# 12 Transformer Design

In the design methods of the previous chapter, copper loss $P_{cu}$ and maximum flux density $B_{max}$ are specified, while core loss $P_{fe}$ is not specifically addressed. This approach is appropriate for a number of applications, such as the filter inductor in which the dominant design constraints are copper loss and saturation flux density. However, in a substantial class of applications, the operating flux density is limited by core loss rather than saturation. For example, in a conventional high-frequency transformer, it is usually necessary to limit the core loss by operating at a reduced value of the peak ac flux density $\Delta B$.

Design of core loss-limited magnetic devices is characterized by finding the ac flux density that minimizes total core plus copper loss. Typically, this optimization problem also involves optimization of the winding geometry to control ac proximity losses, and possibly incorporation of other constraints such as galvanic isolation. Consequently, multiple design iterations are required. In this chapter, the basic design equations are developed, and a first-pass design that minimizes the total core loss plus dc copper loss is found. The winding geometry can then be estimated, and ac proximity losses can be analyzed as described in Sect. 10.4. The design can then be iterated as needed.

This chapter covers the general transformer design problem. It is desired to design a $k$-winding transformer as illustrated in Fig. 12.1. Both copper loss $P_{cu}$ and core loss $P_{fe}$ are modeled. As the operating flux density is increased (by decreasing the number of turns), the copper loss is decreased but the core loss is increased. We will determine the operating flux density that minimizes the total power loss $P_{tot} = P_{fe} + P_{cu}$.

It is possible to generalize the core geometrical constant $K_g$ design method, derived in the previous chapter, to treat the design of magnetic devices when both copper loss and core loss are significant. This leads to the geometrical constant $K_{gfe}$, a measure of the effective magnetic size of core in a transformer design application. Several examples of transformer designs via the $K_{gfe}$ method are given in this chapter. A similar procedure is also derived, for design of single-winding inductors in which core loss is significant.



R. W. Erickson, D. Maksimović, *Fundamentals of Power Electronics*,
https://doi.org/10.1007/978-3-030-43881-4_12

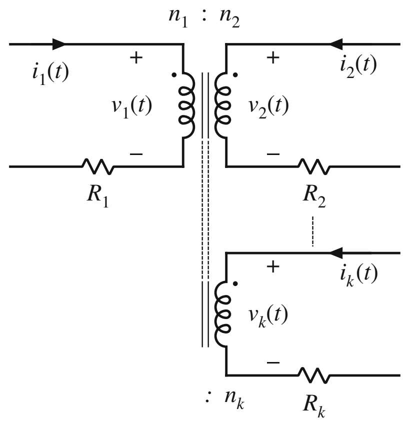


**Fig. 12.1** A $k$-winding transformer, in which both core loss and copper loss are significant

## 12.1 Transformer Design: Basic Constraints

As in the case of the filter inductor design, we can write several basic constraining equations. These equations can then be combined into a single equation for selection of the core size. In the case of transformer design, the basic constraints describe the core loss, flux density, copper loss, and total power loss vs. flux density. The flux density is then chosen to optimize the total power loss.

### 12.1.1 Core Loss

As described in Chap. 10, the total core loss $P_{fe}$ depends on the peak ac flux density $\Delta B$, the operating frequency $f$, and the volume of the core. At a given frequency, we can approximate the core loss by a function of the form

$$P_{fe} = K_{fe}(\Delta B)^{\beta} A_c \ell_m \tag{12.1}$$

Again, $A_c$ is the core cross-sectional area, $\ell_m$ is the core mean magnetic path length, and hence $A_c \ell_m$ is the volume of the core. $K_{fe}$ is a constant of proportionality which depends on the operating frequency. The exponent $\beta$ is determined from the core manufacturer's published data. Typically, the value of $\beta$ for ferrite power materials is approximately 2.6; for other core materials, this exponent lies in the range 2 to 3. Equation (12.1) generally assumes that the applied waveforms are sinusoidal; effects of waveform harmonic content are ignored here.

### 12.1.2 Flux Density

An arbitrary periodic primary voltage waveform $v_1(t)$ is illustrated in Fig. 12.2. The volt-seconds applied during the positive portion of the waveform is denoted $\lambda_1$:

$$\lambda_1 = \int_{t_1}^{t_2} v_1(t)dt \tag{12.2}$$

These volt-seconds, or flux-linkages, cause the flux density to change from its negative peak to its positive peak value. Hence, from Faraday's law, the peak value of the ac component of the flux density is

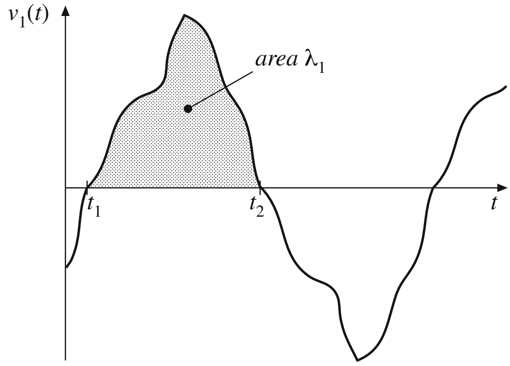


**Fig. 12.2** An arbitrary transformer primary voltage waveform, illustrating the volt-seconds applied during the positive portion of the cycle

$$\Delta B = \frac{\lambda_1}{2n_1A_c} \tag{12.3}$$

Note that, for a given applied voltage waveform and $\lambda_1$, we can reduce $\Delta B$ by increasing the primary turns $n_1$. This has the effect of decreasing the core loss according to Eq. (12.1). However, it also causes the copper loss to increase, since the new windings will be comprised of more turns of smaller wire. As a result, there is an optimal choice for $\Delta B$, in which the total loss is minimized. In the next sections, we will determine the optimal $\Delta B$. Having done so, we can then use Eq. (12.3) to determine the primary turns $n_1$, as follows:

$$n_1 = \frac{\lambda_1}{2\Delta BA_c} \tag{12.4}$$

It should also be noted that, in some converter topologies such as the forward converter with conventional reset winding, the flux density $B(t)$ and the magnetizing current $i_M(t)$ are not allowed to be negative. In consequence, the instantaneous flux density $B(t)$ contains a dc bias. Provided that the core does not approach saturation, this dc bias does not significantly affect the core loss: core loss is determined by the ac component of $B(t)$. Equations (12.2) to (12.4) continue to apply to this case, since $\Delta B$ is the peak value of the ac component of $B(t)$.

### 12.1.3 Copper Loss

As shown in Sect. 11.3.1, the total copper loss is minimized when the core window area $W_A$ is allocated to the various windings according to their relative apparent powers. The total copper loss is then given by Eq. (11.34). This equation can be expressed in the form

$$P_{cu} = \frac{\rho(MLT)n_1^2I_{tot}^2}{W_AK_u} \tag{12.5}$$

where

$$I_{tot} = \sum_{j=1}^{k} \frac{n_j}{n_1} I_j \tag{12.6}$$

is the sum of the rms winding currents, referred to winding 1. Use of Eq. (12.4) to eliminate $n_1$ from Eq. (12.5) leads to

$$P_{cu} = \left(\frac{\rho \lambda_1^2 I_{tot}^2}{4K_u}\right)\left(\frac{(MLT)}{W_A A_c^2}\right)\left(\frac{1}{\Delta B}\right)^2 \tag{12.7}$$

The right-hand side of Eq. (12.7) is grouped into three terms. The first group contains specifications, while the second group is a function of the core geometry. The last term is a function of $\Delta B$, to be chosen to optimize the design. It can be seen that copper loss varies as the inverse square of $\Delta B$; increasing $\Delta B$ reduces $P_{cu}$.

