cm ^x)	A_c (cm ²)	W_A (cm ²)	MLT (cm)	ℓ_m (cm)	R_{th} (°C/W)	(g)
ETD29	0.0978	$8.5 \cdot 10^{-3}$	0.76	0.903	5.33	7.20		30
ETD34	0.193	$13.1 \cdot 10^{-3}$	0.97	1.23	6.00	7.86	19	40
ETD39	0.397	$19.8 \cdot 10^{-3}$	1.25	1.74	6.86	9.21	15	60
ETD44	0.846	$30.4 \cdot 10^{-3}$	1.74	2.13	7.62	10.3	12	94
ETD49	1.42	$41.0 \cdot 10^{-3}$	2.11	2.71	8.51	11.4	11	124

B.5 PQ Core Data

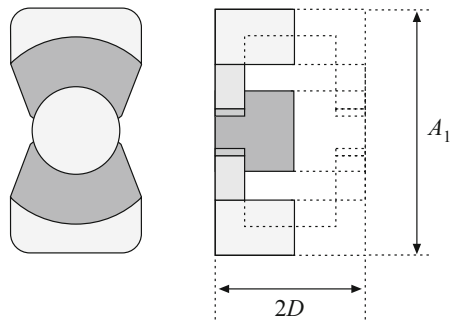


Fig. B.5 PQ core

Core type	Geometrical constant	Geometrical constant	Cross-sectional area	Bobbin winding area	Mean length per turn	Magnetic path length	Core weight
$(A_1/2D)$ (mm)	K_g (cm ⁵)	K_{gfe} (cm ^x)	A_c (cm ²)	W_A (cm ²)	MLT (cm)	ℓ_m (cm)	(g)
PQ20/16	$22.4 \cdot 10^{-3}$	$3.7 \cdot 10^{-3}$	0.62	0.256	4.4	3.74	13
PQ20/20	$33.6 \cdot 10^{-3}$	$4.8 \cdot 10^{-3}$	0.62	0.384	4.4	4.54	15
PQ26/20	$83.9 \cdot 10^{-3}$	$7.2 \cdot 10^{-3}$	1.19	0.333	5.62	4.63	31
PQ26/25	0.125	$9.4 \cdot 10^{-3}$	1.18	0.503	5.62	5.55	36
PQ32/20	0.203	$11.7 \cdot 10^{-3}$	1.70	0.471	6.71	5.55	42
PQ32/30	0.384	$18.6 \cdot 10^{-3}$	1.61	0.995	6.71	7.46	55
PQ35/35	0.820	$30.4 \cdot 10^{-3}$	1.96	1.61	7.52	8.79	73
PQ40/40	1.20	$39.1 \cdot 10^{-3}$	2.01	2.50	8.39	10.2	95

B.6 American Wire Gauge Data

AWG #	Bare area, 10^{-3} cm^2	Resistance, $10^{-6} \Omega/\text{cm}$	Diameter, cm
0000	1072.3	1.608	1.168
000	850.3	2.027	1.040
00	674.2	2.557	0.927
0	534.8	3.224	0.825
1	424.1	4.065	0.735
2	336.3	5.128	0.654
3	266.7	6.463	0.583
4	211.5	8.153	0.519
5	167.7	10.28	0.462
6	133.0	13.0	0.411
7	105.5	16.3	0.366
8	83.67	20.6	0.326
9	66.32	26.0	0.291
10	52.41	32.9	0.267
11	41.60	41.37	0.238
12	33.08	52.09	0.213
13	26.26	69.64	0.190
14	20.02	82.80	0.171
15	16.51	104.3	0.153
16	13.07	131.8	0.137
17	10.39	165.8	0.122
18	8.228	209.5	0.109
19	6.531	263.9	0.0948
20	5.188	332.3	0.0874
21	4.116	418.9	0.0785
22	3.243	531.4	0.0701
23	2.508	666.0	0.0632
24	2.047	842.1	0.0566
25	1.623	1062.0	0.0505
26	1.280	1345.0	0.0452
27	1.021	1687.6	0.0409
28	0.8046	2142.7	0.0366
29	0.6470	2664.3	0.0330

(continued)

AWG #	Bare area, 10^{-3} cm^2	Resistance, $10^{-6} \Omega/\text{cm}$	Diameter, cm
30	0.5067	3402.2	0.0294
31	0.4013	4294.6	0.0267
32	0.3242	5314.9	0.0241
33	0.2554	6748.6	0.0236
34	0.2011	8572.8	0.0191
35	0.1589	10849	0.0170
36	0.1266	13608	0.0152
37	0.1026	16801	0.0140
38	0.08107	21266	0.0124
39	0.06207	27775	0.0109
40	0.04869	35400	0.0096
41	0.03972	43405	0.00863
42	0.03166	54429	0.00762
43	0.02452	70308	0.00685
44	0.0202	85072	0.00635