ME 4470/ESE 4470/ME 5475-02
Wind and Tidal Energy

Problem: Determine pdf from Laramie wind data and show how theoretical distributions fit the distributions

Given: 10 years of wind data from the Laramie Airport at a reference height of 10 meters and converted to a hub height of 80 m

Find:

1. Determine the probability density function (pdf) using the 10 years of wind data. Demonstrate the ability to compute the pdf by showing plots of the pdf for the months of January, April, July, October as well as an pdf for the the year.

2. Using the probability density function, determine the mean and variance values for the periods given above and compare with the values determined in the more conventional way.

3. Overlay Weibull and Rayleigh distributions on the pdfs determined above. Tabulate the parameters needed to determine the distributions. Discuss whether these theoretical distributions are good approximations of the pdfs determined from the data.

Solution: Data from Laramie Airport that was previously converted to the wind speed at 80 m was used for this assignment. 10 years of data (2005-2014) were available for use in determining the pdf. Data from all 10 years was used to determine the pdf for a month or the entire year. The pdf was determined from the wind speed data at 80 m using the following steps.

1. A bin spacing Δu was chosen and a vector of these bins u_i was created. Care was taken to pick a bin spacing that worked with the discrete nature of this data (given at 1 mph increments). An equivalent spacing (or multiples thereof) in m/s was chosen that accounted for both the conversion to metric and the conversion of the data to an 80 m hub height.

2. The histogram h at the velocity bins u_i was determined using data for the appropriate time period.

3. The probability density function was determined from the histogram h

$$\mathrm{pdf(u_i)} = \frac{\mathrm{h}}{\mathrm{N\Delta u}}$$

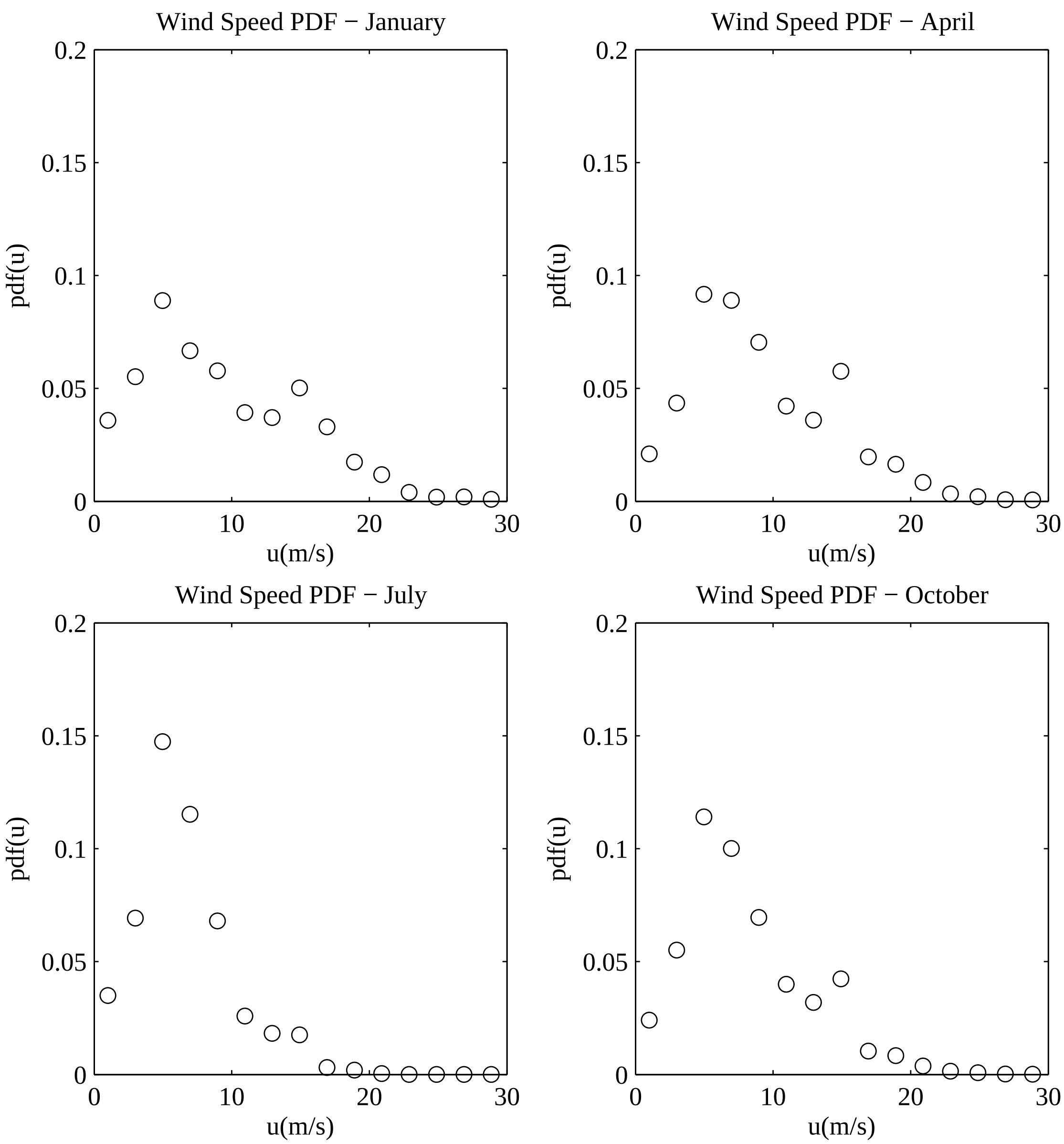
   where N is the total number of wind speed data points used to determine the histogram.

