Wind Turbine Aerodynamics

A. Introduction

LOADS CREATED BY WIND FLOW OVER TURBINE BLADE

CREATES ROTATION

WIND RESOURCE → AERO-LOADS → BLADE ROTATION

ENERGY CONVERSION PROCESS

K.E. IN WIND → ROTATIONAL ENERGY IN HUB → USEFUL ENERGY

INVOLVES AERODYNAMICS

OUR GOAL IS TO EXTRACT WIND'S ENERGY AT A MINIMUM OF COST → JUST A FEW LIGHT BLADES

MODERN WIND TURBINES → AREA OCCUPIED BY BLADES IS AS LOW AS 3% OF SWEPT AREA

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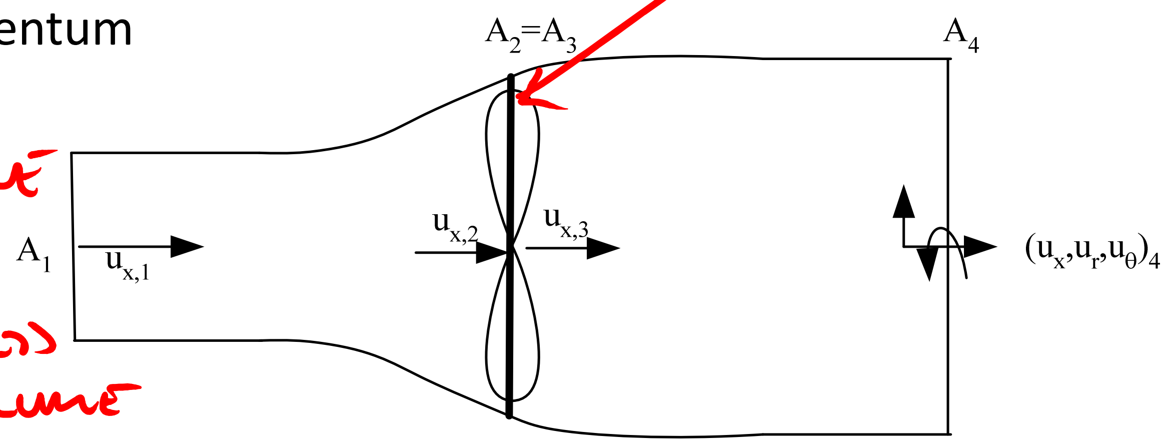
"ACTUATOR DISK"

B. One-Dimensional Momentum Theory



AN ABILITY TO DETERMINE OVERALL BEHAVIOR OF A WIND TURBINE IS PROVIDED BY SIMPLE CONTROL VOLUME ANALYSIS

→ GUIDANCE ON PERFORMANCE

→ LIMITED IN DETAILS

COMES FROM EARLY PREDICTIONS OF SHIP PROP PERFORMANCE

ASSUMPTIONS:

1. STEADY FLOW
2. INCOMPRESSIBLE
3. $u_{x,1} = u_{x,4}$ & CONSTANT ACROSS 1 & 4
4. $u_{r,4}$ & $u_{\theta,4} << u_{x,4}$
5. NO HEAT TRANSFER
6. CONSTANT TEMP
7. HYDROSTATIC TERMS ARE UNIMPORTANT
8. $P_1 = P_4 = P_a$
9. UNIFORM THRUST OVER ROTOR AREA

Bernoulli Equation Along a Streamline:

$$P + 1/2\rho u^2 + \rho g z = C$$

Continuity Equation:

$$\frac{\partial}{\partial t}\int_{CV} \rho dV + \int_{CS} \rho(\vec{v}\cdot\hat{n})dA = 0$$

x-Momentum Equation:

$$\frac{\partial}{\partial t}\int_{CV} \rho u dV + \int_{CS} \rho u(\vec{v}\cdot\hat{n})dA = F_{s,x} + F_{b,x}$$

Energy Equation:

$$\frac{\partial}{\partial t}\int_{CV} (P + 1/2\rho u^2 + \rho g z + \rho u_i) dV + \int_{CS} (P + 1/2\rho u^2 + \rho g z + \rho u_i)(\vec{v}\cdot\hat{n})dA = \dot{Q} + \dot{W}$$

