$$W_t = K_3 \left(\frac{A_p}{K_2} \right)^{0.75} \qquad [5\text{-}20]$$

$$K_w = \frac{K_3}{K_2^{(0.75)}} \qquad [5\text{-}21]$$

The weight-area product, A_p, relationship is therefore:

$$W_t = K_w A_p^{(0.75)} \qquad [5\text{-}22]$$

In which, K_w, is a constant related to core configuration, whose values are given in Table 5-3, These values were obtained by averaging the values from the data taken from the Tables 3-1 through Tables 3-64 in Chapter 3.

Table 5-3. Weight-Area Product Relationship

Weight-Area Product Relationship	
Core Type	K_w
Pot Core	48.0
Powder Core	58.8
Laminations	68.2
C Core	66.6
Single-coil C Core	76.6
Tape-wound Core	82.3

The relationship between weight and area product, A_p, for various core types is graphed in Figures 5-8 through 5-10. The data for Figures 5-8 through 5-10 has been taken from Tables in Chapter 3.

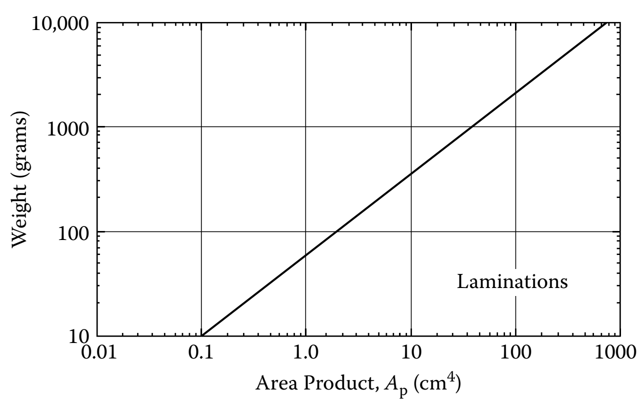

Figure 5-8. Total Weight Versus Area Product, A_p, for EI Laminations.

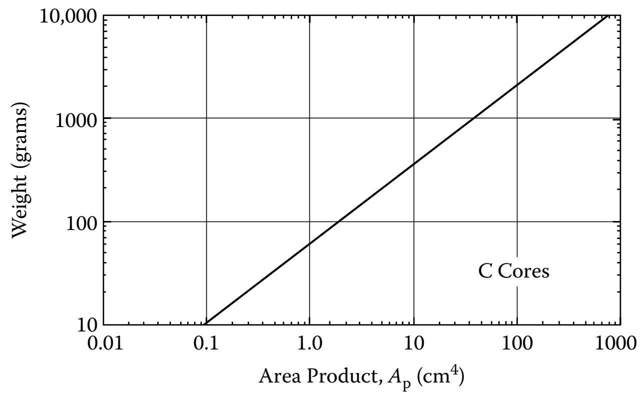

Figure 5-9. Total Weight Versus Area Product, A_p, for C Cores.

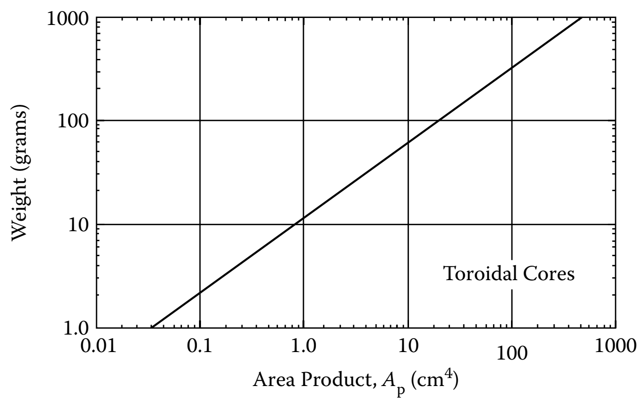

Figure 5-10. Total Weight Versus Area Product, A_p, for Toroidal MPP Cores.

Transformer Surface Area and the Area Product, A_p

The surface area of a transformer can be related to the area product, A_p, of a transformer, treating the surface area, as shown in Figure 5-11 through 5-13. The relationship is derived in accordance with the following reasoning: the surface area varies with the square of any linear dimension (l), whereas the area product, A_p, varies as the fourth power.

$$A_t = K_4 l^2, \quad [\mathrm{cm}^2] \tag{5-23}$$

$$A_p = K_2 l^4, \quad [\mathrm{cm}^4] \tag{5-24}$$

$$l^4 = \frac{A_p}{K_2} \tag{5-25}$$

$$l = \left(\frac{A_p}{K_2}\right)^{(0.25)} \tag{5-26}$$

$$l^2 = \left[\left(\frac{A_p}{K_2}\right)^{0.25}\right]^2 = \left(\frac{A_p}{K_2}\right)^{0.5} \quad [5\text{-}27]$$

$$A_t = K_4\left(\frac{A_p}{K_2}\right)^{0.5} \quad [5\text{-}28]$$

$$K_s = \frac{K_4}{K_2^{(0.5)}} \quad [5\text{-}29]$$

The relationship between surface area, A_t and area product, A_p can be expressed as:

$$A_t = K_s A_p^{(0.5)} \quad [5\text{-}30]$$

In which, K_s, is a constant related to core configuration, whose values are given in Table 5-4. These values were obtained by averaging the values from the data taken from the Tables 3-1 through Tables 3-64 in Chapter 3.

Table 5-4. Surface Area-Area Product Relationship

Surface Area-Area Product Relationship	
Core Type	K_s
Pot Core	33.8
Powder Core	32.5
Laminations	41.3
C Core	39.2
Single-coil C Core	44.5
Tape-wound Core	50.9

The surface area for toroidal type transformers is calculated, as shown below.

$$\text{Top and Bottom Surface} = 2\left(\frac{\pi(OD)^2}{4}\right), \quad [\text{cm}^2]$$

$$\text{Periphery Surface} = \left(\pi(OD)\right)\left(\text{Height}\right), \quad [\text{cm}^2] \quad [5\text{-}31]$$

$$A_t = \frac{\pi(OD)^2}{2} + (\pi(OD)(\text{Height}), \quad [\text{cm}^2]$$

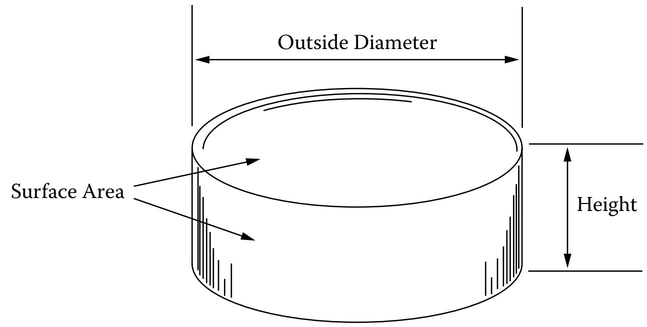

Figure 5-11. Toroidal Transformer Outline Showing the Surface Area.

The surface areas for C cores, Laminations and similar configurations are calculated as shown below. There is a small amount of area that is deducted because the sides and the ends are not a complete square.

$$\begin{aligned}
&\text{End} = (\text{Height})(\text{Length}), \quad [\text{cm}^2] \\
&\text{Top} = (\text{Length})(\text{Width}), \quad [\text{cm}^2] \\
&\text{Side} = (\text{Height})(\text{Width}), \quad [\text{cm}^2] \\
&\text{Surface Area} = 2(\text{End}) + 2(\text{Top}) + 2(\text{Side}), \quad [\text{cm}^2]
\end{aligned} \qquad [5\text{-}32]$$

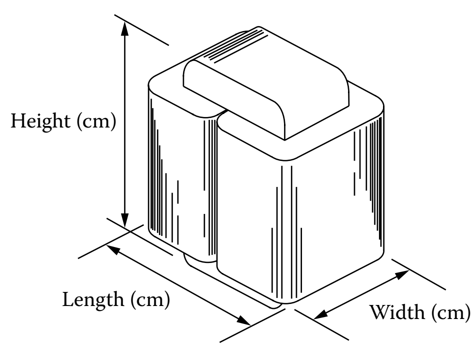

Figure 5-12. C Core Transformer Outline, Showing the Surface Area.

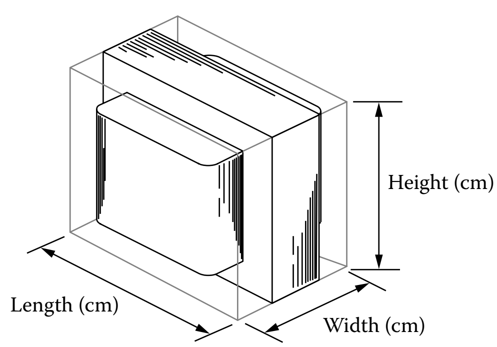

Figure 5-13. Typical EE or EI Transformer Outline, Showing the Surface Area.

The relationship between surface area and area product, A_p, for various core types is graphed in Figures 5-14 through 5-16. The data for these Figures has been taken from Tables in Chapter 3.

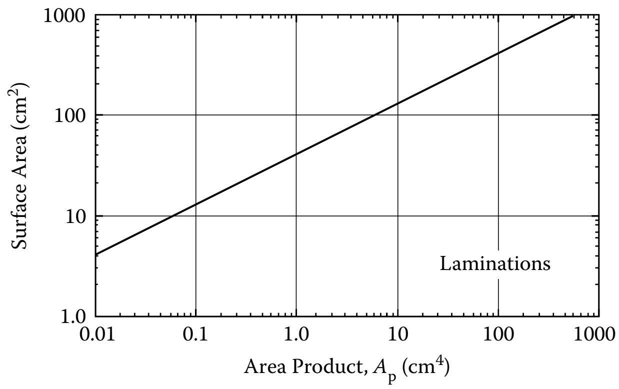

Figure 5-14. Surface Area, A_t, Versus Area Product, A_p, for EI Laminations.

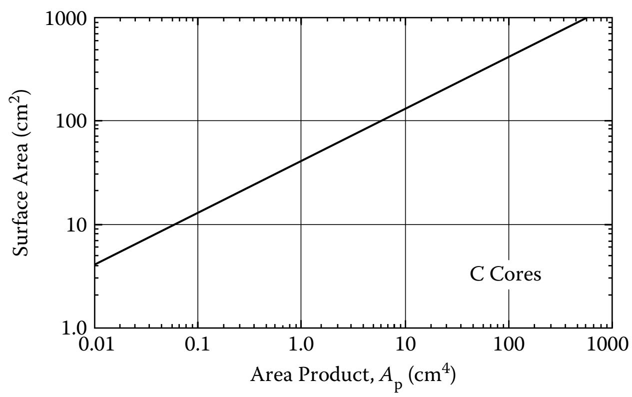

Figure 5-15. Surface Area, A_t, Versus Area Product, A_p, for C Cores.

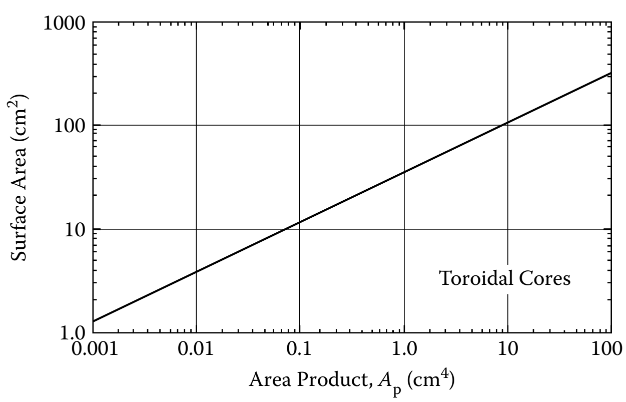

Figure 5-16. Surface Area, A_t, Versus Area Product, A_p, for Toroidal MPP Cores.

Transformer Current Density, J, and the Area Product, A_p

The current density, J, of a transformer can be related to the area product, A_p, of a transformer for a given temperature rise. The relationship can be derived as follows:

$$A_t = K_s A_p^{(0.5)}, \quad [\text{cm}^2] \tag{5-33}$$

$$P_{cu} = I^2 R, \quad [\text{watts}] \tag{5-34}$$

$$I = A_w J, \quad [\text{amps}] \tag{5-35}$$

Therefore,

$$P_{cu} = A_w^2 J^2 R \tag{5-36}$$

And since,

$$R = \frac{\text{MLT}}{A_w} N\rho, \quad [\text{ohms}] \tag{5-37}$$

We have:

$$P_{cu} = A_w^2 J^2 \frac{\text{MLT}}{A_w} N\rho \tag{5-38}$$

$$P_{cu} = A_w J^2 (\text{MLT}) N\rho \tag{5-39}$$

Since MLT has a dimension of length,

$$\text{MLT} = K_5 A_p^{(0.25)} \tag{5-40}$$

$$P_{cu} = A_w J^2 \left(K_5 A_p^{(0.25)}\right) N\rho \tag{5-41}$$

$$A_w N = K_3 W_a = K_6 A_p^{(0.5)} \tag{5-42}$$

$$P_{cu} = \left(K_6 A_p^{(0.5)}\right)\left(K_5 A_p^{(0.25)}\right) J^2 \rho \tag{5-43}$$

Let:

$$K_7 = K_6 K_5 \rho \tag{5-44}$$

Then assuming the core loss is the same as the copper loss for optimized transformer operation: (See Chapter 6),

$$P_{cu} = K_7 A_p^{(0.75)} J^2 = P_{fe} \qquad [5\text{-}45]$$

$$P_\Sigma = P_{cu} + P_{fe} \qquad [5\text{-}46]$$

$$\Delta T = K_8 \frac{P_\Sigma}{A_t} \qquad [5\text{-}47]$$

$$\Delta T = \frac{2K_8 K_7 J^2 A_p^{(0.75)}}{K_s A_p^{(0.5)}} \qquad [5\text{-}48]$$

To simplify, let:

$$K_9 = \frac{2K_8 K_7}{K_s} \qquad [5\text{-}49]$$

Then,

$$\Delta T = K_9 J^2 A_p^{(0.25)} \qquad [5\text{-}50]$$

$$J^2 = \frac{\Delta T}{K_9 A_p^{(0.25)}} \qquad [5\text{-}51]$$

Then, letting:

$$K_{10} = \frac{\Delta T}{K_9} \qquad [5\text{-}52]$$

We have:

$$J^2 = K_{10} A_p^{(0.25)} \qquad [5\text{-}53]$$

The relationship between current density, J, and area product, A_p, can, therefore, be expressed as:

$$J = K_j A_p^{(0.125)} \qquad [5\text{-}54]$$

The constant, K_j, is related to the core configuration, whose values are given in Table 5-5. These values have been derived by averaging the values from the data taken from Tables 3-1 through Tables 3-64 in Chapter 3.

Table 5-5. Constant, K_j, for Temperature Increases of 25°C and 50°C

Temperature Constant, K_j		
Core Type	K_j (Δ25°)	K_j (Δ50°)
Pot Core	433	632
Powder Core	403	590
Laminations	366	534
C Core	322	468
Single-coil C Core	395	569
Tape-wound Core	250	365

The relationship between current density, J, and area product, A_p, for temperature increases of 25°C and 50°C is graphed in Figures 5-17 through 5-19 from data calculated of Tables 3-1 through 3-64 in Chapter 3.

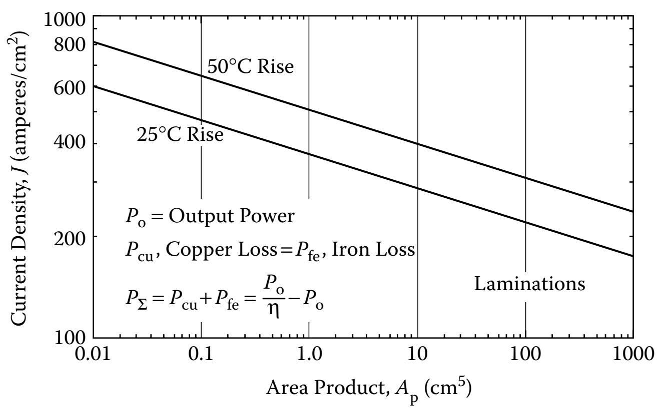

Figure 5-17. Current Density, J, Versus Area Product, A_p, for EI Laminations.

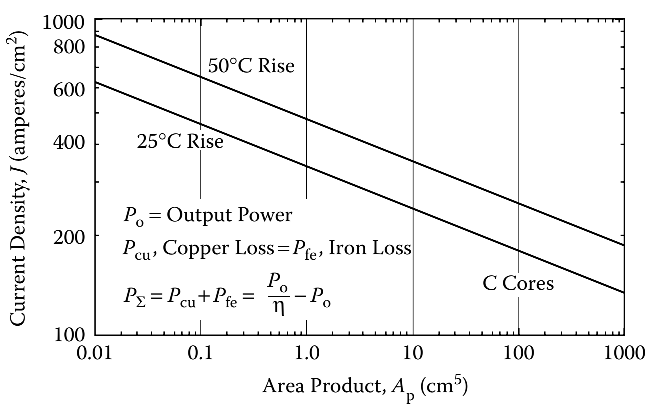

Figure 5-18. Current Density, J, Versus Area Product, A_p, for C Cores.

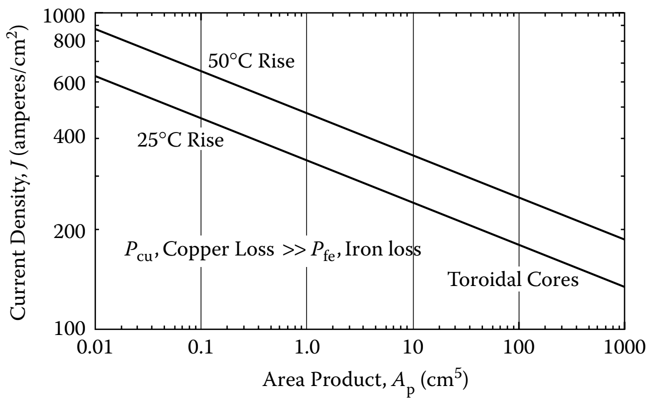

Figure 5-19. Current Density, J, Versus Area Product, A_p, for MPP Cores.

Transformer Core Geometry, K_g, and the Area Product, A_p

The core geometry, K_g, of a transformer can be related to the area product, A_p. The relationship is according to the following: the core geometry, K_g, varies in accordance with the fifth power of any linear dimension, (l), whereas area product, A_p, varies as the fourth power.

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \qquad [5\text{-}55]$$

$$K_g = K_{10} l^5 \qquad [5\text{-}56]$$

$$A_p = K_2 l^4 \qquad [5\text{-}57]$$

From Equation 5-56,

$$l = \left(\frac{K_g}{K_{10}}\right)^{(0.2)} \qquad [5\text{-}58]$$

Then,

$$l^4 = \left[\left(\frac{K_g}{K_{10}}\right)^{(0.2)}\right]^4 = \left(\frac{K_g}{K_{10}}\right)^{(0.8)} \qquad [5\text{-}59]$$

Substituting Equation 5-59 into Equation 5-57,

$$A_p = K_2 \left(\frac{K_g}{K_{10}}\right)^{(0.8)} \qquad [5\text{-}60]$$

Let:

$$K_p = \frac{K_2}{K_{10}^{(0.8)}} \quad [5\text{-}61]$$

Then,

$$A_p = K_p K_g^{(0.8)} \quad [5\text{-}62]$$

The constant, K_p, is related to the core configuration, whose values are given in Table 5-6. These values have been derived by averaging the values from the data taken from Tables 3-1 through Tables 3-64 in Chapter 3.

Table 5-6. Configuration Constant, K_p, for Area Product, A_p, and Core geometry, K_g

Constant, K_p	
Core Type	K_p
Pot Core	8.9
Powder Core	11.8
Laminations	8.3
C Core	12.5
Tape-wound Core	14.0

The relationship between area product, A_p, and core geometry, K_g, is graphed in Figures 5-20 through 5-22, from the data taken from Tables 3-1 through Tables 3-64 in Chapter 3.

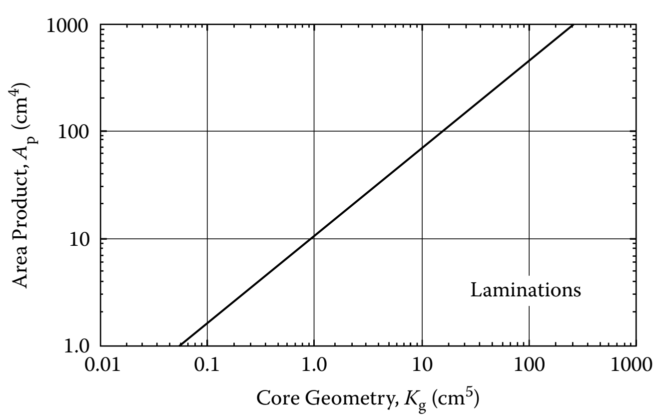

Figure 5-20. Area Product, A_p, Versus Core Geometry, K_g, for EI Laminations.

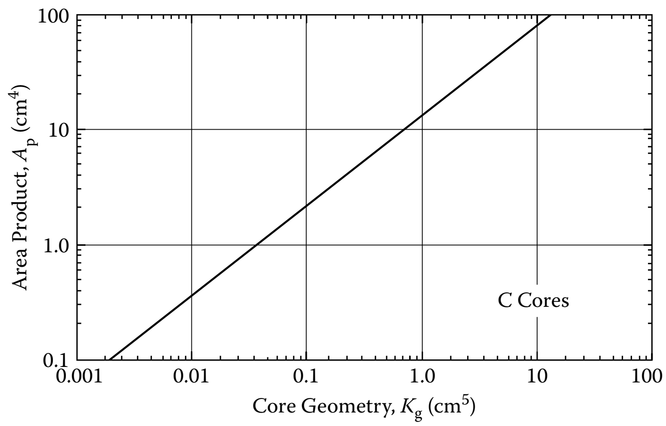

Figure 5-21. Area Product, A_p, Versus Core Geometry, K_g, for C Cores.

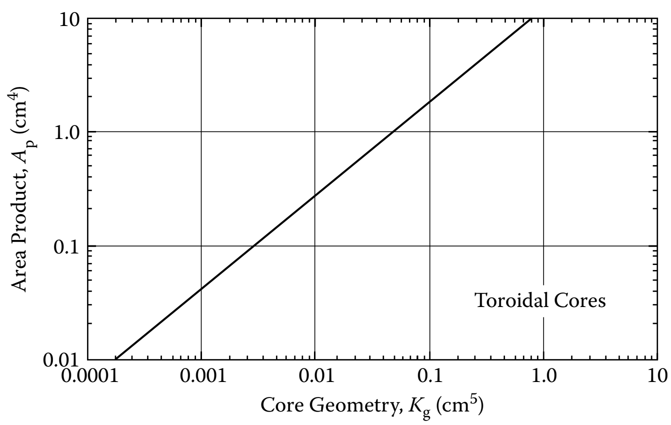

Figure 5-22. Area Product, A_p, Versus Core Geometry, K_g, for MPP Powder Cores.

Weight Versus Transformer Regulation

There are many design tasks where the transformer weight is very important in meeting the design specification. The engineer will raise the operating frequency in order to reduce the size and weight. The magnetic materials will be reviewed for performance at the operating frequency, and at the minimum and maximum temperatures. When the idealized magnetic material has been found and the weight of the transformer is still too high, then the only solution is to change the regulation. The regulation of a transformer versus the weight is shown in Figure 5-23. There are times when the engineer would like to know what the weight impact would be, if the regulation were to be increased or decreased.

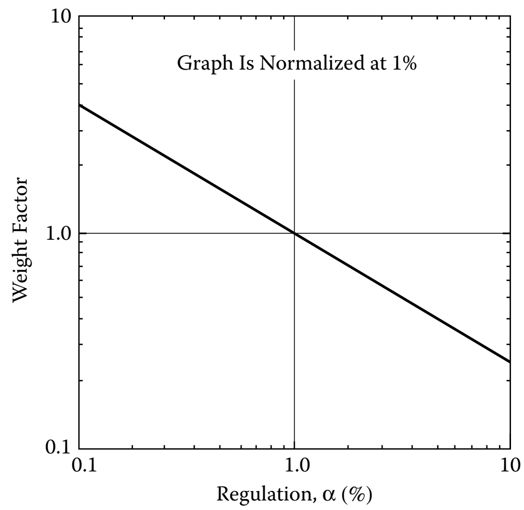

Figure 5-23. Weight Versus Regulation.

References

1. C. McLyman, Transformer Design Trade-offs, Technical Memorandum 33-767 Rev. 1, Jet Propulsion Laboratory, Pasadena, CA.

2. W. J. Muldoon, High Frequency Transformer Optimization, HAC Trade Study Report 2228/1130, May, 1970

3. R. G. Klimo, A. B. Larson, and J. E. Murray, Optimization Study of High Power Static Inverters and Converters, Quarterly report No. 2 NASA-CR-54021, April 20, 1964, Contract NAS 3-2785.

4. F. F. Judd and D. R. Kessler, Design Optimization of Power Transformers, Bell Laboratories, Whippany, New Jersey IEEE Applied Magnetics Workshop, June 5–6, 1975

Chapter 6

Transformer-Inductor Efficiency, Regulation, and Temperature Rise

Table of Contents

Introduction

Transformer efficiency, regulation, and temperature rise are all interrelated. Not all of the input power to the transformer is delivered to the load. The difference between input power and output power is converted into heat. This power loss can be broken down into two components: core loss, P_{fe}, and copper loss, P_{cu}. The core loss is a fixed loss, and the copper loss is a variable loss that is related to the current demand of the load. The copper loss increases by the square of the current and also is termed a quadratic loss. Maximum efficiency is achieved when the fixed loss is equal to the quadratic loss at rated load. Transformer, regulation, α is the copper loss, P_{cu}, divided by the output power, P_o, as shown in Equation [6-1].

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%] \tag{6-1}$$

Transformer Efficiency

The efficiency of a transformer is a good way to measure the effectiveness of the design. Efficiency is defined as the ratio of the output power, P_o, to the input power, P_{in}. The difference between, P_o, and, P_{in}, is due to losses. The total power loss, P_Σ, in the transformer is determined by the fixed losses in the core and the quadratic losses in the windings or copper, as shown in Equation [6-2].

$$P_\Sigma = P_{fe} + P_{cu}, \quad [\text{watts}] \tag{6-2}$$

Where, P_{fe}, is the core loss, and P_{cu}, is the copper loss.

Maximum Efficiency

Maximum efficiency is achieved when the fixed loss is made equal to the quadratic loss, as shown by Equation 6-12. A graph of transformer loss versus output load current is shown in Figure 6-1.

The copper loss increases as the square of the output power, P_o, is multiplied by a constant, K_c:

$$P_{cu} = K_c P_o^2 \tag{6-3}$$

Which may be rewritten as:

$$P_\Sigma = P_{fe} + K_c P_o^2 \tag{6-4}$$

Since:

$$P_{in} = P_o + P_\Sigma \tag{6-5}$$

The efficiency can be expressed as:

$$\eta = \frac{P_o}{P_o + P_\Sigma} \qquad [6\text{-}6]$$

Then, substituting Equation 6-4 into 6-6 gives:

$$\eta = \frac{P_o}{P_o + P_{fe} + KP_o^2} = \frac{P_o}{P_{fe} + P_o + KP_o^2} \qquad [6\text{-}7]$$

And, differentiating with respect to P_o:

$$\frac{d\eta}{dP_o} = \frac{P_{fe} + P_o + KP_o^2 - P_o\left(1 + 2KP_o\right)}{\left(P_{fe} + P_o + KP_o^2\right)^2} \qquad [6\text{-}8]$$

Then, to solve for the maximum, equate Equation 6-8 to 0.

$$\frac{P_{fe} + P_o + KP_o^2 - P_o\left(1 + 2KP_o\right)}{\left(P_{fe} + P_o + KP_o^2\right)^2} = 0 \qquad [6\text{-}9]$$

$$-P_o\left(1 + 2KP_o\right) + \left(P_{fe} + P_o + KP_o^2\right) = 0 \qquad [6\text{-}10]$$

$$-P_o - 2KP_o^2 + P_{fe} + P_o + KP_o^2 = 0 \qquad [6\text{-}11]$$

Therefore,

$$P_{fe} = KP_o^2 = P_{cu} \qquad [6\text{-}12]$$

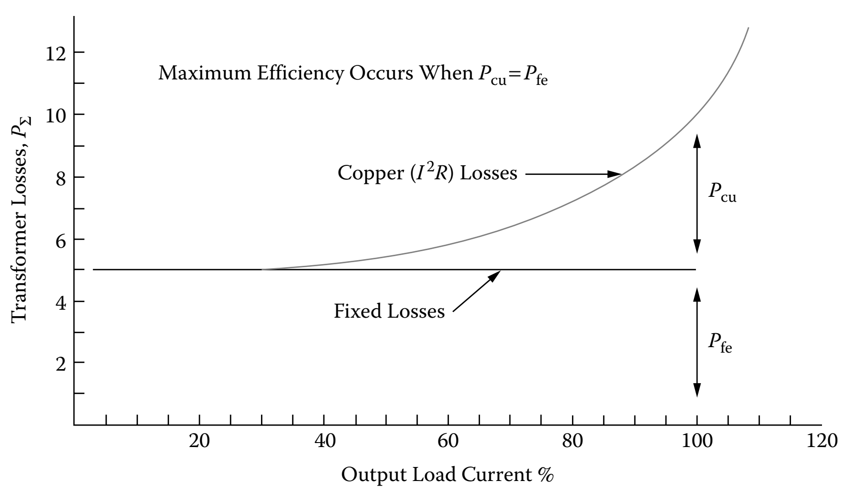

Figure 6-1. Transformer Losses Versus Output Load Current.

Transformer Dissipation, by Radiation and Convection

Temperature rise in a transformer winding cannot be predicted with complete precision, despite the fact that many techniques are described in the literature for its calculation. One reasonable accurate method for open core and winding construction is based upon the assumption that core and winding losses may be lumped together as:

$$P_{\Sigma} = P_{cu} + P_{fe}, \quad \text{[watts]} \qquad [6\text{-}13]$$

And the assumption is made that thermal energy is dissipated uniformly throughout the surface area of the core and winding assembly.

Transfer of heat by thermal radiation occurs when a body is raised to a temperature above its surroundings, and emits radiant energy in the form of waves. In accordance with Stefan-Boltzmann Law, (in Ref. 1) this transfer of heat may be expressed as:

$$W_r = K_r \varepsilon \left(T_2^4 - T_1^4 \right) \qquad [6\text{-}14]$$

Where:

W_r, is watts per square centimeter of surface
$K_r = 5.70(10^{-12})W/(\text{cm}^2/K^4)$
ε, is the emissivity factor
T_2, is the hot body temperature, K (kelvin)
T_1, is the ambient or surrounding temperature, K (kelvin)

Transfer of heat by convection occurs when a body is hotter than the surrounding medium, which is usually air. The layer of air in contact with the hot body that is heated by conduction expands and rises, taking the absorbed heat with it. The next layer, being colder, replaces the risen layer and, in turn, on being heated, also rises. This transfer continues as long as the air, or other medium surrounding the body, is at a lower temperature. The transfer of heat by convection is stated mathematically as:

$$W_c = K_c F \theta^{(\eta)} \sqrt{P} \qquad [6\text{-}15]$$

Where:

W_c, is the watts loss per square centimeter
$K_c = 2.17(10^{-4})$
F, is the air friction factor (unity for a vertical surface)
θ, is the temperature rise, °C

P, is the relative barometic pressure (unity at sea level)

η, is the exponential value, which ranges from 1.0 to 1.25,

depending on the shape and position of the surface being cooled

The total heat dissipated from a plane vertical surface is expressed by Equations 6-13 and 6-15:

$$W = 5.70\left(10^{-12}\right)\varepsilon\left(T_2^4 - T_1^4\right) + 1.4\left(10^{-3}\right)F\theta^{(1.25)}\sqrt{P} \quad [6\text{-}16]$$

Temperature Rise Versus Surface Area, A_t, Dissipation

The temperature rise that can be expected for various levels of power loss is shown in the monograph of Figure 6-2. It is based on Equation 6-16, relying on data obtained from Blume (1938) (Ref. 1) for heat transfer affected by the combination of 55% radiation and 45% convection, from a surface having an emissivity of 0.95, in an ambient temperature of 25°C, at sea level. Power loss (heat dissipation) is expressed in watts per square centimeter of the total surface area. Heat dissipation, by convection from the upper side of a horizontal flat surface, is on the order of 15-20% more than from vertical surface. Heat dissipation, from the underside of a horizontal flat surface, depends upon area and conductivity.

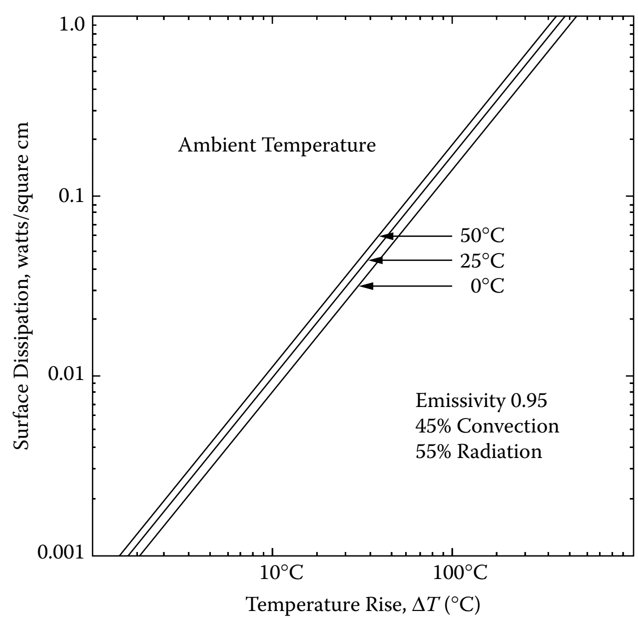

Figure 6-2. Temperature Rise Versus Surface Dissipation. (Adapted from L. F. Blume, *Transformers Engineering*, Wiley, New York, 1938, Figure 7.)

Surface Area, A_t, Required for Heat Dissipation

The effective surface area, A_t, required to dissipate heat, (expressed as watts dissipated per unit area), is:

$$A_t = \frac{P_\Sigma}{\psi}, \quad [\text{cm}^2] \qquad [6\text{-}17]$$

In which ψ is the power density or the average power dissipated per unit area from the surface of the transformer and, P_Σ is the total power lost or dissipated.

The surface area, A_t, of a transformer can be related to the area product, A_p, of a transformer. The straight-line logarithmic relationship, shown in Figure 6-3, has been plotted from the data in Chapter 3. The derivation for the surface area, A_t, and the area product, A_p, is in Chapter 5.

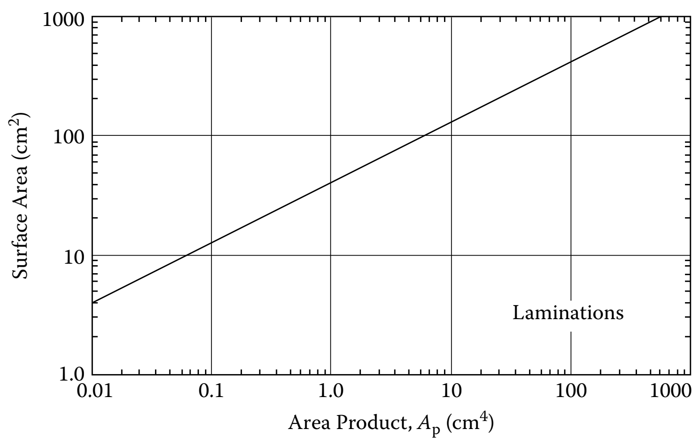

Figure 6-3. Surface Area, A_t Versus Area Product, A_p.

From this surface area, A_t, the following relationship evolves:

$$A_t = K_s \left(A_p\right)^{(0.5)} = \frac{P_\Sigma}{\psi}, \quad [\text{cm}^2] \qquad [6\text{-}18]$$

And from Figure 6-3:

$$\psi = 0.03, \quad [\text{watts-per-cm}^2 \text{ at } 25°\text{C}]$$

$$\psi = 0.07, \quad [\text{watts-per-cm}^2 \text{ at } 50°\text{C}]$$

The temperature rise, T_r, in Equation [6-19] in °C is:

$$T_r = 450\left(\psi\right)^{(0.826)}, \quad [°\text{C}] \qquad [6\text{-}19]$$

Required Surface Area, A_t

There are two common allowable temperature rises for transformers above the ambient temperature. These temperatures are shown in Figure 6-4. The surface area, A_t, required for a 25°C and 50°C rise above the ambient temperature for the total watts, dissipated. The presented data is used as basis for determining the needed transformer surface area, A_t, in cm^2.

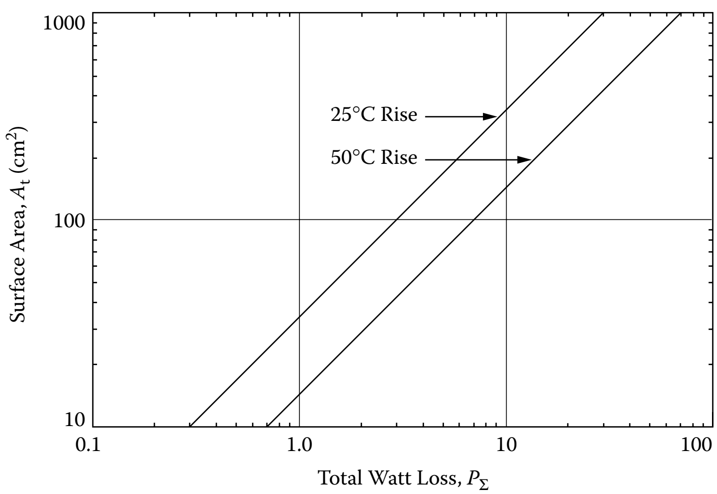

Figure 6-4. Surface Area, A_t Versus Total Watts Loss for Temperature Increases of 25°C and 50°C.

If the transformer is said to be homogeneous, and the thermal energy is dissipated uniformly throughout the surface area of the core and winding assembly, then Figure 6-5 will give a good approximation for the required time constant for a transformer to reach 63% of the final temperature. The temperature rise of a typical transformer is shown in Figure 6-6.

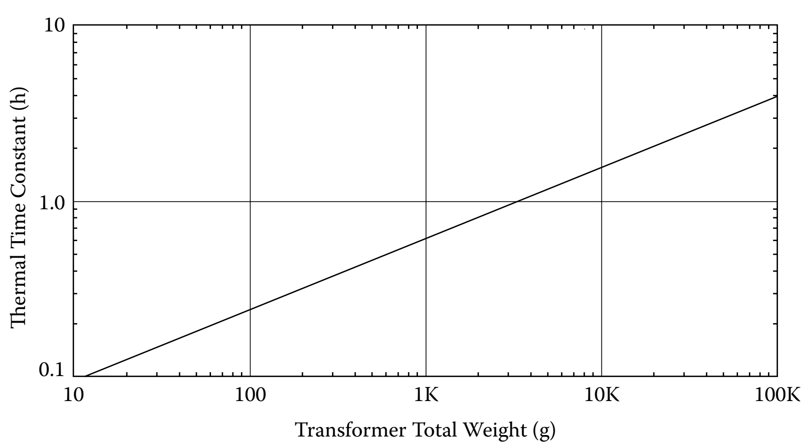

Figure 6-5. Time Required to Reach 63% of Final Temperature.

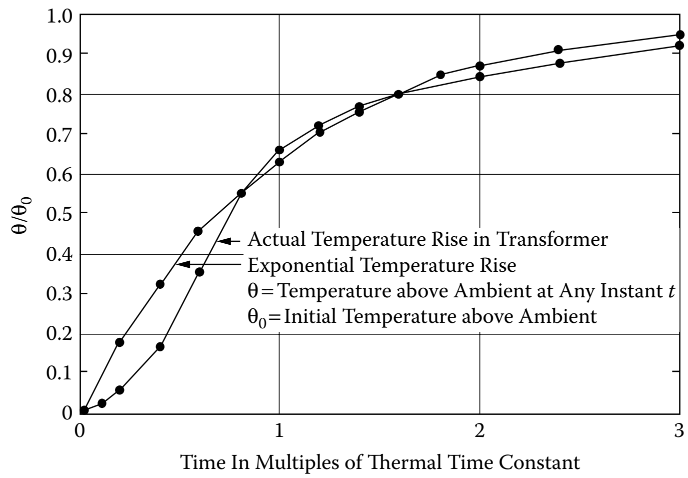

Figure 6-6. Transformer Temperature Rise Time.

Regulation as a Function of Efficiency

The minimum size of a transformer is usually determined either by a temperature rise limit, or by allowable voltage regulation, assuming that size and weight are to be minimized. Figure 6-7 shows a circuit diagram of a transformer with one secondary. Note that α = regulation (%).

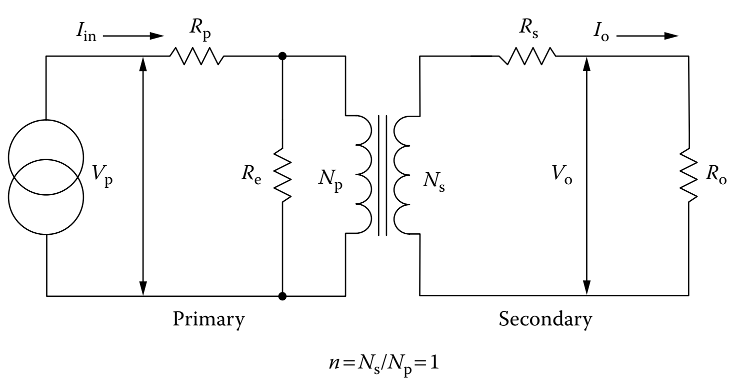

Figure 6-7. Transformer Circuit Diagram.

The assumption is that distributed capacitance in the secondary can be neglected because the frequency and secondary voltage are not excessively high. Also, the winding geometry is designed to limit the leakage inductance to a level, low enough, to be neglected under most operating conditions. The transformer window allocation is shown in Figure 6-8.

$$\frac{W_a}{2} = \text{Primary} = \text{Secondary} \qquad [6\text{-}20]$$

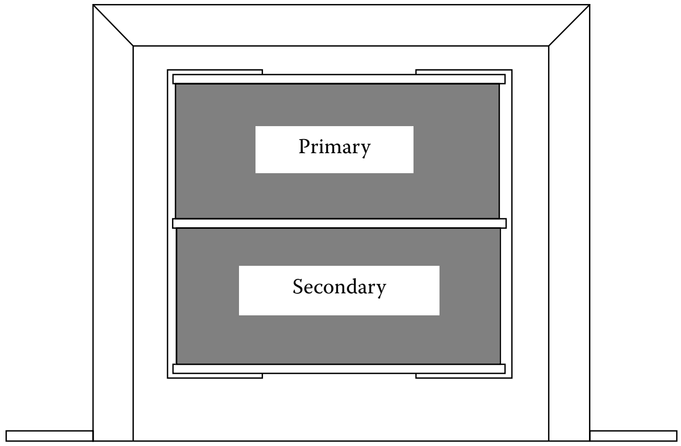

Figure 6-8. Transformer Window Allocation.

Transformer voltage regulation can now be expressed as:

$$\alpha = \frac{V_o(\text{N.L.}) - V_o(\text{F.L.})}{V_o(\text{F.L.})}(100), \quad [\%] \qquad [6\text{-}21]$$

in which, V_o(N.L.), is the no load voltage and, V_o(F.L.), is the full load voltage. For the sake of simplicity, assume the transformer, in Figure 6-5, is an isolation transformer, with a 1:1 turns ratio, and the core impedance, R_e, is infinite.

If the transformer has a 1:1 turns ratio, and the core impedance is infinite, then:

$$I_{in} = I_o, \quad [\text{amps}]$$
$$R_p = R_s, \quad [\text{ohms}] \qquad [6\text{-}22]$$

With equal window areas allocated for the primary and secondary windings, and using the same current density, J:

$$\Delta V_p = I_{in} R_p = \Delta V_s = I_o R_s, \quad [\text{volts}] \qquad [6\text{-}23]$$

Then Regulation is:

$$\alpha = \frac{\Delta V_p}{V_p}(100) + \frac{\Delta V_s}{V_s}(100), \quad [\%] \qquad [6\text{-}24]$$

Multiply the Equation [6-24] by currents, I:

$$\alpha = \frac{\Delta V_p I_{in}}{V_p I_{in}}(100) + \frac{\Delta V_s I_o}{V_s I_o}(100), \quad [\%] \qquad [6\text{-}25]$$

Primary copper loss is:

$$P_p = \Delta V_p I_{in}, \quad \text{[watts]} \qquad [6\text{-}26]$$

Secondary copper loss is:

$$P_s = \Delta V_s I_o, \quad \text{[watts]} \qquad [6\text{-}27]$$

Total copper loss is:

$$P_{cu} = P_p + P_s, \quad \text{[watts]} \qquad [6\text{-}28]$$

Then, the regulation equation can be rewritten to:

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%] \qquad [6\text{-}29]$$

References

1. Blume, L. F., *Transformer Engineering*, John Wiley & Sons Inc. New York, 1938, pp. 272–282.

2. Terman, F. E., *Radio Engineers Handbook*, McGraw-Hill Book Co., Inc., New York, 1943, pp. 28–37.

Chapter 7

Power Transformer Design

Table of Contents

Introduction

The conversion process in power electronics requires the use of transformers and components that are frequently the heaviest and bulkiest item in the conversion circuit. They also have a significant effect upon the overall performance and efficiency of the system. Accordingly, the design of such transformers has an important influence on the overall system weight, power conversion efficiency and cost. Because of the interdependence and interaction of parameters, judicious trade-offs are necessary to achieve design optimization.

The Design Problem Generally

The designer is faced with a set of constraints that must be observed in the design on any transformer. One of these constraints is the output power, P_o, (operating voltage multiplied by maximum current demand). The secondary winding must be capable of delivering to the load within specified regulation limits. Another constraint relates to the minimum efficiency of operation, which is dependent upon the maximum power loss that can be allowed in the transformer. Still another defines the maximum permissible temperature rise for the transformer when it is used in a specified temperature environment.

One of the basic steps in transformer design is the selection of proper core material. Magnetic materials used to design low and high frequency transformers are shown in Table 7-1. Each one of these materials has its own optimum point in the cost, size, frequency and efficiency spectrum. The designer should be aware of the cost difference between silicon-iron, nickel-iron, amorphous and ferrite materials. Other constraints relate to the volume occupied by the transformer and, particularly in aerospace applications, the weight, since weight minimization is an important goal in today's electronics. Finally, cost effectiveness is always an important consideration.

Depending upon the application, certain ones of these constraints will dominate. Parameters affecting others may then be a trade-off, as necessary, to achieve the most desirable design. It is not possible to optimize all parameters in a single design because of their interaction and interdependence. For example, if volume and weight are of greater significance, reductions in both can often be affected, by operating the transformer at a higher frequency, but at a penalty in efficiency. When, the frequency cannot be increased, reduction in weight and volume may still be possible by selecting a more efficient core material, but, at the penalty of increased cost. Thus, judicious trade-offs must be affected to achieve the design goals.

