Introduction

When speaking of a forward converter, the circuit that comes to mind is the single-ended, forward converter circuit, as shown in Figure 14-1. This single-ended, forward converter was developed about 1974 and has become one of the most popular and widely-used topology for powers under 200 W. The single-ended, forward converter gets its name from a family of converters. A description of a forward converter is that when current is flowing in the primary, there is current flowing in the secondary, and in the load. The push-pull converter, full-bridge converter, and half-bridge converter are all, basically, forward converters. The voltage stress on the single-forward converter is the same as it is on the push-pull converter, $2V_{in}$. This circuit's main advantage, that is so appealing to engineers is its simplicity and parts' count.

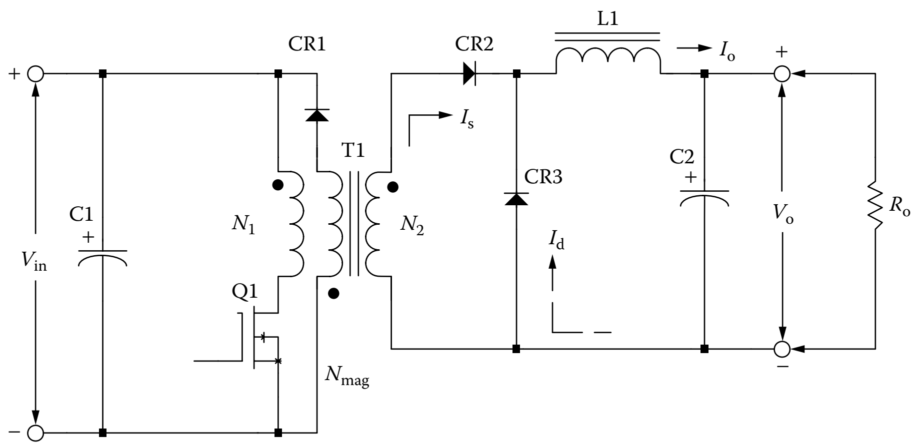

Figure 14-1. Schematic of a Single-Ended Forward Converter.

Circuit Operation

The basic circuit operation of this single-ended, forward converter is as follows: When the drive is applied to, Q1, the secondary current, I_s, will flow through, CR2, and, L1, and into the load. This process is due to transformer action, (T1). At the same time, the magnetizing current begins to build up in the transformer primary. When the base-drive to, Q1, is removed, then, Q1, turns off the magnetizing current that has built up in the primary. The magnetizing current continues to flow through the demagnetizing winding, N_{mag} and CR1. The demagnetizing winding, N_{mag}, has the same number of turns as the primary winding. So, when the magnetizing field collapses, when, Q1, is turned off, diode, CR1, is clamped to the same voltage as the applied voltage during the, t_{on}, time. This means the transistor on time, t_{on}, divided by the total time, T, must not exceed 0.5 or 50%. Otherwise, the forward volt-seconds will exceed the reset volt-second capability and the transformer will saturate. To ensure smooth transfer of the magnetizing current, the primary and demagnetizing winding must be tightly coupled (bifilar). In a push-pull converter, the reset of the core occurs naturally on each alternate half cycle.

Comparing the Dynamic B-H Loops

One of the main reasons for engineers to use the single-ended, forward converter circuit is the problem they have with the push-pull converter, core saturating. The core saturation can be due to an imbalance of the primary or secondary. The dynamic BH loops for the single-ended, forward converter and the push-pull converter are shown in Figure 14-2.

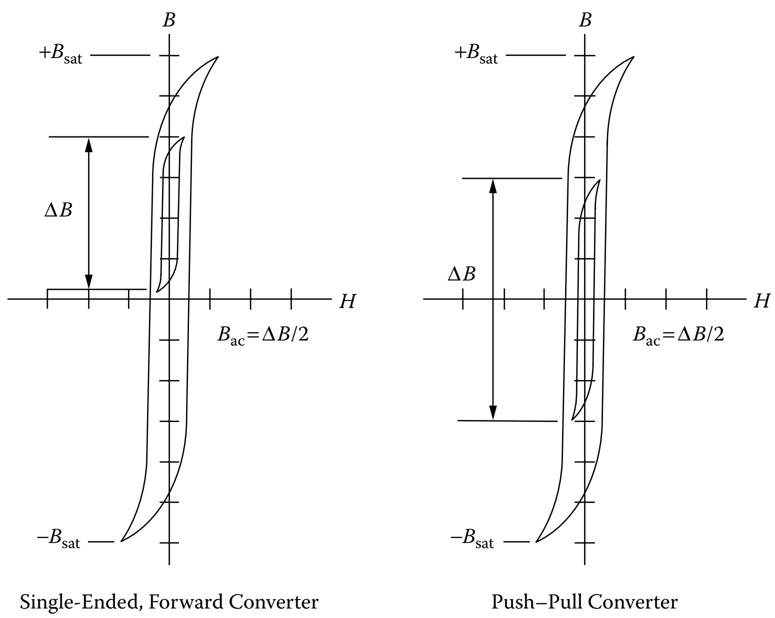

Figure 14-2. The Dynamic BH Loop Comparison.

The average input current for the single-ended, forward converter is about the same as the push-pull converter, but the peak current is always greater than twice the average current. Operating the single-ended, forward converter at low input voltages, the high peak currents could be a component problem. The input filter and output filter for the single-ended, forward converter are always larger than the push-pull converter, because it is operating at the fundamental frequency.

Forward Converter Waveforms

The waveforms, shown in Figure 14-3, are typical waveforms of the single-ended forward converter. The collector current, I_c, is shown in Figure (14-3-A), and the magnetizing current, I_m, is shown in the Figure (14-3-B). The inductor, L1, current, I_L, made up from the rectifier, CR2, and the commutating rectifier, CR3, are shown in Figure (14-3-C). The collector voltage, V_c, is shown in Figure (14-3-D).

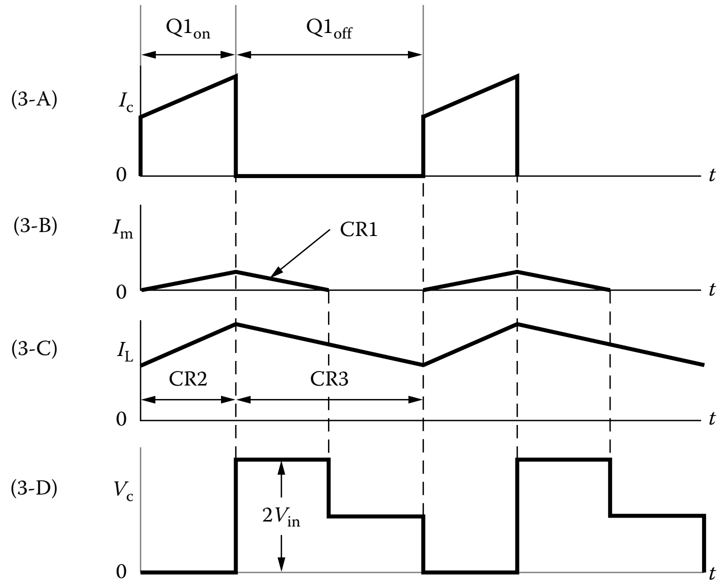

Figure 14-3. Typical Single-Ended Forward, Converter Waveforms.

Another version of the classic, forward converter is the double-ended, forward converter, shown in Figure 14-4. The double-ended, forward converter has two transistors rather than one, compared to the single-ended, forward converter, shown in Figure 14-1. The double-ended forward converter is more complicated than the single-ended forward converter because one of the transistors is on the high side of the input voltage, but it has some significant advantages. The series switching transistors are subjected to only the input voltage, (V_{in}), rather than twice the input voltage, ($2V_{in}$). It also removes the need for a demagnetizing winding. The demagnetizing current now flows through the primary, through, CR1, and CR2, and back to the source, as shown in Figure 14-5. This demagnetizing path also provides a path for the energy stored in the leakage inductance. The resulting spiking voltage, caused from the leakage inductance, is now clamped to the input voltage, plus the two diode drops ($V_{in} + 2V_d$).

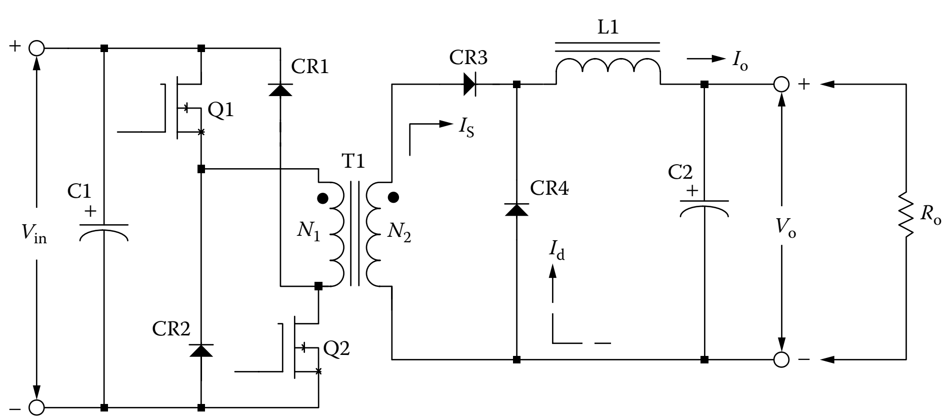

Figure 14-4. Schematic of a Double-Ended Forward Converter.

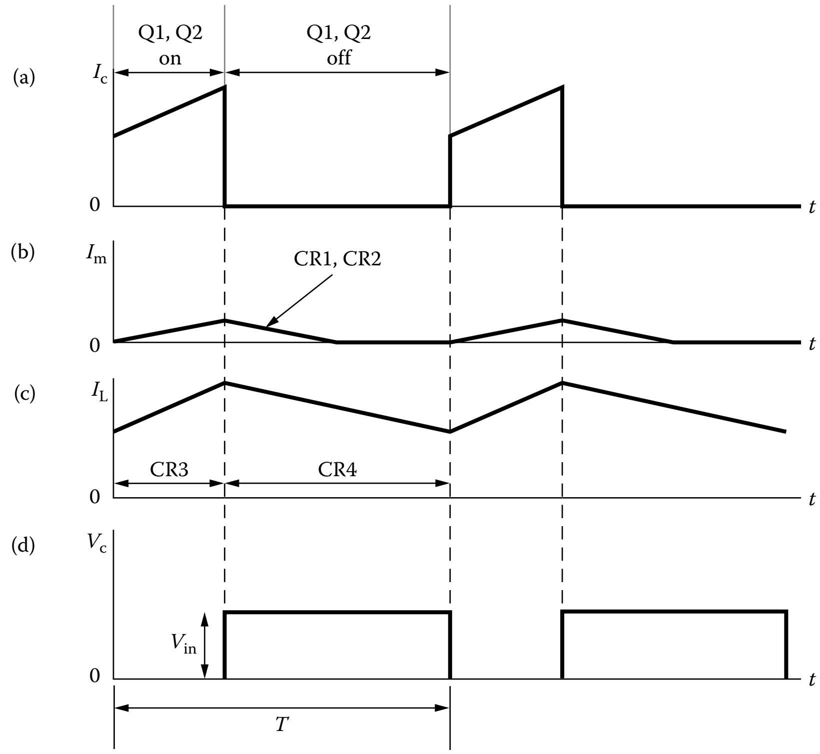

Figure 14-5. Typical Double-Ended Forward, Converter Waveforms.

Note: The Design Equations are from communication and work with the late Dr. J. K. Watson, Professor of Electrical Engineering at the University of Florida.

Electrical coefficient, K_e, is shown in Equation [14-1].

$$K_e = 0.145 f^2 \Delta B^2 \left(10^{-4}\right) \quad [14\text{-}1]$$

Core geometry, K_g, is shown in Equation [14-2].

$$K_g = \frac{P_{in} D_{(\max)}}{\alpha K_e}, \quad [\mathrm{cm}^5] \quad [14\text{-}2]$$

Current density, J, is shown in Equation [14-3].

$$J = \frac{2 P_{in} \sqrt{D_{(\max)}} \left(10^4\right)}{f \Delta B A_c W_a K_u}, \quad [\text{amps per cm}^2] \quad [14\text{-}3]$$

Primary current, I_p, is shown in Equation [14-4].

$$I_p = \frac{P_{in}}{V_{in(\min)} \sqrt{D_{(\max)}}}, \quad [\text{amps}] \quad [14\text{-}4]$$

Transformer Design Using the Core Geometry, K_g, Approach

The following information is the Design specification for a 30 watts, single-ended transformer, operating at 100kHz, using the, K_g, core geometry approach. For a typical design example, assume a single-ended converter circuit, as shown in Figure 14-1, with the following specification:

1. Input voltage, $V_{(min)} = 22$ volts
2. Input voltage, $V_{(nom)} = 28$ volts
3. Input voltage, $V_{(max)} = 35$ volts
4. Output voltage, $V_o = 5.0$ volts
5. Output current, $I_o = 5.0$ amps
6. Frequency, f = 100kHz
7. Efficiency, $\eta = 98\%$
8. Regulation, $\alpha = 0.5\%$
9. Diode voltage drop, $V_d = 1.0$ volt
10. Operating flux density, ΔB, ($B_{ac} = \Delta B/2$) = 0.1 teslas
11. Core Material = ferrite
12. Window utilization, $K_u = 0.3$
13. Temperature rise goal, $T_r = 30°C$
14. Maximum duty ratio, $D_{max} = 0.5$
15. Notes:

    Demag turns ratio, $N_{mag}/N_p = 1$

    Demag power, $P_{mag} = (0.1)P_o$

Select a wire so that the relationship between the ac resistance and the dc resistance is 1:

$$\frac{R_{ac}}{R_{dc}} = 1$$

The skin depth in centimeters is:

$$\varepsilon = \frac{6.62}{\sqrt{f}}, \quad [\text{cm}]$$

$$\varepsilon = \frac{6.62}{\sqrt{100,000}}, \quad [\text{cm}]$$

$$\varepsilon = 0.0209, \quad [\text{cm}]$$

Then, the wire diameter is:

$$\text{Wire Diameter} = 2(\varepsilon), \quad [\text{cm}]$$

$$\text{Wire Diameter} = 2(0.0209), \quad [\text{cm}]$$

$$\text{Wire Diameter} = 0.0418, \quad [\text{cm}]$$

Then, the bare wire area, A_w, is:

$$A_w = \frac{\pi D^2}{4}, \quad [\text{cm}^2]$$

$$A_w = \frac{(3.1416)(0.0418)^2}{4}, \quad [\text{cm}^2]$$

$$A_w = 0.00137, \quad [\text{cm}^2]$$

From the Wire Table, in Chapter 4, Number 26 has a bare wire area of 0.001280 centimeters. This will be the minimum wire size used in this design. If the design requires more wire area to meet the specification, then, the design will use a multifilar of #26. Listed Below are #27 and #28, just in case #26 requires too much rounding off.

Wire AWG	Bare Area	Area Ins.	Bare/Ins.	μΩ/cm
#26	0.001280	0.001603	0.798	1345
#27	0.001021	0.001313	0.778	1687
#28	0.0008046	0.0010515	0.765	2142

Step No. 1: Calculate the transformer output power, P_o.

$$P_o = I_o(V_o + V_d), \quad [\text{watts}]$$

$$P_o = 5(5+1), \quad [\text{watts}]$$

$$P_o = 30, \quad [\text{watts}]$$

Step No. 2: Calculate the input power, P_{in}.

$$P_{in} = \frac{P_o(1.1)}{\eta}, \quad [\text{watts}]$$

$$P_{in} = \frac{30(1.1)}{(0.98)}, \quad [\text{watts}]$$

$$P_{in} = 33.67, \quad [\text{watts}]$$

Step No. 3: Calculate the electrical coefficient, K_e.

$$K_e = 0.145 f^2 \Delta B^2 \left(10^{-4}\right)$$

$$K_e = 0.145(100{,}000)^2(0.1)^2\left(10^{-4}\right)$$

$$K_e = 1450$$

Step No. 4: Calculate the core geometry, K_g.

$$K_g = \frac{P_{in} D_{\max}}{\alpha K_e}, \quad [\text{cm}^5]$$

$$K_g = \frac{(33.67)(0.5)}{(0.5)(1450)}, \quad [\text{cm}^5]$$

$$K_g = 0.0232, \quad [\text{cm}^5]$$

When operating at high frequencies, the engineer has to review the window utilization factor, K_u. When using small bobbin ferrites, the ratio of the bobbin winding area to the core window area is only about 0.6. Operating at 100kHz and having to use a #26 wire, because of the skin effect, the ratio of the bare copper area to the total area is 0.78. Therefore, the overall window utilization, K_u, is reduced. To return the design back to the norm, the core geometry, K_g, is to be multiplied by 1.35, and then, the current density, J, is calculated, using a window utilization factor of 0.29.

$$K_g = 0.0232(1.35), \quad [\text{cm}^5]$$

$$K_g = 0.0313, \quad [\text{cm}^5]$$

Step No. 5: Select a EPC core from Chapter 3, comparable in core geometry K_g.

Core number = EPC-30

Manufacturer = TDK

Magnetic material = PC44

Magnetic path length, MPL = 8.2 cm

Window height, G = 2.6 cm

Core weight, W_{tfe} = 23 grams

Copper weight, W_{tcu} = 22 grams

Mean length turn, MLT = 5.5 cm

Iron area, A_c = 0.61 cm^2

Window area, W_a = 1.118 cm^2

Area product, A_p = 0.682 cm^4

Core geometry, K_g = 0.0301 cm^5

Surface area, A_t = 31.5 cm^2

Millihenrys per 1000 turns, AL = 1570

Step No. 6: Calculate the number of primary turns, N_p.

$$N_p = \frac{V_{in(\min)} D_{(\max)} \left(10^4\right)}{f A_c \Delta B}, \quad [\text{turns}]$$

$$N_p = \frac{(22)(0.5)\left(10^4\right)}{(100,000)(0.61)(0.1)}, \quad [\text{turns}]$$

$$N_p = 18.0, \quad [\text{turns}]$$

Step No. 7: Calculate the current density, J, using a window utilization, $K_u = 0.29$.

$$J = \frac{2P_{in}\sqrt{D_{(\max)}}\left(10^4\right)}{f A_c \Delta B W_a K_u}, \quad [\text{amps/cm}^2]$$

$$J = \frac{2(33.67)(0.707)\left(10^4\right)}{(100,000)(0.61)(0.1)(1.118)(0.29)}, \quad [\text{amps/cm}^2]$$

$$J = 241, \quad [\text{amps/cm}^2]$$

Step No. 8: Calculate the primary rms current, I_p.

$$I_p = \frac{P_{in}}{V_{in(\min)}\sqrt{D_{(\max)}}}, \quad [\text{amps}]$$

$$I_p = \frac{(33.67)}{(22)(0.707)}, \quad [\text{amps}]$$

$$I_p = 2.16, \quad [\text{amps}]$$

Step No. 9: Calculate the primary bare wire area, $A_{wp(B)}$.

$$A_{wp(B)} = \frac{I_p}{J}, \quad [\text{cm}^2]$$

$$A_{wp(B)} = \frac{2.16}{241}, \quad [\text{cm}^2]$$

$$A_{wp(B)} = 0.00896, \quad [\text{cm}^2]$$

Step No. 10: Calculate the required number of primary strands, NS_p.

$$NS_p = \frac{A_{wp(B)}}{\#26}$$

$$NS_p = \frac{0.00896}{0.00128}$$

$$NS_p = 7$$

Step No. 11: Calculate the primary new μΩ per centimeter.

$$(\text{new})\,\mu\Omega/\text{cm} = \frac{\mu\Omega/\text{cm}}{NS_p}$$

$$(\text{new})\,\mu\Omega/\text{cm} = \frac{1345}{7}$$

$$(\text{new})\,\mu\Omega/\text{cm} = 192$$

Step No. 12: Calculate the primary resistance, R_p.

$$R_p = \text{MLT}\left(N_p\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_p = (5.5)(18)(192)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_p = 0.0190, \quad [\text{ohms}]$$

Step No. 13: Calculate the primary copper loss, P_p.

$$P_p = I_p^2 R_p, \quad [\text{watts}]$$

$$P_p = (2.16)^2(0.019), \quad [\text{watts}]$$

$$P_p = 0.0886, \quad [\text{watts}]$$

Step No. 14: Calculate the secondary turns, N_s.

$$N_s = \frac{N_p\left(V_o + V_d\right)}{D_{(\max)}V_{in(\min)}}\left(1 + \frac{\alpha}{100}\right), \quad [\text{turns}]$$

$$N_s = \frac{(18)(5+1)}{(0.5)(22)}\left(1 + \frac{0.5}{100}\right), \quad [\text{turns}]$$

$$N_s = 9.87 \text{ use } 10, \quad [\text{turns}]$$

Step No. 15: Calculate the secondary rms current, I_s.

$$I_s = \frac{I_o}{\sqrt{2}}, \quad [\text{amps}]$$

$$I_s = \frac{5}{1.41}, \quad [\text{amps}]$$

$$I_s = 3.55, \quad [\text{amps}]$$

Step No. 16: Calculate the secondary bare wire area, $A_{ws(B)}$.

$$A_{ws(B)} = \frac{I_s}{J}, \quad [\text{cm}^2]$$

$$A_{ws(B)} = \frac{3.55}{241}, \quad [\text{cm}^2]$$

$$A_{ws(B)} = 0.0147, \quad [\text{cm}^2]$$

Step No. 17: Calculate the required number of secondary strands, NS_s.

$$NS_s = \frac{A_{ws(B)}}{\#26}$$

$$NS_s = \frac{0.0147}{0.00128}$$

$$NS_s = 11.48 \text{ use } 11$$

Step No. 18: Calculate the secondary, new μΩ per centimeter.

$$(\text{new})\mu\Omega / \text{cm} = \frac{\mu\Omega / \text{cm}}{NS_s}$$

$$(\text{new})\mu\Omega / \text{cm} = \frac{1345}{11}$$

$$(\text{new})\mu\Omega / \text{cm} = 122$$

Step No. 19: Calculate the secondary winding resistance, R_s.

$$R_s = \text{MLT}(N_s)\left(\frac{\mu\Omega}{\text{cm}}\right)(10^{-6}), \quad [\text{ohms}]$$

$$R_s = (5.5)(10)(122)(10^{-6}), \quad [\text{ohms}]$$

$$R_s = 0.00671, \quad [\text{ohms}]$$

Step No. 20: Calculate the secondary copper loss, P_s.

$$P_s = I_s^2 R_s, \quad [\text{watts}]$$

$$P_s = (3.55)^2(0.00671), \quad [\text{watts}]$$

$$P_s = 0.0846, \quad [\text{watts}]$$

Step No. 21: Calculate the total primary and secondary copper loss, P_{cu}.

$$P_{cu} = P_p + P_s, \quad [\text{watts}]$$

$$P_{cu} = 0.0886 + 0.0846, \quad [\text{watts}]$$

$$P_{cu} = 0.173, \quad [\text{watts}]$$

Step No. 22: Calculate the transformer regulation, α.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

$$\alpha = \frac{(0.173)}{(30)}(100), \quad [\%]$$

$$\alpha = 0.576, \quad [\%]$$

Step No. 23: Calculate the demag winding inductance, L_{demag}.

$$L_{demag} = L_{1000} N_{demag}^2 (10^{-6}), \quad [\text{mh}]$$

$$L_{demag} = (1570)(18)^2(10^{-6}), \quad [\text{mh}]$$

$$L_{demag} = 0.509, \quad [\text{mh}]$$

Step No. 24: Calculate the time of, Δt. See Figure 14-6.

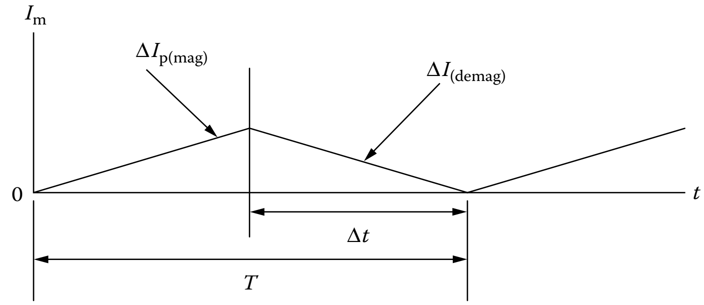

Figure 14-6. Magnetizing Current Waveform.

$$\Delta t = T D_{(max)}, \quad \text{[seconds]}$$

$$T = \frac{1}{f}, \quad \text{[seconds]}$$

$$T = \frac{1}{100,000}, \quad \text{[seconds]}$$

$$T = 10\left(10^{-6}\right), \quad \text{[seconds]}$$

$$\Delta t = \left(10\left(10^{-6}\right)\right)(0.5), \quad \text{[seconds]}$$

$$\Delta t = 5\left(10^{-6}\right), \quad \text{[seconds]}$$

Step No. 25: Calculate the demag, winding delta current, ΔI_{demag}.

$$\Delta I_{demag} = \frac{V_{in}\,\Delta t}{L_{demag}}, \quad \text{[amps]}$$

$$\Delta I_{demag} = \frac{(22)\left(5\left(10^{-6}\right)\right)}{\left(509\left(10^{-6}\right)\right)}, \quad \text{[amps]}$$

$$\Delta I_{demag} = 0.217, \quad \text{[amps]}$$

Step No. 26: Calculate the demag, winding rms current, I_{demag}. This is the rms equation for a saw tooth current.

$$I_{demag} = \Delta I \sqrt{\frac{D_{(max)}}{3}}, \quad \text{[amps]}$$

$$I_{demag} = (0.217)(0.408), \quad \text{[amps]}$$

$$I_{demag} = 0.089, \quad \text{[amps]}$$

Step No. 27: Calculate the required demag, wire area, $A_{w(demag)}$.

$$A_{w(demag)} = \frac{I_{demag}}{J}, \quad [\text{cm}^2]$$

$$A_{w(demag)} = \frac{0.089}{241}, \quad [\text{cm}^2]$$

$$A_{w(demag)} = 0.000369, \quad \approx \#31 \text{ use a } \#26$$

Step No. 28: Calculate the window utilization, K_u.

$$K_u = \frac{N A_{w(B)(\#26)}}{W_a}$$

$$N = \left(N_p NS_p\right) + \left(N_s NS_s\right) + \left(N_{demag} NS_{demag}\right)$$

$$N = (18)(7) + (10)(11) + (18)(1)$$

$$N = 254$$

$$K_u = \frac{(254)(0.00128)}{1.118}$$

$$K_u = 0.291$$

Step No. 29: Calculate the milliwatts per gram, mW/g.

$$\text{mW/g} = 0.000318 (f)^{1.51} (B_{ac})^{2.747}$$

$$\text{mW/g} = 0.000318 (100000)^{1.51} (0.05)^{2.747}$$

$$\text{mW/g} = 3.01$$

Step No. 30: Calculate the core loss, P_{fe}.

$$P_{fe} = \left(mW/g\right)\left(W_{tfe}\right)\left(10^{-3}\right), \quad [\text{watts}]$$

$$P_{fe} = (3.01)(23)\left(10^{-3}\right), \quad [\text{watts}]$$

$$P_{fe} = 0.069, \quad [\text{watts}]$$

Step No. 31: Calculate the total loss, P_Σ.

$$P_\Sigma = P_{cu} + P_{fe}, \quad [\text{watts}]$$

$$P_\Sigma = (0.173) + (0.069), \quad [\text{watts}]$$

$$P_\Sigma = 0.242, \quad [\text{watts}]$$

Step No. 32: Calculate the watts per unit area, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad [\text{watts/cm}^2]$$

$$\psi = \frac{(0.242)}{(31.5)}, \quad [\text{watts/cm}^2]$$

$$\psi = 0.0077, \quad [\text{watts/cm}^2]$$

Step No. 33: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [°C]$$

$$T_r = 450(0.0077)^{(0.826)}, \quad [°C]$$

$$T_r = 8.08, \quad [°C]$$

Forward Converter Output Inductor Design

Part 2 is designing the output inductor, L1, as shown in Figure 14-7. The output filter inductor for Switch-Mode Power Supplies, (SMPS), probably has been designed more times than any other single component. Presented here is a straight-forward approach for selecting the core and the proper wire size to meet the specification.

The losses in the magnetic material will increase significantly when the converter is operating at a higher frequency. However, the core loss in the output inductor of a switching regulator is much lower compared to the core loss in the main converter transformer. The core loss in the output inductor is caused by the change in current or ΔI, which induces a change in flux, as shown in Figure 14-7.

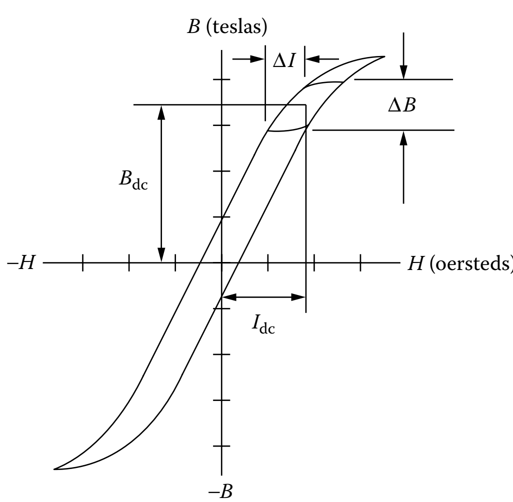

Figure 14-7. Typical Output Inductor BH Loop.

The single-ended, forward converter schematic is shown in Figure 14-8. This topology is appealing to engineers for it simplicity and parts' count. The output filter circuit, shown in Figure 14-8, has three current probes. These current probes monitor the three basic currents in a switch mode, converter output filter. Current probe

A monitors the transformer's secondary current. Current probe B monitors the commutating current through, CR3. Current probe, C, monitors the current through the output inductor, L1.

The typical secondary and filter waveforms of the forward converter are shown in Figure 14-8. The waveforms are shown with the converter operating at a 0.5 duty ratio. The applied voltage, V_1, to the filter, is shown in Figure (14-9-A). The transformer's secondary current is shown in Figure (14-9-B). The commutating current

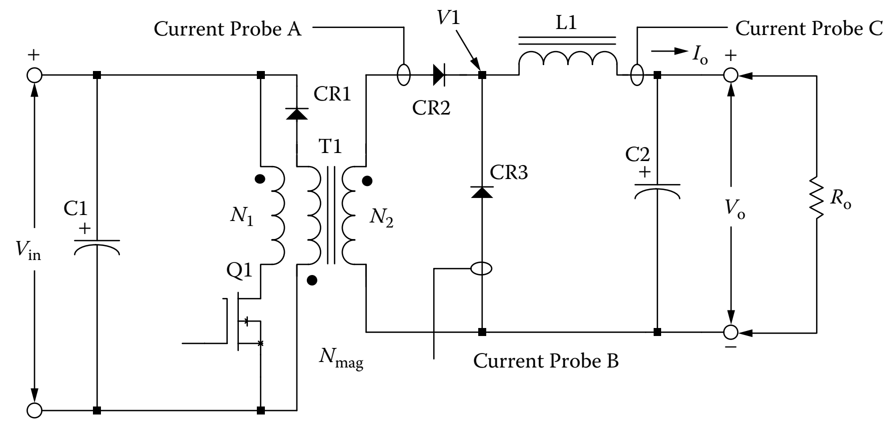

Figure 14-8. Typical Single-ended, Forward Converter.

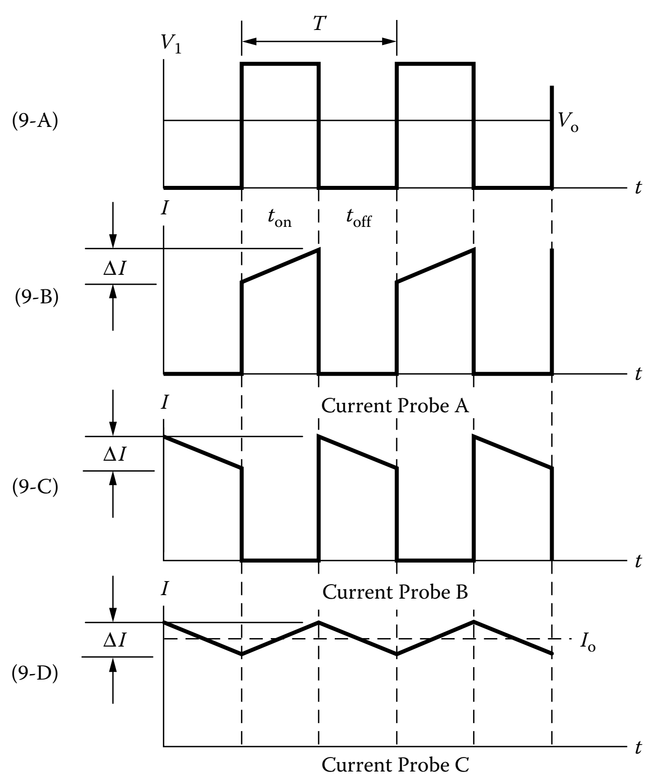

Figure 14-9. Typical Forward Converter Waveforms, Operating at a 0.5 Duty Ratio.

flowing through, CR3, is shown in Figure (14-9-C). The commutating current is the result of, Q1, being turned off, and the field in, L1, collapsing, producing the commutating current. The current flowing through, L1, is shown in Figure (14-9-D). The current flowing through, L1, is the sum of the currents in Figure (14-9-B) and (14-9-C).

The critical inductance current is shown in Figure (14-10-B). The critical inductance current is when the ratio of the delta current to the output load current is equal to $2 = \Delta I / I_o$. If the output load current is allowed to go beyond this point, the current will become discontinuous, as shown in Figure (14-10-D). The applied voltage, V_1, will have ringing at the level of the output voltage, as shown in Figure (14-10-C). When the current in the output inductor becomes discontinuous, as shown in Figure (14-10-D), the response time for a step load becomes very poor.

When designing multiple output converters, the slaved outputs should never have the current in the inductor go discontinuous, or to zero. If the current goes to zero, a slaved output voltage will rise to the value of, V_1. If the current is allowed to go to zero, then, there is not any potential difference between the input and output voltage of the filter. Then the output voltage will rise to equal the peak input voltage.

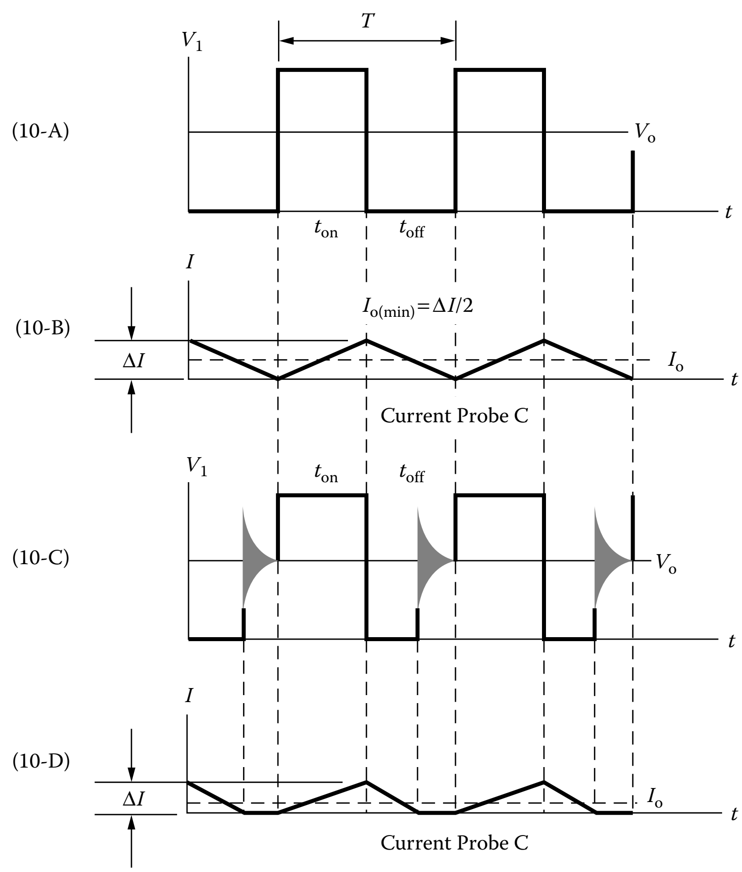

Figure 14-10. Forward Converter; Output Filter Inductor goes from Critical to Discontinuous Operation.

Output Inductor Design Using the Core Geometry, K_g, Approach

The following information is the design specification for a forward converter, 30watt output filter design, operating at 100kHz, using the Kg core geometry approach. For a typical design example, assume an Output Filter Circuit, as shown in Figure 14-1, with the following specifications:

1. Frequency, f = 100kHz
2. Output voltage, $V_o = 5$ volts
3. Output current, $I_{o(max)} = 5.0$ amps
4. Output current, $I_{o(min)} = 0.5$ amps
5. Delta current, $\Delta I = 1.0$ amps
6. Input voltage, $V_{1(max)} = 19$ volts
7. Input voltage, $V_{1(min)} = 12$ volts
8. Regulation, $\alpha = 1.0\%$
9. Output power $(V_o + V_d)$ $(I_{o(max)})$, $P_o = 30$ watts
10. Operating flux density, $B_{pk} = 0.3$ teslas
11. Window utilization, $K_u = 0.4$
12. Diode voltage drop, $V_d = 1.0$ volt

This design procedure will work equally well with all of the various powder cores. Care must be taken regarding maximum flux density with different materials and core loss.

The skin effect on an inductor is the same as a transformer. The main difference is that the ac flux is much lower and does not require the use of the same maximum wire size. The ac flux is caused by the delta current, ΔI, and is normally only a fraction of the dc flux. In this design the ac current and the dc current will be treated the same.

At this point, select a wire so that the relationship between the ac resistance and the dc resistance is 1:

$$\frac{R_{ac}}{R_{dc}} = 1$$

The skin depth, ε, in centimeters, is:

$$\varepsilon = \frac{6.62}{\sqrt{f}}, \quad [\text{cm}]$$

$$\varepsilon = \frac{6.62}{\sqrt{100,000}}, \quad [\text{cm}]$$

$$\varepsilon = 0.0209, \quad [\text{cm}]$$

Then, the wire diameter, D_w, is:

$$D_w = 2(\varepsilon), \quad [\text{cm}]$$

$$D_w = 2(0.0209), \quad [\text{cm}]$$

$$D_w = 0.0418, \quad [\text{cm}]$$

Then, the bare wire area, A_w, is:

$$A_w = \frac{\pi(D_w)^2}{4}, \quad [\text{cm}^2]$$

$$A_w = \frac{(3.1416)(0.0418)^2}{4}, \quad [\text{cm}^2]$$

$$A_w = 0.00137, \quad [\text{cm}^2]$$

From the Wire Table in Chapter 4, Number 27 has a bare wire area of 0.001021 centimeters. This will be the minimum wire size used in this design. If the design requires more wire area to meet the specification, then the design will use a multifilar of #26. Listed Below are #27 and #28, just, in case #26 requires too much rounding off.

Wire AWG	Bare Area	Area Ins.	Bare/Ins.	μΩ/cm
#26	0.001280	0.001603	0.798	1345
#27	0.001021	0.001313	0.778	1687
#28	0.0008046	0.0010515	0.765	2142

Step No. 1: Calculate the total period, T.

$$T = \frac{1}{f}, \quad [\text{seconds}]$$

$$T = \frac{1}{100{,}000}, \quad [\text{seconds}]$$

$$T = 10, \quad [\mu\text{sec}]$$

Step No. 2: Calculate the minimum duty ratio, D_{min}.

$$D_{\min} = \frac{V_o}{V_{1\max}}$$

$$D_{\min} = \frac{5}{19}$$

$$D_{\min} = 0.263$$

Step No. 3: Calculate the required inductance, L.

$$L = \frac{T(V_o + V_d)(1 - D_{\min})}{\Delta I}, \quad \text{[henrys]}$$

$$L = \frac{(10 \times 10^{-6})(5.0 + 1.0)(1 - 0.263)}{(1.0)}, \quad \text{[henrys]}$$

$$L = 44.2, \quad [\mu\text{h}]$$

Step No. 4: Calculate the peak current, I_{pk}.

$$I_{pk} = I_{o(\max)} + \left(\frac{\Delta I}{2}\right), \quad \text{[amps]}$$

$$I_{pk} = (5.0) + \left(\frac{1.0}{2}\right), \quad \text{[amps]}$$

$$I_{pk} = 5.5, \quad \text{[amps]}$$

Step No. 5: Calculate the energy-handling capability in, watt-seconds.

$$\text{Energy} = \frac{L I_{pk}^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Energy} = \frac{(44.2x10^{-6})(5.5)^2}{2}, \quad \text{[watt-seconds]}$$

$$\text{Energy} = 0.000668, \quad \text{[watt-seconds]}$$

Step No. 6: Calculate the electrical conditions, K_e.

$$K_e = 0.145 P_o B_m^2 \times 10^{-4}$$

$$K_e = (0.145)(30)(0.3)^2 \times 10^{-4}$$

$$K_e = 0.0000392$$

Step No. 7: Calculate the core geometry, K_g.

$$K_g = \frac{(\text{Energy})^2}{K_e \alpha} \quad [\text{cm}^5]$$

$$K_g = \frac{(0.000668)^2}{(0.0000392)(1.0)} \quad [\text{cm}^5]$$

$$K_g = 0.01138 \quad [\text{cm}^5]$$

Step No. 8: Select, from Chapter 4, a MPP powder core, comparable in core geometry, K_g.

Core number = MP-55059-A2

Manufacturer = Magnetics

Magnetic path length, MPL = 5.7 cm

Core weight, W_{tfe} = 16.0 grams

Copper weight, W_{tcu} = 15.2 grams

Mean length turn, MLT = 3.2 cm

Iron area, $A_c = 0.331$ cm^2

Window Area, $W_a = 1.356$ cm^2

Area Product, $A_p = 0.449$ cm^4

Core geometry, $K_g = 0.0184$ cm^5

Surface area, $A_t = 28.6$ cm^2

Permeability, $\mu = 60$

Millihenrys per 1000 turns, AL = 43

Step No. 9: Calculate the number of turns, N.

$$N = 1000\sqrt{\frac{L_{(new)}}{L_{(1000)}}}, \quad \text{[turns]}$$

$$N = 1000\sqrt{\frac{0.0442}{43}}, \quad \text{[turns]}$$

$$N = 32, \quad \text{[turns]}$$

Step No. 10: Calculate the rms current, I_{rms}.

$$I_{rms} = \sqrt{I_{o(\max)}^2 + \Delta I^2}, \quad \text{[amps]}$$

$$I_{rms} = \sqrt{(5.0)^2 + (1.0)^2}, \quad \text{[amps]}$$

$$I_{rms} = 5.1, \quad \text{[amps]}$$

Step No. 11: Calculate the current density, J, using a window utilization, $K_u = 0.4$.

$$J = \frac{NI}{W_a K_u}, \quad \text{[amps-per-cm}^2\text{]}$$

$$J = \frac{(32)(5.1)}{(1.36)(0.4)}, \quad \text{[amps-per-cm}^2\text{]}$$

$$J = 300, \quad \text{[amps-per-cm}^2\text{]}$$

Step No. 12: Calculate the required permeability, $\Delta\mu$.

$$\Delta\mu = \frac{B_{pk}(\text{MPL})\left(10^4\right)}{0.4\pi W_a J K_u}, \quad \text{[perm]}$$

$$\Delta\mu = \frac{(0.3)(5.7)\left(10^4\right)}{(1.26)(1.36)(300)(0.4)}, \quad \text{[perm]}$$

$$\Delta\mu = 83.1, \quad \text{use } 60 \quad \text{[perm]}$$

Step No. 13: Calculate the peak flux density, B_{pk}.

$$B_{pk} = \frac{0.4\pi N I_{pk} \mu_r \left(10^{-4}\right)}{(\text{MPL})}, \quad \text{[teslas]}$$

$$B_{pk} = \frac{(1.26)(32)(5.5)(60)\left(10^{-4}\right)}{(5.7)}, \quad \text{[teslas]}$$

$$B_{pk} = 0.233, \quad \text{[teslas]}$$

Step No. 14: Calculate the required bare wire area, $A_{w(B)}$.

$$A_{w(B)} = \frac{I_{rms}}{J}, \quad [\text{cm}^2]$$

$$A_{w(B)} = \frac{5.1}{300}, \quad [\text{cm}^2]$$

$$A_{w(B)} = 0.017, \quad [\text{cm}^2]$$

Step No. 15: Calculate the required number of strands, S_n.

$$S_n = \frac{A_{w(B)}}{\#26}, \quad \text{[strands]}$$

$$S_n = \frac{0.017}{0.00128}, \quad \text{[strands]}$$

$$S_n = 13, \quad \text{[strands]}$$

Step No. 16: Calculate the new, $\mu\Omega$, per centimeter.

$$(\text{new})\mu\Omega / \text{cm} = \frac{\mu\Omega / \text{cm}}{S_n}$$

$$(\text{new})\mu\Omega / \text{cm} = \frac{1345}{13}$$

$$(\text{new})\mu\Omega / \text{cm} = 103$$

Step No. 17: Calculate the winding resistance, R.

$$R = (\text{MLT}) N \left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R = (3.2)(32)(103)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R = 0.0105, \quad \text{[ohms]}$$

Step No. 18: Calculate the winding copper loss, P_{cu}.

$$P_{cu} = I_{rms}^2 R, \quad \text{[watts]}$$

$$P_{cu} = (5.1)^2 (0.0105), \quad \text{[watts]}$$

$$P_{cu} = 0.273, \quad \text{[watts]}$$

Step No. 19: Calculate the magnetizing force in oersteds, H.

$$H = \frac{0.4\pi N I_{pk}}{\text{MPL}}, \quad \text{[oersteds]}$$

$$H = \frac{(1.26)(32)(5.5)}{5.7}, \quad \text{[oersteds]}$$

$$H = 38.9, \quad \text{[oersteds]}$$

Step No. 20: Calculate the ac flux density in teslas, B_{ac}.

$$B_{ac} = \frac{0.4\pi N\left(\frac{\Delta I}{2}\right)\mu_r\left(10^{-4}\right)}{\text{MPL}}, \quad \text{[teslas]}$$

$$B_{ac} = \frac{(1.26)(32)(0.5)(60)\left(10^{-4}\right)}{(5.7)}, \quad \text{[teslas]}$$

$$B_{ac} = 0.0212, \quad \text{[teslas]}$$

Step No. 21: Calculate the regulation, α, for this design.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

$$\alpha = \frac{(0.273)}{(30)}(100), \quad [\%]$$

$$\alpha = 0.91, \quad [\%]$$

Step No. 22: Calculate the watts per kilogram, WK, using MPP 60 perm powder cores coefficients, shown in Chapter 2.

$$WK = 0.551\left(10^{-2}\right) f^{(1.23)} B_{ac}^{(2.12)}, \quad \text{[watts-per-kilogram]}$$

$$WK = 0.551\left(10^{-2}\right)(100000)^{(1.23)}(0.0212)^{(2.12)}, \quad \text{[watts-per-kilogram]}$$

$$WK = 2.203, \quad \text{[watts-per-kilogram]}$$

Step No. 23: Calculate the core loss, P_{fe}.

$$P_{fe} = \left(\frac{\text{milliwatts}}{\text{gram}}\right) W_{tfe}\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = (2.203)(16)\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = 0.0352, \quad \text{[watts]}$$

Step No. 24: Calculate the total loss, P_Σ, core, P_{fe}, and copper, P_{cu}, in watts.

$$P_\Sigma = P_{fe} + P_{cu}, \quad \text{[watts]}$$

$$P_\Sigma = (0.0352) + (0.273), \quad \text{[watts]}$$

$$P_\Sigma = 0.308, \quad \text{[watts]}$$

Step No. 25: Calculate the watt density, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad \text{[watts-per-cm}^2\text{]}$$

$$\psi = \frac{(0.308)}{(28.6)}, \quad \text{[watts-per-cm}^2\text{]}$$

$$\psi = 0.0108, \quad \text{[watts-per-cm}^2\text{]}$$

Step No. 26: Calculate the temperature rise, in, °C.

$$T_r = 450(\psi)^{(0.826)}, \quad [°\text{C}]$$

$$T_r = 450(0.0108)^{(0.826)}, \quad [°\text{C}]$$

$$T_r = 10.7, \quad [°\text{C}]$$

Step No. 27: Calculate the window utilization, K_u.

$$K_u = \frac{N S_n A_{w(B)}}{W_a}$$

$$K_u = \frac{(32)(13)(0.00128)}{(1.356)}$$

$$K_u = 0.393$$

Recognition

The author would like to thank the late **Dr. J. K. Watson**, Professor of Electrical Engineering at the University of Florida for his help with the Forward Converter design equations.

Chapter 15

Input Filter Design

Table of Contents

Introduction

Today, almost all modern equipment uses some sort of power conditioning. There are a lot of different circuit topologies used. When you get to the bottom line, all power conditioning requires some kind of an input filter. The input, LC, filter has become very critical in its design and must be designed not only for EMI, but also for system stability, and for the amount of ac ripple current drawn from the source.

The input voltage supplied to the equipment is also supplied to other users. For this reason, there is a specification requirement regarding the amount of ripple current seen at the source, as shown in Figure 15-1. Ripple currents generated by the user induce a ripple voltage, V_z, across the source impedance. This ripple voltage could impede the performance of other equipment connected to the same bus.

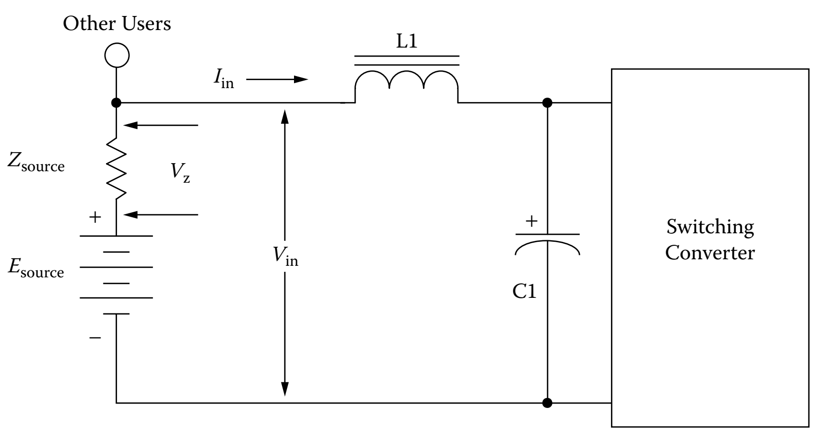

Figure 15-1. Simple, LC, Input Filter.

Capacitor

Switching regulators have required the engineer to put a significantly more analytical effort into the design of the input filter. The current pulse, induced by the switching regulator, has had the most impact on the input capacitor. These current pulses required the use of high quality capacitors with low ESR. The waveforms, induced by the switching regulator, are shown in Figure 15-2. In the input inductor, L1, peak-peak ripple current is, I_L. In the capacitor, C1, peak-peak, ripple current is, I_c. In the capacitor, C1, peak-peak, ripple voltage is, ΔV_c. The equivalent circuit for the capacitor is shown in Figure 15-3. The voltage, ΔV_c, developed across the capacitor, is the sum of two components, the Equivalent Series Resistance, (ESR), and the reactance of the capacitor.

The voltage, V_{CR}, developed across the Equivalent Series Resistance, (ESR), is shown in Equation [15-1].

$$V_{CR} = I_C(\text{ESR}), \quad \text{[volts]} \qquad [15\text{-}1]$$

The voltage, ΔV_{CC}, developed across the capacitance is shown in Equation [15-2].

$$\Delta V_{CC} = I_C\left(\frac{(t_{on})(t_{off})}{(C1)(T)}\right), \quad \text{[volts]} \qquad [15\text{-}2]$$

The sum of the two voltages, ΔV_{CR} and ΔV_{CC}, is shown in Equation [15-3].

$$\Delta V_C = \Delta V_{CR} + \Delta V_{CC}, \quad \text{[volts]} \qquad [15\text{-}3]$$

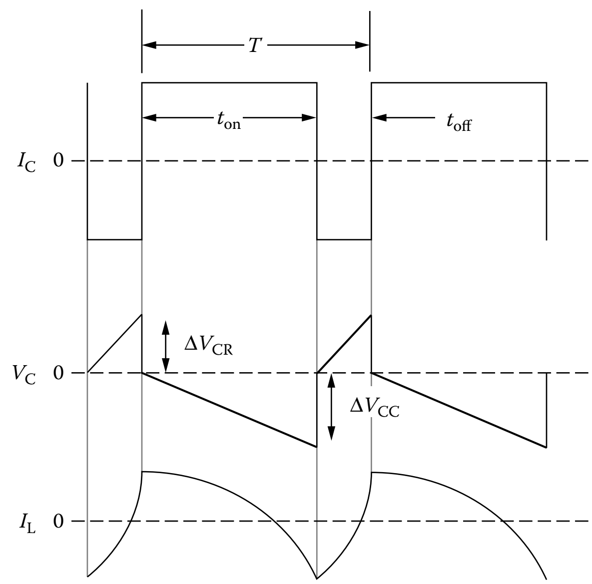

Figure 15-2. Typical Voltage, Current Waveforms.

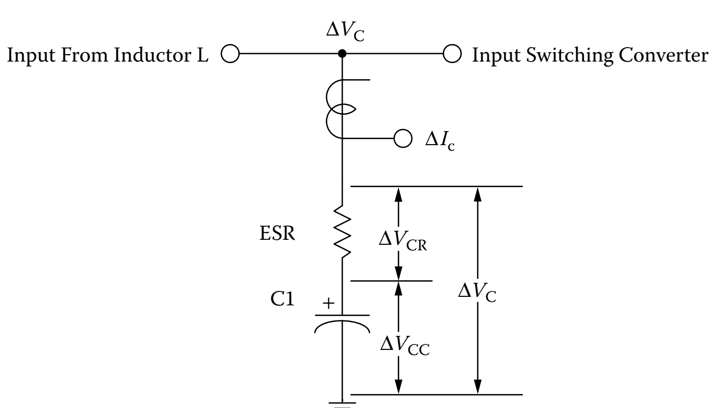

Figure 15-3. Capacitor, Individual Ripple, Components.

Inductor

The input filter inductor is basically a straightforward design. There are four parameters required to achieve a good design: (1) required inductance, (2) dc current, (3) dc resistance, and (4) temperature rise. The requirement for the input inductor is to provide a low ac ripple current to the source. The low ac ripple current in the inductor produces an ac flux at a magnitude of about 0.025 teslas. This resulting low ac flux will keep the core loss to a minimum. The input inductor losses will normally be 80 to 90% copper. A high flux magnetic material is ideally suited in this application. Operating with a high dc flux and a low ac flux, silicon, with its high flux density of 1.6 teslas, will produce the smallest size, as shown in Table 15-1.

Table 15-1. Most Commonly Used Input Filter Material

Magnetic Material Properties		
Material	Operating Flux, B, teslas	Permeability μ_i
Silicon	1.5-1.8	1.5K
Permalloy Powder	0.3	14-550
Iron Power	1.2-1.4	35-90
Ferrite	0.3	1K-15K

Oscillation

The input filter can affect the stability of the associated switching converter. The stability problem results from an interaction between the output impedance of the input filter and the input impedance of the switching converter. Oscillation occurs when the combined positive resistance of the LC filter, and power source exceed the negative dynamic resistance of the regulator's dc input. To prevent oscillation, the capacitor's ESR, and the inductor's resistance must provide sufficient damping. Oscillation will not occur when the conditions are, as shown in Equation [15-4].

$$\left(\frac{\eta(V_{in})^2}{P_o}\right) > \left(\frac{\dfrac{L}{C+(R_L+R_s)(\mathrm{ESR})}}{(R_L+R_s)+(\mathrm{ESR})}\right) \qquad [15\text{-}4]$$

Where, η, is the switching converter efficiency, $V_{in(max)}$, is the input voltage; P_o, is the output power in watts; L, is the input inductor in henrys; where, C, is the filter capacitor in farads, R_L, is inductor series resistance in ohms; R_s, is the source resistance in ohms, and, R_d (ESR), is the equivalent series resistance in ohms. If additional damping, is required, it can be done, by increasing the, R_d (ESR), and/or, R_L. See Figure 15-4. The series resistance, R_d, lowers the, Q, of the filter and kills the potential oscillation.

Applying Power

The inrush current has always been a problem with this simple, LC, input filter. When a step input is applied, such as a relay or switch, S1, as shown in Figure 15-5, there is always a high inrush current.

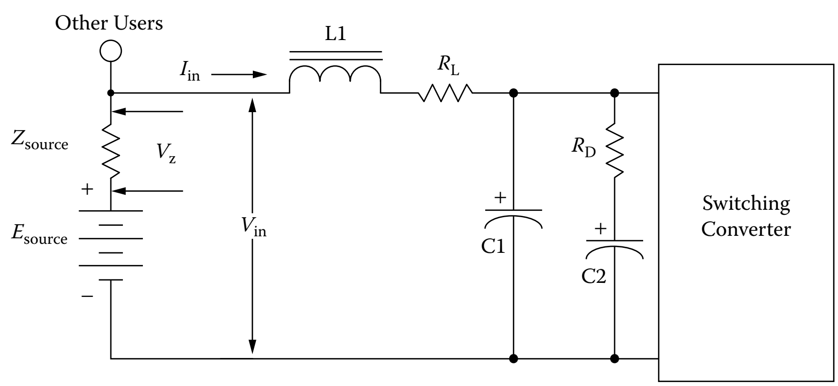

Figure 15-4. Input Filter, with Additional Damping.

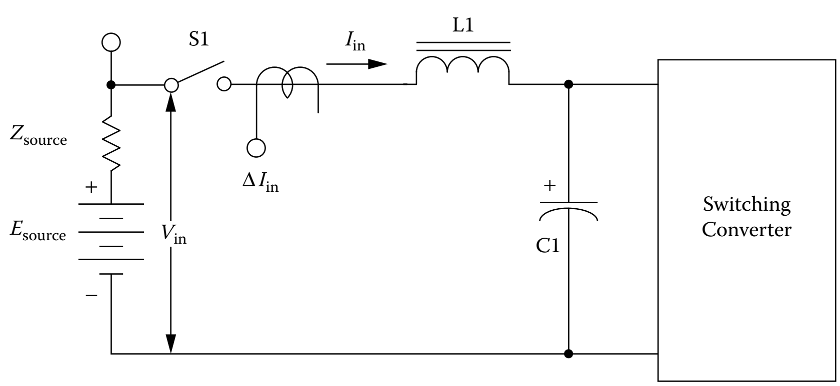

Figure 15-5. Input Filter Inrush Current Measurement.

When, S1, is closed, the full input voltage, V_{in}, is applied directly across the input inductor, L1, because, C1, is discharged. The applied input voltage, V_{in}, (volt-seconds), to the input inductor, L1, and the dc current, (amp-turns), flowing through it is enough to saturate the core. The inductor, L1, is normally designed, using the upper limits of the flux density for minimum size. There are two types of core configurations commonly used for input inductor design: powder cores and gapped cores. Some engineers prefer to design around powder cores because they are simple and less of a hassle, while others design using gapped cores. It is strictly a game of trade-offs. Tests were performed using three different core materials: (1) powder core, (2) ferrite core, and (3) iron alloy. All three materials were designed to have the same inductance and the same dc resistance. The three-inductor designs were tested to compare the inrush current under the same conditions. The inrush current, ΔI, for all three materials is shown in Figure 15-6, using the test circuit, shown in Figure 15-5.

As, shown in Figure 15-6, the inrush current for all three test inductors has about the same general shape and amplitude. The changes in permeability, with dc bias, for both gapped and powder cores are shown in Figure 15-7. Gapped cores have a definitely sharper knee while the powder cores roll off more gradually. The advantage in using a gapped core over the powder core is the ability to use the full flux capacity of the core up to the knee, before the permeability starts to droop.

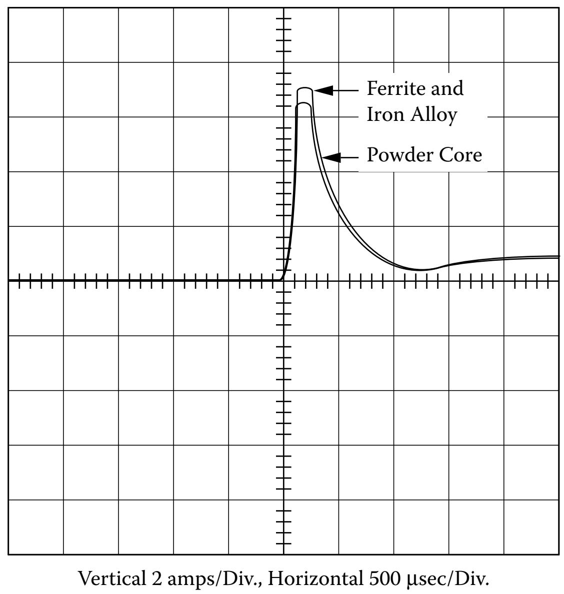

Figure 15-6. Typical, Inrush Current for a Simple Input.

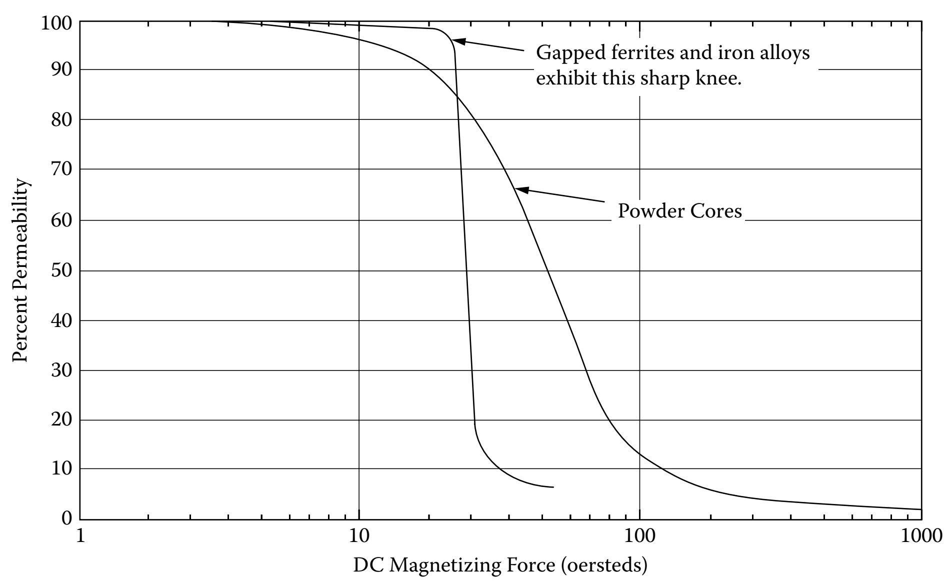

Figure 15-7. Comparing Gapped and Powder Cores, Permeability Change with DC Bias.

Resonant Charge

Most all types of electronic equipment are energized by either a switch or relay. This type of turn-on goes for spacecraft, aircraft, computer, medical equipment, and automobiles. There are some power sources that require some type of current limiting that do not follow the general rule. If the input voltage is applied via a switch or relay to an input filter, as shown in Figure 15-8, a resonant charge condition will develop with, L1 and C1. The resulting resonant charge, with L1 and C1, could put a potential on, C1, which could be as much as twice the applied input voltage, as shown in Figure 15-9. The voltage rating of, C1, must be high enough to sustain this peak voltage without damage. The oscillating voltage is applied to the switching converter.

A simple way to dampen this oscillation is to place one or two diodes across the input choke, as shown in Figure 15-10. The reason for two diodes is the ripple voltage, V_c, might be greater than the threshold voltage of the diode. As the voltage across, C1, rises above the input voltage, V_{in}, due to the oscillation diodes, CR1 and CR2, will become forward-biased, clamping the voltage across, C1, to two diode drops above the input voltage, V_{in}, as shown in Figure 15-11.

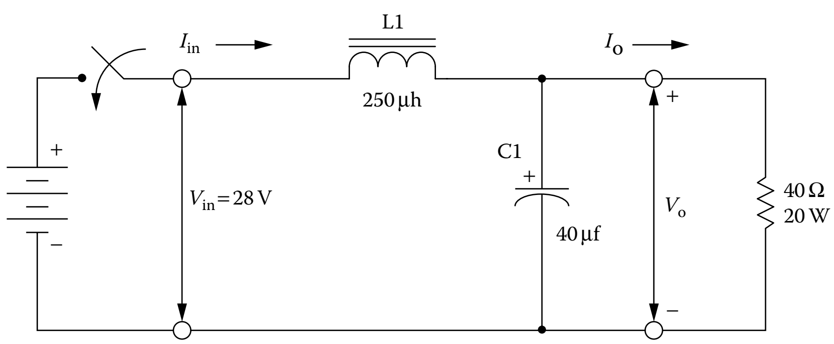

Figure 15-8. Typical, Simple LC, Input Filter.

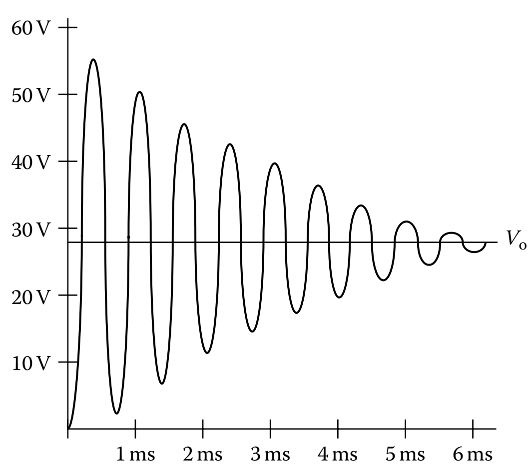

Figure 15-9. Resonating Voltage, across Capacitor, C1.

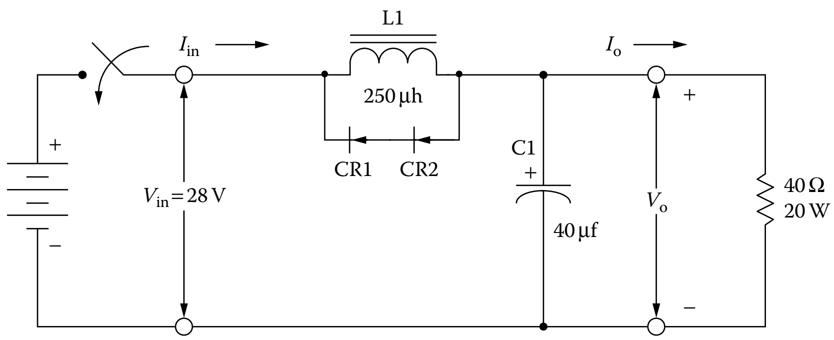

Figure 15-10. Input Inductor with Clamp Diodes.

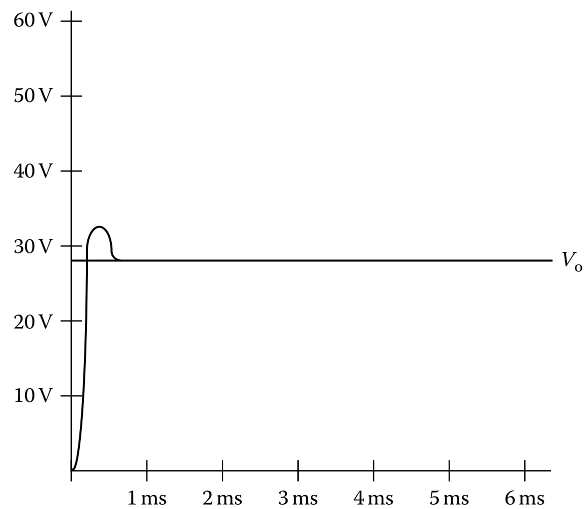

Figure 15-11. DC Voltage Across C1, with the Clamp Diodes.

Input Filter Inductor Design Procedure

The input filter inductor, L1, for this design is shown in Figure 15-12.

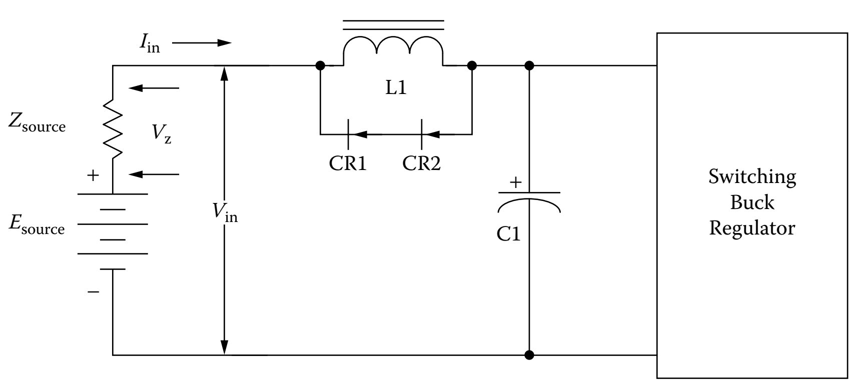

Figure 15-12. Input Filter Circuit.

The ac voltages and currents associated with the input capacitor, C1, are shown in Figure 15-13.

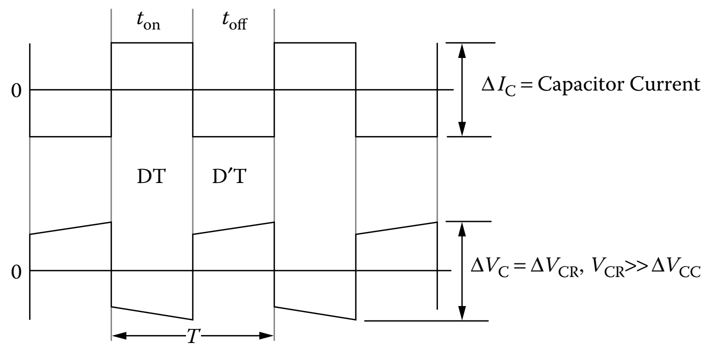

Figure 15-13. Input Capacitor Voltage and Current Ripple.

The ac voltages and currents impressed on the input capacitor, C1, are defined in Figure 15-14.

