The Structures of Human Reasoning

Chapter I: Moral Reasoning &Tautology

Logic. Some of us hate, and some of us love. But to put it simply, it is our very perception of that word and its meaning that attracts or disdains our very interests of that term. When you hear the word: Logic, what do you feel? Hear? Or think? Is it like a cold forbidden feeling of the unknown? Or is a warmth and comfortable feeling like home? What category would you put this word, logic, in? Anyway, there are many ways to express logic. Examples include in mathematics, human reasoning, physics?, etc. However, the ones that we will be discussing are the human reasoning and hopefully in a simpler way, the mathematical way. Let's get started, shall we!

Section 1.1: Propositions

Proposition: The morales of **right** and **wrong**. Just think of **right** as **true**, and **wrong** as **false**. Easy, **right**? Well, the very definition of a proposition is a claim. That claim can be right or wrong, it doesn't matter. As long as it is valid. How do you know if a claim or proposition is valid? Well...

Here are some **valid** propositions:

- It **is right** to smoke in public.
- Toronto **is the** capital of Canada.
- 1+2=3
- 2+2=5

Notice that **all** these are **claims**. You can **determine** if they are **true** or **false**; **right** or **wrong**.

Here are some **In**valid propositions:

- I love you.
- Here's your cheeseburger!
- X+1=2
- X.

Clearly the first two are not claims, or invalid claims. However the third looks like a claim, but in reality it is not. In mathematics, you would solve that equation to find the value of x. However, logically, x is just a letter. It has no meaning. It'll be like 2 plus something equals 3. What is something? Is it a number? Does it mean something? No. It means nothing. And nothing does not equal zero, smarty pants. It has no value assigned to it. A zero has a value of 0, but is that the same as something with no value? Weird, huh? So in truth, the third is an invalid claim, you can't claim it true or false. Also, for the last one, would you be able to make sense of it if some random guy yelled “X.” out? I can't.

For writing claims in a logical form, yay, we would replace parts of the claim with individual symbols or predicates. Example:

It is right to smoke in public. Would become p → q. “It is right” =p, “to”= → , “smoke in public.” =q. Same with mathematical claims: “1+2”=p, “=” = → , and “3”=q. There's also the ↔ ; means if and only if. So p is true if and only if q is false, is the same as: p ↔ q. It's a looping thing, I guess.

Okay so, now that we got that out of the way, let us define some more terms! Yay... It's not so bad. The first on the list is negation, or the opposite of the truthfullness of something. Example: if something is true, then the negation is false. What's the negation of that statement? Why...if something is false, then the negation is true! It's that easy! Now, for symbolic reasons, this term is symbolically defined as ⌐ . We can then use this term or symbol to say that while such n' such is true, the ⌐ of such n' such is false. Get it? Okay, here are some more examples:

- It is not right to smoke in public.
- Toronto is not the capital of Canada.
- ⌐ p → q

The other three are exclusive or, inclusive or, and “and”. ⊕ is exlusive or, ∨ is inclusive or, and ∧ is and! Ok, why do we need to know these symbols and terms, you may ask? Well because they are basically logical operators. You see, we can evaulate claims based on what logical operators are used. Like determining if a claim or statement is true or false. We can evaulate them by using what is called truth tables (Sometimes the best way to explain is pictures):

P	~ P
T	F
F	T

Table 1: Negation of p

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Table 2: the --> or implication.

Exclusive-Or – truth table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Note: in this course any usage of "or" will connote the logical operator $\vee$ as opposed to the exclusive-or.

Lecture 1 28

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

Table 3: Double Implication

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Table 5: Inclusive or

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Table 6: and

You can combine them to form complex logical functions:

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Sigh, so now you should be able to familiarize yourselves with other logical equations and solve them. Also, please note that you should be able to translate them to english and vice versa. It shouldn't be too hard. Anyway, the best way is through practice. So, go online and find something yourselves! You can also make one up. And also, you get to choose which way works best when trying to figure out if a proposition is false or not. If you need help or stuck on anything, consult to your professor or someone who might know the stuff. You can also look up tutorials and or videos. Up to you. Alright! On to the next section!

Section 1.2: Proposition Equivalence & Discrete Variables

Okay so, you know all bout propositions. But did you know that some propositions can be equal to one and another? Well, it's true. We also will be talking about variables called discrete variables or quantifiers.

The first thing that you need to know is some terminology:

Tautology: Basically where all the outcomes, no matter what comes in, are true or right.

Contraction: Everything has a Yin Yang, right? Well so does a tautology! Contraction basically means where all the outcomes, no matter what comes in, are false or wrong.

PROPOSITIONAL EQUIVALENCE

Examples:

- Tautology

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

- Contradiction

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

- Contingency

p	$\sim p$	$p \rightarrow \sim p$
T	F	T
F	T	F

Now let us get introduced to some equivalences!