Transformer designers have used various approaches in arriving at suitable designs. For example, in many cases, a rule of thumb is used for dealing with current density. Typically, an assumption is made that a good working level is 200 amps-per-cm^2 (1000 circular mils-per-ampere). This will work in many instances, but

Table 7-1. Magnetic Materials

Magnetic Material Properties				
Material Name	Trade Name Composition	Initial Permeability μ_i	Flux Density Teslas B_s	Typical Operating Frequency
Silicon	3-97 SiFe	1500	1.5-1.8	50-2k
Orthonol	50-50 NiFe	2000	1.42-1.58	50-2k
Permalloy	80-20 NiFe	25000	0.66-0.82	1k-25k
Amorphous	2605SC	1500	1.5-1.6	250k
Amorphous	2714A	20,000	0.5-6.5	250k
Amorphous	Nanocrystalline	30,000	1.0-1.2	250k
Ferrite	MnZn	0.75-15k	0.3-0.5	10k-2M
Ferrite	NiZn	0.20-1.5k	0.3-0.4	0.2M-100M

the wire size needed to meet this requirement may produce a heavier and bulkier transformer than desired or required. The information presented in this chapter makes it possible to avoid the use of this assumption and other rules of thumb, and to develop a more economical design with greater accuracy.

Power-Handling Ability

For years manufacturers have assigned numeric codes to their cores; these codes represent the power-handling ability. This method assigns to each core a number that is the product of its window area, W_a, and core cross-section area, A_c, and is called the area product, A_p.

These numbers are used by core suppliers to summarize dimensional and electrical properties in their catalogs. They are available for laminations, C-cores, pot cores, powder cores, ferrite toroids, and toroidal tape-wound cores.

The regulation and power-handling ability of a core is related to the core geometry, K_g. Every core has its own inherent, K_g. The core geometry is relatively new, and magnetic core manufacturers do not list this coefficient.

Because of their significance, the area product, A_p, and core geometry, K_g, are treated extensively in this book. A great deal of other information is also presented for the convenience of the designer. Much of the material is in tabular form to assist the designer in making trade-offs, best-suited for his particular application in a minimum amount of time.

These relationships can now be used as new tools to simplify and standardize the process of transformer design. They make it possible to design transformers of lighter weight and smaller volume, or to optimize efficiency, without going through a cut-and-try, design procedure. While developed especially for aerospace applications, the information has wider utility, and can be used for the design of non-aerospace, as well.

Output Power, P_o, Versus Apparent Power, P_t, Capability

Output power, P_o, is of the greatest interest to the user. To the transformer designer, the apparent power, P_t, which is associated with the geometry of the transformer, is of greater importance. Assume, for the sake of simplicity, that the core of an isolation transformer has only two windings in the window area, a primary and a secondary. Also, assume that the window area, W_a, is divided up in proportion to the power-handling capability of the windings, using equal current density. The primary winding handles, P_{in}, and the secondary handles, P_o, to the load. Since the power transformer has to be designed to accommodate the primary, P_{in}, and, P_o, then,

By definition:

$$P_t = P_{in} + P_o, \quad \text{[watts]}$$
$$P_{in} = \frac{P_o}{\eta}, \quad \text{[watts]} \qquad [7\text{-}1]$$

The primary turns can be expressed using Faraday's Law:

$$N_p = \frac{V_p\left(10^4\right)}{A_c B_{ac} f K_f}, \quad \text{[turns]} \qquad [7\text{-}2]$$

The winding area of a transformer is fully utilized when:

$$K_u W_a = N_p A_{wp} + N_s A_{ws} \qquad [7\text{-}3]$$

By definition the wire area is:

$$A_w = \frac{I}{J}, \quad [\text{cm}^2] \qquad [7\text{-}4]$$

Rearranging the Equation shows:

$$K_u W_a = N_p\left(\frac{I_p}{J}\right) + N_s\left(\frac{I_s}{J}\right) \qquad [7\text{-}5]$$

Now, substitute in Faraday's Equation:

$$K_u W_a = \frac{V_p\left(10^4\right)}{A_c B_{ac} f K_f}\left(\frac{I_p}{J}\right)+\frac{V_s\left(10^4\right)}{A_c B_{ac} f K_f}\left(\frac{I_s}{J}\right) \qquad [7\text{-}6]$$

Rearranging shows:

$$W_a A_c = \frac{\left[\left(V_p I_p\right)+\left(V_s I_s\right)\right]\left(10^4\right)}{B_{ac} f J K_f K_u}, \quad [\text{cm}^4] \qquad [7\text{-}7]$$

The output power, P_o, is:

$$P_o = V_s I_s, \quad [\text{watts}] \qquad [7\text{-}8]$$

The input power, P_{in}, is:

$$P_{in} = V_p I_p, \quad [\text{watts}] \qquad [7\text{-}9]$$

Then:

$$P_t = P_{in} + P_o, \quad [\text{watts}] \qquad [7\text{-}10]$$

Substitute in, P_t:

$$W_a A_c = \frac{P_t\left(10^4\right)}{B_{ac} f J K_f K_u}, \quad [\text{cm}^4] \qquad [7\text{-}11]$$

By definition, A_p, equals:

$$A_p = W_a A_c, \quad [\text{cm}^4] \qquad [7\text{-}12]$$

Then:

$$A_p = \frac{P_t\left(10^4\right)}{B_{ac} f J K_f K_u}, \quad [\text{cm}^4] \qquad [7\text{-}13]$$

The designer must be concerned with the apparent power, P_t, and power handling capability of the transformer core and windings. P_t may vary by a factor, ranging from 2 to 2.828 times the input power, P_{in}, depending upon the type of circuit in which the transformer is used. If the current in the rectifier transformer becomes interrupted, its effective RMS value changes. Thus, transformer size is not only determined by the load demand, but also, by application, because of the different copper losses incurred, due to the current waveform.

For example, for a load of one watt, compare the power handling capabilities required for each winding, (neglecting transformer and diode losses, so that $P_{in} = P_o$). For the full-wave bridge circuit of Figure 7-1, the full-wave center-tapped secondary circuit of Figure 7-2, and the push-pull, center-tapped full-wave circuit in Figure 7-3, all the windings have the same number of turns, (N).

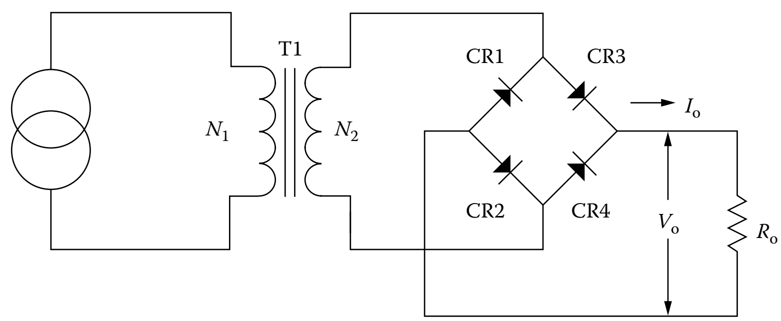

Figure 7-1. Full-Wave Bridge Secondary.

The total apparent power, P_t, for the circuit shown in Figure 7-1 is 2 watts.

This is shown in the following Equation:

$$P_t = P_{in} + P_o, \quad \text{[watts]} \tag{7-14}$$

$$P_t = 2P_{in}, \quad \text{[watts]} \tag{7-15}$$

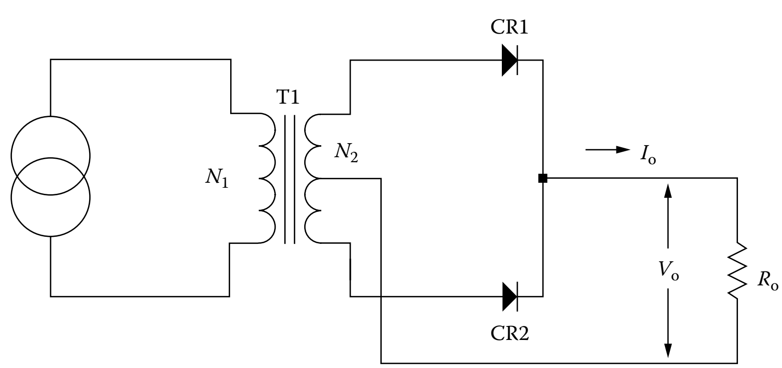

Figure 7-2. Full-Wave, Center-Tapped Secondary.

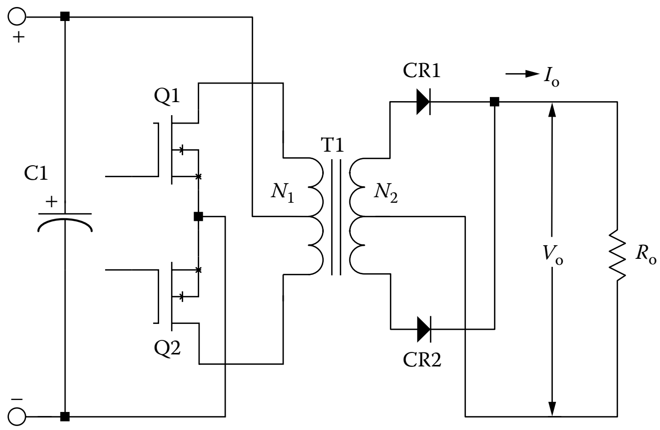

Figure 7-3. Push-Pull Primary, Full-Wave, Center-Tapped Secondary.

The total power, P_t, for the circuit shown in Figure 7-2, increased 20.7%, due to the distorted wave form of the interrupted current flowing in the secondary winding. This is shown in the following Equation:

$$P_t = P_{in} + P_o\sqrt{2}, \quad \text{[watts]} \qquad [7\text{-}16]$$

$$P_t = P_{in}\left(1+\sqrt{2}\right), \quad \text{[watts]} \qquad [7\text{-}17]$$

The total power, P_t, for the circuit is shown in Figure 7-3, which is typical of a dc to dc converter. It increases to 2.828 times, P_{in}, because of the interrupted current flowing in both the primary and secondary windings.

$$P_t = P_{in}\sqrt{2} + P_o\sqrt{2}, \quad \text{[watts]} \qquad [7\text{-}18]$$

$$P_t = 2P_{in}\sqrt{2}, \quad \text{[watts]} \qquad [7\text{-}19]$$

Transformers with Multiple Outputs

This example shows how the apparent power, P_t, changes with a multiple output transformers.

Output	Circuit
5 V @ 10A	center-tapped V_d = diode drop = 1 V
15 V @ 1A	full-wave bridge V_d = diode drop = 2 V
Efficiency = 0.95	

The output power seen by the transformer in Figure 7-4 is:

$$P_{o1}=(V_{o1}+V_d)(I_{o1}),\quad \text{[watts]}$$
$$P_{o1}=(5+1)(10),\quad \text{[watts]} \qquad \text{[7-20]}$$
$$P_{o1}=60,\quad \text{[watts]}$$

And:

$$P_{o2}=(V_{o2}+V_d)(I_{o2}),\quad \text{[watts]}$$
$$P_{o2}=(15+2)(1.0),\quad \text{[watts]} \qquad \text{[7-21]}$$
$$P_{o2}=17,\quad \text{[watts]}$$

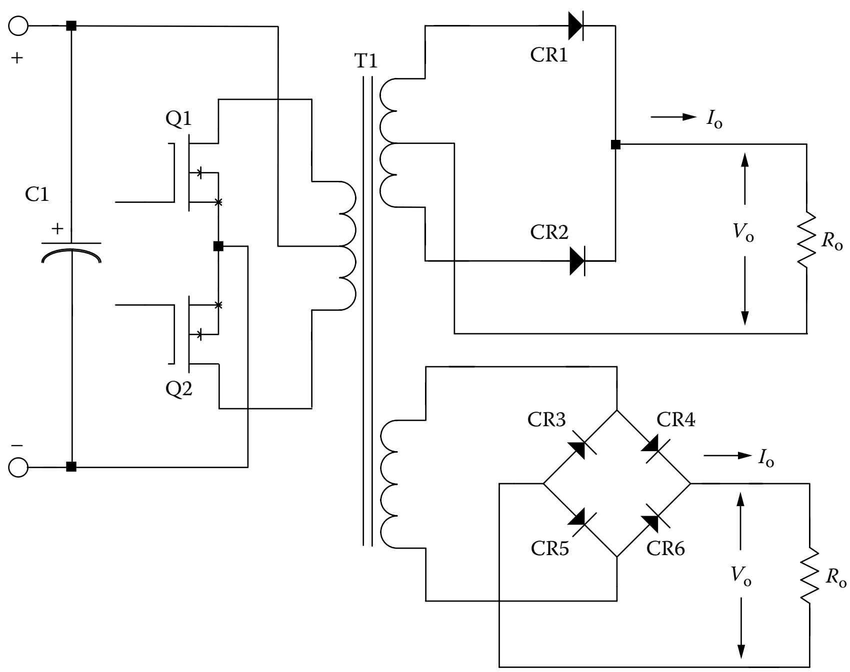

Figure 7-4. Multiple Output Converter.

Because of the different winding configurations, the apparent power, P_t, the transformer outputs will have to be summed to reflect this. When a winding has a center-tap and produces a discontinuous current, then, the power in that winding, be it primary or secondary, has to be multiplied by the factor, U. The factor, U, corrects for the rms current in that winding. If the winding has a center-tap, then the factor, U, is equal to 1.41. If not, the factor, U, is equal to 1.

For an example, summing up the output power of a multiple output transformer, would be:

$$P_{\Sigma}=P_{o1}(U)+P_{o2}(U)+P_n(U)+\cdots \qquad \text{[7-22]}$$

Then:

$$P_\Sigma = P_{o1}(U) + P_{o2}(U), \quad \text{[watts]}$$
$$P_\Sigma = 60(1.41) + 17(1), \quad \text{[watts]} \qquad [7\text{-}23]$$
$$P_\Sigma = 101.6, \quad \text{[watts]}$$

After the secondary has been totaled, then the primary power can be calculated.

$$P_{in} = \frac{P_{o1} + P_{o2}}{\eta}, \quad \text{[watts]}$$
$$P_{in} = \frac{(60)+(17)}{(0.95)}, \quad \text{[watts]} \qquad [7\text{-}24]$$
$$P_{in} = 81, \quad \text{[watts]}$$

Then, the apparent power, P_t, equals:

$$P_t = P_{in}(U) + P_\Sigma, \quad \text{[watts]}$$
$$P_t = (81)(1.41) + (101.6), \quad \text{[watts]} \qquad [7\text{-}25]$$
$$P_t = 215.8, \quad \text{[watts]}$$

Regulation

The minimum size of a transformer is usually determined either by a temperature rise limit, or by allowable voltage regulation, assuming that size and weight are to be minimized. Figure 7-5 shows a circuit diagram of a transformer with one secondary.

Note that α = regulation (%).

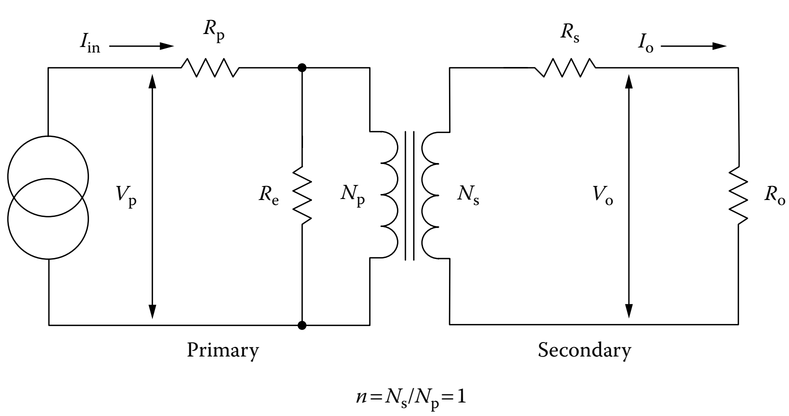

Figure 7-5. Transformer Circuit Diagram.

The assumption is that distributed capacitance in the secondary can be neglected because the frequency and secondary voltage are not excessively high. Also, the winding geometry is designed to limit the leakage inductance to a level, low enough, to be neglected under most operating conditions.

Transformer voltage regulation can now be expressed as:

$$\alpha = \frac{V_o(\text{N.L.}) - V_o(\text{F.L.})}{V_o(\text{F.L.})}(100), \quad [\%] \qquad [7\text{-}26]$$

In which, V_o(N.L.), is the no load voltage and, V_o(F.L.), is the full load voltage. For the sake of simplicity, assume the transformer in Figure 7-5, is an isolation transformer, with a 1:1 turns ratio, and the core impedance, R_e, is infinite.

If the transformer has a 1:1 turns ratio, and the core impedance is infinite, then:

$$\begin{aligned} I_{in} &= I_o, \quad [\text{amps}] \\ R_p &= R_s, \quad [\text{ohms}] \end{aligned} \qquad [7\text{-}27]$$

With equal window areas allocated for the primary and secondary windings, and using the same current density, J:

$$\Delta V_p = I_{in} R_p = \Delta V_s = I_o R_s, \quad [\text{volts}] \qquad [7\text{-}28]$$

Regulation is then:

$$\alpha = \frac{\Delta V_p}{V_p}(100) + \frac{\Delta V_s}{V_s}(100), \quad [\%] \qquad [7\text{-}29]$$

Multiply the Equation by currents, I:

$$\alpha = \frac{\Delta V_p I_{in}}{V_p I_{in}}(100) + \frac{\Delta V_s I_o}{V_s I_o}(100), \quad [\%] \qquad [7\text{-}30]$$

Primary copper loss is:

$$P_p = \Delta V_p I_{in}, \quad [\text{watts}] \qquad [7\text{-}31]$$

Secondary copper loss is:

$$P_s = \Delta V_s I_o, \quad [\text{watts}] \qquad [7\text{-}32]$$

Total copper loss is:

$$P_{cu} = P_p + P_s, \quad \text{[watts]} \tag{7-33}$$

Then, the regulation Equation can be rewritten to:

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%] \tag{7-34}$$

Regulation can be expressed as the power lost in the copper. A transformer, with an output power of 100 watts and a regulation of 2%, will have a 2 watt loss in the copper:

$$P_{cu} = \frac{P_o \alpha}{100}, \quad \text{[watts]} \tag{7-35}$$

$$P_{cu} = \frac{(100)(2)}{100}, \quad \text{[watts]} \tag{7-36}$$

$$P_{cu} = 2, \quad \text{[watts]} \tag{7-37}$$

Relationship, Kg, to Power Transformer Regulation Capability

Although most transformers are designed for a given temperature rise, they can also be designed for a given regulation. The regulation and power-handling ability of a core is related to two constants:

$$\alpha = \frac{P_t}{2K_g K_e}, \quad [\%] \tag{7-38}$$

$$\alpha = \text{Regulation } (\%) \tag{7-39}$$

The constant, K_g, is determined by the core geometry, which may be related by the following Equations:

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \tag{7-40}$$

The constant, K_e, is determined by the magnetic and electric operating conditions, which may be related by the following Equation:

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right) \qquad [7\text{-}41]$$

Where:

K_f = waveform coefficient

4.0 square wave

4.44 sine wave

From the above, it can be seen that factors such as flux density, frequency of operation, and the waveform coefficient have an influence on the transformer size.

Relationship, Ap, to Transformer Power Handling Capability

Transformers

According to the newly developed approach, the power handling capability of a core is related to its area product, A_p, by an equation which may be stated as:

$$A_p = \frac{P_t\left(10^4\right)}{K_f K_u B_m J f}, \quad [\text{cm}^4] \qquad [7\text{-}42]$$

Where:

K_f = waveform coefficient

4.0 square wave

4.44 sine wave

From the above, it can be seen that factors, such as flux density, frequency of operation, and the window utilization factor, K_u, define the maximum space which may be occupied by the copper in the window.

Different Cores, Same Area Product

The area product, A_p, of a core is the product of the available window area, W_a, of the core in square centimeters, (cm^2), multiplied by the effective, cross-sectional area, A_c, in square centimeters, (cm^2), which may be stated as:

$$A_p = W_a A_c, \quad [\text{cm}^4] \qquad [7\text{-}43]$$

Figures 7-6 through Figure 7-9 show, in outline form, three transformer core types that are typical of those shown in the catalogs of suppliers.

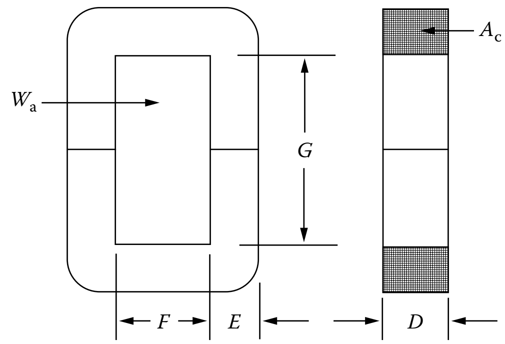

Figure 7-6. Dimensional Outline of a C Core.

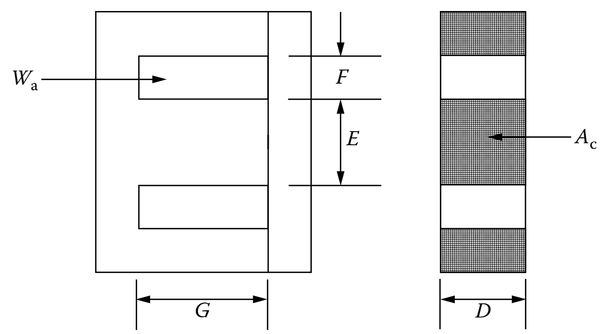

Figure 7-7. Dimensional Outline of a EI Lamination.

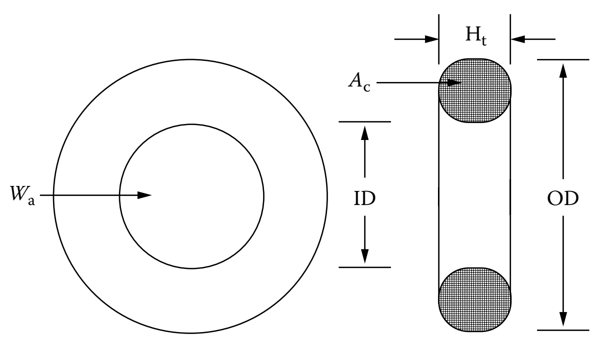

Figure 7-8. Dimensional Outline of a Toroidal Core.

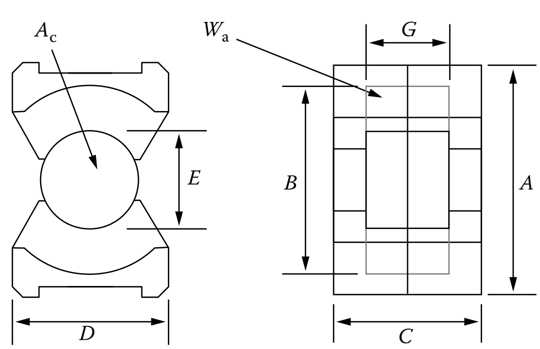

Figure 7-9. Dimensional Outline of a PQ Ferrite Core.

250 Watt Isolation Transformer Design, Using the Core Geometry, K_g, Approach

The following information is the Design specification for a 250 watt isolation transformer, operating at 47 Hz, using the, K_g, core geometry approach. For a typical design example, assume with the following specification:

1. Input voltage, V_{in} = 115 volts
2. Output voltage, V_o = 115 volts
3. Output current, I_o = 2.17 amps
4. Output power, P_o = 250 watts
5. Frequency, f = 47 Hz
6. Efficiency, η = 95%
7. Regulation, α = 5%
8. Operating flux density, B_{ac} = 1.6 teslas
9. Core Material = Silicon M6X
10. Window utilization, K_u = 0.4
11. Temperature rise goal, T_r = 30°C

Step No. 1: Calculate the transformer apparent power, P_t.

$$P_t = P_o\left(\frac{1}{\eta}+1\right), \quad \text{[watts]}$$

$$P_t = 250\left(\frac{1}{0.95}+1\right), \quad \text{[watts]}$$

$$P_t = 513, \quad \text{[watts]}$$

Step No. 2: Calculate the electrical conditions, K_e.

$$K_e = 0.145\left(K_f\right)^2(f)^2(B_m)^2\left(10^{-4}\right)$$

$$K_f = 4.44, \quad \text{[sine wave]}$$

$$K_e = 0.145(4.44)^2(47)^2(1.6)^2\left(10^{-4}\right)$$

$$K_e = 1.62$$

Step No. 3: Calculate the core geometry, K_g.

$$K_g = \frac{P_t}{2K_e\alpha}, \quad \text{[cm}^5\text{]}$$

$$K_g = \frac{(513)}{2(1.62)(5)}, \quad \text{[cm}^5\text{]}$$

$$K_g = 31.7, \quad \text{[cm}^5\text{]}$$

Step No. 4: Select a lamination from Chapter 3, comparable in core geometry, K_g.

Lamination number = EI-150

Manufacturer = Thomas and Skinner

Magnetic path length, MPL = 22.9 cm

Core weight, W_{tfe} = 2.334 kilograms

Copper weight, $W_{tcu,}$ = 853 grams

Mean length turn, MLT = 22 cm

Iron area, A_c = 13.8 cm^2

Window area, W_a = 10.89 cm^2

Area product, $A_{p,}$ = 150 cm^4

Core geometry, K_g = 37.6 cm^5

Surface area, A_t = 479 cm^2

Step No. 5: Calculate the number of primary turns, N_p using Faraday's Law.

$$N_p = \frac{V_{in}\left(10^4\right)}{K_f B_{ac} f A_c}, \quad \text{[turns]}$$

$$N_p = \frac{(115)\left(10^4\right)}{(4.44)(1.6)(47)(13.8)}, \quad \text{[turns]}$$

$$N_p = 250, \quad \text{[turns]}$$

Step No. 6: Calculate the current density, J.

$$J = \frac{P_t\left(10^4\right)}{K_f K_u B_{ac} f A_p}, \quad \text{[amps/cm}^2\text{]}$$

$$J = \frac{513\left(10^4\right)}{(4.44)(0.4)(1.6)(47)(150)}, \quad \text{[amps/cm}^2\text{]}$$

$$J = 256, \quad \text{[amps/cm}^2\text{]}$$

Step No. 7: Calculate the input current, I_{in}.

$$I_{in} = \frac{P_o}{V_{in}\eta}, \quad \text{[amps]}$$

$$I_{in} = \frac{250}{(115)(0.95)}, \quad \text{[amps]}$$

$$I_{in} = 2.28, \quad \text{[amps]}$$

Step No. 8: Calculate the primary bare wire area, $A_{wp(B)}$.

$$A_{wp(B)} = \frac{I_{in}}{J}, \quad [\text{cm}^2]$$

$$A_{wp(B)} = \frac{(2.28)}{256}, \quad [\text{cm}^2]$$

$$A_{wp(B)} = 0.0089, \quad [\text{cm}^2]$$

Step No. 9: Select the wire from the Wire Table, in Chapter 4.

$$AWG = \#18$$

$$A_{wp(B)} = 0.00822, \quad [\text{cm}^2]$$

$$A_{wp} = 0.00933, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 209, \quad [\text{micro-ohm/cm}]$$

Step No. 10: Calculate the primary resistance, R_p.

$$R_p = \text{MLT}\left(N_p\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_p = (22)(250)(209)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_p = 1.15, \quad [\text{ohms}]$$

Step No. 11: Calculate the primary copper loss, P_p.

$$P_p = I_p^2 R_p, \quad [\text{watts}]$$

$$P_p = (2.28)^2(1.15), \quad [\text{watts}]$$

$$P_p = 5.98, \quad [\text{watts}]$$

Step No. 12: Calculate the secondary turns, N_s.

$$N_s = \frac{N_p V_s}{V_{in}}\left(1 + \frac{\alpha}{100}\right), \quad [\text{turns}]$$

$$N_s = \frac{(250)(115)}{(115)}\left(1 + \frac{5}{100}\right), \quad [\text{turns}]$$

$$N_s = 262.5 \text{ use } 263, \quad [\text{turns}]$$

Step No. 13: Calculate the secondary bare wire area, $A_{ws(B)}$.

$$A_{ws(B)} = \frac{I_o}{J}, \quad [\text{cm}^2]$$

$$A_{ws(B)} = \frac{(2.17)}{256}, \quad [\text{cm}^2]$$

$$A_{ws(B)} = 0.00804, \quad [\text{cm}^2]$$

Step No. 14: Select the wire from the Wire Table, in Chapter 4.

$$AWG = \#18$$

$$A_{wp(B)} = 0.00822, \quad [\text{cm}^2]$$

$$A_{wp} = 0.00933, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 209, \quad [\text{micro-ohm/cm}]$$

Step No. 15: Calculate the secondary winding resistance, R_s.

$$R_s = \text{MLT}(N_s)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_s = (22)(263)(209)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_s = 1.21, \quad [\text{ohms}]$$

Step No. 16: Calculate the secondary copper loss, P_s.

$$P_s = I_o^2 R_s, \quad [\text{watts}]$$

$$P_s = (2.17)^2(1.21), \quad [\text{watts}]$$

$$P_s = 5.70, \quad [\text{watts}]$$

Step No. 17: Calculate the total primary and secondary copper loss, P_{cu}.

$$P_{cu} = P_p + P_s, \quad [\text{watts}]$$

$$P_{cu} = 5.98 + 5.7, \quad [\text{watts}]$$

$$P_{cu} = 11.68, \quad [\text{watts}]$$

Step No. 18: Calculate the transformer regulation, α.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

$$\alpha = \frac{(11.68)}{(250)}(100), \quad [\%]$$

$$\alpha = 4.67, \quad [\%]$$

Step No. 19: Calculate the watts per kilogram, W/K. Use the Equation for this material in Chapter 2.

$$W/K = 0.000557(f)^{1.68}(B_{ac})^{1.86}$$

$$W/K = 0.000557(47)^{1.68}(1.6)^{1.86}$$

$$W/K = 0.860$$

Step No. 20: Calculate the core loss, P_{fe}.

$$P_{fe} = (W/K)(W_{tfe})(10^{-3}), \quad [\text{watts}]$$

$$P_{fe} = (0.860)(2.33), \quad [\text{watts}]$$

$$P_{fe} = 2.00, \quad [\text{watts}]$$

Step No. 21: Calculate the total loss, P_Σ.

$$P_\Sigma = P_{cu} + P_{fe}, \quad [\text{watts}]$$

$$P_\Sigma = (11.68) + (2.00), \quad [\text{watts}]$$

$$P_\Sigma = 13.68, \quad [\text{watts}]$$

Step No. 22: Calculate the watts per unit area, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad [\text{watts/cm}^2]$$

$$\psi = \frac{(13.68)}{(479)}, \quad [\text{watts/cm}^2]$$

$$\psi = 0.0286, \quad [\text{watts/cm}^2]$$

Step No. 23: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [°C]$$

$$T_r = 450(0.0286)^{(0.826)}, \quad [°C]$$

$$T_r = 23.9, \quad [°C]$$

Step No. 24: Calculate the total window utilization, K_u.

$$K_u = K_{up} + K_{us}$$

$$K_{us} = \frac{N_s A_{ws(B)}}{W_a}$$

$$K_{us} = \frac{(263)(0.00822)}{(10.89)} = 0.199$$

$$K_{up} = \frac{N_p A_{wp(B)}}{W_a}$$

$$K_{up} = \frac{(250)(0.00822)}{(10.89)} = 0.189$$

$$K_u = (0.189) + (0.199)$$

$$K_u = 0.388$$

38 Watt 100kHz Transformer Design, Using the Core Geometry, K_g, Approach

The following information is the design specification for a 38 watt push-pull transformer, operating at 100kHz, using the K_g core geometry approach. For a typical design example, assume a push-pull, full-wave, center-tapped circuit, as shown in Figure 7-4, with the following specification:

1. Input voltage, $V_{(min)} = 24$ volts
2. Output voltage #1, $V_{(o1)} = 5.0$ volts
3. Output current #1, $I_{(o1)} = 4.0$ amps
4. Output voltage #2, $V_{(o2)} = 12.0$ volts
5. Output current #2, $I_{(o2)} = 1.0$ amps
6. Frequency, f = 100kHz
7. Efficiency, $\eta = 98\%$
8. Regulation, $\alpha = 0.5\%$

9. Diode voltage drop, $V_d = 1.0$ volt
10. Operating flux density, $B_{ac} = 0.05$ teslas
11. Core Material = ferrite
12. Window utilization, $K_u = 0.4$
13. Temperature rise goal, $T_r = 30°C$
14. Maximum duty ratio, $D_{max} = 0.5$
15. Notes:

    Using a center-tapped winding, U = 1.41

    Using a single winding, U = 1.0

At this point, select a wire so that the relationship between the ac resistance and the dc resistance is 1:

$$\frac{R_{ac}}{R_{dc}} = 1$$

The skin depth, ε in centimeters, is:

$$\varepsilon = \frac{6.62}{\sqrt{f}}, \quad [\text{cm}]$$

$$\varepsilon = \frac{6.62}{\sqrt{100{,}000}}, \quad [\text{cm}]$$

$$\varepsilon = 0.0209, \quad [\text{cm}]$$

Then, the wire diameter, D_{AWG}, is:

$$D_{AWG} = 2(\varepsilon), \quad [\text{cm}]$$

$$D_{AWG} = 2(0.0209), \quad [\text{cm}]$$

$$D_{AWG} = 0.0418, \quad [\text{cm}]$$

Then, the bare wire area, A_w, is:

$$A_w = \frac{\pi (D_{AWG})^2}{4}, \quad [\text{cm}^2]$$

$$A_w = \frac{(3.1416)(0.0418)^2}{4}, \quad [\text{cm}^2]$$

$$A_w = 0.00137, \quad [\text{cm}^2]$$

From the Wire Table 4-9, in Chapter 4, Number 27 has a bare wire area of 0.001021 centimeters. This will be the minimum wire size used in this design. If the design requires more wire area to meet the specification, then the design will use a multifilar of #26. Listed Below are #27 and #28, just in case #26 requires too much rounding off.

Wire AWG	Bare Area	Area Ins.	Bare/Ins.	μΩ/cm
#26	0.001280	0.001603	0.798	1345
#27	0.001021	0.001313	0.778	1687
#28	0.0008046	0.0010515	0.765	2142

Step No. 1: Calculate the transformer output power, P_o.

$$P_o = P_{o1} + P_{o2}, \quad \text{[watts]}$$

$$P_{o1} = I_{o1}\left(V_{o1} + V_d\right), \quad \text{[watts]}$$

$$P_{o1} = 4\left(5+1\right), \quad \text{[watts]}$$

$$P_{o1} = 24, \quad \text{[watts]}$$

$$P_{o2} = I_{o2}\left(V_{o2} + V_d\right), \quad \text{[watts]}$$

$$P_{o2} = 1\left(12+2\right), \quad \text{[watts]}$$

$$P_{o2} = 14, \quad \text{[watts]}$$

$$P_o = \left(24+14\right), \quad \text{[watts]}$$

$$P_o = 38, \quad \text{[watts]}$$

Step No. 2: Calculate the total secondary apparent power, P_{ts}.

$$P_{ts} = P_{tso1} + P_{tso2}, \quad \text{[watts]}$$

$$P_{tso1} = P_{o1}\left(U\right), \quad \text{[watts]}$$

$$P_{tso1} = 24\left(1.41\right), \quad \text{[watts]}$$

$$P_{tso1} = 33.8, \quad \text{[watts]}$$

$$P_{tso2} = P_{o2}\left(U\right), \quad \text{[watts]}$$

$$P_{tso2} = 14\left(1\right), \quad \text{[watts]}$$

$$P_{tso2} = 14, \quad \text{[watts]}$$

$$P_{ts} = \left(33.8+14\right), \quad \text{[watts]}$$

$$P_{ts} = 47.8, \quad \text{[watts]}$$

Step No. 3: Calculate the total apparent power, P_t.

$$P_{in} = \left(\frac{P_o}{\eta}\right), \quad \text{[watts]}$$

$$P_{tp} = P_{in}P_a, \quad \text{[watts]}$$

$$P_t = P_{tp} + P_{ts}, \quad \text{[watts]}$$

$$P_t = \left(\frac{38}{0.98}\right)(1.41) + 47.8, \quad \text{[watts]}$$

$$P_t = 102.5, \quad \text{[watts]}$$

Step No. 4: Calculate the electrical conditions, K_e.

$$K_e = 0.145\left(K_f\right)^2(f)^2(B_m)^2\left(10^{-4}\right)$$

$$K_f = 4.0, \quad \text{[square wave]}$$

$$K_e = 0.145(4.0)^2(100000)^2(0.05)^2\left(10^{-4}\right)$$

$$K_e = 5800$$

Step No. 5: Calculate the core geometry, K_g.

$$K_g = \frac{P_t}{2K_e\alpha}, \quad [\text{cm}^5]$$

$$K_g = \frac{(102.5)}{2(5800)0.5}, \quad [\text{cm}^5]$$

$$K_g = 0.0177, \quad [\text{cm}^5]$$

When operating at high frequencies, the engineer has to review the window utilization factor, K_u, in Chapter 4. When using small bobbin ferrites, use the ratio of the bobbin winding area to the core window area which is only about 0.6. Operating at 100kHz and having to use a #26 wire, because of the skin effect, the ratio of the bare copper area to the total area is 0.78. Therefore, the overall window utilization, K_u, is reduced. To return the design back to the norm, the core geometry, K_g, is to be multiplied by 1.35, and then, the current density, J, is calculated, using a window utilization factor of 0.29.

$$K_g = 0.0177(1.35), \quad [\text{cm}^5]$$

$$K_g = 0.0239, \quad [\text{cm}^5]$$

Step No. 6: Select a PQ core from Chapter 3, comparable in core geometry K_g.

Core number = PQ-2020

Manufacturer = TDK

Magnetic material, 2000 um = PC44

Magnetic path length, MPL = 4.5 cm

Window height, G = 1.43 cm

Core weight, W_{tfe} = 15 grams

Copper weight, W_{tcu} = 10.4 grams

Mean length turn, MLT = 4.4 cm

Iron area, $A_c = 0.62\ \text{cm}^2$

Window area, $W_a = 0.658\ \text{cm}^2$

Area product, $A_p = 0.408\ \text{cm}^4$

Core geometry, $K_g = 0.0227\ \text{cm}^5$

Surface area, $A_t = 19.7\ \text{cm}^2$

Millihenrys per 1000 turns, AL = 3150

Step No. 7: Calculate the number of primary turns, N_p, using Faraday's Law.

$$N_p = \frac{V_p\left(10^4\right)}{K_f B_{ac} f A_c}, \quad \text{[turns]}$$

$$N_p = \frac{(24)\left(10^4\right)}{(4.0)(0.05)(100000)(0.62)}, \quad \text{[turns]}$$

$$N_p = 19, \quad \text{[turns]}$$

Step No. 8: Calculate the current density, J, using a window utilization, $K_u = 0.29$.

$$J = \frac{P_t\left(10^4\right)}{K_f K_u B_{ac} f A_p}, \quad \text{[amps/cm}^2\text{]}$$

$$J = \frac{102.5\left(10^4\right)}{(4.0)(0.29)(0.05)(100000)(0.408)}, \quad \text{[amps/cm}^2\text{]}$$

$$J = 433, \quad \text{[amps/cm}^2\text{]}$$

Step No. 9: Calculate the input current, I_{in}.

$$I_{in} = \frac{P_o}{V_{in}\eta}, \quad \text{[amps]}$$

$$I_{in} = \frac{38}{(24)(0.98)}, \quad \text{[amps]}$$

$$I_{in} = 1.61, \quad \text{[amps]}$$

Step No. 10: Calculate the primary bare wire area, $A_{wp(B)}$.

$$A_{wp(B)} = \frac{I_{in}\sqrt{D_{\max}}}{J}, \quad [\text{cm}^2]$$

$$A_{wp(B)} = \frac{(1.61)(0.707)}{433}, \quad [\text{cm}^2]$$

$$A_{wp(B)} = 0.00263, \quad [\text{cm}^2]$$

Step No. 11: Calculate the required number of primary strands, S_{np}.

$$S_{np} = \frac{A_{wp(B)}}{\#26}$$

$$S_{np} = \frac{0.00263}{0.00128}$$

$$S_{np} = 2.05 \text{ use } 2$$

Step No. 12: Calculate the primary new μΩ per centimeter.

$$(\text{new})\mu\Omega/\text{cm} = \frac{\mu\Omega/\text{cm}}{S_{np}}$$

$$(\text{new})\mu\Omega/\text{cm} = \frac{1345}{2}$$

$$(\text{new})\mu\Omega/\text{cm} = 673$$

Step No. 13: Calculate the primary resistance, R_p.

$$R_p = \text{MLT}\left(N_p\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_p = (4.4)(19)(673)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_p = 0.0563, \quad [\text{ohms}]$$

Step No. 14: Calculate the primary copper loss, P_p.

$$P_p = I_p^2 R_p, \quad [\text{watts}]$$

$$P_p = (1.61)^2(0.0563), \quad [\text{watts}]$$

$$P_p = 0.146, \quad [\text{watts}]$$

Step No. 15: Calculate the secondary turns, N_{s1}.

$$N_{s1} = \frac{N_p V_{s1}}{V_{in}}\left(1+\frac{\alpha}{100}\right), \quad \text{[turns]}$$

$$\mathrm{V}_{s1} = V_o + V_d, \quad \text{[volts]}$$

$$\mathrm{V}_{s1} = 5+1, \quad \text{[volts]}$$

$$\mathrm{V}_{s1} = 6, \quad \text{[volts]}$$

$$N_{s1} = \frac{(19)(6)}{(24)}\left(1+\frac{0.5}{100}\right), \quad \text{[turns]}$$

$$N_{s1} = 4.77 \text{ use } 5, \quad \text{[turns]}$$

Step No. 16: Calculate the secondary turns, N_{s2}.

$$N_{s2} = \frac{N_p V_{s1}}{V_{in}}\left(1+\frac{\alpha}{100}\right), \quad \text{[turns]}$$

$$\mathrm{V}_{s2} = V_o + 2V_d, \quad \text{[volts]}$$

$$\mathrm{V}_{s2} = 12+2, \quad \text{[volts]}$$

$$\mathrm{V}_{s2} = 14, \quad \text{[volts]}$$

$$N_{s2} = \frac{(19)(14)}{(24)}\left(1+\frac{0.5}{100}\right), \quad \text{[turns]}$$

$$N_{s2} = 11.1 \text{ use } 11, \quad \text{[turns]}$$

Step No. 17: Calculate the secondary bare wire area, A_{ws1}.

$$A_{ws1} = \frac{I_o\sqrt{D_{\max}}}{J}, \quad [\mathrm{cm}^2]$$

$$A_{ws1} = \frac{(4)(0.707)}{433}, \quad [\mathrm{cm}^2]$$

$$A_{ws1} = 0.00653, \quad [\mathrm{cm}^2]$$

Step No. 18: Calculate the required number of secondary strands, S_{ns1}.

$$S_{ns1} = \frac{A_{ws1(B)}}{\#26}$$

$$S_{ns1} = \frac{0.00653}{0.00128}$$

$$S_{ns1} = 5.1 \text{ use } 5$$

Step No. 19: Calculate the secondary, S_1 new $\mu\Omega$ per centimeter.

$$(\text{new})\mu\Omega/\text{cm} = \frac{\mu\Omega/\text{cm}}{S_{ns1}}$$

$$(\text{new})\mu\Omega/\text{cm} = \frac{1345}{5}$$

$$(\text{new})\mu\Omega/\text{cm} = 269$$

Step No. 20: Calculate the secondary S_1 resistance, R_{s1}.

$$R_{s1} = \text{MLT}(N_{s1})\left(\frac{\mu\Omega}{\text{cm}}\right)(10^{-6}), \quad [\text{ohms}]$$

$$R_{s1} = (4.4)(5)(269)(10^{-6}), \quad [\text{ohms}]$$

$$R_{s1} = 0.0059, \quad [\text{ohms}]$$

Step No. 21: Calculate the secondary copper loss, P_{s1}.

$$P_{s1} = I_{s1}^2 R_{s1}, \quad [\text{watts}]$$

$$P_{s1} = (4.0)^2(0.0059), \quad [\text{watts}]$$

$$P_{s1} = 0.0944, \quad [\text{watts}]$$

Step No. 22: Calculate the secondary bare wire area, A_{ws2}.

$$A_{ws2} = \frac{I_o}{J}, \quad [\text{cm}^2]$$

$$A_{ws2} = \frac{(1)}{433}, \quad [\text{cm}^2]$$

$$A_{ws2} = 0.00231, \quad [\text{cm}^2]$$

Step No. 23: Calculate the required number of secondary strands, S_{ns2}.

$$S_{ns2} = \frac{A_{ws2(B)}}{\#26}$$

$$S_{ns2} = \frac{0.00231}{0.00128}$$

$$S_{ns2} = 1.8 \text{ use } 2$$

Step No. 24: Calculate the secondary, S_2 new $\mu\Omega$ per centimeter.

$$(\text{new})\mu\Omega/\text{cm} = \frac{\mu\Omega/\text{cm}}{S_{ns2}}$$

$$(\text{new})\mu\Omega/\text{cm} = \frac{1345}{2}$$

$$(\text{new})\mu\Omega/\text{cm} = 673$$

Step No. 25: Calculate the secondary, S_2 resistance, R_{s2}.

$$R_{s2} = \text{MLT}(N_{s1})\left(\frac{\mu\Omega}{\text{cm}}\right)(10^{-6}), \quad [\text{ohms}]$$

$$R_{s2} = (4.4)(11)(673)(10^{-6}), \quad [\text{ohms}]$$

$$R_{s2} = 0.0326, \quad [\text{ohms}]$$

Step No. 26: Calculate the secondary, S_2 copper loss, P_{s2}.

$$P_{s2} = I_{s2}^2 R_{s2}, \quad [\text{watts}]$$

$$P_{s2} = (1.0)^2(0.0326), \quad [\text{watts}]$$

$$P_{s2} = 0.0326, \quad [\text{watts}]$$

Step No. 27: Calculate the total secondary copper loss, P_s.

$$P_s = P_{s1} + P_{s2}, \quad [\text{watts}]$$

$$P_s = 0.0944 + 0.0326, \quad [\text{watts}]$$

$$P_s = 0.127, \quad [\text{watts}]$$

Step No. 28: Calculate the total primary and secondary copper loss, P_{cu}.

$$P_{cu} = P_p + P_s, \quad [\text{watts}]$$

$$P_{cu} = 0.146 + 0.127, \quad [\text{watts}]$$

$$P_{cu} = 0.273, \quad [\text{watts}]$$

Step No. 29: Calculate the transformer regulation, α.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

$$\alpha = \frac{(0.273)}{(38)}(100), \quad [\%]$$

$$\alpha = 0.718, \quad [\%]$$

Step No. 30: Calculate the milliwatts per gram, mW/g. Use the Equation for this material in Chapter 2.

$$mW/g = 0.000318(f)^{1.51}(B_{ac})^{2.747}$$
$$mW/g = 0.000318(100000)^{1.51}(0.05)^{2.747}$$
$$mW/g = 3.01$$

Step No. 31: Calculate the core loss, P_{fe}.

$$P_{fe} = (mW/g)(W_{tfe})(10^{-3}), \quad \text{[watts]}$$
$$P_{fe} = (3.01)(15)(10^{-3}), \quad \text{[watts]}$$
$$P_{fe} = 0.045, \quad \text{[watts]}$$

Step No. 32: Calculate the total loss, P_Σ.

$$P_\Sigma = P_{cu} + P_{fe}, \quad \text{[watts]}$$
$$P_\Sigma = (0.273) + (0.045), \quad \text{[watts]}$$
$$P_\Sigma = 0.318, \quad \text{[watts]}$$

Step No. 33: Calculate the watts per unit area, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad \text{[watts/cm}^2\text{]}$$
$$\psi = \frac{(0.318)}{(19.7)}, \quad \text{[watts/cm}^2\text{]}$$
$$\psi = 0.0161, \quad \text{[watts/cm}^2\text{]}$$

Step No. 34: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [°\text{C}]$$
$$T_r = 450(0.0161)^{(0.826)}, \quad [°\text{C}]$$
$$T_r = 14.9, \quad [°\text{C}]$$

Step No. 35: Calculate the total window utilization, K_u.

$$K_u = K_{up} + K_{us}$$

$$K_{us} = K_{us1} + K_{us2}$$

$$K_{us1} = \frac{N_{s1} S_{n1} A_{ws1(B)}}{W_a}$$

$$K_{us1} = \frac{(10)(5)(0.00128)}{(0.658)} = 0.0973$$

$$K_{us2} = \frac{(11)(2)(0.00128)}{(0.658)} = 0.0428$$

$$K_{up} = \frac{N_p S_{np} A_{wp(B)}}{W_a}$$

$$K_{up} = \frac{(38)(2)(0.00128)}{(0.658)} = 0.148$$

$$K_u = (0.148) + (0.0973 + 0.0428)$$

$$K_u = 0.288$$

Chapter 8

DC Inductor Design, Using Gapped Cores

Table of Contents

Introduction

Designers have used various approaches in arriving at suitable inductor designs. For example, in many cases, a rule of thumb used for dealing with current density is that a good working level is 200 amps-per-cm^2 (1000 Cir-Mils-per-amp). This rule is satisfactory in many instances. However, the wire size used to meet this requirement may produce a heavier and bulkier inductor than desired or required. The information presented, herein, will make it possible to avoid the use of this rule and other rules of thumb, and will help to develop an economical and a better design.