The increased copper loss due to the proximity effect is not explicitly accounted for in this design procedure. In practice, the proximity loss must be estimated after the core and winding geometries are known. However, the increased ac resistance due to proximity loss can be accounted for in the design procedure. The effective value of the wire resistivity ρ is increased by a factor equal to the estimated ratio $R_{ac}/R_{dc}$. When the core geometry is known, the engineer can attempt to implement the windings such that the estimated $R_{ac}/R_{dc}$ is obtained. Several design iterations may be needed.

### 12.1.4 Total Power Loss vs. $\Delta B$

The total power loss $P_{tot}$ is found by adding Eqs. (12.1) and (12.7):

$$P_{tot} = P_{fe} + P_{cu} \tag{12.8}$$

The dependence of $P_{fe}$, $P_{cu}$, and $P_{tot}$ on $\Delta B$ is sketched in Fig. 12.3.

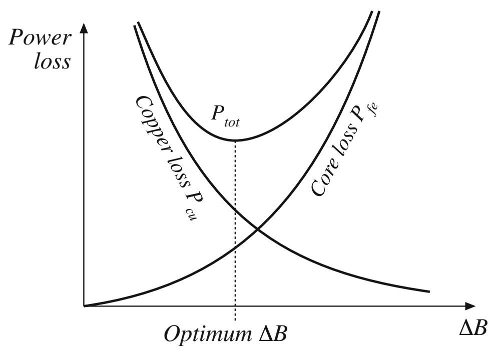


**Fig. 12.3** Dependence of copper loss, core loss, and total loss on peak ac flux density

### 12.1.5 Optimum Flux Density

Let us now choose the value of $\Delta B$ that minimizes Eq. (12.8). At the optimum $\Delta B$, we can write

$$\frac{dP_{tot}}{d(\Delta B)} = \frac{dP_{fe}}{d(\Delta B)} + \frac{dP_{cu}}{d(\Delta B)} = 0 \tag{12.9}$$

Note that the optimum does not necessarily occur where $P_{fe} = P_{cu}$. Rather, it occurs where

$$\frac{dP_{fe}}{d(\Delta B)} = -\frac{dP_{cu}}{d(\Delta B)} \tag{12.10}$$

The derivatives of the core and copper losses with respect to $\Delta B$ are given by

$$\frac{dP_{fe}}{d(\Delta B)} = \beta K_{fe}(\Delta B)^{(\beta-1)} A_c \ell_m \tag{12.11}$$

$$\frac{dP_{cu}}{d(\Delta B)} = -2\left(\frac{\rho\lambda_1^2 I_{tot}^2}{4K_u}\right)\left(\frac{(MLT)}{W_A A_c^2}\right)(\Delta B)^{-3} \tag{12.12}$$

Substitution of Eqs. (12.11) and (12.12) into Eq. (12.10), and solution for $\Delta B$, leads to the optimum flux density

$$\Delta B = \left[\frac{\rho\lambda_1^2 I_{tot}^2}{2K_u}\frac{(MLT)}{W_A A_c^3 \ell_m}\frac{1}{\beta K_{fe}}\right]^{\left(\frac{1}{\beta+2}\right)} \tag{12.13}$$

The resulting total power loss is found by substitution of Eq. (12.13) into (12.1), (12.8), and (12.9). Simplification of the resulting expression leads to

$$P_{tot} = \left[A_c \ell_m K_{fe}\right]^{\left(\frac{2}{\beta+2}\right)} \left[\frac{\rho\lambda_1^2 I_{tot}^2}{4K_u}\frac{(MLT)}{W_A A_c^2}\right]^{\left(\frac{\beta}{\beta+2}\right)} \left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right] \tag{12.14}$$

This expression can be regrouped, as follows:

$$\frac{W_A (A_c)^{(2(\beta-1)/\beta)}}{(MLT)\ell_m^{(2/\beta)}}\left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]^{-\left(\frac{\beta+2}{\beta}\right)} = \frac{\rho\lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4K_u (P_{tot})^{((\beta+2)/\beta)}} \tag{12.15}$$

The terms on the left side of Eq. (12.15) depend on the core geometry, while the terms on the right side depend on specifications regarding the application ($\rho$, $I_{tot}$, $\lambda_1$, $K_u$, $P_{tot}$) and the desired core material ($K_{fe}$, $\beta$). The left side of Eq. (12.15) can be defined as the core geometrical constant $K_{gfe}$:

$$K_{gfe} = \frac{W_A (A_c)^{(2(\beta-1)/\beta)}}{(MLT)\ell_m^{(2/\beta)}}\left[\left(\frac{\beta}{2}\right)^{-\left(\frac{\beta}{\beta+2}\right)} + \left(\frac{\beta}{2}\right)^{\left(\frac{2}{\beta+2}\right)}\right]^{-\left(\frac{\beta+2}{\beta}\right)} \tag{12.16}$$

Hence, to design a transformer, the right side of Eq. (12.15) is evaluated. A core is selected whose $K_{gfe}$ exceeds this value:

$$K_{gfe} \geq \frac{\rho\lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4K_u (P_{tot})^{((\beta+2)/\beta)}} \tag{12.17}$$

The quantity $K_{gfe}$ is similar to the geometrical constant $K_g$ used in the previous chapter to design magnetics when core loss is negligible. $K_{gfe}$ is a measure of the magnetic size of a core, for applications in which core loss is significant. Unfortunately, $K_{gfe}$ depends on $\beta$, and hence the choice of core material affects the value of $K_{gfe}$. However, the $\beta$ of most high-frequency ferrite materials lies in the narrow range 2.6 to 2.8, and $K_{gfe}$ varies by no more than ±5% over this range. Appendix B lists the values of $K_{gfe}$ for various standard ferrite cores, for the value $\beta = 2.7$.

Once a core has been selected, then the values of $A_c$, $W_A$, $\ell_m$, and $MLT$ are known. The peak ac flux density $\Delta B$ can then be evaluated using Eq. (12.13), and the primary turns $n_1$ can be found using Eq. (12.4). The number of turns for the remaining windings can be computed using the desired turns ratios. The various window area allocations are found using Eq. (11.35). The wire sizes for the various windings can then be computed as discussed in the previous chapter,

$$A_{w,j} = \frac{K_u W_A \alpha_j}{n_j} \tag{12.18}$$

where $A_{w,j}$ is the wire area for winding $j$.

## 12.2 A First-Pass Transformer Design Procedure

The procedure developed in the previous sections is summarized below. As in the filter inductor design procedure of the previous chapter, this simple transformer design procedure should be regarded as a first-pass approach. Numerous issues have been neglected, including detailed insulation requirements, conductor eddy current losses, temperature rise, roundoff of number of turns, etc.

The following quantities are specified, using the units noted:

| | | |
|---|---|---|
| Wire effective resistivity | $\rho$ | ($\Omega$-cm) |
| Total rms winding currents, referred to primary | $I_{tot} = \sum_{j=1}^{k} \frac{n_j}{n_i} I_j$ | (A) |
| Desired turns ratios | $n_2/n_1,\ n_3/n_1$, etc. | |
| Applied primary volt-seconds | $\lambda_1 = \int_{\substack{positive\\portion\\of\ cycle}} v_1(t)\,dt$ | (V-sec) |
| Allowed total power dissipation | $P_{tot}$ | (W) |
| Winding fill factor | $K_u$ | |
| Core loss exponent | $\beta$ | |
| Core loss coefficient | $K_{fe}$ | (W/cm$^3$T$^\beta$) |

The core dimensions are expressed in cm:

| | | |
|---|---|---|
| Core cross-sectional area | $A_c$ | (cm$^2$) |
| Core window area | $W_A$ | (cm$^2$) |
| Mean length per turn | $MLT$ | (cm) |
| Magnetic path length | $\ell_m$ | (cm) |
| Peak ac flux density | $\Delta B$ | (Tesla) |
| Wire areas | $A_{w1},\ A_{w2},\ \ldots$ | (cm$^2$) |

The use of centimeters rather than meters requires that appropriate factors be added to the design equations.