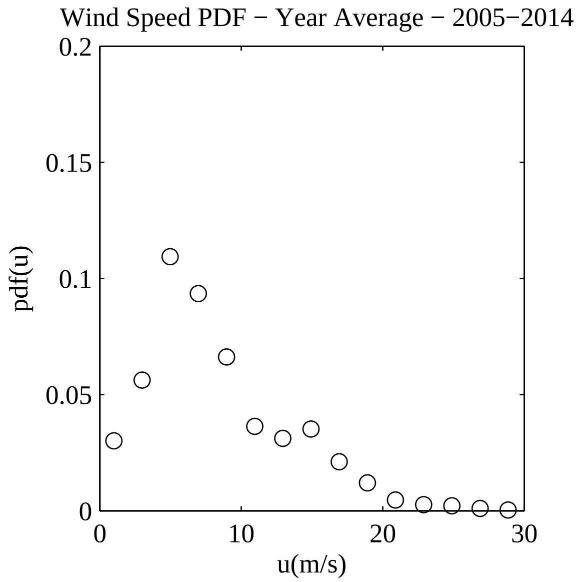
4. Check the result to make sure the pdf is valid

$$\sum_{i=1}^{M} pdf(u_i)\Delta u = 1$$

where M is the number of bins.

a. Applying this approach to the different time periods listed resulted in substantially different results. The pdfs for January, April, and October have considerably more data at the higher wind speeds resulting in a broader pdf than the data from July exhibit. The pdfs are relatively smooth as a bin spacing of the bin spacing equivalent to 3 mph at 10 m converted to 80 m, or 1.99 m/s, was used.





b. As expected, determining the mean directly from the dataset and from the pdf will produce different, but close answers. The results will depend on your exact bin spacing. For the results quoted below, the bin spacing was chosen to be the equivalent of 3 mph at 10 m converted to 80 m or 1.99 m/s (the same spacing were used for the earlier plots). If you use a closer bin spacing, you can get the mean and variance to be nearly exactly that determined the standard way, but the pdf looks noisier.