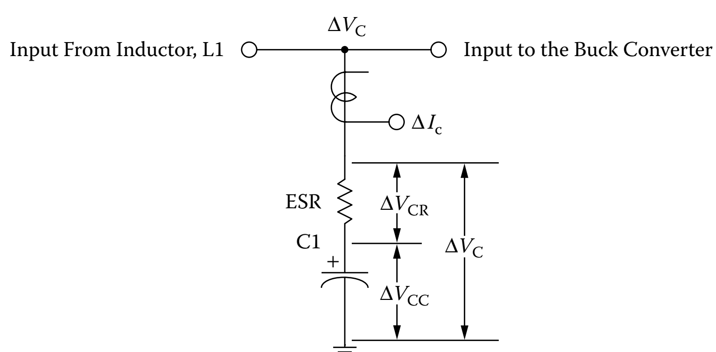

Figure 15-14. Defining the Input Capacitor Voltage and Current Ripple.

The components of the inductor current, due to, ΔV_{CR}, and ΔV_{CC}, are:

$\Delta V_{CR} =$ Peak to Peak component due to capacitor, ESR.

$\Delta V_{CC} =$ Peak to Peak component due to capacitor.

$\Delta I_{LR} =$ Component of the inductor ripple current developed by, ΔV_{CR}.

$\Delta I_{LC} =$ Component of the inductor ripple current developed by, ΔV_{CC}.

$$\Delta I_{LR} = \left(\frac{\Delta V_{CR}}{L}\right)\left(DD'T\right), \quad \text{[amps]}$$
$$\Delta I_{LC} = \left(\frac{\Delta V_{CC}}{2L}\right)\left(\frac{T}{4}\right), \quad \text{[amps]} \qquad [15\text{-}5]$$

It will be considered that, ΔI_{LR}, dominates because of the capacitor, ESR, so:

$$\Delta I_{LR} = \left(\frac{\Delta V_{CR}}{L}\right)\left(DD'T\right), \quad \text{[amps]} \qquad [15\text{-}6]$$

Input Filter Design Specification

1. Peak-Peak ripple voltage, $\Delta V_{cr} = 0.5$ volts
2. Peak-Peak ripple current to the source, $\Delta I_L = 0.010$ amps
3. Period, T = 10 μsec
4. *Converter on-time duty cycle, $D = t_{on}/T = 0.5$
5. *Converter off-time duty cycle, $D' = t_{off}/T = 0.5$
6. Regulation, $\alpha = 0.5\%$
7. Output power drawn from the filter network, $P_o = 50$ watts
8. Maximum current to the load, $\Delta I_c = 4$ amps
9. Average input current, $I_{in} = I_{av} = \Delta I_c D = 2$ amps
10. The ripple frequency, f = 100kHz
11. The core RM ferrite, gapped, $B_{max} = 0.25$ teslas

Step No. 1: Calculate the required inductance, L.

$$L = \frac{\Delta V_{CR}}{\Delta I_L}\left(DD'T\right), \quad \text{[henrys]}$$

$$L = \frac{0.5}{0.01}(0.5)(0.5)\left(10\left(10^{-6}\right)\right), \quad \text{[henrys]}$$

$$L = 0.000125, \quad \text{[henrys]}$$

Step No. 2: Calculate the energy-handling capability.

$$\text{Energy} = \frac{L\left(I_{av}\right)^2}{2}, \quad \text{[watt-second]}$$

$$\text{Energy} = \frac{125\left(10^{-6}\right)(2.0)^2}{2}, \quad \text{[watt-second]}$$

$$\text{Energy} = 0.000250, \quad \text{[watt-second]}$$

Step No. 3: Calculate the electrical coefficient, K_e.

$$K_e = 0.145 P_o B_m^2\left(10^{-4}\right)$$

$$K_e = 0.145(50)(0.25)^2\left(10^{-4}\right)$$

$$K_e = 0.0000453$$

* The worse case time domain is where, D and D′ = 0.5

Step No. 4: Calculate the core geometry, K_g.

$$K_g = \frac{(\text{Energy})^2}{K_e \alpha}, \quad [\text{cm}^5]$$

$$K_g = \frac{(0.00025)^2}{(0.0000453)(0.5)}, \quad [\text{cm}^5]$$

$$K_g = 0.00275, \quad [\text{cm}^5]$$

Step No. 5: Select the comparable core geometry from the RM ferrite cores.

1. Core part number = RM-6
2. Core geometry, K_g = 0.0044 cm^5
3. Core cross-section, A_c = 0.366 cm^2
4. Window area, W_a = 0.260 cm^2
5. Area product, A_p = 0.0953 cm^4
6. Mean length turn, MLT = 3.1 cm
7. Magnetic path length, MPL = 2.86 cm
8. Core weight, W_{tfe} = 5.5 grams
9. Surface area, A_t = 11.3 cm^2
10. Winding Length, G = 0.82 cm
11. Material, P, μ_m = 2500

Step No. 6: Calculate the current density, J, using the area product equation, A_p.

$$J = \frac{2(\text{Energy})(10^4)}{B_m A_p K_u}, \quad [\text{amps-per-cm}^2]$$

$$J = \frac{2(0.00025)(10^4)}{(0.25)(0.0952)(0.4)}, \quad [\text{amps-per-cm}^2]$$

$$J = 525, \quad [\text{amps-per-cm}^2]$$

Step No. 7: Calculate the required bare wire area, $A_{w(B)}$.

$$A_{W(B)} = \frac{I_{avg}}{J}, \quad [\text{cm}^2]$$

$$A_{W(B)} = \frac{(2.0)}{(525)}, \quad [\text{cm}^2]$$

$$A_{W(B)} = 0.00381, \quad [\text{cm}^2]$$

Step No. 8: Select a wire from the Wire Table in Chapter 4. If the area is not within 10%, take the next smallest size. Also record micro-ohms per centimeter.

$$\text{AWG} = \#\,21$$

$$\text{Bare, } A_{W(B)} = 0.00411, \quad [\text{cm}^2]$$

$$\text{Insulated, } A_W = 0.00484, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 419, \quad [\text{micro-ohm/cm}]$$

Step No. 9: Calculate the effective window area, $W_{a(eff)}$. Use the window area found in Step 5. A typical value for, S_3 is 0.75, as shown in Chapter 4.

$$W_{a(eff)} = W_a S_3, \quad [\text{cm}^2]$$

$$W_{a(eff)} = (0.260)(0.75), \quad [\text{cm}^2]$$

$$W_{a(eff)} = 0.195, \quad [\text{cm}^2]$$

Step No. 10: Calculate the number turns possible, N. Use the insulated wire area, A_w, found in Step 8. A typical value for, S_2, is 0.6, as shown in Chapter 4.

$$N = \frac{W_{a(eff)} S_2}{A_W}, \quad [\text{turns}]$$

$$N = \frac{(0.195)(0.60)}{(0.00484)}, \quad [\text{turns}]$$

$$N = 24, \quad [\text{turns}]$$

Step No. 11: Calculate the required gap, l_g.

$$l_g = \frac{0.4\pi N^2 A_c\left(10^{-8}\right)}{L} - \left(\frac{\text{MPL}}{\mu_m}\right), \quad [\text{cm}]$$

$$l_g = \frac{(1.26)(24)^2(0.366)\left(10^{-8}\right)}{(0.000125)} - \left(\frac{2.86}{2500}\right), \quad [\text{cm}]$$

$$l_g = 0.0201, \quad [\text{cm}]$$

Step No. 12: Calculate the equivalent gap in mils.

$$\text{mils} = \text{cm}(393.7)$$

$$\text{mils} = (0.0197)(393.7)$$

$$\text{mils} = 7.91 \text{ use } 8$$

Step No. 13: Calculate the fringing flux factor, F.

$$F = 1 + \frac{l_g}{\sqrt{A_c}} \ln\left(\frac{2G}{l_g}\right)$$

$$F = 1 + \frac{(0.0201)}{\sqrt{0.366}} \ln\left(\frac{2(0.82)}{0.0201}\right)$$

$$F = 1.146$$

Step No. 14: Calculate the new number of turns, N_n, by inserting the fringing flux, F.

$$N_n = \sqrt{\frac{l_g L}{0.4\pi A_c F\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N_n = \sqrt{\frac{(0.0201)(0.000125)}{(1.26)(0.366)(1.146)\left(10^{-8}\right)}}, \quad \text{[turns]}$$

$$N_n = 22, \quad \text{[turns]}$$

Step No. 15: Calculate the winding resistance, R_L. Use the, MLT, from Step 5 and the micro-ohm per centimeter, from Step 8.

$$R_L = (\text{MLT})(N_n)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_L = (3.1)(22)(419)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_L = 0.0286, \quad \text{[ohms]}$$

Step No. 16: Calculate the copper loss, P_{cu}.

$$P_{cu} = I_{avg}^2 R_L, \quad \text{[watts]}$$

$$P_{cu} = (2.0)^2(0.0286), \quad \text{[watts]}$$

$$P_{cu} = 0.114, \quad \text{[watts]}$$

Step No. 17: Calculate the regulation, α.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

$$\alpha = \frac{(0.114)}{(50)}(100), \quad [\%]$$

$$\alpha = 0.228, \quad [\%]$$

Step No. 18: Calculate the ac flux density, B_{ac}.

$$B_{ac} = \frac{0.4\pi N_n F\left(\frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]}$$

$$B_{ac} = \frac{(1.26)(22)(1.14)\left(\frac{0.01}{2}\right)\left(10^{-4}\right)}{(0.0197) + \left(\frac{2.86}{2500}\right)}, \quad \text{[teslas]}$$

$$B_{ac} = 0.000758, \quad \text{[teslas]}$$

Step No. 19: Calculate the watts-per-kilogram, for ferrite, P, material in Chapter 2. Watts per kilogram can be written in milliwatts-per-gram.

$$\text{mW/g} = k\, f^{(m)} B_{ac}^{(n)}$$

$$\text{mW/g} = (0.00198)(100000)^{(1.36)}(0.000758)^{(2.86)}$$

$$\text{mW/g} = 0.0000149$$

Step No. 20: Calculate the core loss, P_{fe}.

$$P_{fe} = (\text{mW/g})\left(W_{tfe}\right)\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = (0.0000149)(5.5)\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = 0.082\left(10^{-6}\right), \quad \text{[watts]}$$

Step No. 21: Calculate the total loss copper plus iron, P_{Σ}.

$$P_{\Sigma} = P_{fe} + P_{cu}, \quad \text{[watts]}$$

$$P_{\Sigma} = (0.000) + (0.114), \quad \text{[watts]}$$

$$P_{\Sigma} = 0.114, \quad \text{[watts]}$$

Step No. 22: Calculate the watt density, ψ. The surface area, A_t, can be found in Step 5.

$$\psi = \frac{P_{\Sigma}}{A_t}, \quad \text{[watts/cm}^2\text{]}$$

$$\psi = \frac{(0.114)}{(11.3)}, \quad \text{[watts/cm}^2\text{]}$$

$$\psi = 0.010, \quad \text{[watts/cm}^2\text{]}$$

Step No. 23: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [°C]$$

$$T_r = 450(0.010)^{(0.826)}, \quad [°C]$$

$$T_r = 10.0, \quad [°C]$$

Step No. 24: Calculate the peak flux density, B_{pk}.

$$B_{pk} = \frac{0.4\pi N_n F\left(I_{dc} + \frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad [\text{teslas}]$$

$$B_{pk} = \frac{(1.26)(22)(1.14)(2.005)\left(10^{-4}\right)}{(0.0197) + \left(\frac{2.86}{2500}\right)}, \quad [\text{teslas}]$$

$$B_{pk} = 0.304, \quad [\text{teslas}]$$

Step No. 25: Calculate the window utilization, K_u.

$$K_u = \frac{A_{w(B)} N_n}{W_a}$$

$$K_u = \frac{(0.00411)(22)}{(0.260)}$$

$$K_u = 0.348$$

Recognition

I would like to thank Jerry Fridenberg, for modeling the circuits in Figure 15-8 and 15-10, on his SPICE program. The modeling results are shown in Figures 15-9 and 15-11.

References

1. T. K. Phelps and W. S. Tate, “Optimizing Passive Input Filter Design,” (no source).

2. David Silber, “Simplifying the Switching Regulator Input Filter,” *Solid-State Power Conversion*, May/June 1975.

3. Dan Sheehan, “Designing a Regulator’s LC Input Filter: ‘Ripple’ Method Prevents Oscillation Woes,” *Electronic Design* 16, August 2, 1979.

Chapter 16

Current Transformer Design

Table of Contents

Introduction

Current transformers are used to measure, or monitor, the current in the lead of an ac power circuit. They are very useful in high-power circuits, where the current is large, i.e., higher than the ratings of so-called self-contained current meters. Other applications relate to overcurrent and undercurrent relaying for power circuit protection, such as, in the power lead of an inverter or converter. Multiturn secondaries then provide a reduced current for detecting overcurrent, undercurrent, peak current, and average current, as shown in Figure 16-1.

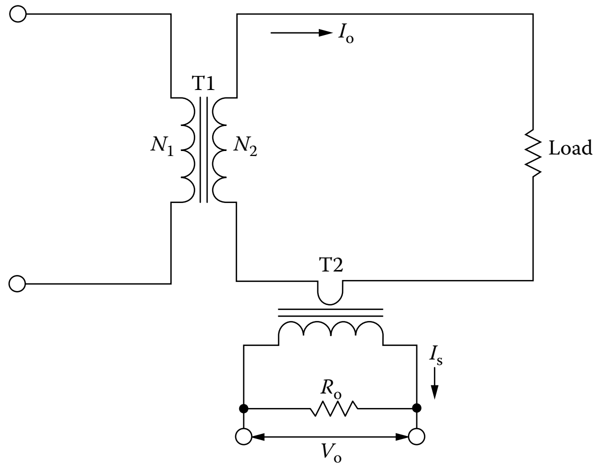

Figure 16-1. Simple, Secondary AC Current Monitor.

In current transformer designs, the core characteristics must be carefully selected because excitation current, I_m, essentially subtracts from the metered current and effects the true ratio and phase angle of the output current.

The simplified equivalent circuit of a current transformer, as shown in Figure 16-2, represents the important elements of a current transformer, where the ratio of primary to secondary turns is shown in the Equation [16-1].

$$n = \frac{N_s}{N_p}, \quad \text{[turns ratio]} \qquad [16\text{-}1]$$

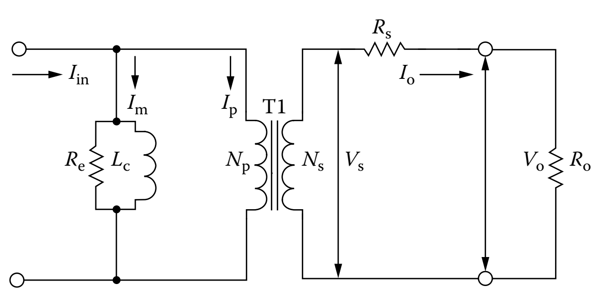

Figure 16-2. Simplified, Equivalent Circuit for a Current Transformer.

Analysis of the Input Current Component

A better understanding of the current transformer behavior may be achieved by considering the applied input current to the primary winding, in terms of various components. Only the ampere-turn component, $I_{in}N_p$, drives the magnetic flux around the core. The ampere-turn, I_mN_p, provides the core loss. The secondary ampere-turns, I_sN_p, balance the remainder of the primary ampere-turns.

The exciting current, I_m, in Figure 16-2, determines the maximum accuracy that can be achieved with a current transformer. Exciting current, I_m, may be defined as the portion of the primary current that satisfies the hysteresis and eddy current losses of the core. If the values of L_c and R_e, in Figure 16-2, are too low because the permeability of the core material is low and the core loss is high, only a part of the current, (I_p/n), will flow in the output load resistor, R_o. The relationship of the exciting current, I_m, to the load current, I_o, is shown in Figure 16-3.

The exciting current, I_m, is equal to:

$$I_m = \frac{H(\text{MPL})}{0.4\pi N}, \quad \text{[amps]} \qquad [16\text{-}2]$$

Where H is the magnetizing force and material dependent, and MPL is the Magnetic Path Length.

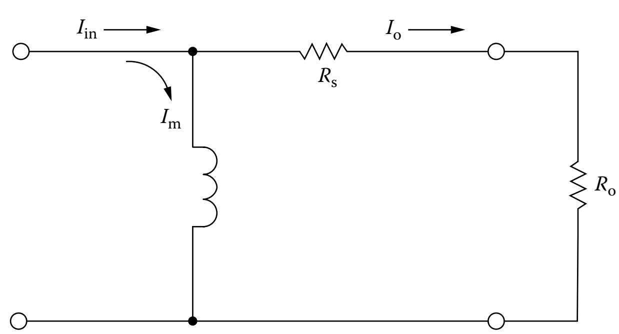

Figure 16-3. Input Current-Output Current Relationship.

The input current is comprised of two components: exciting current, I_m, and the load current, I_o.

$$I_{in}^2 = I_m^2 + I_o^2, \quad \text{[amps]} \qquad [16\text{-}3]$$

Then:

$$I_m^2 = I_{in}^2 - I_o^2, \quad \text{[amps]} \qquad [16\text{-}4]$$

$$I_m = I_{in}\left[1-\left(\frac{I_o}{I_{in}}\right)^2\right]^{1/2}, \quad \text{[amps]} \qquad [16\text{-}5]$$

The above Equation has shown graphically, as in Figure 16-4, that the higher the exciting current, I_m, or core loss, the larger the error. The magnetizing impedance, R_e, determines accuracy, because it shunts part of the input current, I_{in}, away from the primary and thus, produces an error, as shown in Figure 16-4. Core material with the lowest value of, H, achieves the highest accuracy.

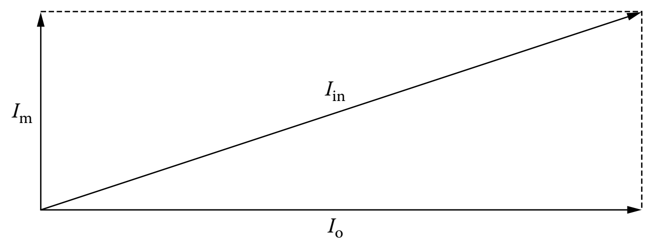

Figure 16-4. Input Current I_{in} Phase Relationship Diagram.

Uniqueness of a Current Transformer

The current transformer function is different than that of a voltage transformer. A current transformer operates with a set primary current and will try to output a constant current to the load, independent of the load. The current transformer will operate into either a short circuit or a resistive load until the voltage induced is enough to saturate the core or cause voltage breakdown. For this reason a current transformer should never operate into an open circuit, as a voltage transformer should never operate into a short circuit. The primary current of a current transformer is not dependent of the secondary load current. The current is really injected into the primary by an external load current, I_{in}. If the load current, I_o, on the current transformer is removed from the secondary winding, while the external load current, I_{in}, is still applied, the flux in the core will rise to a high level, because there is not an opposing current in the secondary winding to prevent this. A very high voltage will appear across the secondary. A current transformer, like any other transformer, must satisfy the amp-turn equation, as shown in Equation [16-6].

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} \qquad [16\text{-}6]$$

The secondary load, R_o, secondary winding resistance, R_s, and secondary load current, I_o, determine the induced voltage of the current transformer, as shown in Equation [16-7].

$$V_s = I_o\left(R_s + R_o\right), \quad \text{[volts]} \qquad [16\text{-}7]$$

If the secondary is designed for dc, then the diode drop must be taken into account, as shown in Equation [16-8].

$$V_s = I_o\left(R_s + R_o\right) + V_d, \quad \text{[volts]} \qquad \text{[16-8]}$$

Simple form:

$$V_s = V_o + V_d, \quad \text{[volts]} \qquad \text{[16-9]}$$

The current ratio will set the turns ratio. The secondary, R_o, load will determine the secondary voltage, V_s. The engineer would use Equation 16-10, to select the required core cross-section, A_c. It is now up to the engineer to pick a core material that would provide the highest permeability at the operating flux density, B_{ac}.

$$A_c = \frac{I_{in}\left(R_s + R_o\right)\left(10^4\right)}{K_f B_{ac} f N_s}, \quad \text{[cm}^2\text{]} \qquad \text{[16-10]}$$

The design requirements would dictate choosing a core material and operating flux density, B_{ac}, that would result in values of, L_c and R_e, as shown in Figure 16-2, values which would be large enough to reduce the current flowing in these elements to satisfy the ratio and phase requirements.

The inductance is calculated from the Equation:

$$L_c = \frac{0.4\pi N_p^2 A_c \Delta\mu\left(10^{-8}\right)}{\text{MPL}}, \quad \text{[henrys]} \qquad \text{[16-11]}$$

R_e is the equivalent core loss, (shunt), resistance. The current is, in phase, with the voltage.

$$R_e = \frac{V_s/n}{P_{fe}}, \quad \text{[ohms]} \qquad \text{[16-12]}$$

Where:

$$\frac{R_e}{n^2} \gg R_s + R_o \qquad \text{[16-13]}$$

And:

$$\frac{2\pi f L_c}{n^2} \gg R_s + R_o \qquad \text{[16-14]}$$

Then:

$$I_p = nI_s \quad [16\text{-}15]$$

Or:

$$I_p N_p = I_s N_s \quad [16\text{-}16]$$

Except for relatively low-accuracy industrial types, current transformers are wound on toroidal cores, which virtually eliminate errors due to leakage inductance. Some errors may be compensated for by adjusting the number of secondary turns.

Current Transformer Circuit Applications

Typical current transformer applications are shown in Figures 16-5 through 16-8.

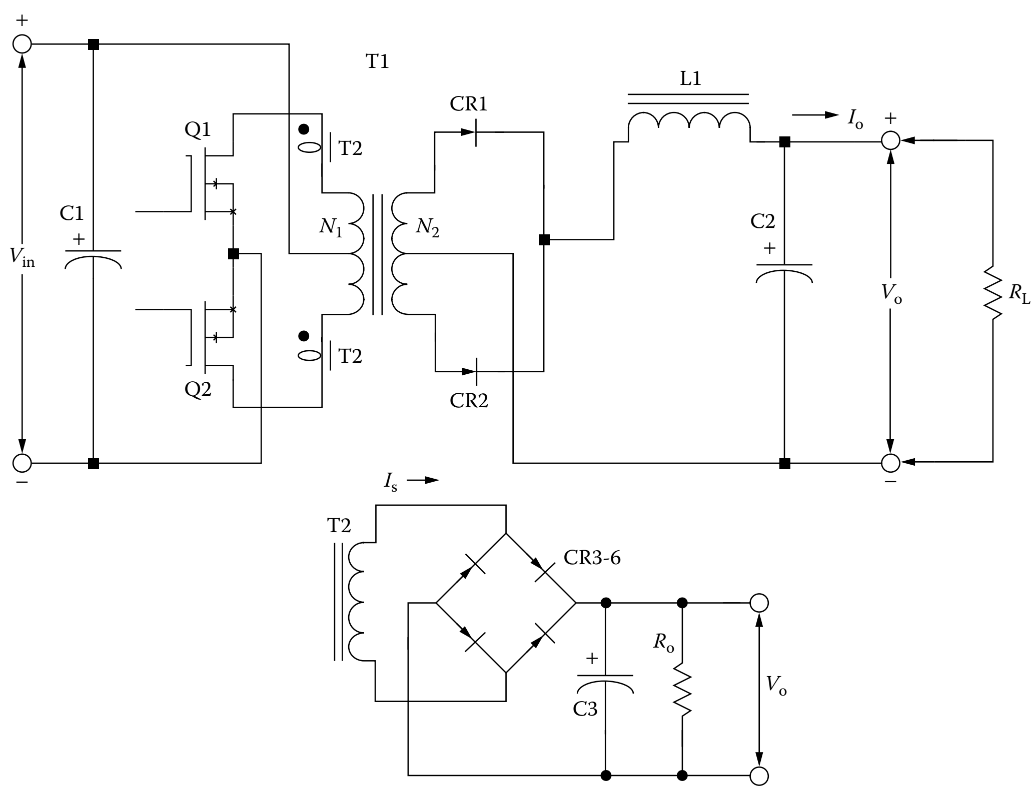

Figure 16-5. Current Transformer, T2, used to Monitor, Q1 and Q2, Drain Current.

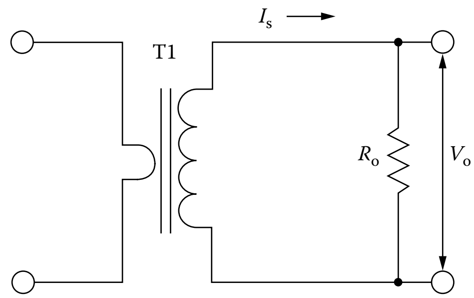

Figure 16-6. Current Transformer, T1, used to Monitor Line Current.

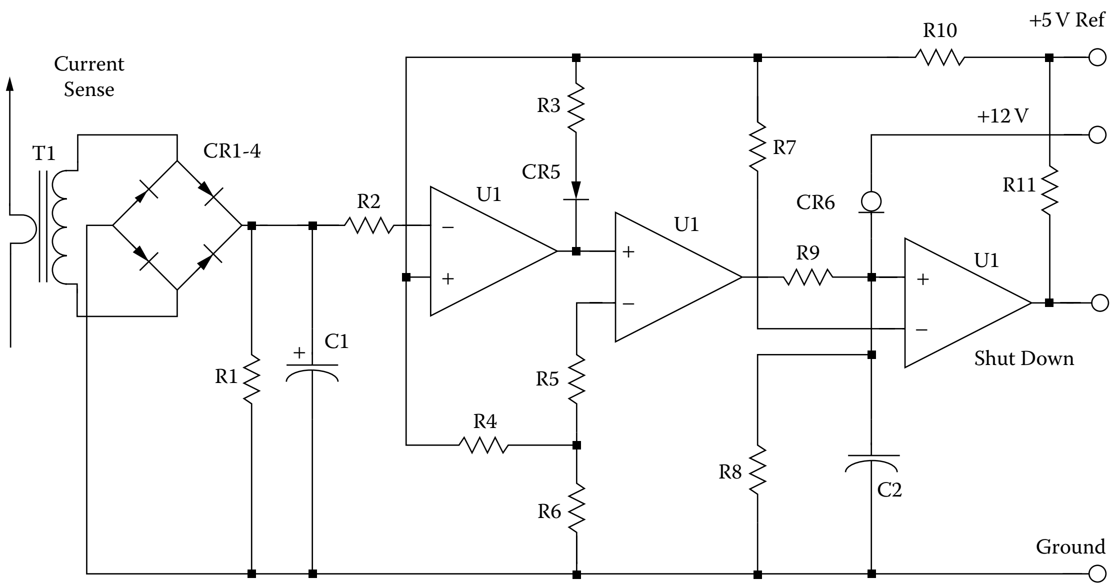

Figure 16-7. Current Transformer, T1, is used as a Level Detector.

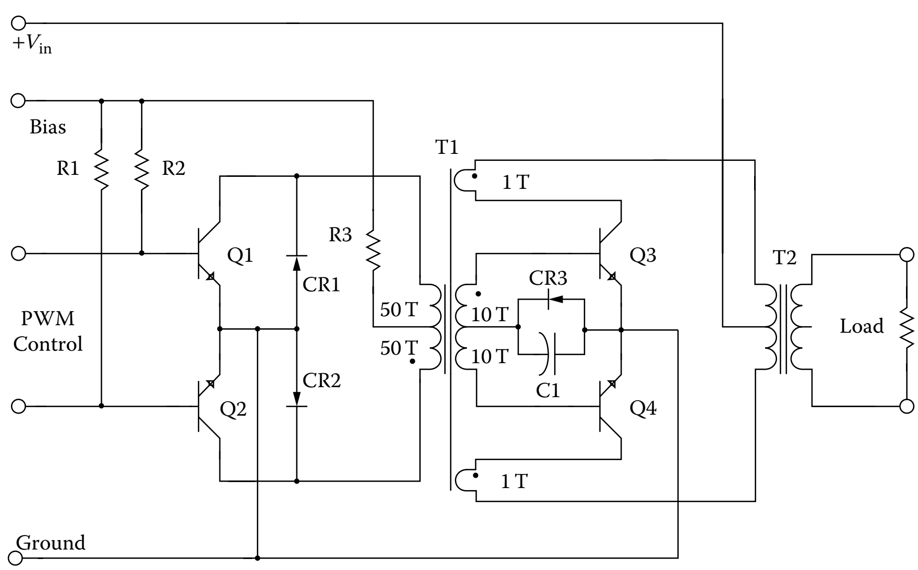

Figure 16-8. Current Transformer, T1, is used for Regenerative Drive.

Current Transformer Design Example

The following information is the design specification for a Current Transformer, as shown in Figure 16-9.

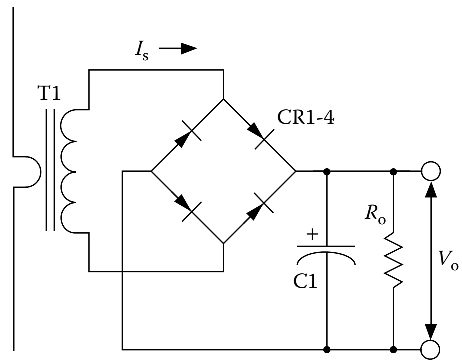

Figure 16-9. Current Monitoring Transformer with dc Output.

1. Primary = 1 turn
2. Input current, I_{in} = 0 – 5 amps
3. Output voltage, V_o = 0 – 5 volts
4. Output load resistance, R_o = 500 ohms
5. Operating frequency, f (square wave) = 2500 hertz
6. Operating flux density, B_{ac} = 0.2 tesla
7. Core loss less than = 3% (error)
8. Diode drop, V_d = 1 volt
9. Magnetic material = Supermalloy 2 mil
10. Waveform factor, K_f = 4.0

Step 1: Calculate the secondary current, I_s.

$$I_s = \frac{V_o}{R_o}, \quad \text{[amps]}$$

$$I_s = \frac{5.0}{500}, \quad \text{[amps]}$$

$$I_s = 0.01, \quad \text{[amps]}$$

Step 2: Calculate the secondary turns, N_s.

$$N_s = \frac{I_p N_p}{I_s}, \quad \text{[turns]}$$

$$N_s = \frac{(5.0)(1.0)}{(0.01)}, \quad \text{[turns]}$$

$$N_s = 500, \quad \text{[turns]}$$

Step 3: Calculate the secondary voltage, V_s.

$$V_s = V_o + 2V_d, \quad \text{[volts]}$$

$$V_s = 5.0 + 2(1.0), \quad \text{[volts]}$$

$$V_s = 7.0, \quad \text{[volts]}$$

Step 4: Calculate the required core iron cross-section, A_c, using Faraday's Equation.

$$A_c = \frac{V_s\left(10^4\right)}{\left(K_f\right)B_{ac}\,f N_s}, \quad \text{[cm}^2\text{]}$$

$$A_c = \frac{(7.0)\left(10^4\right)}{(4.0)(0.2)(2500)(500)}, \quad \text{[cm}^2\text{]}$$

$$A_c = 0.070, \quad \text{[cm}^2\text{]}$$

Step 5: Select a 2mil tape, toroidal core from Chapter 3 with an iron cross-section, A_c, closest to the value calculated.

Toroid = 52000-2F
Manufacturer = Magnetics
Magnetic material = 2mil Supermalloy
Magnetic path length, MPL = 4.99 cm
Core weight, W_{tfe} = 3.3 grams
Copper weight, W_{tcu} = 8.1 grams
Mean length turn, MLT = 2.7 cm
Iron area, A_c = 0.086 cm^2
Window area, W_a = 0.851 cm^2
Area product, A_p = 0.0732 cm^4
Core geometry, K_g = 0.000938 cm^5
Surface area, A_t = 20.6 cm^2

Step 6: Calculate the effective window area, $W_{a(eff)}$. A typical value for, S_3, is 0.75, as shown in Chapter 4.

$$W_{a(eff)} = W_a S_3, \quad \text{[cm}^2\text{]}$$

$$W_{a(eff)} = (0.851)(0.75), \quad \text{[cm}^2\text{]}$$

$$W_{a(eff)} = 0.638, \quad \text{[cm}^2\text{]}$$

Step 7: Calculate the secondary window area, $W_{a\,(sec)}$.

$$W_{a(\sec)} = \frac{W_{a(eff)}}{2}, \quad [\text{cm}^2]$$

$$W_{a(\sec)} = \frac{0.638}{2}, \quad [\text{cm}^2]$$

$$W_{a(\sec)} = 0.319, \quad [\text{cm}^2]$$

Step 8: Calculate the wire area, A_w, with insulation, using a fill factor, S_2 of 0.6.

$$A_w = \frac{W_{a(\sec)} S_2}{N_s}, \quad [\text{cm}^2]$$

$$A_w = \frac{(0.319)(0.6)}{(500)}, \quad [\text{cm}^2]$$

$$A_w = 0.000383, \quad [\text{cm}^2]$$

Step 9: Select a wire area, A_w, with insulation from Wire Table in Chapter 4 for an equivalent AWG wire size. The rule is that when the calculated wire area does not fall within 10% of those listed in the Wire Table, then, the next smaller size should be selected.

AWG No. 33

$$A_w = 0.0003662, \quad [\text{cm}^2]$$

Step 10: Calculate the secondary winding resistance, R_s using the Wire Table in Chapter 4, for μΩ/cm; and Step 5 for the MLT.

$$R_s = \text{MLT}(N_s)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_s = (2.7)(500)(6748)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_s = 9.11, \quad [\text{ohms}]$$

Step 11: Calculate the secondary output power, P_o.

$$P_o = I_s(V_o + 2V_d), \quad [\text{watts}]$$

$$P_o = (0.01)\left(5.0 + 2(1.0)\right), \quad [\text{watts}]$$

$$P_o = 0.070, \quad [\text{watts}]$$

Step 12: Calculate the acceptable core loss, P_{fe}.

$$P_{fe} = P_o\left(\frac{\text{core loss }\%}{100}\right), \quad [\text{watts}]$$

$$P_{fe} = (0.07)\left(\frac{3}{100}\right), \quad [\text{watts}]$$

$$P_{fe} = 0.0021, \quad [\text{watts}]$$

Step 13: Calculate the effective core weight, $W_{tfe(eff)}$. Select the core weight correction factor, K_w, in Chapter 2, for Supermalloy.

$$W_{tfe(eff)} = W_{tfe} K_w, \quad \text{[grams]}$$

$$W_{tfe(eff)} = (3.3)(1.148), \quad \text{[grams]}$$

$$W_{tfe(eff)} = 3.79, \quad \text{[grams]}$$

Step 14: Calculate the allowable core loss, P_{fe}, in milliwatts per gram, mW/g.

$$\text{mW/g} = \frac{P_{fe}}{W_{tfe}}\left(10^3\right), \quad \text{[milliwatts per gram]}$$

$$\text{mW/g} = \frac{(0.0021)}{(3.79)}\left(10^3\right), \quad \text{[milliwatts per gram]}$$

$$\text{mW/g} = 0.554, \quad \text{[milliwatts per gram]}$$

Step 15: Calculate the new flux density using the new core iron, cross-section, A_c.

$$B_{ac} = \frac{V_s\left(10^4\right)}{\left(K_f\right)A_c f N_s}, \quad \text{[teslas]}$$

$$B_{ac} = \frac{(7.0)\left(10^4\right)}{(4.0)(0.086)(2500)(500)}, \quad \text{[teslas]}$$

$$B_{ac} = 0.162, \quad \text{[teslas]}$$

Step 16: Calculate the core loss, P_{fe}, in milliwatts per gram, mW/g.

$$\text{mW/g} = 0.000179(f)^{(1.48)}(B_{ac})^{(2.15)}, \quad \text{[milliwatts per gram]}$$

$$\text{mW/g} = 0.000179(2500)^{(1.48)}(0.162)^{(2.15)}, \quad \text{[milliwatts per gram]}$$

$$\text{mW/g} = 0.382, \quad \text{[milliwatts per gram]}$$

Step 17: Calculate the core loss, P_{fe}, in watts.

$$P_{fe} = W_{tfe}\left(\frac{mW}{g}\right)\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = 3.79(0.382)\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = 0.00145, \quad \text{[watts]}$$

Step 18: Calculate the induced core error in, %.

$$\text{Core loss induced error} = \frac{P_{fe}}{P_o}(100), \quad [\%]$$

$$\text{Core loss induced error} = \frac{0.00145}{.07}(100), \quad [\%]$$

$$\text{Core loss induced error} = 2.07, \quad [\%]$$

Design Performance

A current transformer was built and the data recorded in Table 16-1. It was plotted in Figure 16-10, with an error of 3.4 %. The secondary winding resistance was 6.5 ohms.

Table 16-1.

Current Transformer Electrical Data					
I_{in} amps	I_o volts	I_{in} amps	I_o volts	I_{in} amps	I_o volts
0.250	0.227	1.441	1.377	3.625	3.488
0.500	0.480	2.010	1.929	3.942	3.791
0.746	0.722	2.400	2.310	4.500	4.339
1.008	0.978	2.693	2.593	5.014	4.831
1.262	1.219	3.312	3.181	5.806	5.606

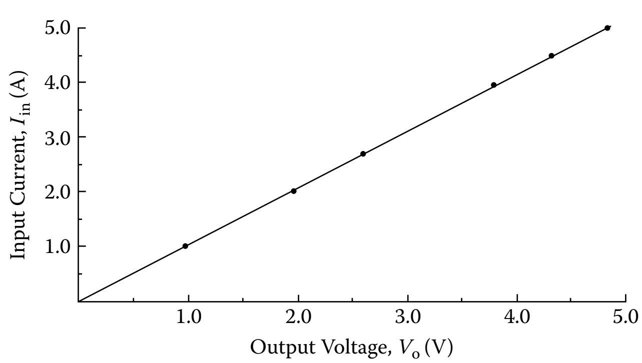

Figure 16-10. Current Transformer, Input Current versus Output Voltage.

Chapter 17

Winding Capacitance and Leakage Inductance

Table of Contents

Introduction

Operation of transformers at high frequencies presents unique design problems due to the increased importance of core loss, leakage inductance, and winding capacitance. The design of high frequency power converters is far less stringent than designing high frequency, wide-band audio transformers. Operating at a single frequency requires fewer turns, and consequently, there is less leakage inductance and less capacitance with which to deal. The equivalent circuit for a two-winding transformer is shown in Figure 17-1.

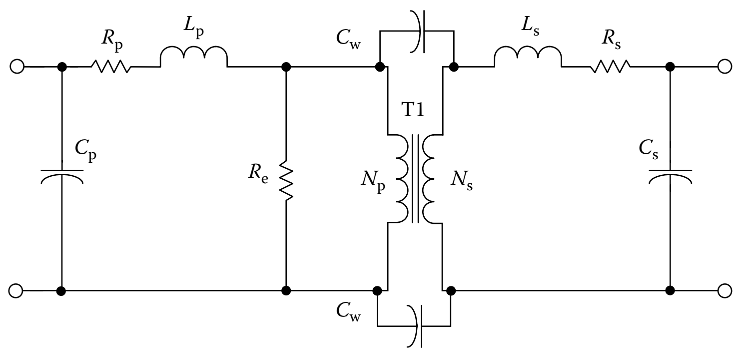

Figure 17-1. Equivalent Transformer Circuit.

High frequency designs require considerably more care in specifying the winding specification. This is because physical orientation and spacing of the windings determine leakage inductance and winding capacitance. Leakage inductance and capacitance are actually distributed throughout the winding in the transformer. However, for simplicity, they are shown as lumped constants, in Figure 17-1. The leakage inductance is represented by L_p for the primary, and L_s for the secondary. The equivalent lumped capacitance is represented by C_p, and, C_s, for the primary and secondary windings. The dc winding resistance is R_p, and, R_s, is for the equivalent resistance for the primary and secondary windings. C_w, is the equivalent lumped, winding-to-winding capacitance. R_e is the equivalent core-loss shunt resistance.

Parasitic Effects

The effects of leakage inductance on switching power supply circuits are shown in Figure 17-2. The voltage spikes, shown in Figure 17-2, are caused by the stored energy in the leakage flux and will increase with load, as shown in Equation 17-1. These spikes will always appear on the leading edge of the voltage switching waveform.

$$Energy = \frac{L_{(Leakage)}\left(I_{(pk)}\right)^2}{2}, \quad \text{[watt-seconds]} \qquad [17\text{-}1]$$

Transformers designed for switching applications are normally designed to have minimum leakage inductance, in order to minimize the voltage spikes, as shown in Figure 17-2. Also, leakage inductance can be observed by the leading edge slope of the trapezoidal current waveform.

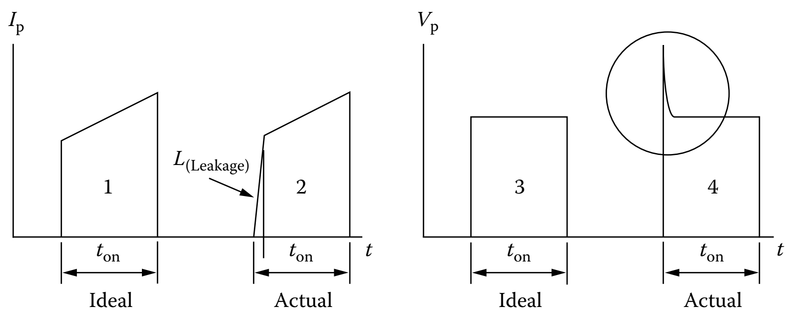

Figure 17-2. Switching Transistor Voltage and Current Waveforms.

Transformers designed for power conversion are normally being driven with a square wave, characterized by fast rise and fall times. This fast transition will generate high current spikes in the primary winding, due to the parasitic capacitance in the transformer. These current spikes, shown in Figure 17-3, are caused by the capacitance in the transformer; they will always appear on the lead edge of the current waveform and always with the same amplitude, regardless of the load. This parasitic capacitance will be charged and discharged every half cycle. Transformer leakage inductance and capacitance have an inverse relationship: if you decrease the leakage inductance, you will increase the capacitance; if you decrease the capacitance, you increase the leakage inductance. These are trade-offs that the power conversion engineer must make to design the best transformer for the application.

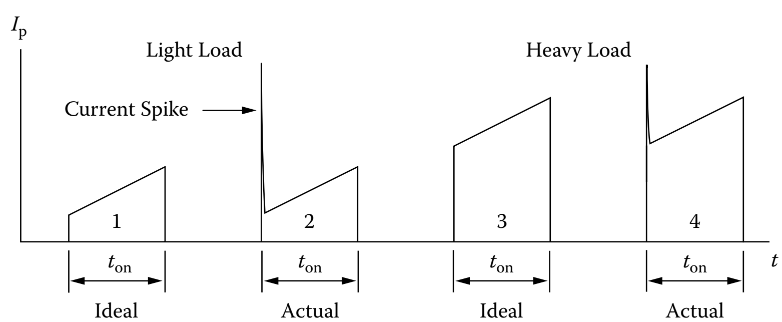

Figure 17-3. Transformer Capacitance Induced Current spike.

Leakage Flux

Leakage inductance is actually distributed throughout the windings of a transformer because of the flux set-up by the primary winding, which does not link the secondary, thus giving rise to leakage inductance in each winding without contributing to the mutual flux, as shown in Figure 17-4.

However, for simplicity, leakage inductance is shown as a lumped constant in Figure 17-1, where the leakage inductance is represented by L_p.

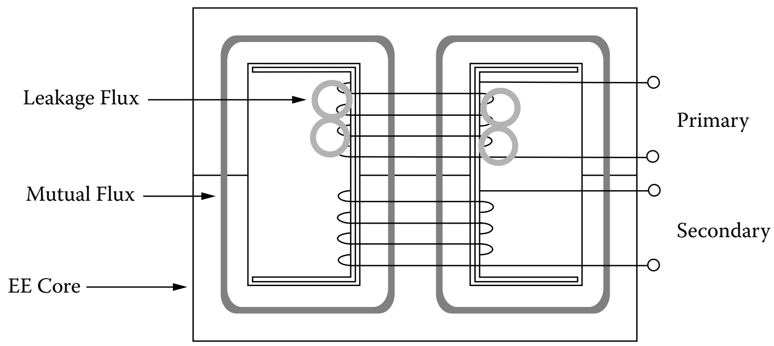

Figure 17-4. Leakage Flux.

In the layer-wound coil, a substantial reduction in leakage inductance, L_p and L_s, is obtained by interweaving the primary and secondary windings. The standard transformer, with a single primary and secondary winding, is shown in Figure 17-5, along with its leakage inductance, Equation [17-2]. Taking the same transformer and splitting the secondary on either side of the primary will reduce the leakage inductance, as shown in Figure 17-6, along with its leakage inductance, Equation [17-3]. The leakage inductance can be reduced even more, by interleaving the primary and secondary, as shown in Figure 17-7, along with its leakage inductance, Equation [17-4]. Transformers can also be constructed using the side-by-side, sectionalized bobbin as shown in Figure 17-8, along with its leakage inductance, Equation [17-5]. The modified three section, side-by-side bobbin is shown in Figure 17-9, along with its leakage inductance Equation [17-6].

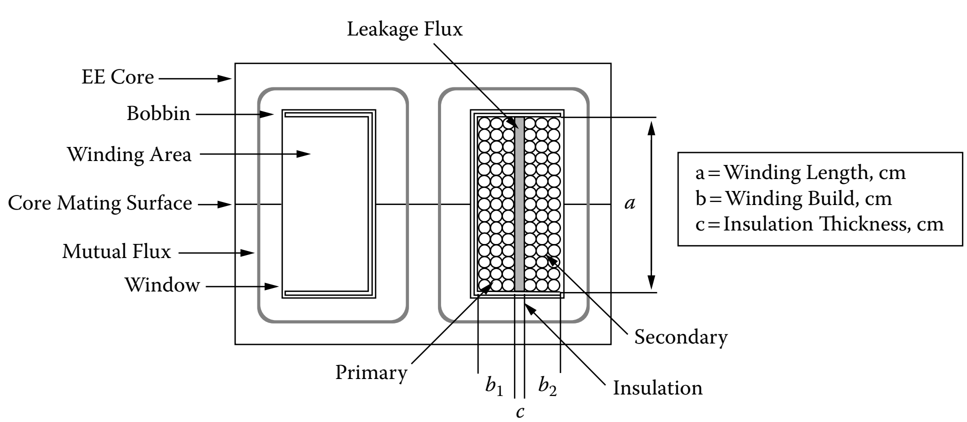

Figure 17-5. Conventional Transformer Configuration.

$$L_p = \frac{4\pi(MLT)N_p^2}{a}\left(c + \frac{b_1 + b_2}{3}\right)\left(10^{-9}\right), \quad \text{[henrys]} \qquad [17\text{-}2]$$

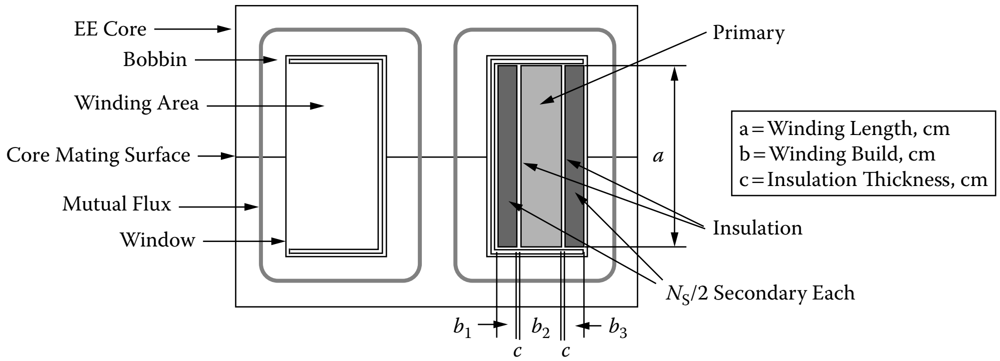

Figure 17-6. Conventional, Transformer Configuration with Simple Interleave.

$$L_p = \frac{\pi(MLT)N_p^2}{a}\left(\Sigma c + \frac{\Sigma b}{3}\right)\left(10^{-9}\right), \quad \text{[henrys]} \qquad [17\text{-}3]$$

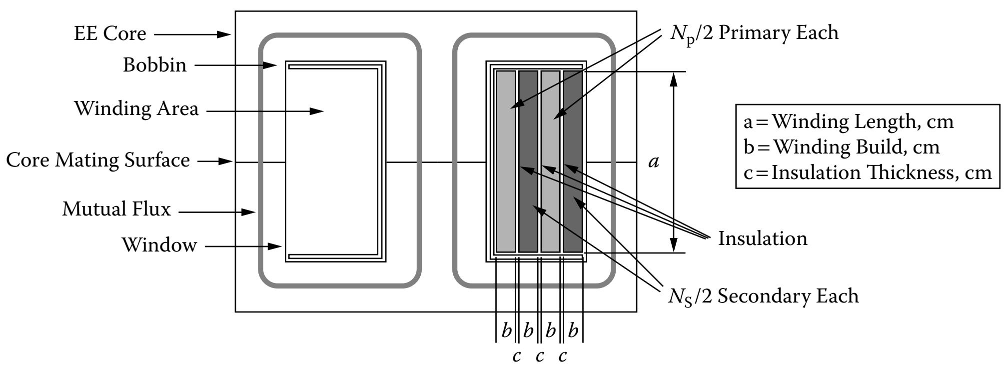

Figure 17-7. Sectionalized, Transformer Configuration Primary and Secondary Interleave.

$$L_p = \frac{\pi(MLT)N_p^2}{a}\left(\Sigma c + \frac{\Sigma b}{3}\right)\left(10^{-9}\right), \quad \text{[henrys]} \qquad [17\text{-}4]$$

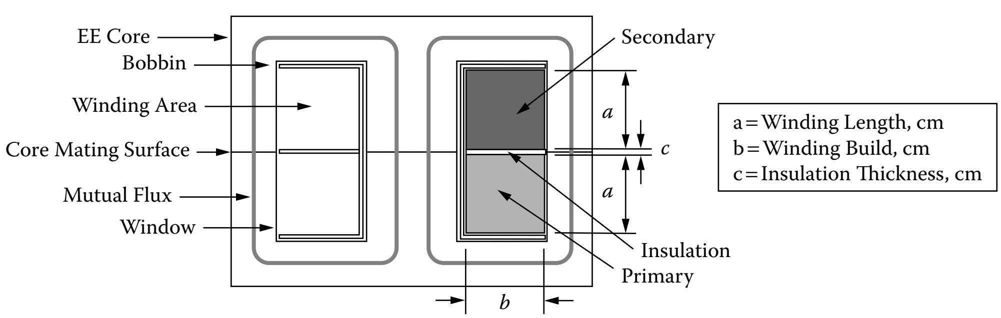

Figure 17-8. Pot Core, Sectionalized Transformer Configuration.

$$L_p = \frac{4\pi(MLT)N_p^2}{b}\left(c + \frac{\Sigma a}{3}\right)\left(10^{-9}\right), \quad \text{[henrys]} \qquad [17\text{-}5]$$

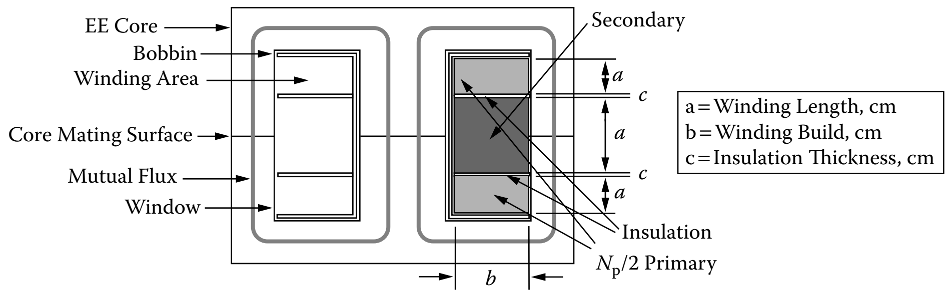

Figure 17-9. Modified, Pot Core Sectionalized, Transformer Configuration.

$$L_p = \frac{\pi (MLT) N_p^2}{b}\left(\Sigma c + \frac{\Sigma a}{3}\right)\left(10^{-9}\right), \quad \text{[henrys]} \qquad [17\text{-}6]$$

Minimizing Leakage Inductance

Magnetic core geometry has a big influence on leakage inductance. To minimize leakage inductance, the primary winding should be wound on a long bobbin, or tube, with the secondary wound as close as possible, using a minimum of insulation. Magnetic cores can have identical rating, but one core will provide a lower leakage inductance than the other. A simple comparison would be two cores with the same window area, but one core has twice the winding length. Only half the winding build is shown in Figure 17-10.

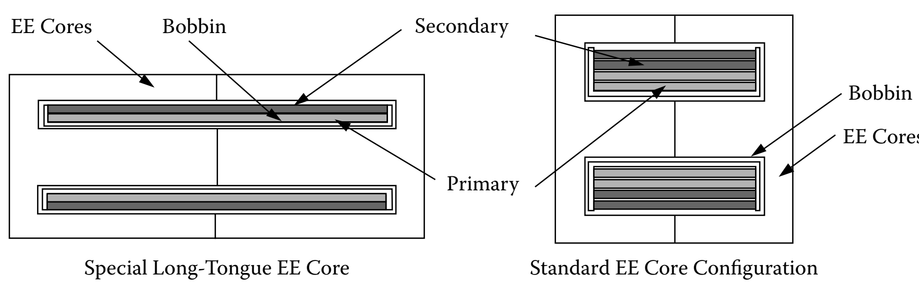

Figure 17-10. Comparing a Standard EE Core and a Special Long Tongue Core.

If layers must be used, the only way to reduce the leakage inductance is to divide the primary winding into sections, and then to sandwich the secondary winding between them, as shown in Figure 17-7. This can pose a real problem when designing around the European VDE specification, because of the required creepage distance and the minimum insulation requirements between the primary and secondary. Minimizing the leakage inductance on a push-pull converter design could be a big problem. A special consideration is required symmetry in both the leakage inductance and dc resistance; this is in order to get a balanced winding for the primary switching circuit to function properly.

The best way to minimize the leakage inductance, and to have a balanced dc resistance in a push-pull or center-tapped winding, is to wind bifilar. Bifilar windings will drastically reduce leakage inductance. This condition also exists on the secondary, when the secondary is a full-wave, center-tapped circuit. A bifilar winding is a pair of insulated wires, wound simultaneously and contiguously, (i.e., close enough to touch each other); Warning: do not use bifilar wire or the capacitance will go out of sight. Each wire constitutes a winding; their proximity reduces leakage inductance by several orders of magnitude, more than ordinary interleaving. This arrangement can be applied to the primary, or to the secondary. It can be applied to the primary and secondary together as well. This arrangement will provide the minimum leakage inductance.

Winding Capacitance

Operating at high frequency presents unique problems in the design of transformers to minimize the effect of winding capacitance. Transformer winding capacitance is detrimental in three ways: (1) winding capacitance can drive the transformer into premature resonance; (2) winding capacitance can produce large primary current spikes when operating from a square wave source, (3) winding capacitance can produce electrostatic coupling to other circuits.

When a transformer is operating, different voltage gradients arise almost everywhere. These voltage gradients are caused by a large variety of capacitance throughout the transformer, due to the turns and how they are placed throughout the transformer. When designing high frequency converters, there are several factors that have a control over the turns: (1) the operating flux density or core loss; (2) the operating voltage levels in the primary and secondary; (3) the primary inductance.

Keeping turns to a minimum will keep the capacitance to a minimum. This capacitance can be separated into four categories: (1) capacitance between turns; (2) capacitance between layers; (3) capacitance between windings; and (4) stray capacitance. The net effect of the capacitance is normally seen by the lumped capacitance, C_p, on the primary, as shown in Figure 17-1. The lumped capacitance is very difficult to calculate by itself. It is much easier to measure the primary inductance and the resonant frequency of the transformer or inductor, as shown in Figure 17-11. Then, calculate the capacitance using Equation [17-7]. The test circuit, in Figure 17-11 functions as follows: The input voltage, V1, is held constant while monitoring the voltage, V2, is swept through the frequency with the power oscillator. When the voltage, V2, rises to a peak, and starts to decay at this peak voltage, the transformer or inductor is in resonance. At this point the phase angle is also 0 degrees at resonance when looking at both the curves of, V1 and V2.

$$C_p = \left(\frac{1}{(\omega_r)^2 L} \right) = \frac{1}{4\pi^2 f_r^2 L}, \quad \text{[farads]} \qquad [17\text{-}7]$$

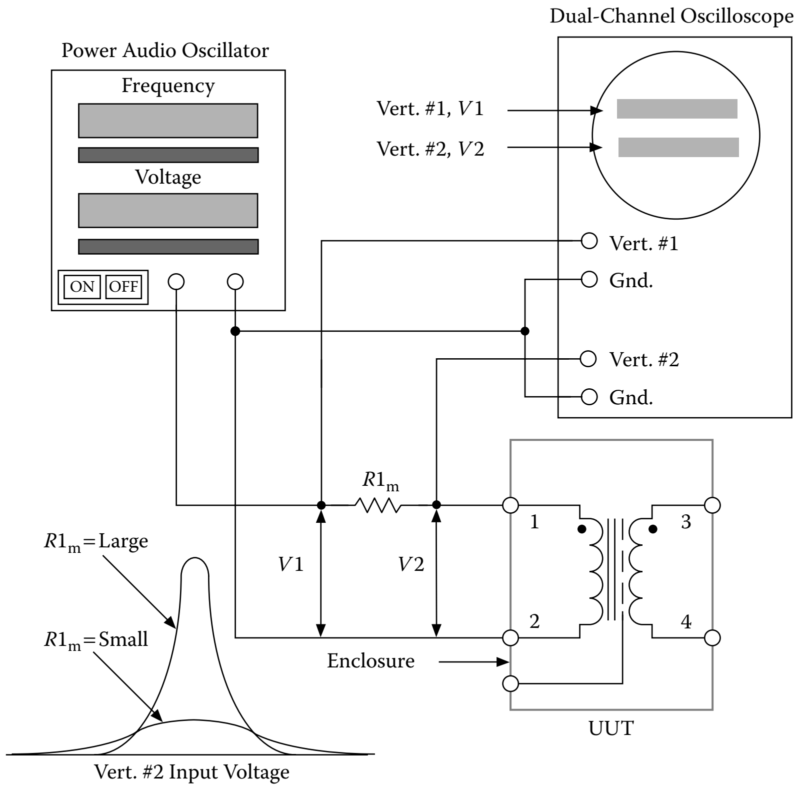

Figure 17-11. Circuit for Measuring either a Transformer or Inductor Self Resonates.

For transformers designed to operate with a square wave, such as a dc-to-dc converter, leakage inductance, L_p, and the lumped capacitance, C_p, should be kept to a minimum. This is because they cause overshoot and oscillate, or ring, as shown in Figure 17-12. The overshoot oscillation, seen in Figure 17-12A, has a resonant frequency, f, that is controlled by, L_p and C_p. This resonant frequency could change and change drastically after potting, depending on the material and its dielectric constant, as shown Figure 17-12B.

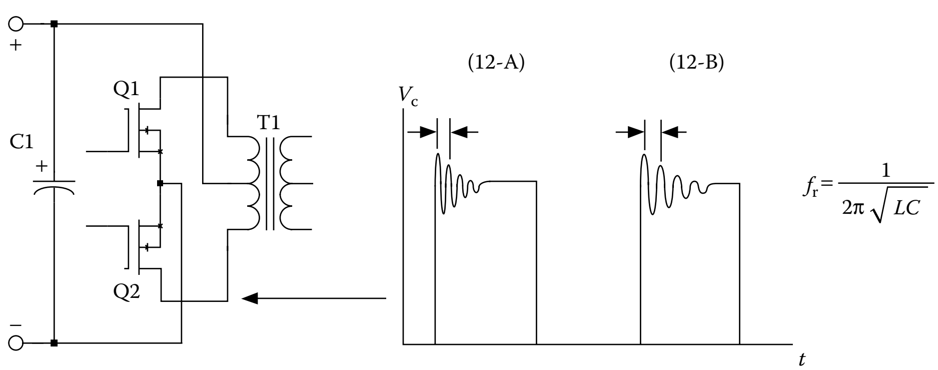

Figure 17-12. Primary Voltage with Leading Edge Ringing.

Winding Capacitance Turn-to-Turn

The turn-to-turn capacitance, C_t, shown in Figure 17-13, should not be a problem if you are operating at high frequency, low voltage power converters, due to the low number of turns. If the turn-to-turn capacitance is important, then change the magnet wire insulation to one with a lower dielectric constant. See Chapter 4.

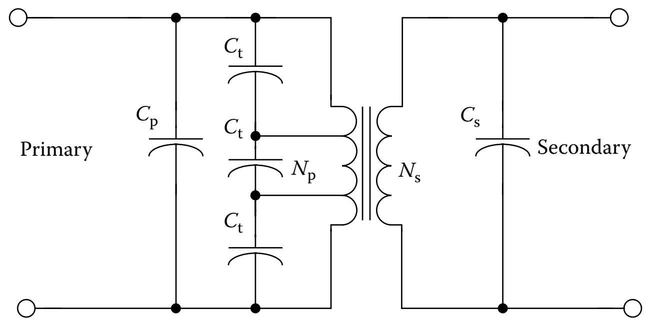

Figure 17-13. Capacitance Turn-to-Turn.

Winding Capacitance Layer-to-Layer

The capacitance between layers on the primary or secondary is the best contributor to the overall, lumped capacitance, C_p. There are three ways to minimize the layer capacitance: (1) Divide the primary and secondary windings into sections, and then sandwich the other winding between them, as shown in Figure 17-7; (2) The foldback winding technique, shown in Figure 17-14, is preferred to the normal U type winding, even though it takes an extra step before starting the next layer. The foldback winding technique will also reduce the voltage gradient between the end of the windings; (3) Increasing the amount of insulation between windings will decrease the amount of capacitance. But remember, this will increase the leakage inductance. If the capacitance is reduced, then the leakage inductance will go up. There is one exception to this rule, and that is, if the windings are sandwiched or interleaved, it will reduce the winding capacitance, but, it will increase the winding-to-winding capacitance.

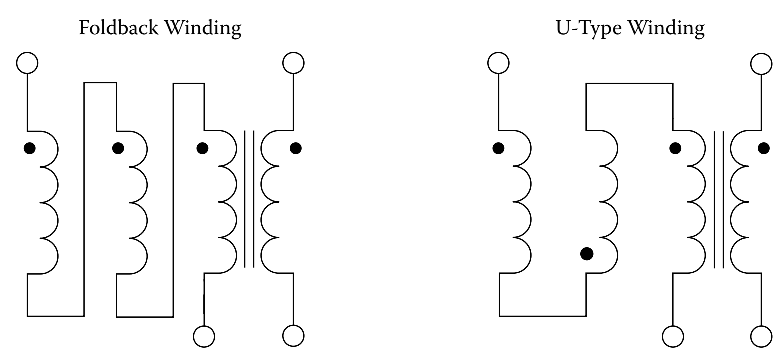

Figure 17-14. Comparing the Foldback to the U Type Winding.

Transformers and inductors wound on toroidal cores can have capacitance problems, just as much if care is not taken in the design at the beginning. It is difficult to control the winding capacitance on a toroidal core because of its odd configuration, but there are ways to control the windings and capacitance. The use of tape barriers to mark a zone for windings, as shown in Figure 17-15, offers a good way to control this capacitance.

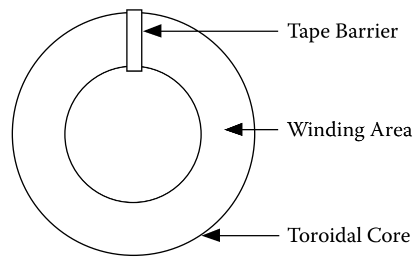

Figure 17-15. Tape Barrier for Winding Toroidal Core.

Another way to help reduce the capacitance effect on toroids is to use the progressive winding technique. The progressive winding technique example is shown in Figure 17-16 and 17-17: Wind five turns forward and wind four turns back, then wind ten turns forward and keep repeating this procedure until the winding is complete.

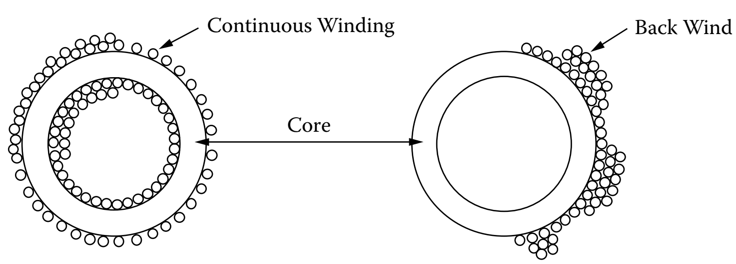

Figure 17-16. Progress Winding Top View.

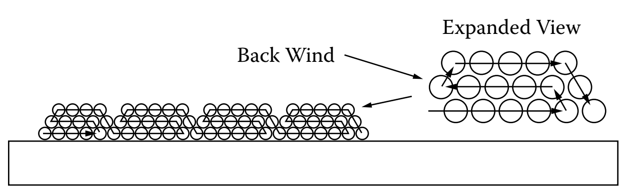

Figure 17-17. Progress Winding Side View.

Capacitance Winding-to-Winding

Balanced windings are very important in keeping down noise and common mode signals that could lead to in-circuit noise and instability problems later on. The capacitance, from winding-to-winding, shown in Figure 17-18, can be reduced, by increasing the amount of insulation between windings. This will decrease the amount of capacitance, but again, this will increase the leakage inductance. The capacitance effect between windings can be reduced, without increasing the leakage inductance noticeably. This can be done, by adding a Faraday Shield or screen, as shown in Figure 17-19, between primary and secondary windings.

A Faraday Shield is an electrostatic shield, usually made of copper foil. The Faraday Shield is normally added along with the insulation between primary and secondary. In some designs, the Faraday Shield can consist of three independent insulated shields or just one. It all depends on the required noise rejection.

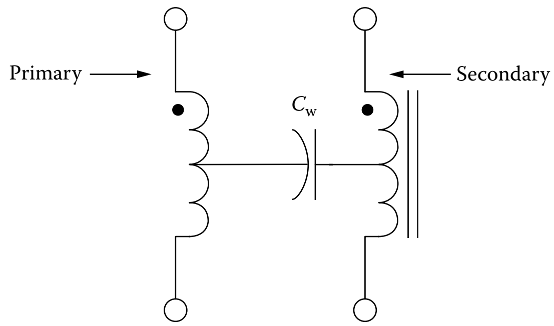

Figure 17-18. Capacitance, C_w, Winding-to-Winding.

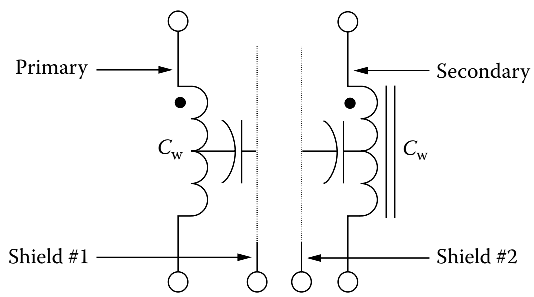

Figure 17-19. Transformer with a Primary and Secondary Shield.

Stray Capacitance

Stray capacitance is very important to minimize because it too, can generate asymmetry currents and could lead to high common mode noise. Stray capacitance is similar to winding-to-winding capacitance except that the capacitance is between the winding next to the core, C_c, and the outer winding next to the surrounding circuitry, C_s, as shown in Figure 17-20. Stray capacitance can be minimized by using a balanced winding, or using a copper shield over the entire winding. A means for measuring leakage current is shown in Figure 17-21. The winding-to-winding capacitance can be calculated, using Equations [17-8] and [17-9].

$$X_c = R1\sqrt{\left(\frac{V_{in}}{V_o}\right)-1}, \quad \text{[ohms]} \qquad [17\text{-}8]$$

$$C_x = \frac{1}{2\pi f X_c}, \quad \text{[farads]} \qquad [17\text{-}9]$$

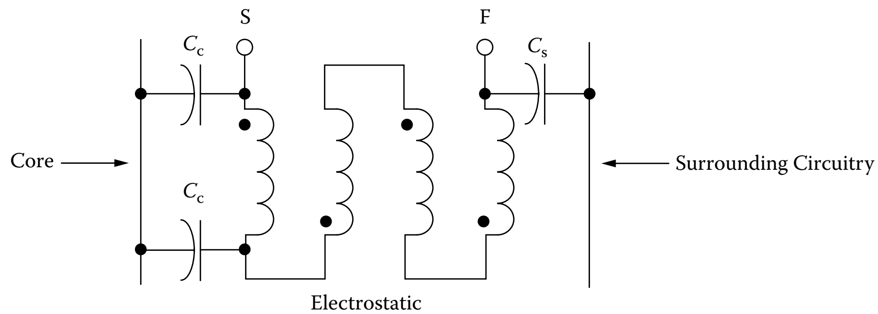

Figure 17-20. Transformer Winding with Stray Capacitance.

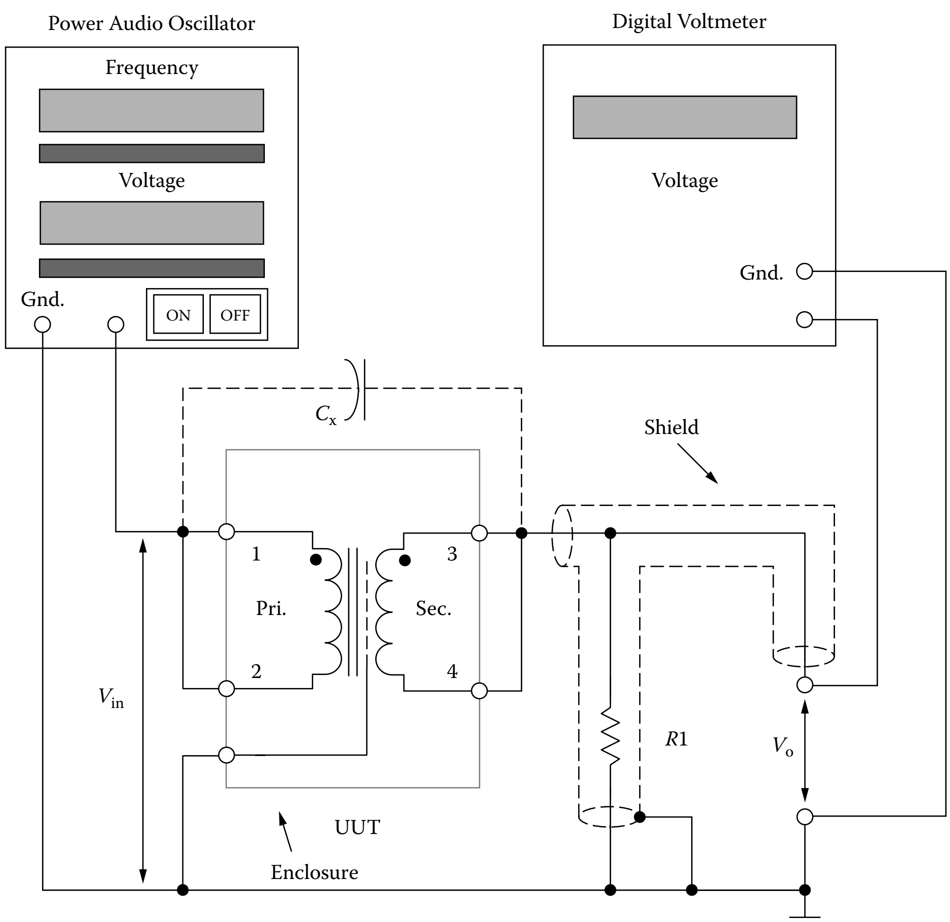

Figure 17-21. Test Circuit for Measuring Primary and Secondary, ac Leakage Current.

