MGCR 341: Finance 1
Winter 2012
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Assignment 1: Solutions

Topic: Time value of money

1) Download "mega example.xlsx" from the assignments folder online. The table is based on the Mega Example from Lecture 3. Fill in the table. See "mega example solutions.xlsx".

Topic: Time value of money

2) Today is month t = 0. A bank is offering you the following financial product:

You make monthly payments to the bank of $500 per month from month t = 25 to month t = 264. In exchange, the bank will give you monthly payments of X from month t = 265 to month t = 504.

The annual interest rate (APR) is 12%, with monthly compounding.

The inflation rate is 0.2% per month.

a) Assuming the contract is fair, what should X be?

If the contract is fair, the PV of what you give the bank equals the PV of what the bank gives you.

The per month interest rate is APR/k = 12%/12 = 1% = 0.01

$$\frac{500}{0.01}\left(1-\frac{1}{1.01^{264-25+1}}\right)\left(\frac{1}{1.01^{24}}\right) = 35{,}763.15 = \frac{X}{0.01}\left(1-\frac{1}{1.01^{504-265+1}}\right)\left(\frac{1}{1.01^{264}}\right)$$

→ **X = 5,446.28**

You could also have used the real approach, but that would have been more complicated.

Suppose that at t = 264, the bank makes you the following offer:

Instead of giving you monthly payments of X from t = 265 to t = 504, the bank will pay you a lump sum of $480,000 at t = 264.

b) Should you accept this lump sum offer?

We can compare the PV of 480,000 to the PV of what the bank will give you (35,763.15).

$$\frac{480{,}000}{1.01^{264}} = 34{,}705.52 < 35{,}763.15$$

So you should not accept the lump sum offer. Another way to see this is to compare 480,000 to the value, as of t = 264, of the bank's annuity

$$\frac{5{,}446.28}{0.01}\left(1-\frac{1}{1.01^{504-265+1}}\right) = 494{,}627.68 > 480{,}000$$

Again, we see that you should not accept the lump sum payment of 480,000.

Suppose your goal in retirement is to consume the same quantity of goods every month from t = 265 to t = 504. After informing the bank of this plan, the bank comes back to you with another offer:

Instead of giving you monthly payments of X from t = 265 to t = 504, the bank will give you a growing annuity, where the first payment is $4,500 at t = 265, and each month the payment grows by 0.2% until the last payment at t = 504.

c) If a hamburger costs $1 at t = 0, what constant amount of hamburgers would the above growing annuity allow you to purchase each month from t = 265 to t = 504?

The growth rate of the annuity is equal to the inflation rate. Thus, each period you will be able to consume the same amount of hamburgers with the money that the bank gives you. We can simply calculate how many hamburgers you could buy at t = 265 with the $4,500 the bank pays at t = 265. We need to convert a nominal amount of $4,500 at t = 265 into the corresponding number of real dollars at t = 265:

$$\frac{4,500}{1.002^{265}} = 2,650.13$$

d) Should you accept the growing annuity instead of the monthly payments of X you found in part a?

We can answer this question by computing the PV of the growing annuity to the PV of the constant annuity:

$$\frac{4,500}{0.01 - 0.002}\left(1 - \frac{1.002^{240}}{1.01^{240}}\right)\left(\frac{1}{1.01^{264}}\right) = 34,639.34 < 35,763.15$$

So you should not accept the growing annuity. We could have obtained the same result by comparing the value of the growing and constant annuity, both as of t = 264:

$$\frac{4,500}{0.01 - 0.002}\left(1 - \frac{1.002^{240}}{1.01^{240}}\right) = 479,084.65 < 494,627.68$$

You could also have solved this question by solving part e first and seeing that the bank's constant annuity allows you to purchase more constant hamburgers per period than the growing annuity.

e) Suppose you decide to stick with the original plan of receiving X from t = 265 to t = 504. What constant amount of hamburgers would you be able to purchase per month from t = 265 to t = 504? (Note that you are not spending X each month.) (Hint: You need to find the value of the constant annuity as of t = 264, and set this equal to the value of the growing annuity as of t = 264, where the first cash flow in the growing annuity is C, and the growth rate is 0.2% per month. Once you find C, you calculate how many hamburgers this allows you to buy at t = 265. This is the amount of hamburgers you can buy each month.)

$$494,627.68 = \frac{C}{0.01 - 0.002}\left(1 - \frac{1.002^{240}}{1.01^{240}}\right)$$

→ C = 4,645.99

At t =265, with $4,645.99 you can buy

$$\frac{4,645.99}{1.002^{265}} = \mathbf{2,736.1\ hamburgers}$$

Note that at t = 265, you are not spending the full 5,446.28 that the bank pays you. You are only spending 4,645.99, and saving the rest. Each period thereafter you spend 1.002 more than the previous period (this allows you to buy the same amount of hamburgers each period). At some point you will have to spend more than the 5,446.28 that the bank is giving you. At this point you start dipping into your savings to make up the difference.

We could also have solved this question using the real approach. In this case, we would have to make sure all quantities are real. The real interest rate is 1.01/1.002 – 1 = 7.984%, and the real growth rate is 0% since the number of hamburgers is constant each period.

$$\frac{494,627.68}{1.002^{264}} = \frac{C}{0.07984}\left(1 - \frac{1}{1.01^{240}}\right)$$

→ C = 2,736.1 hamburgers

Note that in the real approach, the C you back out is already in terms of hamburgers.

f) Based on your answer to part e), how much money are you spending at t = 504?

At t = 504 you are buying 2,736.1 hamburgers, which costs 2,736.1*$(1.002)^{504}$ = $7,489.70

Alternatively, we know that at t = 265 you are spending $4,645.99, and that each period thereafter you are spending 0.2% more. Thus at t = 504 you are spending 4,645.99*$(1.002)^{504-265}$=$7,489.70