	Mean (m/s)		Variance (m^2/s^2)	
Interval	Standard	pdf	Standard	pdf
Jan	8.98	9.25	34.7	32.6
Apr	9.05	9.21	28.2	26.1
Jul	6.23	6.52	13.4	12.0
Oct	7.82	8.04	21.8	20.1
Year	8.01	8.27	26.3	25.0

c. The data are fit with the Rayleigh and Weibull distributions. The pdf for the Rayleigh distribution at velocity u is given by

$$pdf(u) = \frac{\pi}{2}\frac{u}{U}\exp\left[-\frac{\pi}{4}\left(\frac{u}{U}\right)^2\right],$$

where U is the mean wind speed. The pdf for the Weibull distribution is given by

$$pdf(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1}\exp\left[-\left(\frac{u}{c}\right)^k\right],$$

where k, the shape factor, and c, the scale, are parameters that affect the distribution. These last two parameters may be calculated using

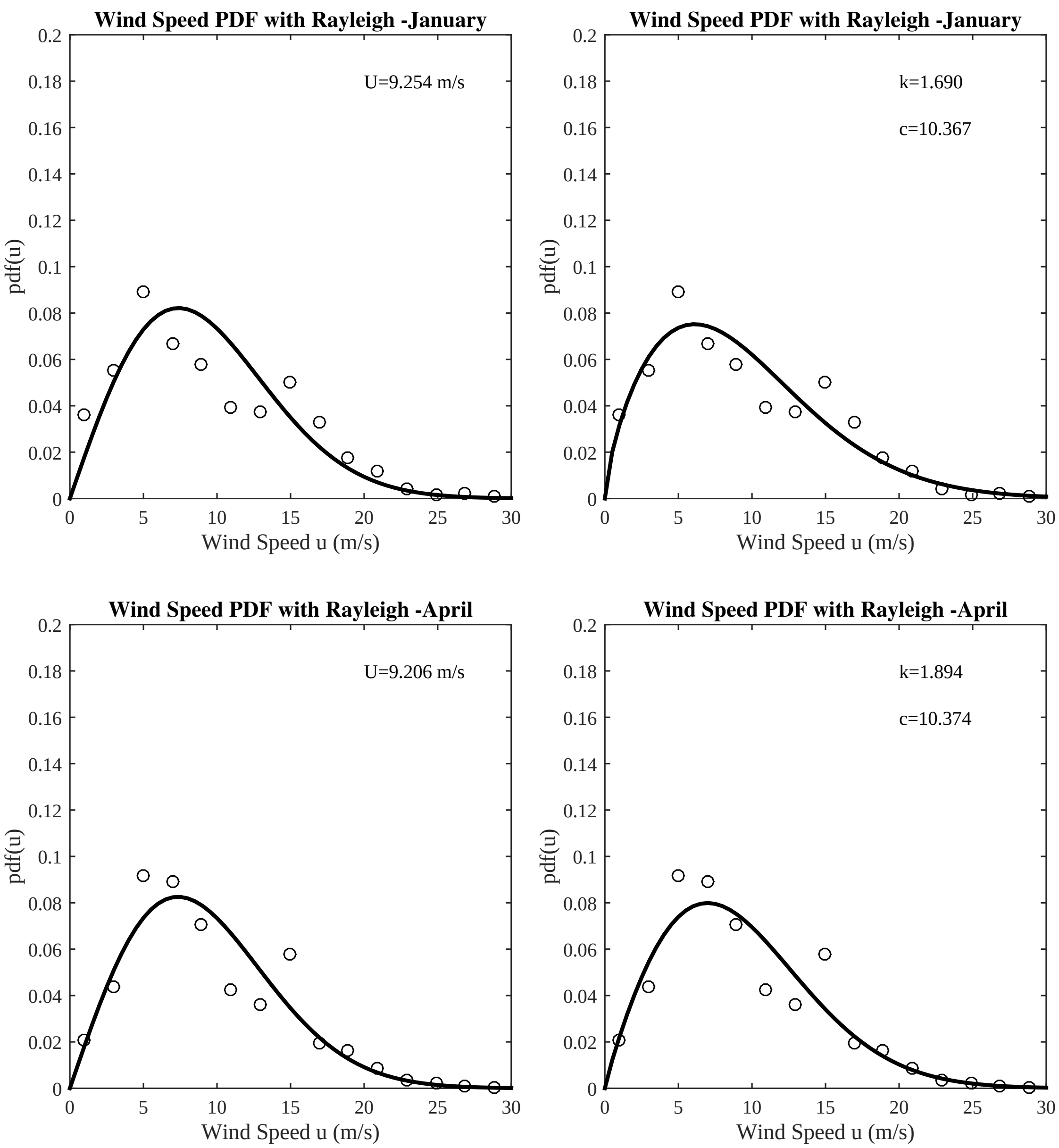
$$k = \left(\frac{\sigma_u}{U}\right)^{-1.086},$$

