MODELING DRIFTING DYNAMICS AND CONTROL IN AUTOMOTIVE ENGINEERING

DENZEL BETHLEY, JOY TOKU, ANDREW EBERBACH

1. INTRODUCTION

Drifting is a thrilling driving skill that has captivated the hearts of motorsport enthusiasts around the globe. It was invented in the 1970's by Kunimitsu Takahashi who used it to corner during the Japanese Touring Car Championship. His techniques greatly inspired generations of street racers and would eventually inspire an entire motorsport. Typically done with front-engine, rear-wheel-drive vehicles, drifting itself is intentional oversteer that allows the driver to slide sideways through corners whilst maintaining control of the vehicle. Unlike the traditional grip run where drivers aim to maintain maximum grip through corners, drifting is all about breaking traction and sliding focus on maintaining momentum through turns.

Beyond its entertainment value, drifting serves as a testing ground for automotive innovation and performance enhancement. Engineers and designers sometimes leverage insights from drifting dynamics to develop advanced vehicle technologies, optimize chassis setups, and refine driver assistance systems. By studying the intricate interactions between tire grip, vehicle dynamics, and driver inputs during drifting maneuvers, researchers can gain valuable insights into vehicle behavior under extreme conditions. This, in turn, can inspire the development of next-generation automotive technologies and enhance overall vehicle performance and safety.

This project seeks to delve into the dynamics and control of drifting, exploring the mathematical models, physical principles, and real-world applications that underpin this exhilarating driving technique. By analyzing the intricate interactions between vehicle dynamics, tire grip, and driver inputs during drifting maneuvers, the project aims to uncover the secrets of high-speed drifting and its potential for enhancing performance and innovation in automotive engineering and motorsport.

2. BACKGROUND INFORMATION

Drifting occurs when a car intentionally oversteers, causing the rear wheels to lose traction and slide sideways. This sideways motion is controlled by the driver to maintain a specific line through a turn. To understand drifting dynamics, we need to explore several key concepts such as vehicle dynamics, traction circle, oversteer and understeer. Vehicle dynamics deals with how vehicles move and respond to external forces. It involves understanding the forces acting on the car, including gravity, friction, and aerodynamics. The traction circle is a graphical representation of the forces a tire can handle before losing grip. It helps in understanding the limits of tire grip and how they affect vehicle behavior during drifting. Oversteer occurs when the rear tires lose traction before the front tires, causing the rear

Date: April 27, 2024.

of the car to slide out. Understeer, on the other hand, happens when the front tires lose traction first, causing the car to push wide through a turn.

Controlling drifting requires precise manipulation of various vehicle parameters to maintain stability and control such as steering input, throttle control, and counter steering. The driver's steering input is necessary in initiating and maintaining a drift. By steering into the slide, the driver can control the angle and direction of the drift. Adjusting the throttle allows the driver to modulate the car's speed and adjust the balance between traction and slip. Proper throttle control is essential for maintaining control throughout the drift. Counter steering is the technique of turning the wheels in the opposite direction of the slide to regain control of the vehicle. It helps stabilize the car and prevent it from spinning out.