### 12.2.1 Procedure

1. *Determine core size.*

$$K_{gfe} \geq \frac{\rho \lambda_1^2 I_{tot}^2 K_{fe}^{(2/\beta)}}{4 K_u (P_{tot})^{((\beta+2)/\beta)}} 10^8 \tag{12.19}$$

Choose a core that is large enough to satisfy this inequality. If necessary, it may be possible to use a smaller core by choosing a core material having lower loss, i.e., smaller $K_{fe}$.

2. *Evaluate peak ac flux density.*

$$\Delta B = \left[ 10^8 \frac{\rho \lambda_1^2 I_{tot}^2}{2K_u} \frac{(MLT)}{W_A A_c^3 \ell_m} \frac{1}{\beta K_{fe}} \right]^{\left(\frac{1}{\beta+2}\right)} \tag{12.20}$$

Check whether $\Delta B$ is greater than the core material saturation flux density. If the core operates with a flux dc bias, then the dc bias plus $\Delta B$ should not exceed the saturation flux density. Proceed to the next step if adequate margins exist to prevent saturation. Otherwise, (1) repeat the procedure using a core material having greater core loss, or (2) use the $K_g$ design method, in which the maximum flux density is specified.

3. *Evaluate primary turns.*

$$n_1 = \frac{\lambda_1}{2\Delta B A_c} 10^4 \tag{12.21}$$

4. *Choose numbers of turns for other windings.*
According to the desired turns ratios:

$$\begin{aligned} n_2 &= n_1 \left( \frac{n_2}{n_1} \right) \\ n_3 &= n_1 \left( \frac{n_3}{n_1} \right) \\ &\vdots \end{aligned} \tag{12.22}$$

5. *Evaluate fraction of window area allocated to each winding.*

$$\begin{aligned} \alpha_1 &= \frac{n_1 I_1}{n_1 I_{tot}} \\ \alpha_2 &= \frac{n_2 I_2}{n_1 I_{tot}} \\ &\vdots \\ \alpha_k &= \frac{n_k I_k}{n_1 I_{tot}} \end{aligned} \tag{12.23}$$

6. *Evaluate wire sizes.*

$$\begin{aligned} A_{w1} &\le \frac{\alpha_1 K_u W_A}{n_1} \\ A_{w2} &\le \frac{\alpha_2 K_u W_A}{n_2} \\ &\vdots \end{aligned} \tag{12.24}$$

Choose wire gauges to satisfy these criteria.

A winding geometry can now be determined, and copper losses due to the proximity effect can be evaluated. If these losses are significant, it may be desirable to further optimize the design by reiterating the above steps, accounting for proximity losses by increasing the effective wire resistivity to the value $\rho_{eff} = \rho_{cu} P_{cu} / P_{dc}$, where $P_{cu}$ is the actual copper loss including proximity effects, and $P_{dc}$ is the copper loss obtained when the proximity effect is negligible.

If desired, the power losses and transformer model parameters can now be checked. For the simple model of Fig. 12.4, the following parameters are estimated:

Magnetizing inductance, referred to winding 1: $L_M = \dfrac{\mu n_1^2 A_c}{\ell_m}$

Peak ac magnetizing current, referred to winding 1: $i_{M,pk} = \dfrac{\lambda_1}{2L_M}$

Winding resistances:

$$R_1 = \frac{\rho n_1 (MLT)}{A_{w1}}$$
$$R_2 = \frac{\rho n_2 (MLT)}{A_{w2}}$$
$$\vdots$$

The core loss, copper loss, and total power loss can be determined using Eqs. (12.1), (12.7), and (12.8), respectively.

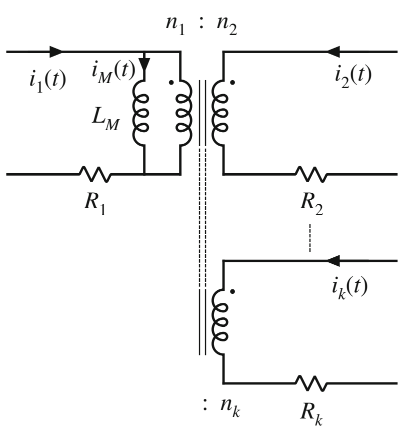


**Fig. 12.4** Computed elements of simple transformer model

## 12.3 Examples

### 12.3.1 Example 1: Single-Output Isolated Ćuk Converter

As an example, let us consider the design of a simple two-winding transformer for the Ćuk converter of Fig. 12.5. This transformer is to be optimized at the operating point shown, corresponding to $D = 0.5$. The steady-state converter solution is $V_{c1} = V_g$, $V_{c2} = V$. The desired transformer turns ratio is $n = n_1/n_2 = 5$. The switching frequency is $f_s = 200$ kHz, corresponding to $T_s = 5\ \mu$s. A ferrite pot core is to be used; at 200 kHz, the chosen ferrite core material is described by the following parameters: $K_{fe} = 24.7\ \mathrm{W/T^{\beta}cm^3}$, $\beta = 2.6$. A fill factor of $K_u = 0.5$ is assumed. Total power loss of $P_{tot} = 0.25$ W is allowed. Copper wire, having a resistivity of $\rho = 1.724 \cdot 10^{-6}\Omega$-cm, is to be used.

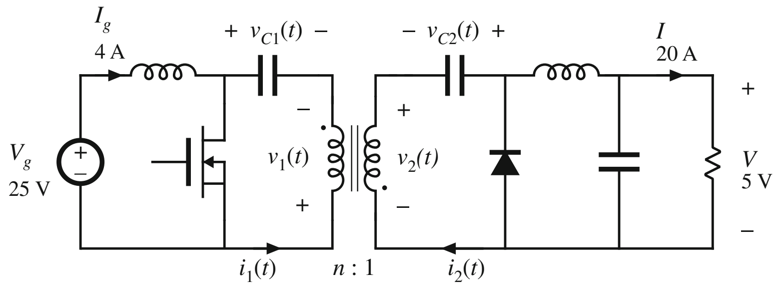


**Fig. 12.5** Isolated Ćuk converter example

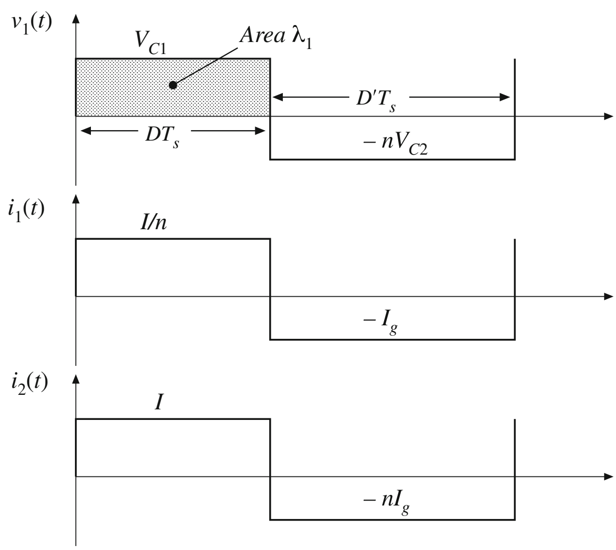


**Fig. 12.6** Waveforms, Ćuk converter transformer design example

Transformer waveforms are illustrated in Fig. 12.6. The applied primary volt-seconds are

$$\begin{aligned} \lambda_1 &= DT_sV_{c1} = (0.5)(5\ \mu\text{sec})(25\ \text{V}) \\ &= 62.5\ \text{V} - \mu\text{sec} \end{aligned} \tag{12.25}$$

