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Parameter Estimations

Q1. Let $(X1, X2, \dots X_n)$ be random sample of size n taken from Normal Population with parameters: mean $= \theta_1$ and variance $= \theta_2$. Find Maximum Likelihood Estimates of these two parameters.

Sol

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f(x) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

Likelihood f^n :-

$$L(X1, X2, \dots X_n; \theta_1, \theta_2) =$$

$$\frac{1}{(2\pi)^{\frac{n}{2}} \theta_2^{\frac{n}{2}}} \exp\left(-\frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2\right)$$

Take natural logarithm of likelihood f^n

$$\ln L(X_1, X_2, \dots X_n; \theta_1, \theta_2) =$$

$$-\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln \theta_2 - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Take derivative with respect to θ_1 and θ_2 and set it equal to zero

$$\frac{d}{d\theta_1} \ln L(X_1, X_2, \dots X_n; \theta_1, \theta_2) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\frac{d}{d\theta_2} \ln L(X_1, X_2, \dots X_n; \theta_1, \theta_2) = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

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Solving equations, Maximum likelihood estimates for θ_1 and θ_2

$$\hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$\boxed{\hat{\theta}_2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \hat{\theta}_1)^2}$$

$\hat{\theta}_1 \rightarrow$ sample mean $\overline{X}$

$\hat{\theta}_1 \rightarrow$ unbiased estimator of mean

$\hat{\theta}_2 \rightarrow$ close to sample variance

$$S^2 = \frac{1}{n-1} \sum_{i=1} (x_i - \overline{X})^2$$

$$\therefore \hat{\theta}_2 = \frac{n-1}{n} S^2$$

$\hat{\theta}_2 \rightarrow$ biased estimator of variance

$$E\,\hat{\theta}_2 = \frac{n-1}{n} \theta_2$$

Q2. Let $X_1, X_2, \ldots X_n$ be random sample from $B(m, \theta)$ distribution where $\theta \in \Theta = (0,1)$ is unknown & $m \rightarrow$ known +ve integer. Compute value of θ using MLE

Sol

$$X_1 = x_1,\ X_2 = x_2, \ldots X_n = x_n$$

Likelihood function

$$L(x_1, x_2, \ldots x_n; \theta) = P_{X_1 X_2 \ldots X_n}(x_1, x_2, \ldots x_n; \theta)$$

$$= \prod_{i=1}^{n} P_{X_i}(x_i; \theta)$$

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$$= \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

$$= \left[\prod_{i=1}^{n} \binom{m}{x_i}\right] \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{mn - \sum_{i=1}^{n} x_i}$$

First term doesn't depend on θ

$\therefore L(x_1, x_2, \ldots x_n; \theta) = C \cdot \theta^{S} (1-\theta)^{mn-S}$

where C doesn't depend on θ

$$S = \sum_{k=1}^{n} x_i$$

Differentiate & set derivative to 0

$$\hat{\theta}_{ML} = \frac{1}{mn} \sum_{k=1}^{n} x_i$$

$$\boxed{\hat{\theta}_{ML} = \frac{1}{mn} \sum_{k=1}^{n} x_i}$$