Wind Turbine Aerodynamics

B. One-Dimensional Momentum Theory

RECALL WE HAD EXPRESSION FOR P

$$P = \dot{m}\left(\frac{u_{x,1}^2 - u_{x,4}^2}{2}\right)$$

$$u_{x,4} = u_{x,1}(1-2a)$$

$$P = \rho u_{x,1}^3 A_2 \, 2a(1-a)^2$$

AREA OF ROTOR

UPSTREAM VELOCITY

EFFICIENCY

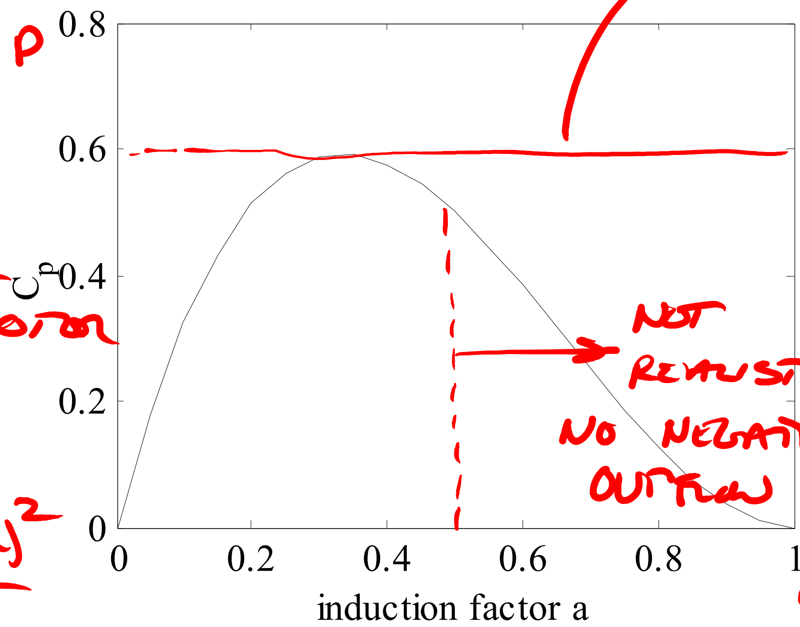
$$\eta = \frac{P}{P_{IN}} = \frac{\rho u_{x,1}^3 A_2 (2a)(1-a)^2}{\frac{1}{2}\rho u_{1,x}^2 \, u_{x,2} \, A_2}$$