The primary rms current is

$$I_1 = \sqrt{D\left(\frac{I}{n}\right)^2 + D'(I_g)^2} = 4\text{A} \tag{12.26}$$

It is assumed that the rms magnetizing current is much smaller than the rms winding currents. Since the transformer contains only two windings, the secondary rms current is equal to

$$I_2 = nI_1 = 20\text{ A} \tag{12.27}$$

The total rms winding current, referred to the primary, is

$$I_{tot} = I_1 + \frac{1}{n} I_2 = 8\text{ A} \tag{12.28}$$

The core size is evaluated using Eq. (12.19):

$$\begin{aligned} K_{gfe} &\geq \frac{(1.724 \cdot 10^{-6})(62.5 \cdot 10^{-6})^2 (8)^2 (24.7)^{(2/26)}}{4(0.5)(0.25)^{(4.6/2.6)}} 10^8 \\ &= 0.00295 \end{aligned} \tag{12.29}$$

The pot core data of Appendix B lists the 2213 pot core with $K_{gfe} = 0.0049$ for $\beta = 2.7$. Evaluation of Eq. (12.16) shows that $K_{gfe} = 0.0047$ for this core, when $\beta = 2.6$. In any event, 2213 is the smallest standard pot core size having $K_{gfe} \leq 0.00295$. The increased value of $K_{gfe}$ should lead to lower total power loss. The peak ac flux density is found by evaluation of Eq. (12.20), using the geometrical data for the 2213 pot core:

$$\begin{aligned} \Delta B &= \left[ 10^8 \frac{(1.724 \cdot 10^{-6})(62.5 \cdot 10^{-6})^2 (8)^2}{2(0.5)} \frac{(4.42)}{(0.297)(0.635)^3 (3.15)} \frac{1}{(2.6)(24.7)} \right]^{(1/4.6)} \\ &= 0.0858\text{Tesla} \end{aligned} \tag{12.30}$$

This flux density is considerably less than the saturation flux density of approximately 0.35 Tesla. The primary turns are determined by evaluation of Eq. (12.21):

$$\begin{aligned} n_1 &= 10^4 \frac{(62.5 \cdot 10^{-6})}{2(0.0858)(0.635)} \\ &= 5.74\text{ turns} \end{aligned} \tag{12.31}$$

The secondary turns are found by evaluation of Eq. (12.22). It is desired that the transformer have a 5:1 turns ratio, and hence

$$n_2 = \frac{n_1}{n} = 1.15\text{ turns} \tag{12.32}$$

In practice, we might select $n_1 = 5$ and $n_2 = 1$. This would lead to a slightly higher $\Delta B$ and slightly higher loss.

The fraction of the window area allocated to windings 1 and 2 are determined using Eq. (12.23):

$$\begin{aligned} \alpha_1 &= \frac{(4\text{A})}{(8\text{A})} = 0.5 \\ \alpha_2 &= \frac{(\frac{1}{5})(20\text{A})}{(8\text{A})} = 0.5 \end{aligned} \tag{12.33}$$

For this example, the window area is divided equally between the primary and secondary windings, since the ratio of their rms currents is equal to the turns ratio. We can now evaluate the primary and secondary wire areas, via Eq. (12.24):

$$A_{w1} = \frac{(0.5)(0.5)(0.297)}{(5)} = 14.8 \cdot 10^{-3}\text{cm}^2$$
$$A_{w2} = \frac{(0.5)(0.5)(0.297)}{(1)} = 74.2 \cdot 10^{-3}\text{cm}^2 \tag{12.34}$$

The wire gauge is selected using the wire table of Appendix B. AWG #16 has area $13.07 \cdot 10^{-3}\text{cm}^2$, and is suitable for the primary winding. AWG #9 is suitable for the secondary winding, with area $66.3 \cdot 10^{-3}\text{cm}^2$. These are very large conductors, and one turn of AWG #9 is not a practical solution! We can also expect significant proximity losses, and significant leakage inductance. In practice, interleaved foil windings might be used. Alternatively, Litz wire or several parallel strands of smaller wire could be employed.

It is a worthwhile exercise to repeat the above design at several different switching frequencies, to determine how transformer size varies with switching frequency. As the switching frequency is increased, the core loss coefficient $K_{fe}$ increases. Figure 12.7 illustrates the transformer pot core size, for various switching frequencies over the range 25 kHz to 1 MHz, for this Ćuk converter example using P material with $P_{tot} < 0.25$ W. Peak flux densities in Tesla are also plotted. For switching frequencies below 250 kHz, increasing the frequency causes the core size to decrease. This occurs because of the decreased applied volt-seconds $\lambda_1$. Over this range, the optimal $\Delta B$ is essentially independent of switching frequency; the $\Delta B$ variations shown occur owing to quantization of core sizes.

For switching frequencies greater than 250 kHz, increasing frequency causes greatly increased core loss. Maintaining $P_{tot} \leq 0.25$W then requires that $\Delta B$ be reduced, and hence the core size is increased. The minimum transformer size for this example is apparently obtained at 250 kHz.

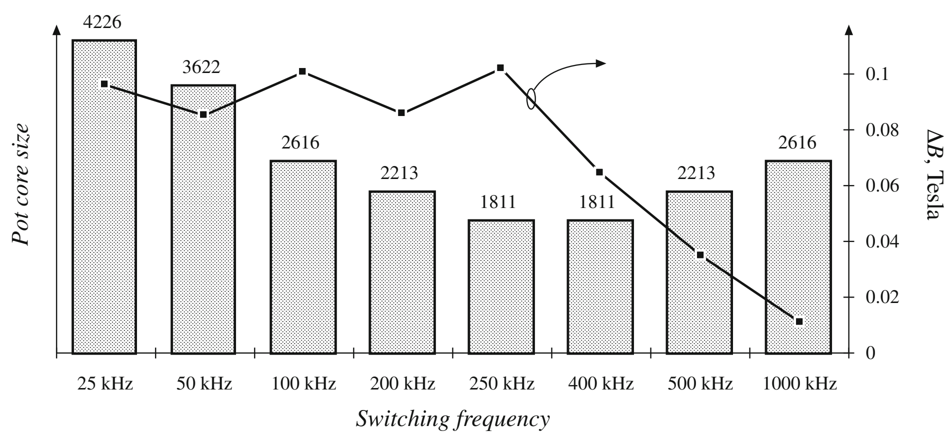


**Fig. 12.7** Variation of transformer size (bar chart) with switching frequency, Ćuk converter example. Optimum peak ac flux density (data points) is also plotted

In practice, several matters complicate the dependence of transformer size on switching frequency. Figure 12.7 ignores the winding geometry and copper losses due to winding eddy currents. Greater power losses can be allowed in larger cores. Use of a different core material may allow higher or lower switching frequencies. The same core material, used in a different application with different specifications, may lead to a different optimal frequency. Nonetheless, examples have been reported in the literature [100–103] in which ferrite transformer size is minimized at frequencies ranging from several hundred kilohertz to several megahertz. More detailed design optimizations can be performed using computer optimization programs [104, 105].