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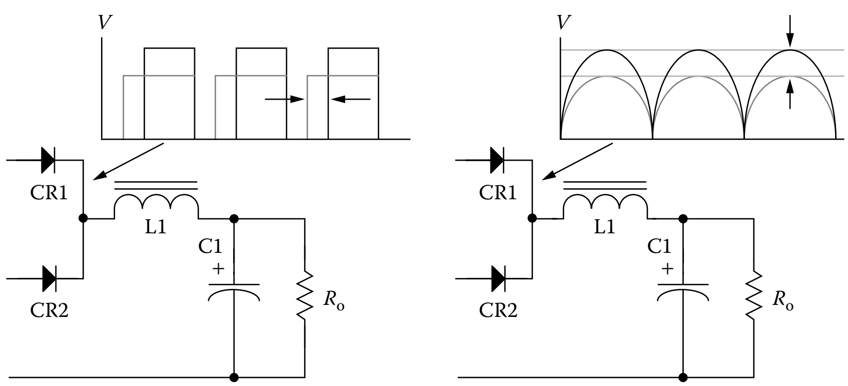
Chapter 18

Quiet Converter Design

Table of Contents

Introduction

A few designers have known about the Resonant Converter described here for many years. This type of Resonant Converter has been built mainly in the range of 200 watts to 2 kilowatts, and has been used as a static inverter. However, it has remained relatively obscure in the general literature. The Quiet Converter was developed at Jet Propulsion Laboratory (JPL), Division 38, to power very sensitive instruments. The Quiet Converter produces a sinusoidal voltage across a parallel resonant tank. The dc output voltage is obtained after rectification and filtering of the sinusoidal secondary voltage. The regulation is achieved by controlling the duty-cycle of the switching transistors. A comparison of the standard type of PWM control with the Quiet Converter and its amplitude modulation (AM), is shown in Figure 18-1. The inherent low noise from this converter is how the nickname, Quiet Converter, came about. The low noise can easily be reduced, even further, by the addition of a Faraday Shield and common-mode inductors. Programs at the Jet Propulsion Laboratory (JPL) that have successfully used the low noise environment of the Quiet Converter are WF/PC-II, Articulated Fold, Mirror Actuators, (Hubbell Space Telescope), MISR (Earth Orbiting System), Raman, and the Mars 05 ONC, CCD Camera.



Square Wave with PWM Control   Sine Wave with Amplitude Control

Figure 18-1. Comparing PWM and Amplitude Control.

The Voltage-fed Converter

The voltage-fed converter circuit is the most widely-used, converter topology. In a voltage-fed converter, the powder source, V_{in}, is connected directly to the transformer through a transistor, Q1, as shown in Figure 18-2. When the transistor, Q1, is switched on, the full source voltage is applied to the transformer, T1, primary, (1-2). The transistor saturation is ignored. Conversely, when Q2 is switched on, the full source voltage is applied to the other half of the transformer, T1, primary, (2-3).

In Figure 18-2, the switching drive circuit alternately saturates and cuts off the semiconductors' switches, Q1 and Q2, causing an alternating voltage to be generated across the primary winding of transformer, T1, and then delivered to the secondary to be rectified and filtered before going to the load. The primary source voltage, V_{in}, is

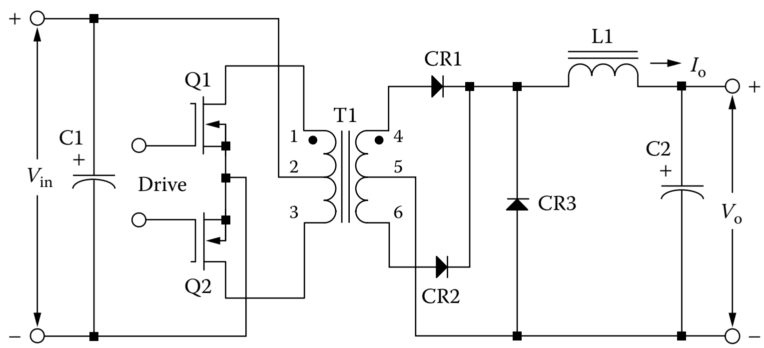


Figure 18-2. Typical, Voltage-fed Power Converter.

directly impressed onto the primary of the transformer, T1, and therefore, the voltage across the transformer, T1, is always a square wave.

Regulating and Filtering

The most effective method of regulation for a voltage-fed converter is Pulse Width Modulation (PWM). A constant output voltage can be obtained for a changing input voltage, by reducing the on time, T_{on}, of Q1 and Q2, as shown in Figure 18-3. The pulse width voltage is applied to the output filter, L1 and C2, averaging circuit to provide the proper output voltage, V_o.

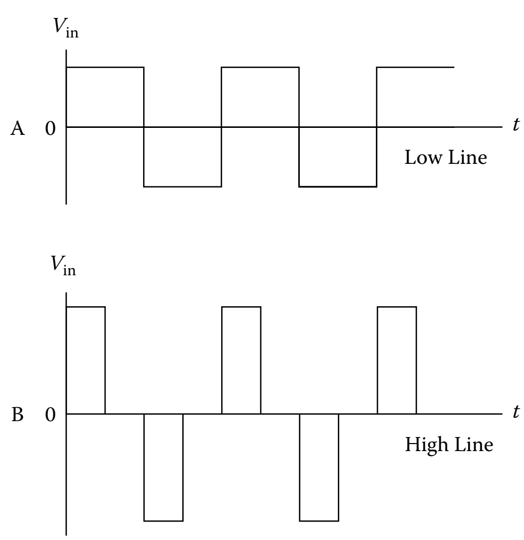


Figure 18-3. Primary Voltage of a PWM Controlled Converter.

The Current-fed Converter

The main difference between a voltage-fed converter and a current-fed converter is the series inductor, L1, shown in Figure 18-4. The inductor, L1, is commonly called a feed-choke or series inductor. It has an inductance large enough in value to maintain a continuous current through the circuit under all conditions of line and load.

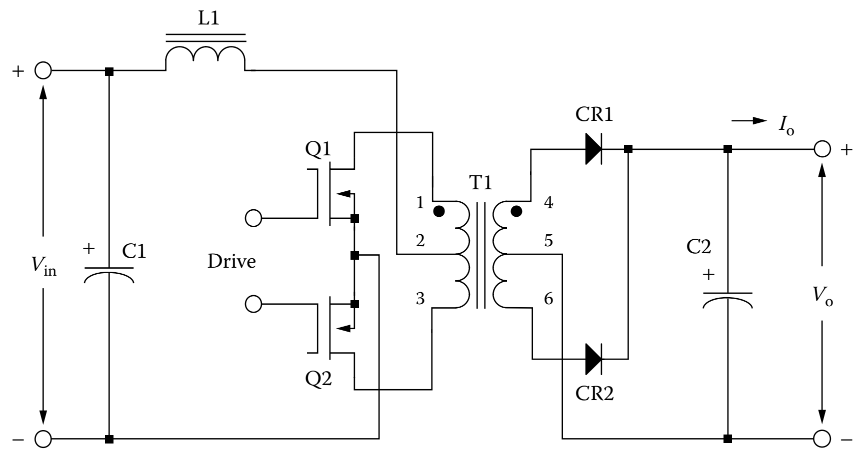


Figure 18-4. Typical Current-fed Power Converter Circuit.

The Quiet Converter

Simple additions to the circuit, in Figure 18-4, change the performance dramatically, and it becomes a whole new converter. The new converter is shown in Figure 18-5. The changes are: 1. The transformer, T1, core material has been changed to Molypermalloy Powder Core, (MPP). The reason for using a powder core is because it has a built-in gap required for the tank circuit and these cores are available with temperature stabilized permeability. The use of a gap ferrite would perform just as well, but the design must be stable over temperature. 2. A commutating winding has been added to the series inductor, L1. 3. A capacitor, C3, was added for the required parallel tuned tank. The tuning capacitor, C3, should be of high quality with a low ESR and stable. The capacitors that were used in the flight power supplies, were plastic film, Type CRH, to MIL-C-83421.

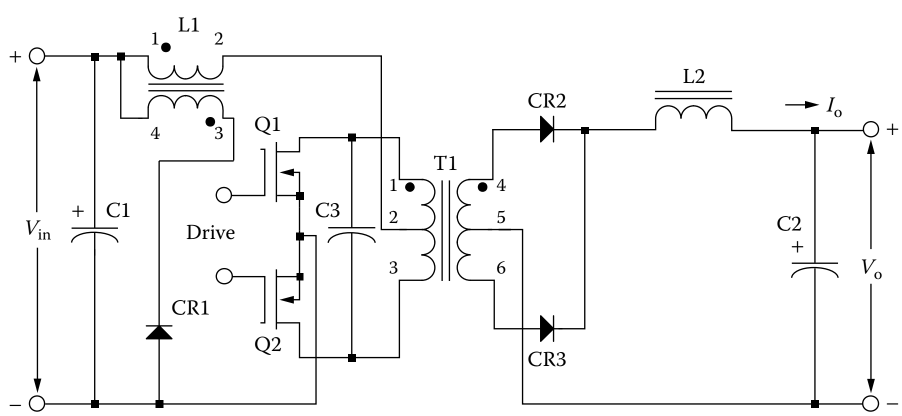


Figure 18-5. Current-fed Parallel Resonant Converter.

With properly designed components, the output voltage of transformer, T1, will always be a sine wave. The sine wave is accomplished by using a tuned parallel resonant tank circuit, (T1C3), to the natural running frequency of the converter. The series inductor, L1, isolates the input dc source from the sine wave voltage across the primary of the transformer, T1.

Regulating and Filtering

The current-fed resonant converter, shown in Figure 18-5, requires a minimum of dead time, (dwell), for the circuit to function properly. The series inductor, L1, when connected, as shown in Figure 18-4, requires continuous conduction of both, Q1 and Q2, along with a small amount of overlap. In this way, there would always be continuous current flowing in, L1. If there is any disruption of current in the series inductor, L1, no matter how small, it would destroy the switching transistors, Q1, and/or Q2.

In order to incorporate Pulse Width Modulation (PWM), or a drive circuit that has inherent dead time that neither transistor is conducting, there must be a means to commutate the current in the series inductor, L1. Adding a winding to the series inductor, L1, is a simple way to commutate the current. When the current flowing in winding, (1-2), is interrupted, the current will now be commutated to the added winding, (3-4). This is done when connected with proper phasing, through a diode CR1, then, back to the dc source to complete the path, as shown in Figure 18-5. Now, when either transistors, Q1 or Q2, are interrupted, the added winding of the series inductor, L1, commutates the current back into the dc source, thus preventing the destruction of the switching transistors, Q1 and Q2.

Quiet Converter Waveforms

The current-fed, sine wave converter waveforms will be referenced from Figure 18-6. In Figure 18-7 through Figure 18-15, refer back to (A)–(J) points in Figure 18-6. The waveforms presented here are copies drawn from an actual photo taken with an oscilloscope camera.

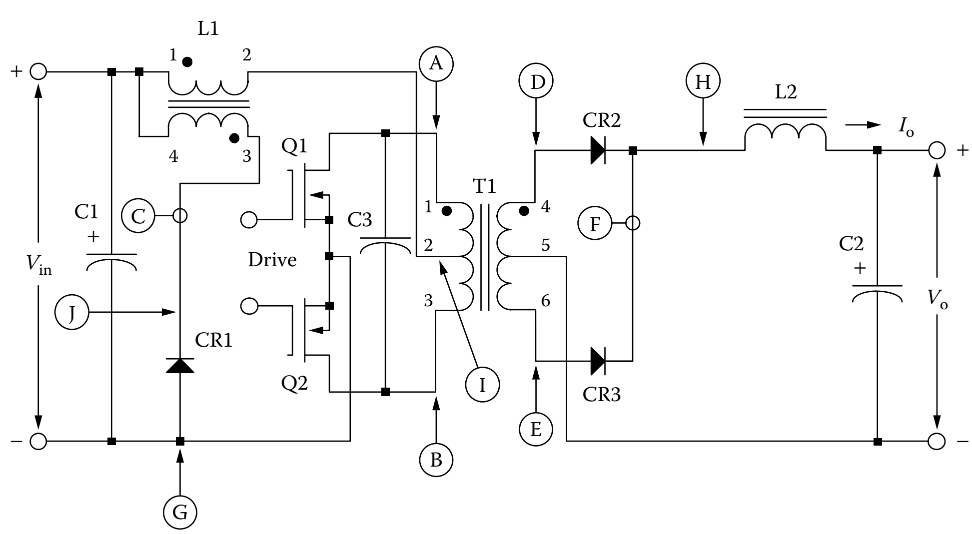


Figure 18-6. Quiet Converter Schematic with Reference Points.

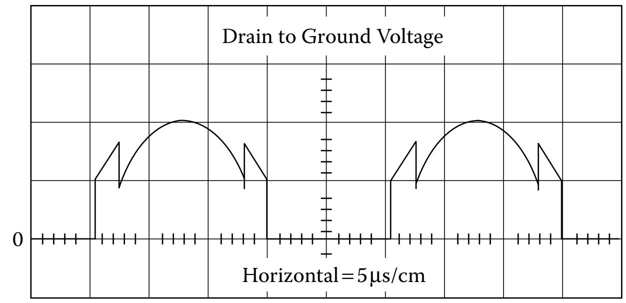


Figure 18-7. Drain to Ground, Voltage Waveform of, Q1 and Q2.

The drain voltage waveform of, Q1, is shown in Figure 18-7. Waveform is taken between points A and G. The converter is properly tuned to the natural frequency.

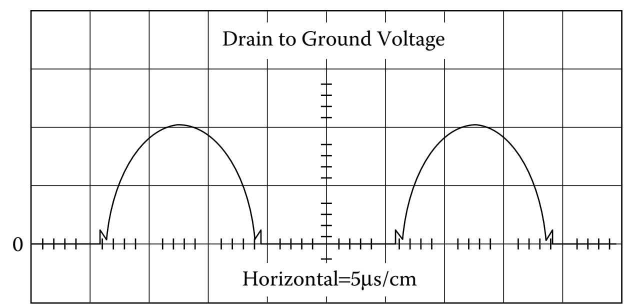


Figure 18-8. Drain to Ground, Voltage Waveform of, Q1 and Q2.

The drain voltage waveform of, Q1, is shown in Figure 18-8, with minimum dead time. Waveform is taken between points A and G. The converter is properly tuned to the natural frequency.

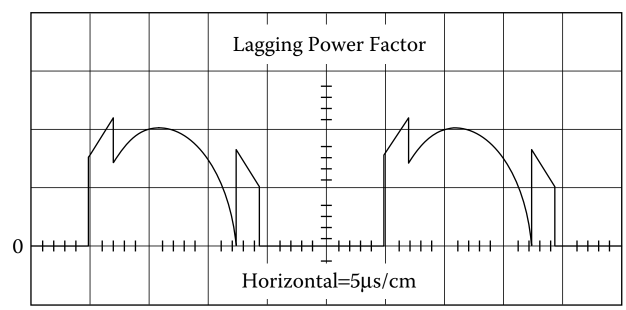


Figure 18-9. Drain to Ground, Voltage Waveform of, Q1 and Q2.

The drain voltage waveform of, Q1, is shown in Figure 18-9. Waveform is taken between points A and G. The converter is improperly tuned to the natural frequency. The resonant tank capacitor is too small in value.

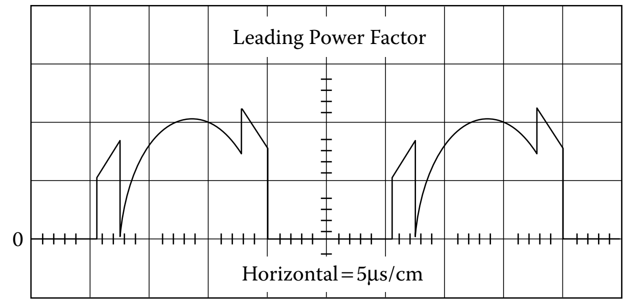


Figure 18-10. Drain to Ground, Voltage Waveform of, Q1 and Q2.

The drain voltage waveform of, Q1, is shown in Figure 18-10. Waveform is taken between points A and G. The converter is improperly tuned to the natural frequency. The resonant tank capacitor is too large in value.

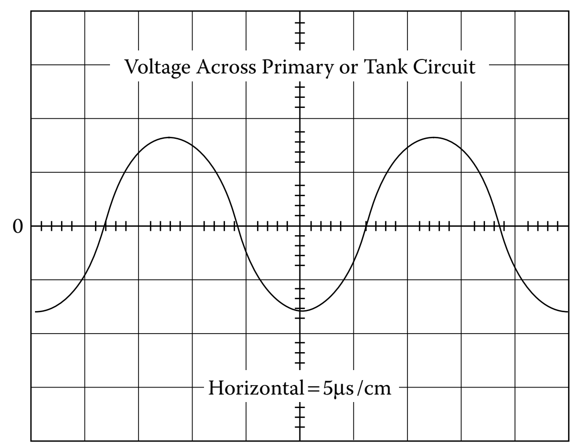


Figure 18-11. Voltage Waveform Across Transformer Primary.

The primary voltage waveform is shown in Figure 18-11, across transformer, T1. Waveform is taken between points A and B. The converter is properly tuned to the natural frequency.

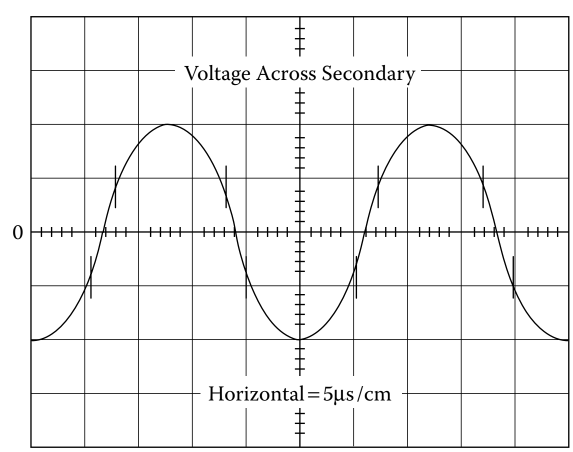


Figure 18-12. Voltage Waveform Across Transformer Secondary.

The secondary voltage waveform of transformer, T1, is shown in Figure 18-12. Waveform is taken between points D and E. The converter is properly tuned to the natural frequency.

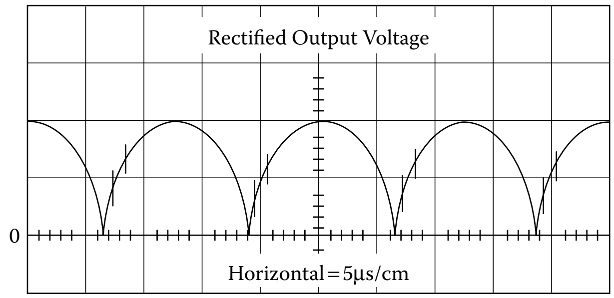


Figure 18-13. Secondary Rectified Voltage Waveform at, CR2 and CR3.

The secondary, rectified voltage waveform, at the cathodes of, CR2 and CR3, is shown in Figure 18-13. Waveform is taken at point H. The converter is properly tuned to the natural frequency.

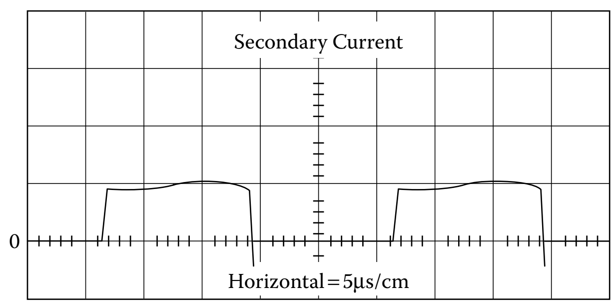


Figure 18-14. Secondary Current Waveform.

The secondary current waveform is shown in Figure 18-14. The current waveform is taken at point F.

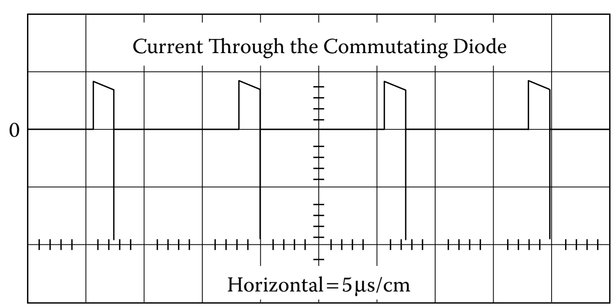


Figure 18-15. Current Through the Commutating Diode, CR1.

Commutating diode current waveform is shown in Figure 18-15. The current is through the series inductor, L1, winding (3-4). Waveform is taken at point C. The converter is properly tuned to the natural frequency.

Technology on the Move

As technology moves ahead, instruments become more sophisticated, smaller in size, and require less power. Less power normally relates to lower current. Lower current requires smaller wire to carry the current. There is a practical point where the wire size can no longer be reduced, even though the current is very small. Reliability is affected when the wire size becomes very small. It becomes a handling and termination problem. If a larger wire size can be tolerated, and it does not impact the size a great deal, then, the larger wire should be used. The smallest wire size that seems to be tolerable, depending on the application, ranges from #35 to #39 AWG and this would be from a specialty house.

Window Utilization Factor, K_u

When designing a transformer or inductor, the window utilization factor, K_u, is the amount of copper that appears in the window area. See Chapter 4. The window utilization factor, K_u, is influenced by five main factors:

1. Wire insulation, S_1.
2. Wire lay fill factor, S_2.
3. Effective window area, S_3.
4. Winding insulation, S_4.
5. Workmanship.

These factors multiplied together will give a normalized window utilization factor of, $K_u = 0.4$.

$$K_u = (S_1)(S_2)(S_3)(S_4) = 0.4 \quad [18\text{-}1]$$

The design of the current-fed sine wave converter is much more detailed and complex, compared to the simple voltage-fed, square wave converter. The sole reason to use the Quiet Converter is because of its inherent low noise, (EMI). The noise of the Quiet Converter can be reduced even further by adding a primary and a secondary Faraday Shield. When a Faraday Shield is added between the primary and secondary, the transformer must be designed to accommodate the shield. Transformer size is mainly determined by the loads. The window utilization, K_u, has to be adjusted during the design to accommodate the Faraday Shield. When the core size is selected for the transformer, it will be a little larger core, do to the added space required by the Faraday Shield.

After the preliminary design, the engineer selects the proper core size for the power transformer. The core geometry, K_g, selects the molypermalloy powder core size. After the molypermalloy powder core size has been selected, the engineer now selects a core with a permeability best-suited for the application. The molypermalloy powder cores come with a range of permeability from 14 to 550, all with the same core geometry, K_g.

Temperature Stability

For the Quiet Converter to function properly over a wide temperature range, the components must be stable over that temperature range. The components that control the oscillator frequency must be stable. The LC tank circuit must be stable and not drift with temperature. Molypermalloy powder cores are offered with stabilized permeability, with code letters M, W, and D from Magnetics, Inc. The W material temperature stability is shown in Figure 18-16.

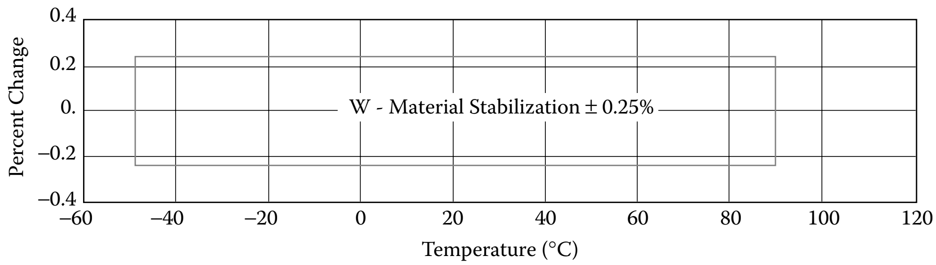


Figure 18-16. Typical, Stabilized Molypermalloy Material.

Calculating the Apparent Power, P_t

The apparent power, P_t, is the power associated with the geometry of the transformer. The designer must be able to make allowances for the rms power in each winding. The primary winding handles, P_{in}, the secondaries handle, P_o, to the load. Since the power transformer has to be designed to accommodate the primary power, P_{in}, and the secondary, P_o, then by definition,

$$P_t = P_{in} + P_\Sigma, \quad \text{[watts]}$$

$$P_\Sigma = P_{o1} + P_{o2} + ...P_{on}$$

$$P_{in} = \frac{P_\Sigma}{\eta}, \quad \text{[watts]} \qquad [18\text{-}2]$$

$$P_t = \frac{P_\Sigma}{\eta} + P_\Sigma, \quad \text{[watts]}$$

$$\eta = \text{efficiency}$$

The designer must be concerned with the apparent power-handling capability, P_t, of the transformer core and winding. The apparent power, P_t, may vary by a factor ranging from 2 to 2.828 times the input power, P_{in}, depending upon the type of circuit in which the transformer is used. If the current in the transformer becomes interrupted, such as a center-tapped secondary or push-pull primary, its effective rms value changes.

Transformer size is thus determined not only by the load demand, but also by application, because of the different copper losses incurred owing to current waveforms.

Because of the different winding configurations, the apparent power, P_t, of the transformer will have to be summed to reflect these differences. When the winding has a center tap and produces a discontinuous current, then the power in that winding, whether it is primary or secondary, has to be multiplied by the factor, U, to correct for the rms current in that winding. If the winding has a center tap, then, U = 1.41; if not, then, U = 1. Summing the output power of a multiple-output transformer would be:

$$P_{\Sigma} = P_{o1}(U) + P_{o2}(U) + ...P_{on}(U) \quad [18\text{-}3]$$

Quiet Converter Design Equations

The transformer secondary voltage, V_s, is shown in Equation [18-4].

V_o = Output voltage

V_d = Diode Drop

$$V_s = (V_o + V_d), \quad \text{[volts]} \quad [18\text{-}4]$$

The maximum secondary true power, $P_{s(max)}$, is shown in Equation [18-5].

$$P_{s(\max)} = V_s\left(I_{o(\max)}\right), \quad \text{[watts]} \quad [18\text{-}5]$$

The minimum secondary true power, $P_{s(min)}$, is shown in Equation [18-6].

$$P_{s(\min)} = V_s\left(I_{o(\min)}\right), \quad \text{[watts]} \quad [18\text{-}6]$$

The secondary apparent power, P_{sa}, is shown in Equation [18-7].

U = 1.41, center tapped winding

U = 1.0, single winding

$$P_{sa} = V_s\left(I_{o(\max)}\right)(U), \quad \text{[watts]} \quad [18\text{-}7]$$

If, there is more than one output, then, sum the total secondary maximum apparent load power, $P_{sa\Sigma}$.

$$P_{sa\Sigma} = P_{sa01} + P_{sa02} + ..., \quad \text{[watts]} \quad [18\text{-}8]$$

If, there is more than one output, then, sum the total secondary maximum load power, $P_{ot(max)}$.

$$P_{ot(\max)} = P_{o01(\max)} + P_{o02(\max)} + ..., \quad \text{[watts]} \quad [18\text{-}9]$$

If, there is more than one output, then, sum the total secondary minimum load power, $P_{ot(min)}$.

$$P_{ot(\min)} = P_{o01(\min)} + P_{o02(\min)} + ..., \quad \text{[watts]} \tag{18-10}$$

The maximum reflected secondary load resistance, $R_{(max)}$, is shown in Equation [18-11].

$R_{(max)}$ = Resistance Value

η = Efficiency

$$R_{(\max)} = \frac{(V_{in})^2(\eta)}{P_{ot(\min)}}, \quad \text{[ohms]} \tag{18-11}$$

The required series inductor inductance, L1, is shown in Equation [18-12].

$\omega = 2\pi f$

f = fundamental frequency

$$L1 = \frac{\left(R_{(\max)}\right)}{3\omega}, \quad \text{[henrys]} \tag{18-12}$$

The total period, T, is shown in Equation [18-13].

$$T = \frac{1}{f}, \quad \text{[seconds]} \tag{18-13}$$

The maximum transistor on time, $t_{on(max)}$, is shown in Equation [18-14].

Transistor drive circuits, such as a Pulse Width Modulator (PWM), will have a minimum of dead time, t_d. Dead time or dwell is shown in Figure 18-17.

$$t_{on(\max)} = \left(\frac{T}{2}\right) - t_d, \quad [\mu\text{sec}] \tag{18-14}$$

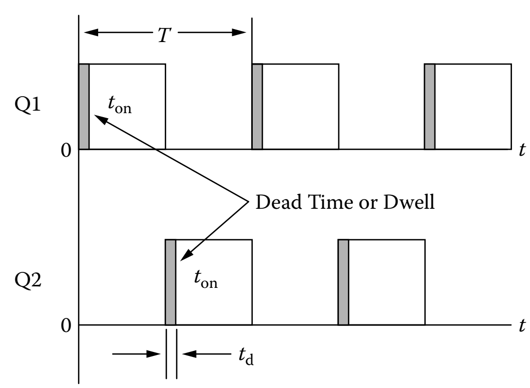


Figure 18-17. Transistor Drive Waveforms, Showing Dead Time or Dwell.

The conversion ratio, K_a, is shown in Equation [18-15].

$$K_a = \frac{\left(4t_{on(\max)} - T\right)}{T\,Sin\left(\frac{t_{on(\max)}180}{T}\right)} \qquad [18\text{-}15]$$

The peak voltage, $V_{c(pk)}$, on the resonant capacitor, C3, as shown in Figure 18-5, is:

$K_b = 2$, center tapped winding.

$K_b = 1$, single winding.

$$V_{c(pk)} = \frac{\pi\left(K_a V_{in} K_b\right)}{2}, \quad \text{[volts]} \qquad [18\text{-}16]$$

The primary rms voltage, $V_{p(rms)}$, is shown in Equation [18-17].

$K_b = 2$, center tapped winding.

$K_b = 1$, single winding.

$$V_{p(rms)} = \frac{0.707\left(V_{c(pk)}\right)}{K_b}, \quad \text{[volts]} \qquad [18\text{-}17]$$

The primary maximum reflected secondary current, I_{ps}, is shown in Equation [18-18].

$$I_{ps} = \frac{P_{ot(\max)}}{V_{p(rms)}\eta}, \quad \text{[amps]} \qquad [18\text{-}18]$$

The secondary reflected loads to the primary, R_{SR}, is shown in Equation [18-19].

$K_b = 2$, center tapped winding.

$K_b = 1$, single winding.

$$R_{SR} = \frac{K_a V_{p(rms)}\left(K_b\right)^2}{I_{sp}}, \quad \text{[ohms]} \qquad [18\text{-}19]$$

Note: The capacitance reactance affects the total percentage of harmonic distortion when:

$$\omega R_{SR} C = 1, \;\approx\; [12\%], \quad \omega R_{SR} C = 2, \;\approx\; [6\%], \quad \omega R_{SR} C = 3, \;\approx\; [4\%]$$

As a general rule:

$$C_x = \frac{2}{2\pi f\left(R_{SR}\right)}, \quad \text{[farads]} \qquad [18\text{-}20]$$

The resonant capacitance, C_x, is shown in Equation [18-21].

Q_T, is a variable that provides the engineer a little latitude with the capacitance value. ($1 < Q_T < 3$)

$$C_x = \frac{Q_T}{2\pi f\left(R_{SR}\right)}, \quad \text{[farads]} \qquad [18\text{-}21]$$

The reactance, X_{cx}, of capacitor, C_x, is shown in Equation [18-22].

Use a standard capacitor.

$$X_{cx} = \frac{1}{2\pi f C_x}, \quad \text{[ohms]} \qquad [18\text{-}22]$$

The capacitor rms current, $I_{cx(rms)}$, is shown in Equation [18-23].

$$I_{cx(rms)} = \frac{(0.707)\left(V_{c(pk)}\right)}{X_{cx}}, \quad \text{[amps]} \qquad [18\text{-}23]$$

The total primary current, $I_{tp(rms)}$, is shown in Equation [18-24].

$$I_{pt(rms)} = \sqrt{\left(\left(I_{p(rms)}\right)^2 + \left(I_{cx(rms)}\right)^2\right)}, \quad \text{[amps]} \qquad [18\text{-}24]$$

The primary tank inductance, L_x, is shown in Equation [18-25].

$$L_x = \frac{1}{\left(2\pi f\right)^2 C_x}, \quad \text{[henrys]} \qquad [18\text{-}25]$$

The total transformer apparent power, P_t, is shown in Equation [18-26].

$$\begin{aligned} P_t &= \left(\text{Primary VA}\right) + \left(\text{Secondary VA}\right) + \left(\text{Capacitor VA}\right), \quad \text{[watts]} \\ P_t &= \left(\frac{P_{ot(\max)}\left(U\right)}{\eta}\right) + \left(P_{sa\Sigma}\right) + \left(K_b V_{p(rms)} I_{cx}\right), \quad \text{[watts]} \end{aligned} \qquad [18\text{-}26]$$

The core geometry, K_g, is shown in Equation [18-27].

K_f is the waveform factor = 4.44

B_{ac} is the operating flux density and its value is an engineering judgment based on the frequency and core material.

$$K_g = \left(\frac{P_t}{0.000029\left(K_f\right)^2 \left(f\right)^2 \left(B_{ac}\right)^2 \alpha}\right), \quad \text{[cm}^5\text{]} \qquad [18\text{-}27]$$

Transformer Design, Using the Core Geometry, K_g, Approach

The following information is the Design specification for a 2.2 watt push-pull transformer, operating at 32kHz, using the K_g, core geometry approach. For a typical design example, assume a push-pull, full wave bridge circuit, with the following specification:

1. Input voltage, $V_{(min)} = 22$ volts
2. Output voltage #1, $V_{s01} = 5.0$ volts
3. Output current #1, $I_{s01(max)} = 0.2$ amps
4. Output current #1, $I_{s01(min)} = 0.1$ amps
5. Output voltage #2, $V_{s02} = 12.0$ volts
6. Output current #2, $I_{s02(max)} = 0.1$ amps
7. Output current #2, $I_{s02(min)} = 0.05$ amps
8. Frequency, f = 32kHz
9. Switching dead time, $t_d = 0.625$ µsec
10. Efficiency, $\eta = 95\%$
11. Regulation, $\alpha = 1.0\%$
12. Diode voltage drop, $V_d = 0.5$ volt
13. Operating flux density, $B_{ac} = 0.05$ tesla
14. Core Material = MPP
15. Window utilization, $K_u = 0.4$
16. Temperature rise goal, $T_r = 15°C$
17. Waveform coefficient, $K_f = 4.44$
18. Notes:

    Using a center tapped winding, U = 1.41

    Using a single winding, U = 1.0

Step 1: Calculate the total secondary voltage, V_s, for each output.

$$V_s = (V_o) + (2V_d), \quad \text{[volts]}$$

$$V_{s01} = (5.0) + (1.0) = 6.0, \quad \text{[volts]}$$

$$V_{s02} = (12) + (1.0) = 13.0, \quad \text{[volts]}$$

Step 2: Calculate the maximum secondary true power, $P_{s(max)}$.

$$P_{s(\max)} = V_s \left(I_{o(\max)}\right), \quad \text{[watts]}$$

$$P_{s01(\max)} = 6.0(0.2) = 1.2, \quad \text{[watts]}$$

$$P_{s02(\max)} = 13.0(0.1) = 1.3, \quad \text{[watts]}$$

Step 3: Calculate the minimum secondary true power, $P_{s(min)}$.

$$P_{s(\min)} = V_s\left(I_{o(\min)}\right), \quad \text{[watts]}$$

$$P_{s01(\min)} = 6.0(0.1) = 0.6, \quad \text{[watts]}$$

$$P_{s02(\min)} = 13.0(0.05) = 0.65, \quad \text{[watts]}$$

Step 4: Calculate the secondary apparent power, P_{sa}.

$$P_{sa} = V_s\left(I_{o(\max)}\right)(U), \quad \text{[watts]}$$

$$P_{sa01} = 6.0(0.2)(1.0) = 1.2, \quad \text{[watts]}$$

$$P_{sa02} = 13.0(0.1)(1.0) = 1.3, \quad \text{[watts]}$$

Step 5: Calculate the secondary total maximum apparent load power, $P_{sa\Sigma}$.

$$P_{sa\Sigma} = P_{sa01} + P_{sa02}, \quad \text{[watts]}$$

$$P_{sa\Sigma} = (1.2) + (1.3), \quad \text{[watts]}$$

$$P_{sa\Sigma} = 2.5, \quad \text{[watts]}$$

Step 6: Calculate the secondary total maximum load power, $P_{ot(max)}$.

$$P_{ot(\max)} = P_{o01(\max)} + P_{o02(\max)}, \quad \text{[watts]}$$

$$P_{ot(\max)} = (1.2) + (1.3), \quad \text{[watts]}$$

$$P_{ot(\max)} = 2.5, \quad \text{[watts]}$$

Step 7: Calculate the secondary total minimum load power, $P_{ot(min)}$.

$$P_{ot(\min)} = P_{o01(\min)} + P_{o02(\min)}, \quad \text{[watts]}$$

$$P_{ot(\min)} = (0.6) + (0.65), \quad \text{[watts]}$$

$$P_{ot(\min)} = 1.25, \quad \text{[watts]}$$

Step 8: Calculate the secondary maximum reflected load resistance, $R_{(max)}$.

$R_{(max)}$ = Resistance Value

η = Efficiency

$$R_{(\max)} = \frac{(V_{in})^2(\eta)}{P_{ot(\min)}}, \quad \text{[ohms]}$$

$$R_{(\max)} = \frac{(22)^2(0.95)}{1.25}, \quad \text{[ohms]}$$

$$R_{(\max)} = 368, \quad \text{[ohms]}$$

Step 9: Calculate the inductance of the series inductor, L1.

$$L1 = \frac{R_{(\max)}}{3\omega}, \quad \text{[henrys]}$$

$$L1 = \frac{(368)}{3(2(3.14)(32{,}000))}, \quad \text{[henrys]}$$

$$L1 = 0.000610, \quad \text{[henrys]}$$

Step 10: Calculate the total period, T.

$$T = \frac{1}{f}, \quad \text{[seconds]}$$

$$T = \frac{1}{32{,}000}, \quad \text{[seconds]}$$

$$T = 31.25, \quad \text{[micro-seconds]}$$

Step 11: Calculate the maximum transistor on time, $T_{on(max)}$. Dead time is shown in Figure 18-17.

$$t_{on(\max)} = \left(\frac{T}{2}\right) - t_d, \quad \text{[usec]}$$

$$t_{on(\max)} = \left(\frac{31.25}{2}\right) - 0.625, \quad \text{[usec]}$$

$$t_{on(\max)} = 15, \quad \text{[usec]}$$

Step 12: Calculate the conversion ratio, K_a.

$$K_a = \frac{\left(4t_{on(\max)} - T\right)}{T\,Sin\left(\frac{t_{on(\max)}180}{T}\right)}$$

$$K_a = \frac{\left(4(15) - (32.25)\right)}{(32.25)Sin\left(\frac{(15)180}{32.25}\right)}$$

$$K_a = 0.866$$

Step 13: Calculate the peak voltage, $V_{c(pk)}$, on the resonant capacitor, C3, as shown in Figure 18-5.

$K_b = 2$, center tapped winding.

$K_b = 1$, single winding.

$$V_{c(pk)} = \frac{\pi\left(K_a V_{in} K_b\right)}{2}, \quad \text{[volts]}$$

$$V_{c(pk)} = \frac{(3.1415)(0.866)(22)(2)}{2}, \quad \text{[volts]}$$

$$V_{c(pk)} = 59.85, \quad \text{[volts]}$$

Step 14: Calculate the primary rms voltage, $V_{p(rms)}$.

$K_b = 2$, center tapped winding.

$K_b = 1$, single winding.

$$V_{p(rms)} = \frac{0.707\left(V_{c(pk)}\right)}{K_b}, \quad \text{[volts]}$$

$$V_{p(rms)} = \frac{0.707(59.85)}{2}, \quad \text{[volts]}$$

$$V_{p(rms)} = 21.2, \quad \text{[volts]}$$

Step 15: Calculate the primary maximum reflected secondary current, I_{ps}.

$$I_{ps} = \frac{P_{ot(\max)}}{V_{p(rms)}\eta}, \quad \text{[amps]}$$

$$I_{ps} = \frac{2.5}{(21.2)(0.95)}, \quad \text{[amps]}$$

$$I_{ps} = 0.124, \quad \text{[amps]}$$

Step 16: Calculate the secondary reflected loads to the primary, R_{SR}.

$K_b = 2$, center tapped winding.

$K_b = 1$, single winding.

$$R_{SR} = \frac{K_a V_{p(rms)} \left(K_b\right)^2}{I_{sp}}, \quad \text{[ohms]}$$

$$R_{SR} = \frac{(0.866)(21.2)(2)^2}{0.124}, \quad \text{[ohms]}$$

$$R_{SR} = 592, \quad \text{[ohms]}$$

Note: The capacitance reactance effects the total percentage of harmonic distortion when:

$$\omega R_{SR} C = 1, \ \approx \ [12\%], \quad \omega R_{SR} C = 2, \ \approx \ [6\%], \quad \omega R_{SR} C = 3, \ \approx \ [4\%]$$

As a general rule:

$$C_x = \frac{2}{2\pi f(R_{SR})}, \quad \text{[farads]}$$

Step 17: Calculate the resonant capacitance, C_x.

$$C_x = \frac{2}{2\pi f(R_{SR})}, \quad \text{[farads]}$$

$$C_x = \frac{2}{(6.28)(32{,}000)(592)}, \quad \text{[farads]}$$

$$C_x = 1.68\left(10^{-8}\right), \quad \text{[farads]}$$

$$C_x = 0.0168 \text{ use a } 0.015, \quad \text{[micro-farads]}$$

Step 18: Calculate the reactance, X_{cx}, of capacitor, C_x. Use a standard capacitor. Let, C_x, equal 0.015 μf.

$$X_{cx} = \frac{1}{2\pi f C_x}, \quad \text{[ohms]}$$

$$X_{cx} = \frac{1}{(6.28)(32{,}000)\left(0.015\left(10^{-6}\right)\right)}, \quad \text{[ohms]}$$

$$X_{cx} = 332, \quad \text{[ohms]}$$

Step 19: Calculate the capacitor current, $I_{cx(rms)}$.

$$I_{cx(rms)} = \frac{(0.707)\left(V_{c(pk)}\right)}{X_{cx}}, \quad \text{[amps]}$$

$$I_{cx(rms)} = \frac{(0.707)(59.85)}{332}, \quad \text{[amps]}$$

$$I_{cx(rms)} = 0.127, \quad \text{[amps]}$$

Step 20: Calculate the total primary current, $I_{tp(rms)}$.

$$I_{pt(rms)} = \sqrt{\left(\left(I_{p(rms)}\right)^2 + \left(I_{cx(rms)}\right)^2\right)}, \quad \text{[amps]}$$

$$I_{pt(rms)} = \sqrt{\left((0.124)^2 + (0.127)^2\right)}, \quad \text{[amps]}$$

$$I_{pt(rms)} = 0.177, \quad \text{[amps]}$$

Step 21: Calculate primary tank inductance, L_x.

$$L_x = \frac{1}{(2\pi)^2 (f)^2 C_x}, \quad \text{[henrys]}$$

$$L_x = \frac{1}{(6.28)^2 (32{,}000)^2 \left(0.015\left(10^{-6}\right)\right)}, \quad \text{[henrys]}$$

$$L_x = 0.00165, \quad \text{[henrys]}$$

Step 22: Calculate the total transformer apparent power, P_t.

$$P_t = (\text{Primary VA}) + (\text{Secondary VA}) + (\text{Capacitor VA}), \quad \text{[watts]}$$

$$P_t = \left(\frac{P_{ot(\max)}(U)}{\eta}\right) + \left(P_{sa\Sigma}\right) + \left(K_b V_{p(rms)} I_{cx}\right), \quad \text{[watts]}$$

$$P_t = \left(\frac{(2.5)(1.41)}{0.95}\right) + (2.5) + \left((2)(21.2)(0.127)\right), \quad \text{[watts]}$$

$$P_t = 11.6, \quad \text{[watts]}$$

Step 23: Calculate the core geometry, K_g. B_{ac} is the operating flux density and its value is an engineering judgment based on the frequency and core material.

$$K_g = \left(\frac{P_t}{0.000029\left(K_f\right)^2 (f)^2 (B_{ac})^2 \alpha}\right), \quad [\text{cm}^5]$$

$$K_g = \left(\frac{11.6}{0.000029(4.44)^2 (32{,}000)^2 (0.05)^2 (1)}\right), \quad [\text{cm}^5]$$

$$K_g = 0.00793, \quad [\text{cm}^5]$$

Design Review

Conversion factor, $K_a = 0.866$

Tank Capacitance, $C_x = 0.015$ µf

Tank Capacitance Peak Voltage, $V_{cx} = 59.85$ volts

Tank Capacitance rms Current, $I_{cx(rms)} = 0.127$ amps

Primary Inductance, $L_x = 0.00165$ henrys

Series Inductor, L1 = 0.000610 henrys

Primary Reflected Current, $I_{ps(rms)} = 0.124$ amps

Primary rms Voltage, $V_{p(rms)} = 21.2$ volts

Primary Total rms Current, $I_{tp(rms)} = 0.177$ amps

Secondary Total Load Power, $P_{ot(max)} = 2.5$ watts

Transformer Total Apparent Power, $P_t = 11.6$ watts

Transformer Core Geometry, $K_g = 0.0107$ cm^2

Step 24: From Chapter 3, select a MPP powder core, comparable in core geometry, K_g.

Core part number = 55848-W4

Manufacturer = Magnetics

Magnetic path length, MPL = 5.09 cm

Core weight, $W_{tfe} = 9.4$ gm

Copper weight, $W_{tcu} = 11.1$ gm

Mean length turn, MLT = 2.8 cm

Iron area, $A_c = 0.226$ cm^2

Window area, $W_a = 1.11$ cm^2

Area product, $A_p = 0.250$ cm^4

Core geometry, $K_g = 0.008$ cm^5

Surface area, $A_t = 22.7$ cm^2

Permeability, $\mu = 60$

Millihenrys per 1000 turns, AL = 32

Step 25: Calculate the total number of primary turns, N_{tp}.

$$N_{tp} = 1000\sqrt{\frac{L_{(new)}}{L_{(1000)}}}, \quad \text{[turns]}$$

$$N_{tp} = 1000\sqrt{\frac{1.65}{32}}, \quad \text{[turns]}$$

$$N_{tp} = 226, \quad \text{round-down [turns]}$$

$$N_p = 113, \quad \text{[each side center tap]}$$

Step 26: Calculate the operating flux density, B_{ac}.

$$B_{ac} = \frac{V_{p(rms)}\left(10^4\right)}{K_f N_p f A_c}, \quad \text{[teslas]}$$

$$B_{ac} = \frac{(21.2)\left(10^4\right)}{(4.44)(113)(32{,}000)(0.226)}, \quad \text{[teslas]}$$

$$B_{ac} = 0.0587, \quad \text{[teslas]}$$

Step 27: Calculate the watts per kilogram, WK, using the MPP 60 perm loss Equation in Chapter 2.

$$WK = 0.788\left(10^{-3}\right)(f)^{(1.41)}\left(B_{ac}\right)^{(2.24)}, \quad \text{[watts/kilogram]}$$

$$WK = 0.788\left(10^{-3}\right)(32{,}000)^{(1.41)}(0.0587)^{(2.24)}, \quad \text{[watts/kilogram]}$$

$$WK = 3.09, \quad \text{[watts/kilogram] or 3.09, [milliwatts/gram]}$$

Step 28: Calculate the core loss, P_{fe}.

$$P_{fe} = \left(\frac{\text{milliwatts}}{\text{grams}}\right) W_{tfe}\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = (3.09)(9.4)\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = 0.0290, \quad \text{[watts]}$$

Step 29: Calculate the volts per turn, $K_{N/V}$.

$$K_{N/V} = \frac{N_p}{V_p}, \quad \text{[turns/volt]}$$

$$K_{N/V} = \frac{(113)}{(21.2)}, \quad \text{[turns/volt]}$$

$$K_{N/V} = 5.33, \quad \text{[turns/volt]}$$

Step 30: Calculate the secondary number of turns N_s. α is regulation in percent. See Chapter 6.

$$K = \left(1 + \frac{\alpha}{100}\right) = 1.01$$

$$N_{s01} = K_{N/V} V_{s01} K = (5.33)(6.0)(1.01) = 32, \quad \text{[turns]}$$

$$N_{s02} = K_{N/V} V_{s02} K = (5.33)(13.0)(1.01) = 70, \quad \text{[turns]}$$

Step 31: Calculate the current density, J, using a window utilization, $K_u = 0.4$.

$$J = \frac{P_t 10^4}{A_p B_m f K_f K_u}, \quad \text{[amps per cm}^2\text{]}$$

$$J = \frac{(11.6)10^4}{(0.25)(0.0587)(32{,}000)(4.44)(0.4)}, \quad \text{[amps per cm}^2\text{]}$$

$$J = 139, \quad \text{[amps per cm}^2\text{]}$$

Step 32: Calculate the secondary required wire area, A_{ws}.

$$A_{ws01} = \frac{I_{s(01)(rms)}}{J} = \frac{0.2}{139} = 1.44 \times 10^{-3}, \quad \text{[cm}^2\text{]}$$

$$A_{ws02} = \frac{I_{s(02)(rms)}}{J} = \frac{0.1}{139} = 0.719 \times 10^{-3}, \quad \text{[cm}^2\text{]}$$

Step 33: Then, select the wire from the Wire Table, in Chapter 4. Record μΩ/cm.

$$A_{ws01} = 1.44 \times 10^{-3}, \text{ use } \#26 = 1.28 \times 10^{-3}, \quad [\text{cm}^2]$$

$$\#26, \frac{\mu\Omega}{cm} = 1345$$

$$A_{ws02} = 0.719 \times 10^{-3}, \text{ use } \#29 = 0.647 \times 10^{-3}, \quad [\text{cm}^2]$$

$$\#29, \frac{\mu\Omega}{cm} = 2664$$

Step 34: Calculate the primary required wire area, A_{wp}.

$$A_{wp} = \frac{I_{tp(rms)}}{J}, \quad [\text{cm}^2]$$

$$A_{wp} = \frac{(0.177)}{139}, \quad [\text{cm}^2]$$

$$A_{wp} = 1.27 \times 10^{-3}, \quad [\text{cm}^2]$$

Step 35: Then, select the wire from the Wire Table, in Chapter 4. Record μΩ/cm.

$$A_{wp} = 1.27 \times 10^{-3}, \text{ use } \#26 = 1.28 \times 10^{-3}, \quad [\text{cm}^2]$$

$$\#26, \frac{\mu\Omega}{cm} = 1345$$

Step 36: Calculate the total secondary window utilization, K_{uts}.

$$K_{us01} = \frac{\left(N_{01} A_{w01}\right)}{W_a} = \frac{(32)(0.00128)}{1.11} = 0.0369$$

$$K_{us02} = \frac{\left(N_{02} A_{w02}\right)}{W_a} = \frac{(70)(0.000647)}{1.11} = 0.0408$$

$$K_{uts} = K_{us01} + K_{us02} = 0.0777$$

Step 37: Calculate the primary window utilization, K_{up}.

$$K_{up} = \frac{\left(N_{tp} A_w\right)}{W_a}$$

$$K_{up} = \frac{(226)(0.00128)}{1.11}$$

$$K_{up} = 0.261$$

Step 38: Calculate the total window utilization, K_u.

$$K_u = K_{up} + K_{uts}$$

$$K_u = 0.261 + 0.0777$$

$$K_u = 0.339$$

Step 39: Calculate the primary, winding resistance, R_p.

$$R_p = \text{MLT}\left(N_p\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_p = 2.80(113)(1345)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_p = 0.426, \quad \text{[ohms]}$$

Step 40: Calculate the primary, copper loss, P_p.

$$P_p = \left(I_{p(rms)}\right)^2 R_p, \quad \text{[watts]}$$

$$P_p = (0.177)^2 (0.426), \quad \text{[watts]}$$

$$P_p = 0.0133, \quad \text{[watts]}$$

Step 41: Calculate the secondary, winding resistance, R_s.

$$R_s = \text{MLT}\left(N_s\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_{s01} = 2.80(32)(1345)\left(10^{-6}\right) = 0.121, \quad \text{[ohms]}$$

$$R_{s02} = 2.80(70)(2664)\left(10^{-6}\right) = 0.186, \quad \text{[ohms]}$$

Step 42: Calculate the secondary, copper loss, P_s.

$$P_s = \left(I_{s(rms)}\right)^2 R_s, \quad \text{[watts]}$$

$$P_{s01} = (0.2)^2 (0.121) = 0.00484, \quad \text{[watts]}$$

$$P_{s02} = (0.1)^2 (0.186) = 0.00186, \quad \text{[watts]}$$

Step 43: Calculate the total secondary, copper loss, P_{ts}.

$$P_{ts} = P_{s01} + P_{s02}, \quad \text{[watts]}$$

$$P_{ts} = 0.00484 + 0.00186, \quad \text{[watts]}$$

$$P_{ts} = 0.0067, \quad \text{[watts]}$$

Step 44: Calculate the total loss, core and copper, P_Σ.

$$P_\Sigma = P_p + P_{ts} + P_{fe}, \quad \text{[watts]}$$

$$P_\Sigma = (0.0133) + (0.0067) + (0.0290), \quad \text{[watts]}$$

$$P_\Sigma = 0.049, \quad \text{[watts]}$$

Step 45: Calculate the watts per unit area, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad \text{[watts per cm}^2\text{]}$$

$$\psi = \frac{(0.049)}{(22.7)}, \quad \text{[watts per cm}^2\text{]}$$

$$\psi = 0.00216, \quad \text{[watts per cm}^2\text{]}$$

Step 46: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [^\circ\text{C}]$$

$$T_r = 450(0.00216)^{(0.826)}, \quad [^\circ\text{C}]$$

$$T_r = 2.83, \quad [^\circ\text{C}]$$

Step 47: Calculate the, Q_t, of the tank.

$$Q_t = 2\pi f C_x R_{SR}$$

$$Q_t = (6.28)(32{,}000)\left(0.015\left(10^{-6}\right)\right)(592)$$

$$Q_t = 1.79$$

For more information, see Equation [18-20].

Recognition

The author would like to thank **Dr. V. Vorperian**, Senior Engineer, Power and Sensor Electronics Group, Jet Propulsion Laboratory (JPL), for his help with the Quiet Converter design equations.

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Chapter 19

Rotary Transformer Design

Table of Contents

Introduction

There are many requirements to transfer signals and power across rotary interfaces. Most things that use slip rings or brushes can be replaced with a rotary transformer. Science instruments, antennas and solar arrays are elements needing rotary power transfer for certain spacecraft (S/C) configurations, such as a spin, stabilized (S/C). Delivery of signals and power has mainly been done by slip rings. There are problems in using slip rings for long life and high reliability: contact wear, noise, and contamination. Contact wear will lead to a conductive path to ground. This conductive path will generate noise and upset the original designed common-mode noise rejection. A simple slip ring assembly and a rotary transformer are shown in Figure 19-1. High data rates and poor slip ring life forced the Galileo (S/C) to replace the signal interface with rotary transformers. The use of a rotary transformer to transfer power on the Galileo (S/C) was contemplated, but it was thought the impact on the (S/C) delivery was too great. The rotary transformers on the Galileo (S/C) lasted the life of the spacecraft, from 1989 to 2003 without a glitch.

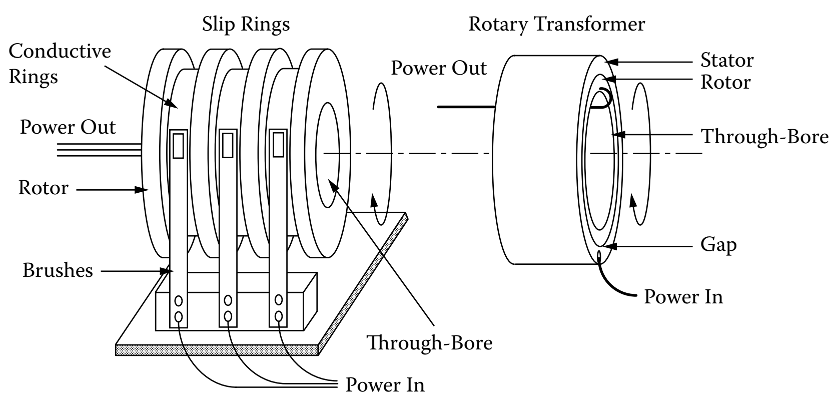

Figure 19-1. Comparing a Slip Ring Assembly and a Rotary Transformer.

Existing approaches to rotary power transfer use square wave converter technology. However, there are problems caused by the inherent gap in a rotary transformer, coupled with the fast rate of change in the square wave voltage. Undue stress is placed on the power electronics and the interface becomes a source of Electromagnetic Interference (EMI) that impacts the overall system's operating integrity.

Basic Rotary Transformer

The rotary transformer is essentially the same as a conventional transformer, except that the geometry is arranged so that the primary and secondary can be rotated, with respect to each other with negligible changes in the electrical characteristics. The most common of the rotary transformers are the axial rotary transformer, shown in Figure 19-2, and the flat plane, (pot core type), rotary transformer, shown in Figure 19-3. The power transfer is accomplished, electro-magnetically, across an air gap. There are no wearing contacts, noise, or contamination problems due to lubrication or wear debris.

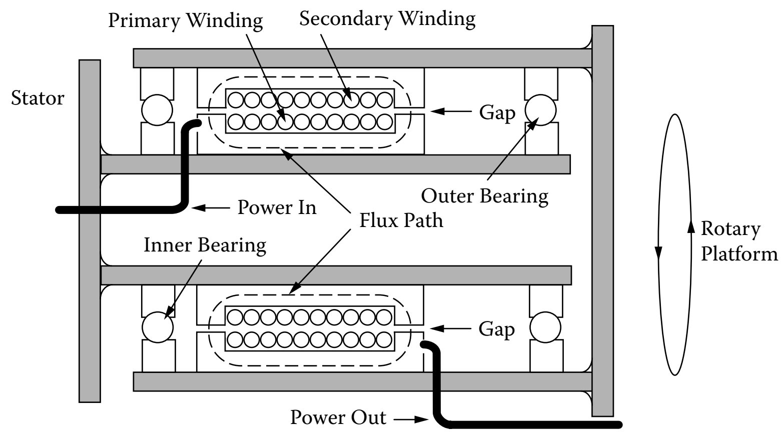

Figure 19-2. Pictorial of an Axial, Type Rotary Transformer.

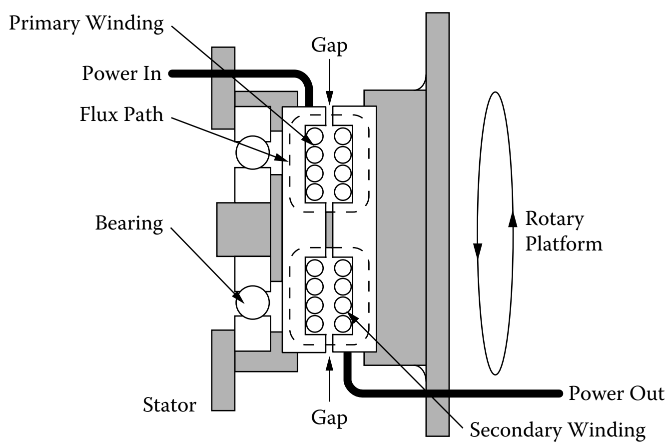

Figure 19-3. Pictorial of a Flat Plane, Type Rotary Transformer.

Square Wave Technology

The ideal converter transformer would have a typical square B-H loop, as shown in Figure 19-4. A converter transformer is normally designed to have a minimum of leakage inductance. The voltage spikes that are normally seen on the primary of a square wave converter transformer are caused by the leakage inductance. To design a converter transformer to have a minimum of leakage inductance, the primary and secondary must have a minimum of distance between them. Minimizing the leakage inductance will reduce the need for power-wasting, snubber circuits. Although there are rotary power transformers designed with the use of square wave converter technology, they are not without problems.

There are two basic problems not found in the normal transformer: (1) the inherent gap in a rotary transformer is one problem, and (2) the required spacing between primary and secondary that leads to large leakage inductance is the other. These problems, along with a square wave drive, are what leads to a high loss, snubber circuit, and become a source of Electromagnetic Interference (EMI) that impacts the adjoining systems

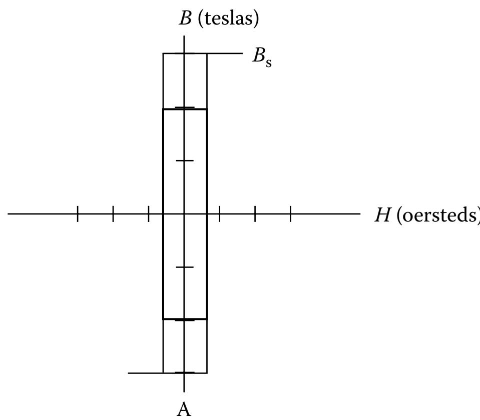

Figure 19-4. Typical, Transformer BH Loop.

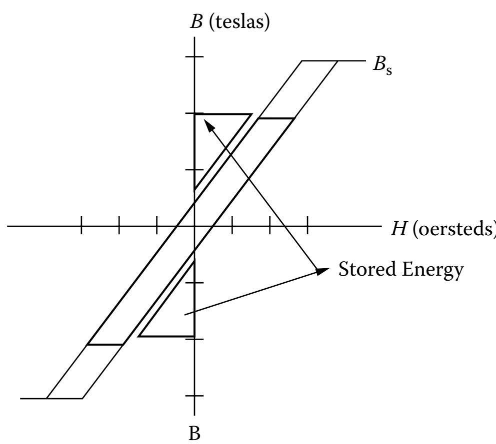

Figure 19-5. Typical, Rotary Transformer BH Loop.

operating integrity. The rotary transformer, because of its inherent gap, has a B-H loop similar to an inductor, as shown in Figure 19-5. Basically, the transformer transforms power, and the inductor stores energy in the gap. The rotary transformer does not have any of the traits of an ideal transformer. It is, more accurately, a trans-inductor having a gap and a secondary, spaced away from the primary.

Rotary Transformer Leakage Inductance

The rotary transformer has an inherent gap and spacing of the primary and secondary. The gap and spacing in the rotary transformer result in a low primary magnetizing inductance. This low primary inductance leads to a high magnetizing current. The leakage inductance, L_p, can be calculated for both axial and flat plane using Equation 19-1. The axial rotary transformer winding dimensions are shown in Figure 19-6. The flat plane rotary transformer winding dimensions are shown in Figure 19-7.

$$L_p = \frac{4\pi(MLT)N_p^2}{a}\left(c + \frac{b_1 + b_2}{3}\right)\left(10^{-9}\right), \quad \text{[henrys]} \qquad [19\text{-}1]$$

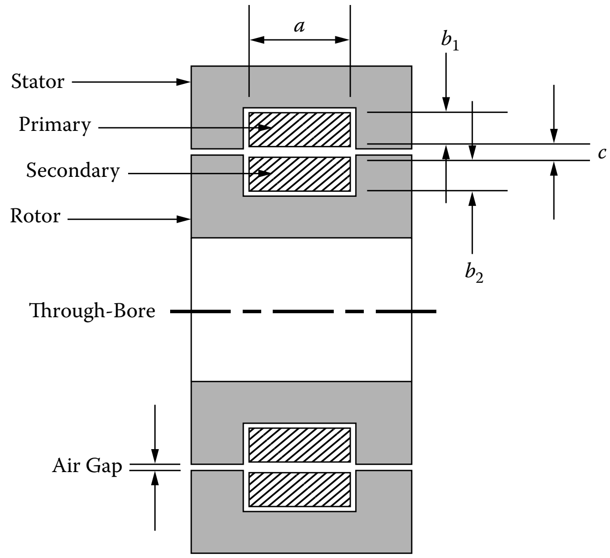

Figure 19-6. Axial Rotary Transformer, Showing Winding Dimensions.

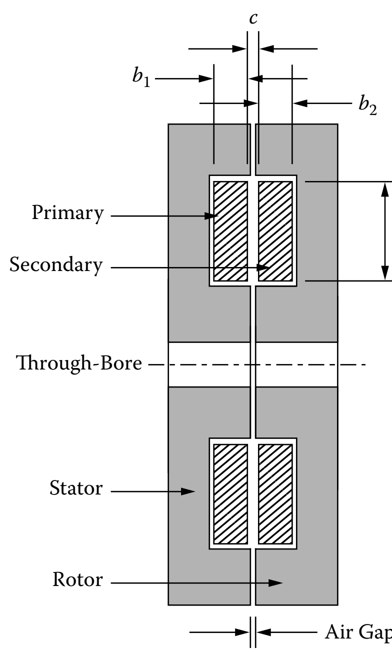

Figure 19-7. Flat Plane Rotary Transformer, Showing Winding Dimensions.

Current-fed Sine Wave Converter Approach

The current-fed, sine wave converter topology is a good candidate to power the rotary transformer. The design would be a current-fed, push-pull, tuned tank converter requiring a gapped transformer. A comparison between a standard, square wave converter, shown in Figure 19-8, and a current-fed, sine wave converter, is shown in Figure 19-9. Using the rotary transformer in this topology, the energy that is stored in the rotary gap that causes so much trouble in the standard square wave driving a rotary transformer, is recovered and is used in the tank circuit. There would not be any need of power-wasting snubbers using the rotary transformer approach. See Chapter 18.

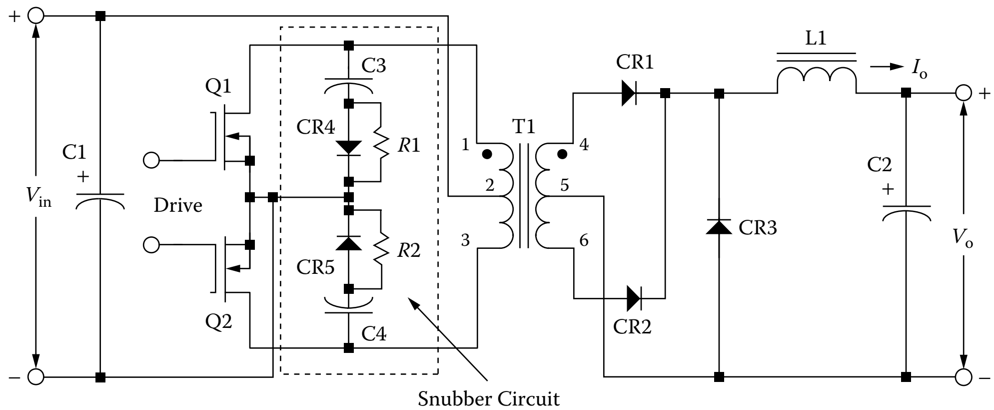

Figure 19-8. Typical, Voltage-fed, Square Wave Converter Circuit with Snubbers.

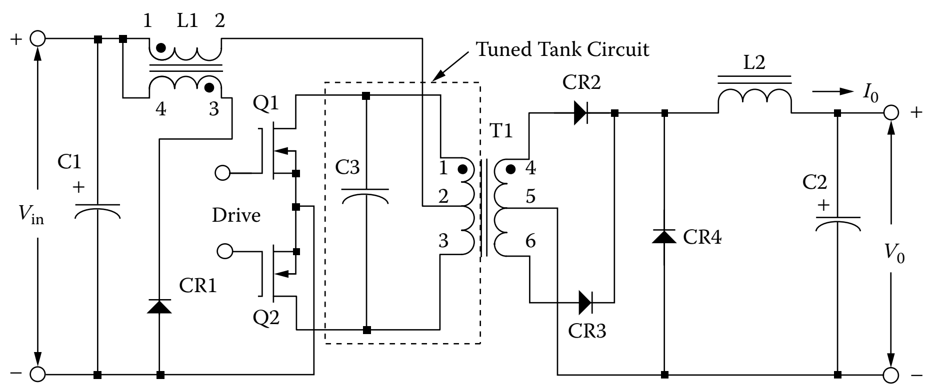

Figure 19-9. Typical, Current-fed, Resonant Converter Circuit.

The current-fed sine wave converter requires a resonant, LC, tank circuit to operate properly. The primary of the rotary transformer would be the ideal inductor, because of the inherent gap of the rotary transformer. There are several advantages to incorporating the resonant tank circuit into the rotary transformer. First, it minimizes the number of components in the power stage. Secondly, the output of the inverter is a natural sine wave, as shown in Figure 19-10, and usually requires no additional filtering. Thirdly, energy stored in the gap of the transformer is released when either power switch is turned off. This energy is commutated in the resonant tank circuit. This provides the capability for direct exchange of power between the tank circuit and the load. There is not a noticeable drive torque in a rotary transformer. The tuning or tank capacitor must be of high quality, stable, and with low ESR.