Theorem 2.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1. *Commutative laws:*	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. *Associative laws:*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. *Distributive laws:*	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. *Identity laws:*	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. *Negation laws:*	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6. *Double negative law:*	$\sim(\sim p) \equiv p$	
7. *Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. *Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. *De Morgan's laws:*	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. *Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. *Negations of* $\mathbf{t}$ *and* $\mathbf{c}$*:*	$\sim\mathbf{t} \equiv \mathbf{c}$	$\sim\mathbf{c} \equiv \mathbf{t}$

Why do they work? Don't ask me, prove them yourselves. Oh wait! There's more!

More Logical Equivalences

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$

$p \to q \equiv \neg q \to \neg p$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg(p \to \neg q)$

$\neg(p \to q) \equiv p \wedge \neg q$

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

With these you could find a counterpart to any claim. Just like there are more than one way to state something, there are more than one way to logically express something!

Moving on...!

So you know about predicates, right? Last time I just gave you a brief introductory of them. Well, now let us use them in the form of math. It's really simple actually:

Q(x , y) is the statement: $x+y=3$. Basically it means that Q(..) is true whenever the statement is true. Like... $1+2=3$. $1=x$, and $2=y$. Get it? Same with math, you can also use that form for english statements too. BTW, the x and y are the predicates of Q or the claim.

Now that you know about predicates, let us move onto the so-called discrete variable. So, what common word comes into mind when you think of the following: *all, some, many, none,* or *few*? Well, they do have numbers in common. But what would you say about the common word? I know one, quantifiers or discrete variables. Basically the independent and dependent as well as the controlled variables are not the only ones out there. The discrete variable is like the number of test subjects in an experiment. Like you can have many or few test subjects. You can also use all plants on earth are test subjects for survival in terms of evolution.

From these common words, we can then categorize them into two categories: the first category is a universal one. That means for all. Like for all plants, or every animal. We call that a universal quantifier. It is represented by the $\forall$. In math or logical equations, we can represent this as follows:

$\forall x$, is for all of x, $x+1<x^2$.

As you can see, we can then formulate whether a statement using the $\forall$ is true or false.

Now, the next category is pretty similar, except it implies for a certain amount or some, or few, or many. This is identified as the existential quantifier. Which means that there exists. Meaning that there has to be at least one. Its symbol is represented as: $\exists$. You should then see how this is used:

$\exists x$, is there exists a x that, $x-1=2$.

Bam! That's basically all the quantifiers you need to know. There is, of course, the quantifier: there exists only one, which is represented as: $\exists!$, but that's about it. Please note that you can also use these with English propositions as well. Basically P will be some predicate and you can quantify whether your statement considers for all of P or for some of it.

"For every person x, if x is a student in this class, then x has visited Mexico or x
$\forall x$ S(x) $\rightarrow$ M(x) V
has visited Canada."
C(x).

You can also put the ¬ symbol in front of a quantifier. Anyway, with these quantifiers, we can nest them into one proposition. Example:

$\forall x \exists y(x + y = 0)$.

For every person in group A, there exists persons in group B that: Person in group A is a family member for Person in group B.

Etc.

We can also do: $\forall(x , y)$ or something similar to signalfy for all x and y.

Section 1.3: Rules of Reasoning and verification:

Okay, let's talk about arguments. There can invalid arguments and valid arguments, depending your reasoning and conclusion. You should be able to tell the difference. If not, ehhh here's an example:

Valid:

If you have money, then you can go shopping.
You have money.
Therefore, you can go shopping.

Invalid:

If you do every problem in this book, then you will know the course.
You know the course.
Therefore, you did every problem in this book.

You can see why this is invalid, right? It is implying that you did every problem in the book, just because you know the course. Invalid arguments are also called fallacies. Arguments can also be logical or mathematical format, just like everything else in this book. See what I did there ;) . There's a form that we follow for arguments, you place the premises or parts of a statement on top of each other, followed by the therefore symbol and conclusion. Example:

Full statement: "If you have access to the network, then you can change your grade."

Premise: "You have access to the network."

Therefore symbol and conclusion: ∴"You can change your grade.

Note: you usually have more than one premise. Now, I want you to tell me whether this is a fallacy or not.

You can also use quantifiers in your arguments as well. Determining whether that argument is valid or not. Which brings us to the next topic. Anyway, off to verifying!

Okay so, verifying. You prolly know about proofs in mathematics and what nots. Well for this book you'll need to know at most, three common verification methods.

The first one is known as direct verifying or direct proof. Basically, we give an obvious way of proving something. Like proving:

2k is even. Prove that 2p is even. You can see how we can prove this. Basically any number, whether it is odd or even is going to become even if multiplied by two. That is basically direct verifying. Straight forward, right? It's basically giving an example of why this is true or false.

Okay, now for some indirect verifying. There are two indirect verifying techniques. The first is known by contraposition, the second is known by contraction. Contraposition is basically taking a statement, and proving that its inverse is true, by proving that the inverse is true we can then assume that the original statement is true, and vice versa. In logic, we can do:

prove that $p \rightarrow q$ is true. Well if the contraposition ($\neg p \rightarrow \neg q$) is true, then the original is true!