### 12.3.2 Example 2: Multiple-Output Full-Bridge Buck Converter

As a second example, let us consider the design of transformer $T_1$ for the multiple-output full-bridge buck converter of Fig. 12.8. This converter has a 5 V and a 15 V output, with maximum loads as shown. The transformer is to be optimized at the full-load operating point shown, corresponding to $D = 0.75$. Waveforms are illustrated in Fig. 12.9. The converter switching frequency is $f_s = 150$ kHz. In the full-bridge configuration, the transformer waveforms have fundamental frequency equal to one-half of the switching frequency, so the effective transformer frequency is 75 kHz. Upon accounting for losses caused by diode forward voltage drops, one finds that the desired transformer turns ratios $n_1 : n_2 : n_3$ are 110:5: 15. A ferrite EE consisting of Magnetics, Inc. P-material is to be used in this example; at 75 kHz, this material is described by the following parameters: $K_{fe} = 7.6\ \mathrm{W/T^{\beta}cm^3}$, $\beta = 2.6$. A fill factor of $K_u = 0.25$ is assumed in this isolated multiple-output application. Total power loss of $P_{tot} = 4$ W, or approximately 0.5% of the load power, is allowed. Copper wire, having a resistivity of $\rho = 1.724 \cdot 10^{-6}\Omega$-cm, is to be used.

The applied primary volt-seconds are

$$\lambda_1 = DT_sV_g = (0.75)(6.67\ \mu\text{sec})(160\ \text{V}) = 800\ \text{V} - \mu\text{sec} \tag{12.35}$$

The primary rms current is

$$I_1 = \left(\frac{n_2}{n_1}I_{5\text{V}} + \frac{n_3}{n_1}I_{15\text{V}}\right)\sqrt{D} = 5.7\text{A} \tag{12.36}$$

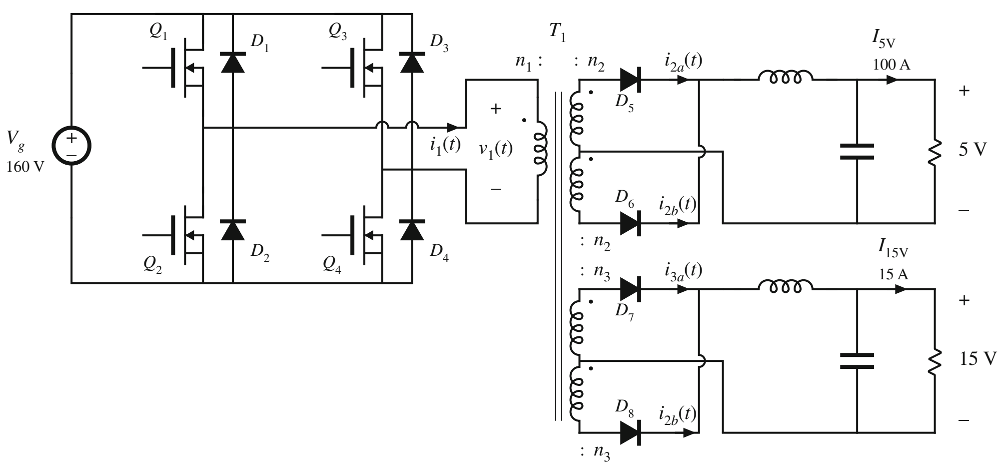


**Fig. 12.8** Multiple-output full-bridge isolated buck converter example

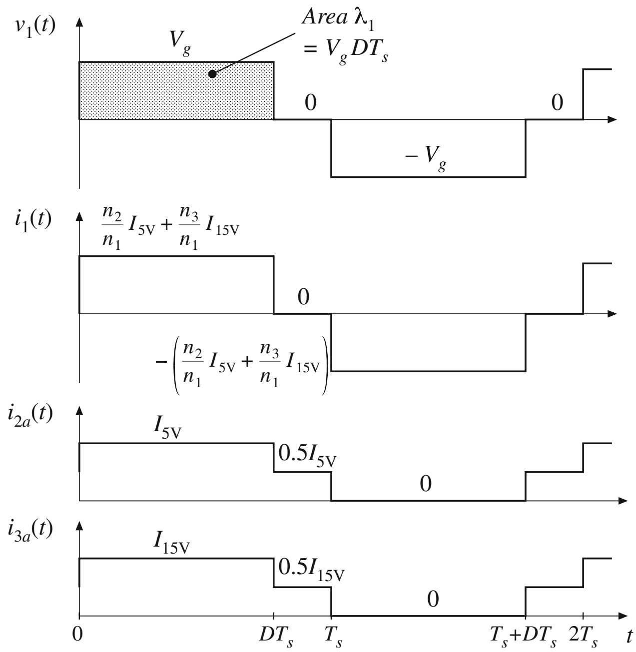


**Fig. 12.9** Transformer waveforms, full-bridge converter example

The 5 V secondary windings carry rms current

$$I_2 = \frac{1}{2} I_{5\,\mathrm{V}} \sqrt{1 + D} = 66.1\mathrm{A} \tag{12.37}$$

The 15 V secondary windings carry rms current

$$I_3 = \frac{1}{2} I_{15\mathrm{V}} \sqrt{1 + D} = 9.9\mathrm{A} \tag{12.38}$$

The total rms winding current, referred to the primary, is

$$\begin{aligned} I_{tot} &= \sum_{\substack{all 5 \\ windings}} \frac{n_j}{n_1} I_j = I_1 + 2\frac{n_2}{n_1} I_2 + 2\frac{n_3}{n_1} I_3 \\ &= 14.4\mathrm{A} \end{aligned} \tag{12.39}$$

The core size is evaluated using Eq. (12.19):

$$\begin{aligned} K_{gfe} &\geq \frac{(1.724 \cdot 10^{-6})(800 \cdot 10^{-6})^2 (14.4)^2 (7.6)^{(2/2.6)}}{4(0.25)(4)^{(4.6/2.6)}} 10^8 \\ &= 0.00937 \end{aligned} \tag{12.40}$$

The EE core data of Appendix B lists the EE40 core with $K_{gfe} = 0.0118$ for $\beta = 2.7$. Evaluation of Eq. (12.16) shows that $K_{gfe} = 0.0108$ for this core, when $\beta = 2.6$. In any event, EE40 is the smallest standard EE core size having $K_{gfe} \le 0.00937$. The peak ac flux density is found by evaluation of Eq. (12.20), using the geometrical data for the EE40 core:

$$\Delta B = \left[ 10^8 \frac{(1.724 \cdot 10^{-6})(800 \cdot 10^{-6})^2 (14.4)^2}{2(0.25)} \frac{(8.5)}{(1.1)(1.27)^3(7.7)} \frac{1}{(2.6)(7.6)} \right]^{(1/46)} \\ = 0.23 \text{ Tesla} \tag{12.41}$$

This flux density is less than the saturation flux density of approximately 0.35 Tesla. The primary turns are determined by evaluation of Eq. (12.21):

$$n_1 = 10^4 \frac{(800 \cdot 10^{-6})}{2(0.23)(1.27)} \\ = 13.7 \text{ turns} \tag{12.42}$$