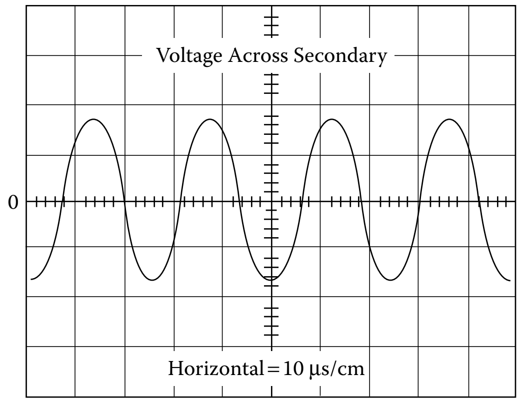

Figure 19-10. Current-fed Converter, Secondary Sine Wave Secondary Voltage.

Rotary Transformer Design Constraints

The rotary transformer requirements pose some unusual design constraints compared to the usual transformer design. The first is the relatively large gap in the magnetic circuit. This gap size depends on the eccentric dimension and the tolerance of the rotating shaft. The gap results in a low primary magnetizing inductance. Secondly, the large space separating primary and secondary windings results in an unusually high primary-to-secondary leakage inductance. Thirdly, the large through-bore requirement results in an inefficient utilization of the core material and copper, due to the fixed mean-length turn. This large diameter results in requiring more copper area for the same regulation. Finally, the core has to be more robust than the normal transformer because of the structural requirement. See Figure 19-11.

Rotary transformer dimensions are usually governed by the mechanical interface, in particular the relatively large gap and the large through-bore, resulting in a long Mean Length Turn (MLT). The rotary transformer is not an ideal magnetic assembly. A toroidal core is an ideal magnetic assembly. Manufacturers use test data, taken from toroidal cores, to present magnetic material characteristics. The magnetic flux in a toroidal core

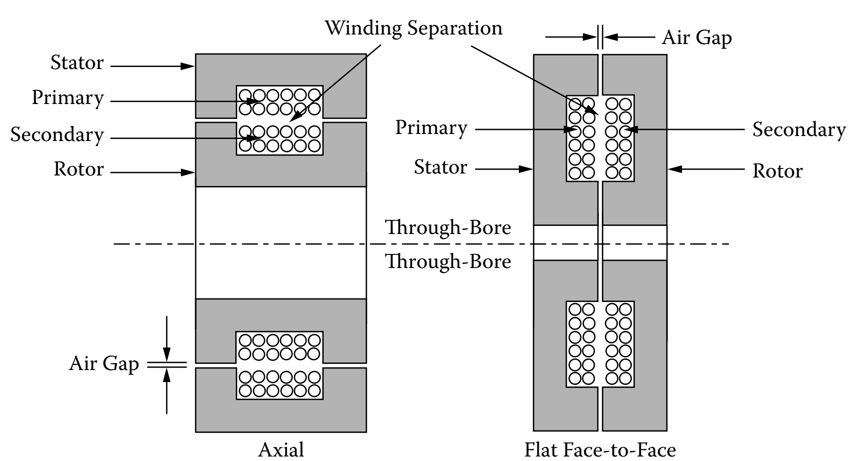

Figure 19-11. Geometries of the Basic Type Rotary Transformers.

travels through a constant core cross-section, A_c, throughout the whole Magnetic Path Length, MPL, as shown in Figure 19-12, and provides ideal magnetic characteristics. It can be seen that the core cross-section throughout the rotary transformers, shown in Figure 19-13 and Figure 19-14, does not provide constant flux density or an ideal magnetic assembly. The rotary transformers for the Galileo spacecraft were about 10 cm in diameter, and manufactured by CMI (Ref. 4.)

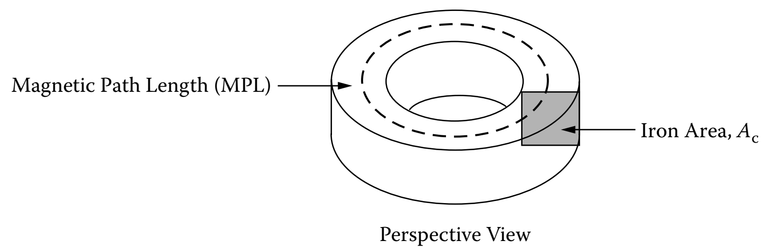

Figure 19-12. Typical Perspective View of a Toroidal Core.

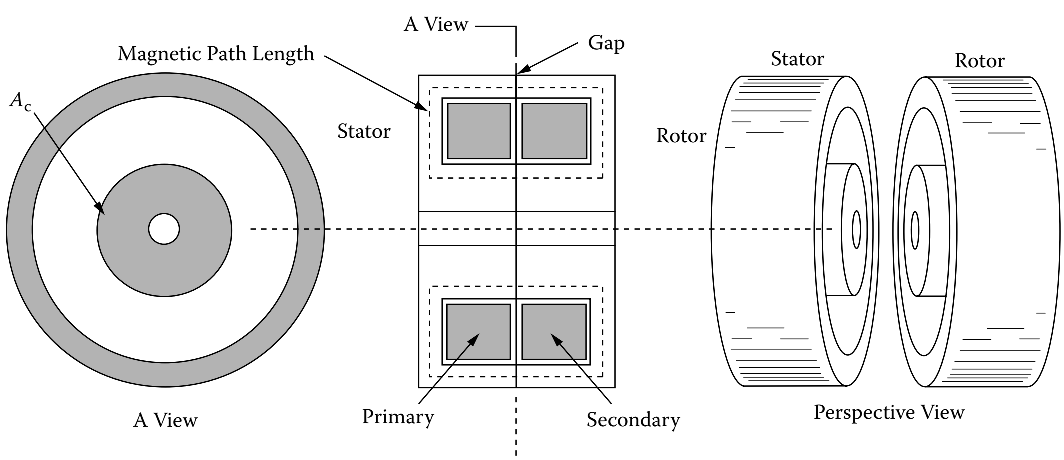

Figure 19-13. Open View of a Flat Plane, Type Rotary Transformers.

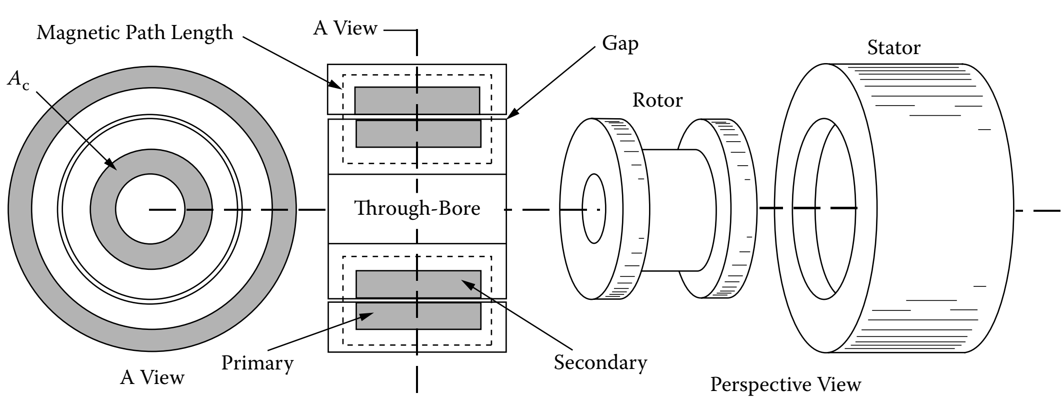

Figure 19-14. Open View of an Axial Type Rotary Transformers.

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Chapter 20

Planar Transformers and Inductors

Table of Contents

Introduction

The planar transformer, or inductor, is a low profile device that covers a large area, whereas, the conventional transformer would be more cubical in volume. Planar Magnetics is the new "buzz" word in the field of power magnetics. It took a few engineers with the foresight to come up with a way to increase the power density, while at the same time increasing the overall performance, and also, making it cost effective. One of the first papers published on planar magnetics was by Alex Estrov, back in 1986. After reviewing this paper, you really get a feeling of what he accomplished. A whole new learning curve can be seen on low profile ferrite cores and printed circuit boards if one is going to do any planar transformer designs. It is an all-new technology for the transformer engineer. The two basic items that made this technology feasible were the power, MOSFETs that increased the switching frequency and enabled the designer to reduce the turns, and the ferrite core, which can be molded and machined into almost any shape. After this paper was written the interest in planar magnetics seems to increase each year.

Planar Transformer Basic Construction

Here, shown in Figure 20-1 through Figure 20-4 are four views of a typical EE core, planar construction method. The assembled planar transformers have very unique characteristics in their finished construction. In the assembled planar transformer, every primary turn is at a precise location, governed by the PC board. The primary is always the same distance from the secondary. This provides a tight control over the primary to secondary leakage inductance. Using the same insulating material will always provide the same capacitance between primary and secondary; in this way, all parasitics will be the same from unit-to-unit. With this type of planar construction, the engineer will have a tight control over leakage inductance, the resonant frequency, and the common-mode rejection. A tight control is necessary on all materials used.

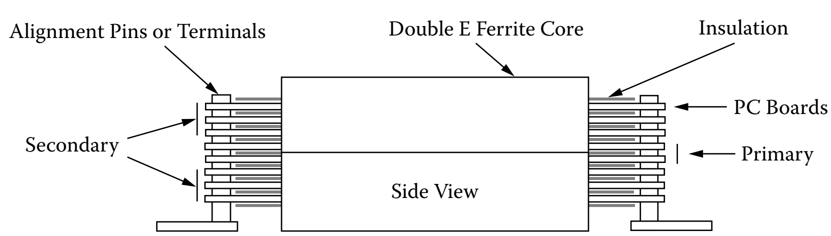

Figure 20-1. Side View of a Typical EE Planar Transformer.

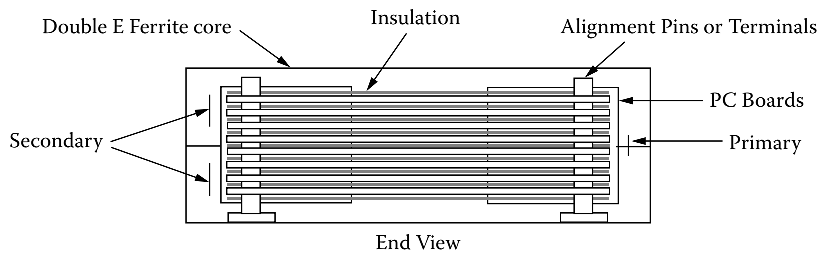

Figure 20-2. End View of a Typical EE Planar Transformer.

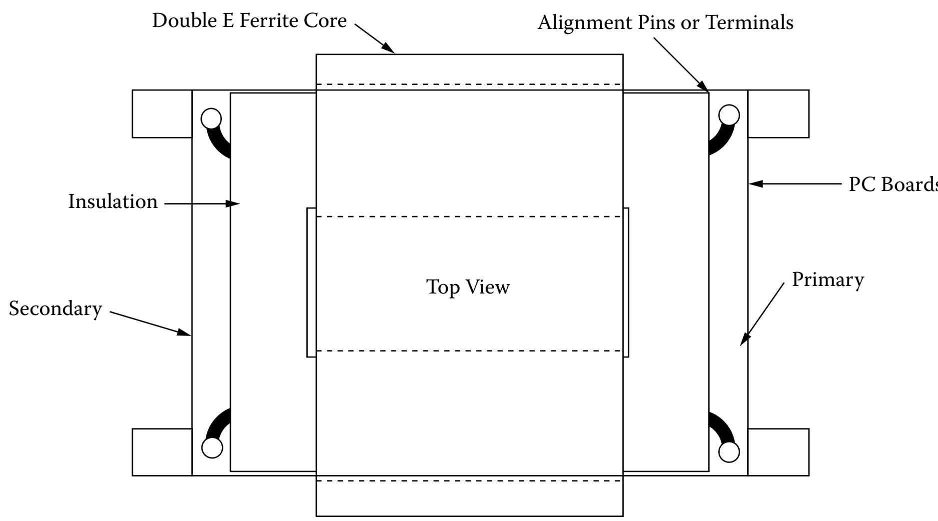

Figure 20-3. Top View of a Typical EE Planar Transformer.

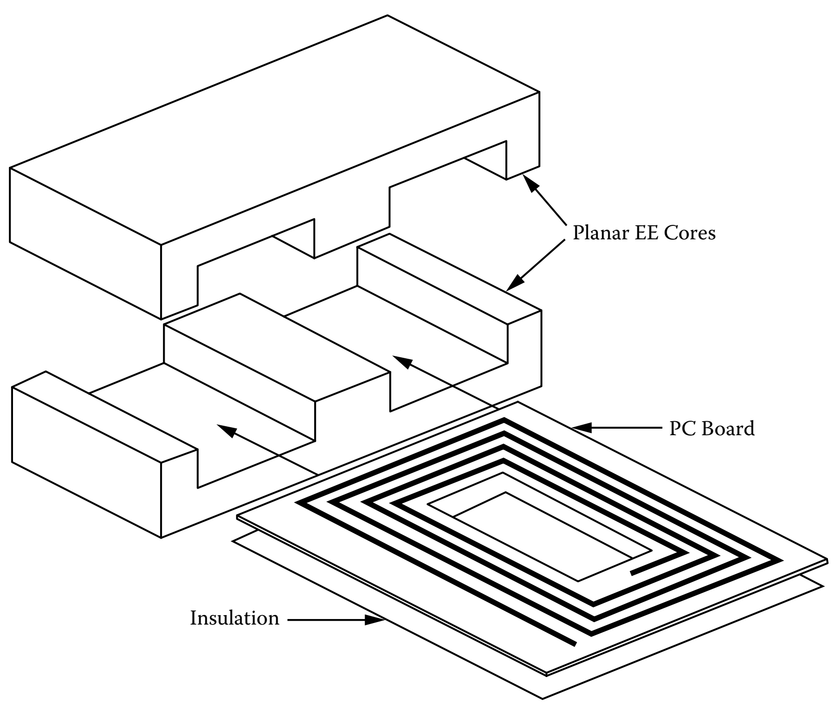

Figure 20-4. A Perspective View of a Typical EE Planar Transformer.

Planar Integrated PC Board Magnetics

Planar transformers and inductors are now being integrated right on the main PC board. Design engineers are pushing the operating frequency higher and higher to where it is commonplace to operate at frequency range between 250-500kHz. As the frequency increases the power supplies are getting smaller and smaller. In order to reduce the size of the power supply even further, engineers are going to planar magnetics that are integrated into the main PC board. An exploded view to show the multi-layers PC board of a planar transformer that has been integrated into the main PC board is shown in Figure 20-5. The final assembly of the same planar transformer is shown in Figure 20-6.

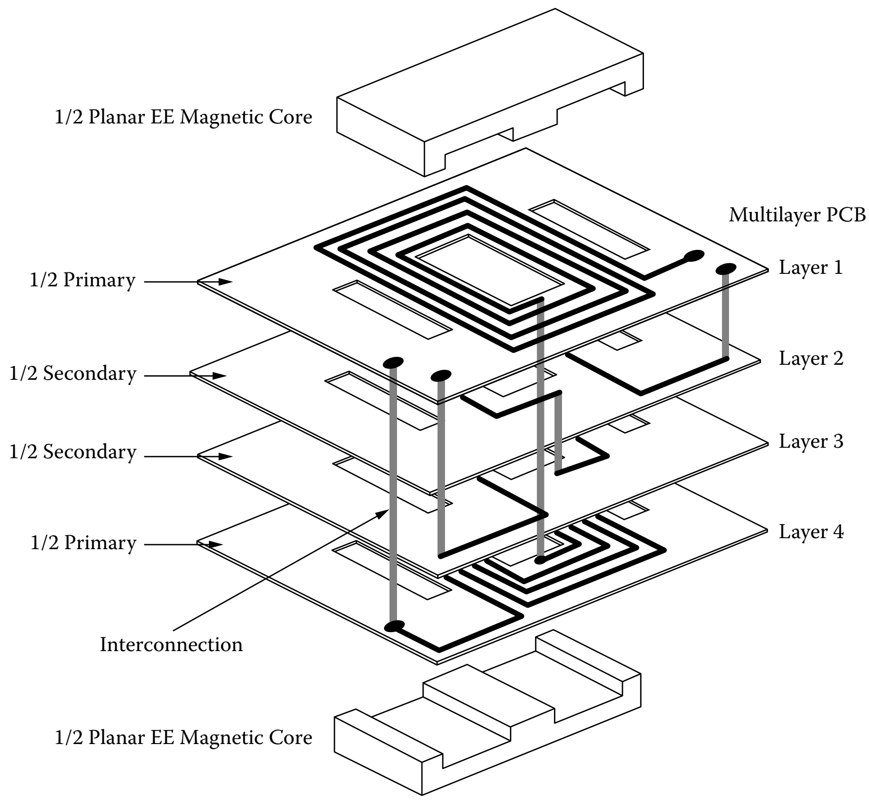

Figure 20-5. A Planar Transformer Integrated into the Main PC Board.

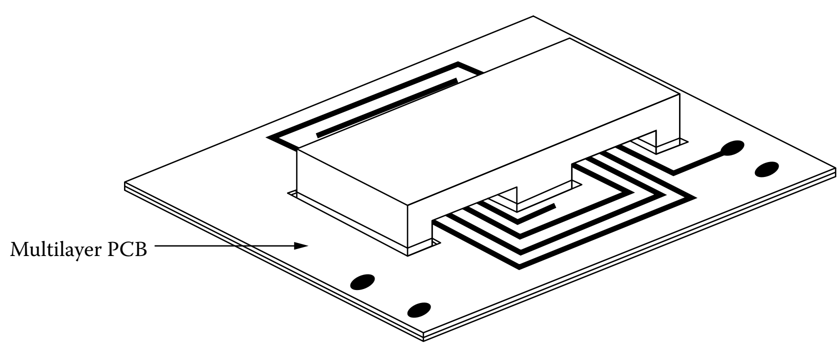

Figure 20-6. PC Board Planar Transformer in Final Assembly.

Core Geometries

The EE and EI are not the only planar geometries now available. There are a few firms in the ferrite industry that offer low profile versions of their standard cores, giving the engineer a few more choices in his design. There are EE and EI cores available from Magnetics Inc. as shown in Figure 20-7; there are ER cores available from Ferroxcube, as shown in Figure 20-8; there are ETD-lp cores available from Ferrite International, as shown in Figure 20-9; there are PQ-lp cores available from Ferrite International, as shown in Figure 20-10; and there are RM-lp cores available from Ferroxcube, as shown in Figure 20-11. There are several advantages, with cores with a round center post, such as PQ-lp, RM-lp, ETD-lp and ER. A round center post results in a more efficient use of copper and a more efficient use of board space. There is a company, Ceramic Magnetics, Inc. (CMI), that can modify any of these cores to your specification or machine a special core for your application. The IEC has a new standard 62313 for planar cores that supercedes standard 61860.

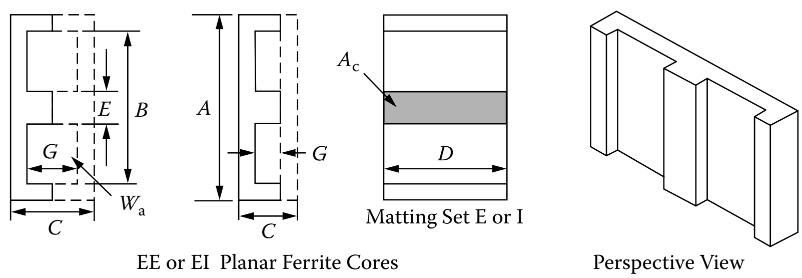

Figure 20-7. Magnetic Inc. EE and EI Low Profile Planar Cores.

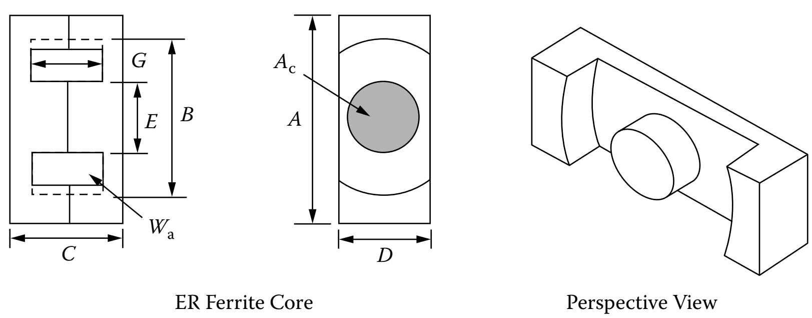

Figure 20-8. Ferroxcube ER Low Profile Planar Cores.

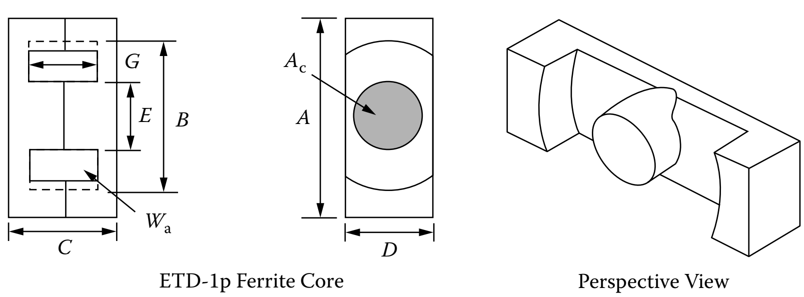

Figure 20-9. Ferrite International ETD Low Profile Planar Cores.

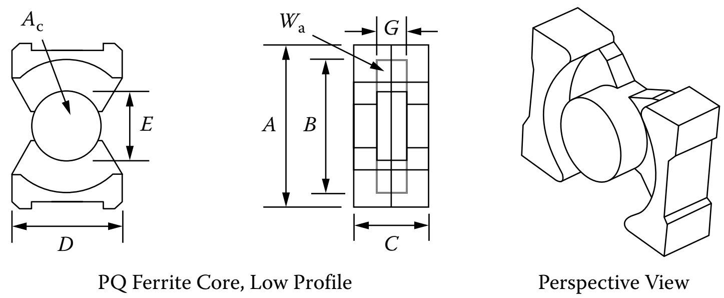

Figure 20-10. Ferrite International PQ Low Profile Planar Cores.

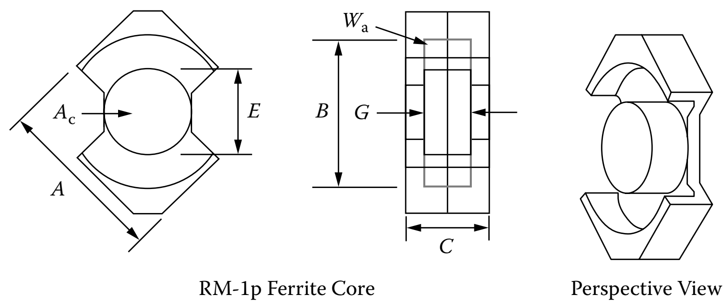

Figure 20-11. Ferroxcube RM Low Profile Planar Cores.

Planar Transformer and Inductor Design Equations

The same design Equations are used, as well as the criteria used to select the proper core, to design a planar transformer as a conventional transformer. Faraday's Law is still used to calculate the required turns, as shown in Equation [20-1].

$$N = \frac{V_p\left(10^4\right)}{K_f f A_c B_{ac}}, \quad \text{[turns]} \qquad [20\text{-}1]$$

The area product, A_p, is the power handling capability of the core for transformers, as shown in Equation [20-2].

$$A_p = \frac{P_t\left(10^4\right)}{K_f K_u f A_c B_{ac} J}, \quad \left[\text{cm}^4\right] \qquad [20\text{-}2]$$

In the gapped inductor Equation, L, is shown in Equation [20-3].

$$L = \frac{0.4\pi N^2 A_c\left(10^{-8}\right)}{l_g + \left(\dfrac{MPL}{\mu_m}\right)}, \quad \text{[henrys]} \qquad [20\text{-}3]$$

The area product, A_p, is the energy handling capability of the core for inductors, as shown in Equation [20-4].

$$A_p = \frac{2\left(\text{Energy}\right)}{K_u B_{ac} J}, \quad \left[\text{cm}^4\right] \qquad [20\text{-}4]$$

Window Utilization, K_u

The window utilization factor in the conventional transformer is about 0.40. This means that 40% of the window is filled with copper. The other 60% of the area is devoted to the bobbin or tube, to the insulation both layer and wire, and to the winding technique. The window utilization is explained, in detail, in Chapter 4. Designing a planar transformer and using the PC winding technique, reduces the window utilization factor even further. The window utilization, K_u, comparison of the two different winding techniques is shown in Figure 20-12.

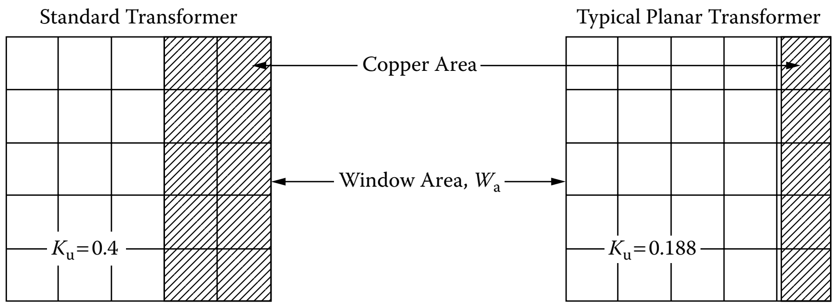

Figure 20-12. Comparing the Window Utilization of a Standard Transformer and a Planar Transformer.

A PC board window utilization, K_u, calculation example will be as follows:

The windings will be placed on a double-sided 2oz PC board 10 mils thick, giving a total thickness of 15.4 mils (0.0391 cm). The Mylar insulation material is between the PC boards, and between the PC boards, and the core will add another 4 mils (0.0102 cm) to the thickness. This will give 19.4 mils (0.0493 cm) per layer. There will be a 20 mil space (margin) between the edge of the board and the copper clad. The copper width will be the window width of 0.551cm, minus 2x the margin of 0.102. This will give a total copper width of 0.449. The window utilization, K_u, will be summed in Table 20-1, using Figure 20-13 as a guide.

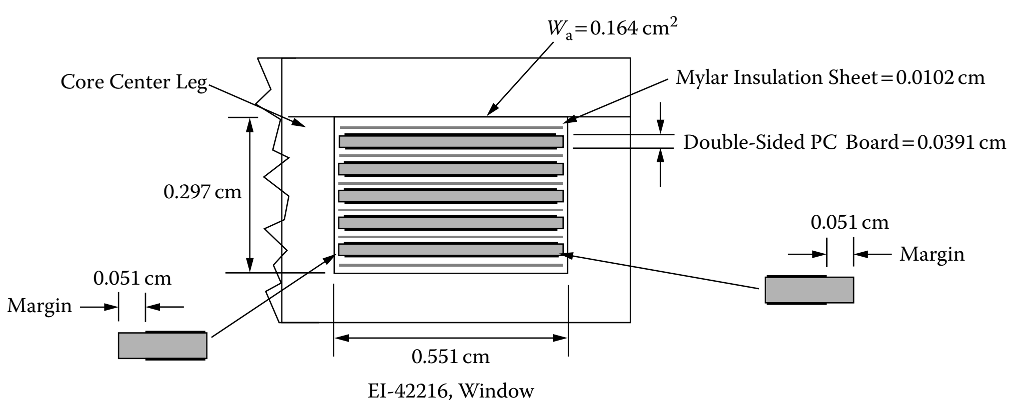

Figure 20-13. Window Utilization of a Typical EI Planar Transformer.

Table 20-1. EI-42216 Window Utilization

EI-42216 Window Utilization	
Window Height, cm	0.2970
Window Width, cm	0.5510
Window Area, cm^2	0.1640
PC Board Thickness with Copper, cm	0.0391
Sheet Insulator, cm	0.0102
Total Insul. 5+1 Layers Thick, cm	0.0612
Total Thickness 5 Layers, cm	0.2570
Copper Thickness 5 Layers, cm	0.0686
Copper Width, cm	0.4494
Total Copper Area, cm^2	0.0308
Window Utilization, K_u	0.1878

Current Density, J

One of the unknown factors in designing planar transformers is the current density, J. The current density controls the copper loss (regulation) and the inherit temperature rise caused by the copper loss. The temperature rise is normally controlled by the surface dissipation of the transformer. The size of a transformer goes up by the cubic law, and the surface area goes up by the square law. Large transformers, such as 60 Hz, are designed with a low current density, while 400 Hz are designed with higher current density for the same temperature rise. There used to be an old rule of thumb, for a large transformer, that you use 1000 circular mils per amp, and for a small transformer, you use 500 circular mils (CM) per amp:

$$500\text{CM/Amp} \approx 400\,\text{Amps/cm}^2, \quad [400\text{ Hertz Aircraft}]$$
$$1000\text{CM/Amp} \approx 200\,\text{Amps/cm}^2, \quad [60\text{ Hertz}]$$

Planar transformer designers handle the current density in a different way. When designing planar transformer PC windings, designers use the same technology used by the printed, circuit board designers, and that is the current rating for a given voltage drop and temperature rise. It is another way of saying the same thing. The printed circuit boards are covered with a copper clad. The thickness of this copper is called out in ounces, such as 1oz, 2oz, and 3oz. The weight in ounces comes from an area of one square foot of material. So 1oz of copper clad would be 1 square foot, and have a thickness of 0.00135 of an inch; 2oz would be 0.0027 of an inch; and 3oz would be 0.00405 of an inch. Tables have been made to show the current capacity for a constant temperature rise with different line width. The design data for 1oz copper is shown in Table 20-2. The 2oz copper is shown in Table 20-3, and 3oz copper is shown in Table 20-4. Planar transformer engineers are using the industrial guidelines for their selection of copper trace thickness and line width, based on temperature rise. For the first effort for a planar transformer, the PC winding should be around:

$$100\text{CM/Amp} \approx 2000\,\text{Amps/cm}^2, \quad [500\text{ kHertz Planar Transformers}]$$

If the current density is based on Table 20-1, with a line width of 0.06 inches, then use:

$$35 \text{CM}/\text{Amp} \approx 5700 \text{Amps}/\text{cm}^2, \quad [500 \text{ kHertz Planar Transformers}]$$

Table 20-2. Design Data for 0.00135 Inch Thick Copper Clad

Printed Circuit Trace Data for 1oz Copper (Based on 10 Inches Long)							
Line Width	Line Width	Resistance micro-ohm	Copper Weight 1oz Thickness 0.00135		Temp. °C Increase above Amb. vs. Current in Amperes		
Inches	mm	per-mm	cm^2	AWG[a]	5°	20°	40°
0.0200	0.51	989.7	0.000174	35	1.00	3.00	4.00
0.0400	1.02	494.9	0.000348	32	2.25	5.00	6.50
0.0600	1.52	329.9	0.000523	30	3.00	6.50	8.00
0.0800	2.03	247.4	0.000697	29	4.00	7.00	9.50
0.1000	2.54	197.9	0.000871	28	4.50	8.00	11.00
0.1200	3.05	165.0	0.001045	27	5.25	9.25	12.00
0.1400	3.56	141.4	0.001219	26	6.00	10.00	13.00
0.1600	4.06	123.7	0.001394	26	6.50	11.00	14.25
0.1800	4.57	110.0	0.001568	25	7.00	11.75	15.00
0.2000	5.08	99.0	0.001742	25	7.25	12.50	16.60

Source: Data From: Handbook of Electronic Packaging.
[a] This is a close approximation to an equivalent AWG wire size.

Table 20-3. Design Data for 0.0027 Inch Thick Copper Clad

Printed Circuit Trace Data for 2oz Copper (Based on 10 Inches Long)							
Line Width	Line Width	Resistance micro-ohm	Copper Weight 2oz Thickness 0.0027		Temp. °C Increase above Amb. vs. Current in Amperes		
Inches	mm	per-mm	cm^2	AWG[a]	5°	20°	40°
0.0200	0.51	494.9	0.000348	32	2.00	4.00	6.25
0.0400	1.02	247.4	0.000697	29	3.25	7.00	9.00
0.0600	1.52	165.0	0.001045	27	4.25	9.00	11.25
0.0800	2.03	123.7	0.001394	26	5.00	10.25	13.25
0.1000	2.54	99.0	0.001742	25	5.25	11.00	15.25
0.1200	3.05	82.5	0.002090	24	5.75	12.25	17.00
0.1400	3.56	70.7	0.002439	23	6.25	13.25	18.50
0.1600	4.06	61.9	0.002787	23	6.50	14.25	20.50
0.1800	4.57	55.0	0.003135	22	7.00	15.25	22.00
0.2000	5.08	49.5	0.003484	22	7.25	16.25	24.00

Source: Data From: Handbook of Electronic Packaging.
[a] This is a close approximation to an equivalent AWG wire size.

Table 20-4. Design Data for 0.00405 Inch Thick Copper Clad

Printed Circuit Trace Data for 3oz Copper (Based on 10 Inches Long)							
Line Width	Line Width	Resistance micro-ohm	Copper Weight 3oz Thickness 0.00405		Temp. °C Increase above Amb. vs. Current in Amperes		
Inches	mm	per-mm	cm^2	AWG[a]	5°	20°	40°
0.0200	0.51	329.9	0.000523	30	2.50	6.00	7.00
0.0400	1.02	165.0	0.001045	27	4.00	8.75	11.00
0.0600	1.52	110.0	0.001568	25	4.75	10.25	13.50
0.0800	2.03	82.5	0.002090	24	5.50	12.00	15.75
0.1000	2.54	66.0	0.002613	23	6.00	13.25	17.50
0.1200	3.05	55.0	0.003135	22	6.75	15.00	19.50
0.1400	3.56	47.1	0.003658	22	7.00	16.00	21.25
0.1600	4.06	41.2	0.004181	21	7.25	17.00	23.00
0.1800	4.57	36.7	0.004703	20	7.75	18.25	25.00
0.2000	5.08	33.0	0.005226	20	8.00	19.75	27.00

Source: Data From: Handbook of Electronic Packaging.
[a] This is a close approximation to an equivalent AWG wire size.

Printed Circuit Windings

There will be a few paths of mystery along the way when engineers first get started in the design of a planar transformer. Therefore, it is easier to start on a simple design and use magnet wire, then convert that into a truly all-planar approach, using a PC winding board design. In this way the engineer will slide up the learning curve slowly. There are several benefits to a printed circuit winding. Once the printed winding board is finished and the layout is fixed, the winding will not vary and all of the parasitics, including the leakage inductance, will be frozen. This is not necessarily true in conventional transformers. There are two basic core configurations available to the engineer for planar design. The first configuration is the EE or EI with the rectangular center post. A typical high current and low current winding PC board for E cores is shown in Figure 20-14.

The second configuration is shown in Figure 20-15. These are four cores with round center legs. Winding PC boards with round center legs are used on PQ-lp, RM-lp, ETD-lp and ER cores. There is an advantage to cores with round center legs. Cores with round center leg will produce a round ID, OD resulting in a more efficient use of copper.

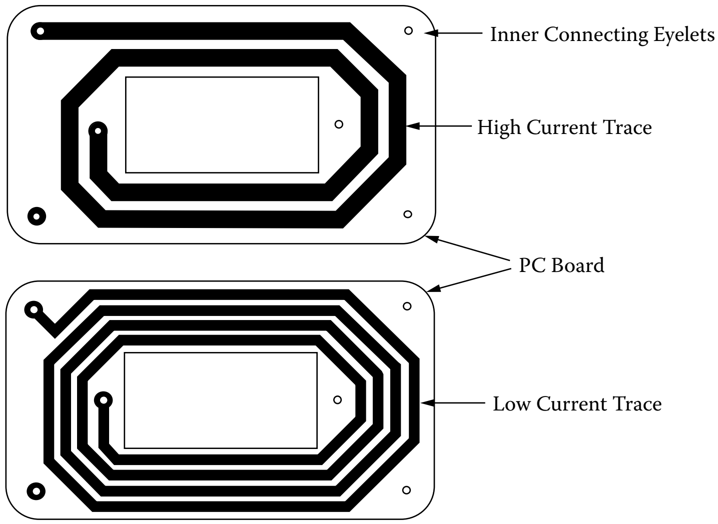

Figure 20-14. Typical Planar E Core Winding PC Board.

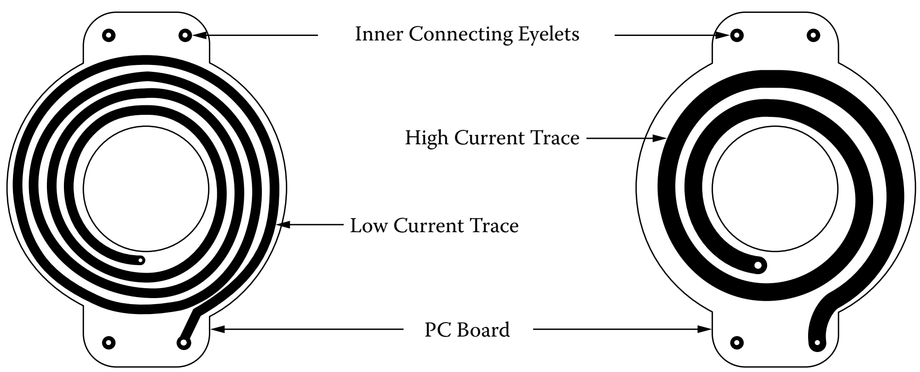

Figure 20-15. Typical Circular Winding PC Board for Cores with Round Center Leg.

Calculating the Mean Length Turn, MLT

The Mean Length Turn (MLT), is required to calculate the dc winding resistance. With the winding resistance known, the winding voltage drop can be calculated at rated load. The winding dimensions, relating to the Mean Length Turn (MLT) for a rectangular winding, is shown in Figure 20-16, along with the MLT equation, and a circular winding is shown in Figure 20-17, along with the MLT equation.

$$\mathrm{MLT} = 2B + 2C + 2.82A, \quad [\mathrm{mm}] \qquad [20\text{-}5]$$

$$\mathrm{MLT} = \frac{\pi(OD + ID)}{2}, \quad [\mathrm{mm}] \qquad [20\text{-}6]$$

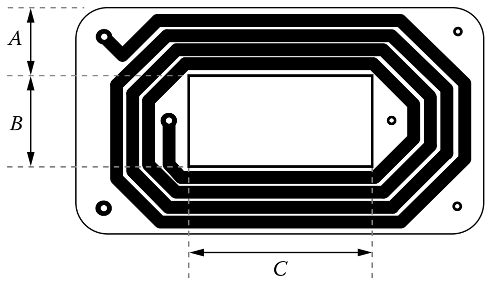

Figure 20-16. Dimensions, Relating to a Rectangular Winding, Mean Length Turn (MLT).

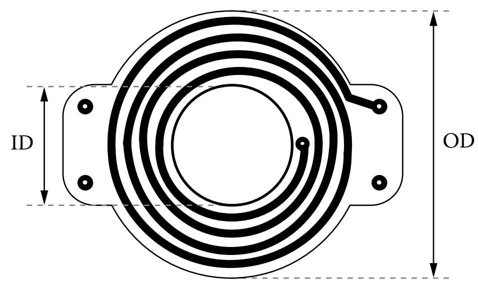

Figure 20-17. Dimensions, Relating to a Circular Winding, Mean Length Turn (MLT).

Winding Resistance and Dissipation

The winding dc resistance and voltage drop will be calculated as follows:

Calculate the Mean Length Turn (MLT) using the winding board configuration and Equation in Figure 20-17. Use the printed winding data in Table 20-5.

Table 20-5. PC Board Winding Data

PC Winding Data		
Item		Units
PC Board Turns Each Side	4	
Winding Trace Thickness	0.0027	inches
Winding Trace Width	2.54	mm
Trace Resistance	99	μΩ/mm
Winding Board, OD	31.5	mm
Winding Board, ID	14.65	mm
Winding Current, I	3	amps
PC Board Thickness	0.5	mm
PC Board Dielectric Constant, K	4.7	

Step 1: Calculate the Mean Length Turn, MLT:

$$\text{MLT} = \frac{\pi(OD + ID)}{2}, \quad [\text{mm}]$$
$$\text{MLT} = \frac{3.14(31.5 + 14.65)}{2}, \quad [\text{mm}]$$
$$\text{MLT} = 72.5, \quad [\text{mm}]$$

Step 2: Calculate the winding resistance, R:

$$R = MLT(N)\left(\frac{\mu\Omega}{\text{mm}}\right)(10^{-6}), \quad [\text{ohms}]$$
$$R = (72.5)(8)(99.0)(10^{-6}), \quad [\text{ohms}]$$
$$R = 0.057, \quad [\text{ohms}]$$

Step 3: Calculate the winding voltage drop, V_w:

$$V_w = IR, \quad [\text{volts}]$$
$$V_w = (3.0)(0.057), \quad [\text{volts}]$$
$$V_w = 0.171, \quad [\text{volts}]$$

Step 4: Calculate the winding dissipation, P_w:

$$P_w = I^2 R, \quad [\text{watts}]$$
$$P_w = (3)^2(0.057), \quad [\text{watts}]$$
$$P_w = 0.513, \quad [\text{watts}]$$

PC Winding Capacitance

The PC winding board trace will have capacitance, to the other side of the board, as shown in Figure 20-18. This capacitance could be to another winding, or to a Faraday shield to ground.

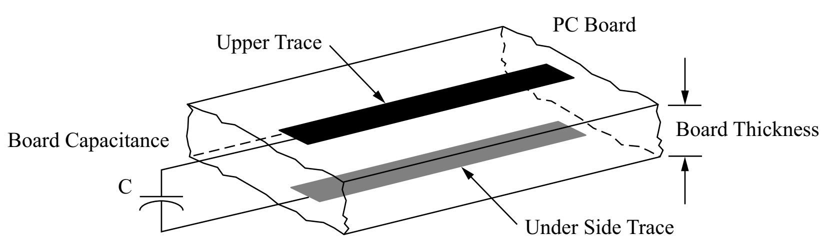

Figure 20-18. PC Board Trace Capacitance.

The formula for calculating the winding trace capacitance, to either another winding trace or ground plane, is given in Equation 20.7.

$$C_p = \frac{0.0085KA}{d}, \quad [\text{pf}] \qquad [20.7]$$

Where:

$C_p =$ capacitance, [pf]
$K =$ dielectric constant
$A =$ area of the trace, [mm^2]
$d =$ thickness of the PC board, [mm]

A typical square wave power converter, operating at 250kHz, will have extremely fast rise and fall times in the order of 0.05 micro-seconds. This fast excursion will generate a fairly high current pulse depending on the capacitance and source impedance.

The calculation of the winding capacitance is as follows:

Use the PC board winding data in Table 20-5, the outline drawing in Figure 20-19, and Equation 20-7:

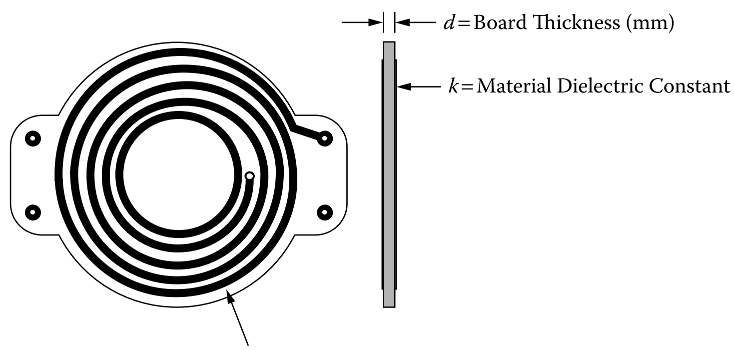

Figure 20-19. PC Board Winding Capacitance.

Step 1: Calculate the winding trace area, A.

$$A = (\text{trace width, mm})(\text{MLT, mm})(\text{turns, N}), \quad [\text{mm}^2]$$
$$A = (2.54)(72.5)(8), \quad [\text{mm}^2]$$
$$A = 1473, \quad [\text{mm}^2]$$

Step 2: Calculate the winding capacitance, C_p.

$$C_p = \frac{0.0085KA}{d}, \quad [\text{pf}]$$
$$C_p = \frac{0.0085(4.7)(1473)}{(0.50)}, \quad [\text{pf}]$$
$$C_p = 118, \quad [\text{pf}]$$

Planar Inductor Design

Planar inductors are designed the same way as the conventional inductors. See Chapter 8. Planar inductors use the same planar cores and PC winding board techniques as the transformers. The main difference is the inductor will have a gap to prevent the dc current from prematurely saturating the core. It is normal to operate planar magnetics at a little higher temperature than conventional designs. It is important to check the maximum operating flux level at maximum operating temperature.

Fringing flux can be severe in any gapped ferrite inductor, but, even more so, on planar construction, because of the printed winding board, as shown in Figure 20-20. When the flux intersects the copper winding, eddy currents are generated, which produces hot spots and reduces the overall efficiency. The use of a PC winding board, (flat traces), can give the eddy currents an added degree of freedom. The resulting loss could be a disaster.

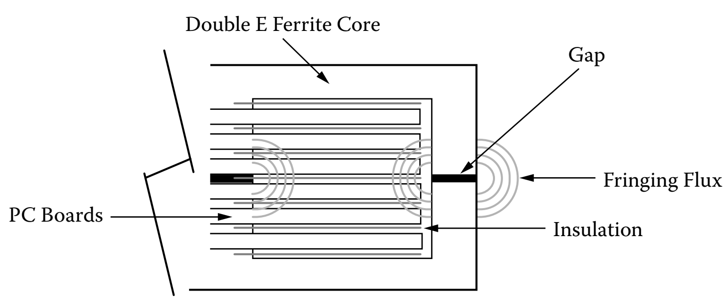

Figure 20-20. Fringing Flux Cutting Across PC Winding Boards.

Winding Termination

Making connections from a planar transformer to the outside world could be very clumsy, if not enough thought is put in for termination. It has to be remembered that this is a high frequency transformer, and skin effect, (ac resistance), has to be addressed. Because of the skin effect it is important the external leads of the planar transformer must be kept as short as possible. Terminations are very important for currents of one amp and above. A poor connection will only get worse. It is recommended to use plated-through holes and eyelets, where possible, but cost will control that. If the transformer has many interconnections, or only a few, there must be provisions made for those connections. When the PC winding boards are stacked, and because of the high density, all connections and interconnections have to be done with extended area pads, as shown in Figure 20-21. The PC winding boards require good artwork registry to make sure the interconnections can be made between boards. Interconnections are usually done, by passing a bus wire through a hole, and at the same time making the connection on the other board. If the solder terminations are to be made on the board, then it is important to leave as much room as possible especially if the connection is to be made with copper foil, as shown in Figure 20-22. When the PC windings have to be paralleled, because of the increased current, the interconnecting jumpers will also have to be increased.

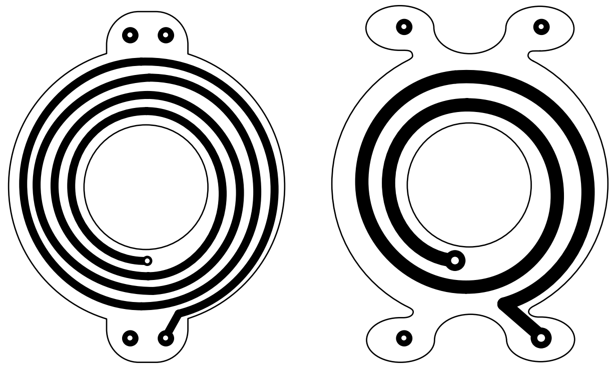

Figure 20-21. PC Winding Boards Showing Butterfly Pads.

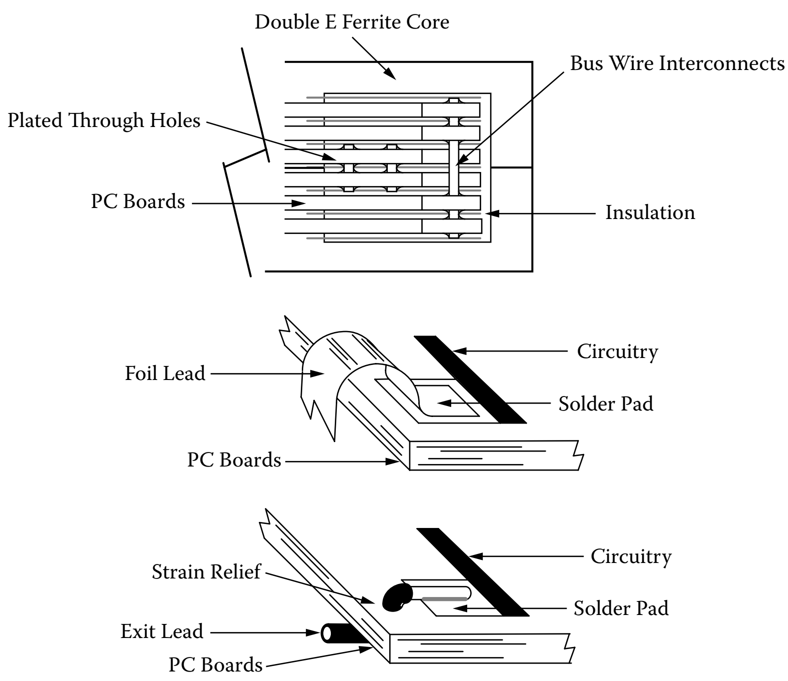

Figure 20-22. PC Winding Boards, Showing Interconnections and Exit Leads.

PC Board Base Materials

PC Board materials are available in various grades, as defined by the National Electrical Manufacturers Association (NEMA). The important properties for PC Board materials are tabulated in Table 20-6. It is very important to choose the correct PC board material for your application. Planar transformers are normally stressed to the last watt for a given temperature rise. This could give rise to hot spots at winding terminations and cause PC Board discoloration. Due to their inherit design Planar transformers will have a wide temperature delta, Δt. It would be wise to stay away from paper/phenolic materials and materials that absorb moisture.

Table 20-6. Properties of Typical Printer Circuit Board Materials

Properties of Typical Printed Circuit Board Materials							
Material/Comments	NEMA Grade						
	FR-1 Paper Phenolic	FR-2 Paper Phenolic	FR-3 Paper Epoxy	FR-4 Glass/ Cloth Epoxy	FR-5 Glass/ Cloth Epoxy	G10 Glass/ Cloth Epoxy	G11 Glass/ Cloth Epoxy
Mechanical Strength	good	good	good	excellent	excellent	excellent	excellent
Moisture Resistant	poor	good	good	excellent	excellent	excellent	excellent
Insulation	fair	good	good	excellent	excellent	excellent	excellent
Arc Resistance	poor	poor	fair	good	good	good	good
Tool Abrasion	good	good	good	poor	poor	poor	poor
Max. Cont. Temp. °C	105	105	105	130	170	130	170
Dielectric Constant, K	4.2	4.2	4.4	4.7	4.3	4.6	4.5

Core Mounting and Assembly

Core assembly and mounting should be strong and stable with temperature. One of the most viable methods for securing core halves together is epoxy adhesive. There is one epoxy adhesive that has been around a long time and that's 3M EC-2216A/B. This bonding technique is shown in Figure 20-23, and it seems to work quite well. When the core halves are properly bonded with epoxy adhesive, there will be little or no effect on the electrical performance. This means the epoxy adhesive added little or no gap to the mating surface. Large temperature excursions are normal in planar magnetics. Care should be taken into account for the coefficients of thermal expansion between the core and mounting surfaces. It has to be remembered ferrite is a ceramic and is very brittle. Planar cores have a low silhouette with thin sections that cannot absorb as much strain as other geometries. After the planar transformer has been assembled, there should be a small amount of play in the PC winding assembly to guarantee there will be a minimum of stress over temperature.

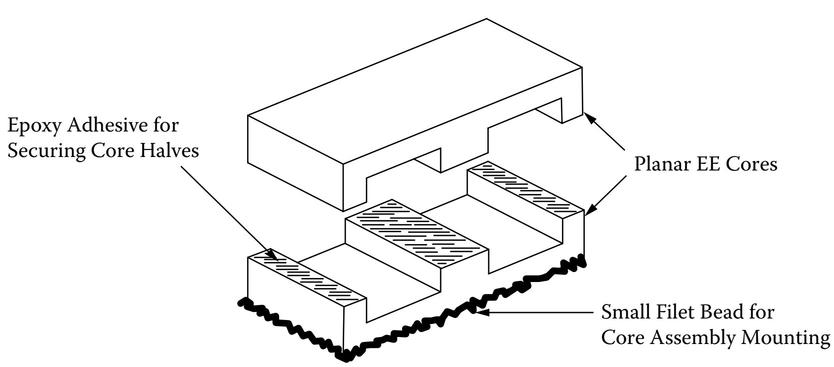

Figure 20-23. Epoxy Adhesive for Securing Transformer Assembly.

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Chapter 21

Derivations for the Design Equations

Table of Contents

Output Power, P_o Versus Apparent Power, P_t Capability

Introduction

Output power, P_o, is of the greatest interest to the user. To the transformer designer, the apparent power, P_t, which is associated with the geometry of the transformer, is of greater importance. Assume, for the sake of simplicity, that the core of an isolation transformer has only two windings in the window area, a primary and a secondary. Also, assume that the window area, W_a, is divided up in proportion to the power-handling capability of the windings, using equal current density. The primary winding handles, P_{in}, and the secondary handles, P_o, to the load. Since the power transformer has to be designed to accommodate the primary, P_{in}, and, P_o, then,

By definition:

$$P_t = P_{in} + P_o, \quad \text{[watts]}$$
$$P_{in} = \frac{P_o}{\eta}, \quad \text{[watts]} \qquad \text{[21-A1]}$$

The primary turns can be expressed using Faraday's Law:

$$N_p = \frac{V_p\left(10^4\right)}{A_c B_{ac} f K_f}, \quad \text{[turns]} \qquad \text{[21-A2]}$$

The winding area of a transformer is fully utilized when:

$$K_u W_a = N_p A_{wp} + N_s A_{ws} \qquad \text{[21-A3]}$$

By definition the wire area is:

$$A_w = \frac{I}{J}, \quad \text{[cm}^2\text{]} \qquad \text{[21-A4]}$$

Rearranging the Equation shows:

$$K_u W_a = N_p\left(\frac{I_p}{J}\right) + N_s\left(\frac{I_s}{J}\right) \qquad \text{[21-A5]}$$

Now, substitute in Faraday's Equation:

$$K_u W_a = \frac{V_p\left(10^4\right)}{A_c B_{ac} f K_f}\left(\frac{I_p}{J}\right) + \frac{V_s\left(10^4\right)}{A_c B_{ac} f K_f}\left(\frac{I_s}{J}\right) \qquad \text{[21-A6]}$$

Rearranging shows:

$$W_a A_c = \frac{\left[\left(V_p I_p\right)+\left(V_s I_s\right)\right]\left(10^4\right)}{B_{ac}\, f\, J\, K_f\, K_u}, \quad [\text{cm}^4] \qquad \text{[21-A7]}$$

The output power, P_o, is:

$$P_o = V_s I_s, \quad [\text{watts}] \qquad \text{[21-A8]}$$

The input power, P_{in}, is:

$$P_{in} = V_p I_p, \quad [\text{watts}] \qquad \text{[21-A9]}$$

Then:

$$P_t = P_{in} + P_o, \quad [\text{watts}] \qquad \text{[21-A10]}$$

Transformer Derivation for the Core Geometry, K_g

Introduction

Although most transformers are designed for a given temperature rise, they can also be designed for a given regulation. The regulation and power-handling ability of a core are related to two constants, K_g and K_e by the equation:

$$P_t = 2K_g K_e \alpha, \quad [\text{watts}] \qquad \text{[21-B1]}$$

Where:

$$\alpha = \text{Regulation}, \quad [\%]$$

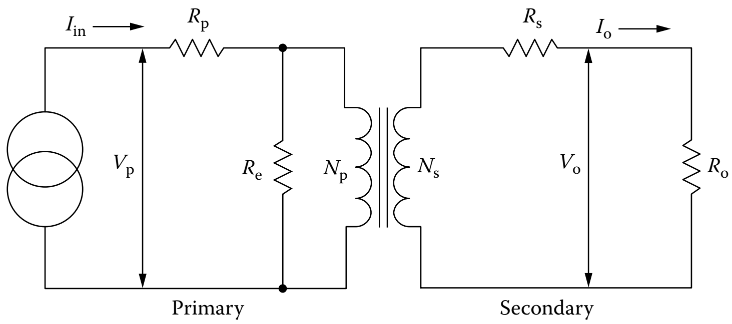
The constant, K_g, is a function of the core geometry:

$$K_g = f\left(A_c, W_a, \text{MLT}\right) \qquad \text{[21-B2]}$$

The constant, K_e, is a function of the magnetic and electrical operating conditions:

$$K_e = g\left(f, B_m\right) \qquad \text{[21-B3]}$$

The derivation of the specific functions for, K_g and K_e, is as follows: First, assume there is a two-winding transformer with equal primary and secondary regulation, as schematically shown in Figure 21-B1. The primary winding has a resistance of, R_p, ohms, and the secondary winding has a resistance of, R_s ohms:



$n = N_s/N_p = 1$

Figure 21-B1. Isolation Transformer.

$$\alpha = \frac{\Delta V_p}{V_p}(100) + \frac{\Delta V_s}{V_s}(100) \qquad [21\text{-B4}]$$

The assumption, for simplicity, is that R_e is infinity (no core loss).

And:

$$I_{in} = I_o \qquad [21\text{-B5}]$$

Then:

$$\Delta V_p = I_p R_p = \Delta V_s = I_s R_s, \quad \text{[volts]} \qquad [21\text{-B6}]$$

$$\alpha = 2\frac{I_p R_p}{V_p}(100) \qquad [21\text{-B7}]$$

Multiply the numerator and denominator by V_p:

$$\alpha = 200\frac{I_p R_p}{V_p}\left(\frac{V_p}{V_p}\right) \qquad [21\text{-B8}]$$

$$\alpha = 200\frac{R_p VA}{V_p^2} \qquad [21\text{-B9}]$$

From the resistivity formula, it is easily shown that:

$$R_p = \frac{(\text{MLT})N_p^2}{W_a K_p}\rho \qquad [21\text{-B10}]$$

Where:

$\rho = 1.724\ (10^{-6})$ ohm cm

K_p is the window utilization factor (primary)

K_s is the window utilization factor (secondary)

$$K_p = \frac{K_u}{2} = K_s \qquad [21\text{-}B11]$$

Faraday's Law expressed in metric units is:

$$V_p = K_f\, f\, N_p\, A_c\, B_m\left(10^{-4}\right) \qquad [21\text{-}B12]$$

Where:

$K_f = 4.0$ for a square wave.

$K_f = 4.44$ for a sine wave.

Substituting Equation 21-B10 and 21-B12, for R_p and V_p, in Equation [21-B13]:

$$VA = \frac{E_p^2}{200R_p}\alpha \qquad [21\text{-}B13]$$

The primary VA is:

$$VA = \frac{\left(K_f\, f\, N_p\, A_c\, B_m\left(10^{-4}\right)\right)\left(K_f\, f\, N_p\, A_c\, B_m\left(10^{-4}\right)\right)}{200\left(\dfrac{(\mathrm{MLT})N_p^2}{W_a K_p}\rho\right)}\alpha \qquad [21\text{-}B14]$$

Simplify:

$$VA = \frac{K_f^2\, f^2\, A_c^2\, B_m^2\, W_a K_p\left(10^{-10}\right)}{2(\mathrm{MLT})\rho}\alpha \qquad [21\text{-}B15]$$

Inserting $1.724(10^{-6})$ for ρ:

$$VA = \frac{0.29K_f^2\, f^2\, A_c^2\, B_m^2\, W_a K_p\left(10^{-4}\right)}{\mathrm{MLT}}\alpha \qquad [21\text{-}B16]$$

Let the primary electrical equal:

$$K_e = 0.29K_f^2\, f^2\, B_m^2\left(10^{-4}\right) \qquad [21\text{-}B17]$$

Let the primary core geometry equal:

$$K_g = \frac{W_a A_c^2 K_p}{\mathrm{MLT}}, \quad [\mathrm{cm}^5] \qquad [21\text{-}B18]$$

The total transformer window utilization factor is:

$$K_p + K_s = K_u$$

$$K_p = \frac{K_u}{2} = K_s \qquad [21\text{-}B19]$$

When this value for, K_p, is put into Equation [21-B16], then:

$$VA = K_e K_g \alpha \qquad [21\text{-B20}]$$

Where:

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right) \qquad [21\text{-B21}]$$

The above VA is the primary power, and the window utilization factor, K_u, includes both the primary and secondary coils.

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \qquad [21\text{-B22}]$$

Regulation of a transformer is related to the copper loss, as shown in Equation [21-B23]:

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%] \qquad [21\text{-B23}]$$

The total VA of the transformer is primary plus secondary:

$$\begin{aligned}&\text{Primary,}\ \ VA = K_e K_g \alpha\\ &\text{plus}\\ &\text{Secondary,}\ \ VA = K_e K_g \alpha\end{aligned} \qquad [21\text{-B24}]$$

The apparent power, P_t, then is:

$$\begin{aligned}P_t &= (\text{Primary}) K_e K_g \alpha + (\text{Secondary}) K_e K_g \alpha\\ P_t &= 2 K_e K_g \alpha\end{aligned} \qquad [21\text{-B25}]$$

Transformer Derivation for the Area Product, A_p

Introduction

The relationship between the power-handling capability of a transformer and the area product, A_p, can be derived as follows:

Faraday's Law expressed in metric units is:

$$V = K_f f N_p A_c B_m \left(10^{-4}\right) \qquad [21\text{-C1}]$$

Where:

K_f = 4.0 for a square wave.

K_f = 4.44 for a sine wave.

The winding area of a transformer is fully utilized when:

$$K_u W_a = N_p A_{wp} + N_s A_{ws} \qquad [21\text{-C2}]$$

By definition the wire area is:

$$A_w = \frac{I}{J}, \quad [\text{cm}^2] \qquad [21\text{-C3}]$$

Rearranging the equation shows:

$$K_u W_a = N_p \left(\frac{I_p}{J}\right) + N_s \left(\frac{I_s}{J}\right) \qquad [21\text{-C4}]$$

Now, substitute in Faraday's Equation:

$$K_u W_a = \frac{V_p \left(10^4\right)}{A_c B_{ac} f K_f} \left(\frac{I_p}{J}\right) + \frac{V_s \left(10^4\right)}{A_c B_{ac} f K_f} \left(\frac{I_s}{J}\right) \qquad [21\text{-C5}]$$

Rearranging shows:

$$W_a A_c = \frac{\left[\left(V_p I_p\right) + \left(V_s I_s\right)\right]\left(10^4\right)}{B_{ac} f J K_f K_u}, \quad [\text{cm}^4] \qquad [21\text{-C6}]$$

The output power, P_o, is:

$$P_o = V_s I_s, \quad [\text{watts}] \qquad [21\text{-C7}]$$

The input power, P_{in}, is:

$$P_{in} = V_p I_p, \quad [\text{watts}] \qquad [21\text{-C8}]$$

Then:

$$P_t = P_{in} + P_o, \quad [\text{watts}] \qquad [21\text{-C9}]$$

Therefore:

$$W_a A_c = \frac{P_t \left(10^4\right)}{B_{ac} f J K_f K_u}, \quad [\text{cm}^4] \qquad [21\text{-C10}]$$

By definition:

$$A_p = W_a A_c \qquad [21\text{-C11}]$$

Then:

$$A_p = \frac{P_t\left(10^4\right)}{B_{ac}\, f\, J\, K_f\, K_u}, \quad [\text{cm}^4] \qquad \text{[21-C12]}$$

Inductor Derivation for the Core Geometry, K_g

Introduction

Inductors, like transformers, are designed for a given temperature rise. They can also be designed for a given regulation. The regulation and energy-handling ability of a core are related to two constants, K_g and K_e, by the equation:

$$\left(\text{Energy}\right)^2 = K_g K_e \alpha, \qquad \text{[21-D1]}$$

Where:

$$\alpha = \text{Regulation}, \quad [\%]$$

The constant, K_g, is a function of the core geometry:

$$K_g = f\left(A_c, W_a, \text{MLT}\right) \qquad \text{[21-D2]}$$

The constant, K_e, is a function of the magnetic and electrical operating conditions:

$$K_e = g\left(P_o, B_m\right) \qquad \text{[21-D3]}$$

The derivation of the specific functions for, K_g and K_e, is as follows: First, assume a dc inductor could be an input or output, as schematically shown in Figure 21-D1. The inductor resistance is R_L.

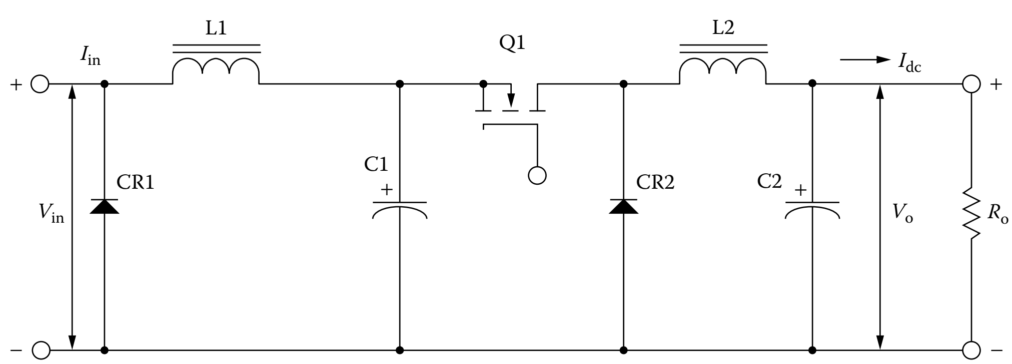


Figure 21-D1. Typical Buck Type Switching Converter.

The output power is:

$$P_o = I_{dc} V_o, \quad \text{[watts]} \qquad \text{[21-D4]}$$

$$\alpha = \frac{I_{dc} R_L}{V_o}(100), \quad [\%] \qquad \text{[21-D5]}$$

The inductance equation is:

$$L = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{l_g}, \quad \text{[henrys]} \qquad \text{[21-D6]}$$

The inductor flux density is:

$$B_{dc} = \frac{0.4\pi N I_{dc} \left(10^{-4}\right)}{l_g}, \quad \text{[teslas]} \qquad \text{[21-D7]}$$

Combine Equations [21-D6] and [21-D7]:

$$\frac{L}{B_{dc}} = \frac{N A_c \left(10^{-4}\right)}{I_{dc}} \qquad \text{[21-D8]}$$

Solve for N:

$$N = \frac{L I_{dc} \left(10^{4}\right)}{B_{dc} A_c}, \quad \text{[turns]} \qquad \text{[21-D9]}$$

From the resistivity formula, it is easily shown that:

$$R_L = \frac{(\text{MLT}) N_p^2}{W_a K_u}\rho, \quad \text{[ohms]} \qquad \text{[21-D10]}$$

Where:

$\rho = 1.724\ (10^{-6})$ ohm cm

Combining Equations [21-D5] and [21-D10]:

$$\alpha = \left(\frac{I_{dc}}{V_o}\right)\left(\frac{(\text{MLT}) N_p^2}{W_a K_u}\rho\right)(100), \quad [\%] \qquad \text{[21-D11]}$$

Take Equation [21-D9] and square it:

$$N^2 = \left(\frac{L I_{dc}}{B_{dc} A_c}\right)^2 \left(10^{8}\right) \qquad \text{[21-D12]}$$

Combine Equations [21-D11] and [21-D12]:

$$\alpha=\left(\frac{I_{dc}(\text{MLT})}{V_o W_a K_u}\rho\right)\left(\frac{L I_{dc}}{B_{dc} A_c}\right)^2\left(10^{10}\right) \qquad \text{[21-D13]}$$

Combine and simplify:

$$\alpha=\left(\frac{I_{dc}(\text{MLT})\left(L I_{dc}\right)^2}{V_o W_a K_u B_{dc}^2 A_c^2}\rho\right)\left(10^{10}\right) \qquad \text{[21-D14]}$$

Multiply the Equation by I_{dc} / I_{dc} and combine:

$$\alpha=\left(\frac{(\text{MLT})\left(L I_{dc}^2\right)^2}{V_o I_{dc} W_a K_u B_{dc}^2 A_c^2}\rho\right)\left(10^{10}\right) \qquad \text{[21-D15]}$$

The energy equation is:

$$\text{Energy}=\frac{L I_{dc}^2}{2}, \quad \text{[watt-seconds]} \qquad \text{[21-D16]}$$

$$2\text{Energy}=L I_{dc}^2$$

Combine and simplify:

$$\alpha=\left(\frac{(2\text{Energy})^2}{P_o B_{dc}^2}\right)\left(\frac{\rho(\text{MLT})}{W_a K_u A_c^2}\right)\left(10^{10}\right) \qquad \text{[21-D17]}$$

The resistivity is:

$$\rho=1.724\left(10^{-6}\right)[\text{ohm cm}] \qquad \text{[21-D18]}$$

Combine the resistivity:

$$\alpha=\left(\frac{6.89(\text{Energy})^2}{P_o B_{dc}^2}\right)\left(\frac{(\text{MLT})}{W_a K_u A_c^2}\right)\left(10^{4}\right) \qquad \text{[21-D19]}$$

Solving for energy:

$$(\text{Energy})^2=0.145 P_o B_{dc}^2\left(\frac{W_a A_c^2 K_u}{\text{MLT}}\right)\left(10^{-4}\right)\alpha \qquad \text{[21-D20]}$$

The core geometry equals:

$$K_g=\frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \qquad \text{[21-D21]}$$

The electrical conditions:

$$K_e = 0.145 P_o B_{dc}^2 \left(10^{-4}\right) \quad [21\text{-D22}]$$

The regulation and energy-handling ability is:

$$\left(\text{Energy}\right)^2 = K_g K_e \alpha \quad [21\text{-D23}]$$

The copper loss is:

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%] \quad [21\text{-D24}]$$

Inductor Derivation for the Area Product, A_p

Introduction

The area product, A_p, can determine the energy-handling capability of an inductor. The area product, A_p, relationship is obtained by the following: (Note that symbols marked with a prime, such as H′, are mks (meter-kilogram-second) units.)

$$E = L\frac{dI}{dt} = N\frac{d\phi}{dt} \quad [21\text{-E1}]$$

Combine and simplify:

$$L = N\frac{d\phi}{dI} \quad [21\text{-E2}]$$

Flux density is:

$$\phi = B_m A_c' \quad [21\text{-E3}]$$

$$B_m = \frac{\mu_o N I}{l_g' + \left(\frac{\text{MPL}'}{\mu_m}\right)} \quad [21\text{-E4}]$$

$$\phi = \frac{\mu_o N I A_c'}{l_g' + \left(\frac{\text{MPL}'}{\mu_m}\right)} \quad [21\text{-E5}]$$

$$\frac{d\phi}{dI} = \frac{\mu_o N A_c'}{l_g' + \left(\frac{\text{MPL}'}{\mu_m}\right)} \qquad [21\text{-E6}]$$

Combine Equations [21-E2] and [21-E6]:

$$L = N\frac{d\phi}{dI} = \frac{\mu_o N^2 A_c'}{l_g' + \left(\frac{\text{MPL}'}{\mu_m}\right)} \qquad [21\text{-E7}]$$

The Energy Equation is:

$$\text{Energy} = \frac{LI^2}{2}, \quad [\text{watt-seconds}] \qquad [21\text{-E8}]$$

Combine Equations [21-E7] and [21-E8]:

$$\text{Energy} = \frac{LI^2}{2} = \frac{\mu_o N^2 A_c' I^2}{2\left(l_g' + \left(\frac{\text{MPL}'}{\mu_m}\right)\right)} \qquad [21\text{-E9}]$$

If B_m is specified:

$$I = \frac{B_m\left(l_g' + \left(\frac{\text{MPL}'}{\mu_m}\right)\right)}{\mu_o N} \qquad [21\text{-E10}]$$

Combine Equations [21-E7] and [21-E10]:

$$\text{Energy} = \frac{\mu_o N^2 A_c'}{2\left(l_g' + \left(\frac{\text{MPL}'}{\mu_m}\right)\right)}\left(\frac{B_m\left(l_g' + \left(\frac{\text{MPL}'}{\mu_m}\right)\right)}{\mu_o N}\right)^2 \qquad [21\text{-E11}]$$

Combine and simplify:

$$Energy = \frac{B_m^2\left(l_g' + \left(\frac{\text{MPL}'}{\mu_m}\right)\right)A_c'}{2\mu_o} \qquad [21\text{-E12}]$$

The winding area of a inductor is fully utilized when:

$$K_u W_a' = N A_w' \qquad [21\text{-E13}]$$

By definition the wire area is:

$$A'_w = \frac{I}{J'} \qquad [21\text{-E14}]$$

Combining Equations [21-E13] and [21-E14]:

$$K_u W'_a = N\left(\frac{I}{J'}\right) \qquad [21\text{-E15}]$$

Solving for I:

$$I = \frac{K_u W_a J}{N} = \frac{B_m\left(l'_g + \left(\frac{\text{MPL}'}{\mu_\text{m}}\right)\right)}{\mu_\text{o} N} \qquad [21\text{-E16}]$$

Rearrange Equation [21-E16]:

$$l'_g + \left(\frac{\text{MPL}'}{\mu_\text{m}}\right) = \frac{K_u W'_a J' \mu_o}{B_m} \qquad [21\text{-E17}]$$

Now, substitute in Energy Equation [21-E11]:

$$\text{Energy} = \frac{B_m^2\left(\frac{K_u W'_a J' \mu_o}{B_m}\right)A'_c}{2\mu_o} \qquad [21\text{-E18}]$$

Rearrange Equation [21-E18]:

$$\text{Energy} = \left(\frac{B_m^2 A'_c}{2\mu_o}\right)\left(\frac{K_u W'_a J' \mu_o}{B_m}\right) \qquad [21\text{-E19}]$$

Combine and simplify:

$$\text{Energy} = \left(\frac{B_m K_u W'_a J' A'_c}{2}\right) \qquad [21\text{-E20}]$$

Now, multiply mks units to return cgs.

$$W'_a = W_a\left(10^{-4}\right)$$

$$A'_c = A_c\left(10^{-4}\right)$$

$$J' = J\left(10^{4}\right)$$

$$\text{MPL}' = \text{MPL}\left(10^{-2}\right)$$

$$l'_g = l_g\left(10^{-2}\right)$$

We can substitute into the energy equation to obtain:

$$\text{Energy} = \frac{B_m K_u W_a J A_c}{2}\left(10^{-4}\right) \qquad [21\text{-E21}]$$

Solve for the area product:

$$A_p = W_a A_c$$

$$A_p = \frac{2(\text{Energy})}{B_m J K_u}, \quad [\text{cm}^4] \qquad [21\text{-E22}]$$

Transformer Regulation

The minimum size of a transformer is usually determined either by a temperature rise limit, or by allowable voltage regulation, assuming that size and weight are to be minimized. Figure 21-F1 shows a circuit diagram of a transformer with one secondary.

