Log-Time Quantum Gravity: Reclocking Quantum Dynamics and Regularizing Early-Time Geometry

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I present *Log-Time Quantum Gravity* (LTQG), a framework that reclocks quantum dynamics using the logarithmic time variable $\sigma = \ln(\tau/\tau_0)$ and pairs it with a conformal rescaling of the background geometry. I establish (i) unitary equivalence of Schrödinger evolution in τ and σ, including time-ordered non-commuting Hamiltonians; (ii) an "asymptotic silence" regime as $\sigma \to -\infty$ in which the effective generator vanishes with finite accumulated phase; and (iii) geometric regularization in flat FLRW cosmology under a Weyl transform $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with $\Omega = 1/t$, yielding an Einstein, constant-curvature spacetime with $\tilde{R} = 12(p-1)^2$ and $\tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} = \tilde{R}^2/4$, $\tilde{K} = \tilde{R}^2/6$.

On the quantum field side, I show numerical equivalence (to tight tolerances using adaptive RK and phase-robust diagnostics) between τ- and σ-time mode evolution for free scalars in expanding backgrounds, while fixed-step tests correctly expose anti-damped σ-regimes as expected. For Schwarzschild backgrounds with spatially varying $\Omega(r,t)$, I compute invariants *directly from* $\tilde{g}_{\mu\nu}$, capturing derivative contributions from the conformal factor.

Disclaimer. I propose LTQG here as a *candidate* resolution strategy for early-time pathologies and clock–geometry alignment. I invite independent verification and stress testing of the mathematics, numerics, and physical implications. A complete open-source validation suite accompanies this work.

I. INTRODUCTION

Reconciling quantum mechanics (QM) with dynamical spacetime remains central to quantum gravity [**?**]. Two persistent challenges are: (i) the role of "time" as a parameter in QM versus a dynamical field in general relativity (GR), and (ii) singular early-time behavior in cosmology and near horizons. I propose *Log-Time Quantum Gravity* (LTQG), which (a) reclocks quantum evolution via $\sigma = \ln(\tau/\tau_0)$ and (b) aligns this reclocking with a Weyl rescaling of the metric. The resulting σ-picture preserves QM while rendering certain geometric and dynamical limits well-posed.

II. LOG-TIME MAPPING AND QUANTUM DYNAMICS

Define $\sigma = \ln(\tau/\tau_0)$ so that $\tau = \tau_0 e^{\sigma}$ and $\partial_\sigma = \tau\,\partial_\tau$. The τ-Schrödinger equation

$$i\hbar\,\partial_\tau\,|\psi(\tau)\rangle = H(\tau)\,|\psi(\tau)\rangle \tag{1}$$

is equivalent to the σ-equation

$$i\hbar\,\partial_\sigma\,|\psi(\sigma)\rangle = H_{\mathrm{eff}}(\sigma)\,|\psi(\sigma)\rangle\,, \qquad H_{\mathrm{eff}}(\sigma) = \tau_0 e^{\sigma}\,H(\tau_0 e^{\sigma}). \tag{2}$$

Proposition 1 (Unitary equivalence). For arbitrary (piecewise continuous) $H(\tau)$, including non-commuting time dependence, the time-ordered evolution operators satisfy

$$\mathcal{T}\exp\Big[-\tfrac{i}{\hbar}\int_{\tau_i}^{\tau_f} H(\tau)\,d\tau\Big] = \mathcal{T}\exp\Big[-\tfrac{i}{\hbar}\int_{\sigma_i}^{\sigma_f} H_{\mathrm{eff}}(\sigma)\,d\sigma\Big], \tag{3}$$

with $\tau_{i,f} = \tau_0 e^{\sigma_{i,f}}$. Hence density matrices and Heisenberg observables coincide in the two clocks.

Asymptotic silence. As $\sigma \to -\infty$, $H_{\mathrm{eff}}(\sigma) \to 0$ and the accumulated phase $\int_{-\infty}^{\sigma} H_{\mathrm{eff}}\,d\sigma' = \tau_0 e^{\sigma}$ is finite, making the σ-past a mathematically tame initialization surface.

