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Preface

Every electrical engineer deals with magnetics, whether as mere inductors, or as transformers, or maybe even with motors and generators. It is hard to imagine any electrical engineering application without magnetic components. Despite magnetics being ubiquitous in electrical engineering, a large proportion of electrical engineers do not have an in-depth understanding of the working of magnetic components. Furthermore, despite the increased availability of documentation related to magnetics provided by manufacturers, it is still difficult to find documentation that enables an engineer to gain in-depth understanding of magnetics from these application notes.

One of the greatest challenges to learning about electromagnetism is that it is a phenomenon that is difficult to perceive. Learning magnetism only through equations and mathematical analysis will not lead to an understanding of electromagnetism. In this book, a visual approach to understanding electromagnetism is chosen by returning to the fundamental laws of physics and imagining the magnetic field produced in the magnetic components used in electrical engineering. This book uses only the basic laws of physics that have been known for close to 200 years to interpret the operation of magnetics.

This book extensively uses simulation as a learning tool. Moreover, by using the free and open source circuit simulator Python Power Electronics, the simulations in the book are universally accessible to a reader in any part of the world. The book uses the fundamental laws of physics to build simulation models for magnetic components. Therefore, through the process of simulations, the book builds a bridge between the world of electrical engineering and electromagnetism. Simulations allow the reader the opportunity to alter simulation models and examine the effects without any fear of physical damage that might occur in hardware.

This book will not be considered "heavy" reading. This is intentional, as the target audience of these books are not just students of power engineering but also practising electrical engineers. This book uses mainly a process of discussion and questioning, rather than mere mathematical analysis to describe the concept of electromagnetism. The book examines in every case different scenarios and variations that help the reader to apply fundamental laws in different conditions and interpret the results. By going back and forth between simulations and theory, the book will provide a hands-on learning experience for both students of electrical engineering and practising engineers.

Hamburg, Germany Shivkumar V. Iyer

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Chapter 1
Introduction

1.1 Why Another Book on Magnetism?

Any reader who glances at the title of this book will immediately ask this question—"Is there any need to publish another book on simulating magnetics?" Either this book will contain a collection of simulations already found in many other books or pulled from the documentation of simulation software that offer ready-made models for magnetic components. This section will address this question. To answer this question, I will describe my own experiences as a power engineer that motivated me to write this book. I strongly believe that many other power engineers have had similar experiences and this book will be of significant use to the power community.

As undergraduates, all electrical engineers learn about magnetic components such as inductors, coupled inductors and transformers in basic courses introducing us to electrical engineering [1, 2]. In these courses, we learn how to analyse electrical circuits and systems using basic network laws. We also learn how to represent an electrical circuit or system by an equivalent network, which can then be analysed using these basic network laws. As we progress to more advanced courses, we apply our basic knowledge to learn about the construction and working of electrical machines such as transformers, motors and generators [3, 4]. Therefore, as undergraduates, we have significant exposure to magnetics in electrical engineering from a theoretical perspective. Additionally, we also learn the practical aspects of machines through laboratory experiments.

Despite all these courses, most electrical engineers struggle with a deeper understanding of magnetic components especially with transformers, motors and generators. This is partly due to the inherent difficulty in perceiving a phenomenon like magnetism when the lines of force (or flux lines) of the magnetic field are a bit tricky to visualize. Nowadays, there are software that allow us to plot the flux lines of magnetic fields [5]. These software are used extensively when designing electrical machines such as motors and generators, as accurate mapping of the magnetic field helps to improve the performance of these machines. These software use fairly

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specialized techniques to arrive at these solutions. Moreover, these software are usually proprietary and not available in all universities and even if they are available, they may not be accessible by undergraduates for course work.

Without diving into the details of magnetism, most simulation software through which one can simulate an electrical circuit offer ready-made simulation models of most magnetic components such as inductors, transformers, motors and generators [6–9]. Advanced simulation software also offer simulation models of fairly advanced machines. Using these software, a student can simulate a circuit that contains an electrical machine without too much effort. All one has to do is to drag-and-drop the component from the library of the simulation software into a circuit schematic. Though simulating circuits with machines is now far simpler, these simulations do not help to understand the inner working of the machines in the simulation.

At this point, I would like to recount my experiences and why I needed to relearn the basics of how magnetism and electrical engineering are interconnected. After my undergraduate studies, I spent many years as a graduate student when I did research in power quality improvement and microgrids. I performed simulations using proprietary simulation software using in-built model of transformers and at times induction motors. All I had to do was enter the parameters of the transformer in a dialog box that contained fields for specifying the power rating, voltage ratings of the windings and also optionally the leakage inductance and magnetizing inductance. Other than that, I just had to connect the transformer to the rest of the circuit. I must have performed close to a hundred simulation studies during my time as a graduate student without ever needing to know about what the simulation model contained as all I needed was the transformation of voltage and current that was provided by the transformer component.

Later when I was working in the R&D department of a company, I was assigned a project where I had to design a 5 kW dc–ac converter that would charge and discharge a 12 V battery from a 120 V ac grid. This dc–ac converter would feature a 20 kHz H-bridge converter that was interfaced to the grid through a high frequency transformer. To begin with, a 5–10 kVA, 12 V:120 V, 20 kHz high frequency transformer could not be directly purchased. I needed to contact a few manufacturers to design and fabricate the transformer. One manufacturer was willing to fabricate the transformer and after a week, submitted the first design for my approval. And this is when I realized I had absolutely no idea how I could verify the design submitted by the manufacturer. If I approved the design without rigorously verifying it, I would be held responsible if the converter did not work and another transformer had to be fabricated.

Thankfully, I could search the internet for articles on transformer design [10–15]. I read many articles written by practising engineers, professors and electronics enthusiasts, and also design notes released by a few companies that manufactured transformer cores. After several days of reading, I finally managed to assemble the formulas needed to design the transformer. Though I managed to verify the manufacturer's design, I had barely managed to understand why those formulas were being used and why they were what they were to begin with. This is when

I realized that if I ever needed a very special design of a transformer in the future, I would be in big trouble. One should not have to follow a set of formulas like a set of commandments to arrive at a design. Since transformers and other magnetics are being used heavily by power engineers, a deeper knowledge of their working was fairly important.

In the recent years, after having become an online teacher and author, I have been gradually revisiting the list of topics that I felt needed more in-depth literature. In this book, I have written about how one can simulate and analyse magnetic components like inductors and coupled inductors, and an electrical machine such as a transformer. The book does not describe the design process of magnetic components as that is quite often closely tied to the application. Rather, the book focuses on understanding how magnetics plays a role in electrical engineering using only the most basic laws of physics that are learned in high school.

How does this book stand apart from other books in this topic? This book has been written by a power engineer and for power engineers. There are books on magnetics that will present detailed solutions to map the magnetic field. This book will not cover these techniques, as for static electrical machines, such a level of detail of the magnetic field is quite often unnecessary. This book will assume uniform magnetic fields and linear magnetic properties that are quite often used in the design of magnetics. This book will describe to the reader how we can build a bridge between electricity and magnetism, by connecting electrical quantities with magnetic quantities and formulating expressions between them.

As electrical engineers, what we are familiar with is the voltage that is applied to energize a system, and the voltage measurable at designated terminals. However, this book will help a reader to fill in the gaps—how does the magnetic field that is produced lead to these induced voltages. With code samples in Python and a detailed description of how simulation models can be gradually developed, this book will give the reader the tools to learn about the working of magnetic components by simulation and example. The most important aspect of this book is how it describes the working of a transformer with several windings, using just a few fundamental laws of physics, and describes to the reader how these laws can be applied to understand the operation of magnetic machines.

1.2 The Importance of Magnetism in Electrical Engineering

In the previous section, I described the reason why I wrote this book. In this section, let us talk about why magnetism is important in electrical engineering. Before we jump into the importance of magnetism, let us broadly talk about how we perceive the appliances that we use every day. In these modern times, everyone is surrounded by a vast number of appliances and systems, and most of us would not realize the nature of these appliances and where magnetism might play a role in them. Magnetism might be critical to the functioning of these appliances and in some cases, any magnetic field produced by them might be a nuisance as well.

For a vast majority of us who do not work in the field, exposure to heavy electrical equipment is limited, let us therefore begin with consumer appliances. Almost everyone nowadays has a smart phone and a few other gadgets such as tablets and laptops. One would not expect a magnetic field to be produced by electronic appliances such as these, as health and safety guidelines are fairly stringent in how strong a magnetic field an appliance can produce [16, 17]. Moreover, we expect these appliances to be adhering to Electromagnetic Compatibility (EMC) standards that require these appliances to not produce a magnetic field strong enough to disrupt other appliances [18]. Despite all these requirements, none of these appliances would function without magnetic components.

Let us consider as an example, our smart phones. Even though we would like to limit the magnetic field produced by our smart phones to the bare minimum, the power needed to run the mobile circuits is derived from dc–dc converters that convert the voltage available from the phone battery to a stable power supply for the mobile circuits. The fact that our mobiles have been getting thinner and lighter despite the fact that they have been getting more computation power is the result of higher density batteries and efficient dc–dc converters. These dc–dc converters will have at least an inductor if not also a high frequency transformer. These inductors are magnetic components, and their fundamental basis of operation is the magnetic field produced in the core of the inductor.

The fact that these mobile phones are EMC approved is due to the shielding that is provided in all these appliances to ensure that magnetic fields produced are limited and localized. This is just one example of a circuit in the mobile phone that uses magnetism to function. This points to the fact that even in appliances where we do not expect a magnetic field to play a role, due to the power conditioning that is necessary to supply the main circuits, almost every appliance produces a magnetic field. Moreover, this magnetic field cannot be said to be a side effect or simply unintentional, rather, it is critical to its functioning. One could say, however, that the magnetic field that is produced should remain local to the circuit where it is necessary and not leak into the surroundings.

Let us now progress from a few sample electronic appliances to the power adaptors or chargers that they use [19, 20]. A mobile phone can be nowadays be charged by any Universal Serial Bus (USB) cable that is connected either to a computer's USB port or to a dedicated charger plugged into a 240 V, 50 Hz (or 120 V, 60 Hz) domestic power outlet. Let us consider the case of the charger plugged into the domestic power outlet. Such a charger usually has an ac–dc converter that converts the 240 V, 50 Hz (or 120 V, 60 Hz) ac voltage to a 5 V dc voltage with which the battery of the mobile can be charged. There are several ways to implement such an ac–dc converter. One possible implementation is to use a flyback converter that comprises of a power device in combination with a high frequency transformer. This will be described in Chap. 5 along with a simulation. In the flyback converter, the high frequency transformer plays a critical role in its operation [19, 20]. Therefore, this is an example of magnetism playing a fundamental role in the operation of the appliance—the mobile phone charger.

We have considered a mobile phone and the charger of the mobile as an example to describe how ubiquitous magnetic components are in our daily lives. Let us examine also a few other cases of appliances in our homes and offices. Almost every home will have several appliances that use electric motors. As an example, a refrigerator has a compressor that compresses freon gas and pumps it around the refrigerator compartment to keep the compartment cool. This compressor has a motor that performs the task of compression and pumping. Depending on the capacity of the refrigerator, the motor can be of different types—single-phase induction motor or three-phase induction motor. Every motor uses a magnetic field to convert electrical energy into mechanical energy. Besides refrigerators, motors are used in a wide variety of other appliances such as fans, mixers/grinders, washing machines, dishwashers etc. The motors used in these appliances will differ as the requirements of the application are different. However, in all cases, the presence of the motor implies the presence of a machine based on magnetism.

Another application of electric motors has been in the steady adoption of electric vehicles (EVs). A decade ago, an EV was the possession of a few enthusiasts. However, in current times, due to the decreasing cost of EVs and the incentives provided for the sale and purchase of EVs, their penetration in the automobile market has steadily increased. Many countries have gone so far as to pledge to eradicate fossil fuel-based cars in a few decades. At the heart of an EV is an electric motor powered by a battery. Popular batteries for EVs are lithium-ion batteries that provide cars a range of several hundreds of kilometres on a full charge. The motor used by the EV once again depends on the type of vehicle. A simple e-bike or e-scooter could use a simple single-phase induction motor, while a full-size Sports Utility Vehicle (SUV) could use a three-phase induction motor or a three-phase synchronous motor. In an EV, magnetism plays a role not only in the motor but also in the converter that supplies this motor and controls the energy needed by the EV to accelerate and decelerate.

Now that we have described a few applications that a non-engineer might be exposed to, let us describe a few industrial applications that are specific to engineering. In industries, close to 70% of the load is considered to be some form of motor load. As an example, motors are used for applications such as crushers, rollers, mixers, centrifuges etc. in industries such as chemical factories, iron and steel factories, pharmaceutical factories, cement factories etc. These motors can range from a few kilowatts (kWs) to several megawatts (MWs). The type of motors used also varies greatly depending on the application. A vast majority of motors used are induction motors with squirrel cage motors being preferred due to their robustness and low maintenance. Wound-rotor induction motors, synchronous motors and switched reluctance motors are several other types of motors used. In an industrial setting, one cannot imagine any industry functioning without motors. This points to how critical magnetic machines are for industry.

Now that we have started our discussion about industrial applications, let us talk about the power system that makes it all possible [21]. More than a hundred years back when industries began to be electrified, every factory had its own dedicated generator. From those rudimentary beginnings, we have now reached the modern power system that is a huge network of transmission lines connecting generating stations scattered over huge landmasses and powering industries and homes in cities over entire continents. One of the most critical components that makes it possible to have such a diverse power system is the transformer, which is described in detail in Chap. 4 [13]. The transformer enables us to raise or lower the voltage by interfacing isolated electrical systems through a magnetic field. This makes it possible to generate power at 690 V using a thermal generator and transmitting it to cities hundreds of kilometres away at hundreds of kilovolts (kVs).

An electrical engineer may find himself or herself working in different domains. He or she could be designing power supplies for consumer appliances, or could be working in motor controls, or could be working in a generating station. Wherever an electrical engineer ends up, he or she is never far away from magnetic components or magnetic machines. Therefore, to understand the basics of how magnetic components and magnetic machines work is essential for any electrical engineer. The detail to which an engineer wishes to learn will vary according to his or her position and the job requirements. However, a basic knowledge of magnetism will help in every case. The purpose of this book is to help an engineer translate fundamental laws of physics into the world of electrical engineering and learn through the process of simulation the role that magnetism plays in electrical engineering. To make these initial steps easier, this book will only consider static magnetic components such as inductors and coupled inductors, and static electrical machines such as transformers.

1.3 Back to the Basic Laws of Physics

The objective of the previous section was to motivate an electrical engineer to learn how magnetism plays a role in electrical engineering either through the magnetic components used in circuits or in the machines that are in use. As already stated in the first section of this chapter, this topic has already been dealt with in dozens if not hundreds of books and other forms of literature. This section will describe the approach used in this book and how that differs from the approach taken in prior art.

This book will describe the working of magnetics through simulations, by formulating simulation models of these magnetics using the basic laws of physics. It is important to emphasize that all engineering analysis in general is a process of expressing physical laws using mathematical equations such that these may be solved and analysed. Therefore, the basic laws of physics are the building blocks of all engineering as we know it. Let us clarify what we mean by the title of this section "Back to the Basic Laws of Physics". During our studies, most engineers learn new concepts only through mathematical equations. Unfortunately, most of us normal human beings do not learn through equations but rather either visually, or through discourse, or through a process of questions and answers. Therefore, using a purely mathematical approach to learning about magnetism will lead to gaps in understanding how things "work under the hood".

In this book, all discussions and analysis of simulation results will always refer to the basic laws of physics rather than engineering equations that have been derived from these. In most cases, engineering equations that are fairly concise as a result of being simplified can be derived from basic physical laws in just a few steps. Therefore, a repeated reference to the basic laws of physics will not increase the burden in creating simulation models. Rather, the application of these basic physical laws and the discourse surrounding their application will lead to a deeper understanding of engineering processes. Besides mathematical expressions and a detailed discussion on them, this book will also use simulations as a tool to accompany learning. The next section will describe the usefulness of simulations.

If one wants to learn about static magnetic components such as inductors or coupled inductors, and static electrical machines such as transformers, a fairly robust understanding can be built using Kirchhoff's Voltage and Current Laws, Ampere's Law, Faraday's Law and Lenz's Law. If we were to progress to moving (or rotating) machines such as motors and generators, a few additional physical laws are needed to understand how mechanical force is produced (motors) or results in electrical energy being produced (generators). However, in this book, we will cover only the modelling and simulation of inductors, coupled inductors and transformers. In this section, we will describe these laws that will be used, such that they can be used directly in the later chapters of this book.

Kirchhoff's Voltage and Current Laws (KVL and KCL) also knows as the Kirchhoff's network laws were described by Gustav Kirchhoff in 1845 [1, 2, 22]. These laws became fundamental laws in circuit analysis and it is hard to imagine electrical engineering without these laws as they are used by every electrical engineer to solve circuits. These laws are applicable to circuits with lumped elements. A lumped element is when an element such as a resistor, inductor or capacitor is represented as an element between terminals across which a definite law can be applied representing the element as a parameter. As an example, the following law is applicable for resistors:

$$v = iR \tag{1.1}$$

where v is the voltage across the resistor terminals, i is the current through the resistor and R is the resistance of the resistor.

We have represented a component by a single parameter, namely the resistance R. However, a resistor can be manufactured in several different ways and using many different materials. Depending on the construction and the nature of materials used, the actual relationship between the voltage v across the terminals and the current i flowing through it may not be representable by a single parameter R. As an example, there will be a maximum current that can be allowed to flow through the resistor. If a current larger than this maximum value flows through the resistor, it will be damaged. If however, we allow a current to flow through the resistor that is fairly close to the maximum limit, the above relationship may not hold true, as a certain degree of damage might already have begun. If one wishes to represent other physical phenomena that occur within the resistor, the above law will not be

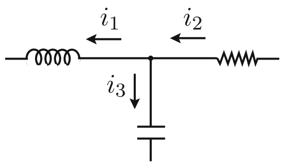


Fig. 1.1 Kirchhoff's Current Law applied at a node in a circuit

applicable. Subsequently, Kirchhoff's laws that use lumped parameters such as the resistance R above will not also be applicable if a circuit does not have lumped elements, but rather, we must express detailed physical phenomena within each element of the circuit.

For most engineering analysis, assuming lumped elements in a circuit is quite reasonable. With this assumption, Kirchhoff's Current Law (KCL) can be stated as—"the algebraic sum of currents in a network of conductors meeting at a point is zero" [1, 2, 22]. Figure 1.1 shows how KCL can be applied in a circuit. In a circuit, the meeting point of more than two conductors in a circuit is called a node. Figure 1.1 shows three conductors meeting at a node. As per KCL, the following expression can be written for the currents i_1, i_2 and i_3 flowing in the three branches meeting at the node:

$$i_1 - i_2 + i_3 = 0 \tag{1.2}$$

Let us arrive at the above result step by step. KCL can be applied at every node in the circuit. In order to apply KCL at a particular node of the circuit, we must assume directions of currents in all the branches that are meeting at the node. While performing circuit analysis, we need to assume directions of currents in every branch of the circuit. If the current in reality is flowing in a direction reverse to what has been assumed, it only means that current will have a negative value. Once we assume directions of all currents in the branches meeting at a node, we consider the current entering at a node to be a negative value and the current leaving a node to be a positive value. KCL states that the sum total of the current entering a node must be equal to the sum total of the current leaving a node. In other words, all the charge that enters a node must leave it, or else there will be charge accumulating at a node. In Fig. 1.1, we could have assumed all the currents entering the node or even all the currents leaving the node. Any random assumption of current directions is acceptable as long as we consider the appropriate sign while writing the KCL equation.

As KCL can be applied at every node in a circuit, we can write N equations for a circuit with N nodes. Typically, one node in a circuit will be a reference node and, therefore, only $N - 1$ equations will be independent, and the Nth equation can be expressed as a linear combination of the remaining equations [1]. If the number of branches in the circuit is B, there will be B current variables. Therefore, using KCL, we can write $N - 1$ equations in B variables for a circuit with N nodes and B branches. These equations can be solved simultaneously to calculate the value of the currents in the circuit. Additionally, if the elements in a branch are known, one

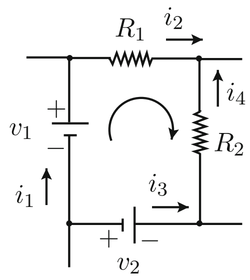


Fig. 1.2 Kirchhoff's Voltage Law applied in a closed loop in a circuit

can express the current in a branch as an expression with respect to the voltages of the nodes at the terminals of the branch. To learn about the various techniques by which KCL can be used to analyse a circuit, the reader is encouraged to read the references [1, 2, 22]. KCL is a very powerful technique in circuit analysis and is used extensively by electrical engineers.

While KCL can be applied at every node in a circuit, Kirchhoff's Voltage Law (KVL) can be applied in every closed loop of a circuit. The statement of KVL is as follows—"the directed sum of the potential differences (voltages) around any closed loop is zero" [1, 2, 22]. Figure 1.2 shows KVL being applied in a closed loop in a circuit. To begin with, let us define a closed loop in a circuit. As shown in Fig. 1.2, a closed loop in a circuit is a closed path that we can trace if we were to start at any one node in a circuit, progress through at least one branch in the circuit and return back to the same node where we began. Quite obviously, for a circuit with several nodes and branches, one could trace several such closed loops if we began at different nodes, passed through different branches connected to the node and eventually find our way back to the same node we started at. In each such closed loop of the circuit, we can apply KVL.

Now let us examine how we can apply KVL in a loop with the loop shown in Fig. 1.2 as an example. We will assume a loop in a particular direction as shown in Fig. 1.2 by the arced arrow inside the loop. A loop can have voltage sources such as v_1 and v_2 as well as passive elements such as the resistors R_1 and R_2. The currents flowing in the branches have been marked as i_1, i_2, i_3 and i_4 and are either known values or merely unknown variables with assumed directions of currents as in the case of KCL before. In order to apply KVL, we need to traverse the loop in some direction. Let us choose a direction as shown in Fig. 1.2 by the arrow inside the loop. When progressing along this loop, any voltage we encounter will be considered as a positive value equal to the value of the voltage if we progress from the negative terminal to the positive terminal and will be considered as a negative value equal to the value of the voltage if we progress from the positive terminal to the negative terminal. Therefore, from Fig. 1.2, voltage source v_1 will be $+v_1$, while voltage source v_2 will be $-v_2$. For passive elements, we will need the relationship between the voltage across the terminals and the current through the element.

We can begin with applying KVL for resistors shown in Fig. 1.2 and then discuss the possibility of other elements. When traversing the closed loop in a given direction, the voltage drop across a passive element will be positive if we traverse the element in a direction opposite to the direction of the current in the branch and will be negative if we traverse the element in the same direction as the direction of current in the branch. With this logic, the voltage drop across R_1 is $-i_2R_1$, while the voltage drop across R_2 is $+i_4R_2$. As per KVL, once we have determined the voltage drops with their polarities, we can assign the sum of these voltage sources and voltage drops to be zero. This results in the following equation:

$$v_1 - i_2R_1 + i_4R_2 - v_2 = 0 \tag{1.3}$$

Just like KCL was an application of the conservation of charge at a node, KVL is an application of the conservation of voltages in a loop. All that KVL states is that the sum total of voltage produced in a loop is equal to the sum total of the voltage drops across all elements in the loop. This only makes sense, as the voltage produced by the sources in a loop has to be dropped across other elements in the loop. When I was an undergraduate, I was confused about one aspect of KVL—if one can imagine numerous closed loops in a circuit, will KVL hold true for each and every closed loop imaginable? Quite surprisingly, the answer is yes. For every closed loop in a circuit, no matter how roundabout a path we choose until we return to the origin node, KVL will hold true.

This might seem almost magical and difficult to believe, but a simple way to interpret it is as follows. Instead of the voltages across each element, let us compute the voltage across each branch in a closed loop such that the voltage across the nodes of a branch is equal to the sum of voltage sources minus the sum of voltage drops across passive elements in that branch. Every branch will have a non-zero voltage across it unless it has been short-circuited. If we consider another branch that is incident at one of the nodes of this branch, and we add the voltages across the two branches together, we will still result in a non-zero voltage as long as the two free nodes of the branches are not short-circuited. However, if we choose any closed loop in a circuit and add branches in the loop one after the other until we close the loop, the net voltage will be zero. This is because the net voltage will be the voltage of the starting node with respect to itself, as the definition of a closed loop is that it ends with the starting node.

Figure 1.2 shows only resistors, but KVL can also be applied if there are other elements with lumped parameters. Let us replace the resistor R_1 with an inductor L_1, and the resistor R_2 by a capacitor C_1. Since we are considering elements with lumped parameters, we can write the following expressions for the voltage across the inductor and capacitor [1, 2]:

$$v_L = L_1 \frac{di_2}{dt} \tag{1.4}$$

$$v_C = \frac{1}{C_1} \int i_4 dt \tag{1.5}$$

The KVL expression for the closed loop can then be expressed as:

$$v_1 - L_1 \frac{di_2}{dt} + \frac{1}{C_1} \int i_4 dt - v_2 = 0 \tag{1.6}$$

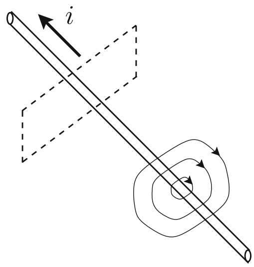
KVL is a very powerful circuit analysis technique that is used as widely as KCL. As a matter of fact, KCL and KVL are used quite often together to analyse complex circuits. In this book, KCL and KVL will be used many times to express the voltage applied across a magnetic element with respect to the voltage drop across resistances, inductances and the emf induced in the magnetic element. These equations will be found in every chapter of the book—Chap. 2 on inductors, Chap. 3 on coupled inductors, Chap. 4 on transformers and Chap. 5 on transformer applications. Now that we have covered the basic network laws that will be used in this book, we can describe the physical laws related to magnetism that will be used in this book.

As we start discussing the basics of electromagnetics, it is important to emphasize that electromagnetics and magnetism in general are very vast domains with a multitude of different laws. However, this book is for a power engineer who wishes to learn about the inner workings of magnetic components. The objective of this book is only to develop a deeper understanding of magnetic components through simulations. The laws used in this book will be only the basic laws that result in functional simulation models. This book will not cover electromagnetism to a great depth, nor will it attempt to understand the finer details of magnetic fields.

The bridge between electricity and magnetism was probably first demonstrated in 1821 by Hans Christian Oersted, who showed that a conductor carrying an electric current produces a magnetic field [23, 24]. This was demonstrated by an experiment where he observed that the compass of a magnetic needle turned and pointed in a direction perpendicular to the current carrying wire. He also observed that the lines of flux of the magnetic field encircled the wire and that these lines of flux were in a plane perpendicular to the wire. Furthermore, the strength of the magnetic field decreased with distance from the wire and also reversed if the direction of current through the wire reversed. After Oersted, several other mathematicians and physicists worked on this phenomenon, and several laws resulted—Ampere's force law, Ampere–Maxwell law and many others [23]. Without diving very deep into what happened two hundred years back, let us concisely list out what we can use as electrical engineers.

One of the simpler visual implementations of many of the laws that resulted from the magnetic field produced by a current carrying conductor was the Right Hand Thumb Rule, which was a combination of Oersted's observations and work done by Ampere [1, 22, 24]. The Right Hand Thumb Rule states that if we were to hold a current carrying conductor by our right hand with the thumb outstretched and pointing in the direction of the current, the direction in which our fingers will encircle the conductor will be the direction of the flux lines of the magnetic field. Figure 1.3 shows the flux lines of the magnetic field for a current i flowing in a conductor in the direction shown. If the reader were to imagine holding this

Fig. 1.3 Right Hand Thumb Rule



conductor with his or her right hand, with the thumb outstretched and pointing upwards in the direction of current, the direction of the magnetic field will be in the direction shown, which is the same as the direction in which the fingers encircle the conductor. The reader must note that the magnetic lines of flux will be in the plane perpendicular to the conductor as shown by the dashed rectangle. These lines of flux will extend indefinitely radially outwards and will be continuous throughout the length of the conductor.

This Right Hand Thumb Rule is applicable to any current carrying conductor, even when it has been twisted or wound as a coil as will be discussed soon. This book extensively uses this rule to determine the direction of the magnetic lines of flux. Besides being able to determine the direction of the magnetic lines of flux, Ampere's Law is extremely useful in expressing the strength of the magnetic field with respect to the current. The following is the expression of Ampere's Law as borrowed from physics [22, 24]:

$$\oint_c H dl = i_{encl} \tag{1.7}$$

where H is the strength of the magnetic field. i_{encl} in the case of a simple current carrying inductor in Fig. 1.3 is merely the current i flowing through the inductor. Soon, we will examine when i_{encl} is different from the current i. The integral notation needs a detailed expression.

The notation $\oint_c$ implies the integral over a closed curve c. In the above expression, we are integrating over one particular closed loop of the magnetic field comprising of one flux line. This closed loop has a magnetic field strength H. As we move radially away from the conductor, the radii of the closed loops of the magnetic flux lines will increase. Therefore, as we move radially away from the conductor, we are integrating over loops of increasing larger lengths. However, every integral of a loop of a magnetic flux line will be equal to the current that is enclosed by the loop—namely i flowing through the conductor. Therefore, as we move radially away from the conductor, with increasing length of the loops, the magnetic field strength H will decrease. Therefore, the above expression relates the magnetic field strength with the distance from the conductor.

A single straight conductor as shown in Fig. 1.3 is not of any particular interest to an electrical engineer. This is due to the fact that the magnetic field produced

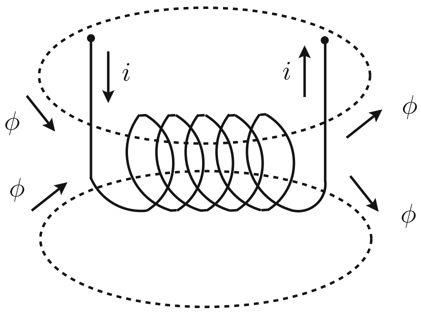


Fig. 1.4 Magnetic field due to the current through a coil

by a single straight conductor carrying a current in the surrounding air will not be significant to put it to any real use. An exception would of course be if we were to examine the magnetic field strength in the surrounding of a high voltage transmission line carrying thousands of amperes of current. In that case, we would need to determine the strength of the magnetic field around the line so as to define a safe distance on either side of the transmission line for the sake of safety to humans. Things begin to get interesting when we take a conductor such as the one in Fig. 1.3 and wind it as a coil.

Figure 1.4 shows the magnetic field produced by a similar conductor as in Fig. 1.3, but now wound in the shape of a coil. All the laws are applicable in this case as well. We can apply Right Hand Thumb Rule to determine the direction of the flux lines of the magnetic field. Since the conductor is wound as a coil with several turns, we can apply the rule to each turn. As a result, there are flux lines of the magnetic field passing through the coil. Though we can apply the Right Hand Thumb Rule to determine the direction of the field, there is another convenient rule that is easier to apply in the case of such coils and is an offshoot of the Right Hand Thumb Rule. The flux lines of the magnetic field will enter the coil face where the current is flowing in the clockwise direction and will leave the coil face where the current is flowing in the counter-clockwise direction. The reader is encouraged to verify this by applying the Right Hand Thumb Rule.

The flux lines of the magnetic field that are passing through the coil will complete their paths through the exterior of the coil as shown in Fig. 1.4. Similar to the case of the single conductor, the flux lines will extend in all directions radially away from the coil. As we move farther away from the coil, the strength of the magnetic field will decrease. We can apply Ampere's Law in this case as well, though with a very interesting twist. The integral over a closed loop of the magnetic flux line of strength H will be equal to the enclosed current. However, in the case of a coil with turns as shown in Fig. 1.4, the enclosed current is no longer merely the current i flowing through the conductor. If the coil has N turns, the enclosed current is Ni leading to this expression [22, 24]:

$$\oint_c Hdl = i_{encl} = Ni \tag{1.8}$$

I struggled for many years to understand this and for that reason, I would like to devote a detailed explanation for the above expression. If one applies the Right Hand Thumb Rule to the coil in Fig. 1.4, it is fairly easy to figure out the direction of the magnetic field. However, it is important to note that we can apply the Right Hand Thumb Rule to every turn of the coil. Therefore, each and every turn of the coil is producing a magnetic field. As a result, there is a cumulative magnetic field due to the current flowing in all the turns of the coil. An accurate map of the magnetic field that can be determined from specialized software is very complex and will include all possible non-linearities and non-uniformities and will not be as neat and clean as shown in Fig. 1.4. However, if we consider only the simplest effects, it is fairly easy to see that the magnetic flux lines due to the current through each turn assist each other especially in the flux lines that pass through the coil. There are other places where there may be a conflict between the flux lines produced by different turns. By and large, by winding a conductor as a coil with N turns, it appears as if the strength of the magnetic field that passes through the coil will be N times stronger than the current through a single straight conductor. The reader is encouraged to verify this visually.

By winding a conductor as a coil with N turns, we seem to have magically enhanced the magnetic field passing through the coil. But the effect of the magnetic field produced by the consecutive turns of the coil assisting each other is very clear. If we use Ampere's Law, we can interpret it as—a single flux line of the magnetic field will enclose N turns of the coil each carrying a current i. Quite obviously, the strength of the magnetic field H will be much stronger in the case of a coil as opposed to a single straight current carrying conductor. This leads to the concept—ampere turns. When we wind a coil with N turns, we define the ampere turns of the coil as Ni [22, 24]. These ampere turns are now responsible for producing the magnetic field and not just the current i as was the case for the straight current carrying conductor.

Since this concept is fairly important and will be used throughout the book, let us state precisely how a reader can interpret these laws. In this book, we will mainly be dealing with coils either in the context of inductors, coupled inductors or transformers. Any coil carrying a current will produce a magnetic field. As per Right Hand Thumb Rule, the flux lines of the magnetic field will pass through the coil. This follows from Oersted's findings in 1821 that the flux lines of the magnetic field will encircle the current carrying conductor [23]. One can determine which face of the coil will be the South pole and which will be the North pole by checking whether the current flows through the coil in that face in the clockwise or the counter-clockwise direction, respectively. Subsequently, we can determine the direction of the flux lines of the magnetic field as they enter the coil at the South pole and leave at the North pole—this is the case with any magnet.

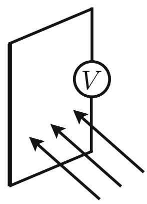
The flux lines of the magnetic field for a current carrying conductor wound as a coil are shown in Fig. 1.4. This is merely mapping the flux lines that pass through the coil with closed loops in the surrounding air. For each flux line, we can apply Ampere's Law. The integral of the magnetic field strength H over the closed loop of a flux line will be equal to the enclosed current, which will be equal to the ampere turns of the coil, i.e. the product of the current in the conductor and the number of turns of the coil. This ampere turns is therefore the driving force that produces the magnetic field and is an extremely important term used throughout the book. In Chap. 2, we will call the ampere turns as Magnetomotive Force (MMF) [25].

Now that we can describe the magnetic field produced by a current flowing through a conductor, which may either be straight or wound as a coil, we can now progress to describing what will be the effect of a magnetic field on a circuit. It should be noted that the magnetic field in this case can be produced by the circuit itself such as a current carrying coil as described above in some part of the circuit or can be an external magnetic field that has no direct relation to the circuit. As stated before, the effects of a magnetic field on a circuit may be very complex and governed by several phenomena that have been studied over the past century by many physicists. However, as electrical engineers trying to build a simulation model of a magnetic component, we can limit ourselves to just a few phenomena and ignore the rest.

The phenomenon of electromagnetic induction was first discovered and reported by Michael Faraday in 1831 [23]. Faraday's Law of electromagnetic induction states that "the electromotive force around a closed path is equal to the negative of the time rate of change of the magnetic flux enclosed by the path" [1, 2, 22, 24]. Let us break this statement up and figure out how we are going to use it. The closed path in the law is any closed loop in a circuit [1, 2]. If we consider a single straight conductor of a finite length, which is open at both ends, this would not qualify as a closed loop, as quite obviously, it is not closed but open at both ends. Suppose we connect a Voltmeter across the two ends of the conductor, we have a closed loop as shown in Fig. 1.5. This is the simplest possible conception of a closed loop for the above law. However, any circuit with at least one closed loop through which current can flow will also fit the above definition.

Before we talk about the "negative of the time rate of change", in simplest terms, if the magnetic field associated with (enclosed by) such a closed path were to change in any way, an electromotive force (emf) will be induced in that closed path. If were to imagine a magnetic field with flux lines passing through the closed path—and this could be the simple connection of a Voltmeter across a conductor—and this magnetic field were to change in any manner, an emf will be induced in the path. This emf induced will cause the Voltmeter to indicate a voltage measurement. A magnetic field may be widespread as we have seen in our discussion above of the magnetic field produced by a current carrying coil with the lines of force spreading

Fig. 1.5 Electromagnetic induction



far and wide even though the strength of the field will decrease as we move away from the coil. This is why the law states the magnetic flux enclosed by the path is what matters, i.e. the flux lines of the magnetic field passing through the enclosed path.

An emf will be induced in the closed path only if the magnetic flux enclosed by the path changes. If there is no change, there will be no induced emf. Furthermore, the law states that the emf induced is equal to the negative rate of change of the magnetic flux enclosed by the path. For a moment, let us drop the term "negative". The magnitude of the induced emf will be equal to the rate of change of magnetic flux passing through the closed path. Therefore, the more rapidly the magnetic flux passing through the closed path changes, the greater will be induced emf. For a single conductor with a Voltmeter connected across it as shown in Fig. 1.5, the magnitude of the induced emf can be expressed mathematically using Faraday's Law as:

$$e = \frac{d\phi}{dt} \tag{1.9}$$

where e is the magnitude of the induced emf and ϕ is the flux in Weber passing through the closed loop.

To delve further into the "enclosed" term in Faraday's Law, let us look at our setup of a single conductor with a Voltmeter across it as shown in Fig. 1.5. The magnetic flux passing through the closed loop is the magnetic flux enclosed by the closed loop. No difference between them and thus the expression above. Things again get interesting if we consider the magnetic flux passing through a coil. In our discussion before, we had considered a current carrying coil and had described the magnetic field produced by it. Now let us imaging the reverse as shown in Fig. 1.6. A coil with no current is in a magnetic field and we are connecting a Voltmeter across the two free terminals of the coil. We thus have a closed path. Suppose, for some reason, the magnetic field to which the coil is exposed as shown in Fig. 1.6 is identical to the magnetic field shown in Fig. 1.4. It is to be noted that we are now assuming no current flowing through the coil. The magnetic field is produced by some external agent.

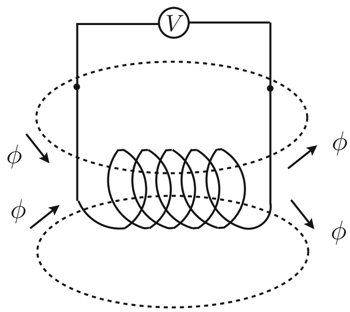


Fig. 1.6 Induced emf in a coil placed in a magnetic field

If the flux lines of the magnetic field passing through the coil were to change, an emf will be induced. This is due to the fact that the coil is connected to the Voltmeter externally and is a closed path. Even if we were to neglect the flux passing through the part of the circuit with the Voltmeter connected across the coil terminals, the flux passing through the coil is still considered to be the flux enclosed by the closed path. By Faraday's Law, the magnitude of the induced emf will be equal to the rate of change (negative to be explained soon) of the flux enclosed by the closed path. In this case, if we assume that the only flux associated with the closed path is the flux passing through the coil, the emf induced will be equal to the magnetic flux enclosed by the coil. And here, because we have a coil, there is difference between the flux passing through the coil and the flux enclosed by the coil.

For a coil with several turns, one could imagine each turn of the coil impacted by the changing magnetic flux passing through the coil. The emf induced across the terminals of the coil is the sum total of the emf induced in each turn of the coil. Therefore, the enclosed magnetic flux is not merely the flux ϕ but is now multiplied by the number of turns N of the coil. Once again, by winding a current carrying conductor as a coil with N turns, we have enhanced the emf induced in the closed path by a factor of N. We can express the magnitude of the induced emf for the circuit of Fig. 1.6 as follows:

$$e = N\frac{d\phi}{dt} = \frac{d}{dt}(N\phi) = \frac{d\psi}{dt} \tag{1.10}$$

In the above equation, the term $N\phi$ is called the flux linkage ψ. The flux linkage ψ of a coil is the product of the flux passing through the coil and the number of turns of the coil and is an indication of the impact of enclosed flux on a coil with more than one turn.

It is fairly easy to see the analogy between ampere turns and flux linkage. In both of them, the number of turns N of the coil appears to take into account the fact that a coil wound with more than one turn will behave differently from a single straight conductor. Therefore, whenever we have a coil with more than one turn, if we are trying to calculate the strength of the magnetic field from the current flowing through the coil, we must calculate the ampere turns Ni and not just consider the current i. Similarly, if we are trying to compute the emf induced in a coil, we must consider the flux linkage $\psi = N\phi$ and not just the flux ϕ passing through the coil.

Now that we have expressed the magnitude of the induced emf, let us talk about the "negative rate of change" that we have so far ignored. The complete mathematical expression of Faraday's Law is as follows:

$$e = -\frac{d\psi}{dt} \tag{1.11}$$

To understand this negative sign, one needs to ask the question—what will be the polarity of the induced emf? Rather than using the negative sign in Faraday's Law, if we wish to determine the polarity of the induced emf, we can use another law—

Lenz's Law. Lenz' Law was formulated by Emil Lenz in 1834 [23] and states that the emf induced due to the change in magnetic flux enclosing a closed path will be such that it will oppose the cause that produces it, i.e. the change in magnetic flux [22, 24]. It is very similar to Newton's Third Law of Motion, which states that the reaction to an action will be opposite to the action.

This book will use Lenz's Law in combination with Faraday's Law to determine the magnitude and polarity of the induced emf. Though Lenz's Law is fairly easy to use, we must take into account a fine detail in the law. The induced emf will oppose the cause that produces it. The cause that produces the induced emf is the *changing* magnetic flux. Therefore, the induced emf will oppose the *changing* magnetic flux. The induced emf will *not* oppose the magnetic flux. This is a distinction one must remember throughout the book and will be repeated over and over again. As an example, a constant magnetic flux will not induce an emf. Therefore, even though there exists a flux, there will be no induced emf. It is a common misconception to assume that the induced emf will always oppose the magnetic flux.

How would one apply Lenz's Law to determine the polarity of the induced emf? The induced emf will be of such a polarity that will either produce or attempt to produce a current flowing in the closed loop, which will oppose the changing magnetic flux. An emf will always be induced and will have a magnitude as expressed above by Faraday's Law when the magnetic flux enclosed changes. However, the closed path can consist of any elements. In our case of Fig. 1.6, the only other component in the closed path besides the coil is a Voltmeter. Therefore, the induced emf will produce a negligible current as the Voltmeter is an extremely high resistance. This negligible current would quite obviously not be able to produce much of a magnetic field to oppose the changing magnetic flux. However, by Lenz's Law, even if the opposing magnetic flux produced by the induced emf is negligible, it will still offer this opposition. Therefore, one can always determine the polarity of the induced emf by asking the question—what polarity should the emf have so that the resulting current produces a magnetic field, which will oppose the change in main magnetic field?

With this rather long and detailed section, we have described the basic laws of physics that will be used to understand magnetism in electrical engineering. We will be using these laws over and over again in all the chapters of this book. Before we end this section, let us pause to note that all these laws were formulated close to 200 years back! These centuries-old laws are all that we as electrical engineers need to develop a basic understanding of magnetics in electrical engineering. Though one can use modern techniques and results to arrive at a much more complex and accurate understanding of the magnetic fields in electrical engineering, for most power engineering applications, such a degree of detail is not necessary. As the reader progresses through this book, he or she will be amazed at how we can produce reasonably functional simulation models with laws that are close to two centuries old.

1.4 Learning Through Simulation

In the previous section, we described in detail the basic laws of physics that will be used in this book to understand the operation of magnetic components. We will be using these basic laws to also produce simulation models that can be used in basic applications in power engineering. In this section, I will talk about why simulation has been used as a tool to learn engineering [6–8] and how the simulations in this book will make the theory presented much more interesting and easier to understand.

Nowadays, computers are available to a vast majority of students. There was a time when a significant proportion of students would still need to visit a computer lab in order to access computational facilities. Nowadays, personal laptops feature sufficiently powerful processors and internal memory so as to provide in our hands a computer good enough to perform simulations. Simulations were once performed mainly by researchers and graduate students who were exploring new technologies and needed to investigate the feasibility of their innovations before a practical implementation. Simulations can now be performed by even undergraduate students as a part of their course work.

Simulations are a very useful learning tool besides also being useful for the purpose of practical engineering and research. A simulation is the next closest thing to building a hardware prototype. Building hardware and laboratory experiments are undoubtedly the best way to learn and also to verify changes in design and control. However, for many students of electrical engineering, a ready access to laboratory facilities is not available. Moreover, the costs associated with building hardware prototypes on a regular basis make it impossible to use hardware implementation as a continuous learning tool. Simulations on the other hand provide a safe platform where an inexperienced engineer can try out new designs, control strategies and custom models without causing any physical damage to a circuit.

To perform simulations, we normally use simulation software for a particular application. For example, as power engineers, we need to perform simulations of electrical circuits, and therefore, we use circuit simulators [9]. A circuit simulator allows us to define a circuit in some convenient manner, which will then be processed by the simulator to define a set of ordinary differential equations that form the mathematical model of the circuit. These are solved by the circuit simulator by a numerical integration technique to provide us with a time-varying behaviour of the circuit. The convenience offered by such a simulator is that we do not need to write equations for every circuit or solve the equations. All we need to do is to represent the circuit in some form of network representation that can be understood by the simulator. Some advanced circuit simulators allow a user to draw a circuit schematic almost similar to a regular electrical circuit, thereby making the process of circuit representation extremely convenient and visually appealing. Some other simulators might need a nodal representation.

There are numerous circuit simulators that a power engineer can use for simulating electrical circuits. Some are proprietary and fairly expensive, while some are free and universally accessible. Many major universities maintain university-wide licenses of scientific software such that any student registered in the university can use these software. A student, therefore, might have access to multiple simulation software. On the other hand, there are a significant proportion of students all over the world who are enrolled in universities that cannot afford to maintain licenses for some of the more expensive scientific software. This has unfortunately resulted in the rise of software piracy. The adoption of free and open source software ensures universal access to software for any interested user in all parts of the world and therefore is one solution to providing software services in a sustainable manner.

Any student who is reading this book can use any circuit simulator that he or she may have access to. The circuit simulator being used must allow the user to create a circuit schematic and create a user-defined component that can contain a custom model of a magnetic component. The user-defined component needs to be interfaced to the rest of the circuit. It needs to receive measured quantities such as voltages and currents as inputs from the main circuit. These inputs will be fed to a mathematical model. The output of the mathematical model will result in changes to the main circuit. Such a closed-loop mechanism must be achievable by the circuit simulator the reader chooses as the simulation tool to accompany this book. A vast number of proprietary and free circuit simulators will be able to perform this task.

This book will use the free and open source circuit simulator Python Power Electronics. Python Power Electronics has been written completely using Python, which is a free and open source high level programming language. Python Power Electronics has been written especially for power engineers who wish to simulate power electronic circuits while implementing control digitally. The homepage of Python Power Electronics can be found at:
https://www.pythonpowerelectronics.com/.

The circuit simulator can be downloaded from the link:
https://www.pythonpowerelectronics.com/?page=softwaredownloads.

The circuit simulator can be used through a web browser by launching a server on the computer. To learn how to install the circuit simulator and on how to use it to simulate power electronic circuits with digital control, the reader can view any of the video series in the following link:
https://www.pythonpowerelectronics.com/?page=videos.

The reader need not use Python Power Electronics to perform the simulations in this course. Most of the code samples and simulations can be achieved using any other circuit simulator. The purpose behind using a free and open source circuit simulator in this book is to promote the use of free and open source software in electrical engineering. Free and open source software not only increases the accessibility of software but also builds a community of users, which enables the sharing of information and knowledge. The reader is encouraged to follow the project on any number of different social media where regular updates are made. For example, a weekly video lecture series can be found on YouTube on the video link above, where different topics in electrical engineering are explored using simulations.

All the simulations in this book have been hosted on GitHub to be freely accessible to everyone. The link for the repository that contains the simulations is: https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics.

The repository contains folders for every chapter. Within each folder are the simulations for that chapter. Each simulation has been arranged in a separate folder and there can also be nested folders in the case of separate cases being simulated separately. Every simulation will have a README file. The reader must first read the README file, which describes what the simulation is about, what are the circuit schematic files used and what are the control files used. In the later chapters, as we describe each simulation, the exact files will be referenced and described in detail.

The reader is strongly encouraged to perform the simulations while reading this book. While describing the simulations, potential modifications to the simulations will also be suggested, and the reader is strongly encouraged to try these out as well. As already stated before, simulations are a safe platform to try out circuits without the fear of circuits blowing up. This gives several opportunities for the learner to look “under the hood” and also poke around with simulation models.

1.5 Outline of the Book

In this final section of the introduction, we will describe the contents of the book and what the reader can expect from each chapter. The book progresses gradually from the basics of magnetic circuits and how these can be used to simulate inductors to finally simulate three-phase transformers and a flyback converter. Each chapter will use the learnings from the previous chapter to add another layer of detail to be able to simulate more complex systems. The reader is strongly encouraged to read the book in the order of the chapters, even if he or she has a specific application that he or she would like to simulate.

Chapter 2 begins with building a simulation model for an inductor [10, 26]. Though it may appear that building a simulation model for an inductor is a silly and useless exercise, the purpose of doing so is to introduce the concept of magnetic circuits and how the magnetic laws can be combined with electrical laws in a simulation model. Since the inductor is the simplest magnetic component imaginable, this will ease us into the process of building a mathematical model using the laws of physics. In Chap. 2, we build an analogy between electrical circuits and magnetic circuits and describe how knowing the construction of an inductor, we can compute the flux in every part of the core of the inductor. Several simulations are presented with different core constructions. The simulations in Chap. 2 can also be used by a power electronics engineer to verify designs of inductors as, quite often, power electronics engineers need to wind custom inductors especially while designing power supplies.

Chapter 3 extends the mathematical model of an inductor described in Chap. 2 to model more than one inductor wound on the same core and, therefore, magnetically coupled to each other [11, 12]. Using the basic laws of physics, we examine several separate scenarios to fully understand the phenomenon of magnetic coupling and also express coupled inductors mathematically. We will introduce the concept of mutual inductance, which will provide us with a manner of expressing the effect of the current flowing in one coil on the flux linking the other coupled coil. The simulation model of an inductor will then be extended to model coupled inductors. Using simulation results, we will verify our theoretical discussions. Since the simulation of magnetically coupled inductors can consist of many inductors wound on the same core, we will convert our simulations model into a flexible and scalable model using matrix equations.

Chapter 4 describes how a transformer is merely a special case of a set of magnetically coupled windings wound on the same core, with the objective being to maximize the transfer of energy from one winding to the others [13, 14]. Since transformers are essentially magnetically coupled coils, the mathematical model of coupled inductors in Chap. 2 will continue to be used. The difference between a transformer and a random set of magnetically coupled coils is that a transformer is an electrical machine that has specifications similar to any other machine like a motor or generator. For example, a transformer will have a rated maximum power and rated maximum voltages that can be applied to each winding. The chapter will describe how using these specifications, an equivalent circuit can be formulated and the parameters of this equivalent circuit can be estimated. Using this equivalent circuit, we will extract the values of inductances that are necessary to use the simulation model of the coupled inductors. We will examine several simulations to understand the working of the transformer. Simulations will examine the magnetizing current and the other components in the no-load current of the transformer. Simulations will also examine the effect of turns ratio in transforming voltages and currents by simulating step-up and step-down transformers.

Chapter 5 describes some of the most common uses of a transformer in electrical engineering while using the simulation models presented in Chap. 4. For a power systems engineer, three-phase transformers are ubiquitous in every system being analysed [27–30, 30, 31]. Therefore, Chap. 5 describes how the transformer simulation models of Chap. 4 can be extended to simulate three-phase star–star and delta–star transformers. To make the contents relevant for a power electronics engineer, the chapter also presents the simulation model of a flyback converter. The chapter presents the theory behind high frequency transformers and the specific application of high frequency transformers in the specific case of a flyback converter [15, 19, 20]. The purpose of describing the operation of the flyback converter is to describe how even in the context of a non-linear power converter such as a flyback converter, the operation of the transformer can still be interpreted using the basic laws of physics.

Chapter 2
Presenting Basic Magnetic Circuits with Inductors

2.1 Introduction

The inductor is probably the most basic magnetic component in electrical engineering [1, 2, 22, 24]. An inductor can appear unintentionally as a parasitic element or can be included intentionally for various reasons. In electrical engineering besides power electronics, an inductor is usually considered to be a parasitic element. As an example, it is normal to consider a distribution feeder as a combination of resistance–inductance (R–L) or resistance–inductance–capacitance (R–L–C) [32]. Such a model is usually to capture the voltage drop across the feeder. When introduced intentionally, an inductor is usually a component in a filter that either blocks the flow of current in a particular path or bypasses current into a path [33, 34].

In power electronics applications, it is difficult to imagine a power converter without at least one inductor [10, 26]. Therefore, despite being an intermediate chapter to the later chapter on transformers, this chapter on inductors is of significant importance to a power electronics engineer. In a power electronic converter, an inductor usually plays a dual role—as a filter and as an energy storage element [35]. The fundamental basis of a static power electronic converter is that power is conditioned by non-linear solid state devices. As a result, the voltage of the input supply is converted into a switched voltage by one or more solid state devices. In order to condition this switched voltage, we need inductors and capacitors besides potentially many other components, to store surplus energy during a particular time interval and transfer the energy from one branch of the circuit to another. Since power electronic converters differ widely in their mode of operation, the manner in which inductors and capacitors condition the output voltage will also vary widely.

In a vast number of power electronics applications, the inductors used are supplied by manufacturers as off-the-shelf components just like resistors, capacitors or power devices. The two basic specifications of an inductor are the inductance and the current rating. Manufacturers are continuously expanding their catalogue of inductors to supply commonly used inductors. However, there are instances when an

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inductor cannot be found available directly as a mass-manufactured component. In such a case, as power electronic engineers, we can have a custom inductor fabricated by a manufacturer for a particular inductance and current rating. There are also a few cases, where due to the uncertain nature of the application, we are forced to wind our own inductors. Many power electronics engineers who work in power supply design and fabrication would at some point of time have wound their own inductors [10, 26].

There are several resources available for winding inductors besides this also being taught in power electronics courses [10, 26]. However, in a vast number of cases, the resources we use are quite confusing resulting in improperly wound inductors. Specifically, the resources that I used never explained the process from fundamentals but rather was just a set of equations. It was for this reason that the very first chapter in this book starts with the simulation of inductors. Though the design of inductors will not be covered in this chapter, the simulation models presented can be used as a very convenient tool to verify if a particular design of an inductor is close to what we expected. Furthermore, with inductors being basic components that are easy to analyse in a circuit, developing a simulation model for an inductor will ease the process for the later chapters where we begin to model magnetic coupling. The reader is encouraged to simulate every example presented in this chapter and also modify those simulations for his or her purposes.

2.2 Revisiting the Basic Inductor

In this section, we will use the basic laws of physics to express the most basic equations of an inductor [1, 2, 10, 22, 24–26]. Modelling and simulating an inductor might seem like a trivial exercise, but this basic understanding of the inductor will demonstrate how it is possible to convert basic physical laws into a simulation model. This section will show, beginning with the inductor, how all that is needed to develop a basic simulation model for any magnetic component are the basic laws of magnetism and induction—Ampere's Law, Faraday's Law and Lenz's Law [10, 22, 24, 26].

The inductor as we know is a coil optionally wound on a core. Most inductors have a core either of iron or ferrite depending on the application [10, 26]. However, it is possible to have an inductor that has no core—also called air-cored inductors. In any case, if a wire is wound as a coil, and if it carries a current, a magnetic field will pass through the coil. This phenomenon is the result of Ampere's Law that states that a current carrying conductor will produce a magnetic field. The direction of the magnetic field can be determined by applying the Right Hand Thumb Rule. If you grab the wire with your right hand with your outstretched thumb pointing in the direction of the flow of current, the direction in which your fingers encircle the wire is the direction of the magnetic field.

In the case of a current passing through a coil, using the Right Hand Thumb Rule produces the following rule for determining the direction of the magnetic field. The

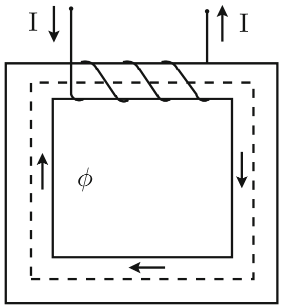


Fig. 2.1 A coil wound on a core

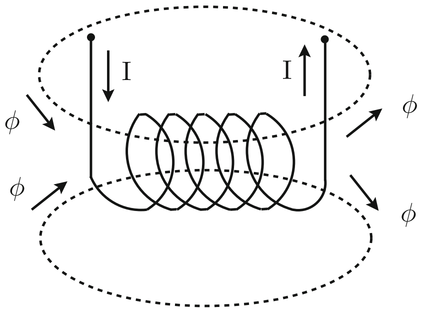


Fig. 2.2 A coil wound in air

face of the coil carrying current in the counter-clockwise direction is the North pole as the magnetic lines of force leave the coil from this side. The face of the coil carrying current in the clockwise direction is the South pole as the magnetic lines of force enter the coil from this side. This rule is just a derivative of the Right Hand Thumb Rule applied specially to the case of a wire wound as a coil. Figures 2.1 and 2.2 show the magnetic lines of force produced by currents flowing through coils wound on a core and without a core.

For an inductor, two physical laws always hold true. Faraday's Law states that when a conductor is placed in changing magnetic field, an emf (electromotive force) is induced across it [22, 24]. The law further states that the induced emf generated is equal to the rate of change of flux linked ψ with the conductor:

$$e = \frac{d\psi}{dt} \tag{2.1}$$

In the case of an inductor, the flux linked with (flowing through) the inductor takes into account the number of turns of the coil and is the product of the magnetic flux ϕ and the number of turns N of the coil:

$$\psi = N\phi \tag{2.2}$$

With the unit of flux being Weber, the unit of flux linkage is merely Weber-turn. Therefore, the induced emf can expressed as:

$$e = N\frac{d\phi}{dt} \tag{2.3}$$

From an engineering perspective, an expression involving the magnetic flux linking an inductor is a fairly abstract equation. To introduce a more practical approach to the above expression, let us introduce inductance of a coil as a physical quantity. The inductance L of a coil is the defined as the flux linkages produced per unit current flowing through it [1, 2, 10, 26]. A coil will have an inductance of 1 Henry, if a current of 1 Ampere flowing through it will produce a flux linkage of 1 Weber-turn:

$$\psi = Li \tag{2.4}$$

Therefore, the induced emf can be expressed with respect to the inductance of the coil as follows:

$$e = L\frac{di}{dt} \tag{2.5}$$

The second law that can be used to model the inductor is Lenz's Law. Lenz's Law states that the induced emf is such so as oppose the cause that produces it [22, 24]. From the above discussion on induced emf being proportional to the rate of change of current, the cause of the induced emf is the change in current. Therefore, this usually means that the induced emf will oppose the change in the current through the inductor as this change in current is what produces the induced emf. Lenz's Law helps to determine the polarity of the induced emf, while Faraday's Law determines the magnitude of the induced emf. It is important to note that the induced emf will oppose the *changing* current, which is the cause that produces it. The induced emf does not oppose the current itself as a resistor does. Further sections and chapters will contain detailed description of cases that will illustrate this concept.

