

# Wind Turbine Mechanics

## C. Static and Dynamic Modeling of Wind Turbines

### 2. Dynamic Modeling

WHY IS DYNAMIC MODELING IMPORTANT?

- TEMPORAL VARIATIONS OF LOADS ARE IMPORTANT FOR FATIGUE
- STABILITY OF DESIGN
- DESIGNING CONTROL STRATEGY

FOR DYNAMIC MODEL, MUST INCLUDE INERTIA

NEEDED IN  
STATIC ANALYSIS

UNSTEADY AERODYNAMICS IS NOW NEEDED

- UNSTEADY BEM FOR INSTANCE
  - UNSTEADY WIND MODEL
- ← COUPLE

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WHAT IS THE BASIS OF A DYNAMIC MODEL?

CONSIDER A SIMPLE DYNAMIC SYSTEM  
MASS SUSPENDED ON A SPRING

CONSIDER STATICS

$$\sum F = 0 = mg - kx$$

A SOLUTION FOR  $x$  YIELDS  
EQUILIBRIUM VALUE

NOW CONSIDER DYNAMICS

$$\sum F = ma = m\ddot{x}$$

$$m\ddot{x} = mg - kx$$

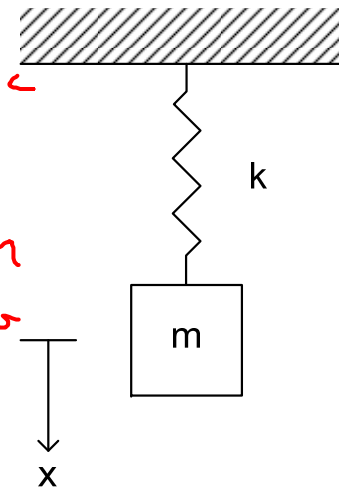
← RESTORING FORCE

↑ gravity

$$m\ddot{x} + kx = mg$$

DAMPING FORCES CAN BE ADDED

→ TEND TO DAMPEN OR RESIST ANY MOTION  
↑ PROPORTIONAL TO  $\dot{x}$



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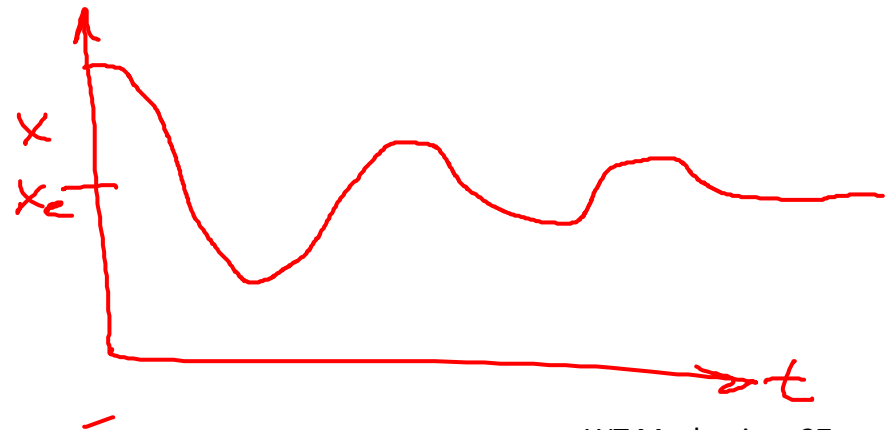
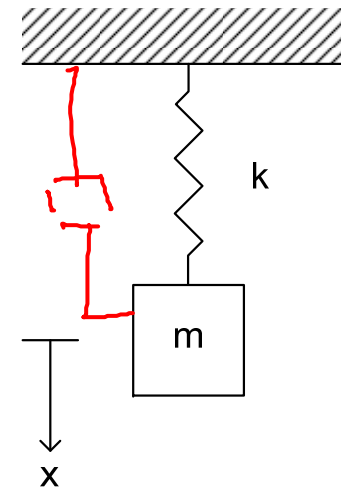
$$m\ddot{x} + c\dot{x} + kx = mg$$

THIS IS THE EQUATION FOR THE "FREE" SYSTEM

$c\dot{x}$  IS DAMPING FORCE

IF SOLVED, THE SOLUTION FOR A MASS DISPLACED FROM EQUILIBRIUM IS DETERMINED  $\rightarrow$  A DAMPING OSCILLATION

$$\left\{ \begin{array}{l} \omega_n = \sqrt{\frac{k}{m}} \Rightarrow \text{NATURAL FREQUENCY} \\ \zeta = \frac{c}{2m\omega_n} \Rightarrow \text{DAMPING RATIO} \end{array} \right. \rightarrow \text{CHARACTERIZES OSCILLATION}$$