Note that α = regulation (%).

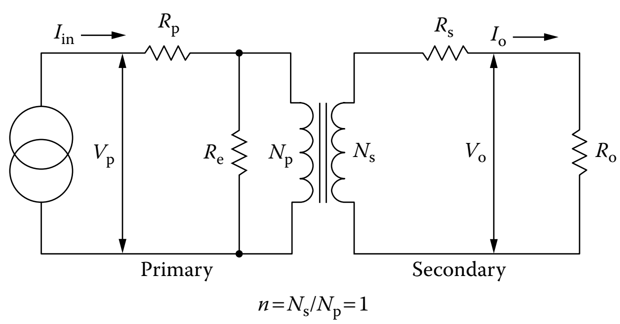


Figure 21-F1. Transformer Circuit Diagram.

The assumption is that distributed capacitance in the secondary can be neglected because the frequency and secondary voltage are not excessively high. Also, the winding geometry is designed to limit the leakage inductance to a level low enough to be neglected under most operating conditions. The transformer window allocation is shown in Figure 21-F2.

$$\frac{W_a}{2} = \text{Primary} = \text{Secondary} \qquad [21\text{-F1}]$$

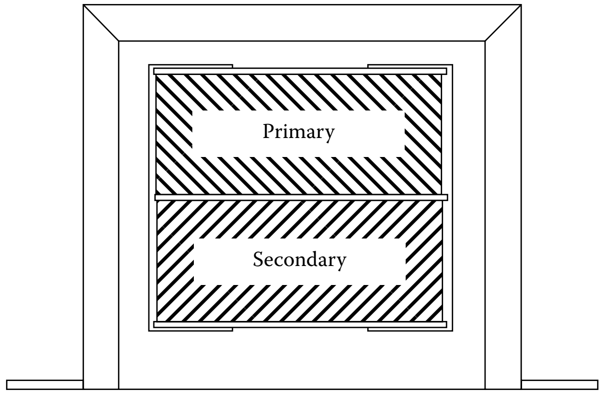


Figure 21-F2. Transformer Window Allocation.

Transformer voltage regulation can now be expressed as:

$$\alpha = \frac{V_o(\text{N.L.}) - V_o(\text{F.L.})}{V_o(\text{F.L.})}(100), \quad [\%] \qquad [21\text{-F2}]$$

In which, V_o(N.L.) is the no load voltage, and V_o(F.L.) is the full load voltage. For the sake of simplicity, assume the transformer in Figure 21-F1 is an isolation transformer, with a 1:1 turns ratio, and the core impedance, R_e, is infinite.

If the transformer has a 1:1 turns ratio and the core impedance is infinite, then:

$$I_{in} = I_o, \quad [\text{amps}]$$

$$R_p = R_s, \quad [\text{ohms}] \qquad [21\text{-F3}]$$

With equal window areas allocated for the primary and secondary windings, and using the same current density, J:

$$\Delta V_p = I_{in} R_p = \Delta V_s = I_o R_s, \quad [\text{volts}] \qquad [21\text{-F4}]$$

Regulation is then:

$$\alpha = \frac{\Delta V_p}{V_p}(100) + \frac{\Delta V_s}{V_s}(100), \quad [\%] \qquad [21\text{-F5}]$$

Multiply the Equation by currents, I_{in}/I_{in}:

$$\alpha = \frac{\Delta V_p I_{in}}{V_p I_{in}}(100) + \frac{\Delta V_s I_o}{V_s I_o}(100), \quad [\%] \qquad [21\text{-F6}]$$

Primary copper loss is:

$$P_p = \Delta V_p I_{in}, \quad [\text{watts}] \qquad [21\text{-F7}]$$

Secondary copper loss is:

$$P_s = \Delta V_s I_o, \quad [\text{watts}] \qquad [21\text{-F8}]$$

Total copper loss is:

$$P_{cu} = P_p + P_s, \quad [\text{watts}] \qquad [21\text{-F9}]$$

Then, the Regulation Equation can be rewritten to:

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%] \qquad [21\text{-F10}]$$

Recognition

The author would like to thank **Richard Ozenbaugh** of Linear Magnetics for his help with the derivations.

Chapter 22

Autotransformer Design

Table of Contents

Introduction

The autotransformer is a unique and special transformer. The autotransformer can provide a step-up voltage or a step-down voltage with respect to the input with good regulation. The biggest advantage of an autotransformer is the reduction in size, weight, and cost compared with an equivalent isolation transformer shown in Figure 22-1, as long as the designed secondary voltage is within limits of the primary voltage. What makes the autotransformer undesirable is that it does not provide isolation between the primary and secondary, as shown in Figure 22-2. A big application for the autotransformer is to boost the line voltage at the end of a long power line, when there is no auxiliary power available.

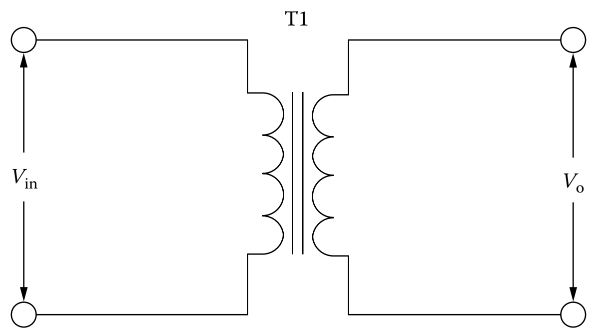


Figure 22-1. Standard Isolation Transformer Schematic.

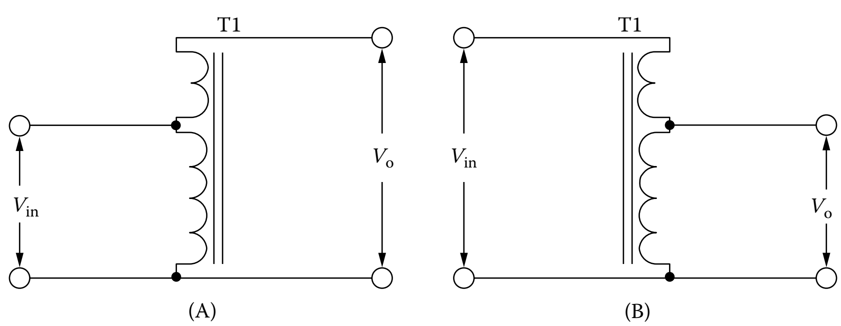


Figure 22-2. (A) is a Step-up and (B) is a Step-down Autotransformer Schematic.

The Voltage and Current Relationship of an Autotransformer

The easiest way to explain the voltage, current and VA rating of an autotransformer is to compare it with a simple two winding isolation transformer shown in Figure 22-3. For this design review the losses will be neglected.

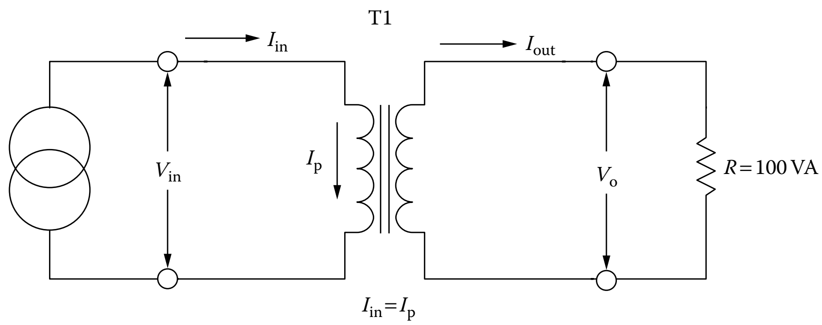


Figure 22-3. Standard Isolation Transformer Schematic.

The output power P_o is of greatest interest to the user. To the transformer designer, the apparent power, P_t, which is associated with the geometry of the transformer, is of greater importance. Assume the core of an isolation transformer has only two windings in the window area, a primary and a secondary, as shown in Figure 22-3. Also assume that the window area, W_a, is divided up in proportion of the power-handling capability of the windings using equal current density. The primary winding handles, P_{in}, and the secondary winding handles, P_o, to the load.

The output power, P_o, is:

$$P_o = V_o I_o, \quad \text{[watts]} \tag{22-1}$$

The input power, P_{in}, is:

$$P_{in} = V_{in} I_{in}, \quad \text{[watts]} \tag{22-2}$$

The apparent power, P_t, is:

$$P_t = P_{in} + P_o, \quad \text{[watts]} \tag{22-3}$$

The apparent power, P_t, for an output, P_o, of 100 watts is:

$$\begin{aligned} P_t &= P_{in} + P_o, \quad \text{[watts]} \\ P_t &= 100 + 100, \quad \text{[watts]} \\ P_t &= 200, \quad \text{[watts]} \end{aligned} \tag{22-4}$$

The apparent power, P_t, is required for calculating both, area product, A_p, in Equation [22-5] and core geometry K_g, in Equation [22-6] when the proper core size is selected, for a design.

The area product, A_p, is:

$$A_p = \frac{P_t(10^4)}{K_u K_f B_m f J}, \quad \text{[cm}^4\text{]} \tag{22-5}$$

$$A_p = W_a A_c, \quad \text{[cm}^4\text{]}$$

The core geometry, K_g, is:

$$K_g = \frac{P_t}{2K_e\alpha}, \quad [\text{cm}^5]$$
$$K_e = 0.145K_f^2 f^2 B_m^2 \left(10^{-4}\right) \qquad [22\text{-}6]$$
$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5]$$

The regulation or copper loss, α for an isolation transformer is shown in Equation [22-7].

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%] \qquad [22\text{-}7]$$

Autotransformer Step-up or Boost

The voltage, current and VA rating of a step-up autotransformer is the same as an isolation transformer. The main difference is, the apparent power P_t, calculation and the fact that there is no electrical isolation between primary and secondary. The turns-ratio for primary and secondary is the same as the isolation transformer. The step-up autotransformer schematic is shown in Figure 22-4 and the Design Equations follow.

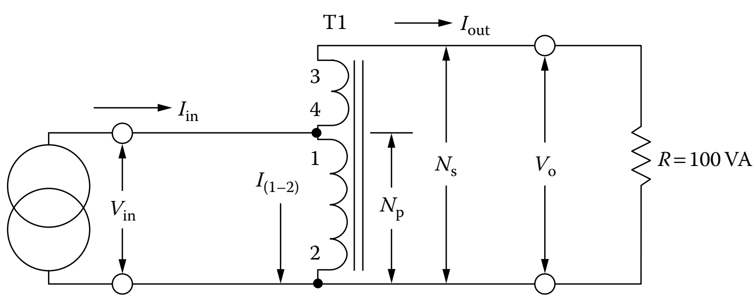


Figure 22-4. A 100 watt Step-up or Boost Autotransformer.

It is assumed for this explanation, that there is not a core loss or copper loss for the autotransformer.

The autotransformer turns ratio is:

$$\frac{N_p}{N_s} = \frac{V_{in}}{V_o}, \quad [\text{volts}] \qquad [22\text{-}8]$$

The output voltage, V_o, is:

$$V_o = \frac{N_s}{N_p}V_{in}, \quad [\text{volts}] \qquad [22\text{-}9]$$

The input output power relationship, P_{in}, is:

$$V_{in}I_{in} = V_o I_o, \quad [\text{VA}] \qquad [22\text{-}10]$$

The autotransformer current, $I_{(1-2)}$, is:

$$I_{(1-2)} = I_{in} - I_o, \quad \text{[amps]} \tag{22-11}$$

The volt-amp rating for a boost autotransformer, VA, is:

$$V_{in} I_{(1-2)} = (V_o - V_{in}) I_o, \quad \text{[VA]} \tag{22-12}$$

The autotransformer input power volt-amps, P_{tin}, is:

$$P_{tin} = V_{in} I_{(1-2)} = V_{in}(I_{in} - I_o), \quad \text{[VA]} \tag{22-13}$$

The autotransformer boost output power volt-amps, P_{AT}, is:

$$P_{AT} = (V_o - V_{in}) I_o, \quad \text{[VA]} \tag{22-14}$$

The autotransformer apparent power, P_t, is:

$$P_t = V_{in}(I_{in} - I_o) + (V_o - V_{in}) I_o, \quad \text{[VA]} \tag{22-15}$$

$$P_t = P_{AT} + P_{tin}, \quad \text{[VA]} \tag{22-16}$$

It can be seen quite easily from Equation [22-14], when the difference between the output voltage V_o, and the input voltage V_{in}, becomes very small the apparent power P_t, also becomes very small.

The core geometry, K_g, is:

$$\begin{aligned} K_g &= \frac{P_t}{2K_e\alpha}, \quad [\text{cm}^5] \\ K_e &= 0.145 K_f^2 f^2 B_m^2 (10^{-4}) \\ K_g &= \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \end{aligned} \tag{22-17}$$

The copper loss ratio, α in percent for an autotransformer is shown in Equation [22-18].

$$\alpha = \frac{P_{cu}}{P_{AT}}(100), \quad [\%] \tag{22-18}$$

Autotransformer Step-down or Buck

The voltage, current and VA rating of a buck autotransformer is the same as an isolation transformer. The main difference is the apparent power, P_t, calculation and there is no electrical isolation between primary and secondary.

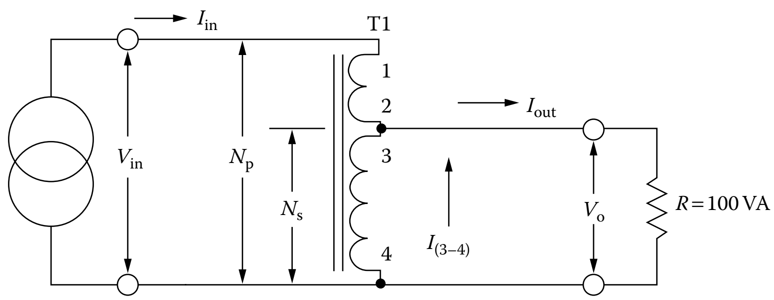


Figure 22-5. A 100 watt Step-down or Buck Autotransformer.

The turns-ratio for primary and secondary is the same as the isolation transformer. The step-down or buck autotransformer schematic is shown in Figure 22-5.

It is assumed for this explanation, that there is not a core loss or copper loss for the autotransformer.

The autotransformer turns ratio is:

$$\frac{N_p}{N_s} = \frac{V_{in}}{V_o}, \quad \text{[volts]} \qquad [22\text{-}19]$$

The output voltage, V_o, is:

$$V_o = \frac{N_s}{N_p} V_{in}, \quad \text{[volts]} \qquad [22\text{-}20]$$

The input output power relationship, P_o, is:

$$V_{in} I_{in} = V_o I_o, \quad \text{[VA]} \qquad [22\text{-}21]$$

The autotransformer winding (3-4) current, $I_{(3\text{-}4)}$, is:

$$I_{(3-4)} = I_o - I_{in}, \quad \text{[amps]} \qquad [22\text{-}22]$$

The volt-amp rating for a buck autotransformer, VA, is:

$$V_o I_{(3-4)} = I_{in}\left(V_{in} - V_o\right), \quad \text{[VA]} \qquad [22\text{-}23]$$

The autotransformer buck winding (1-2) volt-amp, P_{AT}, is:

$$P_{AT} = I_{in}\left(V_{in} - V_o\right), \quad \text{[VA]} \qquad [22\text{-}24]$$

The autotransformer winding (3-4) volt-amps, P_{tin}, is:

$$P_{tin} = V_o I_{(3-4)}, \quad \text{[VA]} \qquad [22\text{-}25]$$

The autotransformer apparent power P_t, is:

$$P_t = (V_{in} - V_o) I_{in} + V_o (I_o - I_{in}), \quad \text{[VA]} \qquad [22\text{-}26]$$

$$P_t = P_{AT} + P_{tin}, \quad \text{[VA]} \qquad [22\text{-}27]$$

It can be seen quite easily from Equation [22-24], that when the difference between the output voltage, V_o, and the input voltage, V_{in}, become very small, the apparent power, P_t, also becomes very small.

The core geometry, K_g, is:

$$K_g = \frac{P_t}{2K_e \alpha}, \quad [\text{cm}^5]$$
$$K_e = 0.145 K_f^2 f^2 B_m^2 (10^{-4}) \qquad [22\text{-}28]$$
$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5]$$

The copper loss ratio, α for an autotransformer is shown in Equation [22-29].

$$\alpha = \frac{P_{cu}}{P_{AT}}(100), \quad [\%] \qquad [22\text{-}29]$$

250 Watt Step-up Autotransformer Design, (Using the Core Geometry, K_g, Approach)

The following information is the Design specification for a 250 watt autotransformer, as shown in Figure 22-6, operating at 60 Hz, using the, K_g, core geometry approach. For a typical design example, assume the design with the following specifications:

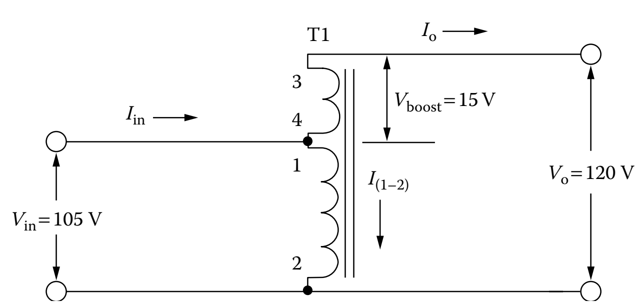


Figure 22-6. Step-up Autotransformer.

Electrical Design Specification

1. Input voltage, V_{in} = 105 volts
2. Output voltage, V_o = 120 volts
3. Boost voltage, V_{bost} = 15 volts
4. Output current, I_o = 2.08 amps
5. Output power, P_o = 250 watts
6. Frequency, f = 60Hz
7. Efficiency, η = 95%
8. Copper loss ratio, α = 5%
9. Operating flux density, B_{ac} = 1.4 teslas
10. Core Material = Silicon M6X
11. Window utilization, K_u = 0.4
12. Temperature rise goal, T_r < 20°C

Step No. 1: Calculate the input current, I_{in}.

$$I_{in} = \frac{P_o}{V_{in}\eta}, \quad \text{[amps]}$$
$$I_{in} = \frac{250}{(105)(0.95)}, \quad \text{[amps]}$$
$$I_{in} = 2.51, \quad \text{[amps]}$$

Step No. 2: Calculate the autotransformer current in winding (1–2), $I_{(1\text{-}2)}$.

$$I_{(1-2)} = I_{in} - I_o, \quad \text{[amps]}$$
$$I_{(1-2)} = 2.51 - 2.08, \quad \text{[amps]}$$
$$I_{(1-2)} = 0.43, \quad \text{[amps]}$$

Step No. 3: Calculate the autotransformer volt-amps in winding (1–2), P_{tin}.

$$P_{tin} = V_{in} I_{(1-2)}, \quad \text{[watts]}$$
$$P_{tin} = (105)(0.43), \quad \text{[watts]}$$
$$P_{tin} = 45.1, \quad \text{[watts]}$$

Step No. 4: Calculate the autotransformer boost winding volt-amps in winding (3–4), P_{AT}.

$$P_{AT} = (V_o - V_{in}) I_o, \quad \text{[watts]}$$
$$P_{AT} = (120 - 105)(2.08), \quad \text{[watts]}$$
$$P_{AT} = 31.2, \quad \text{[watts]}$$

Step No. 5: Calculate the autotransformer apparent power, P_t.

$$P_t = P_{tin} + P_{AT}, \quad \text{[watts]}$$
$$P_t = 45.1 + 31.2, \quad \text{[watts]}$$
$$P_t = 76.3, \quad \text{[watts]}$$

Step No. 6: Calculate the electrical conditions, K_e.

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right)$$
$$K_e = 0.145(4.44)^2(60)^2(1.4)^2\left(10^{-4}\right)$$
$$K_e = 2.02$$

Step No. 7: Calculate the core geometry, K_g.

$$K_g = \frac{P_t}{2K_e\alpha}, \quad \text{[cm}^5\text{]}$$
$$K_g = \frac{76.3}{2(2.02)(5)}, \quad \text{[cm}^5\text{]}$$
$$K_g = 3.78, \quad \text{[cm}^5\text{]}$$

Step No. 8: Select a lamination from Chapter 3, comparable to core geometry, K_g.

1. Lamination = EI-100
2. Manufacturer = Temple
3. D Dimension = 2.54 cm
4. E Dimension = 2.54 cm
5. F Dimension = 1.27 cm
6. Core Geometry, K_g = 4.93 cm^5
7. Area Product, A_p = 29.7 cm^4
8. Core Weight, W_t = 676 grams
9. Surface Area, A_t = 213 cm^2
10. Iron Area, A_c = 6.13 cm^2
11. Window Area, W_a = 4.84 cm^2
12. Magnetic Material, 14 mil = Si-Fe
13. Efficiency = 95%
14. Mean Length Turn, MLT = 14.8 cm

Step No. 9: Calculate the number of turns required for winding (1–2), $N_{(1\text{-}2)}$.

$$N_{(1-2)} = \frac{V_{in}\left(10^4\right)}{K_f B_{ac} f A_c}, \quad \text{[turns]}$$
$$N_{(1-2)} = \frac{105\left(10^4\right)}{(4.44)(1.4)(60)(6.13)}, \quad \text{[turns]}$$
$$N_{(1-2)} = 459, \quad \text{[turns]}$$

Step No. 10: Calculate the current density, J.

$$J = \frac{P_t(10^4)}{K_u K_f B_{ac} f A_p}, \quad [\text{amps/cm}^2]$$
$$J = \frac{76.3(10^4)}{(0.4)(4.44)(1.4)(60)(29.7)}, \quad [\text{amps/cm}^2]$$
$$J = 172, \quad [\text{amps/cm}^2] \quad \text{use, } J = 200$$

Reviewing, K_u, in Step 27, a current density, J, of 200 amps/cm² would be more feasible.

Step No. 11: Calculate the autotransformer bare wire area, $A_{w(1\text{-}2)(B)}$.

$$A_{w(1-2)(B)} = \frac{I_{(1-2)}}{J}, \quad [\text{cm}^2]$$
$$A_{w(1-2)(B)} = \frac{(0.43)}{200}, \quad [\text{cm}^2]$$
$$A_{w(1-2)(B)} = 0.00215, \quad [\text{cm}^2]$$

Step No. 12: Select the wire from the Wire Table in Chapter 4.

$$AWG = \#24$$
$$A_{w(1-2)(B)} = 0.00205, \quad [\text{cm}^2]$$
$$A_{w(1-2)} = 0.00251, \quad [\text{cm}^2]$$
$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 842, \quad [\text{micro-ohm/cm}]$$

Step No. 13: Calculate the autotransformer winding (1-2) resistance, $R_{(1\text{-}2)}$.

$$R_{(1-2)} = \text{MLT}\left(N_{(1-2)}\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$
$$R_{(1-2)} = (14.8)(459)(842)\left(10^{-6}\right), \quad [\text{ohms}]$$
$$R_{(1-2)} = 5.72, \quad [\text{ohms}]$$

Step No. 14: Calculate the autotransformer winding (1–2) copper loss, $P_{(1\text{-}2)}$.

$$P_{(1-2)} = I_{(1-2)}^2 R_{(1-2)}, \quad [\text{watts}]$$
$$P_{(1-2)} = (0.43)^2(5.72), \quad [\text{watts}]$$
$$P_{(1-2)} = 1.058, \quad [\text{watts}]$$

Step No. 15: Calculate the secondary boost winding (3-4) turns, $N_{(3\text{-}4)}$.

$$N_{(3-4)} = \frac{N_{(1-2)} V_{bost}}{V_{in}}\left(1 + \frac{\alpha}{100}\right), \quad [\text{turns}]$$
$$N_{(3-4)} = \frac{(459)(15)}{(105)}\left(1 + \frac{5}{100}\right), \quad [\text{turns}]$$
$$N_{(3-4)} = 68.8 \text{ use } 69, \quad [\text{turns}]$$

Step No. 16: Calculate the autotransformer boost winding (3-4) bare wire area, $A_{w(3\text{-}4)(B)}$.

$$A_{w(3-4)(B)} = \frac{I_o}{J}, \quad [\text{cm}^2]$$
$$A_{w(3-4)(B)} = \frac{(2.08)}{200}, \quad [\text{cm}^2]$$
$$A_{w(3-4)(B)} = 0.0104, \quad [\text{cm}^2]$$

Step No. 17: Select the wire from the Wire Table in Chapter 4.

$$AWG = \#17$$
$$A_{w(3-4)(B)} = 0.0104, \quad [\text{cm}^2]$$
$$A_{w(3-4)} = 0.0117, \quad [\text{cm}^2]$$
$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 166, \quad [\text{micro-ohm/cm}]$$

Step No. 18: Calculate the boost winding (3-4) resistance, $R_{(3\text{-}4)}$.

$$R_{(3-4)} = \text{MLT}\left(N_{(3-4)}\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$
$$R_{(3-4)} = (14.8)(69)(166)\left(10^{-6}\right), \quad [\text{ohms}]$$
$$R_{(3-4)} = 0.170, \quad [\text{ohms}]$$

Step No. 19: Calculate the boost winding (3-4) copper loss, $P_{(3\text{-}4)}$.

$$P_{(3-4)} = I_o^2 R_{(3-4)}, \quad [\text{watts}]$$
$$P_{(3-4)} = (2.08)^2(0.170), \quad [\text{watts}]$$
$$P_{(3-4)} = 0.735, \quad [\text{watts}]$$

Step No. 20: Calculate the total copper loss, P_{cu}.

$$P_{cu} = P_{(1-2)} + P_{(3-4)}, \quad [\text{watts}]$$
$$P_{cu} = 1.058 + 0.735, \quad [\text{watts}]$$
$$P_{cu} = 1.793, \quad [\text{watts}]$$

Step No. 21: Calculate the autotransformer regulation, α.

Use, $P_{AT} = 31.2$ calculated in Step 4.

$$\alpha = \frac{P_{cu}}{P_{AT}}(100), \quad [\%]$$
$$\alpha = \frac{(1.793)}{(31.2)}(100), \quad [\%]$$
$$\alpha = 5.7, \quad [\%]$$

Step No. 22: Calculate the watts per kilogram, W/K. Use the Equation for this material presented in Chapter 2.

$$W/K = 0.000557(f)^{1.68}(B_{ac})^{1.86}$$
$$W/K = 0.000557(60)^{1.68}(1.4)^{1.86}$$
$$W/K = 1.011$$

Step No. 23: Calculate the core loss, P_{fe}.

$$P_{fe} = (W/K)(W_{tfe})(10^{-3}), \quad \text{[watts]}$$
$$P_{fe} = (1.011)(0.676), \quad \text{[watts]}$$
$$P_{fe} = 0.683, \quad \text{[watts]}$$

Step No. 24: Calculate the total loss, P_Σ.

$$P_\Sigma = P_{cu} + P_{fe}, \quad \text{[watts]}$$
$$P_\Sigma = (1.793) + (0.683), \quad \text{[watts]}$$
$$P_\Sigma = 2.476, \quad \text{[watts]}$$

Step No. 25: Calculate the watts per unit area, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad \text{[watts/cm}^2\text{]}$$
$$\psi = \frac{(2.476)}{(213)}, \quad \text{[watts/cm}^2\text{]}$$
$$\psi = 0.0116, \quad \text{[watts/cm}^2\text{]}$$

Step No. 26: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [°\text{C}]$$
$$T_r = 450(0.0116)^{(0.826)}, \quad [°\text{C}]$$
$$T_r = 11.3, \quad [°\text{C}]$$

Confirming the Window Utilization

Step No. 27: Calculate the total window utilization, K_u.

$$K_u = K_{u(1-2)} + K_{u(3-4)}$$
$$K_{u(3-4)} = \frac{N_{(3-4)} A_{w(3-4)(B)}}{W_a}$$
$$K_{u(3-4)} = \frac{(69)(0.0104)}{(4.84)} = 0.148$$
$$K_{u(1-2)} = \frac{N_{(1-2)} A_{w(1-2)(B)}}{W_a}$$
$$K_{u(1-2)} = \frac{(459)(0.00205)}{(4.84)} = 0.194$$
$$K_u = (0.194) + (0.148)$$
$$K_u = 0.342$$

250 Watt Step-up Autotransformer Design Test Data (Using the Core Geometry, K_g, Approach)

The following information is the test data for the above 250 watt Step-up Autotransformer, as shown in Figure 22-7, operating at 60 Hz, using the, K_g, core geometry approach.

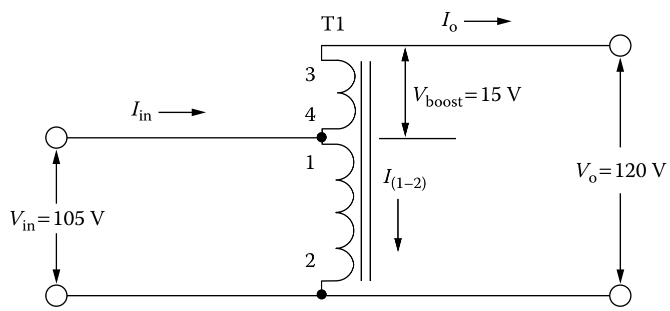


Figure 22-7. Step-up Autotransformer.

Test Data

1. Input voltage, V_{in} = 105 volts
2. Output voltage, V_o, No Load = 121 volts
3. Output voltage, V_o, Full Load = 120.3 volts
4. Output current, I_o = 2.08 amps
5. Output power, P_o = 250 watts
6. Current, $I_{(1-2)}$ = 0.351 amps
7. Current, I_{in} = 2.43 amps
8. Resistance, $R_{(1-2)}$ = 4.916 ohms
9. Resistance, $R_{(3-4)}$ = 0.170 ohms
10. Temperature rise, T_r = 14.2°C

Comparing the Step-up Autotransformer Design With a Standard Isolation Transformer

When comparing the Step-up Autotransformer Design with a Standard Isolation Transformer, use the same electrical requirements.

The output power P_o, is:

$$P_o = V_o I_o, \quad \text{[watts]}$$

The input power P_{in} is:

$$P_{in} = \frac{V_o I_o}{\eta}, \quad \text{[watts]}$$

Step No. 1: Calculate the apparent power P_t for an output P_o, of 250 watts:

$$P_t = P_o\left(\frac{1}{\eta}+1\right), \quad \text{[watts]}$$

$$P_t = (250)\left(\frac{1}{0.95}+1\right), \quad \text{[watts]}$$

$$P_t = 513, \quad \text{[watts]}$$

Step No. 2: Calculate the electrical conditions, K_e.

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right)$$

$$K_e = 0.145(4.44)^2(60)^2(1.4)^2\left(10^{-4}\right)$$

$$K_e = 2.02$$

Step No. 3: Calculate the core geometry, K_g.

$$K_g = \frac{P_t}{2K_e\alpha}, \quad [\text{cm}^5]$$

$$K_g = \frac{513}{2(2.02)(5)}, \quad [\text{cm}^5]$$

$$K_g = 25.4, \quad [\text{cm}^5]$$

Step No. 4: Select a lamination from Chapter 3, comparable to core geometry, K_g.

1. Lamination = EI-138
2. Core Geometry, K_g = 24.5 cm^5
3. Core Weight, W_t = 1786 grams
4. Copper Weight, W_{tcu} = 653 cm^2
5. Iron Area, A_c = 11.59 cm^2
6. Window Area, W_a = 9.148 cm^2

250-Watt Step-down Autotransformer Design (Using the Core Geometry, K_g, Approach)

The following information is the Design specification for a 250-Watt Step-down Autotransformer, as shown in Figure 22-8, operating at 60 Hz, using the, K_g, core geometry approach. For a typical design example, assume the design with the following specifications:

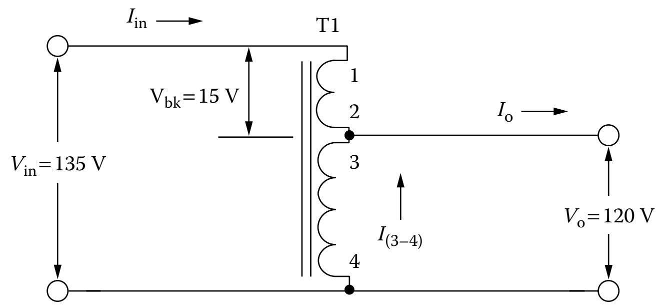


Figure 22-8. Step-down Autotransformer.

Electrical Design Specification

1. Input voltage, V_{in} = 135 volts
2. Output voltage, V_o = 120 volts
3. Buck voltage, V_{bk} = 15 volts
4. Output current, I_o = 2.08 amps
5. Output power, P_o = 250 watts
6. Frequency, f = 60 Hz
7. Efficiency, η = 95%
8. Copper loss, α = 5 %
9. Operating flux density, B_{ac} = 1.4 teslas
10. Core Material = Silicon M6X
11. Window utilization, K_u = 0.4
12. Temperature rise goal, T_r < 20°C

Step No. 1: Calculate the input current, I_{in}.

$$I_{in} = \frac{P_o}{V_{in}\eta}, \quad \text{[amps]}$$
$$I_{in} = \frac{250}{(135)(0.95)}, \quad \text{[amps]}$$
$$I_{in} = 1.95, \quad \text{[amps]}$$

Step No. 2: Calculate the autotransformer winding (3-4) current, $I_{(3-4)}$.

$$I_{(3-4)} = I_o - I_{in}, \quad \text{[amps]}$$
$$I_{(3-4)} = 2.08 - 1.95, \quad \text{[amps]}$$
$$I_{(3-4)} = 0.13, \quad \text{[amps]}$$

Step No. 3: Calculate the autotransformer winding (3-4) volt-amp rating VA, P_{to}.

$$P_{to} = V_o I_{(3-4)}, \quad \text{[watts]}$$
$$P_{to} = (120)(0.13), \quad \text{[watts]}$$
$$P_{to} = 15.6, \quad \text{[watts]}$$

Step No. 4: Calculate the autotransformer buck winding (1-2) volt-amps rating, P_{AT}.

$$P_{AT} = I_{in}\left(V_{in} - V_o\right), \quad \text{[watts]}$$
$$P_{AT} = (1.95)(135 - 120), \quad \text{[watts]}$$
$$P_{AT} = 29.25, \quad \text{[watts]}$$

Step No. 5: Calculate winding (1-2) and winding (3-4) apparent power, P_t.

$$P_t = P_{AT} + P_{to}, \quad \text{[watts]}$$
$$P_t = 29.25 + 15.6, \quad \text{[watts]}$$
$$P_t = 44.85, \quad \text{[watts]}$$

Step No. 6: Calculate the electrical conditions, K_e.

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right)$$
$$K_e = 0.145(4.44)^2(60)^2(1.4)^2\left(10^{-4}\right)$$
$$K_e = 2.02$$

Step No. 7: Calculate the core geometry, K_g.

$$K_g = \frac{P_t}{2K_e\alpha}, \quad [\text{cm}^5]$$
$$K_g = \frac{44.85}{2(2.02)(5)}, \quad [\text{cm}^5]$$
$$K_g = 2.22, \quad [\text{cm}^5]$$

Step No. 8: Select a lamination from Chapter 3, comparable to core geometry, K_g.

1. Lamination = EI-875
2. Manufacturer = Temple
3. D Dimension = 2.22 cm
4. E Dimension = 2.22 cm
5. F Dimension = 1.11 cm
6. Core Geometry, K_g = 2.513 cm^5
7. Area Product, A_p = 17.4 cm^4
8. Core Weight, W_t = 457 grams
9. Surface Area, A_t = 163 cm^2
10. Iron Area, A_c = 4.69 cm^2
11. Window Area, W_a = 3.705 cm^2
12. Magnetic Material, 14 mil = Si-Fe
13. Efficiency = 95%
14. Mean Length Turn, MLT = 13.0 cm

Step No. 9: Calculate the number of turns for winding (3-4), $N_{(3\text{-}4)}$.

$$N_{(3-4)} = \frac{V_o\left(10^4\right)}{K_f B_{ac} f A_c}, \quad \text{[turns]}$$

$$N_{(3-4)} = \frac{120\left(10^4\right)}{(4.44)(1.4)(60)(4.69)}, \quad \text{[turns]}$$

$$N_{(3-4)} = 686, \quad \text{[turns]}$$

Step No. 10: Calculate the current density, J.

$$J = \frac{P_t\left(10^4\right)}{K_u K_f B_{ac} f A_p}, \quad \text{[amps/cm}^2\text{]}$$

$$J = \frac{44.85\left(10^4\right)}{(0.4)(4.44)(1.4)(60)(17.4)}, \quad \text{[amps/cm}^2\text{]}$$

$$J = 173, \quad \text{[amps/cm}^2\text{]} \quad \text{use, } J = 200$$

Reviewing, K_u, in Step 27, a current density, J, of 200 amps/cm² would be more feasible.

Step No. 11: Calculate the bare wire area for winding (3-4), $A_{w(3\text{-}4)(B)}$.

$$A_{w(3-4)(B)} = \frac{I_{(3-4)}}{J}, \quad \text{[cm}^2\text{]}$$

$$A_{w(3-4)(B)} = \frac{(0.13)}{200}, \quad \text{[cm}^2\text{]}$$

$$A_{w(3-4)(B)} = 0.00065, \quad \text{[cm}^2\text{]}$$

Step No. 12: Select the wire from the Wire Table, in Chapter 4.

$$AWG = \#29$$

$$A_{w(3-4)(B)} = 0.000647, \quad \text{[cm}^2\text{]}$$

$$A_{w(3-4)} = 0.000855, \quad \text{[cm}^2\text{]}$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 2664, \quad \text{[micro-ohm/cm]}$$

Step No. 13: Calculate winding (3-4) resistance, $R_{(3\text{-}4)}$.

$$R_{(3-4)} = \text{MLT}\left(N_p\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_{(3-4)} = (13)(686)(2664)\left(10^{-6}\right), \quad \text{[ohms]}$$

$$R_{(3-4)} = 23.8, \quad \text{[ohms]}$$

Step No. 14: Calculate winding (3-4) copper loss, $P_{(3\text{-}4)}$.

$$P_{(3-4)} = I_{(3-4)}^2 R_{(3-4)}, \quad \text{[watts]}$$

$$P_{(3-4)} = (0.13)^2(23.8), \quad \text{[watts]}$$

$$P_{(3-4)} = 0.402, \quad \text{[watts]}$$

Step No. 15: Calculate the buck winding (1-2) turns, $N_{(1\text{-}2)}$.

$$N_{(1\text{-}2)} = \frac{N_{(3-4)}V_{bk}}{V_o}, \quad \text{[turns]}$$
$$N_{(1\text{-}2)} = \frac{(686)(15)}{(120)}, \quad \text{[turns]}$$
$$N_{(1\text{-}2)} = 85.7 \ \text{ use } 86, \quad \text{[turns]}$$

Step No. 16: Calculate the buck winding (1-2) bare wire area, $A_{w(1\text{-}2)(B)}$.

$$A_{w(1-2)(B)} = \frac{I_{in}}{J}, \quad [\text{cm}^2]$$
$$A_{w(1-2)(B)} = \frac{(1.95)}{200}, \quad [\text{cm}^2]$$
$$A_{w(1-2)(B)} = 0.00975, \quad [\text{cm}^2]$$

Step No. 17: Select the wire from the Wire Table, in Chapter 4.

$$AWG = \#18$$
$$A_{w(1-2)(B)} = 0.00823, \quad [\text{cm}^2]$$
$$A_{w(1-2)} = 0.00933, \quad [\text{cm}^2]$$
$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 210, \quad \text{[micro-ohm/cm]}$$

Step No. 18: Calculate the buck winding (1-2) resistance, $R_{(1\text{-}2)}$.

$$R_{(1-2)} = \text{MLT}\left(N_{(1-2)}\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$
$$R_{(1-2)} = (13)(86)(210)\left(10^{-6}\right), \quad \text{[ohms]}$$
$$R_{(1-2)} = 0.235, \quad \text{[ohms]}$$

Step No. 19: Calculate the buck winding (1-2) copper loss, $P_{(1\text{-}2)}$.

$$P_{(1-2)} = I_{in}^2 R_{(1-2)}, \quad \text{[watts]}$$
$$P_{(1-2)} = (1.95)^2(0.235), \quad \text{[watts]}$$
$$P_{(1-2)_s} = 0.894, \quad \text{[watts]}$$

Step No. 20: Calculate winding (1-2) and winding (3-4) total copper loss, P_{cu}.

$$P_{cu} = P_{(1-2)} + P_{(3-4)}, \quad \text{[watts]}$$
$$P_{cu} = 0.894 + 0.402, \quad \text{[watts]}$$
$$P_{cu} = 1.296, \quad \text{[watts]}$$

Step No. 21: Calculate the transformer regulation, α.

Use, $P_{AT} = 29.25$ calculated in Step 4.

$$\alpha = \frac{P_{cu}}{P_{tin}}(100), \quad [\%]$$
$$\alpha = \frac{(1.296)}{(29.25)}(100), \quad [\%]$$
$$\alpha = 4.43, \quad [\%]$$

Step No. 22: Calculate the watts per kilogram, W/K. Use the Equation for this material found in Chapter 2.

$$W/K = 0.000557(f)^{1.68}(B_{ac})^{1.86}$$
$$W/K = 0.000557(60)^{1.68}(1.4)^{1.86}$$
$$W/K = 1.01$$

Step No. 23: Calculate the core loss, P_{fe}.

$$P_{fe} = (W/K)(W_{tfe})(10^{-3}), \quad [\text{watts}]$$
$$P_{fe} = (1.01)(0.457), \quad [\text{watts}]$$
$$P_{fe} = 0.462, \quad [\text{watts}]$$

Step No. 24: Calculate the total loss, P_{Σ}.

$$P_{\Sigma} = P_{cu} + P_{fe}, \quad [\text{watts}]$$
$$P_{\Sigma} = (1.296) + (0.462), \quad [\text{watts}]$$
$$P_{\Sigma} = 1.758, \quad [\text{watts}]$$

Step No. 25: Calculate the watts per unit area, ψ.

$$\psi = \frac{P_{\Sigma}}{A_t}, \quad [\text{watts/cm}^2]$$
$$\psi = \frac{(1.758)}{(163)}, \quad [\text{watts/cm}^2]$$
$$\psi = 0.0108, \quad [\text{watts/cm}^2]$$

Step No. 26: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [^\circ\text{C}]$$
$$T_r = 450(0.0108)^{(0.826)}, \quad [^\circ\text{C}]$$
$$T_r = 10.7, \quad [^\circ\text{C}]$$

Confirming the Window Utilization

Step No. 27: Calculate the total window utilization, K_u.

$$K_u = K_{u(1-2)} + K_{u(3-4)}$$

$$K_{u(1-2)} = \frac{N_{(1-2)} A_{w(1-2)(B)}}{W_a}$$

$$K_{u(1-2)} = \frac{(86)(0.00823)}{(3.705)} = 0.191$$

$$K_{u(3-4)} = \frac{N_{(3-4)} A_{w(3-4)(B)}}{W_a}$$

$$K_{up} = \frac{(686)(0.000647)}{(3.705)} = 0.120$$

$$K_u = (0.191) + (0.120)$$

$$K_u = 0.311$$

250 Watt Step-down Autotransformer Design Test Data (Using the Core Geometry, K_g, Approach)

The following information is the test data for the above 250 watt buck autotransformer, as shown in Figure 22-9, operating at 60 Hz, using the, K_g, core geometry approach.

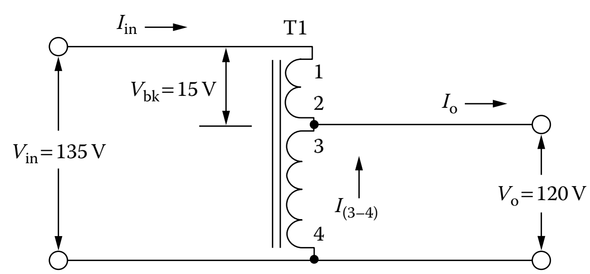


Figure 22-9. Step-down Autotransformer.

Test Data

1. Input voltage, V_{in} = 135 volts
2. Output voltage, V_o, No Load = 120.3 volts
3. Output voltage, V_o, Full Load = 119.6 volts
4. Output current, I_o = 2.08 amps
5. Output power, P_o = 250 watts
6. Current, $I_{(3-4)}$ = 0.24 amps
7. Current, I_{in} = 1.86 amps
8. Resistance, $R_{(1-2)}$ = 0.237 ohms
9. Resistance, $R_{(3-4)}$ = 19.72 ohms
10. Temperature rise, T_r = 19.1°C

Comparing the Autotransformer Design With a Standard Isolation Transformer

When comparing the Step-up Autotransformer Design with a standard isolation transformer use the same electrical requirements.

The output power P_o, is:

$$P_o = V_o I_o, \quad \text{[watts]}$$

The input power P_{in} is:

$$P_{in} = \frac{V_o I_o}{\eta}, \quad \text{[watts]}$$

Step No. 1: Calculate the apparent power P_t for an output P_o, of 250 watts:

$$P_t = P_o\left(\frac{1}{\eta}+1\right), \quad \text{[watts]}$$

$$P_t = (250)\left(\frac{1}{0.95}+1\right), \quad \text{[watts]}$$

$$P_t = 513, \quad \text{[watts]}$$

Step No. 2: Calculate the electrical conditions, K_e.

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right)$$

$$K_e = 0.145(4.44)^2(60)^2(1.4)^2\left(10^{-4}\right)$$

$$K_e = 2.02$$

Step No. 3: Calculate the core geometry, K_g.

$$K_g = \frac{P_t}{2K_e\alpha}, \quad [\text{cm}^5]$$

$$K_g = \frac{513}{2(2.02)(5)}, \quad [\text{cm}^5]$$

$$K_g = 25.4, \quad [\text{cm}^5]$$

Step No. 4: Select a lamination from Chapter 3, comparable to core geometry, K_g.

1. Lamination = EI-138
2. Core Geometry, K_g = 24.5 cm^5
3. Core Weight, W_t = 1786 grams
4. Copper Weight, W_{tcu} = 653 cm^2
5. Iron Area, A_c = 11.59 cm^2
6. Window Area, W_a = 9.148 cm^2

Engineering Note

The copper loss regulation, α, for a standard transformer, as shown in Figure 22-1, has to do with the total copper loss, P_{cu}, and the output power, P_o, as shown in Equation [22-30].

$$\alpha = \frac{P_{cu}}{P_o}(100) \quad [\%] \qquad [22\text{-}30]$$

$$P_{cu} = P_p + P_s, \quad [\text{watts}] \qquad [22\text{-}31]$$

$$P_o = V_o I_o, \quad [\text{watts}] \qquad [22\text{-}32]$$

The copper loss, α, for an Autotransformer, as shown in Figure 22-2, has to do with the total copper loss, P_{cu}, and the volt-amps, and the VA of the boost or buck winding, as shown in Equation [22-33].

$$\alpha = \frac{P_{cu}}{P_{AT}}(100) \quad [\%] \qquad [22\text{-}33]$$

$$P_{cu} = P_{(1-2)} + P_{(3-4)}, \quad [\text{watts}] \qquad [22\text{-}34]$$

Autotransformer Boost

$$P_{AT} = (V_o - V_{in})I_o, \quad [\text{watts}] \qquad [22\text{-}35]$$

Autotransformer Buck

$$P_{AT} = (V_{in} - V_o)I_{in}, \quad [\text{watts}] \qquad [22\text{-}36]$$

Recognition

I would like to give thanks to Charles Barnett, an engineer at Leightner Electronics Inc., for building and testing the 250 watt step-up and step-down autotransformer design examples.

Leightner Electronics Inc.
1501 S. Tennessee St.
McKinney, TX. 75069

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2. Lee, R., *Electronic Transformers and Circuits*, John Wiley & Sons, New York, N.Y., 1958, pp. 250–252.

3. Nordenberg, H.M., *Electronic Transformers*, Reinhold Publishing Corporation, New York, 1964, pp. 147–149.

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Chapter 23

Common-Mode Inductor Design

Table of Contents

Introduction

Switch Mode Power Supplies (SMPS) are normally one of the biggest generators of noise. The SMPS generates two types of noise: common mode and differential mode. Common mode noise occurs simultaneously on both lines of a conductor pair with respect to a common ground chassis, whereas differential noise occurs between the input conductor paths. The common mode filters are generally relied upon to suppress line conducted common mode noise, whereas the input LC filter is used to minimize the differential noise (ripple current) back to the source.

Differential Mode Noise

The differential mode noise diagram is shown in Figure 23-1. The differential mode noise follows the same path as the input power. A typical SMPS generating differential mode noise is shown in Figure 24-2.

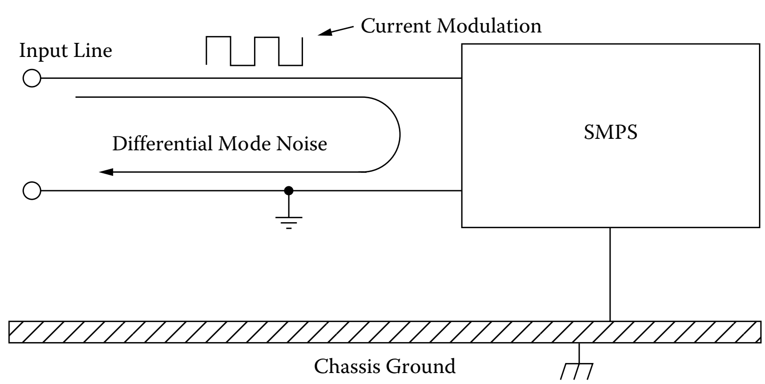


Figure 23-1. The Differential Mode Noise Path Diagram from a SMPS.

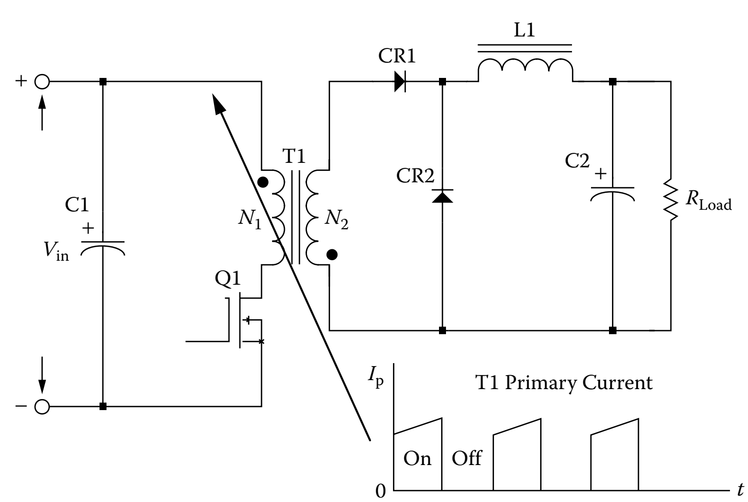


Figure 23-2. Input Current Modulation from a Flyback Converter.

A simple LC filter can normally reduce the noise to a reasonable limit, as shown in Figure 23-3.

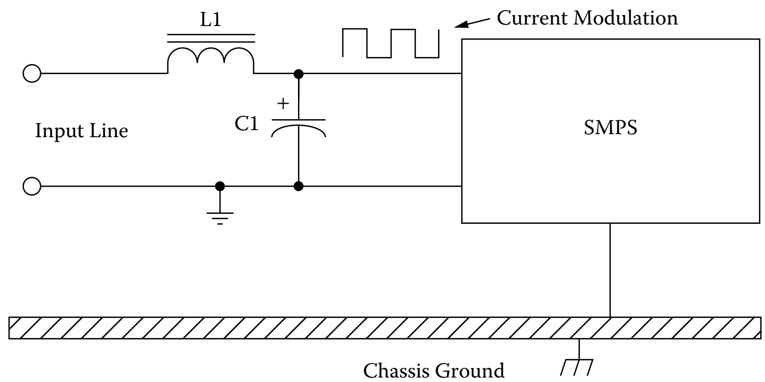


Figure 23-3. Typical Differential Mode Noise Input Filter.

The input LC filter is designed to reduce the input current modulation at the power source. The input current modulation generates a noise voltage within power source, as shown in Figure 23-4. It is shown in Figure 23-4 that any noise $V_{(noise)}$ generated by the Switch Mode Power Supply (SMPS) can be seen by the other users of the power source. The magnitude of the noise $V_{(noise)}$ is shown in Equation [23-1]. The ripple current $I_{L(pk\text{-}pk)}$ is calculated, using Equation [23-2]. R_X is an external non-inductive resistor that is used to measure the ripple voltage $V_{I(ripple)}$ to calculate the ripple current $I_{L(pk\text{-}pk)}$.

$$V_{(noise)} = I_{L(pk-pk)} R_Z, \quad \text{[volts]} \tag{23-1}$$

$$I_{L(pk-pk)} = \frac{V_{I(ripple)}}{R_X}, \quad \text{[amps]} \tag{23-2}$$

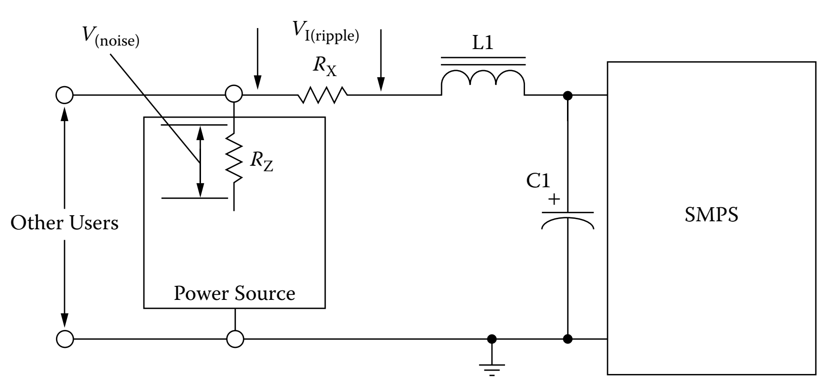


Figure 23-4. Noise Disturbances Generated in the Power Source.

The differential mode (DM) noise, $V_{(noise)}$ will generally be designed to have a magnitude of less than 100 millivolts, peak to peak. The common mode (CM) noise can have an amplitude of several volts and give erroneous measurements. The test setup should be clean without ground loops and the test equipment should have good common mode (CM) rejection.

Common Mode Noise

A simple common mode noise diagram is shown in Figure 23-5. The common mode noise occurs simultaneously on both lines of the input conductor pair with respect to a common ground (chassis). The primary source of this noise is the charging and discharging of stray capacitance, C_s from the high switching rate within the power supply, as shown in Figure 23-6.

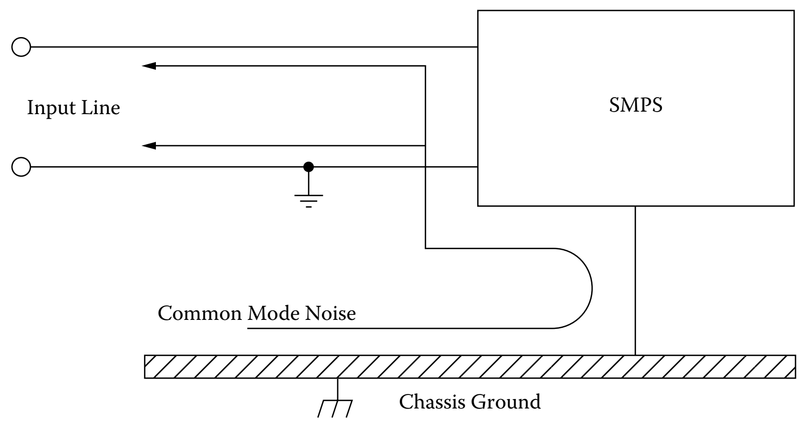


Figure 23-5. Common Mode Noise and Its Trail.

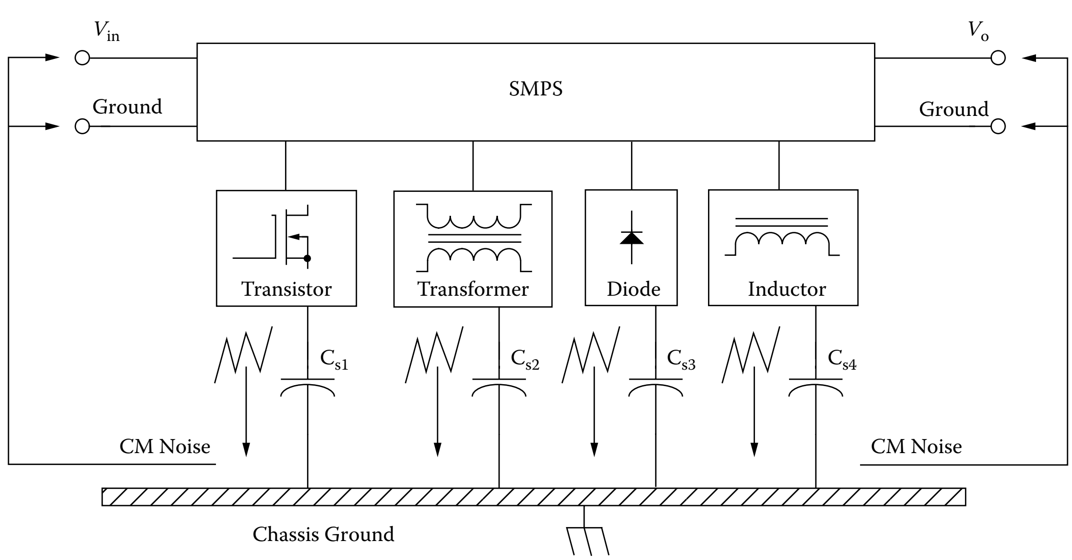


Figure 23-6. The Common Mode Noise Can Travel Down Both Input and Output Lines.

The main source of common mode noise in the SMPS is the stray capacitance to chassis ground that can be found in switching power transformer, inductors, transistors and diodes connected to the heat sink and any wiring carrying high ac currents. The primary source of this noise is the di/dt and dv/dt of the fast switching power MOSFET transistors. See Figure 23-7. This sub-microsecond rise and fall time causes fast charging and discharging of the stray parasitic capacitance, C_s, which creates high pulse currents to flow. The delta current, ΔI can be calculated using Equation [23-3].

$$\Delta I = \frac{C_s V_{(pk)}}{\Delta t}, \quad \text{[amps]} \qquad [23\text{-}3]$$

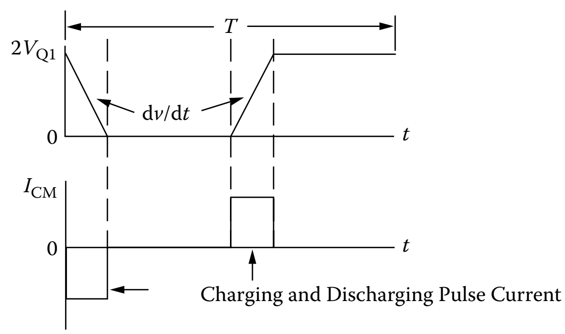


Figure 23-7. Charging and Discharging Stray Capacitance in Switch Mode Power Supply.

Semiconductors Common Mode Noise Source

The primary noise generators that couple noise into the chassis are transistors and diodes. This is because they are bolted to a heat sink and the heat sink is connected to the chassis. If the transistors and diodes are not mounted on a heat sink, then this stray capacitance, C_s problem is minimized. To a lesser extent power transformers and switching inductors that are bolted to the chassis provide their share of common mode noise. There are some simple ways to reduce the common mode noise at the noise source. The power switching transistor connected to the heat sink generating common mode noise through the stray capacitance, C_s is shown in Figure 23-8. The noise generated is from the current charging and discharging the stray capacitance, C_s created by the insulating hardware used to mount the power transistor to the heat sink. This noise current that is generated with the stray capacitance, C_s must return to the point of origin. The only way for the current, I_s to return is through the input lines, unless there is an alternate path provided.

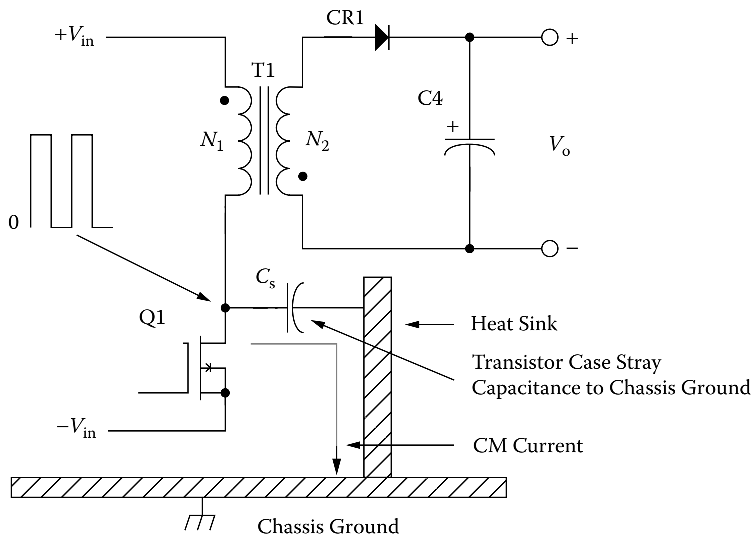


Figure 23-8. Charging and Discharging Stray Capacitance in Switch Mode Power Supply.

The noise current, I_s, that flows through the stray capacitance, C_s, can be diverted to the input ground. Doing this bypass will keep the noise current within the power supply, as shown in Figure 23-9. Using insulated copper foil between the transistor and the heat sink and then grounding the foil can do this. The output power rectifiers have the same stray capacitance, C_s, mounting problem as the transistors, as shown in Figure 23-10. The noise current, I_s, that flows through the stray capacitance, C_s, can be diverted to ground also, but this time it is the output ground, as shown in Figure 23-11. Using insulated copper foil between the rectifiers and the heat sink, followed by grounding the foil to the output ground can accomplish this goal.

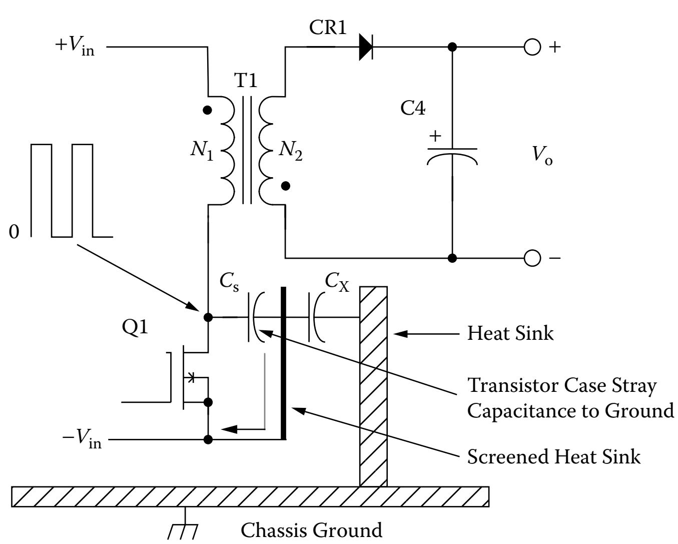


Figure 23-9. Transistor Mounting to Bypass the Stray Capacitance to the Input Ground.

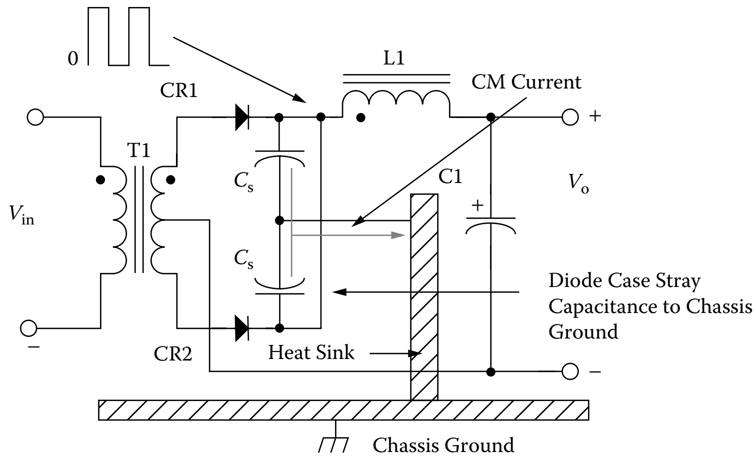


Figure 23-10. Rectifier Mounting Stray Capacitance, C_s, to the Heat sink.

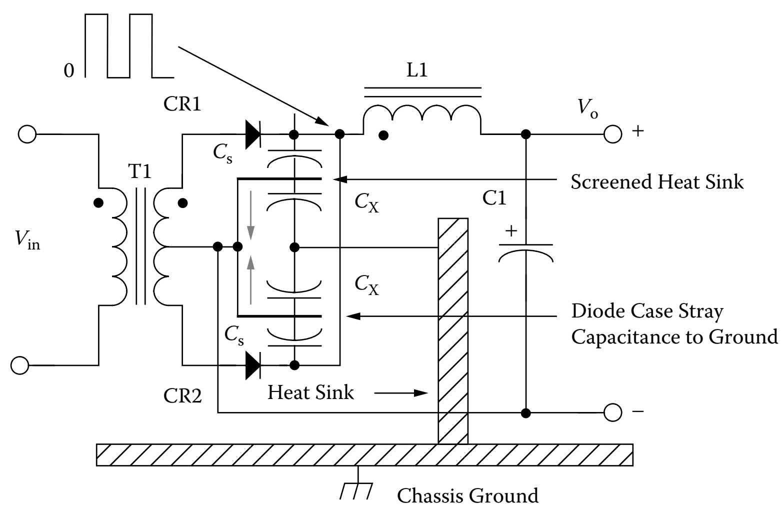


Figure 23-11. Mounting the Rectifier to Bypass the Stray Capacitance, C_s, to the Output Ground.

