**ME 4470/ESE 4470/ME 5475-02**
**Wind and Tidal Energy**

**Problem:** Determine the aerodynamic loads on a blade using Blade Element Momentum Theory

**Given:** Blade airfoil properties given below as well as lift/drag data for each airfoil

- Blade and chord distribution:

| $r$ (m) | $c$ (m) | $\theta_p$(deg) | Shape |
|---|---|---|---|
| 4 | 1.5 | 26.0 | DU-97-W-300 |
| 6 | 1.4 | 16.0 | DU-97-W-300 |
| 8 | 1.3 | 10.0 | DU-97-W-300 |
| 10 | 1.2 | 5.7 | DU-91-W-250 |
| 12 | 1.0 | 3.7 | DU-91-W-250 |
| 14 | 0.8 | 2.5 | DU-91-W-250 |
| 16 | 0.6 | 2.0 | DU-95-W-180 |
| 18 | 0.4 | 1.5 | DU-95-W-180 |
| 20 | 0.2 | 1.0 | DU-95-W-180 |

- Blade minimum radius: 3 m
- Blade tip radius: 21 m
- Rotational speed: 30 rpm
- Number of blades: 3

**Find:** Develop a BEM code and apply to this blade to determine the following for a 10 m/s wind at sea level ($\rho = 1.23$ kg/m$^3$).

a. Plot the local angle of attack $\alpha$ in degrees as a function of distance from the hub with and without the Prandtl tip correction.

b. Plot the local Normal and Tangential Force Coefficients with and without the Prandtl tip correction as a function of distance from the hub. and without it. Where does the correction have the most effect?

c. Determine the total thrust, torque, and power experienced by the turbine blades from the results determined using the Prandtl tip correction. Since these quantities require integration over the blade, make sure you explain how you evaluate each quantity.

**Solution:** Using BEM, the following equations are derived.

$$\tan\phi = \frac{V}{U} \tag{1}$$

$$\alpha = \phi - \theta_p \tag{2}$$

$$C_N = C_L \cos(\phi) + C_D \sin(\phi) \tag{3}$$

$$C_T = C_L \sin(\phi) - C_D \cos(\phi) \tag{4}$$

$$F_N = C_N \frac{1}{2} \rho W^2 c \tag{5}$$

$$F_T = C_T \frac{1}{2} \rho W^2 c \tag{6}$$

$$\sigma = \frac{cB}{2\pi r} \tag{7}$$

$$a = \left(1 + \frac{4 \sin^2(\phi)}{\sigma C_N}\right)^{-1} \tag{8}$$

$$a' = \left(\frac{4 \sin(\phi) \cos(\phi)}{\sigma C_T} - 1\right)^{-1} \tag{9}$$

The Prandtl tip corrections are given by the following relations.

$$a = \left(1 + \frac{4F \sin^2(\phi)}{\sigma C_N}\right)^{-1} \tag{10}$$

$$a' = \left(\frac{4F \sin(\phi) \cos(\phi)}{\sigma C_T}\right)^{-1} \tag{11}$$

$$F = \frac{2}{\pi} \cos^{-1}(e^{-f}) \tag{12}$$

$$f = \frac{B}{2} \frac{R - r}{r \sin \phi} \tag{13}$$

A BEM code was written that carried out the following steps.

1. Initialize $a$ and $a'$ to 0.
2. Compute flow angle $\phi$ at each $r_i$ using Eq. 1 with $U$ and $V$.
3. Determine $W$ from $U$ and $V$.
4. Determine $\alpha$ using Eq. 2 with $\phi$ and $\theta_p$.
5. Determine $C_l$ and $C_d$ using the appropriate tabulated airfoil data.
6. Determine $C_N$ and $C_T$ using Eqns. 3 and 4 with $C_l$ and $C_d$.

7. Determine the induction factors $a$ and $a'$ using Eqns. 8 and 9 (no tip correction) or Eqns. 10 and 11 (with tip correction).

8. If induction factors have changed significantly, update with

$$a_{diff} = a - a_{previous}$$

$$a_{new} = a_{previous} + a_{diff}/2$$

and repeat from step 2.

9. Once the induction factors converge, determine $F_N$ and $F_T$ using Eqns. 5 and 6.

Having determined the force distributions ($F_N(r)$ and $F_T(r)$), the thrust, torque and power can be determined using the following equations.