3. ANALYTICAL SOLUTION

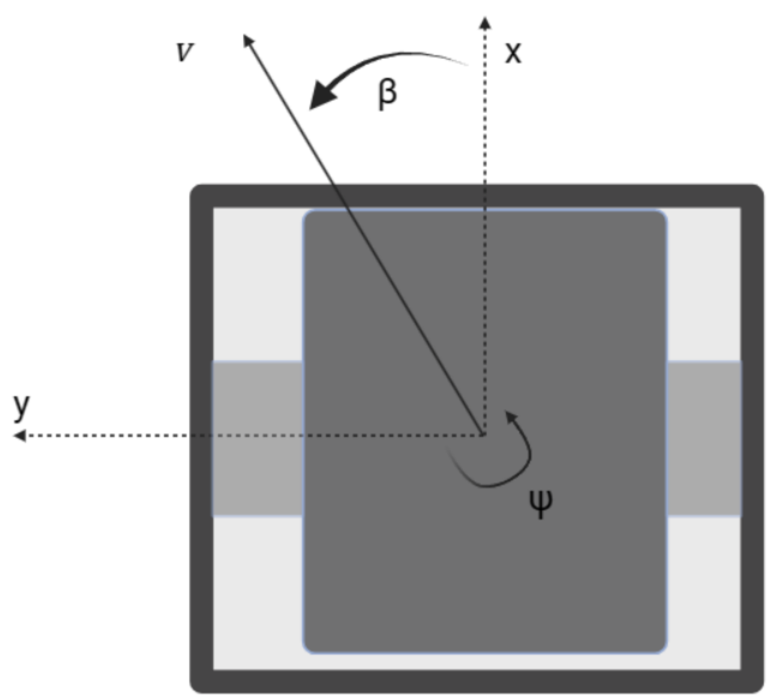

Figure 1. Mono-wheeled vehicle.

For our analytical solution, we are going to focus on doing the similar process that we will need to do for our numerical solution, but in a simpler case. For instance, for the analytical solution, we will be going over a mono-wheeled vehicle, but implementing the same math that will be covered in the numerical solution, with some assumptions that will be listed below. We will be going over a mono-wheeled vehicle for the analytical solution, which will be monostatic in the way of not falling over by slight changes to the center of gravity being either right above or the center of the wheel. The second assumption is that there will be enough mass in the implemented body that will not bring any inaccuracies on the differential equation.

However, we would like to review how we will solve the analytical and numerical conceptually. We will find the inertial acceleration equation. We are doing the inertial acceleration equation by implementing what we will need for our project. Drifting is a way of overcoming the inertia of the tires. The tires go the way the forces implemented on the tires result in. The equation for inertial acceleration with the variables that we will be using is

$$\begin{bmatrix}\dot{x}_{in}\\ \dot{y}_{in}\end{bmatrix} = v_{cg}\begin{bmatrix}\cos(\beta+\psi)\\ \sin(\beta+\psi)\end{bmatrix},$$

which represents the rate of change of the yaw rates and the rate of change of the angle of slip [1].

Then, after finding the initial acceleration equation, we will multiply it by a matrix that will incorporate a missing variable needed for our equation to be accurate. The matrix that we are talking about is [1]:

$$\begin{aligned}\begin{bmatrix}\ddot{x}_{cg}\\ \ddot{y}_{cg}\end{bmatrix} &= R^b_{in}(\psi)]\cdot\begin{bmatrix}\ddot{x}_{in}\\ \ddot{y}_{in}\end{bmatrix}\\ &= v_{cg}\left(\dot{\beta}+\dot{\psi}\right)\begin{bmatrix}-\sin\beta\\ \cos\beta\end{bmatrix} + \dot{v}_{cg}\begin{bmatrix}\cos\beta\\ \sin\beta\end{bmatrix}.\end{aligned}$$

This matrix is a way to interplant the rotation of inertial acceleration onto the car's body. But for the analytical solution only. Alternatively, a better way to say it is we are making a more accurate drifting model of a car by putting the forces of the initial acceleration onto the body into the equation by this matrix.

To this point, we were calculating and forming an equation for acceleration of inertia. However, to do the differential for drifting, we will need to have the forces implemented on the tires and then the car's center of gravity later. To rewrite the differential equation that we have formed till now, we will use Newton's second law. To rewrite our acceleration equation as a force equation in matrix format, the only thing we will need to do is divide the force equation by the mass of the vehicle in question. An iteration of this equation is shown below [1]:

$$v_{cg}\left(\dot{\beta}+\dot{\psi}\right)\begin{bmatrix}-\sin\beta\\ \cos\beta\end{bmatrix} + \dot{v}_{cg}\begin{bmatrix}\cos\beta\\ \sin\beta\end{bmatrix} = \frac{1}{m_{cg}}\cdot\begin{bmatrix}F_X\\ F_Y\end{bmatrix}.$$

Next, the following equation uses algebra to get two equations. One of them is the acceleration at the center of gravity and the rate of change slip angle. That is written in the equations below [1]:

$$\begin{aligned}\dot{v}_{cg} &= \frac{1}{m_{cg}\cos\beta}F_X + v_{cg}\left(\dot{\beta}+\dot{\psi}\right)\tan\beta\\ \dot{\beta} &= \frac{1}{m_{cg}v_{cg}\cos\beta}(F_Y - m_{cg}\dot{v}_{cg}\sin\beta) - \dot{\psi}.\end{aligned}$$

The rate of change of velocity equation is the equation needed to know. The acceleration on the car would be implemented as force later in the differential equation. While the rate of change of slip angle will also be used later in the equation to relate, the angle of where the car is going to the angle of tire. These two equations in addition to an equation that represents the yaw moment that will be implemented to find a function. That is, what we need is a multiplier to find the force that will be implemented on the tires of the vehicle.