Critical Inductance for Sine Wave Rectification

The LC filter is the basic method of reducing ripple levels. The two basic rectifier circuits are the full-wave center-tap, as shown in Figure 8-1, and the full-wave bridge, as shown in Figure 8-2. To achieve normal inductor operation, it is necessary that there be a continuous flow of current through the input inductor, L1.

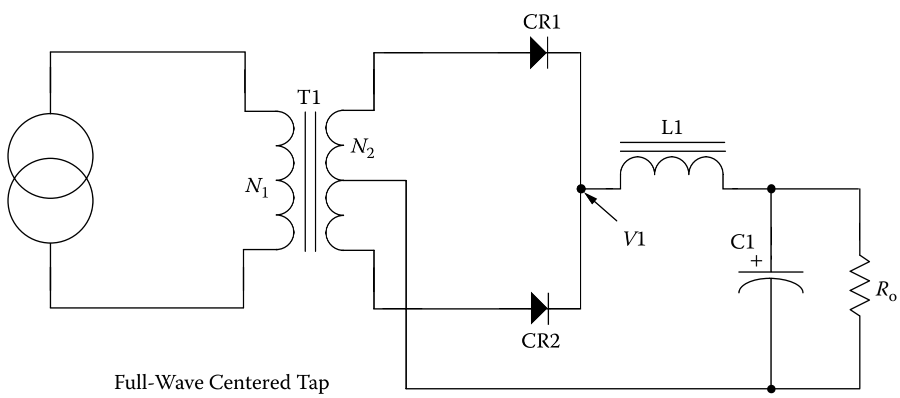

Figure 8-1. Full-Wave Center Tap with an LC filter.

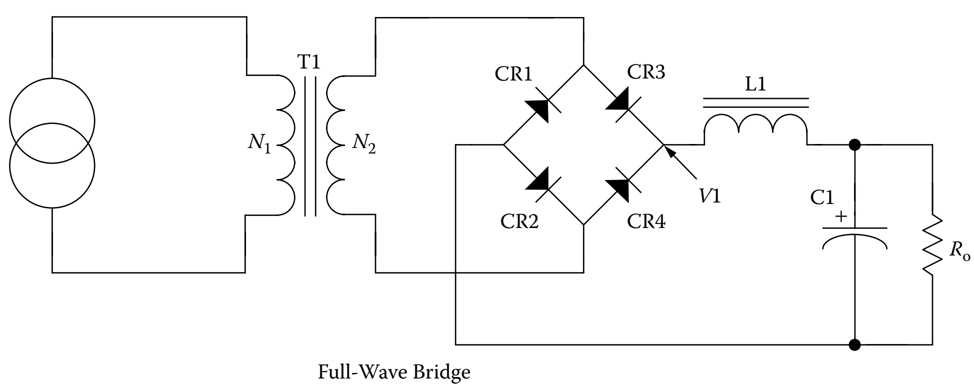

Figure 8-2. Full-Wave Bridge with an LC filter.

The value for minimum inductance called critical inductance, $L_{(crt)}$ is:

$$L_{(crt)} = \frac{R_{o(\max)}}{3\omega}, \quad \text{[henrys]} \qquad [8\text{-}1]$$

Where:

$\omega = 2\pi f$

$f =$ line frequency

The higher the load resistance, R_o, (i.e., the lower the dc load current), the more difficult it is to maintain a continuous flow of current. The filter inductor operates in the following manner: When, R_o, approaches infinity, under an unloaded condition, (no bleeder resistor), $I_o = 0$, the filter capacitor will charge to, $V1_{pk}$, the peak voltage. Therefore, the output voltage will be equal to the peak value of the input voltage, as shown in Figure 8-3.

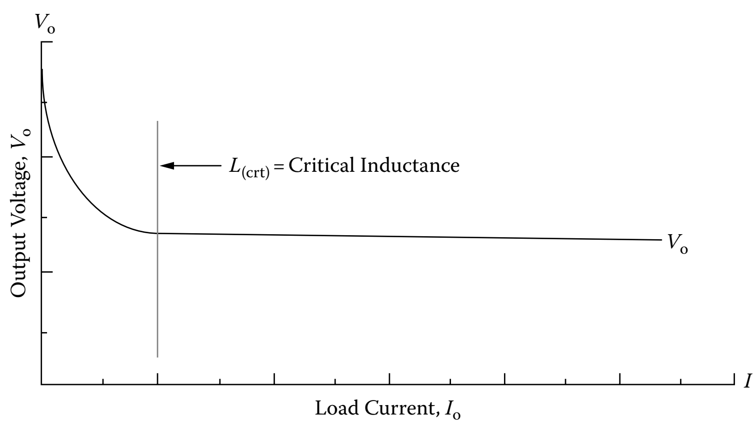

Figure 8-3. Critical Inductance Point.

The ripple reduction from a single stage LC filter can be calculated, using Equation [8-2] and Figure 8-4.

$$V_{r(pk)} = V_{in(pk)}\left(\frac{1}{(2\pi f)^2 L1C1}\right), \quad \text{[volts-peak]} \qquad [8\text{-}2]$$

Where:

L = henrys

C = micro-farads

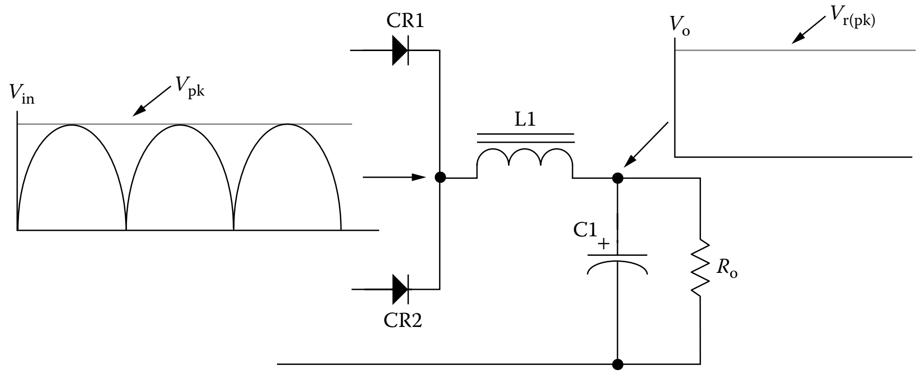

Figure 8-4. LC Filter Ripple Reduction.

Critical Inductance for Buck Type Converters

The buck type converter schematic is shown in Figure 8-5, and the buck type dc-to-dc converter is shown in Figure 8.6. The buck regulator filter circuit shown in Figure 8-5 has three current probes. These current probes monitor the three basic currents in a switch mode, buck output filter. Current probe A monitors the power

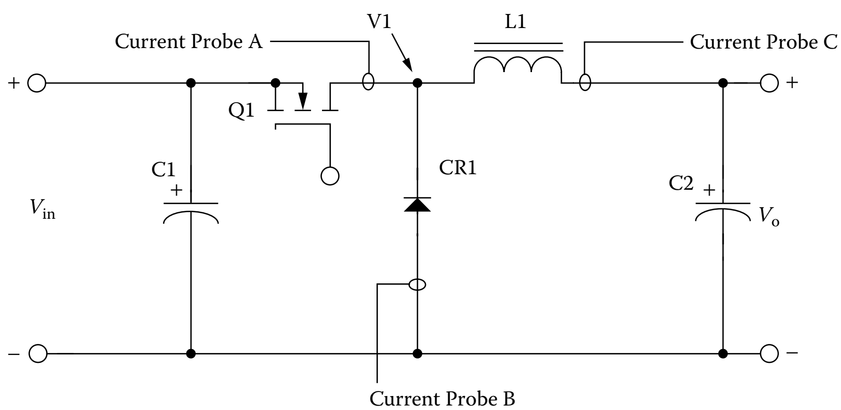

Figure 8-5. Buck Regulator Converter.

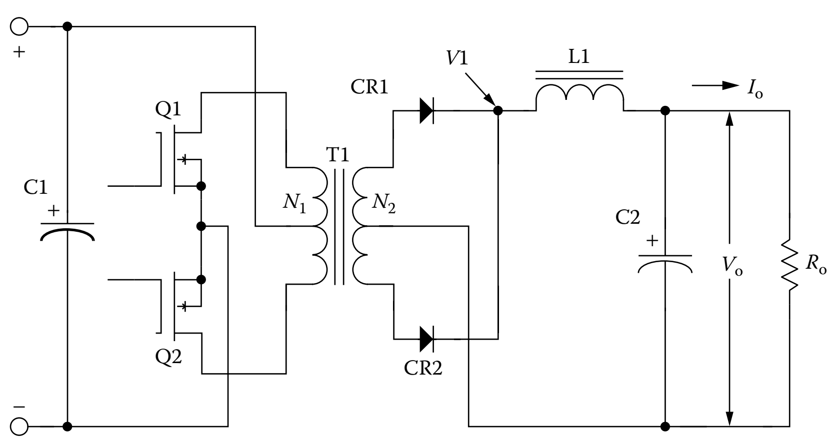

Figure 8-6. Push-Pull Buck Type Converter.

MOSFET, Q1, switching current. Current probe B monitors the commutating current through CR1. Current probe C monitors the current through the output inductor, L1.

The typical filter waveforms of the buck converter are shown in Figure 8-7. The waveforms are shown with the converter operating at a 0.5 duty ratio. The applied voltage, V1, to the filter, is shown in Figure 8-7A. The power MOSFET, Q1, current is shown in Figure 8-7B. The commutating current flowing through CR1 is shown in Figure 8-7C. The commutating current is the result of, Q1, being turned off, and the field in L1 collapsing, producing the commutating current. The current flowing through, L1, is shown in Figure 8-7D. The current flowing through, L1, is the sum of the currents in Figure 8-7B and 8-7C.

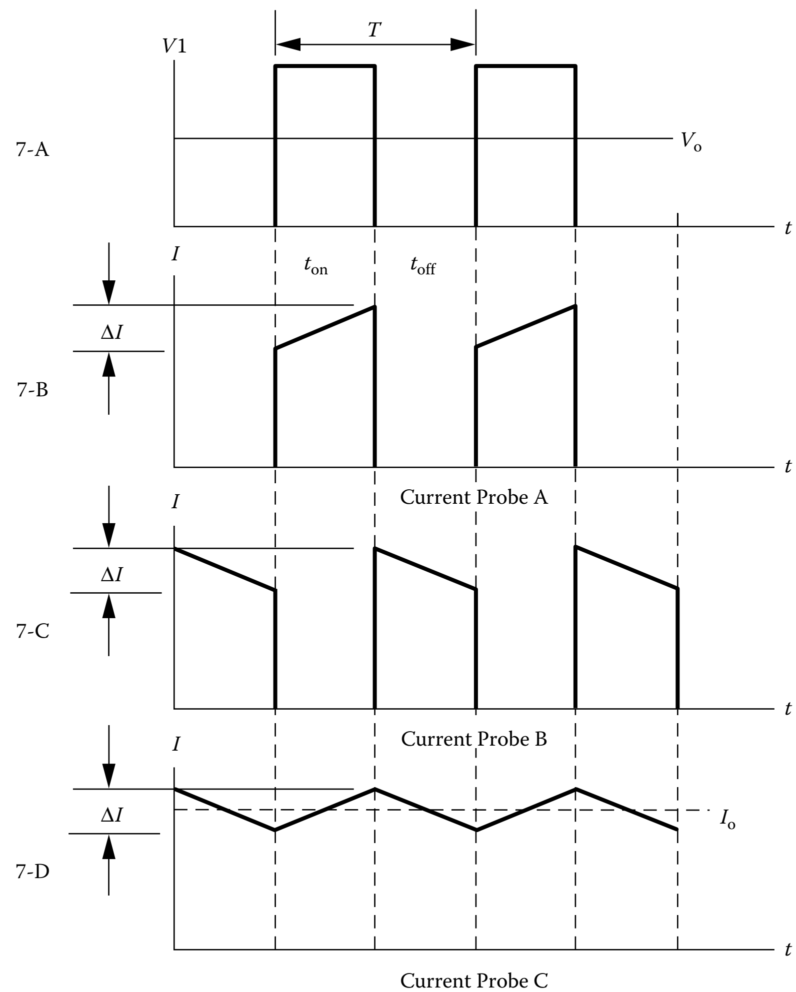

Figure 8-7. Typical Buck Converter Waveforms, Operating at a 0.5 Duty Ratio.

The critical inductance current is shown in Figure 8-8, 8-B and is realized in Equation 8-3. The critical inductance current is when the ratio of the delta current to the output load current is equal to $2 = \Delta I / I_o$. If the output load current is allowed to go beyond this point, the current will become discontinuous, as shown in Figure 8-8, 8-D. The applied voltage, V1, will have ringing at the level of the output voltage, as shown in Figure 8-8, 8-C. When the current in the output inductor becomes discontinuous, as shown in Figure 8-8, 8-D, the response time for a step load becomes very poor.

$$L_{(critical)} = \frac{V_o T\left(1 - D_{(\min)}\right)}{2I_{o(\min)}}, \quad \text{[henrys]} \qquad [8\text{-}3]$$

$$D_{(\min)} = \frac{V_o}{\left(\eta V_{in(\max)}\right)} \qquad [8\text{-}4]$$

When designing multiple output converters similar to Figure 8-6, the slaved outputs should never have the current in the inductor go discontinuous or to zero. If the current goes to zero, a slaved output voltage will rise to the value of, V1. If the current is allowed to go to zero, then, there is no potential difference between the input and output voltage of the filter. Then the output voltage will rise to equal the peak input voltage.

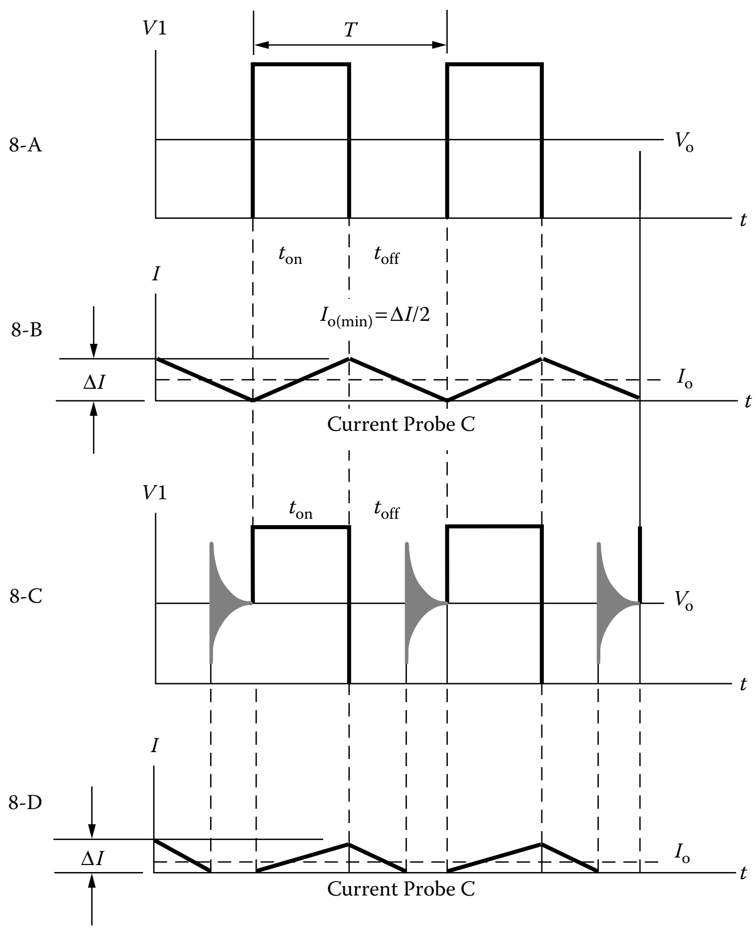

Figure 8-8. Buck Converter, Output Filter Inductor Goes from Critical to Discontinuous Operation.

Core Materials, Used in PWM Converters

Designers have routinely tended to specify Molypermalloy powder materials for filter inductors used in high-frequency, power converters and pulse-width-modulators (PWM) switched regulators, because of the availability of manufacturers' literature containing tables, graphs, and examples that simplify the design task. Use of these cores may result in an inductor design, not optimized for size and weight. For example, as shown in Figure 8-9, Molypermalloy powder cores, operating with a dc bias of 0.3T, have only about 80% of the original inductance, with very rapid falloff at higher flux densities. When size is of greatest concern then, magnetic materials with high flux saturation, B_s, would be the first choice. Materials, such as silicon or some amorphous materials, have approximately four times the useful flux density compared to Molypermalloy powder cores. Iron alloys retain 90% of their original inductance, at greater than 1.2T. Iron alloys when designed correctly and used in the right application, will perform well at frequencies up to 100kHz. When operating above 100kHz, then the only material is a ferrite. Ferrite materials have a negative temperature coefficient regarding flux density. The operating temperature and temperature rise should be used to calculate the maximum flux density.

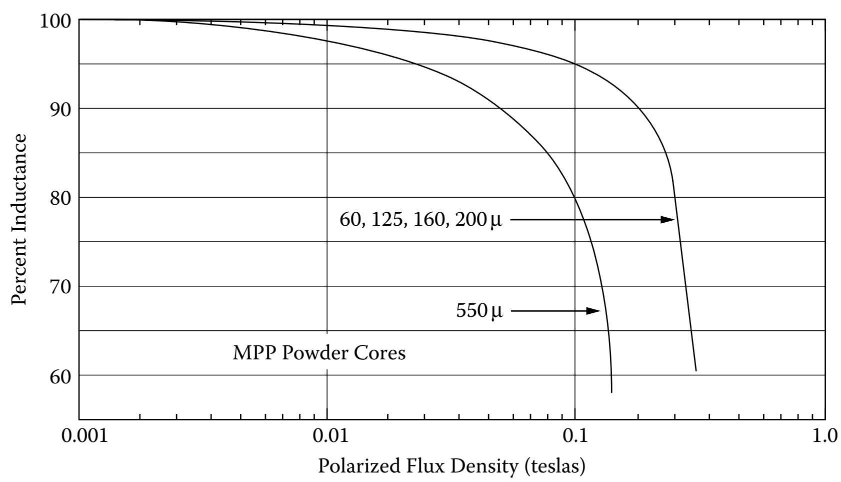

Figure 8-9. Inductance Versus dc Bias.

To get optimum performance, together with size, the engineer must evaluate the materials for both, B_s, and, B_{ac}. See Table 8-1. The operating dc flux has only to do with I^2R losses, (copper). The ac flux, B_{ac}, has to do with core loss. This loss depends directly on the material. There are many factors that impact a design: cost, size, temperature rise and material availability.

There are significant advantages to be gained by the use of iron alloys and ferrites in the design of power inductors, despite certain disadvantages, such as the need for banding and gapping materials, banding tools, mounting brackets, and winding mandrels.

Table 8-1. Magnetic Material Properties

Magnetic Material Properties					
Material Name	Composition	Initial Permeability μ_i	Flux Density Teslas B_s	Curie Temp. °C	Density grams/cm^3 δ
Silicon	3-97 SiFe	1500	1.5-1.8	750	7.63
Orthonol	50-50 NiFe	2000	1.42-1.58	500	8.24
Permalloy	80-20 NiFe	25000	0.66-0.82	460	8.73
Amorphous	81-3.5 FeSi	1500	1.5-1.6	370	7.32
Amorphous	66-4 CoFe	800	0.57	250	7.59
Amorphous(μ)	73-15 FeSi	30000	1.0-1.2	460	7.73
Ferrite	MnZn	2500	0.5	>230	4.8

Iron alloys and ferrites provide greater flexibility in the design of high frequency power inductors, because the air gap can be adjusted to any desired length, and because the relative permeability is high, even at high, dc flux density.

Fundamental Considerations

The design of a linear reactor depends upon four related factors:

1. Desired inductance, L
2. Direct current, I_{dc}
3. Alternating current, ΔI
4. Power loss and temperature, T_r

With these requirements established, the designer must determine the maximum values for, B_{dc}, and, B_{ac}, that will not produce magnetic saturation. The designer must make trade-offs that will yield the highest inductance for a given volume. It should be remembered the peak operating flux, B_{pk}, depends upon, $B_{dc} + B_{ac}$, in the manner in Figure 8-10.

$$B_{pk} = B_{dc} + \frac{B_{ac}}{2}, \quad \text{[teslas]} \qquad \text{[8-5]}$$

$$B_{dc} = \frac{0.4\pi N I_{dc}\left(10^{-4}\right)}{l_g + \left(\dfrac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]} \qquad \text{[8-6]}$$

$$B_{ac} = \frac{0.4\pi N\left(\frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]} \qquad [8\text{-}7]$$

$$B_{pk} = \frac{0.4\pi N\left(I_{dc} + \frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]} \qquad [8\text{-}8]$$

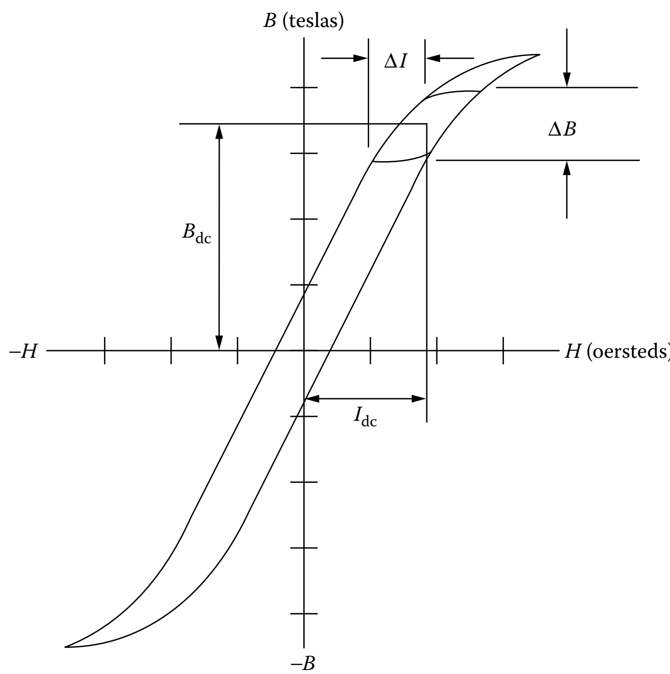

Figure 8-10. Inductor Flux Density Versus $I_{dc} + \Delta I$ Current.

The inductance of an iron-core inductor carrying direct current and having an air gap may be expressed as:

$$L = \frac{0.4\pi N^2 A_c\left(10^{-8}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[henrys]} \qquad [8\text{-}9]$$

This equation shows that inductance is dependent on the effective length of the magnetic path, which is the sum of the air gap length, l_g, and the ratio of the core mean length to the material permeability, MPL/μ_m. When the core air gap, l_g, is large compared to the ratio, MPL/μ_m, because of material permeability, μ_m, variations in,

μ_m, do not substantially affect the total effective Magnetic Path Length or the inductance. Then the inductance Equation [8-9] reduces to:

$$L = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{l_g}, \quad \text{[henrys]} \qquad [8\text{-}10]$$

Final determination of the air gap size requires consideration of the effect of fringing flux, which is a function of gap dimension, the shape of the pole faces, and the shape, size, and location of the winding. Its net effect is to shorten the air gap. Because of the fringing flux it is wise to lower the initial operating flux density, 10 to 20%.

Fringing Flux

Fringing flux decreases the total reluctance of the magnetic path and therefore, increases the inductance by a factor, F, to a value greater than that calculated from Equation [8-10]. Fringing flux is a larger percentage of the total for the larger gaps.

The fringing factor is:

$$F = 1 + \frac{l_g}{\sqrt{A_c}} \ln\left(\frac{2G}{l_g}\right) \qquad [8\text{-}11]$$

Where G is the winding length, defined in Chapter 3. This equation is valid for laminations, C cores and cut ferrites. Equation [8-11] is plotted in Figure 8-11.

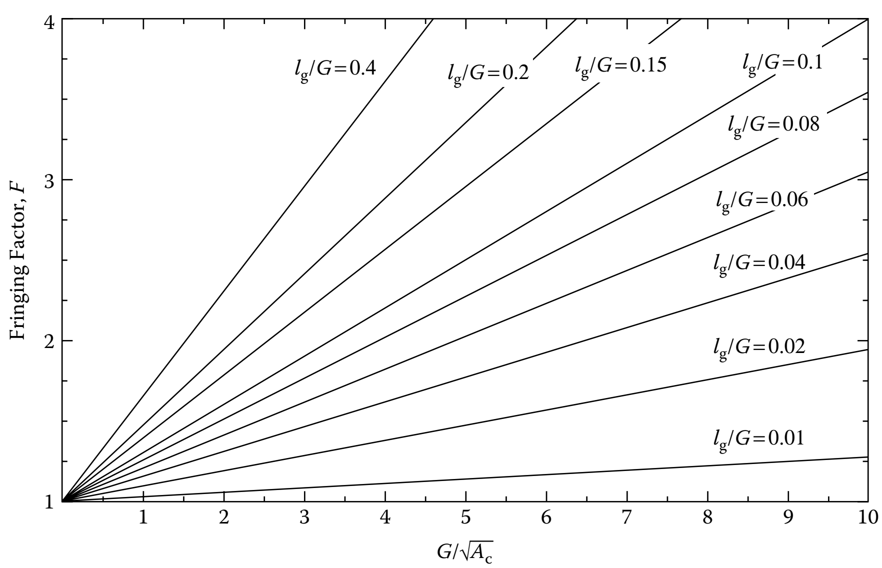

Figure 8-11. Increase of Inductance with Fringing Flux at the Gap.

As the air gap increases, the flux across the gap fringes more and more. Some of the fringing flux strikes the core, perpendicular to the strip or tape, and sets up eddy currents, which cause additional losses in the core. If the gap dimension gets too large, the fringing flux will strike the copper winding and produce eddy currents, generating heat, just like an induction heater. The fringing flux will jump the gap and produce eddy currents, in both the core and winding, as shown in Figure 8-12.

The inductance, L, computed in Equation [8-10], does not include the effect of the fringing flux. The value of inductance, L′, corrected for fringing flux is:

$$L' = \frac{0.4\pi N^2 F A_c \left(10^{-8}\right)}{l_g}, \quad \text{[henrys]} \qquad \text{[8-12]}$$

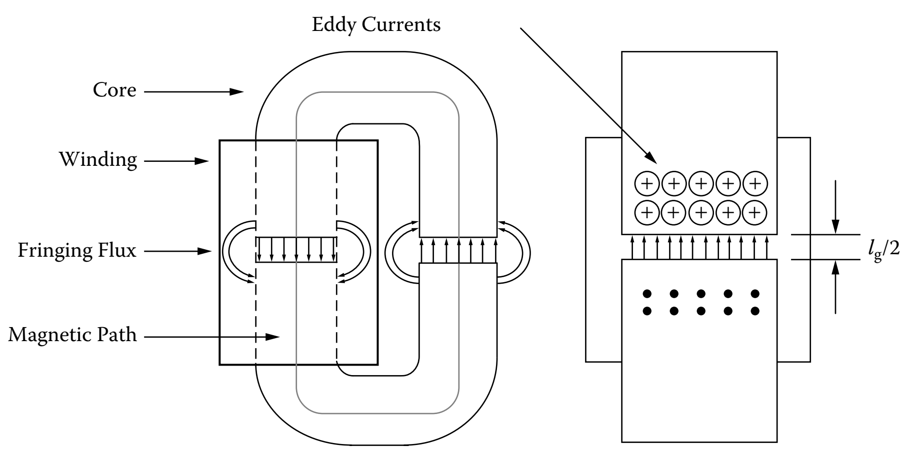

Figure 8-12. Fringing Flux Around the Gap of an Inductor.

The effect permeability may be calculated from the following Equation:

$$\mu_e = \frac{\mu_m}{1 + \left(\frac{l_g}{\text{MPL}}\right)\mu_m} \qquad \text{[8-13]}$$

Where, μ_m, is the material permeability.

Inductors

Inductors that carry direct current are used frequently in a wide variety of ground, air, and space applications. Selection of the best magnetic core for an inductor frequently involves a trial-and-error type of calculation.

The author has developed a simplified method of designing optimum, dc carrying inductors with gapped cores. This method allows the engineer to select the proper core that will provide correct copper loss, and make allowances for fringing flux, without relying on trial-and-error and the use of the cumbersome Hanna's curves.

Rather than discuss the various methods used by transformer designers, the author believes it is more useful to consider typical design problems, and to work out solutions using the approach based upon newly formulated relationships. Two gapped core designs will be compared. To compare their merits, the first design example will use the core geometry, K_g, and the second design will use the area product, A_p.

Inductors, designed in this handbook, are banded together with phosphor bronze banding material, or held together with aluminum brackets. The use of steel banding material, or brackets that bridge the gap are not recommended, because the use of steel across the gap is called shorting the gap. When the gap is shorted, the inductance will increase from the calculated value.

Relationship of, A_p, to Inductor's Energy-Handling Capability

The energy-handling capability of a core is related to its area product, A_p, by the Equation:

$$A_p = \frac{2(\text{Energy})(10^4)}{B_m J K_u}, \quad [\text{cm}^4] \qquad [8\text{-}14]$$

Where: Energy is in watt-seconds.

B_m is the flux density, teslas.

J is the current density, amps-per-cm^2.

K_u is the window utilization factor. (See Chapter 4)

From the above, it can be seen that factors, such as flux density, B_m, window utilization factor, K_u, (which defines the maximum space that may be used by the copper in the window), and the current density, J, which controls the copper loss, all impact the area product, A_p. The energy-handling capability of a core is derived from:

$$\text{Energy} = \frac{LI^2}{2}, \quad [\text{watt-seconds}] \qquad [8\text{-}15]$$

Relationship of, K_g, to Inductor's Energy-Handling Capability

Inductors, like transformers, are designed for a given temperature rise. They can also be designed for a given regulation. The regulation and energy handling ability of a core is related to two constants:

$$\alpha = \frac{(\text{Energy})^2}{K_g K_e}, \quad [\%] \qquad [8\text{-}16]$$

Where, α, is the regulation, %:

The constant, K_g, is determined by the core geometry:

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \qquad [8\text{-}17]$$

The constant, K_e, is determined by the magnetic and electrical operating conditions:

$$K_e = 0.145 P_o B_{pk}^2 \left(10^{-4}\right) \qquad [8\text{-}18]$$

The peak operating flux density, B_{pk}, is:

$$B_{pk} = B_{dc} + \frac{B_{ac}}{2}, \quad [\text{teslas}] \qquad [8\text{-}19]$$

From the above, it can be seen that the flux density, B_{pk}, is the predominant factor governing size.

The output power, P_o, is defined in Figure 8-13.

$$P_{o(L1)} = V_{(01)} I_{(01)} \qquad P_{o(L2)} = V_{(02)} I_{(02)} \qquad [8\text{-}20]$$

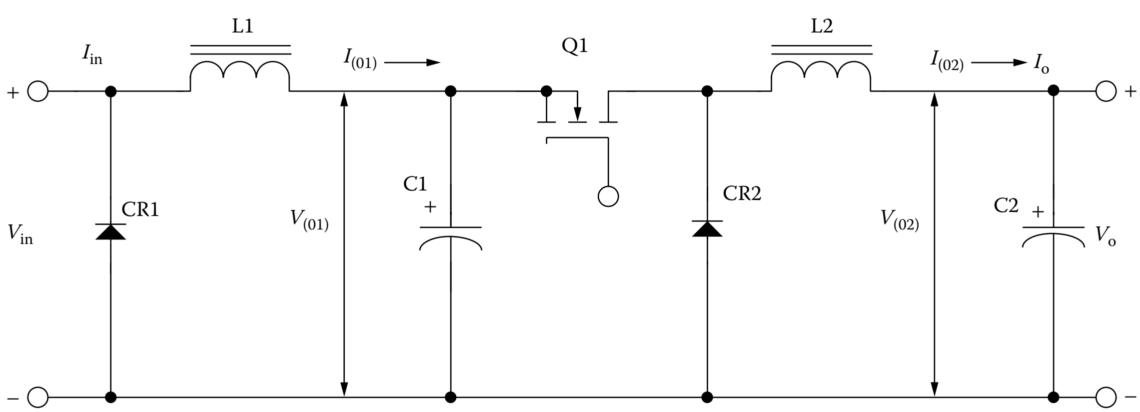

Figure 8-13. Defining the Inductor Output Power.

Gapped Inductor Design Example Using the Core Geometry, K_g, Approach

Step No. 1: Design a linear dc inductor with the following specifications:

1. Inductance, L = 0.0025 henrys
2. dc current, I_o = 1.5 amps
3. ac current, ΔI = 0.2 amps
4. Output power, P_o = 100 watts
5. Regulation, α = 1.0 %
6. Ripple Frequency = 200kHz
7. Operating flux density, B_m = 0.22 tesla
8. Core Material = ferrite
9. Window utilization, K_u = 0.4
10. Temperature rise goal, T_r = 25°C

Step No. 2: Calculate the peak current, I_{pk}.

$$I_{pk} = I_o + \frac{\Delta I}{2}, \quad \text{[amps]}$$

$$I_{pk} = (1.5) + \frac{(0.2)}{2}, \quad \text{[amps]}$$

$$I_{pk} = 1.6, \quad \text{[amps]}$$

Step No. 3: Calculate the energy-handling capability.

$$\text{Energy} = \frac{LI_{pk}^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Energy} = \frac{(0.0025)(1.6)^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Energy} = 0.0032, \quad \text{[watt-seconds]}$$

Step No. 4: Calculate the electrical conditions coefficient, K_e.

$$K_e = 0.145 P_o B_m^2 \left(10^{-4}\right)$$

$$K_e = 0.145(100)(0.22)^2 \left(10^{-4}\right)$$

$$K_e = 0.0000702$$

Step No. 5: Calculate the core geometry coefficient, K_g.

$$K_g = \frac{(\text{Energy})^2}{K_e \alpha}, \quad [\text{cm}^5]$$

$$K_g = \frac{(0.0032)^2}{(0.0000702)(1.0)}, \quad [\text{cm}^5]$$

$$K_g = 0.146, \quad [\text{cm}^5]$$

Step No. 6: Select an ETD ferrite core from Chapter 3. The data listed is the closest core to the calculated core geometry, K_g.

1. Core Number = ETD-39
2. Magnetic Path Length, MPL = 9.22 cm
3. Core Weight, W_{tfe} = 60 grams
4. Mean Length Turn, MLT = 8.3 cm
5. Iron Area, A_c = 1.252 cm^2
6. Window Area, W_a = 2.34 cm^2
7. Area Product, A_p = 2.93 cm^4
8. Core Geometry, K_g = 0.177 cm^5
9. Surface Area, A_t = 69.9 cm^2
10. Material, P = 2500μ
11. Millihenrys-per-1000 Turns, AL = 3295 mh
12. Winding Length, G = 2.84 cm

Step No. 7: Calculate the current density, J, using the area product Equation, A_p.

$$J = \frac{2(Energy)(10^4)}{B_m A_p K_u}, \quad [\text{amps-per-cm}^2]$$

$$J = \frac{2(0.0032)(10^4)}{(0.22)(2.93)(0.4)}, \quad [\text{amps-per-cm}^2]$$

$$J = 248, \quad [\text{amps-per-cm}^2]$$

Step No. 8: Calculate the rms current, I_{rms}.

$$I_{rms} = \sqrt{I_o^2 + \Delta I^2}, \quad [\text{amps}]$$

$$I_{rms} = \sqrt{(1.5)^2 + (0.2)^2}, \quad [\text{amps}]$$

$$I_{rms} = 1.51, \quad [\text{amps}]$$

Step No. 9: Calculate the required bare wire area, $A_{w(B)}$.

$$A_{W(B)} = \frac{I_{rms}}{J}, \quad [\text{cm}^2]$$

$$A_{W(B)} = \frac{(1.51)}{(248)}, \quad [\text{cm}^2]$$

$$A_{W(B)} = 0.00609, \quad [\text{cm}^2]$$

Step No. 10: Select a wire from the Wire Table in Chapter 4. If the area is not within 10%, take the next smallest size. Also, record the micro-ohms per centimeter.

$$\text{AWG} = \#19$$

$$\text{Bare, } A_{W(B)} = 0.00653, \quad [\text{cm}^2]$$

$$\text{Insulated, } A_W = 0.00754, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 264, \quad [\text{micro-ohm/cm}]$$

Step No. 11: Calculate the effective window area, $W_{a(eff)}$, using the window area found in Step 6. A typical value for, S_3, is 0.75, as shown in Chapter 4.

$$W_{a(eff)} = W_a S_3, \quad [\text{cm}^2]$$

$$W_{a(eff)} = (2.34)(0.75), \quad [\text{cm}^2]$$

$$W_{a(eff)} = 1.76, \quad [\text{cm}^2]$$

Step No. 12: Calculate the number turns possible, N, using the insulated wire area, A_w, found in Step 10. A typical value for, S_2, is 0.6, as shown in Chapter 4.

$$N = \frac{W_{a(eff)} S_2}{A_W}, \quad [\text{turns}]$$

$$N = \frac{(1.76)(0.60)}{(0.00754)}, \quad [\text{turns}]$$

$$N = 140, \quad [\text{turns}]$$

Step No. 13: Calculate the required gap, l_g.

$$l_g = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{L} - \left(\frac{\text{MPL}}{\mu_m}\right), \quad \text{[cm]}$$

$$l_g = \frac{(1.26)(140)^2(1.25)\left(10^{-8}\right)}{(0.0025)} - \left(\frac{9.22}{2500}\right), \quad \text{[cm]}$$

$$l_g = 0.120, \quad \text{[cm]}$$

Step No. 14: Calculate the equivalent gap in mils.

$$\text{mils} = cm(393.7)$$

$$\text{mils} = (0.120)(393.7)$$

$$\text{mils} = 47.2 \text{ use } 50$$

Step No. 15: Calculate the fringing flux factor, F.

$$F = 1 + \frac{l_g}{\sqrt{A_c}} \ln\left(\frac{2G}{l_g}\right)$$

$$F = 1 + \frac{(0.120)}{\sqrt{1.25}} \ln\left(\frac{2(2.84)}{0.120}\right)$$

$$F = 1.41$$

Step No. 16: Calculate the new number of turns, N_n, by inserting the fringing flux, F.

$$N_n = \sqrt{\frac{l_g L}{0.4\pi A_c F\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N_n = \sqrt{\frac{(0.120)(0.0025)}{(1.26)(1.25)(1.41)\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N_n = 116, \quad \text{[turns]}$$

Step No. 17: Calculate the winding resistance, R_L. Use the MLT, from Step 6, and the micro-ohm per centimeter from Step 10.

$$R_L = (\text{MLT})(N_n)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_L = (8.3)(116)(264)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_L = 0.254, \quad \text{[ohms]}$$

Step No. 18: Calculate the copper loss, P_{cu}.

$$P_{cu} = I_{rms}^2 R_L, \quad \text{[watts]}$$

$$P_{cu} = (1.51)^2 (0.254), \quad \text{[watts]}$$

$$P_{cu} = 0.579, \quad \text{[watts]}$$

Step No. 19: Calculate the regulation, α.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

$$\alpha = \frac{(0.579)}{(100)}(100), \quad [\%]$$

$$\alpha = 0.579, \quad [\%]$$

Step No. 20: Calculate the ac flux density, B_{ac}.

$$B_{ac} = \frac{0.4\pi N_n F\left(\frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]}$$

$$B_{ac} = \frac{(1.26)(116)(1.41)\left(\frac{0.2}{2}\right)\left(10^{-4}\right)}{(0.120) + \left(\frac{9.22}{2500}\right)}, \quad \text{[teslas]}$$

$$B_{ac} = 0.0167, \quad \text{[teslas]}$$

Step No. 21: Calculate the watts per kilogram for ferrite, P, material in Chapter 2. Watts per kilogram can be written in milliwatts per gram.

$$\text{mW/g} = k f^{(m)} B_{ac}^{(n)}$$

$$\text{mW/g} = (0.00004855)(200000)^{(1.63)}(0.0167)^{(2.62)}$$

$$\text{mW/g} = 0.468$$

Step No. 22: Calculate the core loss, P_{fe}.

$$P_{fe} = (\text{mW/g})(W_{tfe})\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = (0.468)(60)\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = 0.0281, \quad \text{[watts]}$$

Step No. 23: Calculate the total loss, copper plus iron, P_{Σ}.

$$P_{\Sigma} = P_{fe} + P_{cu}, \quad \text{[watts]}$$

$$P_{\Sigma} = (0.0281) + (0.579), \quad \text{[watts]}$$

$$P_{\Sigma} = 0.607, \quad \text{[watts]}$$

Step No. 24: Calculate the watt density, ψ. The surface area, A_t, can be found in Step 6.

$$\psi = \frac{P_{\Sigma}}{A_t}, \quad \text{[watts/cm}^2\text{]}$$

$$\psi = \frac{(0.607)}{(69.9)}, \quad \text{[watts/cm}^2\text{]}$$

$$\psi = 0.00868, \quad \text{[watts/cm}^2\text{]}$$

Step No. 25: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [°\text{C}]$$

$$T_r = 450(0.00868)^{(0.826)}, \quad [°\text{C}]$$

$$T_r = 8.92, \quad [°\text{C}]$$

Step No. 26: Calculate the peak flux density, B_{pk}.

$$B_{pk} = \frac{0.4\pi N_n F\left(I_{dc} + \frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]}$$

$$B_{pk} = \frac{(1.26)(116)(1.41)(1.6)\left(10^{-4}\right)}{(0.127) + \left(\frac{9.22}{2500}\right)}, \quad \text{[teslas]}$$

$$B_{pk} = 0.252, \quad \text{[teslas]}$$

Note: The big advantage in using the core geometry design procedure is that the wire current density is calculated. When using the area product design procedure, the current density is an estimate, at best. In this next design the same current density will be used as in the core geometry design.

Gapped Inductor Design Example Using the Area Product, A_p, Approach

Step No. 1: Design a linear dc inductor with the following specifications:

1. Inductance, L = 0.0025 henrys
2. dc current, I_o = 1.5 amps
3. ac current, ΔI = 0.2 amps
4. Output power, P_o = 100 watts
5. Current Density, J = 250 amps-per-cm^2
6. Ripple Frequency = 200kHz
7. Operating flux density, B_m = 0.22 tesla
8. Core Material = ferrite
9. Window utilization, K_u = 0.4
10. Temperature rise goal, T_r = 25°C

Step No. 2: Calculate the peak current, I_{pk}.

$$I_{pk} = I_o + \frac{\Delta I}{2}, \quad \text{[amps]}$$

$$I_{pk} = (1.5) + \frac{(0.2)}{2}, \quad \text{[amps]}$$

$$I_{pk} = 1.6, \quad \text{[amps]}$$

Step No. 3: Calculate the energy-handling capability.

$$\text{Energy} = \frac{LI_{pk}^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Energy} = \frac{(0.0025)(1.6)^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Energy} = 0.0032, \quad \text{[watt-seconds]}$$

Step No. 4: Calculate the area product, A_p.

$$A_p = \frac{2(\text{Energy})(10^4)}{B_m J K_u}, \quad [\text{cm}^4]$$

$$A_p = \frac{2(0.0032)(10^4)}{(0.22)(248)(0.4)}, \quad [\text{cm}^4]$$

$$A_p = 2.93, \quad [\text{cm}^4]$$

Step No. 5: Select an ETD ferrite core from Chapter 3. The data listed is the closest core to the calculated area product, A_p.

