

Wind Turbine Aerodynamics

C. Wake Rotation Effects

1. Theory

NON-DIMENSIONALIZE TO DETERMINE
INCREMENTAL POWER COEFFICIENT

$$dC_p = \frac{dP}{\frac{1}{2} \rho A_2 u_{x,2}^3}$$

$$dC_p = 8a'(1-a) \frac{dr^3}{\lambda^2} d\lambda r \quad \leftarrow \text{INTEGRATE TO DETERMINE } C_p$$

NEED RELATIONSHIPS AMONG a' , a , λ

OUR INTEREST IS MAXIMUM POWER \rightarrow MAXIMIZE $(1-a)a'$

APPROACH 1) DEFINE $f = a'(1-a)$

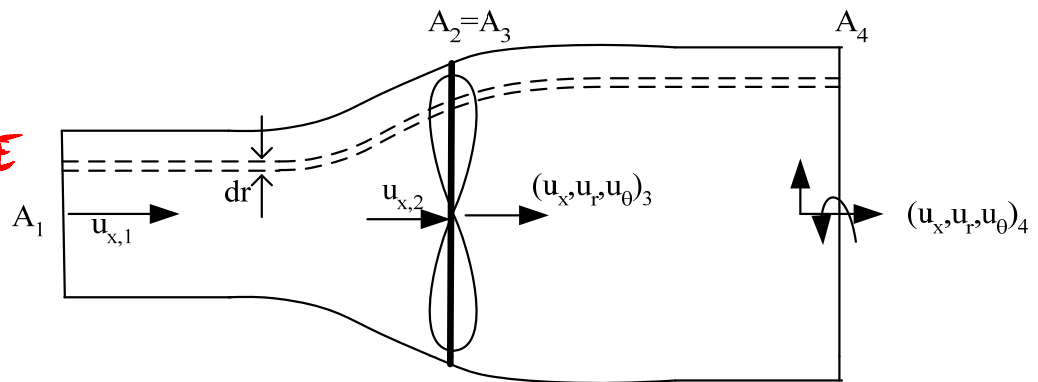
2) DIFFERENTIATE
w.r.t. a
& SET = 0

$$\frac{df}{da} = (1-a) \frac{da'}{da} - a' = 0$$

3) NEED $\frac{da'}{da} \rightarrow$ USE $\lambda^2 a' (1+a') = a(1-a)$

YIELDS

$$\frac{da'}{da} = \frac{1-2a}{1+2a'} \frac{1}{\lambda^2}$$



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4) SUBSTITUTE INTO EQ. FOR $\frac{dC_p}{da}$

$$a' = \frac{1-3a}{4a-1}$$

↑ CONDITION FOR MAXIMIZING $a'(1-a)$

5) SUBSTITUTE INTO λ^2 RELATION TO WRITE λ IN TERMS OF a

$$\lambda^2 = \frac{(1-a)}{(1-3a)} (4a-1)^2$$

6) DIFFERENTIATE TO GET $d\lambda$

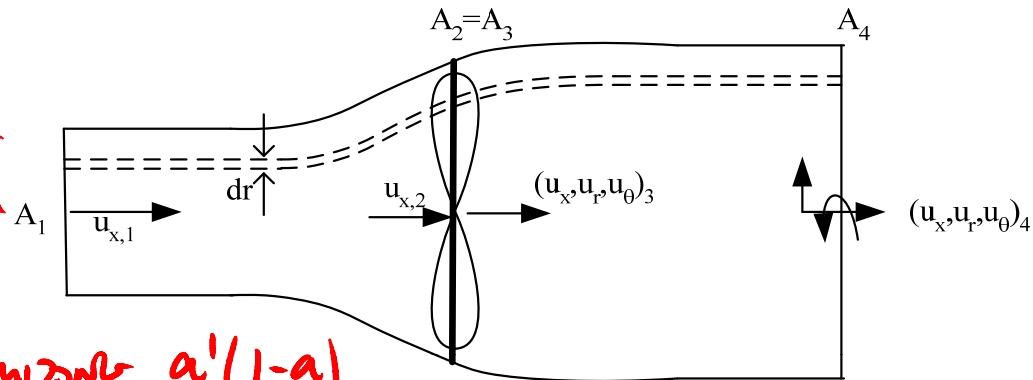
7) SUBSTITUTE INTO dC_p EQUATION
& INTEGRATE

