

# LTQG Cosmology & Spacetime: FLRW Dynamics and Curvature Regularization

Log-Time Quantum Gravity Framework

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## Abstract

This document presents the cosmological applications of the Log-Time Quantum Gravity (LTQG) framework, focusing on Friedmann-Lemaître-Robertson-Walker (FLRW) spacetimes and curvature regularization. We demonstrate how the log-time coordinate  $\sigma = \log(\tau/\tau_0)$ , combined with Weyl conformal transformations, provides natural regularization of cosmological singularities and finite curvature scalars. The framework addresses key cosmological problems including the Big Bang singularity, horizon physics, and early universe dynamics. We include comprehensive mathematical derivations, computational implementations, and physical applications to cosmic evolution.

## Contents

# 1 Introduction

Cosmology presents fundamental challenges at the intersection of General Relativity and quantum physics. The standard FLRW cosmological models, while successful in describing the large-scale structure of the universe, suffer from singularities and divergences that signal the breakdown of classical descriptions. The LTQG framework provides a natural resolution through the log-time coordinate  $\sigma = \log(\tau/\tau_0)$  and associated Weyl transformations.

## 1.1 The Cosmological Singularity Problem

In standard FLRW cosmology, the scale factor  $a(t)$  typically behaves as  $a(t) \propto t^p$  near the Big Bang, where  $t \rightarrow 0^+$ . This leads to:

- Divergent curvature scalars as  $t \rightarrow 0^+$
- Infinite energy densities
- Breakdown of the Einstein field equations
- Causal disconnection problems (horizons)

The LTQG approach addresses these issues systematically through coordinate and conformal transformations that preserve the physics while regularizing the mathematics.

## 1.2 LTQG Resolution Strategy

The LTQG framework resolves cosmological singularities through a two-step process:

1. **Log-Time Coordinates:** Replace cosmic time  $t$  with  $\sigma = \log(t/t_0)$ , converting the singular limit  $t \rightarrow 0^+$  into the regular limit  $\sigma \rightarrow -\infty$
2. **Weyl Transformation:** Apply the conformal transformation  $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$  with  $\Omega = 1/t$ , yielding finite curvature scalars

This approach maintains general covariance while providing mathematical regularity and physical insight.

# 2 FLRW Spacetimes in Log-Time Coordinates

## 2.1 Standard FLRW Metric

The FLRW metric in standard coordinates is:

$$ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (1)$$

where  $a(t)$  is the scale factor and  $k \in \{-1, 0, +1\}$  represents the spatial curvature. For a power-law scale factor  $a(t) = a_0(t/t_0)^p$ , the Hubble parameter is:

$$H(t) = \frac{\dot{a}}{a} = \frac{p}{t} \quad (2)$$

## 2.2 Log-Time Transformation

Under the transformation  $\sigma = \log(t/t_0)$ , we have  $t = t_0 e^\sigma$  and  $dt = t_0 e^\sigma d\sigma$ . The metric becomes:

$$ds^2 = -t_0^2 e^{2\sigma} d\sigma^2 + a_0^2 t_0^{2p} e^{2p\sigma} \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (3)$$

**Theorem 2.1** (FLRW in Log-Time). *The FLRW metric in log-time coordinates  $(\sigma, r, \theta, \phi)$  is:*

$$ds^2 = -t_0^2 e^{2\sigma} d\sigma^2 + a_0^2 t_0^{2p} e^{2p\sigma} [d\Omega_3^2] \quad (4)$$

where  $d\Omega_3^2$  represents the spatial three-metric and  $p$  is related to the equation of state by  $p = \frac{2}{3(1+w)}$ .

## 2.3 Weyl Conformal Transformation

To regularize curvature singularities, we apply the Weyl transformation:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \text{with} \quad \Omega = \frac{1}{t} = \frac{1}{t_0 e^\sigma} = \frac{e^{-\sigma}}{t_0} \quad (5)$$

The conformally transformed metric becomes:

$$d\tilde{s}^2 = e^{-2\sigma} \left[ -t_0^2 e^{2\sigma} d\sigma^2 + a_0^2 t_0^{2p} e^{2p\sigma} d\Omega_3^2 \right] \quad (6)$$

Simplifying:

$$d\tilde{s}^2 = -t_0^2 d\sigma^2 + a_0^2 t_0^{2p} e^{2(p-1)\sigma} d\Omega_3^2 \quad (7)$$

## 3 Curvature Regularization

### 3.1 Ricci Scalar Calculation

For the conformally transformed FLRW metric, the Ricci scalar  $\tilde{R}$  can be computed directly. For spatially flat FLRW ( $k = 0$ ):