1. Core Number = ETD-39
2. Magnetic Path Length, MPL = 9.22 cm
3. Core Weight, W_{tfe} = 60 grams
4. Mean Length Turn, MLT = 8.3 cm
5. Iron Area, $A_c = 1.252\ \text{cm}^2$
6. Window Area, $W_a = 2.34\ \text{cm}^2$
7. Area Product, $A_p = 2.93\ \text{cm}^4$
8. Core Geometry, $K_g = 0.177\ \text{cm}^5$
9. Surface Area, $A_t = 69.9\ \text{cm}^2$
10. Material, P = 2500μ
11. Millihenrys-per-1k, AL = 3295 mh
12. Winding Length, G = 2.84 cm

Step No. 6: Calculate the rms current, I_{rms}.

$$I_{rms} = \sqrt{I_o^2 + \Delta I^2}, \quad \text{[amps]}$$

$$I_{rms} = \sqrt{(1.5)^2 + (0.2)^2}, \quad \text{[amps]}$$

$$I_{rms} = 1.51, \quad \text{[amps]}$$

Step No. 7: Calculate the required bare wire area, $A_{w(B)}$.

$$A_{W(B)} = \frac{I_{rms}}{J}, \quad [\text{cm}^2]$$

$$A_{W(B)} = \frac{(1.51)}{(248)}, \quad [\text{cm}^2]$$

$$A_{W(B)} = 0.00609, \quad [\text{cm}^2]$$

Step No. 8: Select a wire from the Wire Table in Chapter 4. If the area is not within 10%, take the next smallest size. Also, record micro-ohms per centimeter.

$$\text{AWG} = \#19$$

$$\text{Bare, } A_{W(B)} = 0.00653, \quad [\text{cm}^2]$$

$$\text{Insulated, } A_W = 0.00754, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 264, \quad \text{[micro-ohm/cm]}$$

Step No. 9: Calculate the effective window area, $W_{a(eff)}$. Use the window area found in Step 6. A typical value for, S_3, is 0.75, as shown in Chapter 4.

$$W_{a(eff)} = W_a S_3, \quad [\text{cm}^2]$$

$$W_{a(eff)} = (2.34)(0.75), \quad [\text{cm}^2]$$

$$W_{a(eff)} = 1.76, \quad [\text{cm}^2]$$

Step No. 10: Calculate the number turns possible, N, using the insulated wire area, A_w, found in Step 8. A typical value for, S_2, is 0.6, as shown in Chapter 4.

$$N = \frac{W_{a(eff)} S_2}{A_W}, \quad [\text{turns}]$$

$$N = \frac{(1.76)(0.60)}{(0.00754)}, \quad [\text{turns}]$$

$$N = 140, \quad [\text{turns}]$$

Step No. 11: Calculate the required gap, l_g.

$$l_g = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{L} - \left(\frac{\text{MPL}}{\mu_m}\right), \quad [\text{cm}]$$

$$l_g = \frac{(1.26)(140)^2(1.25)\left(10^{-8}\right)}{(0.0025)} - \left(\frac{9.22}{2500}\right), \quad [\text{cm}]$$

$$l_g = 0.120, \quad [\text{cm}]$$

Step No. 12: Calculate the equivalent gap in mils.

$$\text{mils} = cm(393.7)$$

$$\text{mils} = (0.120)(393.7)$$

$$\text{mils} = 47.2 \text{ use } 50$$

Step No. 13: Calculate the fringing flux factor, F.

$$F = 1 + \frac{l_g}{\sqrt{A_c}} \ln\left(\frac{2G}{l_g}\right)$$

$$F = 1 + \frac{(0.120)}{\sqrt{1.25}} \ln\left(\frac{2(2.84)}{0.120}\right)$$

$$F = 1.41$$

Step No. 14: Calculate the new number of turns, N_n, by inserting the fringing flux, F.

$$N_n = \sqrt{\frac{l_g L}{0.4\pi A_c F\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N_n = \sqrt{\frac{(0.120)(0.0025)}{(1.26)(1.25)(1.41)\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N_n = 116, \quad \text{[turns]}$$

Step No. 15: Calculate the winding resistance, R_L. Use the, MLT, from Step 5, and the micro-ohm per centimeter, from Step 10.

$$R_L = (\text{MLT})(N_n)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_L = (8.3)(116)(264)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_L = 0.254, \quad \text{[ohms]}$$

Step No. 16: Calculate the copper loss, P_{cu}.

$$P_{cu} = I_{rms}^2 R_L, \quad \text{[watts]}$$

$$P_{cu} = (1.51)^2(0.254), \quad \text{[watts]}$$

$$P_{cu} = 0.579, \quad \text{[watts]}$$

Step No. 17: Calculate the regulation, α.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

$$\alpha = \frac{(0.579)}{(100)}(100), \quad [\%]$$

$$\alpha = 0.579, \quad [\%]$$

Step No. 18: Calculate the ac flux density, B_{ac}.

$$B_{ac} = \frac{0.4\pi N_n F\left(\frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]}$$

$$B_{ac} = \frac{(1.26)(116)(1.41)\left(\frac{0.2}{2}\right)\left(10^{-4}\right)}{(0.120) + \left(\frac{9.22}{2500}\right)}, \quad \text{[teslas]}$$

$$B_{ac} = 0.0167, \quad \text{[teslas]}$$

Step No. 19: Calculate the watts per kilogram for ferrite, P, material in Chapter 2. Watts per kilogram can be written in milliwatts per gram.

$$\text{mW/g} = k\, f^{(m)} B_{ac}^{(n)}$$

$$\text{mW/g} = (0.00004855)(200000)^{(1.63)}(0.0167)^{(2.62)}$$

$$\text{mW/g} = 0.468$$

Step No. 20: Calculate the core loss, P_{fe}.

$$P_{fe} = (\text{mW/g})(W_{tfe})(10^{-3}), \quad [\text{watts}]$$

$$P_{fe} = (0.468)(60)(10^{-3}), \quad [\text{watts}]$$

$$P_{fe} = 0.0281, \quad [\text{watts}]$$

Step No. 21: Calculate the total loss copper plus iron, P_Σ.

$$P_\Sigma = P_{fe} + P_{cu}, \quad [\text{watts}]$$

$$P_\Sigma = (0.0281) + (0.579), \quad [\text{watts}]$$

$$P_\Sigma = 0.607, \quad [\text{watts}]$$

Step No. 22: Calculate the watt density, ψ. The surface area, A_t, can be found in Step 5.

$$\psi = \frac{P_\Sigma}{A_t}, \quad [\text{watts/cm}^2]$$

$$\psi = \frac{(0.607)}{(69.9)}, \quad [\text{watts/cm}^2]$$

$$\psi = 0.00868, \quad [\text{watts/cm}^2]$$

Step No. 23: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [^\circ\text{C}]$$

$$T_r = 450(0.00868)^{(0.826)}, \quad [^\circ\text{C}]$$

$$T_r = 8.92, \quad [^\circ\text{C}]$$

Step No. 24: Calculate the peak flux density, B_{pk}.

$$B_{pk} = \frac{0.4\pi N_n F\left(I_{dc} + \frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]}$$

$$B_{pk} = \frac{(1.26)(116)(1.41)(1.6)\left(10^{-4}\right)}{(0.127) + \left(\frac{9.22}{2500}\right)}, \quad \text{[teslas]}$$

$$B_{pk} = 0.252, \quad \text{[teslas]}$$

Step No. 25: Calculate the effective permeability, μ_e. Knowing the effective permeability, the ETD-39 ferrite core can be ordered with a built-in gap.

$$\mu_e = \frac{\mu_m}{1 + \left(\frac{l_g}{\text{MPL}}\right)\mu_m}$$

$$\mu_e = \frac{(2500)}{1 + \left(\frac{(0.120)}{9.22}\right)(2500)}$$

$$\mu_e = 74.5 \text{ use } 75$$

Step No. 26: Calculate the window utilization, K_u.

$$K_u = \frac{N_n A_{w(B)}}{W_a}$$

$$K_u = \frac{(116)(0.00653)}{(2.34)}$$

$$K_u = 0.324$$

Chapter 9

DC Inductor Design, Using Powder Cores

Table of Contents

Introduction

Powder cores are manufactured from very fine particles of magnetic materials. The powder is coated with an inert insulation to minimize eddy current losses and to introduce a distributed air gap into the core structure. The insulated powder is then compacted into toroidal and EE cores. The magnetic flux in a toroidal powder core can be contained inside the core more readily than in a lamination or C core, as the winding covers the core along the entire Magnetic Path Length. The design of an inductor also frequently involves consideration of the effect of its magnetic field on devices near where it is placed. This is especially true in the design of high-current inductors for converters and switching regulators used in spacecraft.

Toroidal powder cores are widely used in high-reliability military and space applications because of their good stability over wide temperature ranges, and their ability to withstand high levels of shock, vibration, and nuclear radiation without degradation. Other applications for these cores are:

1. Stable, high-Q filters operating in the frequency range of 1kHz to 1MHz.
2. Loading coils used to cancel out the distributed capacitance in telephone cables.
3. Pulse transformers.
4. Differential mode EMI noise filters.
5. Flyback transformers.
6. Energy storage, or output inductors, in circuits with large amounts of dc current flowing.

Molybdenum Permalloy Powder Cores (MPP)

Molybdenum Permalloy Powder Cores (MPP) are manufactured from very fine particles of an 81% nickel, 17% iron, and a 2% molybdenum alloy. The insulated powder is then compacted into, EE, and toroidal cores. The toroidal cores range in size from 0.1 inch (0.254 cm) to 5 inches (12.7 cm) in the outside diameter. MPP cores are available in permeabilities ranging from 14 up to 550. See Table 9-1.

High Flux Powder Cores (HF)

High Flux Powder Cores (HF) are manufactured from very fine particles of a 50% nickel, and 50% iron. The insulated powder is then compacted into, EE, and toroidal cores. The toroidal cores range in size from 0.25 inch (0.635 cm) to 3 inches (7.62 cm) in the outside diameter. HF cores are available in permeabilities ranging from 14 up to 160. See Table 9-1.

Sendust Powder Cores (Magnetics Kool Mμ)

Sendust powder cores are manufactured from very fine particles of an 85% iron, 9% silicon, and 6% aluminum. The insulated powder is then compacted into EE and toroidal cores. The toroidal cores range in size from 0.14 inch (0.35 cm) to 3 inches (7.62 cm) in the outside diameter. Sendust cores are available in permeabilities ranging from 26 up to 125. See Table 9-1.

Iron Powder Cores

The low cost iron powder cores are typically used in today's low and high frequency power switching conversion applications for differential-mode, input and output, power inductors. The distributed air gap characteristic of iron powder produces a core with permeability ranging from 10 to 100. This feature, in conjunction with the inherent high saturation point of iron, makes it very difficult to saturate. While iron powder cores may be limited in their use because of low permeability, or rather high core loss at high frequency, they have become a very popular choice in either, EE, or toroidal as a core material for high-volume commercial applications. They are popular due to their low cost compared with other core materials. The toroidal cores range in size from 0.3 inch (0.76 cm) to 6.5 inches (16.5 cm) in the outside diameter. See Table 9-1.

Table 9-1. Standard Powder Core Permeability

Standard Powder Core Permeabilities				
Powder Material	MPP	High Flux	Sendust (Kool Mμ)	Iron Powder
Initial Permeability, μ_i				
10				X
14	X	X		
26	X	X	X	
35				X
40			X	
55				X
60	X	X	X	X
75			X	X
90			X	
100				X
125	X	X	X	
147	X	X		
160	X	X		
173	X			
200	X			
300	X			
550	X			

Inductors

Inductors that carry direct current are used frequently in a wide variety of ground, air, and space applications. Selection of the best magnetic core for an inductor frequently involves a trial-and-error type of calculation.

The design of an inductor also frequently involves consideration of the effect of its magnetic field on other devices in the immediate vicinity. This is especially true in the design of high-current inductors for converters and switching regulators used in spacecraft, which may also employ sensitive magnetic field detectors. For this type of design problem, it is frequently imperative that a toroidal core be used. The magnetic flux in a powder core can be contained inside the core more readily than in a lamination or C core, as the winding covers the core along the entire Magnetic Path Length. The author has developed a simplified method of designing optimum, dc carrying inductors with powder cores. This method allows the correct core permeability to be determined without relying on the trial and error method.

Relationship of, A_p, to Inductor's Energy-Handling Capability

The energy-handling capability of a core is related to its area product, A_p, by the Equation:

$$A_p = \frac{2(\text{Energy})(10^4)}{B_m J K_u}, \quad [\text{cm}^4] \qquad [9\text{-}1]$$

Where: Energy is in watt-seconds.

B_m is the flux density, teslas.

J is the current density, amps-per-cm^2.

K_u is the window utilization factor. (See Chapter 4)

From the above factors, such as flux density, B_m, window utilization factor, K_u, (which defines the maximum space that may be used by the copper in the window), and the current density, J, which controls the copper loss can be seen. The energy-handling capability of a core is derived from:

$$\text{Energy} = \frac{LI^2}{2}, \quad [\text{watt-seconds}] \qquad [9\text{-}2]$$

Relationship of, K_g, to Inductor's Energy-Handling Capability

Inductors, like transformers, are designed for a given temperature rise. They can also be designed for a given regulation. The regulation and energy handling ability of a core is related to two constants:

$$\alpha = \frac{(\text{Energy})^2}{K_g K_e}, \quad [\%] \qquad [9\text{-}3]$$

Where, α, is the regulation in, %.

The constant, K_g, is determined by the core geometry:

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \qquad [9\text{-}4]$$

The constant, K_e, is determined by the magnetic and electrical operating conditions:

$$K_e = 0.145 P_o B_m^2 \left(10^{-4}\right) \qquad [9\text{-}5]$$

The output power, P_o, is defined in Figure 9-1.

$$P_{o(L1)} = V_{(01)} I_{(01)} \quad P_{o(L2)} = V_{(02)} I_{(02)} \qquad [9\text{-}6]$$

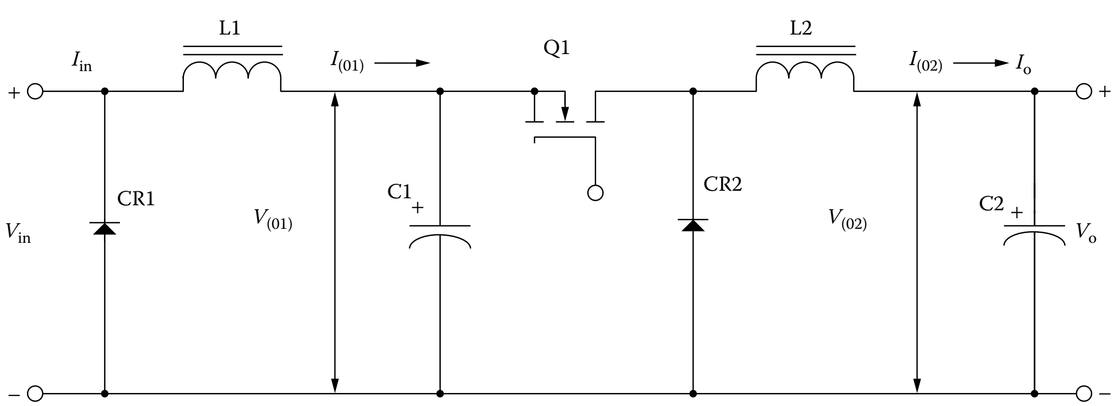

Figure 9-1. Defining the Inductor Output Power.

The operating flux density, B_m, is:

$$B_m = B_{dc} + \frac{B_{ac}}{2}, \quad [\text{teslas}] \qquad [9\text{-}7]$$

From the above, it can be seen that the flux density, B_m, is the predominant factor in governing size.

Fundamental Considerations

The design of a linear reactor depends upon four related factors:

1. Desired inductance, L.
2. Direct current, I_{dc}.
3. Alternating current, ΔI.
4. Power loss and temperature, T_r.

With these requirements established, the designer must determine the maximum values for, B_{dc}, and B_{ac}, that will not produce magnetic saturation and must make trade-offs that will yield the highest inductance for a given volume. The core permeability chosen dictates the maximum dc flux density that can be tolerated for a given design.

If an inductance is to be constant with the increasing direct current, there must be a negligible drop in inductance over the operating current range. The maximum, H, (magnetizing force) then is an indication of a core's capability, as shown in Figure 9-2.

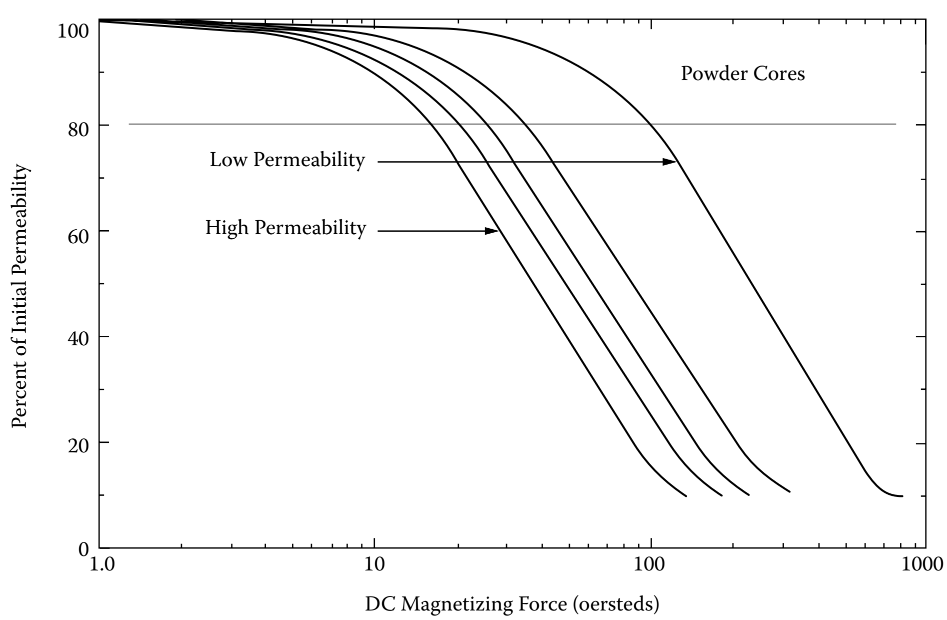

Figure 9-2. Typical Permeability Versus dc Bias Curves for Powder Cores.

Most manufacturers give the dc magnetizing force, H in oersteds:

$$H = \frac{0.4\pi NI}{\text{MPL}}, \quad \text{[oersteds]} \qquad [9\text{-}8]$$

Some engineers prefer amp-turns:

$$NI = 0.8H(\text{MPL}), \quad \text{[amp-turns]} \qquad [9\text{-}9]$$

Inductance decreases with increasing flux density, B, and magnetizing force, H, for various materials of different values of permeability. The selection of the correct permeability for a given design is made using Equation [9-10].

$$\mu_\Delta = \frac{B_m(\text{MPL})\left(10^4\right)}{0.4\pi W_a J K_u} \qquad [9\text{-}10]$$

It should be remembered that the maximum flux, B_m, depends upon, $B_{dc} + B_{ac}$, in the manner shown in Figure 9-3.

$$B_m = B_{dc} + \frac{B_{ac}}{2}, \quad \text{[teslas]} \qquad [9\text{-}11]$$

$$B_{dc} = \frac{0.4\pi N I_{dc}\mu\left(10^{-4}\right)}{\text{MPL}}, \quad \text{[teslas]} \qquad [9\text{-}12]$$

$$B_{ac} = \frac{0.4\pi N\left(\frac{\Delta I}{2}\right)\mu\left(10^{-4}\right)}{\text{MPL}}, \quad \text{[teslas]} \qquad [9\text{-}13]$$

$$B_{pk} = \frac{0.4\pi N\left(I_{dc} + \frac{\Delta I}{2}\right)\mu\left(10^{-4}\right)}{\text{MPL}}, \quad \text{[teslas]} \qquad [9\text{-}14]$$

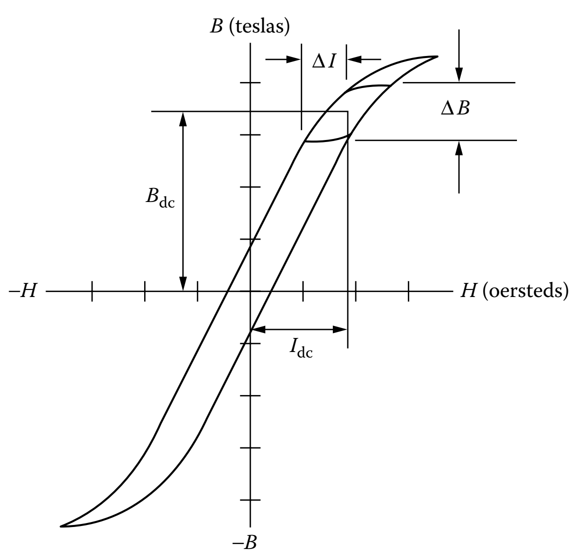

Figure 9-3. Inductor Flux Density Versus, $I_{dc} + \Delta I$ Current.

The flux density for the initial design for Molypermalloy powder cores should be limited to 0.3T maximum for, $B_{dc} + B_{ac}$, as shown in Figure 9-4.

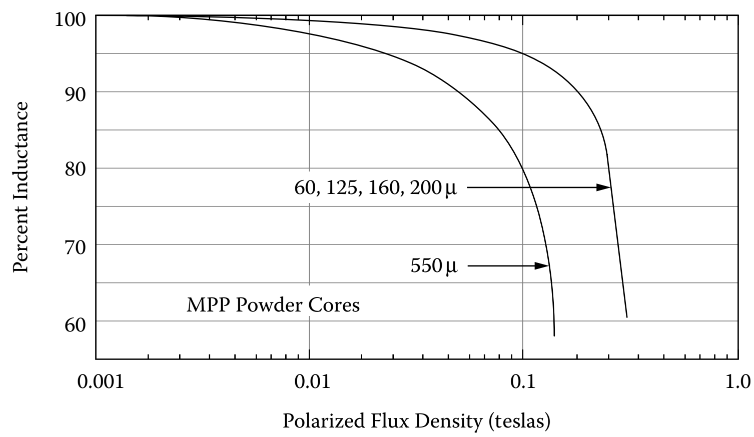

Figure 9-4. Inductance Versus dc Bias.

Toroidal Powder Core Design Using the Core Geometry, K_g, Approach

This design procedure will work with all powder cores.

Step No. 1: Design a linear dc inductor with the following specifications:

1. Inductance, L = 0.0025 henrys
2. dc current, I_o = 1.5 amps
3. ac current, ΔI = 0.2 amps
4. Output power, P_o = 100 watts
5. Regulation, α = 1.0 %
6. Ripple Frequency = 20kHz
7. Operating flux density, B_m = 0.3 tesla
8. Core Material = MPP
9. Window utilization, K_u = 0.4
10. Temperature rise goal, T_r = 25°C

Step No. 2: Calculate the peak current, I_{pk}.

$$I_{pk} = I_o + \frac{\Delta I}{2}, \quad \text{[amps]}$$

$$I_{pk} = (1.5) + \frac{(0.2)}{2}, \quad \text{[amps]}$$

$$I_{pk} = 1.6, \quad \text{[amps]}$$

Step No. 3: Calculate the energy-handling capability.

$$\text{Energy} = \frac{LI_{pk}^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Energy} = \frac{(0.0025)(1.6)^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Energy} = 0.0032, \quad \text{[watt-seconds]}$$

Step No. 4: Calculate the electrical conditions coefficient, K_e.

$$K_e = 0.145 P_o B_m^2 \left(10^{-4}\right)$$

$$K_e = 0.145(100)(0.3)^2\left(10^{-4}\right)$$

$$K_e = 0.0001305$$

Step No. 5: Calculate the core geometry coefficient, K_g.

$$K_g = \frac{(\text{Energy})^2}{K_e \alpha}, \quad [\text{cm}^5]$$

$$K_g = \frac{(0.0032)^2}{(0.0001305)(1.0)}, \quad [\text{cm}^5]$$

$$K_g = 0.0785, \quad [\text{cm}^5]$$

Step No. 6: Select a MPP powder core from Chapter 3. The data listed is the closest core to the calculated core geometry, K_g.

1. Core Number = 55586
2. Magnetic Path Length, MPL = 8.95 cm
3. Core Weight, W_{tfe} = 34.9 grams
4. Mean Length Turn, MLT = 4.40 cm
5. Iron Area, A_c = 0.454 cm^2
6. Window Area, W_a = 3.94 cm^2
7. Area Product, A_p = 1.79 cm^4
8. Core Geometry, K_g = 0.0738 cm^5
9. Surface Area, A_t = 64.4 cm^2
10. Permeability, μ = 60
11. Millihenrys-per-1k, AL = 38 mh

Step No. 7: Calculate the current density, J, using the area product Equation, A_p.

$$J = \frac{2(Energy)(10^4)}{B_m A_p K_u}, \quad \text{[amps-per-cm}^2\text{]}$$

$$J = \frac{2(0.0032)(10^4)}{(0.3)(1.79)(0.4)}, \quad \text{[amps-per-cm}^2\text{]}$$

$$J = 298, \quad \text{[amps-per-cm}^2\text{]}$$

Step No. 8: Calculate the rms current, I_{rms}.

$$I_{rms} = \sqrt{I_o^2 + \Delta I^2}, \quad \text{[amps]}$$

$$I_{rms} = \sqrt{(1.5)^2 + (0.2)^2}, \quad \text{[amps]}$$

$$I_{rms} = 1.51, \quad \text{[amps]}$$

Step No. 9: Calculate the required bare wire area, $A_{w(B)}$.

$$A_{W(B)} = \frac{I_{rms}}{J}, \quad \text{[cm}^2\text{]}$$

$$A_{W(B)} = \frac{(1.51)}{(298)}, \quad \text{[cm}^2\text{]}$$

$$A_{W(B)} = 0.00507, \quad \text{[cm}^2\text{]}$$

Step No. 10: Select a wire from the Wire Table in Chapter 4. If the area is not within 10%, take the next smallest size. Also, record the micro-ohms per centimeter.

$$\text{AWG} = \#20$$

$$\text{Bare, } A_{W(B)} = 0.00519, \quad \text{[cm}^2\text{]}$$

$$\text{Insulated, } A_W = 0.00606, \quad \text{[cm}^2\text{]}$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 332, \quad \text{[micro-ohm/cm]}$$

Step No. 11: Calculate the effective window area, $W_{a(eff)}$. Use the window area found in Step 6. A typical value for, S_3, is 0.75, as shown in Chapter 4.

$$W_{a(eff)} = W_a S_3, \quad \text{[cm}^2\text{]}$$

$$W_{a(eff)} = (3.94)(0.75), \quad \text{[cm}^2\text{]}$$

$$W_{a(eff)} = 2.96, \quad \text{[cm}^2\text{]}$$

Step No. 12: Calculate the number turns possible for, N. Use the insulated wire area, A_w, found in Step 10. A typical value for, S_2, is 0.6, as shown in Chapter 4.

$$N = \frac{W_{a(eff)} S_2}{A_W}, \quad \text{[turns]}$$

$$N = \frac{(2.96)(0.60)}{(0.00606)}, \quad \text{[turns]}$$

$$N = 293, \quad \text{[turns]}$$

Step No. 13: Calculate the required core permeability, μ.

$$\mu_\Delta = \frac{B_m (\text{MPL})\left(10^4\right)}{0.4\pi W_a J K_u}$$

$$\mu_\Delta = \frac{(0.30)(8.95)\left(10^4\right)}{(1.26)(3.94)(298)(0.4)}$$

$$\mu_\Delta = 45.4$$

Note: The permeability of 45.4 is close enough to use a 60μ core. Also note that there are other permeabilities available, See Table 9-1. Because of size, Chapter 3 has listed only 60μ Tables for MPP, High Flux, Sendust and a 75μ Table for Iron powder. For cores, with other than 60μ, use the manufacturer's catalog.

Step No. 14: Calculate the number of turns, N_L, required.

$$N_L = 1000\sqrt{\frac{L}{L_{(1000)}}}, \quad \text{[turns]}$$

$$N_L = 1000\sqrt{\left(\frac{2.5}{38}\right)}, \quad \text{[turns]}$$

$$N_L = 256, \quad \text{[turns]}$$

Step No. 15: Calculate the winding resistance, R_L. Use the MLT from Step 6 and the micro-ohm per centimeter from Step 10.

$$R_L = (\text{MLT})(N_L)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_L = (4.4)(256)(332)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_L = 0.374, \quad \text{[ohms]}$$

Step No. 16: Calculate the copper loss, P_{cu}.

$$P_{cu} = I_{rms}^2 R_L, \quad \text{[watts]}$$

$$P_{cu} = (1.51)^2(0.374), \quad \text{[watts]}$$

$$P_{cu} = 0.853, \quad \text{[watts]}$$

Step No. 17: Calculate the regulation, α.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

$$\alpha = \frac{(0.853)}{(100)}(100), \quad [\%]$$

$$\alpha = 0.853, \quad [\%]$$

Step No. 18: Calculate the ac flux density, B_{ac}.

$$B_{ac} = \frac{0.4\pi N_L\left(\frac{\Delta I}{2}\right)\mu\left(10^{-4}\right)}{\text{MPL}}, \quad [\text{teslas}]$$

$$B_{ac} = \frac{(1.25)(256)\left(\frac{0.2}{2}\right)(60)\left(10^{-4}\right)}{(8.95)}, \quad [\text{teslas}]$$

$$B_{ac} = 0.0215, \quad [\text{teslas}]$$

Step No. 19: Calculate the watts per kilogram for the appropriate MPP powder core material in Chapter 2. Watts per kilogram can be written in milliwatts per gram.

$$\text{mW/g} = k\,f^{(m)}B_{ac}^{(n)}$$

$$\text{mW/g} = (0.000788)(20000)^{(1.41)}(0.0215)^{(2.24)}$$

$$\text{mW/g} = 0.168$$

Step No. 20: Calculate the core loss, P_{fe}.

$$P_{fe} = (\text{mW/g})\left(W_{tfe}\right)\left(10^{-3}\right), \quad [\text{watts}]$$

$$P_{fe} = (0.168)(34.9)\left(10^{-3}\right), \quad [\text{watts}]$$

$$P_{fe} = 0.00586, \quad [\text{watts}]$$

Step No. 21: Calculate the total loss copper plus iron, P_Σ.

$$P_\Sigma = P_{fe} + P_{cu}, \quad [\text{watts}]$$

$$P_\Sigma = (0.00586) + (0.853), \quad [\text{watts}]$$

$$P_\Sigma = 0.859, \quad [\text{watts}]$$

Step No. 22: Calculate the watt density, ψ. The surface area, A_t can be found in Step 6.

$$\psi = \frac{P_\Sigma}{A_t}, \quad [\text{watts/cm}^2]$$

$$\psi = \frac{(0.859)}{(64.4)}, \quad [\text{watts/cm}^2]$$

$$\psi = 0.0133, \quad [\text{watts/cm}^2]$$

Step No. 23: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [^\circ\text{C}]$$

$$T_r = 450(0.0133)^{(0.826)}, \quad [^\circ\text{C}]$$

$$T_r = 12.7, \quad [^\circ\text{C}]$$

Step No. 24: Calculate the dc magnetizing force, H.

$$H = \frac{0.4\pi N_L I_{pk}}{\text{MPL}}, \quad [\text{oersteds}]$$

$$H = \frac{(1.26)(256)(1.6)}{(8.95)}, \quad [\text{oersteds}]$$

$$H = 57.7, \quad [\text{oersteds}]$$

Step No. 25: Calculate the window utilization, K_u.

$$K_u = \frac{N_{L(new)} A_{w(B)\#20}}{W_a}$$

$$K_u = \frac{((256)(0.00519))}{(3.94)}$$

$$K_u = 0.337$$

Note: The big advantage in using the core geometry design procedure is that the current density is calculated. Using the area product design procedure, the current density is an estimate, at best. In this next design the same current density will be used as in core geometry.

Toroidal Powder Core Inductor Design, Using the Area Product, A_p, Approach

Step No. 1: Design a linear dc inductor with the following specifications:

1. Inductance, L = 0.0025 henrys.
2. dc current, I_o = 1.5 amps
3. ac current, ΔI = 0.2 amps
4. Output power, P_o = 100 watts
5. Current Density, J = 300 amps-per-cm^2
6. Ripple Frequency = 20kHz
7. Operating flux density, B_m = 0.3 tesla
8. Core Material = MPP
9. Window utilization, K_u = 0.4
10. Temperature rise goal, T_r = 25°C

Step No. 2: Calculate the peak current, I_{pk}.

$$I_{pk} = I_o + \frac{\Delta I}{2}, \quad \text{[amps]}$$

$$I_{pk} = (1.5) + \frac{(0.2)}{2}, \quad \text{[amps]}$$

$$I_{pk} = 1.6, \quad \text{[amps]}$$

Step No. 3: Calculate the energy-handling capability.

$$\text{Energy} = \frac{L I_{pk}^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Energy} = \frac{(0.0025)(1.6)^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Energy} = 0.0032, \quad \text{[watt-seconds]}$$

Step No. 4: Calculate the area product, A_p.

$$A_p = \frac{2(\text{Energy})(10^4)}{B_m J K_u}, \quad [\text{cm}^4]$$

$$A_p = \frac{2(0.0032)(10^4)}{(0.3)(300)(0.4)}, \quad [\text{cm}^4]$$

$$A_p = 1.78, \quad [\text{cm}^4]$$

Step No. 5: Select a MPP powder core from Chapter 3. The data listed is the closest core to the calculated core geometry, K_g.

1. Core Number = 55586
2. Magnetic Path Length, MPL = 8.95 cm
3. Core Weight, W_{tfe} = 34.9 grams
4. Mean Length Turn, MLT = 4.40 cm
5. Iron Area, $A_c = 0.454$ cm^2
6. Window Area, $W_a = 3.94$ cm^2
7. Area Product, $A_p = 1.79$ cm^4
8. Core Geometry, $K_g = 0.0742$ cm^5
9. Surface Area, $A_t = 64.4$ cm^2
10. Permeability, $\mu = 60$
11. Millihenrys-per-1k, AL = 38 mh

Step No. 6: Calculate the rms current, I_{rms}.

$$I_{rms} = \sqrt{I_o^2 + \Delta I^2}, \quad \text{[amps]}$$

$$I_{rms} = \sqrt{(1.5)^2 + (0.2)^2}, \quad \text{[amps]}$$

$$I_{rms} = 1.51, \quad \text{[amps]}$$

Step No. 7: Calculate the required bare wire area, $A_{w(B)}$.

$$A_{W(B)} = \frac{I_{rms}}{J}, \quad [\text{cm}^2]$$

$$A_{W(B)} = \frac{(1.51)}{(298)}, \quad [\text{cm}^2]$$

$$A_{W(B)} = 0.00507, \quad [\text{cm}^2]$$

Step No. 8: Select a wire from the Wire Table in Chapter 4. If the area is not within 10%, take the next smallest size. Also, record the micro-ohms per centimeter.

$$\text{AWG} = \#20$$

$$\text{Bare, } A_{W(B)} = 0.00519, \quad [\text{cm}^2]$$

$$\text{Insulated, } A_W = 0.00606, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 332, \quad \text{[micro-ohm/cm]}$$

Step No. 9: Calculate the effective window area, $W_{a(eff)}$. Use the window area found in Step 5. A typical value for, S_3, is 0.75, as shown in Chapter 4.

$$W_{a(eff)} = W_a S_3, \quad [\text{cm}^2]$$

$$W_{a(eff)} = (3.94)(0.75), \quad [\text{cm}^2]$$

$$W_{a(eff)} = 2.96, \quad [\text{cm}^2]$$

Step No. 10: Calculate the number turns possible, N. Use the insulated wire area, A_w, found in Step 8. A typical value for, S_2, is 0.6, as shown in Chapter 4.

$$N = \frac{W_{a(eff)} S_2}{A_W}, \quad [\text{turns}]$$

$$N = \frac{(2.96)(0.60)}{(0.00606)}, \quad [\text{turns}]$$

$$N = 293, \quad [\text{turns}]$$

Step No. 11: Calculate the required core permeability, μ.

$$\mu_\Delta = \frac{B_m(\text{MPL})(10^4)}{0.4\pi W_a J K_u}$$

$$\mu_\Delta = \frac{(0.30)(8.95)(10^4)}{(1.26)(3.94)(298)(0.4)}$$

$$\mu_\Delta = 45.4$$

Note: The permeability of 45.4 is close enough to use a 60μ core. Also note, there are other permeabilities available. See Table 9-1. Because of size, Chapter 3 has listed only 60μ tables for MPP, High Flux, Sendust and 75μ table for Iron powder. For cores, with other than 60μ, use the manufacturer's catalog.

Step No. 12: Calculate the number of turns, N_L, required.

$$N_L = 1000\sqrt{\frac{L}{L_{(1000)}}}, \quad [\text{turns}]$$

$$N_L = 1000\sqrt{\left(\frac{2.5}{38}\right)}, \quad [\text{turns}]$$

$$N_L = 256, \quad [\text{turns}]$$

Step No. 13: Calculate the winding resistance, R_L. Use the MLT, from Step 5, and the micro-ohm per centimeter, from Step 8.

$$R_L = (\text{MLT})(N_L)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_L = (4.4)(256)(332)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_L = 0.374, \quad [\text{ohms}]$$

Step No. 14: Calculate the copper loss, P_{cu}.

$$P_{cu} = I_{rms}^2 R_L, \quad [\text{watts}]$$

$$P_{cu} = (1.51)^2(0.374), \quad [\text{watts}]$$

$$P_{cu} = 0.853, \quad [\text{watts}]$$

Step No. 15: Calculate the ac flux density, B_{ac}.

$$B_{ac} = \frac{0.4\pi N_L\left(\frac{\Delta I}{2}\right)\mu\left(10^{-4}\right)}{\text{MPL}}, \quad [\text{teslas}]$$

$$B_{ac} = \frac{(1.25)(256)\left(\frac{0.2}{2}\right)(60)\left(10^{-4}\right)}{(8.95)}, \quad [\text{teslas}]$$

$$B_{ac} = 0.0215, \quad [\text{teslas}]$$

Step No. 16: Calculate the watts per kilogram for the appropriate MPP powder core material in Chapter 2. Watts per kilogram can be written in milliwatts per gram.

$$\text{mW/g} = k f^{(m)} B_{ac}^{(n)}$$

$$\text{mW/g} = (0.000788)(20000)^{(1.41)}(0.0215)^{(2.24)}$$

$$\text{mW/g} = 0.168$$

Step No. 17: Calculate the core loss, P_{fe}.

$$P_{fe} = (\text{mW/g})\left(W_{tfe}\right)\left(10^{-3}\right), \quad [\text{watts}]$$

$$P_{fe} = (0.168)(34.9)\left(10^{-3}\right), \quad [\text{watts}]$$

$$P_{fe} = 0.00586, \quad [\text{watts}]$$

Step No. 18: Calculate the total copper loss plus iron, P_Σ.

$$P_\Sigma = P_{fe} + P_{cu}, \quad \text{[watts]}$$

$$P_\Sigma = (0.00586) + (0.853), \quad \text{[watts]}$$

$$P_\Sigma = 0.859, \quad \text{[watts]}$$

Step No. 19: Calculate the watt density, ψ. The surface area, A_t can be found in Step 5.

$$\psi = \frac{P_\Sigma}{A_t}, \quad \text{[watts/cm}^2\text{]}$$

$$\psi = \frac{(0.859)}{(64.4)}, \quad \text{[watts/cm}^2\text{]}$$

$$\psi = 0.0133, \quad \text{[watts/cm}^2\text{]}$$

Step No. 20: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [^\circ\text{C}]$$

$$T_r = 450(0.0133)^{(0.826)}, \quad [^\circ\text{C}]$$

$$T_r = 12.7, \quad [^\circ\text{C}]$$

Step No. 21: Calculate the dc magnetizing force, H.

$$H = \frac{0.4\pi N_L I_{pk}}{\text{MPL}}, \quad \text{[oersteds]}$$

$$H = \frac{(1.26)(256)(1.6)}{(8.95)}, \quad \text{[oersteds]}$$

$$H = 57.7, \quad \text{[oersteds]}$$

Step No. 22: Calculate the window utilization, K_u.

$$K_u = \frac{N_{L(new)} A_{w(B)\#20}}{W_a}$$

$$K_u = \frac{((256)(0.00519))}{(3.94)}$$

$$K_u = 0.337$$

Chapter 10

AC Inductor Design

Table of Contents

Introduction

The design of an ac inductor is quite similar to that of a transformer. If there is no dc flux in the core, the design calculations are straightforward. The apparent power, P_t, of an inductor is the VA of the inductor. As shown in Equation [10-1] it is the product of the excitation voltage and the current through the inductor.

$$P_t = VA, \quad \text{[watts]} \tag{10-1}$$

Requirements

The design of the ac inductor requires the calculation of the volt-amp (VA) capability. In some applications the inductance is specified, and in others, the current is specified. If the inductance is specified, then, the current has to be calculated. If the current is specified, then the inductance has to be calculated. A series, ac inductor, L1, being used in a Ferroresonant Voltage Stabilizer is shown in Figure 10-1.

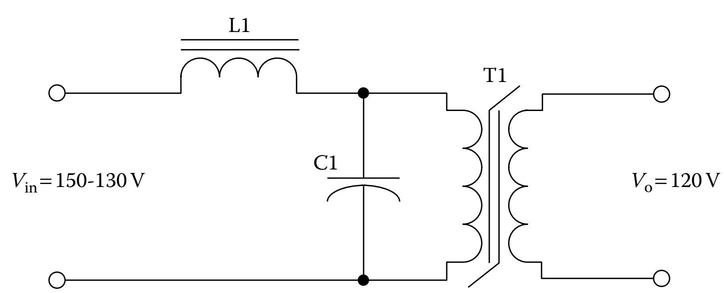

Figure 10-1. Series ac Inductor, L1, as used in a Ferroresonant Voltage Stabilizer.

Relationship of, A_p, to the Inductor Volt-Amp Capability

The volt-amp capability of a core is related to its area product, A_p, as shown in Equation [10-2].

$$A_p = \frac{VA\left(10^4\right)}{K_f K_u B_{ac} f J}, \quad [\text{cm}^4] \tag{10-2}$$

Where:

K_f = wave form factor.

K_u = window utilization factor.

B_{ac} = operating flux density, T, teslas

f = operating frequency, Hz

J = current density, amps/cm^2

From the above, it can be seen that factors such as flux density, B_{ac}, the window utilization factor, K_u, (which defines the maximum space occupied by the copper in the window), and the current density, J, all have an influence on the inductor area product, A_p.

Relationship, K_g, to the Inductor Volt-Amp Capability

Although most inductors are designed for a given temperature rise, they can also be designed for a given regulation. The regulation and volt-amp ability of a core is related to two constants, as shown in Equation [10-3].

$$\alpha = \frac{VA}{K_g K_e}, \quad [\%] \qquad [10\text{-}3]$$

$$\alpha = \text{Regulation } (\%) \qquad [10\text{-}4]$$

The constant, K_g, is determined by the core geometry, which may be related, as shown in Equation [10-5].

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \qquad [10\text{-}5]$$

The constant, K_e, is determined by the magnetic and electric operating conditions, which may be related, as shown in Equation [10-6].

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right) \qquad [10\text{-}6]$$

Where:

K_f = waveform coefficient

4.0 square wave

4.44 sine wave

From the above, it can be seen that factors such as flux density, frequency of operation, and the waveform coefficient have an influence on the transformer size.

Fundamental Considerations

The design of a linear ac inductor depends upon five related factors:

1. Desired inductance
2. Applied voltage, (across inductor)
3. Frequency
4. Operating Flux density
5. Temperature Rise

With these requirements established, the designer must determine the maximum values for, B_{ac}, which will not produce magnetic saturation, and then make trade-offs that will yield the highest inductance for a given volume. The core material selected determines the maximum flux density that can be tolerated for a given design. Magnetic materials and their operating flux levels are given in Chapter 2.

The ac inductor like a transformer, must support the applied voltage, V_{ac}. The number of turns is calculated from Faraday's Law, as shown in Equation [10-7].

$$N = \frac{V_{ac}\left(10^4\right)}{K_f B_{ac} f A_c}, \quad \text{[turns]} \qquad [10\text{-}7]$$

The inductance of an iron-core inductor, with an air gap, may be expressed, as shown in Equation [10-8].

$$L = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{l_g + \left(\dfrac{\text{MPL}}{\mu_m}\right)}, \quad \text{[henrys]} \qquad [10\text{-}8]$$

Inductance is seen to be inversely dependent on the effective Magnetic Path Length, MPL, which is the sum of the air gap length, l_g, and the ratio of the Magnetic Path Length, MPL, to material permeability, μ_m.

When the core air gap, l_g, is larger compared to the ratio, MPL/μ_m, because of the high material permeability, μ_m, variations in, μ_m, do not substantially affect the total effective Magnetic Path Length, MPL, or the inductance, L. The inductance Equation then reduces to Equation [10-9].

$$L = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{l_g}, \quad \text{[henrys]} \qquad [10\text{-}9]$$

Rearranging the Equation to solve for the gap is shown in Equation [10-10].

$$l_g = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{L}, \quad \text{[cm]} \qquad [10\text{-}10]$$

Fringing Flux

Final determination of the air gap requires consideration of the effect of fringing flux, which is a function of gap dimension, the shape of the pole faces, and the shape, size, and location of the winding, as shown in Figure 10-2 and Figure 10-3. Its net effect is to make the effective air gap less than its physical dimension.

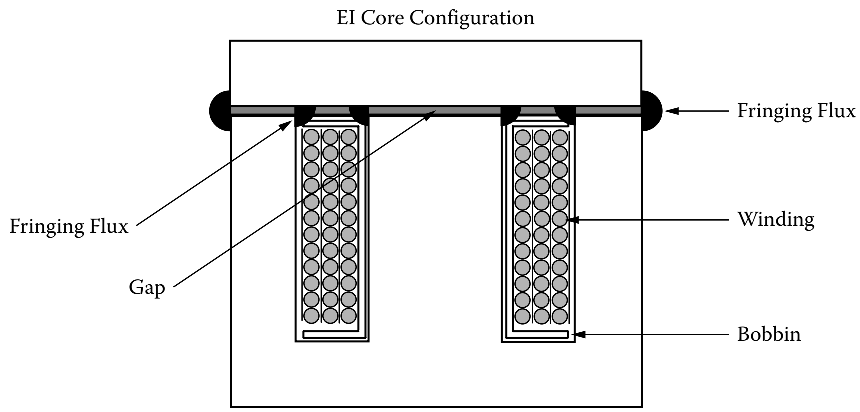

Figure 10-2. Fringing Flux Location on an EI Core Configuration.

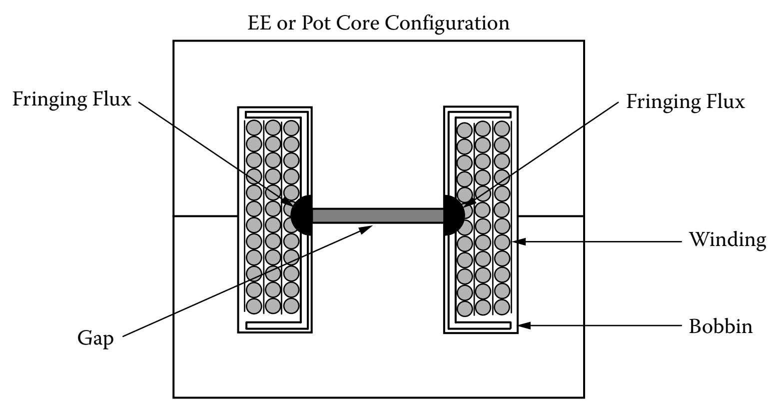

Figure 10-3. Fringing Flux Location on an EE or Pot Core Configuration.