Transformers and Inductors Common Mode Noise Source

Transformers and multi-winding inductors have two paths for their common mode noise with their stray capacitance, C_s. The two paths are winding-to-winding and winding to the core. The stray capacitance, C_s, between the winding and the core is much easier to handle than the stray capacitance, C_s, between the primary and secondary. A buck type flyback inductor, L1, is shown with the stray capacitance, C_s, from the coil to the core in Figure 23-12. A flyback transformer, T1, with its stray capacitance, C_s, between the coil and the core is shown in Figure 23-13. If the transformer is not mounted directly on the chassis, but on a printed circuit board, it might be a good idea to ground it, just in case. A way to minimize the stray capacitance, C_s to the core is to add an insulated copper shield, as shown in Figure 23-14. This copper shield would be placed around the center leg and the outer surface of the coil, being very careful not to create a shorted turn, as shown in Figure 23-14. The copper shield should be terminated at the center to eliminate voltage gradient effects, as shown.

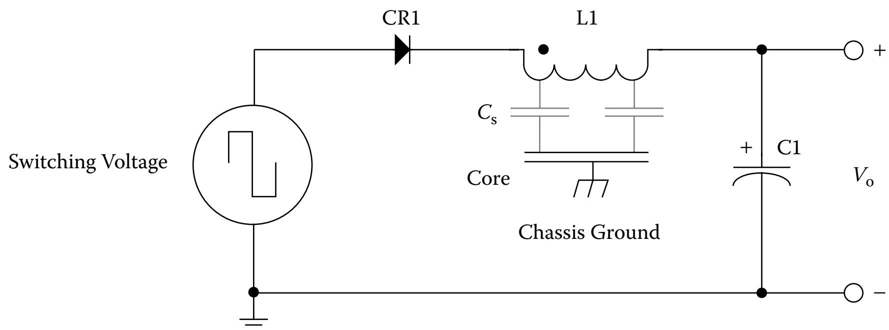


Figure 23-12. Stray Capacitance, C_s from the Inductor Winding to the Core.

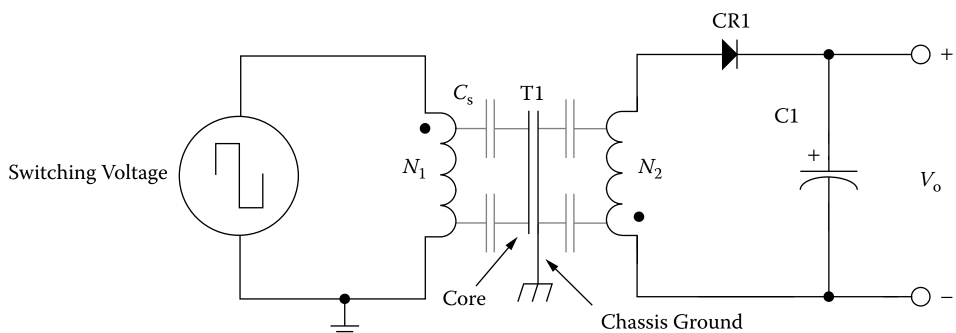


Figure 23-13. Stray Capacitance, C_s from a Flyback Transformer Windings to the Core.

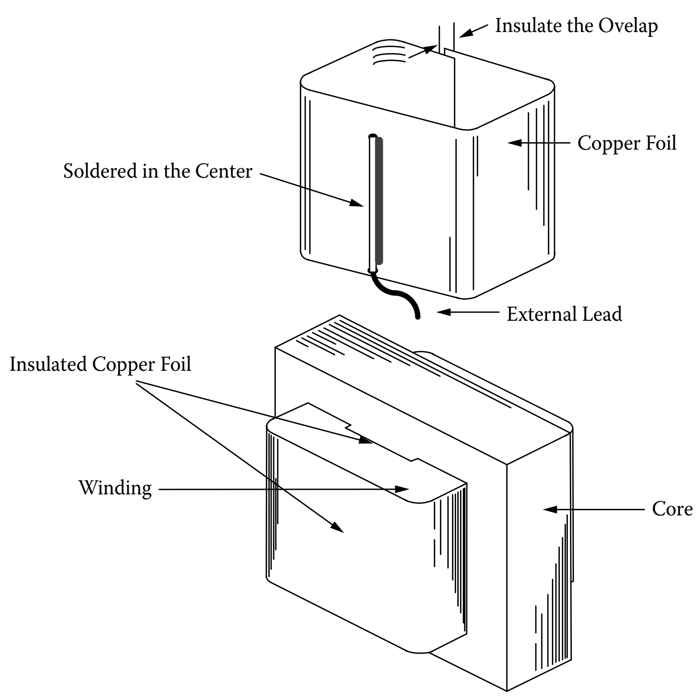


Figure 23-14. Shielding Stray Capacitance, C_s from the Power Transformer Windings to the Core.

Faraday Shield

The stray capacitance, C_s, between primary and secondary, is shown in Figure 23-15, on Side A. This stray capacitance, C_s, between winding and winding is much harder to handle, because, as the stray capacitance, C_s is decreased, the leakage inductance, L_s, will increase. The performance of a high frequency converter power transformer and/or inductor is normally designed to have a very tight coupling between the primary and secondary to minimize the leakage inductance. Designing a transformer to minimize the stray capacitance, C_s, between primary and secondary will greatly increase the leakage inductance, L_s, which has an undesirable side effect.

The common mode noise produced by the stray capacitance, C_s can be reduced with the addition of a Faraday copper shield, as shown in Figure 23-15, Side B. This copper shield would be placed between the primary and the secondaries, as shown in Figure 23-15B. The copper shield would be installed the same way as in Figure 23-14, and should be terminated at the center to eliminate voltage gradient effects, as shown. To reduce the common mode noise even more, multiple Faraday shields can be added, as shown in Figure 23-16.

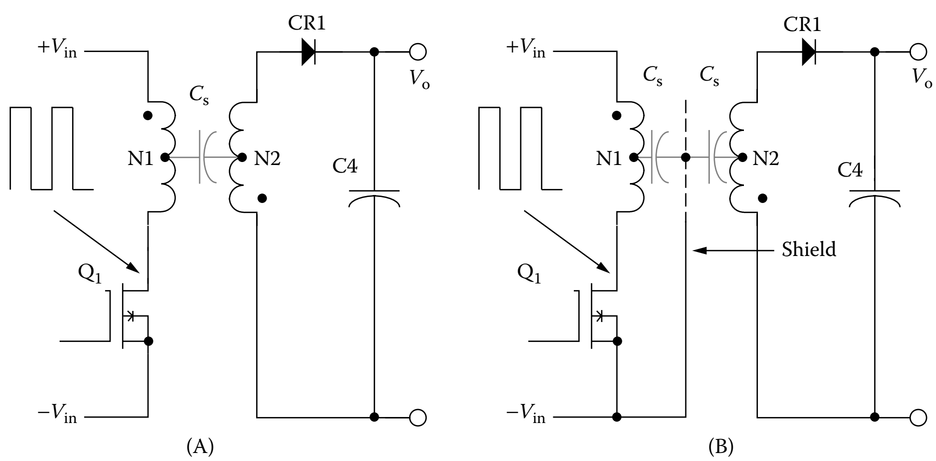


Figure 23-15. A Faraday Shield is Added to Bypass the Common Mode Noise to Ground.

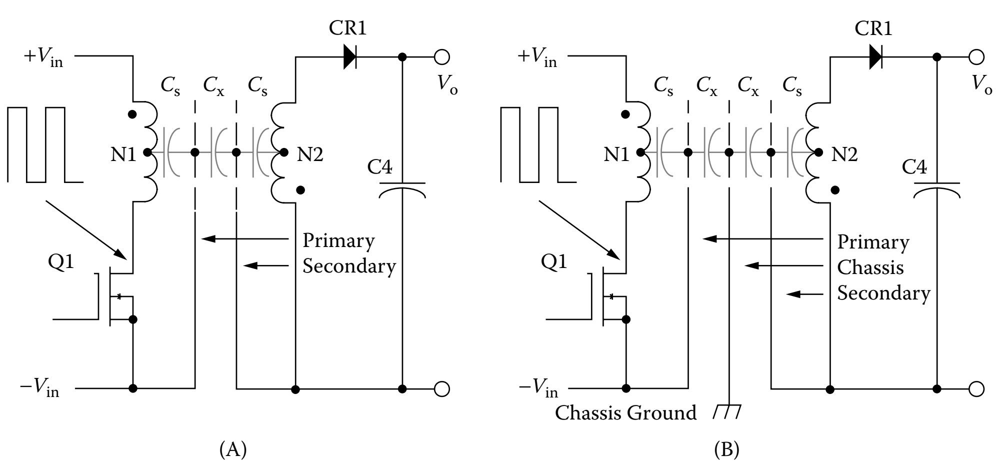


Figure 23-16. Multiple Faraday Shields to Get Better Common Mode Rejection.

The Common Mode Filter

The typical input common mode filter components; L1, C2 and C3 are shown in Figure 23-17. The common mode inductor, L1, has two identical windings, N1 and N2. The input current from the power source goes into lead #1 of L1 and comes out of lead 3 of L1. Since both leads are starts, the net flux change is zero. This is because the amp-turn of N1 is equal to the amp-turn of N2, and they cancel each other. The capacitors, C2 and C3 are of high quality with low ESR, and are used to bypass the noise current created from the stray capacitance, C_s, within the power supply. The capacitors, C2 and C3, keep the common mode noise current created by the stray capacitance, C_s within the power supply. The output common mode filter is shown in Figure 23-18. The output common mode filter components are, L4, C4 and C5. The output common mode filter performs the same way as the input common mode filter.

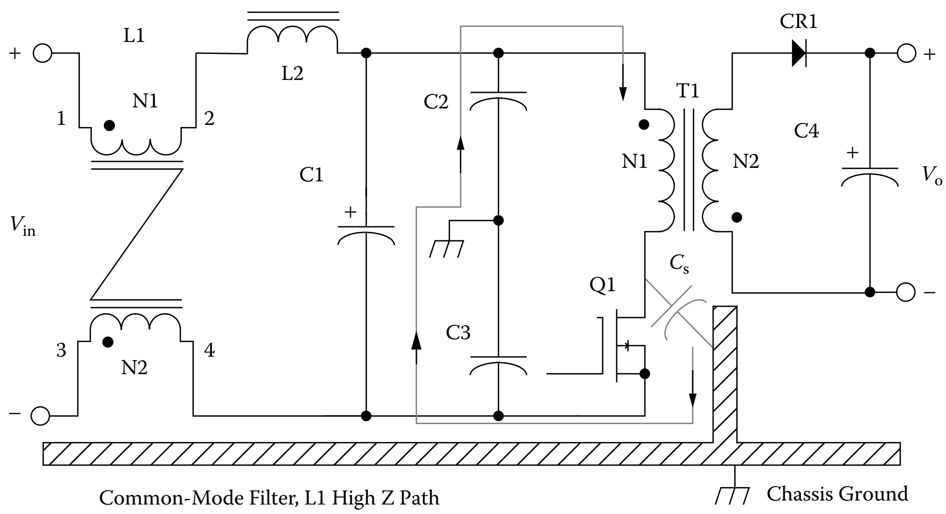


Figure 23-17. Input Common Mode Filter with an Alternate Path for the Stray Capacitance Current.

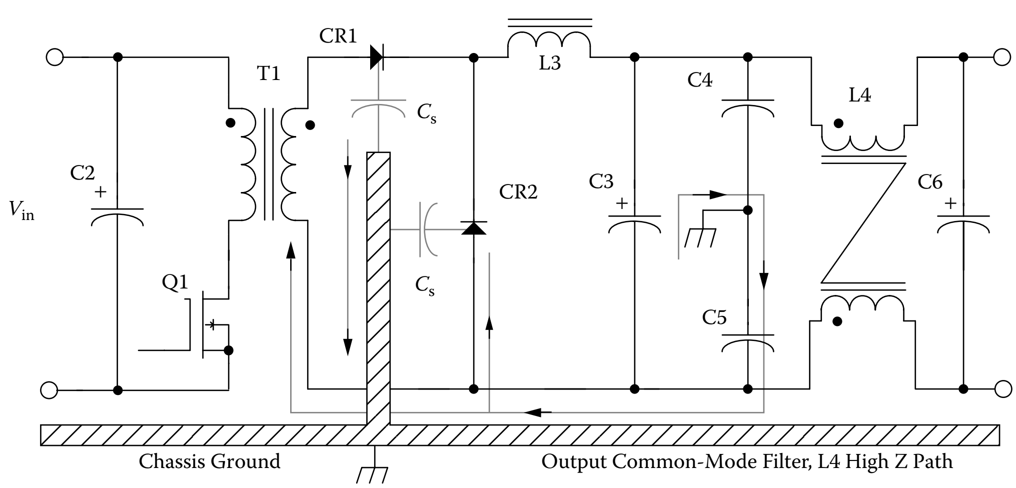


Figure 23-18. Output Common Mode Filter with an Alternate Path for the Stray Capacitance Current.

The Common Mode Filter Inductor

The toroidal core is the most popular for the common mode filter inductor, because of its high permeability and usually requires very few turns, as shown in Figure 23-19. The windings are placed on the toroidal core with the alignment spacer. The alignment spacer helps provide a tight control of the windings space and provides a good control of the parasitics. As mentioned earlier, the differential current or amp-turns through the common mode filter inductor cancel and produce zero net flux because of the phasing of N1 to N2, as shown in Figure 23-20. Because of the phasing on the common mode inductor the differential current produces zero net flux. This means the common mode inductor can be designed more effectively with a core of high permeability.

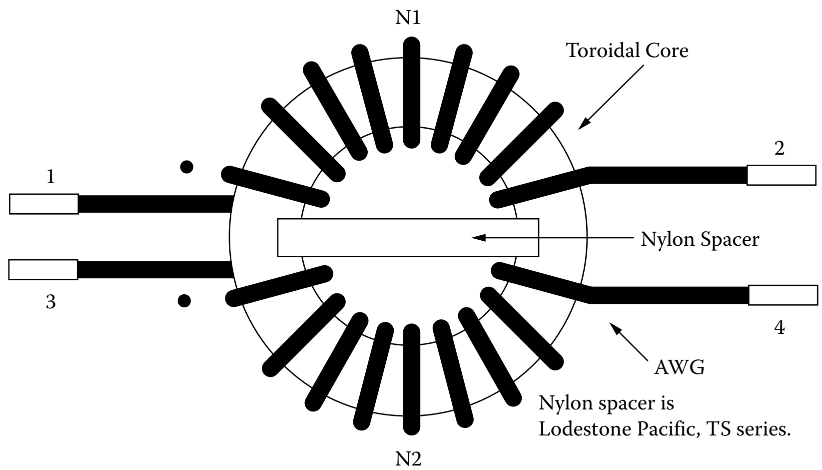


Figure 23-19. Outline of a Toroidal Common Mode Filter Inductor.

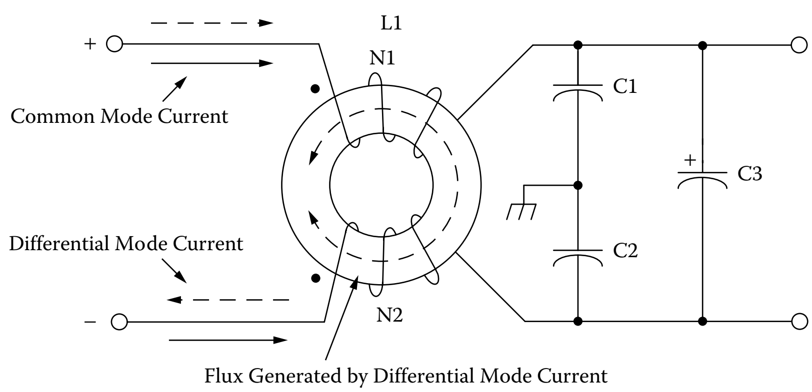


Figure 23-20. A Common Mode Filter Showing the Applied Currents.

Choosing the Magnetic Material

The engineer will usually know how much space is available for the common mode inductor. In the norm, the design of common mode filters is done after the design is finished. The space left will go to the filter. The engineer also knows how much insertion loss can be tolerated. This insertion loss can be in the form of power or voltage drop. The engineer can control the watts loss, the temperature rise, and/or the voltage drop by selecting the proper cores and wire size.

The noise spectrum generated by switching power supplies usually runs anywhere from 10kHz to 50MHz. To provide proper attenuation, the impedance of the inductor must be sufficiently high over this frequency range. The magnetic material must have a high initial permeability and a low cost. High permeability ferrites, such as Magnetics J with 5,000u and W with 10,000u, are the materials of choice for most common mode inductors. There is an amorphous material by Vacuumschmelze, (VAC) which is successful on the common mode inductor market with a nanocrystalline material called Vitroperm 500F. VAC's Vitroperm 500F can be compared with ferrites in Table 23-1.

Table 23-1. Magnetic Material Properties

Magnetic Material Properties					
Manufacturer Name	Material Name	Trade Name Composition	Initial Permeability μ_i	Flux Density Teslas B_s	Typical Operating Frequency
Magnetics	Ferrite J	MnZn	5,000	0.43	10k-2M
Magnetics	Ferrite W	MnZn	10,000	0.43	10k-2M
CMI	CMD5005	NiZn	1600	0.3	0.2M-100M
VAC	Vitroperm 500F	Nanocrystalline	30,000	1.2	10k-2M

Ferrite Temperature Characteristics

Ferrite materials with a medium to high permeability, μ_m do have a "down" side. Ferrites are very temperature-sensitive regarding permeability, μ_m, as shown in Figure 23-21. As can be seen in Figure 23-21, the permeability, μ_m changes quite readily with temperature. The inductance is directly proportional permeability, μ_m, as shown by Equation [23-4]. When an inductor is designed with a gap, then the permeability, μ_m, changes to the relative permeability, μ_r. The relative permeability, μ_r, is about 600 or less depending on the gap, l_g, dimension, as shown in Equation [23-5]. The gap, l_g, will stabilize the inductance changes with temperature. Temperature also effects the saturation, or B_{max}, of a ferrite core. The flux density, B_{max}, will decrease, as shown in Figure 23-22. As the temperature goes up the saturation flux goes down. This definitely has a negative effect when designing for high temperature.

$$L = \frac{0.4\pi N^2 A_c \mu_m (10^{-8})}{\text{MPL}}, \quad \text{[henrys]} \qquad [23\text{-}4]$$

$$\mu_r = \frac{\mu_m}{1 + \left(\frac{l_g}{\text{MPL}}\right)\mu_m}, \quad \text{[relative perm]} \qquad [23\text{-}5]$$

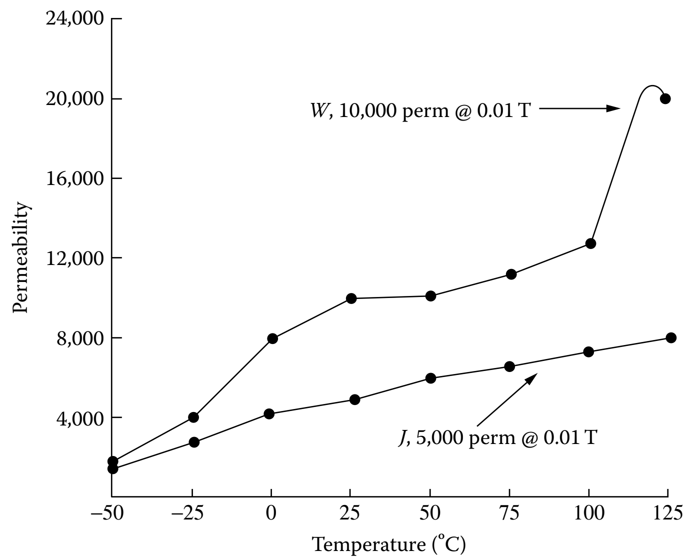


Figure 23-21. Changes in Permeability with Temperature.

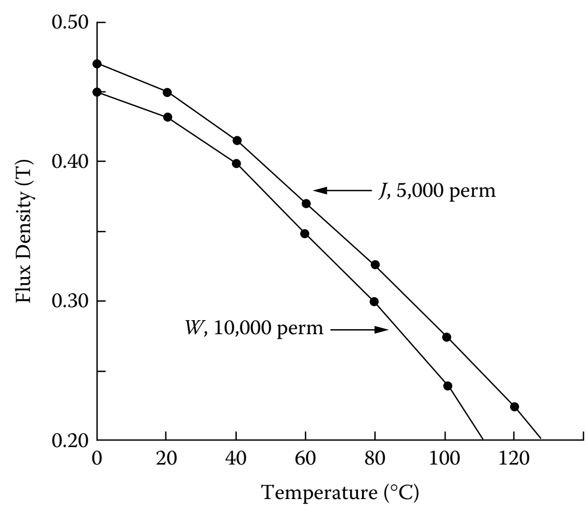


Figure 23-22. Changes in Flux Density with Temperature.

Ferrite Stress Characteristics

Ferrite Materials are susceptible to mechanical stress, both in compression and tension. High permeability materials are particularly affected with negative changes in permeability under moderate stresses. If it is important that the inductance be fairly stable in operation, then, after the units are built, temperature cycling will help relieve the pressure and stabilize the core. Also, if the turns are kept to a minimum and the epoxy encapsulant is brushed on with a very thin coating, it will help. The ferrite core should be checked before using for cracks and fissures. Below are two conditions to remember when using ferrites:

1. An encapsulant will cause stresses.
2. Winding directly on the core will cause stresses.

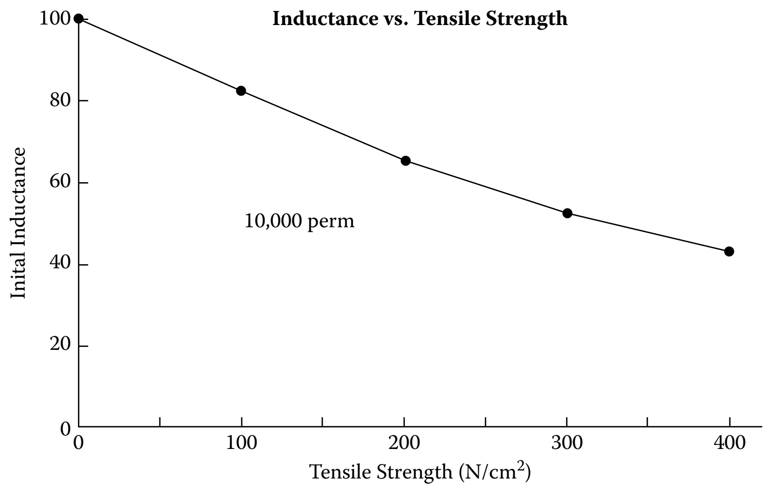


Figure 23-23. Toroidal Common Mode Filter Inductor

Core Saturation

It has been shown that some amount of differential flux exits the core from each winding. This leakage flux is proportional to both the line current and leakage inductance of the winding. Because the leakage flux leaves the core and is not canceled, it is then possible to saturate the core under high currents. Care must be taken to reduce the leakage inductance. The windings on the core should be wound identical so they look like a mirror image of each other. When selecting a core and you have a choice, always pick a core that has the highest permeability. A core selected with the highest permeability requires fewer turns for the required inductance. A design with fewer turns means less leakage flux.

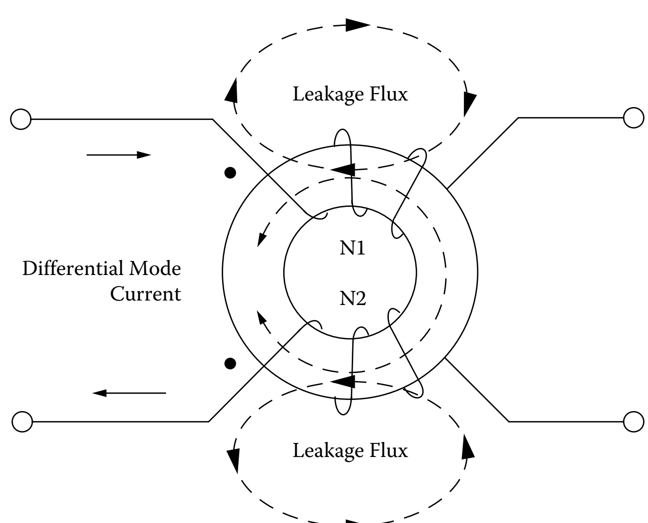


Figure 23-24. Toroidal Common Mode Filter Inductor with Leakage Flux.

Common Mode Filter Inductor Design Specification

1. Core Configuration = Toroid
2. Core Material = Ferrite
3. Material Permeability, W (+/–30) = 10,000 u
4. Line Current, I_{in} = 0.5 amps
5. Coil Reactance at 10 kHz, X_L = 100 ohms
6. Current Density = 400 per cm^2
7. It will use a Nylon Spacer = LP, TS series*
8. Mounting = LP, Vertical*
9. Core Selected P/N = TC-41605
10. Outside Diameter, O.D = 1.664 cm
11. Inside Diameter, I.D = 0.812 cm
12. Core Height, Ht = 0.521 cm

* See Reference #9

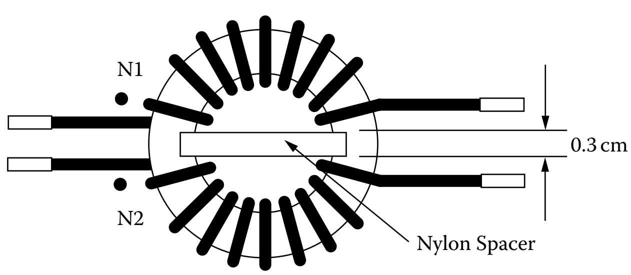
Step No. 1: Calculate the bare wire area, $A_{w(B)}$.

$$A_{w(B)} = \frac{I_{in}}{J}, \quad [\text{cm}^2]$$
$$A_{w(B)} = \frac{0.5}{400}, \quad [\text{cm}^2]$$
$$A_{w(B)} = 0.00125, \quad [\text{cm}^2]$$

Step No. 2: Go to the wire Table in Chapter 4, Table 4-9, column 2 and select a wire area with the closest calculated bare wire area, $A_{w(B)}$ from Step Number 1. Then, record the wire size AWG from Column 1, the bare wire area from Column 2, and the resistance in micro-ohms from Column 4 and the wire diameter with insulation from Column 7.

Column 1, #26 [AWG]
Column 2, 0.00128 cm^2 [Wire Area]
Column 4, 1345 [μohms per cm]
Column 7, 0.0452 cm [Diameter with Insulation]

Step No. 3: The design will have a nylon spacer to separate the coils, as shown.



Step No. 4: Calculate the minimum inductance, L.

$$L_{(\min)} = \frac{X_L}{2\pi f}, \quad \text{[henrys]}$$
$$L_{(\min)} = \frac{100}{2(3.14)(10000)}, \quad \text{[henrys]}$$
$$L_{(\min)} = 0.00159, \quad \text{[henrys]}$$

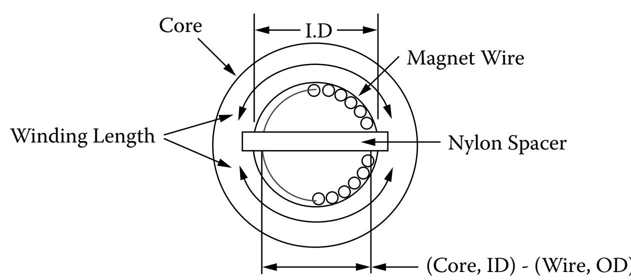
Step No. 5: Choose a core size from Chapter 3, Table 3-52. Then record ID and the AL value for the 1 Kperm core. Choosing the correct core on the first or second go-around takes a little skill and luck.

$$\text{Core} = \text{TC-41605}$$
$$ID = 0.812, \quad \text{[cm]}$$
$$\text{A}_\text{L}\text{, for a 1000 perm, } \frac{\text{mh}}{1000\text{T}} = 548, \quad \text{[factor]}$$

Step No. 6: Calculate the millihenrys per 1000 turns for W material in kiloperm.

$$A_L = \left(\frac{mh}{1000\text{T}}\right)\left(W_{kiloperm}\right), \quad \text{[mh per 1000 turns]}$$
$$A_L = (548)(10), \quad \text{[mh per 1000 turns]}$$
$$A_L = 5480, \quad \text{[mh per 1000 turns]}$$

Step No. 7: Calculate the total winding mean length.



$$\text{Winding Length} = \pi(\text{Core, ID} - \text{Wire, OD}) - 2(\text{Spacer, Width})$$
$$\text{Winding Length} = 3.14(0.812 - 0.0452) - 2(0.3)$$
$$\text{Winding Length} = 1.81, \quad \text{[cm]}$$

Step No. 8: Calculate the winding length per coil.

$$\text{Winding Length per coil} = \frac{\text{Total Winding Length}}{2}, \quad \text{[cm]}$$
$$\text{Winding Length per coil} = \frac{1.81}{2}, \quad \text{[cm]}$$
$$\text{Winding Length per coil} = 0.905, \quad \text{[cm]}$$

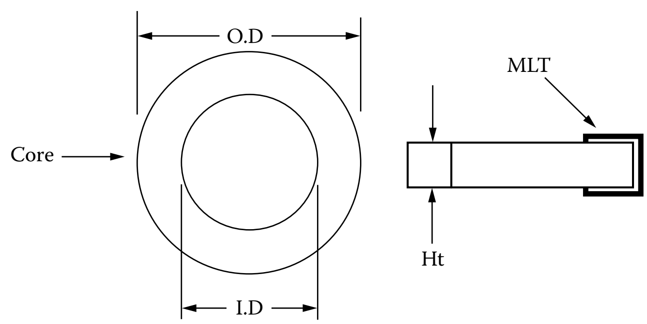
Step No. 9: Calculate the number of turns possible using #26 for each coil.

$$\text{Turns} = \frac{\text{Winding Length (cm)}}{\text{Wire Diameter (cm)}}, \quad \text{[turns]}$$
$$\text{Turns} = \frac{0.905}{0.0452}, \quad \text{[turns]}$$
$$\text{Turns} = 20, \quad \text{[turns]}$$

Step No. 10: Calculate the inductance, L using the low end of the permeability (–30%).

$$L = \left(\frac{mh}{1000T}\right) N^2 \left(10^{-6}\right), \quad \text{[millihenrys]}$$
$$L = (3836)(20)^2 \left(10^{-6}\right), \quad \text{[millihenrys]}$$
$$L = 1.53, \quad \text{[millihenrys]}$$

Step No. 11: Calculate the mean length turn.



$$\text{MLT} = (OD - ID) + 2(Ht), \quad \text{[cm]}$$
$$\text{MLT} = (1.664 - 0.812) + 2(0.521), \quad \text{[cm]}$$
$$\text{MLT} = 1.894, \quad \text{[cm]}$$

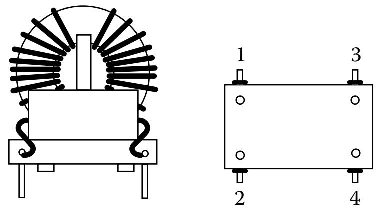
Step No. 12: Calculate the winding resistance of a coil, R.

$$R = \text{MLT}(N1)\left(\frac{\mu\Omega}{\text{cm}}\right) x10^{-6}, \quad \text{[ohms]}$$
$$R = 1.89(20)(1345) x10^{-6}, \quad \text{[ohms]}$$
$$R = 0.0508, \quad \text{[ohms]}$$

Step No. 13: Calculate the total copper loss for both coils, P_{cu}.

$$P_{cu} = I_{in}^2 \, 2(R), \quad \text{[watts]}$$
$$P_{cu} = (0.5)^2 \, 2(0.0508), \quad \text{[watts]}$$
$$P_{cu} = 0.0254, \quad \text{[watts]}$$

Step No. 14: The common mode inductor is assembled in the vertical mount and ready for test.



Vertical Toriodal Mount

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2. Magnetics Technical Bulletin, FC-S2, 1995.
3. Magnetics Ferrite Catalog, FC-601, 2006.
4. Sebranig, Steve and Leonard Crane, *Guide for Common Mode Filter Design Coilcraft*, 1985.
5. Kociecki, John, Predicting the Performance of Common-Mode Inductors, Data General Corporation, n.d.
6. Nave, Mark, A Novel Differential Mode Rejection Network for Conducted Emissions Diagnostics, IEEE, 1089.
7. Leonard Crane, Sebranig, Steve, *Common Mode Filter Inductor Analysis*, Coilcraft Publication, 1985.
8. Ozenbaugh, Richard, *EMI Filter Design*, CRC Press, New York, 2001.
9. Lodestone Pacific, VTM Series, Vertical Toroid Mount, 2002.

Chapter 24

Series Saturable Reactor Design

Table of Contents

Introduction

The saturable reactor is a magnetic device that had great popularity in 1950 and 1960, but when the transistor and silicon controlled rectifier came along, the saturable reactor nearly fell out of sight. The saturable reactor is still being used in motor control, power supplies and current transducers. It is also rugged and provides good electrical isolation between input and output.

The Series Saturable Reactor

The saturable reactor circuit diagram is shown in Figure 24-1. The saturable reactor being used as a current transducer is shown in Figure 24-2. The saturable reactor has two gate windings connected in series opposing and a single control winding. The saturable reactor can be constructed using two toroidal cores, DU or EI lamination. The saturable reactor works best when using a square loop and high permeability magnetic material. The saturable reactor is an amp-turn device. Amp-turns in the control winding will produce amp-turns in the gate winding. The amp-turn Equation is [24-1]:

$$N_{c(5-6)} I_c = N_{g(1-2)} I_g \quad [24\text{-}1]$$

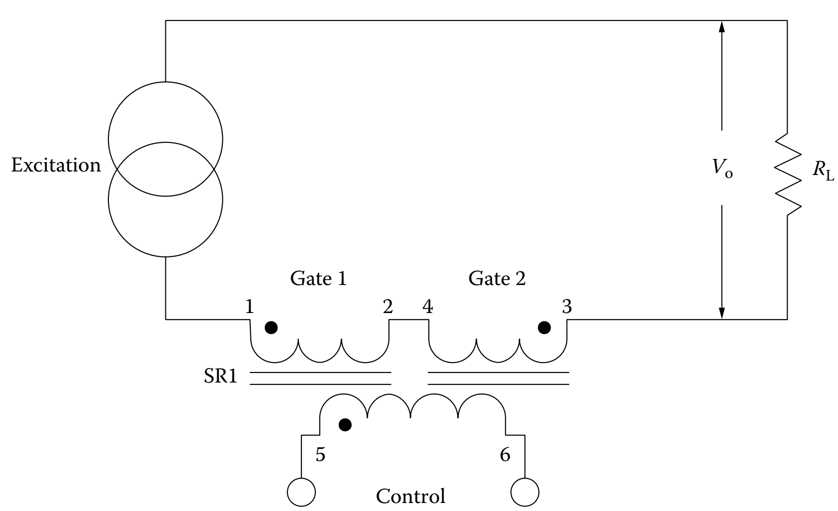

Figure 24-1. The Basic Two Core Series Saturable Reactor.

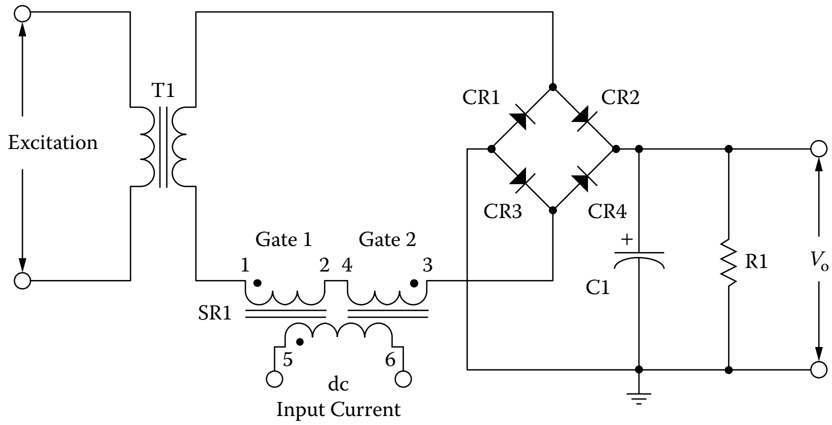

Figure 24-2. Series Saturable Reactor used as a Current Transducer.

Basic Operation

To understand the saturable reactor operation, we will go through the functional steps of the operation using an EI core, as shown in Figure 24-3. The core material is a square-loop, grain-oriented type 50-50 Nickel-Iron (Orthonol) with a high relative permeability. The gate windings, N_{g1}, and N_{g2}, on the outer legs of the EI core both have the same number of turns. The control winding, N_c, is wound on the center leg of the EI core. As shown in Figure 24-3, a dc current is applied to the control winding, N_c, via the battery, E_c. Under these conditions a magnetic flux is generated in the core. As can be seen by the arrows, the flux travels up in the center leg and down in the outer legs. If the dc control current is reversed, the direction of the flux is reversed, but nothing else changes.

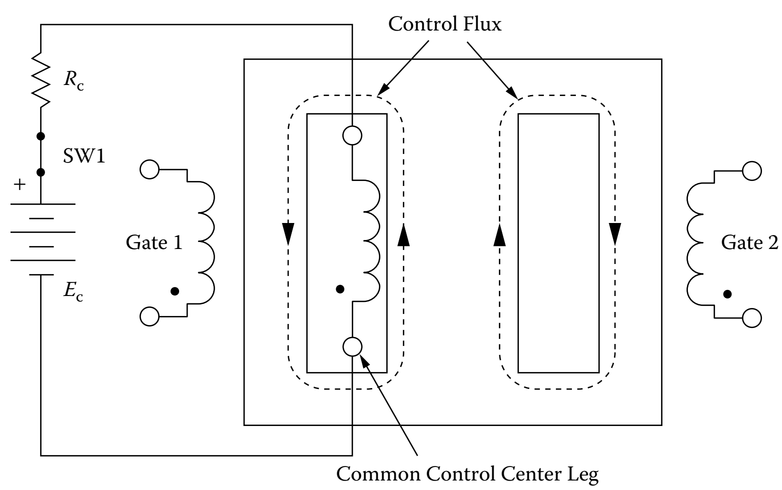

Figure 24-3. Magnetic Flux Produced by the Control Winding Alone.

The ac excitation will now be applied to the gate windings connected in series, which are opposing along with the load resistor, R_L. The dc current will be removed from the control winding, as shown in Figure 24-4. Each gate is designed to support ½ of the applied ac excitation voltage. The direction of the magnetic flux generated by the gate windings is shown in Figure 24-4. The flux is in the downward direction in the right leg, and in an upward direction in the left leg of the EI core, as indicated by the arrows. The flux generated by the gate windings is opposite and equal, and is relatively independent of each other. The flux in each gate is in a static mode when there is no dc control current, as shown in Figure 24-5.

When the gates are supporting all of the applied voltage and the control winding dc current is zero, then the output voltage, V_o, measured across the load resistor, R_L, will be caused by the magnetizing current, I_m, as shown in Figure 24-6. The magnetizing current, I_m, can be calculated using Equation [24-2]. If the magnetizing current, I_m, is to be reduced the engineer have two alternatives: reduce the operating flux density, B_m, or change the magnetic material to a material that requires less drive.

$$I_m = \frac{V_o}{R_L}, \quad \text{[amps]} \tag{24-2}$$

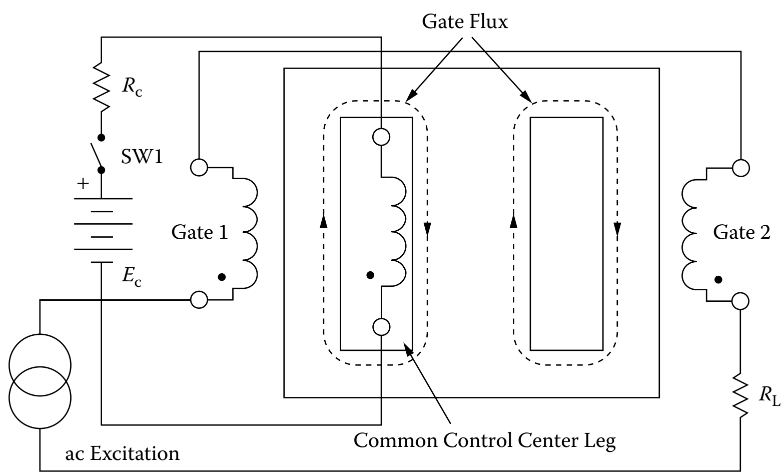

Figure 24-4. Magnetic Flux Produced by Both Gate Windings.

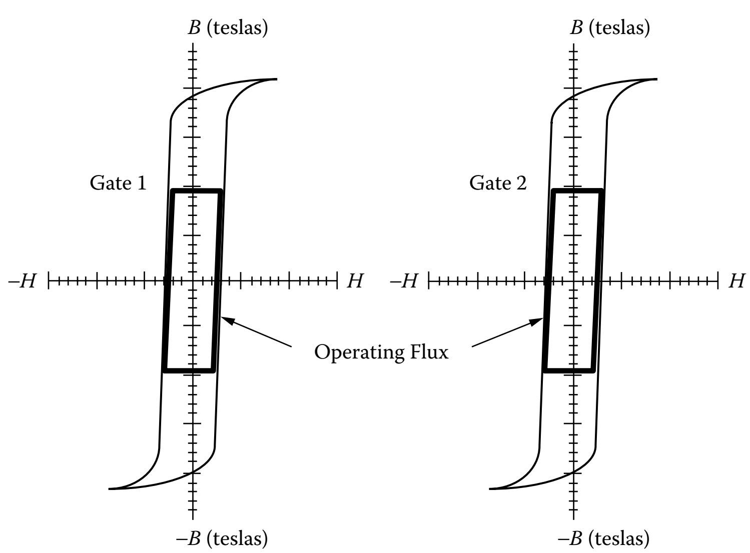

Figure 24-5. Gate Operating Flux with No dc Control Current.

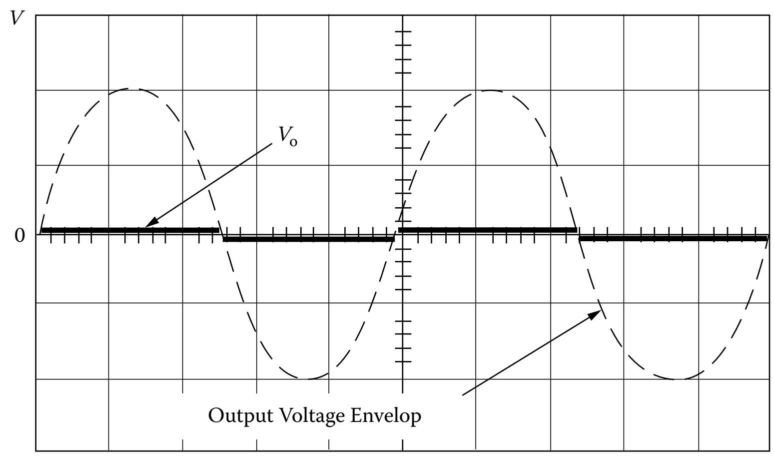

Figure 24-6. Output Voltage Caused by the Magnetizing Current I_m.

How the Series Saturable Reactor Operates

Now both conditions can be combined. Enough current needs to be applied in the control winding to produce a 50% duty cycle and ac excitation then is applied to the gates, as shown in Figure 24-7. The control winding produces flux in the center leg and in both outer legs in the same direction. The flux, produced by Gate 1 in the left leg, is in the opposite direction to the flux produced by the control winding in the center leg. The flux, produced by Gate 2 in the right leg, is in the same direction as the flux produced by the control winding in the center leg. The net result is the amp-turns (N_cI_c) in the control winding will bias the core into saturation to satisfy the Equation [24-3]. This saturation of the core leg will create an output voltage across the load resistor, R_L as shown in Figure 24-8.

$$I_{g2} = \frac{N_c I_c}{N_{g2}}, \qquad [24\text{-}3]$$

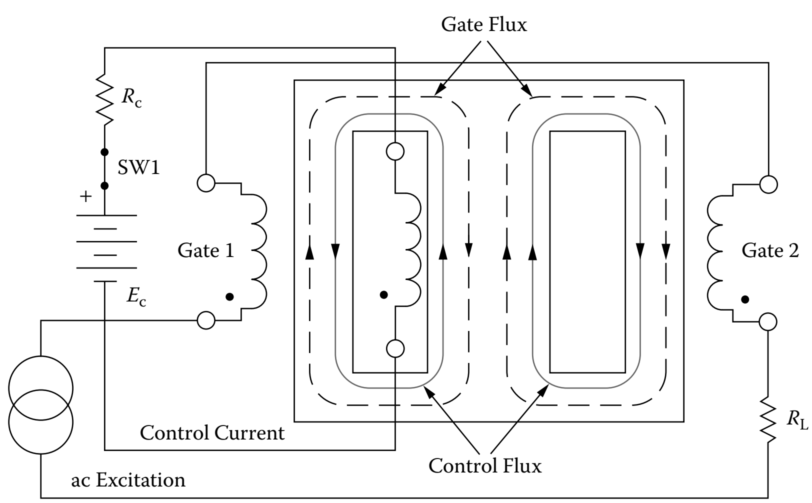

Figure 24-7. The Magnetic Flux for Is Shown for Both Gate Windings and the Control Winding.

The saturable reactor gates are connected in series. Whatever current is flowing in Gate 2 is also flowing in Gate 1, but the amp-turns flowing in the control winding is in the opposite direction to Gate 1 and the net result is they cancel each other, as shown in Equation [24-4].

$$0 = N_{g1} I_{g1} - N_c I_c \qquad [24\text{-}4]$$

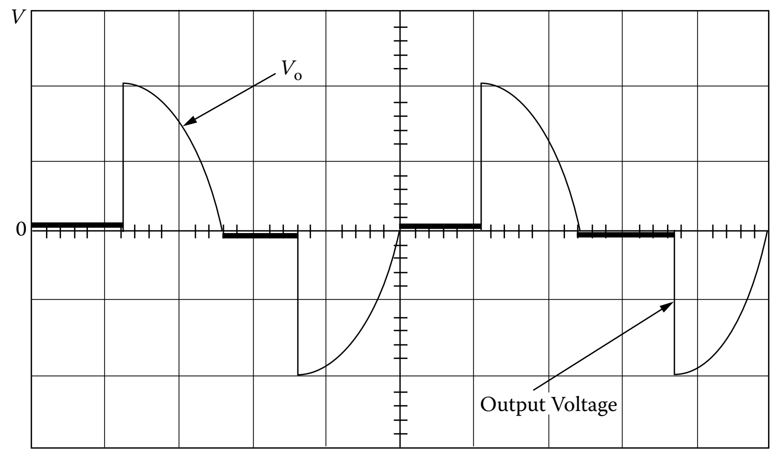

Figure 24-8. Output Voltage Across the Load Operating at a 50% Duty Cycle.

This condition reverses every half cycle and Gate 1 will go into saturation, while Gate 2 will come out of saturation. The B-H loops in Figure 24-5 will now change, when the current is flowing in the control winding. The flux in Gate 1 is still symmetrical around the origin, while Gate 2 goes into saturation, as shown in Figure 24-9. When the input ac excitation reverses on the next half cycle, the gate flux also reverses and Gate 1 goes into saturation and Gate 2 goes out of saturation, as shown in Figure 24-10.

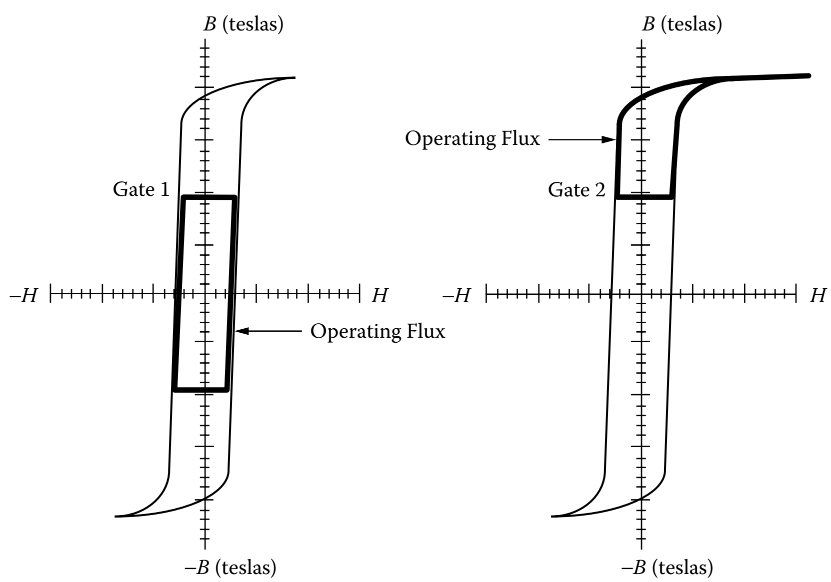

Figure 24-9. The Control Current Is Forcing Gate 2 into Saturation.

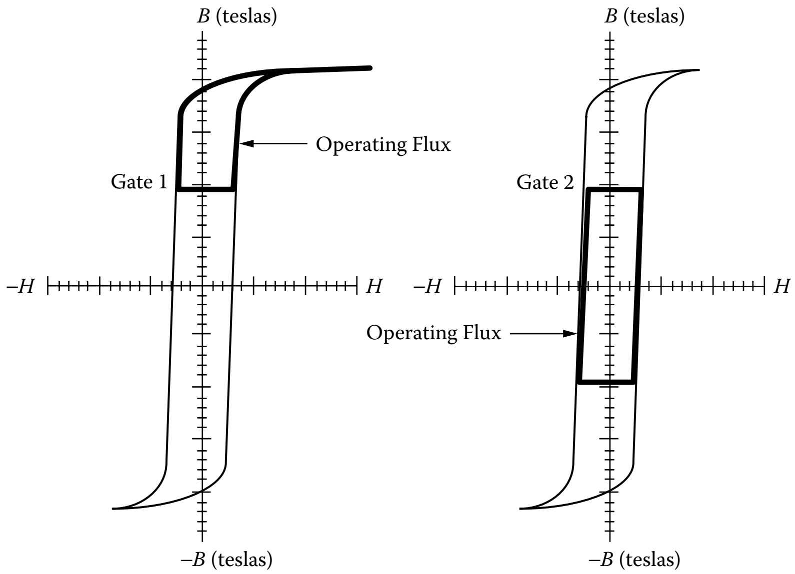

Figure 24-10. The Control Current Is Forcing Gate1 into Saturation.

Control Winding

With the ac excitation applied to the gates and without any dc control current applied, the net change in flux through the control winding is zero, making the induced voltage in the control winding zero. This condition is true because the flux generated by the opposing gates in the center leg of the EI core cancels. It can only be true if there are equal turns on each gate and the permeability of the outer legs of the EI core is equal. To minimize the unbalance that can occur because of the gate windings and core material, there is usually a test specification written to control the gate turns and the permeability of the material.

The induced voltage in the control winding is from the above example operating at a 50% duty cycle, as shown in Figure 24-11. If the ac excitation to the gates were a square wave then the induced voltage would be a square

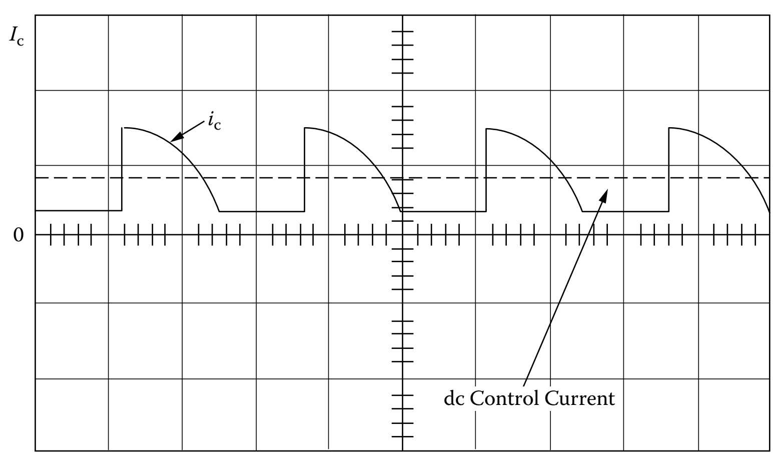

Figure 24-11. The Induced Voltage in the Control Winding Is from a Sine Wave Excitation.

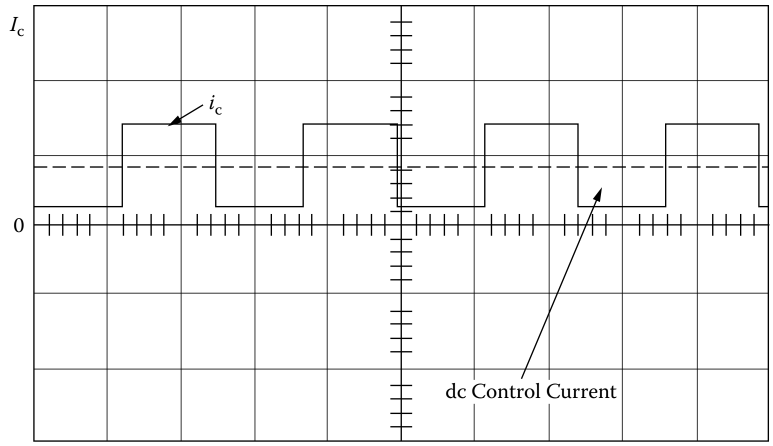

Figure 24-12. The Induced Voltage in the Control Winding Is from a Square Wave Excitation.

wave, as shown in Figure 24-12. The induced voltage in the control winding is always the second harmonic of the operating frequency. The second harmonic is caused by the changing flux in the core not at saturation.

Saturated Inductance and Winding Resistance

After the saturable reactor has been built it would be wise to measure all of the parameters before designing the rest of the circuit. This is because the output voltage at the load could be lower than the design voltage, as shown in Figure 24-13. This discrepancy is because of the winding resistance and the saturated inductance of the saturable reactor. The saturated inductance is a built-in ac resistor that will support voltage after the magnetic core has been saturated with the control amp-turns. If the magnetic material is ideal and the core is saturated, the permeability will drop to zero. All you will have then is an air core inductor. The saturated

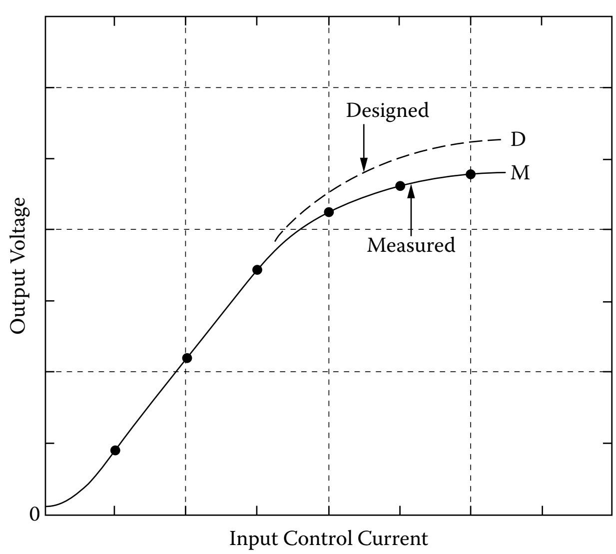

Figure 24-13. Comparing the Design and Measured Output Voltage at the Load.

inductance is very dependent on the magnetic material and its configuration. Depending on the design the engineer has the choice of laminations, C cores or toroids. Toroidal cores, by their construction, have the minimum of air gap. Both laminations and C cores have air gaps and this results in shearing over the hysteresis loop. See Chapter 2. To minimize the saturated inductance pick a material and construction that will provide a minimum of air gap, high permeability and a high flux density, if possible.

The magnetic materials provide two types of B-H loops, a high permeability square loop and a round loop to do the design, as shown in Figure 24-14. The square B-H loop found in Figure 24-14 is typical of what is found in toroidal cores. The X marks the B_{sat} of the magnetic material. If the excitation is increased, the horizontal line above the X will just become longer and B_{sat} remains constant. The round B-H loop found Figure 24-14 is typical of what is found with laminations because of the small air-gaps. The Y marks the, B_{sat}, for the high permeability magnetic material within the core. If the excitation is increased the diagonal line above the Y will also increase upward. The diagonal line above Y shows, B_{sat}, has not been completely reached because of the small air gaps and will take more H to accomplish, B_{sat}.

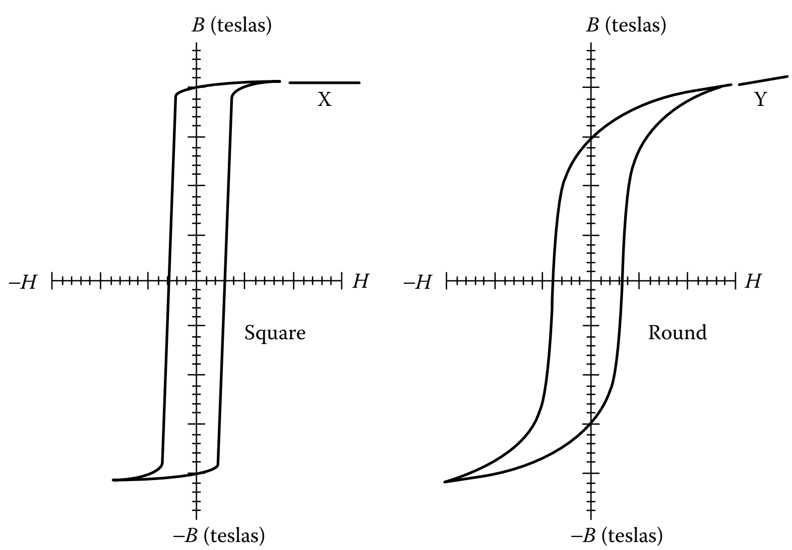

Figure 24-14. Comparing Square and Round B-H Loops.

Saturable Reactor Power Gain

One of the qualities of a saturable reactor is based on its power gain. The power gain is dependent to a great deal upon the design, type, and physical size of the saturable reactor. Power gain of a saturable reactor is the ratio of the input control power, P_c, to the output or load power, P_o. The input control power is the power dissipated in the control winding. The control power, P_c, is the power dissipated in the control winding, as shown in Equation [24-5]. The control current is, I_c, and the winding resistance is, R_c. The output power, P_o, is the power delivered to the load, as shown in Equation [24-6]. The current flowing in the gate winding is, I_g, and the load resistance is, R_L. The power gain then is the output power, P_o, divided by the control power, P_c, as shown in Equation [24-7].

$$P_c = I_c^2 R_c, \quad \text{[watts] [control]} \qquad [24\text{-}5]$$

$$P_o = I_o^2 R_L, \quad \text{[watts] [load]} \qquad [24\text{-}6]$$

$$P_{(gain)} = \frac{\left(I_o^2 R_L\right)}{\left(I_c^2 R_c\right)}, \quad \text{[power gain]} \qquad [24\text{-}7]$$

The currents in the saturable reactor are inversely proportional to the turn's ratio, so Equation [24-7] can be rewritten to Equation [24-8]. The control turns are, N_c, and the gate turns (single gate) are, N_g.

$$P_{(gain)} = \frac{\left(N_{c(5-6)}^2 R_L\right)}{\left(N_{g(1-2)}^2 R_c\right)}, \quad \text{[power gain]} \qquad [24\text{-}8]$$

The above two Equations [24-7] and [24-8] are without feedback.

Response Time for Saturable Reactors

The response time, t_r, for a saturable reactor is very slow for a step change in the control signal. This is because the control winding is very inductive. Any step change in current in the control winding will take time to reach its final value. The response time, t_r, is given as a time constant of the control circuit. The response time is the time it takes for the load current to reach 63 percent of its final value for a step input change as shown in Figure 24-15. It can be seen in Figure 24-15, when there is an input step change, t_1, in the control circuit, the output current, I_1, will rise from its initial value at, t_1, to its final value of, I_3, at time, t_3. The value of, I_2, is the current at 63 percent of the final current, I_3, which is a one time constant. The time constant is shown in Equation [24-9].

$$t_r = \frac{L_c}{R_c}, \quad \text{[seconds]} \qquad [24\text{-}9]$$

t_r = [seconds], [time constant]

L_c = [henries], [contral winding inductance]

R_c = [ohms], [contral winding resistance]

This concept was derived from linear systems. It yields sufficiently accurate results for most purposes.

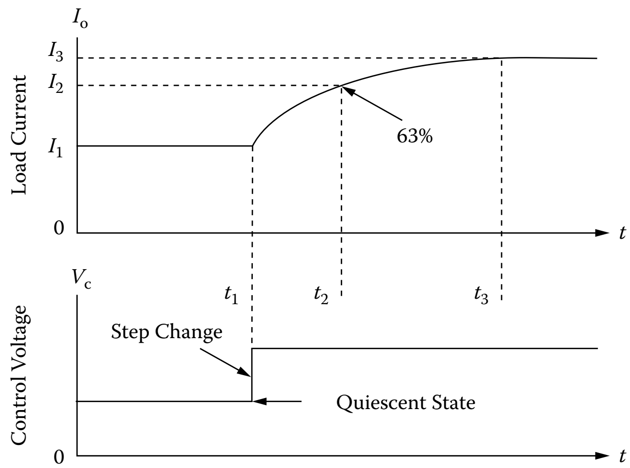

Figure 24-15. Output Response Time for an Input Signal Step Change.

Saturable Reactor Apparent Power, P_t

Each gate of the saturable reactor, SR1, will support half of the output voltage to the load. The volt-amps, VA, of the saturable reactor, SR1, are equal to the volt-amps, VA, of the load. The amp-turns in the control winding will produce amp-turns in the gate winding, as shown in Equation [24-10]. If the volt-amps of the gates are equal to the volt-amps of the control, then the window area is divided equally between the gate and the control, as shown in Figure 24-16.

$$N_{c(5-6)}I_c = N_{g(1-2)}I_g \qquad [24\text{-}10]$$

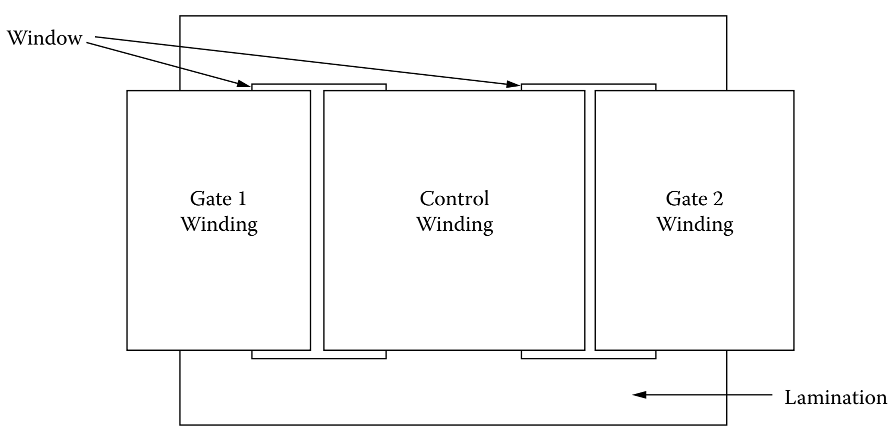

Figure 24-16. Series Saturable Reactor Showing Gate and Control Window Allocation.

To have full control of the output voltage from, T1, as shown in Figure 24-2, the volt-amps of the saturable reactor, SR1, must equal the volt-amps of the load, P_o. The saturable reactor, SR1, has a gate winding and a common control winding. Both are equal to one-half the load power, P_o, as shown in Equation [24-11].

$$P_t = 0.5P_o\left(\frac{1_{(gate)}}{\eta} + 1_{(control)}\right), \quad \text{[watts]} \qquad [24\text{-}11]$$

The apparent power, P_t, calculated in Equation 24-11 is for each core of a saturable reactor using two cores, such as a toroid, as shown in Figure 24-18. The total apparent power, P_t, is shown in Equation [24-12]. The Equation [24-12] would be used for designs using laminations. It must be remembered that the gates are wound on the outer legs of the lamination where the cross-section, A_c, is normally only half the center leg.

$$P_t = P_o\left(\frac{1_{(gate)}}{\eta} + 1_{(control)}\right), \quad \text{[watts]} \qquad [24\text{-}12]$$

The electrical conditions, K_e, for the core geometry, K_g, are shown in Equation [24-13].

$$K_e = 0.145K_f^2 f^2 B_m^2 (10^{-4}) \qquad [24\text{-}13]$$

The calculation for the core geometry, K_g, is shown in Equation [24-14].

$$K_g = \frac{P_t}{2K_e\alpha}, \quad [\text{cm}^5] \qquad [24\text{-}14]$$

Note: Alpha, α, is the combined copper loss of the saturable reactor control and gate windings. The core calculation for the core geometry, K_g, is shown in Equation [24-15].

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \qquad [24\text{-}15]$$

Mean Length Turn for E Cores

The Mean Length Turn, (MLT), is required to calculate both winding resistance and weight for given winding. Using the EI lamination in a saturable reactor design requires the calculation of the resistance of both control and gate windings. The winding dimensions, relating to the Mean Length Turn, (MLT) for a tube or bobbin coil, are shown in Figure 24-17. The Mean Length Turn, (MLT) for the control winding is shown in Equation 24-16 and the Mean Length Turn, (MLT) for the gate windings, is shown in Equation [24-17].

$$\text{MLT}_{(control)} = 2(D+2B) + 2(E+2B) + 2\pi\left(\frac{F}{4}\right), \quad \text{[cm]} \qquad [24\text{-}16]$$

$$\text{MLT}_{(gate)} = 2(D+2B) + 2\left(\frac{E}{2}+2B\right) + 2\pi\left(\frac{F}{4}\right), \quad \text{[cm]} \qquad [24\text{-}17]$$

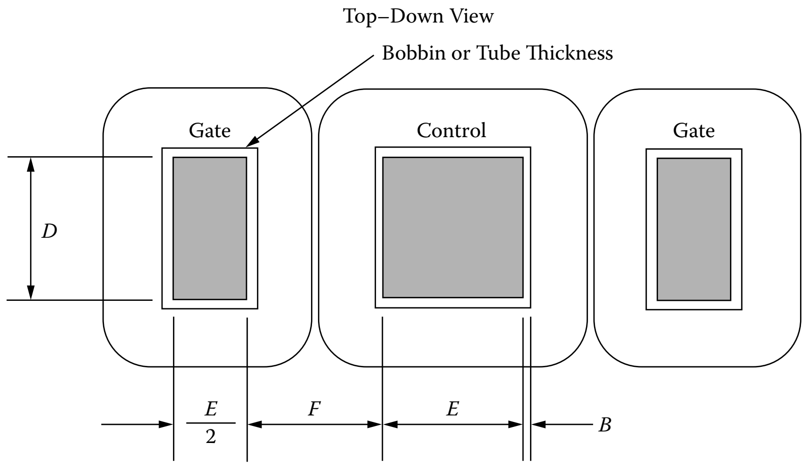

Figure 24-17. Dimensions, Relating to the Mean Length Turn, (MLT).

Calculating, MLT for Toroidal Cores

In order to wind the toroidal core, there has to be room to allow free passage of the wire shuttle. The wire shuttle could be a machine shuttle or a hand wind shuttle. Winding a saturable reactor is not any different than winding any other toroidal core. Half of the inside diameter will be set aside for the shuttle clearance. This will leave 75% of the window, W_a, for the windings. It is very difficult to calculate the Mean Length Turn (MLT) for a toroidal core used in saturable reactor design that would satisfy all conditions. The saturable reactor has two cores and each core has its own gate winding. The gate winding is wound first. After the gate winding is wound, the cores are tested and matched, then they are assembled for the common control winding to be wound. The assembly of the toroidal cores can be seen in Figure 24-18. The fabrication of a toroidal design is weighted heavily on the skill of the winder. The toroidal core saturable reactor requires special attention, when wound by hand or wound by machine, because of the angle it imposes when two cores are stacked. The Mean Length Turn, (MLT) for a toroidal saturable reactor is very difficult to obtain, because its shape is very non-linear. A good approximation for a toroidal core, Mean Length Turn (MLT) for the gate winding is Equation [24-18] and Equation [24-19] for the control winding, shown in Figure 24-18. There is a correction factor of 0.85 for the gate winding and a correction factor of 0.70 for the control winding.

Nomenclature clarification:

1. A = Core strip width including the case. Core Height
2. B = Core build including the case. (OD – ID) / 2
3. C = Is the build of both gate and control winding = W/4
4. D = Is the build of the gate, = W/8
5. W = Is the inside diameter of the core including the case.

$$\text{MLT} = (2A) + (2B) + \pi 0.125W)(0.85), \quad \text{approximation [Gate]} \qquad [24\text{-}18]$$

$$\text{MLT} = (4A + 0.25W) + (2B) + 2\pi(0.375W)(0.70), \quad \text{approximation [Control]} \qquad [24\text{-}19]$$

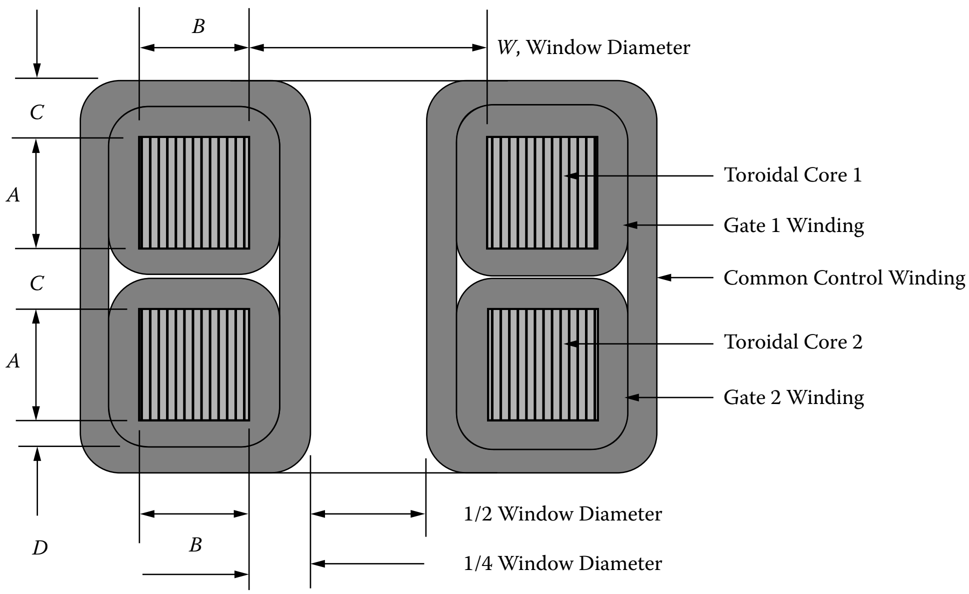

Figure 24-18. Toroidal Core Saturable Reactor Mean Length Turn, (MLT) is a Good Approximation.