The secondary turns are found by evaluation of Eq. (12.22). It is desired that the transformer have a 110:5:15 turns ratio, and hence

$$n_2 = \frac{5}{110} n_1 = 0.62 \text{ turns} \tag{12.43}$$

$$n_3 = \frac{5}{110} n_1 = 1.87 \text{ turns} \tag{12.44}$$

In practice, we might select $n_1 = 22$, $n_2 = 1$, and $n_3 = 3$. This would lead to a reduced $\Delta B$ with reduced core loss and increased copper loss. Since the resulting $\Delta B$ is suboptimal, the total power loss will be increased. According to Eq. (12.3), the peak ac flux density for the EE40 core will be

$$\Delta B = \frac{(800 \cdot 10^{-6})}{2(22)(1.27)} 10^4 = 0.143 \text{ Tesla} \tag{12.45}$$

The resulting core and copper loss can be computed using Eqs. (12.1) and (12.7):

$$P_{fe} = (7.6)(0.143)^{2.6}(1.27)(7.7) = 0.47 \text{ W} \tag{12.46}$$

$$P_{cu} = \frac{(1.724 \cdot 10^{-6})(800 \cdot 10^{-6})^2 (14.4)^2}{4(0.25)} \frac{(8.5)}{(1.1)(1.27)^2} \frac{1}{(0.143)^2} 10^8 \\ = 5.4 \text{ W} \tag{12.47}$$

Hence, the total power loss would be

$$P_{tot} = P_{fe} + P_{cu} = 5.9 \text{ W} \tag{12.48}$$

Since this is 50% greater than the design goal of 4 W, it is necessary to increase the core size. The next larger EE core is the EE50 core, having $K_{gfe}$ of 0.0284. The optimum ac flux density for this core, given by Eq. (12.3), is $\Delta B = 0.14$ T; operation at this flux density would require

$n_1 = 12$ and would lead to a total power loss of 2.3 W. With $n_1 = 22$, calculations similar to Eqs. (12.45) to (12.48) lead to a peak flux density of $\Delta B = 0.08$ T. The resulting power losses would then be $P_{fe} = 0.23$ W, $P_{cu} = 3.89$ W, $P_{tot} = 4.12$ W.

With the EE50 core and $n_1 = 22$, the fraction of the available window area allocated to the primary winding is given by Eq. (12.23) as

$$\alpha_1 = \frac{I_1}{I_{tot}} = \frac{5.7}{14.4} = 0.396 \tag{12.49}$$

The fraction of the available window area allocated to each half of the 5 V secondary winding should be

$$\alpha_2 = \frac{n_2 I_2}{n_1 I_{tot}} = \frac{5}{110}\frac{66.1}{14.4} = 0.209 \tag{12.50}$$

The fraction of the available window area allocated to each half of the 15 V secondary winding should be

$$\alpha_3 = \frac{n_3 I_3}{n_1 I_{tot}} = \frac{15}{110}\frac{9.9}{14.4} = 0.094 \tag{12.51}$$

The primary wire area $A_{w1}$, 5 V secondary wire area $A_{w2}$, and 15 V secondary wire area $A_{w3}$ are then given by Eq. (12.24) as

$$\begin{aligned}
A_{w1} &= \frac{\alpha_1 K_u W_A}{n_1} = \frac{(0.396)(0.25)(1.78)}{(22)} = 8.0 \cdot 10^{-3}\text{cm}^2 \\
&\Rightarrow \text{AWG\#19} \\
A_{w2} &= \frac{\alpha_2 K_u W_A}{n_2} = \frac{(0.209)(0.25)(1.78)}{(1)} = 930 \cdot 10^{-3}\text{cm}^2 \\
&\Rightarrow \text{AWG\#8} \\
A_{w3} &= \frac{\alpha_3 K_u W_A}{n_3} = \frac{(0.094)(0.25)(1.78)}{(3)} = 13.9 \cdot 10^{-3}\text{cm}^2 \\
&\Rightarrow \text{AWG\#16}
\end{aligned} \tag{12.52}$$

It may be preferable to wind the 15 V outputs using two #19 wires in parallel; this would lead to the same area $A_{w3}$ but would be easier to wind. The 5 V windings could be wound using many turns of smaller paralleled wires, but it would probably be easier to use a flat copper foil winding. If insulation requirements allow, proximity losses could be minimized by interleaving several thin layers of foil with the primary winding.

## 12.4 AC Inductor Design

The transformer design procedure of the previous sections can be adapted to handle the design of other magnetic devices in which both core loss and copper loss are significant. A procedure is outlined here for design of single-winding inductors whose waveforms contain significant high-frequency ac components (Fig. 12.10). An optimal value of $\Delta B$ is found, which leads to minimum total core plus copper loss. The major difference is that we must design to obtain a given inductance, using a core with an air gap. The constraints and a step-by-step procedure are briefly outlined below.

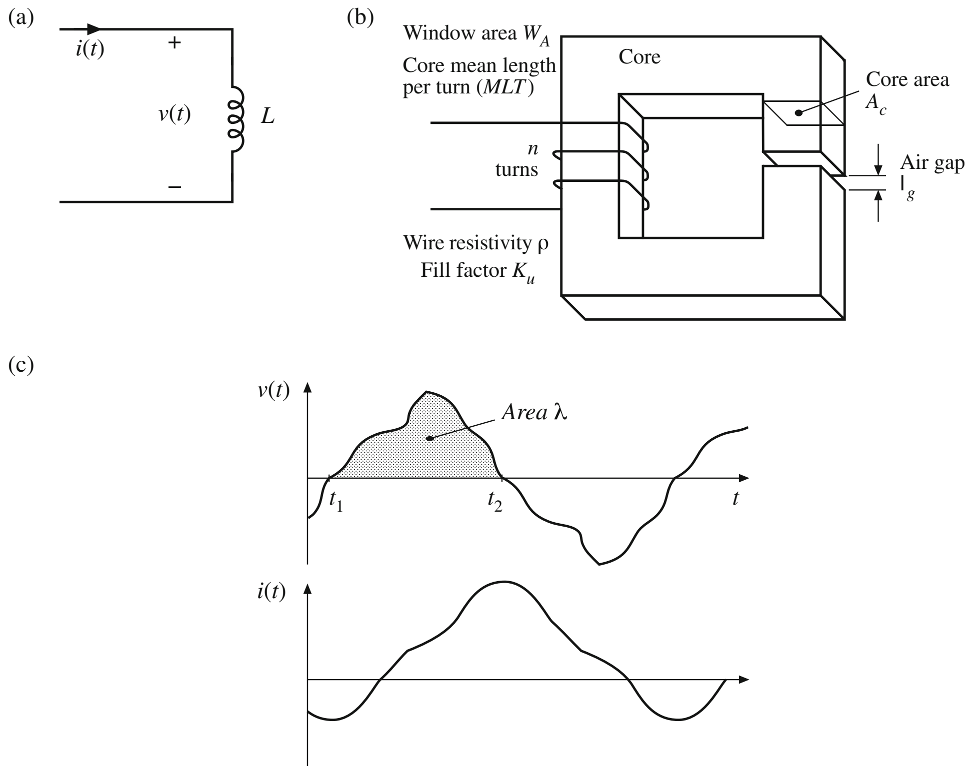


**Fig. 12.10** Ac inductor, in which copper loss and core loss are significant: (**a**) definition of terminal quantities, (**b**) core geometry, (**c**) arbitrary terminal waveforms

### 12.4.1 Outline of Derivation

As in the filter inductor design procedure of the previous chapter, the desired inductance $L$ must be obtained, given by

$$L = \frac{\mu_0 A_c n^2}{\ell_g} \tag{12.53}$$

The applied voltage waveform and the peak ac component of the flux density $\Delta B$ are related according to

$$\Delta B = \frac{\lambda}{2nA_c} \tag{12.54}$$

The copper loss is given by

$$P_{cu} = \frac{\rho n^2 (MLT)}{K_u W_A} I^2 \tag{12.55}$$

where $I$ is the rms value of $i(t)$. The core loss $P_{fe}$ is given by Eq. (12.1).