Mathematically, if expressing all quantities as phasors, one could combine the two laws to result in the following expression for the induced emf:

$$\bar{e} = -L\frac{d\bar{i}}{dt} \tag{2.6}$$

However, the above equation is applicable for a phasor representation, which is not well suited for a simulation model. Therefore, we will not use the negative sign in the equation but rather include it in the circuit as we build our simulation model.

In this section, we have described how Faraday's Law and Lenz's Law determine the emf induced in an inductor when the current flowing through the inductor changes. The laws link the change in magnetic flux linking the coil with the emf induced in the coil. However, using our definition of inductance, we can include inductance into the expression of induced emf. In the next section, we will demonstrate how this expression of induced emf can be expanded to an inductor in a circuit.

2.3 Inductor Model

In the previous section, we examined how Faraday's Law can be used to express the emf induced in the inductor with respect to the rate of change of flux linkages, and therefore with respect to the change of current through the inductor. However, the expressions in the previous section were for the inductor in isolation. In this section, we will examine how the expressions from the previous section can be used when the inductor is a part of a larger circuit.

While simulating a circuit, the inductor is modelled by substituting the following expression into the network equations for the circuit:

$$L\frac{di}{dt} \tag{2.7}$$

The network equations can be written for any circuit based on Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL). By substituting the above term, the resultant differential equations that represent the circuit can be solved numerically in a simulation [9].

However, this considers the inductor as a constant and does not consider any of the magnetic details of the inductor. The assumption used to arrive at the above expression in the previous section was

$$\phi \propto i \tag{2.8}$$

In reality, this is not the case. The relationship between ϕ and i depends on the core material, the structure of the core and most importantly is not even a simple linear function [10, 22, 24, 26]. In most cases, it is simpler to neglect the non-linear nature of the relationship between ϕ and i unless we wish to simulate in detail the effect of the core magnetic material.

It is possible to include details of the magnetic properties of elements such as an inductor in a simulation [5]. The level of detail that we wish to include will determine the complexity of the model. Very complex and detailed models might need techniques such as finite element methods, which are out of the scope of this book. However, for those who would like an inductor model that is more than a mere constant L, it is possible to do so with basic concepts of magnetics from physics. In this section, we will introduce the basic concepts of magnetism in order to create an inductor model. However, these concepts can be extended later to any magnetic component such as coupled inductors and transformers that will be covered in later chapters [10, 26].

Before we get into the details of magnetic characteristics of the inductor core, let us review the basics of magnetics. In the previous section, we had considered two inductors in Figs. 2.1 and 2.2. The flux paths shown in the two figures was the result of applying the Right Hand Thumb Rule. Let us first consider the air-cored inductor in Fig. 2.2. Let us suppose that this coil has N turns and is carrying a current I. From Ampere's Law, any current carrying conductor will produce a magnetic field. In the case of a coil of N turns, the current flowing through each turn will augment the magnetic field produced by the others. The very first step to calculating the magnetic field is to define a term called the magneto motive force (MMF) of the coil as [25]:

$$\text{MMF} = NI \tag{2.9}$$

In simple terms, the MMF is basically how much of an effect the current flowing has in terms of the ability of the coil to produce a magnetic field. Since it is a coil, the current's effect increases by a factor equal to the number of turns. For a single straight wire, the MMF would be just the current I.

Now that we have defined the MMF as the driving force that produces the magnetic field, the next question is what else do we need to know to calculate the flux? Since this coil of Fig. 2.2 is wound without a core, the lines of flux pass through the coil and complete their closed paths on all sides through the air. The wider flux paths will have decreasing field strengths until the field strength or the magnitude of flux becomes negligible. Therefore, when calculating flux, we need to define the path of the flux. In the case of an air-cored inductor of Fig. 2.2, the path is difficult to assign unless we assume the shortest possible path around the coil through which the flux can complete its path. To make the understanding of flux paths easier, let us consider the inductor of Fig. 2.1 where the coil is wound around an iron core.

Before we begin calculating the magnitude of the magnetic flux, what is the difference between the two inductors? As already stated, the magnetic flux completes a path just like current does. This flux path can be through a single medium or through multiple mediums. Every medium offers a certain level of opposition to the flux just like conductors offer opposition in the form of resistance to the flow of current through them. The opposition offered by a medium to the flow of magnetic flux through it is called reluctance [1, 2, 22, 24]. The reluctance offered by the medium is dependent on the length of the path, cross-sectional area of the path and the permeability of the medium:

$$\rho = \frac{l}{\mu A} \tag{2.10}$$

The above formula is very similar to the formula for calculating the resistance of a current-carrying conductor.

In the expression (2.10) for reluctance, the intrinsic property of the medium that offers opposition to the flow of flux through it is the permeability μ, which has a unit of Henry per metre (H/m) [22, 24]. This permeability is further expressed as:

$$\mu = \mu_0 \mu_r \tag{2.11}$$

where μ_0 is the permeability of free space and is equal to $4\pi \times 10^{-7}$ H/m. Free space (or air) is considered the base in terms of measuring permeability of a medium towards the flow of flux through it. μ_r is the relative permeability of a medium with respect to that of free space and is merely a constant. From (2.10), the larger is the permeability of the medium, the lower will be the reluctance of the particular path to the flow of flux.

Magnetism is a subject that has been studied intensively for many decades and close to a century. The magnetic properties of many different materials have been studied under various conditions and the interested reader can find an almost inexhaustible pile of information on this topic [22–24]. However, we need to restrict our discussion in this book to specific applications with respect to power electronics. In power electronics, the most popular core materials used are iron and ferrite. The actual fabrication of the core might need the iron to be laminated or the ferrite to be a composite or a polymer. In some rare applications, there could be no core as well. In the case of no core or air core, the permeability of the flux path is μ_0 itself. In the case of iron, the relative permeability is in the range of 1000–5000 depending on the presence of other metals in cases of alloys. In the case of ferrite, the relative permeability is usually in the range of 300–500 depending on the presence of other metals in cases of alloys.

With this background on the permeability of iron and ferrite cores, it is fairly evident that the reluctance of the core is far less than that of the surrounding air. Therefore, a vast majority of the magnetic lines of flux will flow through the core and only a negligible amount of flux will leak into the surrounding air [10, 26]. If we neglect the leakage of flux into the air, we can now determine the path through which the magnetic flux will flow in the core and thereby calculate the reluctance of the path. In the case of Fig. 2.1, we can define the average length of the flux path (shown as a dotted line) as l and the cross-sectional area (imagine a depth to the core) of A. We can use (2.10) to calculate the reluctance of the flux path. The magnitude of the flux can be expressed as [22, 24]:

$$\phi = \frac{\text{MMF}}{\rho} = \frac{NI\mu A}{l} \tag{2.12}$$

From the above expression, the flux is directly proportional to the current flowing through the coil, the permeability of the core and the cross-sectional area of the core. The flux on the other hand is inversely proportional to the mean length of the flux path through the core. Subsequently, using the definition of the inductance, the inductance can be calculated as:

$$L = \frac{N\phi}{I} = \frac{N^2\mu A}{l} \tag{2.13}$$

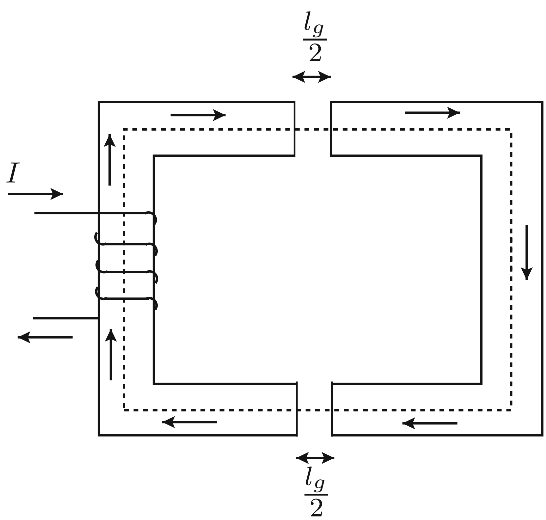
Cores can be more complicated than just a single closed path as shown in Fig. 2.1 or there might be a core with multiple windings. Therefore, the question arises as to how we will calculate the flux using an expression such as (2.12). The good news is that the path of magnetic flux can be dealt with as a magnetic circuit in a manner very similar to how an electric circuit is used to compute the current flowing through different branches of the circuit. This will be described in the next section.

2.4 Magnetic Circuits

In the previous section, we had used the basic physical laws of magnetism to write an expression for the magnetic flux given a particular construction of an inductor. The construction of the inductor is with respect to the construction of the core (cross-sectional area and length), the permeability of the core and the number of turns of the coil wound on the core [10, 26]. Figure 2.1 was a sample inductor with a core that we had chosen for our derivation. In this section, we will extend our discussion to calculate the flux in case the core is a bit more complex than the one in Fig. 2.1.

A core such as the one in Fig. 2.1 will have a magnetic flux ϕ expressed by (2.12). In order to increase the magnetic flux ϕ for that particular core, we could increase the number of turns of the coil. In order to decrease ϕ, we could decrease the number of turns. Another technique that is used to adjust the flux produced by a given number of turns is to introduce an air gap as shown in Fig. 2.3 [10, 26]. For the purpose of illustration, the air gap has been shown to be large. However, when introducing an air gap to decrease the flux in the core, this air gap is usually much less than a millimetre. In order to introduce this air gap, two half cores are chosen and joined

Fig. 2.3 Iron core with two air gaps



together while separating them by some kind of insulating material [10, 26]. The gap introduced by the insulating material is usually equivalent to an air gap that has a permeability equal to that of μ_0.

By introducing this air gap, we are introducing a segment that has a much higher reluctance than that of the iron core. In Fig. 2.3, we have shown the two air gaps to be of length $\frac{l_g}{2}$. If the length of the complete path shown as the dotted line passing through the centre of the core were to be denoted by l, this would imply the length of the flux path through the iron core was $l - l_g$. It should be noted that at the air gap, the flux would experience a fringing effect and would not be as uniform as it was while passing through the iron core [5]. However, this detail is very difficult to express and therefore, we will neglect this fringing effect and assume the flux to be as uniform as it was while passing through the iron core. The reluctances of the paths through the iron core and the air gap can be expressed as:

$$\rho_c = \frac{l - l_g}{\mu_0 \mu_r A} \tag{2.14}$$

$$\rho_g = \frac{l_g}{\mu_0 A} \tag{2.15}$$

The flux flowing due to the MMF generated by the current I flowing through N turns of the coil will face the sum of the reluctances due to the iron core and the air gap [1, 2]:

$$\phi = \frac{NI}{\rho_c + \rho_g} \tag{2.16}$$

Therefore, by introducing an extremely small air gap in the core, the flux through the core can be adjusted. At times, this form of adjustment results in bringing the inductance of this component closer to the desired value as opposed to adjusting the number of turns. The inductance can be calculated as:

$$L = \frac{N\phi}{I} = \frac{N^2}{\rho_c + \rho_g} \tag{2.17}$$

From the above case, it is quite clear that magnetic flux behaves in a manner quite similar to electric current. The reluctances encountered by the flux path add up when mediums are in series in exactly the same way resistances add up to oppose the flow of current when different conducting materials are connected in series. In the case of parallel limbs in the iron core, the magnetic flux has parallel paths to flow and again just like electric current, a greater proportion of the flux will flow through the path of least reluctance. As an example, consider the same coil wound on the iron

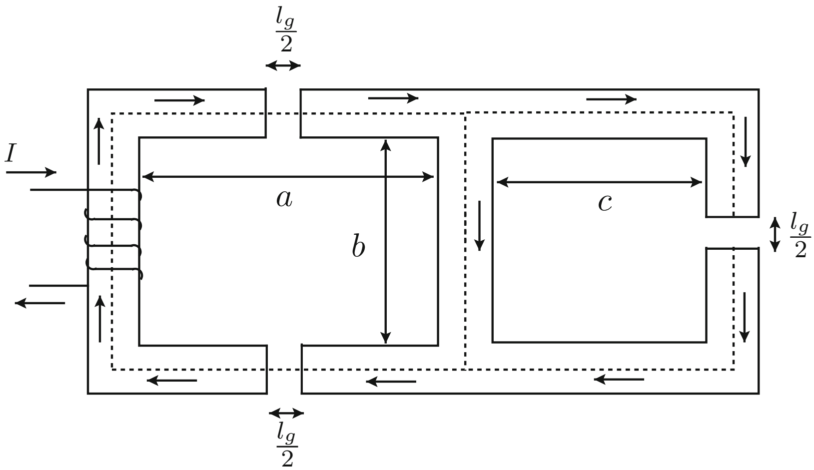


Fig. 2.4 Iron core with multiple legs and air gaps

core of Fig. 2.4. If the width of the core was w, we can calculate the reluctance of each leg as follows:

$$
\begin{aligned}
\rho_1 &= \frac{2a + b + 3w - l_g}{\mu_0 \mu_r A} \\
\rho_{g1} &= \frac{l_g}{\mu_0 A} \\
\rho_2 &= \frac{b + w}{\mu_0 \mu_r A} \\
\rho_3 &= \frac{2c + b + 3w - \frac{l_g}{2}}{\mu_0 \mu_r A} \\
\rho_{g2} &= \frac{l_g}{2\mu_0 A}
\end{aligned}
\tag{2.18}
$$

In the calculation of the reluctance of every limb of the iron core, we have used the mean length of the flux path. The main flux produced by the coil (flowing in the left limb) can be expressed as:

$$
\phi = \frac{NI}{\rho_1 + \rho_{g1} + \frac{\rho_2(\rho_3 + \rho_{g2})}{\rho_2 + \rho_3 + \rho_{g2}}}
\tag{2.19}
$$

In the above expression, the parallel paths available to the main flux result in an equivalent reluctance that is the parallel combination of the reluctances ρ_2 and $\rho_3 +$

ρ_{g2}. This parallel combination of reluctances can be computed in exactly the same manner as for a parallel combination of resistances.

This analogy with electric circuits enables us to compute the magnetic flux for fairly complex core constructions. Extending our analogy, we can also use superposition theorem to calculate the flux due to multiple windings on different legs of the iron core. These relations will be extremely useful when we model electrical machines such as transformers, which have multiple windings wound on the same core. Before we use the above expressions to model inductors, we need to understand that the above expressions contain approximations. In reality, the magnetic properties of the core material are not constants but can change due to various reasons. In the next section, we will describe the *B–H* relationship, which is one of the most widely used expressions for the magnetic properties of a core.

2.5 *B–H* Relationship

In the past section, we established a method to calculate the flux in a magnetic circuit. In order to perform this calculation, we need to know the nature of the magnetic material, the dimensions of the core, the length and the cross-sectional area. The only drawback to such a calculation is that it contains the dimensions of the magnetic material, which makes it a little inconvenient to use as the computation is coupled to a particular core construction. In this section, we will introduce two new physical quantities so as to express the flux calculation in a form that is independent of the core.

The magnetic field strength H for a magnetic circuit is defined from Ampere's Law as [22, 24]:

$$H = \frac{NI}{l} \tag{2.20}$$

The strength of the magnetic field is therefore the magneto motive force (MMF) per unit length of the magnetic path. The next term defined is the flux density B, which is the flux per unit cross-sectional area of the magnetic path:

$$B = \frac{\phi}{A} \tag{2.21}$$

With these two definitions, we can rewrite the magnetic flux equation as:

$$\frac{\phi}{A} = \mu \frac{NI}{l} \tag{2.22}$$

From our definitions of magnetic field strength and flux density:

$$B = \mu H \tag{2.23}$$

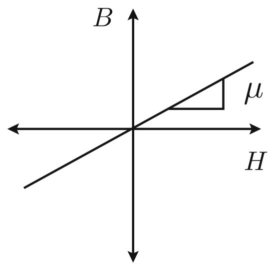


Fig. 2.5 Relationship between B and H with constant μ

In Eq. (2.23), we have eliminated the physical dimensions of the magnetic core. Instead we have expressed the flux density (flux per unit area) with respect to the magnetic field strength (MMF per unit length) and the permeability of the core. This equation therefore establishes the magnetic behaviour of any magnetic material as μ is a property of the material. This equation is a very powerful relation that sets the base for most of magnetic design in electrical engineering. Though this relation is extremely useful by itself, it assumes that the permeability μ is a constant. Every constant related to a material is subject to change with environmental conditions such as temperature and many others. However, given all environmental conditions to remain fixed, we might be tempted to use (2.23) as the absolute truth with respect to determining the flux density B in terms of the magnetic field strength H or vice versa. In simple terms, we expect the relationship between B and H to be as shown in Fig. 2.5.

Such a linear relationship between B and H with the slope of the line being the permeability μ would make a very convenient design tool. Such a tool can be used to calculate how much would be the flux density B in a core made of the material if the magnetic field strength H were to be of a given value. We could quickly determine what would be the effect of a given number of turns N of a coil wound on the core carrying a current I if we assume a mean length of the flux path in the core. Unfortunately, magnetism is a phenomenon that is much more complex than a mere straight line as depicted in Fig. 2.5. To capture detailed aspects of magnetism is not something that will be attempted in this book as our objective is to model magnetic components for power electronic applications. However, a few details are worth considering as they do affect us power engineers.

The first complication to Fig. 2.5 is the effect of saturation [22, 24]. As the magnitude of H increases, the magnitude of B will increase proportionally. However, for values of H greater than a particular value, the increase in B will no longer be proportional and will not obey (2.23). This effect is saturation. To visualize this, if the core is thought of as a container and flux as a fluid, saturation is when the container is filled to the capacity with this fluid called flux and cannot further take large quantities of it. Saturation is depicted as shown in Fig. 2.6. The two large dots in the curve indicate the limits of the linear part of the B–H relationship. Either these points or two different points within the linear part of the curve are called the knee points similar to the knee of a human leg. The values of B at these knee

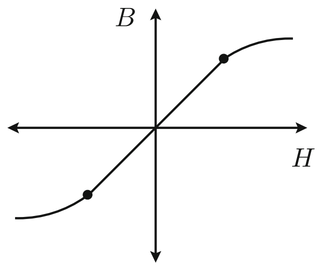


Fig. 2.6 Relationship between B and H showing saturation

points are taken as the maximum value of flux density that a core can be subjected to without causing the core to saturate.

A simulation of the effect of saturation is quite complex and will not be included in this book [5]. However, at this stage of the discussion, let us discuss two aspects of saturation—the effect and the relationship. If a very large current were to flow through a coil wound on a core such that the magnetic field strength H produced by it was larger than the value of H corresponding to the knee point, the core of the inductor is said to be saturated. The flux density B will no longer be proportional to the magnetic field strength H. If a current carrying conductor (wire) is forced to carry a current larger than the value it is safely allowed to carry, it will experience greater losses that could potentially damage the conductor. The resultant heating effect could cause the surrounding insulation to get damaged and if the current was very high, the wire might simply burn. In a similar manner, if the flux density B in a core was to be much higher than the knee point value, such a core might experience large losses that could cause the magnetic properties of the core to deteriorate. From basic physics, excessive heat can damage a magnet and deteriorate the magnetic properties of an electromagnet.

The next question is how do we study the behaviour of the core in this region of saturation if we can no longer count on the permeability μ of the core material to be a constant? Magnetic materials available as cores for most engineering applications are usually accompanied by datasheets where the manufacturers provide B–H curves under a number of different operating conditions [10, 26]. Therefore, to study the behaviour of the core when saturated, we must rely on experimental data provided by the manufacturer of the core material. In order to use these non-linear characteristics in a simulation, we could rely on techniques such as curve fitting where a non-linear expression is derived so that it matches a given curve. Therefore, in such cases, we would need curve-fitting techniques to estimate a function f such that:

$$B = f(H) \tag{2.24}$$

The other aspect of the B–H relationship that deviates from the ideal linear characteristics of Fig. 2.5 is the fundamental nature of magnetic materials. When a core made of a magnetic material such as iron is placed in a magnetic field, it

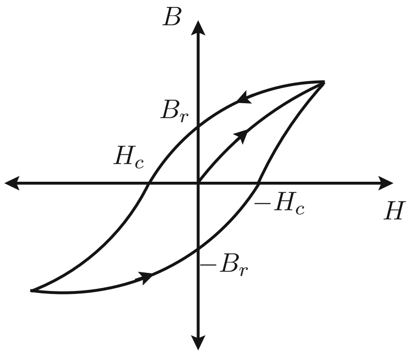


Fig. 2.7 Practical relationship between B and H without saturation

will become magnetized. If the magnetic field were to be removed, the core will continue to retain some of its magnetism even if the magnetic field produced by it is now weaker than as compared to when it was in the external magnetic field. In order to completely demagnetize the core, a reverse magnetic field will need to be applied. This is true for a coil wound on a core as well. This retention of magnetization by the core causes the B–H relationship to become non-linear and follow a path as shown by arrows in Fig. 2.7 [22, 24].

In Fig. 2.5, if the current I flowing through the coil increases, this will cause the magnetic field strength H to increase. Subsequently, we expect the flux density B to increase proportionally. However, in reality, the increase in B will not be linear but rather will be a curve. This is due to the fact that as the magnetic field strength increases and the core gets magnetized, the core is sluggish to respond to the increase in H with a proportional increase in B. One can think of this behaviour as a form of friction. The result is the B–H curve, which is as shown in Fig. 2.7. If the current flowing through the coil is an alternating current (ac), when it reaches its peak, it will decrease back to zero following a sine waveform with respect to time. As the current decreases to zero, H will decrease until it reaches zero. One would expect B to decrease proportionally to zero along the straight line of Fig. 2.5. However, this is where the retentivity of the core as a magnetic material plays a role. It "'fights" to retain some of the old B as H decreases.

Once current starts decreasing from its peak causing H to decrease as well, the operating point follows another curve. This is due to the property of the core to retain the magnetic flux and therefore, the decrease in B is slower than the decrease in H. When current becomes zero causing H to be zero, the flux density B has a positive non-zero value. This value B_r is called the retentive flux density. To force B to zero, the current has to become negative resulting in a negative H. The value of $H = H_c$ for which $B = 0$ is called the coercive magnetic field strength. Symmetric values of $-B_r$ and $-H_c$ can be found during the negative half cycle of the current. As with the case of saturation, these curves are usually provided by the manufacturer of the core for various operating conditions such as temperature.

In this section, we have described the relationship between the magnetic field strength H and the flux density B in the core. As already stated, this relationship between B and H allows us to define the behaviour of a magnetic core independent of the core dimensions as we found in the previous section. Depending on our requirements, we can model the core to the detail we require. We could neglect saturation and retentivity and use a linear relationship as shown in Fig. 2.5. Or we could include saturation while neglecting retentivity as shown in Fig. 2.6. Or we could include both saturation and retentivity—this curve is not shown but the phenomenon of retentivity extends also into the saturation zone. In the next section, we will talk about how we can put together all these different expressions described over the past few sections to form a simulation model.

2.6 Simulating an Inductor

In the past few sections, we have examined the different physical laws and phenomena that determine the operation of the inductor. We examined how Faraday's Law and Lenz's Law can be used to determine the relationship between flux and applied voltage. We examined how a magnetic circuit can be solved to determine the flux due to the current flowing in a coil. We also examined the B–H curve of a core and how saturation impacts this characteristic. In this section, let us put all this together to create a simulation model for an inductor.

When creating a simulation model for an electrical component (or even a machine), we need to determine the inputs and outputs [6–8]. An inductor is a two-terminal component as shown in Fig. 2.8 and is typically connected in series with other components. In the case of such a two terminal component, there are two electrical quantities that can be measured with respect to the terminals—the current flowing into one terminal and out of the other and the voltage across the terminals. For our simulation model of the inductor to be accurate in the electrical sense, the current flowing through the terminals and the voltage across the terminals must conform to the physical laws established for the inductor. If we were to express the voltage and current as functions with respect to each other:

$$v = L\frac{di}{dt} = f(i) \tag{2.25}$$

$$i = \frac{1}{L}\int v dt = g(v) \tag{2.26}$$

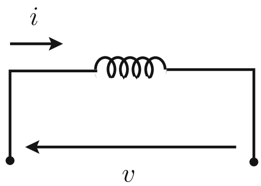


Fig. 2.8 The inductor as a two-terminal component

In the above equations, f and g would be the detailed magnetic models that we have discussed in the previous sections. However, the above equations show that whichever functional representation we choose, one variable will be independent and the other dependent. In the first equation, the current through the terminals is the independent variable and the voltage across the terminals is the dependent variable. In the second equation, the voltage across the terminals is the independent variable and the current through the terminals is the dependent variable. From an electrical sense, what does it mean if the voltage is independent or the current is dependent? To answer this question, we need to ask—is the component voltage driven or current driven? If we turn on a circuit with an inductor, are we applying a voltage across the inductor or are we passing a current through it? In most cases, the voltage is the independent input that is applied to a circuit while the current that flows is dependent on the components. Therefore, we can begin by labelling the voltage across the inductor terminals as the independent variable while the current needs to be calculated using our mathematical model.

Our simulation model must accept the voltage across the terminals as the input, use this voltage to calculate the current drawn from the mathematical model based on the physical laws and finally ensure that this current passes through the terminals. This is achieved using Fig. 2.9 where a Voltmeter measures the voltage V across the terminals and feeds it to the mathematical model $g(V)$. The output of the mathematical model is the current i. To realize this current i, a controlled voltage source is connected in series with a variable resistor between the terminals. The output of the controlled voltage source and the value of the variable resistor are adjusted in relation to the measured voltage across the terminals, such that the current flowing through the terminals is equal to the value calculated by the mathematical model.

One could argue that to produce a variable current equal to the value produced by the mathematical model, all we need is either a controllable voltage source or a variable resistor. When modelling an inductor and for that matter, any other electrical component using detailed models, even though we need only voltage to

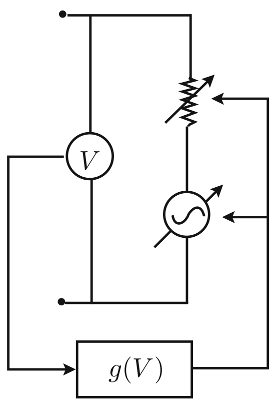


Fig. 2.9 Simulation model of the inductor

be variable, it is still advisable to choose the resistor as a variable resistor so that the model can be applied to systems of different ratings. This is so as to be able to perform impedance matching between our model and the system to which we are connecting the model. This will be better explained in the later section when we simulate our custom model. Therefore, even if the value of the variable resistor is calculated and set initially, choosing a value calculated from system ratings will result in a more stable simulation model.

In this section, we have described how a mathematical model of an inductor can be implemented in a simulation to produce a customizable component. We have not yet derived the mathematical model of the inductor, as we must decide the extent to which we would like to model the magnetic circuit of an inductor. The objective of this section was to define the interface of the simulation model of the inductor in terms of inputs and outputs before we formulate the mathematical model. In the next few sections, we will present a few cases where increasing details of the magnetic circuit of an inductor are simulated.

2.7 Simulations of an Inductor

In the previous section, we described the interface for the model that we can use for simulation of an inductor. In this section, let us begin with basic simulations where the equations pertaining to the magnetic model can be included. The simulations will gradually increase in complexity as the detail to which the magnetic model is simulated is increased. All simulations will be performed using the free and open source circuit simulator Python Power Electronics with all code written in Python.

Figure 2.10 shows the sample circuit that we can use to test our inductor models. In this circuit, we have connected a voltage source to a resistor R_1 and inductor L_1 connected in series. The resistor r_s is the parasitic resistance of the voltage source. We are measuring the current supplied by the voltage source and the voltage across the series combination R_1–L_1. We could choose either an ac voltage source of 50 or 60 Hz or we could choose a dc voltage source. For simplicity, let us choose a dc voltage source as it will be easier to interpret the simulation results with a dc

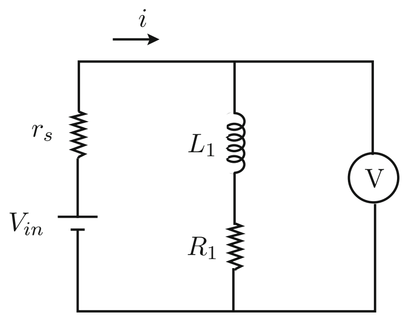


Fig. 2.10 Test circuit for inductor simulation

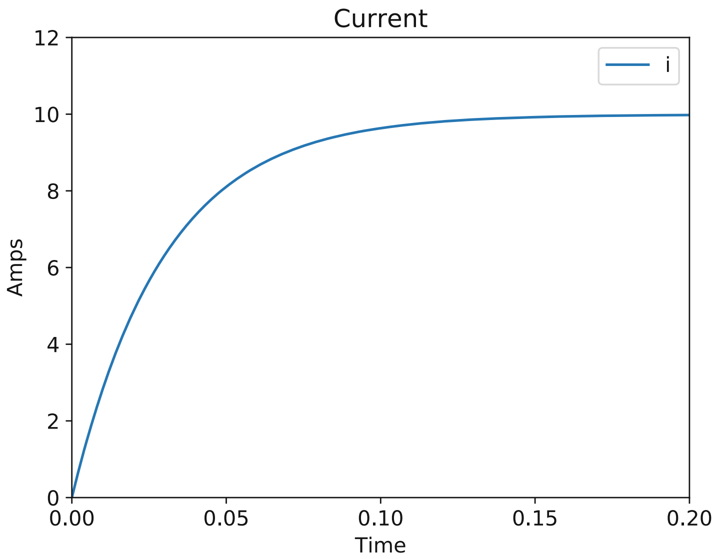


Fig. 2.11 Simulation result with in-built inductor model

voltage. We can choose some sample values for the simulation—a dc voltage of 100 V, $r_s = 0.01\,\Omega$, $R_1 = 10\,\Omega$ and $L_1 = 0.3$H. These are just random values and the reader is welcome to change these. It should be noted that in this test circuit, we are simulating the inductor L_1. To begin with, let us use the in-built model of the inductor, which contains no details of the magnetic circuit. This simulation can be found in the folder `inbuilt_inductor` within `chapter2_inductors` in the following link in the simulation repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics.

Figure 2.11 shows the plot of the current supplied by the voltage source. The time constant of the circuit is $\tau = \frac{L_1}{R_1} = 0.03$ s. Therefore, the settling time of the circuit is expected to be $4\tau = 0.12$ s. The settling time of a system is the time taken for the system to settle to within 2% of its steady state following a disturbance. In this particular case, the initial application of the dc voltage at time $t = 0$ is the disturbance. From the result of Fig. 2.11, the settling time of the circuit is approximately 0.12 s. The steady state will be the current $\frac{100}{R_1} = 10$ A. This of course is a fairly trivial simulation. However, our only purpose of showing this result is to compare it with the results we obtain from the simulation of a custom inductor.

Let us repeat the above simulation while replacing the in-built inductor in Fig. 2.10 with the custom model of Fig. 2.9. This simulation can be found in the folder `basic_Ldibydt_equation` within the folder `magnetic_model` in `chapter2_inductors` inside the repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics.

We will need a controllable voltage source and a variable resistor that can be attained by using the ControlledVoltageSource element and the VariableResistor element, respectively. Moreover, we also need a control logic for the function $g(v)$ shown in Fig. 2.9. Repeating the expression of $g(v)$ from the previous section:

$$i = \frac{1}{L}\int v dt = g(v) \tag{2.27}$$

The above expression needs to be realized in a program and used to regulate the voltage produced by the ControlledVoltageSource element such that a current equal to i flows in Fig. 2.9.

Before describing how the program corresponding to $g(v)$ can be written, there needs to be a change in the expression for $g(v)$ due to an issue specific to the numerical implementation of the integration in $g(v)$. In $g(v)$, we are integrating the voltage v applied across the inductor. The ideal case is that v will be a sinusoid of the fundamental frequency with some arbitrary phase angle. Integrating a sinusoid will produce a cosinusoid along with an integration offset:

$$\int V_m \sin(\omega t + \phi) dt = -\frac{\cos(\omega t + \phi)}{\omega} + C \tag{2.28}$$

To compute the value of the integration offset, we would use the initial conditions. However, in a physical system such as an electrical circuit, such an integration offset will imply a dc component in the current. In practical cases, this dc component that arises at the time the circuit is excited will gradually decay to zero due to the resistance in the circuit. However, in the expression for $g(v)$ above, we have considered a pure inductor without any resistance. Therefore, any integration offset that is produced will remain and not decay. However, this cannot be allowed to occur particularly if the inductor is used in ac circuits.

To ensure that any integration offset that appears eventually decays to zero, we need to add a parasitic resistance to the inductance. Such a parasitic resistance could represent the winding resistance of the inductor as well as any magnetic losses in the inductor iron core. Therefore, the modified expression for $g(v)$ is

$$i = \frac{1}{L}\int (v - ir) dt = g(v) \tag{2.29}$$

It is important to note that the resistance r in the expression for $g(v)$ above is not the same as the series resistance in Fig. 2.9. The series resistance in Fig. 2.9 is a much larger resistance that is needed to ensure that the difference between the output of the controlled voltage source and the voltage measured across the inductor terminals results in the current i as computed.

With this change in the expression for inductor current, we can now begin writing the control program. It is always best to begin with a skeletal code.

```
dt = 1.0e-6
L = 0.3
r = 0.01
if t_clock >= t1:
    # Inductor model
    t1 += dt
```

In the above block of code, we have defined three variables `dt`, `L` and `r` at the head of the file. `dt` is the time interval at which this code needs to be executed. In Python Power Electronics, control functions can be executed at designated time instants by defining time events in the configuration of the control file. In the above code, `t1` is the time event variable, which is compared with `t_clock`. `t_clock` is the present simulation time instant and is provided and updated by the circuit simulator. The inductor model will be executed when the time instant of simulation is greater than or equal to the time event. At the end of the inductor model computation, the time event variable is incremented with the time interval `dt`, which ensures that the inductor model is computed at the interval of `dt`.

Since we have not begun simulating power electronic circuits, a 1 μs integration time step is sufficient for a stable and accurate simulation. In the case of the custom inductor model, the control file is in reality a model computation file that should be executed at preferably the same time step as the simulation integration time step. In contrast, control files that run control algorithms and change the state of components run at a much larger time step as compared to the simulation integration time step. In later chapters, the time step `dt` of the control file and the integration time step will be reduced to smaller values when power electronic circuits will be combined with magnetic components.

The code in a control file such as the block above will be inserted into a function by the circuit simulator at run time. Therefore, all the rules for Python variables apply. As an example, every variable must be defined either at the head of the file such as `dt`, `L` and `r` or must be defined as special variables in the configuration of the control file such as the time event variable `t1`. The only exception is `t_clock`, which is an internal variable of the circuit simulator and it is advisable to not alter this. It is advisable to define constants and parameters at the head of the file such as the case with `dt`, `L` and `r`. All other variables that can change as the simulation proceeds should be defined in the configuration of the control file. For now, we have started with a definition of the time event variable `t1`.

With this basic background on the setup of a control file in Python Power Electronics, we can now include the integral equation $g(v)$ in the placeholder "Inductor model". The expression $g(v)$ contains the voltage measured by the Voltmeter across the component terminals. In the configuration of the control file, we can define an input variable `vmeas` and connect it to the measurement of the voltmeter. Numerically, the integral equation will be realized as a summation:

$$i[n] = \sum_{k=0}^{n} \frac{1}{L}(v[k] - i[k-1]r)dt \tag{2.30}$$

In the above equation, consecutive samples are separated by a time interval equal to dt. In order to calculate the current at a sample n, we must perform the summation from $k = 0$ to $k = n$ if we assume the circuit to be at rest at $t = 0$ and all variables to be 0. However, the current i also appears on the right hand side within the summation. In such a case, the sample of current within the summation will be the previous sample $k - 1$. The summation can be simplified to the following recursive form:

$$i[n] = i[n-1] + \frac{1}{L}(v[n] - i[n-1]r)dt \tag{2.31}$$

We have already defined the voltage v in the program as `vmeas`. We need to define the current i as a variable. From a programming perspective, we need any Python variable such as `ind_current` that can be initialized to a value of 0 at time $t = 0$ and will be updated with every new value of `vmeas`. As for the current in the right hand side, it will hold the previous value of the computation at the previous time instant. In order to achieve this, we can define a static variable `ind_current` and initialize it to 0. The Python code corresponding to $g(v)$ will be

```
ind_current += (1/L) * (vmeas - ind_current*r)*dt
```

We must now use this value of current to compute a voltage that will be generated by the ControlledVoltageSource. The voltage generated by the ControlledVoltage-Source must take into account the resistance of the VariableResistor in series with it. Let us now create two output variables in the configuration of the control file. The first will be connected to the ControlledVoltageSource and will be called `vsrc` and the second will be connected to the VariableResistor and will be called `series_res`. The initial value of `vsrc` can be 0 since at time $t = 0$ we would like the system to start from rest. The initial value of `series_res` on the other hand can be set to a value such as 100 Ω as the resistance in series with the voltage source cannot be zero but must be a value that can limit the current when the main voltage is switched on.

With the inductor current already calculated, the voltage to be generated by the ControlledVoltageSource can be computed by a simple application of Kirchhoff's Voltage Law in the closed loop with the Voltmeter, ControlledVoltageSource and VariableResistor:

```
vsrc = vmeas - ind_current*series_res
```

With this, we have now configured our control file with an input being the Voltmeter and outputs being the ControlledVoltageSource and VariableResistor. We have created a time event variable to regulate the intervals at which our model equations are computed and static variables have been defined, which will update as the simulation progresses.

Before we conclude on this basic simulation model of an inductor, we need to elaborate on the numerical integration technique used in the simulation. The inductor model above uses the Backward Euler integration method. In this simulation, we arrived at the results we expected and for a simulation as simple as this, Backward Euler provides sufficient accuracy and stability. However, in later chapters, we will simulate more complex magnetic circuits where accuracy and stability could play a major role in the final results obtained. Therefore, before we proceed to the next section, let us replace the Backward Euler integration method with the Runge–Kutta Fourth Order integration method. For a detailed description of the Runge–Kutta Fourth Order method, the reader is advised to read the following reference [36]. We could replace the single statement with multiple statements as follows:

```
    k1 = (1/L) * (vmeas - ind_current*r)
    k2 = (1/L) * (vmeas - (ind_current + dt*k1/2.0)*r)
    k3 = (1/L) * (vmeas - (ind_current + dt*k2/2.0)*r)
    k4 = (1/L) * (vmeas - (ind_current + dt*k3)*r)
    k = (k1 + k2*2 + k3*2 + k4)*dt/6.0
    ind_current += k
```

As can be seen, the Runge–Kutta Fourth Order method calculates four slopes based on the derivative of current and the final increment is the weighted average of the four slopes. The complete control code can be listed as follows:

```
dt = 1.0e-6
L = 0.3
r = 0.01
if t_clock >= t1:
    # Inductor model
    k1 = (1/L) * (vmeas - ind_current*r)
    k2 = (1/L) * (vmeas - (ind_current + dt*k1/2.0)*r)
    k3 = (1/L) * (vmeas - (ind_current + dt*k2/2.0)*r)
    k4 = (1/L) * (vmeas - (ind_current + dt*k3)*r)
    k = (k1 + k2*2 + k3*2 + k4)*dt/6.0
    ind_current += k
    vsrc = vmeas - ind_current*series_res
    t1 += dt
```

In this section, we have now introduced a control function that can execute a custom inductor model. The model currently does not contain any details of the magnetic circuit and is merely a numerical integration of the regular inductor equation. However, with this simple model, we now have a controller that can compute the current that is required to flow through the inductor given that a particular voltage is applied across it. In the next section, we will begin with introducing details of the magnetic circuit of the inductor coil.

2.8 Simulating an Inductor with the Magnetic Circuit

In the previous section, we had created a custom simulation model of an inductor using the control interface provided by Python Power Electronics. However, we had solved the basic $L\frac{dt}{dt}$ equation of the inductor. In this section, we will use the control interface setup in the previous section and extend the mathematical model to include the magnetic circuit of the inductor. We will begin with basic simulations that compute the flux in the iron core and later take into account the shape of the core and air gaps.

To include the magnetic circuit of the inductor, we need to break up our computation into several steps. We will still continue to use the model of Fig. 2.9 with the change being that the function $g(v)$ will be replaced by many other functions. To begin with, we will use Faraday's Law to express the induced emf with respect to the applied voltage. The induced emf in turn can be expressed using Kirchhoff's Voltage Law:

$$e = v - ir \tag{2.32}$$

where r is the parasitic resistance of the inductor as described in the previous section and is a combination of the winding resistance and also represents the core losses.

The induced emf can then be expressed as the rate of change of flux linkages of the inductor coil using Faraday's Law:

$$e = \frac{d\psi}{dt} = N\frac{d\phi}{dt} = v - ir \tag{2.33}$$

We have simplified the above expression further since flux linkage ψ of a coil is merely the product of flux ϕ associated with the coil and the number of turns N of the coil. The physical significance of the above expression is that since the resistance drop ir is usually negligible, the applied voltage defines the flux in the core that passes through the coil.

In the past section dealing with the basics of magnetics, we have already discussed how the flux in the core can be expressed in terms of the MMF of the coil and the reluctance ρ of the core flux path:

$$\phi = \frac{\text{MMF}}{\rho} \tag{2.34}$$

The above expression assumes a linear magnetic relationship such as shown in Fig. 2.5. We can calculate the reluctances of the different segments of the core and express the core flux as:

$$\phi = \frac{\text{MMF}}{\rho_{eq}} = \frac{Ni}{\rho_{eq}} \tag{2.35}$$

where ρ_{eq} is the equivalent reluctance of the core in case it contains an air gap (Fig. 2.3) or has more than one limb (Fig. 2.4).

From the above expression, calculating the inductor current i results in

$$i = \frac{\phi \rho_{eq}}{N} \tag{2.36}$$

In the above expression, we have included details of the inductor core. With this model, it is now possible to simulate a multi-limb core inductor with an air gap in one or more limbs as long as the structural details (length of limbs, air gap, cross-sectional area) are known and the number of turns of the inductor coil is known. Since, in the case of inductors wound for special purposes, these are usually known, the above model can be used simulate it directly.

Even at this stage, one can argue that the above model is still fairly simple since the equivalent inductance of a coil wound on a multi-limb core can be calculated anyway. However, we are now one step ahead of our basic $L\frac{di}{dt}$ inductor model. For now, let us use this simulation model with an example of the coil wound on a simple single limb core and then on a multi-limb core with air gaps. Let us consider a core similar to Fig. 2.3. At first, let us neglect the air gap and consider the core to be a continuous iron medium. This simulation can be found in the folder `rectangular_core_inductor` within the folder `magnetic_model` in `chapter2_inductors` inside the repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics.

Let us assume the core to have the following dimensions—cross-sectional area $A_c = 9\,\text{cm}^2$ and mean length of flux path $l = 15\,\text{cm}$. Let us assume that the iron core has a relative permeability of $\mu_r = 1000$. Let us assume that the coil wound on the core has 200 turns. This can be translated to the following Python code:

```python
import math
dt = 1.0e-6
r = 0.1
no_of_turns = 200
cs_area = 9.0e-4
length_iron = 15.0e-2
mu_0 = 4*math.pi*1.0e-7
mu_r = 1000.0
```

Since in this core, there is single limb without an air gap, the inductance can be calculated by the formula described before:

$$L = \frac{N^2 \mu A_c}{l} = \frac{200^2 \times 1000 \times 4\pi \times 10^{-7} \times 9 \times 10^{-4}}{15 \times 10^{-2}} = 0.3016\text{H} \tag{2.37}$$

This design of an inductor should result in a value of inductance very close to what was used in the previous simulation (0.3 Henry). Our simulation results should, therefore, not be very different. This would make the subsequent simulations interesting when we consider cores with multiple limbs and air gaps.

The model computations will however be in multiple steps instead of a single one. To begin with, we will compute the flux linkages and subsequently the core flux:

```
k1 = (vmeas - ind_current*r)
k2 = (vmeas - (ind_current + dt*k1/2.0)*r)
k3 = (vmeas - (ind_current + dt*k2/2.0)*r)
k4 = (vmeas - (ind_current + dt*k3)*r)
k = (k1 + k2*2 + k3*2 + k4)*dt/6.0
flux_linkage += k
flux = flux_linkage / no_of_turns
```

Here, `flux_linkage` will need to be defined as a Static variable in the control configuration to ensure that it can be accumulated between simulation iterations as required by the process of integration. We are merely integrating the induced emf, which by Faraday's Law is the rate of the change of flux linkages of the coil. The core flux is merely the flux linkages divided by the number of turns of the coil.

We can now use the core dimensions and the magnetic properties defined at the head of the file:

```
R_iron = length_iron / (mu * cs_area)
```

For now, in the absence of an air gap, the reluctance of the iron core is fairly simple to calculate. The inductor current can then be updated as:

```
ind_current = flux * R_iron / no_of_turns
```

The entire code can be summarized as:

```
import math
dt = 1.0e-6
r = 0.01
no_of_turns = 200
cs_area = 9.0e-4
length_iron = 15.0e-2
mu_0 = 4*math.pi*1.0e-7
mu_r = 1000.0
mu = mu_0 * mu_r

if t_clock >= t1:
    # Runge--Kutta Fourth Order method
    k1 = (vmeas - ind_current*r)
    k2 = (vmeas - (ind_current + dt*k1/2.0)*r)
    k3 = (vmeas - (ind_current + dt*k2/2.0)*r)
    k4 = (vmeas - (ind_current + dt*k3)*r)
    k = (k1 + k2*2 + k3*2 + k4)*dt/6.0
    flux_linkage += k
    flux = flux_linkage / no_of_turns
    R_iron = length_iron / (mu * cs_area)
    ind_current = flux * R_iron / no_of_turns
```

```
vsrc = vmeas − ind_current*series_res

indmodel_flux = flux
indmodel_emf = vmeas − ind_current*r

t1 += dt
```

Simulating with the above model will result in an inductor current that is very close to the waveform in Fig. 2.11. However, to understand what we have achieved by introducing a few intermediate computations, we can examine a few other quantities that we had no access to before. For example, we can define VariableStorage elements in the control configuration by the names of `indmodel_flux` and `indmodel_emf`. Let us assign the `flux` to `indmodel_flux` and plot this variable as shown in Fig. 2.12. Figure 2.12 shows how the core flux follows the same wave shape as the current since the magnetic flux is directly proportional to the current flowing through the coil. Figure 2.12 shows the flux being 0 at time $t = 0$ and attaining a steady state of approximately 15 milli Weber. We could also in the Python control code compute the flux density B. This will be left as an exercise to the reader. Plotting the flux density can be useful to know what would be the maximum flux density that is attained to ensure that this is below the saturation limit.

Another advantage of computing these in a simulation is that it helps to understand the theory behind electrical components such as inductors, and in later chapters, machines such as transformers. Figures 2.11 and 2.12 show how the current through the inductor and the flux in the core increases gradually until they

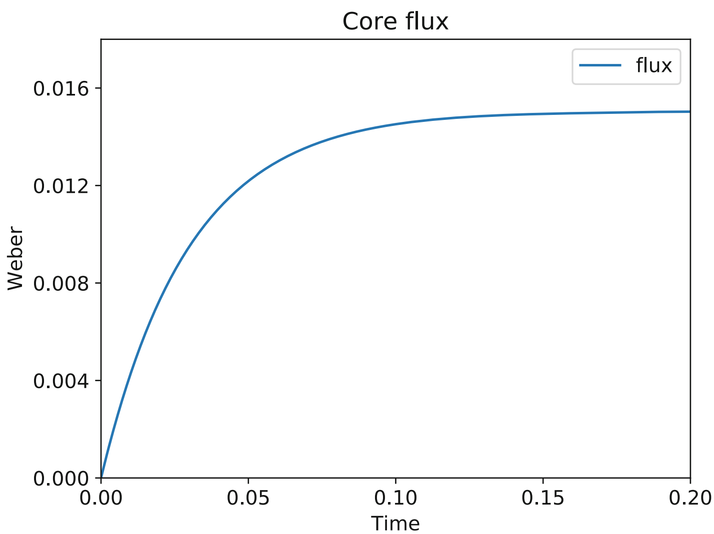


Fig. 2.12 Flux in the inductor core

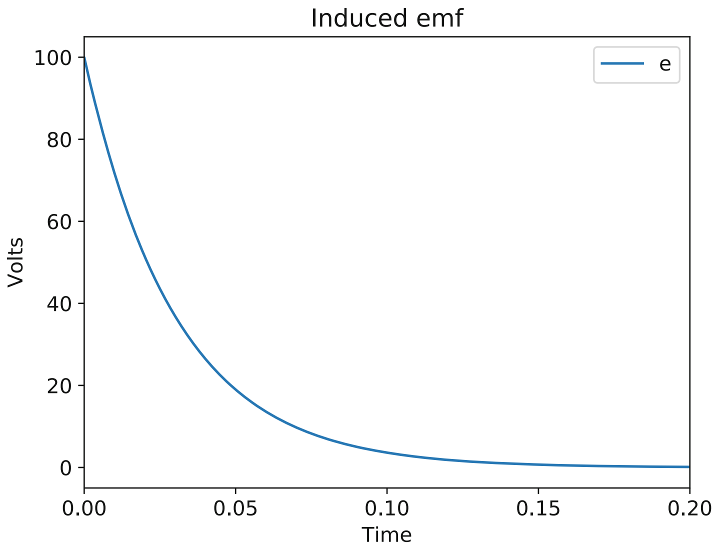


Fig. 2.13 Induced emf produced

reach constant values. Most of us who simulate circuits with inductors rarely stop to examine what happens at the fundamental level explained using the laws of physics. To begin with, we are applying a voltage of 100 V to a resistor–inductor combination at time $t = 0$. This is a step input to the circuit. However, the current starts at 0. The reader is encouraged to verify that if the inductor is removed, the current will jump to the steady state value of 10 Ampere.

The fundamental property of the inductor is that it opposes any abrupt change to the current flowing through it. This opposition can be inferred from Faraday's Law, which states that an emf is induced with a magnitude equal to the rate of change of flux linkages of the inductor coil and from Lenz's Law, which states that this induced emf will oppose the cause that produces it. In the Python code, we have plotted the induced emf by defining a VariableStorage element `indmodel_emf`. This induced emf is shown in Fig. 2.13. As can be seen, the induced emf starts at an initial value of 100 V. This induced emf of a magnitude equal to the applied voltage and with a polarity such that it will oppose the flow of current ensures that the current does not jump at the instant of applying the voltage.

As the current gradually increases, the induced emf gradually decreases until at steady state, the induced emf is zero. This is fairly intuitive as the highest rate of change of current is when the current has still not started to flow, while as it increases, the rate of change will decrease. Eventually, the induced emf is zero, and the current is limited only by the external resistor of 10 Ω shown in Fig. 2.10. Since all we have is a dc voltage source connected to a resistor, the result is a dc current, which does not change. The inductor plays no role in the circuit except for perhaps its negligible winding resistance and core losses.

The simulation results are also helpful in dispelling some misconceptions with respect to magnetic components. One misconception is that an inductor will offer an impedance to the flow of current equal to the inductive reactance $2\pi f L$. However, it is to be noted that the inductive reactance is frequency dependent and in a dc system, the inductive reactance is 0 in the steady state. It is important to note that an emf is induced only when the flux linkages of the inductor coil changes, which in turn implies that the current flowing through it must change. An emf will not be induced due to the flow of current through it.

Another misconception is that applying a dc voltage to an inductor will cause it to saturate. In Fig. 2.12, the core flux attains a constant value in steady state. However, whether this flux in the core will cause saturation can be determined by computing the flux density in different parts of the core. As shown in Fig. 2.6, every magnetic material has a value of flux density that defines the saturation level of a core formed of that magnetic material. Unless the flux density in the core exceeds the value of flux density at the boundary of the linear portion of the *B*–*H* relationship, the inductor cannot be said to be saturated. Application of a dc voltage alone will not force the inductor to be saturated. The current that flows through the inductor is also determined by the external impedance in the circuit. The flux in the core will be determined by the current flowing through the coil and the reluctance of the core.

In this section, we have gradually included the mathematical model of the inductor based on the basic laws of physics. Most importantly, we translated these laws to Python code and used it to simulate a basic inductor as a coil wound on an iron core. Such a simulation model would help an engineer verify the design of an inductor as it can also be used to plot quantities such as the flux or flux density in the core. In the next section, we will examine how the model can be augmented to account for air gaps and multi-limb cores.

2.9 Practical Cores with Air Gaps and Multiple Limbs

In the previous section, we began programming the basic laws of physics that can be used to model a magnetic component like an inductor. We had started with the simplest possible core—a rectangular core with a uniform cross-sectional area and no air gap. In many cases, inductor cores will have air gaps or multiple limbs. In this section, we will describe how to include air gaps and multiple limbs into the simulation model.

Let us simulate an inductor wound as a coil on a rectangular iron core with an air gap as shown in Fig. 2.3. This simulation can be found in the folder `rectangular_core_with_air_gap` within the folder `magnetic_model` in `chapter2_inductors` inside the repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics.

The core of Fig. 2.3 has two air gaps of length $\frac{l_g}{2}$. However, since it is a single magnetic flux path, we could assume a single air gap of length l_g. The code can be modified to have the following parameters at the head of the file:

```
length_core = 15.0e-2
length_airgap = 0.2e-3
length_iron = length_core - length_airgap
```

We have assumed an air gap length of 0.2 mm. Therefore, the length of the iron segment of the core can be calculated from the total length of the flux path and the air gap.

The calculation of the flux linkages and the resultant flux in the segment of the core in which the coil is wound remains unchanged. The calculation of the MMF and the resultant current through the coil will now be with respect to the total reluctance of the iron segment and the air gap:

```
    R_iron = length_iron / (mu * cs_area)
    R_airgap = length_airgap / (mu_0 * cs_area)
    ind_current = flux * (R_iron + R_airgap) / no_of_turns
```

It should be noted that though the air gap is only 0.2 mm, the permeability of the air gap is 1000 times lower than the permeability of iron.

This simulation will yield very different results. Figure 2.14 shows the inductor current. The rate of rise of the current can be seen to be very different. The reader is advised to calculate the inductance from the calculations presented in the previous section and verify that the settling time is equal to $4\frac{L}{R}$. Since a dc voltage is applied

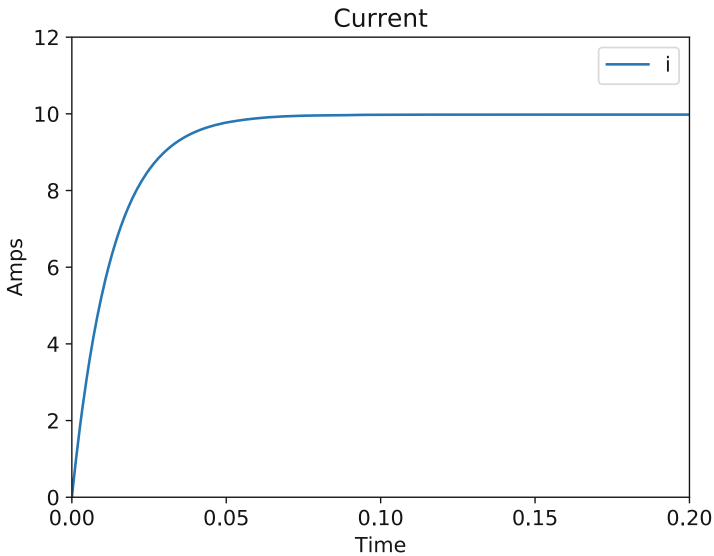


Fig. 2.14 Inductor current

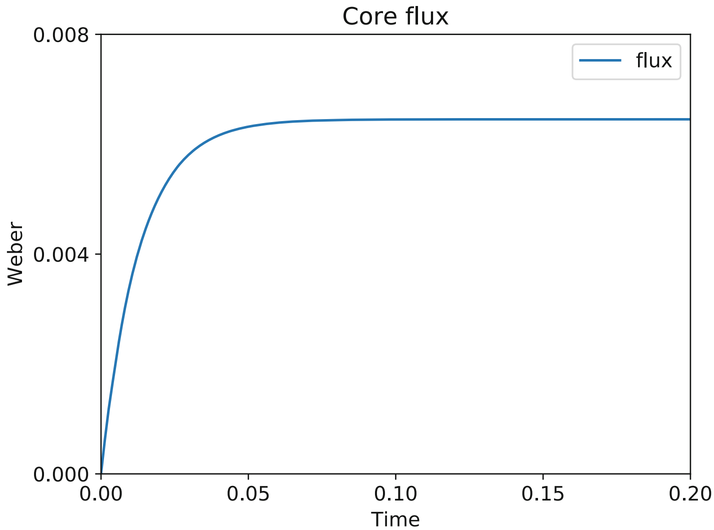


Fig. 2.15 Flux in the core with air gap

to the R–L combination, the steady state current value is not affected and is still 10 Ampere.

Figure 2.15 shows the flux in the core. The maximum value of flux at steady state has decreased. This is due to the fact that the steady state value of current is determined by the $10\,\Omega$ resistance. However, the total reluctance of the iron core is much greater due to the air gap. Therefore, even though the MMF generated is the same in steady state, the resultant flux is smaller due to the larger equivalent reluctance of the magnetic circuit. The reader can further expand the results by plotting the MMF across the air gap and the MMF across the iron segment of the core to compare how total MMF produced by the coil is used in the magnetic circuit. The reader is also encouraged to plot the emf and verify that it falls to zero at the same rate as the flux rises to its steady state value.

Before we proceed to the next simulation, it is worth discussing the simulation results. To calculate the flux linkages and the flux, we have used Faraday's Law. Even though Faraday's Law equates the induced emf to the rate of change of flux linkages, for most practical inductor coils with a negligible winding resistance, the applied voltage is approximately equal to the induced emf. Therefore, at time $t = 0$, we are integrating the applied voltage to compute the flux. As the flux increases from zero with the integral operation in the computation, the current is calculated according to the MMF needed to overcome the reluctance of the core and the air gap. As the length of the air gap increases, the MMF and, therefore, the current will also increase. This accounts for the rapid rise in the current as the MMF needed to produce the flux is much greater in the presence of the air gap. However, it should be noted that the current also flows in the external circuit that has a $10\,\Omega$ resistor producing a larger voltage drop across the external resistor, due to which the applied voltage across the inductor will also fall sharply. This lower applied voltage in turn results in lower flux linkages and core flux that eventually results in a lower steady state value of core flux.

Now that we have simulated an iron core with an air gap, let us consider a more complex core such as that of Fig. 2.4. This simulation can be found in the folder `multi_limb_core_with_air_gap` within the folder `magnetic_model` in `chapter2_inductors` inside the repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics.
The core has multiple legs and more than one air gap. In the past section, where the inductor of Fig. 2.4 was presented, the reluctance of the different legs of the core was also derived from the geometry of the core. We can translate these to Python code at the head of the control file:

```
length_a = 5.0e-2
length_b = 4.5e-2
length_c = 4.0e-2
length_w = 3.0e-2
length_lg_2 = 0.1e-3
```

These definitions are similar to the dimensions marked in Fig. 2.4.

