Two-Experiment Plan: σ-Uniform vs. τ-Uniform Metrology in QFT

Protocol Addendum to LTQG

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Principle

Observables are invariant under monotone reparameterizations of time, but *measurement protocols* need not be. Uniform sampling in $\sigma = \log(\tau/\tau_0)$ redistributes resolution towards early proper times. We propose two numerically reproducible experiments with acceptance criteria.

1 Experiment A: Bogoliubov Coefficients in Expanding FLRW

Model. Real scalar field on spatially flat FLRW with $a(\tau) = (\tau/\tau_*)^p$, $p \in \{\frac{1}{2}, \frac{2}{3}\}$. Use canonical variable $v = a^{3/2}u$ with mode equation

$$v_k'' + \omega_k(\tau)^2 v_k = 0, \qquad \omega_k^2 = k^2 a^{-2} + m^2 - \frac{a''}{a} - \frac{3}{2}\left(\frac{a'}{a}\right)^2, \tag{1}$$

primes denoting τ-derivatives. Impose $W[v_k, \bar{v}_k] = i$ at a shared early-time slice.

Protocol. Integrate modes over a fixed physical interval in the scale factor (e.g. $a \in [a_i, a_f]$). Construct instantaneous adiabatic vacuum and extract (α_k, β_k) via the KG inner product on common slices. Repeat with sampling grids: (i) N samples uniform in τ, (ii) N samples uniform in σ.

Hypothesis. β_k estimates converge to the same continuum limit, but *σ-uniform sampling reduces bias/variance* at fixed N for spectra with early-time structure.

Acceptance criteria. For k in a fixed decade and N fixed,

$$\max_k \left| \, |\beta_k|^2_{\tau\text{-grid}} - |\beta_k|^2_{\sigma\text{-grid}} \right| \le \varepsilon_1(N), \tag{2}$$

$$\mathrm{MSE}_\sigma(N) < \mathrm{MSE}_\tau(N) \quad \text{by a factor } \ge 1 + \delta, \tag{3}$$

with typical targets $\varepsilon_1 \sim 10^{-6}$, $\delta \sim 0.2$.

2 Experiment B: Near-Horizon Phase Accumulation (Unruh–DeWitt Detector)

Model. Static Schwarzschild exterior with detector on a stationary worldline at areal radius $r > r_s$. Proper time increment is $d\tau = \sqrt{1 - r_s/r}\, dt$. Consider a detector with switching function $\chi(\cdot)$ and energy gap Ω; the response function is

$$\mathcal{F} \propto \int d\tau\, d\tau'\, \chi(\tau)\chi(\tau') e^{-i\Omega(\tau-\tau')} W(\tau, \tau'), \tag{4}$$

with W the pullback of the Wightman function.

Protocol. Compare two families of compactly supported switching functions of equal L^1-mass: (i) Uniform windows in τ of width $\Delta\tau$; (ii) Uniform windows in $\sigma = \log(\tau/\tau_0)$ mapped back to τ with Jacobian $d\tau = \tau\, d\sigma$ (thus earlier τ receive finer weight). Evaluate $\mathcal{F}$ numerically (or via near-horizon Rindler approximation).

Hypothesis. For windows that extend towards smaller τ (closer to the horizon or early-time segments), the σ-uniform protocol *stabilizes* phase accumulation and exhibits reduced sensitivity to ultraviolet transients, consistent with the "asymptotic silence" scaling.

Acceptance criteria. For matching support endpoints,

$$|\mathcal{F}_\sigma - \mathcal{F}_\tau| \le \varepsilon_2 \quad \text{(invariance)}, \tag{5}$$

$$\mathrm{Var}[\mathcal{F}_\sigma] < \mathrm{Var}[\mathcal{F}_\tau] \quad \text{over grid refinements, by } \ge 1 + \delta. \tag{6}$$

3 Reproducibility Notes

- Use shared random seeds and identical physical slices for comparisons.
- Track Wronskian drift and KG norm as sanity checks: target $\le 10^{-8}$.
- Publish code and config files; report $(N, \text{grid type})$ and wall-clock, to quantify efficiency gains from σ-uniform sampling.