DEPENDS ON TURBINE

DEFINE POWER COEFFICIENT

$$C_p = \frac{P}{\frac{1}{2}\rho u_{1,x}^3 A_2}$$

$$C_p = \frac{P}{1/2\rho u_{x,1}^3 A_2} = 4a(1-a)^2$$



BETZ LIMIT 0.5926

NOT REALISTIC

NO NEGATIVE OUTFLOW

Wind Turbine Aerodynamics

C. Wake Rotation Effects

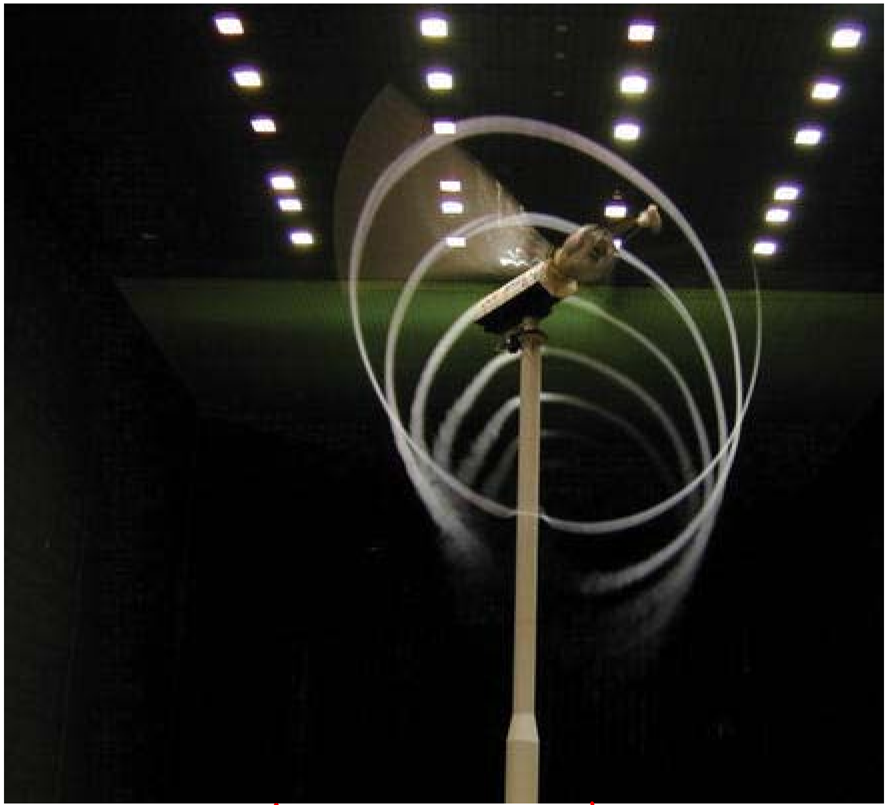
IN PREVIOUS ANALYSIS

$$u_{\theta,4} \ll u_{x,4}$$

WE KNOW THE WIND PRODUCES A TORQUE ON BLADES → BLADE EXERTS TORQUE ON FLOW

FLOW BEHIND TURBINE ROTATES IN DIRECTION OPPOSITE TO BLADE ROTATION