Fringing flux decreases the total reluctance of the magnetic path, and therefore increases the inductance by a factor, F, to a value greater than that calculated from Equation [10-9]. Fringing flux is a larger percentage of the total for larger gaps. The fringing flux factor is shown in Equation [10-11].

$$F=\left(1+\frac{l_g}{\sqrt{A_c}}\ln\left(\frac{2G}{l_g}\right)\right) \qquad [10\text{-}11]$$

G is the winding length as defined in Chapter 3. Equation [10-11] is valid for cut C cores, laminations and cut ferrite cores.

The inductance, L, computed in Equation [10-9] does not include the effect of fringing flux, F. The value of inductance, L', in Equation [10-12] does correct for fringing flux:

$$L'=\frac{0.4\pi N^2 A_c F\left(10^{-8}\right)}{l_g}, \quad \text{[henrys]} \qquad [10\text{-}12]$$

Now that the fringing flux, F, has been calculated, it is necessary to recalculate the number of turns using the fringing flux, Factor F, as shown in Equation [10-13].

$$N_{(new)} = \sqrt{\frac{Ll_g}{0.4\pi A_c F\left(10^{-8}\right)}}, \quad \text{[turns]} \qquad [10\text{-}13]$$

After the new turns, $N_{(new)}$, have been calculated, then, use Equation [10-13] with the new turns, $N_{(new)}$, and solve for B_{ac}. This check will provide the operating flux density, in order to calculate the core loss, P_{fe}, and will also provide a check on core saturation margin, as shown in Equation [10-14].

$$B_{ac} = \frac{V_{ac}\left(10^4\right)}{K_f N_{(new)} f A_c}, \quad \text{[teslas]} \qquad [10\text{-}14]$$

The losses in an ac inductor are made up of three components:

1. Copper loss, P_{cu}
2. Iron loss, P_{fe}
3. Gap loss, P_g

The copper loss, P_{cu}, is I^2R and is straightforward, if the skin effect is minimal. The iron loss, P_{fe}, is calculated from core manufacturers' data. Gap loss, P_g, is independent of core material strip thickness and permeability. Maximum efficiency is reached in an inductor, as in a transformer, when the copper loss, P_{cu}, and the iron loss, P_{fe}, are equal, but only when the gap of the core is zero. The gap loss does not occur in the air gap, itself, but is caused by magnetic flux, fringing around the gap, and reentering the core in a direction of high loss. As the air gap increases, the flux across the gap fringes more and more, and some of the fringing flux strikes the core, perpendicular to the laminations, and sets up eddy currents which cause additional losses called gap loss, P_g. Also distribution of the fringing flux is affected by other aspects of the core geometry, the proximity of the coils turns to the core, and whether there are turns on both legs. (See Table 10-1). Accurate prediction of the gap loss depends on the amount of fringing flux. The gap loss Equation [10-15] is from an article written by Ruben, L., and Stephens, D. (See the Reference at the end of this Chapter.)

$$P_g = K_i E l_g f B_{ac}^2, \quad \text{[watt]} \qquad [10\text{-}15]$$

Where, E (as defined in Chapter 3) is the strip or tongue width, in cm.

Table 10-1. Gap Loss Coefficient

Configuration	K_i
Two-coil C core	0.0388
Single-coil C core	0.0775
Lamination	0.1550

When designing inductors where there is a choice of cores, always pick the core with the smallest ratio between window area, W_a and iron area, A_c, as shown in Equation [10-16].

$$\frac{W_a}{A_c} = \text{[smallest ratio]} \quad [10\text{-}16]$$

Comparing two cores with identical area products, A_p, for the same design specification, the core with a minimum of window area will generate a minimum of fringing flux. If there is a design change and it requires the use of the next larger core, it would be far more beneficial to double up on the core being used, than to pick a larger core, as shown in Figure 10-4.

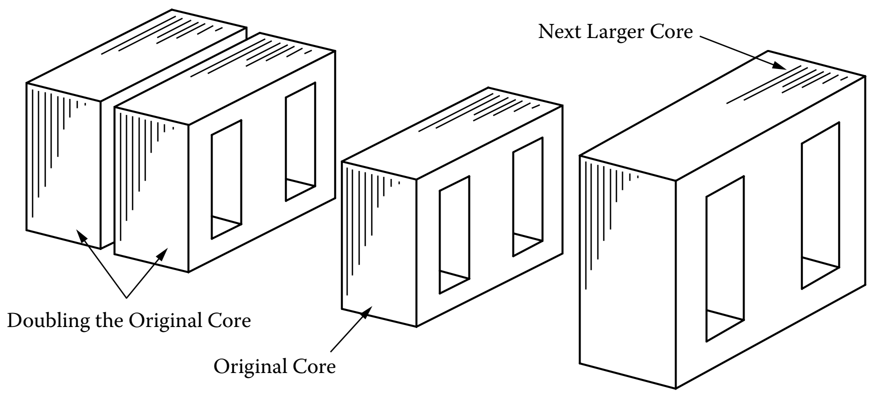

Figure 10-4. Comparing Core Configurations.

For example, if the next larger core was selected, normally all of the core proportions will increase. This means, the window, W_a, and the iron cross-section, A_c, would have both increased. (A larger core should not be used, as the fringing flux would also increase.) If you want to keep the fringing flux to a minimum, then double up on the original core. Therefore the iron area, A_c, would double, but the window area, W_a, will remain the same. This will reduce the, W_a/A_c, ratio, as shown in Equation 10-16. With an increase in iron cross-section, A_c, the turns would have to decrease for the same window area, W_a. With a decrease in turns, the gap would also decrease, resulting in less fringing flux.

When designing a transformer, the engineer will push the flux density as far he can without saturating the core. That cannot be done with an ac inductor because you must leave a margin for the fringing flux factor. One of the biggest problems in designing ac inductors is keeping the gap to a minimum. This problem becomes acute when designing high frequency ac inductors. The problem in designing high frequency inductors is the required turns to support the applied voltage, then gapping to provide the proper inductance. This problem is minimized when using powder cores if the right permeability can be found.

AC Inductor Design Example

Step No. 1: Design a linear ac inductor with the following specifications.

1. Applied voltage, V_L= 120 volts
2. Line current, I_L = 1.0 amps
3. Line frequency = 60 hertz.
4. Current density, J = 300 amp/cm^2
5. Efficiency goal, η(100) = 90%
6. Magnetic material = Silicon
7. Magnetic material permeability, μ_m = 1500
8. Flux density, B_{ac} = 1.4 teslas
9. Window utilization, K_u = 0.4
10. Waveform factor, K_f = 4.44
11. Temperature rise goal, T_r = 50°C

Step No. 2: Calculate the apparent power, P_t or VA of the inductor, L.

$$VA = V_L I_L, \quad \text{[watts]}$$

$$VA = (120)(1.0), \quad \text{[watts]}$$

$$VA = 120, \quad \text{[watts]}$$

Step No. 3: Calculate the area product, A_p.

$$A_p = \frac{VA(10^4)}{K_f K_u f B_{ac} J}, \quad [\text{cm}^4]$$

$$A_p = \frac{(120)(10^4)}{(4.44)(0.4)(60)(1.4)(300)}, \quad [\text{cm}^4]$$

$$A_p = 26.8, \quad [\text{cm}^4]$$

Step No. 4: Select an EI lamination from Chapter 3. The closest lamination to the calculated area product, A_p, is the EI-100.

1. Core Number = EI-100
2. Magnetic Path Length, MPL = 15.2 cm
3. Core Weight, W_{tfe} = 676 grams
4. Mean Length Turn, MLT = 14.8 cm
5. Iron Area, A_c = 6.13 cm^2

6. Window Area, $W_a = 4.84$ cm^2
7. Area Product, $A_p = 29.7$ cm^4
8. Core Geometry, $K_g = 4.93$ cm^5
9. Surface Area, $A_t = 213$ cm^2
10. Winding length, G = 3.81 cm
11. Lamination tongue, E = 2.54 cm

Step No. 5: Calculate the number of inductor turns, N_L.

$$N_L = \frac{V_L\left(10^4\right)}{K_f B_{ac} f A_c}, \quad \text{[turns]}$$

$$N_L = \frac{(120)\left(10^4\right)}{(4.44)(1.4)(60)(6.13)}, \quad \text{[turns]}$$

$$N_L = 525, \quad \text{[turns]}$$

Step No. 6: Calculate the inductive reactance, X_L.

$$X_L = \frac{V_L}{I_L}, \quad \text{[ohms]}$$

$$X_L = \frac{(120)}{(1.0)}, \quad \text{[ohms]}$$

$$X_L = 120, \quad \text{[ohms]}$$

Step No. 7: Calculate the required inductance, L.

$$L = \frac{X_L}{2\pi f}, \quad \text{[henrys]}$$

$$L = \frac{120}{2(3.14)(60)}, \quad \text{[henrys]}$$

$$L = 0.318, \quad \text{[henrys]}$$

Step No. 8: Calculate the required gap, l_g.

$$l_g = \left(\frac{0.4\pi N_L^2 A_c\left(10^{-8}\right)}{L}\right) - \left(\frac{\text{MPL}}{\mu_m}\right), \quad \text{[cm]}$$

$$l_g = \left(\frac{(1.26)(525)^2(6.13)\left(10^{-8}\right)}{0.318}\right) - \left(\frac{15.2}{1500}\right), \quad \text{[cm]}$$

$l_g = 0.0568, \quad$ [cm] or $l_g = 22.4, \quad$ [mils]: This would be in 10 mils each leg.

Step No. 9: Calculate the fringing flux, F.

$$F = \left(1 + \frac{l_g}{\sqrt{A_c}} \ln \frac{2(G)}{l_g}\right)$$

$$F = \left(1 + \frac{0.0568}{\sqrt{6.13}} \ln \frac{2(3.81)}{0.0568}\right)$$

$$F = 1.112$$

Step No. 10: Using the fringing flux, recalculate the series inductor turns, $N_{L(new)}$.

$$N_{L(new)} = \sqrt{\frac{l_g L}{0.4\pi A_c F(10^{-8})}}, \quad \text{[turns]}$$

$$N_{L(new)} = \sqrt{\frac{(0.0568)(0.318)}{(1.26)(6.13)(1.112)(10^{-8})}}, \quad \text{[turns]}$$

$$N_{L(new)} = 459, \quad \text{[turns]}$$

Step No. 11: Using the new turns, recalculate the flux density, B_{ac}.

$$B_{ac} = \frac{V_L(10^4)}{K_f N_{L(new)} A_c f}, \quad \text{[teslas]}$$

$$B_{ac} = \frac{(120)(10^4)}{(4.44)(459)(6.13)(60)}, \quad \text{[teslas]}$$

$$B_{ac} = 1.6, \quad \text{[teslas]}$$

Step No. 12: Calculate the inductor bare wire area, $A_{wL(B)}$.

$$A_{wL(B)} = \frac{I_L}{J}, \quad [\text{cm}^2]$$

$$A_{wL(B)} = \frac{(1.0)}{(300)}, \quad [\text{cm}^2]$$

$$A_{wL(B)} = 0.00333, \quad [\text{cm}^2]$$

Step No. 13: Select a wire from the Wire Table in Chapter 4.

$$AWG = \#22$$

$$A_{w(B)} = 0.00324, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 531, \quad \text{[micro-ohm/cm]}$$

Step No. 14: Calculate the inductor winding resistance, R_L. Use the MLT from the core data found in Step 4 and the micro-ohm per centimeter, found in Step 13.

$$R_L = (\text{MLT})(N_s)\left(\frac{\mu\Omega}{\text{cm}}\right)(10^{-6}), \quad \text{[ohms]}$$

$$R_L = (14.8)(459)(531)(10^{-6}), \quad \text{[ohms]}$$

$$R_L = 3.61, \quad \text{[ohms]}$$

Step No. 15: Calculate the inductor winding copper loss, P_L.

$$P_L = (I_L)^2 R_L, \quad \text{[watts]}$$

$$P_L = (1.0)^2(3.61), \quad \text{[watts]}$$

$$P_L = 3.61, \quad \text{[watts]}$$

Step No. 16: Calculate the watts-per-kilograms, W/K, for the appropriate core material. See Chapter 2.

$$W/K = 0.000557 f^{(1.68)} B_s^{(1.86)}, \quad \text{[watts-per-kilogram]}$$

$$W/K = 0.000557(60)^{(1.68)}(1.6)^{(1.86)}, \quad \text{[watts-per-kilogram]}$$

$$W/K = 1.30, \quad \text{[watts-per-kilogram]}$$

Step No. 17: Calculate the core loss in watts, P_{fe}.

$$P_{fe} = (W/K)W_{tfe}, \quad \text{[watts]}$$

$$P_{fe} = (1.30)(0.676), \quad \text{[watts]}$$

$$P_{fe} = 0.878, \quad \text{[watts]}$$

Step No. 18: Calculate the gap loss, P_g.

$$P_g = K_i E l_g f B_{ac}^2, \quad \text{[watts]}$$

$$P_g = (0.155)(2.54)(0.0568)(60)(1.6)^2, \quad \text{[watts]}$$

$$P_g = 3.43, \quad \text{[watts]}$$

Step No. 19: Calculate the total inductor losses, P_Σ.

$$P_\Sigma = P_{cu} + P_{fe} + P_g, \quad \text{[watts]}$$

$$P_\Sigma = (3.61) + (0.878) + (3.43), \quad \text{[watts]}$$

$$P_\Sigma = 7.92, \quad \text{[watts]}$$

Step No. 20: Calculate the inductor surface area watt density, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad [\text{watts-per-cm}^2]$$

$$\psi = \frac{(7.92)}{(213)}, \quad [\text{watts-per-cm}^2]$$

$$\psi = 0.0372, \quad [\text{watts-per-cm}^2]$$

Step No. 21: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [^{\circ}\text{C}]$$

$$T_r = 450(0.0372)^{(0.826)}, \quad [^{\circ}C]$$

$$T_r = 29.7, \quad [^{\circ}\text{C}]$$

Step No. 22: Calculate the window utilization, K_u.

$$K_u = \frac{N_{L(new)} A_{w(B)\#22}}{W_a}$$

$$K_u = \frac{((459)(0.00324))}{(4.84)}$$

$$K_u = 0.307$$

Reference

1. Ruben, L., and Stephens, D. Gap Loss in Current-Limiting Transformer. *Electromechanical Design*, April, 1973, pp. 24–126.

Chapter 11

Constant Voltage Transformer (CVT)

Table of Contents

Introduction

The Constant-Voltage Transformer (CVT) has a wide application, particularly where reliability and inherent regulating ability against line voltage changes are of prime importance. The output of a Constant-Voltage Transformer is essentially a square wave, which is desirable for rectifier output applications while, also having good circuit characteristics. The main disadvantage to a Constant-Voltage Transformer is efficiency and regulation for frequency and load. The equations presented here for designing a Constant-Voltage Transformers at line frequency have been used at 400Hz on aircraft, and as high as 20kHz.

Constant-Voltage Transformer, Regulating Characteristics

The basic two-component (CVT) Ferroresonant regulator is shown in Figure 11-1. The inductor, L1, is a linear inductor and is in series with, C1, across the input line. The voltage across capacitor, C1, would be considerably greater than the line voltage, because of the resonant condition between L1 and C1.

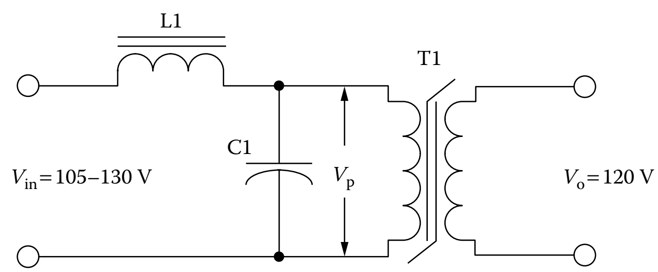

Figure 11-1. Two Component Ferroresonant Voltage Stabilizer.

The voltage, V_p, can be limited to a predetermined amplitude by using a self-saturating transformer, T1, which has high impedance, until a certain level of flux density is reached. At that flux density, the transformer saturates and becomes a low-impedance path, which prevents further voltage buildup across the capacitor. This limiting action produces a voltage waveform that has a fairly flat top characteristic, as shown in Figure 11-2 on each half-cycle.

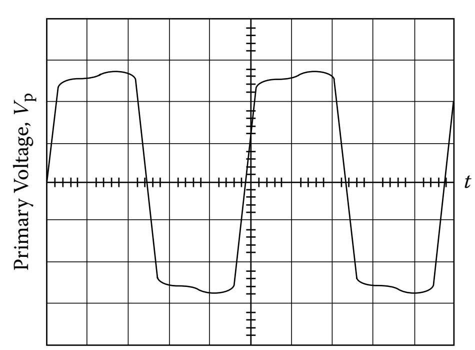

Figure 11-2. Primary Voltage Waveform of a Constant Voltage Transformer.

Electrical Parameters of a CVT Line Regulator

When the Constant-Voltage Transformer is operating as a line regulator, the output voltage will vary as a function of the input voltage, as shown in Figure 11-3. The magnetic material used to design transformer, T1, has an impact on line regulation. Transformers designed with a square B-H loop will result in better line regulation. If the output of the line regulator is subjected to a load power factor (lagging) with less than unity, the output will change, as shown in Figure 11-4.

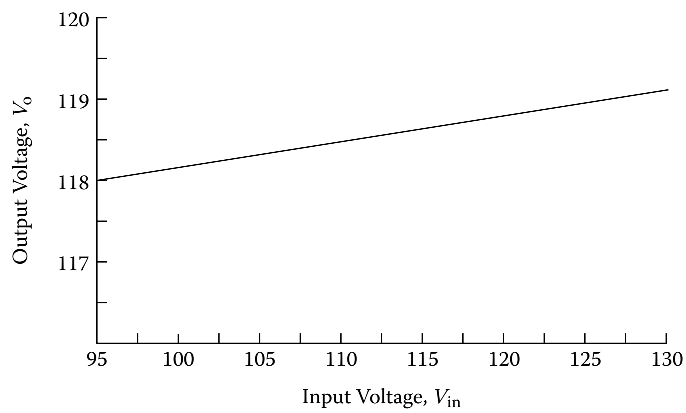

Figure 11-3. Output Voltage Variation, as a Function of Input Voltage.

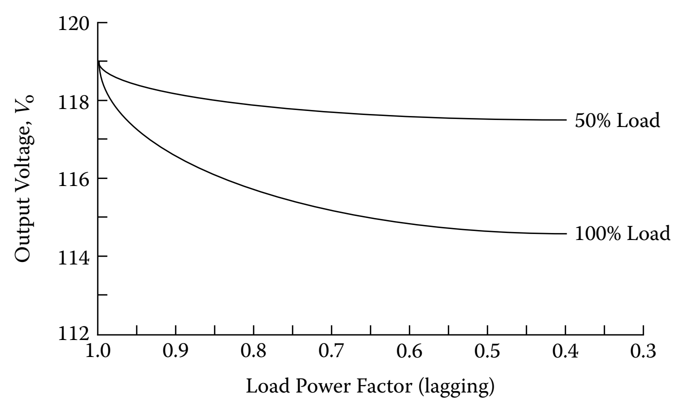

Figure 11-4. Output Voltage Variation, as a Function of Load Power Factor.

If the Constant-Voltage Transformer is subjected to a line voltage frequency change, the output voltage will vary, as shown in Figure 11-5. The regulation of a Constant-Voltage Transformer can be designed to be better than a few percent. Capability for handling a short circuit is an inherent feature of a Constant-Voltage Transformer. The short-circuit current is limited and set by the series inductance, L. The regulation characteristics, at various lines and loads, are shown in Figure 11-6. It should be noted that a dead short, corresponding to zero output voltage, does not greatly increase the load current; whereas for most transformers, this dead short would be destructive.

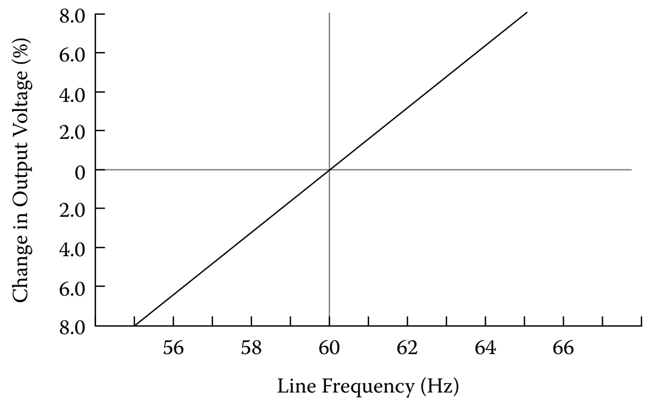

Figure 11-5. Output Voltage Variation, as a Function of Line Frequency Change.

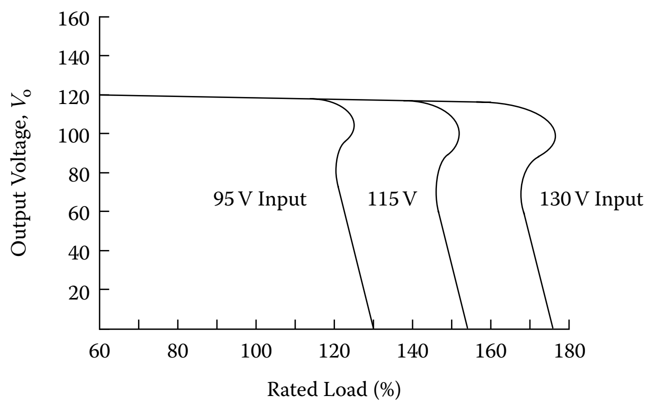

Figure 11-6. Output Voltage Variation, as a Function of Output Voltage vs. Load.

Constant-Voltage Transformer, Design Equations

Proper operation and power capacity of a Constant-Voltage Transformer (CVT) depends on components, L1 and C1, as shown in Figure 11-7. Experience has shown that the, LC, relationship is shown in Equation [11-1].

$$LC\omega^2 = 1.5 \quad [11\text{-}1]$$

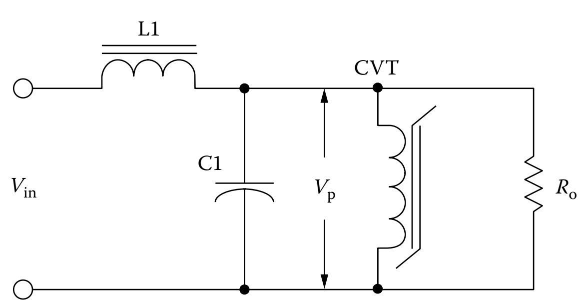

Figure 11-7. Basic, Constant Voltage Transformer Circuit.

The inductance can be expressed, as shown in Equation [11-2].

$$L = \frac{R_{o(R)}}{2\omega}, \quad \text{[henrys]} \qquad [11\text{-}2]$$

The capacitance can be expressed, as shown in Equation [11-3].

$$C = \frac{1}{0.33\omega R_{o(R)}}, \quad \text{[farads]} \qquad [11\text{-}3]$$

Referring to Figure 11-7, assume there is a sinusoidal input voltage, an ideal input inductor, L1, and a series capacitor, C1. All voltage and currents are rms values. V_{in} is the voltage value just before the circuit starts to regulate at full load; $R_{o(R)}$, is the reflected resistance back to the primary, including efficiency; η is the efficiency, and, P_o, is the output power, as shown in Equations [11-4] and [11-5].

$$P_o = \frac{V_s^2}{R_o}, \quad \text{[watts]} \qquad [11\text{-}4]$$

$$R_{o(R)} = \frac{\left(V_p\right)^2 \eta}{P_o}, \quad \text{[ohms]} \qquad [11\text{-}5]$$

It is common practice for the output to be isolated from the input and to connect C1 to a step-up winding on the Constant-Voltage Transformer (CVT). In order to use smaller capacitor values, a step-up winding must be added, as shown in Figure 11-8. The penalty for using a smaller capacitor requires the use of a step-up winding. This step-up winding increases the, VA, or size of the transformer. This can be seen in Equation [11-6]. The energy in a capacitor is:

$$\text{Energy} = \frac{CV^2}{2}, \quad \text{[watt-seconds]}$$

$$C = \frac{2\left(\text{Energy}\right)}{V^2}, \quad \text{[farads]} \qquad [11\text{-}6]$$

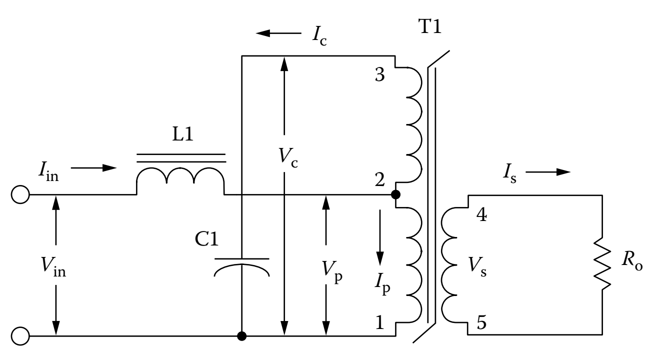

Figure 11-8. CVT, with a Capacitor Step-up Winding.

The secondary current, I_s, can be expressed, as shown in Equation [11-7].

$$I_s = \frac{P_o}{V_s}, \quad \text{[amps]} \qquad [11\text{-}7]$$

With the step-up winding, the primary current, I_p, is related to the secondary current by the following Equation [11-8] [Ref. 4].

$$I_p = \frac{I_s\left(V_{s(4-5)}\right)}{\eta\left(V_{p(1-2)}\right)}\left(1+\sqrt{\frac{V_{p(1-2)}}{V_{c(1-3)}}}\right), \quad \text{[amps]} \qquad [11\text{-}8]$$

The current, I_c, through the capacitor, is increased by, K_c, because of the effective higher frequency. Due to the quasi-voltage waveform, as shown in Figure 11-2, the equivalent ac impedance of the resonant capacitor is reduced to some value lower than its normal sine wave value. This is due to an increase in odd harmonics, as shown in Equation [11-9].

$$I_c = K_c V_c \omega C, \quad \text{[amps]} \qquad [11\text{-}9]$$

Where, K_c, can vary from 1.0 to 1.5.

Empirically, it has been shown that, for good performance, the primary operating voltage should be as shown in Equation [11-10].

$$V_p = V_{in}(0.95), \quad \text{[volts]} \qquad [11\text{-}10]$$

When the resonating capacitor is connected across a step-up winding, as is Figure 11-8, both the value of the capacitor and the volume can be reduced. C_n, is the new capacitance value, and, V_n, is the new voltage across the capacitor.

$$C_n V_n^2 = C_{(1-2)} V_{(1-2)}^2 \qquad [11\text{-}11]$$

The apparent power, P_t, is the sum of each winding, VA, as shown in Equation [11-12].

$$P_t = \left(VA_{(1-2)}\right) + \left(VA_{(2-3)}\right) + \left(VA_{(4-5)}\right), \quad \text{[watts]} \qquad [11\text{-}12]$$

The line voltage regulation of a constant-voltage transformer is shown in Equation [11-13].

$$\Delta V_p = 4.44 \Delta B_s A_c f N_p \left(10^4\right), \quad \text{[volts]} \qquad [11\text{-}13]$$

The output voltage regulation of a constant-voltage transformer, for a change in line voltage, is a function of the squareness of the B-H loop, as shown in Figure 11-9. The saturation flux density, B_s, is dependent on the

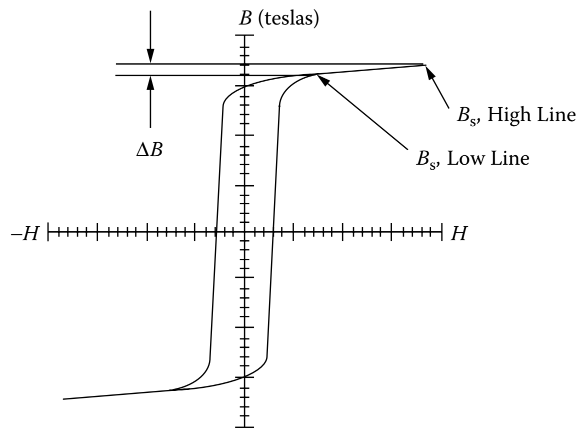

Figure 11-9. B-H Loop of a CVT at High and Low Line.

annealing process of the magnetic material. It seems that each manufacturer has his own annealing process, which has an impact on the saturation flux density, B_s.

Constant-Voltage Transformer, Design Example

Design a Constant-Voltage Transformer (CVT) line regulator with the following specifications:

1. Input voltage = 105-129 volts
2. Line frequency = 60 hertz.
3. Output voltage, V_s = 120 volts
4. Output VA = 250 watts.
5. Transformer current density, J = 300 amps/cm^2
6. Capacitor voltage, V_c = 440 volts
7. Capacitor coefficient, K_c = 1.5
8. Efficiency goal, $\eta(100)$ = 85%
9. Magnetic Material = Silicon
10. Saturating flux density, B_s = 1.95 teslas
11. Window utilization, K_u = 0.4
12. Temperature rise goal, T_r = 50°C

Step No. 1: Calculate the primary voltage, V_p.

$$V_p = V_{in(\min)}(0.95), \quad \text{[volts]}$$

$$V_p = (105)(0.95), \quad \text{[volts]}$$

$$V_p = 99.75, \quad \text{[volts]}$$

Step No. 2: Calculate the reflected resistance, $R_{o(R)}$ back to the primary, including efficiency η.

$$R_{o(R)} = \frac{(V_p)^2 \eta}{P_o}, \quad \text{[ohms]}$$

$$R_{o(R)} = \frac{(99.75)^2 (0.85)}{250}, \quad \text{[ohms]}$$

$$R_{o(R)} = 33.8, \quad \text{[ohms]}$$

Step No. 3: Calculate the required capacitance, C1.

$$C = \frac{1}{0.33\omega R_{o(R)}}, \quad \text{[farads]}$$

$$C = \frac{1}{0.33(377)(33.8)}, \quad \text{[farads]}$$

$$C = 238\left(10^{-6}\right), \quad \text{[farads]}$$

Step No. 4: Calculate the new capacitance value, using the higher voltage, V_c.

$$C_{(1-3)} = \frac{C_{(1-2)}\left(V_{(1-2)}\right)^2}{\left(V_{(1-3)}\right)^2}, \quad \text{[farads]}$$

$$C_{(1-3)} = \frac{\left(238\left(10^{-6}\right)\right)(99.75)^2}{(440)^2}, \quad \text{[farads]}$$

$$C_{(1-3)} = 12.3\left(10^{-6}\right), \quad \text{[farads]}$$

A standard motor run capacitor is a 12.5 µf/440v.

Step No. 5: Calculate the capacitor current, I_c.

$$I_c = 1.5V_c\omega C, \quad \text{[amps]}$$

$$I_c = 1.5(440)(377)\left(12.5\left(10^{-6}\right)\right), \quad \text{[amps]}$$

$$I_c = 3.11, \quad \text{[amps]}$$

Step No. 6: Calculate the secondary current, I_s.

$$I_s = \frac{P_o}{V_s}, \quad \text{[amps]}$$

$$I_s = \frac{250}{120}, \quad \text{[amps]}$$

$$I_s = 2.08, \quad \text{[amps]}$$

Step No. 7: Calculate the primary current, I_p.

$$I_p = \frac{I_s\left(V_{s(4-5)}\right)}{\eta\left(V_{p(1-2)}\right)}\left(1+\sqrt{\frac{V_{p(1-2)}}{V_{c(1-3)}}}\right), \quad \text{[amps]}$$

$$I_p = \frac{2.08(120)}{(0.85)(99.75)}\left(1+\sqrt{\frac{99.75}{440}}\right), \quad \text{[amps]}$$

$$I_p = 4.35, \quad \text{[amps]}$$

Step No. 8: Calculate the apparent power P_t.

$$P_t = \left(VA_{(1-2)}\right)+\left(VA_{(2-3)}\right)+\left(VA_{(4-5)}\right), \quad \text{[watts]}$$

$$VA_{(1-2)} = V_p I_p = (99.75)(4.35) = 434, \quad \text{[watts]}$$

$$VA_{(2-3)} = \left(V_c - V_p\right)I_c = (340)(3.11) = 1057, \quad \text{[watts]}$$

$$VA_{(4-5)} = V_s I_s = (120)(2.08) = 250, \quad \text{[watts]}$$

$$P_t = (434)+(1057)+(250), \quad \text{[watts]}$$

$$P_t = 1741, \quad \text{[watts]}$$

Step No. 9: Calculate the area product, A_p.

$$A_p = \frac{P_t\left(10^4\right)}{K_f K_u f B_s J}, \quad \text{[cm}^4\text{]}$$

$$A_p = \frac{(1742)\left(10^4\right)}{(4.44)(0.4)(60)(1.95)(300)}, \quad \text{[cm}^4\text{]}$$

$$A_p = 279, \quad \text{[cm}^4\text{]}$$

Step No. 10: Select an, EI, lamination from Chapter Three. The closest lamination to the calculated area product, A_p, is the EI-175.

1. Core Number = EI-175
2. Magnetic Path Length, MPL = 26.7 cm
3. Core Weight, W_{tfe} = 3.71 kilograms
4. Mean Length Turn, MLT = 25.6 cm
5. Iron Area, A_c = 18.8 cm^2
6. Window Area, W_a = 14.8 cm^2
7. Area Product, A_p = 278 cm^4
8. Core Geometry, K_g = 81.7 cm^5
9. Surface Area, A_t = 652 cm^2

Step No. 11: Calculate the number of primary turns, N_p.

$$N_p = \frac{V_p\left(10^4\right)}{K_f B_s f A_c}, \quad \text{[turns]}$$

$$N_p = \frac{(99.75)\left(10^4\right)}{(4.44)(1.95)(60)(18.8)}, \quad \text{[turns]}$$

$$N_p = 102, \quad \text{[turns]}$$

Step No. 12: Calculate the primary bare wire area, $A_{wp(B)}$.

$$A_{wp(B)} = \frac{I_p}{J}, \quad [\text{cm}^2]$$

$$A_{wp(B)} = \frac{(4.35)}{(300)}, \quad [\text{cm}^2]$$

$$A_{wp(B)} = 0.0145, \quad [\text{cm}^2]$$

Step No. 13: Select a wire from the Wire Table in Chapter 4.

$$AWG = \#16$$

$$A_{w(B)} = 0.0131, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 132, \quad \text{[micro-ohm/cm]}$$

Step No. 14: Calculate the primary resistance, R_p. Use the MLT from the core data and the micro-ohm per centimeter found in Step 13.

$$R_p = (\text{MLT})\left(N_p\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_p = (25.6)(102)(132)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_p = 0.345, \quad \text{[ohms]}$$

Step No. 15: Calculate the primary copper loss, P_p.

$$P_p = \left(I_p\right)^2 R_p, \quad \text{[watts]}$$

$$P_p = (4.35)^2(0.345), \quad \text{[watts]}$$

$$P_p = 6.53, \quad \text{[watts]}$$

Step No. 16: Calculate the required turns for the step-up capacitor winding, N_c.

$$N_c = \frac{N_p\left(V_c - V_p\right)}{V_p}, \quad \text{[turns]}$$

$$N_c = \frac{(102)(440-99.75)}{99.75}, \quad \text{[turns]}$$

$$N_c = 348, \quad \text{[turns]}$$

Step No. 17: Calculate the capacitor step-up winding bare wire area, $A_{wc(B)}$.

$$A_{wc(B)} = \frac{I_c}{J}, \quad [\text{cm}^2]$$

$$A_{wc(B)} = \frac{(3.11)}{(300)}, \quad [\text{cm}^2]$$

$$A_{wc(B)} = 0.0104, \quad [\text{cm}^2]$$

Step No. 18: Select a wire from the Wire Table in Chapter 4.

$$AWG = \#17$$

$$A_{w(B)} = 0.0104, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 166, \quad \text{[micro-ohm/cm]}$$

Step No. 19: Calculate the capacitor winding resistance, R_c. Use the MLT from the core data and the micro-ohm per centimeter found in Step 18.

$$R_c = (\text{MLT})(N_c)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_c = (25.6)(348)(166)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_c = 1.48, \quad \text{[ohms]}$$

Step No. 20: Calculate the capacitor step-up winding copper loss, P_c.

$$P_c = \left(I_c\right)^2 R_c, \quad \text{[watts]}$$

$$P_c = (3.11)^2(1.48), \quad \text{[watts]}$$

$$P_c = 14.3, \quad \text{[watts]}$$

Step No. 21: Calculate the turns for the secondary, N_s.

$$N_s = \frac{N_p V_s}{V_p}, \quad \text{[turns]}$$

$$N_s = \frac{(102)(120)}{99.75}, \quad \text{[turns]}$$

$$N_s = 123, \quad \text{[turns]}$$

Step No. 22: Calculate the secondary bare wire area, $A_{ws(B)}$.

$$A_{ws(B)} = \frac{I_s}{J}, \quad [\text{cm}^2]$$

$$A_{ws(B)} = \frac{(2.08)}{(300)}, \quad [\text{cm}^2]$$

$$A_{ws(B)} = 0.00693, \quad [\text{cm}^2]$$

Step No. 23: Select a wire from the Wire Table in Chapter 4.

$$AWG = \#19$$

$$A_{w(B)} = 0.00653, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 264, \quad \text{[micro-ohm/cm]}$$

Step No. 24: Calculate the secondary winding resistance, R_s. Use the MLT from the core data and the micro-ohm per centimeter found in Step 23.

$$R_s = (\text{MLT})(N_s)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_s = (25.6)(123)(264)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_s = 0.831, \quad \text{[ohms]}$$

Step No. 25: Calculate the secondary winding copper loss, P_s.

$$P_s = (I_s)^2 R_s, \quad \text{[watts]}$$

$$P_s = (2.08)^2(0.831), \quad \text{[watts]}$$

$$P_s = 3.59, \quad \text{[watts]}$$

Step No. 26: Calculate the total copper loss, P_{cu}.

$$P_{cu} = P_p + P_s + P_c, \quad \text{[watts]}$$

$$P_{cu} = (6.53) + (3.59) + (14.3), \quad \text{[watts]}$$

$$P_{cu} = 24.4, \quad \text{[watts]}$$

Step No. 27: Calculate the watts-per-kilogram, W/K, for the appropriate core material. See Chapter 2.

$$W/K = 0.000557 f^{(1.68)} B_s^{(1.86)}, \quad \text{[watts-per-kilogram]}$$

$$W/K = 0.000557(60)^{(1.68)}(1.95)^{(1.86)}, \quad \text{[watts-per-kilogram]}$$

$$W/K = 1.87, \quad \text{[watts-per-kilogram]}$$

Step No. 28: Calculate the core loss in watts, P_{fe}.

$$P_{fe} = (W/K) W_{tfe}, \quad \text{[watts]}$$

$$P_{fe} = (1.87)(3.71), \quad \text{[watts]}$$

$$P_{fe} = 6.94, \quad \text{[watts]}$$

Step No. 29: Calculate the total losses, P_{Σ}

$$P_{\Sigma} = P_{cu} + P_{fe}, \quad \text{[watts]}$$

$$P_{\Sigma} = (24.4) + (6.94), \quad \text{[watts]}$$

$$P_{\Sigma} = 31.34, \quad \text{[watts]}$$

Step No. 30: Calculate the transformer surface watt density, ψ.

$$\psi = \frac{P_{\Sigma}}{A_t}, \quad \text{[watts-per-cm}^2\text{]}$$

$$\psi = \frac{(31.34)}{(652)}, \quad \text{[watts-per-cm}^2\text{]}$$

$$\psi = 0.0481, \quad \text{[watts-per-cm}^2\text{]}$$

Step No. 31: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [^\circ\text{C}]$$

$$T_r = 450(0.0481)^{(0.826)}, \quad [^\circ\text{C}]$$

$$T_r = 36.7, \quad [^\circ\text{C}]$$

Step No. 32: Calculate the transformer efficiency, η

$$\eta = \frac{P_o}{(P_o + P_\Sigma)}(100), \quad [\%]$$

$$\eta = \frac{(250)}{(250 + 31.3)}(100), \quad [\%]$$

$$\eta = 88.9, \quad [\%]$$

Step No. 33: Calculate the window utilization, K_u.

$$K_u = \frac{N_p A_{wp(B)\#16} + N_c A_{wc(B)\#17} + N_s A_{ws(B)\#19}}{W_a}$$

$$K_u = \frac{((102)(0.0131)) + ((348)(0.0104)) + ((123)(0.00653))}{(14.6)}$$

$$K_u = 0.394$$

Series AC Inductor, Design Example

(Also, see Chapter 9.)

Step No. 34: Design a series, linear ac inductor with the following specifications:

1. Applied voltage = 129 volts
2. Line frequency = 60 hertz
3. Current density, J = 300 amp/cm^2
4. Efficiency goal, η(100) = 85%
5. Magnetic material = Silicon
6. Magnetic material permeability, μ_m = 1500
7. Flux density, B_{ac} = 1.4 teslas
8. Window utilization, K_u = 0.4
9. Waveform factor, K_f = 4.44
10. Temperature rise goal, T_r = 50°C

Step No. 35: Calculate the required series inductance, L1. See Figure 11-8.

$$L1 = \frac{R_{o(R)}}{2\omega}, \quad [\text{henrys}]$$

$$L1 = \frac{(33.8)}{2(377)}, \quad [\text{henrys}]$$

$$L1 = 0.0448, \quad [\text{henrys}]$$

Step No. 36: Calculate the inductor reactance, X_L.

$$X_L = 2\pi f L1, \quad \text{[ohms]}$$

$$X_L = (6.28)(60)(0.0448), \quad \text{[ohms]}$$

$$X_L = 16.9, \quad \text{[ohms]}$$

Step No. 37: Calculate the short-circuit current, I_L.

$$I_L = \frac{V_{in(\max)}}{X_L}, \quad \text{[amps]}$$

$$I_L = \frac{(129)}{(16.9)}, \quad \text{[amps]}$$

$$I_L = 7.63, \quad \text{[amps]}$$

Step No. 38: Calculate the apparent power, P_t or VA, of the input series inductor, L1. Use the high line voltage of 129 volts and the normal running current, I_p, from Step 7.

$$VA = V_{in(\max)} I_{L(n)}, \quad \text{[watts]}$$

$$VA = (129)(4.35), \quad \text{[watts]}$$

$$VA = 561, \quad \text{[watts]}$$

Step No. 39: Calculate the area product, A_p.

$$A_p = \frac{VA\left(10^4\right)}{K_f K_u f B_{ac} J}, \quad [\text{cm}^4]$$

$$A_p = \frac{(561)\left(10^4\right)}{(4.44)(0.4)(60)(1.4)(300)}, \quad [\text{cm}^4]$$

$$A_p = 125, \quad [\text{cm}^4]$$

Step No. 40: Select an EI lamination from Chapter Three with the closest calculated area product, A_p.

1. Core Number = EI-138
2. Magnetic Path Length, MPL = 21 cm
3. Core Weight, W_{tfe} = 1.79 kilograms
4. Mean Length Turn, MLT = 20.1 cm
5. Iron Area, A_c = 11.6 cm^2
6. Window Area, W_a = 9.15 cm^2
7. Area Product, A_p = 106 cm^4
8. Core Geometry, K_g = 24.5 cm^5
9. Surface Area, A_t = 403 cm^2
10. Winding length, G = 5.24 cm
11. Lamination tongue, E = 3.49 cm

Step No. 41: Calculate the number of inductor turns, N_L.

$$N_L = \frac{V_{in(\max)}\left(10^4\right)}{K_f B_{ac} f A_c}, \quad \text{[turns]}$$

$$N_L = \frac{(129)\left(10^4\right)}{(4.44)(1.4)(60)(11.6)}, \quad \text{[turns]}$$

$$N_L = 298, \quad \text{[turns]}$$

Step No. 42: Calculate the required gap, L_g.

$$l_g = \left(\frac{0.4\pi N_L^2 A_c\left(10^{-8}\right)}{L}\right) - \left(\frac{\text{MPL}}{\mu_m}\right), \quad \text{[cm]}$$

$$l_g = \left(\frac{(1.26)(298)^2(11.6)\left(10^{-8}\right)}{0.0448}\right) - \left(\frac{21}{1500}\right), \quad \text{[cm]}$$

$l_g = 0.276, \quad$ [cm] or $l_g = 0.109, \quad$ [mils]: This would be 50 mils in each leg.