The flux linkages of the core will still be computed in exactly the same manner as until now, as the rate of change of flux linkages will continue to be governed by Faraday's Law. Subsequent to the computation of flux linkages, the flux will also be calculated by dividing the flux linkages by the number of turns of the coil. However, this flux that has been calculated is the flux passing through the coil. From Fig. 2.4, this flux is in the left limb of the core on which the coil has been wound. To determine the MMF that would produce this flux, we can solve the magnetic circuit as shown in Fig. 2.16.

The flux ϕ can be expressed as:

$$\phi = \frac{\text{MMF}}{\rho_{eq}} = \frac{\text{MMF}}{\rho_1 + \frac{\rho_2 \rho_3}{\rho_2 + \rho_3}} \tag{2.38}$$

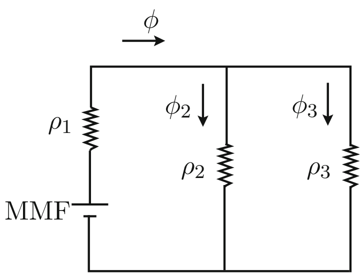


Fig. 2.16 Equivalent magnetic circuit of multi-limb core inductor

The reluctances of the different segments of the core have already been described in the past section and can be translated to Python code as:

```
R1 = (2*length_a + length_b + 2.0*length_w - 2*length_lg_2) / (mu * cs_area)
Rg1 = 2*length_lg_2 / (mu_0 * cs_area)
R2 = (length_b + length_w) / (mu * cs_area)
R3 = (2*length_c + length_b + 2.0*length_w - 2*length_lg_2) / (mu * cs_area)
Rg2 = length_lg_2 / (mu_0 * cs_area)
R_eq = R1 + Rg1 + (R2 * (R3 + Rg2) / (R2 + R3 + Rg2))
```

A potential advantage of calculating the reluctances of each limb of the core is that we can now compute the flux (and flux density) flowing in each limb. The computation of the inductor current will now take into account the equivalent reluctance `R_eq`:

```
ind_current = flux * R_eq / no_of_turns
indmodel_flux = flux
indmodel_flux2 = flux*(R2 * (R3 + Rg2)/(R2 + R3 + Rg2))/R2
indmodel_flux3 = flux*(R2 * (R3 + Rg2)/(R2 + R3 + Rg2))/R3
```

We can create two more VariableStorage elements `indmodel_flux2` and `indmodel_flux3` and assign to them the values of flux in the central limb and the right limb, respectively.

Figures 2.17 and 2.18 show the simulation results with the inductor current and the fluxes in the three limbs. The current continues to have a much faster settling time than the first result of Fig. 2.11. This is due to the smaller inductance owing to the air gaps in the left and right limbs of the core. Figure 2.18 shows the fluxes in the three limbs, with the flux ϕ in the left most limb being the highest as the coil

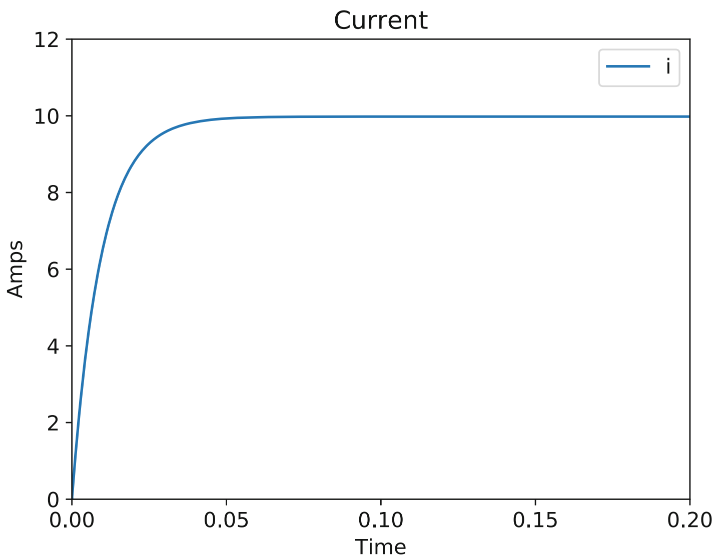


Fig. 2.17 Inductor current

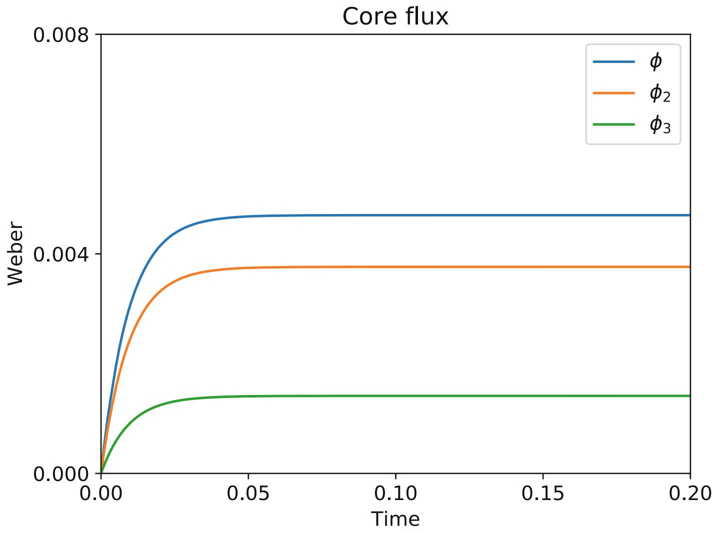


Fig. 2.18 Fluxes in different core limbs

is wound on that limb. The flux ϕ_2 in the central limb is greater than the flux ϕ_3 in the right limb as the right limb has an air gap leading to it being the path of greater reluctance. The reader is also encouraged to verify that $\phi = \phi_2 + \phi_3$ as the flux ϕ in the left most limb divides into ϕ_2 and ϕ_3.

With these simulations, we have simulated inductors wound on cores with multiple limbs with air gaps. The purpose of these simulations was to demonstrate how a simulation model can be formulated based on the laws of physics. Such a simulation can be used to verify the design of an inductor that might be wound for any given application, as the simulation takes into account the number of turns of the inductor coil and the construction of the inductor core. The simulation enables us to plot variables such as the flux in different segments of the core and thereby determine if there is a possibility of the core to be saturated.

2.10 Conclusions

In this chapter, we have introduced the concept of magnetic circuit modelling by simulating inductors. Besides a theoretical introduction to magnetic circuits, this chapter provided simulations of circuits with inductors. This chapter might seem trivial and useless as in most cases, one cannot imagine a need to simulate the magnetic circuit of an inductor. Therefore, in concluding this chapter, let us discuss what has been gained from this chapter as well as what are its shortcomings.

Magnetism as a subject has been studied extensively and this chapter has merely used some of the most basic laws and expressions related to magnetic fields in electric circuits. We used Ampere's Law to state that the inductor coil will produce a magnetic field and used the Right Hand Thumb Rule to determine the direction of the magnetic lines of flux. For a few types of inductor cores, we illustrated the path of the magnetic flux. It is important to note that determining the magnetic flux is fairly complex and requires a very different approach. In this chapter, we assumed the magnetic field to be uniform and neglected all other effects—non-uniformity, fringing etc. The purpose of describing the magnetic field produced in an inductor core was to introduce the concept of the magnetic circuit.

With a few examples, we had shown how a magnetic circuit can be solved in a very similar manner to an electric circuit. We had introduced the concept of magneto motive force (MMF), reluctance of the flux path, magnetic field strength and flux density. We had shown how for the case of a simple coil wound on a core, we could use the MMF generated along with the reluctances of the core segments to determine the flux flowing in all the segments of the core. With flux being analogous to current, we found that the reluctances of the core segments can be treated in exactly the same way as resistances in an electric circuit. Despite the fact that we have neglected all the non-linearity in the magnetic field, to be able to determine the flux in every part of the core given a particular current flowing through a coil leads to a better picture of the operation of an inductor.

Following a discussion on magnetic circuits, we had described how the magnetic circuit of an inductor can be simulated. Eventually, we need an inductor model that can be inserted into a circuit. The chapter described how the inductor can be modelled as a combination of a Voltmeter, a controlled voltage source and a variable resistor. We described how the control loop takes as an input the voltage across the inductor terminals and solves a mathematical model that produces a current that will flow through the inductor. This current is produced by modifying the output of the controlled voltage source with respect to the voltage measured across the terminals. Several simulations were presented where we gradually increased the complexity of the mathematical model of the inductor, until the final simulation presented the solution of the magnetic circuit whereby the flux and subsequently the flux density in every part of the core can be computed.

With this chapter, we have described how the basic laws of physics can be used to produce a simulation model for the inductor. It is important to emphasize that this simulation model does not model the non-linear aspects of magnetism. However, for most power electronics applications, a basic insight into the magnetic circuit of an inductor will be sufficient to determine the operation of an inductor in certain applications. As an example, in power supply applications, engineers find themselves winding inductors by hand. The simulation models in this chapter can be used to roughly estimate how such hand-wound inductors would behave in power converters. Besides being able to determine the current flowing through the inductor, other details such as the flux density in the core can be determined as well.

The main purpose of this chapter, however, was to serve as a gradual introduction to the more complex magnetic circuits that will be found in machines such as in transformers that will be covered in the later chapters. By beginning the discussion on magnetic fields, we have laid the foundation for the concept of energy transfer through the magnetic field. In the next chapter, we will begin the discussion on magnetic coupling between multiple coils wound on a core. The mathematical model of coupled inductors will be an extension of the single inductor presented in this chapter using the same basic laws of physics presented in this chapter. The next chapter on coupled inductors is also an intermediate chapter before we progress to simulating transformers. By meandering through this concept of magnetic circuits, we wish to provide the reader an insight into the importance of magnetism in electrical engineering, and how it can be understood and modelled in a fairly simple manner using only the most basic laws of physics.

Chapter 3
Simulating Magnetically Linked Circuits

3.1 Introduction

In the previous chapter, we described how we can simulate a basic inductor while including some details of the construction with respect to the core structure and the number of turns of the inductor coil. With inductors being one of the most fundamental components in any power electronic converter, the ability to simulate an inductor in detail will be beneficial to power engineers who might have to wind their own inductors for specific applications. In this chapter, we will extend our knowledge of magnetic circuits and the flux produced in the core due to current flowing in the inductor coil to examine the behaviour of magnetically coupled coils.

A vast number of machines in electrical engineering use the concept of magnetic coupling to achieve transfer of power [1–4]. The next chapter will be devoted to transformers, which will describe how a transformer can achieve transfer of power from one isolated electrical system to another [10, 26]. Besides transformers, motors and generators also use the concept of magnetic coupling to achieve conversion of electrical energy into mechanical energy and vice versa [3, 4]. Therefore, before describing the operation of transformers, this chapter is devoted to describing the concept of magnetic coupling. Even though magnetic coupling is a mere extension of the basic laws of physics already described in the previous chapter, there are several nuances to how magnetically coupled circuits can behave. In this chapter, using a simple setup of two coils wound on a core, the nature of magnetic coupling is described using both theory and simulations [11].

All the basic laws of physics (Ampere's Law, Faraday's Law and Lenz's Law) are applicable in the case of more than one coil wound on a core [22, 24]. We will describe how the fundamental nature of magnetic coupling is the fact that emfs will be induced in all the coils wound on a core through which a magnetic flux is flowing [11]. Therefore, by energizing a single coil wound on a core, energy can be transferred to other coils wound on the core. Furthermore, by using Faraday's Law and Lenz's Law, we can decipher the nature of the emfs induced in all the

S. V. Iyer, *Modeling and Python Simulation of Magnetics for Power Electronics Applications*, https://doi.org/10.1007/978-3-030-96768-0_3

coils and subsequently, the current that will flow through them. We can continue to translate these systems to magnetic circuits and solve them to determine the system operation. A number of examples are presented to illustrate these concepts in detail.

In order to be able to express magnetically coupled circuits in a concise manner, we define the mutual inductance between two coupled coils [1, 2, 10, 26]. We describe how the mutual inductance between coils along with the self-inductances of the coils results in concise expressions of flux linkage and subsequently of induced emfs. To eliminate the need to determine the polarity of induced emfs based on the physical winding sense of coils, we describe the dot polarity convention [1, 2, 10, 26]. Using the dot polarity convention, we describe how magnetically coupled systems can be represented using simple schematics with the only addition being dots on the terminals of inductors to indicate the nature of coupling between inductors. Using self and mutual inductances along with dot polarities, we describe how detailed simulation models can be developed for several cases of coupled inductors.

Several simulations will be described where the above concepts are translated to practical Python code. Simulation results are analysed so as to compare the simulated behaviour with what we expect from theoretical results. To conclude this chapter, we will present a scalable simulation model using matrix equations with which we can simulate any number of coupled inductors. In the next chapter, this scalable simulation model will be used to simulate transformers. The next chapter based on transformers will examine the practical application of transfer of energy through magnetic coupling. This chapter will serve as an intermediary chapter where magnetic coupling is formulated mathematically extending on our knowledge of magnetic circuits presented in the previous chapter on inductors. As in the previous chapter, theory will be supported by simulations and code samples that the reader can implement and tinker with.

3.2 The Concept of Magnetic Coupling

In the previous chapter on simulating inductors, we have examined how a coil wound on an iron core produces a flux that flows through the entire core [22, 24]. We defined the reluctance of a magnetic medium, which we showed was dependent on the permeability of the magnetic material, the length of the flux path through the medium and the cross-sectional area of the flux path. We also examined how flux can be calculated using magnetic circuits just like the current in an electric circuit, with the MMF being the driving force and the reluctance of the different parts of the magnetic core perceived as series/parallel combinations.

In the previous chapter, when we had described Faraday's and Lenz's Laws to determine the magnitude and polarity of the emf induced in an inductor, we had only considered the current through the inductor, which was responsible for producing

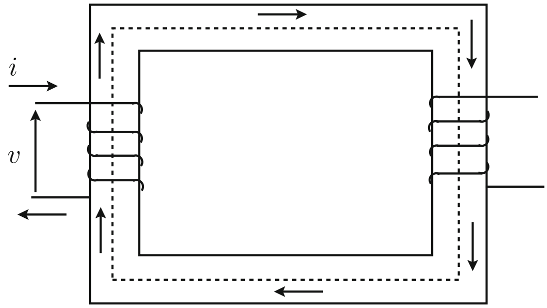


Fig. 3.1 Two coils wound on the same core

the flux. In this chapter, and this section, we will get started with the concept of magnetically coupled coils [11]. As an example, consider the same rectangular core as in the previous chapter as shown in Fig. 3.1. However, let us assume that another coil is wound on the other leg. The coil wound on the left limb is connected to an external circuit, which is not shown in the figure for simplicity, but this external circuit results in a current i flowing through the coil. The second coil wound on the right limb is left open-circuited without any connection either to the coil on the left limb or to any other external circuit.

As before, the current flowing through the first coil will produce a magnetic flux that flows through the core. The direction of the flux shown by arrows within the core can be determined by using the Right Hand Thumb Rule with respect to the current flowing through the coil. The face of the coil through which current flows in the counter-clockwise direction forms the North pole of the coil and the other face of the coil forms the South pole. This flux flows not only through the coil wound on the left limb that produces it but also with the second new coil wound on the right limb. The two coils that have no electrical connection are now said to be magnetically linked. Faraday's and Lenz's Laws apply to both coils. If the current through the first coil is changing, the resultant flux flowing through the core will also be changing. Therefore, this changing flux will induce an emf in the first coil as well as the second coil, with the magnitudes of the emfs determined by Faraday's Law, *i.e.* directly proportional to the rate of change of flux linking the coils.

Before we progress to developing the mathematical model for the entire system, let us express the magnetic coupling mathematically [10, 11, 26]. If the flux in the core is denoted by ϕ, the coil of the left limb has N_1 turns and the coil on the right limb has N_2 turns, the flux linkages of the two coils can be expressed as:

$$\psi_1 = N_1\phi \tag{3.1}$$

$$\psi_2 = N_2\phi \tag{3.2}$$

The magnitude of the emfs induced in the coils is determined by Faraday's Law:

$$e_1 = \frac{d\psi_1}{dt} = N_1 \frac{d\phi}{dt} \tag{3.3}$$

$$e_2 = \frac{d\psi_2}{dt} = N_2 \frac{d\phi}{dt} \tag{3.4}$$

As can be observed from the above equations, both the induced emfs e_1 and e_2 are proportional to the rate of change of core flux $\frac{d\phi}{dt}$. Therefore, the magnitude of the induced emfs only differ by the number of turns N_1 and N_2 of the two coils. This fact shows the nature of coupling between the two coils. Each coil has an induced emf with a magnitude that follows the rate of change of core flux. So far we have examined the magnitude of the induced emfs, but what about the polarity? As in the case of inductors in the previous chapter, it is better to determine the polarity of the induced emfs in the circuit rather than express it in the equations. The polarity of the induced emfs can be determined by using Lenz's Law, which we have already used in the previous chapter while simulating inductors. The induced emf in a coil will be in such a sense that it opposes the cause that produces it. The cause of the induced emf is the changing core flux passing through the coil. Therefore, both e_1 and e_2 will oppose the change in flux ϕ.

With the case of the simple inductor in the previous chapter, we never had to delve deep into the polarity of the induced emf. The induced emf was merely one of the intermediate computations while calculating the current flowing through the inductor. However, in the case of coupled coils, we have many interesting cases as will be examined soon. Therefore, the polarity of the induced emf is now fairly important, particularly in the second coil wound on the right limb. To determine the polarity of the induced emf, we need to ask the question—how can the induced emf oppose the change in core flux that produces it? Answer—the induced emfs have to produce core fluxes that oppose the change in the core flux. Here again, it is important to emphasize that the induced emf will oppose the change in the core flux and not the core flux itself.

To determine the induced emfs in the two coils, we note that the coil wound on the left limb is energized by an external voltage v, while the coil wound on the right limb is left open-circuited. Therefore, for the coil wound on the left limb, the following expression can be written as:

$$v - ir_1 = e_1 \tag{3.5}$$

where r_1 is the winding resistance of the coil wound on the left limb. In the case of the coil wound on the right limb, since it is open-circuited, there will be no current flowing, nor is there an external voltage source connected to it. However, if the voltage across the terminals of the coil wound on the right limb is measured, it will be the same as the induced emf e_2 as there is no resistive drop with the current being zero.

From (3.5), it appears as though the induced emf e_1 opposes the current i flowing through the coil wound on the left limb. This is true as Kirchhoff's Voltage Law is always applicable in any closed loop, and the fundamental basis of the law is that the sum of voltage sources and voltage drops in a closed loop will be zero. Since the resistive drop ir_1 is usually negligible for most practical coils, the main opposition to the applied voltage is the induced emf. It is the induced emf that limits the flow of current through the coil when a voltage is applied across it. Even despite the fact the induced emf seems to be the only opposition to the flow of current, it is important to emphasize that the induced emf will not always be there to oppose the flow of current. The induced emf will *only* be present when the magnetic flux associated with the coil changes with respect to time. As an example, if the applied voltage were to be a dc voltage, in steady state, the induced emf will be zero once the current rises and settles to the steady state value. We have examined this in the previous chapter using simulations.

Equation (3.5) can still be used to answer the question—how will the induced emf oppose the cause that produces it (the changing core flux)? If we were to remove the applied voltage v by short-circuiting it, we now can examine the effect of the induced emf e_1 alone. Figure 3.2 shows the effect of the induced emf alone. Since the supply voltage has been replaced by a short-circuit, the direction of current has reversed. This current is denoted by i', as it is not the same as the actual coil current in Fig. 3.1. Due to the reversal of the current, the core flux has also reversed direction with respect to the core flux in Fig. 3.1. This can be verified by applying the Right Hand Thumb Rule. Therefore, the induced emf attempts to oppose the changing core flux by producing a changing flux that flows through the core in the reverse direction.

A deeper investigation will show the true nature of the induced emf [10, 26]. According to Faraday's Law, the induced emf is directly proportional to the rate of change of core flux $\frac{d\phi}{dt}$. If this $\frac{d\phi}{dt}$ were positive, it would mean that the core flux is increasing in the direction shown in Fig. 3.1. This would imply that the induced emf e_1 would also have a rate of change that is positive, *i.e.* $\frac{de_1}{dt} > 0$. From Fig. 3.2, such an induced emf will produce a current i', which will also have a positive rate of

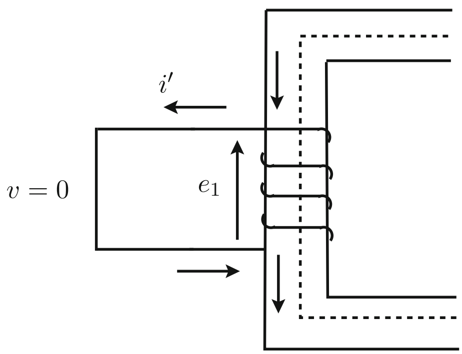


Fig. 3.2 Effect of induced emf on the core flux

change, *i.e.* $\frac{di'}{dt} > 0$. An increasing current i' will produce an increasing flux in the downward direction. Therefore, the increasing core flux is opposed by an increasing induced emf that tries to produce an increasing core flux in the reverse direction. The reader is encouraged to try out the above steps in the case when the core flux in Fig. 3.1 were to be decreasing.

In the above case of examining the effect of the induced emf, we ended up combining Faraday's Law and Kirchhoff's Voltage Law to prove that Lenz's Law is applicable—that the induced emf will be such that it opposes the cause that produces it. We began with the argument that if an external voltage source is connected to the coil, by Kirchhoff's Voltage Law, the induced emf will oppose the applied voltage as the only voltages in the closed loop are the applied voltage, the resistive drop and the induced emf. If we wish to determine the nature of induced emf in the coil wound on the right limb, we can no longer use Kirchhoff's Voltage Law, as the coil on the right limb is left open-circuited. However, even if the coil is open-circuited, the induced emf in the coil will abide by Faraday's Law and Lenz's Law.

We have already expressed the equation for the magnitude of the induced emf as directly proportional to the rate of change of core flux. We need only to determine the polarity of the induced emf. Figure 3.3 shows the coil wound on the right limb. To determine how the induced emf can oppose the cause that produces it, we need make an assumption. This is due to the fact that when the coil is open-circuited, no current can flow through the coil and therefore, nothing can be done to oppose the change in core flux, which is the cause that produces it. Let us assume that the coil is short-circuited as shown by dashed lines. If we assume the coil to be closed, a current can now flow through the coil and this current can produce a flux that opposes the change in core flux. From Fig. 3.1, the core flux passes through the coil in the downward direction. Therefore, in order to oppose the change in the core flux, the current flowing through the coil must be such that it produces a flux that passes through the coil in the upward direction as shown in Fig. 3.3. This is possible only if the induced emf e_2 has a polarity as shown in Fig. 3.3, which will result in the current i_2'.

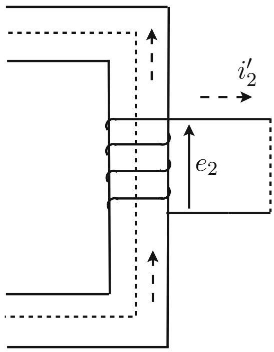


Fig. 3.3 Induced emf in the coupled coil

The induced emf e_2 can be quantified with respect to the rate of change of core flux $\frac{d\phi}{dt}$ as done for the coil wound on the left limb of the core. Let us consider what happens when the core flux is decreasing, *i.e.* $\frac{d\phi}{dt} < 0$. In this case, the induced emf e_2 will have a magnitude that is negative. With the polarity of e_2 as shown in Fig. 3.3, an induced emf of negative magnitude would result in a current that is negative. A negative coil current would try to produce a flux in the core that is negative, which would imply that for the direction shown in Fig. 3.3, the flux would be in the downward direction. This is the same direction as the core flux. This might at first glance seem like a mistake, as we expect the induced emf e_2 to produce a flux that will oppose the core flux. We would like to reiterate that the induced emf will not oppose the core flux, but instead it will oppose the change in core flux. The core flux is decreasing and therefore, the induced emf will attempt to produce a flux that will cause it to increase. Therefore, the induced emf attempts to produce a flux in the same direction as the core flux, thereby opposing the decrease in the core flux. The reader is encouraged to examine the case when the core flux is increasing, *i.e.* $\frac{d\phi}{dt} > 0$.

In this section, we examined in detail how the basic laws of physics can be used to show how two coils whose magnetic fields interact with each other end up being magnetically coupled. Due to one of the coils being energized by a voltage source, an emf is induced in both coils. The nature of magnetic coupling might seem confusing to begin with. In the next section, we will examine several different cases with the same example of two coils wound on a rectangular core. Through these cases, the concept of magnetic coupling will be clearer to the reader after which we can proceed to developing the simulation model.

3.3 The Nature of Magnetic Coupling

In the previous section, we had examined how two coils wound on the same core become magnetically coupled because the magnetic field produced by one of them interacts with the other. We had used one specific case to examine how emf is induced in the two coils. As we progress to transformers in the next chapter, the nature of magnetic coupling can become a bit tricky to fully understand. Therefore, it would be better to examine through a number of different examples how coupled coils behave. We will consider the same case of the two coils wound on the rectangular core in Fig. 3.1 as in the previous section, but we will look at several different scenarios.

For each of the cases in this section, we will use the basic laws of physics to determine the nature of coupling, specifically, Faraday's Law and Lenz's Law. The reader should be familiar with these laws and most importantly the nuances related to these laws [22, 24]. Faraday's Law states that a changing magnetic field associated with a coil (or conductor) results in an emf induced in the coil, and the induced emf is equal to the rate of change of flux linkages of the coil. It is important to emphasize

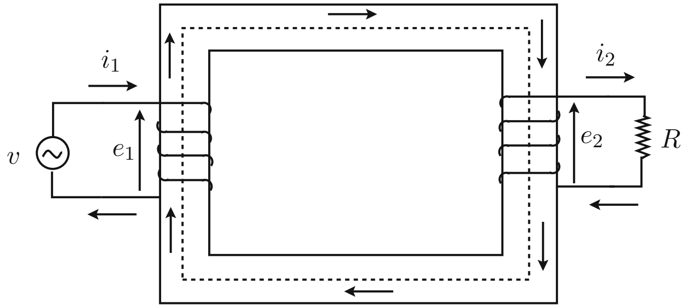


Fig. 3.4 Coupled coils

that the magnetic field must change for the emf to be induced. Lenz's Law states that the induced emf will be such so as to oppose the cause that produces it. It is again important to emphasize that the cause of the induced emf is the changing magnetic field and not the magnetic field itself. Armed with these two laws, let us now examine several interesting cases with the two coils wound on a rectangular core.

Let us expand on Fig. 3.1 by adding some components as shown in Fig. 3.4. The coil on the left limb is connected to an external voltage source v. There could also be a series impedance R-L-C circuit with an impedance of Z_1 along with the voltage source, but for simplicity, we have neglected the equivalent impedance for now. The coil on the right limb is connected to a resistor that can be thought of as a dump resistor. The reader can repeat the logic used in the previous section to arrive at similar conclusions. Emfs e_1 and e_2 are induced in the left and right limb coils, respectively. The currents i_1 and i_2 will be flowing through the two coils. The flux in the core will be flowing in the direction shown by arrows.

At first glance, by energizing the coil on the left limb, we are causing energy to be produced in the coil on the right limb, which is dissipated in the dump resistor R. Through magnetic coupling between the coils, energy is transferred from one coil to the other through the medium of the magnetic field. We can write Kirchhoff's Voltage Law for the coil on the left limb:

$$v - i_1 r_1 - e_1 = 0 \tag{3.6}$$

If we assume the circuit to be completely de-energized initially (at $t = 0$), the current i_1 at $t = 0$ is 0 and therefore, the induced emf $e_1 = v$. Another approach to this relation is to use the fact that the coil has an inductance that will not allow the current through to it to change rapidly. Therefore, the current i_1 through the coil will change gradually from 0. As a result, the induced emf e_1 will be initially equal to the applied voltage v and will gradually differ as the current flows through the coil.

As the current i_1 flows through the coil in the left limb, a flux will build up in the core flowing in the direction shown. How can we determine this flux? We could use the magnetic circuit that we used in the previous chapter while simulating inductors. The net MMF will be $N_1 i_i - N_2 i_2$. If we know the dimensions of the core and the relative permeability of the core material, we can calculate the reluctance of the core and subsequently the flux. However, instead of using this method, we will use another method that will be more convenient as we deal with multiple coupled coils. According to Faraday's Law, the induced emf is equal to the rate of change of flux linkages:

$$e_1 = \frac{d\psi_1}{dt} = N_1 \frac{d\phi}{dt} \tag{3.7}$$

Therefore, the flux in the core can be expressed as:

$$\phi = \frac{1}{N_1} \int e_1 dt = \frac{1}{N_1} \int (v - i_1 r_1) dt \tag{3.8}$$

In the above equation, since the resistor r_1 is a negligible winding resistance of the coil, the quantity that primarily influences the core flux is the applied voltage v. At this point, we must remember that we have neglected any equivalent impedance in series with the applied voltage. If there was such an impedance (denoted by say Z_1), the applied voltage v in the above equation would be expanded to $v - i_1 Z_1$. In the simulations of the inductor in the previous chapter, we had considered a resistance connected in series with the inductance being simulated. As the equivalent impedance Z_1 becomes larger, the influence of the applied voltage v will decrease.

For now, let us assume the applied voltage v to be an arbitrary voltage—not necessarily a pure dc or a pure ac sine waveform of a particular frequency. If the flux ϕ due to the above integral equation is not constant but is changing, by Faraday's Law, an emf will be induced in the coil in the right limb due to the flux flowing through it. This induced emf can be expressed as:

$$e_2 = N_2 \frac{d\phi}{dt} \tag{3.9}$$

This induced emf will result in a current flowing through the coil in the right limb expressed by Kirchhoff's Voltage Law as:

$$e_2 - i_2 r_2 - i_2 R = 0 \tag{3.10}$$

As we have already discussed in the previous section, according to Lenz's Law, the induced emf e_2 will be of such a nature that it will oppose the cause that produces it. In the previous section, however, we had considered the coil in the right limb to be open-circuited due to which, even though an emf was induced in it, no current flowed through it. In the circuit of Fig. 3.4, the coil in the right limb feeds a dump resistor and therefore, a current is flowing through it. The reader is encouraged to use the same logic as in the previous section to verify that the polarity of the induced emf e_2 is as shown in Fig. 3.4. However, now an important question needs to be asked—if the induced emf e_2 is of such a nature that it opposes the change in core flux and results in a current i_2 flowing, how will the core flux be affected?

The MMF produced by the coils in the left limb and right limb are $N_1 i_1$ and $N_2 i_2$, respectively. Given the direction of currents flowing in the two coils, they would be trying to produce fluxes flowing in the opposite directions. The reader is encouraged to verify this fact by applying the Right Hand Thumb Rule to the two coils with their currents in the direction shown. It is very important to emphasize that we cannot conclude using Lenz's Law that the fluxes produced by the currents in the two coils will oppose each other. According to Lenz's Law, the induced emfs and subsequently the currents will oppose the *changing* core flux and not the flux itself. The fact that the fluxes produced by the two coil currents oppose each other is the conclusion of using the magnetic circuit once the directions of currents in the coils have been determined.

Now that we have established that the currents in the two coils will tend to produce opposing fluxes in the core, it is clear that the current i_2 flowing in the coil wound on the right limb will tend to demagnetize the core by opposing the flux produced by current i_1 flowing in the left limb. For this magnetic circuit, the net MMF will be $N_1 i_1 - N_2 i_2$. It is therefore quite natural to expect the flux in the core to decrease due to this decreased MMF. However, we had expressed the core flux using (3.8). If the voltage v remains the same, the core flux will also remain approximately the same (neglecting the drop $i_1 r_1$). Here arises a confusion—if the core flux will not change, what effect will the current i_2 have?

We need to remember that when coils are wound together in such a manner that their magnetic fields interact with each other, energy is transferred from one coil to the other [10, 11, 26]. Energy is not produced in the coil wound on the right limb just because it is affected by the magnetic field produced by the coil wound on the left limb. This can only happen if we account for the energy transferred from the coil on the left limb to the coil on the right limb. If the flux in the core is not going to change, the net MMF must remain at the value of $N_1 i_1$. The MMF in the coil on the right limb $N_2 i_2$ must additionally flow through the coil on the left limb to ensure that the net MMF remains the same. The net MMF can be expressed as:

$$N_1 i_1 + N_2 i_2 - N_2 i_2 = N_1 i_1 + N_1 \frac{N_2}{N_1} i_2 - N_2 i_2 \tag{3.11}$$

As a result, the current flowing through the coil wound on the left limb has increased to:

$$i_1' = i_1 + \frac{N_2}{N_1} i_2 \tag{3.12}$$

This increase in the current in the coil on the left limb is understandable as it supplies the energy consumed by the dump resistor R in the coil on the right limb. The

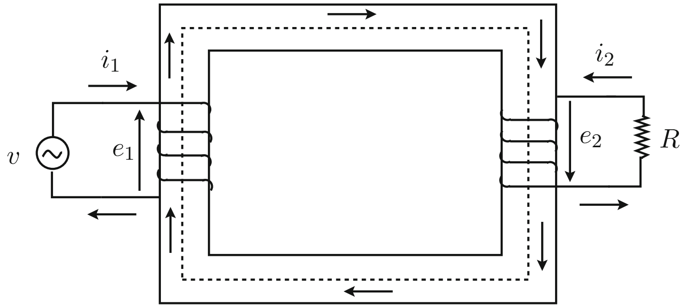


Fig. 3.5 Changing the sense of winding of one of the coils

second term in the above equation is called the transformed current [10, 26]. Since the current in the coil wound on the left limb has increased, by (3.8), the core flux will decrease. How much the core flux will decrease will depend on the current i_1' and if there is an equivalent impedance Z_1 in series with the applied voltage v.

Let us examine a few variations to the above coupled coils to dig deeper into the nature of magnetic coupling. In Fig. 3.4, we used Lenz's Law to determine the polarity of induced emf e_2 in the coil wound on the right limb. What if we change the sense in which the coil on the right limb is wound as shown in Fig. 3.5? As before, applying Lenz's Law, we can state that the induced emf e_2 must be such that it opposes the change in core flux. Let us consider the case of the core flux to be increasing, *i.e.* $\frac{d\phi}{dt} > 0$. In that case, the induced emf e_2 will be increasing in magnitude. As a result, the current i_2 that will flow through the coil on the right limb will also be increasing in magnitude. The flux produced by this current will be an increasing flux. By Lenz's Law, this flux produced by i_2 must oppose the change in the core flux. In order for this opposition to occur, the flux produced by i_2 must be in a direction opposite to that of the main core flux. For this to happen, the polarity of the current i_2 and therefore the induced emf e_2 will be as shown in Fig. 3.5. These polarities are exactly the opposite of the polarities in Fig. 3.4. Therefore, by changing the sense of winding of a coil, the polarity of the voltage supplied to the dump resistor R can be reversed.

Let us consider another case where we use the same coupled coils in Fig. 3.4, but instead of connecting a dump resistor R to the coil on the right limb, we short-circuit the coil instead. This is shown in Fig. 3.6. The equations related to the coil on the left limb will remain the same. However, the equations related to the coil on the right limb will change as follows. Applying Kirchhoff's Voltage Law on the coil on the right limb gives us

$$e_2 - i_2 r_2 = 0 \tag{3.13}$$

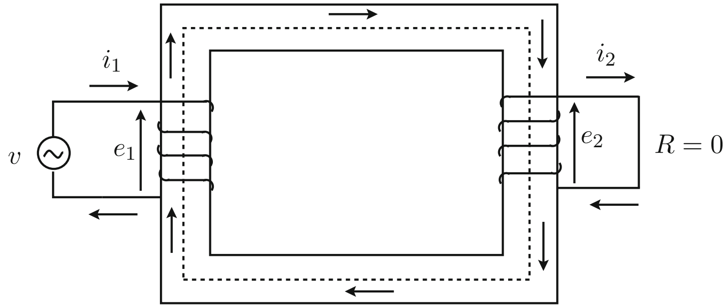


Fig. 3.6 Short-circuiting one of the coils

With the winding resistance of most practical coils being negligible, the above expression can be approximated as:

$$e_2 \approx 0 \tag{3.14}$$

The equations that link the two coils will also remain the same as the nature of their coupling has not changed. Therefore, the induced emfs of the coils can still be expressed by Faraday's Law as (3.7) and (3.9). The nature of the coupling of the coils is exhibited by the fact that both induced emfs are now proportional to the rate of change of core flux. If one of the emfs were to be zero, the other emf would also be zero. This would imply

$$e_1 \approx 0 \tag{3.15}$$

Therefore, by short-circuiting the coil on the right limb, we have effectively short-circuited the coil on the left limb as well. The current i_1 will be limited only by the equivalent impedance Z_1 in series with the applied voltage. If this equivalent series impedance Z_1 were to be negligible, the current i_1 would be dangerously high in magnitude and would usually burn the wire of the coil wound on the left limb. If, however, there were a substantial equivalent series impedance Z_1, the current i_1 would be limited. A very interesting question to be asked in this case would be—what would the core flux be? The core flux would still be determined using Faraday's Law and can be expressed by (3.8). If the flux is the integral of the induced emf that is negligible, the core flux would therefore be negligible as well.

Let us examine another case. In the coupled coils of Fig. 3.4, let us remove the iron core leaving the coils wound in air as shown in Fig. 3.7. By removing the core, we have removed the magnetic medium that was providing a relatively low permeability path for the flux to flow and link the two coils. A core made of laminated iron would have a relative permeability (μ_r) of approximately 1000 compared to the relative permeability of air being 1. With the permeability of the medium decreasing by a factor of 1000, the flux produced will drastically decrease.

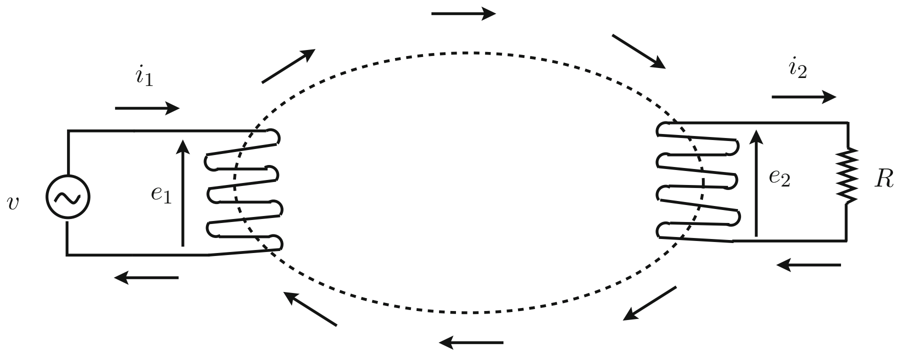


Fig. 3.7 Removing the core

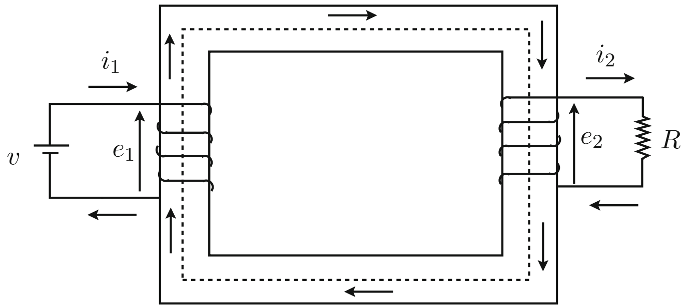


Fig. 3.8 Applying a dc voltage

For a given external voltage applied to the coil on the left, the induced emf will much lower due to the fact that the rate of change of flux will also be much lower at a diminished flux. Moreover, depending on the physical separation between the two coils, the flux linking the right coil will be lower and in most cases, almost negligible. As a result, a negligible emf e_2 will be induced in the coil on the right and a very large current will flow in the coil on the left due to e_1 being smaller in magnitude. Unless the two coils are almost merged together, the two coils can be said to have no practical coupling.

Let us examine one last case before we end this section. In the case of Fig. 3.4, let us connect a dc voltage such as a dc battery to the coil wound on the left limb as shown in Fig. 3.8. As before, we can use (3.8) to determine the flux produced. Since we are integrating $v - i_1 r1$, for a practical coil with a negligible winding resistance r_1, we are integrating the applied voltage. If v is a dc voltage, it is quite obvious that the flux will very soon increase to values that will saturate the core. The induced emf e_1 starts with a value equal to the applied voltage v as the coil will not allow an abrupt rise in the current through it. As the core begins to saturate, the induced emf will decrease as it is proportional to the rate of the change of flux, thereby causing the current i_1 to increase. Eventually, the current drawn will increase until it will burn the coil.

There is, however, an interesting case, if we consider the applied voltage v to be in series with an equivalent series winding Z_1. In such a case, the effective applied voltage to the coil changes from v to $v - i_1 Z_1$. The core flux will increase due to the applied dc voltage and the current i_1 flowing through the coil will increase as well in order to produce the MMF $N_1 i_1$ that will sustain the core flux. As the current i_1 increases, the voltage $v - i_1 Z_1$ applied to the coil decreases. If Z_1 is significantly large, the applied voltage will reach a negligible value, and the flux will stop increasing and remain constant.

If this core flux does not cause the core to saturate, this will not cause harm to the coil or the core. If, on the other hand, Z_1 is not very large, the core flux will increase to values that cause the core to saturate. The flux may settle at some value in the saturation zone of the B–H curve and the current drawn will be proportionate in order to sustain this flux. Even if such a current does not result in the coil being damaged, such a state of operation where the core is saturated and currents are high is not recommended as the excessive losses can deteriorate the magnetic properties of the core permanently. Therefore, applying a steady dc voltage on a coil is not recommended and if one wishes to do so, a series impedance must be connected to ensure that the core does not saturate. In either case, whether the core saturates or not, the flux will attain a constant value at steady state, and the induced emfs e_1 and e_2 will become zero. Therefore, in the case of a dc voltage applied to a coil, no energy will be transferred to a coil that is magnetically coupled to it.

In this section, we examined several possible cases with the two magnetically coupled coils. The purpose of this section was to understand in detail the nature of magnetic coupling using the basic laws of physics. The purpose behind including this section was for the reader to get comfortable with the usage of physical laws to understand how coupled coils behave. In the later sections and also the later chapters, this fundamental nature of coupling will form the basis of transformers. In the next section, we will derive a generalized set of equations for magnetically coupled coils.

3.4 Mutual Inductance

In the previous section, we had used the basic laws of physics to understand how magnetically coupled coils behave. We were able to understand how the coils would behave under different conditions from mathematical equations based on these basic laws of physics. However, in order to develop a simulation model of coupled inductors, we need a rigorous set of equations that will capture all these physical phenomena and that can be applied to any possible condition. In this section, we will begin to express the magnetic coupling between inductors mathematically, which in turn can be used to develop a simulation model in the later sections.

In the previous chapter, for an inductor, we can express the flux linkage ψ with respect to the current i using the property of the inductance as [1, 2]:

$$\psi = Li \tag{3.16}$$

This is just how the property of inductance is defined. An inductance of 1 Henry is the inductance that will produce a flux linkage of 1 Wb-turn when a current of 1 Ampere is flowing through the inductor. Since we are relating the current to the flux linkage, we are taking everything into account—number of turns, permeability of the core, mean length of flux path, cross-sectional area of the core. This makes inductance a very convenient property as it enables us to represent many variables with just one. Because once we have wound an inductor on a core, you do not change anything physically, only the current through the coil changes.

In the case of coupled coils, the above equation will change as for a particular coil, the flux linkages are impacted not only by the current through that coil but also by the current through the other coil that is magnetically coupled to this coil. For now, we will consider the example of two coils wound on a rectangular core as shown in Fig. 3.9. The flux linkage of the coil wound on the left limb (let us call it coil 1) is expressed as:

$$\psi_1 = L_1 i_1 \pm f(i_2) \tag{3.17}$$

The first term is exactly the same as before—inductance of coil 1 and current i_1 through coil 1 affecting the flux linkages of the coil. However, because this coil 1 is magnetically linked to the coil wound on the right limb (let us call it coil 2), any current i_2 flowing through coil 2 will also impact the flux linkages of coil 1. In the previous section, we had considered specific cases with coil 1 energized by an external voltage and coil 2 not being independently energized. In this section, we are assuming the two coils to have their own independent external circuits that result in currents of i_1 and i_2 to flow through them. We have included the effect of the current i_2 in coil 2 as $\pm f(i_2)$—some arbitrary function f of the current in coil 2. Moreover, we do not know if the magnetic field produced by the current in coil 2 will assist or oppose the magnetic field produced by the current in coil 1. Though

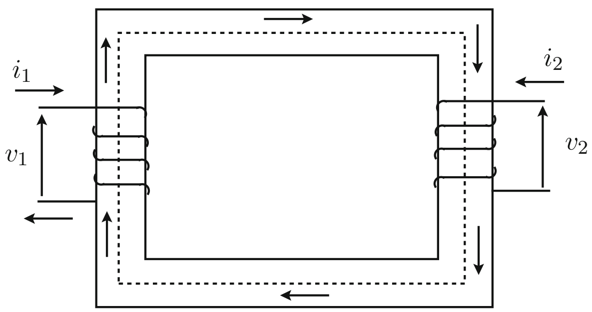


Fig. 3.9 Magnetically coupled coils

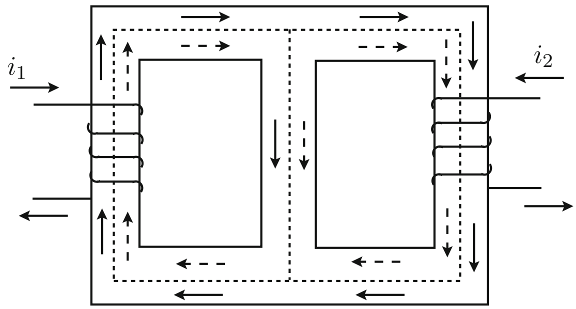
in Fig. 3.9, we can determine if the magnetic fields produced by the currents will assist or oppose each other from the sense of their winding, let us ignore the actual winding details for now.

Before we proceed to detailed mathematical modelling, we need to clarify about the circuits connected to the two coils. In the previous section, coil 1 was the primary coil that would result in an induced emf in coil 2, thereby resulting in transfer of energy from coil 1 to coil 2. This is a very specific case. However, to make the discussion in this section more general, we would like to bring in the condition where any circuit could be connected to coil 1 and coil 2. Both of them could be energized by voltage sources and could have other interfacing components as well. What we are interested in is expressing through equations the impact of the current in one coil on the other coupled coil, due to the fact that they are magnetically coupled.

In such a case, where some arbitrary currents i_1 and i_2 are flowing through the coils, coil 1 and coil 2 are both trying to produce fluxes in the core. The direction of the fluxes can be determined for the currents flowing in coil 1 and coil 2 using the Right Hand Thumb Rule. In Fig. 3.9, for the manner in which the two coils are wound, if the currents are flowing in the two coils as shown, the result is that the fluxes produced by the two coils end up flowing in the same direction through the core. One could imagine another case, where the current flowing through one of the coils is reversed. What would happen in that case? The fluxes produced by the coils would be in the opposite directions. The stronger coil wins and it ends up pushing a flux through the other. Exactly the same way in an electrical circuit with multiple voltage sources where one voltage source would force a current through the other. In this simple case of a rectangular core as shown in Fig. 3.9, the stronger coil would be the one with the larger MMF—the product of the current and the number of turns. It is very important to stress that in a more complex core with multiple limbs, the stronger coil is not just determined by the MMF of the coil. Eventually, we must solve the magnetic circuit as a whole taking into account the MMF of the coils and the reluctance of every limb of the core.

To illustrate how we can determine the flux in the core by solving the magnetic circuit, let us use Fig. 3.10. We could determine the flux in every limb of the core by considering only one coil at a time and calculating the equivalent reluctance. This

Fig. 3.10 Magnetic circuit solution for two coils wound on a three-limb core



will result in two fluxes as shown by solid arrows (for the coil wound on the left limb) and dashed arrows (for the coil wound on the right limb). The flux in the core due to both coils will be the sum of the two fluxes by superposition theorem. This will result in the fluxes expressed as functions of the current flowing in the coils as follows. In the following equations, ρ_1, ρ_2 and ρ_3 are the reluctances of the left limb, right limb and central limb, respectively.

The flux flowing in the left limb is expressed as:

$$\phi_1 = \frac{N_1 i_1}{\rho_1 + \frac{\rho_2\rho_3}{\rho_2+\rho_3}} + \frac{N_2 i_2}{\rho_2 + \frac{\rho_1\rho_3}{\rho_1+\rho_3}} \frac{\frac{\rho_1\rho_3}{\rho_1+\rho_3}}{\rho_1} \tag{3.18}$$

The flux flowing the in right limb is expressed as:

$$\phi_2 = \frac{N_1 i_1}{\rho_1 + \frac{\rho_2\rho_3}{\rho_2+\rho_3}} \frac{\frac{\rho_2\rho_3}{\rho_2+\rho_3}}{\rho_2} + \frac{N_2 i_2}{\rho_2 + \frac{\rho_1\rho_3}{\rho_1+\rho_3}} \tag{3.19}$$

Further simplifications are possible with the above two equations though expressing them in the above form makes it very simple to understand how the fluxes can be calculated as the effect of both coils.

Subsequently, the flux linkages ψ_1 and ψ_2 can be determined as follows:

$$\psi_1 = \frac{N_1^2 i_1}{\rho_1 + \frac{\rho_2\rho_3}{\rho_2+\rho_3}} + \frac{N_1 N_2 i_2}{\rho_2 + \frac{\rho_1\rho_3}{\rho_1+\rho_3}} \frac{\frac{\rho_1\rho_3}{\rho_1+\rho_3}}{\rho_1} \tag{3.20}$$

$$\psi_2 = \frac{N_1 N_2 i_1}{\rho_1 + \frac{\rho_2\rho_3}{\rho_2+\rho_3}} \frac{\frac{\rho_2\rho_3}{\rho_2+\rho_3}}{\rho_2} + \frac{N_2^2 i_2}{\rho_2 + \frac{\rho_1\rho_3}{\rho_1+\rho_3}} \tag{3.21}$$

It is fairly easy to observe that (3.20) is of the same structure as (3.17). We have expressed the flux linkage ψ_1 as a sum of a function of the current i_1 in coil 1 and a function of the current i_2 in coil 1.

The reader is encouraged to determine that the first term in (3.20) and the second term in (3.21) are merely the inductances L_1 and L_2 of the coils if we ignore the effect of the other coil. This calculation can be found in the previous chapter. We will now rename these inductances L_1 and L_2 as the self-inductances of the coil [1, 2]. The self-inductance of a coil expresses the flux linkages of the coil due to a current flowing through the coil itself ignoring any other coils that may be magnetically coupled to it. We can now introduce another term—mutual inductance. The mutual inductance expresses the flux linkage of a coil due to the current flowing in another coil magnetically coupled to it [1, 2]. This mutual inductance is a quantity that expresses the relationship between two coils and we normally use the letter M to denote it.

The mutual inductance M_{12} will define the flux linkage of coil 1 due to the current flowing in coil 2:

$$M_{12} = \frac{N_1 N_2}{\rho_2 + \frac{\rho_1 \rho_3}{\rho_1 + \rho_3}} \frac{\rho_3}{\rho_1 + \rho_3} \tag{3.22}$$

The mutual inductance M_{21} will define the flux linkage of coil 2 due to the current flowing in coil 1:

$$M_{21} = \frac{N_1 N_2}{\rho_1 + \frac{\rho_2 \rho_3}{\rho_2 + \rho_3}} \frac{\rho_3}{\rho_2 + \rho_3} \tag{3.23}$$

It can be observed from the above equations that the mutual inductances will be equal when $\rho_1 = \rho_2$. It is usually assumed that when two coils are magnetically coupled, their effect on each other will be identical or that the mutual inductance will be same. However, that is not the case—in general, when two entities affect each other, the effect that one has on the other need not be the same.

The flux linkage equations for the two coils can be written as [1, 2]:

$$\psi_1 = L_1 i_1 + M_{12} i_2 \tag{3.24}$$

$$\psi_2 = L_2 i_2 + M_{21} i_1 \tag{3.25}$$

The term with the mutual inductance has a positive sign due to the sense of winding of the coils and the directions of currents chosen. If the sense of winding of one of the coils was changed or the direction of current was reversed, the term with the mutual inductance would have a negative sign. The reader is encouraged to verify this.

Theoretically, we could imagine many strange shapes of cores and could imagine coils wound in various different limbs. In such cases, the mutual inductance M_{xy} and M_{yx} between two coupled coils x and y would not be the same. However, the practical reason for winding coils on the same core such that they are magnetically linked is usually to either achieve transfer of energy from one coil to the other (transformers) or to augment/decrease the effective inductance. In such cases, practical cores have fairly regular shapes, due to which two magnetically coupled coils x and y will have the same mutual inductance $M_{xy} = M_{yx}$. We can substitute $\rho_1 = \rho_2$ in the expressions for the mutual inductances M_{12} and M_{21} above. This would imply that the left limb and the right limb of the core have identical dimensions—mean length, cross-sectional area and relative permeability—which is quite reasonable for a practical core.

Besides the convenience of $M_{12} = M_{21}$, assuming a symmetrical core results in another very interesting relation. The self-inductances L_1 and L_2 can be written as:

$$L_1 = \frac{N_1^2}{\rho_1 + \frac{\rho_1\rho_3}{\rho_1+\rho_3}} \tag{3.26}$$

$$L_2 = \frac{N_2^2}{\rho_1 + \frac{\rho_1\rho_3}{\rho_1+\rho_3}} \tag{3.27}$$

In the above equations, $\rho_1 = \rho_2$ has been assumed to reduce the number of variables.

The product of the self-inductances in the above expressions results in the following expression:

$$L_1L_2 = \frac{N_1^2N_2^2}{\left(\rho_1 + \frac{\rho_1\rho_3}{\rho_1+\rho_3}\right)^2} \tag{3.28}$$

If we take the product of the mutual-inductances M_{12} and M_{21}, we arrive at the following expression:

$$M_{12}M_{21} = \frac{N_1^2N_2^2}{\left(\rho_1 + \frac{\rho_1\rho_3}{\rho_1+\rho_3}\right)^2}\left(\frac{\rho_3}{\rho_1+\rho_3}\right)^2 \tag{3.29}$$

Comparing the above expressions, we arrive at the following relation [1, 2]:

$$M_{12}M_{21} = L_1L_2\left(\frac{\rho_3}{\rho_1+\rho_3}\right)^2 \tag{3.30}$$

From the above expression, we have now found a way to relate the mutual inductances between two coils with respect to the self-inductances of the coils. Since $M_{12} = M_{21}$, we can express the above equation as:

$$M_{12} = M_{21} = \frac{\rho_3}{\rho_1+\rho_3}\sqrt{L_1L_2} = k\sqrt{L_1L_2} \tag{3.31}$$

In the above equation, we have expressed the mutual inductance between two coils with respect to the self-inductances of the coils in addition to a factor k. This k is called the coupling factor between the coils. k is the ratio of the flux produced by a coil that links with the other coupled coil. In the case of Fig. 3.10, due to the presence of the central limb, only a fraction of the flux produced by a coil links with the other. In the case of the simple rectangular core of Fig. 3.9, the coupling factor $k = 1$ because all the flux produced by a coil links with the other coil.

In the above discussion of coupling factor k, we have assumed that all the flux produced by a coil will flow through the iron core. Usually this is not the case and there will always be a certain amount of flux that leaks into the air and does not flow through the core. Since the iron core has a permeability that is approximately a 1000 times greater than the surrounding air, the flux that leaks into the air will be negligible. It is normal to further multiply the coupling factor k above with another coupling factor to take into account the flux that leaks into the air. This coupling factor can be denoted by k_{air} and can be between 0.995 to 0.98, which would result in a leakage between 0.5% and 2%, respectively.

In this section, we have quantified the effect of the current flowing through a coil on the flux linkages of another magnetically coupled coil through the property of mutual inductance. Furthermore, we derived the mutual inductances between coupled coils with respect to the self-inductances of the coils and the ratio of the flux produced by one coil that links the other coil. This flux linkage equation is a generalized equation that can be written for any number of coils that are magnetically coupled. In the next section as well as the next chapters, we will use this equation as the basis of the simulation model for coupled coils.

3.5 Approach to Simulating Coupled Coils

In this section, we will use the concept of mutual inductance and self-inductance to expand on the method used to simulate a single inductor, which was described in the previous chapter. We had modelled inductors as variable voltage sources in series with variable resistors of such values that the current drawn by the inductor is equal to that computed from the mathematical model. While modelling magnetically coupled coils, each coil will be represented as a variable voltage source in series with a variable resistance and their instantaneous values will be such so as to draw currents as computed from the mathematical model. In this section, we will describe how we will use this mathematical model in a simulation.

Figure 3.11 shows how we can expand the voltage in series with a resistance model used for simulating a single inductor to the coupled coils of Fig. 3.9. The two coils will have independent electrical circuits that are connected to their separate terminals. The voltage across each coil terminal is measured and fed to the mathematical model that describes the magnetic coupling between the two coils. The output of the mathematical model will be the value of the voltages to be produced by the controllable voltage sources and optionally the value of the resistances in series with the voltage sources. As can be seen from Fig. 3.11, there is a single mathematical model for both coils and this mathematical model will be based on the equations derived in the previous section.

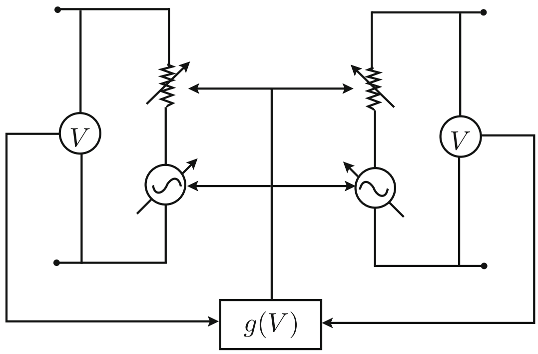


Fig. 3.11 Simulation model of coupled coils

Let the voltages measured across the terminals of the left and the right coils be v_1 and v_2, respectively. We can independently apply Kirchhoff's Voltage Law to the two coils as follows:

$$e_1 = v_1 - i_1 r_1 \tag{3.32}$$

$$e_2 = v_2 - i_2 r_2 \tag{3.33}$$

Here e_1 and e_2 are emfs induced in the two coils, while r_1 and r_2 are the winding resistances of the two coils. The directions of currents i_1 and i_2 have been assumed to be the same as in Fig. 3.9—entering the upper terminals of each coil.

The emfs induced in the two coils can be expressed by Faraday's Law as:

$$e_1 = \frac{d\psi_1}{dt} \tag{3.34}$$

$$e_2 = \frac{d\psi_2}{dt} \tag{3.35}$$

Subsequently, the flux linkages can be expressed as integrals of the induced emfs:

$$\psi_1 = \int e_1 dt \tag{3.36}$$

$$\psi_2 = \int e_2 dt \tag{3.37}$$

The flux linkages of a coil can be expressed as the cumulative effect of the current through the coil and the current through the magnetically linked coil using self and mutual inductances as described in the previous section:

$$\psi_1 = L_1 i_1 \pm M_{12} i_2 \tag{3.38}$$

$$\psi_2 = L_2 i_2 \pm M_{21} i_1 \tag{3.39}$$

In the above equations, we have maintained all the generalizations possible. The $\pm$ sign with the mutual inductance term takes into account the fact that the coils can be wound in such a sense that the fluxes produced by the coils could be in the same direction or in the opposite directions. Moreover, the mutual inductances M_{12} and M_{21} between the coils could be different depending on core construction even though they are the same in the case of the rectangular core of Fig. 3.9.