Toroidal Saturable Reactor Surface Area

The data such as height and outside diameter required for the surface area calculation, was taken from Figure 24-18. The height, Ht, of the saturable reactor is shown in Equation [24-20] and the outside diameter, OD, is shown in Equation [24-21]. The saturable reactor outline is shown in Figure 24-19.

$$Ht = (3C + 2A)$$
$$OD = (2C + 2B + W)$$

$$A = (\text{Core Height, Ht}),\ B = (\text{Core Build, (OD-ID)}/2),\ C = (\text{Winding Build, } W/4 = ID/4)$$

$$SR_{Ht} = \left(3\left(\frac{ID}{4}\right) + 2Ht\right) \qquad [24\text{-}20]$$

$$SR_{OD} = \left(\left(\frac{ID}{2}\right) + (OD - ID) + ID\right) \qquad [24\text{-}21]$$

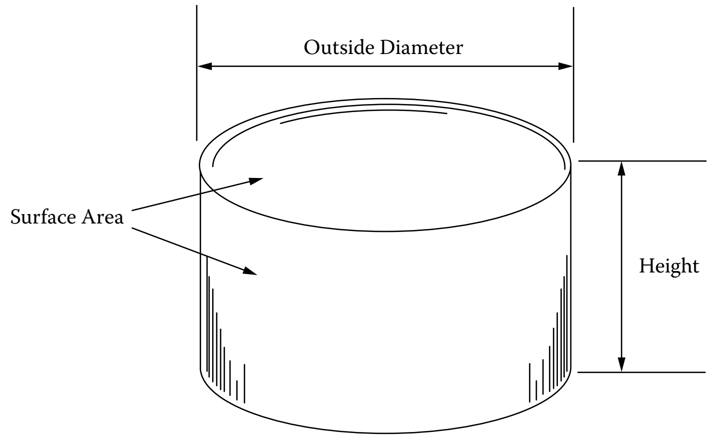

Figure 24-19. The Outline of a Toroidal Wound Saturable Reactor.

The surface area of the saturable reactor can be calculated using Equation, [24-24].

$$\text{Top and Bottom Surface} = 2\left(\frac{\pi\left(SR_{OD}\right)^2}{4}\right), \quad [\text{cm}^2] \qquad [24\text{-}22]$$

$$\text{Periphery Surface} = \left(\pi\left(SR_{OD}\right)\right)\left(\text{SR}_{\text{Ht}}\right), \quad [\text{cm}^2] \qquad [24\text{-}23]$$

$$A_t = \frac{\pi\left(SR_{OD}\right)^2}{2} + \left(\pi\left(SR_{OD}\right)\right)\left(\text{SR}_{\text{Ht}}\right), \quad [\text{cm}^2] \qquad [24\text{-}24]$$

E Core Saturable Reactor Surface Area

The data for height, width and length required for the surface area calculation was taken from Chapter 3. The height, Ht, is (E+G), the width, is (D+F), and length is (2E+3F). The saturable reactor outline is shown in Figure 24-20. The surface area for an E core saturable reactor can be calculated using Equation, [24-28].

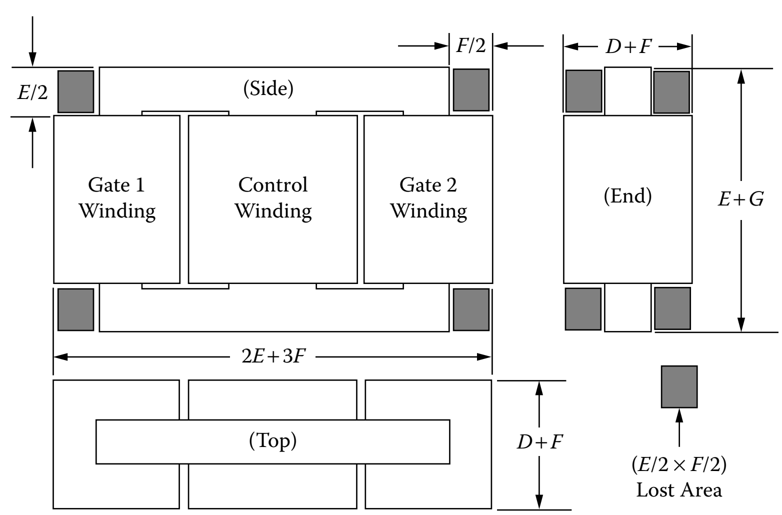

Figure 24-20. The Outline of an E Core Wound Saturable Reactor.

$$\text{Side Area} = (2E+3F)(E+G)-(EF), \quad [\text{cm}^2] \qquad [24\text{-}25]$$

$$\text{End Area} = (E+G)(D+F)-(EF), \quad [\text{cm}^2] \qquad [24\text{-}26]$$

$$\text{Top Area} = (2E+3F)(D+F), \quad [\text{cm}^2] \qquad [24\text{-}27]$$

$$A_t = 2(\text{Side Area})+2(\text{End Area})+2(\text{Top Area}), \quad [\text{cm}^2] \qquad [24\text{-}28]$$

Designing with Toroidal Tape Cores

The performance of a saturable reactor using tape toroidal cores, as shown in Figure 24-21, is far more superior to using laminations. The tape toroidal core comes in a lot of sizes and shapes. The tape toroidal cores with a large window are known for their high power gain. The tape toroidal cores have a much higher permeability and can be matched to a much tighter tolerance. The tape toroidal cores, because of their construction, do not have an air gap. A core without an air gap will provide a minimum of magnetizing current. The net result is a minimum of offset voltage on the output when the control current is zero. Also, having cores with high permeability and operating at a high frequency does have a drawback. When magnetic components with high permeability cores are operating at high frequency, special care must be taken so the magnetic components do not go into self-resonates.

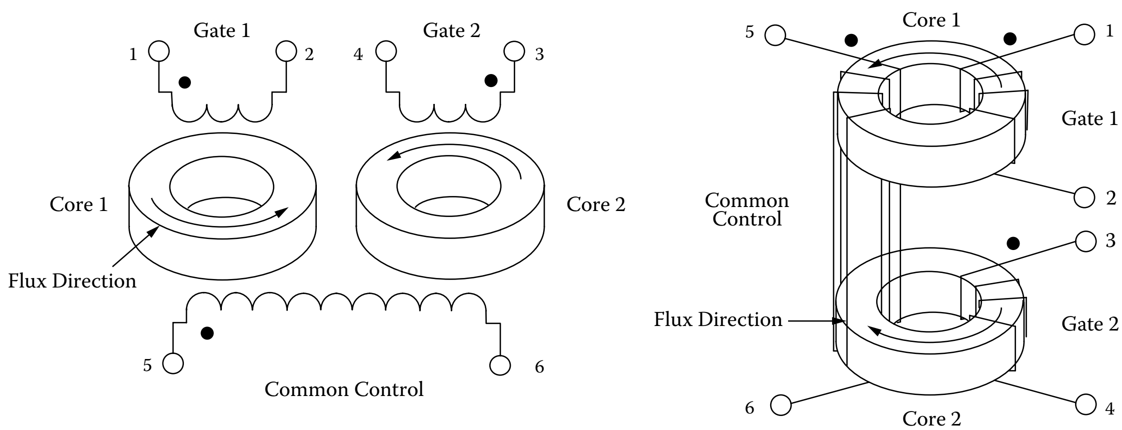

Figure 24-21. Saturable Reactor Designed Using Toroidal Cores.

Comparing the Toroidal Tape Cores with the Laminations

The schematics of a saturable reactor, using toroidal tape cores and a saturable reactor using laminations are shown in Figure 24-22. There is truly not much difference between the saturable reactor designed with laminations or toroidal tape cores, but it does bring out why the toroidal cores have to be matched.

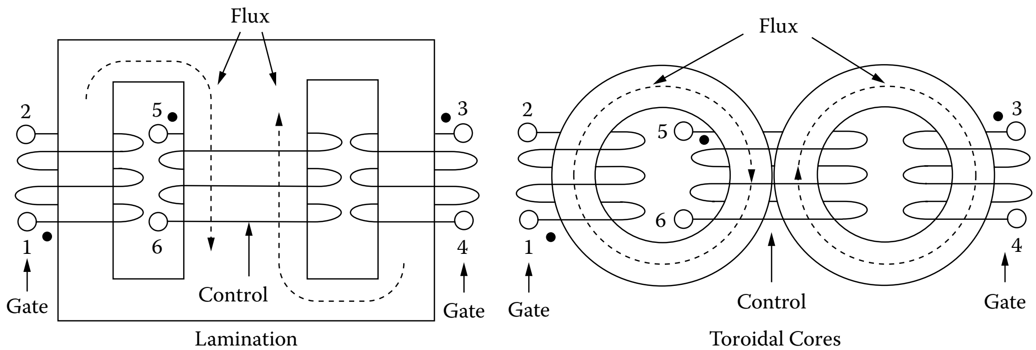

Figure 24-22. Comparing Saturable Reactors with Toroidal Cores and Laminations.

Series Saturable Reactor Design Example

The saturable reactor will control the voltage of 12.6 volts at 1.0 amps to the power an incandescent lamp. The load resistor, R1, as shown in Figure 24-23, will be 12 ohms. The load resistor, R1, is used in place of the incandescent lamp for testing. The reason for using a resistor in place of the lamp is because the lamp is a non-linear load and the transfer function looks much cleaner and more representative of the saturable reactive, SR1, output.

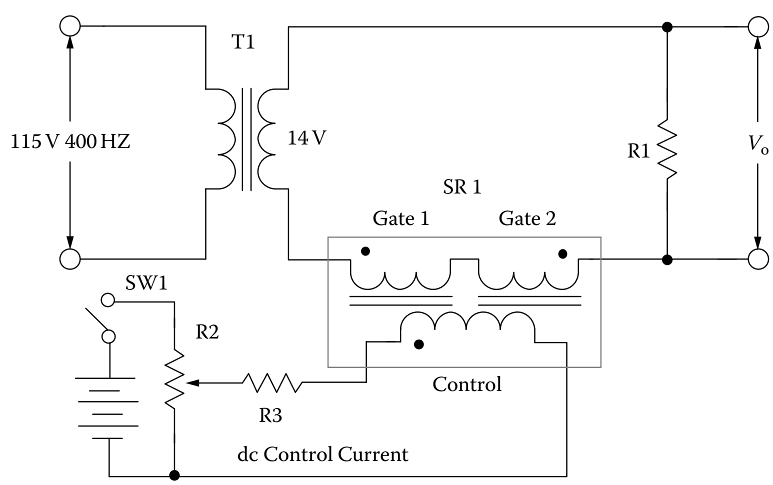

Figure 24-23. AC Power Supply being controlled by a Saturable Reactor, SR1.

Specification and Design

1. Transformer, T1 Output Voltage = 14 volts ac
2. Operating Frequency = 400 Hz
3. Load voltage, V_o = 12.6 volts ac
4. Load Current, I_o = 1.0 amps
5. Magnetic Core Type = Toroid
6. Magnetic Material, 4 mil = 50-50 NiFe
7. Efficiency, η = 95%
8. Regulation (copper loss), α = 5%
9. Operating Flux Density, B_{ac} = 1.0 T
10. Control Current, I_c = 0.10 amps

Step No. 1: Calculate the load power, P_o.

$$P_o = V_o I_o, \quad \text{[watts]}$$
$$P_o = (12.6)(1.0), \quad \text{[watts]}$$
$$P_o = 12.6, \quad \text{[watts]}$$

Step No. 2: Calculate the saturable reactor apparent power, P_t, for a single core.

$$P_t = (0.5)P_o\left(\frac{1_{(gate)}}{\eta} + 1_{(control)}\right), \quad \text{[watts]}$$
$$P_t = (0.5)(12.6)\left(\frac{1}{0.95} + 1\right), \quad \text{[watts]}$$
$$P_t = 12.9, \quad \text{[watts]}$$

Step No. 3: Calculate the electrical conditions, K_e.

$$K_e = 0.145 K_f^2 f^2 B_m^2 (10^{-4})$$
$$K_f = 4.44$$
$$K_e = 0.145(4.44)^2(400)^2(1.0)^2(10^{-4})$$
$$K_e = 45.7$$

Step No. 4: Calculate the core geometry, K_g.

$$K_g = \frac{P_t}{2K_e\alpha}, \quad [\text{cm}^5]$$
$$K_g = \frac{12.9}{2(45.7)(5)}, \quad [\text{cm}^5]$$
$$K_g = 0.0282, \quad [\text{cm}^5]$$

Step No. 5: This core was selected from "Magnetics Tape Wound Cores Catalog," TWC 500, comparable to core geometry, K_g.

1. Toroid = 52029-4A
2. Manufacturer = Magnetics
3. OD Dimension = 3.78 cm
4. ID Dimension = 2.25 cm
5. Ht Dimension = 0.978 cm
6. Core Geometry, $K_g = 0.0256$ cm^5
7. Area Product, $A_p = 1.08$ cm^4
8. Core Weight, $W_t = 19.66$ grams
9. Iron Area, $A_c = 0.272$ cm^2
10. Window Area, $W_a = 3.97$ cm^2
11. Magnetic Path Length, MPL = 9.47 cm
12. Magnetic Material, 4 mil = Orthonol
13. Efficiency = 95%
14. Copper Loss, $\alpha = 5\%$

Step No. 6: Calculate the window area for the gate winding, A_{cg}.

$$W_{ag} = \frac{W_a}{2}, \quad [\text{cm}^2]$$

$$W_{ag} = \frac{3.97}{2}, \quad [\text{cm}^2]$$

$$W_{ag} = 1.98, \quad [\text{cm}^2]$$

Step No. 7: Calculate the number of turns for the gate winding, N_g.

$$N_g = \frac{0.5V_o(10^4)}{K_f B_{ac} f A_c}, \quad [\text{turns}]$$

$$N_g = \frac{0.5(14)(10^4)}{(4.44)(1.0)(400)(0.272)}, \quad [\text{turns}]$$

$$N_g = 145, \quad [\text{turns}]$$

Step No. 8: Calculate the current density, J.

$$J = \frac{P_t(10^4)}{K_u K_f B_{ac} f A_p}, \quad [\text{amps/cm}^2]$$

$$J = \frac{(12.9)(10^4)}{(0.4)(4.44)(1.0)(400)(1.08)}, \quad [\text{amps/cm}^2]$$

$$J = 168, \quad [\text{amps/cm}^2]$$

Step No. 10: Calculate the gate bare wire area, $A_{wg(B)}$.

$$A_{wg(B)} = \frac{I_o}{J}, \quad [\text{cm}^2]$$
$$A_{wg(B)} = \frac{1.0}{168}, \quad [\text{cm}^2]$$
$$A_{wg(B)} = 0.00595, \quad [\text{cm}^2]$$

Step No. 11: Select the wire from the Wire Table in Chapter 4.

$$AWG = 20$$
$$A_{w(B)} = 0.005188, \quad [\text{cm}^2]$$
$$A_{wg} = 0.006065, \quad [\text{cm}^2]$$
$$\left(\frac{\mu\Omega}{cm}\right) = 332, \quad [\text{micro-ohm/cm}]$$

Step No. 12: Calculate the Mean Length Turn, (MLT) for the gate. Use Figure 24-18 for a reference.

A = Core with case Ht Dimension = 0.978

B = Core with case Build = (OD-ID)/2 = 0.765

W = Core with case Inside Diameter = 2.25

$$\text{MLT} = ((2A) + (2B) + (\pi 0.125W))(0.85), \quad [\text{cm}]$$
$$\text{MLT} = ((2(0.978)) + (2(0.765)) + (\pi 0.125(2.25)))(0.85), \quad [\text{cm}]$$
$$\text{MLT} = 3.71, \quad [\text{cm}]$$

Step No. 13: Calculate the gate winding resistance, R_g.

$$R_g = \text{MLT}(N_g)\left(\frac{\mu\Omega}{\text{cm}}\right)(10^{-6}), \quad [\text{ohms}]$$
$$R_g = 3.71(145)(332)(10^{-6}), \quad [\text{ohms}]$$
$$R_g = 0.179, \quad [\text{ohms}]$$

Step No. 14: Calculate the gate winding copper loss, P_g.

$$P_g = I_g^2 R_g, \quad [\text{watts}]$$
$$P_g = (1.0)^2(0.179), \quad [\text{watts}]$$
$$P_g = 0.179, \quad [\text{watts}]$$

Step No. 15: Calculate the required control turns, N_c.

$$N_c = \left(\frac{N_g I_g}{I_c}\right), \quad [\text{turns}]$$
$$N_c = \left(\frac{(145)(1.0)}{0.1}\right), \quad [\text{turns}]$$
$$N_c = 1450, \quad [\text{turns}]$$

Step No. 16: Calculate the control bare wire area, $A_{w(B)}$.

$$A_{wc(B)} = \frac{I_c}{J}, \quad [\text{cm}^2]$$
$$A_{wc(B)} = \frac{0.10}{168}, \quad [\text{cm}^2]$$
$$A_{wc(B)} = 0.000595, \quad [\text{cm}^2]$$

Step No. 17: Select the wire from the Wire Table, in Chapter 4.

$$AWG = 30$$
$$A_{wc(B)} = 0.000507, \quad [\text{cm}^2]$$
$$A_{wc} = 0.000678, \quad [\text{cm}^2]$$
$$\left(\frac{\mu\Omega}{cm}\right) = 3402, \quad [\text{micro-ohm/cm}]$$

Step No. 18: Calculate the Mean Length Turn, (MLT) for the control winding. Use Figure 24-18 for reference.

A = Core with case Ht Dimension = 0.978

B = Core with case Build = (OD-ID)/2 = 0.765

W = Core with case Inside Diameter = 2.25

$$\text{MLT} = ((4A + 0.25W) + (2B) + 2\pi(0.375W))(0.70), \quad [\text{cm}]$$
$$\text{MLT} = (4(0.978) + 0.25(2.25) + 2(0.765) + 2\pi(0.375(2.25)))(0.70), \quad [\text{cm}]$$
$$\text{MLT} = 7.91, \quad [\text{cm}]$$

Step No. 19: Calculate the control winding resistance, R_c.

$$R_c = \text{MLT}(N_c)\left(\frac{\mu\Omega}{\text{cm}}\right)(10^{-6}), \quad [\text{ohms}]$$
$$R_c = 7.91(1450)(3402)(10^{-6}), \quad [\text{ohms}]$$
$$R_c = 39.0, \quad [\text{ohms}]$$

Step No. 20: Calculate the control winding copper loss, P_c.

$$P_c = I_c^2 R_c, \quad [\text{watts}]$$
$$P_c = (0.1)^2 (39.0), \quad [\text{watts}]$$
$$P_c = 0.39, \quad [\text{watts}]$$

Step No. 21: Calculate the total copper loss for the gates and control, P_{cu}.

$$P_{cu} = P_{g1} + P_{g2} + P_c, \quad [\text{watts}]$$
$$P_{cu} = (0.179) + (0.179) + (0.39), \quad [\text{watts}]$$
$$P_{cu} = 0.748, \quad [\text{watts}]$$

Step No. 22: Calculate the percent of total copper loss, α.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$
$$\alpha = \frac{(0.748)}{(12.6)}(100), \quad [\%]$$
$$\alpha = 5.94, \quad [\%]$$

Step No. 23: Calculate the watts per kilogram, W/K. Use the Equation for this material in Chapter 2.

$$W/K = 0.000618(f)^{1.48}(B_{ac})^{1.44}$$
$$W/K = 0.000618(400)^{1.48}(1.0)^{1.44}$$
$$W/K = 4.38$$

Step No. 24: Calculate the core loss, P_{fe}.

$$P_{fe} = (W/K)2(W_{tfe})(10^{-3}), \quad [\text{watts}]$$
$$P_{fe} = (4.38)2(19.7)(10^{-3}), \quad [\text{watts}]$$
$$P_{fe} = 0.173, \quad [\text{watts}]$$

Step No. 25: Calculate the total loss, P_Σ.

$$P_\Sigma = P_{cu} + P_{fe}, \quad [\text{watts}]$$
$$P_\Sigma = (0.748) + (0.173), \quad [\text{watts}]$$
$$P_\Sigma = 0.921, \quad [\text{watts}]$$

Step No. 26: Calculate the saturable reactor height, Ht using Figure 24-19 for reference. See Chapter 5.

$$SR_{Ht} = \left(3\left(\frac{ID}{4}\right) + 2Ht\right), \quad [\text{cm}]$$
$$SR_{Ht} = \left(3\left(\frac{2.25}{4}\right) + 2(0.978)\right), \quad [\text{cm}]$$
$$SR_{Ht} = 3.64, \quad [\text{cm}]$$

Step No. 27: Calculate the saturable reactor outside diameter, OD using Figure 24-19 for reference. See Chapter 5.

$$SR_{OD} = \left(\left(\frac{ID}{2}\right) + (OD - ID) + ID\right), \quad [\text{cm}]$$
$$SR_{OD} = \left(\left(\frac{2.25}{2}\right) + (3.78 - 2.25) + 2.25\right), \quad [\text{cm}]$$
$$SR_{OD} = 4.91, \quad [\text{cm}]$$

Step No. 28: Calculate the saturable reactor surface area using Figure 24-19 for reference. See Chapter 5.

$$A_t = \frac{\pi(SR_{OD})^2}{2} + (\pi(SR_{OD})(\text{SR}_{\text{Ht}})), \quad [\text{cm}^2]$$
$$A_t = \frac{\pi(4.91)^2}{2} + (\pi(4.91)(3.64)), \quad [\text{cm}^2]$$
$$A_t = 85.8, \quad [\text{cm}^2]$$

Step No. 29: Calculate the watts per unit area, ψ.

$$\psi = \frac{P_{\Sigma}}{A_t}, \quad [\text{watts/cm}^2]$$
$$\psi = \frac{0.921}{85.8}, \quad [\text{watts/cm}^2]$$
$$\psi = 0.0107, \quad [\text{watts/cm}^2]$$

Step No. 30: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{0.826}, \quad [°\text{C}]$$
$$T_r = 450(0.0107)^{0.826}, \quad [°\text{C}]$$
$$T_r = 10.6, \quad [°\text{C}]$$

Step No. 31: Calculate the window utilization, K_u, for both control and gate.

$$K_{uc} = \left(\frac{N_c A_{wc(B)}}{\frac{W_a}{2}} \right), \quad K_{uc} = \frac{(1450)(0.000507)}{(1.98)} = 0.371$$

$$K_{ug} = \left(\frac{N_g A_{wg(B)}}{\frac{W_a}{2}} \right), \quad K_{ug} = \frac{(145)(0.005188)}{(1.98)} = 0.380$$

Series Saturable Reactor Design Test Data (Core Geometry, K_g, Approach)

Summary

The above series saturable reactor was built, and tested. The following information is the test data for the above series saturable reactor design. The input control current versus the output voltage test data is shown in Table 24-1. The input current versus output voltage transfer function is shown in Figure 24-24. The author hopes that this design, with its step-by-step approach, helps the readers understand the design of a series saturable reactor.

Test Data

1. Frequency, f = 400 Hz
2. Output voltage, V_o = 12.7 volts
3. Output current, I_o = 1.0 amps
4. Control current, I_c = 0.10 amps
5. Gate resistance, R_g = 0.176 ohms
6. Control resistance, R_c = 38.4 ohms
7. Temperature Rise, T_r = 10.4°C

Table 24-1. Series Saturable Reactor Showing Input Current versus Output Voltage

Saturable Reactor		
Steps	Control Current Amps	Output Voltage Volts
1	0.00	0.200
2	0.01	1.600
3	0.02	2.900
4	0.03	4.400
5	0.04	5.700
6	0.05	7.100
7	0.06	8.400
8	0.07	9.900
9	0.08	11.400
10	0.09	12.500
11	0.10	12.700
12	0.11	12.700

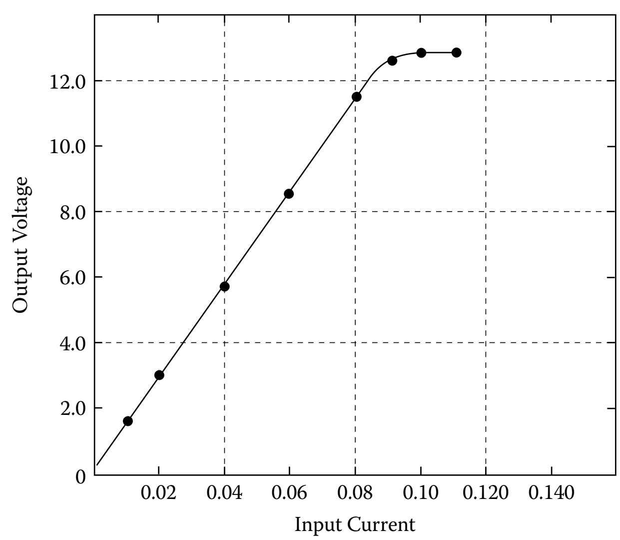

Figure 24-24. Transfer Function Using the Data in Table 24-1.

Ultra Low Power 0-15 Amp Current Transducer (Saturable Reactor)

Introduction

A unique, ultra low power current transducer was developed to measure the charge and discharge current on the Mariner Mark II battery. It provides current detection and isolation, via a series connected saturable reactor. The current transducer, shown in Figure 24-25, provides a 0-3 volt output for a 0-15 amps input. The power source is +5 volts and consumes 22 milliwatts of power. The current transducer performance can best be seen by looking at the transfer function of input current versus output voltage, as shown in Figure 24-26. The data for Figure 24-26 is taken from Table 24-2.

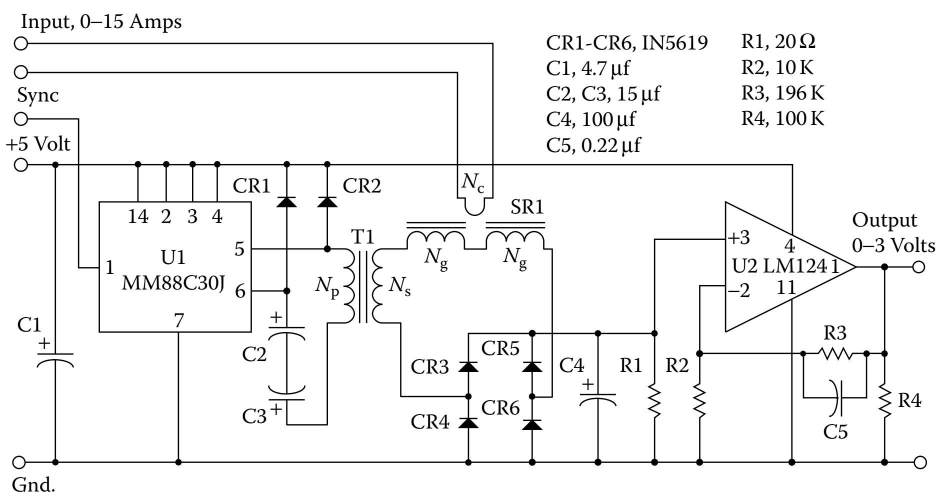

Figure 24-25. Schematic for the Ultra Low, Power Current Transducer.

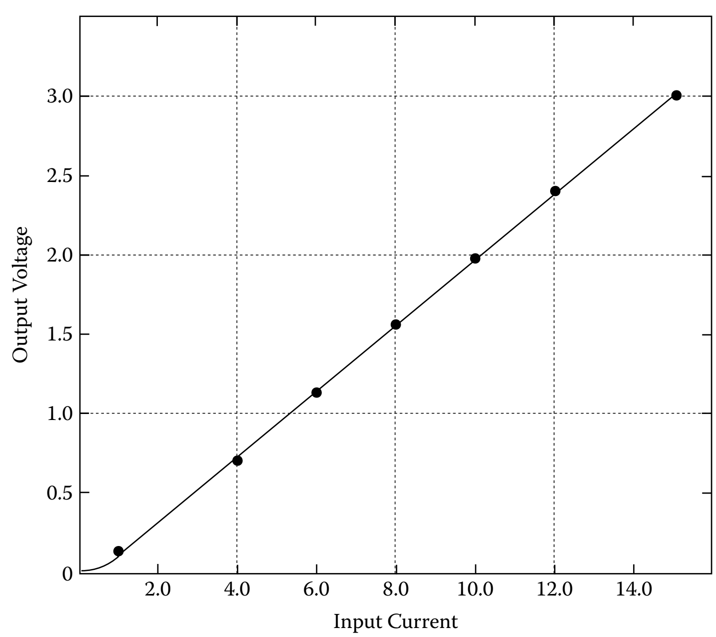

Figure 24-26. Input Current versus Output Voltage.

Table 24-2.

Milliwatt Current Transducer		
Steps	Input Current Amps	Output Voltage Volts
1	0.00	0.020
2	0.50	0.051
3	1.00	0.136
4	2.00	0.298
5	3.00	0.475
6	4.00	0.669
7	5.00	0.875
8	6.00	1.089
9	7.00	1.306
10	8.00	1.525
11	9.00	1.744
12	10.00	1.962
13	11.00	2.178
14	12.00	2.392
15	13.00	2.603
16	14.00	2.814
17	15.00	3.022

The saturable reactor can be used for both power and signal. The typical output power transfer function is shown in Figure 24-27A. The output power transfer function becomes nonlinear at the higher output voltages, as shown in Figure 24-27A. This non-linearity is fine for controlling lamps, heaters and motor control, but will not cut it as a transducer. The input-output transfer function for a telemetry transducer must be very linear. To get a good linear transfer function from a saturable reactor it is wise to use only 40 percent of its output capability, as shown in Figure 24-27B. The selection of a magnetic material with a square BH loop will also help.

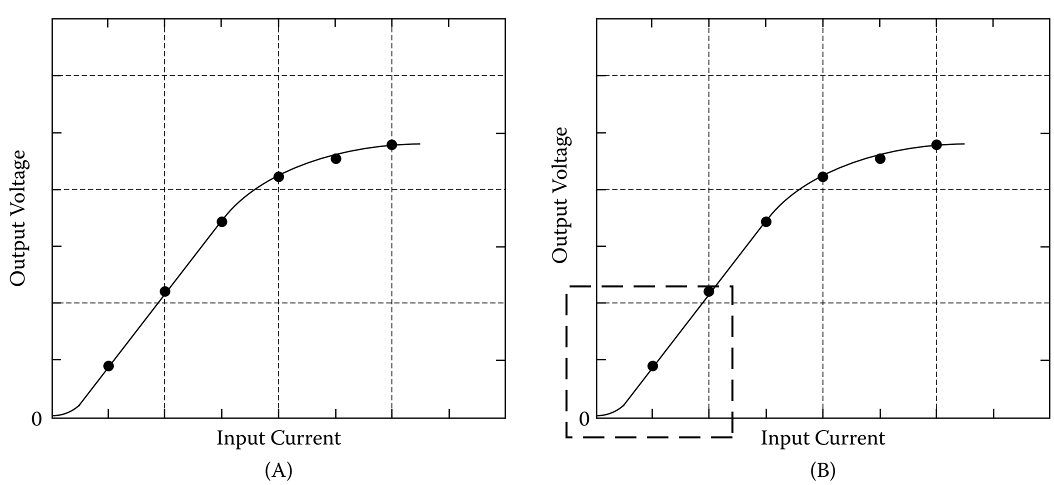

Figure 24-27. Typical Saturable Reactor Input-Output Transfer Function.

Circuit Description

The current detection and isolation is accomplished with a saturable reactor, (SR1). The line driver, (U1) provides the excitation power for the transducer. The inverter transformer, (T1), supplies power to the current-sensing, series saturable reactor, SR1. The output of the saturable reactor is then rectified with diodes (CR3-CR6) and provides the signal to the operational amplifier, (U2). The operational amplifier has a gain of 20 and produces a corresponding output of 0-3 volt for a 0-15 amp input. The transducer is powered from a 5-volt dc power source.

Specification

1. Input Current, I_{in} = 0-15 amps dc
2. Output voltage, V_o = 0-3 volts dc
3. Supply Voltage, V_{in} = 5 volts dc
4. Maximum Supply Power @ 10 amps Input Current = 50 milliwatts
5. Operating Frequency, f = 2.3kHz
6. External Drive Signal = 0-5 volts
7. Power Source Line Driver = MM78C30

Design Discipline

Power Transformer, (T1)

The power transformer, (T1), was designed for high efficiency and minimum capacitance for optimum performance. To minimize core loss, the transformer was designed, using an 80/20 nickel-iron core. The core is a Magnetics type, 52056-2D. The primary and secondary windings were interleaved and progressively wound 350° to minimize the capacitance. The winding procedure is shown in Figure 24-28. The primary has 408 turns of number #34 AWG and the secondary has 660 turns of number #34 AWG.

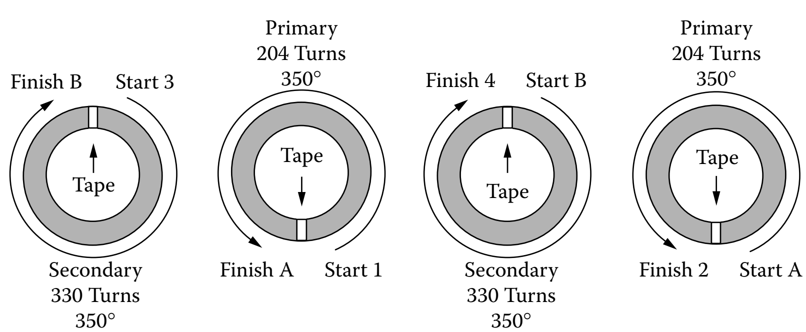

Figure 24-28. Transformer, T1, Winding Configuration.

Saturable Reactor, (SR1)

The saturable reactor was designed to have a one-turn, current sense winding. The design was to have an output current set at approximately 10 ma at 15 amps input. This process was done in order to keep the power consumption to a minimum. In the design of a saturable reactor, the amp-turns in the control winding must equal the amp-turns in the gate windings, as shown in Equation [24-29].

$$
\begin{aligned}
I_g N_g &= I_c N_c \\
N_g &= \frac{I_c N_c}{I_g}, \quad \text{[turns]} \\
N_g &= \frac{(15)(1)}{(0.01)}, \quad \text{[turns]} \\
N_g &= 1500, \quad \text{[turns]}
\end{aligned} \qquad [24\text{-}29]
$$

The saturable reactor has two opposing gates with a common control, as shown in Figure 24-28. With 1500 turns on the gates and a high permeability core, an effort must be made to minimize the capacitance in the gate winding in order to remove any chance of resonance. In this design the gate windings were wound progressively with 750 turns, 350° then insulated, and another 750 turns, 350° were wound progressively. The windings are rotated 180° to each other on the core. The cores are Magnetics 52057-2D, 80/20 nickel-iron, with a standard 5 percent, sine current match.

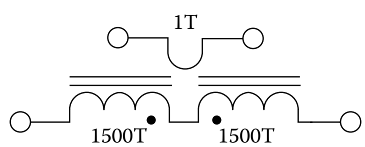

Figure 24-29. Simple Schematic of a Series Saturable Reactor.

Line Driver, (U1)

The line driver, IC, is a National Semiconductor MM78C30 dual differential line driver, which provides sufficient power to drive transformer, T1. The input signal to the line driver is a 2.3 kHz square wave, 0-5 volts.

Operational Amplifier (U2)

The operational amplifier is a National Semiconductor, LM124. The LM124 was chosen because of its input common-mode characteristics and its output voltage can swing to ground while operating from a single power supply.

Commutating Diodes (CR1, CR2)

The commutating diodes are 1N5619, and they are used to commutate the reactive current in the primary of T1, back to the source. The reactive current on the primary is caused by the reactive component of the saturable reactors (SR1) gates connected on the secondary. These two diodes commutate the reactive current back to the source on each half cycle. Without these diodes there would be large voltage spikes on the primary that waste power and could destroy the line driver. The input current to the transducer operating in a quiescent state is about 1.2 ma. With a signal current of 10amps the input current rose to 4.4 ma and 7.2 ma at 15 amps. The input power with a 10amp signal is 22 milliwatts, which is well within the specification of 50-mw requirement.

Rectifier Diodes (CR3-CR6)

These rectifier diodes are 1N5619 and are used to rectify the output of the saturable reactor.

Capacitor (C1)

The filter capacitor C1 is 4.7uf. It is required to store the energy from CR1 and CR2.

Capacitors (C2 and C3)

The line driver, U1, requires a 2.3kHz, 0-5 volt signal at pin 1 to function. If this signal is removed the line driver will go into an unknown state, and will not provide an ac source to the transformer, T1. The capacitors, C2 and C3, are used for blocking dc into the transformer. If the line driver can be assured of always getting a drive signal, then capacitors, C2 and C3, can be eliminated.

Summary

As you can see, the performance of this current transducer is adequate for most applications. The circuit is simple and does not require a lot of calibration. The circuit can be easily modified to handle other input currents. The line driver circuit can be designed and/or modified to become a self-contained oscillator or an oscillator can be simply added. The reason for operating at such a low frequency is because of the saturable reactors. The gate inductance of the saturable reactors is extremely high. This high inductance requires the operation frequency to keep well below the self-resonate frequency of the saturable reactors.

Recognition

I would like to give thanks to Charles Barnett, an engineer at Leightner Electronics Inc. for building and testing the 12watt, saturable reactor design example.

Leightner Electronics Inc.
1501 S. Tennessee St.
McKinney, TX. 75069

I would like to give thanks to Zack Cataldi, a Senior Applications Engineer at Magnetics for supplying the cores for the saturable reactor design example.

Magnetics
110 Delta Drive
Pittsburgh, PA 15238

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2. Lee, R., *Electronic Transformers and Circuits*, John Wiley & Sons, New York, 1958, pp. 259–291.

3. Platt, S., *Magnetic Amplifiers Theory and Application*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1958, Chapter 4.

4. National Aeronautics and Space Administration, New Technology, NPO-16888, Low Power 0-15 Amp Current Transducer.

Chapter 25

Self-Saturating, Magnetic Amplifiers

Table of Contents

Introduction

The self-saturating magnetic amplifier had great popularity in 1950 and 1960 like the saturable reactors, but when the transistor and silicon controlled rectifier came along, the magnetic amplifier nearly became obsolete. One of the early successes for the magnetic amplifier was power supplies for military aircraft operating at 400 Hz. The magnetic amplifier is still being used in motor control, power supplies. It also provides good electrical isolation between input and output. Magnetic amplifiers are inherently rugged, with a long life, and good reliability. Magnetic amplifiers are still being used in very remote locations. Only the operational and design highlights of the basic self-saturating, two-core magnetic amplifier will be discussed in this chapter. It would take a book to cover all of the attributes of a magnetic amplifier. A review of Chapter 2, "Magnetic Materials and Their Characteristics", will help one to understand how the magnetic core operates.

Self-Saturating, Magnetic Amplifier Overview

The basic self-saturating two-core magnetic amplifier with a controllable dc output is shown in Figure 25-1, and the two-core, self-saturating magnetic amplifier with a controllable ac output is shown in Figure 25-2. An overview description of the self-saturating magnetic amplifier is done with a resistive load, R_L.

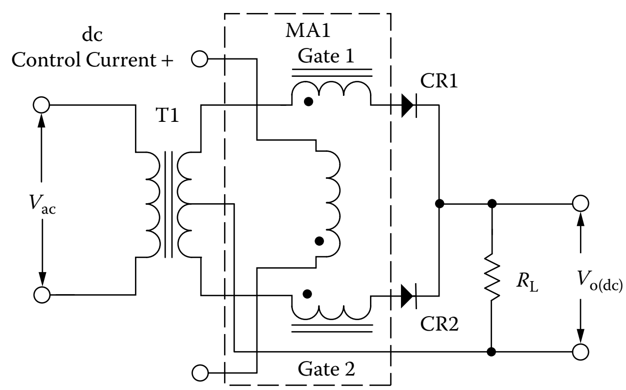

Figure 25-1. Magnetic Amplifier Circuit with a Controllable dc Voltage Output.

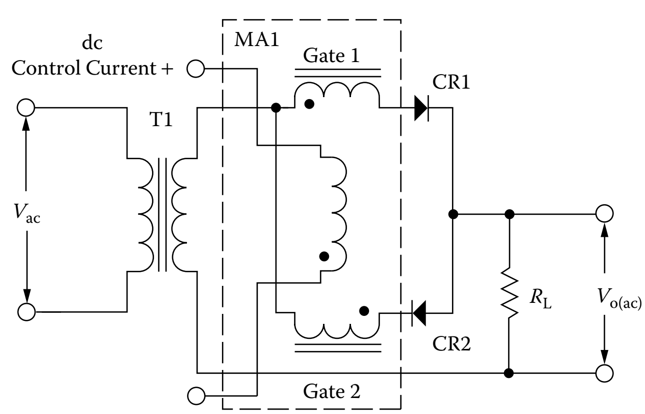

Figure 25-2. Magnetic Amplifier Circuit with a Controllable ac Voltage Output.

The performance and design complexity of a self-saturating magnetic amplifier design is based on the magnetic material used in the magnetic amplifier. The engineer can choose a magnetic material that has a square or round B-H loop, as shown in Figure 25-3 for the design. A good percentage of the magnetic amplifiers designed, will be designed on a toroidal core and have a magnetic material with a square loop and a high permeability at the operating conditions.

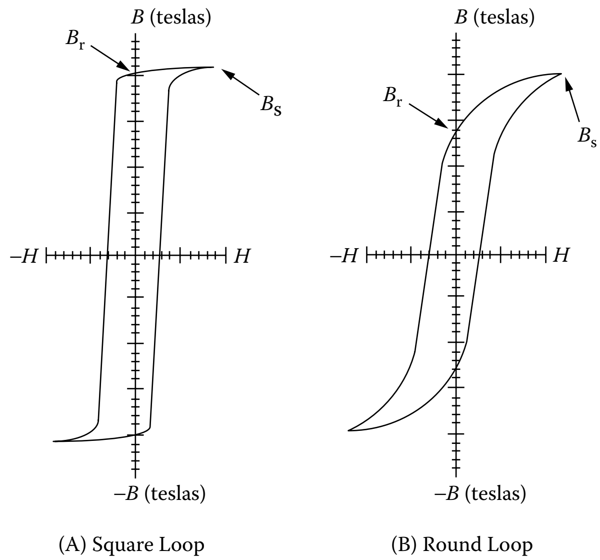

Figure 25-3. Comparing the (A) Square and (B) Round B-H Loop.

Basic Operation of the Self-Saturating, Mag-Amp

The core material and configuration used in the design of a self-saturating magnetic amplifier controls the overall performance of the magnetic amplifier. The gate from a magnetic amplifier is shown in Figure 25-4A, with the ac excitation from the transformer, T1 into a resistance load, R_L.

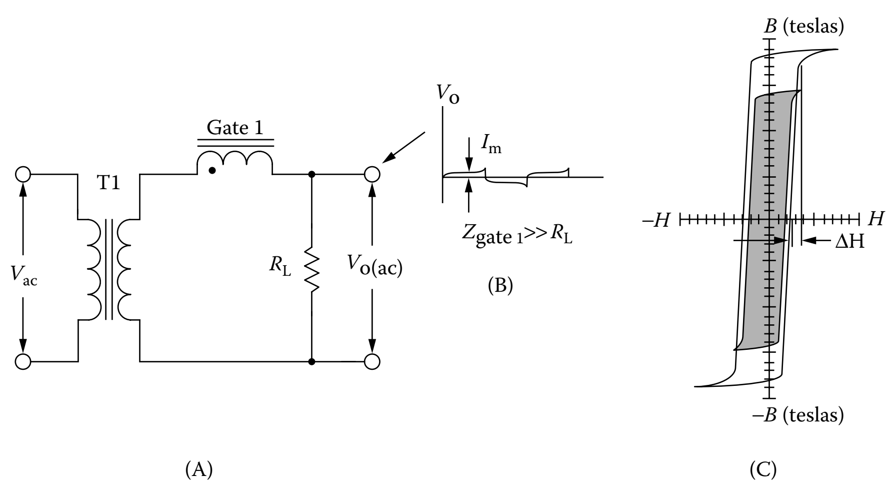

Figure 25-4. Magnetic Amplifier Gate Being Excited, Having a Square B-H Loop.

The core configuration for Gate-1, shown in Figure 25-4A, is a toroid. The magnetic material is a square loop with high permeability. The impedance of the gate, $Z_{(gate)}$, is much greater than the load resistance, R_L ($Z_{(gate)}$ >> R_L). The magnetizing current, shown in Figure 25-4B, is that of a magnetic material with a square B-H loop. In Figure 25-C, note how the magnetizing current follows the outline of the B-H loop.

The ΔH, shown in Figure 25-4C, is the magnetizing force required to drive the core into saturation, or out of saturation. The magnetizing force, H, can be in amp-turns, as shown in Equation [25-1], or oersteds, as shown in Equation [25-2]. The magnetizing force, H, is different for all magnetic materials, and it will change with frequency and material thickness. To change amp-turns/meter to oersteds, multiply (amp-turns × 0.01256) and to change oersteds to amp-turns/meter, divide (oersteds/0.01256).

$$H = \frac{NI}{(\mathrm{MPL})}, \quad \text{[amp-turns/meter]} \qquad [25\text{-}1]$$

$$H = \frac{0.4\pi NI}{(\mathrm{MPL})}, \quad \text{[oersteds]} \qquad [25\text{-}2]$$

The basic two-core self-saturating magnetic amplifier is shown in Figure 25-5. The self-saturating magnetic amplifier gets its name because the rectifier diodes, CR1 and CR2. The diodes are in series with the gates and put a unidirectional current into Gates 1 and 2 of the magnetic amplifier, which drive the cores to saturation. The output of the magnetic amplifier, MA1, can be seen in Figure 25-5B. With the Gates 1 and 2 saturated, the output is that of a center-tap, full wave rectifier. The dc current flowing into the Gates 1 and 2 drives the cores into saturation, as shown in Figure 25-5C.

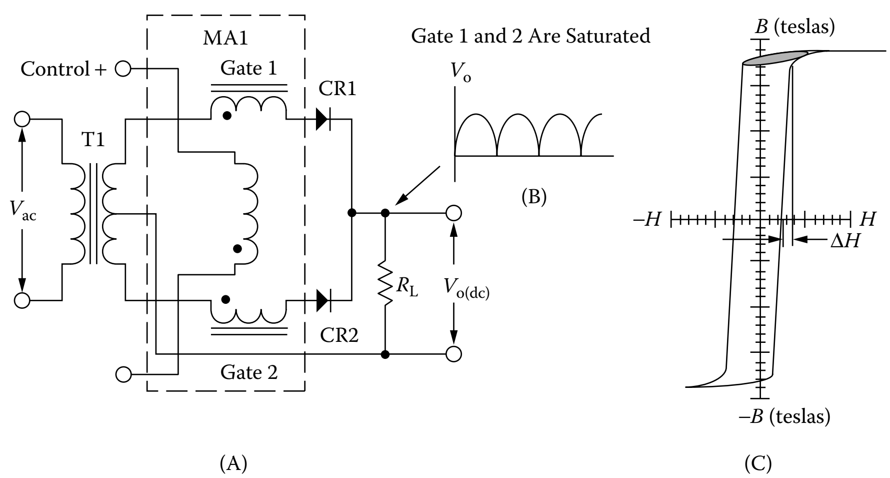

Figure 25-5. Two-Core, Self-Saturating Magnetic Amplifier with dc Output.

To make the gates operate as a variable switch, a current must be applied to the control winding in reverse to the amp-turns in the gate winding. The current in the gate winding is flowing into the dot, as shown in Figure 25-5A. The current in the control winding must have the current flowing out of the dot. It can be seen in Figure 25-6 as the control magnetizing current is increased from core saturation in Figure 25-6A to a 50 percent duty cycle in

Figure 25-6B, and then increases even further to where the gate is absorbing the total applied voltage in Figure 25-6C. This can only be done if the gates are designed to support the full voltage from the secondary of T1. There are some magnetic amplifier design circuits where the gate does not support the total secondary voltage.

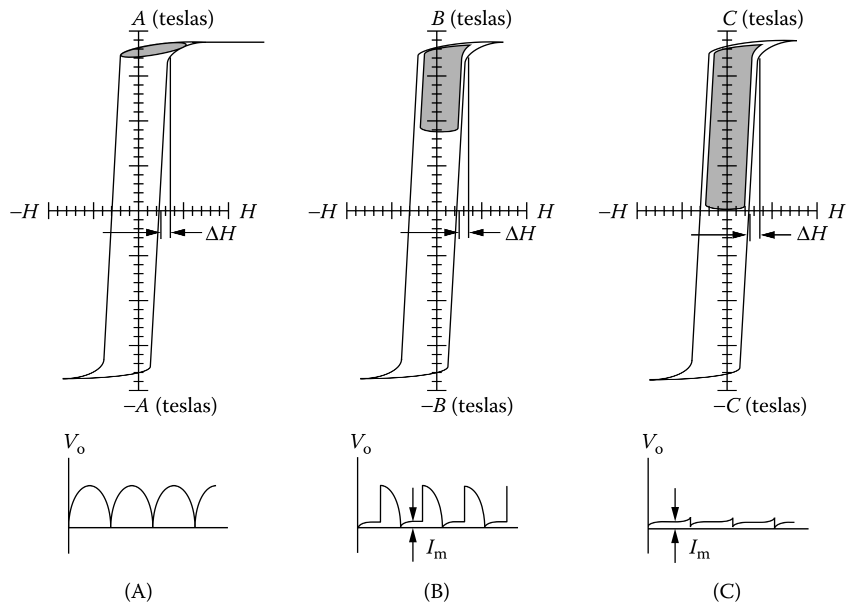

Figure 25-6. Two-Core, Self-Saturating Magnetic Amplifier with dc Output.

Square and Round B-H Loop Performance

The functional difference between the square B-H loop and the round B-H loop can best be shown by looking at the output voltage waveform across the load resistor, R_L with the control winding open (no dc current). This condition is best illustrated when the self-saturating magnetic amplifier is driven by a square wave, as shown in Figure 25-7.

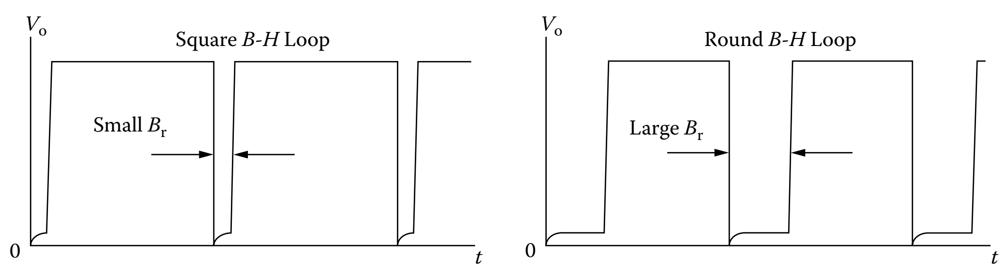

Figure 25-7. Comparing the Performance of Round and Square B-H Loop Magnetic Materials.

It can be seen in Figure 25-7 that the round magnetic material has a large gap between half cycles. What happens is that every time the transformer, T1 switches, (See Figure 25-1), diode CR1 turns off and diode, CR2 turns on. This cycle causes the flux in Gate 1 to fall back to residual flux, B_r, and then, on the next half cycle, the flux starting point will be at the residual flux, B_r, as shown in Figure 25-3. Round or square B-H loop magnetic materials will always fall back to the residual flux, B_r, when the excitation is removed. It is easy to see that the round loop material is not being used to its full flux capacity. There are times when round loop magnetic material is a requirement because of the cost or the other assets of the material.

Adding the Bias Winding

If the round loop magnetic material is the choice then the B-H loop will have to be shifted over to get the full core capability. A typical transfer function of a self-saturating magnetic amplifier is shown in Figure 25-8, showing magnetic materials with both square and round B-H loops.

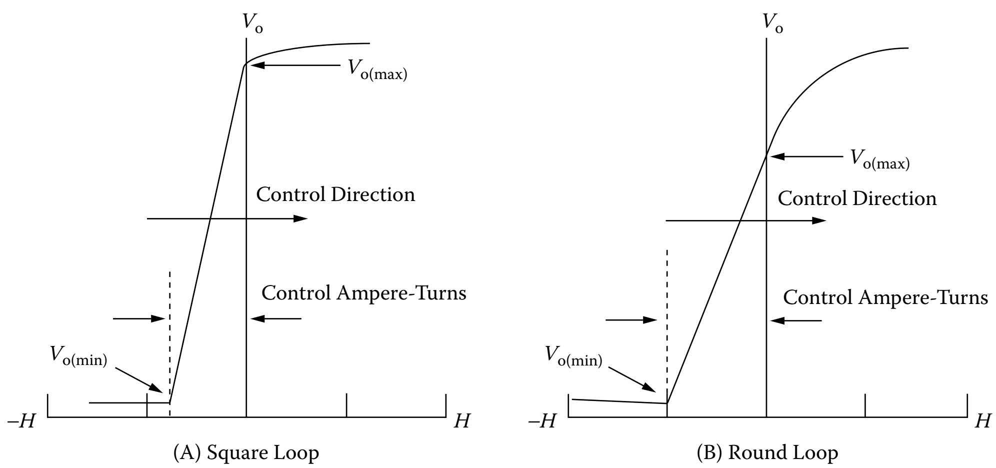

Figure 25-8. Comparing Transfer Function for Both Round and Square Magnetic Material.

As seen in Figure 25-8 the control amp-turns can bias the core, as shown by the arrow in the control direction, to get the output voltage down to minimum, $V_{o(min)}$. A bias winding can be added to the magnetic amplifier, shown in Figure 25-1, to bias the core with amp-turns in the opposite direction to the control amp-turns, as shown in Figure 25-9.

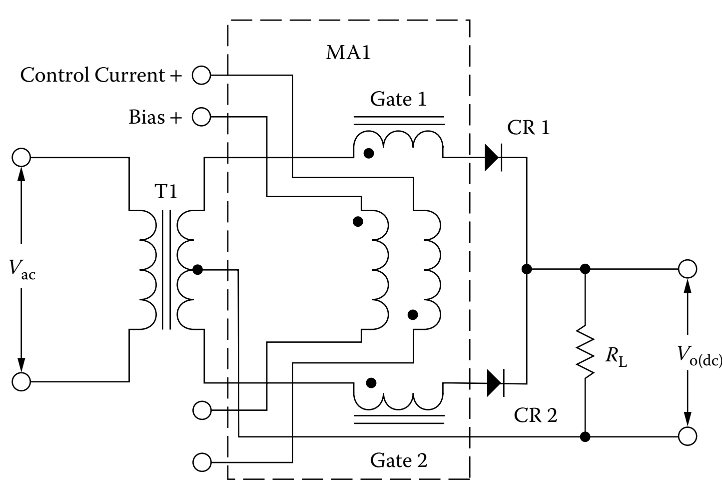

Figure 25-9. A Magnetic Amplifier Showing a Control and Bias Winding.

The new bias winding amp-turns will bias the gates in the same direction as the rectifiers, CR1 and CR2. The output load current travels into the start (dot) of the gate windings. The bias winding will also have the current travel into the start (dot). The current in the bias winding will shift both cores and hold the flux even though the switching load current has been removed from CR1 and CR2 by transformer action. The control current now controls the full range of the output voltage from minimum, $V_{o(min)}$ to maximum, $V_{o(max)}$ and uses the full capacity of the core, as shown in Figure 25-10. Comparing the square B-H loop in Figure 25-8A with the biased round B-H loop in Figure 25-10B, both have the same output voltage range, but the round B-H loop material requires more control current, I_c. This is because the control current, I_c, now has overcome the bias amp-turns to get the same output voltage, V_o, range.

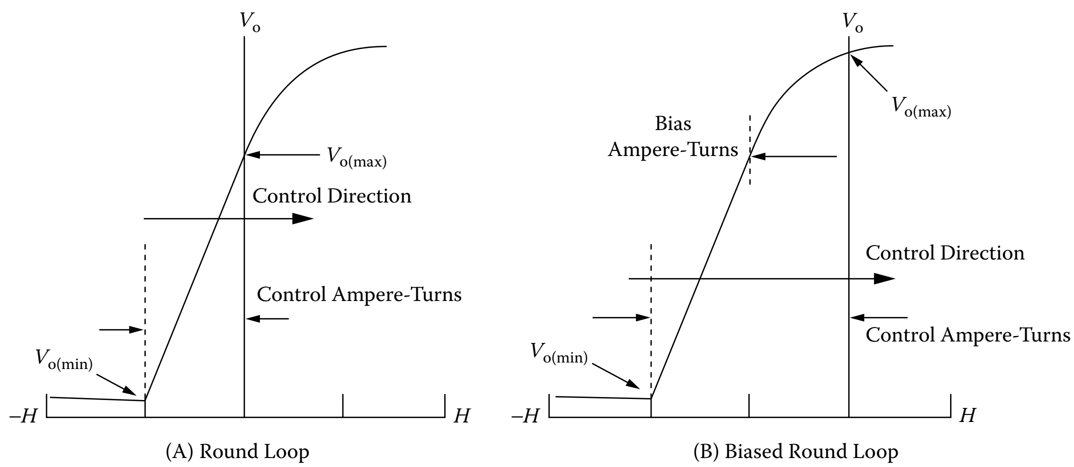

Figure 25-10. Showing How the B-H Loop can be shifted with the Bias Winding.

Control Winding and Rectifiers

The window area of the magnetic amplifier, MA1, has to accommodate both power and control windings. The rectifiers have a big impact on the control winding. Back in the 50's and 60's the mag-amp engineer had a lot of problems just with the diode rectifiers, CR1 and CR2, as shown in Figure 25-9. The rectifiers that were available at this time were selenium, germanium, and the new silicon. At that time the forward voltage drop and the reverse leakage were not tightly controlled. The semiconductor manufacturers were just beginning to get their process down. The problem caused by the reverse leakage current in the rectifiers is shown in Figure 25-11. The reverse leakage current would reset the core with amp-turns every half cycle. To be able to get full use of the core, a bias winding would have to be added to overcome this reverse rectifier leakage current. The rectifiers have to be well-balanced in both forward and reverse directions to keep the ac, unbalanced noise generated in the control winding to a minimum. Early self-saturating magnetic amplifiers required a lot of room in the window for the control and bias windings, because of the loss of gain caused by the reverse leakage current.

As time went on the manufacturers of semiconductors got their processes down and produced rectifiers with very tight specification for both forward and reverse conditions. A well-balanced magnetic amplifier has a minimum

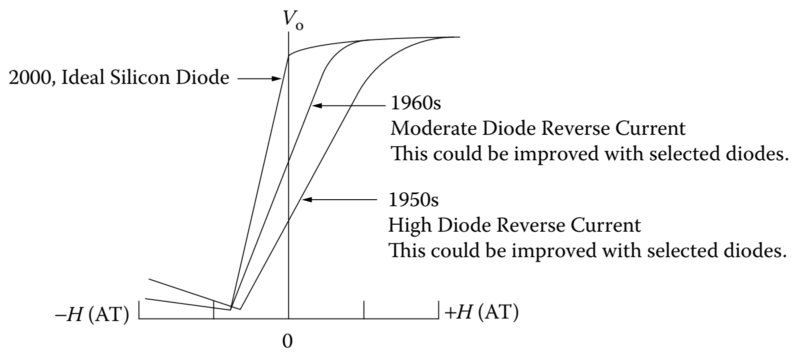

Figure 25-11. How the Transfer Function can be shifted with Reverse Diode Leakage.

of disturbance in the control winding. The engineers would use a transistor to drive the magnetic amplifier and would not depend on the gain of the mag-amp itself. This reduced the number of turns required for the control. The window area required for the control winding is up to the engineer on how to incorporate the control.

Self-Saturating Magnetic Amplifier Apparent Power, P_t

The designer must be concerned with the apparent power, P_t, and power handling capability of the mag-amp core and windings, when selecting the correct core. The mag-amp has two gate windings and a control winding and could include a bias winding, as shown in Figure 25-12. For more information on apparent, power, P_t, and power-handling capability, see Chapters 7 and 21. Each gate of the self-saturating magnetic amplifier, MA1, is designed to support the full output voltage, V_o. The current, I_g, in each gate is interrupted every half cycle. This affects the RMS value and changes the volt-amps, VA, of the gate winding, as shown in Equation [25-3] and Equation [25-4]. The apparent power, P_t, for the two gate windings is the same as the apparent power, P_t for the transformer, T1 secondary.

$$I_{g(rms)} = 0.707\left(I_o\right), \quad [25\text{-}3]$$

$$VA_{(gates)} = P_o\sqrt{2}, \quad [25\text{-}4]$$

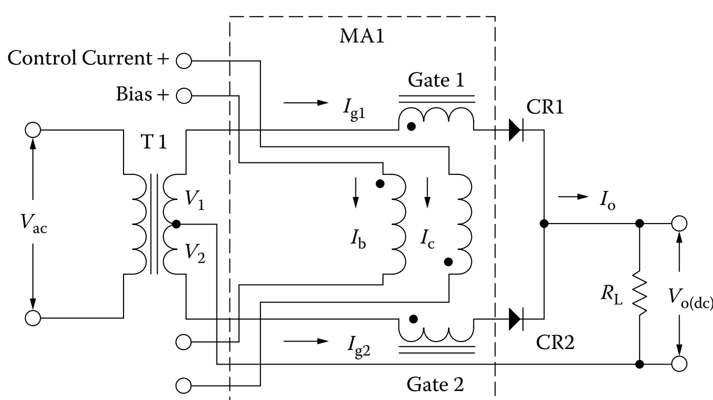

Figure 25-12. Self-Saturating Magnetic Amplifier Showing Gate and Control Current.

There must be area in the window assigned to the control and bias windings when a core is selected. The Equation for the apparent power, P_t, for a transformer is shown in Equation [25-5]. The Equation [25-5] for the transformer apparent power, P_t can be simplified to Equation [25-6].

$$P_t = \text{Primary Power} + \text{Secondary Power}, \quad [\text{watts}] \qquad [25\text{-}5]$$

$$P_t = P_{in} + P_o \quad [\text{watts}] \qquad [25\text{-}6]$$

The apparent power, P_t, Equation [25-6] can be modified for the magnetic amplifier by substituting the input power, P_{in}, with the gate volt-amps, $VA_{(gates)}$. The control and bias windings coefficient, K_{cw}, can be substituted for the output power, P_o. The new apparent power, P_t, for a magnetic amplifier is shown in Equation [25-7].

$$P_t = P_o\left(\left(\sqrt{2}\right)_{(gate)} + K_{cw}\right), \quad [\text{watts}] \qquad [25\text{-}7]$$

The engineer can adjust the apparent power, P_t, to accommodate the control and bias windings by using the winding coefficient, K_{cw}. The window is divided up to where the gates get 60% and the control and bias windings will get 40% of the available winding area when the winding coefficient, $K_{cw} = 1$, as shown in Figure 25-13. This is a good starting point and not a hard fast rule. There will be many designs in which that ratio will change.

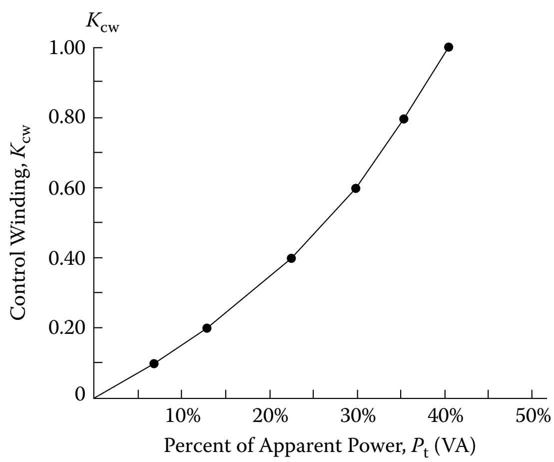

Figure 25-13. How the Control Winding Coefficient, K_{cw}, affects the apparent power, P_t, in percent.

The apparent power, P_t, calculated in Equation [25-8] is for each core of a magnetic amplifier using two cores, such as a toroid or a double stack DU lamination.

$$P_t = 0.5P_o\left(\sqrt{2}_{(gate)} + K_{cw}\right), \quad [\text{watts}] \qquad [25\text{-}8]$$

The electrical conditions, K_e, are shown in Equation [25-9].

$$K_e = 0.145K_f^2 f^2 B_m^2\left(10^{-4}\right) \qquad [25\text{-}9]$$

The calculation for the core geometry, K_g, is shown in Equation [25-10].

$$K_g = \frac{P_t}{2K_e\alpha}, \quad [\text{cm}^5] \qquad [25\text{-}10]$$

Note: Alpha, α, is the combined copper loss for the control and gate windings of the magnetic amplifier. The core calculation for the core geometry, K_g, is shown in Equation [25-11].

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \qquad [25\text{-}11]$$

Magnetic Amplifier Power Gain

One of the qualities of a magnetic amplifier is based on its power gain. The power gain is dependent to a great deal upon the design, type, and physical size of the magnetic amplifier. The power gain of a self-saturating magnetic amplifier is normally very high compared to a saturable reactor. Power gain of a magnetic amplifier is the ratio of the input control power, P_c to the output or load power, P_o. The input control power is the power dissipated in the control winding. The control power, P_c is the power dissipated in the control winding, as shown in Equation [25-12]. The control current is, I_c and the winding resistance is, R_c. The output power, P_o is the power delivered to the load, as shown in Equation [25-13]. The current flowing in the gate winding is, I_g and the load resistance is, R_L. The power gain then is the output power, P_o divided by the control power, P_c as shown in Equation [25-14].

$$P_c = I_c^2 R_c, \quad [\text{watts}]\ [\text{control}] \qquad [25\text{-}12]$$

$$P_o = I_o^2 R_L, \quad [\text{watts}]\ [\text{load}] \qquad [25\text{-}13]$$

$$P_{(gain)} = \frac{\left(I_o^2 R_L\right)}{\left(I_c^2 R_c\right)}, \quad [\text{power gain}] \qquad [25\text{-}14]$$

Self-Saturating Magnetic Amplifier Response Time

The response time, t_r for a magnetic amplifier is very slow for a step change in the control signal. This is because the control winding is very inductive. Any step change in current in the control winding will take time to reach its final value. The response time, t_r is given as a time constant of the control circuit. The response time is the time it takes for the load current to reach 63 percent of its final value for a step input change, as shown in Figure 25-14. It can be seen in Figure 25-14, when there is an input step change, t_1, in the control circuit. The output current, I_1, will rise from its initial value at, t_1, to its final value of, I_3, at the time, t_3. The value of I_2 is the current at 63 percent of the final current, I_3, which is one time constant. The time constant is shown in Equation [25-15].

$$t_r = \frac{L_c}{R_c}, \quad \text{[seconds]}$$

$t_r =$ [seconds], [time constant] [25-15]

$L_c =$ [henries], [control winding inductance]

$R_c =$ [ohms], [control winding resistance]

This concept was derived from linear systems. It yields sufficiently accurate results for most purposes.

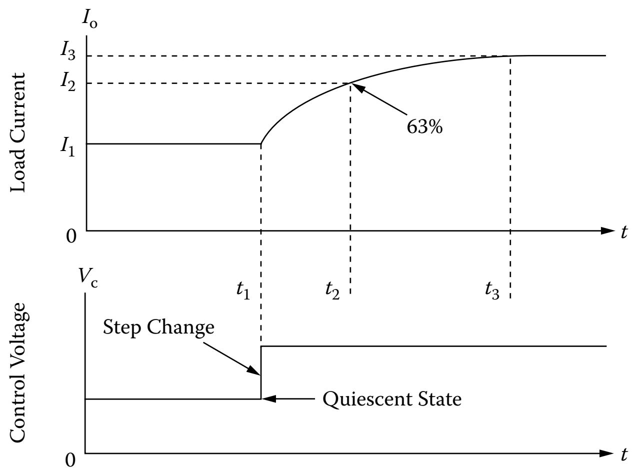

Figure 25-14. Output Response Time for a Input Signal Step Change.

Mean Length Turn for DU Laminations

The Mean Length Turn, (MLT), is required to calculate both winding resistance and weight for a given winding. Using the DU lamination in a magnetic amplifier design requires the calculation of the resistance of both control and gate windings. The winding dimensions, relating to the Mean Length Turn, (MLT) for a tube or bobbin coil, are shown in Figure 25-15. The Mean Length Turn, (MLT) for the control winding is shown in Equation [25-16] and the Mean Length Turn, (MLT) for the gate windings, is shown in Equation [25-17].

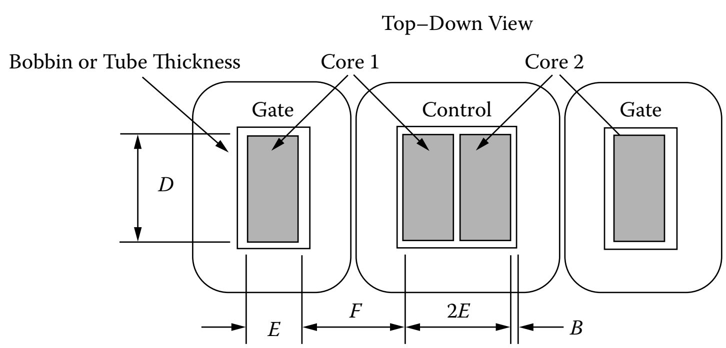

Figure 25-15. Dimensions, Relating to the Mean Length Turn, (MLT) for a DU Lamination.

$$\text{MLT}_{(control)} = 2(D+2B)+4(E+B)+\pi\left(\frac{F}{2}\right), \quad [\text{cm}] \qquad [25\text{-}16]$$

$$\text{MLT}_{(gate)} = 2(D+2B)+2(E+2B)+\pi\left(\frac{F}{2}\right), \quad [\text{cm}] \qquad [25\text{-}17]$$

Calculating, MLT for Toroidal Cores

In order to wind the toroidal core, there has to be room to allow free passage of the wire shuttle. The wire shuttle could be a machine shuttle or a hand-wind shuttle. Winding a magnetic amplifier is not any different than winding any other toroidal core. Half of the inside diameter will be set aside for the shuttle clearance. This will leave 75% of the window, W_a, for the windings. It is very difficult to calculate the Mean Length Turn, (MLT) for a toroidal core used in magnetic amplifier design that would satisfy all conditions. The magnetic amplifier has two cores and each core has its own gate winding. The gate windings are wound first. After the gate windings are wound the cores are tested and matched. Then they are assembled for the common control winding to be wound over both. The assembly of the toroidal cores can be seen in Figure 25-16. The fabrication of a toroidal design is weighted heavily on the skill of the winder. The toroidal core magnetic amplifier requires special attention, when wound by hand or wound by machine, because of the angle it imposes when two cores are stacked. The Mean Length Turn, (MLT) for a toroidal magnetic amplifier is very difficult to obtain, because its shape is very nonlinear. A good approximation for a toroidal core, Mean Length Turn (MLT) for the gate winding is Equation [25-18] and Equation [25-19] for the control winding that is shown in Figure 25-16. There is a correction factor of 0.85 for the gate winding and a correction factor of 0.70 for the control winding.