Step No. 43: Calculate the fringing flux, F.

$$F = \left(1 + \frac{l_g}{\sqrt{A_c}} \ln \frac{2(G)}{l_g}\right)$$

$$F = \left(1 + \frac{0.276}{\sqrt{11.6}} \ln \frac{2(5.24)}{0.276}\right)$$

$$F = 1.29$$

Step No. 44: Using the fringing flux, recalculate the series inductor turns, $N_{L(new)}$.

$$N_{L(new)} = \sqrt{\frac{l_g L}{0.4\pi A_c F\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N_{L(new)} = \sqrt{\frac{(0.276)(0.0448)}{(1.26)(11.6)(1.29)\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N_{L(new)} = 256, \quad \text{[turns]}$$

Step No. 45: Using the new turns, recalculate the flux density, B_{ac}.

$$B_{ac} = \frac{V_{in(\max)}\left(10^4\right)}{K_f N_{L(new)} A_c f}, \quad \text{[teslas]}$$

$$B_{ac} = \frac{(129)\left(10^4\right)}{(4.44)(256)(11.6)(60)}, \quad \text{[teslas]}$$

$$B_{ac} = 1.63, \quad \text{[teslas]}$$

Step No. 46: Calculate the inductor bare wire area, $A_{wL(B)}$.

$$A_{wL(B)} = \frac{I_{L(n)}}{J}, \quad [\text{cm}^2]$$

$$A_{wL(B)} = \frac{(4.35)}{(300)}, \quad [\text{cm}^2]$$

$$A_{wL(B)} = 0.0145, \quad [\text{cm}^2]$$

Step No. 47: Select a wire from the Wire Table in Chapter 4.

$$AWG = \#16$$

$$A_{w(B)} = 0.01307, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 132, \quad [\text{micro-ohm/cm}]$$

Step No. 48: Calculate the inductor winding resistance, R_L. Use the MLT from the core data and the micro-ohm per centimeter found in Step 40.

$$R_L = (\text{MLT})(N_s)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_L = (20.1)(256)(132)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_L = 0.679, \quad [\text{ohms}]$$

Step No. 49: Calculate the inductor winding copper loss, P_L.

$$P_L = (I_L)^2 R_L, \quad [\text{watts}]$$

$$P_L = (4.35)^2(0.679), \quad [\text{watts}]$$

$$P_L = 12.8, \quad [\text{watts}]$$

Step No. 50: Calculate the watts-per-kilogram, W/K, for the appropriate core material. See Chapter 2.

$$W/K = 0.000557 f^{(1.68)} B_s^{(1.86)}, \quad [\text{watts-per-kilogram}]$$

$$W/K = 0.000557(60)^{(1.68)}(1.63)^{(1.86)}, \quad [\text{watts-per-kilogram}]$$

$$W/K = 1.34, \quad [\text{watts-per-kilogram}]$$

Step No. 51: Calculate the core loss in watts, P_{fe}.

$$P_{fe} = (W/K)W_{tfe}, \quad \text{[watts]}$$

$$P_{fe} = (1.34)(1.79), \quad \text{[watts]}$$

$$P_{fe} = 2.4, \quad \text{[watts]}$$

Step No. 52: Calculate the gap loss, P_g.

$$P_g = K_i E l_g f B_{ac}^2, \quad \text{[watts]}$$

$$P_g = (0.155)(3.49)(0.276)(60)(1.63)^2, \quad \text{[watts]}$$

$$P_g = 23.8, \quad \text{[watts]}$$

Step No. 53: Calculate the total losses, P_Σ

$$P_\Sigma = P_{cu} + P_{fe} + P_g, \quad \text{[watts]}$$

$$P_\Sigma = (12.8) + (2.4) + (23.8), \quad \text{[watts]}$$

$$P_\Sigma = 39, \quad \text{[watts]}$$

Step No. 54: Calculate the inductor surface area watt density, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad \text{[watts-per-cm}^2\text{]}$$

$$\psi = \frac{(39)}{(403)}, \quad \text{[watts-per-cm}^2\text{]}$$

$$\psi = 0.0968, \quad \text{[watts-per-cm}^2\text{]}$$

Step No. 55: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [°\text{C}]$$

$$T_r = 450(0.0968)^{(0.826)}, \quad [°\text{C}]$$

$$T_r = 65, \quad [°\text{C}]$$

Step No. 56: Calculate the window utilization, K_u.

$$K_u = \frac{N_{L(new)} A_{w(B)\#16}}{W_a}$$

$$K_u = \frac{((256)(0.0131))}{(9.15)}$$

$$K_u = 0.367$$

References

1. Ruben, L., and Stephens, D. Gap Loss in Current-Limiting Transformer. *Electromechanical Design*, April, 1973, pp. 24–126.

2. H. P. Hart and R. J. Kakalec, "The Derivation and Application of Design Equations for Ferroresonant Voltage Regulators and Regulated Rectifiers," *IEEE Trans. Magnetics*, vol. Mag-7, No.1, March, 1971, pp. 205–211.

3. I. B. Friedman, "The Analysis and Design of Constant Voltage Regulators," *IRE Trans. Component Parts*, vol. CP-3, March, 1956, pp. 11–14.

4. S. Lendena, "Design of a Magnetic Voltage Stabilizer." *Electronics Technology*, May, 1961, pp. 154–155.

Chapter 12

Three-Phase Transformer Design

Table of Contents

Introduction

Three-phase power is used almost exclusively for generation, transmission, and distribution, as well as for all industrial uses. It is also used on aircraft, both commercial and military. It has many advantages over single-phase power. The transformer can be made smaller and lighter for the same power handling capability, because the copper and iron are used more effectively. In circuitry, for conversion from ac to dc, the output contains a much lower ripple amplitude, and a higher frequency component, which is 3 times and 6 times the line frequency, and which, in turn, requires less filtering.

Primary Circuit

The two most commonly used primary circuits for three-phase transformers are the Star, or Y connection, as shown in Figure 12-1, and the other being known as the Delta (Δ) connection, as shown in Figure 12-2. The design requirement for each particular job dictates which method of connection will be used.

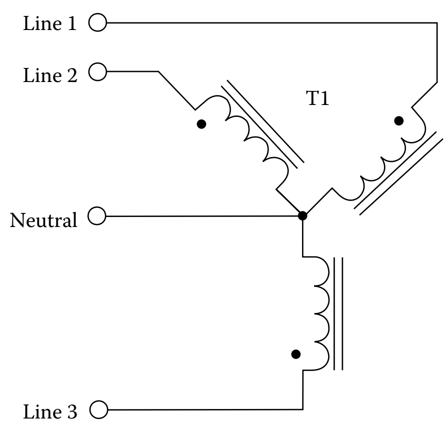

Figure 12-1. Three-Phase Transformer, Connected in Star.

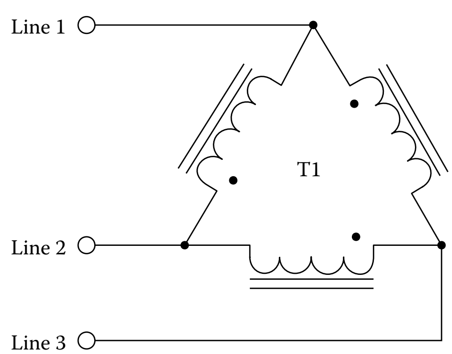

Figure 12-2. Three-Phase Transformer, Connected in Delta.

Comparing Transformer, Physical Size

The schematic diagram in Figure 12-3, shows the connection of three single-phase transformers: (a) Operating from a three-phase power source and a single three-phase transformer; and (b) Operating from a three-phase power source connected in a delta-delta configuration. The single three-phase transformer, T4, would be lighter and smaller than a bank of three single-phase transformers of the same total rating. Since the windings of the three-phase transformer are placed on a common magnetic core, rather than on three independent cores, the consolidation results in an appreciable savings in the copper, the core, and the insulating materials.

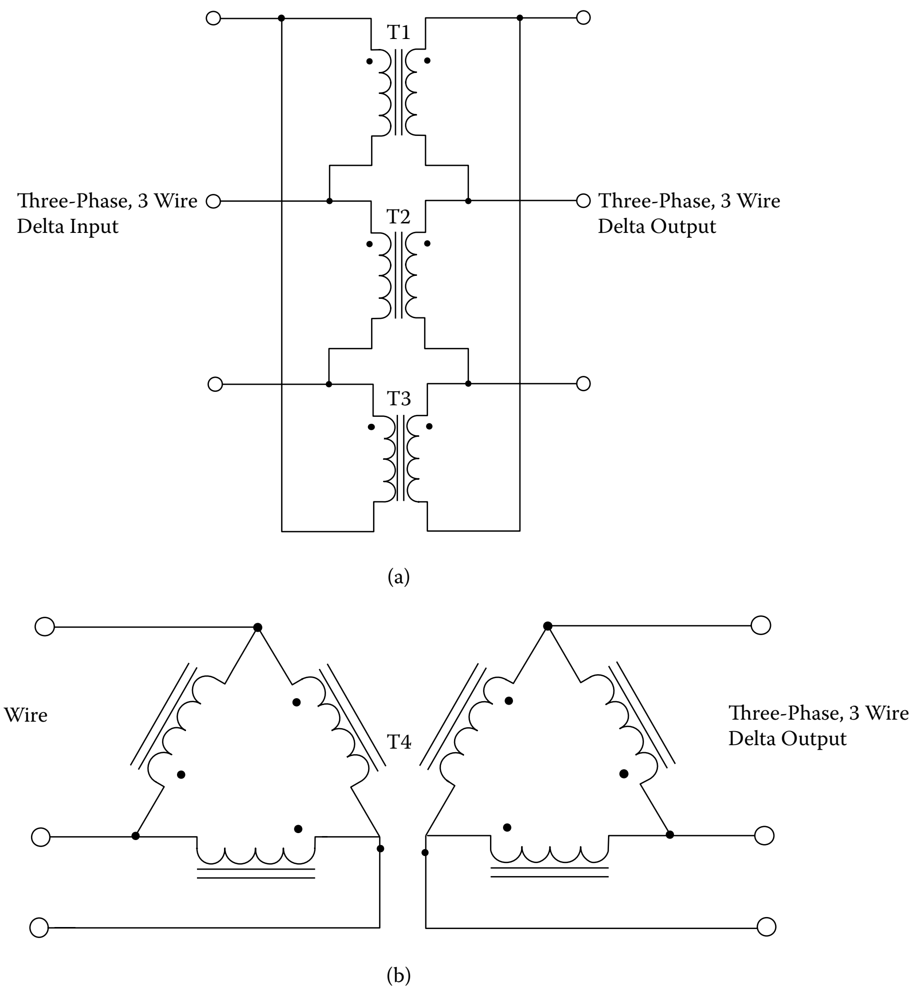

Figure 12-3. Comparing Three Single-Phase Transformers Connected in Three-Phase Delta.

A cutaway view of a single-phase transformer, showing the window area and iron area of two types of core configuration, is shown in Figure 12-4 and Figure 12-5. The EI lamination, shown in Figure 12-4, is known as a shell type, because it looks like the core surrounds the coil. The C core, shown in Figure 12-5, is known as a core type, because it looks like the coil surrounds the core.

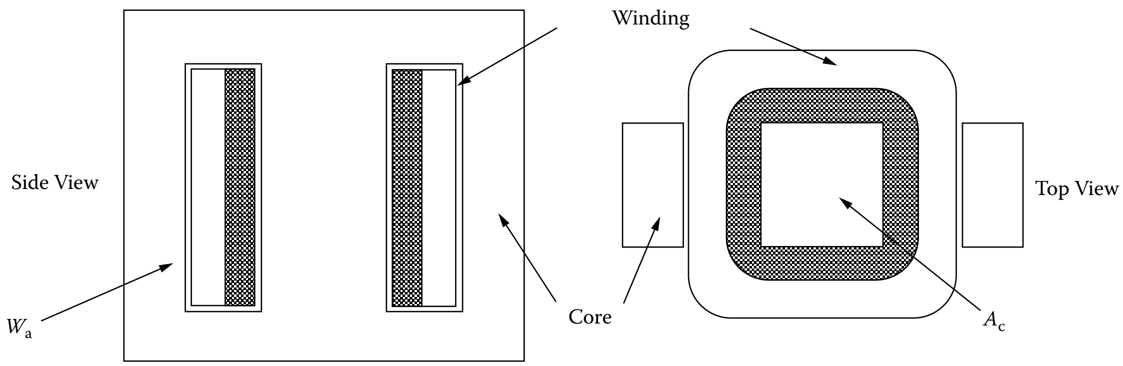

Figure 12-4. Illustrating a Shell Type Transformer.

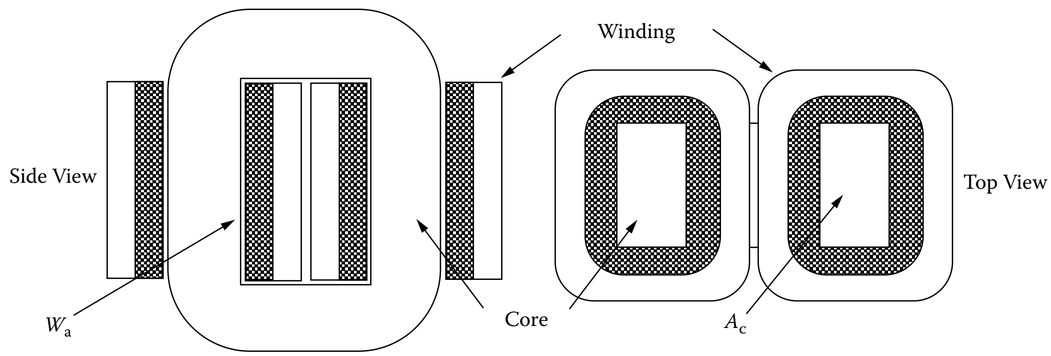

Figure 12-5. Illustrating a Core Type Transformer.

Cutaway views of a three-phase transformer are shown in Figure 12-6. These cross-sectional views show the window and iron areas. The three-legged core is designed to take advantage of the fact that, with balanced voltages impressed, the flux in each phase leg, adds up to zero. Therefore, no return leg is needed under normal conditions. When the transformer is subjected to unbalanced loads, or unbalanced line voltages, it may be best to use three single-phase transformers, because of the high-circulating currents.

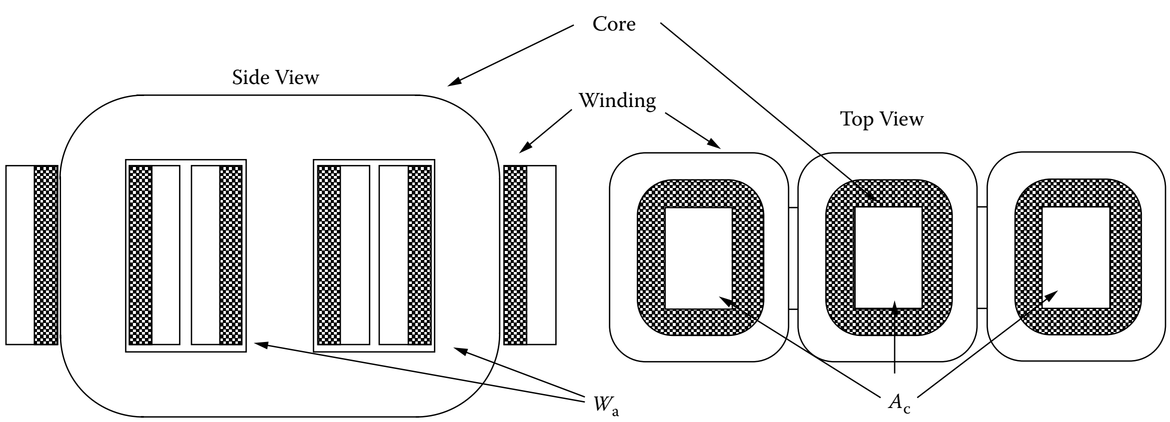

Figure 12-6. Cutaway View of a Three-Phase Transformer.

Phase Current, Line Current, and Voltage in a Delta System

In a Three-Phase Delta Circuit, such as the one shown in Figure 12-7, the line voltage and line current are commonly called phase voltage and phase current. The line voltage, $E_{(Line)}$, will be the same as the actual winding voltage of the transformer. However, the line current, $I_{(Line)}$, is equal to the phase current, $I_{(Phase)}$, times the square root of 3, as shown in Equation [12-1].

$$I_{(line)} = I_{(phase)}\sqrt{3}, \quad \text{[amps]} \qquad [12\text{-}1]$$

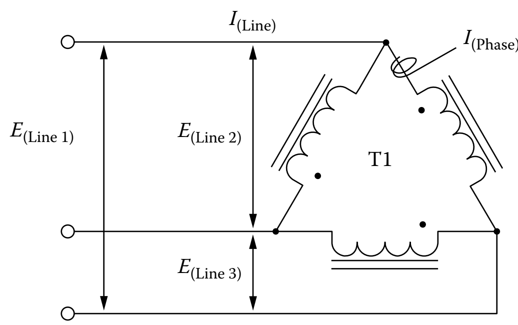

Figure 12-7. Voltage and Current Relationship of a Three-Phase Delta Circuit.

Phase Voltage, Line Voltage, and Current in a Wye System

The relationship between the Line voltage, Line current and the winding, or Phase voltage and Phase current, in a Three-Phase Wye Circuit can be seen in Figure 12-8. In a Wye System, the voltage between any two wires in the line will always be the square root of three times the phase voltage, $E_{(Phase)}$, between the neutral, and any one of the lines, as shown in Equation [12-2].

$$E_{(phase)} = \frac{E_{(line)}}{\sqrt{3}}, \quad \text{[volts]} \qquad [12\text{-}2]$$

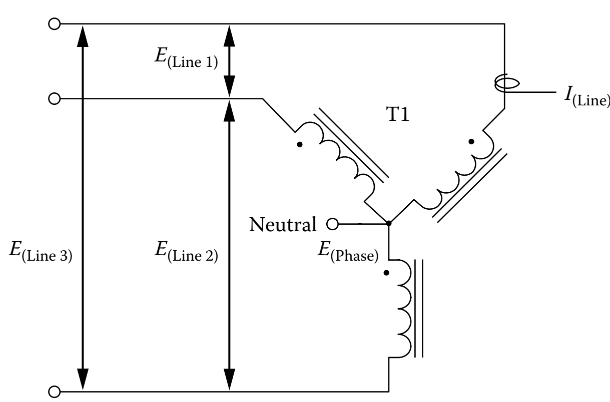

Figure 12-8. Voltage and Current Relationship for a Three-Phase Wye Circuit.

Comparing Multiphase and Single-Phase Power

Three-phase power distribution has a significant advantage over the single-phase. Most high power equipment and industrial complexes will use three-phase power. One of the biggest advantages in using three-phase power distribution has to do with smaller magnetic components handling the same power as single-phase. This can be seen in aircraft and shipboard equipment, as well as fixed ground installations. One of the basic reasons for selecting three-phase is the transformer size. Another reason is, if dc is a requirement, the capacitor and inductor filtering components are both smaller. The odd shape of a three-phase transformer could be troublesome, as well as the keeping of balanced loads to minimize circulating currents.

The single-phase, full wave bridge circuit is shown in Figure 12-9. The ripple voltage frequency is always twice the line frequency. Only 50% of the total current flows through each rectifier. The three-phase, Delta full wave bridge circuit is shown in Figure 12-10. The ripple voltage frequency is always 6 times the line frequency. Only 33% of the total current flows through each rectifier. Looking at the ripple in Figure 12-10, it is obvious the LC components will be smaller.

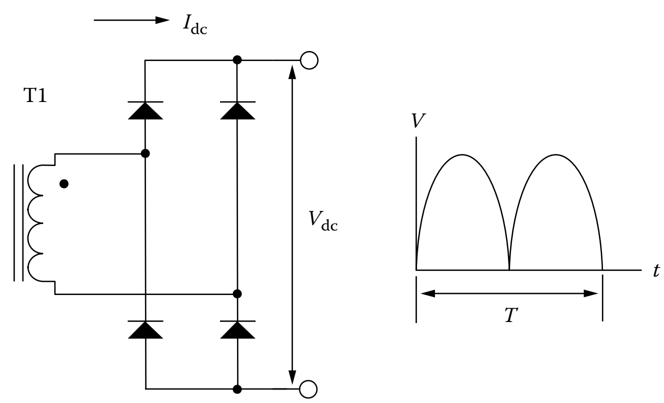

Figure 12-9. Single-Phase Full Wave Bridge.

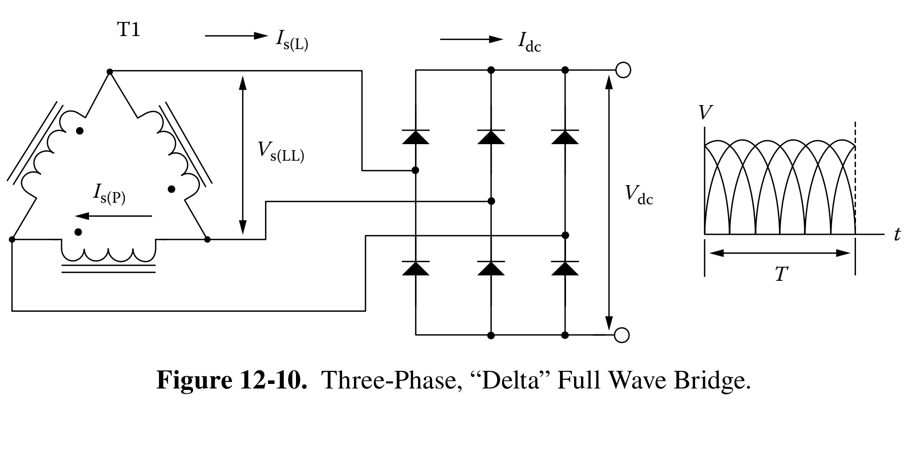

Figure 12-10. Three-Phase, "Delta" Full Wave Bridge.

Multiphase Rectifier Circuits

Table 12-1 lists voltage and current ratios for the circuits, shown in Figures 12-11 through Figures 12-14 for inductive output filters. These ratios apply for sinusoidal ac input voltages. The values that are shown do not take into consideration voltage drops, which occur in the power transformer or rectifier diodes.

Table 12-1. Three-Phase Voltage and Current Ratios for Rectifier Circuits

Three Phase Rectifier Circuit Data			
Delta-Delta Full Wave, Figure 12-11			
Item	Factor		
Primary VA	1.050	×	dc watts output
Secondary V/leg	0.740	×	average dc output voltage
Secondary I/leg	0.471	×	average dc output current
Secondary VA	1.050	×	dc watts output
Ripple Voltage %	4.200		
Ripple Frequency	6f		
Delta-Wye Full Wave, Figure 12-12			
Item	Factor		
Primary VA	1.050	×	dc watts output
Secondary Line to Line	0.740	×	average dc output voltage
Secondary V/leg	0.428 to Neutral	×	average dc output voltage
Secondary I/leg	0.817	×	average dc output current
Secondary VA	1.050	×	dc watts output
Ripple Voltage %	4.200		
Ripple Frequency	6f		
Delta-Wye Half Wave, Figure 12-13			
Item	Factor		
Primary VA	1.210	×	dc watts output
Secondary Line to Line	0.740	×	average dc output voltage
Secondary V/leg	0.855 to Neutral	×	average dc output voltage
Secondary I/leg	0.577	×	average dc output current
Secondary VA	1.480	×	dc watts output
Ripple Voltage %	18.000		
Ripple Frequency	3f		
Delta-Wye 6 Phase Half Wave, Figure 12-14			
Item	Factor		
Primary VA	1.280	×	dc watts output
Secondary Line to Line	1.480	×	average dc output voltage
Secondary V/leg	0.740 to Neutral	×	average dc output voltage
Secondary I/leg	0.408	×	average dc output current
Secondary VA	1.810	×	dc watts output
Ripple Voltage %	4.200		
Ripple Frequency	6f		
Root mean square values to the average dc. Sine-wave, infinite inductance, no transformer or rectifier losses.			

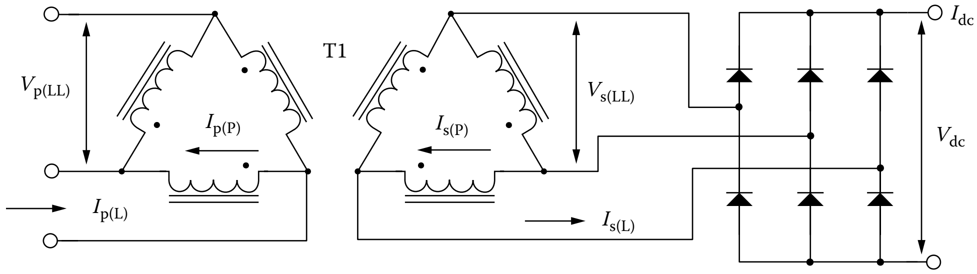

Figure 12-11. Three-Phase, "Delta-Delta" Full Wave Bridge.

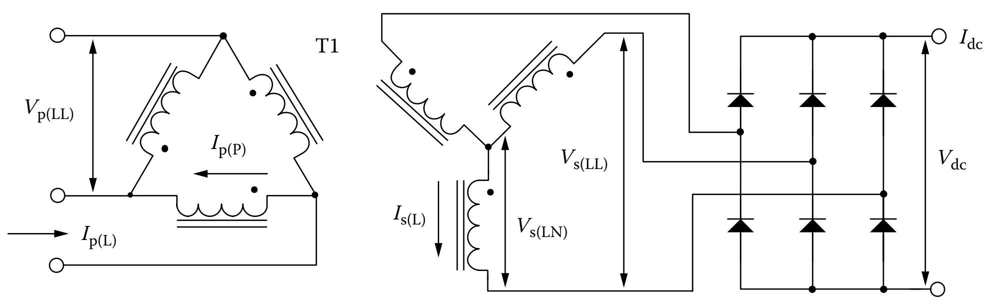

Figure 12-12. Three-Phase, "Delta-Wye" Full Wave Circuit.

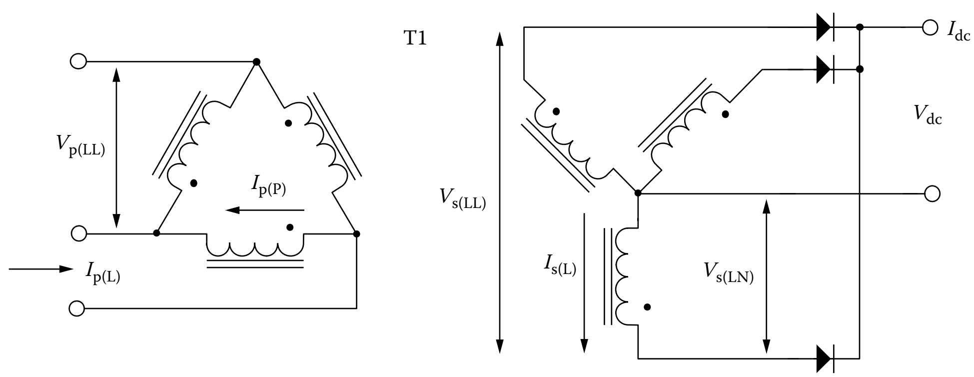

Figure 12-13. Three-Phase, "Delta-Wye" Half Wave Bridge.

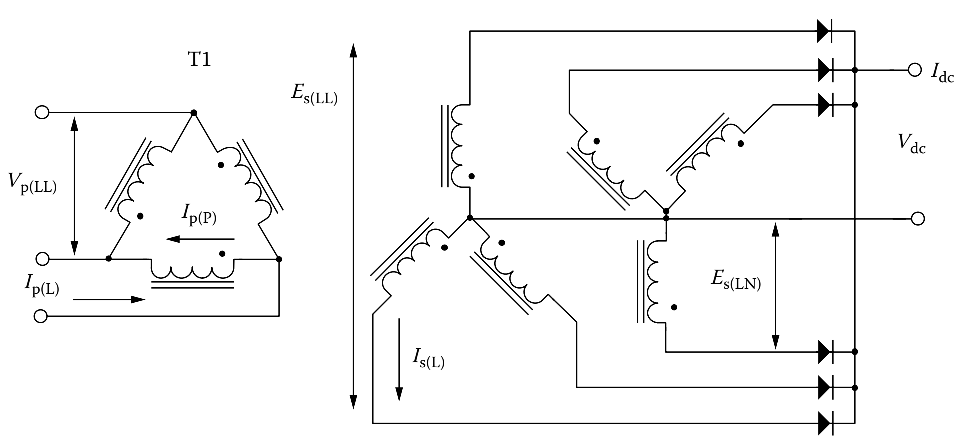

Figure 12-14. Three-Phase, "Delta-Wye" Six Phase Star.

Area Product, A_p, and Core Geometry, K_g, for Three Phase Transformers

The area product, A_p, of a three-phase core is defined differently than that for a single-phase core. The window area, W_a, and iron area, A_c, for a single-phase transformer are shown in Figure 12-4 and 12-5. The window area, W_a, and iron area, A_c, for a three-phase transformer are shown in Figure 12-6. The area product, A_p, of a core is the product of the available window area, W_a, of the core in square centimeters, (cm^2), multiplied by the effective, cross-section area, A_c, in square centimeters, (cm^2), which may be stated as shown, in Equation [12-3].

$$\text{Single-phase: } A_p = W_a A_c, \quad [\text{cm}^4] \tag{12-3}$$

This is correct for a single-phase transformer. For three-phase transformers, because there are basically two window areas, W_a, and three iron areas, A_c, the window utilization is different, and the area product, A_p, changes to the Equation [12-4].

$$\text{Three-phase: } A_p = 3\left(\frac{W_a}{2} A_c\right), \quad [\text{cm}^4] \tag{12-4}$$

This reduces to Equation [12-5].

$$A_p = 1.5\left(W_a A_c\right), \quad [\text{cm}^4] \tag{12-5}$$

It is basically the same thing for the core geometry, K_g, for a single-phase transformer and the core geometry, K_g, for a three-phase transformer. The core geometry, K_g, for a single-phase transformer is shown in Equation [12-6].

$$\text{Single-phase: } K_g = \left(\frac{W_a A_c^2 K_u}{\text{MLT}}\right), \quad [\text{cm}^5] \tag{12-6}$$

In the three-phase transformer, core geometry, K_g, is shown in Equation [12-7].

$$\text{Three-phase: } K_g = 3\left(\left(\frac{W_a}{2}\right)\frac{A_c^2 K_u}{\text{MLT}}\right), \quad [\text{cm}^5] \tag{12-7}$$

This reduces to Equation [12-8].

$$K_g = 1.5\left(\frac{W_a A_c^2 K_u}{\text{MLT}}\right), \quad [\text{cm}^5] \tag{12-8}$$

Output Power Versus Apparent Power, P_t, Capability

The apparent power, P_t, is described in detail in Chapter 7. The apparent power, P_t, of a transformer is the combined power of the primary and secondary windings, which handle, P_{in} and P_o, to the load, respectively. Since the power transformer has to be designed to accommodate the primary, P_{in}, and the secondary, P_o the apparent power is shown in Equation [12-9].

$$P_t = P_{in} + P_o, \quad \text{[watts]} \tag{12-9}$$

$$P_{in} = \frac{P_o}{\eta}, \quad \text{[watts]} \tag{12-10}$$

Substituting:

$$P_t = \frac{P_o}{\eta} + P_o, \quad \text{[watts]} \tag{12-11}$$

$$P_t = P_o\left(\frac{1}{\eta} + 1\right), \quad \text{[watts]} \tag{12-12}$$

The designer must be concerned with the apparent power handling capability, P_t, of the transformer core and winding. The apparent power, P_t, varies with the type of circuit in which the transformer is used. If the current in the rectifier is interrupted, its effective rms value changes. Transformer size is thus determined, not only by the load demand, but also by current wave shape. An example of the primary and secondary, VA, is shown to compare the power-handling capability required by each three-phase rectifier circuit in Table 12-1 and Figures 12-11 through 12-14. This comparison will negate transformer and diode losses so that $P_{in} = P_o$ ($\eta = 1$) for all three-phase rectifier circuits.

1. The apparent power, P_t, for a Delta-Delta, Full Wave, from Figure 12-11 is shown in Equation [12-13].

$$\begin{aligned} P_t &= P_o\left(\frac{P_{VA}}{\eta} + S_{VA}\right), \quad \text{[watts]} \\ P_t &= P_o\left(\frac{1.05}{1} + 1.05\right), \quad \text{[watts]} \\ P_t &= P_o(2.1), \quad \text{[watts]} \end{aligned} \tag{12-13}$$

2. The apparent power, P_t, for a Delta-Wye, Full Wave, from Figure 12-12 is shown in Equation [12-14].

$$\begin{aligned} P_t &= P_o\left(\frac{P_{VA}}{\eta} + S_{VA}\right), \quad \text{[watts]} \\ P_t &= P_o\left(\frac{1.05}{1} + 1.05\right), \quad \text{[watts]} \\ P_t &= P_o(2.1), \quad \text{[watts]} \end{aligned} \tag{12-14}$$

3. The apparent power, P_t, for a Delta-Wye, Half Wave, from Figure 12-13 is shown in Equation [12-15].

$$P_t = P_o\left(\frac{P_{VA}}{\eta} + S_{VA}\right), \quad \text{[watts]}$$
$$P_t = P_o\left(\frac{1.21}{1} + 1.48\right), \quad \text{[watts]} \qquad [12\text{-}15]$$
$$P_t = P_o(2.69), \quad \text{[watts]}$$

4. The apparent power, P_t, for a Delta-Wye, 6 Phase Wave, from Figure 12-14 is shown in Equation [12-16].

$$P_t = P_o\left(\frac{P_{VA}}{\eta} + S_{VA}\right), \quad \text{[watts]}$$
$$P_t = P_o\left(\frac{1.28}{1} + 1.81\right), \quad \text{[watts]} \qquad [12\text{-}16]$$
$$P_t = P_o(3.09), \quad \text{[watts]}$$

Relationship, K_g, to Power Transformer Regulation Capability

Although most transformers are designed for a given temperature rise, they can also be designed for a given regulation. The regulation and power-handling ability of a core are related to two constants:

$$\alpha = \frac{P_t}{2K_g K_e}, \quad [\%] \qquad [12\text{-}17]$$

$$\alpha = \text{Regulation } (\%) \qquad [12\text{-}18]$$

The constant, K_g, is determined by the core geometry, which is shown in Equation [12-19].

$$K_g = 1.5\left(\frac{W_a A_c^2 K_u}{\text{MLT}}\right) = \frac{P_t}{2K_e\alpha}, \quad [\text{cm}^5] \qquad [12\text{-}19]$$

The constant, K_e, is determined by the magnetic and electric operating conditions, which is shown in Equation [12-20].

$$K_e = 2.86 f^2 B^2 \left(10^{-4}\right) \qquad [12\text{-}20]$$

From the above, it can be seen that factors, such as flux density, frequency of operation, and waveform coefficient, have an influence on the transformer size.

Relationship, A_p, to Transformer Power Handling Capability

According to the newly developed approach, the power handling capability of a core is related to its area product, A_p, for a single phase as shown in Equation [12-21], and a three phase as shown in Equation [12-22].

$$A_p = \frac{P_t\left(10^4\right)}{K_f K_u B_m J f}, \quad [\text{cm}^4] \qquad [12\text{-}21]$$

$$A_p = 1.5\left(W_a A_c\right) = \frac{P_t\left(10^4\right)}{K_f K_u B_m J f}, \quad [\text{cm}^4] \qquad [12\text{-}22]$$

Where:

K_f = waveform coefficient

K_f = 4.0 square wave

K_f = 4.44 sine wave

From the above, it can be seen that factors such as flux density, frequency of operation, and window utilization factor, K_u, define the maximum space, which may be occupied by the copper in the window.

Three-Phase, Transformer Design Example

The following information is the Design specification for a three-phase, isolation transformer, using the K_g, core geometry approach.

Design specification:

1. Input voltage, V_{in} = 208 V, 3 Wire
2. Output voltage, V_o = 28 V
3. Output Current, I_o = 10 amps
4. Output Circuit = Full Bridge
5. Input / Output = Delta / Delta
6. Frequency, Three Phase, f = 60 hertz
7. Efficiency, $\eta(100)$ = 95 %
8. Regulation, α = 5 %
9. Flux Density, B_{ac} = 1.4 teslas
10. Magnetic Material = Silicon M6X
11. Window Utilization $K_u = (K_{up} + K_{us}) = 0.4$
12. Diode Drop, V_d = 1.0 volt

Step No. 1: Calculate the apparent power, P_t.

$$P_t = P_o\left(\frac{1.05}{\eta}+1.05\right), \quad \text{[watts]}$$

$$P_o = I_o\left(V_o+2V_d\right)=(10)(30)=300, \quad \text{[watts]}$$

$$P_t = 300\left(\frac{1.05}{0.95}+1.05\right), \quad \text{[watts]}$$

$$P_t = 647, \quad \text{[watts]}$$

Step No. 2: Calculate the electrical conditions, K_e.

$$K_e = 2.86 f^2 B^2\left(10^{-4}\right)$$

$$K_e = 2.86(60)^2(1.4)^2\left(10^{-4}\right)$$

$$K_e = 2.02$$

Step No. 3: Calculate the core geometry, K_g.

$$K_g = \frac{P_t}{2K_e\alpha}, \quad \text{[cm}^5\text{]}$$

$$K_g = \frac{647}{2(2.02)(5)}, \quad \text{[cm}^5\text{]}$$

$$K_g = 32, \quad \text{[cm}^5\text{]}$$

Step No. 4: This data is taken from Chapter 3. The section is on, EI, Three-Phase Laminations.

Core number = 100EI-3P

Iron weight, W_{tfe} = 2.751 kilograms

Mean length turn, MLT = 16.7 cm

Iron area, A_c = 6.129 cm^2

Window area, W_a = 29.0 cm^2

Area product, A_p = 267cm^4

Core geometry, K_g = 39 cm^5

Surface area, A_t = 730 cm^2

Step No. 5: Calculate the number of primary turns, N_p, using Faraday's Law.

$$N_p = \frac{V_{p(Line)}\left(10^4\right)}{4.44 B_{ac} A_c f}, \quad \text{[turns]}$$

$$N_p = \frac{208\left(10^4\right)}{4.44(1.4)(6.129)(60)}, \quad \text{[turns]}$$

$$N_p = 910, \quad \text{[turns]}$$

Step No. 6: Calculate the primary line current, $I_{p(Line)}$.

$$I_{p(Line)} = \frac{P_o}{3V_{p(Line)}\eta}, \quad [\text{amps}]$$

$$I_{p(Line)} = \frac{300}{3(208)(0.95)}, \quad [\text{amps}]$$

$$I_{p(Line)} = 0.506, \quad [\text{amps}]$$

Step No. 7: Calculate the primary phase current, $I_{p(phase)}$.

$$I_{p(Phase)} = \frac{I_{p(Line)}}{\sqrt{3}}, \quad [\text{amps}]$$

$$I_{p(Phase)} = \frac{0.506}{1.73}, \quad [\text{amps}]$$

$$I_{p(Phase)} = 0.292, \quad [\text{amps}]$$

Step No. 8: Calculate the primary bare wire area, $A_{wp(B)}$. The window area available for the primary is, W_a / 4. The primary window utilization is, $K_{up} = 0.2$.

$$A_{wp(B)} = \left(\frac{K_{u(p)} W_a}{4N_p}\right), \quad [\text{cm}^2]$$

$$A_{wp(B)} = \left(\frac{(0.2)(29.0)}{4(910)}\right), \quad [\text{cm}^2]$$

$$A_{wp(B)} = 0.00159, \quad [\text{cm}^2]$$

Step No. 9: The selection of the wire would be from the Wire Table, in Chapter 4.

$$AWG\ \#25$$

$$A_{w(B)} = 0.001623, \quad [\text{cm}^2]$$

$$A_{w(Ins)} = 0.002002, \quad [\text{cm}^2]$$

$$\frac{\mu\Omega}{\text{cm}} = 1062$$

Step No. 10: Calculate the primary winding resistance. Use the MLT, from Step 4, and the micro-ohm, per centimeter, found in Step 9.

$$R_p = \text{MLT}(N_p)\left(\frac{\mu\Omega}{\text{cm}}\right)(10^{-6}), \quad [\text{ohms}]$$

$$R_p = (16.7)(910)(1062)(10^{-6}), \quad [\text{ohms}]$$

$$R_p = 16.1, \quad [\text{ohms}]$$

Step No. 11: Calculate the total primary copper loss, P_p.

$$P_p = 3\left(I_{p(phase)}\right)^2 R_p, \quad \text{[watts]}$$

$$P_p = 3(0.292)^2(16.1), \quad \text{[watts]}$$

$$P_p = 4.12, \quad \text{[watts]}$$

Step No. 12: Calculate the secondary turns, N_s.

$$N_s = \frac{N_p V_s}{V_p}\left(1+\frac{\alpha}{100}\right), \quad \text{[turns]}$$

$$V_s = (0.740)(V_o + 2V_d) = (0.740)(28+2) = 22.2$$

$$N_s = \frac{(910)(22.2)}{(208)}\left(1+\frac{5}{100}\right), \quad \text{[turns]}$$

$$N_s = 102, \quad \text{[turns]}$$

Step No. 13: Calculate the secondary bare wire area, $A_{ws(B)}$.

$$A_{ws(B)} = \left(\frac{K_{u(s)} W_a}{4N_s}\right), \quad [\text{cm}^2]$$

$$A_{ws(B)} = \left(\frac{(0.2)(29.0)}{4(102)}\right), \quad [\text{cm}^2]$$

$$A_{ws(B)} = 0.0142, \quad [\text{cm}^2]$$

Step No. 14: The selection of the wire will be from the Wire Table in Chapter 4.

$$AWG\ \#16$$

$$A_{w(B)} = 0.01307, \quad [\text{cm}^2]$$

$$A_{w(Ins)} = 0.01473, \quad [\text{cm}^2]$$

$$\frac{\mu\Omega}{\text{cm}} = 132$$

Step No. 15: Calculate the secondary winding resistance, R_s. Use the MLT, from Step 4, and the micro-ohm per centimeter, found in Step 14.

$$R_s = \text{MLT}(N_s)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_s = (16.7)(102)(132)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_s = 0.225, \quad \text{[ohms]}$$

Step No. 16: Calculate the secondary line current, $I_{s(line)}$.

$$I_{s(line)} = (0.471)I_o, \quad \text{[amps]}$$

$$I_{s(line)} = (0.471)(10), \quad \text{[amps]}$$

$$I_{s(line)} = 4.71, \quad \text{[amps]}$$

Step No. 17: Calculate the secondary phase current, $I_{s(phase)}$.

$$I_{s(phase)} = \frac{I_{s(line)}}{\sqrt{3}}, \quad \text{[amps]}$$

$$I_{s(phase)} = \frac{4.71}{1.73}, \quad \text{[amps]}$$

$$I_{s(phase)} = 2.72, \quad \text{[amps]}$$

Step No. 18: Calculate the total secondary copper loss, P_s.

$$P_s = 3\left(I_{s(phase)}\right)^2 R_s, \quad \text{[watts]}$$

$$P_s = 3(2.72)^2(0.225), \quad \text{[watts]}$$

$$P_s = 4.99, \quad \text{[watts]}$$

Step No. 19: Calculate the transformer regulation, α.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

$$P_{cu} = P_p + P_s, \quad \text{[watts]}$$

$$P_{cu} = 4.12 + 4.99, \quad \text{[watts]}$$

$$P_{cu} = 9.11, \quad \text{[watts]}$$

$$\alpha = \left(\frac{9.11}{300}\right)(100), \quad [\%]$$

$$\alpha = 3.03, \quad [\%]$$

Step No. 20: Calculate the watts per kilogram.

$$\text{Watts/kilogram} = K f^{(m)} B_{ac}^{(n)}$$

$$\text{Watts/kilogram} = 0.000557(60)^{(1.68)}(1.40)^{(1.86)}$$

$$\text{Watts/kilogram} = 1.01$$

Step No. 21: Calculate the core loss, P_{fe}. Core weight, W_{tfe}, is found in Step 4.

$$P_{fe} = \text{Watts/Kilogram}\left(\text{W}_{\text{tfe}}\right), \quad [\text{watts}]$$

$$P_{fe} = 1.01(2.751), \quad [\text{watts}]$$

$$P_{fe} = 2.78, \quad [\text{watts}]$$

Step No. 22: Summarize the total transformer losses, P_{Σ}.

$$P_{\Sigma} = P_p + P_s + P_{fe}, \quad [\text{watts}]$$

$$P_{\Sigma} = 4.12 + 4.99 + 2.78, \quad [\text{watts}]$$

$$P_{\Sigma} = 11.89, \quad [\text{watts}]$$

Step No. 23: Calculate the transformer efficiency, η.

$$\eta = \frac{P_o}{P_o + P_{\Sigma}}(100), \quad [\%]$$

$$\eta = \frac{300}{300 + 11.89}(100), \quad [\%]$$

$$\eta = 96.2, \quad [\%]$$

Step No. 24: Calculate the watts per unit area, ψ. The surface area, A_t, is found in Step 4.

$$\psi = \frac{P_{\Sigma}}{A_t}, \quad [\text{watts per cm}^2]$$

$$\psi = \frac{(11.89)}{(730)}, \quad [\text{watts per cm}^2]$$

$$\psi = 0.0163, \quad [\text{watts per cm}^2]$$

Step No. 25: Calculate the temperature rise, T_r. The watts per unit area ψ is found in Step 24.

$$T_r = 450(\psi)^{0.826}, \quad [^{\circ}\text{C}]$$

$$T_r = 450(0.0163)^{0.826}, \quad [^{\circ}\text{C}]$$

$$T_r = 15, \quad [^{\circ}\text{C}]$$

Step No. 26: Calculate the total window utilization, K_u. The window area is found in Step 4.

$$K_u = K_{up} + K_{us}$$

$$K_u = \frac{4 N_p A_{wp(B)(25)}}{W_a} + \frac{4 N_s A_{ws(B)(16)}}{W_a}$$

$$K_u = \frac{4(910)(0.001623)}{29} + \frac{4(102)(0.01307)}{29}$$

$$K_u = (0.204) + (0.184)$$

$$K_u = 0.388$$

Chapter 13

Flyback Converters, Transformer Design

Table of Contents

Introduction

The principle behind Flyback converters is based on the storage of energy in the inductor during the charging, or the "on period", t_{on}, and the discharge of the energy to the load during the "off period", t_{off}. There are four basic types that are the most common, energy storage, inductor type converter circuits:

1. Step down, or buck converter.
2. Step up, or boost converter.
3. Inverting, buck-boost converter.
4. Isolated, buck-boost converter.

Energy Transfer

Two distinct modes of operation are possible for the Flyback switching converters, shown in Figure 13-1:

Discontinuous Mode All energy stored in the inductor is transferred to an output capacitor and load circuit before another charging period occurs. This topology results in a smaller inductor size, but puts a larger stress on the capacitor and switching device.

Continuous Mode Energy stored in the inductor is not completely transferred to the output capacitor and load circuit before another charging period occurs.

The total period is shown in Equation [13-1].

$$T = \frac{1}{f} \qquad [13\text{-}1]$$

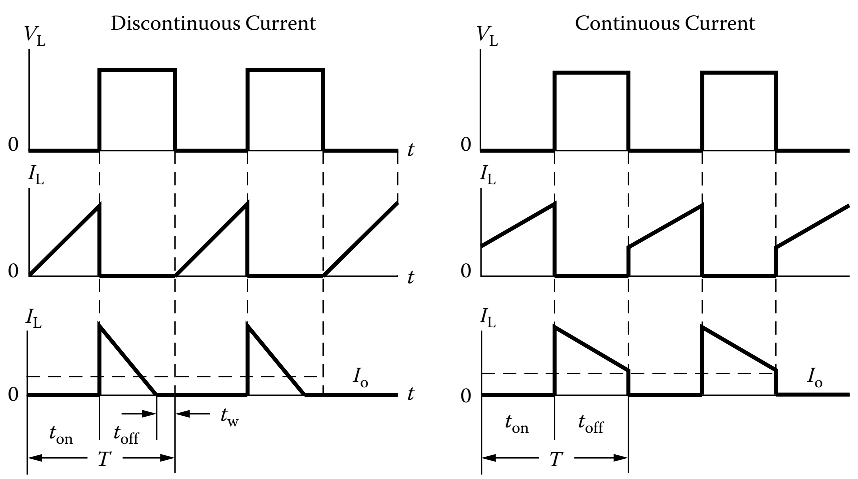

Figure 13-1. Comparing Discontinuous and Continuous Current Waveforms.

Discontinuous Current Mode

In the Discontinuous Mode, a smaller inductance is required, but the penalty results in higher peak currents in the switching transistor. As a consequence, the winding losses are increased because of the higher rms values, due to the higher peak currents. This also results in a higher ripple current and ripple voltage in the input and output capacitor, and gives added stress to the switching transistor. The advantage of this circuit, other than having a smaller inductor, is that when the switching device is turned on, the initial current is zero. A zero current means the output diode has completely recovered, and the switching device does not momentarily turn on into a short. This diode recovery reduces the EMI radiation. The discontinuous mode converter does not exhibit the right half plane zero. Without the right half plane zero, the loop is easy to stabilize.