In Eqs. (3.38) and (3.39), the flux linkages ψ_1 and ψ_2 are computed from the applied voltages and the currents already flowing through the coils. This concept has been described in the previous chapter where the simulation model for the inductor was developed. Therefore, (3.38) and (3.39) are simultaneous equations with the variables being i_1 and i_2 and upon solving them, we would obtain the updated values of the currents i_1 and i_2 flowing through the two coils. Solving simultaneous equations is fairly simple and for this specific case of two coupled coils, we could solve them by mere observation as:

$$i_1 = \frac{\psi_1 \mp M_{12} i_2}{L_1} \tag{3.40}$$

$$i_2 = \frac{\psi_2 \mp \frac{M_{21}}{L_1}\psi_1}{L_2 \mp \frac{M_{12}M_{21}}{L_1}} \tag{3.41}$$

From the above equations, it is fairly obvious that a few simplifications will make the solution much cleaner. To begin with, there could be any number of coils that are magnetically coupled in which case such a solution will be very inconvenient. In a later section, we will generalize the solution by expressing the equations in matrix form and solving them using matrix manipulations. The second messy aspect of the above equations is the fact that we have included the $\pm$ term to take into account the sense of winding of the two coils. Though it is essential to include the sense of winding of the coils in the model, including it in the above equations results in equations that can be different for every set of coupled coils. If we can write the flux linkage equations with only positive signs for every mutual inductance term, we would result in a set of equations that would be identical for any magnetically coupled system. In the next section, we will introduce the concept of the dot polarity, which introduces a visual method of depicting the sense of winding of coils while leaving the equations to be the identical in every case.

3.6 Dot Polarity of Coupled Coils

In the previous section, we had developed a simulation model for the coupled inductors seen in Fig. 3.9. The equations for the flux linkages of each coil take into account the mutual inductances between the coils with the sense of winding of the coils appearing in the equations as positive terms in case the fluxes flow in

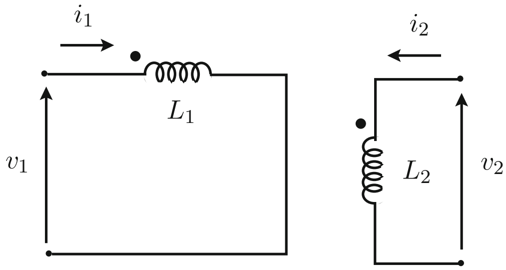


Fig. 3.12 Dot polarity to indicate the sense of winding of coupled coils with assisting fluxes

the same direction and as negative terms in case the fluxes flow in the opposite direction. This is a bit inconvenient as there is a disconnect between the circuit and the equations. When we represent coupled coils, we would typically not show the coils wound on a core as this will make circuit diagrams complicated. We would like to show the coupled coils as mere inductors and denote their sense of winding in some manner visually on a circuit. In this section, we will introduce the concept of the dot polarity, which is a convenient manner of marking the relative sense of windings of two coupled coils [1, 2].

Figure 3.12 shows how the coupled coils of Fig. 3.9 can be represented as mere inductors without any details of the core or how the coils are wound on the core. As can be noticed, there is a “dot” on each inductor (coil)—on the left terminal for the left inductor and on the top terminal for the right inductor [1, 2]. In more complex circuits with several coupled coils wound on separate cores, we could use a number of different shapes such as triangles, rectangles, ellipses etc. to indicate that coils are coupled together [1, 2]. Moreover, several inductors can be coupled since there could be several coils wound on the same core, in which case, they will all have a similar shape (such as a dot) on one of their terminals.

By using dot polarities, we have eliminated the need to draw the core and the coil wound on the core as we did in the previous sections. This makes circuit diagrams much simpler. The dot polarity indicates that the two inductors shown are magnetically coupled. Conversely, the presence of shapes such as a dot on inductors anywhere in a circuit indicates that they are magnetically coupled. In Fig. 3.12, the inductors have been deliberately drawn such that one is horizontal while the other is vertical. In complex circuits, inductors can be far apart in the circuit diagram, but a dot polarity indicates that they are still magnetically coupled. Now that we have described how dot polarities are used, let us examine what they imply for the nature of magnetic coupling between them and how the sense of winding can be inferred.

In simple terms, when a dot polarity exists on two inductors as shown in Fig. 3.12, it implies that currents flowing through the inductors while entering at the terminals marked by dots will produce fluxes in the core flowing in the same direction [1, 2]. Moreover, the flux produced in one coil due to the current flowing in the other will be in the same direction as the flux produced due to the current flowing in the coil itself as long as both currents are entering the terminals marked by dots. With this

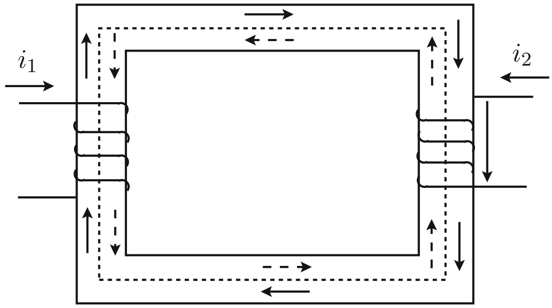


Fig. 3.13 Coils wound on a core that produce opposing fluxes in the core

definition, the reader can verify that the inductors with the dot polarities of Fig. 3.12 precisely depict the coupled coils wound on a rectangular core shown in Fig. 3.9.

Once we have established this basic definition, a number of other effects can also be deduced in the manner similar to that performed in the previous section. If an increasing current flows through one of the inductors while entering at a terminal marked by a dot, the emfs induced in all the inductors will be such that the positive polarity of the emfs will appear at all the terminals marked by dots. Conversely, if a decreasing current flows through one of the inductors while entering at a terminal marked by a dot, the emfs induced in all the inductors will be such that the negative polarity of the emfs will appear at all the terminals marked by dots. One can also examine cases when increasing or decreasing currents flow through the inductors such that they leave a terminal marked by a dot. The reader is encouraged to verify these and use Fig. 3.9 with the coils on the core so that Faraday's Law, Lenz's Law and Right Hand Thumb Rule can be applied in each case.

To complete the explanation on how dot polarities can be used to denote the sense of winding of inductors that are magnetically coupled, let us consider the case of Fig. 3.13 in which the coil on the right limb is wound in a sense that is opposite to that of Fig. 3.9. By using the Right Hand Thumb Rule, for the current i_2 flowing through the right limb coil as shown, the flux produced will be in a direction that is opposite to the flux produced by the current flowing in the left limb coil. Such a set of coils can also be conveniently represented using mere inductors with dot polarities. We need to only ask the question—which terminal of the inductors (coils) would the currents have to enter while flowing through the inductors, that the fluxes in the core due to the currents would be in the same direction? From Fig. 3.13, the current would have to enter the right limb coil at the lower terminal in order to produce a flux in the same direction as the current entering the left limb coil at the upper terminal. Therefore, as shown in Fig. 3.14, the dot polarities are placed such that a dot appears in the left terminal of the left inductor and at the lower terminal of the right inductor. It is important to understand that these dots are merely a convention, and we can place them on the other terminals of either inductor as well, *i.e.* on the right terminal of the left inductor and at the upper terminal of the right inductor.

By using dot polarities to depict the sense of winding of magnetically coupled coils, we can now remove the sense of coupling from the flux linkage equations and

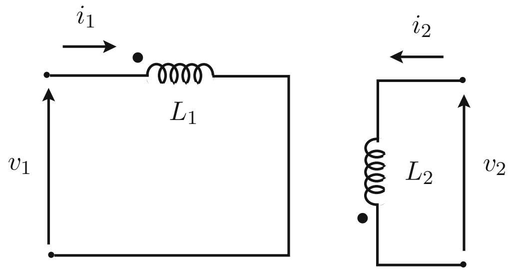


Fig. 3.14 Dot polarity to indicate the sense of winding of coupled coils with opposing fluxes

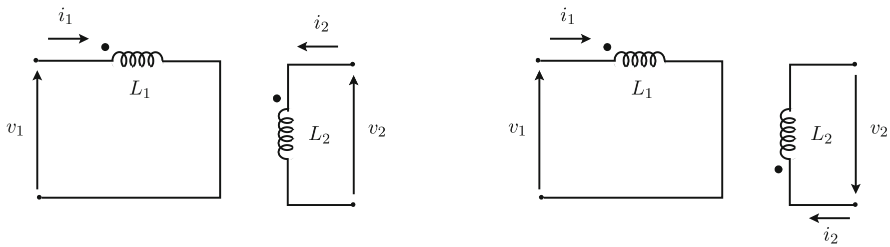


Fig. 3.15 Comparison of dot polarity representation of coupled coils with assisting and opposing fluxes

instead visually depict how coupled coils are connected in their respective circuits. We must, however, make an assumption while using dot polarities in coupled coils. We assume that the current through the coil enters the terminal marked by a dot. We also assume that the voltage measured across a coil is such that the terminal with the dot has the positive polarity. We depict these assumptions for both cases as shown in Fig. 3.15. For the set of inductors on the right, the direction of current i_2 and measured voltage v_2 have been reversed. Since the currents i_1 and i_2 are entering the terminals marked by dots, the flux linkage equations will be the same in both cases and can be expressed as:

$$\psi_1 = L_1 i_1 + M_{12} i_2 \tag{3.42}$$

$$\psi_2 = L_2 i_2 + M_{21} i_1 \tag{3.43}$$

In this section, we have described a convenient manner to depict magnetically coupled coils as inductors while visually depicting how they produce fluxes in the core. This method can also be used in developing simulation models so as to simulate any number of coils that are magnetically coupled. Furthermore, by choosing convenient directions for current flowing through a coil and the voltage measured across the coil terminals, we can write the flux linkage equations without having to think about the nature of coupling between the coils. In the next section as well as in the next chapters, these assumptions help in being able to model several coils that are magnetically coupled in the simplest possible manner.

3.7 Simulating Magnetically Coupled Coils

In the past few sections, we introduced several concepts that we can now use to simulate magnetically coupled coils. We had examined how the coupling between coils can be quantified by calculating the mutual inductance between the coupled coils. We had also described how dot polarities can be used to visually depict the effect that a current flowing in a coil has on the flux flowing through another magnetically coupled coil. In this section, we will use all these concepts to simulate magnetically coupled coils wound on an iron core. The basis for this simulation will be very similar to the simulations of inductors performed in the previous chapter.

Figure 3.16 shows the topology of the circuit that will be chosen for simulation. This simulation can be found in the folder `basic_concept` within `chapter3_magnetic_coupling` in the following link in the simulation repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics.
We have used the dot polarity convention to depict the nature of magnetic coupling between the coils. Each coil is connected to its own circuit that may comprise of voltage sources v_{s1} and v_{s2} in series with feeder impedances Z_{s1} and Z_{s2}. Furthermore, the circuits may also be feeding loads such as Z_{L1} and Z_{L2}. The voltage sources v_{s1} and v_{s2} can be dc voltages sources, ac voltage sources or even power converter output voltages. At this point, we will begin our simulations with the simple case of the voltage sources being dc voltages. Let us also make the assumptions that the feeder impedances Z_{s1} and Z_{s2} are mere feeder resistances R_{s1} and R_{s2} and the loads Z_{L1} and Z_{L2} are mere load resistors R_{L1} and R_{L2}. Let us choose $R_{s1} = R_{s2} = 0.1\Omega$ and $R_{L1} = R_{L2} = 10\Omega$. Let us begin with the voltage source $v_{s1} = 24$ Volts and $v_{s2} = 0$, implying only the circuit on the left is energized.

We must now define the coils and the coupling between them. We could define the coils in terms of the number of turns in which case we would also need the dimensions of the iron core in order to define the magnetic circuit. It is always possible to start every simulation of any magnetic component such as an inductor, coupled inductors or even machines such as transformers, motors and

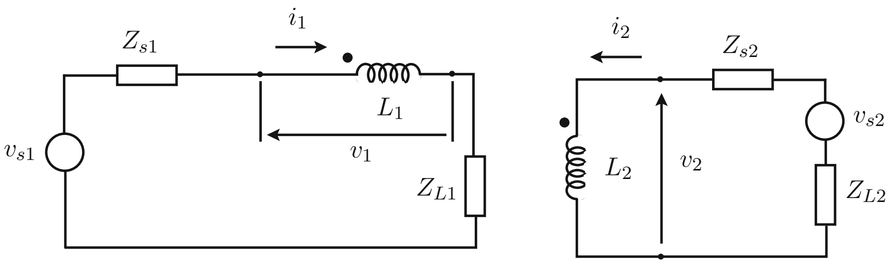


Fig. 3.16 Circuit topology with coupled coils

generators, with every detail of their construction. However, unless the objective of the simulation is to determine the detailed state of the component or machine such as the flux in different parts of the core, using the construction of the component or machine in a simulation is inconvenient. Therefore, for simplicity, while simulating coupled inductors and in later chapters transformers, we will represent them as an equivalent circuit with inductances and resistances.

In the case of the coupled coils shown in Fig. 3.16, we only need their self-inductances L_1 and L_2, the mutual inductance $M_{12} = M_{21} = M$ between them and their winding resistances r_1 and r_2. As already discussed in the previous section pertaining to mutual inductance, for a simple rectangular core, the mutual inductance of one coil with respect to the other will be the same as that of the other. In most practical cases, core shapes are taken to be as simple as possible and therefore, if two coils need to be magnetically coupled, there is no need to choose a core of a very complex shape. Such an assumption also simplifies the computation of the mutual inductance with respect to the self-inductances as $M = k\sqrt{L_1 L_2}$ with k being the coupling factor. In the case of the simple rectangular core, k accounts for the leakage flux and therefore can be chosen to be 0.99 as this implies 1% of the flux generated by a coil does not link the other coil but leaks instead.

With the above definitions, we can get started with the control code for the mutual inductor magnetic model with the following parameter definitions:

```
import math
dt = 1.0e-6
L1 = 0.3
L2 = 0.3
r1 = 0.1
r2 = 0.1
coupling_factor = 0.99
M = coupling_factor*math.sqrt(L1 * L2)
res_output1 = 100.0
res_output2 = 100.0
if t_clock >= t1:
    # Model
    t1 += dt
```

As with the case of simulation of inductors, since we are going to solve a mathematical model, which in turn involves numerical integration, we need to run the code at a time interval. In the above code, `t1` is the time event variable that needs to be defined in the configuration of the control file. In the above code, the other two variables `res_output1` and `res_output2` are the variable resistors in series with the controllable voltage sources as shown in Fig. 3.11. Since the values of these variables need to be linked to the resistance of the VariableResistor elements in the circuit, we need to define these variables as Output Variables in the configuration of the control file. Since this is only a basic simulation proving the concept of mutual inductances, we can choose an integration time step of 1 µs and we can also choose the interval at which the above code runs to be the same by setting `dt` to be 1 µs.

We can now start adding code to the model, which will be solved iteratively. We need to compute the induced emfs of the two coupled coils and subsequently the flux linkages of the coils:

```
if t_clock >= t1:
    e1 = v1 - ind_curr1*r1
    e2 = v2 - ind_curr2*r2
    psi1 += e1*dt
    psi2 += e2*dt
    ...
    t1 += dt
```

In the above statements, we are using the voltages `v1` and `v2` measured across the inductor terminals as shown in Figs. 3.11 and 3.16. These variables `v1` and `v2` need to be defined as Input Variables in the configuration file and their input source needs to be defined as the voltages measured by the Voltmeters shown in Fig. 3.11. The currents `ind_curr1` and `ind_curr2` need to be defined as static variables in the control configuration as they will be computed by the model. `psi1` and `psi2` are flux linkages computed by integrating the induced emfs and therefore, will also need to be defined as static variables in the control configuration. The induced emfs `e1` and `e2` are temporary variables in this model and they can remain as regular local Python variables that will be created and destroyed in every iteration. However, if we wish to plot these variables or perform any other computations, it is advisable to define `e1` and `e2` also as static variables in the control configuration. To ensure that the Python compiler does not throw any unexpected errors due to referencing a variable that does not exist, every variable must be defined in every iteration by either defining them at the head of the file in case of parameters or using the special variables in the control configuration.

Now that the flux linkages `psi1` and `psi2` have been computed, we can calculate the currents using (3.40) and (3.41):

```
if t_clock >= t1:
    ...
    ind_curr2 = (psi2 - (M * psi1 / L1)) / (L2 - (M * M / L1))
    ind_curr1 = (psi1 - (M * ind_curr2)) / L1
    ...
    t1 += dt
```

We are merely solving a simultaneous set of equations with the variables being `ind_curr1` and `ind_curr2`. Later in the section, we will examine a more elegant solution using matrix equations.

Now that we have computed the currents that must flow through the two coils, we need only to adjust the output voltages of the controllable voltages in Fig. 3.11. We can define two Output Variables `vout1` and `vout2` in the control configuration and specify the target of the variables to be the controllable voltage sources in Fig. 3.11.

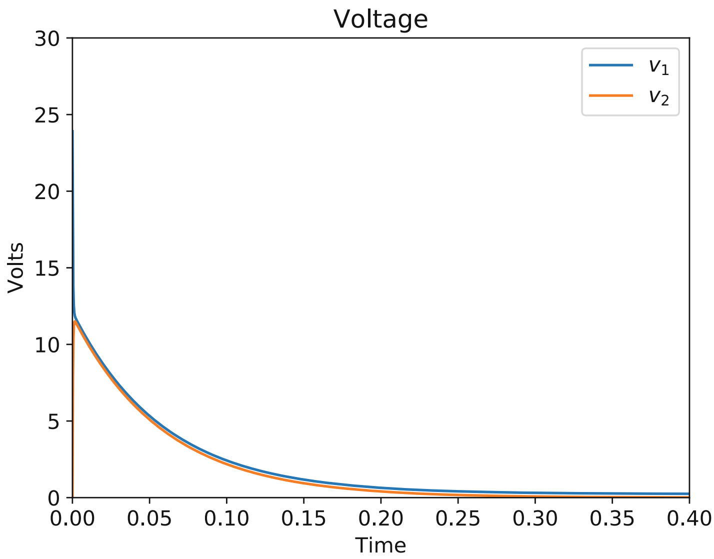


Fig. 3.17 Coil terminal voltages in the case of coupled coils producing assisting fluxes

We can compute the output voltages using Kirchhoff's Voltage Law applied either to Fig. 3.11 or the sub-circuit of Fig. 3.16 as follows:

```
if t_clock >= t1:
    ...
    vout1 = v1 - ind_curr1*res_output1
    vout2 = v2 - ind_curr2*res_output2
    ...
    t1 += dt
```

Figures 3.17, 3.18, 3.19, and 3.20 show the simulation results. Figure 3.17 shows the terminal voltages v_1 and v_2 across the coils and can be seen to decay from a value of approximately 12V to zero at steady state. To understand this behaviour of v_1 and v_2, let us examine the induced emfs in Fig. 3.19, which shows that the induced emfs are also decaying from a value of around 12 V to zero at steady state. From Figs. 3.17 and 3.19, it can be observed that there is a very short-lived transient when the circuit is energized. Figure 3.20 shows this transient in the induced emfs. The induced emf e_1 at time $t = 0$ is equal to the dc source of 24 V. This is understandable, since the current through the inductor cannot abruptly rise but has to increase gradually. However, e_1 drops drastically soon after and the induced emf e_2 in coil 2 increases equally drastically to attain a value close to e_1. This behaviour may seem very confusing but can be understood if we follow the basic laws as duone before.

Due to the mutual coupling between the two coils, their induced emf will be proportional to the number of turns. In this case, we have considered two coils with the same self-inductance, which implies that they have the same number of turns assuming the parts of the core on which they are wound have the same dimensions.

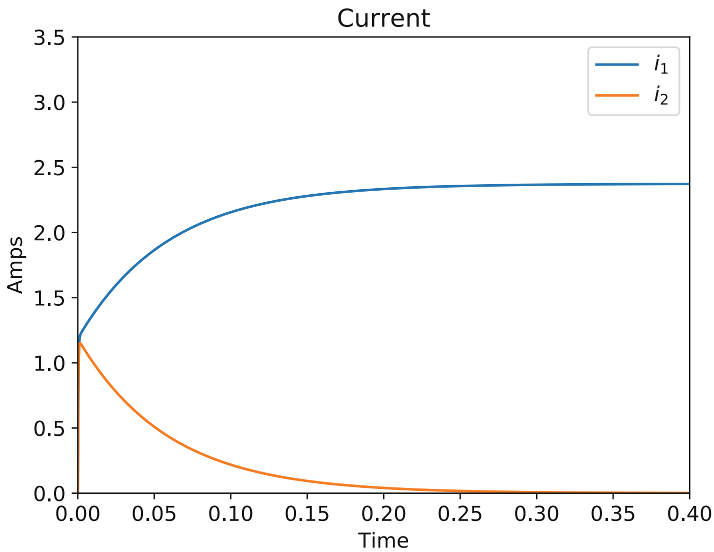


Fig. 3.18 Coil currents in the case of coupled coils producing assisting fluxes

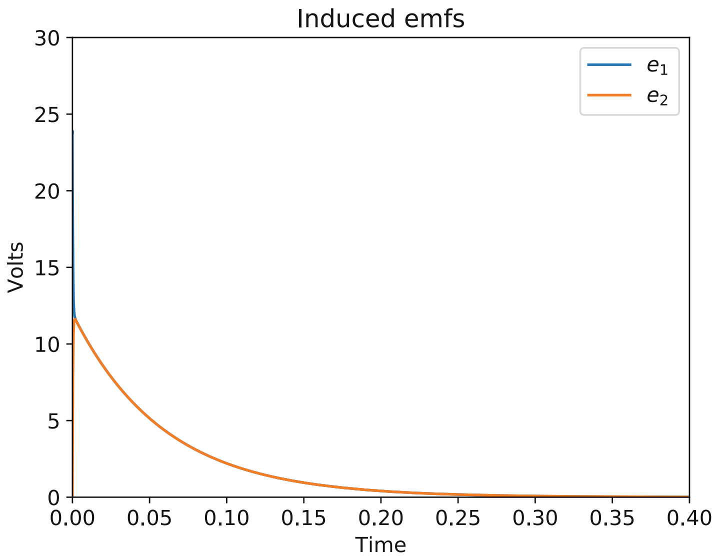


Fig. 3.19 Coil-induced emfs in the case of coupled coils producing assisting fluxes

Therefore, once e_1 is produced in coil 1, a proportional e_2 develops in coil 2 as well. Once e_2 increases in coil 2, this results in a terminal voltage v_2 developed across coil 2 as shown in Fig. 3.17. Therefore, the current i_2 flowing through coil 2 will increase depending on the value of the load resistor R_{L2}. This can be seen from Fig. 3.18. The

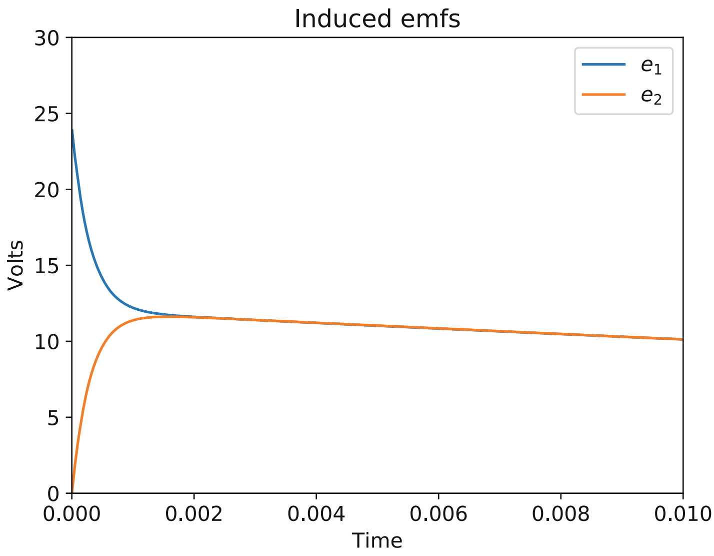


Fig. 3.20 Coil-induced emfs during circuit energization

reader is encouraged to revise the previous section, which described the different possibilities with coupled coils in which we saw how the current flowing in one coil is transferred to the coupled coil. Therefore, the current i_2 flowing in coil 2 will be translated to coil 1.

With a current i_1 flowing through coil 1, the terminal voltage v_1 across coil 1 will fall due to the load resistor R_{L1}. This results in the fall of the induced emf e_1 as shown in Fig. 3.20. After this initial transient, the two induced emfs follow the same path. As the current i_1 through coil 1 increases and reaches a steady state, the flux in the core will also reach a steady state and therefore, e_1 and e_2 decay to zero. With e_2 decaying to zero, the current i_2 flowing through coil 2 will also decay to zero. It can be seen from Fig. 3.17 that the voltage v_1 across coil 1 is not zero in steady state but had a small non-zero value. This is merely due to the drop across the coil winding resistance with a steady state current i_1 flowing through coil 1.

The source used to energize coil 1 in the above simulation has been a dc source. However, the reader is welcome to repeat the simulation with an ac supply. Moreover, the reader can also change the parameters of the coupled inductors such as the self-inductances and the coupling factor. Since the later chapters will be dedicated to the operation of transformers energized by ac supplies, these simulation results will not be presented in this chapter. However, we will repeat the simulation for the case when the dot polarities of the coupled coils are as shown in right sub-figure of Fig. 3.15. The circuit being simulated can be redrawn as shown in Fig. 3.21. It is important to note from Fig. 3.21 that the current i_2 in coil 2 has been reversed in direction such that it enters the terminal marked by the dot. Moreover, the voltage v_2 across coil 2 terminals has also been reversed such that the positive

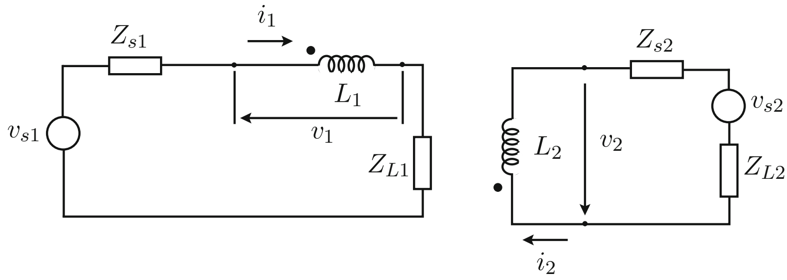


Fig. 3.21 Circuit topology with coupled coils while reversing the sense of winding

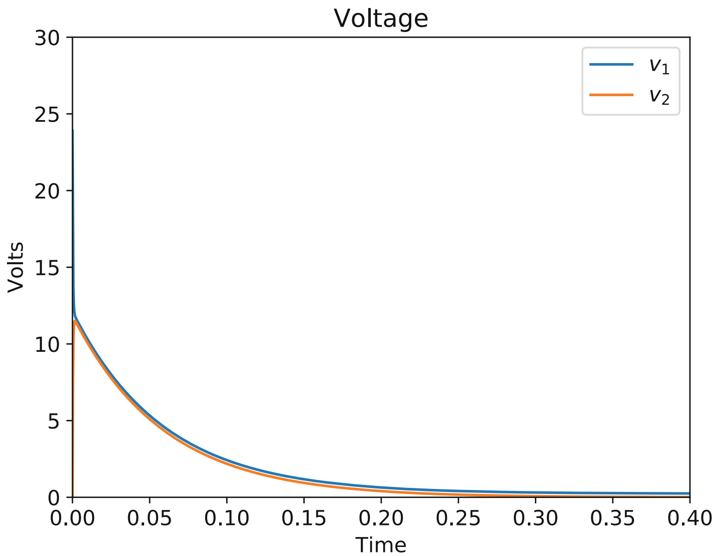


Fig. 3.22 Coil terminal voltages in the case of coupled coils producing opposing fluxes

polarity is at the terminal marked by the dot. These conventions ensure that the flux linkage equations always have positive signs and therefore the code representing the mathematical model does not change. To adapt to these dot conventions, the polarity of the Voltmeter and the controlled voltage source on the right in Fig. 3.11 will be reversed.

Figures 3.22, 3.23, 3.24, and 3.25 show the simulation results for the circuit of Fig. 3.21. Figures 3.22 and 3.23 show the coil terminal voltages and the induced emf in the coils. Strangely, these are exactly the same as the previous simulation. However, it is important to remember that because coil 2 does not have a voltage source, the terminal voltage will be the same as the induced emf. From the definition of dot polarity, an increasing current entering one of the coils at a terminal marked by a dot will produce an induced emf in the other coil such that the polarity of the

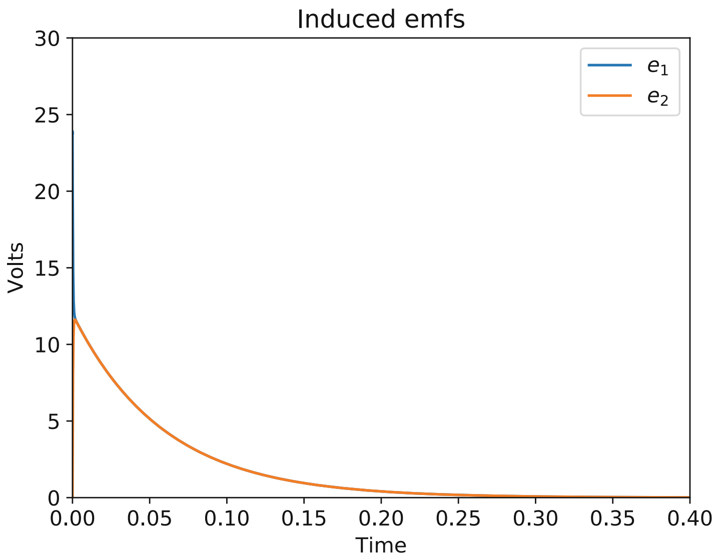


Fig. 3.23 Coil-induced emfs in the case of coupled coils producing opposing fluxes

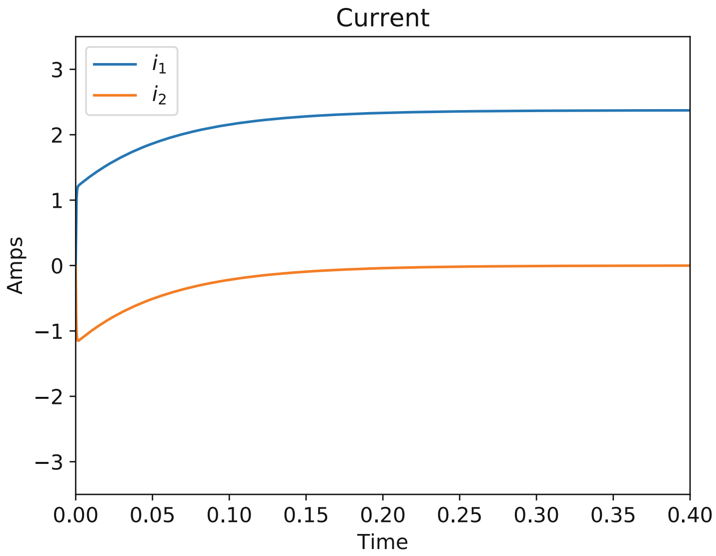


Fig. 3.24 Coil currents in the case of coupled coils producing opposing fluxes

emf will be positive at the terminal marked by the corresponding dot. Therefore, since current i_1 is initially increasing and entering coil 1 at a terminal marked by a dot, the induced emfs and the coil voltages will be the same as in the previous simulation.

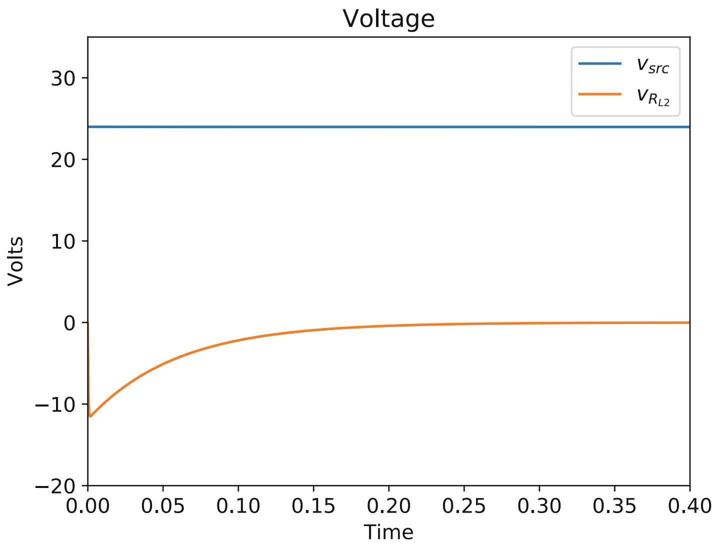


Fig. 3.25 Supply voltage and voltage across load R_{L2}

The difference between the simulations will be known when we plot the coil currents as shown in Fig. 3.24. The current i_2 in coil 2 has the opposite polarity as in the previous simulation. This is due to the fact that the induced emf e_2 in coil 2 has a polarity such that it is positive at the terminal marked by the dot. As a result, the current flowing through the load resistor R_{L2} will be in the reverse direction. To get a better understanding of the voltages produced by the coils, Fig. 3.25 shows the supply voltage of 24 V and the voltage appearing across the load resistor R_{L2}. The supply voltage is a constant 24 V as expected, while the voltage across R_{L2} rises to approximately −12 V and then decays to zero once the current i_1 through coil 1 reaches a constant value at steady state.

In this section, we have used the theory described in the past few sections to develop simulation models for coupled coils in a very basic circuit. The purpose of this section was to describe how using Python code, we can simulate coupled conductors while also including the sense of winding of the coils using dot polarities. The simulation results support our theoretical discussions and the reader is encouraged to extend these simulations with variations in the circuit topology. In the next section, we will discuss a more practical case of when coupled inductors need to be simulated. The next chapter based on transformers will describe in detail a practical application of magnetic coupling.

3.8 Ćuk Converter

In the previous section, we simulated magnetically coupled coils that were connected to two independent isolated electric circuits. When we wish to transfer electrical energy from one circuit to another isolated circuit, we would use a transformer, which is a special case of magnetically coupled coils designed in such a manner so as to achieve maximum power transfer with the minimum possible leakage and losses. In undergraduate courses, as electrical engineers, we often analyse random circuits that contain coupled coils that are usually marked using dot polarities. In such circuits, the coupled inductors may not be in isolated subcircuits. In this section, we will examine such a case of where coupled inductors could be used in a power electronic converter [12, 37, 38].

Besides using magnetic coupling to transfer energy from one isolated circuit to another, another reason for magnetic coupling between inductors is that they are wound on the same core. In an inductor, the core results in the inductor being bulky and occupying space in a converter. If a converter is being used as a power supply, we would like to decrease the size as much as possible to make the power supply portable and light. In power converters, inductors and capacitors are integral components as they are energy storage elements that allow us to condition power while producing a particular output for a given input. If a power converter has multiple inductors, if each inductor needs a separate core, this would result in the size of the converter increasing and potentially becoming bulky. On the other hand, if more than one inductor could be wound on the same core, the size of the converter could be decreased. Once coils are wound on the same core, there will be a mutual inductance between the coils. In the circuit equations, the mutual inductance will appear in addition to the self-inductance of the coils.

Besides the circuit equations needing to be modified, winding more than one inductor on the same core is practical only under some circumstances. This is due to the fact that flux in the core is now dependent on the current flowing through both coils. As a result, the induced emfs in the coils will also be affected by the rate of change of current flowing through both coils. If a circuit is so designed, that the current flowing through two or more inductors will have the same pattern—increase during the same intervals and decrease during the same intervals, these inductors could be wound on the same core. However, if the nature of current flowing through inductors is very different and not at all correlated, winding them on the same core could result in the operation of the circuit being altered and eventually cause the circuit to malfunction. This concept can be best described using the example of the Ćuk converter [37, 38].

Figure 3.26 shows the topology of a Ćuk converter named after Slobodan Ćuk who first presented its design as an improvement over the basic buck-boost converter [37]. The Ćuk converter can achieve an output voltage v_o that is either higher or lower than the input V_{in} (buck-boost) though the output is inverted just like with a conventional buck-boost converter. The main energy storage device in the converter is the capacitor C_1. This is in contrast to the conventional buck-boost converter that

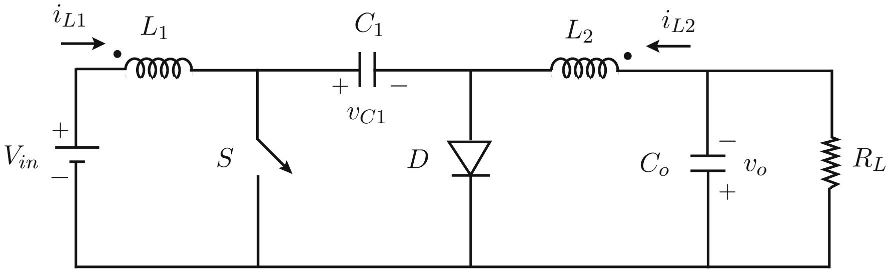


Fig. 3.26 Ćuk converter

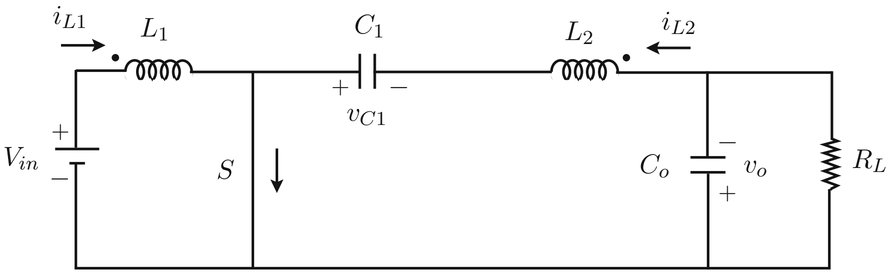


Fig. 3.27 Ćuk converter operation when switch *S* is conducting

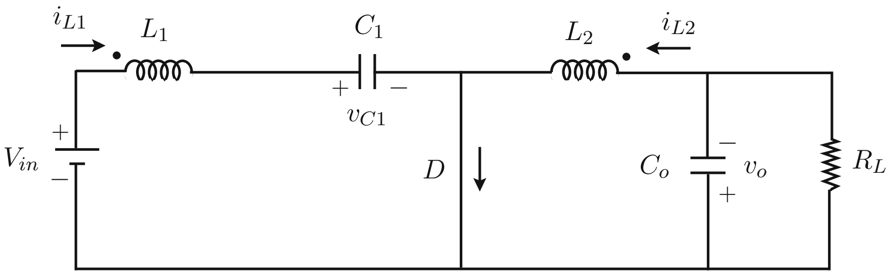


Fig. 3.28 Ćuk converter operation when diode *D* is conducting

uses the inductor as the energy storage device, which results in either high ripple currents or a bulky converter if a large inductor is chosen to decrease the current ripple. In the Ćuk converter, the inductors L_1 and L_2 experience much lower ripple current and additionally their currents i_{L1} and i_{L2} are continuous making closed loop control design simpler.

The working of the Ćuk converter can be depicted by Figs. 3.27 and 3.28. When the switch S is turned ON, the supply V_{in} energizes the inductor L_1 and the current i_{L1} increases. Initially, when all currents and voltages are zero with the circuit being at rest, the very first action will be the inductor L_1 energizing. When the switch S is turned OFF, the current i_{L1} will freewheel through the diode D and will therefore charge the capacitor C_1 due to which the voltage v_{C1} will increase with the polarity shown. Since the energy in the inductor L_1 charges the capacitor C_1, the current i_{L1} will decrease. When the switch S is turned ON in the next cycle, the current i_{L1} will increase as before with energy being stored in the inductor. However, now with the capacitor C_1 being charged, this capacitor will charge inductor L_2 and current i_{L2} will increase. As a result, energy has been transferred from the supply V_{in} to C_1 through the inductor L_1 and now the capacitor C_1 transfers energy to L_2. Along with energizing the inductor L_2, when the switch is conducting, the capacitor C_1 also transfers energy to the output capacitor C_o and the output voltage v_o will increase. When the switch is turned OFF, inductor L_1 transfers energy to the capacitor C_1. With the inductor L_2 also energized, the current i_{L2} will freewheel and continue to supply the output capacitor. Since both inductors are freewheeling through the diode D, the currents i_{L1} and i_{L2} will decrease.

From the working of the Ćuk converter, a few inferences can be made. When the switch is turned ON, the currents i_{L1} and i_{L2} increase simultaneously—inductor L_1 energized by the supply and inductor L_2 energized by the capacitor C_1. When the switch is turned OFF, the currents i_{L1} and i_{L2} decrease simultaneously—inductor L_1 transfers energy to capacitor C_1 and inductor L_2 transfers energy to the output capacitor C_o. Therefore, the Ćuk converter uses two inductors L_1 and L_2 whose currents increase and decrease simultaneously during the same intervals. Moreover, the function of these inductors is merely to transfer energy to and from the capacitor C_1. Ripples in the currents i_{L1} and i_{L2} will be present due to the transfer of energy. However, to decrease electromagnetic interference, it is advisable to decrease the ripple as much as possible. The simplest way to decrease current ripple is to increase the size of the inductors. However, this would result in a more bulky converter, which is undesirable. Therefore, a convenient manner of decreasing the ripple while also limiting the size of the inductors is to wind them on the same core so as to introduce a mutual inductance between them.

In Fig. 3.26, the inductors L_1 and L_2 have been marked by dot polarities on their terminals to indicate that they are magnetically coupled. It is important to note that polarities are such that currents i_{L1} and i_{L2} are entering the terminals marked by the dot polarities, which implies that the fluxes produced in the core due to the currents will be in the same direction. The flux linkage equation for L_1 and L_2 can be expressed as:

$$\psi_1 = L_1 i_1 + M i_2 \tag{3.44}$$

$$\psi_2 = L_2 i_2 + M i_1 \tag{3.45}$$

In the above equations, there is now a mutual inductance M term. This term results in the flux linkages of both inductors being larger than they would have been if the inductors had been isolated and wound on separate cores. Subsequently, the rate of change of flux linkages will also be larger resulting in large induced emfs in the inductors. Since the induced emf in the inductor is the primary opponent of the change in the current flowing through the inductor, this will result in reduced current ripple.

The simulation of the Ćuk converter can be found in the folder `cuk_converter` within `chapter3_magnetic_coupling` in the following link in the simulation repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics
To simulate the Ćuk converter of Fig. 3.26, we need one additional component of control to generate gate signals to the switch S. Since we need only to demonstrate the basic operation of the Ćuk converter, we choose a simple constant frequency operation of the converter in which the switch S is turned ON for a duration of the switching time period [35]. Therefore, if T is the time period of a switching cycle and t_{on} is the time interval for which the switch S is turned ON, we define the duty ratio d as:

$$d = \frac{t_{on}}{T} \tag{3.46}$$

Therefore, d lies between 0 and 1. By varying the duty ratio and therefore varying the time interval for which the switch S is turned ON, the output voltage v_o can be varied. To regulate the output voltage v_o to a desired reference, we vary the duty ratio d.

In this simulation, we will not implement output voltage regulation as closed loop control is a fairly vast topic. Instead, we will assume a value of d between 0 and 1 (say 0.3). To convert this value of duty ratio to a gate signal, we use the Pulse Width Modulation (PWM) technique where we compare the duty ratio with a fixed frequency carrier waveform. For a dc–dc converter, the fixed frequency carrier waveform is usually a sawtooth waveform [35]. The PWM technique can be implemented using the following code block:

```
dt = 1.0e-8
f_sw = 20000.0
T_sw = 1 / f_sw
carr_slope = 1 / T_sw
if t_clock >= t1:
    carr_wave += carr_slope*dt
    if carr_wave > 1.0:
        carr_wave = 0.0
    duty_ratio = 0.3
    if duty_ratio > carr_wave:
        s1gate = 1.0
    else:
        s1gate = 0.0
    pwm_carr = carr_wave
    pwm_dutyratio = duty_ratio
    pwm_gate = s1gate
    t1 += dt
```

In the above code block, we have chosen the switching frequency `f_sw` to be 20 kHz or 20,000 Hz. This is typical for a dc–dc converter such as the Ćuk converter. As with other control codes, this has a Time Event variable `t1`, which is updated at a constant interval at which we would like the control code to be executed. This is usually the case when we implement control in hardware using platforms such as microcontrollers. The control code is executed at a regular interval by configuring a timer. Since our switching frequency `f_sw` is 20,000 Hz, we choose the time interval of the control to be 10 nanoseconds. This is to ensure that we are able to generate accurate switching gate signals within each switching time period.

The carrier waveform is denoted by the variable `carr_wave`. This carrier waveform is compared with the fixed duty ratio of 0.3 (the reader is encouraged to try out other values). When the duty ratio is greater than the carrier waveform, the switch S is turned ON by providing a gate signal of 1 while when the duty ratio is lower than the carrier waveform, the switch is turned OFF by providing a gate signal of 0. This can be depicted in Fig. 3.29. As can be seen, using the technique of PWM, the duty ratio of 0.3 is translated to an ON time for the switch for 30% of the time period of a switching cycle.

Figure 3.30 shows the currents i_{L1} and i_{L2} through the inductors L_1 and L_2. The currents can be seen to increase when the switch S is turned ON and conducting and can be seen to decrease when the switch S is turned OFF and the diode D conducts instead. Importantly, the currents rise and fall simultaneously during the same interval. Figure 3.31 shows the output voltage. When the switch S is turned ON, the output voltage appears to decrease which is contrary to what we expect from our theoretical discussion. However, we have measured the output voltage with such

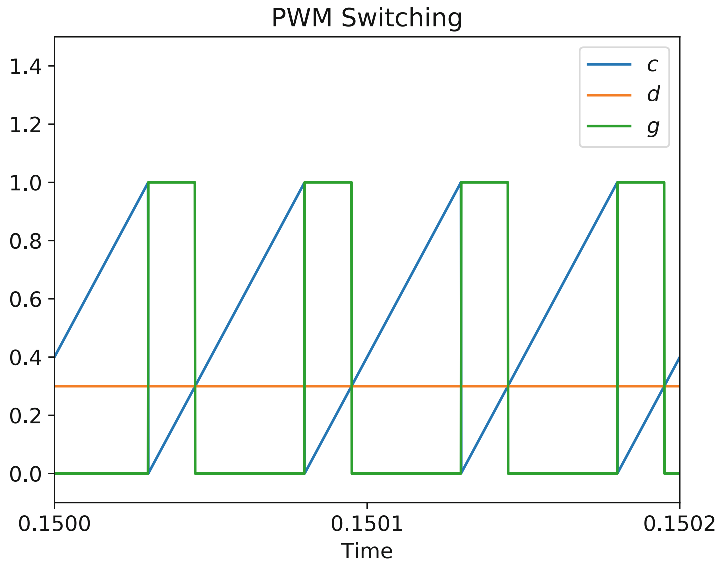


Fig. 3.29 Gate signals produced by PWM

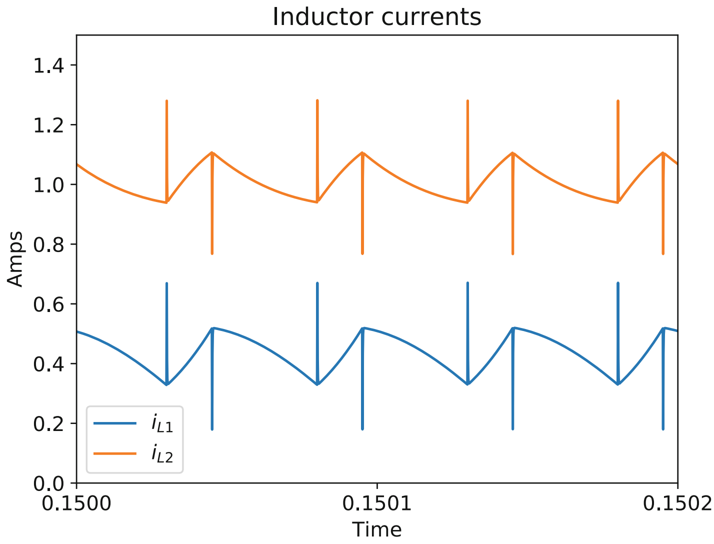


Fig. 3.30 Inductor currents i_{L1} and i_{L2}

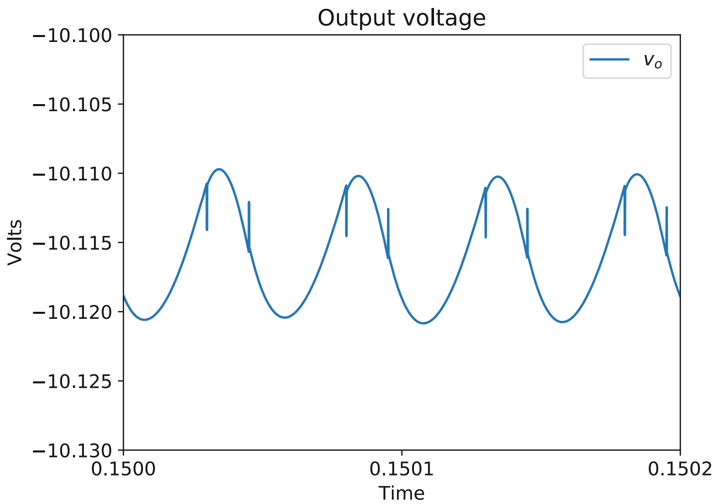


Fig. 3.31 Output voltage v_o

a polarity that it is a negative voltage. Therefore, in reality, when the voltage seems to decrease, the magnitude is increasing but in the negative sense.

The simulation of the Ćuk converter thereby shows a use case when we can wind inductors on the same core due to which they will have a mutual inductance. The cumulative effect of the self-inductance and the mutual inductance decreases the current ripple. The reader is encouraged to repeat the simulation with either separate inductors or by setting the mutual inductance M to be zero in the mathematical model of the coupled inductors. Therefore, in such a case, winding inductors on the same core has a double benefit—decreases the size of the converter as there is only one core and also decreases the current ripple without having to increase the size of the inductor.

With the simulation of the Ćuk converter, we have presented a practical case of when magnetic coupling between two inductors is beneficial even though the intention is not to achieve power transfer from one coil to the other. In a vast number of cases, intentional magnetic coupling between coils is usually to achieve power being transferred from one coil to the other, which will be discussed in the next chapter. Before bringing this chapter to a close, we will address one other issue in the mathematical model of coupled inductors. The model is specifically for two coils and the currents have been obtained by solving simultaneous equations of the flux linkages of the two coils. However, there could be any number of coils wound on the same core resulting in many inductors being magnetically coupled. In order to express the currents through all the coils, we would need to solve the flux linkage equations as matrix equations. This will be discussed in the next section.

3.9 Scalable Mathematical Models with Magnetic Coupling

In the previous sections, we had expressed the mathematical model for two coupled coils using simultaneous equations. Such a description is very convenient to analyse and understand the phenomenon of magnetic coupling. However, in the next chapter where we begin to model and simulate transformers, we can have many coils that are magnetically coupled since in many cases transformers have multiple windings with a voltage source connected to one winding supplying loads in all the others. In such cases, using the method of manually solving simultaneous equations will not be feasible. The sensible way of solving equations in such cases is by expressing them as matrix equations and using matrix manipulation techniques to simplify them.

Let us start with the same case as the coupled coils with dot polarities such that currents are entering the terminals of both coils marked by dots due to which the fluxes produced by the currents will be in the same direction through the core.

$$v_1 = r_1 i_1 + e_1 = r_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \tag{3.47}$$

$$v_2 = r_2 i_2 + e_2 = r_2 i_2 + M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \tag{3.48}$$

We have merely expanded the induced emf terms e_1 and e_2 with the rate of changes of flux linkages ψ_1 and ψ_2, which in turn have been expressed using currents and inductances.

When we encounter such a system of simultaneous equations, we can begin to express them using matrices by defining a few vectors. We can define a voltage vector **v** and a current vector **i** as:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad , \quad \mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{3.49}$$

We can express the simultaneous equations using these vectors and two more matrices as follows:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{3.50}$$

We have directly written the differential equations for the currents that we wish to compute without the intermediate computation of coil flux linkages. We can solve these differential equations using any numerical integration technique to obtain the currents for any sample k with respect to the voltages in the same sample k and the currents from the previous sample $k - 1$. Before progressing to the step of numerical integration, we will need to simplify the matrix equation. This is due to the fact that in the current state, both equations are coupled and if we integrate either equation, we end up integrating both currents. Luckily, by using matrices to express the equation, we have at our convenience many manipulation techniques using which the matrices can be transformed to another structure, which makes the solution of the equations much simpler [39].

Consider this matrix that contains the self and mutual inductances:

$$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \tag{3.51}$$

This matrix couples the derivatives of the inductor currents. If this matrix can be converted to either one of these forms:

$$\begin{bmatrix} L_1' & M' \\ 0 & L_2' \end{bmatrix} \qquad \begin{bmatrix} 0 & M' \\ M'' & L_2' \end{bmatrix} \tag{3.52}$$

we would have achieved a certain level of decoupling as now the derivative of one of the currents can be directly integrated without any influence of the derivative of the other current. To elaborate, in the matrix on the left $\frac{di_2}{dt}$ can be integrated to calculate i_2, while for the matrix on the right $\frac{di_1}{dt}$ can be integrated to calculate i_1. Once the derivative of one of the currents can be calculated, the derivative of the other current can be calculated by substituting the previously computed derivative. Therefore, in either of the matrix forms, a solution exists for computing all the currents.

The matrix on the left above is called an upper triangular matrix, while the matrix on the right above is called a lower triangular matrix [39]. The former is a matrix that has all elements below the diagonal to be zero, while the latter is a matrix in which all the elements above the diagonal are zero. Either of these can be chosen and the technique to produce this transformation will be the same in either case except that the precise steps will vary. Let us choose to transform the matrix to an upper triangular form as an example. To begin our solution, let us rewrite our matrix equation as follows:

$$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = - \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{3.53}$$

We have re-arranged our equation into a left and right hand side, with the objective of simplifying the matrix equation such that the left hand side of $\left[\frac{di_1}{dt}, \frac{di_2}{dt}\right]^T$ can be computed and subsequently integrated to provide the currents i_1 and i_2. From basic mathematics, we can perform a number of operations to the left and right hand side of the equation, which will not alter the solution as long as the same operation is performed on both sides of the equation [39]. As an example, we could multiply one row of all matrices on both sides of the equation by a scalar constant (say 5) without changing the solution:

$$\begin{bmatrix} 5L_1 & 5M \\ M & L_2 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = - \begin{bmatrix} 5r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} 5v_1 \\ v_2 \end{bmatrix} \tag{3.54}$$

We could also add one row of every matrix to another row of every matrix of both sides of the equation without changing the solution. For example, if we add the first row to the second row:

$$\begin{bmatrix} 5L_1 & 5M \\ M + 5L_1 & L_2 + 5M \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = - \begin{bmatrix} 5r_1 & 0 \\ 5r_1 & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} 5v_1 \\ v_2 + 5v_1 \end{bmatrix} \tag{3.55}$$

These operations are no different from the manner in which simultaneous equations are solved by hand. The reader is encouraged to verify this.

Instead of choosing the random value of 5 to multiply the first row with, if we choose the value of $-\frac{M}{L_1}$, the matrix equation turns out to be

$$\begin{bmatrix} L_1 & M \\ 0 & L_2 - \frac{M^2}{L_1} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = - \begin{bmatrix} r_1 & 0 \\ r_1 - \frac{Mr_1}{L_1} & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 - \frac{Mv_1}{L_1} \end{bmatrix} \tag{3.56}$$

In the above equation, we have retained the first row in its original form without multiplying it by any constants but have added to the second row terms containing the first row multiplied by the constant $-\frac{M}{L_1}$. As can be observed, we have transformed the matrix on the left hand side of the equation to an upper triangular form. It should be noted that this transformation was not coincidental, rather deliberate by choosing a multiplying factor that would eliminate the element in the second row and first column for the matrix on the left hand side.

The beauty of this technique lies in the fact that it can be applied to any matrix that represents any system and can be formulated as an algorithm to be solved by a computer. For example, let us suppose there were three coils wound on the same core with self-inductances L_1, L_2 and L_3. Let us suppose the mutual inductance between coils 1 and 2 was M_{12}, the mutual inductance between coils 2 and 3 was M_{23} and the mutual inductance between coils 1 and 3 was M_{13}. The new set of equations can be represented in the matrix form as follows:

$$\begin{bmatrix} L_1 & M_{12} & M_{13} \\ M_{12} & L_2 & M_{23} \\ M_{13} & M_{23} & L_3 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = - \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \tag{3.57}$$

To transform the matrix on the left hand side of the above equation to an upper triangular form, we need to reduce three elements to zero—element (2, 1) (=M_{12}), element (3, 1) (=M_{13}) and element (3, 2) (=M_{23}). We can add to the second row of all matrices on both sides of the equation the first row of the same matrix multiplied by the product $-\frac{M_{12}}{L_1}$. This will eliminate element (2, 1) of the left hand side matrix while altering the elements in the second row of all matrices on both sides of the equation. Furthermore, we can add to the third row of all matrices on both sides of the equation the first row of the same matrix multiplied by the product $-\frac{M_{13}}{L_1}$. This will eliminate element (3, 1) of the left hand side matrix while altering the elements in the third row of all matrices on both sides of the equation. We can eliminate the element (3, 2) of the left hand side matrix by adding the second row multiplied by an appropriate factor to the third row of all matrices on both sides of the equation. This factor will have to be determined after the calculation carried out in the previous step as the second row has already changed.