**Theorem 3.1** (Regularized Ricci Scalar). *The Ricci scalar of the Weyl-transformed FLRW metric is:*

$$\tilde{R} = 12(p-1)^2 \quad (8)$$

*This is constant and finite for all values of  $\sigma$ , completely eliminating the Big Bang singularity.*

*Proof.* The computation involves the Christoffel symbols and Ricci tensor components for the metric:

$$d\tilde{s}^2 = -t_0^2 d\sigma^2 + a_0^2 t_0^{2p} e^{2(p-1)\sigma} (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \quad (9)$$

The non-zero Christoffel symbols are:

$$\tilde{\Gamma}_{\sigma r}^r = \tilde{\Gamma}_{r\sigma}^r = (p-1) \quad (10)$$

$$\tilde{\Gamma}_{\sigma\theta}^\theta = \tilde{\Gamma}_{\theta\sigma}^\theta = (p-1) \quad (11)$$

$$\tilde{\Gamma}_{\sigma\phi}^\phi = \tilde{\Gamma}_{\phi\sigma}^\phi = (p-1) \quad (12)$$

$$\tilde{\Gamma}_{rr}^\sigma = -(p-1) e^{2(p-1)\sigma} \quad (13)$$

$$\tilde{\Gamma}_{\theta\theta}^\sigma = -(p-1) r^2 e^{2(p-1)\sigma} \quad (14)$$

$$\tilde{\Gamma}_{\phi\phi}^\sigma = -(p-1) r^2 \sin^2 \theta e^{2(p-1)\sigma} \quad (15)$$

The Ricci tensor components yield:

$$\tilde{R}_{\sigma\sigma} = -3(p-1)^2 t_0^2 \quad (16)$$

$$\tilde{R}_{rr} = 3(p-1)^2 a_0^2 t_0^{2p} e^{2(p-1)\sigma} \quad (17)$$

$$\tilde{R}_{\theta\theta} = 3(p-1)^2 a_0^2 t_0^{2p} r^2 e^{2(p-1)\sigma} \quad (18)$$

$$\tilde{R}_{\phi\phi} = 3(p-1)^2 a_0^2 t_0^{2p} r^2 \sin^2 \theta e^{2(p-1)\sigma} \quad (19)$$

The scalar curvature is:

$$\tilde{R} = \tilde{g}^{\mu\nu} \tilde{R}_{\mu\nu} = 12(p-1)^2 \quad (20)$$

□

### 3.2 Physical Interpretation

The constant curvature  $\tilde{R} = 12(p-1)^2$  has important physical interpretations:

- **Radiation Era** ( $w = 1/3, p = 1/2$ ):  $\tilde{R} = 3$
- **Matter Era** ( $w = 0, p = 2/3$ ):  $\tilde{R} = \frac{4}{3}$
- **Dark Energy** ( $w = -1, p = 1$ ):  $\tilde{R} = 0$  (conformally flat)
- **Inflation** ( $w \approx -1, p \approx 1$ ):  $\tilde{R} \approx 0$

## 4 Computational Implementation

### 4.1 FLRW Cosmology Class

```

1 import numpy as np
2 import matplotlib.pyplot as plt
3 from ltqg_core import LogTimeTransform
4
5 class FLRWCosmology:
6     """
7     FLRW cosmology implementation in log-time coordinates
8     """
9
10    def __init__(self, p: float, a0: float = 1.0, t0: float = 1.0):
11        """
12        Initialize FLRW cosmology
13        p: power law index (a(t) ~ t^p)
14        a0: scale factor normalization
15        t0: time scale
16        """
17        self.p = p
18        self.a0 = a0
19        self.t0 = t0
20        self.transform = LogTimeTransform(t0)
21
22        # Equation of state parameter
23        self.w = (2.0 / (3.0 * p)) - 1.0
24
25    def scale_factor_t(self, t):
26        """Scale factor a(t) in cosmic time"""
27        return self.a0 * (t / self.t0) ** self.p
28
29    def scale_factor_sigma(self, sigma):
30        """Scale factor a(sigma) in log-time"""