The value of $\Delta B$ that minimizes the total power loss $P_{tot} = P_{cu} + P_{fe}$ is found in a manner similar to the transformer design derivation. Equation (12.54) is used to eliminate $n$ from the expression for $P_{cu}$. The optimal $\Delta B$ is then computed by setting the derivative of $P_{tot}$ to zero. The result is

$$\Delta B = \left[\frac{\rho \lambda^2 I^2}{2K_u} \frac{(MLT)}{W_A A_c^3 \ell_m} \frac{1}{\beta K_{fe}}\right]^{\left(\frac{1}{\beta+2}\right)} \tag{12.56}$$

which is essentially the same as Eq. (12.13). The total power loss $P_{tot}$ is evaluated at this value of $\Delta B$, and the resulting expression is manipulated to find $K_{gfe}$. The result is

$$K_{gfe} \geq \frac{\rho \lambda^2 I^2 K_{fe}^{(2/\beta)}}{2K_u (P_{tot})^{((\beta+2)/\beta)}} \tag{12.57}$$

where $K_{gfe}$ is defined as in Eq. (12.16). A core that satisfies this inequality is selected.

### 12.4.2 First-Pass AC Inductor Design Procedure

The units of Sect. 12.2 are employed here.

1. *Determine core size.*

$$K_{gfe} \geq \frac{\rho \lambda^2 I^2 K_{fe}^{(2/\beta)}}{2K_u (P_{tot})^{((\beta+2)/\beta)}} 10^8 \tag{12.58}$$

   Choose a core that is large enough to satisfy this inequality. If necessary, it may be possible to use a smaller core by choosing a core material having lower loss, that is, smaller $K_{fe}$.

2. *Evaluate peak ac flux density.*

$$\Delta B = \left[10^8 \frac{\rho \lambda^2 I^2}{2K_u} \frac{(MLT)}{W_4 A_c^3 \ell_m} \frac{1}{\beta K_{fe}}\right]^{\left(\frac{1}{\beta+2}\right)} \tag{12.59}$$

3. *Number of turns.*

$$n = \frac{\lambda}{2\Delta B A_c} 10^4 \tag{12.60}$$

4. *Air gap length.*

$$\ell_g = \frac{\mu_0 A_c n^2}{L} 10^{-4} \tag{12.61}$$

   with $A_c$ specified in $cm^2$ and $\ell_g$ expressed in meters. Alternatively, the air gap can be indirectly expressed via $A_L$(mH/1000 turns):

$$A_L = \frac{L}{n^2} 10^9 \tag{12.62}$$

5. *Check for saturation.*
   If the inductor current contains a dc component $I_{dc}$, then the maximum total flux density $B_{max}$ is greater than the peak ac flux density $\Delta B$. The maximum total flux density, in Tesla, is given by

$$B_{max} = \Delta B + \frac{L I_{dc}}{n A_c} 10^4 \tag{12.63}$$

   If $B_{max}$ is close to or greater than the saturation flux density $B_{sat}$, then the core may saturate. The filter inductor design procedure of the previous chapter should then be used, to operate at a lower flux density.

6. *Evaluate wire size.*

$$A_w \le \frac{K_u W_A}{n} \tag{12.64}$$

A winding geometry can now be determined, and copper losses due to the proximity effect can be evaluated. If these losses are significant, it may be desirable to further optimize the design by reiterating the above steps, accounting for proximity losses by increasing the effective wire resistivity to the value $\rho_{eff} = \rho_{cu} P_{cu}/P_{dc}$, where $P_{cu}$ is the actual copper loss including proximity effects, and $P_{dc}$ is the copper loss predicted when the proximity effect is ignored.

7. *Check power loss.*

$$\begin{aligned} P_{cu} &= \frac{\rho n (MLT)}{A_w} I^2 \\ P_{fe} &= K_{fe} (\Delta B)^{\beta} A_c \ell_m \\ P_{tot} &= P_{cu} + P_{fe} \end{aligned} \tag{12.65}$$

## 12.5 Summary

1. In a multiple-winding transformer, the low-frequency copper losses are minimized when the available window area is allocated to the windings according to their apparent powers, or ampere-turns.
2. As peak ac flux density is increased, core loss increases while copper losses decrease. There is an optimum flux density that leads to minimum total power loss. Provided that the core material is operated near its intended frequency, then the optimum flux density is less than the saturation flux density. Minimization of total loss then determines the choice of peak ac flux density.
3. The core geometrical constant $K_{gfe}$ is a measure of the magnetic size of a core, for applications in which core loss is significant. In the $K_{gfe}$ design method, the peak flux density is optimized to yield minimum total loss, as opposed to the $K_g$ design method where peak flux density is a given specification.

## Problems

**12.1** Forward converter inductor and transformer design. The objective of this problem set is to design the magnetics (two inductors and one transformer) of the two-transistor, two-output forward converter shown in Fig. 12.11. The ferrite core material to be used for all three devices has a saturation flux density of approximately 0.3 T at 120°C. To provide a safety margin for your designs, you should use a maximum flux density $B_{max}$ that is no greater than 75% of this value. The core loss at 100 kHz is described by Eq. (12.1), with the parameter values $\beta = 2.6$ and $K_{fe} = 50 \mathrm{W/T^{\beta} cm^3}$. Calculate copper loss at 100°C.

*Steady-state converter analysis and design*. You may assume 100% efficiency and ideal lossless components for this section.

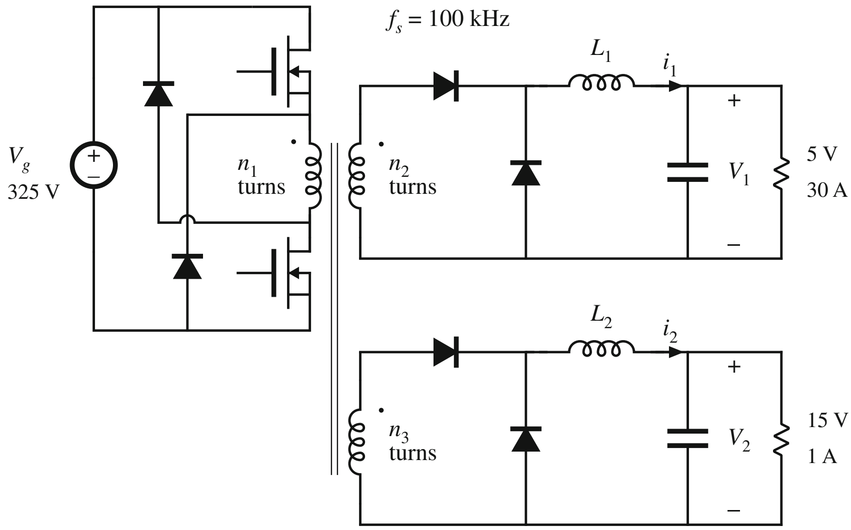


**Fig. 12.11** Two-output forward converter of Problem 12.1

(**a**) Select the transformer turns ratios so that the desired output voltages are obtained when the duty cycle is $D = 0.4$.
(**b**) Specify values of $L_1$ and $L_2$ such that their current ripples $\Delta i_1$ and $\Delta i_2$ are 10% of their respective full-load current dc components $I_1$ and $I_2$.
(**c**) Determine the peak and rms currents in each inductor and transformer winding.