Though the above process may seem tedious, it is a mechanical process that can be automated using code as follows:

```
L = [[L1, M], [M, L2]]
R = [[r1, 0.0], [0.0, r2]]
V = [24.0, 0.0]
for count1 in range(len(L)):
    if not L[count1][count1]:
        for count2 in range(count1+1, len(L)):
            if L[count2][count1]:
                L[count1], L[count2] = L[count2], L[count1]
                R[count1], R[count2] = R[count2], R[count1]
                V[count1], V[count2] = V[count2], V[count1]
                break
    if L[count1][count1]:
        for count2 in range(count1+1, len(L)):
            comm_factor = L[count2][count1]/L[count1][count1]
            for count3 in range(len(L[count1])):
                L[count2][count3] -= L[count1][count3]*comm_factor
                R[count2][count3] -= R[count1][count3]*comm_factor
            V[count2] -= V[count1]*comm_factor
```

From the code block, it is clear how all we need to do is define the matrices and the process of triangularization that follows is a mechanical process. The reader is encouraged to choose larger matrices for **L**, **R** and **V** and verify that the process is able to convert the matrix **L** to an upper triangular form in every case. The only important factor is that the matrices **L** and **R** should be square and have the same dimensions, while the matrix **V** should have the same number of rows as **L** and **R**. Subsequent to triangularization, the differential equations can be solved in the reverse order as follows:

```
dibydt = [0.0, 0.0]
for count1 in range(len(L)-1, -1, -1):
    if L[count1][count1]:
        k = [0.0, 0.0, 0.0, 0.0]
        for k_count in range(len(k)):
            for count2 in range(len(B[count1])):
                k[k_count] += B[count1][count2]*coil_voltages[count2]
            for count2 in range(len(A[count1])):
                if (k_count==0):
                    k[k_count] -= A[count1][count2]*ind_current[count2]
                elif count2>count1:
                    k[k_count] -= A[count1][count2]*(ind_current[count2] + \
                            dt*dibydt[count2])
                if (k_count==1):
                    if (count2 == count1):
                        k[k_count] -= A[count1][count2]*(ind_current[count2] + \
                            dt*k[0]/2.0)
                    elif count2>count1:
                        k[k_count] -= A[count1][count2]*(ind_current[count2] + \
                            dt*dibydt[count2])
                    else:
                        k[k_count] -= A[count1][count2]*ind_current[count2]
                if (k_count==2):
                    if (count2 == count1):
                        k[k_count] -= A[count1][count2]*(ind_current[count2] + \
                            dt*k[1]/2.0)
                    elif count2>count1:
                        k[k_count] -= A[count1][count2]*(ind_current[count2] + \
                            dt*dibydt[count2])
                    else:
                        k[k_count] -= A[count1][count2]*ind_current[count2]
                if (k_count==3):
                    if (count2 == count1):
                        k[k_count] -= A[count1][count2]*(ind_current[count2] + \
                            dt*k[2]/2.0)
                    elif count2>count1:
                        k[k_count] -= A[count1][count2]*(ind_current[count2] + \
                            dt*dibydt[count2])
                    else:
                        k[k_count] -= A[count1][count2]*ind_current[count2]
```

```
for count2 in range(len(L[count1])):
            if not count2==count1:
               k[k_count] -= L[count1][count2]*dibydt[count2]
         k[k_count] = k[k_count] / L[count1][count1]
      dibydt[count1] = (k[0] + 2*k[1] + 2*k[2] + k[3])/6.0
      ind_current[count1] += dibydt[count1]*dt
   else:
      ind_current[count1] = V[count1]
      for count2 in range(count1+1, len(L)):
         ind_current[count1] −= L[count1][count2] * dibydt[count2]
      for count2 in range(count1+1, len(R)):
         if not count2==count1:
            ind_current[count1] −= R[count1][count2] * ind_current[count2]
            ind_current[count1] = ind_current[count1] / R[count1][count1]
```

In the above code segment, the numerical integration has been performed using Runge–Kutta Fourth Order method, which provides greater accuracy as compared to the simple integration by Backward Euler's method used in the previous simulations. The reader is encouraged to include these two code segments into the simulation of the coupled inductors presented previously in this chapter as well as the simulation of the Ćuk converter. In the next chapter, system equations will be expressed in the matrix form and integrated using Runge–Kutta Fourth Order method as shown above.

3.10 Conclusions

In this chapter, we used our knowledge of magnetic circuits gained from the previous chapter to develop simulation models for magnetically coupled circuits. We began with our assertion that all the basic laws of physics applicable to an inductor (single coil wound on a core) are applicable for the case of multiple coils wound on the same core. Such systems can also be represented by magnetic circuits comprised of the MMFs produced by each coil and the reluctances of each limb of the core. The emfs induced in the coils can be expressed with respect to the rate of change of flux linkages of the coils. The chapter began with a simple explanation of the concept of magnetic coupling between two coils wound on a rectangular core with only one coil being energized.

As the concept of magnetic coupling forms the basis of transformer operation, which will be dealt with in the next chapter, we examined in detail various possibilities that can arise when multiple coils are wound on a single core. We have expressed equivalent magnetic circuits and have performed a mathematical analysis for different conditions. Initially, we have used only the basic laws of physics such as Ampere's Law, Faraday's Law and Lenz's Law to determine the behaviour of the magnetically coupled coils. The scenarios helped us to understand how energy is transferred from one coil to another magnetically coupled coil. The objective behind these discussions was to understand how the basic laws of physics can continue to be applied even though the behaviour of a magnetically coupled system differs from a single inductor.

Following our discussions on understanding the concept of magnetic coupling, we proceeded to formulate a mathematical model for a magnetically coupled system. We introduced the concept of mutual inductance in addition to self-inductance to express the flux linkages of a coil with respect to the currents flowing in all coils of the magnetically coupled system. We also described how dot polarities can be used to indicate the sense of windings of coils such that magnetically coupled coils can be represented as simple inductors with dots on one of their terminals instead of depicting the detailed windings on the core. Combining mutual inductances and dot polarities, concise flux linkage equations could then be written for any magnetically coupled system.

The chapter consolidates the theoretical discussion with a simulation of two coils wound on the same core. The simulation has two separate cases where the dot polarity of the coils are such that in one case, the fluxes produced by the coils are in the same direction while in the other case, the fluxes produced by the coils are in the opposite direction. To introduce a practical aspect to the discussion on magnetic coupling, we described the operation of a Ćuk converter in which magnetically coupled inductors are often used. Using the Ćuk converter, we described when it is beneficial to wind multiple inductors on the same core. Simulations of the Ćuk converter showed how similar patterns of currents through the inductors make it possible to wind them on the same core, thereby decreasing the size of the converter and also decreasing the current ripple.

To generalize the simulation of a magnetically coupled system, the simulation model is converted to a matrix form such that any number of coils wound on a core can be modelled. We described the advantage of a matrix representation with the concise matrix equation that was the result. To simplify the matrix equation, we described a process whereby the inductance matrix could be converted to an upper triangular form and presented the code block for doing so. Subsequent to the triangularization, the process of numerical integration was modified for the matrix representation. Such a matrix representation results in a convenient and expandable simulation model as will be seen in the next chapter when we simulate transformers.

In the next chapter, we will model and simulate transformers using the fundamentals of magnetic coupling described in this chapter. Transformers are a specific application of coupled coils wound on a core so as to achieve the maximum possible transfer of energy from one coil to the other. However, this chapter will help the reader to understand the fundamental nature of magnetic coupling that makes this possible. Moreover, this nature of magnetic coupling can also be extended to rotating machines such as motors and generators, though this book will not cover them.

Chapter 4
Modelling and Simulating Transformers

4.1 Introduction

In the previous chapters, we gradually built up the discussion on energy transfer through magnetic coupling. The objective of the previous chapters was to build a bridge between magnetism as dealt in physics and simulation models of components such as inductors that use magnetism. The topic of magnetism is vast and modelling the magnetic field accurately is very complex that requires special techniques and solutions. However, by assuming uniform magnetic fields and linear magnetic properties, we were able to formulate concise mathematical models that captured the phenomenon of magnetic coupling to a degree acceptable for most electrical engineering applications. Since magnetism as a concept is a bit tricky and translating physical laws related to magnetism into electrical engineering can result in confusion, the past two chapters introduced the modelling of magnetic circuits with the objective of understanding the behaviour of any arbitrary magnetic circuit.

In this chapter, we finally arrive at the main course of this book—an electrical machine based on the transfer of energy through magnetic coupling. The transformer is a static machine very commonly used in electrical engineering for interfacing different electrical systems together [10, 11, 13, 14]. There are numerous applications of transformers, and it is impossible to cover all the different applications in a single book [13, 14]. The objective of this chapter is to introduce some of the most common applications of transformers that an electrical engineer might encounter and present simulation models for these applications. As before, even though the basic underlying concept of transformer simulations remains the same for most applications, for an in-depth understanding of transformer operation, this chapter examines different applications over separate sections.

Under the hood, a transformer is just a set of magnetically coupled coils. Therefore, before reading this chapter, the reader is advised to be familiar with all the nuances of magnetically coupled coils described in the previous chapter. What makes a transformer a special case of magnetically coupled coils is that a

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transformer in most cases is designed to achieve maximum transfer of power from one coil to other coils. Therefore, even though the underlying simulation model is similar to simple magnetically coupled coils, arriving at the base model from the specifications of a transformer as a machine will be described by representing the equivalent circuit of a transformer. The equivalent circuit of a transformer is dealt with during undergraduate studies [13, 14]. However, this chapter will relate the transformer equivalent circuit to the simulation model. Moreover, the equivalent circuit will be built up gradually as we add details to the transformer model.

During my undergraduate years, I faced a lot of confusion related to the detailed operation of a transformer. Particularly, understanding how the turns ratio of a transformer relates to the manner in which the winding currents of the transformer are expressed was usually merely assumed as a simple formula. On the whole, to use a set of formulae to figure out the operation of an equipment or machine without a physical understanding of its working was something that I found to be a hindrance when I became a graduate student. This is an obstacle most engineers face. The depth to which they learn transformers and machines in general was inadequate for application either during their professional careers or when pursuing research.

As I found the need to relearn electrical machines from a different perspective later, I also found the motivation to learn not just through equations but also through simulations. Simulations enable us to recreate the physical phenomena behind the operation of a machine that enable us to understand how the different physical laws are related [6]. Even though the basic laws of physics are known to us from high school courses, to be able to put them together such that the operation of a machine can be understood can be a little tricky and will need some back and forth between these laws. This approach has been taken in Chap. 2 while introducing magnetic circuits in the context of inductors and in Chap. 3 while extending the magnetic circuits to understand coupled magnetic circuits.

In this chapter, following the extension of the simulation model of coupled inductors to transformers, several sections are dedicated to describing the operation of a transformer through simulation results [13, 14]. The concept of current transformation and MMF equalization are explained in detail using both theory and simulations. The chapter will present simulations of step-up and step-down transformers and will describe the impact of turns ratio on the winding voltages and currents. Finally, the chapter will also describe how multi-winding transformers (with more than two windings) can be modelled and simulated. This will lay the foundation for the next chapter, which will discuss how three-phase transformers can be modelled and simulated.

Just like the previous chapters, this chapter will try to combine theory and simulations by referring to the basic laws of physics while describing simulation results. The reader might find some of the discussions repetitive. However, the purpose of these long discussions is to provide a framework where the reader can try to form connections between the different laws of physics to interpret simulation results. Such an approach will be not only useful in understanding the operation of transformers, but can later also be extended to other electrical machines such as motors and generators. The reader is encouraged to try out the simulations described in the chapter and also to attempt some of the modifications that have been suggested as exercises.

4.2 Background

Transformers are one of the most widely used electrical machines, and without them the modern electrical grid would not be possible [21]. The different uses of transformers and the solutions offered by them are unfortunately not fully appreciated by most electrical engineers [13]. For a power engineer, an in-depth understanding of transformer operation is extremely important. Quite unfortunately, very few power engineers bother to fully understand how a transformer operates and stick to the most basic equations that are used to mathematically express transformer operation. As already described in the previous chapters, all that is needed to fully understand a static magnetic component or machine are a few basic laws of physics—Ampere's Law, Faraday's Law, and Lenz's Law. In this chapter, we will extend our knowledge of magnetic coupling described in the previous chapter to examine in detail the working of transformers.

Transformers have numerous uses and applications. Before we dive into the details of the operation of a transformer, let us look at some of the major uses of transformers [13]. As already stated, the modern electrical grid would not be possible without transformers. To elaborate on this, we need to appreciate that modern electrical grids in most parts of the world are gigantic interconnected systems similar to the network of roads and railways that connect different parts of a land mass. Electrical energy is produced at generating stations that could be based on thermal (coal, gas or nuclear), hydro-electric and nowadays even renewable energy sources (solar, wind, etc.). Except for renewable energy generation that is possible at the load centre itself in the form of rooftop solar for example, other forms of energy generation are usually located in remote areas. The remote location of generating stations is for a number of reasons—safety (for example nuclear) or the proximity to the energy source (hydro-electric for example).

The electrical energy generated in remote generating stations needs to be transferred to the load centres, which are cities and industrial areas that may be several hundreds or thousands of kilometres away through transmission systems. The generators at the generating stations produce electrical energy at a voltage in the range of 500 V to 11 kV, and a large generation station (a thermal power station for example) could produce thousands of megawatts of power. If such large amounts of power were to be transmitted over long distances at the generated voltage, the currents that will flow will result in very large ohmic losses leading to a very inefficient transmission system. In order to transmit large amounts of power over long distances, the voltage level needs to be increased. With the power being the product of voltage and current, a higher voltage level will result in lower currents and therefore lower ohmic losses. This operation of increasing the level of voltage from the generating station to the transmission system is performed using transformers. Using transformers, transmission systems have been energized to voltage levels of several hundreds of kilovolts with the highest voltage levels being around 800 kV.

To understand how electrical power transmission systems work, a good analogy is with the network of roads that connect a country or region. Within a city or town, there are usually a number of different types of roads with different speed limits. In residential areas of the city, roads might be two lane or four lane roads with a speed limit of 40 km/hr or 50 km/hr. There are special zones such as schools and hospitals where the roads will have reduced speed limits of 20 km/hr or 30 km/hr with excessive speeds being prevented by speed bumps. On the other hand, arterial roads that connect different parts of the city together might have six lanes and higher speed limits of 60 km/hr to 80 km/hr to enable drivers to move around faster. Highways that connect distant parts of the city or different cities together can have speed limits in the excess of 100 km/hr, and driving too slow will not be permitted. The philosophy being planning the road network is to enable drivers to move around as quickly as possible while ensuring safety of drivers and also pedestrians and surroundings. Driving too fast in a residential area is a danger to pedestrians, and road safety planners will implement features such as speed traps and speed cameras to spot violators. However, in a highway, speed limits when they exist will be to ensure safety of drivers but otherwise highways exist to facilitate rapid movement of vehicles.

In exactly the same manner, there exist a number of different voltage levels in an electric grid. At homes and offices, voltage levels are typically either 240 V or 120 V (in the USA and Canada) with most appliances rated for these voltages. The reason is that power consumed by domestic appliances is rarely larger than a few thousand watts and at 120V or 240V, the current drawn by these appliances will be at most a few tens of amperes. Currents larger than a few tens of amperes in a domestic situation can be dangerous and cause fire safety issues. It is important to note that a large number of house fires are caused due to short-circuits that produce large currents in the order of several tens or hundreds of amperes. The voltage level at which apartment buildings and houses are supplied however is usually much higher. This is due to the fact that a single feeder will supply a row of houses or apartment buildings, and therefore the overall power supplied by this feeder maybe in the range of several thousands of watts. For example, a row of apartment buildings might be supplied by a feeder with a voltage range of 3.3 kV to 11 kV. The higher voltage level makes it possible to supply this augmented power at lower current levels. Similarly, as we progress upwards in the grid with segments of the city supplied by a feeder, the voltage levels of the feeder can be in the range of 11 kV to 33 kV or even higher. Finally, the main transmission lines that supply the city will be at far higher voltage levels in the range of 110 kV to 400 kV.

Therefore, comparing the electric grid and the road network, the power supplied by a transmission line or feeder in the electric grid is equivalent to the volume or number of vehicles passing through a highway or road in the road network. The voltage level of a transmission line or feeder in the electric grid is equivalent to the speed limit of a highway or a road in the road network. The current flowing through a transmission line or feeder in the electric grid is equivalent to the density of vehicles in a highway or a road in the road network. Transformers along with a number of other equipment ensure that we are able to transfer large amounts of power over long distances while being able to convert them from one voltage level to another to ensure safety of the system. Without transformers, we would have just a single voltage level across the entire grid, which would be either too low to be totally inefficient or too high to be potentially dangerous. Therefore, one needs to take a moment to appreciate the role played by transformers in the modern power system.

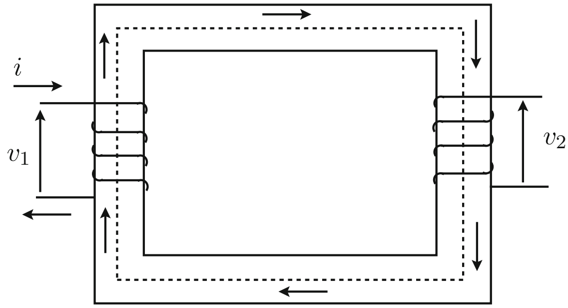
Besides being a critical component in the modern power system for transforming voltage levels, transformers are also used for numerous other purposes in isolating circuits for a number of different reasons [10, 13, 14, 26]. As an example, when designing control circuits, different parts of a controller may need to be electrically isolated to ensure they do not interfere with each other. This is quite often the case in a power converter with multiple power devices, with the gate driver of each power device energized by an isolated power supply. The power supplies may need to be isolated to ensure that there is no circulating current that can accidentally cause a power device to conduct when it should not conduct. Transformers are also used in certain power converter topologies such as cascaded converters to ensure that circulating currents between modules is not possible [10, 26]. Some of these examples will be simulated to show the role played by transformers.

The idea behind this basic background was to throw some light on the importance of transformers in electrical engineering. It is difficult for a power engineer to not encounter a transformer at some point of time during a project. Though there are transformer-less appliances that are designed specifically without a transformer to decrease the size and cost, a vast number of appliances and systems will need transformers to function. In the next section, we will gradually ease into the modelling of transformers from the very basic concepts of physics as we did in the previous chapters for inductors and coupled inductors.

4.3 Transformer Basics

In this section, we will get started with the basic operating principle of a transformer [13, 14, 27, 28]. We will leave the detailed model and simulation for a later section. As we did in the previous chapter with coupled coils, we will gradually meander through the concept of how a transformer comes into being and how it operates. The previous chapter covered in detail the modelling and simulation of magnetically coupled coils. Under the hood, a transformer is merely a set of magnetically coupled coils. A transformer is a special case of magnetically coupled coils, which is so designed to maximize the transfer of energy from one coil to the other while minimizing losses and leakage. This is essential to ensure that the power system remains efficient with minimal losses while voltage levels are being raised or lowered using transformers.

Fig. 4.1 Two coils wound on the same core



Let us start with the basic setup of two coils wound on an iron core as shown in Fig. 4.1. A detailed discussion can be found in the previous chapter. Let us quickly repeat some of the basic expressions so as to extend our discussion to be more specific to transformers. The voltage v_1 applied to the coil wound on the left limb (let us call it coil 1) can be expressed as

$$v_1 = e_1 + i_1 r_1 \tag{4.1}$$

where r_1 is the winding resistance, e_1 is the induced emf and i_1 is the current in coil 1. This expression is merely Kirchhoff's Voltage Law applied to coil 1. Since the coil wound on the right limb (let us call it coil 2) is left open-circuited, no current i_2 will flow through it and therefore,

$$v_2 = e_2 \tag{4.2}$$

The voltage v_2 across the terminals is equal to the emf e_2 induced in the coil.

Using Faraday's Law, the emf induced in the coils is equal to the rate of change of flux linkages of the coils:

$$e_1 = \frac{\psi_1}{dt} = v_1 - i_1 r_1 \tag{4.3}$$

$$e_2 = \frac{\psi_2}{dt} = v_2 \tag{4.4}$$

If L_1 and L_2 are the self-inductances of coil 1 and 2, respectively, and M is the mutual inductance between the coils, the flux linkages can be expressed as

$$\psi_1 = L_1 i_1 \tag{4.5}$$

$$\psi_2 = M i_1 \tag{4.6}$$

These equations are for the special case of Fig. 4.1 where coil 2 is open-circuited. Therefore, the flux linkages of coils 1 and 2 are only due to the current flowing in coil 1. The concept of mutual inductance has been described in detail in the previous chapter, and the reader is encouraged to revisit the previous chapter if the above expression is unclear.

Due to the current i_1 flowing in coil 1, the flux produced in the core links with coil 2 to produce flux linkages ψ_2. If the current i_1 changes, the flux linkages ψ_1 and ψ_2 will change as well due to which emfs e_1 and e_2 will be induced in the coils. The emf e_2 induced in coil 2 produces a voltage v_2 across the terminals of the coil. Since coil 2 is open-circuited, the voltage produced across the coil terminals will not result in any power consumed. But, if we do connect a load such as a simple resistive load, a current i_2 will flow through coil 2. With this, we have now achieved transfer of energy from coil 1 to coil 2 without any electrical connection between the coils. This phenomenon of energy transfer has been described in detail with several different possible variations in the previous chapter. At a very basic level, this transfer of energy is the basis of operation of a transformer.

Following this very basic discussion viewing a transformer as merely coils that are magnetically coupled, we can now extend the discussion to the specifics of a transformer. In the previous chapter, we had considered the coils in Fig. 4.1 to be either connected to two independent isolated circuits or in the case of the Ćuk converter, to different parts of the same circuit. In the case of a transformer, unless in very specific cases, the different coils are usually connected to independent isolated circuits. Furthermore, since the objective of a transformer is to transfer power from one isolated circuit to another, the primary specification of a transformer is the maximum power that can be transferred from one isolated circuit to other isolated circuits [13, 14, 27, 28]. This is in contrast to a set of mutually coupled coils that can be specified only with respect to their self- and mutual inductances.

Therefore, while all the equations written above involving the self- and mutual inductances hold true for a transformer just like they did for magnetically coupled coils, the approach for arriving at these equations for transformers is quite different. Along with the primary specification of the transformer being the maximum power rating, the next important specifications are the maximum rated voltages that can be applied to each coil of the transformer [13, 14, 27, 28]. At this point, we need to rename the coils wound on the core as the windings of the transformer as that is the terminology that is commonly used with transformers. These windings of the transformer may or may not be connected to an energized system. It is important to note that more than one winding of a transformer could be energized as it is common to use a transformer to interface two different systems at different voltages. There may be cases where only one winding is energized since the other winding might feed a load or another passive circuit.

Before we continue with our description, let us introduce an example with some specifications so as to make the discussion easier to follow. Let us consider a transformer with two windings and a maximum power rating of 10,000 Volt-Amperes or 10,000 VA or 10 kVA. Volt-Amperes is the rating for apparent power, which includes the active power in Watts as well as the reactive power in Volt-Ampere-Reactive (VAR). Let us suppose that this transformer has two windings and both windings have maximum voltage ratings of 240 V. Moreover, we can also

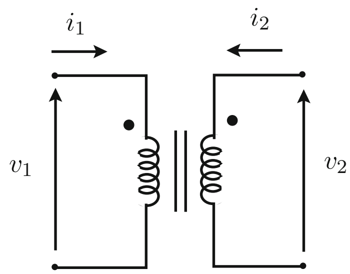


Fig. 4.2 Simple winding depiction of a transformer

specify that this transformer should be operated at a frequency of 50 Hz or 60 Hz (in USA or Canada). Quite a simple transformer, but to begin with, this should be sufficient to help us understand the transformer parameters. Such a transformer can transfer a maximum of 10 kVA of apparent power from any one winding to the other. A maximum of 240 V can be applied to either or both windings. For example, we could connect terminals of one winding of the transformer to the live and neutral of a domestic power outlet that supplies 240 V, 50 Hz single-phase ac supply and connect to the other winding a simple load resistor.

Figure 4.2 shows how a transformer can be depicted in a circuit [13, 14]. In comparison to the previous chapter, it is quite clear that the depiction of the transformer is very similar to magnetically coupled coils. In most circuit diagrams, a transformer contains the vertical lines between the windings that indicate the core of the transformer and that the windings are wound on the same core. However, the windings will not always have the dot polarity marked on their windings in which case it is assumed that the upper terminals of the windings are marked by dots. However, if one of the windings is wound in the opposite sense, dots marking one of the terminals of all the windings are usually present. Simulations will make this more clear in the later sections.

With respect to the sample specifications of the transformer, the voltages v_1 and v_2 that are applied to the windings can have a maximum RMS value of 240 V. Since the maximum power rating of the transformer is 10 kVA, the currents i_1 and i_2 should be limited to a maximum RMS value of 10,000/240 = 41.667 A. Applying excessive voltage to either winding could result in saturation of the transformer core and will destroy/burn the windings in the case of voltages much higher than the maximum rated voltage [13, 14, 27, 28]. If the circuit connected to a winding draws a current much greater than the maximum rated current, the ohmic losses will increase resulting in an increased operating temperature that is detrimental to the insulation of the wires and other parts of the transformer [13, 14, 27, 28].

In order to develop a simulation model for a transformer, we need to apply the equations involving L_1, L_2 and M. For this, we need to build a transformer equivalent circuit [13, 14, 27, 28]. In practice, for a particular transformer, if we wish to analyse or simulate it, we will also build the transformer equivalent circuit. If we have a physical transformer and can perform experiments on it, there are tests such as the short-circuit test and the open circuit test using which most of the significant parameters of the equivalent circuit can be determined [13, 14, 27, 28]. However, without conducting experiments, it is also possible to determine the approximate parameters of the equivalent circuit by the process of estimation. This has the added advantage of being able to examine the behaviour of the transformer for variations in the equivalent circuit parameters. Let us develop the equivalent circuit of the transformer step by step.

Let us suppose that the transformer of Fig. 4.2 is energized on winding 1 with a voltage v_1 applied to it. Let us suppose winding 2 is left open-circuited. We have already applied Kirchhoff's Voltage Law to winding 1:

$$v_1 = e_1 + i_1 r_1 \tag{4.7}$$

If winding 1 were to have an inductance of L_1, the above equation can be rewritten as

$$v_1 = L_1 \frac{di_1}{dt} + i_1 r_1 \tag{4.8}$$

Winding 1 being a coil has an inductance L_1 and a winding resistance r_1 and the voltage applied across the winding is opposed by the impedance offered by the winding. The RMS value of the current through winding 1 can be expressed as

$$I_1 = \frac{V_1}{\sqrt{(\omega L_1)^2 + r_1^2}} \tag{4.9}$$

where I_1 and V_1 are the RMS values of current i_1 and voltage v_1, respectively, while $\omega = 2\pi f$ is the angular frequency of the system.

We have chosen the maximum voltage rating of both windings of the transformer to be 240 V. While building the equivalent circuit of the transformer, let us assume that the voltage applied to winding 1 is equal to the maximum rated voltage of 240 V. If an RMS value I_1 is known, the impedance of winding 1 can be calculated. To be able to assume a value for I_1, we need to ask the question, in the assumed state of the transformer with a voltage applied to winding 1 and winding 2 open-circuited, what happens in the transformer? Essentially, we are performing the open circuit test on the transformer where the rated voltage is applied across a winding and the other winding is left open-circuited.

In the previous chapter, we had discussed many conditions in which magnetically coupled coils can be energized. We can quickly repeat the final result and the reader is encouraged to revise the previous chapter if needed. With winding 1 alone energized, a current i_1 will flow through it and current i_2 is zero since winding 2 is open-circuited. This current i_1 will establish a magnetic flux in the core. Since the voltage v_1 applied across winding 1 is an ac voltage, the core flux will be an ac flux as well. By Faraday's Law, emfs e_1 and e_2 will be induced in the two windings. The only effect of the current i_1 is to set up the magnetic flux in the core. This current i_1 is called the magnetizing current of the transformer. It is important to remember that we could do exactly the same test on winding 2—apply a voltage v_2 across winding 2 with an RMS value equal to the maximum rated value and leave winding 1 open-circuited. In that case, no current i_1 will flow in winding 1 and the current i_2 flowing in winding 2 will also be the magnetizing current.

The transformer is a magnetic equipment as the fundamental basis of operation of the transformer is the magnetic flux in the core. Therefore, the magnetizing current flowing in the energized winding is absolutely essential [13, 14, 27, 28]. Since one winding is open-circuited, the transformer is not transferring any power from one winding to the other. The transformer is still consuming power since there is ohmic loss in winding 1 and core losses (eddy current and hysteresis losses) since there is a magnetic flux in the core. These losses are called the no-load losses of the transformer [13, 14, 27, 28]. These no-load losses are an extremely small fraction of the maximum power rating of the transformer. This is fairly obvious since we would like the transformer to be as efficient as possible. At no load the ohmic losses in winding 1 are usually negligible compared to the core losses.

With this background, we make an assumption and also use commonly known data related to transformers. The magnetizing current flowing through a winding is usually around 1–5% of the rated maximum current of the winding [13, 14, 27, 28]. If we assume the magnetizing current to be 2% of the rated maximum current, the winding impedance can be calculated as

$$\sqrt{(\omega L_1)^2 + r_1^2} = 50\frac{V_{1rated}}{I_{1rated}} \tag{4.10}$$

We have already assumed that the voltage v_1 applied across winding 1 is equal to the maximum rated voltage. Therefore, the above expression provides the impedance with respect to the winding ratings.

In the impedance of the transformer winding, the winding resistance will be negligible as otherwise the no-load losses will be significant even if the current drawn was equal to the magnetizing current. Therefore, a further approximation can be made as follows:

$$L_1 \approx 50\frac{V_{1rated}}{\omega_{rated} I_{1rated}} \tag{4.11}$$

ω_{rated} is merely the nominal frequency of the system corresponding to 50 Hz or 60 Hz. The frequency plays a role in determining the flux density in the core, which we will discuss later. However, without going into the details of the magnetic circuit, we have obtained the self-inductance of winding 1. We can use this expression for winding 2 as well:

$$L_2 \approx 50\frac{V_{2rated}}{\omega_{rated} I_{2rated}} \tag{4.12}$$

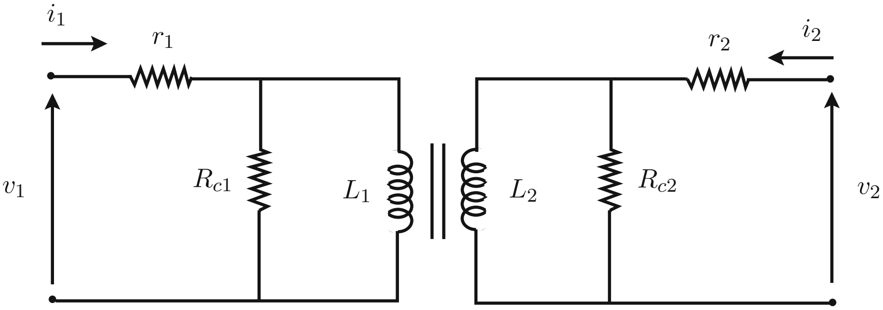


Fig. 4.3 Basic transformer equivalent circuit with self-inductances

Now that we have computed the self-inductances L_1 and L_2 of winding 1 and 2, respectively, we can draw a very basic equivalent circuit of the transformer as shown in Fig. 4.3. The reader should observe that the dot polarities have been omitted from the windings. When a transformer is depicted without dot polarities, it can be assumed that the dots are on the upper terminals of the windings as shown in Fig. 4.2. From Fig. 4.3, the induced emfs in the windings appear across the self-inductances and are the difference between the applied voltages and the ohmic drop across winding resistances r_1 and r_2. We have included two more resistances—R_{c1} and R_{c2}. These resistances represent the core losses in the transformer. In practice, we would estimate the core losses from the open circuit test as the no-load losses are largely due to the core losses. However, for the simulation model, we can estimate the core losses by assuming that the core losses are 1% of the maximum rated power of the transformer [13, 14, 27, 28]. Therefore,

$$R_{c1} \approx \frac{V_{1rated}^2}{0.01P_{rated}} \tag{4.13}$$

$$R_{c2} \approx \frac{V_{2rated}^2}{0.01P_{rated}} \tag{4.14}$$

To complete the basic equivalent circuit of Fig. 4.3, we need values for the winding resistances r_1 and r_2. In practice, the winding resistances are determined by performing a short-circuit test on the transformer in which one of the windings of the transformer is short-circuited and the other winding is energized with a voltage such that the currents flowing in the windings are equal to the rated winding currents. In such a case, the losses in the transformer are primarily the ohmic losses as the core losses are negligible due to the very low voltage that is needed to be applied when one of the windings is short-circuited. For a simulation model, we can estimate the winding resistances r_1 and r_2 using values that are normally found in transformers. We can assume the winding resistances r_1 and r_2 to be such that if a rated current were to be flowing through the windings, the voltage drop across the winding resistances would be approximately 0.5 to 1% of the rated winding voltage [13, 14, 27, 28]. Therefore,

$$r_1 \approx 0.01 \frac{V_{1rated}}{I_{1rated}} \tag{4.15}$$

$$r_2 \approx 0.01 \frac{V_{2rated}}{I_{2rated}} \tag{4.16}$$

With the equivalent circuit of Fig. 4.3, we can use the equations once we have a value for the mutual inductance M. The mutual inductance can be computed as before with respect to the self-inductances as follows:

$$M = k\sqrt{L_1 L_2} \tag{4.17}$$

where k is the coupling factor between the transformer windings. This expression was used in the previous chapter when modelling and simulating magnetically coupled coils and takes into account the fact that not all of the flux generated by the current flowing through a winding will link with the other winding. However, a transformer is so designed as to minimize the leakage of flux produced by a winding. Therefore, instead of choosing a value of the coupling factor k, the leakage in the transformer windings can be better modelled in another manner.

Instead of representing the inductance of the windings by the self-inductances L_1 and L_2 alone, we can separate the inductances into two parts—magnetizing inductances L_{m1} and L_{m2} that produce the flux that flows through the core and links with the other winding and leakage inductances L_{l1} and L_{l2} that will produce fluxes that leak into the air [13, 14, 27, 28]. This is shown in Fig. 4.4 [13, 14, 27, 28]. The magnetizing inductances L_{m1} and L_{m2} take the place of the self-inductances L_1 and L_2 as now these inductances are responsible for producing the core flux and therefore the induced emfs in the windings. The leakage inductances L_{l1} and L_{l2} are in series with the winding resistances r_1 and r_2, and the voltage drops across them result in a decreased induced emf.

We can estimate the leakage inductances in the same way as we did for many of the other parameters. The leakage inductances will merely decrease the induced emf

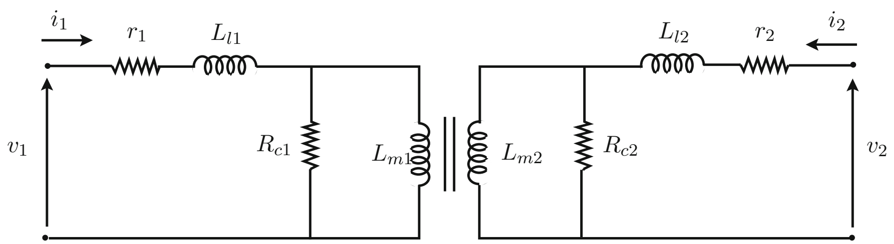


Fig. 4.4 Basic transformer equivalent circuit with magnetizing and leakage inductances

due to the leakage of flux. Therefore, we assume the leakage inductances to be such that when rated currents are flowing through the windings, the voltage drop across the leakage inductances will be 1–2% of the rated winding voltage [13, 14, 27, 28]. As a result, we arrive at the following expressions:

$$L_{l1} \approx 0.02 \frac{V_{1rated}}{\omega_{rated} I_{1rated}} \tag{4.18}$$

$$L_{l2} \approx 0.02 \frac{V_{2rated}}{\omega_{rated} I_{2rated}} \tag{4.19}$$

The mutual inductance M is merely expressed with respect to the magnetizing inductances as follows:

$$M = \sqrt{L_{m1} L_{m2}} \tag{4.20}$$

The coupling factor k has been removed as the leakage of flux has been accounted for separately through the leakage inductances. After all, as per definition, the magnetizing inductances L_{m1} and L_{m2} produce fluxes that flow through the core and link completely with other windings. However, there is one last complication—we have computed the self-inductances L_1 and L_2 as well as the leakage inductances L_{l1} and L_{l2}. We need to find expressions for the magnetizing inductances L_{m1} and L_{m2} in order to compute the mutual inductance M.

The magnetizing inductances L_{m1} and L_{m2} can be computed rigorously using network laws applied to the equivalent circuit of Fig. 4.4. However, a simplification to the equivalent circuit will make the computation much simpler without much difference to the end result. Figure 4.5 shows the simplification in which the core loss resistors R_{c1} and R_{c1} are directly connected across the transformer terminals rather than across the magnetizing inductances [13, 14, 27, 28]. This will result in higher core losses since the core losses are due to the magnetic flux in the core of the transformer, which is related to the induced emfs rather than the terminal voltages. The rise in core losses will however be negligible since the voltage drop across the winding resistances and the leakage inductances is the range of 1 to 3% of the rated winding voltages. The simplification on the other hand brings the leakage

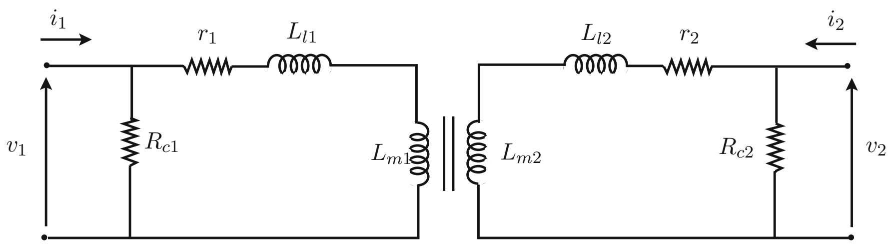


Fig. 4.5 Simplified transformer equivalent circuit

inductances in series with the magnetizing inductances. With this, we can express the self-inductances as the sum of the leakage inductances and the magnetizing inductances, resulting in the following expressions [13, 14, 27, 28]:

$$L_{m1} = L_1 - L_{l1} \tag{4.21}$$

$$L_{m2} = L_2 - L_{l2} \tag{4.22}$$

We now have an equivalent circuit for the transformer along with expressions to estimate all the parameters of the equivalent circuit. Most importantly, all estimates use only two specifications of the transformer—the maximum rated power in VA and the maximum rated voltages of the windings. As will be shown in the next section, the above equivalent circuit along with the parameter estimation provides us with a very flexible yet realistic model with which a transformer can be simulated in a number of different applications.

4.4 Simulating the Transformer

In the previous section, we described step by step how the equivalent circuit of the transformer can be developed using the basic laws of physics. As already stated before, under the hood, a transformer is just a set of magnetically coupled windings. Therefore, the approach to simulation will be very similar to the approach taken in the previous chapter. The main difference lies in being able to translate the manufacturer specifications of the transformer into parameters of the equivalent circuit so as to be able to solve the magnetic circuit [13, 14, 27, 28]. In this section, we will begin with the simulation of a simple two winding transformer with both windings having the same maximum voltage rating.

Figure 4.6 shows the circuit used for the simulation of the transformer. This simulation can be found in the folder `two_winding` within `chapter4_transformers` in the following link in the simulation repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics

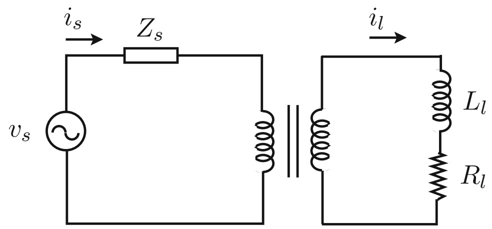


Fig. 4.6 Circuit for basic simulation of a transformer

Let us consider a transformer with the same sample specifications as chosen in the previous section—a maximum power rating of 10 kVA with both windings having a maximum voltage rating of 240 V RMS and let us choose the operating frequency to be 50 Hz. Let us energize winding 1 with a supply voltage v_s having an RMS value of 240 V and frequency of 50 Hz. The supply has a feeder Z_s with a parasitic impedance. Winding 2 of the transformer is supplying a load that is a resistor–inductor load with resistance R_l and inductance L_l, which could be typical of an industrial load such as a motor. The transformer internally has the equivalent circuit of Fig. 4.5.

The transformer will be modelled as two voltage sources in series with resistances as in the previous chapter while simulating magnetically coupled coils. Therefore, with the external circuit of Fig. 4.6 connected to the voltage source in series with resistance model of the transformer, all that needs to be done is to write the control code that will model the transformer and generate the voltages to be fed to the voltage sources. To begin with, let us compute the parameters of the transformer equivalent circuit from the transformer specifications that are usually provided by the manufacturer:

```
import math
dt = 1.0e-6
VArated = 10000.0
V1rated = 240.0
V2rated = 240.0
frated = 50.0
omega_rated = 2*math.pi*frated
```

Since we are simulating a 50 Hz transformer and a 50 Hz ac system, a control time step of 1 µs is sufficiently small to ensure accuracy. When we simulate high frequency transformers in the next chapter, this time step will need to be decreased.

The remaining parameters of the transformer equivalent circuit can be computed from these manufacturer specifications as described in the previous section. In the control code:

```
I1rated = VArated / V1rated
I2rated = VArated / V2rated
Z1rated = V1rated / I1rated
Z2rated = V2rated / I2rated
L1 = 50.0 * Z1rated / omega_rated
L2 = 50.0 * Z2rated / omega_rated
Ll1 = 0.02 * Z1rated / omega_rated
Ll2 = 0.02 * Z2rated / omega_rated
Lm1 = L1 - Ll1
Lm2 = L2 - Ll2
M12 = math.sqrt(Lm1 * Lm2)
r1 = 0.01 * Z1rated
r2 = 0.01 * Z2rated
Rc1 = V1rated * V1rated / (0.01 *VArated)
Rc2 = V2rated * V2rated / (0.01 *VArated)
```

The reader is encouraged to simulate a transformer with different equivalent circuit parameters such as increased or decreased leakage inductances or core loss resistors. If the reader has access to a laboratory and a physical transformer, performing open circuit and short-circuit tests can provide more realistic values for the equivalent circuit parameters.

The above computations result in the following parameters of the equivalent circuit:

```
L1 = L2 = 0.91673
Lm1 = Lm2 = 0.91636
M12 = 0.91636
L11 = L12 = 0.00036
r1 = r2 = 0.0576
Rc1 = Rc1 = 576.0
```

Let us make a few observations of these parameters. The self-inductances of the windings are close to 1H. In the chapter on simulating inductors, we had simulated sample inductors considering the number of turns of the coil, the area of cross-section of the core and the mean length of the core flux path. The reader can use the expression for the inductance to arrive at a probable number of turns N_1 and N_2 of the transformer windings. Moreover, the reactance of the windings will be such that the magnetizing current drawn by the windings will be limited to around 2% of the rated current when the rated voltages are applied at the windings. The resistors R_{c1} and R_{c2} that represent the core losses of the transformer can be included in the magnetic model. However, since they are mere resistors, these can be connected as Variable Resistor components in the transformer model by connecting resistors across the Voltmeters as shown in Fig. 4.7. The remaining parameters of the equivalent circuit—the self-inductances, mutual inductance and winding resistance will feature in the mathematical model $g(V)$.

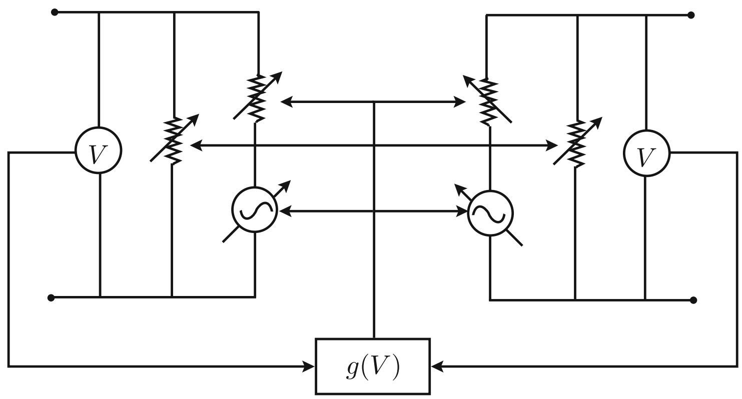


Fig. 4.7 Simulation model of transformer windings

The control code for the transformer model will need some interface parameters as was the case for inductors and coupled coils in the previous chapters. The most important interface variable is always the Time Event variable `t1`. This is quite often neglected but when designing digital controllers or solving models digitally, the time step at which the model needs to be solved must be accurately controlled. Next, we need to define the inputs. As with coupled coils, there are two inputs—the measurements of the Voltmeters shown in Fig. 4.7 that we can call `v1` and `v2`. When we simulate multi-winding transformers, we will have more than two windings and therefore, the measurement of applied voltage across each winding will be an input. Finally, we can define the outputs. One set of outputs will be for the Variable Resistor components. The Variable Resistor components in series with the voltage source can be called `res_output1` and `res_output2`. The Variable Resistor components in parallel with the Voltmeters represent the core losses, and these have already been computed as `Rc1` and `Rc2` and so these variables can be used to define the outputs. The last set of outputs will be the voltages of the Controlled Voltage sources and we can define them as `vout1` and `vout2`.

After defining the inputs and outputs of the controller, we can define Static Variables and Variable Storage elements. Variable Storage elements are useful to plot internal control variables but are not essential for a working controller if the controller is an isolated component. Variable Storage elements are also useful for connecting different controllers, which is not the case in this simulation. Therefore, to keep the simulation to the bare minimum, we will skip defining Variable Storage elements. Static Variables are those control variables that need to be stored between successive iterations of the simulation. As the magnetic model of the transformer is merely the same as that of coupled coils, the only essential Static Variable that needs to be defined is `winding_currents`. We will solve the magnetic model as matrix equations as described in the previous chapter. Therefore, this static variable will be initialized as a Python list as follows:

```
if t_clock <= dt:
    winding_currents = [0.0, 0.0]
```

Following these computations and initializations, the transformer model can be solved iteratively as follows:

```
if t_clock >= t1:
    # Model
    t1 += dt
```

Since the solution of the transformer mathematical model involves numerical integration, the above conditional block is critical to ensure that the numerical integration occurs once every 1 μs. The mathematical model of the transformer is identical to that of coupled coils and can be repeated as follows:

```
L = [[L1, M12], [M12, L2]]
R = [[r1, 0.0], [0.0, r2]]
V = [v1, v2]
for count1 in range(len(L)):
    if not L[count1][count1]:
        for count2 in range(count1+1, len(L)):
            if L[count2][count1]:
                L[count1], L[count2] = L[count2], L[count1]
                R[count1], R[count2] = R[count2], R[count1]
                V[count1], V[count2] = V[count2], V[count1]
                break
    if L[count1][count1]:
        for count2 in range(count1+1, len(L)):
            comm_factor = L[count2][count1]/L[count1][count1]
            for count3 in range(len(L[count1])):
                L[count2][count3] -= L[count1][count3]*comm_factor
                R[count2][count3] -= R[count1][count3]*comm_factor
            V[count2] -= V[count1]*comm_factor
dibydt = [0.0, 0.0]
for count1 in range(len(L)-1, -1, -1):
    if L[count1][count1]:
        k = [0.0, 0.0, 0.0, 0.0]
        for k_count in range(len(k)):
            k[k_count] = V[count1]
            for count2 in range(count1+1, len(L)):
                k[k_count] -= L[count1][count2]*dibydt[count2]
            for count2 in range(len(R)):
                if k_count==0:
                    if count2>count1:
                        k[k_count] -= R[count1][count2]*(winding_currents[count2] + \
                                dibydt[count2]*dt)
                    else:
                        k[k_count] -= R[count1][count2]*winding_currents[count2]
                if k_count==1:
                    if count2==count1:
                        k[k_count] -= R[count1][count2]*(winding_currents[count2] + \
                                k[0]*dt/2.0)
                    elif count2>count1:
                        k[k_count] -= R[count1][count2]*(winding_currents[count2] + \
                                dibydt[count2]*dt)
                    else:
                        k[k_count] -= R[count1][count2]*winding_currents[count2]
                if k_count==2:
                    if count2==count1:
                        k[k_count] -= R[count1][count2]*(winding_currents[count2] + \
                                k[1]*dt/2.0)
                    elif count2>count1:
                        k[k_count] -= R[count1][count2]*(winding_currents[count2] + \
                                dibydt[count2]*dt)
                    else:
                        k[k_count] -= R[count1][count2]*winding_currents[count2]
```

```
                if k_count==3:
                    if count2==count1:
                        k[k_count] -= R[count1][count2]*(winding_currents[count2] + \
                                k[2]*dt)
                    elif count2>count1:
                        k[k_count] -= R[count1][count2]*(winding_currents[count2] + \
                                dibydt[count2]*dt)
                    else:
                        k[k_count] -= R[count1][count2]*winding_currents[count2]
            k[k_count] = k[k_count]/L[count1][count1]
        dibydt[count1] = (k[0] + 2.0*k[1] + 2.0*k[2] + k[3])/6.0
        winding_currents[count1] += dibydt[count1]*dt
    else:
        winding_currents[count1] = V[count1]
        for count2 in range(count1+1, len(L)):
            winding_currents[count1] -= L[count1][count2]*dibydt[count2]
        for count2 in range(count1+1, len(R)):
            if not count2==count1:
                winding_currents[count1] -= R[count1][count2]*winding_currents[count2]
        winding_currents[count1] = winding_currents[count1] / R[count1][count1]

vout1 = v1 - winding_currents[0]*res_output1
vout2 = v2 - winding_currents[1]*res_output2
```

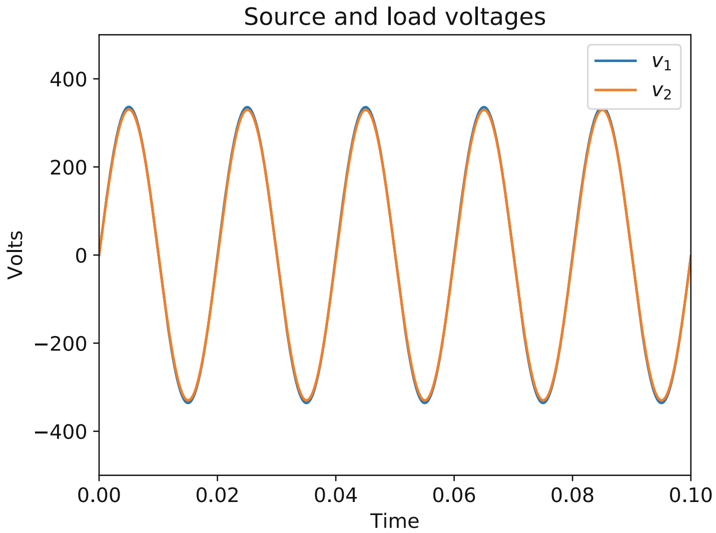


Fig. 4.8 Transformer winding voltages

The results of the simulation are shown in Figs. 4.8 and 4.9. Since, we are simulating a transformer with the same voltage ratings on both windings (also called a 1:1 transformer), the voltages at both windings in Fig. 4.8 appear to be overlapping. The voltage across the load will be lesser than the voltage at the source by the ohmic drop across the winding resistances and leakage inductances. Figure 4.9 shows the currents in the transformer windings that at first glance appear to have a few anomalies. The currents are not overlapping in the same manner as

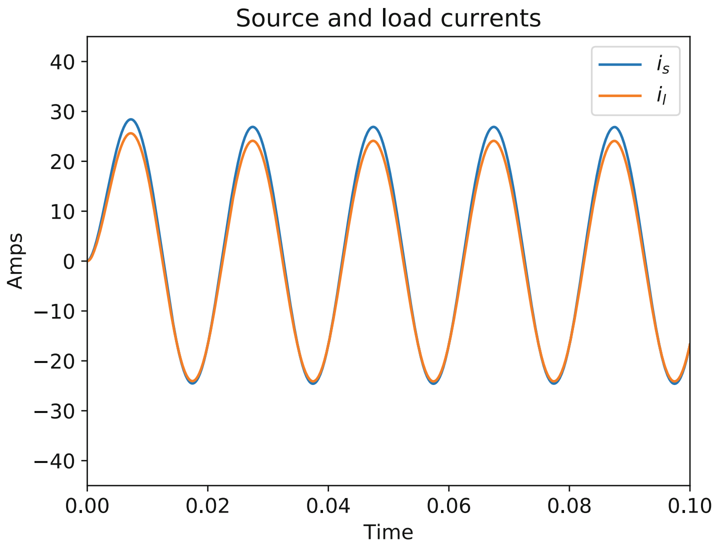


Fig. 4.9 Transformer winding currents

the winding voltages in Fig. 4.8. This is due to the fact that the primary winding (winding 1) that is energized by the voltage source draws a magnetizing current that is around 2% of the maximum rated current of the winding. Since only one winding is energized by the transformer, the magnetizing current will be drawn from winding 1, resulting in the current in winding 1 being noticeably larger than the current in winding 2. There is another anomaly that seems like an error but in reality can be explained—the current in winding 1 appears to have a dc offset.

As already stated before, as per Faraday's Law, the induced emfs e_1 and e_2 in the two windings will be directly proportional to the rate of change of core flux. Therefore, the core flux will be directly proportional to the integral of the induced emfs. Since, the voltage drops across the winding resistance and leakage inductance is a mere 2–3% of the rated winding voltage, the induced emfs can be approximated to the applied voltages. Therefore, the following equation can be written:

$$\phi \propto \int v_1 dt \tag{4.23}$$

The integral of a function will produce an integration offset, which will depend on the nature of the function being integrated and the initial conditions (at $t = 0$). The voltage source from Fig. 4.8 can be seen to be sine waveform. As we know, the integral of a sine waveform is the sum of a cosine waveform and an integration offset as follows:

$$\phi \propto \int V_m \sin \omega t dt = -\frac{V_m}{\omega} \cos \omega t + C \tag{4.24}$$

The integration offset can be determined by applying initial conditions. If we assume the transformer to be not energized at time $t = 0$, the flux ϕ in the core is zero at

$t = 0$. The flux in the core cannot abruptly increase with the application of a voltage and therefore, the flux will gradually increase. This implies

$$0 = -\frac{V_m}{\omega} + C \implies C = \frac{V_m}{\omega} \tag{4.25}$$

The above equation implies that the core flux has a dc offset. This might seem confusing as we had said that the flux cannot change abruptly upon application of a voltage and will gradually increase. But, the dc offset merely ensures that the waveform of the core flux starts as zero as core flux is expressed as the sum of a cosine waveform and a dc offset.

$$\phi \propto -\frac{V_m}{\omega} \cos \omega t + \frac{V_m}{\omega} \tag{4.26}$$

The cosine waveform alone would have resulted in the core flux abruptly increasing to the negative maximum value immediately upon the application of the voltage. The dc offset on the other hand ensures that despite being a cosine waveform, the flux must conform to the physical constraint of not changing abruptly.

Therefore, the core flux has a dc offset. The magnetizing current drawn by winding 1 is responsible for maintaining this core flux. The core flux is directly proportional to the current in winding 1 since the core flux is merely the ampere turns of winding 1 against the reluctance of the flux path. This in turn implies that the magnetizing current has a dc offset. It is this dc offset that is evident in Fig. 4.9. The next question that would follow would be—how is it acceptable that a transformer connected to an ac circuit is drawing a current that has a dc component? This dc offset in the magnetizing current is only temporary and will decay with the time constant of the circuit. The time constant of winding 1 is

$$\tau = \frac{L_1}{r_1} = \frac{0.91673}{0.0576} = 15.92\,\text{s} \tag{4.27}$$

The settling time of a circuit is 4 times the time constant of the circuit, which implies that the dc offset will disappear in 63.7 s.

To verify this effect, we can repeat the simulation while choosing the applied voltage at winding 1 to be a cosine waveform instead of a sine waveform as follows:

$$v_s = \sqrt{2} \times 240 \cos 100\pi t \tag{4.28}$$

In this case, the core flux being approximately the integral of the applied voltage can be expressed with a zero integration offset as merely (Fig. 4.10):

$$\phi \propto \frac{V_m}{\omega} \sin \omega t + 0 \tag{4.29}$$

Figure 4.11 shows the winding currents when the source voltage v_s still has a magnitude of 240 V RMS but is a cosine waveform of 50 Hz frequency as shown in

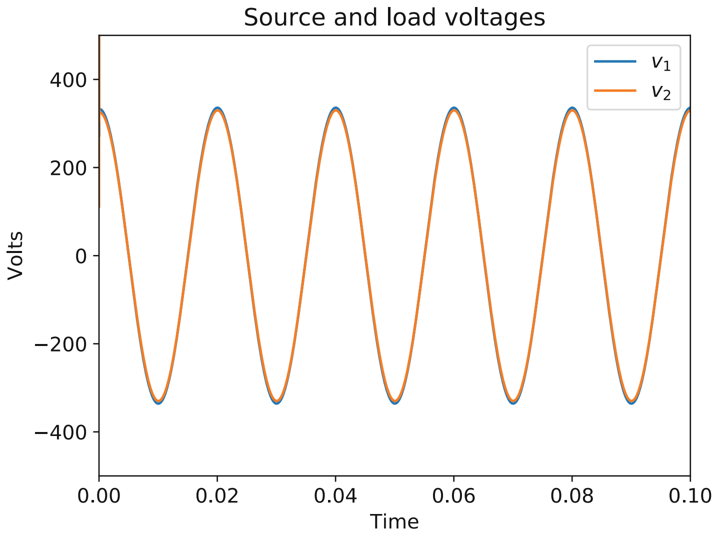


Fig. 4.10 Transformer winding voltages with a cosine source voltage

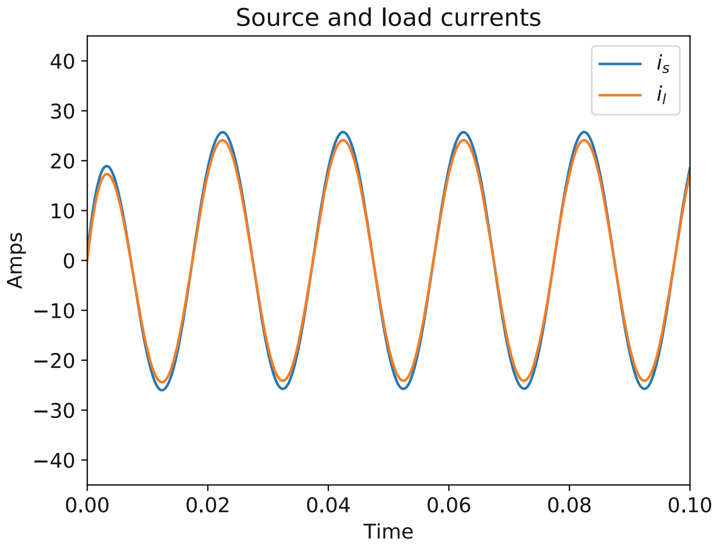


Fig. 4.11 Transformer winding currents with a cosine source voltage

Fig. 4.10. The currents do not have a dc offset, though there is a one cycle transient which is due to the time constant of the load connected at winding 2. As can be seen, current i_1 in winding 1 is still noticeably larger than current i_2 in winding 2, due to the magnetizing current being drawn by winding 1. The reader is encouraged to try out different initial phase angles to the source voltage waveform to examine the dc offset in the current in winding 1.

There are several variations to this basic simulation that the reader can attempt. As an example, the reader can energize winding 2 instead of winding 1 and observe that the results change only to the extent that now the magnetizing current is drawn by winding 2. On the other hand, there are several nuances to the operation of a transformer that most often confuses a beginner. In the next few sections, we will gradually introduce several more simulations that will dive deeper into the operation of the transformer. The reader is advised to review the basic discussion section of the previous chapter on magnetically coupled coils, as the next few sections will use those concepts repeatedly.

4.5 Understanding Transformer Winding Currents

In the previous section, we had started off with a basic simulation of a two winding transformer. We had energized one of the windings and had connected a load to the other winding. The simulation results showed the load being supplied by a voltage approximately equal to that applied to the other winding and the load current being supplied by the source in addition to a magnetizing current. This simple simulation got us started to understanding transformer operation. However, to fully understand how the current in one winding affects the current in the other, it would be interesting to dig a little deeper into the working of the transformer using simulations [13, 14, 27, 28]. We can continue with the same simulation of a two winding transformer as in the previous section, and just alter some details as necessary.