The equation is going to represent the yawing moment. We do not derive nor differentiate for this equation. We get the following equation from Jakobsen, from which we base this paper:

$$I_z = \ddot{\psi} = \frac{l_r}{2}F_w + \frac{l_r}{2}F_w.$$

This occasion represents the rate of change of the yawing moment, which is when on the rotation of the z-axis has a moment of torque that results in forces implemented on the tires in this case. If let out, it will affect the accuracy of the predicted drift of a four-wheeled vehicle in question by the rotation of the car or the point of tire, where inertia is taking it being different than assumed.

We now have the three variables in question to make that differential equation. We are now going to focus on the forces that are going to be implemented on the tires and the longitudinal and lateral axis that would inspire drift or the overcoming of frictional force on tire. We first must state the equation that we are going to use to represent the force that will be implemented on the longitudinal and lateral axis, which will be the equation below:

$$F_W = \begin{bmatrix} F_{XW} \\ F_{YW} \end{bmatrix} = \begin{bmatrix} F_{XW} + F_{\text{wind}} \\ F_{YW} \end{bmatrix}.$$

This is a 2×1 matrix that states the longitudinal forces as F_X plus a coefficient of force that would be implemented on the vehicle of question, and then the F_Y is going to be the force on the lateral axis.

Next, we introduce the steering angle into equation as a multiplier of the forces in the longitudinal and lateral axes. We then separate the equation in the matrices even more by splitting the forces into longitudinal and sidewheel forces, which is shown below:

$$\begin{bmatrix} F_{LW}\cos\delta - F_{SW}\sin\delta + F_{x,\text{wind}} \\ F_{LW}\cos\delta + F_{SW}\sin\delta \end{bmatrix}.$$

We now rewrite the sidewheel forces as a new equation that states that the sidewheel force is equal to the force coefficient multiplied by the difference of the inertia is taking a car by the wheel angle:

$$\begin{bmatrix} F_{LW}\cos\delta - \alpha c_W\sin\delta + F_{x,\text{wind}} \\ F_{LW}\cos\delta + \alpha c_W\sin\delta \end{bmatrix}.$$

For the coefficient of force, we will need a constant for the frictional force that the road will ensue. Jakobsen states that the force coefficient c_{ij} can be found as the gradient of the friction curve evaluated at zero slip. This, however, requires knowledge of the conditions at the road surface. To avoid this, Kececi and Tao suggest that the force coefficient can instead be described by a tire constant and the friction parameter which then can be adapted by an online parameter estimation scheme [1]. With that, it will make the full equation to be:

$$\begin{bmatrix} F_{LW}\cos\delta - \alpha\mu c_W\sin\delta + F_{x,\text{wind}} \\ F_{LW}\cos\delta + \alpha\mu c_W\sin\delta \end{bmatrix}.$$

Having all the independent and dependent variables found, we can make a 3-variable function matrix that represents drifting a mono-wheeled vehicle. With the assumptions that we made prior in the analytical solution, the resulting matrix will yield 3 functions that will be the solution for the analytical solution.