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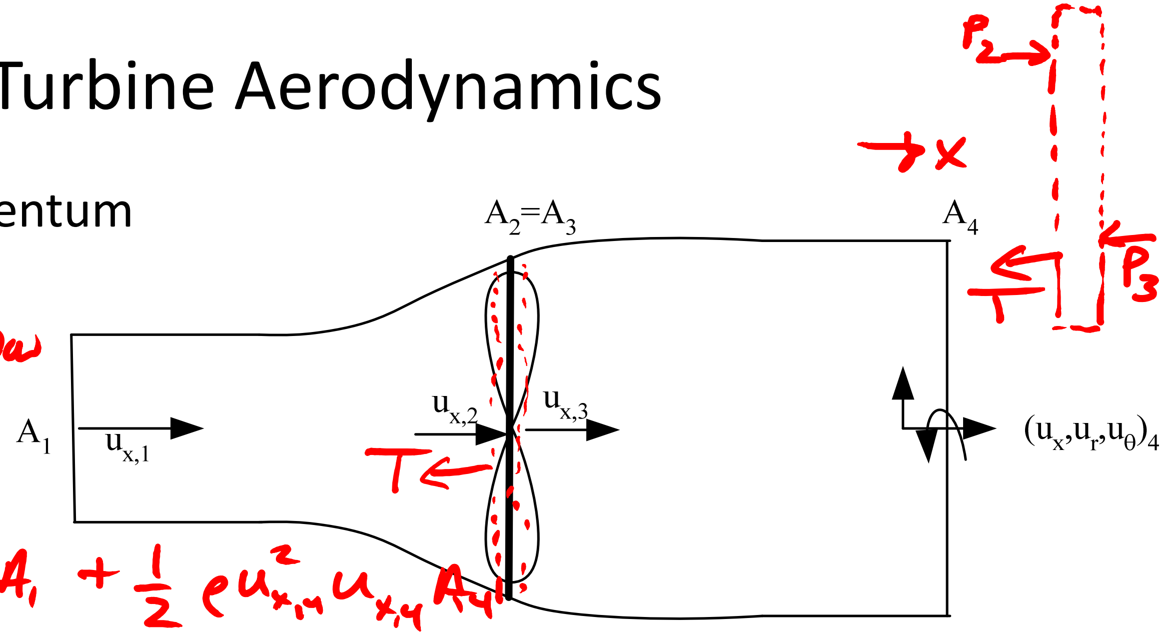

CONSIDER ENERGY EQUATION BETWEEN INLET & OUTLET ① & ④

$$\dot{W} = -\frac{1}{2}\rho u_{x,1}^2 u_{x,1} A_1 + \frac{1}{2}\rho u_{x,4}^2 u_{x,4} A_4$$

CONTINUITY BETWEEN ① & ④

$$\rho u_{x,1} A_1 = \rho u_{x,4} A_4 = \dot{m}$$

X-MOMENTUM BETWEEN ① & ④

$$-\rho u_{x,1} u_{x,1} A_1 + \rho u_{x,4} u_{x,4} A_4 = -T$$

X-MOMENTUM BETWEEN ② & ③

$$0 = P_2 A_2 - P_3 A_3 - T$$

CONSIDER BERNOULLI FROM ① → ②

$$P_1 + \frac{1}{2}\rho u_{1,x}^2 = P_2 + \frac{1}{2}\rho u_{x,2}^2$$

FROM ③ → ④

$$P_3 + \frac{1}{2}\rho u_{x,3}^2 = P_4 + \frac{1}{2}\rho u_{x,4}^2$$

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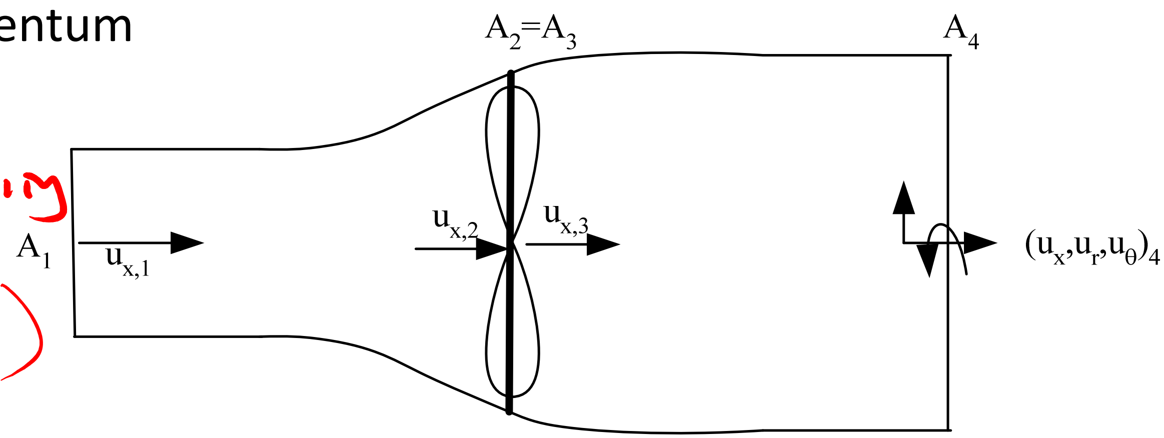