To begin with, let us increase the load resistor R_l in Fig. 4.6 to a very large value of around $10\,\text{k}\Omega$ such that the transformer is now operating at a very light load and therefore approximately no load. The currents in the transformer windings are shown in the simulation results of Fig. 4.12. Even though the load has such a large resistance that the current drawn by the load should be negligible, the currents in both windings appear to have non-negligible values. At no load, the transformer will draw currents to supply the core losses in the transformer that have been represented by the two resistors R_{c1} and R_{c2} [13, 14, 27, 28]. The current i_l in winding 2 shown in Fig. 4.12 is the current drawn by the load R_l and L_l. The current i_s on the other hand is the current drawn by the loss resistors R_{c1}, R_{c2}, load current i_l and also the magnetizing current of the transformer [13, 14, 27, 28].

It might be a bit confusing to understand the different currents. Let us compare the simulation results of Fig. 4.12 with the simulation results of Fig. 4.13, which show the winding currents i_1 and i_2 (note the direction of i_2) as depicted in the equivalent circuit of Fig. 4.5. The winding currents are clearly different from the source and load currents. The load current i_l in Fig. 4.12 is the current drawn by the load R_l-L_l alone and is negligible. The current i_2 is the current drawn by the loss resistor R_{c2} and the load current. This explains how the current i_2 in Fig. 4.13 is larger than the current i_l in Fig. 4.12. The current i_1 on the other hand is the sum of the currents drawn by the loss resistor R_{c2}, the load current i_l and the magnetizing

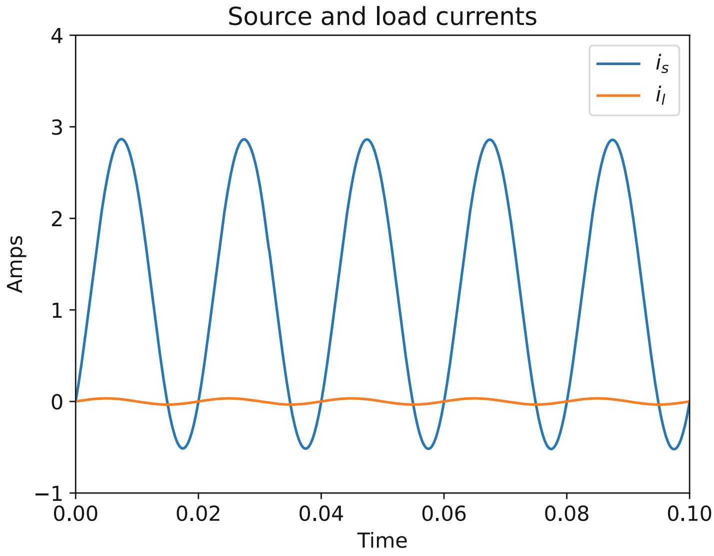


Fig. 4.12 Transformer source and load currents at no load

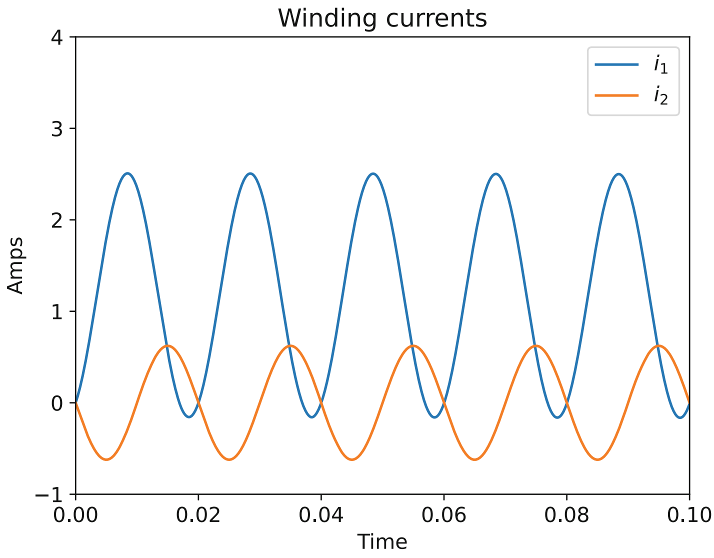


Fig. 4.13 Transformer winding currents at no load

current. All the currents can be found on the different branches of the simplified transformer equivalent circuit of Fig. 4.5 except for the magnetizing current.

Let us now talk about this mysterious magnetizing current that appears in i_1 and i_s. The magnetizing current is the current drawn by the transformer to maintain the magnetic flux in the core. Since the magnetic flux in the transformer core is fundamental to its operation, the magnetizing current is also fundamental to the operation of the transformer [13, 14, 27, 28]. An analogy to the magnetizing current can be made to the altitude an airplane needs to reach after it takes off. A normal commercial airplane will probably rise to an altitude of 20,000 feet from sea level and maintain that height until it reaches its destination. Whether the airplane is booked to capacity with passengers or is almost empty, the airplane still needs to rise to this height. This is the intrinsic cost of any commercial aerospace flight. Similarly, when a transformer is energized and is able to serve a load on one or more of its windings, it needs to draw a magnetizing current to establish the magnetic flux in its core that makes it possible to serve loads on multiple windings. Let us now examine in depth, how this magnetizing current comes into being.

Even when the transformer is operating at no load, it will continue to draw a magnetizing current. This magnetizing current is essential to set up the core flux of the transformer. When any one winding of the transformer is energized, a core flux will be established. This can be deduced by the combination of Kirchhoff's Voltage Law and Faraday's Law to winding 1:

$$v_1 = N_1 \frac{d\phi}{dt} + i_1 r_1 \tag{4.30}$$

where the flux linkage ψ_1 of winding 1 is $N_1\phi$ with N_1 being the number of turns of winding 1. The core flux is therefore [13, 14, 27, 28]:

$$\phi = \frac{1}{N_1} \int (v_1 - i_1 r_1) dt \tag{4.31}$$

The core flux links with winding 2 as well inducing an emf in it and therefore producing a voltage across the terminals of winding 2. In this no-load case, the current drawn by the load connected to winding 2 is approximately zero. However, in order that the flux ϕ flows in the transformer core, a current must be drawn by one or both of the windings. This follows from our discussion of magnetic circuits in the previous chapters, where one coil or multiple coils wound on a core can be solved as a magnetic circuit with the ampere turns of the coils being the sources against the reluctance offered by the core. In this case, if we assume a very simple core (such as a rectangular core) [13, 14, 27, 28]:

$$\phi = \frac{N_1 i_1 + N_2 i_2}{\rho} \tag{4.32}$$

where ρ is the reluctance of the core:

$$\rho = \frac{l}{\mu_0 \mu_r A_c} \tag{4.33}$$

The reason why the net ampere turns in the magnetic circuit have been chosen as the sum of the ampere turns of the two windings is due to the sense in which the windings have been wound and the directions we have chosen for the currents i_1 and i_2 in Fig. 4.5. For this, the reader is advised to review the previous chapter where a detailed explanation is provided of the nature of induced emfs due to the sense of windings and also of the dot polarity convention. Since, for the transformer of Fig. 4.6, the dot polarities have been omitted, this implies that the dots can be placed at the upper terminals of the two windings.

In this particular simulation case, since a passive load is connected to winding 2, the current i_2 will be such that it decreases the net ampere turns, and therefore demagnetizes the core. With winding 1 being the only winding being energized by a source, the current i_1 flowing in winding 1 is the current that sets up the magnetic flux in the core. If we neglect the currents drawn by the loss resistors R_{c1} and R_{c2}, we could express the net ampere turns as

$$\phi \approx \frac{N_1 i_s - N_2 i_l}{\rho} \tag{4.34}$$

Given the direction of the supply/source current i_s and the load current i_l in Fig. 4.6, it is fairly obvious that the current drawn by the load will demagnetize the core. However, since in this no-load condition, the current i_l is negligible, we can further make the approximation:

$$\phi \approx \frac{N_1 i_s}{\rho} \tag{4.35}$$

What can we conclude from the above discussion? When the transformer is operating at close to no load, the transformer draws a current from the supply due to the core losses, which have been represented by resistors R_{c1} and R_{c2}. Besides the core loss component, the transformer will also draw a current called the magnetizing current that is necessary to maintain the flux in the core. In the no-load case, these are the only currents that are flowing in the transformer with respect to the simplified equivalent circuit of Fig. 4.5 [13, 14, 27, 28].

Let us now bring back the load connected to winding 2 by restoring $R_l = 10\,\Omega$. The simulation results have been presented in the previous section, so let us try to analyse the condition in this section. In this load condition, the three currents from the no-load condition will continue to flow—the core loss currents and the magnetizing current [13, 14, 27, 28]. In addition, the current i_l drawn by the load will no longer be negligible but will be quite significant. So, therefore, if we return back to the approximate expression for flux of (4.34), it would appear that the load current would significantly demagnetize the core leaving the core flux at a very low value.

This is quite often a source of confusion for students and it was for me as well when I was an undergraduate. However, we must remember that when winding 1 of the transformer is energized with a supply, the flux in the core is determined by (4.31) [13, 14, 27, 28]. Though (4.31) contains a voltage drop term dependent on the current i_1, with the winding resistance r_1 being negligible, the flux will therefore not change significantly even if the current i_1 changes from the minimal value equal to the magnetizing current to the maximum value equal to the rated current of winding 1. Therefore, the core flux is determined by the magnitude of the supply voltage with which the transformer is energized [13, 14, 27, 28].

Equations (4.31) and (4.34) appear to be contradictory. Equation (4.31) describes a flux that will change minimally with changes in the winding current. Equation (4.34) on the other hand states that the flux will decrease as the load current i_l increases. Both equations are correct and we merely need to understand them as cause and effect [13, 14, 27, 28]. Equation (4.31) is the primary equation as the supply voltage is the independent input that energizes the transformer, and therefore, with this expression, we can arrive at the core flux. Equation (4.34) merely states that the difference of the ampere turns of the windings will produce the core flux.

If the flux ϕ in the core will be unaffected but the load current i_l has a demagnetizing effect, this points to only one possibility. The demagnetizing effect of the load current i_l is cancelled out by another current that has a magnetizing effect [13, 14, 27, 28]. In the no-load case, only the magnetizing current was flowing in winding 1 of the transformer. However, to nullify the demagnetizing effect of i_l, a current must flow in winding 1 such that ϕ is unaltered. Let us write this expression as

$$\phi \approx \frac{N_1 i_m + N_1 i_l^{'} - N_2 i_l}{\rho} \tag{4.36}$$

In the above expression for flux, the ampere turns $N_1 i_m$ produced the core flux in the no-load case. Therefore, it follows that the second and third terms must be equal and cancel each other out resulting in [13, 14, 27, 28]

$$N_1 i_l^{'} = N_2 i_l \tag{4.37}$$

The current $i_l^{'}$ is the load current flowing in winding 2 referred to winding 1. Any current flowing in any winding can be referred to another winding by using the constraint that the referred current must produce the same ampere turns as the original current [13, 14, 27, 28].

As a result, for the load current i_l flowing in winding 2, the current supplied by the source in winding 1 will be

$$i_s = i_m + i_l^{'} \tag{4.38}$$

This follows from simple energy conservation as the power consumed by the load must be provided by the source in addition to the power lost in the transformer. The transformer is merely a machine that transfers electrical energy from one winding to another through a magnetic field thereby allowing the windings to be electrically isolated. The transformer does not produce any energy, and therefore the energy supplied at the load cannot be greater than the energy supplied at the source.

The referred current $i_l^{'}$ leads us to another term that is extremely important for transformers [13, 14, 27, 28]:

$$i_l^{'} = \frac{N_2}{N_1} i_l \tag{4.39}$$

The ratio $\frac{N_2}{N_1}$ is called the turns ratio of the transformer and is the ratio of the number of turns of one winding with respect to the number of turns of the other [13, 14, 27, 28]. This turns ratio is also used to express the terminal voltage of one winding with respect to the other.

We have already written the expression for the terminal voltage using Kirchhoff's Voltage Law and Faraday's Law:

$$v_1 = N_1 \frac{d\phi}{dt} - i_1 r_1 \approx N_1 \frac{d\phi}{dt} \tag{4.40}$$

$$v_2 = N_2 \frac{d\phi}{dt} - i_2 r_2 \approx N_2 \frac{d\phi}{dt} \tag{4.41}$$

The above expressions can be rearranged and equated as

$$\frac{v_1}{N_1} = \frac{v_2}{N_2} \tag{4.42}$$

To produce an expression for the voltages with respect to the turns ratio as follows:

$$v_1 = \frac{N_1}{N_2} v_2 \tag{4.43}$$

The above expression can be used to determine the turns ratio of the transformer as the maximum rated voltages of the windings are specified by the manufacturer [13, 14, 27, 28]. Therefore,

$$\frac{N_1}{N_2} = \frac{v_{1rated}}{v_{2rated}} \tag{4.44}$$

With the knowledge of the turns ratio from the manufacturer specifications, we can use the turns ratio to refer voltages and currents from one winding to another. This provides a very convenient tool for mathematical analysis.

In the circuit topology of Fig. 4.6 chosen for the above simulations, only one winding (winding 1) has been energized by an ac voltage source while the other winding (winding 2) had a passive load connected to its terminals. In such a case, the flow of power is fairly obvious—from the voltage source in winding 1 to the load in winding 2. Moreover, the magnetizing current is drawn by winding 1 from

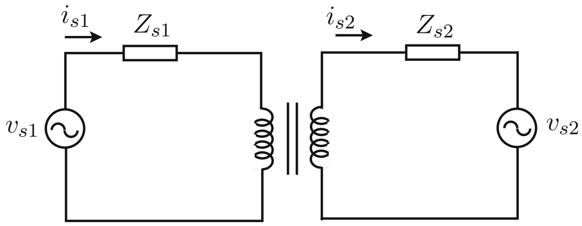


Fig. 4.14 Circuit topology with more than one voltage source

the voltage source to maintain the core flux as it is the only energy source connected to the transformer. In many cases, more than one winding of the transformer will be energized as the transformer is quite often used for transferring power from one circuit to another. Figure 4.14 shows the circuit of the same transformer as simulated so far but with a voltage source connected to each winding.

For the simulation, let us assume the feeder impedances Z_{s1} and Z_{s2} to be equal and with a nominal value of

$$r_{s1} = r_{s2} = 0.02\Omega \quad , \quad L_{s1} = L_{s2} = 0.0001\text{H}$$

While choosing the voltage sources for the simulation, let us choose them to have the same phase angle but introduce a very small difference in their RMS magnitudes of 2V as follows:

$$v_{s1} = 240\sqrt{2}\cos(100\pi t)$$

$$v_{s2} = 242\sqrt{2}\cos(100\pi t)$$

Figure 4.15 shows the currents i_{s1} and i_{s2} flowing between the two voltage sources. Since we have chosen the waveforms of the voltage sources to be cosines, we do not see a dc offset and this makes interpreting the results a little easier. In Fig. 4.15, the source voltage v_{s1} has been superimposed while being scaled down by a factor of 15 to better understand the nature of currents i_{s1} and i_{s2}. By comparing with the directions of currents i_{s1} and i_{s2} in Fig. 4.14, and knowing that the magnitude of v_{s2} is greater than v_{s1}, it is clear that power is flowing from v_{s2} to v_{s1}. This is due to the fact that the currents i_{s1} and i_{s2} are leading the voltage v_{s1}, which implies that the currents are flowing in the reverse direction. Furthermore, it can be observed that the magnitude of i_{s2} is greater than i_{s1} as now winding 2 is the winding that is being primarily energized and therefore winding 2 draws the magnetizing current from v_{s2}.

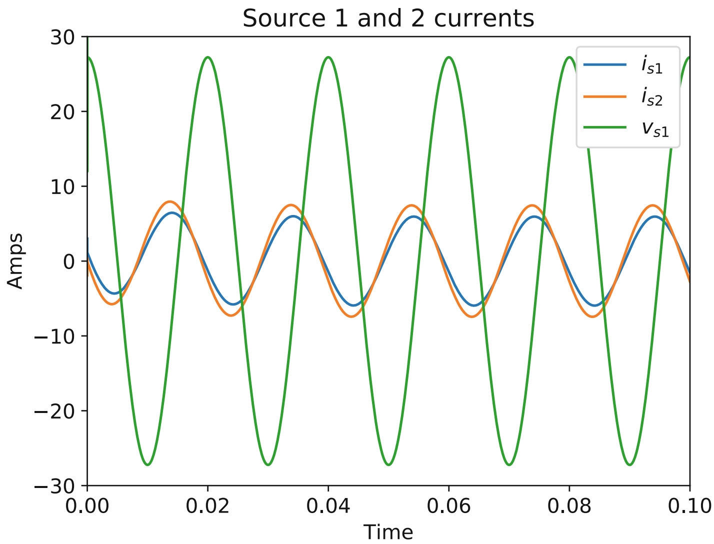


Fig. 4.15 Source and load currents with dual voltage sources

The above simulation with a voltage source connected to each winding can be interpreted analytically in exactly the same way we did for the no load and the load simulations. We can now write expressions for both windings using Kirchhoff's Voltage Law and Faraday's Law as follows:

$$v_1 = N_1 \frac{d\phi}{dt} + i_1 r_1 \tag{4.45}$$

$$v_2 = N_2 \frac{d\phi}{dt} + i_2 r_2 \tag{4.46}$$

From the above expressions, it is very clear that the transformer core flux will be determined by the voltage source that has the larger magnitude. Not only will the magnitude of the core flux be determined by the voltage source that has the larger magnitude, but also the direction of the core flux as was described in the previous chapter [13, 14, 27, 28].

When a transformer is used to interface two different energized circuits in such a manner, it is very similar to connecting voltage sources in series with the transformer winding resistances and leakage inductances as interfacing impedances. In the above case, since both windings had the same rated maximum voltages and therefore equal number of turns, it was fairly simple to analyse the results and label one of the voltage sources to be the "stronger" voltage source [13, 14, 27, 28]. However, the most common use of a transformer is to be the interface between two systems at a very different voltage as is required in the power system. This will be discussed in the next section where we will consider transformers with a turns ratio other than 1.

In this section, we examined in detail how currents flow in the windings of the transformer. In order to analyse simulation results, we used only the basic laws of physics. The analysis helps to understand how the transformer operates as a magnetic machine in comparison to merely solving the equivalent circuit. The analysis has been simple due to the presence of only two windings in the transformer and both windings having identical maximum voltage ratings. In the next section, we will simulate and analyse step-up and step-down transformers.

4.6 Transformer Turns Ratio

In the previous sections, we had simulated a two winding transformer where both windings had identical maximum voltage ratings. In this section, let us get started with the practical transformer where the windings can have very different maximum voltage ratings. As an example, let us consider transformers to have voltage ratings of 240 V/11 kV or 240 V/24 V—this first type usually is a distribution transformer that steps down the voltage from a distribution feeder and supplies an apartment building or house while the second type is usually a control transformer, which one can use to generate a power supply of 5 V, 9 V or 12 V [13, 14]. The analysis in this section will be very similar to that carried out in the previous section and we will be using the very same basic laws of physics.

To simulate the 240 V/11 kV transformer, we can use the same circuit as Fig. 4.6 with winding 1 energized by a 240V RMS, 50Hz voltage source while winding 2 is supplying a load. This simulation can be found in the folder `step_up_transformer` within `chapter4_transformers` in the following link in the simulation repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics
In most cases, such step-up transformers are usually three-phase transformers, which will be covered in the next chapter. However, for now, to simulate a step-up transformer, we could choose any voltage ratings for winding 1 and winding 2. Since this is a step-up transformer that could potentially connect to a high voltage distribution feeder, let us choose a higher maximum power rating of 100 kVA. Since the load comprising of R_l and L_l will be connected on winding 2 having a maximum voltage rating of 11 kV, let us choose $R_l = 1000\Omega$ and $L_l = 30$H. As for the feeder Z_s connecting the source voltage to the winding 1 terminals, we can continue with the parameters in the previous sections of $L_s = 0.0001$H and $r_s = 0.02\Omega$.

To change the transformer magnetic model of the previous section, the only change that needs to be made is with respect to the parameters defined at the head of the file:

```
import math
dt = 1.0e-6
VArated = 100000.0
V1rated = 240.0
```

```
V2rated = 11000.0
frated = 50.0
omega_rated = 2*math.pi*frated
```

Once these parameters have been changed, the remaining computations of the transformer equivalent circuit parameters will use these parameters, and therefore no changes need to be made in those computations unless we wish to change our estimates for any of the parameters.

With the above transformer specifications, the parameters of the transformer equivalent circuit will be

```
L1 = 0.09167, L2 = 192.577
Lm1 = 0.09163, Lm2 = 192.5
M12 = 4.2
Ll1 = 36.67e-6, Ll2 = 0.077
r1 = 0.00576, r2 = 12.1
Rc1 = 57.6, Rc1 = 1210000
```

A quick look at the above parameters gives us an idea of how different the two systems are that are being interfaced by the transformer. As an example, the winding resistance of the 240 V winding 1 is a mere 576 mΩ while the winding resistance of the 11 kV winding 2 is 12.1 Ω. This is equivalent to comparing the tail of a mouse and an elephant!

Besides the parameters of the transformer equivalent circuit being drastically different for the two windings, we must also remember that the transformer is merely an energy conversion device and does not generate any energy. Therefore, the power available at winding 1 will differ from the power available at winding 2 by the losses of the transformer [13, 14, 27, 28]. The rated currents of the two windings are

```
I1rated = 416.67 , I2rated = 9.09
```

The maximum rated current of winding 1 (the low voltage winding) is much larger than the maximum current rating of winding 2 (the high voltage winding).

This might at first appear confusing as one would expect the high voltage system to have a larger current rating. However, once again, it is important to remember that the transformer is merely the interface between two systems at a different voltage [13, 14, 27, 28]. If this transformer were supplying power to apartment buildings from a 11 kV distribution feeder, there would be such a transformer for every building in a street or an area. Therefore, each 240 V winding of the transformer would be supplying the building itself while the 11 kV winding is merely one branch of the distribution feeder. The distribution feeder may be supplying numerous apartment buildings due to which the total current in the feeder could also be in the range of hundreds of amperes. The total power capacity of the distribution feeder could be in the range of several hundreds of kVAs or even a few MVAs (mega Volt-Amperes). The transformer is merely supplying one branch from this distribution feeder to an apartment building that we have chosen to have a maximum power rating of 100 kVA.

The advantage of building the transformer simulation model in the manner described in the previous section is that it results in a mathematical model that is easily modifiable as the system changes. Once we change the maximum power rating and maximum voltage ratings of the windings, the rest of the simulation model—the matrix equation, the triangularization and the numerical integration—can remain the same. In the next section, we will examine how we can simulate a transformer with more than two windings by a simple modification. The resultant transformer model is extremely flexible and can be used to simulate some fairly complex transformers as will be demonstrated in the next chapter.

Figures 4.16 and 4.17 show the simulation results. The results show how drastically different the quantities on the two different windings are. To be able to better interpret the results, let us scale one of the quantities in each plot so that the waveforms can be compared. Figure 4.18 shows the voltages with the load voltage scaled down by a factor of 10 and Fig. 4.19 show the currents with the source current scaled down by a factor of 10. Due to the difference in the windings of the transformer, it is no longer a simple matter to determine which winding is supplying the magnetizing current. However, we could undertake a similar exercise as in the previous section to further analyse the results.

To begin with, once the maximum rated voltage of the windings has been specified, we can establish the turns ratio of the transformer:

$$\frac{N_2}{N_1} = \frac{v_{2rated}}{v_{1rated}} = \frac{11000}{240} = 45.83 \tag{4.47}$$

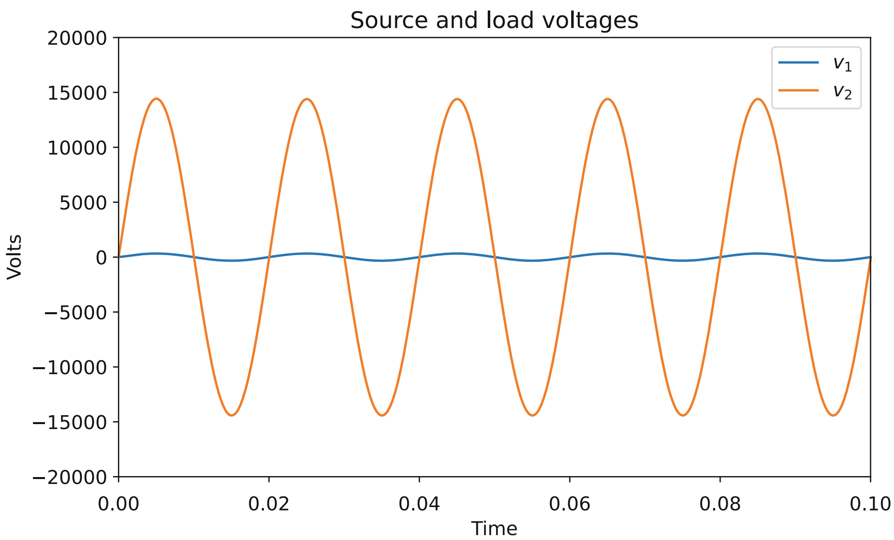


Fig. 4.16 Step up transformer source and load voltages

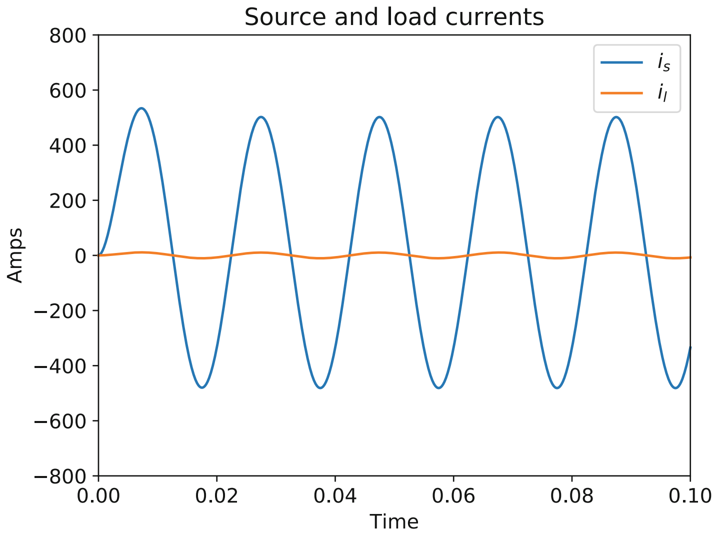


Fig. 4.17 Step up transformer source and load currents

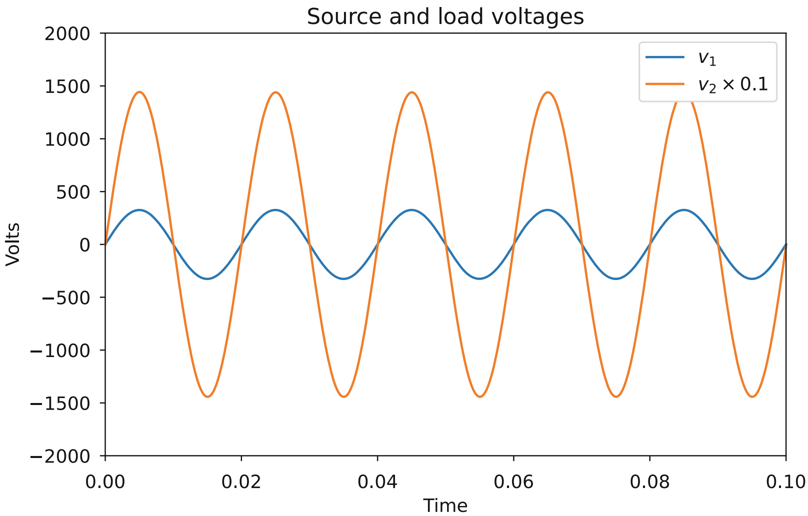


Fig. 4.18 Step up transformer source and load scaled voltages

The turns ratio of a transformer can be defined either as $\frac{N_2}{N_1}$ or $\frac{N_1}{N_2}$ [13, 14, 27, 28]. Either is acceptable and in this case, we have chosen the former as it results in a more convenient number. All that we must remember is that the ratio of the winding turns is equal to the ratio of the voltage ratings of the windings.

As already described in the previous section, the significance of the turns ratio of the transformer lies in the ratio of the induced emfs of the windings, which are

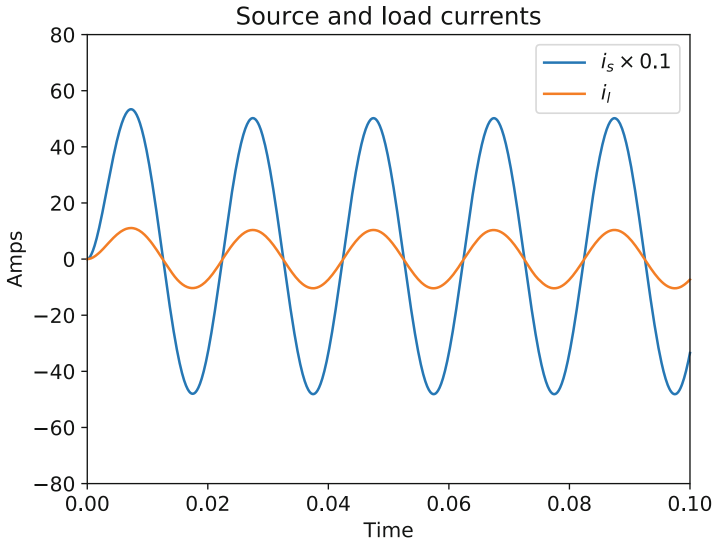


Fig. 4.19 Step up transformer source and load scaled currents

equal to the rate of change of flux linkages of the windings. Assuming a simple core construction where the flux linking the windings is the same, the flux linkage of a winding is equal to the product of the flux and the number of turns of the winding:

$$\psi_1 = N_1 \phi \tag{4.48}$$

$$\psi_2 = N_2 \phi \tag{4.49}$$

The induced emfs of the windings can be expressed as

$$e_1 = \frac{d\psi_1}{dt} = N_1 \frac{d\phi}{dt} \tag{4.50}$$

$$e_2 = \frac{d\psi_2}{dt} = N_2 \frac{d\phi}{dt} \tag{4.51}$$

Which implies that for a given core flux ϕ, the induced emfs will also follow the transformer turns ratio. Since the winding voltages differ from the induced emfs by the voltage drop across the winding resistance, the winding voltages are also constrained by the turns ratio:

$$v_1 = e_1 + i_1 r_1 \approx N_1 \frac{d\phi}{dt} \tag{4.52}$$

$$v_2 = e_2 + i_2 r_2 \approx N_2 \frac{d\phi}{dt} \tag{4.53}$$

As in the previous section, these few equations alone can be used to understand the transformer winding voltages. In this specific simulation, winding 1 is energized, which leads to the core flux being determined as in the previous section:

$$\phi = \frac{1}{N_1} \int (v_1 - i_1 r_1)\, dt \tag{4.54}$$

With this core flux linking winding 2, the emf induced in the winding is

$$e_2 = N_2 \frac{d\phi}{dt} = \frac{N_2}{N_1} e_1 = \frac{N_2}{N_1} (v_1 - i_1 r_1) \tag{4.55}$$

The induced emf in winding 2 produces a terminal voltage:

$$v_2 \approx e_2 - i_l r_2 = \frac{N_2}{N_1} (v_1 - i_1 r_1) - i_l r_2 \tag{4.56}$$

It is important to note that we have used the load current i_l in the expression for v_2 rather than the internal winding 2 current i_2 as the direction of current i_2 is the opposite of i_l (Fig. 4.5). With these equations, it is fairly clear how a voltage applied to the terminals of winding 1 eventually produces a voltage across winding 2, which has been multiplied by the turns ratio $\frac{N_2}{N_1}$. This is exactly the behaviour we observe in Figs. 4.16 and 4.18 where the load voltage is a multiple of the source voltage and both winding voltages have approximately the same phase angle. In this simulation, only a passive load was connected to winding 2. However, we could think of several variations to the simulation. First, we could connect a source voltage to winding 2 and a passive load to winding 1. Second, we could connect voltage sources to both winding 1 and winding 2.

The first case is fairly simple. All we need to keep in mind is that the voltage source connected to winding 2 must remain within the maximum rating of 11 kV. The load connected to winding 1 must be such that the current drawn by the load must be within the rated maximum current for the transformed voltage $\frac{N_1}{N_2} v_2$ that appears across the terminals of winding 1. We can progress through the equations in reverse—starting from the terminal voltage expression for winding 2, the core flux expression, the expression for induced emf in winding 1 and finally the expression for terminal voltage in winding 1 [13, 14, 27, 28]. The reader is encouraged to try out this simulation.

The second case is a bit more interesting. If there were voltage sources connected to both windings, we could think of it as a case when a generator is being interfaced to the high voltage transmission line that is already energized by other generators. At the terminals of the two windings, we can write the following expressions:

$$v_1 = N_1 \frac{d\phi}{dt} + i_1 r_1 \tag{4.57}$$

$$v_2 = N_2 \frac{d\phi}{dt} + i_2 r_2 \tag{4.58}$$

We had examined a similar case in the previous section except that the transformer had the same number of turns in each winding. In this section, the transformer windings have different number of turns. Therefore, to determine which winding behaves as a source and which winding behaves as a load, we need to take into account not only the voltage at the winding but also the number of turns of the winding [13, 14, 27, 28]. As an example, let us suppose that $\frac{v_1}{N_1}$ was larger than $\frac{v_2}{N_2}$. This would result in the flux in the core flowing in such a manner that winding 1 was magnetizing the core and winding 2 would be demagnetizing the core as winding 1 becomes the "stronger" winding. As a result, in the above equations, the current i_1 would be positive while the current i_2 will be negative. Power flows from winding 1 to winding 2. If a generator interfaced to the power system would need to inject power into the transmission system, the voltage applied to the low voltage winding would merely have to be such that the ratio of voltage and number of turns of the winding makes the winding appear as a stronger winding.

A note of caution must be exercised at this point. Solving the above voltage equations directly is being done in the simulation as we express the voltage equations as matrix equations and triangularize them. However, due to the derivative of the flux, we cannot equate the above equations to two voltage sources connected together in series with an impedance. The analogy to voltage sources interconnected by an impedance is relevant when we consider a magnetic circuit where the ampere turns of coils/windings and the reluctance of the core on which they are wound can be solved to determine the core flux [13, 14, 27, 28]. However, a direct analogy to the voltages applied to the coils/windings will not be correct. We can, however, use the equations to estimate the nature of power flow and understand the basic physical operation of the transformer.

Now that we have had a discussion on the winding voltages of the transformer, we can now direct our attention to the winding currents of the transformer. As with the case of the winding voltages, the winding currents can also be understood in the same manner as we did in the previous section. To begin with, let us start with the magnetizing current. Every transformer will need to draw a magnetizing current in order to maintain the flux in the core. In order to simulate the condition where the winding currents are approximately equal to the magnetizing current, we can use the same strategy as in the previous section—make the load resistance very large so as to make the load current negligible. If winding 1 is energized and the load is connected to winding 2, we can increase the load resistance from $R_l = 1000\Omega$ to $R_l = 1,000,000\,\Omega$.

Figure 4.20 shows the no-load source and load currents of the transformer. The load current i_l is negligible as we have increased the load resistance to $1\,\text{M}\Omega$. The source current i_s supplies the loss in resistors R_{c1} and R_{c2} and the magnetizing current i_m of the transformer. The no-load current has a dc offset as we have chosen the source voltage to have a sine waveform. The source current i_s has a peak of around 30A, which at first might seem unusual. Taking into account the dc offset in the source current, the ac peak of the source current is around 15 A, which corresponds to an RMS value of around 10.6 A. However, the rated current in the primary winding has an RMS value of 416.67 A, which implies that the no-load

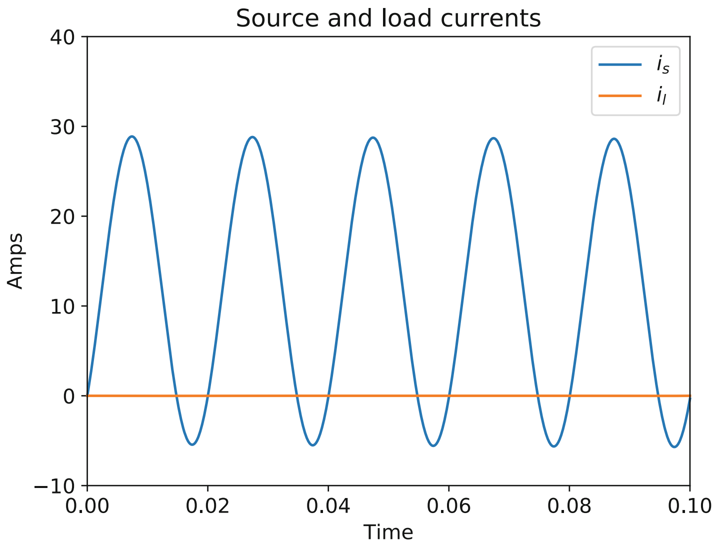


Fig. 4.20 No-load current of the transformer

current is merely 2.5% of the rated current. Since the self-inductance of transformer winding is so designed that the current drawn will be 2% of the rated current, these numbers are only expected. The reader is encouraged to repeat the simulation while energizing winding 2 with a 11 kV RMS voltage source and connecting a large resistance in winding 1. The no-load current should also be around 2–2.5% of the rated current (9.09 A) of winding 2.

If we restore the load resistor connected to winding 2 back to its normal value of 1000 Ω, the currents will be as shown in Figs. 4.17 and 4.19. The current i_l drawn by the load will result in a winding current i_2. From Figs. 4.6 and 4.5, this winding current will have a demagnetizing effect on the core flux. This has been discussed in detail in the previous chapter while describing dot polarities and the magnetic coupling. However, as in the previous section, the core flux is primarily determined by the voltage v_1 that energizes winding 1 of the transformer:

$$\phi = \frac{1}{N_1} \int (v_1 - i_1 r_1)\, dt \approx \frac{1}{N_1} \int v_1 dt \tag{4.59}$$

The above approximation implies that the flux will not change significantly with the winding current i_1 but will remain roughly unaffected by current flowing in the winding. Just like in the previous section, the core flux can be expressed as

$$\phi = \frac{N_1 i_s - N_2 i_l}{R} \tag{4.60}$$

where R is the reluctance of the core. In the previous no-load case, with the source current being approximately equal to the magnetizing current:

$$\phi \approx \frac{N_1 i_m}{R} \tag{4.61}$$

If the load current despite having a demagnetizing effect on the core does not change the core flux, this would imply that the demagnetizing effect of the load current is cancelled by another component in the source current:

$$\phi = \frac{N_1 i_m + N_1 i_l^{'} - N_2 i_l}{R} \tag{4.62}$$

In the above expression, the current $i_l^{'}$ is the load current i_l referred to winding 1. As discussed in the previous section, a referred current is merely the current in a winding being referred to another winding such that the ampere turns of the referred current is the same as the original current [13, 14, 27, 28]. Therefore,

$$N_1 i_l^{'} = N_2 i_l \tag{4.63}$$

In the previous section, we had arrived at this result. However, since both windings had identical number of turns, in the previous section, the referred load current $i_l^{'}$ was the same as i_l. In this simulation case, with a turns ratio of $\frac{N_2}{N_1} = 45.83$, the referred load current $i_l^{'}$ will be 45.83 times the load current i_l. As a result, even though the load current is merely around 9A due to the values of $R_l = 1000\Omega$ and $L_l = 3$H, the referred load current is approximately 450 A. From Figs. 4.17 and 4.19, the source current can now be explained.

To complete the discussion on transformers with a turns ratio, let us also simulate a step-down transformer where the maximum rated voltage of winding 2 is lower than that of winding 1. Though such transformers can be used for a wide number of applications, let us consider a step-down transformer used in control applications quite often to produce power supplies for control circuits used in power converters [10, 26]. Let us suppose that winding 1 of the transformer is energized by an ac supply of 240 V, 50 Hz. As an example, let us suppose that we wish to use this transformer to produce a 5 V power supply. In such a case, winding 2 of the transformer can have a maximum rating of 12 V. To this winding, we could connect a simple diode rectifier that converts the ac voltage at the winding terminals to an uncontrolled dc to be further regulated by any dc–dc converter such as a buck converter, flyback converter or a forward converter. At this stage, we will not simulate the power electronics but will simulate just the transformer. This simulation can be found in the folder `step_down_transformer` within `chapter4_transformers` in the following link in the simulation repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics

As with the simulation of the step-up transformer, let us define the parameters of the step-down transformer. Since this step-down transformer will be used for control purposes, we do not need a large power rating. A maximum power rating of 100 VA should be sufficient for most control circuits used in power electronic applications. We can use the circuit topology of Fig. 4.6. The feeder impedance that connects the source voltage to winding 1 can remain at the same value as the previous simulation at $R_s = 0.02\Omega$, $L_s = 0.0001$H. The load connected to winding 2 needs to be such a value that power drawn by the load will be less than 100 VA of a terminal voltage of 12 V RMS. We can choose the load impedance to be $R_l = 1\Omega$, $L_l = 0.003$H.

As with the simulation of the step-up transformer, the only change that needs to be made in the transformer model are the parameters defined at the head of the file:

```
import math
dt = 1.0e-6
VArated = 100.0
V1rated = 240.0
V2rated = 12.0
frated = 50.0
omega_rated = 2*math.pi*frated
```

The calculation of the equivalent circuit parameters, the simplification of the matrix equations and the numerical integration will remain the same.

Figures 4.21 and 4.22 show the simulation results with the step-down transformer. As can be seen from Fig. 4.21, the voltage v_2 available at the winding 2 is a fraction of the voltage v_1 available at winding 1. On the other hand, the current drawn by the load i_l is referred to winding 1 and can be calculated as

$$i_l^{'} = \frac{N_2}{N_1} i_l \tag{4.64}$$

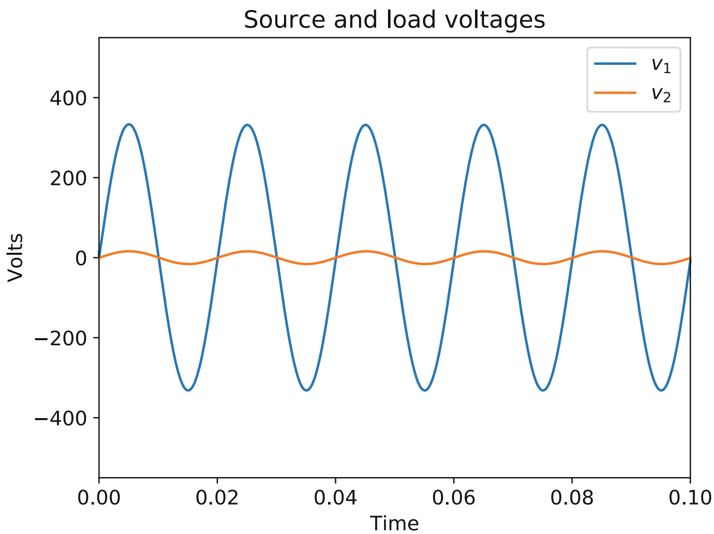


Fig. 4.21 Step down transformer source and load voltages

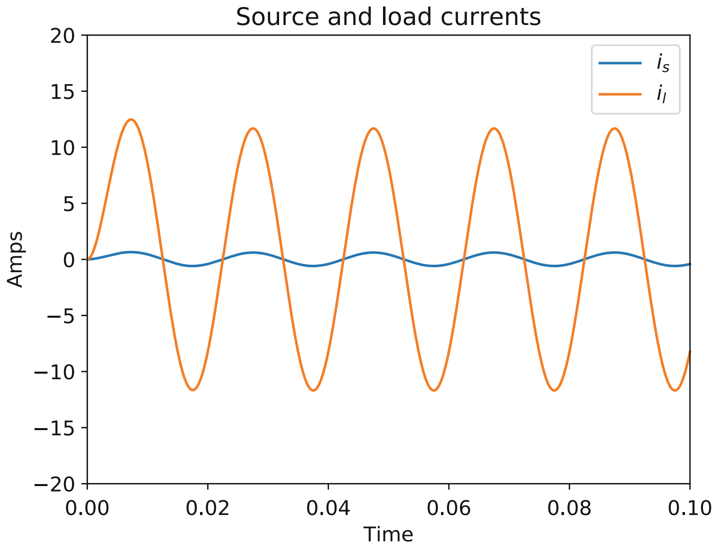


Fig. 4.22 Step down transformer source and load currents

As the transformer is a step-down transformer, $\frac{N_2}{N_1} < 1$, and therefore, the source current i_s is a mere fraction of the load current i_l. The analysis carried out for the step-up transformer can be repeated for the step-down transformer as well. This exercise will be left to the reader.

In this section, we simulated transformers where the windings did not have the same maximum voltage rating and therefore had turns ratios different from 1. In a vast number of applications, transformers are used to either step-up the voltage from a generator or step-down the voltage from a feeder to a load. Therefore, understanding the operation of step-up and step-down transformers from the basic laws of physics will make this very common application of the transformer clearer. Until now, the transformers simulated always had two windings. However, in a number of cases, transformers can have more than two windings. In such cases, a single supply can be used to feed loads at very different voltages. This will be discussed in the next section.

4.7 Multi-winding Transformers

In the past few sections, we had simulated a number of transformers and analysed them in detail. The purpose of these simulations was to show how the basic laws of physics can be used to both simulate transformers and understand their behaviour. In this section, we will simulate a transformer with more than two windings. Such multi-winding transformers are commonly used when designing power supplies for control circuits as most control circuits need multiple voltages such as 5 V, 9 V, 12 V, etc. [13, 14, 27, 28]. An example of a flyback converter will be described in the next

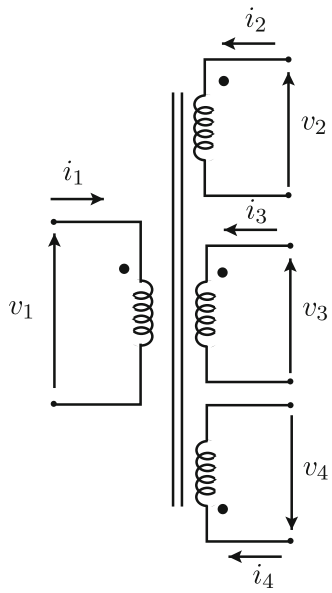


Fig. 4.23 Transformer with four windings

chapter where a single source can be used to produce a number of isolated power supplies. Besides power supplies, simulating multi-winding transformers is essential for simulating three-phase transformers, which form the bulk of transformers used in the power system [13, 14, 27, 28].

The simulation of a multi-winding transformer is a mere extension of the simulation of a two winding transformer. Figure 4.23 shows a transformer with four windings. It is important to note that the arrangement of the windings in Fig. 4.23 is not significant. We could have arranged the four windings in any manner—all in a vertical line, two on the left and two on the right, etc. Physically, these are coils wound on a core and the exact construction details might be very different. This is due to the fact that the main objective in a transformer is to achieve maximum power transfer between windings. The schematic of Fig. 4.23 is merely for the convenience of analysing a transformer in a circuit.

We could use Kirchhoff's Voltage Law for each winding:

$$v_1 - i_1 r_1 - \frac{d\psi_1}{dt} = 0 \tag{4.65}$$

$$v_2 - i_2 r_2 - \frac{d\psi_2}{dt} = 0 \tag{4.66}$$

$$v_3 - i_3 r_3 - \frac{d\psi_3}{dt} = 0 \tag{4.67}$$

$$v_4 - i_4 r_4 - \frac{d\psi_4}{dt} = 0 \tag{4.68}$$

From Fig. 4.23, the fourth winding has a dot on the lower terminal in contrast with the other windings. However, the KVL equation can still be written in the same manner due to the directions of current i_4 and voltage v_4.

The flux linkages of the windings can be expressed in terms of the self-inductances of the windings and the mutual inductances between windings as follows:

$$\psi_1 = L_1 i_1 + M_{12} i_2 + M_{13} i_3 + M_{14} i_4 \tag{4.69}$$

$$\psi_2 = M_{12} i_1 + L_2 i_2 + M_{23} i_3 + M_{24} i_4 \tag{4.70}$$

$$\psi_3 = M_{13} i_1 + M_{23} i_2 + L_3 i_3 + M_{34} i_4 \tag{4.71}$$

$$\psi_4 = M_{14} i_1 + M_{24} i_2 + M_{34} i_3 + L_4 i_4 \tag{4.72}$$

In the previous simulation, we had a single mutual inductance for the windings, which was defined as the flux linkage produced in a winding due to the current flowing in another winding. Therefore, we can define mutual inductances for every pair of windings x and y and we can continue to use the expression:

$$M_{xy} = k\sqrt{L_x L_y} \tag{4.73}$$

In the above expression k is the coupling factor between the two windings. This coupling factor does not take into consideration the leakage of flux as we will be computing a separate leakage inductance that is a part of the self-inductance. The coupling factor is merely an indication of the fraction of the flux produced by a winding that links with another winding. In three-phase transformers, which will be discussed in the next chapter, windings may be wound on separate limbs of the transformer, due to which only a fraction of the flux produced by the winding may link with another winding. At this point, we can assume a simple rectangular core construction where all the flux produced by a winding links completely with the other windings, due to which the coupling factor k is equal to 1 for all mutual inductances.

We could rewrite the voltage equations as a matrix equation as follows:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} + \begin{bmatrix} L_1 & M_{12} & M_{13} & M_{14} \\ M_{12} & L_2 & M_{23} & M_{24} \\ M_{13} & M_{23} & L_3 & M_{34} \\ M_{14} & M_{24} & M_{34} & L_4 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \tag{4.74}$$

If we were to compare this matrix equation with the matrix equation for the two winding transformer, it is fairly clear that the model is merely an extension of the two winding transformer with larger matrices. Moreover, the inductance matrix has a fairly well-defined construction—symmetric with the diagonal elements being

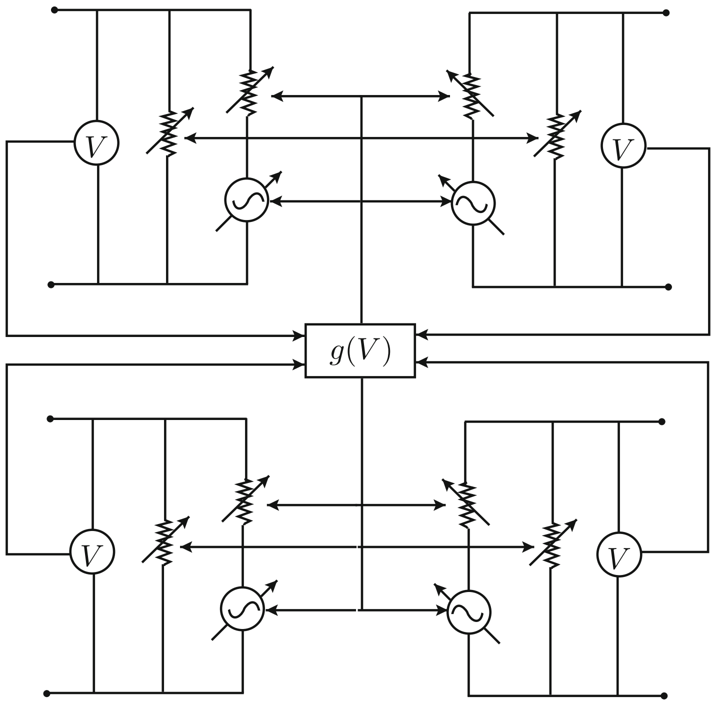


Fig. 4.24 Simulation model of the four winding transformer

the self-inductances and the off-diagonal elements being the mutual inductances. With this matrix equation, we can simulate fairly complex transformers with several windings without much alterations to the mathematical model and its solution.

In the mathematical model, we will be solving the above matrix equation and the resultant currents would then be represented by the usual combination of controllable voltage sources in series with variable resistors. Figure 4.24 shows how each transformer winding is modelled as a block comprising of a controllable voltage source in series with a variable resistance and the winding terminal voltage measured by a Voltmeter. The model receives as inputs the winding terminal voltages v_1, v_2, v_3, v_4 and produces signals for the windings currents i_1, i_2, i_3, i_4. These winding currents are achieved by adjusting the voltage outputs of the controllable voltage sources with respect to the measured winding terminal voltages and the series resistances.

Let us describe the details in simulating a multi-winding transformer by performing a simulation of a four winding transformer as shown in Fig. 4.23. This simulation can be found in the folder `multiwinding_transformer` within `chapter4_transformers` in the following link in the simulation repository: https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics

Let us consider the following maximum rated voltages—240 V RMS for winding 1, 240 V RMS for winding 2, 480 V RMS for winding 3 and 120 V RMS for winding 4. Therefore, we have a 1:1 turns ratio between winding 1 and 2, 1:2 turns ratio between winding 1 and 3, and 2:1 turns ratio between winding 1 and winding 4. Let us energize winding 1 with a 240 V, 50 Hz ac single-phase supply and connect loads to all the other windings. Therefore, the objective of such a transformer would be to supply different loads at different voltage levels. Since we have three loads altogether, let us choose the maximum power rating of this transformer to be 30 kVA—three times the two winding transformer simulated before.

We can define these specifications of the transformer at the head of the Python file as follows:

```
import math
dt = 1.0e-6
VArated = 30000.0
V1rated = 240.0
V2rated = 240.0
V3rated = 480.0
V4rated = 120.0
frated = 50.0
omega_rated = 2*math.pi*frated
```

We have merely added the ratings for the third and fourth winding. We can proceed to calculate the maximum current ratings and the rated impedance for all the windings:

```
I1rated = VArated / V1rated
I2rated = VArated / V2rated
I3rated = VArated / V3rated
I4rated = VArated / V4rated
Z1rated = V1rated / I1rated
Z2rated = V2rated / I2rated
Z3rated = V3rated / I3rated
Z4rated = V4rated / I4rated
```

From the above computations, it needs to be emphasized that the maximum current rating of each winding is calculated assuming that the winding is going to be transferring the maximum rated power. In our specific application, that is not the case, as only winding 1 is supplying the maximum rated power while all other windings are supplying a load that is only a fraction of the total power rating of the transformer. However, when a transformer is designed and simulated, we do not usually differentiate between windings as source and load windings. Any winding could be energized and supplying loads connected to the other windings. Therefore, the maximum rated current for each winding is calculated assuming that the total maximum rated power of the transformer will be supplied by the winding.

With these base computations, the parameters of the transformer equivalent circuit can be computed as follows:

```
L1 = 50.0 * Z1rated / omega_rated
L2 = 50.0 * Z2rated / omega_rated
L3 = 50.0 * Z3rated / omega_rated
```

```
L4 = 50.0 * Z4rated / omega_rated
Ll1 = 0.02 * Z1rated / omega_rated
Ll2 = 0.02 * Z2rated / omega_rated
Ll3 = 0.02 * Z3rated / omega_rated
Ll4 = 0.02 * Z4rated / omega_rated
Lm1 = L1 - Ll1
Lm2 = L2 - Ll2
Lm3 = L3 - Ll3
Lm4 = L4 - Ll4
M12 = math.sqrt(Lm1 * Lm2)
M13 = math.sqrt(Lm1 * Lm3)
M14 = math.sqrt(Lm1 * Lm4)
M23 = math.sqrt(Lm2 * Lm3)
M24 = math.sqrt(Lm2 * Lm4)
M34 = math.sqrt(Lm3 * Lm4)
r1 = 0.01 * Z1rated
r2 = 0.01 * Z2rated
r3 = 0.01 * Z3rated
r4 = 0.01 * Z4rated
Rc1 = V1rated * V1rated / (0.01 *VArated)
Rc2 = V2rated * V2rated / (0.01 *VArated)
Rc3 = V3rated * V3rated / (0.01 *VArated)
Rc4 = V4rated * V4rated / (0.01 *VArated)
res_output1 = 100.0 * Z1rated
res_output2 = 100.0 * Z2rated
res_output3 = 100.0 * Z3rated
res_output4 = 100.0 * Z4rated
```

With the parameters of the transformer equivalent circuit, the matrices in the matrix equation can be written as follows:

```
winding_currents = [0.0, 0.0, 0.0, 0.0]
L = [
    [L1, M12, M13, M14],
    [M12, L2, M23, M24],
    [M13, M23, L3, M34],
    [M14, M24, M34, L4]
]
R = [
    [r1, 0.0, 0.0, 0.0],
    [0.0, r2, 0.0, 0.0],
    [0.0, 0.0, r3, 0.0],
    [0.0, 0.0, 0.0, r4]
]
V = [v1, v2, v3, v4]
```

After the definition of the matrices of the equation, the code for simplification and numerical integration are exactly the same as before. After the winding currents have been updated, the output voltages can be computed as

```
vout1 = v1 - winding_currents[0]*res_output1
vout2 = v2 - winding_currents[1]*res_output2
vout3 = v3 - winding_currents[2]*res_output3
vout4 = v4 - winding_currents[3]*res_output4
```

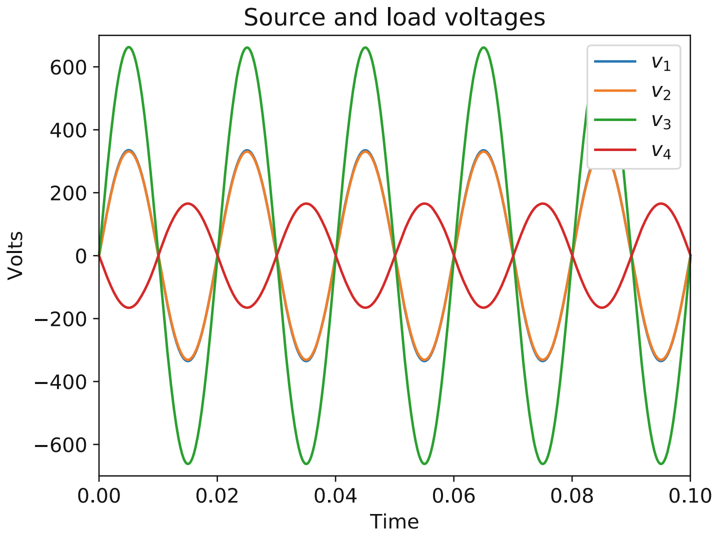


Fig. 4.25 Transformer winding voltages

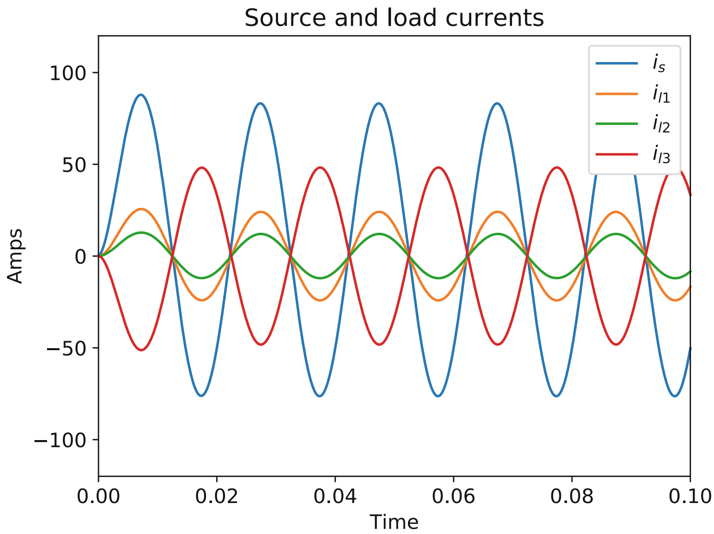


Fig. 4.26 Transformer winding currents

Figures 4.25 and 4.26 show the simulation results of the four winding transformer. The voltages v_1 and v_2 at windings 1 and 2 are approximately the same as both windings have the same turns ratio. The voltage v_3 at winding 3 has a magnitude that is twice that of v_1 as the turns ratio between winding 3 and winding 1 is 2:1. The voltage v_4 at winding 4 has a magnitude that is half of that of v_1 as the turns ratio between winding 4 and 1 is 1:2. Moreover, the voltage v_4 is phase shifted by 180° with respect to v_1 due to the dot polarity of winding 4 being the opposite of the other windings. The currents in Fig. 4.26 can also be explained with respect to the turns ratios between the transformer windings. The current i_s supplied by the load will be the sum of the transferred currents i'_{l1}, i'_{l2}, i'_{l3} drawn by the loads connected in windings 2, 3 and 4.