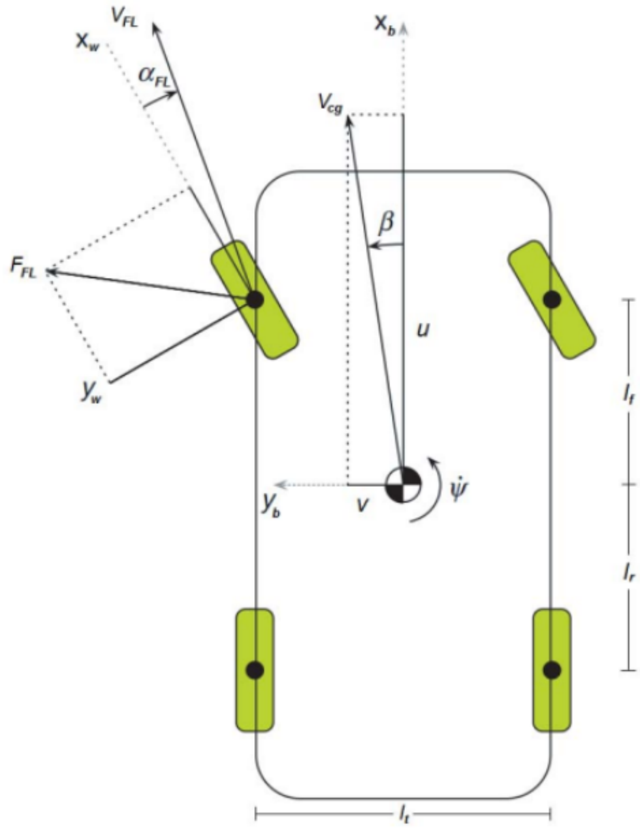

FIGURE 2. Illustration of the vehicle and its variables [1].

4. NUMERICAL SIMULATION FOR DRIFTING DYNAMICS

As in the analytical solution, the numerical solution will be focused on the forming of the equation for drifting. While it's the same equation, instead of it being a mono-wheeled vehicle, it is a four-wheeled vehicle. Furthermore, the numerical simulation for drifting dynamics focuses on drifting a four-wheeled car, or the change of velocity, slip-angle, and yaw moments of a four-wheeled vehicle.

Recall the equation

$$\begin{bmatrix} \dot{x}_{in} \\ \dot{y}_{in} \end{bmatrix} = v_{cg} \begin{bmatrix} \cos(\beta + \psi) \\ \sin(\beta + \psi) \end{bmatrix}.$$

The forces acting in the horizontal plane consist of

$$\begin{bmatrix} F_X \\ F_Y \end{bmatrix} = \begin{bmatrix} F_{x,FL} + F_{x,FR} + F_{x,RL} + F_{x,RR} + F_{x,\text{wind}} \\ F_{y,FL} + F_{y,FR} + F_{y,RL} + F_{y,RR} \end{bmatrix},$$

where $F_{x,ij}, F_{y,ij}$ are the wheel forces and $F_{x,\text{wind}}$ is the aerodynamic drag

$$F_{x,\text{wind}} = -\frac{1}{2} c_{\text{air},x} A_{\text{front}} \rho_{\text{air}} v_{cg}^2.$$

The wheel forces act in the wheel frame. For the front wheels, this frame is rotated by the steering angle δ from the body frame [1]. The expressions for the transformed wheel forces acting in the longitudinal direction of the vehicle are written as

$$\begin{aligned}
F_{x,FL} &= F_{L,FL}\cos\delta - F_{S,FL}\sin\delta \\
F_{x,FR} &= F_{L,FR}\cos\delta - F_{S,FR}\sin\delta \\
F_{x,RL} &= F_{L,RL} \\
F_{x,RR} &= F_{L,RR},
\end{aligned}$$

and the wheel forces in the lateral direction are written as

$$\begin{aligned}
F_{y,FL} &= F_{S,FL}\cos\delta + F_{L,FL}\sin\delta \\
F_{y,FR} &= F_{S,FR}\cos\delta + F_{L,FR}\sin\delta \\
F_{y,RL} &= F_{S,RL} \\
F_{y,RR} &= F_{S,RR}.
\end{aligned}$$

