Log-Time Quantum Gravity: Reclocking Quantum Dynamics and Regularizing Early-Time Geometry

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We present *Log-Time Quantum Gravity* (LTQG), a framework that reclocks quantum dynamics using the logarithmic time variable $\sigma = \ln(\tau/\tau_0)$ and pairs it with a conformal rescaling of the background geometry. We establish (i) unitary equivalence of Schrödinger evolution in τ and σ, including time-ordered non-commuting Hamiltonians; (ii) an "asymptotic silence" regime as $\sigma \to -\infty$ in which the effective generator vanishes with finite accumulated phase; and (iii) geometric regularization in flat FLRW cosmology under a Weyl transform $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with $\Omega = 1/t$, yielding an Einstein, constant-curvature spacetime with $\tilde{R} = 12(p-1)^2$ and $\tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} = \tilde{R}^2/4$, $\tilde{K} = \tilde{R}^2/6$.

On the quantum field side, we show numerical equivalence (to tight tolerances using adaptive RK and phase-robust diagnostics) between τ- and σ-time mode evolution for free scalars in expanding backgrounds, while fixed-step tests correctly expose anti-damped σ-regimes as expected. For Schwarzschild backgrounds with spatially varying $\Omega(r,t)$, we compute invariants *directly from* $\tilde{g}_{\mu\nu}$, capturing derivative contributions from the conformal factor.

Disclaimer. LTQG is proposed here as a *candidate* resolution strategy for early-time pathologies and clock–geometry alignment. We invite independent verification and stress testing of the mathematics, numerics, and physical implications. A complete open-source validation suite accompanies this work.

I. INTRODUCTION

Reconciling quantum mechanics (QM) with dynamical spacetime remains central to quantum gravity [?]. Two persistent challenges are: (i) the role of "time" as a parameter in QM versus a dynamical field in general relativity (GR), and (ii) singular early-time behavior in cosmology and near horizons. We propose *Log-Time Quantum Gravity* (LTQG), which (a) reclocks quantum evolution via $\sigma = \ln(\tau/\tau_0)$ and (b) aligns this reclocking with a Weyl rescaling of the metric. The resulting σ-picture preserves QM while rendering certain geometric and dynamical limits well-posed.

II. LOG-TIME MAPPING AND QUANTUM DYNAMICS

Define $\sigma = \ln(\tau/\tau_0)$ so that $\tau = \tau_0 e^\sigma$ and $\partial_\sigma = \tau\,\partial_\tau$. The τ-Schrödinger equation

$$i\hbar\,\partial_\tau\,|\psi(\tau)\rangle = H(\tau)\,|\psi(\tau)\rangle \tag{1}$$

is equivalent to the σ-equation

$$i\hbar\,\partial_\sigma\,|\psi(\sigma)\rangle = H_{\text{eff}}(\sigma)\,|\psi(\sigma)\rangle, \qquad H_{\text{eff}}(\sigma) = \tau_0 e^\sigma\,H(\tau_0 e^\sigma). \tag{2}$$

Proposition 1 (Unitary equivalence). For arbitrary (piecewise continuous) $H(\tau)$, including non-commuting time dependence, the time-ordered evolution operators satisfy

$$\mathcal{T}\exp\Big[-\tfrac{i}{\hbar}\int_{\tau_i}^{\tau_f} H(\tau)\,d\tau\Big] = \mathcal{T}\exp\Big[-\tfrac{i}{\hbar}\int_{\sigma_i}^{\sigma_f} H_{\text{eff}}(\sigma)\,d\sigma\Big], \tag{3}$$

with $\tau_{i,f} = \tau_0 e^{\sigma_{i,f}}$. Hence density matrices and Heisenberg observables coincide in the two clocks.

Asymptotic silence. As $\sigma \to -\infty$, $H_{\text{eff}}(\sigma) \to 0$ and the accumulated phase $\int_{-\infty}^{\sigma} H_{\text{eff}}\,d\sigma' = \tau_0 e^\sigma$ is finite, making the σ-past a mathematically tame initialization surface.

III. WEYL RESCALING AND GEOMETRIC REGULARIZATION

We pair the clock map with a conformal rescaling $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$. For flat FLRW $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$, $a(t) = t^p$, choose $\Omega = 1/t$. Using the 4D Weyl identity,

$$\tilde{R} = \Omega^{-2}\Big(R - 6\,\square\ln\Omega - 6\,(\nabla\ln\Omega)^2\Big), \tag{4}$$

one finds

$$\tilde{R} = 12(p-1)^2, \qquad \tilde{R}_{\mu\nu} = \frac{\tilde{R}}{4}\,\tilde{g}_{\mu\nu}, \qquad \tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu} = \frac{\tilde{R}^2}{4}, \quad \tilde{K} = \frac{\tilde{R}^2}{6}. \tag{5}$$

Thus the transformed spacetime is Einstein and constant curvature; all curvature scalars are finite constants for any p.