Before we complete this section and also the chapter, let us optimize the mathematical model of the transformer. Though the code that we have been using is functional, it is inefficient as we are performing several computations repeatedly in every simulation instant when they need to be performed only once at the beginning of the simulation. The transformer model is largely decided by construction as the equivalent circuit of a transformer will not change significantly. As the transformer ages, the parameters will change since component parameters do change with age. However, these changes will not be very significant and therefore, assuming the transformer equivalent circuit to remain the same is not very unreasonable and will simplify the model to a great extent. This simulation can be found in the folder `simplified_transformer_model` within `chapter4_transformers` in the following link in the simulation repository:
https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics

We have been simplifying and integrating the matrix equation of (4.74). Let us make a modification to the equation as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} + \begin{bmatrix} L_1 & M_{12} & M_{13} & M_{14} \\ M_{12} & L_2 & M_{23} & M_{24} \\ M_{13} & M_{23} & L_3 & M_{34} \\ M_{14} & M_{24} & M_{34} & L_4 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \tag{4.75}$$

We have multiplied the vector of winding voltages by an identity matrix and therefore, the equation remains unchanged. However, by doing so, the row operations in the process of triangularization will now be performed on the identity matrix rather than the vector of winding voltages:

```
B = [
    [1.0, 0.0, 0.0, 0.0],
    [0.0, 1.0, 0.0, 0.0],
    [0.0, 0.0, 1.0, 0.0],
    [0.0, 0.0, 0.0, 1.0]
]
for count1 in range(len(L)):
    if not L[count1][count1]:
        for count2 in range(count1+1, len(L)):
            if L[count2][count1]:
                L[count1], L[count2] = L[count2], L[count1]
                R[count1], R[count2] = R[count2], R[count1]
                B[count1], B[count2] = B[count2], B[count1]
                break
```

```
    if L[count1][count1]:
        for count2 in range(count1+1, len(L)):
            comm_factor = L[count2][count1]/L[count1][count1]
            for count3 in range(len(L[count1])):
                L[count2][count3] -= L[count1][count3]*comm_factor
                R[count2][count3] -= R[count1][count3]*comm_factor
                B[count2][count3] -= B[count1][count3]*comm_factor
```

The matrices **L**, **R**, **B** are constants as we are assuming the parameters of the transformer equivalent circuit to be constants that do not change after design. Therefore, once the matrix equation has been simplified, the matrices in the simplified equation will also remain constant. Therefore, the simplification needs to be performed only once in the simulation, thereby reducing the computation burden significantly. At each simulation instant, the only change that needs to be made is that we must compute the initial value during integration, rather than initializing it to the voltage:

```
# k[k_count] = V[count1]
k[k_count] = 0.0
for count2 in range(len(B)):
    k[k_count] += B[count1][count2]*V[count2]
    .....
    .....
# winding_currents[count1] = V[count1]
winding_currents[count1] = 0.0
for count2 in range(len(B)):
    winding_currents[count1] += B[count1][count2]*V[count2]
```

With these changes made to ensure that the solution is unaffected, we need to ensure that the simplification of the matrices occurs only once in the simulation. For this, we can define a static variable `init_simulation` and provide an initial value of 1, which signifies that initially the parameters need to be computed. We can now perform all initial computations within this conditional check:

```
if init_simulation > 0:
    I1rated = VArated / V1rated
    I2rated = VArated / V2rated
    I3rated = VArated / V3rated
    I4rated = VArated / V4rated
    Z1rated = V1rated / I1rated
    Z2rated = V2rated / I2rated
    Z3rated = V3rated / I3rated
    Z4rated = V4rated / I4rated
    L1 = 50.0 * Z1rated / omega_rated
    L2 = 50.0 * Z2rated / omega_rated
    L3 = 50.0 * Z3rated / omega_rated
    L4 = 50.0 * Z4rated / omega_rated
    Ll1 = 0.02 * Z1rated / omega_rated
    Ll2 = 0.02 * Z2rated / omega_rated
    Ll3 = 0.02 * Z3rated / omega_rated
    Ll4 = 0.02 * Z4rated / omega_rated
    Lm1 = L1 - Ll1
    Lm2 = L2 - Ll2
```

```
Lm3 = L3 - L13
Lm4 = L4 - L14
M12 = math.sqrt(Lm1 * Lm2)
M13 = math.sqrt(Lm1 * Lm3)
M14 = math.sqrt(Lm1 * Lm4)
M23 = math.sqrt(Lm2 * Lm3)
M24 = math.sqrt(Lm2 * Lm4)
M34 = math.sqrt(Lm3 * Lm4)
r1 = 0.01 * Z1rated
r2 = 0.01 * Z2rated
r3 = 0.01 * Z3rated
r4 = 0.01 * Z4rated
Rc1 = V1rated * V1rated / (0.01 *VArated)
Rc2 = V2rated * V2rated / (0.01 *VArated)
Rc3 = V3rated * V3rated / (0.01 *VArated)
Rc4 = V4rated * V4rated / (0.01 *VArated)
res_output1 = 100.0 * Z1rated
res_output2 = 100.0 * Z2rated
res_output3 = 100.0 * Z3rated
res_output4 = 100.0 * Z4rated
winding_currents = [0.0, 0.0, 0.0, 0.0]
L = [
    [L1, M12, M13, M14],
    [M12, L2, M23, M24],
    [M13, M23, L3, M34],
    [M14, M24, M34, L4]
]
R = [
    [r1, 0.0, 0.0, 0.0],
    [0.0, r2, 0.0, 0.0],
    [0.0, 0.0, r3, 0.0],
    [0.0, 0.0, 0.0, r4]
]
B = [
    [1.0, 0.0, 0.0, 0.0],
    [0.0, 1.0, 0.0, 0.0],
    [0.0, 0.0, 1.0, 0.0],
    [0.0, 0.0, 0.0, 1.0]
]
for count1 in range(len(L)):
    if not L[count1][count1]:
        for count2 in range(count1+1, len(L)):
            if L[count2][count1]:
                L[count1], L[count2] = L[count2], L[count1]
                R[count1], R[count2] = R[count2], R[count1]
                B[count1], B[count2] = B[count2], B[count1]
                break
    if L[count1][count1]:
        for count2 in range(count1+1, len(L)):
            comm_factor = L[count2][count1]/L[count1][count1]
            for count3 in range(len(L[count1])):
                L[count2][count3] -= \
                        L[count1][count3]*comm_factor
                R[count2][count3] -= \
```

```
                        R[count1][count3]*comm_factor
                B[count2][count3] -= \
                        B[count1][count3]*comm_factor

    init_simulation = -1
```

The computation of the parameters of the transformer equivalent circuit will now be performed only once at the very first simulation instant at $t = 0$ when `init_simulation` is equal to 1. After computing the parameters and simplifying the matrices, we are setting the value of `init_simulation` to -1 to ensure that the computations are not performed subsequently. With this change, however, many of the variables will not exist after the first simulation instant. This is due to fact that in Python, a variable is created when it is assigned a value for the very first time. Therefore, as an example, the variable `I1rated` will be created in the very first simulation instant and assigned a value equal to `VArated / V1rated`. In subsequent iterations, when the conditional block will not be executed, `I1rated` will no longer be created and therefore will not appear anywhere in the control file. This is a problem for certain variables such as the matrices `L`, `R` and `B` as these are used in the numerical integration and therefore, must exist and retain the computed value for the entire simulation. In order to ensure that they retain their computed values, they can be defined as static variables and assigned dummy initial values of 0. In Python, when a variable is defined a value but redefined as another value, the old object is destroyed and a new object is created. Therefore, defining `L`, `R` and `B` as static variables with initial values of 0 but later computing them as lists will imply that for the rest of the simulation, they will retain their values as lists.

In this section, we have shown how the mathematical model can be expanded fairly easily to simulate transformers with several windings. Most computations related to the parameters of the equivalent circuit are merely replicates of the basic computations for the base two winding transformer. The final matrices in the matrix equation have a well-defined structure—matrix `R` is a diagonal matrix with the diagonal elements being the winding resistances and `B` is an identity matrix. Matrix `L` is a symmetric matrix with the diagonal elements being the self-inductances of the windings and the off-diagonal elements being the mutual inductances between the windings. The concluding simplification to the model by minimizing the computations by performing initialization only once at the beginning of the simulation helps to improve the speed of the simulation. This simulation model of the transformer will be used in the next chapter to simulate a few chosen advanced applications of transformers.

4.8 Conclusions

In this chapter, we have extended the simulation models presented in the previous chapters to a simulation model of a transformer. In Chap. 2, we converted the basic laws of physics into a very basic simulation model of the inductor, which is one of the simplest magnetic components in electrical engineering. In Chap. 3, we extended the basic magnetic model of an inductor to simulate magnetically coupled inductors. A transformer, in its most fundamental form, is just a set of magnetically coupled inductors. Therefore, the underlying simulation model of magnetically coupled coils presented in the previous chapter is applicable for transformers as well. The difference lies in the transformer having a different set of specifications since it is an electrical machine quite often designed, fabricated and manufactured for a particular application.

The chapter begins with an overview of transformers, their importance in modern electrical engineering and some of the major applications of transformers. The chapter describes how the transformer as a machine can be translated into an equivalent circuit. The parameters of this equivalent circuit can either be determined through laboratory experiments such as the open circuit test and the short-circuit test, or can be estimated as shown in the chapter. Though this might result in approximate values of the parameters of the transformer equivalent circuit, reasonable estimates can produce a fairly usable simulation model of a transformer. Furthermore, the process of estimating the transformer equivalent circuit has an added advantage—we can potentially change these estimates to determine how a transformer behaves as it might change due to ageing.

Once the equivalent circuit of the transformer is determined, it is clear that this equivalent circuit can be thought of as coupled inductors. Therefore, the parameters of the equivalent circuit can be transferred to the simulation model of coupled inductors already derived in the previous chapter. Simulations of several transformers are presented such that in different cases, we have not only translated the specifications of the transformer to the simulation model, but we have also examined the impact of the transformer specifications on the nature of parameters. This is particularly the case of step-up and step-down transformers, in which it is clear that the electrical systems on the different windings of the transformer are totally different. This provides us with an insight into the role of transformers in the modern power system, where transformers interface the generation, transmission and distribution systems.

Using simulations, we have delved deep into the operation of transformers. We use simulations to examine the no-load operation of the transformer where we differentiate between the core loss components of current and the magnetizing current of the transformer. We use theory and simulations to describe why a transformer needs to draw a magnetizing current at all times. We also examined how the core flux of the transformer is determined primarily by the energizing voltage and to a much smaller extent by the winding currents. Furthermore, we also examine the impact of the winding currents on the core flux and how winding currents can be transformed using the turns ratio of the transformer. We emphasize on the fact that besides the magnetizing current of the transformer, any other current flowing in any other winding will result in a transformed current flowing in the other winding due to the necessity for the MMFs of the windings to be balanced.

To emphasize on how the simulation models used are flexible and scalable, we have simulated a number of different type of transformers. We have shown how simulating step-up and step-down transformers needs nothing more than modified transformer specifications in the simulation model with all other computations remaining unaffected. Furthermore, we have also simulated multi-winding transformers and have shown how a transformer with several windings requires merely larger system matrices with most of these matrices being intuitive and having a well-defined structure. In the previous chapter, we had already expressed the final differential equation of magnetically coupled systems as a matrix equation that allows us to implement a repeatable algorithm to simplify the matrices and integrate them numerically.

Though we have simulated a number of different transformers in this chapter, in the next chapter we will use these simulation models to simulate some transformers that occur often in electrical engineering. A few examples of these are three-phase transformers that form the bulk of the modern power system. In addition, we will also examine the application of high frequency transformers in a power electronics application. We will consider a power supply consisting of a flyback converter and will show through simulations, how even taking into consideration the highly non-linear behaviour of power electronic converters, the underlying principle of transformer operation remains the same.

Chapter 5
Applications of Transformers

5.1 Introduction

In the previous chapter, we had presented simulation models for transformers and analysed their operation through simulation results. However, the transformers studied in detail were two winding transformers that are typically used for single-phase applications. The major application of transformers is as three-phase transformers in the power system [13, 14, 21]. Though the simulation models of the previous chapter can be extended to simulate three-phase transformers, there are a few aspects of three-phase transformers that are not immediately obvious. In this chapter, we will cover the simulation of three-phase transformers to make the material in this book more complete.

The simulation of three-phase transformers is an extension of the simulation of multi-winding transformers already described in the previous chapter. However, to simulate any three-phase machine, whether a transformer or a generator, a certain detail of construction needs to be known or assumed [27–31]. In this chapter, we will introduce the concept of three-phase systems and how the windings of a three-phase transformer can be connected. Moreover, we will also examine how the windings of the transformer can be wound on the core and how this will affect the simulation model of the transformer.

We will simulate different three-phase transformers having different connection types to cover some of the most popularly used transformers in the power system [27–31]. In all the simulations, the underlying simulation model is the same as the basic simulation model of the transformer presented in the previous chapter, except that the specifications of the transformer will change. We will simulate the star–star and delta–star transformers. We will also discuss how distribution systems that need a neutral wire to support single-phase appliances can be fed by a delta–star transformer.

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In addition to simulating three-phase transformers, this chapter will introduce high frequency transformers used in power electronic applications [10, 11, 40, 41]. This is of great relevance to a power electronics engineer, as a vast number of dc–dc converters contain transformers and provide isolated outputs. We will begin the discussion with the effect of frequency on the transformer construction. The notable differences in a high frequency transformer are decreased size and the use of ferrite cores as opposed to laminated iron cores [15].

We will examine a very popular converter used in power supplies—the flyback converter [19, 20]. The flyback converter contains a high frequency transformer and we will use the basic laws of physics to understand how the transformer behaves during converter operation. Even though the operation of the power converter forces the transformer to operate in a manner drastically different from regular grid frequency transformers, we will show how the basic laws of physics still hold true to describe the operation of the converter. We will simulate a flyback converter with three isolated outputs using a four winding high frequency transformer to examine the operation through simulation results.

This chapter will not present any new theory or discussion related to transformers, but will merely use the discussions already put forth in the previous chapter. The reader is advised to be well-versed with transformer simulations and applying the basic laws of physics to understand transformer operation. As with the previous chapters, there will be numerous references of simulations in this chapter. The reader is advised to try out the simulations and also attempt modifications as suggested.

5.2 Three-Phase Systems

In the domestic (home and office) power system, equipment and appliances are usually single-phase and rated for either 120 V, 60 Hz (USA, Canada) or 240 V, 50 Hz. Outside the home and office, a vast majority of equipment and appliances are usually three-phase as multi-phase systems allow for greater energy density—more power at relatively lower size as compared to the equivalent single-phase equipment [1, 21]. Therefore, as the power rating of generators, motors and other loads increases, it is usual for these loads to be designed to receive three-phase ac supply. Most of the power system is also three phase except for the feeders that feed homes and offices. The transformers that interconnect different segments of the power system are also usually three-phase transformers. Before we get to three-phase transformers, let us talk about three-phase systems in general.

Let us start by how a three-phase voltage is generated. The three phases are three windings wound on the generator stator or rotor. These windings are wound in such a manner that the voltages produced in the windings will such that they will be phase shifted with respect to each other [1]. For a multi-phase machine with x phases, the voltages will have a phase shift of $360^\circ/x$. For a three-phase machine, this phase shift will therefore be 120°. Let us name the generated output voltages of the three-phase machine v_a, v_b, v_c, and assume that they have RMS magnitudes of 240 V and frequencies of 50 Hz. We could express the three-phase voltages as follows [1]:

$$\begin{aligned} v_a &= 240\sqrt{2}\cos(100\pi t) \\ v_b &= 240\sqrt{2}\cos\left(100\pi t - \frac{2\pi}{3}\right) \\ v_c &= 240\sqrt{2}\cos\left(100\pi t - \frac{4\pi}{3}\right) \end{aligned} \tag{5.1}$$

Figure 5.1 shows the three-phase voltage waveforms for a few cycles. As can be seen, all three voltages are waveforms that have the same peak as they have the same RMS magnitudes, but are phase shifted by 120°. This is one of the critical aspects of three-phase (or multi-phase) systems—electrical quantities such as voltages and currents that have the same magnitudes in all phases and are phase shifted from each other by 120°. In such cases, the three-phase system is called a balanced system [1, 21]. In practical cases, three-phase systems can and will be unbalanced, either by having unequal magnitudes in the three phases or the phase shift between them may not be 120° [1, 21]. There is vast theory in power systems studies on three-phase systems and also on handling unbalanced systems [21]. However, let us begin with balanced three-phase systems as our goal is to simulate three-phase transformers.

One of the most popular methods of analysing three-phase systems is by expressing them as phasors [1, 21]. A phasor is a rotating vector with a magnitude

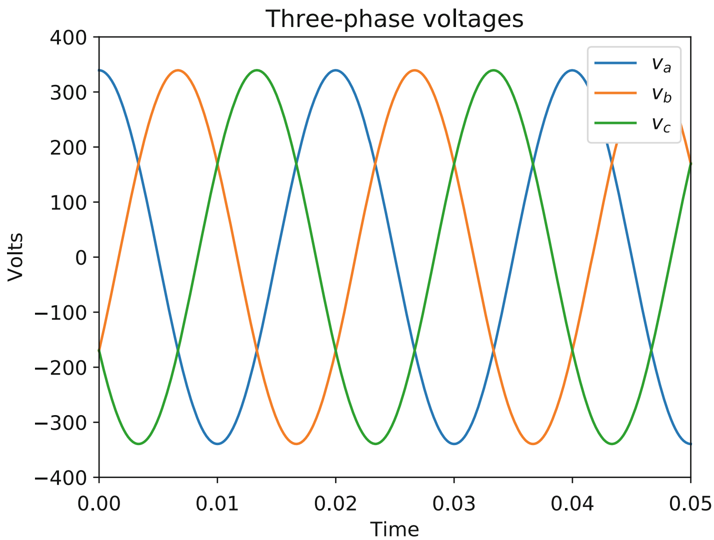

Fig. 5.1 Sample three-phase voltages

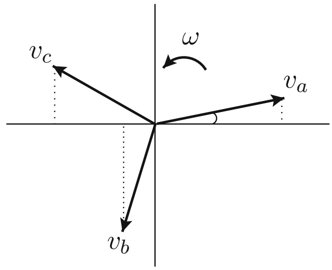

Fig. 5.2 Three-phase voltages as phasors

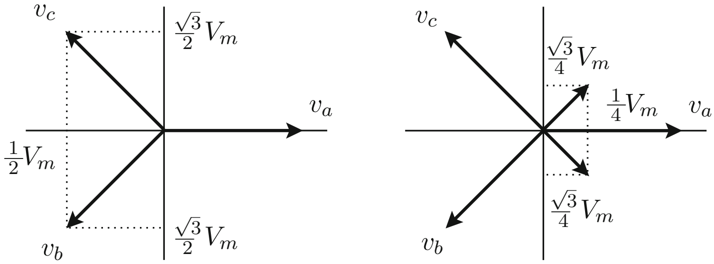

Fig. 5.3 The use of phasors for computations

and a frequency of rotation. This concept is best described with a diagram such as Fig. 5.2. Figure 5.2 shows the three-phase voltages as phasors rotating at an angular speed of $\omega = 2\pi f$. The magnitude of the phasors is equal to the RMS magnitudes of the voltages. At any given instant, the phase angle of a phasor is the angle made by the phasor with the positive horizontal axis being measured in the counter-clockwise sense as shown for the phasor v_a. If the voltages are cosine waveforms as assumed in our case, the projections of the phasors on the horizontal axis will be their instantaneous values as shown in Fig. 5.2. If on the other hand, the voltages were assumed to be sine waveforms, the projections of the phasors on the vertical axis will be their instantaneous values. It can be observed from Fig. 5.2 that the phasor v_b lags behind phasor v_a by 120° and phasor v_c lags behind phasor v_b by 120°. The reader is encouraged to verify that these phasors would result in the voltage waveforms that have been assumed.

This phasor representation is applicable for all ac quantities such as three-phase currents. Such a phasor representation of three-phase ac quantities is extremely popular as we can use it not only for visualization but also for performing basic computations. An example of phasor computations can be demonstrated by performing a couple of sample computations such as $v_a + v_b + v_c$ and $v_a - \frac{v_b}{2} - \frac{v_c}{2}$. These computations are shown in Fig. 5.3. Each resultant phasor can be resolved into projections on the horizontal and vertical axes, and subsequently the arithmetic operation yields the projections of the resultant phasor on the same horizontal and vertical axes [1, 21]. For convenience, the phasor v_a has been chosen along the horizontal axis and therefore has a zero projection on the vertical axis. The other phasors v_b and v_c have projections on the horizontal and vertical axes as shown by the dashed lines. It is quite clear that $v_a + v_b + v_c = 0$ and $v_a - \frac{v_b}{2} - \frac{v_c}{2} = \frac{3}{2}V_m$ with $V_m = 240\sqrt{2}$. The reader is encouraged to verify the computations on their own.

With this basic overview of three-phase systems, we can now begin to talk about three-phase transformers [13]. A three-phase transformer will interface two or more three-phase systems of either the same or different voltage levels. As in the previous chapter, we will begin with the simple example of two three-phase systems being interconnected by a transformer. Therefore, the transformer will have a primary consisting of three windings that will be connected to the a, b, c terminals of a three-phase supply and will have a secondary consisting of three windings that will be connected to the a, b, c terminals of another three-phase supply. As in the previous chapter, we could have only one of the three-phase systems being energized or both of the three-phase systems being energized. The next question to be asked is how will the windings of the transformer be connected and what are the connections available in the three-phase systems being interfaced.

There are several ways in which the three phases of a transformer's windings can be connected, but a few are the most widely used—the star (or wye) with neutral, the isolated star (or wye) and the delta connection [13]. Figure 5.4 shows these connections for the three different cases. The neutral terminal is a fourth terminal in the case of the star winding with the common interconnection node being brought out. Such a terminal is extremely useful in domestic distribution systems as each phase terminal along with the neutral forms a single-phase supply as is available in the power outlets of our homes and offices [1, 13]. However, in transmission systems, the neutral terminal and wire are not convenient as this would result in increased costs since we now need an extra wire. Therefore, transmission systems are usually devoid of the neutral wire. This results in three-wire and four-wire

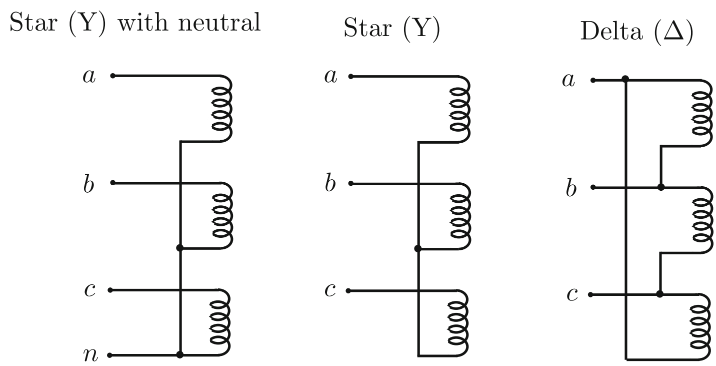

Fig. 5.4 Star and delta connection of transformer windings

systems with the difference being the absence or presence of the neutral wire. To achieve a four-wire system with a neutral wire, the star connection with the neutral terminal is usually the preferred option. To achieve a three-wire system, though both the isolated star and the delta are possible, the delta connection quite often has advantages over the star.

With these transformer connections, it is clear that the voltages across a winding may not be the same as the voltages between two terminals. In Fig. 5.4, for star connected windings, the voltage between terminals a and b will not be the same as the voltage across the phase a or phase b winding. The voltage between terminals a and b can be expressed as

$$v_{ab} = v_{an} - v_{bn} \tag{5.2}$$

In the above equation, the voltage v_{ab} is called the line voltage as it is the voltage available between the terminals (or lines) a and b, while voltages v_{an} and v_{bn} are called the phase voltages as these are the voltages across the windings connected between a terminal and the neutral thereby forming a phase [1].

We can define three line voltages as follows:

$$\begin{aligned} v_{ab} &= v_{an} - v_{bn} \\ v_{bc} &= v_{bn} - v_{cn} \\ v_{ca} &= v_{cn} - v_{an} \end{aligned} \tag{5.3}$$

We can use the phasor computation method similar to Fig. 5.3, to compute these line voltages. The reader is encouraged to perform these computations as an exercise. The line voltage v_{ab} will have a magnitude that is $\sqrt{3}$ times the phase voltage v_{an} and will lead the phase voltage by 30° [1]. Similar relations exist between v_{bc} and v_{bn}, and v_{ca} and v_{cn}. These line voltages can be written as the following trigonometric equations:

$$\begin{aligned} v_{ab} &= 240\sqrt{3}\sqrt{2}\cos\left(100\pi t + \frac{\pi}{6}\right) \\ v_{bc} &= 240\sqrt{3}\sqrt{2}\cos\left(100\pi t - \frac{2\pi}{3} + \frac{\pi}{6}\right) \\ v_{ca} &= 240\sqrt{3}\sqrt{2}\cos\left(100\pi t - \frac{4\pi}{3} + \frac{\pi}{6}\right) \end{aligned} \tag{5.4}$$

On the other hand, for delta connected windings such as that shown in Fig. 5.4, the line voltages are the same as the phase voltages. With reference to the delta connected winding in Fig. 5.4, the line voltages can be written as [1]

$$\begin{aligned} v_{ab} &= v_a = 240\sqrt{2}\cos(100\pi t) \\ v_{bc} &= v_b = 240\sqrt{2}\cos\left(100\pi t - \frac{2\pi}{3}\right) \\ v_{ca} &= v_c = 240\sqrt{2}\cos\left(100\pi t - \frac{4\pi}{3}\right) \end{aligned} \tag{5.5}$$

Along with the definition of line voltages for three-phase systems, there also exists the definition of line currents as the currents flowing in or out of the terminals of the three-phase system. The currents in the windings can be expressed as i_a, i_b, i_c and are called the phase currents in a three-phase system. The currents flowing into or out of the a, b and c terminals of the three-phase system can be called i_{La}, i_{Lb}, i_{Lc}, with the subscript L indicating that the current is flowing in the line. For a star connected system as shown in Fig. 5.4, the line currents are equal to the phase currents [1]:

$$\begin{aligned} i_{La} &= i_a \\ i_{Lb} &= i_b \\ i_{Lc} &= i_c \end{aligned} \tag{5.6}$$

On the other hand, for the delta connected system shown in Fig. 5.4, the line currents are differences of the phase currents as follows:

$$\begin{aligned} i_{La} &= i_a - i_b \\ i_{Lb} &= i_b - i_c \\ i_{Lc} &= i_c - i_a \end{aligned} \tag{5.7}$$

Phasor computations can be performed for the line currents in delta connected systems.

When we wish to provide the voltage specifications of a three-phase system, the convention is to specify the RMS line voltages [1]. As an example, if we specify the voltage of a three-phase domestic distribution system to be 400 V, we imply that the RMS line voltage is 400 V. As we have seen before, the relationship between the line voltages and the phase voltages can differ depending on whether the three-phase system is a star connected or a delta connected system. Therefore, along with the specification of the voltage level, it is always customary to specify the connection type of the three-phase system. As an example, a domestic distribution system could be a three-phase star with neutral having a voltage level of 400 V. This implies that the line voltage has an RMS value of 400 V and the phase voltage (between a line terminal and the neutral) has an RMS value of 230 V. On the other hand, a 11 kV delta connected three-phase system will have both line and phase voltages to have the same RMS value of 11 kV.

With the above discussion, we are now ready to modify the specifications of the single-phase transformers of the previous chapter and begin modelling three-phase transformers. If the reason for having several different types of connection of three-phase systems is confusing at this point, the simulations in the later sections will show their advantages and therefore, their applications. Before going further to the next section, the reader must be clear about the difference between line voltage and phase voltage in a three-phase system. The reader is encouraged to use the phasor computation method to arrive at phasors for line voltages with respect to phase voltages for both star and delta connected three-phase systems.

5.3 Simulation Model of a Three-Phase Transformer

In the previous section, we have provided a basic overview of three-phase systems, which was essential in order to describe some of the winding connections of three-phase transformers. Besides the star and delta connections, there are a few other types that have their uses in special applications [13, 27, 28]. In this section, we will begin simulating three-phase transformers while choosing a few connection possibilities to cover some of the broad three-phase transformer types. High power rating transformers with several windings will need to some extent a basic understanding of the construction of transformers in order to arrive at the parameters of the transformer equivalent circuit. Subsequently, we will extend the transformer models developed in the previous chapter, and it will be clear, that as with multi-winding transformers, simulating a three-phase transformer involves merely extending the two winding transformer model.

Let us start by simulating a 500 kVA transformer with the primary and the secondary being star connected set of windings. This simulation can be found in the folder `transformer_star_star` within `chapter5_transformer_applications` in the following link in the simulation repository: https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics

Such a three-phase transformer can be called a star–star (Y-Y) transformer to indicate the method in which the windings are connected. Let us connect a three-phase voltage source to the primary and a three-phase resistive-inductive load to the secondary. Figure 5.5 shows the topology of the circuit that will be simulated. Let the voltage levels on the primary and the secondary of the transformer be 415 V and 11 kV, respectively. From our discussion in the previous section, these voltage levels being three-phase voltage levels are the RMS magnitudes of the line voltages. Each phase of the voltage source in the primary can have an RMS magnitude of 240 V.

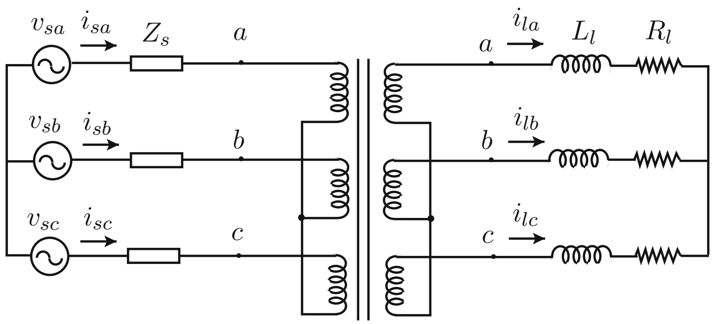

Fig. 5.5 Circuit for simulation of a star–star transformer

The voltage sources can be assumed to form a balanced three-phase supply and have the following expressions:

$$
\begin{aligned}
v_a &= 240\sqrt{2}\sin(100\pi t)\\
v_b &= 240\sqrt{2}\sin\left(100\pi t - \frac{2\pi}{3}\right)\\
v_c &= 240\sqrt{2}\sin\left(100\pi t - \frac{4\pi}{3}\right)
\end{aligned}
\tag{5.8}
$$

The three-phase transformer that we are going to simulate will have six windings numbered 1–6. Each winding will be modelled by the voltage source in series with resistance and with the voltage across the terminals measured by a Voltmeter. We much choose the indexes to denote a particular winding so as to compute the parameters of the equivalent circuit. Let us choose the odd numbers 1, 3, and 5 to denote the primary windings of the transformer and the even numbers 2, 4, and 6 to denote the secondary windings of the transformer. Therefore, windings 1 and 2 will represent phase *a* of the transformer with 1 being the primary phase *a* winding and the 2 being the secondary phase *a* winding. Windings 3 and 4 will represent phase *b* of the transformer with 3 being the primary phase *b* winding and the 4 being the secondary phase *b* winding. Windings 5 and 6 will represent phase *c* of the transformer with 5 being the primary phase *c* winding and the 6 being the secondary phase *c* winding. The reader is free to choose another convention instead of this.

With this convention, we can begin with the transformer specifications:

```
import math
dt = 1.0e-6
Vrated_primary = 415.0 / math.sqrt(3.0)
Vrated_secondary = 11000.0 / math.sqrt(3.0)
# Phase a - Windings 1 and 2
# Phase b - Windings 3 and 4
# Phase c - Windings 5 and 6
VArated = 500000.0 / 3
```

```
# Primary windings
V1rated = Vrated_primary
V3rated = Vrated_primary
V5rated = Vrated_primary
# Secondary windings
V2rated = Vrated_secondary
V4rated = Vrated_secondary
V6rated = Vrated_secondary
frated = 50.0
omega_rated = 2*math.pi*frated
```

In the above block, `Vrated_primary` and `Vrated_secondary` are the phase voltage ratings of the transformer windings. These are chosen to be $\frac{1}{\sqrt{3}}$ of the RMS values of the line voltages as both primary and secondary of the transformer are connected in star. This step is quite often skipped and results in errors. It is important to remember that in a three-phase system, any voltage level specified is the RMS magnitude of the line voltage and not the phase voltage. When computing the parameters of the transformer, we must compute all parameters per-phase, and therefore, the line voltage needs to be translated to the phase voltage for the chosen connection type. Besides converting voltage ratings from line voltage levels to phase voltage levels, we need to compute the per-phase maximum rated power by dividing the total maximum power rating by a factor of 3.

We can use the power and the voltage ratings to compute the rated impedance and therefore, the self-, leakage and magnetizing inductances of each winding:

```
I1rated = VArated / V1rated
I2rated = VArated / V2rated
I3rated = VArated / V3rated
I4rated = VArated / V4rated
I5rated = VArated / V5rated
I6rated = VArated / V6rated
Z1rated = V1rated / I1rated
Z2rated = V2rated / I2rated
Z3rated = V3rated / I3rated
Z4rated = V4rated / I4rated
Z5rated = V5rated / I5rated
Z6rated = V6rated / I6rated
L1 = 50.0 * Z1rated / omega_rated
L2 = 50.0 * Z2rated / omega_rated
L3 = 50.0 * Z3rated / omega_rated
L4 = 50.0 * Z4rated / omega_rated
L5 = 50.0 * Z5rated / omega_rated
L6 = 50.0 * Z6rated / omega_rated
Ll1 = 0.02 * Z1rated / omega_rated
Ll2 = 0.02 * Z2rated / omega_rated
Ll3 = 0.02 * Z3rated / omega_rated
Ll4 = 0.02 * Z4rated / omega_rated
Ll5 = 0.02 * Z5rated / omega_rated
Ll6 = 0.02 * Z6rated / omega_rated
Lm1 = L1 - Ll1
Lm2 = L2 - Ll2
Lm3 = L3 - Ll3
```

```
Lm4 = L4 - L14
Lm5 = L5 - L15
Lm6 = L6 - L16
```

All we have done is replicate the computation for each winding. For computing the self- and leakage inductances of a winding, we do not need any association or linkage with any other winding. Therefore, these computations are independent for each winding. The next step would be to compute the mutual inductances. At this point, we need to either know or make assumptions about the transformer connection.

The construction of transformers is a very detailed domain and transformers can be wound in various different ways according to the need of the application [13, 14, 28]. However, in most applications that need transformers, the purpose of the transformer is to achieve the maximum possible transfer of power with the minimal possible size of the transformer. In terms of the size of the transformer, the core of the transformer usually contributes the most to the bulk of the transformer. Cores used for transformers have different shapes and use different materials to decrease magnetic losses such as hysteresis and eddy current losses. In the previous chapter, we had assumed the transformer core to be a simple rectangular core. In the chapter on inductors and coupled inductors, we had also considered the three-limbed core. In a similar manner, it is possible to have multi-limbed cores with a number of different limbs on which windings can be wound.

In most practical transformers, if the purpose is to transfer power from one winding to another, it would be obvious that they be wound in such a manner that flux produced by one winding would link with the other winding with the minimal possible leakage. On the other hand, it is also essential to ensure that there is sufficient isolation between the windings to ensure that the two windings remain completely isolated in the electrical sense. Therefore, in the case of transformers with large turns ratio, the windings would also need to be sufficiently isolated. This introduces a constraint on the space needed for a winding. Here lies the challenge in transformer design—achieving maximum efficiency at lowest possible size while ensuring acceptable electrical isolation.

With this background, let us choose a sample construction for our three-phase transformer. Let us assume that the transformer core consists of three limbs as shown in Fig. 5.6 [13, 14]. The primary and the secondary windings of a phase are wound on one limb. Figure 5.6 is just a simplified schematic, and the actual construction will be much more complicated taking into account the isolation between the primary and the secondary. By assuming such a construction, we are using only the basic logic in transformer construction—maximizing the coupling between windings of a phase while allowing for sufficient space for windings and ensuring electrical isolation. As an example, the windings of phase $a - 1$ and 2 are wound on the same limb ensuring maximum coupling between them and therefore minimal leakage of flux. By winding all three phases on the same core, we are maximizing the use of the iron core and therefore, such a three-phase transformer will be less bulky as compared to three single-phase transformers.

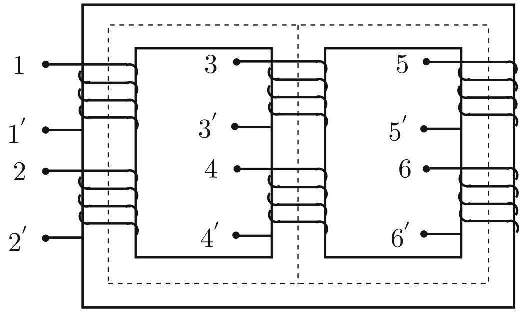

Fig. 5.6 Three-phase transformer wound on a three-limbed core

If the three-phase transformer had been constructed as three single-phase transformers, there would have been no magnetic coupling between the windings of different phases. However, with all three phases wound on the same core, there will be some coupling between the windings of different phases [27]. From Fig. 5.6, the primary and the secondary windings of a phase have been wound on the same limb, and therefore all the flux generated by one of the windings will completely link with the other except for the leakage flux. However, the flux generated by a winding of one of the phases will not completely link with the winding of another phase wound on another limb. The reader is encouraged to refer the previous chapters, where the magnetic circuit has been solved to determine the core flux. If we assume the reluctance of each limb of the core to be the same, the flux generated by a winding in one limb will divide equally into the other two limbs.

The mutual inductance between two windings can be expressed as

$$M_{xy} = k\sqrt{L_x L_y} \tag{5.9}$$

with k being the proportion of the flux generated by a winding that links with the other winding. It is important to remember that k does not include the leakage flux as leakage is accounted by the leakage inductance of the winding. Therefore, between winding 1 and 2 of phase a, the coupling factor is $k = 1$ while between winding 1 of phase a and winding 3 of phase b, the coupling factor is $k = 0.5$. We are making the assumption that all three limbs of the core have the same reluctances. If we choose to calculate the reluctance of each limb with precise dimensions, the left and the right limbs might have a slightly larger reluctance than the central limb. In such a case, the coupling factor k will not be 0.5 but will be larger than 0.5 if we consider the mutual inductance between a winding on one of the extreme limbs and the central limb; but it will be smaller than 0.5 if we consider the mutual inductance between windings on the extreme limbs. For simplicity, we will assume all three limbs to have the same reluctance leading to a convenient value of $k = 0.5$.

The above discussion on transformer construction was essential to understand how to compute the mutual inductance between the windings of the transformer. In the case of high power transformers with several windings, it might be a bit confusing to determine the extent of magnetic coupling between the windings. All that needs to be known is the basic construction of the transformer core and manner in which the windings are wound on the core. The coupling factor between two windings is merely the ratio of the flux produced by a winding that links with the other. This coupling factor can be determined by knowledge of the reluctances of the core segments and solving the magnetic circuit with the reluctances as series–parallel combinations. However, for a complete simulation model, it is important that we take into account all mutual inductances. It is tempting to assume that the windings between phases are not magnetically coupled. However, such an assumption is equivalent to simulating three single-phase transformers with separate cores for each phase.

With the above background, we can now compute the mutual inductances between the windings as

```
# Mutual inductance within windings of a phase
M12 = math.sqrt(Lm1 * Lm2)
M34 = math.sqrt(Lm3 * Lm4)
M56 = math.sqrt(Lm5 * Lm6)
# Mutual inductance between phases
# Half of the flux generated in one limb
# links with a winding in another limb
k_factor = 0.5
M13 = k_factor * math.sqrt(Lm1 * Lm3)
M14 = k_factor * math.sqrt(Lm1 * Lm4)
M15 = k_factor * math.sqrt(Lm1 * Lm5)
M16 = k_factor * math.sqrt(Lm1 * Lm6)
M23 = k_factor * math.sqrt(Lm2 * Lm3)
M24 = k_factor * math.sqrt(Lm2 * Lm4)
M25 = k_factor * math.sqrt(Lm2 * Lm5)
M26 = k_factor * math.sqrt(Lm2 * Lm6)
M35 = k_factor * math.sqrt(Lm3 * Lm5)
M36 = k_factor * math.sqrt(Lm3 * Lm6)
M45 = k_factor * math.sqrt(Lm4 * Lm5)
M46 = k_factor * math.sqrt(Lm4 * Lm6)
```

With the self-, leakage and mutual inductances computed, we can compute the series resistance, core loss resistance and the interfacing resistance for each winding:

```
r1 = 0.01 * Z1rated
r2 = 0.01 * Z2rated
r3 = 0.01 * Z3rated
r4 = 0.01 * Z4rated
r5 = 0.01 * Z5rated
r6 = 0.01 * Z6rated
Rc1 = V1rated * V1rated / (0.01 *VArated)
Rc2 = V2rated * V2rated / (0.01 *VArated)
Rc3 = V3rated * V3rated / (0.01 *VArated)
Rc4 = V4rated * V4rated / (0.01 *VArated)
Rc5 = V5rated * V5rated / (0.01 *VArated)
Rc6 = V6rated * V6rated / (0.01 *VArated)
res_output1 = 100.0 * Z1rated
res_output2 = 100.0 * Z2rated
res_output3 = 100.0 * Z3rated
res_output4 = 100.0 * Z4rated
```

```
res_output5 = 100.0 * Z5rated
res_output6 = 100.0 * Z6rated
```

All the parameters of the transformer equivalent circuit have now been computed. We can now define the matrices of the matrix equation, which will be simplified and solved using numerical integration:

```
L = [
    [L1, M12, M13, M14, M15, M16],
    [M12, L2, M23, M24, M25, M26],
    [M13, M23, L3, M34, M35, M36],
    [M14, M24, M34, L4, M45, M46],
    [M15, M25, M35, M45, L5, M56],
    [M16, M26, M36, M46, M56, L6]
]
R = [
    [r1, 0.0, 0.0, 0.0, 0.0, 0.0],
    [0.0, r2, 0.0, 0.0, 0.0, 0.0],
    [0.0, 0.0, r3, 0.0, 0.0, 0.0],
    [0.0, 0.0, 0.0, r4, 0.0, 0.0],
    [0.0, 0.0, 0.0, 0.0, r5, 0.0],
    [0.0, 0.0, 0.0, 0.0, 0.0, r6]
]
B = [
    [1.0, 0.0, 0.0, 0.0, 0.0, 0.0],
    [0.0, 1.0, 0.0, 0.0, 0.0, 0.0],
    [0.0, 0.0, 1.0, 0.0, 0.0, 0.0],
    [0.0, 0.0, 0.0, 1.0, 0.0, 0.0],
    [0.0, 0.0, 0.0, 0.0, 1.0, 0.0],
    [0.0, 0.0, 0.0, 0.0, 0.0, 1.0]
]
```

As can be seen from all the computations above, most of them are merely repetitions. Only the computation of the mutual inductances need some knowledge of the transformer construction. Subsequently, construction of the matrices of the matrix equation that represents the mathematical model of the transformer is a mechanical process that can be easily extended for large transformers with many windings. The matrix `R` is a diagonal matrix with the elements of the diagonal being the winding resistances while the matrix `B` is an identity matrix. The inductance matrix `L` is a well structured matrix with the diagonal elements being the self-inductance of the winding and the off-diagonal elements being the mutual inductances between two windings. Once these matrices have been defined, their simplification so as to convert `L` to an upper triangular form and the numerical integration is the same as before.

Once the winding currents have been computed by numerical integration, the voltages of the controllable voltage sources that are used to represent each winding can then be computed as

```
vout1 = v1 - winding_currents[0]*res_output1
vout2 = v2 - winding_currents[1]*res_output2
vout3 = v3 - winding_currents[2]*res_output3
vout4 = v4 - winding_currents[3]*res_output4
```

```
vout5 = v5 − winding_currents[4]*res_output5
vout6 = v6 − winding_currents[5]*res_output6
```

In this section, we have demonstrated how a three-phase transformer can be simulated by merely extending the basic transformer model. Furthermore, a fairly robust simulation model can be generated by taking into account some basic aspects of the transformer construction that will affect the coupling between the transformer windings. However, the transformer can be simulated using only the basic specifications provided by the manufacturer—the maximum rated power in VA, the maximum line voltage rating of each interface of the transformer and the connection type of the transformer windings. The reader can either refer to construction details provided by the manufacturer to compute parameters such as the mutual inductances between windings or can make reasonable assumptions. In the next section, let us examine the simulation results of the above model and further discuss how with a few minor modifications, three-phase transformers with other winding connections can also be simulated.

5.4 Simulation Results of Three-Phase Transformers

In the previous section, we examined in detail how a simulation model for a three-phase transformer can be developed with only the basic manufacturer specifications. In this section, we will examine the results of the simulation model in the previous section while also examining how transformers with different winding connections can be simulated. As will be seen, the modifications are surprisingly minor allowing us to use this model for simulating a wide number of three-phase transformers used in the modern power system.

With the transformer model developed in the previous section, we can simulate the circuit of Fig. 5.5. Figures 5.7 and 5.8 show the line voltages in the primary and secondary of the transformer. The peaks of the voltages can be seen to correspond to RMS values of 415 V for the primary and 11 kV for the secondary. The voltage sources connected to the primary have RMS magnitudes of 240 V. It is due to the star connection that the line voltages applied to the primary of the transformer are $\sqrt{3}$ times the RMS magnitude of the voltage sources connected in the phases. By computing `Vrated_primary` and `Vrated_secondary` to be $\frac{1}{\sqrt{3}}$ of the line voltages, the rated voltages of the windings are maintained equal to the RMS magnitudes of the phase voltages instead of the line voltages. As stated in the previous section, this computation is very important as any three-phase system voltage magnitude is usually specified as the RMS value of the line voltage, but to compute transformer equivalent circuit parameters, the rated voltage of the windings must be calculated as phase voltages.

Figures 5.9 and 5.10 show the primary and secondary phase voltages, respectively. In contrast to line voltages in Figs. 5.7 and 5.8, the RMS magnitudes of the phase voltages are 240 V for the primary and 6.35 kV for the secondary. The reader

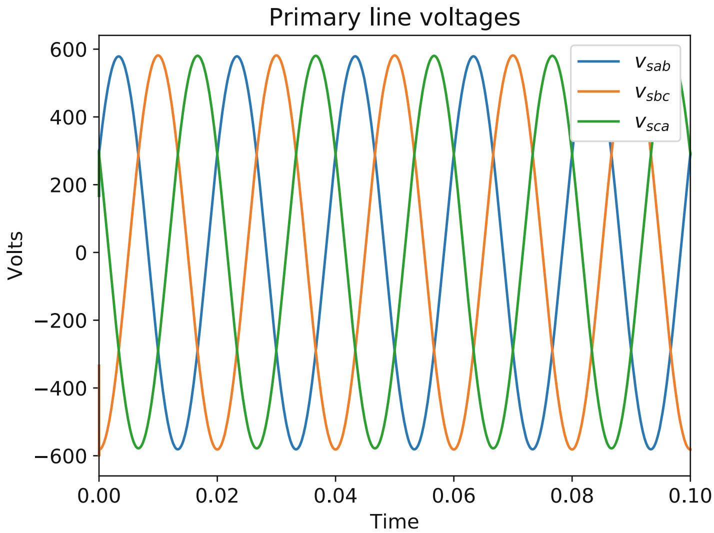

Fig. 5.7 Transformer primary line voltages

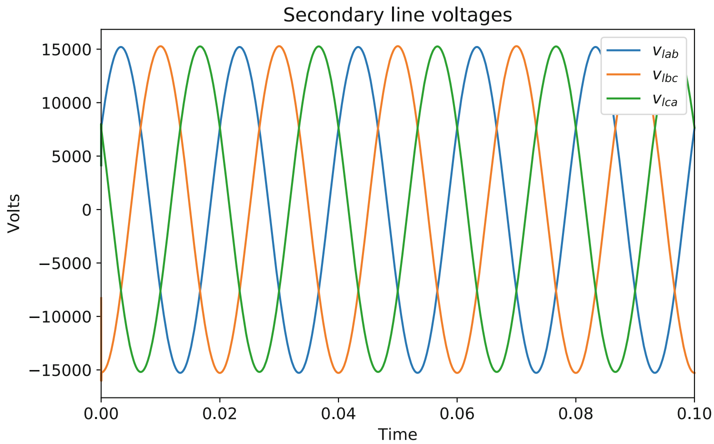

Fig. 5.8 Transformer secondary line voltages

is encouraged to plot the phase voltage v_{sa} and the line voltage v_{sab} together to verify that v_{sab} leads v_{sa} by a phase angle of 30°. Figures 5.11 and 5.12 show the primary and secondary currents, respectively. The primary currents have a dc offset due to the magnetizing currents having a dc offset with the primary voltages being sine waveforms.

The reader is encouraged to apply different voltages to the primary of the transformer and verify the secondary voltage that is produced is according to the turns ratio specified. The three-phase transformer that has been simulated is a

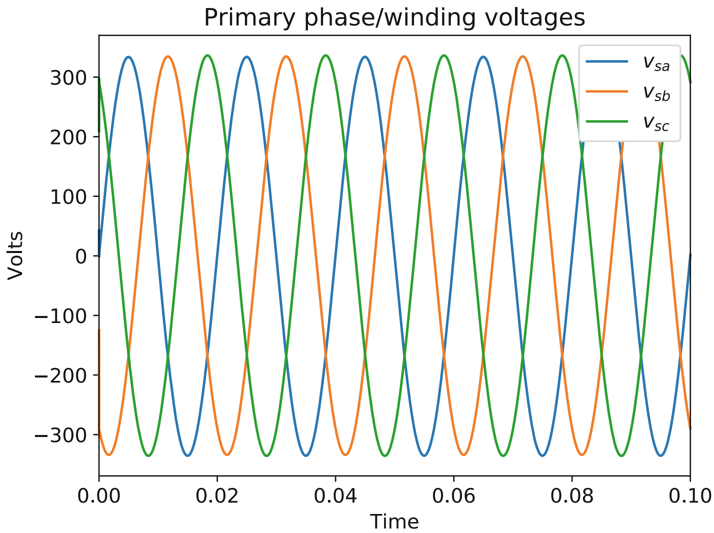

Fig. 5.9 Transformer primary phase voltages

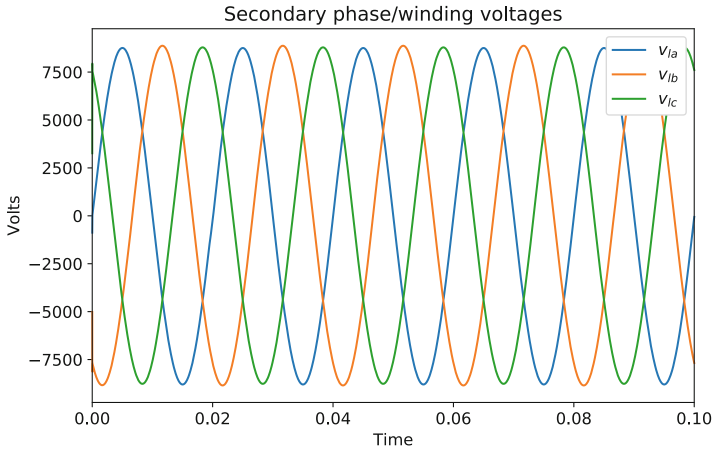

Fig. 5.10 Transformer secondary phase voltages

star–star transformer, where both the primary and the secondary windings of the transformer are connected in star. Moreover, we have been dealing with balanced three-phase systems where the RMS magnitude of all three-phase voltages is the same, phase b lags behind phase a by 120° and phase c lags behind phase b by 120°. In domestic distribution systems, where each phase of a three-phase system might be supplying loads in different parts of a house or an office, the chances are that the three-phase system will be unbalanced. This is due to the fact that the

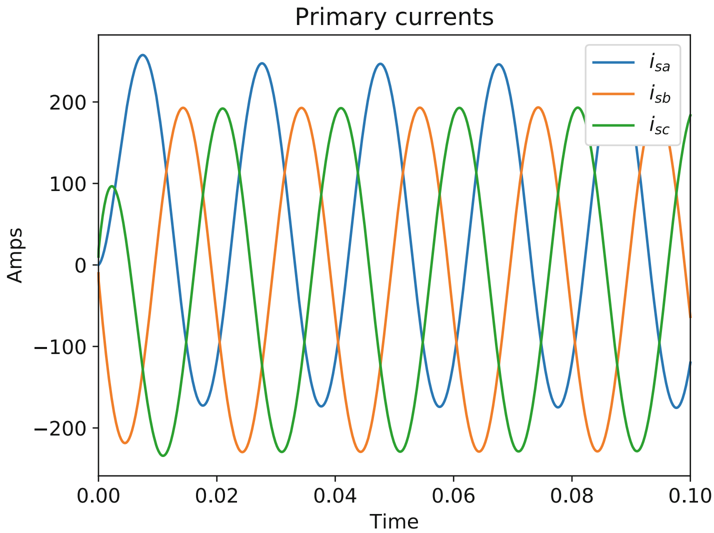

Fig. 5.11 Transformer primary currents

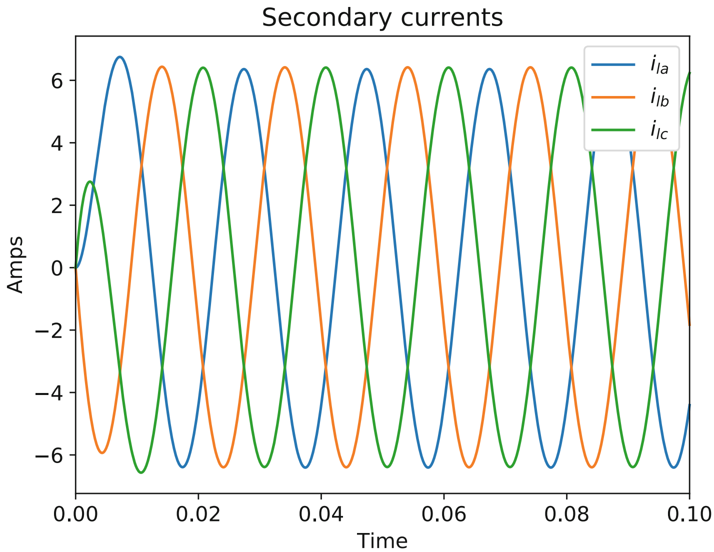

Fig. 5.12 Transformer secondary currents

currents drawn by unbalanced loads will in turn produce unbalanced voltages due to the unbalanced voltage drops across feeders.

A very popular connection strategy with respect to three-phase transformers is when either the primary or the secondary is connected in star and the other is connected in delta [13]. These transformers are called star–delta or delta–star transformers. These transformers are very popular when one of the subsystems needs to be a four-wire system with a neutral but the other need not [1]. An example is of a distribution transformer that supplies a house or building at 415 V from a 11 kV or a 22 kV distribution feeder. The three-phase supply at the house will need to have a neutral connection as domestic power supplies are single-phase and therefore, most appliances will be connected between a phase and a neutral. The 415 V three-phase supply will need to be star connected with a neutral. However, the three-phase system on the 11 kV or 22 kV does not need a neutral connection as that feeder is merely supplying power and is not feeding any loads that need a neutral. In such a case, connecting the 11 kV or 22 kV feeder in star with a neutral would unnecessarily require a four-wire feeder. If instead, we choose to connect the 11 kV or 22 kV windings of the transformer in delta, we would have a three-wire feeder, which would decrease the cost.

A four-wire system is required in certain cases such as while supplying loads that will need a neutral connection. However, in such cases, where the proportion of single-phase loads is high, a four-wire system is usually unbalanced. A current will flow in the neutral wire besides also the three-phase wires. This flow of current cannot be avoided as that is the requirement of the load. The question then arises—how would four currents map to three currents in a delta–star transformer, with the delta connected windings not having a neutral wire similar to the star connected windings? The current in the neutral on the star connected windings will circulate within the delta connected windings [13]. The line currents on the delta connected side will be devoid of this neutral current. This might seem a little confusing and will be demonstrated through a simulation soon.

Figure 5.13 shows a circuit for simulating a three-phase delta–star transformer. This simulation can be found in the folder `transformer_delta_star` within `chapter5_transformer_applications` in the following link in the simulation repository: https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics

The primary windings of the transformer are connected in delta and supplied by a three-phase voltage source. The secondary of the transformer is connected

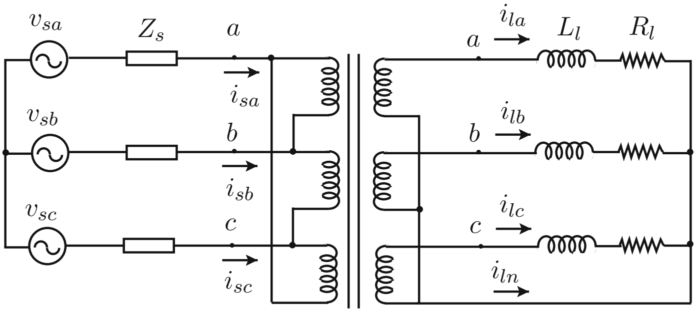

Fig. 5.13 Circuit for simulation of a delta–star transformer

in star and is supplying a three-phase load that is connected in star. To begin with, let us consider a balanced load, and therefore, the neutral connection on the secondary carries no current. Soon, we will introduce an unbalance in the load and repeat the simulation. Let us choose the transformer to have a maximum power rating of 50 kVA. Let us choose the primary (delta) of the transformer to have a maximum voltage rating of 11 kV and the secondary (star) of the transformer to have a maximum voltage rating of 415 V. With these basic specifications, let us simulate the circuit of Fig. 5.13.