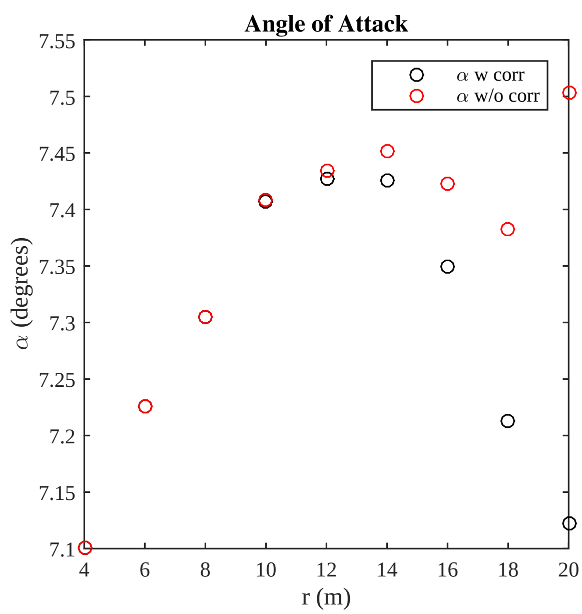
$$T = B \int_{r_min}^{R} F_N dr$$

$$T_x = B \int_{r_min}^{R} r F_T dr$$

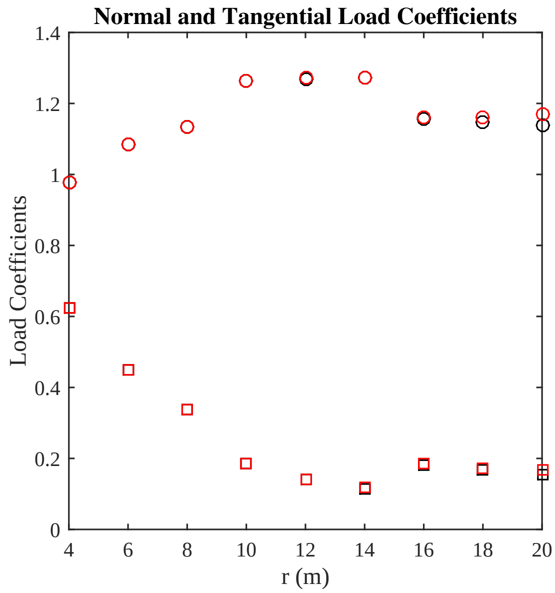
$$P = T_x * \Omega$$

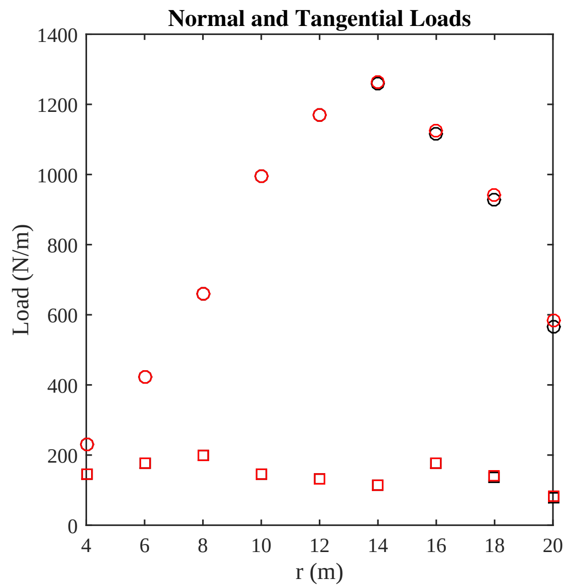
Using this approach, the following results were determined.

a. The angle of attack across the blade is shown in the figure below.




Note that the correction has the most effect in the outboard section of the blade. The absolute changes in angle of attack are small.




b. The force coefficients and force distributions are shown in the figures above. In accordance with the angle of attack, the only significant changes are observed outboard.

c. The Thrust, torque, and power with and without corrections are listed below.

| | Thrust kN | Torque kN*m | Power kW |
|---|---|---|---|
| No Correction | 43.1 | 86.0 | 270.0 |
| With Correction | 42.9 | 84.7 | 266.0 |

Note that again, the changes to the predicted thrust, torque, and power are small. The largest (<2%) are for the torque and power as the outboard part of the blade is weighted heavier in these results.