NUMERICAL INTEGRATE
FOR DIFFERENT λ^5

$$C_{p,max} = \frac{24}{\lambda^2} \int_{a_1}^{a_2} \left[\frac{(1-a)(1-2a)(1-4a)}{1-3a} \right] da$$

a_1 IS INDUCTION FACTOR FOR $\lambda_r = 0$

a_2 IS INDUCTION FACTOR FOR $\lambda_r = 1$
 $r=0$
 $r=R$



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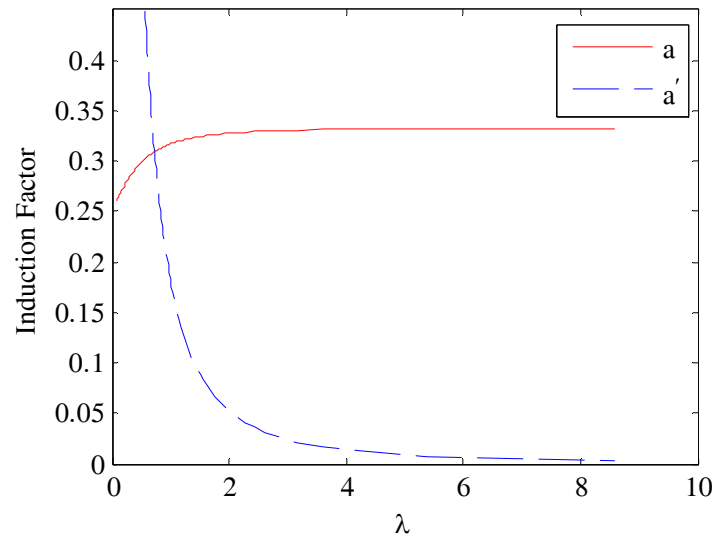
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OBSERVATIONS