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WHAT HAPPENS IF UNSTEADY FORCE ACTS ON THE SYSTEM

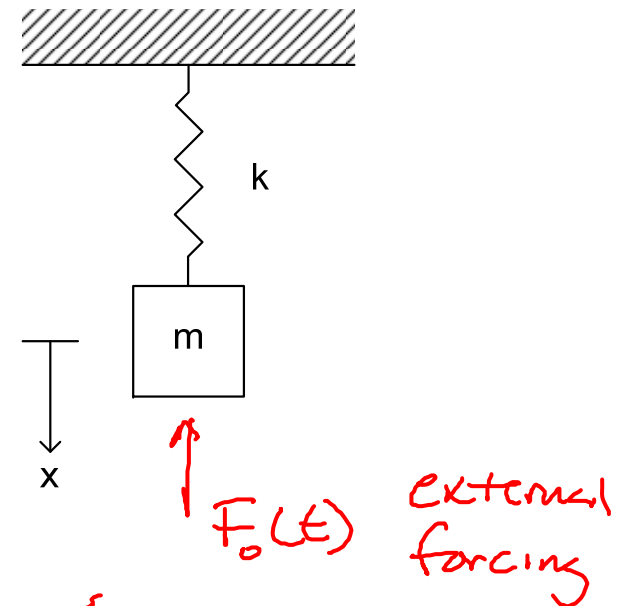
$$m\ddot{x} + c\dot{x} + kx = F_0(t)$$

IT IS THE INTERACTION OF THE FORCING & THE SYSTEM THAT YIELD THE RESULTING MOTION

CONSIDER A SINUSOIDAL FORCING

$$F_0(t) = A \sin(\omega t)$$

WHAT IS SYSTEM RESPONSE



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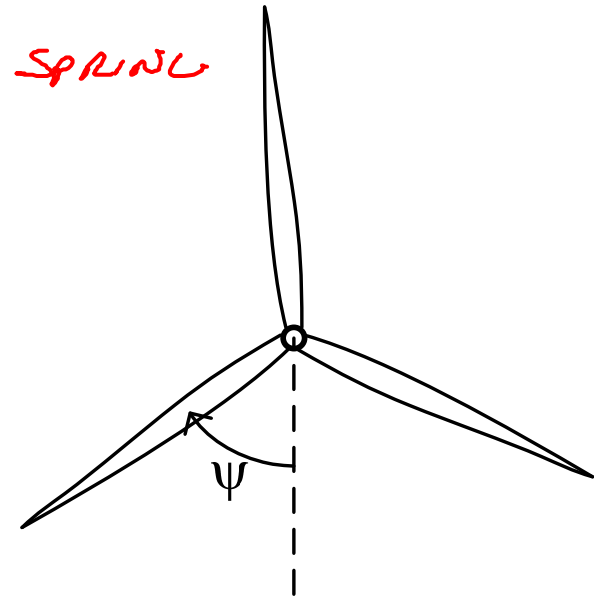
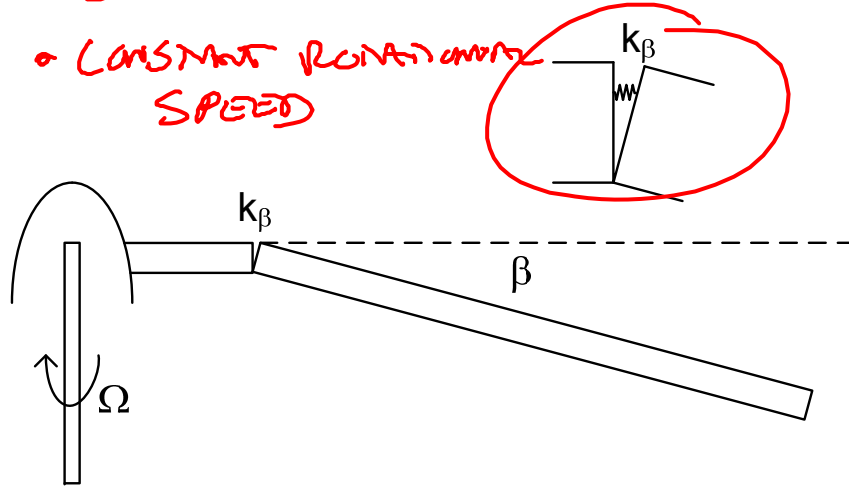
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CONSIDER APPLYING SIMPLE CONCEPTS  
DEVELOPED TO A TURBINE BLADE } HINGE-SPRING  
MODEL IS  
SIMPLEST

IN ITS SIMPLEST FORM

- BLADE HAS UNIFORM CROSS SECTION
- BLADE IS RIGID
- COMPLIANCE IS REPRESENTED BY A SPRING
- CONSTANT ROTATIONAL SPEED



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GENERAL FORM OF DYNAMIC EQUATION  $\ddot{\beta} + c\dot{\beta} + k\beta = F(t)$

(INERTIA)  $\ddot{\beta}$  DAMPING  $c\dot{\beta}$  RESTORING  $k\beta$  FORCING  $F(t)$

AFTER ACCOUNTING FOR FORCES WE'VE DISCUSSED

$$\ddot{\beta} + \left[ \underbrace{1 + \epsilon}_{\text{CENTRIFUGAL}} + \underbrace{\frac{G}{\Omega^2} \cos \psi}_{\text{GRAVITY}} + \underbrace{\frac{k_{\beta}}{\Omega^2 I_b}}_{\text{BLADE SPRING}} \right] \beta = \underbrace{\frac{M_{\beta}}{\Omega^2 I_b}}_{\text{AERO DYNAMIC LOAD}} - \underbrace{2 \bar{g} \cos(\psi)}_{\text{YAW MOTION}}$$

FREE RESPONSE                      FORCING