The longitudinal wheel forces can be controlled by the driving torque from the engine and the breaks and therefore considered as the control input. Only the lateral wheel forces are then undecided. These forces can be approximated by assuming a linear relationship between the wheel slip angle α and the resulting force. The wheel slip angle is found from the geometrical relationship between the steering angle, vehicle sideslip, and rotational velocity [1]:

$$\begin{aligned}
F_{S,FL} &= c_{FL}\alpha_{FL} = c_{FL}\left(\delta - \beta - \frac{l_f\dot{\psi}}{v_{cg}}\right) \\
F_{S,FR} &= c_{FR}\alpha_{FR} = c_{FR}\left(\delta - \beta - \frac{l_f\dot{\psi}}{v_{cg}}\right) \\
F_{S,RL} &= c_{RL}\alpha_{RL} = c_{RL}\left(-\beta + \frac{l_r\dot{\psi}}{v_{cg}}\right) \\
F_{S,RR} &= c_{RR}\alpha_{RR} = c_{RR}\left(-\beta + \frac{l_r\dot{\psi}}{v_{cg}}\right).
\end{aligned}$$

The force coefficient c_{ij} can be found as the gradient of the friction curve evaluated at zero slip [1]. This, however, requires knowledge of the conditions at the road surface. To avoid this issue, Kececi and Tao suggest that the force coefficient can instead be described by a tire constant and the friction parameter which then can be adapted by an online parameter estimation scheme [1]:

$$c_{ij} = c\mu_{ij}.$$

The yawing moment is described by Jakobsen in the following way [1]:

$$I_z\ddot{\psi} = (F_{y,FR} + F_{y,FL})l_f - (F_{y,RR} + F_{y,RL})l_r + (F_{x,RR} - F_{x,RL})\frac{l_t}{2} + (F_{x,FR} - F_{x,FL})\frac{l_t}{2},$$

where l_f is the distance from center of gravity to the front axle, l_r is the distance from center of gravity to the rear axle, and l_t is the wheel tread [1]. The following equations form the basis for the vehicle model [1]:

$$\begin{aligned}
\dot{v}_{cg} &= \frac{\cos\beta}{m_{cg}} F_X + \frac{\sin\beta}{m_{cg}} F_Y \\
\dot{\beta} &= -\frac{\sin\beta}{m_{cg} v_{cg}} F_X + \frac{\cos\beta}{m_{cg} v_{cg}} F_Y - \dot{\psi} \\
I_z \ddot{\psi} &= (F_{y,FR} + F_{y,FL}) l_f - (F_{y,RR} + F_{y,RL}) l_r \\
&\quad + (F_{x,RR} - F_{x,RL}) \frac{l_t}{2} + (F_{x,FR} - F_{x,FL}) \frac{l_t}{2}.
\end{aligned}$$

Inserting the expressions for the wheel forces completes the nonlinear two-track vehicle model [1]:

$$\dot{x} = \begin{bmatrix} \dot{v}_{cg} \\ \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} f_1(\vec{x}, \vec{u}) \\ f_2(\vec{x}, \vec{u}) \\ f_3(\vec{x}, \vec{u}) \end{bmatrix}$$

with the state and input vectors

$$\vec{x} = \begin{bmatrix} v_{cg} \\ \beta \\ \dot{\psi} \end{bmatrix}; \qquad \vec{u} = \begin{bmatrix} F_{L,FL} \\ F_{L,FR} \\ F_{L,RL} \\ F_{L,RR} \\ \delta \end{bmatrix}.$$

The full expression for the nonlinear function $f_1(\vec{x}, \vec{u})$ is

$$\begin{aligned}
f_1(\vec{x}, \vec{u}) = \frac{1}{m_{cg}} \Bigg[&(F_{L,FL} + F_{L,FR}) \cos(\delta - \beta) - (c_{FL} + c_{FR}) \left(\delta - \beta - \frac{l_f \dot{\psi}}{v_{cg}} \right) \sin(\delta - \beta) \\
&+ \left(F_{L,RL} + F_{L,RR} - \frac{1}{2} c_{\mathrm{air},x} A_{\mathrm{front}} \rho_{\mathrm{air}} v_{cg}^2 \right) \cos\beta \\
&+ (c_{RL} + c_{RR})(-\beta + l_r \dot{\psi} / v_{cg}) \sin\beta \Bigg];
\end{aligned}$$