COMBINE ENERGY & CONTINUITY

$$-\dot{W} = P = \dot{m}\left(\frac{u_{x,1}^2 - u_{x,4}^2}{2}\right)$$

ELIMINATE $u_{x,4}$

USE X-MOMENTUM & BERNOULLI

$$T = A_2(P_2 - P_3) \leftarrow \text{X-MOM}$$

$P_1 = P_4$ $\qquad u_{x,2} = u_{x,3}$

$$P_2 - P_3 = \cancel{P_1} + \tfrac{1}{2}\rho u_{x,1}^2 - \cancel{\tfrac{1}{2}\rho u_{x,2}^2} - \cancel{P_4} - \tfrac{1}{2}\rho u_{x,4}^2 + \cancel{\tfrac{1}{2}\rho u_{x,3}^2}$$

$$P_2 - P_3 = \tfrac{1}{2}\rho u_{x,1}^2 - \tfrac{1}{2}\rho u_{x,4}^2$$

SUBSTITUTE $P_2 - P_3$ INTO X-MOM ABOVE

$$T = \tfrac{1}{2}\rho A_2\left(u_{x,1}^2 - u_{x,4}^2\right) = \tfrac{1}{2}\rho A_2(u_{x,1}+u_{x,4})(u_{x,1}-u_{x,4})$$

SET EQUAL TO OTHER THRUST EQUATION (X-MOM FROM 1→4)

$$T = \rho u_{x,1} u_{x,1} A_1 - \rho u_{x,4} u_{x,4} A_4$$

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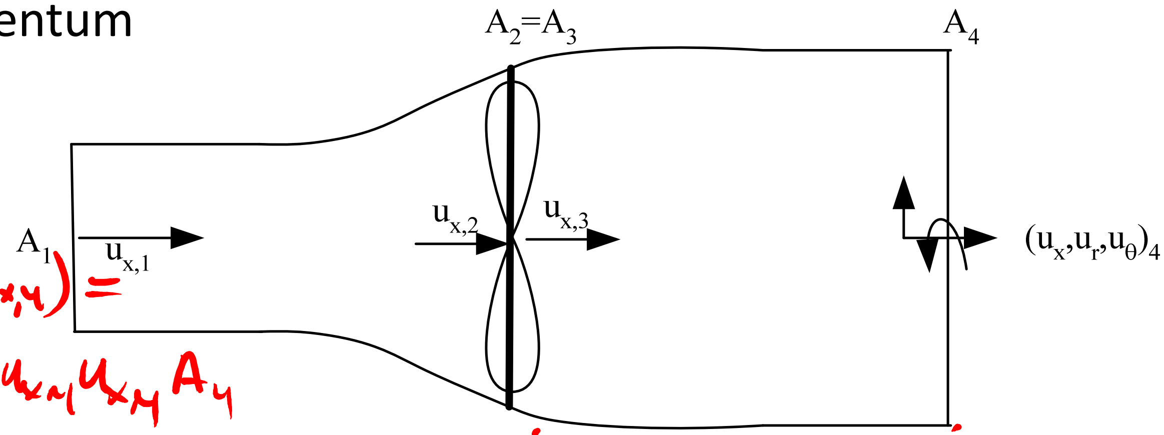

SET T=T

$$\frac{1}{2}\rho A_2 (u_{x,1}+u_{x,4})(u_{x,1}-u_{x,4}) = \rho u_{x,1} u_{x,1} A_1 - \rho u_{x,4} u_{x,4} A_4$$

$$\frac{1}{2}\overbrace{\rho A_2 u_{x,2}}^{\dot m}\frac{1}{u_{x,2}}(u_{x,1}+u_{x,4})(u_{x,1}-u_{x,4}) = \overbrace{\rho u_{x,1} A_1}^{\dot m} u_{x,1} - \overbrace{\rho u_{x,4} A_4}^{\dot m} u_{x,4}$$

$$\frac{1}{2}\frac{(u_{x,1}+u_{x,4})}{u_{x,2}}(\cancel{u_{x,1}-u_{x,4}}) = \cancel{u_{x,1}-u_{x,4}}$$

$$u_{x,2} = \frac{1}{2}(u_{x,1}+u_{x,4})$$ ← TURBINE VELOCITY IS THE AVERAGE OF INLET & OUTLET VELOCITIES

NOW DEFINE a, THE INDUCTION FACTOR

$$u_{x,2} = u_{x,1}(1-a)$$
$$u_{x,4} = u_{x,1}(1-2a)$$