KINETIC ENERGY IN ROTATION RESULTS IN LESS ENERGY EXTRACTED FROM THE FLOW



SLOW ROTATION TURBINES (LOW SPEED/HIGH TORQUE) → MORE ROTATION LESS EFFICIENT THAN FAST ROTATION/LOW TORQUE TURBINE

Wind Turbine Aerodynamics

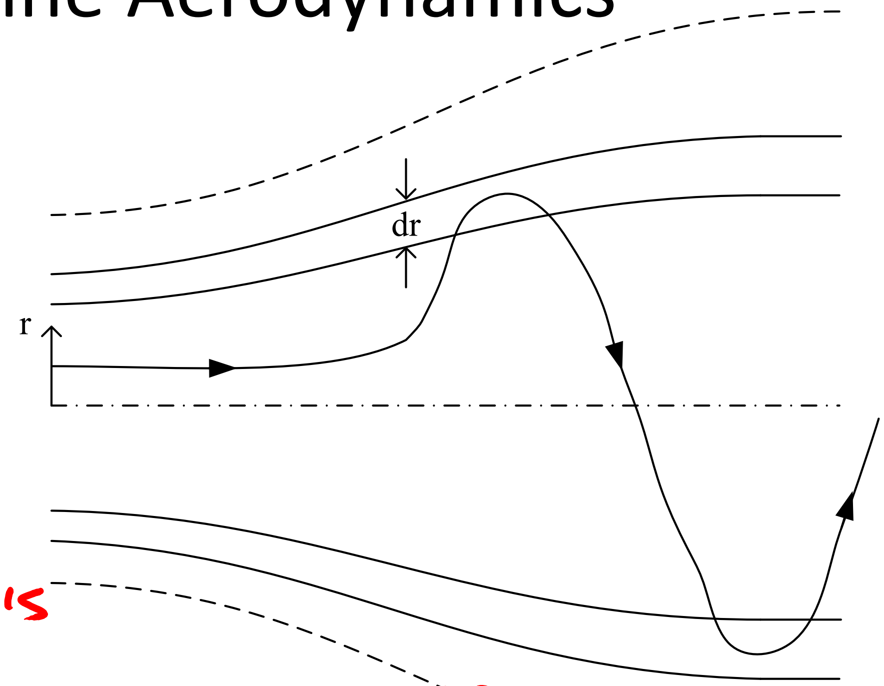
C. Wake Rotation Effects

1. Theory

CONSIDER TURBINE ROTATING WITH ANGULAR VELOCITY Ω

AN ANGULAR VELOCITY IS IMPARTED TO FLOW ω

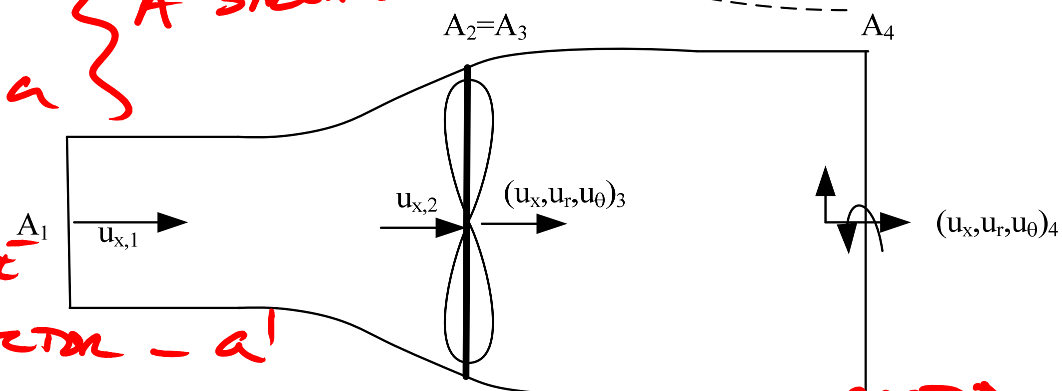
THE ROTATION IMPARTED IS NOW A FUNCTION OF r