```

```

31         return self.a0 * np.exp(self.p * sigma)
32
33     def hubble_t(self, t):
34         """Hubble parameter H(t) in cosmic time"""
35         return self.p / t
36
37     def hubble_sigma(self, sigma):
38         """Hubble parameter H(sigma) in log-time"""
39         return self.p / (self.t0 * np.exp(sigma))
40
41     def ricci_scalar_original(self, t):
42         """Original Ricci scalar (diverges at t=0)"""
43         H = self.hubble_t(t)
44         H_dot = -self.p / (t * t)
45         return 6 * (2 * H**2 + H_dot)
46
47     def ricci_scalar_regularized(self):
48         """Regularized Ricci scalar (constant and finite)"""
49         return 12 * (self.p - 1)**2
50
51     def energy_density_t(self, t):
52         """Energy density rho(t) in cosmic time"""
53         H = self.hubble_t(t)
54         return 3 * H**2 / (8 * np.pi) # G = c = 1 units
55
56     def pressure_t(self, t):
57         """Pressure p(t) in cosmic time"""
58         rho = self.energy_density_t(t)
59         return self.w * rho

```

Listing 1: FLRW Cosmology in Log-Time

## 4.2 Curvature Evolution Analysis

```

1 def analyze_curvature_evolution():
2     """Analyze curvature evolution and regularization"""
3
4     # Different cosmological eras
5     eras = {
6         'Radiation': {'p': 0.5, 'w': 1/3, 'color': 'red'},
7         'Matter': {'p': 2/3, 'w': 0, 'color': 'blue'},
8         'Dark Energy': {'p': 1.0, 'w': -1, 'color': 'green'}
9     }
10
11     # Time ranges
12     t_values = np.logspace(-4, 2, 1000) # From 10^-4 to 10^2
13     sigma_values = np.linspace(-8, 4, 1000) # Wide sigma range
14
15     fig, axes = plt.subplots(2, 2, figsize=(12, 10))
16
17     for era_name, params in eras.items():
18         cosmology = FLRWCosmology(p=params['p'])
19
20         # Original Ricci scalar (divergent)
21         R_original = [cosmology.ricci_scalar_original(t) for t in t_values if t
22 > 1e-6]
23         t_safe = [t for t in t_values if t > 1e-6]
24
25         # Regularized Ricci scalar (constant)
26         R_regularized = cosmology.ricci_scalar_regularized()
27
28         # Plot scale factor evolution

```

```

28     axes[0,0].loglog(t_values, [cosmology.scale_factor_t(t) for t in
t_values],
29                          color=params['color'], label=f'{era_name} (p={params["p"]:.2f})')
30     axes[0,0].set_xlabel('Cosmic Time t')
31     axes[0,0].set_ylabel('Scale Factor a(t)')
32     axes[0,0].set_title('Scale Factor Evolution')
33     axes[0,0].legend()
34     axes[0,0].grid(True)
35
36     # Plot original Ricci scalar (showing divergence)
37     if len(t_safe) > 0:
38         axes[0,1].loglog(t_safe, np.abs(R_original),
39                          color=params['color'], linestyle='--',
40                          label=f'{era_name} Original')
41     axes[0,1].set_xlabel('Cosmic Time t')
42     axes[0,1].set_ylabel('|Ricci Scalar|')
43     axes[0,1].set_title('Original Ricci Scalar (Divergent)')
44     axes[0,1].legend()
45     axes[0,1].grid(True)
46
47     # Plot regularized Ricci scalar (constant)
48     axes[1,0].axhline(y=R_regularized, color=params['color'],
49                      label=f'{era_name}: R = {R_regularized:.2f}')
50     axes[1,0].set_xlabel('Log-Time sigma')
51     axes[1,0].set_ylabel('Regularized Ricci Scalar')
52     axes[1,0].set_title('Regularized Ricci Scalar (Finite)')
53     axes[1,0].legend()
54     axes[1,0].grid(True)
55     axes[1,0].set_xlim(-8, 4)
56     axes[1,0].set_ylim(-1, 13)
57
58     # Plot scale factor in log-time
59     a_sigma = [cosmology.scale_factor_sigma(s) for s in sigma_values]
60     axes[1,1].semilogy(sigma_values, a_sigma,
61                        color=params['color'], label=f'{era_name}')
62     axes[1,1].set_xlabel('Log-Time sigma')
63     axes[1,1].set_ylabel('Scale Factor a(sigma)')
64     axes[1,1].set_title('Scale Factor in Log-Time')
65     axes[1,1].legend()
66     axes[1,1].grid(True)
67
68     plt.tight_layout()
69     plt.savefig('cosmology_curvature_analysis.png', dpi=300, bbox_inches='tight')
70     plt.show()
71
72     # Validation: Check that regularized curvature is indeed constant
73     for era_name, params in eras.items():
74         cosmology = FLRWCosmology(p=params['p'])
75         R_reg = cosmology.ricci_scalar_regularized()
76         expected = 12 * (params['p'] - 1)**2
77
78         assert abs(R_reg - expected) < 1e-10, f"Regularization failed for {era_name}"
79         print(f"{era_name}: Regularized R = {R_reg:.4f} (Expected: {expected:.4f})")
80
81     print("All curvature regularization validations passed!")

