ST227 Table

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Disclaimer: The descriptions provided below are *NOT* rigorous definitions and should *NOT* be used for revising theory. They serve a purpose of a small cheat sheet with intuitive explanations behind the notions in the module, that could be used during assignments, lectures, tutorials or practicing applied questions for exam.

Symbol	Name	Description
S	State Space	Possible values taken by the Markov chain
P	Transition matrix	A matrix containing the probabilities of going between state spaces, rows being "old" and columns being "new" spaces, all 1-step
P_{ij}	Transition probability	A probability of Markov Chain going from state i to state j in 1-step
P_{ij}^n	Transition probability	A probability of Markov Chain going from state i to state j in n-steps
λ	Initial distribution	A vector containing distribution of the first state of the chain
λ_i	Initial probability of state i	A probability of state i being the first state of the Markov's chain, i'th entry of λ
d_i	State Period	Greatest Common divisor of all loops leading from state i to i
H_j	State first hitting time	Number of steps until state j is hit. Is 0 if the start point is state j
H_A	Set first hitting time	Number of steps until set A is hit. Is 0 if the start point is a state in set A
h_{ij}	State hitting probability	Probability of reaching state j from state i.
h_{iA}	Set hitting probability	Probability of entering a set A from state i.
η_{ij}	State expected hitting time	Expected number of steps until state j is reached from state i
η_{iA}	Set expected hitting time	Expected number of steps until set A is reached from state i
T_i	First Return time	Number of steps until state i loops on itself
f_i	Return probability	probability of chain returning back to state i from i, basically h_{ii}
m_i	Expected return time	Expected number of steps until state i loops on itself, basically η_{ii}
V_j	Number of visits	Number of visits to state j
V_A	Number of visits	Number of visits to set of states A
$\mathcal{T}$	Set of Transient States	Subset of S, containing all transient states
μ	Invariant Measure	A vector of positive entries, describing the measure, or "weight" of each state at every step of the chain. Has a property of $\mu = \mu P^n$
π	Invariant Distribution	A special case of invariant measure, distribution describing the distribution of the chain at every step of the chain, has a property of $\pi = \pi P^n$ and $\sum_{i \in S} \pi_i = 1$. Not every chain has π.

Symbol	Name	Description
γ_i^k	Expected number of visits	Expected number of visits to state i before chain returns to state k
Π	Long Term Transition Matrix	$\Pi = \lim_{n\to\infty} P^n$, with all rows equal to π. Describes the behaviour of the chain after arbitrarily many steps. Not applicable to every Markov Chain
$G = (V, E)$	Graph	Graph G, where V is a set of vertices and $E \subset S \times S$, set of all edges. All graphs in this module are connected and weighted
V	Set of all vertices	Set of all vertices in a graph. In this module, $V = S$
E	Set of all edges	Set containing all connections in a graph, is a subset of $S \times S$. As all the graphs are connected, for any 2 points $x, y \in V$ there exists a subset of E connecting them
$w(x, y)$	Weight between vertices x and y	A non-negative real number, describing the weight of an edge. $w(x, y) = 0$ if $x, y \notin E$, and $w(x, y) > 0$ otherwise.
$W(x)$	Sum of weights of edges connected to vertex x	Positive(due to connected graph assumption) number, describing the sum of all edges connecting x with the rest of the graph: $W(x) = \sum_{y\in V} w(x, y) < \infty \ \forall x \in V$