SO PRESSURE $P_2 - P_3$
AXIAL INDUCTION FACTOR a
} NEED TO ANALYZE A STREAM TUBE

WILL ALSO VARY WITH r

INTRODUCE A NEW VARIABLE
ANGULAR INDUCTION FACTOR $- a'$
← MEASURE OF ANGULAR VELOCITY IMPARTED TO FLOW



Wind Turbine Aerodynamics

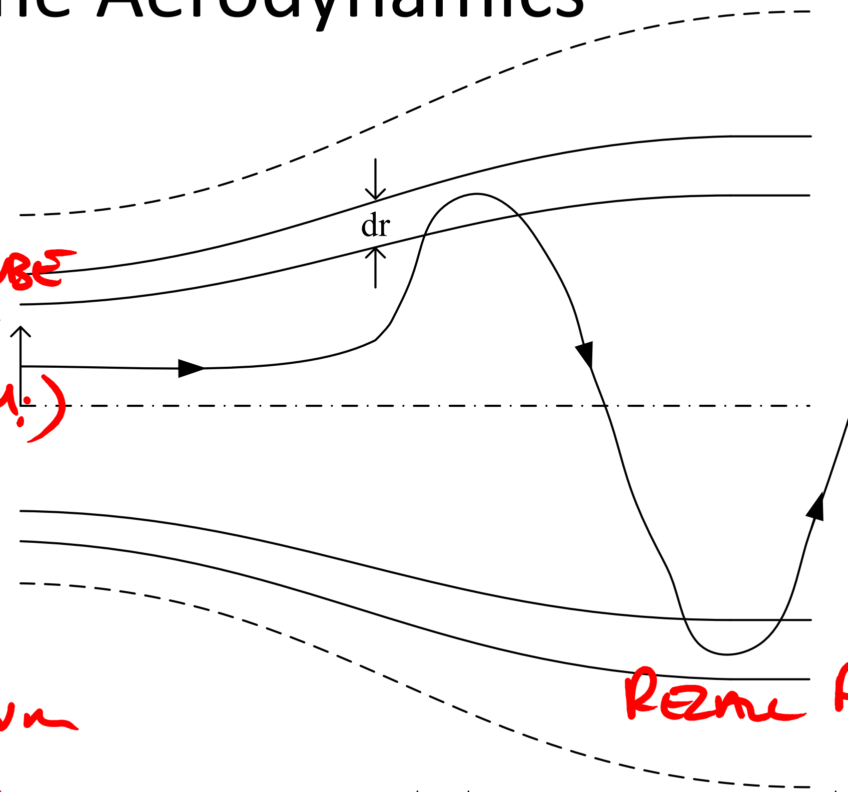
C. Wake Rotation Effects

1. Theory

ENERGY APPLIED TO STREAM TUBE

$$\int_{CV} \rho \left(P/\rho + \tfrac{1}{2}\left(u_x^2 + u_\theta^2 + \cancel{u_r^2}\right) + \cancel{gz} + u_i \right)(\vec{v}\cdot\hat{n})\,dA = \dot{W} + \cancel{\dot{Q}}$$

$$w = \frac{P_2 - P_3}{\rho} - \frac{1}{2}u_{\theta,3}^2$$



RECALL P(r)

CONSIDER ANGULAR MOMENTUM

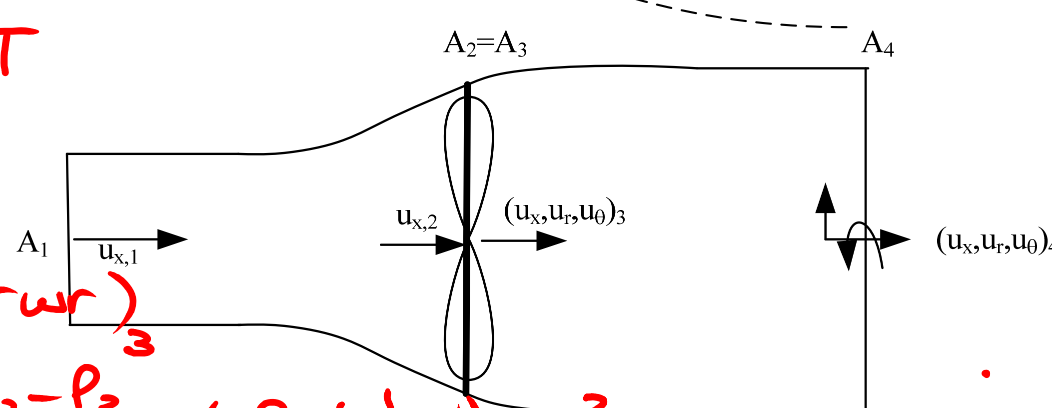
$$\int_{CV} \rho (\vec{r}\times\vec{v})(\vec{v}\cdot\hat{n})\,dA = T$$

& USE $\dot{W} = T\omega$

$$w = \frac{\dot{W}}{\dot{m}}$$

$$w = -(\Omega r \underbrace{\cancel{\omega r}}_{0})_2 + (\Omega r \omega r)_3$$

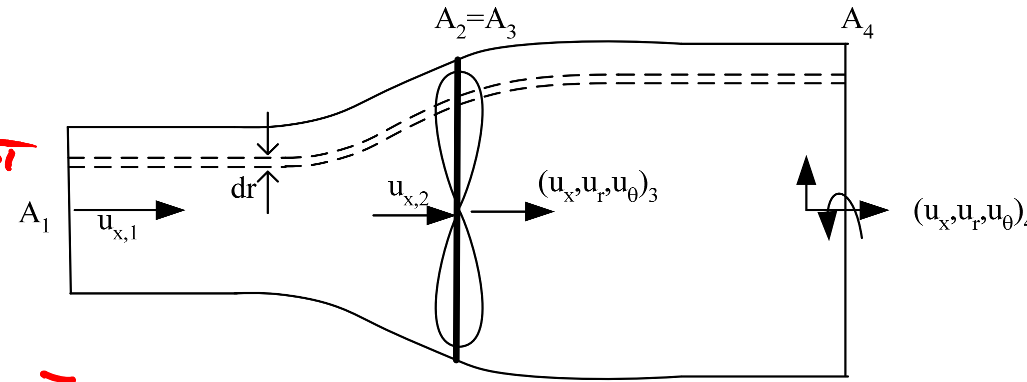
$$\frac{P_2 - P_3}{\rho} = \left(\Omega + \tfrac{1}{2}\omega\right)\omega r^2$$