IV. QFT IN THE σ-PICTURE

For a free scalar mode u_k, the τ-equation maps to

$$u_k'' + (1-3p)\,u_k' + t(\sigma)^2\,\Omega_k^2\big(t(\sigma)\big)\,u_k = 0, \qquad t(\sigma) = \tau_0 e^\sigma, \tag{6}$$

where prime is ∂_σ. Using complex adiabatic initial data, integrating factors, and adaptive RK45, we verify:

1. Phase-robust equivalence between τ- and σ-evolution via Wronskian conservation, energy checks, and Bogoliubov coefficients (α_k, β_k).

2. Fixed-step RK stress tests in anti-damped σ-regimes display large amplitude error *by design*; adaptive solvers remove this artifact.

V. BLACK HOLES WITH SPATIALLY VARYING $\Omega(r,t)$

For Schwarzschild $ds^2 = -(1 - r_s/r)dt^2 + (1 - r_s/r)^{-1}dr^2 + r^2 d\Omega_2^2$ and a clock-based $\Omega(r,t) = 1/(t\sqrt{1 - r_s/r})$, we *compute all invariants directly from* $\tilde{g}_{\mu\nu}$. Because Ω has spatial dependence, $\tilde{R}, \tilde{R}_{\mu\nu}\tilde{R}^{\mu\nu}, \tilde{K}$ receive derivative contributions from Ω. Near the horizon $r \to r_s^+$, $\tilde{R}$ remains finite and the others have analytic limits that differ from naive Ω^{-4} scaling, as expected.

VI. ACTION, CLOCK DYNAMICS, AND CONSTRAINTS

Treating the clock τ as a scalar with action $S[g, \tau, \Phi] = \int d^4x \sqrt{-g}\,[\frac{1}{2\kappa}R - \frac{1}{2}(\nabla\tau)^2 - V(\tau) + \mathcal{L}_\Phi]$, variation yields Einstein equations with $T_{\mu\nu}^{(\tau)} = \nabla_\mu\tau\nabla_\nu\tau - \frac{1}{2}g_{\mu\nu}(\nabla\tau)^2 - g_{\mu\nu}V(\tau)$ and the clock equation $\Box\tau - V'(\tau) = 0$. In FLRW these reduce to $\ddot{\tau} + 3H\dot{\tau} + V'(\tau) = 0$ and the standard Hamiltonian constraint, matching our symbolic derivations.

VII. NUMERICAL AND SYMBOLIC VALIDATION

We provide a comprehensive Python suite that checks:

- Invertibility and chain rule of the log-time map;

- Unitary equivalence for constant and non-commuting $H(\tau)$, plus Heisenberg consistency;

- Asymptotic silence: $H_{\text{eff}} \to 0$ with finite phase;

- FLRW curvature via Weyl identity and directly from $\tilde{g}_{\mu\nu}$ (constant-curvature identities asserted);

- Schwarzschild invariants from $\tilde{g}_{\mu\nu}$ with symmetry and near-horizon checks;

- QFT mode equivalence with adiabatic ICs, integrating factor, adaptive RK, Wronskian and Bogoliubov diagnostics.

VIII. DISCUSSION AND OUTLOOK

LTQG leaves QM intact while choosing a clock and geometry that regularize difficult regimes and stabilize computation. Immediate applications include: robust state preparation in early-universe QFT, controlled analysis of quenches in time-dependent Hamiltonians, and near-horizon field evolution with spatially varying conformal factors.

Limitations and open work. We have focused on free fields and classical backgrounds. Interactions, backreaction, nontrivial topologies, and observational signatures remain to be explored. For black holes, alternative Ω choices may better capture near-horizon physics in specific setups.

Community validation (disclaimer). **This paper proposes LTQG as a candidate framework and reports a complete set of mathematical and numerical checks supporting it. We explicitly invite independent replication and scrutiny of our derivations and code.** The validation suite is available (see Code Availability).

CODE AVAILABILITY

All scripts to reproduce the symbolic and numerical results are available at: *(https://github.com/DenzilGreenwood/Log_Time_v2.git*

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