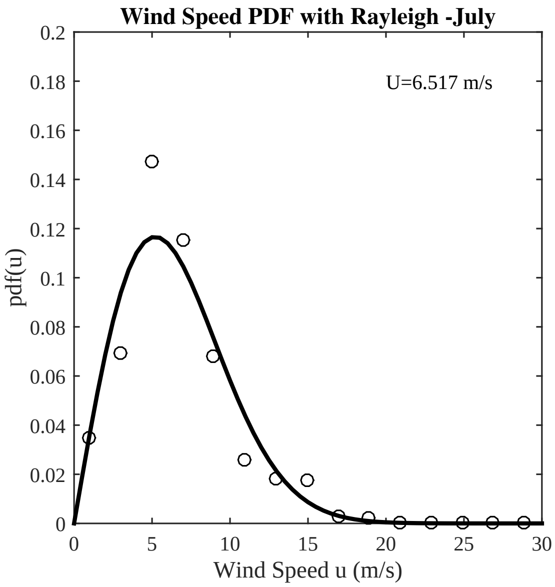
and

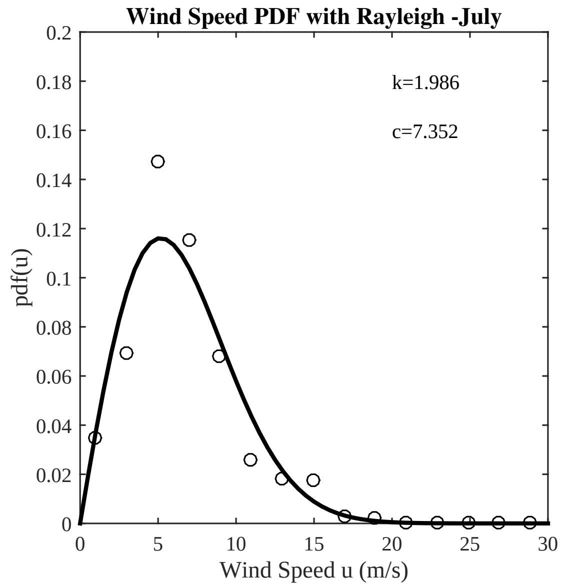
$$c = \frac{U}{\Gamma\left(1 + \frac{1}{k}\right)},$$

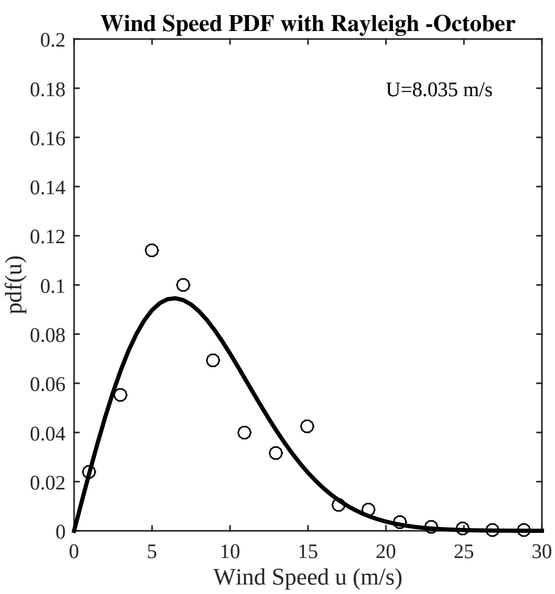
where Γ is the gamma function.