the full expression for the nonlinear function $f_2(\vec{x}, \vec{u})$ is

$$\begin{aligned}
f_2(\vec{x}, \vec{u}) = \frac{1}{m_{cg} v_{cg}} \Bigg[&(c_{FL} + c_{FR}) \left(\delta - \beta - \frac{l_f \dot{\psi}}{v_{cg}} \right) \cos(\delta - \beta) \\
&+ (F_{L,FL} + F_{L,FR}) \sin(\delta - \beta) \\
&- \left(F_{L,RL} + F_{L,RR} - \frac{1}{2} c_{\mathrm{air},x} A_{\mathrm{front}} \rho_{\mathrm{air}} v_{cg}^2 \right) \sin\beta \\
&+ (c_{RL} + c_{RR}) \left(-\beta + \frac{l_r \dot{\psi}}{v_{cg}} \right) \cos\beta \Bigg] - \dot{\psi};
\end{aligned}$$

and the full expression for the nonlinear function $f_3(\vec{x}, \vec{u})$ is

$$f_3(\vec{x},\vec{u}) = \frac{1}{I_z}\Bigg[l_f(F_{L,FL}+F_{L,FR})\sin\delta + l_f(c_{FL}+c_{FR})\left(\delta-\beta-\frac{l_f\dot{\psi}}{v_{cg}}\right)\cos\delta$$
$$+\frac{l_t}{2}(F_{L,FR}-F_{L,FL})\cos\delta - \frac{l_t}{2}(c_{FR}-c_{FL})\left(\delta-\beta-\frac{l_f\dot{\psi}}{v_{cg}}\right)\sin\delta$$
$$-l_r(c_{RL}+c_{RR})\left(-\beta+\frac{l_r\dot{\psi}}{v_{cg}}\right)+\frac{l_t}{2}(F_{L,RR}-F_{L,RL})\Bigg].$$

5. DIFFERENTIAL EQUATIONS FOR DRIFTING DYNAMICS

For the application of the differential we solved, we are generalizing the differential equations we saw before in the analytical and numerical solutions. To make a 3-dimensional vector field of a vehicle, the example of our choosing will be Mazada MX-5 Miata. For the near 50% rear length and front length ratio, the historical accuracy of it being a car for drifting and prior knowledge that we have on vehicle. For the 3-dimensional vector plane, we will be using the equations that we have gathered by doing the multiplication of the 1×3 matrix and 1×5 matrices shown below:

$$\vec{x} = \begin{bmatrix} v_{cg} \\ \beta \\ \dot{\psi} \end{bmatrix}; \qquad \vec{u} = \begin{bmatrix} F_{L,FL} \\ F_{L,FR} \\ F_{L,RL} \\ F_{L,RR} \\ \delta \end{bmatrix}.$$

We chose a 3-dimensional vector field because the goal of this project is to have a way to implement the equations that we differentiated above to drift a car. We will put enough constraints on the equation to be solved for. We can have a vector field where a driver of the car can look for a prediction of the up-and-coming drift that he is trying to attempt for safety and planning of route. But to do that we will need to generalize the formulae.

Assumptions and Constants. First, assumptions that we have made for the equation that we differentiated is that the rear tires and the back tires have the same force. Or that the difference between left and right of the vehicle is minute. It can be neglected. Then the coefficient of force in the equation will be equal for the reasoning being the equation's generalization. Which will make the new matrix that form the equations to be:

$$\vec{x} = \begin{bmatrix} v_{cg} \\ \beta \\ \dot{\psi} \end{bmatrix}; \qquad \vec{u} = \begin{bmatrix} F_{LR} \\ F_{LF} \\ \delta \end{bmatrix}.$$

The constants that we will use are based on the dynamics of the Mazda MX-5 Miata. The weight of the Miata that we have used in this equation is $2,116$ lbs. The coefficient of drag that we have used in this equation is 0.35 and because the length of the matter is rounded up to 154 in. For the car of our choosing, Mazda MX-5 Miata, it has a near perfect weight distribution. As the center of mass is almost perfectly in the middle of the car. The difference between the lengths of the length of rear to center gravity and length from front to center gravity is negligible. By being that, we assume they're the same (1300 mm). Applying all the constants to the equation, we get the revised equation to be:

$$\begin{aligned}
\dot{v}_{cg} = \frac{1}{980}\Bigg(&2F_{LF}\cos(\delta-\beta) - 4\left(\delta-\beta-\frac{1.3m\dot{\psi}}{v_{cg}}\right)\sin(\delta-\beta)\\
&+\left(2F_{CR}-\frac{1}{2}(0.35)A(1.293)v_{cg}^2\right)\cos(\beta) + 8(-\beta)\left(\frac{1.3m\dot{\psi}}{v_{cg}}\right)\sin(\beta)\Bigg)\\
\dot{\beta} = \frac{1}{980}\Bigg(&4\left(\delta-\beta-\frac{1.3\dot{\psi}}{v_{cg}}\right)\sin(\delta-\beta) + (2F_{LF})\cos(\delta-\beta)\\
&-\left(2F_{LR}-\frac{1}{2}(0.35)A(1.293)v_{cg}^2\right)\sin(\beta) + 4\left(-\beta+\frac{1.3\dot{\psi}}{v_{cg}}\right)\cos(\beta)\Bigg)\\
\dot{\psi} = \frac{1}{I_z}\Bigg(&1.3(2F_{LF})\sin(\delta) + 1.3(4)(\delta-\beta-\frac{1.3\dot{\psi}}{v_{cg}}\Bigg)\cos(\delta) + 0.65(2F_{LF})\cos(\delta)\\
&-0.65(4)\left(\delta-\beta-\frac{1.3\dot{\psi}}{v_{cg}}\right) - 1.3(4)\left(-\beta+\frac{1.3\dot{\psi}}{v_{cg}}\right) + 0.65(2F_{LR}))
\end{aligned}$$

The equations above give a differential for the drifting of the specially picked car Miata with the use of these equations, we can form more graphs or more equations to represent the rate of change of steering angle, slip, angle, or velocity of the given car.

6. REAL WORLD APPLICATION

When cars drift, drivers need to stay in control even while sliding sideways. This requires quick reflexes and precise steering. These skills can be translated into making cars safer on slippery roads or in emergency situations. By understanding how drivers handle drifts, engineers can develop better driver assistance systems to prevent accidents and keep everyone safe on the road. In industries like manufacturing or logistics, efficient movement of vehicles is crucial for productivity. Drifting techniques can be applied to optimize the movement of forklifts, trucks, and other vehicles within warehouses or production facilities.

By mastering controlled slides, operators can navigate tight spaces more effectively, saving time and reducing the risk of accidents. In emergency situations, such as search and rescue operations in difficult ground, traditional driving techniques may not be sufficient. Drifting skills can be valuable for drivers navigating rugged landscapes or treacherous conditions. By learning how to control a drifting vehicle, rescue teams can reach remote locations more safely and efficiently and save more lives in critical situations. Drifting techniques can also be useful in agriculture. Farmers often need to navigate their vehicles through tight spaces or around obstacles in the field. By learning how to control a drifting car, farmers can improve their maneuverability, making tasks like plowing, seeding, and harvesting more efficient.

7. CONCLUSION

Drifting happens when drivers intentionally make their cars slide sideways through corners, while still staying in control. It is different from regular racing where the goal is to keep the tires gripping the road as much as possible. Instead, drifting is all about breaking traction and sliding around while keeping up speed. Engineers and designers watch how cars drift to learn how to make them faster and safer. They look at things like how the tires grip the road, how the car moves, and what the driver does while drifting. All this helps them make cars that work even better, especially in tough situations.

So, drifting is not just a sport—it is a way to make cars smarter and safer for everyone. In this project, we looked at how cars drift and why they drifted. We used math and science to understand how cars move when they drift and how the driver makes it happen. By studying this, we hope to learn more about how to make cars better and faster. This can lead to innovative ideas for making cars that are more advanced and easier to use in the future. So, by understanding drifting, we can make cars and racing even more fun and exciting for everyone involved.

Acknowledgments

We would like to give a special thanks to our advisor C. Scott Alons (the Cook), Jakob Lieng Jakobsen (the person who wrote the paper upon which we base our research), Semeion Walker (the bystander who proof-read a part of our paper), and Victor Gomez.

References

[1] J. L. Jakobsen. Autonomous drifting of a 1:5 scale model car. Master's Thesis in Engineering Cybernetics, June 2011.