*Inductor design*. Allow copper loss of 1 W in $L_1$ and 0.4 W in $L_2$. Assume a fill factor of $K_u = 0.5$. Use ferrite EE cores—tables of geometrical data for standard EE core sizes are given in Appendix B. Design the output filter inductors $L_1$ and $L_2$. For each inductor, specify:

(*i*) EE core size
(*ii*) Air gap length
(*iii*) Number of turns
(*iv*) AWG wire size

*Transformer design*. Allow a total power loss of 1 W. Assume a fill factor of $K_u = 0.35$ (lower than for the filter inductors, to allow space for insulation between the windings). Use a ferrite EE core. You may neglect losses due to the skin and proximity effects, but you should include core and copper losses. Design the transformer, and specify the following:

(*i*) EE core size
(*ii*) Turns $n_1$, $n_2$, and $n_3$
(*iii*) AWG wire size for the three windings

Check your transformer design:

(*iv*) Compute the maximum flux density. Will the core saturate?
(*v*) Compute the core loss, the copper loss of each winding, and the total power loss

**12.2** A single-transistor forward converter operates with an input voltage $V_g = 160$ V, and supplies two outputs: 24 V at 2 A, and 15 V at 6 A. The duty cycle is $D = 0.4$. The turns ratio between the primary winding and the reset winding is 1:1. The switching frequency is 100 kHz. The core material loss equation parameters are $\beta = 2.7$, $K_{fe} = 50$. You may assume a fill factor of 0.25. Do not allow the core maximum flux density to exceed 0.3 T. Design a transformer for this application, having a total power loss no greater than 1.5 W at 100°C. Neglect proximity losses. You may neglect the reset winding. Use a ferrite PQ core. Specify: core size, peak ac flux density, wire sizes, and number of turns for each winding. Compute the core and copper losses for your design.

**12.3** Flyback/SEPIC transformer design. The "transformer" of the flyback and SEPIC converters is an energy storage device, which might be more accurately described as a multiple-winding inductor. The magnetizing inductance $L_p$ functions as an energy-transferring inductor of the converter, and therefore the "transformer" normally contains an air gap. The converter may be designed to operate in either the continuous or discontinuous conduction mode. Core loss may be significant. It is also important to ensure that the peak current in the magnetizing inductance does not cause saturation.

A flyback transformer is to be designed for the following two-output flyback converter application:

| | |
|---|---|
| Input: | 160 Vdc |
| Output 1: | 5 Vdc at 10 A |
| Output 2: | 15 Vdc at 1 A |
| Switching frequency: | 100 kHz |
| Magnetizing inductance $L_p$: | 1.33 mH, referred to primary |
| Turns ratio: | 160: 5: 15 |
| Transformer power loss: | Allow 1 W total |

(**a**) Does the converter operate in CCM or DCM? Referred to the primary winding, how large are (*i*) the magnetizing current ripple $\Delta i$, (*ii*) the magnetizing current dc component $I$, and (*iii*) the peak magnetizing current $I_{pk}$?

(**b**) Determine (*i*) the rms winding currents, and (*ii*) the applied primary volt-seconds $\lambda_1$. Is $\lambda_1$ proportional to $I_{pk}$?

(**c**) Modify the transformer and ac inductor design procedures of this chapter, to derive a general procedure for designing flyback transformers that explicitly accounts for both core and copper loss, and that employs the optimum ac flux density that minimizes the total loss.

(**d**) Give a general step-by-step design procedure, with all specifications and units clearly stated.

(**e**) Design the flyback transformer for the converter of part (a), using your step-by-step procedure of Part (d). Use a ferrite EE core, with $\beta = 2.7$ and $K_{fe} = 50\mathrm{W/T^{\beta}cm^3}$. Specify: core size, air gap length, turns, and wire sizes for all windings.

(**f**) For your final design of part (e), what are (*i*) the core loss, (*ii*) the total copper loss, and (*iii*) the peak flux density?

**12.4** Over the intended range of operating frequencies, the frequency dependence of the core loss coefficient $K_{fe}$ of a certain ferrite core material can be approximated using a monotonically increasing fourth-order polynomial of the form

$$K_{fe}(f) = K_{fe0}\left(1 + a_1\left(\frac{f}{f_0}\right) + a_2\left(\frac{f}{f_0}\right)^2 + a_3\left(\frac{f}{f_0}\right)^3 + a_4\left(\frac{f}{f_0}\right)^4\right)$$

where $K_{fe0}$, $a_1$, $a_2$, $a_3$, $a_4$, and $f_0$ are constants. In a typical converter transformer application, the applied primary volt-seconds $\lambda_1$ varies directly with the switching period $T_s = 1/f$. It is desired to choose the optimum switching frequency such that $K_{gfe}$, and therefore the transformer size, are minimized.

(**a**) Show that the optimum switching frequency is a root of the polynomial

$$1 + a_1\left(\frac{\beta-1}{\beta}\right)\left(\frac{f}{f_0}\right) + a_2\left(\frac{\beta-2}{\beta}\right)\left(\frac{f}{f_0}\right)^2 + a_3\left(\frac{\beta-3}{\beta}\right)\left(\frac{f}{f_0}\right)^3 + a_4\left(\frac{\beta-4}{\beta}\right)\left(\frac{f}{f_0}\right)^4$$

Next, a core material is chosen whose core loss parameters are

$$\begin{aligned} \beta &= 2.7 & K_{fe0} &= 7.6 \\ f_0 &= 100\,\text{kHz} & & \\ a_1 &= -1.3 & a_2 &= 5.3 \\ a_3 &= -0.5 & a_4 &= 0.075 \end{aligned}$$

The polynomial fits the manufacturer's published data over the range 10 kHz<f<1 MHz.

(**b**) Sketch $K_{fe}$ vs. $f$

(**c**) Determine the value of $f$ that minimizes $K_{gfe}$.

(**d**) Sketch $K_{gfe}(f)/K_{gfe}(100\,\text{kHz})$, over the range 100 kHz $\le f \le$ 1 MHz. How sensitive is the transformer size to the choice of switching frequency?

**12.5** Transformer design to attain a given temperature rise. The temperature rise $\Delta T$ of the center leg of a ferrite core is directly proportional to the total power loss $P_{tot}$ of a transformer: $\Delta T = R_{th}P_{tot}$, where $R_{th}$ is the thermal resistance of the transformer under given environmental conditions. You may assume that this temperature rise has minimal dependence on the distribution of losses within the transformer. It is desired to modify the $K_{gfe}$ transformer design method, such that temperature rise $\Delta T$ replaces total power loss $P_{tot}$ as a specification. You may neglect the dependence of the wire resistivity $\rho$ on temperature.

(**a**) Modify the $n$-winding transformer $K_{gfe}$ design method, as necessary. Define a new core geometrical constant $K_{th}$ that includes $R_{th}$.

(**b**) Thermal resistances of ferrite EC cores are listed in Sect. B.3 of Appendix B. Tabulate $K_{th}$ for these cores, using $\beta = 2.7$.

(**c**) A 750 W single-output full-bridge isolated buck dc–dc converter operates with converter switching frequency $f_s$ = 200 kHz, dc input voltage $V_g$ = 400 V, and dc output voltage $V$ = 48 V. The turns ratio is 6:1. The core loss equation parameters at 100 kHz are $K_{fe}$ = 10 W/T$^{\beta}$cm$^3$ and $\beta = 2.7$. Assume a fill factor of $K_u = 0.3$. You may neglect proximity losses. Use your design procedure of parts (a) and (b) to design a transformer for this application, in which the temperature rise is limited to 20°C. Specify: EC core size, primary and secondary turns, wire sizes, and peak ac flux density.