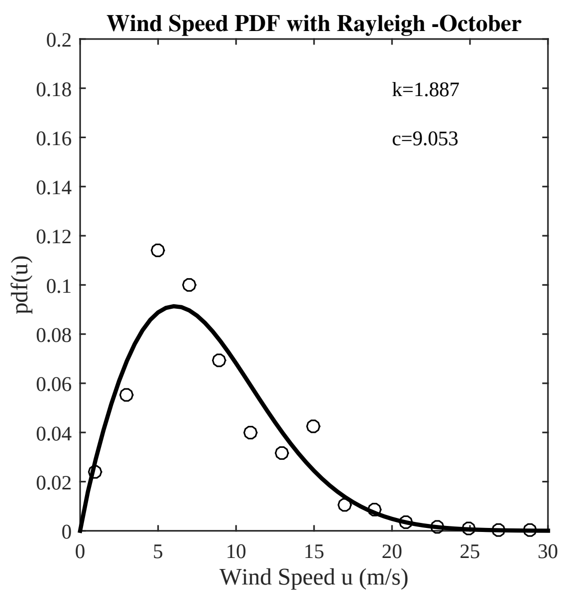
The results of determining the fit parameters are shown in the figures that follow. To help evaluate the goodness of fit, the theoretical distributions are plotted on top of the experimentally determined pdfs. The Weibull fits appear to be slightly better than the Rayleigh fits as might be expected with the Weibull distribution being a two parameter expression as opposed to the one parameter expression given by the Rayleigh distribution. The parameters determined for these fits are listed in the table below.

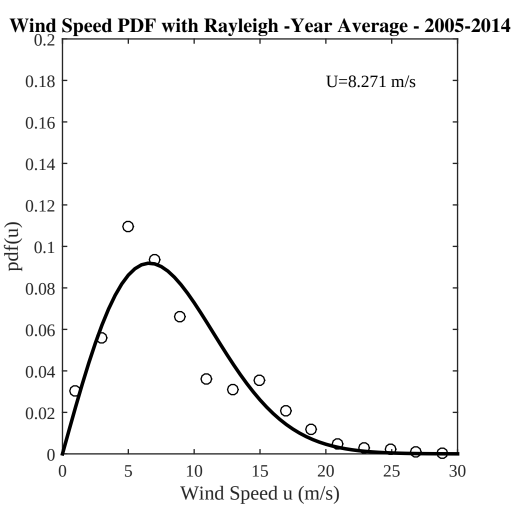


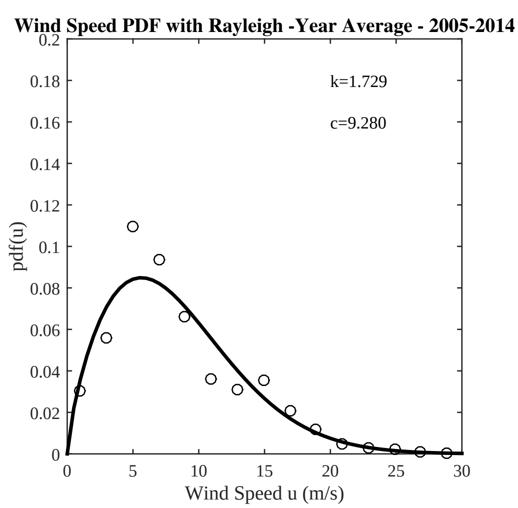

Wind Speed PDF with Rayleigh -July
U=6.517 m/s
pdf(u)
Wind Speed u (m/s)


Wind Speed PDF with Rayleigh -July
k=1.986
c=7.352
pdf(u)
Wind Speed u (m/s)


Wind Speed PDF with Rayleigh -October
U=8.035 m/s
pdf(u)
Wind Speed u (m/s)


Wind Speed PDF with Rayleigh -October
k=1.887
c=9.053
pdf(u)
Wind Speed u (m/s)


Wind Speed PDF with Rayleigh -Year Average - 2005-2014
U=8.271 m/s
pdf(u)
Wind Speed u (m/s)


Wind Speed PDF with Rayleigh -Year Average - 2005-2014
k=1.729
c=9.280
pdf(u)
Wind Speed u (m/s)

	Rayleigh	Weibull	
Period	U	c	k
January	9.53	10.37	1.69
April	9.21	10.37	1.89
July	6.52	7.35	1.99
October	8.04	9.05	1.89
Yearly	8.27	9.28	1.73