We can get started with translating the transformer power and voltage rating to the simulation model of the transformer:

```
import math
dt = 1.0e-6
Vrated_primary = 11000.0
Vrated_secondary = 415.0 / math.sqrt(3.0)
# Phase a - Windings 1 and 2
# Phase b - Windings 3 and 4
# Phase c - Windings 5 and 6
VArated = 50000.0 / 3     # Volt-Amperes
# Primary windings
V1rated = Vrated_primary
V3rated = Vrated_primary
V5rated = Vrated_primary
# Secondary windings
V2rated = Vrated_secondary
V4rated = Vrated_secondary
V6rated = Vrated_secondary
frated = 50.0        # Hz
omega_rated = 2*math.pi*frated
```

The per-phase voltage rating of the primary is the same as the voltage level of the three-phase system because the primary windings of the transformer are connected in delta due to which the phase and line voltages have the same RMS magnitude. On the other hand, the per-phase voltage rating of the secondary is $\frac{1}{\sqrt{3}}$ times the voltage level of the three-phase system because the secondary windings are connected in star. Just as in the previous simulation, we need to compute the maximum per-phase power rating by dividing the total three-phase maximum power rating by 3. The above changes are the only changes that need to be made to the simulation model. All other computations will remain the same as the computation of all other parameters are made on a per-phase basis using these basic power and voltage ratings.

Figures 5.14 and 5.15 show the primary and the secondary line voltages of the three-phase transformer. The peaks of the waveforms correspond to the line voltage ratings of the primary and the secondary of the transformer. Figures 5.16 and 5.17 show the primary and secondary line currents of the three-phase transformer. As in the previous simulation, the currents in the primary have dc offsets due to the magnetizing currents drawn by the primary winding. The reader is encouraged to plot the waveforms of the other measured quantities such as the phase voltages and phase currents.

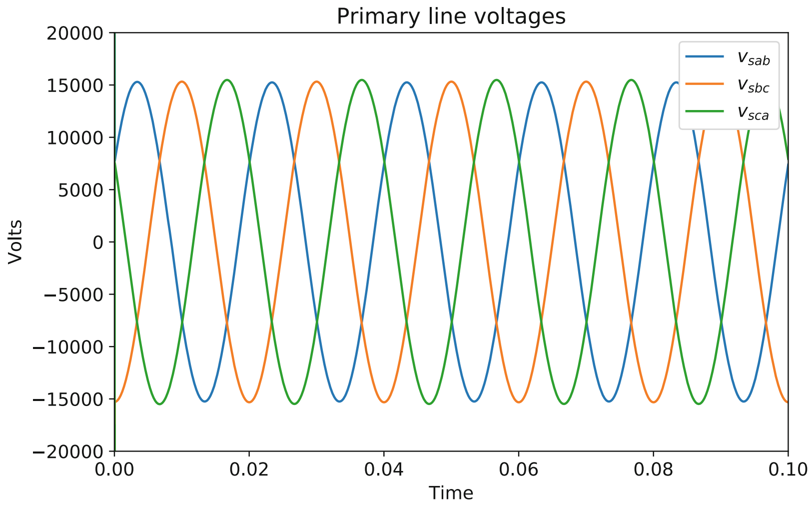

Fig. 5.14 Transformer primary line voltages

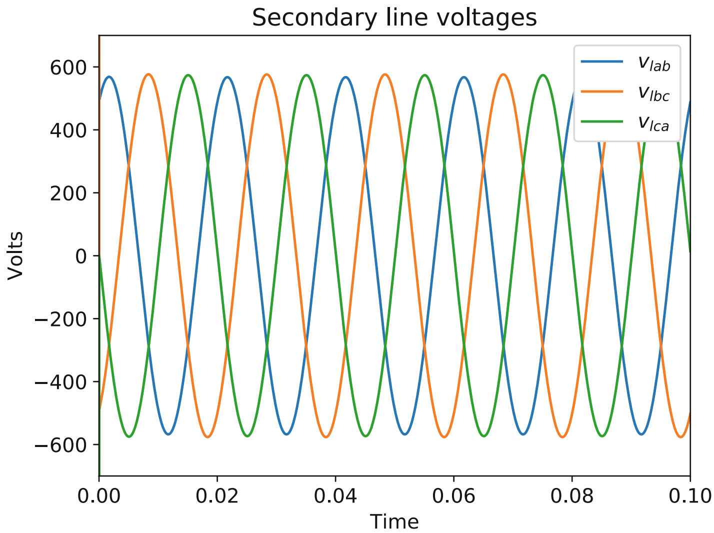

Fig. 5.15 Transformer secondary line voltages

The operation of the delta–star transformer is most interesting when we consider the case of the load on the star connected windings being unbalanced. Let us make the load impedance in phase b to be 1.5 times the load impedance in phase a, and the load impedance in phase c to be 2 times the load impedance in phase a. We have now introduced a fairly severe unbalance between the three phases of the load.

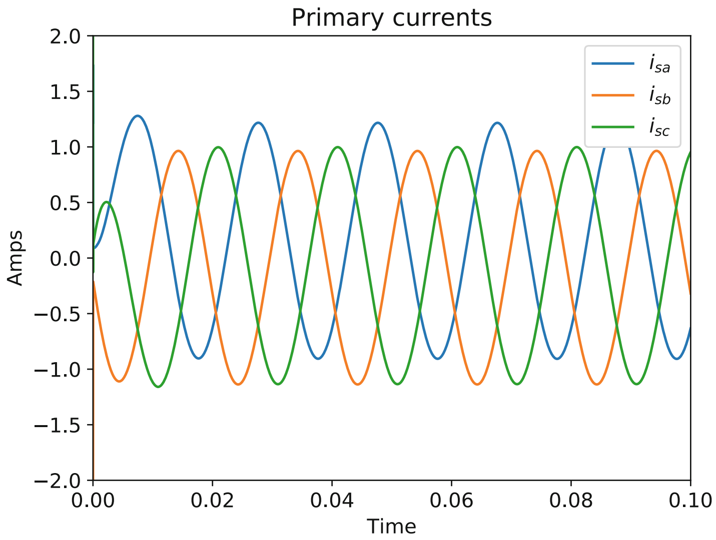

Fig. 5.16 Transformer primary line currents

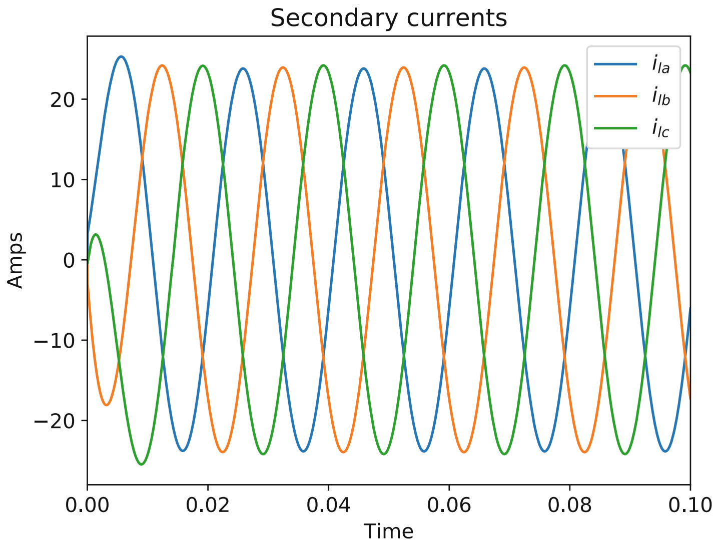

Fig. 5.17 Transformer secondary line currents

Under these unbalanced conditions, the current i_{ln} will now be a non-negligible value from the expression:

$$i_{ln} = -(i_{la} + i_{lb} + i_{lc}) \tag{5.10}$$

Since the secondary of the transformer is connected in star, both the phase and line currents in the secondary will be unbalanced and will have a component $\frac{i_{ln}}{3}$, which is called the zero sequence component. The zero sequence component can only flow in the secondary transformer windings if a referred current $\frac{N_2}{N_1}\frac{i_{ln}}{3}$ ($\frac{N_2}{N_1}$—transformer turns ratio) can flow in the primary windings as well [1]. This is fundamental to the operation of any transformer and follows from the theory described in the previous chapter. If the voltage applied to the primary windings is approximately constant, the core flux will remain approximately constant as well. Therefore, any current flowing in the secondary winding due to a load being connected will have a demagnetizing effect. This demagnetizing effect has to be neutralized by a current flowing in the primary winding that has a nullifying magnetizing effect.

If the secondary windings of the transformer were to be connected in star with a neutral wire, that alone does not imply that a current can flow in the neutral wire due to the load being unbalanced [1]. If a current flows in the neutral wire, a zero sequence component must also flow in the three phases of the secondary. And this can only be possible if a referred zero sequence component flows in the primary windings as well. Not all transformer connections will permit this. For example, the star–star transformer of Fig. 5.5 that has no neutral wire in the primary or secondary windings cannot support a zero sequence current. Even if the secondary windings of Fig. 5.5 had a neutral wire that was connected to the neutral of the load, such a connection would not result in the flow of a neutral current. This is due to the fact that the neutral current will require zero sequence components to flow in the secondary windings as well. With the primary of the transformer not having a neutral wire, referred zero sequence components cannot flow in the primary windings.

In the case of the delta–star transformer in Fig. 5.13, a very interesting case follows. Due to the primary of the transformer being connected in delta, the three-phase system on the primary cannot have a neutral wire. Therefore, it is easy to jump to the conclusion that a delta–star transformer will not support zero sequence currents just like a star–star transformer without any neutral on the primary. However, the zero sequence currents can circulate within the delta connection as shown in Fig. 5.18. Due to the nature of connection of the primary windings, a zero sequence component can flow as

$$ki_{in} = i_{wa} + i_{iwb} + i_{wc} \tag{5.11}$$

The currents in each winding i_{wa}, i_{wb}, i_{wc} will have zero sequence components equal to $\frac{i_{ln}}{3}$. However, the line currents i_{sa}, i_{sb}, i_{sc} will not have these zero sequence

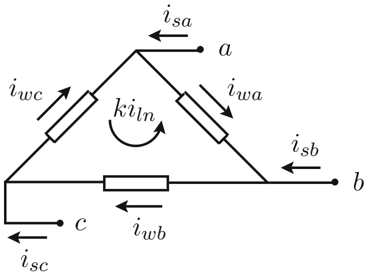

Fig. 5.18 Zero sequence component circulating in delta connected windings

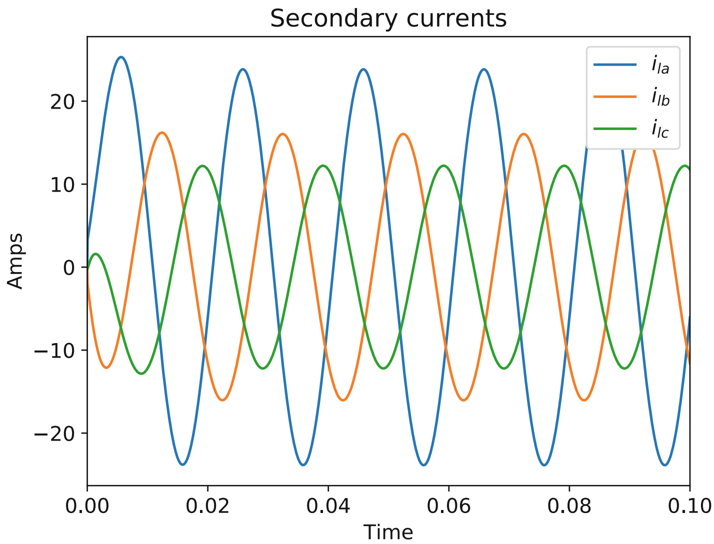

Fig. 5.19 Transformer secondary line currents—unbalanced

components as the zero sequence components in the windings will cancel out as follows:

$$
\begin{aligned}
i_{sa} &= i_{wa} - i_{wc} \\
i_{sb} &= i_{wb} - i_{wa} \\
i_{sc} &= i_{wc} - i_{wb}
\end{aligned}
\tag{5.12}
$$

Therefore, a delta–star (or a star–delta) transformer is a very useful transformer when one three-phase system must support an unbalanced system but the other three-phase system need not. Let us examine this with a modified simulation.

Figure 5.19 shows the simulation results with the load currents in the star connected secondary being unbalanced sine waveforms of different RMS magnitudes.

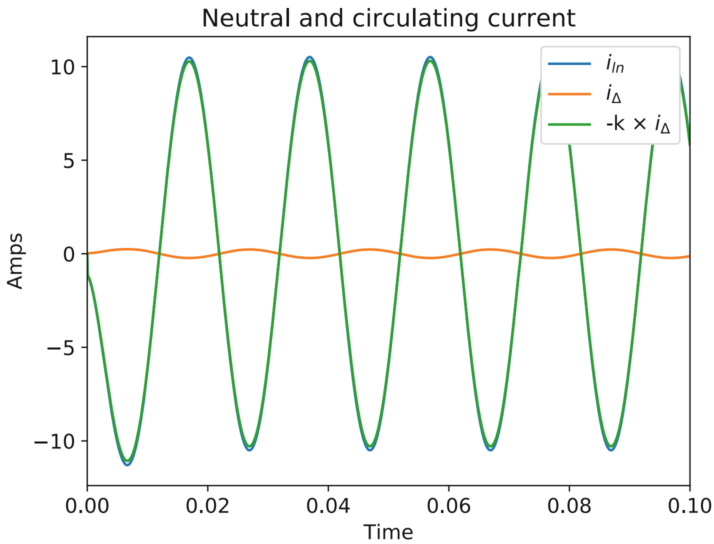

Fig. 5.20 Secondary neutral current and circulating current in the primary

The neutral current in the secondary can be measured using the component Ammeter_loadneutral. However, the circulating current in the delta connected primary cannot be directly measured by an Ammeter as this circulating current is due to the zero sequence component present in the winding currents. We can calculate this circulating current by computing in the simulation model the sum of the currents measured by Ammeter_T1W1, Ammeter_T1W3 and Ammeter_T1W5. We will need to configure the control file with three additional inputs that are connected to the measurements of Ammeter_T1W1, Ammeter_T1W3 and Ammeter_T1W5 and designated by the input variables `iprim1`, `iprim2` and `iprim3`, respectively.

Figure 5.20 shows the current in the secondary neutral along with the circulating current i_Δ in the delta connected primary expressed as

$$i_\Delta = i_{wa} + i_{wb} + i_{wc} \tag{5.13}$$

The circulating current in the delta connected primary is much smaller than the neutral current in the secondary due to the turns ratio of the transformer. To compare the circulating current with the neutral current, the circulating current is multiplied by a value of 45 close to the turns ratio of the transformer 45.83. This is so that the waveforms are not exactly superimposed but are close enough to visually verify that they are equal. Furthermore, since the neutral current is the negative sum of the three secondary line currents, the circulating current is inverted. From Fig. 5.20, it is very clear that the neutral current in the secondary is circulating in the delta connected primary windings.

In this section, we have examined in detail the simulation of two different three-phase transformers. It is clear that the transformer model developed in the previous chapter is extremely flexible and can be modified to simulate different types of transformers. Besides the mathematical model, the only other change that needs to be made is the physical connection in the circuit schematics between transformer windings. The reader is encouraged to try simulating other combinations of three-phase transformers such as the star–delta transformer or the star–star transformer with and without a neutral wire.

5.5 High Frequency Transformers

Until now, all the transformers that have been simulated have been line frequency (50 Hz or 60 Hz) transformers. These transformers are critical in the power system and enable efficient bulk power transmission over long distances by stepping up the voltage to hundreds of kilovolts. With the proliferation of portable electronic appliances such as laptops, mobiles, tablets, etc., the power supplies for these have been steadily getting smaller and lighter. Within these power supplies are transformers that would have been completely unnoticed unless the power supplies are taken apart for repair [10, 26]. This application of the transformer is also extremely interesting, especially for a power electronics engineer. In this section, we will introduce the concept of these power supply transformers and describe how they are different from line frequency transformers.

Before we describe transformers used in power supply applications that contain power electronic converters, let us describe the impact of high frequencies on transformers [1, 13]. When a winding of a transformer is energized with an ac voltage of some frequency f, a magnetic flux will flow in the core of the transformer that will also be an ac quantity of the same frequency f. Let us express this core flux as the following sinusoid:

$$\phi = \phi_m \sin 2\pi f t \tag{5.14}$$

With the core flux being an ac quantity, emfs will be induced in all the windings of the transformer, which will be directly proportional to the rate of the change of core flux:

$$e_x = N_x \frac{d\phi}{dt} = 2\pi N_x \phi_m f \cos 2\pi f t \tag{5.15}$$

From the above expression, for a given winding with N_x turns, the product $\phi_m f$ contains the variables that impact the magnitude of the induced emf as frequency changes. For a fixed magnitude ϕ_m of core flux, the induced emf will increase as the frequency increases.

Let us consider the simple case of a transformer being energized by a single supply whose voltage magnitude is equal to the maximum rated voltage of the winding being energized. As we have already described in the previous chapter, the emf induced in this winding will be expressed as

$$e_x = v - i_x r_x \approx v \tag{5.16}$$

As it is a fair approximation to neglect the voltage drop across the winding resistance, the induced emf is approximately equal to the voltage applied across the winding.

From the above equations, it follows that the peak V_m of the applied voltage can be expressed as

$$V_m \approx 2\pi N_x \phi_m f \tag{5.17}$$

Subsequently, we can ignore the constants and the number of turns N_x of the winding as a construction constant, to result in the following expression:

$$V_m \propto \phi_m f \tag{5.18}$$

This final proportionality expression describes the impact of frequency on a transformer. As the frequency f increases, the magnitude of the core flux ϕ_m will decrease. For a particular construction of the core and a given cross-sectional area A_c of the core, the flux density B will decrease as a result of the increase in frequency. In Chap. 2, we had described the B-H curve of magnetic materials. Such B-H curves are available for most commonly used magnetic materials besides also being available from the manufacturer supplying the core or the magnetic material for the core. For every magnetic material, there exists for a particular temperature and other conditions, a value of flux density B beyond which the core saturates.

If the flux in the core is such that the flux density B exceeds this saturation value, the resultant core saturation will cause increased losses and raised temperatures, which in turn can cause the magnetic properties of the core to deteriorate. However, it is also advisable to allow the maximum flux in the core to attain a value such that the flux density will be slightly lower than the saturation value. In this manner, the transformer core is fully utilized resulting in an optimized design [13]. Due to increased frequency, the core flux decreases in magnitude. Instead of allowing the flux density in the core to drop to a very low value, the size of the core can be decreased such that the maximum flux density will continue to be slightly lower than the saturation value of flux. Therefore, with increased frequency, the size of the core decreases resulting in smaller transformers and thereby smaller power supplies [10, 26].

There are a few other effects of high frequency operation that require high frequency transformers to be very different in construction with respect to line frequency transformers. The ac flux in the transformer core results in an induced emf in the windings of the transformer. However, besides the windings, emfs are induced in the core itself as core materials such as iron are conductors. This is due to the fact that Faraday's Law is applicable to any conductor that is subjected to a changing magnetic field and this includes the core and not just the windings. Due to the emfs induced in the core, currents called eddy currents flow in the core [13]. These currents have circular paths and flow in planes perpendicular to the direction of the magnetic flux. The direction of these currents can be deciphered by using Lenz's Law—they will be in such a direction that they oppose the cause (changing flux) that produces them.

These eddy currents are quite obviously undesirable as there is no benefit in currents flowing in the core—only the voltages at the windings are useful. These eddy currents result in ohmic losses in the core and result in increased core temperatures. In our simulation model, we have captured eddy current losses using the core loss resistor R_c. To minimize the heating effect of these eddy currents, the iron core is divided into sheets or laminations that are coated with insulating materials. Therefore, instead of using a solid block of iron, the transformer core is comprised of a number of iron laminations insulated from one another with a thin layer of insulation [13]. This increases the resistance for eddy currents to flow and therefore decreases eddy current losses.

Another major core loss component in transformers is the hysteresis loss [13]. In Chap. 2, we have described the B-H curve. We have described how the ideal linear B-H curve does not exist in practical cores as when a core is magnetized, it tends to retain its magnetism. This implies that as the magnetic field strength H increases, the flux density B will not increase linearly but will increase at a lower rate. Similarly, when H decreases, B will not follow linearly as the core will retain some of its magnetism and therefore will decrease at a lower rate. This retention of the core produces a loss that is called as the hysteresis loss. In the simulation model, the hysteresis loss was included in the loss resistor R_c.

Since eddy currents are produced due to the changing magnetic flux in the core, as the frequency of the supply increases, the eddy currents will increase in magnitude and so will eddy current losses [13]. The same applies for hysteresis loss as well [13]. With increased frequency, the loss resulting from the retention of the core increases. As we progress to very high frequency operation in thousands of Hertz, eddy current and hysteresis losses will be unacceptably high even in iron cores that are laminated. For this reason, high frequency transformers use cores made from ferrite that is a ceramic composed of iron oxide and several other metals [15]. Ferrite cores have magnetic properties similar to iron cores but are not conductive, which results in decreased eddy current losses at high frequencies.

Ferrite cores are available in numerous shapes (circular, rectangular, C-shaped, E-shaped, etc.) and sizes for ready use in winding inductors or transformers [15]. In addition, a number of high frequency inductors and transformers of standard ratings are available for standard power supply applications. A power electronics engineer can either purchase a ready-made inductor or transformer, or can use ferrite cores available in different shapes and construct custom-designed transformers and inductors. All concepts in design for high frequency inductors and transformers are exactly the same as for line frequency inductors and transformers. An engineer need only remember three laws—Kirchhoff's Voltage Law, Faraday's Law and Lenz's law to translate the windings and the core to a magnetic circuit.

In this section, we have described the basic differences between high frequency transformers and grid frequency transformers. The major advantage of a high frequency transformer is the decreased size due to the lower flux magnitude, which results in a smaller core. On the other hand, high frequency operation needs the transformer core to be composed of a special material such as ferrite to minimize core losses that are proportional to frequency. In the next section, we will describe the operation of the high frequency transformer using a very popular converter called the flyback converter.

5.6 Flyback Converter

In the previous section, we described some of the broad differences between grid frequency transformers and high frequency transformers. The philosophy behind the operation of a high frequency transformer is the same as that for a grid frequency transformer. The difference lies in the application, whereby high frequency transformers are largely used in conjunction with power electronic converters due to which they are energized by a switched voltage in contrast to grid frequency transformers that are energized by a voltage that has roughly a sine waveform [10, 26]. Depending on the power electronic converter, the type of switched voltage a high frequency transformer is subjected to can vary drastically. In this section, we will examine a very popularly used converter—the flyback converter [19, 20].

Figure 5.21 shows the topology of the flyback converter [19, 20]. What makes a flyback converter such an interesting power converter is, that at first glance, one might think such a converter will never work. The flyback converter consists of a dc voltage source such as a battery energizing the primary winding of a high frequency transformer through a power device S such as a MOSFET. The gate signals to

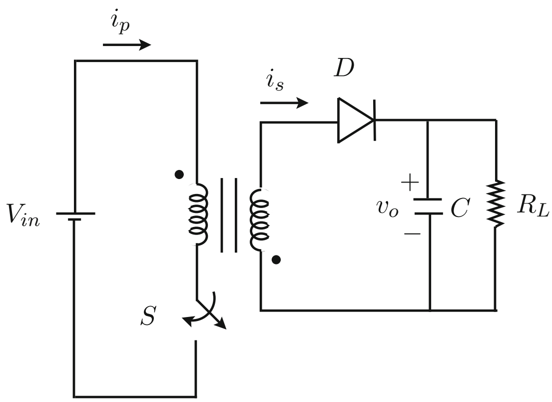

Fig. 5.21 Topology of a flyback converter

the power device can be turned on and off such that the voltage applied across the primary winding is a switched voltage. It is extremely important to notice the dot polarities of the transformer windings—for the primary the dot is on the upper terminal while for the secondary the dot is on the lower terminal. The secondary of the transformer feeds a load R_L connected across an output capacitor C through a Diode D. It is again extremely important to notice that the anode of the diode is connected to the terminal of the secondary that does not have a dot.

As with all power converters, in order to regulate the output voltage produced, the conduction and non-conduction time intervals of the power device S need to be regulated. The literature of control strategies proposed for the flyback converter is vast [19, 20] and the simulation covered in this chapter will not include closed-loop control. Since our purpose is merely to understand the operation of the high frequency transformer, all we need is to implement an open-loop switching strategy. We already covered an example of a Ćuk converter in Chap. 3 where the power device was operated at a constant switching frequency during which it was provided gate signals for a fraction of the switching time period such that the device was conducting and for the remaining part of the switching time period, gate signals are removed and the device does not conduct. This is one of the simplest switching strategies and can be implemented as a Pulse Width Modulation (PWM) scheme with a constant frequency carrier waveform and a duty ratio. There have been many variations proposed to this scheme, and the reader is encouraged to try and implement them.

Figure 5.22 shows the operation of the flyback converter through the waveforms of the three main quantities—the current in the primary winding, the current in the secondary winding and the output voltage. Let us assume for simplicity that the input voltage V_{in} is constant. In Fig. 5.22, the time interval for which the power

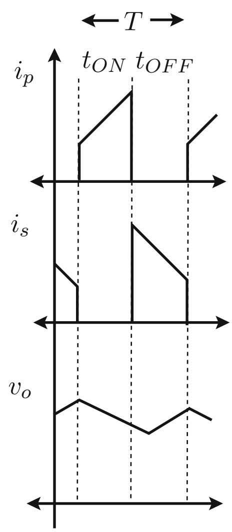

Fig. 5.22 Working of a flyback controller

device is conducting is marked as t_{ON} and the time interval when non-conducting is marked as t_{OFF}. When the power device is conducting, the input voltage appears across the primary winding of the transformer. As with any transformer, an induced emf will be produced in the winding that will be equal and opposite to the applied voltage. Therefore, the induced emf will have a polarity such that the upper terminal of the primary has a positive polarity with respect to the lower terminal.

Due to the dot polarities of the transformer windings, the emf induced in the secondary terminal will be such that the voltage at the terminal with the corresponding dot (lower terminal) will be positive with respect to the upper terminal. The reader should use the theory presented in Chaps. 3 and 4 to verify this concept. As a result, the diode will be reverse biased and will not conduct. Since only one winding of the transformer is conducting, the current drawn by the primary winding is the magnetizing current alone, which increases as shown in Fig. 5.22. During this interval (t_{ON}), the current i_s in the secondary is zero. The output voltage v_o across capacitor C will decrease as it discharges through the load resistor R_L.

When the gate signals to the power device S are withdrawn and the power device stops conducting, we are now in the interval marked by t_{OFF}. One might think that nothing will happen during this interval as the input voltage V_{in} has been disconnected from the transformer. However, we must remember that in the previous interval t_{ON}, due to the flow of the magnetizing current, a flux will flow in the transformer core and energy is stored in the magnetic field of the transformer. This stored energy will try to find a way to continue to flow. The reason—Faraday's Law states that any change in the magnetic field associated with a conductor (winding) will produce an induced emf. Therefore, if the power device has stopped conducting, and the primary is no longer energized by the input voltage V_{in}, the core flux will fall due to the loss of the energy source. This will result in emfs induced in both the primary and the secondary.

The next question—what will be the nature or polarity of the induced emf? For this, we use Lenz's Law—the emfs induced will be such so as to oppose the cause that produced them. The cause in this case is the falling core flux, which implies that the emfs will attempt to restore the core flux. The reader is again advised to review the theory behind dot polarity conventions. The dots on the terminals of the transformer indicate that currents flowing in the two windings will have the same effect if they were both to enter (or both to leave) the terminals with the dots. During the period t_{ON} of conduction, the current i_p was flowing in the primary winding such that it entered the primary at the upper terminal marked by the dot. During the period, the magnetic flux rose in the transformer core as the magnetizing current i_p increased. This implies, that if the core flux were to be made to increase, a current must flow in either winding such that it enters one of the dots.

With the power device S being made to stop conducting by withdrawing the gate signals, a current cannot flow in the primary. This implies the only way to restore the core flux is for a current to flow in the secondary winding. If the induced emf in the secondary winding is such that the upper terminal of the secondary that does not have a dot has a positive voltage with respect to the lower terminal of the secondary, the diode will be forward biased if the induced emf is greater than the output voltage v_o. If the diode is forward biased and begins to conduct, a current i_s will flow in the secondary charging the capacitor C. This current i_s will flow into the lower terminal of the secondary marked by the dot thereby opposing the fall in core flux. Since the output capacitor C is being charged by the secondary current i_s, the output voltage v_o will increase. As the energy stored in the magnetic field is dissipating during this interval, the secondary current begins at a non-zero value and decreases, and the energy in the field dissipates.

As can be seen from the operation of the flyback converter described above, there are stark differences with respect to the normal operation of a transformer even though the basic laws of physics governing the operation of the transformer are the same. When the primary of the transformer is energized by the input voltage during t_{ON}, the Kirchhoff's Voltage Law expressed at the primary winding is as follows:

$$V_{in} = e_p + i_p r_p \tag{5.19}$$

where r_p is the primary winding resistance and e_p is the primary winding induced emf.

Faraday's Law applies to the high frequency transformer as well, and therefore induced emf can be expressed as the rate of change of primary winding flux linkages:

$$V_{in} = N_p \frac{d\phi}{dt} + i_p r_p \tag{5.20}$$

From the above expression, the transformer core flux will increase as it is approximately equal to the integral of the applied voltage. Subsequently, the primary current i_p will increase so as to provide the ampere turns necessary for the core flux to increase.

All the above expressions were very similar to the case of the grid frequency transformer. However, there is now a drastic difference between the flyback transformer and a normal grid frequency transformer. When the primary is energized and the core flux increases, the load is disconnected from the secondary winding. This is due to the dot polarity of the windings and the placement of the diode that causes it to be reverse biased. In a regular grid frequency transformer, the secondary usually supplies the load and therefore, a current flows in the secondary winding. This current in the secondary winding has a demagnetizing effect on the core flux and results in a current component drawn by the primary winding such so as to restore the core flux. In the case of the flyback transformer, with no current flowing in the secondary, the primary current and the core flux increase without any relation to the secondary.

This might almost seem like a violation, as until now, we had described how turns ratio can be used to equate the primary and the secondary currents of the transformer:

$$N_p i_p = N_s i_s \tag{5.21}$$

When the power device is conducting and $i_s = 0$, it might seem like the above equation is violated. However, we must remember that a transformer must draw a magnetizing current in order to maintain the magnetic field in the core. This magnetizing current is drawn by the winding that is energized and is unrelated to the current flowing in any other winding. In the case of the flyback transformer, the primary current is the magnetizing current of the transformer and is needed for the core flux to increase.

There is one more riddle in the operation of the flyback converter. How is it that the induced emf in the secondary winding attains a polarity such so as to cause the diode to conduct during the t_{OFF} interval? We must remember that, emfs are induced whenever the core flux changes. Therefore, an emf is induced in the secondary winding during the t_{ON} period as well when the core flux is increasing, and the magnitude of the secondary winding induced emf can be expressed as

$$e_s = N_s \frac{d\phi}{dt} \tag{5.22}$$

During the t_{ON} period, the induced emf in the primary winding is such so as to oppose the increase in the core flux. Since the dot polarities of the windings are chosen in a reverse fashion, the induced emf in the secondary winding will have such a polarity that it will attempt to produce a secondary current that will also oppose the increasing core flux. As per our theory of dot polarities, for the secondary current to oppose the increase in core flux, it would have to be in a direction opposite to that shown in Fig. 5.21. Due to the presence of the diode, such a current cannot flow. Therefore, the induced emf in the secondary winding though present has no effect.

During the t_{OFF} period, when the primary is no longer energized by the source V_{in}, the core flux will tend to decrease. The resultant induced emf in both the primary and the secondary windings will now be exactly the reverse of what they were during the t_{ON} period as now the induced emfs will such so as to oppose the decreasing core flux. During this period, it is the induced emf in the primary that has no effect while the induced emf in the secondary winding will cause the diode to be forward biased. Once again, with only the current in one winding being non-zero, it is the magnetizing current that is flowing in the secondary. Therefore, in reality, the flyback converter is a very clever design that uses a transformer to store energy in the magnetic field from the input source in one interval and dissipate this stored energy in the load in the other interval.

In this section, we described the operation of a flyback converter using the basic laws of physics as was done in all the cases before. The transformer used in a flyback converter behaves very differently from a regular grid frequency transformer. However, every state of operation of the flyback converter and the transformer complies with the basic laws of physics, and therefore, a flyback transformer under the hood is just another transformer. In the next section, we will present the simulation of a flyback converter.

5.7 Simulation of a Flyback Converter

In the previous section, we described the operation of the flyback converter. The reader will find several arguments on the internet about whether the transformer used in the flyback converter is indeed a transformer. This is due to the fact that the flyback converter has a pulsed transfer of power through the transformer unlike most other transformer applications, where a load is continuously served by the transformer. From the previous section, it is clear that with all the laws of physics that are applicable to the flyback transformer, we can continue to model and simulate the transformer for the flyback converter using the simulation model developed so far. In this section, we will describe the changes needed to simulate a flyback converter and present the simulation results.

Figure 5.23 shows the topology of the flyback converter that will be simulated. This simulation can be found in the folder `flyback_converter` within `chapter5_transformer_applications` in the following link in the sim-

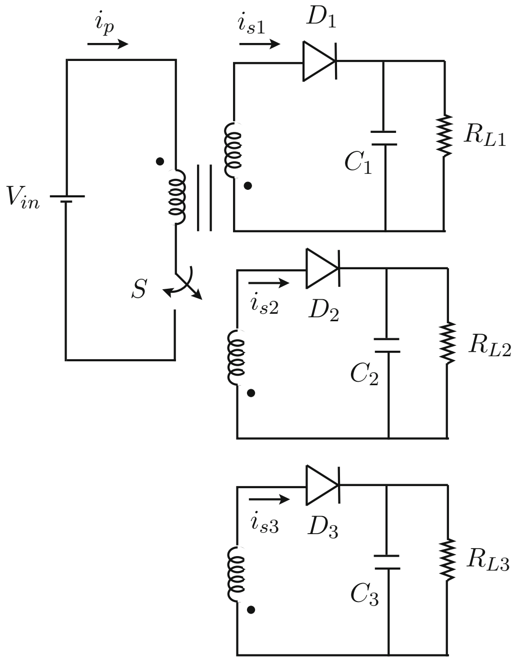

Fig. 5.23 Flyback converter with multiple outputs

ulation repository: https://github.com/opensourceelectrical/simulating-magnetics-for-power-electronics

One of the advantages of the flyback converter is that due to the necessity of the transformer, it is also possible to use a transformer with several secondary windings to produce multiple outputs. Such a topology has a few very important benefits. Since we have several windings, we can have different turns ratios between the secondary windings and the primary windings so as to produce outputs at different levels. This is extremely useful when a control circuit requires multiple power supplies such as 3.3 V, 5 V, 9 V, etc. Another benefit of having several windings is that each output is isolated from the other. This is very important in gate drivers used for power electronic converters, when the gate drivers for the different power devices in the converter need isolated power supplies to ensure that there is no interference between them that could cause a false conduction state of a power device.

In the topology of the flyback converter seen in Fig. 5.23, it is important to note that we are maintaining the reverse dot polarities for all three secondary windings. This is to ensure that none of the diodes D_1, D_2 or D_3 are forward biased and conducting when the power device S in the primary is conducting. Let us suppose that the input voltage V_{in} were the output voltage of a 24 V battery. Let us suppose we wish to produce output voltages across capacitors C_1, C_2 and C_3 to be 5 V, 3.3 V and 12 V, respectively. We must now choose the rated voltages of the windings of the transformer. It is important to remember that the power device S is conducting for a period of time during a switching time period. During this period, the primary winding is energized by the battery. When the power device S is not conducting, the secondary windings are transferring power to the outputs. The duty ratio d of the power device is the ratio $\frac{t_{ON}}{T_s}$, which will always be less than 1.

There is a vast amount of literature related to various possible control and operation strategies of the flyback converter [19, 20]. In this simulation, we will not delve into the details of the various ways in which the flyback converter can be operated as our focus is to examine the behaviour of the high frequency transformer in the converter. Therefore, we will choose a fairly simple mode of operation. We will not implement a closed-loop control scheme whereby one of the output voltages can be regulated to a desired reference. We will choose a fixed duty ratio d and implement an open-loop control scheme. Let us choose a duty ratio of 0.25 such that the power device is conducting only for 25% of the switching time period. Since we have chosen this value of duty ratio, let us choose the rated voltages of the primary winding of the transformer to be 24 V and the secondary windings of the transformer to be 15 V, 9 V and 36 V, respectively. Due to the open-loop operation of the converter, the output voltages will only be close to the values that are desired.

To define the specifications of the transformer in the flyback converter, we need to specify the maximum power rating of the transformer. Since we have three isolated outputs that would be used for control purposes, we could choose the maximum power rating of the transformer to be 500 VA. The main specifications of the transformer in the simulation model can be

```
import math
dt = 1.0e-8
VArated = 500.0
V1rated = 24.0
V2rated = 15.0
V3rated = 9.0
V4rated = 36.0
frated = 10000.0
omega_rated = 2*math.pi*frated
```

We have chosen the rated frequency of the transformer to be 10 kHz as we will choose this frequency to be the switching frequency of the converter. Flyback converters usually have a switching frequency range between 10 kHz and 50 kHz. We have also chosen the time step `dt` of the control file to be 10 ns as we are simulating a high frequency transformer instead of a grid frequency transformer. This decrease in the control time step can be seen to be essential when we examine the parameters of the transformer equivalent circuit. As an example, the parameters of winding 1 are listed:

```
L1 = 0.0009167
Lm1 = 0.0009163
M12 = 0.0005727
Ll1 = 366.7e-9
r1 = 0.01152
Rc1 = 115.2
```

Due to the decreased values of inductances, the time constants ($\frac{L}{R}$) of all the branches in the transformer sub-circuit have also decreased. In order to simulate a circuit with much smaller time constants in a stable manner, the integration time step of the simulation also needs to be decreased. By decreasing the control time step to 10 ns, we can force the integration time step to be 10 ns as well. Additionally, we can also decrease the integration time step in the simulation parameters to 10 ns along with the data storage time step so that the plots of simulation waveforms can capture all events in the simulation. Once these have been defined, all other computations of the parameters of the equivalent circuit, the formulation of the matrix equation and the numerical integration will remain the same as the previous simulations.

To complete the simulation, we need to choose the output bus capacitors C_1, C_2 and C_3 and the load resistors R_{L1}, R_{L2} and R_{L3}. The purpose of these output bus capacitors are to produce a stable voltage for the load. In order to do so, we should calculate the maximum possible ripple current in the secondary windings and the largest possible load so as to determine the ripple in the output voltage [19, 20]. However, since this is not a detailed converter design, we can choose the output bus capacitors to be 100 μF each. The load resistors will typically resemble the current drawn by the particular control circuit that is supplied by the particular output. Again, for the sake of simplicity, we can choose all three load resistors to be 10 Ω each.

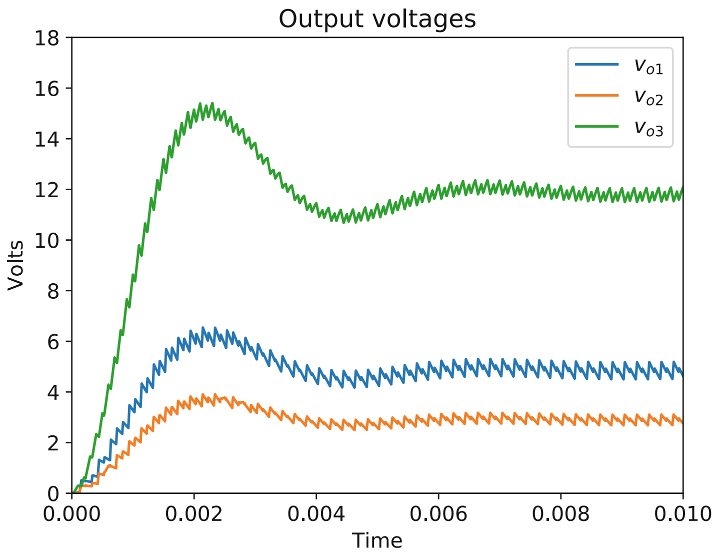

Fig. 5.24 Output voltages of the flyback converter

Figure 5.24 shows the output voltages v_{o1}, v_{o2}, and v_{o3} across the capacitors C_1, C_2 and C_3 in Fig. 5.23. After initial transients, the output voltages settle approximately to 5 V, 3.3 V and 12 V. The output voltages can be seen to have a ripple component. This ripple component can be decreased by increasing the value of the capacitors C_1, C_2, and C_3. The reader is encouraged to zoom into the simulation result and examine the fall of the output voltages when the power device S is conducting and the rise of the output voltages when S stops conducting.

Figure 5.25 shows the current i_p in the primary winding and the currents i_{s1}, i_{s2} and i_{s3} in the secondary windings for one switching time period. When the power device is conducting, i_p is increasing while the currents i_{s1}, i_{s2} and i_{s3} are zero. When the power device stops conducting, i_p is zero while the currents i_{s1}, i_{s2} and i_{s3} are flowing. The currents fall to zero at different rates as the output voltages in the three windings are different, and therefore, the load currents supplied by the windings are different. The currents i_{s1} and i_{s2} fall to zero before the power device starts conducting at the beginning of the next switching time period. The current i_{s3} however continues to flow until the end of the switching time period. As a result, the current i_p is seen to start from a non-zero value as in the beginning of the next switching time period; the transformer core flux has not completely fallen to zero. This mode of operation is called the continuous conduction mode. When all three secondary currents fall to zero before the start of the next switching time period, the converter is said to operate in discontinuous conduction mode.

Figure 5.26 shows the flux linkages of the primary winding for the same time interval as in Fig. 5.25. The flux linkage increases when the power device is conducting, as the transformer is being magnetized by the input voltage. The flux

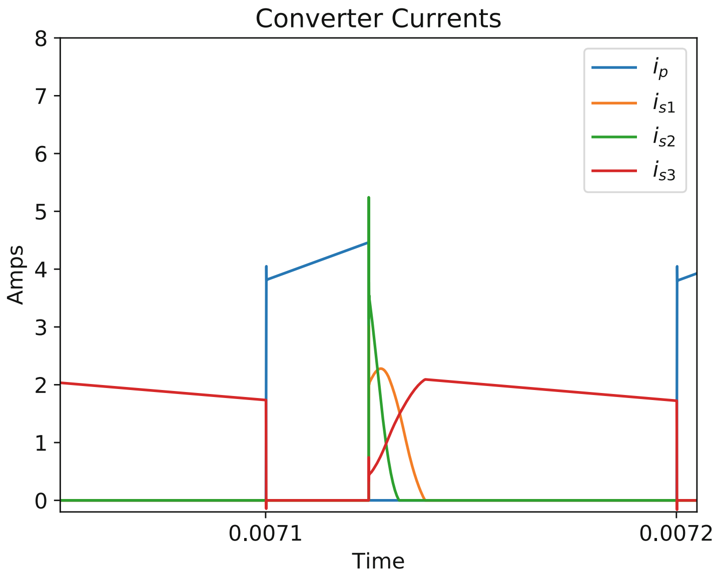

Fig. 5.25 Primary and secondary winding currents

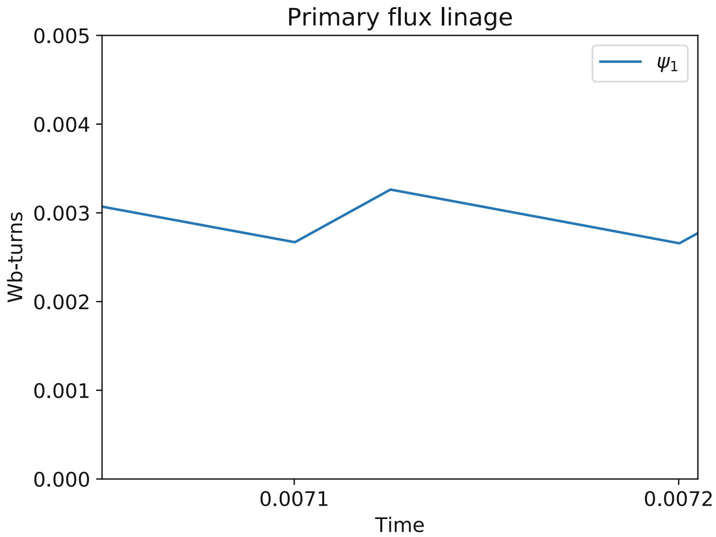

Fig. 5.26 Flux linkages of the primary winding

linkage decreases when the power device stops conducting, as the energy in the magnetic field is transferred to the loads in the secondary windings. It should be noted that this is the flux linkage of the primary winding, which is expressed as

$$\psi_1 = L_1 i_p + M_{12} i_{s1} + M_{13} i_{s2} + M_{14} i_{s3} = N_p \phi \tag{5.23}$$

With this simulation of the flyback converter, we conclude on the simulation of high frequency transformers. Though the behaviour of transformers varies with the operation strategy of power converters, it is always possible to use the basic laws of physics. Subsequently, one can use the simulation model of the transformer with basic modifications in the specifications of the transformer. The reader is encouraged to use the simulation model presented for simulating other power electronic converters such as the forward converter or the push-pull converter. With this simulation, we can also conclude this chapter on the applications of transformers in practical engineering systems.

5.8 Conclusions

The use of transformers in electrical engineering is too vast and varied to cover in a single book. However, a power engineer will deal with both single-phase and three-phase transformers as he or she works on projects. The purpose of this chapter was to provide a set of simulation examples that a power engineer can use and further extend as required by the application. In this chapter, we introduced the concept of three-phase systems and how a three-phase transformer can be conceived. A simple solution to achieving a three-phase transformer is to use three single-phase transformers. However, in this chapter, we have provided along with the basics of three-phase systems, a glimpse into the possible construction of a three-phase transformer and how that can be translated to a simulation model.

Three-phase systems differ widely in their application, and this also requires three-phase transformers to have varied construction and connection arrangements. In this chapter, we have introduced some of the most popular connection arrangements of three-phase transformer windings—the star and delta connections. We have also described the significance of these connections with respect to the requirements of the three-phase system. We have described how the star connection provides a neutral wire and terminal, which are essential in distribution systems for domestic applications where single-phase appliances are connected between a phase and neutral. Furthermore, we have described how three-phase domestic distribution systems can be unbalanced due to the demand of loads divided into the three phases.

The simulation models in this chapter are mere extensions of the simulation models of transformers already presented in the previous chapter. The manner in which the simulation models have been used with minimal modifications points to how flexible and scalable the models are. By translating a potential construction of a three-phase transformer to the change required in the simulation model, we have provided the reader with an approach that he or she can take if a special transformer needs to be modelled. This will be of use if the reader wishes to simulate transformers for applications such as High Voltage DC (HVDC) transmission systems where the transformers can be significantly modified.

The simulation results presented in the chapter describe not only the basic performance of three-phase transformers by taking the example of star–star and delta–star transformers, but also enable us to understand some of the trickier concepts in three-phase systems. We had considered the simulation of an unbalanced domestic distribution system fed by a delta–star transformer. We had examined how the zero sequence component in the star connected transformer windings will circulate within the delta connected windings. This approach will enable the reader to attempt to understand the circulating currents that can appear in systems with multiple transformer windings that may occur in applications such as cascaded converters.

With this chapter, we will bring the main contents of this book to an end. The purpose of these chapters is to enable the reader to not only simulate magnetic components, but also to dissect them and understand the phenomena that enable their operation. Though transformers are considered basic machines since they do not rotate, systems that contain transformers can be fairly complex and without "looking under the hood," an engineer may not fully understand every mode of operation of the system. Therefore, with the repeated use of the basic laws of physics, an engineer can simulate and analyse systems that he or she may encounter in their professional careers. The advantage of returning to the basic laws of physics is that an engineer need only to remember very basic laws that he or she has learned as a high-school student to approach any engineering system.

Chapter 6
Conclusions

6.1 Usefulness of the Book

As we conclude this book, it is worthwhile to reflect on what this book might have achieved and also potential pit-falls that a reader might want to avoid. Writing this book has helped me to re-learn many of the basic concepts of electromagnetism. At the same time, it has also made me aware of how vast this domain is, and how foolish it would be for any single book to even attempt to completely cover it. Therefore, this section will talk about how a reader can use the contents of the book, and cases where it might be advisable for a reader to not use the book.

In the introduction of the book, I had narrated my own experiences and why I decided to write this book. I had never bothered to learn about the inner working or design of magnetic components until I was faced with the task of approving a transformer design submitted to me. It was then that I realized that as an electrical engineer, I could not afford to not understand how magnetic components are designed. In order to understand how they are designed, I needed to understand how they work. To understand how magnetic components work, all one has to do is revise the basic laws of magnetism, which one learns in high school. In reality, this entire book has been merely a revision of high school physics.

In this book, I had used a few of the basic laws of physics related to electromagnetism. And here, it is important to emphasize the word "few." Magnetism is a vast domain and there are numerous laws to understand the finer detail of magnetic fields. This book has not considered the complexity of magnetic fields. The magnetic fields have been assumed to be uniform in most cases. This is rarely ever the case. Non-uniformity in magnetic fields and effects such as fringing occur fairly commonly in every magnetic component. However, these effects will not drastically change our results, especially since we have considered static magnetic components. In the case of rotating magnetic machines, non-linearity in the magnetic field can result in increased vibration, which might not be acceptable.

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The reader must note that this book has not dealt with the advanced topics in electromagnetism. If a reader does need an accurate map of the magnetic field in his or her design or analysis, the reader should seek specialized software that can map the magnetic field of an electromagnet in detail. Such software is used in machine design by researchers who are working on improvements to motors and generators. Such applications are quite common in the domain of electric vehicles and wind turbines, where we are constantly pushing the boundaries in terms of increasing the range of vehicles or increasing the capacity of offshore wind turbine generators.

One of the objectives of this book is to demystify the role of magnetism in the operation of magnetic components that electrical engineers use on a regular basis. This could be the case of a power supply designer that uses high frequency inductors and transformers, or a power system engineer that might be working at a substation with a high voltage transformer. The purpose of this book is to go beyond the very basic equations that most electrical engineers use to analyse circuits. Though the equations and equivalent circuits that are commonly used are extremely useful, mathematical equations alone do not help in understanding the inner workings of any system. For that purpose, this book uses a theory-and-simulation approach. In this book, the basic laws of physics are described and these are then compared with simulation results.

It is also fairly important to emphasize what this book does not suggest a reader to do. The book does not recommend a reader to build simulation models from scratch for every purpose. Simulations are a great way to learn, and in some cases, when simulation models do not exist, we are forced to build our own. However, building simulation models from scratch when such models exist is simply re-inventing the wheel. Not only is that a waste of time, it is important to remember that when a scientific group releases a simulation model, a great deal of research has been invested in building that model. As already stated, physical phenomena surrounding any particular application may be quite complex and in order to build a mathematical model, one may need to use several laws and relationships. As an example, if one wishes to simulate the impact of saturation on a transformer, the effect of saturation may be fairly complex and to accurately simulate it, it is advisable to look for specialized models. To attempt to build a custom model might not only be an unnecessary waste of time, but may also yield inaccurate results that could completely derail a project.

To conclude, this book should help an electrical engineer understand how magnetic components work. It should also provide an insight into how to combine the basic laws of physics with electrical engineering, to produce functional simulation models. However, do not get too enthusiastic and attempt to build complex simulation models using basic laws. If one needs to model complexity, it is best to place our faith in the work of experts. Please do remember—moderation is always the best approach, whether it is for drinking alcohol or simulations.

6.2 A Summary of the Course Contents

In this section, I will describe the main highlights of each chapter of this book. I will also describe what are the main learnings from each chapter and how one should perceive the lessons learned from every chapter. This book has dealt with a very narrow topic in electrical engineering, and most texts on basic electrical engineering would normally dedicate just a chapter to magnetics.

In Chap. 1, I have provided an introduction that described the need for the book and the motivation for writing this book. This book was on a list of topics that I had marked off when I was working in industry as one topic that needed detailed documentation, and more importantly, written and presented in a manner that is understandable to a young engineer. While writing this book, I kept in the back of my mind my own experiences and frustrations when trying to understand magnetics. During my interaction with other electrical engineers, my own experiences were not isolated or even rare. Since the purpose of the book was to build an understanding of magnetics from the basic laws of physics, the introduction covers these basic laws that will be used in the book. The laws are Kirchhoff's circuit laws, Ampere's Law, Faraday's Law and Lenz's Law. One would learn these laws in high school. The purpose of describing them in the introduction was to provide a quick reference for the reader, and also to specifically describe how these laws are applicable in electrical engineering.

In Chap. 2, I have started with analysis and simulation of inductors. As we start with a simple magnetic component, it is much easier to understand how to apply the basic laws of physics in understanding how inductors behave and how we can build a simulation model for an inductor. We introduce the concept of a magnetic circuit and describe how a magnetic circuit is analogous to an electric circuit. We examined a few sample inductor cores and represented the magnetic circuits for each one, while also describing how these can be solved. With this theory, we presented a simulation model of an inductor. We simulated several inductors, and in each simulation, we gradually increased the complexity of the inductor construction. We analysed the simulation results to understand how the inductor behaved at a deeper level than merely the $L\frac{di}{dt}$ equation. Most importantly, in this chapter, we had built that bridge between the world of magnetism and electrical engineering.

In Chap. 3, we had begun with the concept of magnetic coupling. We had shown with the simple example of two coils wound on the same core, how the flux produced by one coil links the other, therefore, laying the foundation of a magnetic link between two circuits that had no electrical link. We had examined several cases of these two coils to fully understand the nature of magnetic coupling, while expressing each case mathematically in order to explicitly describe how each case adheres to the basic laws of physics. We introduced the concept of mutual inductance and the dot polarity convention to depict the nature of magnetic coupling. We then extended the simulation model of an inductor to include a magnetically coupled inductor. Using simulations, we examined the nature of magnetic coupling and analysed the results to compare them with our theoretical discussions.

In Chap. 4, we began with the discussion on transformers by describing some of the broad applications of transformers in electrical engineering. We stated that transformers are merely magnetically coupled coils that have been so designed to achieve the maximum possible transfer of power from one winding to another. We described how we can translate the machine specifications of a transformer to an equivalent circuit. We also described how the parameters of the equivalent circuit can be estimated. Subsequently, this equivalent circuit can be expressed as magnetically coupled coils, and therefore, the simulation model in Chap. 3 can be used. We simulated a few simple transformers and analysed the results to examine the working of the transformer in greater detail, by examining the magnetizing current, and the effect of turns ratio on the transformation of voltage and current. We had shown how the transformer model is flexible and scalable, and can be used to simulate a transformer with several windings.

In Chap. 5, we had described how the simulation model of the transformer presented in Chap. 4 can be used in practical scenarios. We had simulated three-phase transformers that would be useful for a power engineer working in distribution systems, and a high frequency transformer in a flyback converter that would be useful for a power electronics engineer working in power supply design. We show how the simulation models in Chap. 4 can be easily extended to simulate a three-phase transformer whether the windings are connected in star or delta. We simulated a star–star transformer and a delta–star transformer. We also simulated an unbalanced four-wire load fed by a delta–star transformer to describe how the concept of circulating zero sequence components can be simulated as well. By simulating a high frequency transformer for a flyback converter, we show how the simulation of a high frequency transformer is merely a change in the transformer specifications. By describing in detail the working of the flyback converter, we show how even in the context of a non-linear power converter, the operation of the transformer can still continue to be interpreted by the basic laws of physics.

6.3 The Difference in Approach

If you are reading this conclusion chapter after reading the rest of the book, it would be fairly clear that the approach to writing this book is completely different from that of a conventional text or for that matter other technical books. This approach in writing this book, and also used in previous books, is in fact my philosophy behind writing books and teaching engineering as an online teacher. In this section, I would like to describe why I chose this approach and what I hope to achieve that the conventional approach might not be able to.

As already stated, what motivated me to write this book has been my frustration with most of conventional books in helping young engineers understand basic concepts. In reality, most textbooks are written as accompanying material for students who are enrolled at a university. It is assumed that a teacher would help a student through the process of raising doubts, and solving assignments. However, as we are well aware, there has been an acute shortage of teachers all over the world. The increasing opportunities in engineering have resulted in fewer qualified engineers choosing to become teachers. For that matter, I too had declined teaching positions following my PhD and chose to remain in industry. Therefore, to assume that all engineering students will receive the guidance of a qualified and motivated teacher is now no longer true. Books nowadays will have to be written such that students can learn independently, rather than need a teacher.

Which brings us to the question—how do we learn? This question falls more under the domain of psychology and neuroscience than engineering. However, almost all of us can attest to the fact that our best learning experiences have been either visually, or through a series of questions and answers or by a process of experimentation. What do we mean by a visual learning experience? A visual learning experience is when we perceive a physical phenomenon as occurring right before our eyes. When trying to learn topics such as electricity and magnetism, the greatest challenge is that these are not phenomena that can be seen unless there is a mechanical output as in the case of a motor. Therefore, to understand magnetism, we need to visualize the magnetic field.

In this book, by understanding the magnetic field produced by an inductor, a coupled set of coils or a transformer, we are trying to visualize the magnetic field produced. In a conventional text, one would expect merely equations or at the most equivalent circuits. However, most of us would not directly be able to form the connection between a circuit or an equation and the underlying physical phenomenon. Furthermore, when one considers different scenarios and possibilities, that helps in understanding a phenomenon. In this book, whether it is for a mere inductor or coupled inductors or transformers, the book describes many different possibilities that help to improve the learning experience for the reader.

Simulations are being rapidly adopted nowadays as a learning tool, and this book utilizes simulations to a great extent. In this book, we go back and forth between simulations and theory. We simulate several different variations of a scenario and use our theory to interpret the results. Moreover, by creating our own simulation models, we can make modifications and attempt to simulate special conditions. This “poking around” is greatly beneficial to a student, especially since a simulation carries no risk of a circuit being physically damaged.

Finally, the target audience of this book is not merely students of electrical engineering, but also practising engineers, who have regular jobs in the power industry. Therefore, this book has attempted to break the topic of magnetism in electrical engineering in such a manner that it can be read and understood by someone who is not a full-time student. With the minimal use of equations, and simulations and discussions to describe concepts, this book attempts to project magnetism in an easily understandable manner to a varied audience.

6.4 Scope for Future Work

To conclude this book, I would like to talk about the road ahead in this last section of the last chapter. This will be my third book ever since I decided to become a teacher/author. As a teacher and an author, I draw from my own experiences from when I was an undergraduate student, a graduate student and a researcher employed in a company. I teach and write about topics that I found I needed to learn about and struggled to understand. In the words of the great Richard Feynman—“if you want to master something, teach it!”

Becoming a teacher has forced me to become a student again. Writing this book has made me go back and relearn concepts that I had either long forgotten or had taken for granted. Writing an equation over and over again had made me stop thinking—why is this equation the way it is? Could it not be another way? And if not, why not? In this book, while trying to explain magnetism, I have used this questioning approach—of examining different scenarios and uncovering potential contradictions. In most cases, it is always possible through a process of questioning and seeking for answers, to apply basic laws and arrive at the final results.

For the reader, I would suggest that if you find the contents of this book interesting, look at the internet homepage of the project Python Power Electronics: https://www.pythonpowerelectronics.com/

A reader can find vast resources on various topics in power electronics. This project is ongoing and continuous, and I add new material almost every week. Most importantly, most of the content related to this project is free and open source in order to encourage community engagement and support. The objective is to achieve the level of collaboration and growth that is present in the software community through open source content and good documentation.

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