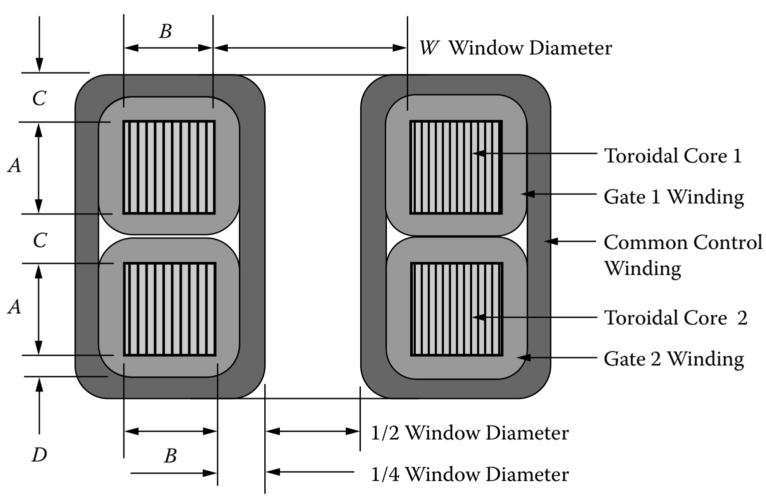

Figure 25-16. Toroidal Core Mag-Amp Mean Length Turn, (MLT) is a Good Approximation.

$$\text{MLT} = \left((2A)+(2B)+(\pi 0.125W)\right)(0.85), \quad \text{Approximation [Gate]} \qquad [25\text{-}18]$$

$$\text{MLT} = \left(4A+0.25W+2B+2\pi(0.375W)\right)(0.70), \quad \text{Approximation [Control]} \qquad [25\text{-}19]$$

Nomenclature clarification:

1. A = Core strip width including the case.
2. B = Core build including the case.
3. C = Is the build of both gate and control winding = W/4
4. D = Is the build of the gate, = W/8
5. W = Is the inside diameter of the core including the case.

Toroidal Magnetic Amplifier Surface Area

The data such as height and outside diameter, required for the surface area calculation was taken from Figure 25-16. The Height, Ht, of the magnetic amplifier is shown in Equation [25-20] and the Outside Diameter, OD, is shown in Equation [25-21]. The magnetic amplifier outline is shown in Figure 25-17.

$$Ht = (3C + 2A)$$

$$OD = (2C + 2B + W)$$

$$A = (\text{Core Height, Ht}),\ B = \left(\text{Core Build, } (\text{OD} - \text{ID})/2\right),\ C = \left(\text{Winding Build, } W/4 = ID/4\right)$$

$$MA_{Ht} = \left(3\left(\frac{ID}{4}\right) + 2Ht\right), \qquad [25\text{-}20]$$

$$MA_{OD} = \left(\left(\frac{ID}{2}\right) + (OD - ID) + ID\right), \qquad [25\text{-}21]$$

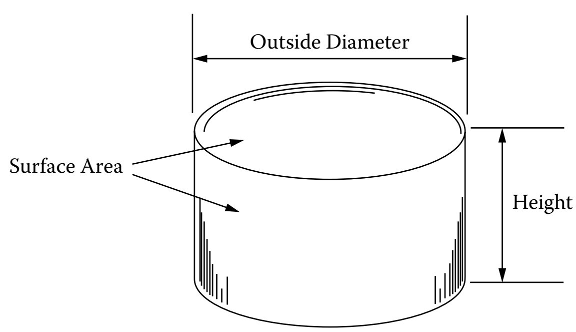

Figure 25-17. The Outline of a Toroidal Wound Magnetic Amplifier.

The surface area of a Toroidal Magnetic Amplifier can be calculated using Equation, [25-24].

$$\text{Top and Bottom Surface} = 2\left(\frac{\pi (MA_{OD})^2}{4}\right), \quad [\text{cm}^2], \qquad [25\text{-}22]$$

$$\text{Periphery Surface} = \left(\pi\left(MA_{OD}\right)\right)\left(MA_{\text{Ht}}\right), \quad [\text{cm}^2], \qquad [25\text{-}23]$$

$$A_t = \frac{\pi\left(MA_{OD}\right)^2}{2} + \left(\pi\left(MA_{OD}\right)\right)\left(MA_{\text{Ht}}\right), \quad [\text{cm}^2] \qquad [25\text{-}24]$$

DU Lamination Magnetic Amplifier Surface Area

The data for height, width and length required for the surface area calculation was taken from Chapter 3. The height, Ht, is (H + G), the width, is (D + F), and length is (4E + 3F). The magnetic amplifier outline is shown in Figure 25-18. The surface area for a DU lamination magnetic amplifier can be calculated using Equation, [24-28].

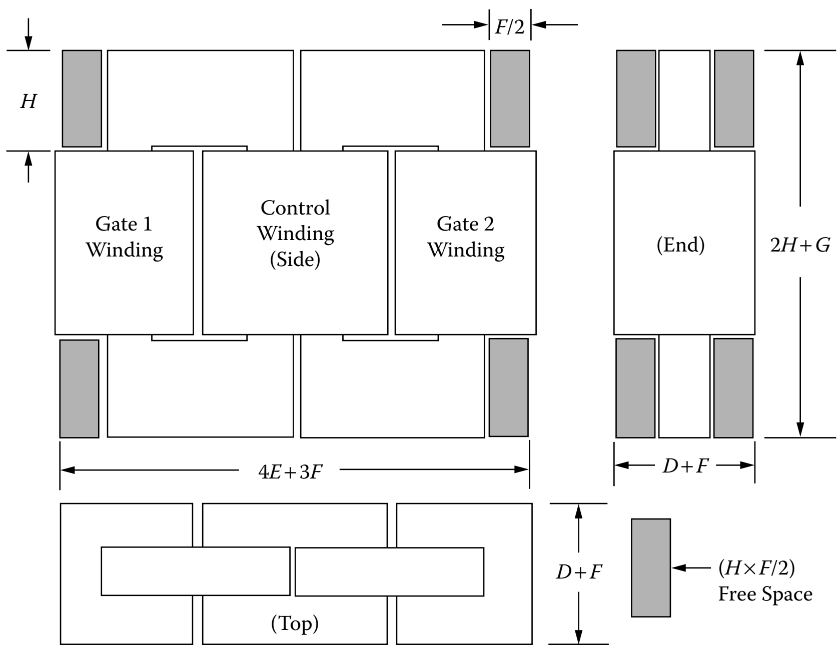

Figure 25-18. The Outline of a DU Lamination Core Wound Magnetic Amplifier.

$$\text{Side Area} = (4E + 3F)(H + G) - 2(HF), \quad [\text{cm}^2] \qquad [25\text{-}25]$$

$$\text{End Area} = (2H + G)(D + F) - 2(HF), \quad [\text{cm}^2] \qquad [25\text{-}26]$$

$$\text{Top Area} = (4E + 3F)(D + F), \quad [\text{cm}^2] \qquad [25\text{-}27]$$

$$A_t = 2(\text{Side Area}) + 2(\text{End Area}) + 2(\text{Top Area}), \quad [\text{cm}^2] \qquad [25\text{-}28]$$

Control Winding Calculation

The control winding must have enough amp-turns to pull the gates out of saturation to where the gates are supporting the total applied voltage, as shown in Figure 25-6. Each magnetic material type has its own electrical characteristic. The magnetic materials that are used in laminations and tape cores are supplied in different thicknesses, so the engineer can design and get best performance at the operating frequency. See Chapter 3. The area within, the hysteresis loop are losses. As the frequency is increased the eddy current loss in the magnetic material increases, and the hysteresis loop gets wider as the losses go up, as shown in Figure 25-19.

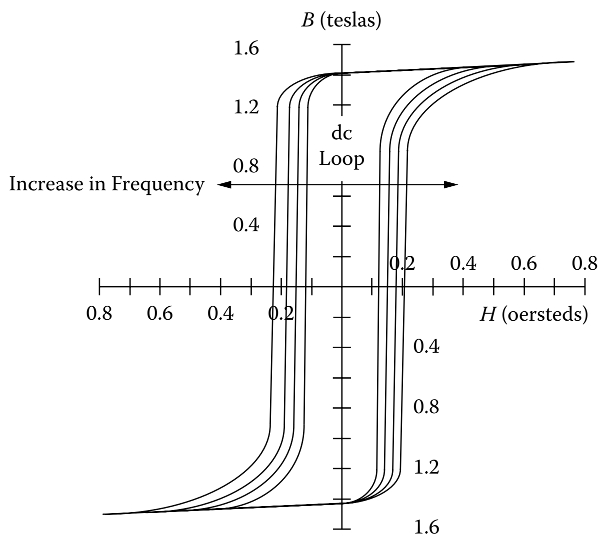

Figure 25-19. Showing How the B-H Loop Changes with Frequency.

The widening of the hysteresis loop has to be taken into consideration when calculating the control turns. When calculating the number of turns required for a control current of 5 ma and a Magnetic Path Length, (MPL), of 9.5 cm, use Figure 25-20 as a guide. The value for A in Figure 25-20 is 0.26 oersteds. The tolerance set by the manufacturers for the magnetizing force (oersteds) is +/–25%. The turns can be calculated using Equation [25-29] for oersteds and Equation [25-30] for amp-turns per meter.

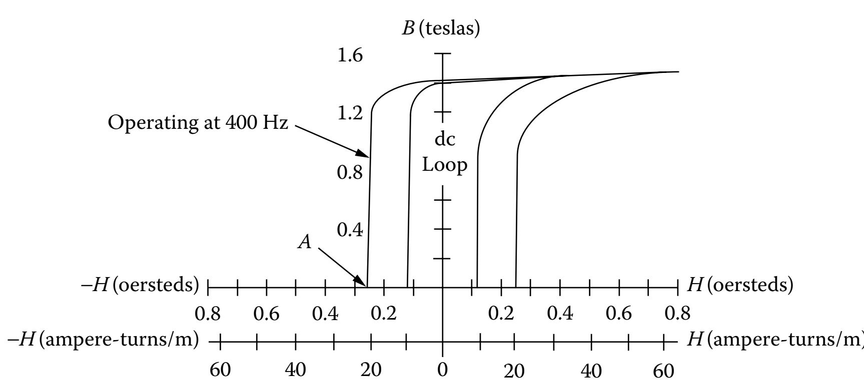

Figure 25-20. The Required Magnetizing Force, A, Operating at 400 Hertz.

$$N = \frac{H(\text{MPL})}{0.4\pi I_c}, \quad \text{[turns]}$$

$$N = \frac{(0.26)(9.5)}{(0.4)(3.415)(0.005)}, \quad \text{[turns]} \qquad [25\text{-}29]$$

$$N = 393, \quad \text{[turns]}$$

$$N = \frac{H(\text{MPL})\left(10^{-2}\right)}{I_c}, \quad \text{[turns]}$$

$$N = \frac{(20.7)(9.5)\left(10^{-2}\right)}{0.005} \qquad [25\text{-}30]$$

$$N = 393, \quad \text{[turns]}$$

Bias Winding Calculation

The bias winding must have enough amp-turns to shift the B-H loop over to get the full core capability, as shown in Figure 25-21. The magnetic material, shown in Figure 25-22, is Square Permalloy 80 in a toroidal configuration. Square Permalloy 80 is considered a square loop material even though it is a little rounded at the top. Although a bias winding could improve its performance, it is not considered mandatory.

Normally the round B-H loop is caused by laminations that introduce a small amount of air-gap. Even a small amount of air-gap will shear the B-H loop over, and this produces the round B-H loop. The amount rounding or shearing of the B-H loop is hard to determine. If the manufacturer does not have the curves or data, it might be wise to wind a sample unit to get the actual data. Then, take a look at the output voltages as shown in Figure 25-7. This will show what is really required and what will be gained with the bias winding. There is another way and that is to look at the B-H loop. Looking at the B-H loop can be done by using one of the cores

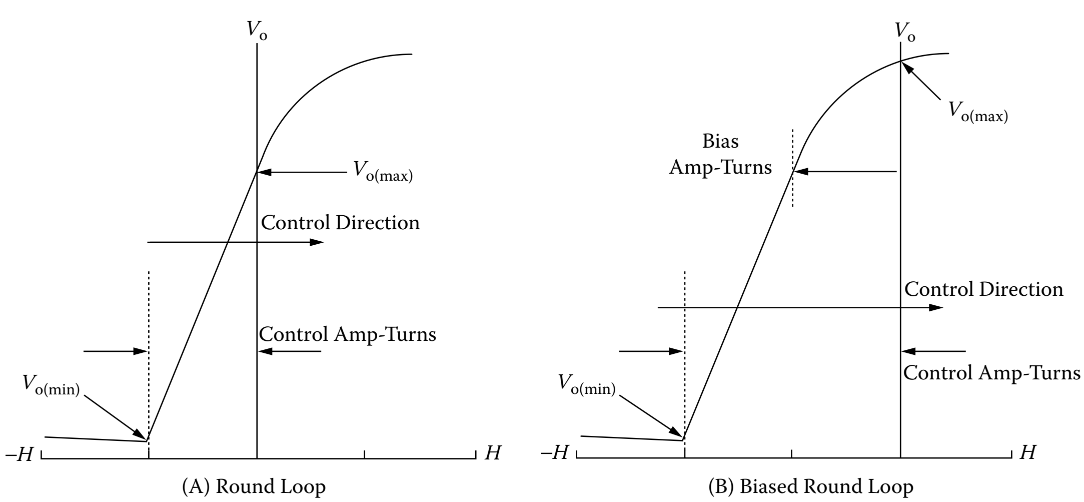

Figure 25-21. Showing How the Bias Amp-Turns Shifts the B-H Loop.

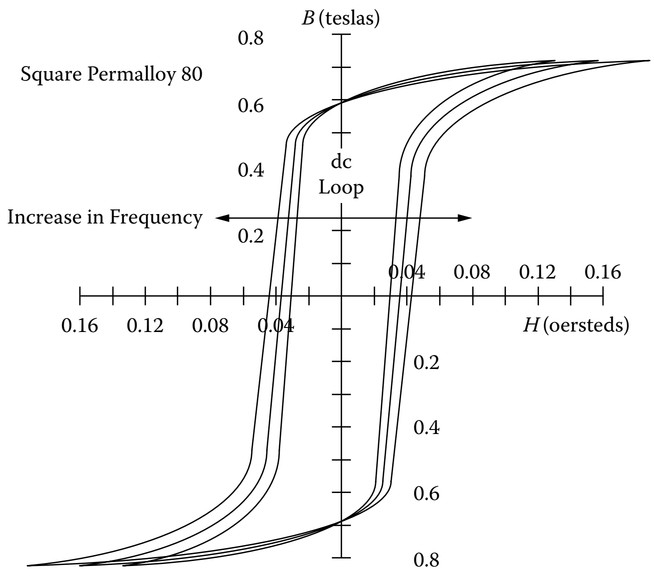

Figure 25-22. Square Permalloy 80 B-H Loop with Increasing Frequency.

and putting on a few turns to see the rounding or shearing over. See Chapter 2. If the self-saturating magnetic amplifier is connected, as shown in Figure 25-12, then the bias current goes into the dot. The amp-turn bias required to shift the B-H loop is shown in Equation [25-31].

$$H = \frac{NI}{\text{MPL}}, \quad \text{[amp-turns/meter]} \qquad [25\text{-}31]$$

Control Winding Precautions

If good design procedures are used, where the cores are matched and high quality diodes are used, the fundamental a-c voltage induced in the control winding by the gates will cancel each other. There could easily be a high voltage induced in the control winding that could require very special attention. This high voltage condition could easily happen if the voltage on the gate was high and there was a large turns ratio between the gate and control winding, as shown in Equation [25-32]. It would be wise to take this precaution taking steps to insulate the control winding to minimize the chances of voltage breakdown. There are conditions that can happen that could be unforeseen at the time, such as wiring error, phase reversing and or a bad component.

$$V_c = \frac{N_c V_g}{N_g}, \quad \text{[volts]} \qquad [25\text{-}32]$$

Self-Saturating Magnetic Amplifier Design Example

The design example will just be the magnetic amplifier used to control and regulate a 12.0 volt at 1.0 amp power supply. The magnetic amplifier test schematic is shown in Figure 25-23. The design will operate from a 14 volt, 400 Hz power source. The core configuration will be toroidal in shape. The cores are matched using the Constant Current Flux Reset test method. A description of this method can be found in Magnetics manual, "Tape Wound Cores Design Manual TWC-500". The magnetic material will be 50-50 Ni-Fe with a material thickness of 4 mils. The B-H loop for this magnetic material is shown in Figure 25-24. The B-H loop is very square and will not require a bias winding. The magnetic amplifier control current, I_c, is 3 ma with the current going out of the dot. The winding coefficient is $K_{cw} = 1$. In this way the window will be divided up to where the gates get 60% and the control winding will get 40% of the available winding area, as shown in Figure 25-13.

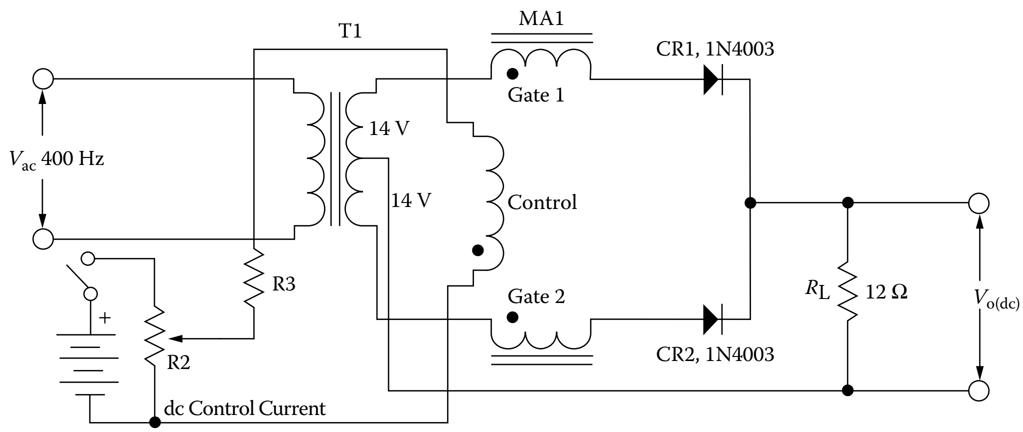

Figure 25-23. Schematic of a Self-Saturating Magnetic Amplifier Test Circuit.

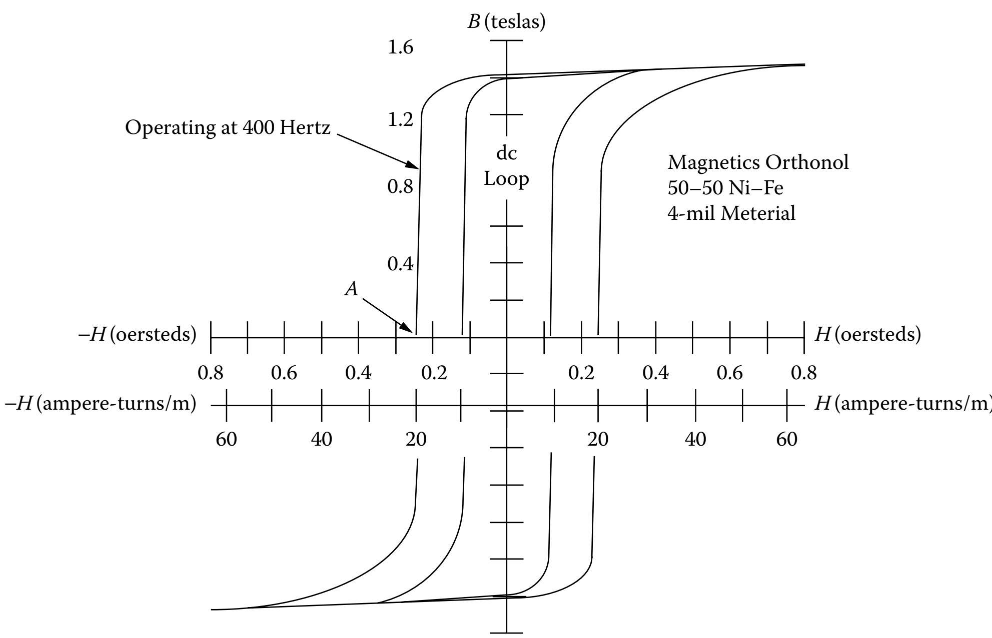

Figure 25-24. Showing the B-H Loop for 50-50 Ni-Fe 4 mil Magnetic Material.

Specification and Design

1. Transformer, T1 Output Voltage, V_t = 14 volts ac
2. Operating Frequency = 400 Hz
3. Load voltage, V_o = 12.0 volts dc
4. Load Current, I_o = 1.0 amps
5. Magnetic Core Type = Toroid
6. Magnetic Material, 4 mil = 50-50 NiFe
7. Efficiency, η = 95%
8. Regulation (copper loss), α = 5%
9. Operating Flux Density, B_{ac} = 1.1T
10. Control Current, I_c = 0.003 amps
11. Magnetizing Force, H = 0.26 oersteds
12. Control Winding Coefficient, K_{cw} = 1.0

Step No. 1: Calculate the load power, P_o.

$$P_o = V_o I_o, \quad \text{[watts]}$$

$$P_o = (12.0)(1.0), \quad \text{[watts]}$$

$$P_o = 12, \quad \text{[watts]}$$

Step No. 2: Calculate the magnetic amplifier, MA1 apparent power, P_t, for a single core. Let, K_{cw} = 1.

$$P_t = (0.5) P_o \left(\sqrt{2_{(gate)}} + K_{cw}\right), \quad \text{[watts]}$$

$$P_t = (0.5)(12)(1.41 + 1), \quad \text{[watts]}$$

$$P_t = 14.5, \quad \text{[watts]}$$

Step No. 3: Calculate the electrical conditions, K_e.

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right)$$

$$K_f = 4.44$$

$$K_e = 0.145(4.44)^2 (400)^2 (1.1)^2 \left(10^{-4}\right)$$

$$K_e = 55.3$$

Step No. 4: Calculate the core geometry, K_g.

$$K_g = \frac{P_t}{2 K_e \alpha}, \quad \text{[cm}^5\text{]}$$

$$K_g = \frac{14.5}{2(55.3)(5)}, \quad \text{[cm}^5\text{]}$$

$$K_g = 0.0262, \quad \text{[cm}^5\text{]}$$

Step No. 5: This core was selected from Magnetics Tape Wound Cores catalog, TWC 500, comparable to core geometry, K_g.

1. Toroid = 52029-4A
2. Manufacturer = Magnetics
3. OD Dimension = 3.78 cm
4. ID Dimension = 2.25 cm
5. Ht Dimension = 0.978 cm
6. Core Geometry, K_g = 0.0256 cm^5
7. Area Product, A_p = 1.08 cm^4
8. Core Weight, W_t = 19.66 grams
9. Iron Area, A_c = 0.272 cm^2
10. Window Area, W_a = 3.97 cm^2
11. Magnetic Path Length, MPL = 9.47 cm
12. Magnetic Material, 4 mil = Orthonol

Step No. 6: Calculate the window area for both gate and control winding.

$$W_{ag} = W_a(0.60) = (3.97)(0.60) = 2.38, \quad [\text{cm}^2] \ [\text{Gate}]$$

$$W_{ac} = W_a(0.40) = (3.97)(0.40) = 1.59, \quad [\text{cm}^2] \ [\text{Control}]$$

Step No. 7: Calculate the number of turns for the gate winding, N_g.

$$N_g = \frac{(V_t - V_d)(10^4)}{K_f B_{ac} f A_c}, \quad [\text{turns}]$$

$$N_g = \frac{(14-1)(10^4)}{(4.44)(1.1)(400)(0.272)}, \quad [\text{turns}]$$

$$N_g = 245, \quad [\text{turns}]$$

Step No. 8: Calculate the gate rms current, $I_{g(rms)}$.

$$I_{g(rms)} = (0.707)I_o, \quad [\text{amps}]$$

$$I_{g(rms)} = (0.707)(1.0), \quad [\text{amps}]$$

$$I_{g(rms)} = (0.707), \quad [\text{amps}]$$

Step No. 9: Calculate the gate wire area, A_{wg}.

$$A_{wg} = \frac{K_u W_{ag}}{N_g}$$

$$A_{wg} = \frac{(0.4)(2.38)}{245}$$

$$A_{wg} = 0.00389$$

Step No. 10: Select the wire from the Wire Table in Chapter 4.

$$AWG = 22$$

$$A_{w(B)} = 0.00324, \quad [\text{cm}^2]$$

$$A_{wg} = 0.00386, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{cm}\right) = 531, \quad [\text{micro-ohm/cm}]$$

Step No. 11: Calculate the Mean Length Turn, (MLT) for the gate. Use Figure 25-16 for a reference.

A = Core with case Ht Dimension = 0.978, [cm]

B = Core with case Build = (OD − ID)/2 = 0.765, [cm]

W = Core with case Inside Diameter = 2.25, [cm]

$$\text{MLT} = (2A + 2B + \pi 0.125W)(0.85), \quad [\text{cm}]$$

$$\text{MLT} = \big(2(0.978) + 2(0.765) + \pi 0.125(2.25)\big)(0.85), \quad [\text{cm}]$$

$$\text{MLT} = 3.71, \quad [\text{cm}]$$

Step No. 12: Calculate the gate winding resistance, R_g.

$$R_g = \text{MLT}\left(N_g\right)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_g = 3.71(245)(531)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_g = 0.483, \quad [\text{ohms}]$$

Step No. 13: Calculate the gate winding copper loss, P_g.

$$P_g = I_g^2 R_g, \quad [\text{watts}]$$

$$P_g = (0.707)^2(0.483), \quad [\text{watts}]$$

$$P_g = 0.241, \quad [\text{watts}]$$

Step No. 14: Calculate the required control turns, N_c.

$$N_c = \left(\frac{H(\text{MPL})}{0.4\pi I_c}\right), \quad [\text{turns}]$$

$$N_c = \left(\frac{(0.260)(9.47)}{(1.256)(0.003)}\right), \quad [\text{turns}]$$

$$N_c = 653 \text{ use } 660, \quad [\text{turns}]$$

Step No. 15: Calculate the control wire area, A_{wc}.

$$A_{wc} = \frac{K_u W_{ac}}{N_c}$$

$$A_{wc} = \frac{(0.4)(1.59)}{660}$$

$$A_{wc} = 0.000964$$

Step No. 16: Select the wire from the Wire Table in Chapter 4.

$$AWG = 28$$

$$A_{wc(B)} = 0.000805, \quad [\text{cm}^2]$$

$$A_{wc} = 0.00105, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{cm}\right) = 2142, \quad [\text{micro-ohm/cm}]$$

Step No. 17: Calculate the Mean Length Turn, (MLT) for the control winding. Use Figure 25-16 for reference.

A = Core with case Ht Dimension = 0.978

B = Core with case Build = (OD − ID)/2 = 0.765

W = Core with case Inside Diameter = 2.25

$$\text{MLT} = \left((4A + 0.25W) + (2B) + 2\pi(0.375W)\right)(0.70), \quad [\text{cm}]$$

$$\text{MLT} = \left(4(0.978) + 0.25(2.25)\right) + \left(2(0.765)\right) + 2\pi\left(0.375(2.25)\right)(0.70), \quad [\text{cm}]$$

$$\text{MLT} = 7.91, \quad [\text{cm}]$$

Step No. 18: Calculate the control winding resistance, R_c.

$$R_c = \text{MLT}(N_c)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_c = 7.91(660)(2142)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R_c = 11.2, \quad [\text{ohms}]$$

Step No. 19: Calculate the control winding copper loss, P_c.

$$P_c = I_c^2 R_c, \quad [\text{watts}]$$

$$P_c = (0.003)^2(11.2), \quad [\text{watts}]$$

$$P_c = 0.0001, \quad [\text{watts}]$$

Step No. 20: Calculate the total copper loss for the gates and control, P_{cu}.

$$P_{cu} = P_{g1} + P_{g2} + P_c, \quad \text{[watts]}$$

$$P_{cu} = (0.241) + (0.241) + (0.0001), \quad \text{[watts]}$$

$$P_{cu} = 0.482, \quad \text{[watts]}$$

Step No. 21: Calculate the percent of total copper loss, α.

$$\alpha = \frac{P_{cu}}{P_o}(100), \quad [\%]$$

$$\alpha = \frac{(0.482)}{(12.0)}(100), \quad [\%]$$

$$\alpha = 4.02, \quad [\%]$$

Step No. 22: Calculate the watts per kilogram, W/K. Use the equation for this material in Chapter 2.

$$W/K = 0.000618(f)^{1.48}(B_{ac})^{1.44}$$

$$W/K = 0.000618(400)^{1.48}(1.1)^{1.44}$$

$$W/K = 5.03$$

Step No. 23: Calculate the core loss, P_{fe}.

$$P_{fe} = (W/K)2(W_{tfe})(10^{-3}), \quad \text{[watts]}$$

$$P_{fe} = (5.03)2(19.7)(10^{-3}), \quad \text{[watts]}$$

$$P_{fe} = 0.198, \quad \text{[watts]}$$

Step No. 24: Calculate the total loss, P_{Σ}.

$$P_{\Sigma} = P_{cu} + P_{fe}, \quad \text{[watts]}$$

$$P_{\Sigma} = (0.482) + (0.198), \quad \text{[watts]}$$

$$P_{\Sigma} = 0.68, \quad \text{[watts]}$$

Step No. 25: Calculate the magnetic amplifier height, Ht, using Figure 25-17 for reference, See Chapter 5.

$$MA_{Ht} = \left(3\left(\frac{ID}{4}\right) + 2Ht\right), \quad \text{[cm]}$$

$$MA_{Ht} = \left(3\left(\frac{2.25}{4}\right) + 2(0.978)\right), \quad \text{[cm]}$$

$$MA_{Ht} = 3.64, \quad \text{[cm]}$$

Step No. 26: Calculate the magnetic amplifier outside diameter, OD using Figure 25-17 for reference. See Chapter 5.

$$MA_{OD} = \left(\left(\frac{ID}{2}\right) + (OD - ID) + ID\right), \quad \text{[cm]}$$

$$MA_{OD} = \left(\left(\frac{2.25}{2}\right) + (3.78 - 2.25) + 2.25\right), \quad \text{[cm]}$$

$$MA_{OD} = 4.91, \quad \text{[cm]}$$

Step No. 27: Calculate the magnetic amplifier surface area using Figure 25-17 for reference, See Chapter 5.

$$A_t = \frac{\pi(MA_{OD})^2}{2} + (\pi(MA_{OD})(MA_{\text{Ht}}), \quad [\text{cm}^2]$$

$$A_t = \frac{\pi(4.91)^2}{2} + (\pi(4.91)(3.64), \quad [\text{cm}^2]$$

$$A_t = 85.8, \quad [\text{cm}^2]$$

Step No. 28: Calculate the watts per unit area, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad [\text{watts/cm}^2]$$

$$\psi = \frac{0.68}{85.8}, \quad [\text{watts/cm}^2]$$

$$\psi = 0.00793, \quad [\text{watts/cm}^2]$$

Step No. 29: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{0.826}, \quad [°\text{C}]$$

$$T_r = 450(0.00793)^{0.826}, \quad [°\text{C}]$$

$$T_r = 8.28, \quad [°\text{C}]$$

Step No. 30: Calculate the window utilization, K_u, for both control and gate.

$$K_{uc} = \left(\frac{N_c A_{wc(B)}}{\frac{W_a}{2}}\right), \quad K_{uc} = \frac{(660)(0.000805)}{(1.59)} = 0.334$$

$$K_{ug} = \left(\frac{N_g A_{wg(B)}}{\frac{W_a}{2}}\right), \quad K_{ug} = \frac{(245)(0.00324)}{(2.38)} = 0.334$$

Self-Saturating, Magnetic Amplifier Design Test Data

Summary

The above self-saturating, magnetic amplifier was built, and tested. The following information is the test data for the above magnetic amplifier design. The input control current versus the output voltage test data is shown in Table 25-1. The input current versus output voltage transfer function is shown in Figure 25-25. The author hopes that this design, with its step-by-step approach, helps the readers understand the design of a self-saturating, magnetic amplifier.

Test Data

1. Frequency, f = 400 Hz
2. Output voltage, V_o = 12.05 volts
3. Output current, I_o = 1.0 amps
4. Control current, I_c = 0.003 amps
5. Gate resistance, R_g = 0.475 ohms
6. Control resistance, R_c = 10.56 ohms
7. Temperature Rise, T_r = 8.1°C

Table 25-1. Self-Saturating, Magnetic Amplifier, Showing Input Current versus Output Voltage

Self-Saturating Magnetic Amplifier		
Steps	Control Current ma	Output Voltage Volts
1	0.000	12.050
2	0.250	12.020
3	0.500	12.000
4	0.750	11.880
5	1.000	11.550
6	1.250	10.350
7	1.500	8.770
8	1.750	7.130
9	2.000	5.220
10	2.250	3.570
11	2.500	2.360
12	2.750	1.270
13	3.000	0.480
14		

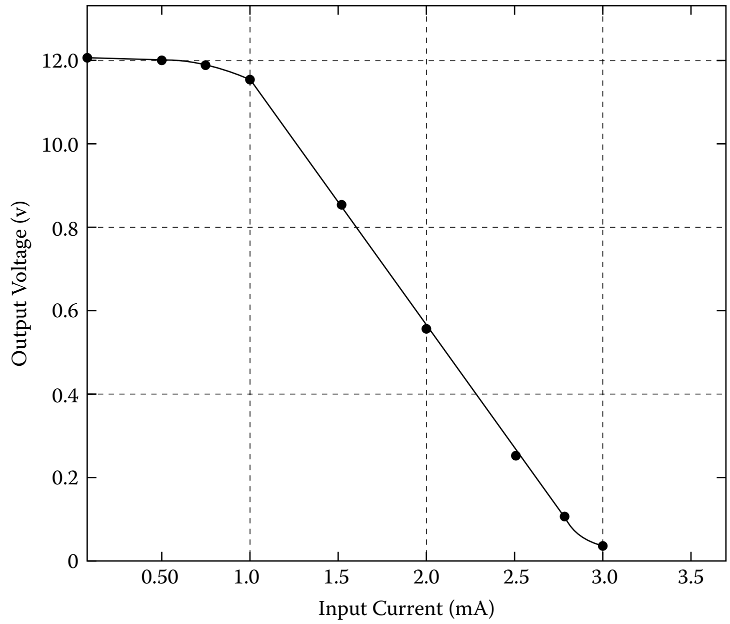

Figure 25-25. Transfer Function Using the Data in Table 25-1.

Recognition

I would like to thank Charles Barnett, an engineer at Leightner Electronics Inc., for building and testing the magnetic amplifier design example.

Leightner Electronics Inc.
1501 S. Tennessee St.
McKinney, TX. 75069

I would like to thank Zack Cataldi, a Senior Applications Engineer at Magnetics, for supplying the cores for the magnetic amplifier design example.

Magnetics
110 Delta Drive
Pittsburgh, PA 15238

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3. Platt, S., *Magnetic Amplifiers Theory and Application*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1958, Chapter 9.

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Chapter 26

Designing Inductors for a Given Resistance

Table of Contents

Introduction

There are times when the engineer is required to design around a black box. This means the engineer does not get an overview of the complete picture. The parameters given to the engineer are inductance, dc current, ac current, dc resistance, temperature rise, and volume. If the engineer is lucky, a sample unit will show up. With a design example in hand the engineer can get more of a feel for what core was used in the design. I have been asked many times, by engineers, to write a step-by-step approach using my equations to design an inductor for a given dc resistance. An inductor with its internal equivalent resistance is shown in Figure 26-1.

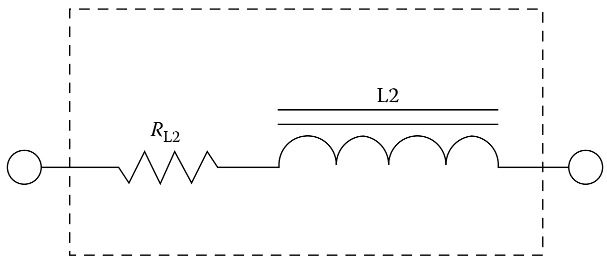

Figure 26-1. Output Inductor, L2, Showing the Internal Equivalent Resistance.

Design Overview

This chapter will present a straight forward approach for selecting the core and the proper wire size to meet the specification for the winding resistance that can be used for both an input inductor, L1, and an output inductor, L2, as shown in Figure 26-2.

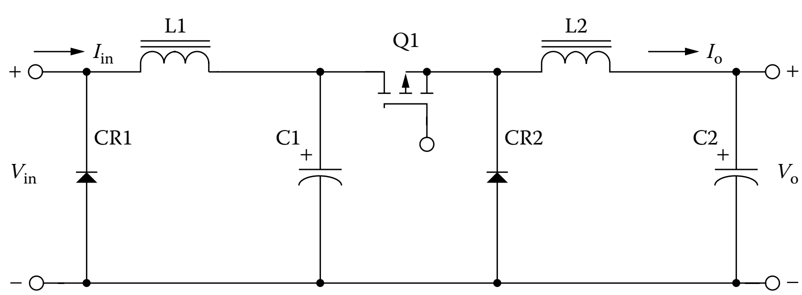

Figure 26-2. Schematic of a Simple Buck Regulator.

Designing to meet a given resistance is very difficult and requires more than several iterations using the area product, A_p, method. The core geometry, K_g, approach is very precise and will lead you to the correct core, with a minimum of time.

The losses in the magnetic material will increase significantly when the converter is operating at a higher frequency. However, the core loss in the output inductor of a switching regulator is much lower compared to the core loss in the main converter transformer. The core loss in the output inductor is caused by the change in current or ΔI, which induces a change in flux, B_{ac}, as shown in Figure 26-3.

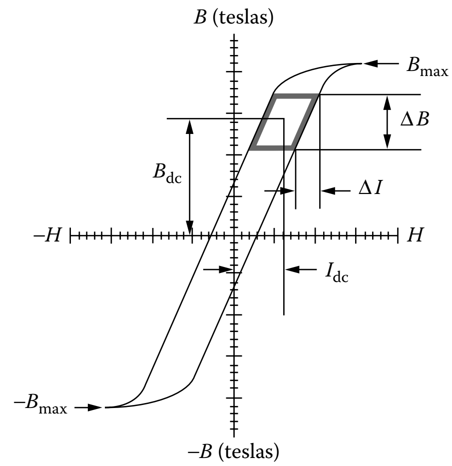

Figure 26-3. Typical, Output Inductor, B-H Loop.

Powder Core Inductor Design Example (Core Geometry, K_g, Approach)

The magnetic cores and wire data have been taken from Chapters 3 and 4. A powder core inductor design example, assume an output filter, L2, as shown in Figure 26-2, with the following specifications:

1. Frequency, f = 100kHz
2. Inductance, L = 50 µh
3. Output current, I_o = 5.0 amps
4. Delta current, ΔI = 1.0 amps
5. DC Resistance, R = 0.01 ohms
6. Temperature Rise, T_r = <20°C
7. Window utilization, K_u = 0.4
8. Operating Flux, B_m = 0.3 T

This design procedure will work equally well with all of the various powder cores. Care must be taken regarding maximum flux density with different materials and core loss.

Step No. 1: Calculate the peak current, I_{pk}.

$$I_{pk} = I_{o(\max)} + \left(\frac{\Delta I}{2}\right) \quad \text{[amps]}$$

$$I_{pk} = (5.0) + \left(\frac{1.0}{2}\right) \quad \text{[amps]}$$

$$I_{pk} = 5.5 \quad \text{[amps]}$$

Step No. 2: Calculate the energy-handling capability in watt-seconds, w-s.

$$\text{Energy} = \frac{LI_{pk}^2}{2} \quad \text{[w-s]}$$

$$\text{Energy} = \frac{\left(50\times10^{-6}\right)(5.5)^2}{2} \quad \text{[w-s]}$$

$$\text{Energy} = 0.000756 \quad \text{[w-s]}$$

Step No. 3: Calculate the electrical conditions, K_e.

$$K_e = \frac{345L}{B_m^2 R_{(L)}}$$

$$K_e = \frac{345\left(50\times10^{-6}\right)}{(0.3)^2(0.01)}$$

$$K_e = 19.2$$

Step No. 4: Calculate the core geometry, K_g.

$$K_g = K_e\left(\text{Energy}\right) \quad [\text{cm}^5]$$

$$K_g = (19.2)(0.000756) \quad [\text{cm}^5]$$

$$K_g = 0.0145 \quad [\text{cm}^5]$$

Step No. 5: Select a MPP powder core, from Chapter 3, comparable in core geometry, K_g.

Core number = 55059-A2

Manufacturer = Magnetics

Magnetic path length, MPL = 5.7 cm

Core weight, W_{tfe} = 15.0 grams

Copper weight, W_{tcu} = 15.2 grams

Mean length turn, MLT = 3.2 cm

Iron area, A_c = 0.331 cm^2

Window Area, W_a = 1.356 cm^2

Area Product, A_p = 0.449 cm^4

Core geometry, K_g = 0.0186 cm^5

Surface area, A_t = 28.6 cm^2

Permeability, μ = 60

Millihenrys per 1000 turns, AL = 43

Step No. 6: Calculate the number of turns, N.

$$N = 1000\sqrt{\frac{L_{(new)}}{L_{(1000)}}} \quad \text{[turns]}$$

$$N = 1000\sqrt{\frac{.05}{43}} \quad \text{[turns]}$$

$$N = 34 \quad \text{[turns]}$$

Step No. 7: Calculate the rms current, I_{rms}.

$$I_{rms} = \sqrt{I_{o(\max)}^2 + \Delta I^2} \quad \text{[amps]}$$

$$I_{rms} = \sqrt{(5)^2 + (1)^2} \quad \text{[amps]}$$

$$I_{rms} = 5.1 \quad \text{[amps]}$$

Step No. 8: Calculate the current density, J, using a window utilization, $K_u = 0.4$.

$$J = \frac{NI}{W_a K_u} \quad \text{[amps/cm}^2\text{]}$$

$$J = \frac{(34)(5.1)}{(1.356)(.4)} \quad \text{[amps/cm}^2\text{]}$$

$$J = 320 \quad \text{[amps/cm}^2\text{]}$$

Step No. 9: Calculate the required permeability, $\Delta\mu$.

$$\Delta\mu = \frac{(B_m)(MPL)\times 10^4}{0.4\pi(W_a)(J)(K_u)}$$

$$\Delta\mu = \frac{(0.3)(5.7)\times 10^4}{(1.256)(1.356)(320)(0.4)}$$

$$\Delta\mu = 78.4 \text{ use 60 perm}$$

Step No. 10: Calculate the peak flux density, B_m.

$$B_m = \frac{0.4\pi(N)(I_{pk})(\mu_r)\times 10^{-4}}{MPL} \quad \text{[teslas]}$$

$$B_m = \frac{1.256(34)(5.5)(60)\times 10^{-4}}{(5.7)} \quad \text{[teslas]}$$

$$B_m = 0.247 \quad \text{[teslas]}$$

Step No. 11: Calculate the required bare wire area, $A_{w(B)}$.

$$A_{w(B)} = \frac{I_{rms}}{J} \quad [\text{cm}^2]$$

$$A_{w(B)} = \frac{5.1}{320} \quad [\text{cm}^2]$$

$$A_{w(B)} = 0.0159 \quad [\text{cm}^2]$$

Step No. 12: Select a wire size with the required area from the Wire Table in Chapter 4. If the area is not within a few percent of required area, then go to the next smallest size.

$$\text{AWG} = \#15$$

$$A_{w(B)} = 0.0165$$

$$\mu\Omega / cm = 104$$

Step No. 13: Calculate the winding resistance, R.

$$R = \text{MLT}(N)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R = (3.2)(34)(104)\left(10^{-6}\right), \quad [\text{ohms}]$$

$$R = 0.0113, \quad [\text{ohms}]$$

Step No. 14: Calculate the winding copper loss, P_{cu}.

$$P_{cu} = I^2 R, \quad [\text{watts}]$$

$$P_{cu} = (5.1)^2(0.0113), \quad [\text{watts}]$$

$$P_{cu} = 0.294, \quad [\text{watts}]$$

Step No. 15: Calculate the magnetizing force in oersteds, H.

$$H = \frac{(0.4\pi)NI_{pk}}{MPL} \quad [\text{oersteds}]$$

$$H = \frac{(1.256)(34)(5.5)}{5.7} \quad [\text{oersteds}]$$

$$H = 41.2 \quad [\text{oersteds}]$$

Step No. 16: Calculate the ac flux density in teslas, B_{ac}.

$$B_{ac} = \frac{(0.4\pi)(N)\left(\frac{\Delta I}{2}\right)(\mu_r)\times 10^{-4}}{MPL} \quad \text{[teslas]}$$

$$B_{ac} = \frac{(1.256)(34)(0.5)(60)\times 10^{-4}}{5.7} \quad \text{[teslas]}$$

$$B_{ac} = 0.0225 \quad \text{[teslas]}$$

Step No. 17: Calculate the watts per kilogram, WK, using MPP 60 perm powder cores, as shown in Chapter 7.

$$WK = 0.788\times 10^{-3}\left(f\right)^{(1.41)}\left(B_{ac}\right)^{(2.24)} \quad \text{[watts/kilogram]}$$

$$WK = 0.788\times 10^{-3}\left(100000\right)^{(1.41)}\left(0.0225\right)^{(2.24)} \quad \text{[watts/kilogram]}$$

$$WK = 1.80 \quad \text{[watts/kilogram] or [milliwatts/gram]}$$

Step No. 18: Calculate the core loss, P_{fe}.

$$P_{fe} = \left(\frac{\text{milliwatts}}{\text{gram}}\right)W_{tfe}\times 10^{-3} \quad \text{[watts]}$$

$$P_{fe} = (1.8)(15)\times 10^{-3} \quad \text{[watts]}$$

$$P_{fe} = 0.027 \quad \text{[watts]}$$

Step No. 19: Calculate the total loss P_{Σ}, core P_{fe} and copper P_{cu}, in watts.

$$P_{\Sigma} = P_{fe} + P_{cu} \quad \text{[watts]}$$

$$P_{\Sigma} = (0.027)+(0.294) \quad \text{[watts]}$$

$$P_{\Sigma} = 0.321 \quad \text{[watts]}$$

Step No. 20: Calculate the watt density, ψ.

$$\psi = \frac{P_{\Sigma}}{A_t} \quad \text{[watts/cm}^2\text{]}$$

$$\psi = \frac{0.321}{28.6} \quad \text{[watts/cm}^2\text{]}$$

$$\psi = 0.0112 \quad \text{[watts/cm}^2\text{]}$$

Step No. 21: Calculate the temperature rise in degrees C.

$$T_r = 450(\psi)^{(0.826)} \quad \text{[degrees C]}$$

$$T_r = 450(0.0112)^{(0.826)} \quad \text{[degrees C]}$$

$$T_r = 11.0 \quad \text{[degrees C]}$$

Step No. 22: Calculate the window utilization, K_u.

$$K_u = \frac{NA_{w(B)}}{W_a}$$

$$K_u = \frac{(34)(0.0165)}{(1.356)}$$

$$K_u = 0.414$$

Powder Core Inductor Design Test Data (Core Geometry, K_g, Approach)

Summary

The following information is the test data for the above powder core inductor. It is difficult to design and achieve the required resistance with just one wire and also have a compact design. If the engineer is required to get closer to the required resistance called out in the design specification, the inductor could be wound bifilar using a large wire and a smaller wire. It would be like having two resistors in parallel to get the correct resistance. The author hopes that this chapter, with its step-by-step approach, helps the readers understand the design of a filter inductor with the required resistance. The above example of this inductor was built, and tested. It meets the intent of the specification and shows the reader a typical design.

Test Data

1. Frequency, f = 100kHz
2. Inductance, L = 54 µh
3. Inductance, L, (at 5.0 amps) = 50 µh
4. Output current, I_o = 5.0 amps
5. DC Resistance, R_L = 0.0119 ohms
6. Temperature Rise, T_r = 12.5°C
7. Window utilization, K_u (See Chapter 4) = 0.404
8. Max. Operating Flux, B_m = 0.247 T

Gapped Ferrite Inductor Design Example (Core Geometry, K_g, Approach)

The magnetic cores and wire data have been taken from Chapters 3 and 4. For a gapped inductor design example, assume an output filter, L2, as shown in Figure 26-2, with the following specifications:

Step No. 1: Design Specifications.

1. Frequency, f = 100kHz
2. Inductance, L = 50 µh
3. Output current, I_o = 4.0 amps
4. Delta current, ΔI = 0.4 amps
5. DC Resistance, R = 0.01 ohms
6. Temperature Rise, T_r = <20°C
7. Window utilization, K_u (See Chapter 4) = 0.313
8. Operating Flux, B_m = 0.25 T

Step No. 2: Calculate the peak current, I_{pk}.

$$I_{pk} = I_{o(\max)} + \left(\frac{\Delta I}{2}\right) \quad \text{[amps]}$$

$$I_{pk} = (4.0) + \left(\frac{0.4}{2}\right) \quad \text{[amps]}$$

$$I_{pk} = 4.2 \quad \text{[amps]}$$

Step No. 3: Calculate the energy-handling capability in watt-seconds, w-s.

$$\text{Energy} = \frac{LI_{pk}^2}{2} \quad \text{[w-s]}$$

$$\text{Energy} = \frac{\left(50\times10^{-6}\right)(4.2)^2}{2} \quad \text{[w-s]}$$

$$\text{Energy} = 0.000441 \quad \text{[w-s]}$$

Step No. 4: Calculate the electrical conditions, K_e.

$$K_e = \frac{345L}{B_m^2 R_{(L)}}$$

$$K_e = \frac{345\left(50\times10^{-6}\right)}{(0.25)^2(0.01)}$$

$$K_e = 27.6$$

Step No. 5: Calculate the core geometry, K_g. Since this design is using a ferrite core the core geometry, K_g, has to be increased approximately 20%. See Chapter 4 on window utilization, K_u, for bobbin ferrites.

$$K_g = K_e(\text{Energy})(1.20) \quad [\text{cm}^5]$$

$$K_g = (27.6)(0.000441)(1.20) \quad [\text{cm}^5]$$

$$K_g = 0.0146 \quad [\text{cm}^5]$$

Step No. 6: Select a Pot core from Chapter 7, comparable in core geometry, K_g.

Core number = PC-42213
Manufacturer = Magnetics
Magnetic path length, MPL = 3.12 cm
Core weight, W_{tfe} = 13 grams
Copper weight, W_{tcu} = 6.2 grams
Mean length turn, MLT = 4.4 cm
Iron area, A_c = 0.634 cm^2
Window Area, W_a = 0.414 cm^2
Area Product, A_p = 0.262 cm^4
Core geometry, K_g = 0.0151 cm^5
Winding Length, G = 0.940 cm
Surface area, A_t = 16.4 cm^2
Permeability, μ = 2500 (P)
Millihenrys per 1000 turns, AL = 4393

Step No. 7: Calculate the current density, J, using the area product Equation, A_p.

$$J = \frac{2(\text{Energy})(10^4)}{B_m A_p K_u}, \quad [\text{amps-per-cm}^2]$$

$$J = \frac{2(0.000441)(10^4)}{(0.25)(0.262)(0.4)}, \quad [\text{amps-per-cm}^2]$$

$$J = 337, \quad [\text{amps-per-cm}^2]$$

Step No. 8: Calculate the rms current, I_{rms}.

$$I_{rms} = \sqrt{I_o^2 + \Delta I^2}, \quad [\text{amps}]$$

$$I_{rms} = \sqrt{(4.0)^2 + (0.4)^2}, \quad [\text{amps}]$$

$$I_{rms} = 4.02, \quad [\text{amps}]$$

Step No. 9: Calculate the required bare wire area, $A_{w(B)}$.

$$A_{W(B)} = \frac{I_{rms}}{J}, \quad [\text{cm}^2]$$

$$A_{W(B)} = \frac{(4.02)}{(337)}, \quad [\text{cm}^2]$$

$$A_{W(B)} = 0.0119, \quad [\text{cm}^2]$$

Step No. 10: Select a wire from the Wire Table in Chapter 4. If the area is not within a few percent, take the next smallest size. Also, record the micro-ohms per centimeter.

$$\text{AWG} = \#17$$

$$\text{Bare, } A_{W(B)} = 0.01039, \quad [\text{cm}^2]$$

$$\text{Insulated, } A_W = 0.0117, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 166, \quad [\text{micro-ohm/cm}]$$

Step No. 11: Calculate the effective window area, $W_{a(eff)}$. Use the window area found in Step 6. A typical value for, S_3, is 0.6, as shown in Chapter 4.

$$W_{a(eff)} = W_a S_3, \quad [\text{cm}^2]$$

$$W_{a(eff)} = (0.414)(0.60), \quad [\text{cm}^2]$$

$$W_{a(eff)} = 0.248, \quad [\text{cm}^2]$$

Step No. 12: Calculate the number turns possible, N, using the insulated wire area, A_w, found in Step 10. A typical value for, S_2, is 0.61, as shown in Chapter 4.

$$N = \frac{W_{a(eff)} S_2}{A_W}, \quad [\text{turns}]$$

$$N = \frac{(0.248)(0.61)}{(0.0117)}, \quad [\text{turns}]$$

$$N = 12.9 \text{ use } 13, \quad [\text{turns}]$$

Step No. 13: Calculate the required gap, l_g.

$$l_g = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{L} - \left(\frac{\text{MPL}}{\mu_m}\right), \quad [\text{cm}]$$

$$l_g = \frac{(1.26)(13)^2(0.634)\left(10^{-8}\right)}{(0.00005)} - \left(\frac{3.12}{2500}\right), \quad [\text{cm}]$$

$$l_g = 0.0257, \quad [\text{cm}]$$

Step No. 14: Calculate the equivalent gap in mils.

$$\text{mils} = \text{cm}(393.7)$$
$$\text{mils} = (0.0257)(393.7)$$
$$\text{mils} = 10, \text{ then cm} = 0.0254$$

Step No. 15: Calculate the fringing flux factor, F.

$$F = 1 + \frac{l_g}{\sqrt{A_c}} \ln\left(\frac{2G}{l_g}\right)$$
$$F = 1 + \frac{(0.0254)}{\sqrt{0.634}} \ln\left(\frac{2(0.940)}{0.0254}\right)$$
$$F = 1.14$$

Step No. 16: Calculate the new number of turns, N_n, by inserting the fringing flux, F.

$$N_n = \sqrt{\frac{l_g L}{0.4\pi A_c F\left(10^{-8}\right)}}, \quad \text{[turns]}$$
$$N_n = \sqrt{\frac{(0.0254)(0.00005)}{(1.26)(0.634)(1.14)\left(10^{-8}\right)}}, \quad \text{[turns]}$$
$$N_n = 12, \quad \text{[turns]}$$

Step No. 17: Calculate the winding resistance, R_L. Use the MLT from Step 6 and the micro-ohm per centimeter from Step 10.

$$R_L = (\text{MLT})(N_n)\left(\frac{\mu\Omega}{\text{cm}}\right)\left(10^{-6}\right), \quad \text{[ohms]}$$
$$R_L = (4.4)(12)(166)\left(10^{-6}\right), \quad \text{[ohms]}$$
$$R_L = 0.0088, \quad \text{[ohms]}$$

Step No. 18: Calculate the copper loss, P_{cu}.

$$P_{cu} = I_{rms}^2 R_L, \quad \text{[watts]}$$
$$P_{cu} = (4.02)^2(0.0088), \quad \text{[watts]}$$
$$P_{cu} = 0.142, \quad \text{[watts]}$$

Step No. 19: Calculate the ac flux density, B_{ac}.

$$B_{ac} = \frac{0.4\pi N_n F\left(\frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]}$$

$$B_{ac} = \frac{(1.26)(12)(1.14)\left(\frac{0.4}{2}\right)\left(10^{-4}\right)}{(0.0254) + \left(\frac{3.12}{2500}\right)}, \quad \text{[teslas]}$$

$$B_{ac} = 0.0129, \quad \text{[teslas]}$$

Step No. 20: Calculate the watts per kilogram for ferrite, P, material in Chapter 2. Watts per kilogram can be written in milliwatts per gram.

$$\text{mW/g} = k f^{(m)} B_{ac}^{(n)}$$

$$\text{mW/g} = (0.00004855)(100000)^{(1.63)}(0.0129)^{(2.62)}$$

$$\text{mW/g} = 0.0769$$

Step No. 21: Calculate the core loss, P_{fe}.

$$P_{fe} = (\text{mW/g})\left(W_{tfe}\right)\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = (0.0769)(13)\left(10^{-3}\right), \quad \text{[watts]}$$

$$P_{fe} = 0.001, \quad \text{[watts]}$$

Step No. 22: Calculate the total loss, copper plus iron, P_Σ.

$$P_\Sigma = P_{fe} + P_{cu}, \quad \text{[watts]}$$

$$P_\Sigma = (0.001) + (0.265), \quad \text{[watts]}$$

$$P_\Sigma = 0.266, \quad \text{[watts]}$$

Step No. 23: Calculate the watt density, ψ. The surface area, A_t, can be found in Step 6.

$$\psi = \frac{P_\Sigma}{A_t}, \quad \text{[watts/cm}^2\text{]}$$

$$\psi = \frac{(0.266)}{(16.4)}, \quad \text{[watts/cm}^2\text{]}$$

$$\psi = 0.0162, \quad \text{[watts/cm}^2\text{]}$$

Step No. 24: Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [°\text{C}]$$

$$T_r = 450(0.0162)^{(0.826)}, \quad [°\text{C}]$$

$$T_r = 14.9, \quad [°\text{C}]$$

Step No. 25: Calculate the peak flux density, B_{pk}.

$$B_{pk} = \frac{0.4\pi N_n F\left(I_{dc} + \frac{\Delta I}{2}\right)\left(10^{-4}\right)}{l_g + \left(\frac{\text{MPL}}{\mu_m}\right)}, \quad \text{[teslas]}$$

$$B_{pk} = \frac{(1.26)(12)(1.14)(4.2)\left(10^{-4}\right)}{(0.0254) + \left(\frac{3.12}{2500}\right)}, \quad \text{[teslas]}$$

$$B_{pk} = 0.271, \quad \text{[teslas]}$$

Step No. 26: Calculate the total window utilization, K_u.

$$K_u = \frac{NA_{w(B)}}{W_a}$$

$$K_u = \frac{(12)(0.01039)}{(0.414)}$$

$$K_u = 0.301$$

Gapped, Ferrite Inductor Design Test Data (Core Geometry, K_g, Approach)

Summary

The following information is the test data for the above gapped ferrite inductor. It is difficult to design and achieve the required resistance with just one wire and also have a compact design. If the engineer is required to get closer to the required resistance called out in the design specification, the inductor could be wound bifilar using a large wire and a smaller wire. It would be like having two resistors in parallel to get the correct resistance. The author hopes that this chapter, with its step-by-step approach, helps the readers understand the design of a filter inductor with the required resistance. The above example of this inductor was built, and tested. It meets the intent of the specification and shows the reader a typical design.

Test Data

1. Frequency, f = 100kHz
2. Inductance, L = 52 µh
3. Inductance, L, (at 4.0 amps) = 49 µh
4. Output current, I_o = 4.0 amps
5. DC Resistance, R_L = 0.0088ohms
6. Temperature Rise, T_r = 12.1°C
7. Window utilization, K_u (See Chapter 4) = 0.301
8. Max. Operating Flux, B_m = 0.271 T

Powder Core, Input Inductor Design Example (Core Geometry, K_g, Approach)

For a typical design example, assume an input filter circuit, as shown in Figure 26-2, with the following specifications:

1. Frequency, f = 100kHz
2. Inductance, L = 125 uh
3. Input current, I_{in} = 2.5 amps
4. Delta current, ΔI = 0.010 amps
5. DC Resistance, R = 0.05 ohms
6. Temperature Rise, T_r = <20°C
7. Window utilization, K_u = 0.4
8. Operating Flux, B_m = 0.3

This design procedure will work equally well with all of the various powder cores. Care must be taken regarding maximum flux density with different materials and core loss.

Step No. 1: Calculate the peak current, I_{pk}.

$$I_{pk} = I_{in(\max)} + \left(\frac{\Delta I}{2}\right) \quad \text{[amps]}$$

$$I_{pk} = (2.5) + \left(\frac{0.01}{2}\right) \quad \text{[amps]}$$

$$I_{pk} = 2.505 \quad \text{[amps]}$$

Step No. 2: Calculate the energy-handling capability in watt-seconds, w-s.

$$\text{Energy} = \frac{LI_{pk}^2}{2} \quad \text{[w-s]}$$

$$\text{Energy} = \frac{\left(125\times10^{-6}\right)(2.505)^2}{2} \quad \text{[w-s]}$$

$$\text{Energy} = 0.000392 \quad \text{[w-s]}$$

Step No. 3: Calculate the electrical conditions, K_e.

$$K_e = \frac{345L}{B_m^2 R_{(L)}}$$

$$K_e = \frac{345\left(125\times10^{-6}\right)}{(0.3)^2(0.05)}$$

$$K_e = 9.58$$

Step No. 4: Calculate the core geometry, K_g.

$$K_g = K_e(\text{Energy}) \quad [\text{cm}^5]$$

$$K_g = (9.58)(0.000392) \quad [\text{cm}^5]$$

$$K_g = 0.00376 \quad [\text{cm}^5]$$

Step No. 5: Select a Sendust powder core from Chapter 3 comparable in core geometry, K_g.

Core number = 77121-A7
Manufacturer = Magnetics
Magnetic path length, MPL = 4.11 cm
Core weight, W_{tfe} = 5.524 grams
Copper weight, W_{tcu} = 6.10 grams
Mean length turn, MLT = 2.5 cm
Iron area, A_c = 0.192 cm^2
Window Area, W_a = 0.684 cm^2
Area Product, A_p = 0.131 cm^4
Core geometry, K_g = 0.00403 cm^5
Surface area, A_t = 16.0 cm^2
Permeability, μ = 60
Millihenrys per 1000 turns, AL = 35

Step No. 6: Calculate the number of turns, N.

$$N = 1000\sqrt{\frac{L_{(new)}}{L_{(1000)}}} \quad [\text{turns}]$$

$$N = 1000\sqrt{\frac{.125}{35}} \quad [\text{turns}]$$

$$N = 59.7 \text{ use } 60 \quad [\text{turns}]$$

Step No. 7: Calculate the rms current, I_{rms}.

$$I_{rms} = \sqrt{I_{o(\max)}^2 + \Delta I^2} \quad [\text{amps}]$$

$$I_{rms} = \sqrt{(2.5)^2 + (0.01)^2} \quad [\text{amps}]$$

$$I_{rms} = 2.50 \quad [\text{amps}]$$

Step No. 8: Calculate the current density, J, using a window utilization, $K_u = 0.4$.

$$J = \frac{NI}{W_a K_u} \quad [\text{amps/cm}^2]$$

$$J = \frac{(60)(2.5)}{(0.684)(.4)} \quad [\text{amps/cm}^2]$$

$$J = 458 \quad [\text{amps/cm}^2]$$

Step No. 9: Calculate the required permeability, Δμ.

$$\Delta\mu = \frac{(B_m)(MPL)\times 10^4}{0.4\pi(W_a)(J)(K_u)}$$

$$\Delta\mu = \frac{(0.3)(4.11)\times 10^4}{(1.256)(0.684)(458)(0.4)}$$

$$\Delta\mu = 78.3 \text{ use 60 perm}$$

Step No. 10: Calculate the peak flux density, B_m.

$$B_m = \frac{0.4\pi(N)(I_{pk})(\mu_r)\times 10^{-4}}{MPL} \quad [\text{teslas}]$$

$$B_m = \frac{1.256(60)(2.5)(60)\times 10^{-4}}{(4.11)} \quad [\text{teslas}]$$

$$B_m = 0.275 \quad [\text{teslas}]$$

Step No. 11: Calculate the required bare wire area, $A_{w(B)}$.

$$A_{w(B)} = \frac{I_{rms}}{J} \quad [\text{cm}^2]$$

$$A_{w(B)} = \frac{2.5}{458} \quad [\text{cm}^2]$$

$$A_{w(B)} = 0.00546 \quad [\text{cm}^2]$$

Step No. 12: Select a wire size with the required area from the Wire Table in Chapter 4. If the area is not within 10% of required area, then go to the next smallest size.

$$\text{AWG} = \#20$$
$$A_{w(B)} = 0.00519$$
$$\mu\Omega / cm = 332$$

Step No. 13: Calculate the winding resistance, R.

$$R = \text{MLT}(N)\left(\frac{\mu\Omega}{\text{cm}}\right)(10^{-6}), \quad [\text{ohms}]$$

$$R = (2.5)(60)(332)(10^{-6}), \quad [\text{ohms}]$$

$$R = 0.0498, \quad [\text{ohms}]$$

Step No. 14: Calculate the winding copper loss, P_{cu}.

$$P_{cu} = I^2 R, \quad \text{[watts]}$$

$$P_{cu} = (2.5)^2(0.0498), \quad \text{[watts]}$$

$$P_{cu} = 0.311, \quad \text{[watts]}$$

Step No. 15: Calculate the magnetizing force in oersteds, H.

$$H = \frac{(0.4\pi)NI_{pk}}{MPL} \quad \text{[oersteds]}$$

$$H = \frac{(1.256)(60)(2.5)}{4.11} \quad \text{[oersteds]}$$

$$H = 45.8 \quad \text{[oersteds]}$$

Step No. 16: Calculate the ac flux density in teslas, B_{ac}.

$$B_{ac} = \frac{(0.4\pi)(N)\left(\frac{\Delta I}{2}\right)(\mu_r)\times 10^{-4}}{MPL} \quad \text{[teslas]}$$

$$B_{ac} = \frac{(1.256)(60)(0.005)(60)\times 10^{-4}}{4.11} \quad \text{[teslas]}$$

$$B_{ac} = 0.00055 \quad \text{[teslas]}$$

Note: Normally the ac flux is very low for the input filter inductor.

Step No. 17: Calculate the watts per kilogram, WK, using Sendust 60 perm powder cores, as shown in Chapter 2.

$$WK = 0.634\times 10^{-3}(f)^{(1.46)}(B_{ac})^{(2.0)} \quad \text{[watts/kilogram]}$$

$$WK = 0.634\times 10^{-3}(100000)^{(1.46)}(0.00055)^{(2.0)} \quad \text{[watts/kilogram]}$$

$$WK = 0.00383 \quad \text{[watts/kilogram] or [milliwatts/gram]}$$

Step No. 18: Calculate the core loss, P_{fe}.

$$P_{fe} = \left(\frac{\text{milliwatts}}{\text{gram}}\right)W_{tfe}\times 10^{-3} \quad \text{[watts]}$$

$$P_{fe} = (0.00383)(5.524)\times 10^{-3} \quad \text{[watts]}$$

$$P_{fe} = 4.5\left(10^{-7}\right) \quad \text{[watts]}$$

Step No. 19: Calculate the total loss, P_Σ core P_{fe} and copper P_{cu}, in watts.

$$P_\Sigma = P_{fe} + P_{cu} \quad \text{[watts]}$$

$$P_\Sigma = (0.000) + (0.311) \quad \text{[watts]}$$

$$P_\Sigma = 0.311 \quad \text{[watts]}$$

Step No. 20: Calculate the watt density, ψ.

$$\psi = \frac{P_\Sigma}{A_t} \quad \text{[watts/cm}^2\text{]}$$

$$\psi = \frac{0.311}{16} \quad \text{[watts/cm}^2\text{]}$$

$$\psi = 0.0194 \quad \text{[watts/cm}^2\text{]}$$

Step No. 21: Calculate the temperature rise in degrees C.

$$T_r = 450(\psi)^{(0.826)} \quad \text{[degrees C]}$$

$$T_r = 450(0.0194)^{(0.826)} \quad \text{[degrees C]}$$

$$T_r = 17.3 \quad \text{[degrees C]}$$

Step No. 22: Calculate the window utilization, K_u.

$$K_u = \frac{NA_{w(B)}}{W_a}$$

$$K_u = \frac{(60)(0.00519)}{(0.684)}$$

$$K_u = 0.455$$

Powder Core, Input Inductor Design Test Data (Core Geometry, K_g, Approach)

Summary

The following information is the test data for the above powder core inductor. It is difficult to design and achieve the required resistance with just one wire and also have a compact design. If the engineer is required to get closer to the required resistance called out in the design specification, the inductor could be wound bifilar using a large wire and a smaller wire. It would be like having two resistors in parallel to get the correct resistance. The author hopes that this chapter, with its step-by-step approach, helps the readers understand the design of a filter inductor with the required resistance. The above example of this inductor was built, and tested. It meets the intent of the specification and shows the reader a typical design.

Test Data

1. Frequency, f = 100kHz
2. Inductance, L = 125 µh
3. Inductance, L, (at 2.5 amps) = 103 µh
4. Output current, I_o = 2.5 amps
5. DC Resistance, R_L = 0.0503 ohms
6. Temperature Rise, T_r = 16.6°C
7. Window utilization, K_u (See Chapter 4) = 0.455
8. Max. Operating Flux, B_m = 0.275 T

Recognition

I would like to give thanks to Charles Barnett, an engineer at Leightner Electronics Inc., for building and testing the inductor design examples.

Leightner Electronics Inc.
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The Author would like to thank Paul A. Levin for his work he did to develop these Equations from my original Core Geometry (Kg) Equations.

Index

G

H

I

N

O

P

Q

R

S