Continuous Current Mode

In the Continuous Mode, a larger inductor is required; this results in a lower peak current at the end of the cycle than in a discontinuous system of equivalent output power. The Continuous Mode demands a high current flowing through the switch during turn-on, and can lead to high switch dissipation. The continuous mode converter does exhibit the right half-plane zero. With the right half-plane zero, the loop becomes very difficult to stabilize for a wide range of input voltage. The relationship between the B-H loops for continuous and discontinuous operation is shown in Figure 13-2.

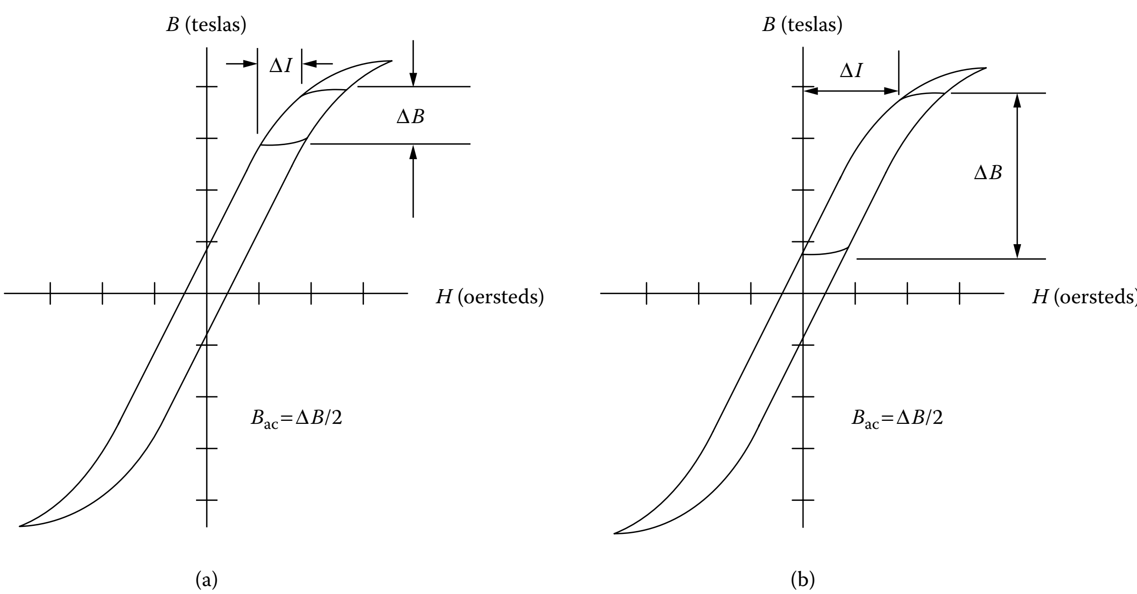

Figure 13-2. Continuous (A) and Discontinuous (B), B-H Loops, Showing ΔB and ΔI.

Continuous and Discontinuous Boundary

When the load current increases, the control circuit causes the transistor to increase the "on time", t_{on}. The peak current in the inductor will increase, resulting in a steady reduction in the dwell time, t_w. When the load current increases to a critical level, t_w becomes zero, and the discontinuous boundary is reached. If the load current is further increased, the inductor current will no longer discharge to zero on every cycle, and continuous current operation results.

The Buck Converter

The Buck Converter is shown in Figure 13-3. The output voltage of this converter is always less than the input voltage. In the buck circuit, the transistor switch, Q1, is placed in series with the dc input voltage. The transistor, Q1, interrupts the dc input voltage, providing a variable-width pulse, (duty ratio), to a simple averaging, LC, filter. When the transistor switch, Q1, is closed, the dc input voltage is applied across the output filter inductor, L1, and the current flows through the inductor to the load. When the switch is open, the energy, stored in the field of the inductor, L1, maintains the current through the load. The discontinuous voltage and current waveforms are shown in Figure 13-4, and the continuous waveforms in Figure 13-5.

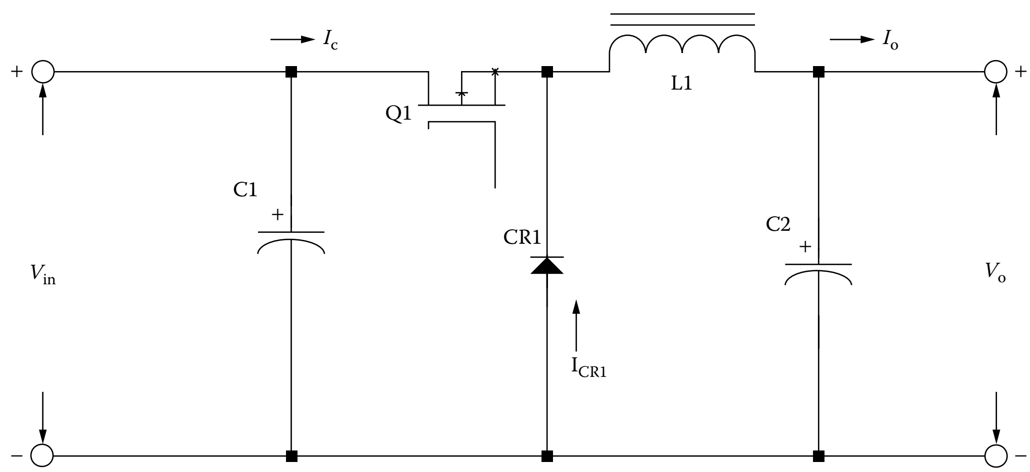

Figure 13-3. Schematic of a Buck Switching Converter.

Discontinuous Current, Buck Converter Design Equations

The maximum Inductance, L_{max}, is shown in Equation [13-2].

$$L_{max} = \frac{(V_o + V_d)T(1 - D_{max} - D_w)}{2I_{o(max)}}, \quad \text{[henrys]} \qquad [13\text{-}2]$$

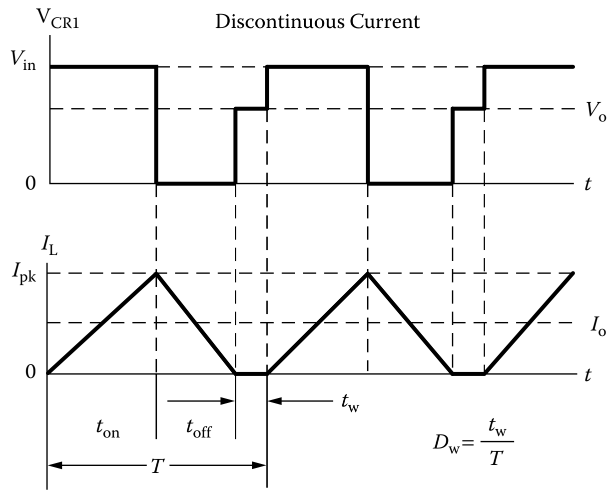

Figure 13-4. Discontinuous Current, Buck Converter Waveforms.

Maximum duty ratio, $D_{(max)}$ is shown in Equation [13-3].

$$D_{(\max)} = \frac{V_o(1 - D_w)}{(\eta V_{in(\min)})} \qquad [13\text{-}3]$$

Maximum on time, $t_{on(max)}$, is shown in Equation [13-4].

$$t_{on(\max)} = T D_{\max} \qquad [13\text{-}4]$$

Maximum off time, $t_{off(max)}$, is shown in Equation [13-5].

$$t_{off(\max)} = T(1 - D_{\min}) \qquad [13\text{-}5]$$

The inductor peak current, $I_{(pk)}$ is shown in Equation [13-6].

$$I_{(pk)} = \frac{2I_{o(\max)}}{(1 - D_w)} \qquad [13\text{-}6]$$

Continuous Current, Buck Converter Design Equations

Inductance, L, is shown in Equation [13-7].

$$L = \frac{V_o T(1 - D_{\min})}{2I_{o(\min)}}, \quad \text{[henrys]} \qquad [13\text{-}7]$$

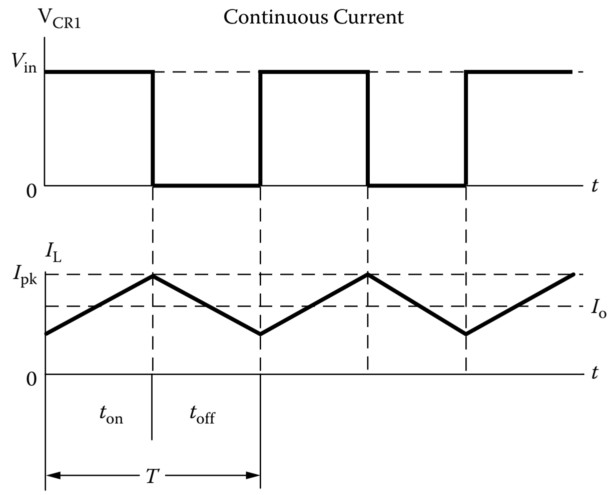

Figure 13-5. Continuous Current, Buck Converter Waveforms.

Maximum duty ratio, $D_{(max)}$, is shown in Equation [13-8].

$$D_{(\max)} = \frac{V_o}{\left(\eta V_{in(\min)}\right)} \qquad [13\text{-}8]$$

Minimum duty ratio, $D_{(min)}$, is shown in Equation [13-9].

$$D_{(\min)} = \frac{V_o}{\left(\eta V_{in(\max)}\right)} \qquad [13\text{-}9]$$

Maximum on time, $t_{on(max)}$ is shown in Equation [13-10].

$$t_{on(\max)} = T D_{\max} \qquad [13\text{-}10]$$

Maximum off time, $t_{off(max)}$ is shown in Equation [13-11].

$$t_{off(\max)} = T\left(1 - D_{\max}\right) \qquad [13\text{-}11]$$

The inductor delta current, ΔI, is shown in Equation [13-12].

$$\Delta I = \frac{\left(T V_{in(\max)} D_{(\min)}\right)\left(1 - D_{(\min)}\right)}{L} \qquad [13\text{-}12]$$

The inductor peak current, $I_{(pk)}$, is shown in Equation [13-13].

$$I_{(pk)} = I_{o(\max)} + \frac{\Delta I}{2} \qquad [13\text{-}13]$$

The Boost Converter

The Boost Converter is shown in Figure 13-6. The output voltage of this converter is always greater than the input voltage. The boost converter stores energy in the inductor, L1, and then, delivers the stored energy along with the energy from the dc source to the load. When the transistor switch, Q1, is closed, current flows through inductor, L1, and the transistor switch, Q1, charging inductor, L1, but does not deliver any current to the load. When the switch is open, the voltage across the load equals the dc input voltage plus the energy stored in inductor, L1. The energy is stored in, L1, then discharges, delivering current to the load. The discontinuous voltage and current waveforms are shown in Figure 13-7, and the continuous waveforms in Figure 13-8.

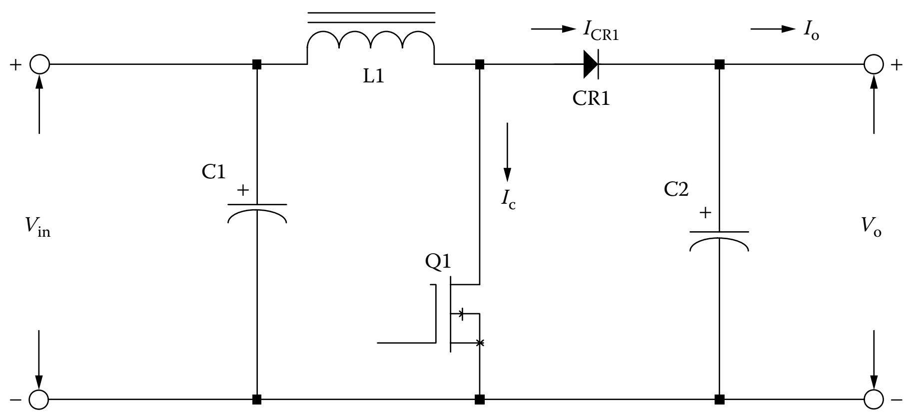

Figure 13-6. Schematic of a Boost Switching Converter.

Discontinuous Current, Boost Converter Design Equations

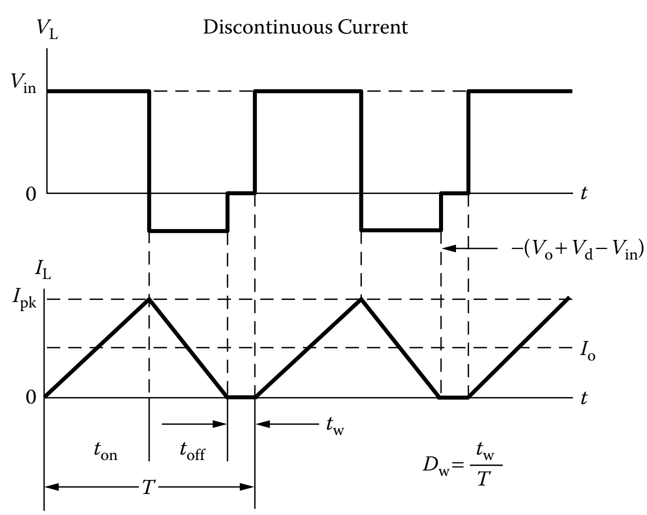

Figure 13-7. Discontinuous Current, Boost Converter Waveforms.

The maximum inductance, L_{max}, is shown in Equation [13-14].

$$L_{\max}=\frac{(V_o+V_d)T\,D_{(\max)}(1-D_{\max}-D_w)^2}{2I_{o(\max)}},\quad [\text{henrys}] \tag{13-14}$$

Maximum duty ratio, $D_{(max)}$, is shown in Equation [13-15].

$$D_{(\max)}=(1-D_w)\left(\frac{V_o-V_{in(\min)}+V_d}{V_o}\right) \tag{13-15}$$

Minimum duty ratio, $D_{(min)}$, is shown in Equation [13-16].

$$D_{(\min)}=(1-D_w)\left(\frac{V_o-V_{in(\max)}+V_d}{V_o}\right) \tag{13-16}$$

Maximum on time, $t_{on(max)}$, is shown in Equation [13-17].

$$t_{on(\max)}=T\,D_{\max} \tag{13-17}$$

Maximum off time, $t_{off(max)}$, is shown in Equation [13-18].

$$t_{off(\max)}=T(1-D_{\min}) \tag{13-18}$$

The inductor peak current, $I_{(pk)}$, is shown in Equation [13-19].

$$I_{(pk)}=\frac{2P_{o(\max)}}{\eta V_{in}(1-D_w)} \tag{13-19}$$

Continuous Current, Boost Converter Design Equations

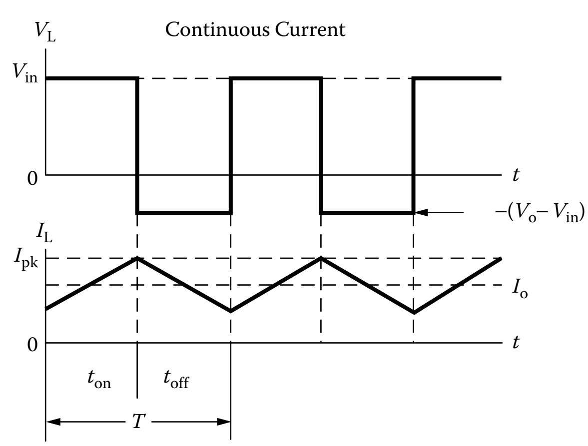

Figure 13-8. Continuous Current, Boost Converter Waveforms.

Inductance, L, is shown in Equation [13-20].

$$L = \frac{(V_o + V_d) T D_{(\min)} (1 - D_{\min})^2}{2 I_{o(\min)}}, \quad \text{[henrys]} \qquad [13\text{-}20]$$

Maximum duty ratio, $D_{(\max)}$ is shown in Equation [13-21].

$$D_{(\max)} = 1 - \left(\frac{V_{in(\min)} \eta}{V_o} \right) \qquad [13\text{-}21]$$

Minimum duty ratio, $D_{(\min)}$, is shown in Equation [13-22].

$$D_{(\min)} = 1 - \left(\frac{V_{in(\max)} \eta}{V_o} \right) \qquad [13\text{-}22]$$

Maximum on time, $t_{on(\max)}$, is shown in Equation [13-23].

$$t_{on(\max)} = T D_{\max} \qquad [13\text{-}23]$$

Maximum off time, $t_{off(\max)}$, is shown in Equation [13-24].

$$t_{off(\max)} = T(1 - D_{\min}) \qquad [13\text{-}24]$$

The inductor delta current, ΔI, is shown in Equation [13-25].

$$\Delta I = \frac{\left(T V_{in(\max)} D_{(\min)} \right)}{L} \qquad [13\text{-}25]$$

The inductor peak current, $I_{(pk)}$, is shown in Equation [13-26].

$$I_{(pk)} = \left(\frac{I_{o(\max)}}{1 - D_{(\max)}} \right) + \left(\frac{\Delta I}{2} \right) \qquad [13\text{-}26]$$

The Inverting Buck-Boost Converter

The inverting buck boost converter is shown in Figure 13-9. It is a variation of the boost circuit. The inverting converter delivers only the energy stored by the inductor, L1, to the load. The output voltage of the inverting converter can be greater, or less than, the input voltage. When the transistor switch, Q1, is closed, the inductor is storing energy, but no current is delivered to the load because diode, CR1, is back-biased. When the transistor switch, Q1, is open, the blocking diode is forward-biased and the energy stored in the inductor, L1, is transferred to the load. The discontinuous voltage and current waveforms are shown in Figure 13-10 and the continuous waveforms in Figure 13-11.

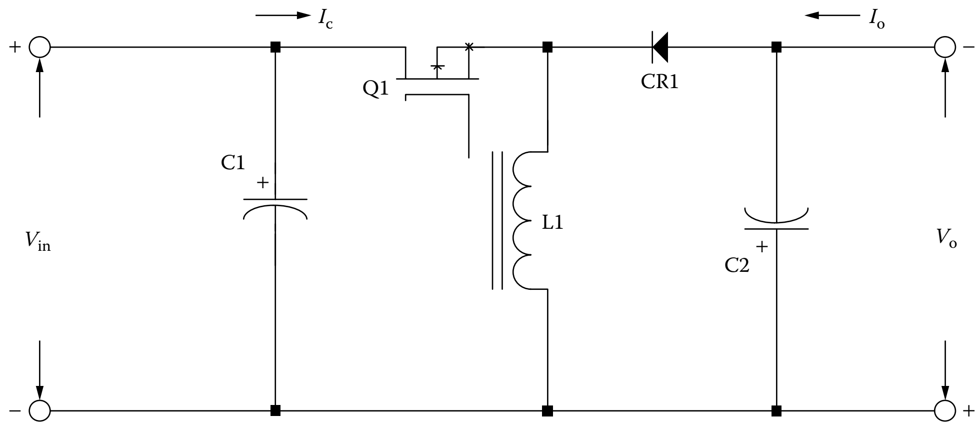

Figure 13-9. Schematic of an Inverting Buck-Boost Switching Converter.

Discontinuous Current, Inverting Buck-Boost Design Equations

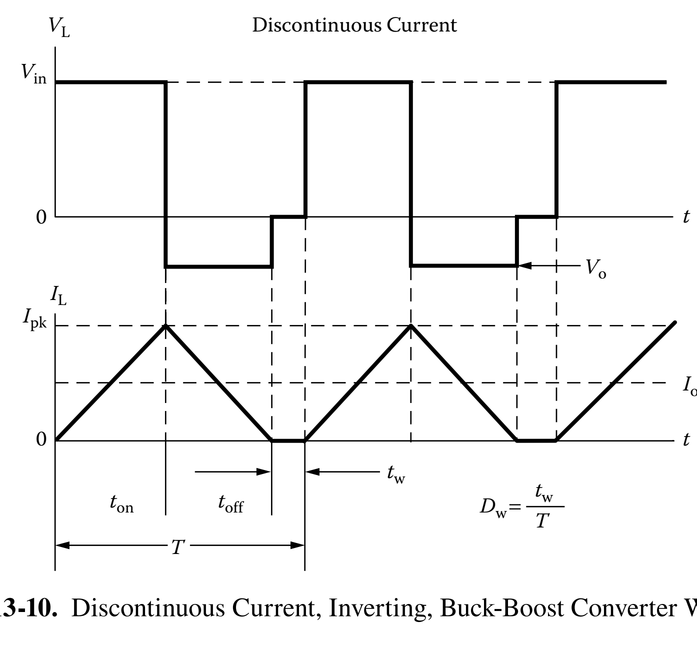

Figure 13-10. Discontinuous Current, Inverting, Buck-Boost Converter Waveforms.

The maximum inductance, L_{max}, is shown in Equation [13-27].

$$L_{max} = \frac{(V_o + V_d)T(1 - D_{max} - D_w)^2}{2I_{o(max)}}, \quad \text{[henrys]} \qquad [13\text{-}27]$$

Maximum duty ratio, D_{max}, is shown in Equation [13-28].

$$D_{\max} = \frac{\left(V_o + V_d\right)\left(1 - D_w\right)}{\left(V_o + V_d + V_{in(\min)}\right)} \qquad [13\text{-}28]$$

Minimum duty ratio, D_{min}, is shown in Equation [13-29].

$$D_{\min} = \frac{\left(V_o + V_d\right)\left(1 - D_w\right)}{V_o + V_d + V_{in(\max)}} \qquad [13\text{-}29]$$

Maximum on time, $t_{on(max)}$, is shown in Equation [13-30].

$$t_{on(\max)} = T\,D_{\max} \qquad [13\text{-}30]$$

Maximum off time, $t_{off(max)}$, is shown in Equation [13-31].

$$t_{off(\max)} = T\left(1 - D_{\min} - D_w\right) \qquad [13\text{-}31]$$

The inductor peak current, $I_{(pk)}$, is shown in Equation [13-32].

$$I_{(pk)} = \frac{2P_{o(\max)}}{\left(D_{(\max)} V_{in(\min)} \eta\right)} \qquad [13\text{-}32]$$

Continuous Current, Inverting Buck-Boost Design Equations

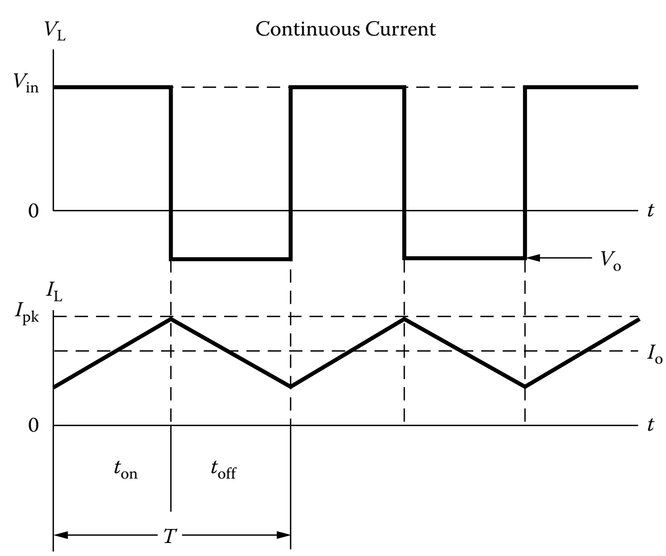

Figure 13-11. Continuous Current, Inverting, Buck-Boost Converter Waveforms.

Inductance, L, is shown in Equation [13-33].

$$L = \frac{(V_o + V_d)T(1 - D_{\min})^2}{2I_{o(\min)}}, \quad \text{[henrys]} \tag{13-33}$$

Maximum duty ratio, D_{max}, is shown in Equation [13-34].

$$D_{\max} = \frac{V_o}{V_o + \left(\eta V_{in(\min)}\right)} \tag{13-34}$$

Minimum duty ratio, D_{min}, is shown in Equation [13-35].

$$D_{\min} = \frac{V_o}{V_o + \left(\eta V_{in(\max)}\right)} \tag{13-35}$$

Maximum on time, $t_{on(max)}$, is shown in Equation [13-36].

$$t_{on(\max)} = T D_{\max} \tag{13-36}$$

Maximum off time, $t_{off(max)}$, is shown in Equation [13-37].

$$t_{off(\max)} = T(1 - D_{\min}) \tag{13-37}$$

The inductor delta current, ΔI, is shown in Equation [13-38].

$$\Delta I = \frac{\left(T V_{in(\max)} D_{(\min)}\right)}{L} \tag{13-38}$$

The inductor peak current, $I_{(pk)}$, is shown in Equation [13-39].

$$I_{(pk)} = \left(\frac{I_{o(\max)}}{1 - D_{\max}}\right) + \left(\frac{\Delta I}{2}\right) \tag{13-39}$$

The Isolated, Buck-Boost Converter

The Isolated Buck-Boost Converter is shown in Figure 13-12. This converter can provide line isolation, and also has the capability of multiple outputs, which require only a diode and a capacitor; the filter inductor is built-in. The isolated buck-boost converter is quite popular in low power applications because of simplicity and low cost. This converter does not lend itself to the VDE specification because of the required voltage insulation between primary and secondary. Care must be taken because this leakage inductance could generate high voltage spikes on the primary. The discontinuous voltage and current waveforms are shown in Figure 13-13, and the continuous waveforms in Figure 13-14.

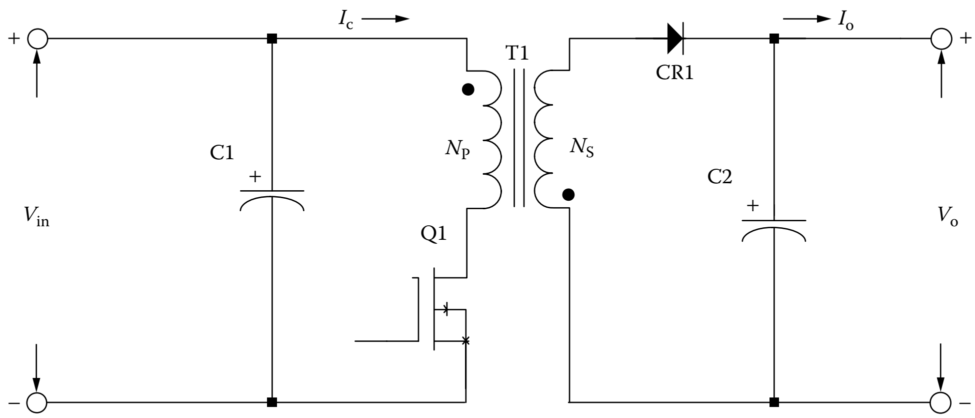

Figure 13-12. Schematic of an Isolated Buck-Boost Switching Converter.

Discontinuous Current, Isolated Buck-Boost Design Equations

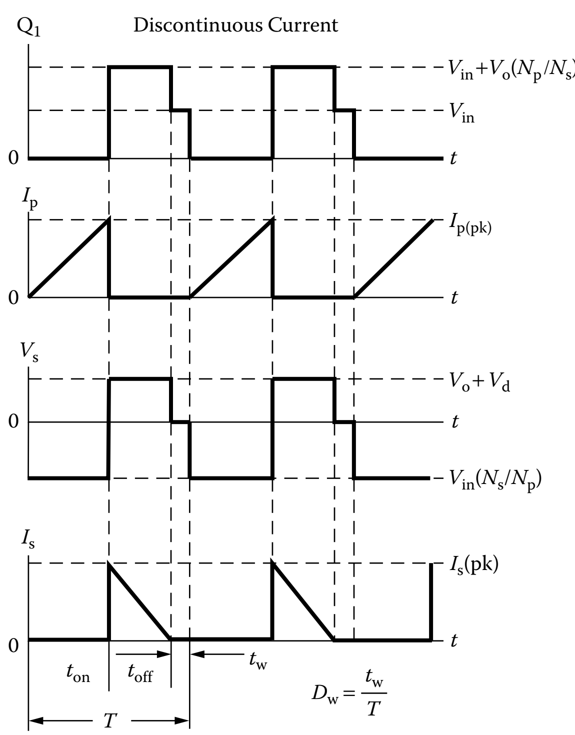

Figure 13-13. Discontinuous Current, Isolated Buck-Boost Converter Waveforms.

Primary maximum inductance, $L_{p(max)}$, is shown in Equation [13-40].

$$L_{p(\max)} = \frac{\left(R_{in(equiv.)}\right) T \left(D_{\max}\right)^2}{2}, \quad \text{[henrys]} \qquad [13\text{-}40]$$

Maximum on time, $t_{on(max)}$, is shown in Equation [13-41].

$$t_{on(\max)} = T D_{\max} \qquad [13\text{-}41]$$

Maximum off time, $t_{off(max)}$, is shown in Equation [13-42].

$$t_{off(\max)} = T\left(1 - D_{\min} - D_w\right) \qquad [13\text{-}42]$$

Total output power, $P_{o(max)}$ is shown in Equation [13-43].

$$P_{o(\max)} = I_{o1(\max)}\left(V_{o1} + V_d\right) + I_{o2(\max)}\left(V_{o2} + V_d\right) + \ldots\ldots\ldots\ldots \qquad [13\text{-}43]$$

Maximum input power, $P_{in(max)}$ is shown in Equation [13-44].

$$P_{in(\max)} = \frac{P_{o(\max)}}{\eta} \qquad [13\text{-}44]$$

Equivalent input resistance, $R_{in(equiv)}$, is shown in Equation [13-45].

$$R_{in(equv.)} = \frac{\left(V_{in(\min)}\right)^2}{P_{in(\max)}} \qquad [13\text{-}45]$$

The primary peak current, $I_{p(pk)}$ is shown in Equation [13-46].

$$I_{p(pk)} = \frac{2 P_{in(\max)} T}{T_{on(\max)} V_{in(\min)}} \qquad [13\text{-}46]$$

Continuous Current, Isolated Buck-Boost Design Equations

Inductance, L, is shown in Equation [13-47].

$$L = \frac{\left(V_{in(\max)} D_{(\min)}\right)^2 T}{2 P_{in(\min)}}, \quad \text{[henrys]} \qquad [13\text{-}47]$$

Minimum duty ratio, D_{min}, is shown in Equation [13-48].

$$D_{\min} = \left(\frac{V_{in(\min)}}{V_{in(\max)}}\right) D_{(\max)} \qquad [13\text{-}48]$$

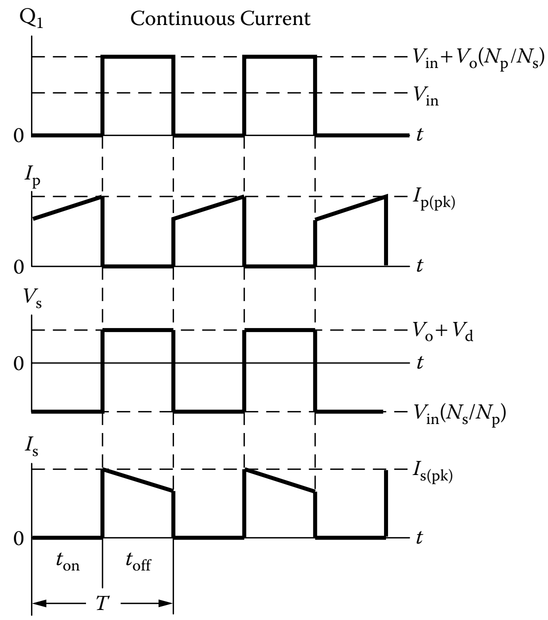

Figure 13-14. Continuous Current, Isolated Buck-Boost Converter Waveforms.

Maximum on time, $t_{on(max)}$, is shown in Equation [13-49].

$$t_{on(\max)} = T D_{\max} \qquad [13\text{-}49]$$

Maximum off time, $t_{off(max)}$, is shown in Equation [13-50].

$$t_{off(\max)} = T\left(1 - D_{\min}\right) \qquad [13\text{-}50]$$

Minimum output power, $P_{o(min)}$, is shown in Equation [13-51].

$$P_{o(\min)} = I_{o1(\min)}\left(V_{o1} + V_d\right) + I_{o2(\min)}\left(V_{o2} + V_d\right) + ... \qquad [13\text{-}51]$$

Minimum input power, $P_{in(min)}$, is shown in Equation [13-52].

$$P_{in(\min)} = \frac{P_{o(\min)}}{\eta} \qquad [13\text{-}52]$$

The inductor delta current, ΔI, is shown in Equation [13-53].

$$\Delta I = \frac{\left(T V_{in(\min)} D_{(\max)}\right)}{L} \qquad [13\text{-}53]$$

The inductor peak current, $I_{(pk)}$, is shown in Equation [13-54].

$$I_{(pk)} = \left(\frac{I_{in(\max)}}{D_{\max}}\right) + \left(\frac{\Delta I}{2}\right) \qquad [13\text{-}54]$$

Design Example, Buck-Boost Isolated Converter Discontinuous Current

1. Input voltage nominal, V_{in} = 28 volts
2. Input voltage minimum, $V_{in(min)}$ = 24 volts
3. Input voltage maximum, $V_{in(max)}$ = 32 volts
4. Output voltage, V_{o1} = 5 volts
5. Output current, I_{o1} = 2 amps
6. Output voltage, V_{o2} = 12 volts
7. Output current, I_{o2} = 0.5 amps
8. *Window utilization, K_u = 0.29
9. Frequency, f = 100kHz
10. Converter efficiency, η = 90%
11. Maximum duty ratio, $D_{(max)}$ = 0.5
12. Dwell time duty ratio, $D_{(w)}$ = 0.1
13. Regulation, α = 1.0%
14. Operating flux density, B_m = 0.25 tesla
15. Diode voltage, V_d = 1.0 volt

*When operating at high frequencies, the engineer has to review the window utilization factor, K_u. When using a small bobbin ferrite, the ratio of the bobbin winding area to the core window area is only about 0.6. Operating at 100kHz and having to use a #26 wire, because of the skin effect, the ratio of the bare copper area is 0.78. Therefore, the overall window utilization, K_u, is reduced. The core geometries, K_g, in Chapter 3 have been calculated with a window utilization, K_u, of 0.4. To return the design back to the norm, the core geometry, K_g is to be multiplied by 1.35, and then, the current density, J, is calculated, using a window utilization factor of 0.29. See Chapter 4.

Skin Effect

The skin effect on an inductor is the same as a transformer. In the normal dc inductor, the ac current (ac flux), is much lower, and does not require the use of the same, maximum wire size. This is not the case in the discontinuous, current type, flyback converter, where all of the flux is ac and without dc. In the discontinuous, flyback design, the skin effect has to be treated just like a high frequency transformer.

There are times when the larger wire is just too difficult to wind. Large wire is not only hard to handle, but it does not give the proper lay. It is easier to wind with bi-filar or quad-filar wire, with the equivalent cross-section.

Select a wire so that the relationship between the ac resistance and the dc resistance is 1 is shown in Equation [13-55].

$$\frac{R_{ac}}{R_{dc}} = 1 \qquad [13\text{-}55]$$

The skin depth in centimeters is:

$$\varepsilon = \frac{6.62}{\sqrt{f}}, \quad [\text{cm}]$$

$$\varepsilon = \frac{6.62}{\sqrt{100,000}}, \quad [\text{cm}]$$

$$\varepsilon = 0.0209, \quad [\text{cm}]$$

Then, the wire diameter is:

$$\text{Wire Diameter} = 2(\varepsilon), \quad [\text{cm}]$$

$$\text{Wire Diameter} = 2(0.0209), \quad [\text{cm}]$$

$$\text{Wire Diameter} = 0.0418, \quad [\text{cm}]$$

Then, the bare wire area A_w is:

$$A_w = \frac{\pi D^2}{4}, \quad [\text{cm}^2]$$

$$A_w = \frac{(3.1416)(0.0418)^2}{4}, \quad [\text{cm}^2]$$

$$A_w = 0.00137, \quad [\text{cm}^2]$$

From the Wire Table in Chapter 4, Number 26 has a bare wire area of 0.00128 centimeters. This will be the minimum wire size used in this design. If the design requires more wire area to meet the specification, then the design will use a multifilar of #26. Listed below are #27 and #28, just in case #26 requires too much rounding off.

Wire AWG	Bare Area	Area Ins.	Bare/Ins.	μΩ/cm
#26	0.001280	0.001603	0.798	1345
#27	0.001021	0.001313	0.778	1687
#28	0.0008046	0.0010515	0.765	2142

Step No. 1: Calculate the total period, T.

$$T = \frac{1}{f} \quad [\text{seconds}]$$

$$T = \frac{1}{100000} \quad [\text{seconds}]$$

$$T = 10 \quad [\mu\text{sec}]$$

Step No. 2: Calculate the maximum transistor on time, t_{on}.

$$t_{on} = TD_{\max} \quad [\mu\text{sec}]$$

$$t_{on} = \left(10\times10^{-6}\right)\left(0.5\right) \quad [\mu\text{sec}]$$

$$t_{on} = 5.0 \quad [\mu\text{sec}]$$

Step No. 3: Calculate the secondary load power, P_{o1}.

$$P_{o1} = I_{o1}\left(V_{o1} + V_d\right) \quad [\text{watts}]$$

$$P_{o1} = (2)(5+1) \quad [\text{watts}]$$

$$P_{o1} = 12 \quad [\text{watts}]$$

Step No. 4: Calculate the secondary load power, P_{o2}.

$$P_{o2} = I_{o2}\left(V_{o2} + V_d\right) \quad [\text{watts}]$$

$$P_{o2} = (0.5)(12+1) \quad [\text{watts}]$$

$$P_{o2} = 6.5 \quad [\text{watts}]$$

Step No. 5: Calculate the total secondary load power, $P_{o(max)}$.

$$P_{o(\max)} = P_{o1} + P_{o2} \quad [\text{watts}]$$

$$P_{o(\max)} = 12 + 6.5 \quad [\text{watts}]$$

$$P_{o(\max)} = 18.5 \quad [\text{watts}]$$

Step No. 6: Calculate the maximum input current, $I_{in(max)}$.

$$I_{in(\max)} = \frac{P_{o(\max)}}{V_{in(\min)}\eta} \quad [\text{amps}]$$

$$I_{in(\max)} = \frac{18.5}{(24)(0.9)} \quad [\text{amps}]$$

$$I_{in(\max)} = 0.856 \quad [\text{amps}]$$

Step No. 7: Calculate the primary peak current, $I_{p(pk)}$.

$$I_{p(pk)} = \frac{2P_{o(\max)}T}{\eta V_{in(\min)}t_{on(\max)}} \quad [\text{amps peak}]$$

$$I_{p(pk)} = \frac{2\left(18.5\right)\left(10\times10^{-6}\right)}{\left(0.9\right)\left(24\right)\left(5\times10^{-6}\right)} \quad [\text{amps peak}]$$

$$I_{p(pk)} = 3.43 \quad [\text{amps peak}]$$

Step No. 8: Calculate the primary rms current, $I_{p(pk)}$.

$$I_{p(rms)} = I_{p(pk)}\sqrt{\frac{t_{on}}{3T}} \quad \text{[amps]}$$

$$I_{p(rms)} = 3.43\sqrt{\frac{5}{3(10)}} \quad \text{[amps]}$$

$$I_{p(rms)} = 1.40 \quad \text{[amps]}$$

Step No. 9: Calculate the maximum input power, $P_{in(max)}$.

$$P_{in(\max)} = \frac{P_{o(\max)}}{\eta} \quad \text{[watts]}$$

$$P_{in(\max)} = \frac{18.5}{0.9} \quad \text{[watts]}$$

$$P_{in(\max)} = 20.6 \quad \text{[watts]}$$

Step No. 10: Calculate the equivalent input resistance, $R_{in(equiv)}$.

$$R_{in(equiv)} = \frac{\left(V_{in(\min)}\right)^2}{P_{in(\max)}}, \quad \text{[ohms]}$$

$$R_{in(equiv)} = \frac{(24)^2}{20.6}, \quad \text{[ohms]}$$

$$R_{in(equiv)} = 28, \quad \text{[ohms]}$$

Step No. 11: Calculate the required primary inductance, L.

$$L = \frac{\left(R_{in(equiv)}\right)T\left(D_{\max}\right)^2}{2} \quad \text{[henry]}$$

$$L = \frac{(28)\left(10\times10^{-6}\right)(0.5)^2}{2} \quad \text{[henry]}$$

$$L = 35 \quad [\mu\text{h}]$$

Step No. 12: Calculate the energy-handling capability in watt-seconds, w-s.

$$\text{Energy} = \frac{LI_{p(pk)}^2}{2} \quad \text{[w-s]}$$

$$\text{Energy} = \frac{\left(35\times10^{-6}\right)(3.43)^2}{2} \quad \text{[w-s]}$$

$$\text{Energy} = 0.000206 \quad \text{[w-s]}$$

Step No. 13: Calculate the electrical conditions, K_e.

$$K_e = 0.145 P_o B_m^2 \times 10^{-4}$$

$$K_e = (0.145)(18.5)(0.25)^2 \times 10^{-4}$$

$$K_e = 0.0000168$$

Step No. 14: Calculate the core geometry, K_g. See the design specification for, window utilization factor, K_u.

$$K_g = \frac{(\text{Energy})^2}{K_e \alpha} \quad [\text{cm}^5]$$

$$K_g = \frac{(0.000206)^2}{\left(16.8\left(10^{-6}\right)\right)(1.0)} \quad [\text{cm}^5]$$

$$K_g = 0.00253 \quad [\text{cm}^5]$$

$$K_g = 0.00253(1.35), \quad [\text{cm}^5]$$

$$K_g = 0.00342, \quad [\text{cm}^5]$$

Step No. 15: Select, from Chapter 3, an EFD core comparable in core geometry, K_g.