Wind Turbine Aerodynamics

C. Wake Rotation Effects

1. Theory



NOW CONSIDER THE THRUST IN EACH STREAM TUBE

$$dT = (P_2 - P_3)\,dA$$

$$dT = \left[\rho\left(\Omega + \tfrac{1}{2}\omega\right) r^2 \omega\right] 2\pi r\,dr$$

DEFINE ANGULAR INDUCTION FACTOR

$$a' = \frac{\omega}{2\Omega}$$

SUBSTITUTE INTO dT

$$dT = 2a'(1+a')\rho\Omega^2 r^2 2\pi r\,dr$$

RECALL FROM EARLIER ANALYSIS

$$dT = \tfrac{1}{2}\rho\left(u_{x,1}^2 - u_{x,4}^2\right) dA$$

SUB AXIAL INDUCTION FACTOR

$$a = \frac{u_{1,x} - u_{2,x}}{u_{1,x}}$$

$$dT = \tfrac{1}{2}\rho u_{1,x}^2\, 4a(1-a) 2\pi r\,dr$$

SET EQUAL

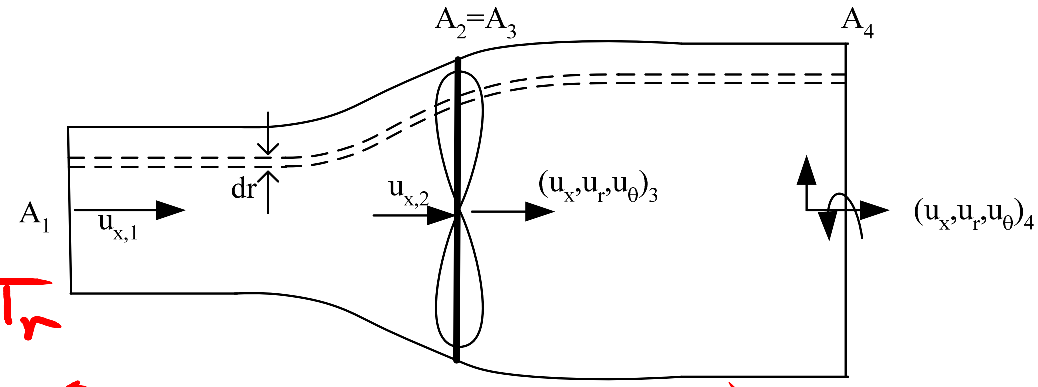
$$\frac{a(1-a)}{a'(1+a')} = \frac{\Omega^2 r^2}{u_{1,x}^2} = \lambda_r^2$$

$$\lambda_r = \frac{\text{ROTATION VELOCITY}}{\text{INFLOW VELOCITY}} \rightarrow \text{LOCAL SPEED RATIO}$$

Wind Turbine Aerodynamics

C. Wake Rotation Effects

1. Theory



ONCE MORE BACK TO ANGULAR MOMENTUM

$$\int_{CV} \rho(\vec{r}\times\vec{v})(\vec{v}\cdot\hat{n})\,dA = T_r$$

APPLY BETWEEN ② & ③ ON ANNULAR REGION

$$dT_r = \rho\, r\, u_{\theta,3}\, u_{x,3}\, 2\pi r\, dr$$

SUBSTITUTE INDUCTION FACTOR

$$dT_r = 4a'(1-a)\tfrac{1}{2}\rho\, u_{1,x}\, \Omega r^2\, 2\pi r\, dr$$

INCREMENTAL POWER FROM ANNULUS, $dP = \Omega\, dT_r$

$$dP = \tfrac{1}{2}\rho A_2 u_{1,x}^3 \left[8a'(1-a)\frac{\lambda_r^3}{\lambda^2} d\lambda_r \right) \qquad \lambda = \frac{\Omega R}{u_{1,x}}$$

SO POWER DEPENDS ON a', a & λ $\qquad \lambda_r = f(\lambda, r)$