a' HIGHEST FOR LOW λ

a LEVELS OFF WITH INCREASING λ



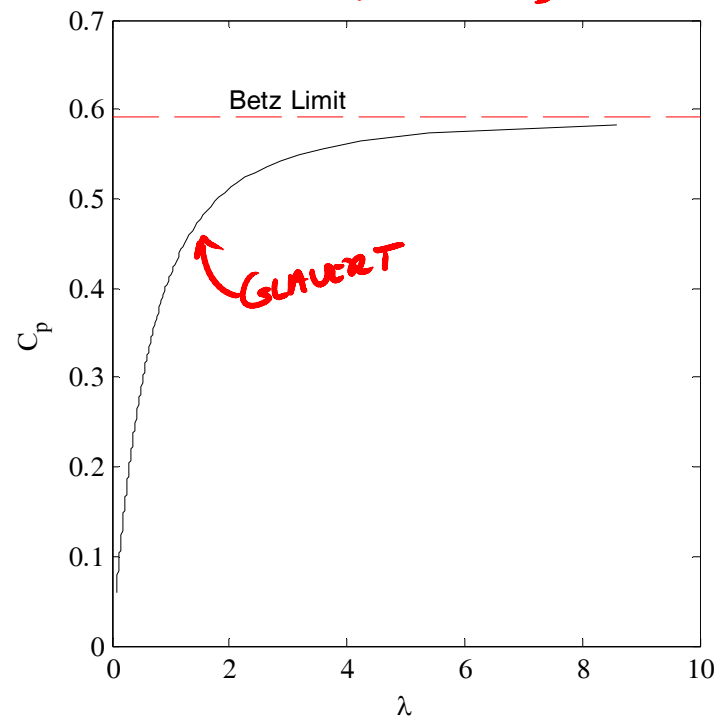
INCREASE TIP SPEED RATIO TO

→ DECREASE WAKE ROTATION

→ INCREASE POWER AVAILABLE TO BLADE ROTATION

C_p CURVE DEMONSTRATES THE IMPROVEMENT WITH INCREASING λ

C_p APPROACHES BETZ LIMIT ONLY AT HIGH TIP SPEEDS



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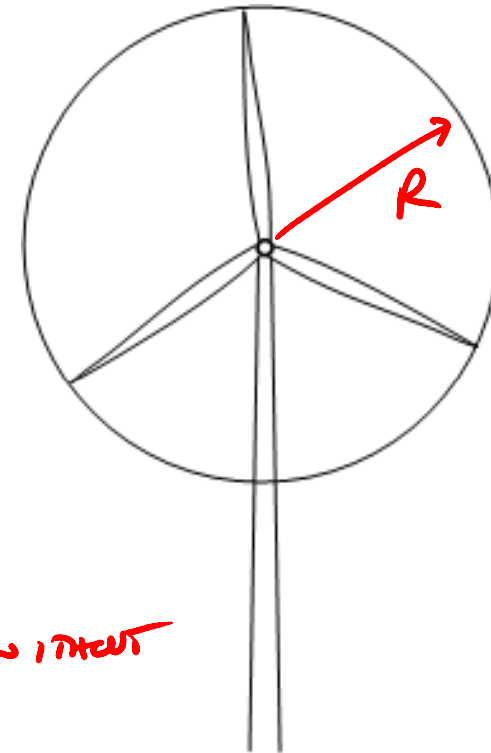
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2. Practical Consideration

CONSIDER A MULTI-BLADED WIND TURBINE

EFFICIENCY DECREASES IF

- BLADES ROTATE SO QUICKLY THAT BLADES PASS THROUGH TURBULENT WAKE OF PRECEDING BLADE
- BLADES ROTATE SO SLOWLY SUCH THAT MUCH OF THE AIR PASSES BY BLADES WITHOUT INTERACTING WITH THEM



DEFINE AN IDEAL TIP SPEED RATIO

$$\lambda_0 = \frac{2\pi R}{k n R}$$

R - ROTOR RADIUS
n - NUMBER OF B

kR - DISTANCE
DISTANCE

TRAIL DISTANCE BELOW WHICH
BLADE WILL NOT FEEL EFFECTS OF
PREVIOUS BLADE

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$$\lambda_0 = \frac{2\pi}{kn}$$