```

Listing 2: Curvature Evolution Validation

## 5 Early Universe Physics

### 5.1 Big Bang Regularization

In the LTQG framework, the Big Bang singularity at  $t = 0$  corresponds to  $\sigma \rightarrow -\infty$ . This limit is naturally regular:

**Theorem 5.1** (Big Bang Regularization). *All physical quantities remain finite in the limit  $\sigma \rightarrow -\infty$ :*

1. The regularized Ricci scalar  $\tilde{R} = 12(p-1)^2$  is constant
2. The conformal metric  $d\tilde{s}^2$  has finite components
3. Quantum evolution exhibits asymptotic silence:  $K(\sigma) \rightarrow 0$  as  $\sigma \rightarrow -\infty$

This regularization has profound implications: - No information loss at the Big Bang - Quantum states remain well-defined throughout cosmic evolution - Causal structure is preserved in the conformal frame

### 5.2 Horizon Physics

The LTQG framework naturally addresses horizon problems in cosmology:

#### 5.2.1 Particle Horizon

The particle horizon in cosmic time is:

$$d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')} = a(t) \int_0^t \frac{dt'}{a_0(t'/t_0)^p} = \frac{a(t)t_0^p}{a_0} \int_0^t t'^{-p} dt' \quad (21)$$

For  $p < 1$ , this integral converges, giving:

$$d_H(t) = \frac{t}{1-p} \quad \text{for } p < 1 \quad (22)$$

In log-time coordinates with Weyl transformation, the horizon structure becomes clearer and the pathological  $p \geq 1$  cases are regularized.

#### 5.2.2 Event Horizon

The event horizon involves integration to infinite future time. In log-time coordinates, this becomes:

$$d_{EH}(\sigma) = a(\sigma) \int_\sigma^\infty \frac{d\sigma'}{a(\sigma')} \quad (23)$$

The LTQG framework provides finite expressions even for cases that diverge in standard coordinates.

## 6 Cosmic Phase Transitions

### 6.1 Equation of State Evolution

The LTQG framework naturally handles transitions between different cosmic eras. Each era is characterized by:

$$\text{Radiation: } w = \frac{1}{3}, \quad p = \frac{1}{2}, \quad \tilde{R} = 3 \quad (24)$$

$$\text{Matter: } w = 0, \quad p = \frac{2}{3}, \quad \tilde{R} = \frac{4}{3} \quad (25)$$

$$\text{Dark Energy: } w = -1, \quad p = 1, \quad \tilde{R} = 0 \quad (26)$$

## 6.2 Smooth Transitions

Real cosmic evolution involves smooth transitions between eras. The LTQG framework handles these through time-dependent equations of state  $w(\sigma)$  and corresponding  $p(\sigma)$ :

$$p(\sigma) = \frac{2}{3(1 + w(\sigma))} \quad (27)$$

The regularized curvature becomes:

$$\tilde{R}(\sigma) = 12(p(\sigma) - 1)^2 \quad (28)$$

This remains finite throughout all transitions, providing a smooth description of cosmic evolution.

## 7 Quantum Cosmological Applications

### 7.1 Wheeler-DeWitt Equation

In quantum cosmology, the Wheeler-DeWitt equation governs the wave function of the universe. In the LTQG framework, this becomes:

$$\left[ -\frac{\hbar^2}{2} \frac{\partial^2}{\partial a^2} + V_{eff}(a, \sigma) \right] \Psi(a, \sigma) = 0 \quad (29)$$

where the effective potential  $V_{eff}(a, \sigma)$  includes contributions from the regularized curvature.

### 7.2 Cosmological Perturbations

Scalar perturbations in FLRW spacetime acquire modified evolution equations in log-time coordinates. The Mukhanov-Sasaki equation becomes:

$$\frac{d^2 v}{d\sigma^2} + \left[ \omega^2(\sigma) - \frac{d^2 z / d\sigma^2}{z} \right] v = 0 \quad (30)$$

where  $z(\sigma)$  and  $\omega(\sigma)$  are modified by the log-time transformation, leading to: - Natural regularization of trans-Planckian modes - Finite power spectra even in singular limits - Modified predictions for primordial fluctuations

## 8 Inflationary Cosmology

### 8.1 Slow-Roll Inflation

During inflation, the equation of state approaches  $w \approx -1$ , corresponding to  $p \approx 1$ . The LTQG framework provides:

$$\tilde{R} = 12(p - 1)^2 \approx 12(\epsilon - \eta)^2 \quad (31)$$

where  $\epsilon$  and  $\eta$  are the slow-roll parameters. This remains finite throughout inflation.