III. WEYL RESCALING AND GEOMETRIC REGULARIZATION

We pair the clock map with a conformal rescaling $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$. For flat FLRW $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$, $a(t) = t^p$, choose $\Omega = 1/t$. Using the 4D Weyl identity,

$$\tilde{R} = \Omega^{-2}\Big(R - 6,\ln\Omega - 6,(\nabla\ln\Omega)^2\Big), \tag{4}$$

one finds

$$\tilde{R} = 12(p-1)^2, \qquad \tilde{R}_{\mu\nu} = \frac{\tilde{R}}{4}\,\tilde{g}_{\mu\nu}, \qquad \tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} = \frac{\tilde{R}^2}{4}, \quad \tilde{K} = \frac{\tilde{R}^2}{6}. \tag{5}$$

Thus the transformed spacetime is Einstein and constant curvature; all curvature scalars are finite constants for any p.

IV. QFT IN THE σ-PICTURE

For a free scalar mode u_k, the τ-equation maps to

$$u_k'' + (1-3p)\,u_k' + t(\sigma)^2\,\Omega_k^2\big(t(\sigma)\big)\,u_k = 0, \qquad t(\sigma) = \tau_0 e^{\sigma}, \tag{6}$$

where prime is ∂_σ. Using complex adiabatic initial data, integrating factors, and adaptive RK45, I verify:

1. Phase-robust equivalence between τ- and σ-evolution via Wronskian conservation, energy checks, and Bogoliubov coefficients (α_k, β_k).
2. Fixed-step RK stress tests in anti-damped σ-regimes display large amplitude error *by design*; adaptive solvers remove this artifact.

V. BLACK HOLES WITH SPATIALLY VARYING $\Omega(r,t)$

For Schwarzschild $ds^2 = -(1-r_s/r)dt^2 + (1-r_s/r)^{-1}dr^2 + r^2 d\Omega_2^2$ and a clock-based $\Omega(r,t) = 1/(t\sqrt{1-r_s/r})$, I *compute all invariants directly from* $\tilde{g}_{\mu\nu}$. Because Ω has spatial dependence, $\tilde{R}, \tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu}, \tilde{K}$ receive derivative contributions from Ω. Near the horizon $r \to r_s^+$, $\tilde{R}$ remains finite and the others have analytic limits that differ from naive Ω^{-4} scaling, as expected.

VI. ACTION, CLOCK DYNAMICS, AND CONSTRAINTS

Treating the clock τ as a scalar with action $S[g,\tau,\Phi] = \int d^4x\sqrt{-g}\,[\frac{1}{2\kappa}R - \frac{1}{2}(\nabla\tau)^2 - V(\tau) + \mathcal{L}_\Phi]$, variation yields Einstein equations with $T^{(\tau)}_{\mu\nu} = \nabla_\mu\tau\nabla_\nu\tau - \frac{1}{2}g_{\mu\nu}(\nabla\tau)^2 - g_{\mu\nu}V(\tau)$ and the clock equation $\tau - V'(\tau) = 0$. In FLRW these reduce to $\ddot{\tau} + 3H\dot{\tau} + V'(\tau) = 0$ and the standard Hamiltonian constraint, matching my symbolic derivations.

VII. NUMERICAL AND SYMBOLIC VALIDATION

I provide a comprehensive Python suite that checks:

- Invertibility and chain rule of the log-time map;
- Unitary equivalence for constant and non-commuting $H(\tau)$, plus Heisenberg consistency;
- Asymptotic silence: $H_{\text{eff}} \to 0$ with finite phase;
- FLRW curvature via Weyl identity and directly from $\tilde{g}_{\mu\nu}$ (constant-curvature identities asserted);
- Schwarzschild invariants from $\tilde{g}_{\mu\nu}$ with symmetry and near-horizon checks;
- QFT mode equivalence with adiabatic ICs, integrating factor, adaptive RK, Wronskian and Bogoliubov diagnostics.

VIII. DISCUSSION AND OUTLOOK

LTQG leaves QM intact while choosing a clock and geometry that regularize difficult regimes and stabilize computation. Immediate applications include: robust state preparation in early-universe QFT, controlled analysis of quenches in time-dependent Hamiltonians, and near-horizon field evolution with spatially varying conformal factors.

Limitations and open work. I have focused on free fields and classical backgrounds. Interactions, backreaction, nontrivial topologies, and observational signatures remain to be explored. For black holes, alternative Ω choices may better capture near-horizon physics in specific setups.

Community validation (disclaimer). **I propose LTQG in this paper as a candidate framework and report a complete set of mathematical and numerical checks supporting it. I explicitly invite independent replication and scrutiny of my derivations and code.** The validation suite is available (see Code Availability).

CODE AVAILABILITY

All scripts to reproduce the symbolic and numerical results are available at: *(https://github.com/DenzilGreenwood/Log_Time_v2.gi*

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