CONSIDER A 2-BLADED MACHINE

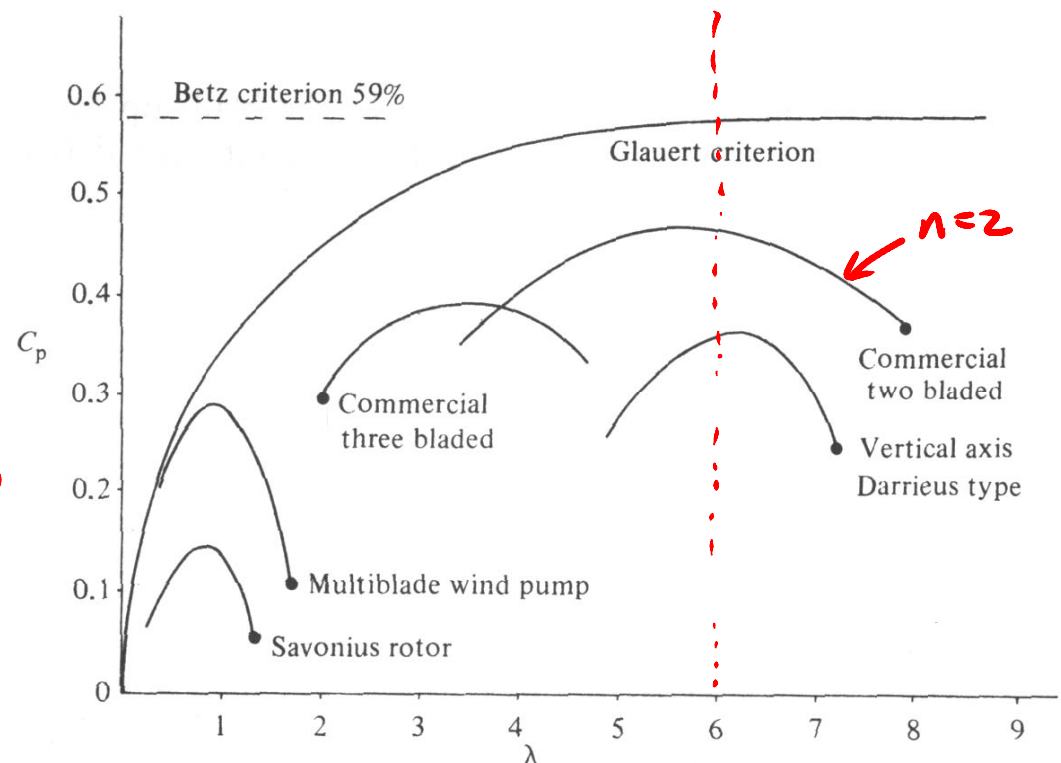
$$k = \frac{1}{2} \quad n = 2$$

$$\lambda_0 = 2\pi \sim 6$$

WITH CAREFULLY DESIGNED AIRFOILS,
THE BEST TIP SPEED RATIO
CAN BE INCREASED

$$\lambda_{\text{best}} \sim 1.3 \lambda_0$$

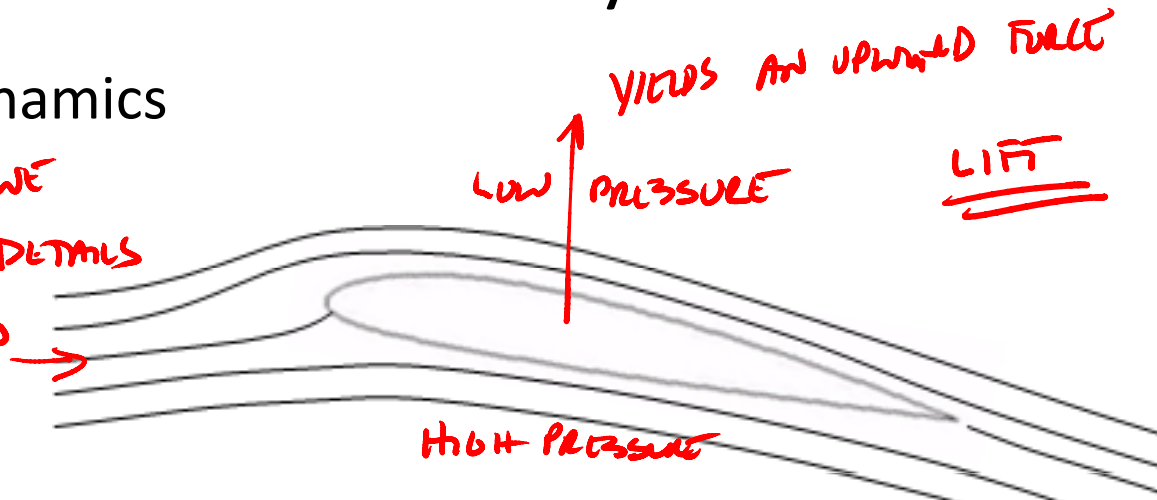
LEADS TO UNDERSTANDING OF WHY WE SEE 2 & 3 BLADE
MACHINES



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D. Blade Aerodynamics

UP UNTIL THIS POINT, WE
HAVE NOT CONSIDERED DETAILS
OF WHAT GOES ON IN
ROTAL PLANE



THE FLOW AROUND AIRFOIL CREATES FORCES & MOMENTS ON THE BLADE

THESE FORCES & MOMENTS ARE USED TO PRODUCE USEFUL MOTION

WT BLADE → LIFT COMPONENT PRODUCES ROTATION

AIRPLANE → LIFT TO PROVIDE FORCE TO OVERCOME GRAVITY

WE MUST DISCUSS THESE FORCES & MOMENTS & HOW THEY
ARE CREATED & MANAGED IF WE ARE TO BUILD
EFFECTIVE BLADES

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D. Blade Aerodynamics

1. Terminology