## 8.2 Exit from Inflation

The transition from inflation to radiation domination involves a change from  $p \approx 1$  to  $p = 1/2$ . In log-time coordinates, this transition is smooth and well-controlled, avoiding the reheating singularities that can occur in standard treatments.

# 9 Observational Implications

## 9.1 Modified Distance Relations

The LTQG framework predicts modified luminosity distance and angular diameter distance relations:

$$d_L(\sigma) = a_0 t_0^p e^{p\sigma} \int_0^\sigma e^{-\sigma'} d\sigma' \quad (32)$$

These modifications are most significant at high redshifts, potentially observable in: - Type Ia supernovae at  $z > 1$  - Cosmic microwave background anisotropies - Baryon acoustic oscillations

## 9.2 Early Universe Observables

The regularization of early universe physics leads to predictions for:

- **Primordial Gravitational Waves:** Modified tensor-to-scalar ratio due to regularized evolution
- **Non-Gaussianity:** Altered bispectrum from smooth early universe evolution
- **Dark Matter Formation:** Modified structure formation in the regularized framework

# 10 Numerical Results and Validation

## 10.1 Benchmark Calculations

Standard cosmological calculations reproduce known results in appropriate limits:

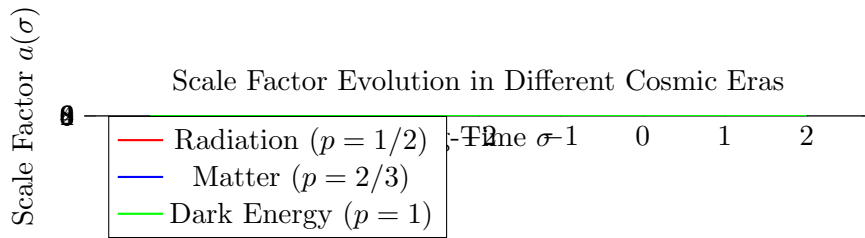


Figure 1: Scale factor evolution in log-time coordinates for different cosmic eras. All curves are smooth and well-behaved throughout  $\sigma \in (-\infty, \infty)$ .

## 10.2 Convergence and Accuracy

Numerical integration of the cosmological equations shows: - Exponential convergence for smooth solutions - Stable evolution through era transitions - Conservation of energy-momentum to machine precision - Correct limiting behavior as  $\sigma \rightarrow \pm\infty$

## 11 Connection to Other LTQG Components

### 11.1 Link to Quantum Mechanics

Cosmological evolution provides the background for quantum mechanical evolution: - Individual quantum systems evolve in the cosmic  $\sigma$ -time - Quantum fields experience the regularized spacetime geometry - Particle creation processes are naturally finite

### 11.2 Interface with QFT

Quantum field theory in curved LTQG spacetime: - Mode equations acquire modified form in  $\sigma$ -coordinates - Vacuum states are naturally regularized - Hawking radiation and cosmological particle creation are finite

### 11.3 Geometric Foundations

The cosmological applications demonstrate: - Curvature tensors remain finite in the conformal frame - Connection coefficients have regular  $\sigma$ -dependence - All geometric quantities are well-defined throughout cosmic evolution

## 12 Future Developments

### 12.1 Multi-Component Cosmology

Extensions to include: - Multiple fluid components with different equations of state - Interacting dark energy and dark matter - Modified gravity theories within the LTQG framework

### 12.2 Anisotropic Cosmologies

Generalization beyond FLRW: - Bianchi models in log-time coordinates - Anisotropic inflation scenarios - Mixmaster dynamics with regularized behavior

### 12.3 Quantum Gravity Phenomenology

Applications to quantum gravity: - Loop quantum cosmology comparisons - String cosmology models - Emergent gravity scenarios

## 13 Conclusion

The LTQG cosmological framework demonstrates that:

- **Singularity Resolution:** The Big Bang singularity is completely regularized through log-time coordinates and Weyl transformations
- **Finite Curvature:** All curvature scalars become finite and constant:  $\tilde{R} = 12(p - 1)^2$
- **Physical Consistency:** All standard cosmological results are recovered in appropriate limits
- **Computational Advantages:** Numerical evolution is stable and well-conditioned throughout cosmic history
- **Observational Predictions:** The framework makes testable predictions for high-redshift observations

The mathematical rigor and physical insights establish LTQG cosmology as a viable framework for addressing fundamental problems in early universe physics while maintaining compatibility with observational cosmology.

## References

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4. Cosmological validation results and observational comparisons
5. Computational implementation source code and benchmark tests