Core number = EFD-20

Manufacturer = Ferroxcube

Material = 3C85

Magnetic path length, MPL = 4.7 cm

Core weight, W_{tfe} = 7.0 grams

Copper weight, W_{tcu} = 6.8 grams

Mean length turn, MLT = 3.80 cm

Iron area, A_c = 0.31 cm^2

Window Area, W_a = 0.501 cm^2

Area Product, A_p = 0.155 cm^4

Core geometry, K_g = 0.00507 cm^5

Surface area, A_t = 13.3 cm^2

Core Permeability = 2500

Winding Length, G = 1.54 cm

Step No. 16: Calculate the current density, J, using a window utilization, K_u = 0.29.

$$J = \frac{2(\text{Energy})\left(10^4\right)}{B_m A_p K_u}, \quad [\text{amps/cm}^2]$$

$$J = \frac{2(0.000206)\left(10^4\right)}{(0.25)(0.155)(0.29)}, \quad [\text{amps/cm}^2]$$

$$J = 367, \quad [\text{amps/cm}^2]$$

Step No. 17: Calculate the primary wire area, $A_{pw(B)}$.

$$A_{pw(B)} = \frac{I_{prms}}{J} \quad [\text{cm}^2]$$

$$A_{pw(B)} = \frac{1.4}{367} \quad [\text{cm}^2]$$

$$A_{pw(B)} = 0.00381 \quad [\text{cm}^2]$$

Step No. 18: Calculate the required number of primary strands, S_{np}.

$$S_{np} = \frac{A_{wp(B)}}{\#26\ (\text{bare area})}$$

$$S_{np} = \frac{(0.00381)}{(0.00128)}$$

$$S_{np} = 2.97 \ \text{ use } 3$$

Step No. 19: Calculate the number of primary turns, N_p. Half of the available window is primary, $W_{ap}/2$. Use the number of strands, S_{np}, and the area for #26.

$$W_{ap} = \frac{W_a}{2} = \frac{0.501}{2} = 0.250, \quad [\text{cm}^2]$$

$$N_p = \frac{K_u W_{ap}}{3(\#26(\text{Bare Area}))}, \quad [\text{turns}]$$

$$N_p = \frac{(0.29)(0.25)}{3(0.00128)}, \quad [\text{turns}]$$

$$N_p = 18.9 \text{ use } 19, \quad [\text{turns}]$$

Step No. 20: Calculate the required gap, l_g.

$$l_g = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{L} - \left(\frac{\text{MPL}}{\mu_m}\right), \quad [\text{cm}]$$

$$l_g = \frac{(1.26)(19)^2(0.31)\left(10^{-8}\right)}{(0.000035)} - \left(\frac{4.7}{2500}\right), \quad [\text{cm}]$$

$$l_g = 0.0384, \quad [\text{cm}]$$

Step No. 21: Calculate the equivalent gap in mils.

$$\text{mils} = \text{cm}(393.7)$$

$$\text{mils} = (0.0384)(393.7)$$

$$\text{mils} = 15$$

Step No. 22: Calculate the fringing flux factor, F.

$$F = 1 + \frac{l_g}{\sqrt{A_c}} \ln\left(\frac{2G}{l_g}\right)$$

$$F = 1 + \frac{(0.0384)}{\sqrt{0.31}} \ln\left(\frac{2(1.54)}{0.0384}\right)$$

$$F = 1.30$$

Step No. 23: Calculate the new number of turns, N_{np}, by inserting the fringing flux, F.

$$N_{np} = \sqrt{\frac{l_g L}{0.4\pi A_c F(10^{-8})}}, \quad \text{[turns]}$$

$$N_{np} = \sqrt{\frac{(0.0384)(0.000035)}{(1.26)(0.31)(1.3)(10^{-8})}}, \quad \text{[turns]}$$

$$N_{np} = 16, \quad \text{[turns]}$$

Step No. 24: Calculate the peak flux density, B_{pk}.

$$B_{pk} = \frac{0.4\pi N_{np} F\left(I_{p(pk)}\right)(10^{-4})}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]}$$

$$B_{pk} = \frac{(1.26)(16)(1.3)(3.43)(10^{-4})}{(0.0384) + \left(\frac{4.7}{2500}\right)}, \quad \text{[teslas]}$$

$$B_{pk} = 0.223, \quad \text{[teslas]}$$

Step No. 25: Calculate the primary, the new μΩ/cm.

$$(\text{new})\mu\Omega / cm = \frac{\mu\Omega / cm}{S_{np}}$$

$$(\text{new})\mu\Omega / cm = \frac{1345}{3}$$

$$(\text{new})\mu\Omega / cm = 448$$

Step No. 26: Calculate the primary winding resistance, R_p.

$$R_p = MLT\left(N_{np}\right)\left(\frac{\mu\Omega}{cm}\right)\times 10^{-6} \quad \text{[ohms]}$$

$$R_p = \left(3.8\right)\left(16\right)\left(448\right)\times 10^{-6} \quad \text{[ohms]}$$

$$R_p = 0.0272 \quad \text{[ohms]}$$

Step No. 27: Calculate the primary copper loss, P_p.

$$P_p = I_p^2 R_p \quad \text{[watts]}$$

$$P_p = (1.4)^2\,(.0272) \quad \text{[watts]}$$

$$P_p = 0.0533 \text{ [watts]}$$

Step No. 28: Calculate the secondary turns, N_{s01}.

$$N_{s01} = \frac{N_{np}\left(V_{01}+V_d\right)\left(1-D_{\max}-D_w\right)}{\left(V_p D_{\max}\right)} \quad \text{[turns]}$$

$$N_{s01} = \frac{16(5+1)(1-0.5-0.1)}{(24)(0.5)} \quad \text{[turns]}$$

$$N_{s01} = 3.2 \text{ use } 3 \quad \text{[turns]}$$

Step No. 29: Calculate the secondary peak current, $I_{s01(pk)}$.

$$I_{s01(pk)} = \frac{2I_{01}}{\left(1-D_{\max}-D_w\right)} \quad \text{[amps]}$$

$$I_{s01(pk)} = \frac{2(2.0)}{(1-0.5-0.1)} \quad \text{[amps]}$$

$$I_{s01(pk)} = 10 \quad \text{[amps]}$$

Step No. 30: Calculate the secondary rms current, $I_{s0(rms)}$.

$$I_{s01(rms)} = I_{s01(pk)}\sqrt{\frac{\left(1-D_{\max}-D_w\right)}{3}} \quad \text{[amps]}$$

$$I_{s01(rms)} = (10)\sqrt{\frac{(1-0.5-0.1)}{3}} \quad \text{[amps]}$$

$$I_{s01(rms)} = 3.65 \quad \text{[amps]}$$

Step No. 31: Calculate the secondary wire area, $A_{sw01(B)}$.

$$A_{sw01(B)} = \frac{I_{s01(rms)}}{J} \quad [\text{cm}^2]$$

$$A_{sw01(B)} = \frac{3.65}{367} \quad [\text{cm}^2]$$

$$A_{sw01(B)} = 0.00995 \quad [\text{cm}^2]$$

Step No. 32: Calculate the required number of secondary strands, S_{ns01}.

$$S_{ns01} = \frac{A_{sw01(B)}}{\text{wire}_A}$$

$$S_{ns01} = \frac{(0.00995)}{(0.00128)}$$

$$S_{ns01} = 7.8 \ \text{ use } 8$$

Step No. 33: Calculate the, S_{01}, secondary, μΩ/cm.

$$(S_{01})\mu\Omega / cm = \frac{\mu\Omega / cm}{S_{ns01}}$$

$$(S_{01})\mu\Omega / cm = \frac{1345}{8}$$

$$(S_{01})\mu\Omega / cm = 168$$

Step No. 34: Calculate the winding resistance, R_{s01}.

$$R_{s01} = MLT\left(N_{s01}\right)\left(\frac{\mu\Omega}{cm}\right) \times 10^{-6} \quad [\text{ohms}]$$

$$R_{s01} = 3.8(3)(168) \times 10^{-6} \quad [\text{ohms}]$$

$$R_{s01} = 0.00192 \quad [\text{ohms}]$$

Step No. 35: Calculate the secondary copper loss, P_{s01}.

$$P_{s01} = I_{s01}^2 R_{s01} \quad [\text{watts}]$$

$$P_{s01} = (3.65)^2 (.00192) \quad [\text{watts}]$$

$$P_{s01} = 0.0256 \ [\text{watts}]$$

Step No. 36: Calculate the secondary turns, N_{s02}.

$$N_{s02} = \frac{N_{np}(V_{02}+V_d)(1-D_{max}-D_w)}{(V_p D_{max})} \quad \text{[turns]}$$

$$N_{s02} = \frac{16(12+1)(1-0.5-0.1)}{(24)(0.5)} \quad \text{[turns]}$$

$$N_{s02} = 6.9 \text{ use } 7 \quad \text{[turns]}$$

Step No. 37: Calculate the secondary peak current, $I_{s02(pk)}$.

$$I_{s02(pk)} = \frac{2I_{02}}{(1-D_{max}-D_w)} \quad \text{[amps]}$$

$$I_{s02(pk)} = \frac{2(0.5)}{(1-0.5-0.1)} \quad \text{[amps]}$$

$$I_{s02(pk)} = 2.5 \quad \text{[amps]}$$

Step No. 38: Calculate the secondary rms current, $I_{s02(rms)}$.

$$I_{s02(rms)} = I_{s02(pk)}\sqrt{\frac{(1-D_{max}-D_w)}{3}} \quad \text{[amps]}$$

$$I_{s02(rms)} = (2.5)\sqrt{\frac{(1-0.5-0.1)}{3}} \quad \text{[amps]}$$

$$I_{s02(rms)} = 0.913 \quad \text{[amps]}$$

Step No. 39: Calculate the secondary wire area, $A_{sw02(B)}$.

$$A_{sw02(B)} = \frac{I_{s02(rms)}}{J} \quad \text{[cm}^2\text{]}$$

$$A_{sw02(B)} = \frac{0.913}{367} \quad \text{[cm}^2\text{]}$$

$$A_{sw02(B)} = 0.00249 \quad \text{[cm}^2\text{]}$$

Step No. 40: Calculate the required number of secondary strands, S_{ns02}.

$$S_{ns02} = \frac{A_{sw02(B)}}{wire_A}$$

$$S_{ns02} = \frac{(0.00249)}{(0.00128)}$$

$$S_{ns02} = 1.95 \text{ use } 2$$

Step No. 41: Calculate the, S_{02}, secondary, μΩ/cm.

$$(S_{02})\mu\Omega / cm = \frac{\mu\Omega / cm}{S_{ns02}}$$

$$(S_{02})\mu\Omega / cm = \frac{1345}{2}$$

$$(S_{02})\mu\Omega / cm = 672$$

Step No. 42: Calculate the winding resistance, R_{s02}.

$$R_{s02} = MLT\left(N_{s02}\right)\left(\frac{\mu\Omega}{cm}\right)\times 10^{-6} \quad \text{[ohms]}$$

$$R_{s02} = 3.8(7)(672)\times 10^{-6} \quad \text{[ohms]}$$

$$R_{s02} = 0.0179 \quad \text{[ohms]}$$

Step No. 43: Calculate the secondary copper loss, P_{s02}.

$$P_{s02} = I_{s02}^2 R_{s02} \quad \text{[watts]}$$

$$P_{s2} = (0.913)^2(.0179) \quad \text{[watts]}$$

$$P_{s2} = 0.0149 \text{ [watts]}$$

Step No. 44: Calculate the window utilization, K_u.

$$\text{[turns]} = \left(N_p S_{np}\right) \quad \text{[primary]}$$

$$\text{[turns]} = (16)(3) = 48 \quad \text{[primary]}$$

$$\text{[turns]} = \left(N_{s01} S_{ns01}\right) \quad \text{[secondary]}$$

$$\text{[turns]} = (3)(8) = 24 \quad \text{[secondary]}$$

$$\text{[turns]} = \left(N_{s02} S_{ns02}\right) \quad \text{[secondary]}$$

$$\text{[turns]} = (7)(2) = 14 \quad \text{[secondary]}$$

$$N_t = 86 \text{ turns \#26}$$

$$K_u = \frac{N_t A_w}{W_a} = \frac{(86)(0.00128)}{(0.501)}$$

$$K_u = 0.220$$

Step No. 45: Calculate the total copper loss, P_{cu}.

$$P_{cu} = P_p + P_{s01} + \text{P}_{s02} \quad \text{[watts]}$$

$$P_{cu} = (0.0533) + (0.0256) + (0.0149) \quad \text{[watts]}$$

$$P_{cu} = 0.0938 \quad \text{[watts]}$$

Step No. 46: Calculate the regulation, α, for this design.

$$\alpha = \frac{P_{cu}}{P_o} \times 100 \quad [\%]$$

$$\alpha = \frac{(0.0938)}{(18.5)} \times 100 \quad [\%]$$

$$\alpha = 0.507 \quad [\%]$$

Step No. 47: Calculate the ac flux density, B_{ac}.

$$B_{ac} = \frac{0.4\pi N_{np} F\left(\frac{I_{p(pk)}}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad [\text{teslas}]$$

$$B_{ac} = \frac{(1.26)(16)(1.3)(1.72)\left(10^{-4}\right)}{(0.0384) + \left(\frac{4.7}{2500}\right)}, \quad [\text{teslas}]$$

$$B_{ac} = 0.111, \quad [\text{teslas}]$$

Step No. 48: Calculate the watts per kilogram, WK.

$$WK = 4.855\left(10^{-5}\right)(f)^{(1.63)}(B_{ac})^{(2.62)} \quad [\text{watts/kilogram}]$$

$$WK = 4.855\left(10^{-5}\right)(100000)^{(1.63)}(0.111)^{(2.62)} \quad [\text{watts/kilogram}]$$

$$WK = 21.6 \quad [\text{watts/kilogram}] \text{ or } [\text{milliwatts/gram}]$$

Step No. 49: Calculate the core loss, P_{fe}.

$$P_{fe} = \left(\frac{\text{milliwatts}}{\text{gram}}\right) W_{tfe} \times 10^{-3} \quad [\text{watts}]$$

$$P_{fe} = (21.6)(7) \times 10^{-3} \quad [\text{watts}]$$

$$P_{fe} = 0.151 \quad [\text{watts}]$$

Step No. 50: Calculate the total loss, core P_{fe} and copper P_{cu}, in watts, P_Σ.

$$P_\Sigma = P_{fe} + P_{cu} \quad [\text{watts}]$$

$$P_\Sigma = (0.151) + (0.0938) \quad [\text{watts}]$$

$$P_\Sigma = 0.245 \quad [\text{watts}]$$

Step No. 51: Calculate the watt density, ψ

$$\psi = \frac{P_\Sigma}{A_t} \quad [\text{watts/cm}^2]$$

$$\psi = \frac{0.245}{13.3} \quad [\text{watts/cm}^2]$$

$$\psi = 0.0184 \quad [\text{watts/cm}^2]$$

Step No. 52: Calculate the temperature rise, T_r, in, °C.

$$T_r = 450(\psi)^{(0.826)} \quad [°\text{C}]$$

$$T_r = 450(0.0184)^{(0.826)} \quad [°\text{C}]$$

$$T_r = 16.6 \quad [°\text{C}]$$

Design Example, Boost Converter, Discontinuous Current

1. Input voltage nominal, V_{in} = 28 volts
2. Input voltage minimum, $V_{in(min)}$ = 26 volts
3. Input voltage maximum, $V_{in(max)}$ = 32 volts
4. Output voltage, V_{o1} = 50 volts
5. Output current, I_{o1} = 1 amps
6. *Window utilization, K_u = 0.29
7. Frequency, f = 100kHz
8. Converter efficiency, η = 92 %
9. Dwell time duty ratio, $D_{(w)}$ = 0.1
10. Regulation, α = 1.0%
11. Operating flux density, B_m = 0.25 tesla
12. Diode voltage, V_d = 1.0 volts

*When operating at high frequencies, the engineer has to review the window utilization factor, K_u. When using a small bobbin ferrite, the ratio of the bobbin winding area to the core window area is only about 0.6. Operating at 100kHz and having to use a #26 wire, because of the skin effect, the ratio of the bare copper area is 0.78. Therefore, the overall window utilization, K_u, is reduced. The core geometries, K_g, in Chapter 3 have been calculated with a window utilization, K_u, of 0.4. To return the design back to the norm, the core geometry, K_g is to be multiplied by 1.35, and then, the current density, J, is calculated, using a window utilization factor of 0.29. See Chapter 4.

Skin Effect

The skin effect on an inductor is the same as a transformer. In the normal dc inductor, the ac current, (ac flux), is much lower and does not require the use of the same maximum wire size. This is not the case in the discontinuous, current type, flyback converter, where all of the flux is ac and without dc. In the discontinuous, flyback design, the skin effect has to be treated just like a high frequency transformer.

There are times when the larger wire is just too difficult to wind. Large wire is not only hard to handle, but it does not give the proper lay. It is easier to wind with bi-filar or quad-filar, wire with the equivalent cross-section.

At this point, select a wire so that the relationship between the ac resistance and the dc resistance is 1, as is shown in Equation [13-56].

$$\frac{R_{ac}}{R_{dc}} = 1 \qquad [13\text{-}56]$$

The skin depth in centimeters is:

$$\varepsilon = \frac{6.62}{\sqrt{f}}, \quad [\text{cm}]$$

$$\varepsilon = \frac{6.62}{\sqrt{100,000}}, \quad [\text{cm}]$$

$$\varepsilon = 0.0209, \quad [\text{cm}]$$

Then, the wire diameter is:

$$\text{Wire Diameter} = 2(\varepsilon), \quad [\text{cm}]$$

$$\text{Wire Diameter} = 2(0.0209), \quad [\text{cm}]$$

$$\text{Wire Diameter} = 0.0418, \quad [\text{cm}]$$

Then, the bare wire area A_w is:

$$A_w = \frac{\pi D^2}{4}, \quad [\text{cm}^2]$$

$$A_w = \frac{(3.1416)(0.0418)^2}{4}, \quad [\text{cm}^2]$$

$$A_w = 0.00137, \quad [\text{cm}^2]$$

From the Wire Table in Chapter 4, Number 26 has a bare wire area of 0.001028 centimeters. This will be the minimum wire size used in this design. If the design requires more wire area to meet the specification, then the design will use a multifilar of #26. Listed Below are #27 and #28, just in case #26 requires too much rounding off.

Wire AWG	Bare Area	Area Ins.	Bare/Ins.	μΩ/cm
#26	0.001280	0.001603	0.798	1345
#27	0.001021	0.001313	0.778	1687
#28	0.0008046	0.0010515	0.765	2142

Step No. 1: Calculate the total period, T.

$$T = \frac{1}{f}, \quad \text{[seconds]}$$

$$T = \frac{1}{100{,}000}, \quad \text{[seconds]}$$

$$T = 10, \quad [\mu\text{sec}]$$

Step No. 2: Calculate the maximum output power, P_o.

$$P_o = (V_o + V_d)(I_o), \quad \text{[watts]}$$

$$P_o = (50 + 1.0)(1.0), \quad \text{[watts]}$$

$$P_o = 51, \quad \text{[watts]}$$

Step No. 3: Calculate the maximum input current, $I_{in(max)}$.

$$I_{in(\max)} = \frac{P_o}{V_{in(\min)}\eta}, \quad \text{[amps]}$$

$$I_{in(\max)} = \frac{(51)}{(26)(0.92)}, \quad \text{[amps]}$$

$$I_{in(\max)} = 2.13, \quad \text{[amps]}$$

Step No. 4: Calculate the maximum duty ratio, $D_{(max)}$.

$$D_{(\max)} = (1 - D_w)\left(\frac{V_o - V_{in(\min)} + V_d}{V_o}\right)$$

$$D_{(\max)} = (1 - 0.1)\left(\frac{(50) - (26) + (1.0)}{50}\right)$$

$$D_{(\max)} = 0.45$$

Step No. 5: Calculate the minimum duty ratio, $D_{(min)}$.

$$D_{(\min)} = (1 - D_w)\left(\frac{V_o - V_{in(\max)} + V_d}{V_o}\right)$$

$$D_{(\min)} = (1 - 0.1)\left(\frac{(50) - (32) + (1.0)}{50}\right)$$

$$D_{(\min)} = 0.342$$

Step No. 6: Calculate the required inductance, L_{max}.

$$L_{\max} = \frac{(V_o + V_d)T D_{(\max)}(1 - D_{\max} - D_w)^2}{2I_{o(\max)}}, \quad \text{[henrys]}$$

$$L_{\max} = \frac{(50 + 1.0)\left(10\left(10^{-6}\right)\right)(0.45)(1 - 0.45 - 0.1)^2}{2(1.0)}, \quad \text{[henrys]}$$

$$L_{\max} = 23.2 \text{ use } 23, \quad [\mu\text{h}]$$

Step No. 7: Calculate the peak current, I_{pk}. In a discontinuous current boost the peak current is, $I_{(pk)} = \Delta I$.

$$I_{(pk)} = \frac{2P_{o(\max)}}{\eta\left(V_o D_{(\min)}\right)}, \quad \text{[amps]}$$

$$I_{(pk)} = \frac{2(51)}{(0.92)\left((50)(0.342)\right)}, \quad \text{[amps]}$$

$$I_{(pk)} = 6.48, \quad \text{[amps]}$$

Step No. 8: Calculate the rms current, $I_{(rms)}$.

$$I_{(rms)} = I_{pk}\sqrt{\frac{TD_{(\max)}}{3T}}, \quad \text{[amps]}$$

$$I_{(rms)} = (6.48)\sqrt{\frac{\left(10 \times 10^{-6}\right)(0.45)}{3\left(10 \times 10^{-6}\right)}}, \quad \text{[amps]}$$

$$I_{(rms)} = 2.51, \quad \text{[amps]}$$

Step No. 9: Calculate the total energy-handling capability in watt-seconds, w-s.

$$\text{Energy} = \frac{LI_{pk}^2}{2}, \quad \text{[w-s]}$$

$$\text{Energy} = \frac{\left(23\times10^{-6}\right)\left(6.48\right)^2}{2}, \quad \text{[w-s]}$$

$$\text{Energy} = 0.000483, \quad \text{[w-s]}$$

Step No. 10: Calculate the electrical conditions, K_e.

$$K_e = 0.145 P_o B_m^2 \left(10^{-4}\right)$$

$$K_e = 0.145(51)(0.25)^2\left(10^{-4}\right)$$

$$K_e = 0.0000462$$

Step No. 11: Calculate the core geometry, K_g.

$$K_g = \frac{(\text{Energy})^2}{K_e \alpha}, \quad [\text{cm}^5]$$

$$K_g = \frac{(0.000483)^2}{(0.0000462)(1.0)}, \quad [\text{cm}^5]$$

$$K_g = 0.00505, \quad [\text{cm}^5]$$

$$K_g = 0.00505(1.35), \quad [\text{cm}^5]$$

$$K_g = 0.00682, \quad [\text{cm}^5]$$

Step No. 12: From Chapter 3, select a core that is comparable in core geometry, K_g.

Core number = RM-6

Manufacturer = TDK

Magnetic path length, MPL = 2.86 cm

Core weight, W_{tfe} = 5.5 grams

Copper weight, W_{tcu} = 2.9 grams

Mean length turn, MLT = 3.1 cm

Iron area, A_c = 0.366 cm^2

Window Area, W_a = 0.260 cm^2

Area Product, A_p = 0.0952 cm^4

Core geometry, K_g = 0.0044 cm^5

Surface area, A_t = 11.3 cm^2

Permeability, μ_m = 2500

Winding Length, G = 0.82

Step No. 13: Calculate the current density, J, using a window utilization, $K_u = 0.29$.

$$J = \frac{2(Energy)(10^4)}{B_m A_p K_u}, \quad [\text{amps/cm}^2]$$

$$J = \frac{2(0.000483)(10^4)}{(0.25)(0.0952)(0.29)}, \quad [\text{amps/cm}^2]$$

$$J = 1398, \quad [\text{amps/cm}^2]$$

Step No. 14: Calculate the wire area, $A_{w(B)}$.

$$A_{w(B)} = \frac{I_{rms}}{J} \quad [\text{cm}^2]$$

$$A_{w(B)} = \frac{2.51}{1398} \quad [\text{cm}^2]$$

$$A_{w(B)} = 0.00179 \quad [\text{cm}^2]$$

Step No. 15: Calculate the required number of strands, S_n.

$$S_n = \frac{A_{w(B)}}{\#26\ (\text{bare area})}$$

$$S_n = \frac{(0.00180)}{(0.00128)}$$

$$S_n = 1.41 \ \text{ use 2}$$

Step No. 16: Calculate the number of turns, N, using the number of strands, S_n, and the area for #26.

$$N = \frac{K_u W_a}{S_n \#26}, \quad [\text{turns}]$$

$$N = \frac{(0.29)(0.26)}{2(0.00128)}, \quad [\text{turns}]$$

$$N = 29.5 \text{ use } 30, \quad [\text{turns}]$$

Step No. 17: Calculate the required gap, l_g.

$$l_g = \frac{0.4\pi N^2 A_c(10^{-8})}{L} - \left(\frac{\text{MPL}}{\mu_m}\right), \quad [\text{cm}]$$

$$l_g = \frac{(1.26)(30)^2(0.366)(10^{-8})}{(0.000023)} - \left(\frac{2.86}{2500}\right), \quad [\text{cm}]$$

$$l_g = 0.179, \quad [\text{cm}]$$

Step No. 19: Calculate the equivalent gap in mils.

$$\text{mils} = \text{cm}(393.7)$$

$$\text{mils} = (0.179)(393.7)$$

$$\text{mils} = 70$$

Step No. 20: Calculate the fringing flux factor, F.

$$F = 1 + \frac{l_g}{\sqrt{A_c}} \ln\left(\frac{2G}{l_g}\right)$$

$$F = 1 + \frac{(0.179)}{\sqrt{0.366}} \ln\left(\frac{2(0.82)}{0.179}\right)$$

$$F = 1.66$$

Step No. 21: Calculate the new number of turns, N_{np}, by inserting the fringing flux, F.

$$N_{np} = \sqrt{\frac{l_g L}{0.4\pi A_c F\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N_{np} = \sqrt{\frac{(0.179)(0.000023)}{(1.26)(0.366)(1.66)\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N_{np} = 23, \quad \text{[turns]}$$

Step No. 22: Calculate the peak flux density, B_{pk}.

$$B_{pk} = \frac{0.4\pi N_n F\left(I_{(pk)}\right)\left(10^{-4}\right)}{l_g + \left(\dfrac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]}$$

$$B_{pk} = \frac{(1.26)(23)(1.66)(6.48)\left(10^{-4}\right)}{(0.179) + \left(\dfrac{2.86}{2500}\right)}, \quad \text{[teslas]}$$

$$B_{pk} = 0.177, \quad \text{[teslas]}$$

Step No. 23: Calculate the new, μΩ/cm.

$$(\text{new})\mu\Omega / \text{cm} = \frac{\mu\Omega / \text{cm}}{S_n}$$

$$(\text{new})\mu\Omega / \text{cm} = \frac{1345}{2}$$

$$(\text{new})\mu\Omega / \text{cm} = 673$$

Step No. 24: Calculate the primary winding resistance, R.

$$R = MLT\left(N_n\right)\left(\frac{\mu\Omega}{cm}\right)\times 10^{-6} \quad \text{[ohms]}$$

$$R = \left(3.1\right)\left(23\right)\left(673\right)\times 10^{-6} \quad \text{[ohms]}$$

$$R = 0.0480 \quad \text{[ohms]}$$

Step No. 25: Calculate the copper loss, P_{cu}.

$$P_{cu} = I_{rms}^2 R \quad \text{[watts]}$$

$$P_{cu} = (2.51)^2(.0480) \quad \text{[watts]}$$

$$P_{cu} = 0.302 \text{ [watts]}$$

Step No. 26: Calculate the regulation, α, for this design.

$$\alpha = \frac{P_{cu}}{P_o}\times 100, \quad [\%]$$

$$\alpha = \frac{\left(0.302\right)}{\left(50\right)}\times 100, \quad [\%]$$

$$\alpha = 0.604, \quad [\%]$$

Step No. 27: Calculate the ac flux density in teslas, B_{ac}.

$$B_{ac} = \frac{0.4\pi N_n F\left(\frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g+\left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]}$$

$$B_{ac} = \frac{(1.26)(23)(1.66)(3.24)\left(10^{-4}\right)}{(0.179)+\left(\frac{2.86}{2500}\right)}, \quad \text{[teslas]}$$

$$B_{ac} = 0.0869, \quad \text{[teslas]}$$

Step No. 28: Calculate the watts per kilogram, WK.

$$WK = 4.855\left(10^{-5}\right)\left(f\right)^{(1.63)}\left(B_{ac}\right)^{(2.62)}, \quad \text{[watts/kilogram]}$$

$$WK = 4.855\left(10^{-5}\right)\left(100000\right)^{(1.63)}\left(0.0869\right)^{(2.62)}, \quad \text{[watts/kilogram]}$$

$$WK = 11.39, \quad \text{[watts/kilogram] or [milliwatts/gram]}$$

Step No. 29: Calculate the core loss, P_{fe}.

$$P_{fe} = \left(\frac{\text{milliwatts}}{\text{gram}}\right) W_{tfe} \times 10^{-3}, \quad [\text{watts}]$$

$$P_{fe} = (11.39)(5.5) \times 10^{-3}, \quad [\text{watts}]$$

$$P_{fe} = 0.0626, \quad [\text{watts}]$$

Step No. 30: Calculate the total loss, P_Σ, core, P_{fe}, and copper, P_{cu}.

$$P_\Sigma = P_{fe} + P_{cu}, \quad [\text{watts}]$$

$$P_\Sigma = (0.0626) + (0.302), \quad [\text{watts}]$$

$$P_\Sigma = 0.365, \quad [\text{watts}]$$

Step No. 31: Calculate the watt density, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad [\text{watts/cm}^2]$$

$$\psi = \frac{0.365}{11.3}, \quad [\text{watts/cm}^2]$$

$$\psi = 0.0323, \quad [\text{watts/cm}^2]$$

Step No. 32: Calculate the temperature rise, T_r, in, °C.

$$T_r = 450(\psi)^{(0.826)}, \quad [°\text{C}]$$

$$T_r = 450(0.0323)^{(0.826)}, \quad [°\text{C}]$$

$$T_r = 26.4, \quad [°\text{C}]$$

Designing Boost Inductors for Power Factor Correction (PFC)

Historically, the standard power supplies designed for electronic equipment have had a notoriously poor power factor in the area of (0.5-0.6), and a correspondingly, high, harmonic current content. This design approach utilizes a simple rectifier capacitor input filter that results in large current pulses drawn from the line, that cause distorting of the line voltage and create large amounts of EMI and noise.

The regulating bodies, IEC in Europe and IEEE in the United States, have been working to develop a standard for limiting harmonic current, in off-line equipment. The German standardization bodies have established IEC 1000-2, and it is generally accepted as the standard for limiting harmonic currents in off-line equipment.

Many new electronic products are required to have a near unity power factor and a distortion free, current input waveform. The conventional ac-dc converters usually employ a full wave, rectifier-bridge, with a simple filter to draw power from the ac line. The typical, rectifier capacitor, input bridge filter and associated waveforms, as shown in Figure 13-15, are no longer good enough.

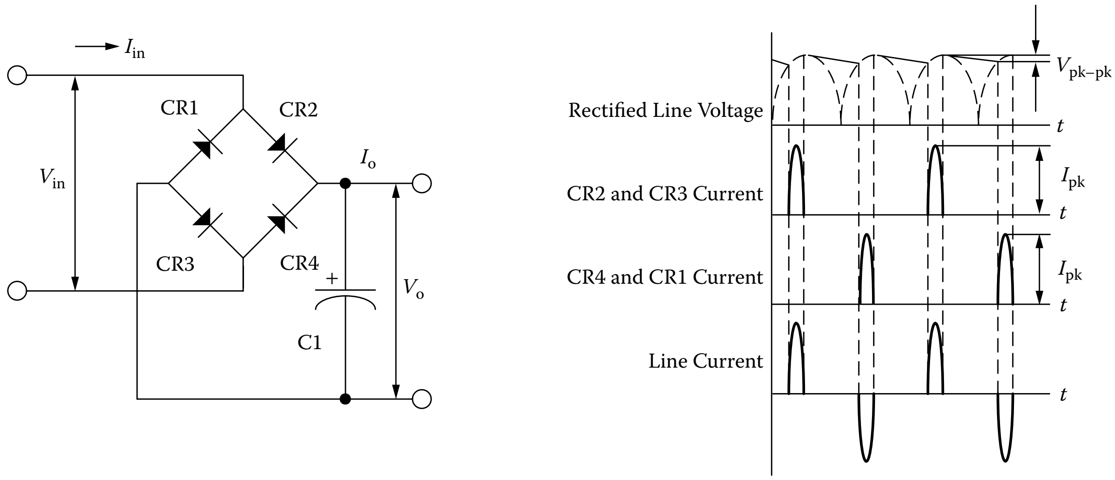

Figure 13-15. Typical, Capacitor Input Bridge Rectifier Filter.

The line current waveform for equipment that utilizes off-line rectifier capacitor input filter, is shown in Figure 13-15. The line current is supplied in narrow pulses. Consequently, the power factor is poor (0.5 – 0.6), due to a high harmonic distortion of the current waveform. The power supply can be designed with a power factor approaching unity, by the addition of an input inductor, as shown in Figure 13-16. The reasons why the input inductors are not designed into power supplies is very simple: cost, weight and bulk. The inductance Equation for, L1, is shown in Equation [13-57].

$$L1 = \frac{V_o}{3\omega I_{o(\min)}}, \quad \text{[henrys]} \qquad [13\text{-}57]$$

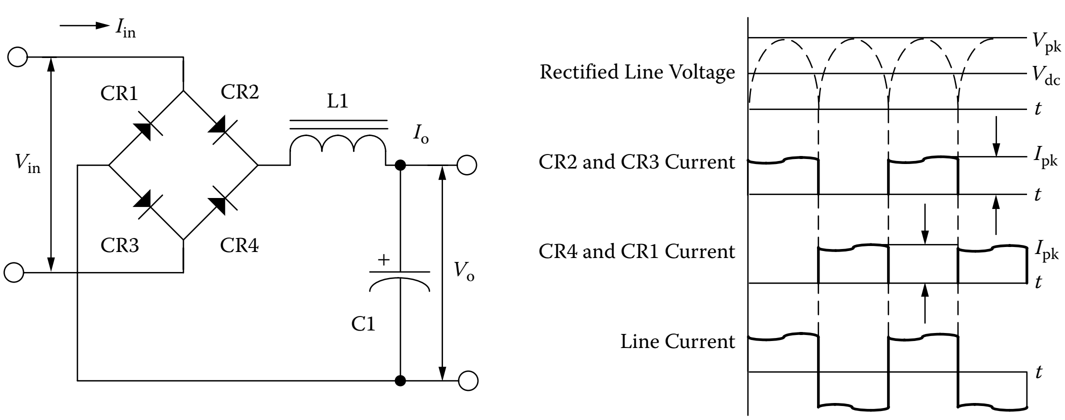

Figure 13-16. A Typical, Inductor Input Bridge Rectifier Filter.

Standard Boost Flyback Converter

The standard dc-to-dc boost flyback converter is shown in Figure 13-6, along with the voltage and current waveforms, shown in Figures 13-7 and 13-8. The boost converter has become the choice of many engineers as the power stage in the active power factor corrector design. The basic circuit can be operated in either the continuous or discontinuous mode.

Boost PFC Converter

The boost Power Factor Correction (PFC) converter is shown in Figure 13-17. The boost converter is the most popular of the power factor pre-regulators. The boost converter can operate in two modes, continuous and discontinuous. The current through the inductor, L1, is shown in Figure 13-18, for both continuous and discontinuous operation. After examining the schematic, the advantages and disadvantages of the boost converter can readily be seen. The disadvantage is the high output voltage to the load circuit and therefore the current limit cannot be implemented. The advantage is that the circuit requires a minimum of parts and the gate drive to, Q1, is referenced to ground.

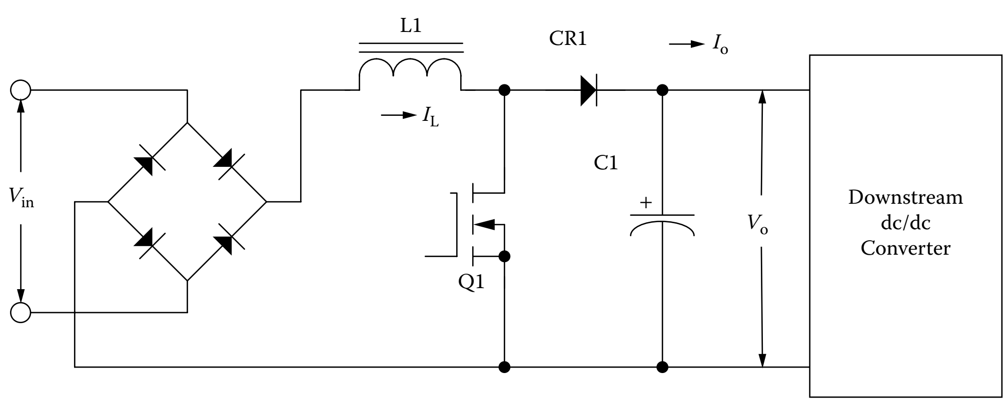

Figure 13-17. Boost PFC Converter.

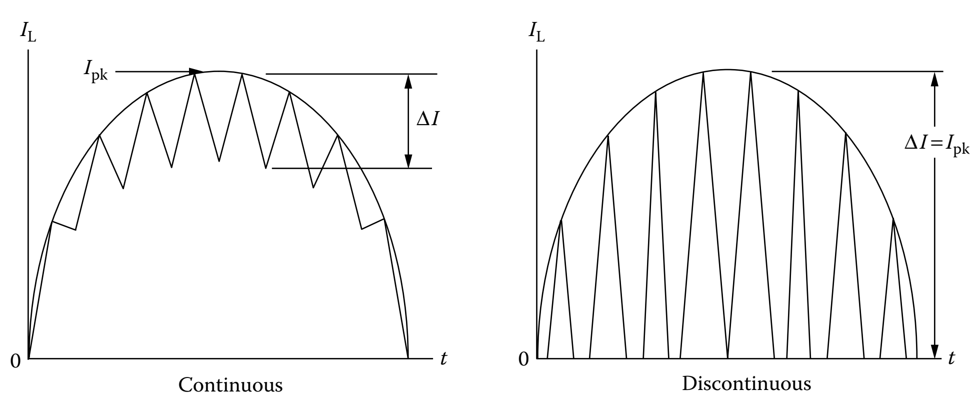

Figure 13-18. Current Through Inductor L1.

Design Example, (PFC) Boost Converter, Continuous Current

The following pages describe a step-by-step procedure for designing a continuous current boost inductor for a Power Factor Correction (PFC) converter, as shown in Figure 13-17, with the following specifications:

1. Output power, $P_o = 250$ watts
2. Input voltage range, $V_{in} = 90 - 270$ volts
3. Line frequency, $f_{(line)} = 47 - 65$ Hz
4. Output voltage, $V_o = 400$ volts
5. Switching frequency, f = 100kHz
6. Inductor ripple current, $\Delta I = 20\%$ of I_{pk}
7. Magnetic core = ETD
8. Magnetic material = R
9. Converter efficiency, $\eta = 95\%$
10. Inductor regulation, $\alpha = 1\%$
11. *Window utilization, $K_u = 0.29$
12. Operating Flux, $B_m = 0.25$ tesla

*When operating at high frequencies, the engineer has to review the window utilization factor, K_u. When using a small bobbin ferrite, the ratio of the bobbin winding area to the core window area is only about 0.6. Operating at 100kHz and having to use a #26 wire, because of the skin effect, the ratio of the bare copper area is 0.78. Therefore, the overall window utilization, K_u, is reduced. The core geometries, K_g, in Chapter 3 have been calculated with a window utilization, K_u, of 0.4. To return the design back to the norm, the core geometry, K_g, is to be multiplied by 1.35, and then, the current density, J, is calculated, using a window utilization factor of 0.29. See Chapter 4.

Skin Effect

The skin effect on an inductor is the same as a transformer. In the normal dc inductor, the ac current (ac flux), is much lower, and does not require the use of the same, maximum wire size. This is not the case in the discontinuous, current type, flyback converter, where all of the flux is ac and no dc. In the discontinuous, flyback design, the skin effect has to be treated just like a high frequency transformer.

There are times when the larger wire is just too difficult to wind. Large wire is not only hard to handle, but it does not give the proper lay. It is easier to wind with bi-filar or quad-filar wire, with the equivalent cross-section.

Select a wire so that the relationship between the ac resistance and the dc resistance is 1:

$$\frac{R_{ac}}{R_{dc}} = 1$$

The skin depth in centimeters is:

$$\varepsilon = \frac{6.62}{\sqrt{f}}, \quad [\text{cm}]$$

$$\varepsilon = \frac{6.62}{\sqrt{100,000}}, \quad [\text{cm}]$$

$$\varepsilon = 0.0209, \quad [\text{cm}]$$

Then, the wire diameter is:

$$\text{Wire Diameter} = 2(\varepsilon), \quad [\text{cm}]$$

$$\text{Wire Diameter} = 2(0.0209), \quad [\text{cm}]$$

$$\text{Wire Diameter} = 0.0418, \quad [\text{cm}]$$

Then, the bare wire area, A_w, is:

$$A_w = \frac{\pi D^2}{4}, \ [\text{cm}^2]$$

$$A_w = \frac{(3.1416)(0.0418)^2}{4}, \ [\text{cm}^2]$$

$$A_w = 0.00137, \ [\text{cm}^2]$$

From the Wire Table in Chapter 4, Number 26 has a bare wire area of 0.00128 centimeters. This will be the minimum wire size used in this design. If the design requires more wire area to meet the specification, then, the design will use a multifilar of #26.

Wire AWG	Bare Area	Area Ins.	Bare/Ins.	μΩ/cm
#26	0.001280	0.001603	0.798	1345

Step No. 1: Calculate the input power, P_{in}.

$$P_{in} = \frac{P_o}{\eta}, \quad [\text{watts}]$$

$$P_{in} = \frac{250}{0.95}, \quad [\text{watts}]$$

$$P_{in} = 263, \quad [\text{watts}]$$

Step No. 2: Calculate the peak input current, I_{pk}.

$$I_{pk} = \frac{P_{in}\sqrt{2}}{V_{in(\min)}}, \quad [\text{amps}]$$

$$I_{pk} = \frac{(263)(1.41)}{90}, \quad [\text{amps}]$$

$$I_{pk} = 4.12, \quad [\text{amps}]$$

Step No. 3: Calculate the input ripple current, ΔI.

$$\Delta I = 0.2 I_{pk}, \quad \text{[amps]}$$

$$\Delta I = 0.2(4.12), \quad \text{[amps]}$$

$$\Delta I = 0.824, \quad \text{[amps]}$$

Step No. 4: Calculate the maximum duty ratio, $D_{(max)}$.

$$D_{(\max)} = \frac{\left(V_o - \left(V_{in(\min)}\sqrt{2}\right)\right)}{V_o}$$

$$D_{(\max)} = \frac{\left(400 - \left(90\sqrt{2}\right)\right)}{400}$$

$$D_{(\max)} = 0.683$$

Step No. 5: Calculate the required boost inductance, L.

$$L = \frac{\left(V_{in(\min)}\sqrt{2}\right) D_{(\max)}}{\Delta I\, f}, \quad \text{[henrys]}$$

$$L = \frac{(126.9)(0.683)}{(0.824)(100000)}, \quad \text{[henrys]}$$

$$L = 0.00105, \quad \text{[henrys]}$$

Step No. 6: Calculate the Energy required, Eng.

$$\text{Eng} = \frac{L I_{pk}^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Eng} = \frac{(0.00105)(4.12)^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Eng} = 0.00891, \quad \text{[watt-seconds]}$$

Step No. 7: Calculate the electrical coefficient, K_e.

$$K_e = 0.145 P_o B_m^2 \left(10^{-4}\right)$$

$$K_e = 0.145(250)(0.25)^2 \left(10^{-4}\right)$$

$$K_e = 0.000227$$

Step No. 8: Calculate the core geometry coefficient, K_g.

$$K_g = \frac{(\text{Eng})^2}{K_e \alpha}, \quad [\text{cm}^5]$$

$$K_g = \frac{(0.00891)^2}{(0.000227)(1)}, \quad [\text{cm}^5]$$

$$K_g = 0.35, \quad [\text{cm}^5]$$

$$K_g = 0.35(1.35), \quad [\text{cm}^5] \text{ Corrected}$$

$$K_g = 0.47, \quad [\text{cm}^5]$$

Step No. 9: From Chapter 3, select an ETD ferrite core, comparable in core geometry, K_g.

Core number = ETD-44

Manufacturer = Ferroxcube

Magnetic path length, MPL = 10.3 cm

Core weight, W_{tfe} = 94 grams

Copper weight, W_{tcu} = 93.2 grams

Mean length turn, MLT = 9.4 cm

Iron area, A_c = 1.74 cm^2

Window Area, W_a = 2.79 cm^2

Area Product, A_p = 4.85 cm^4

Core geometry, K_g = 0.360 cm^5

Surface area, A_t = 87.9 cm^2

Material 3F3, μ_m = 2000

Millihenrys per 1000 turns, AL = 3364

Winding Length, G = 3.22

Step No. 10: Calculate the current density, J.

$$J = \frac{2(\text{Eng})(10^4)}{B_m A_p K_u}, \quad [\text{amps/cm}^2]$$

$$J = \frac{2(0.00891)(10^4)}{(0.25)(4.85)(0.29)}, \quad [\text{amps/cm}^2]$$

$$J = 507, \quad [\text{amps/cm}^2]$$

Step No. 11: Calculate the rms current, I_{rms}.

$$I_{rms} = \frac{I_{pk}}{\sqrt{2}}, \quad [\text{amps}]$$

$$I_{rms} = \frac{4.12}{\sqrt{2}}, \quad [\text{amps}]$$

$$I_{rms} = 2.91, \quad [\text{amps}]$$

Step No. 12: Calculate the required bare wire area, $A_{w(B)}$.

$$A_{w(B)} = \frac{I_{rms}}{J}, \quad [\text{cm}^2]$$

$$A_{w(B)} = \frac{2.91}{507}, \quad [\text{cm}^2]$$

$$A_{w(B)} = 0.00574, \quad [\text{cm}^2]$$

Step No. 13: Calculate the required number of strands, S_n.

$$S_n = \frac{A_{w(B)}}{\#26(\text{bare area})}, \quad [\text{cm}^2]$$

$$S_n = \frac{(0.00574)}{(0.00128)}, \quad [\text{cm}^2]$$

$$S_n = 4.48 \text{ use } 5, \quad [\text{cm}^2]$$

Step No. 14: Calculate the required number of turns, N, using the number of strands, S_n, and the area for #26.

$$N = \frac{W_a K_u}{S_n \#26}, \quad [\text{turns}]$$

$$N = \frac{(2.79)(0.29)}{5(0.00128)}, \quad [\text{turns}]$$

$$N = 126, \quad [\text{turns}]$$

Step No. 15: Calculate the required gap, l_g.

$$l_g = \left(\frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{L}\right), \quad [\text{cm}]$$

$$l_g = \left(\frac{(1.257)(126)^2(1.74)\left(10^{-8}\right)}{0.00105}\right), \quad [\text{cm}]$$

$$l_g = 0.331, \quad [\text{cm}]$$

Change the gap to mils: 0.331 × 393.7 = 130 mils center or 65 mils per each outer leg.

Step No. 16: Calculate the fringing flux factor, F.

$$F = \left(1 + \left(\frac{l_g}{\sqrt{A_c}}\right)\ln\left(\frac{2G}{l_g}\right)\right)$$

$$F = \left(1 + \left(\frac{0.331}{1.32}\right)\ln\left(\frac{6.44}{0.331}\right)\right)$$

$$F = 1.74$$

Step No. 17: Calculate the new turns using the fringing flux.

$$N = \sqrt{\frac{l_g L}{0.4\pi A_c F\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N = \sqrt{\frac{(0.331)(0.00105)}{(1.257)(1.74)(1.74)\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N = 96, \quad \text{[turns]}$$

Step No. 18: Calculate the peak flux, B_{pk}.

$$B_{pk} = F\left(\frac{0.4\pi N I_{pk}\left(10^{-4}\right)}{l_g}\right), \quad \text{[teslas]}$$

$$B_{pk} = 1.74\left(\frac{(1.257)(96)(4.12)\left(10^{-4}\right)}{0.331}\right), \quad \text{[teslas]}$$

$$B_{pk} = 0.261, \quad \text{[teslas]}$$

Step No. 19: Calculate the new, μΩ/cm.

$$(\text{new})\mu\Omega / \text{cm} = \frac{\mu\Omega / \text{cm}}{S_n}$$

$$(\text{new})\mu\Omega / \text{cm} = \frac{1345}{5}$$

$$(\text{new})\mu\Omega / \text{cm} = 269$$

Step No. 20: Calculate the winding resistance, R.

$$R = (\text{MLT})N\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R = (9.4)(96)(269)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R = 0.243, \quad \text{[ohms]}$$

Step No. 21: Calculate the winding copper loss, P_{cu}.

$$P_{cu} = I_{rms}^2 R, \quad \text{[watts]}$$

$$P_{cu} = (2.91)^2(0.243), \quad \text{[watts]}$$

$$P_{cu} = 2.06, \quad \text{[watts]}$$

Step No. 22: Calculate the regulation, α.

$$\alpha = \frac{P_{cu}}{P_o} 100, \quad [\%]$$

$$\alpha = \frac{(2.06)}{(250)} 100, \quad [\%]$$

$$\alpha = 0.824, \quad [\%]$$

Step No. 23: Calculate the ac flux density, B_{ac}.

$$B_{ac} = \frac{0.4\pi N\left(\frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g}, \quad [\text{teslas}]$$

$$B_{ac} = \frac{(1.257)(96)(0.412)\left(10^{-4}\right)}{0.331}, \quad [\text{teslas}]$$

$$B_{ac} = 0.0150, \quad [\text{teslas}]$$

Step No. 24: Calculate the watts per kilogram, W/K, using R material in Chapter 2.

$$W / K = 4.316\left(10^{-5}\right)(f)^{1.64}(B_{ac})^{2.68}, \quad [\text{watts per kilogram}]$$

$$W / K = 4.316\left(10^{-5}\right)(100000)^{1.64}(0.0150)^{2.68}, \quad [\text{watts per kilogram}]$$

$$W / K = 0.0885, \quad [\text{watts per kilogram}]$$

Step No. 25: Calculate the core loss, P_{fe}.

$$P_{fe} = W_{tfe}\left(10^{-3}\right)(W / K), \quad [\text{watts}]$$

$$P_{fe} = (94)\left(10^{-3}\right)(0.0885), \quad [\text{watts}]$$

$$P_{fe} = 0.0083, \quad [\text{watts}]$$

Step No. 26: Calculate the total loss core loss, P_{fe} and copper loss, P_{cu}.

$$P = P_{cu} + P_{fe}, \quad [\text{watts}]$$

$$P = (2.03) + (0.0083), \quad [\text{watts}]$$

$$P = 2.04, \quad [\text{watts}]$$

Step No. 27: Calculate the watt density, ψ.

$$\psi = \frac{P}{A_t}, \quad [\text{watts per cm}^2]$$

$$\psi = \frac{2.04}{87.9}, \quad [\text{watts per cm}^2]$$

$$\psi = 0.023, \quad [\text{watts per cm}^2]$$

Step No. 28: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{0.826}, \quad [°C]$$

$$T_r = 450(0.023)^{0.826}, \quad [°C]$$

$$T_r = 19.9, \quad [°C]$$

Step No. 29: Calculate the window utilization, K_u.

$$K_u = \frac{N S_n A_{w(B)}}{W_a}$$

$$K_u = \frac{(95)(5)(0.00128)}{(2.79)}$$

$$K_u = 0.218$$

Recognitions

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Chapter 14

Forward Converter, Transformer Design, and Output Inductor Design

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