Wind Turbine Mechanics

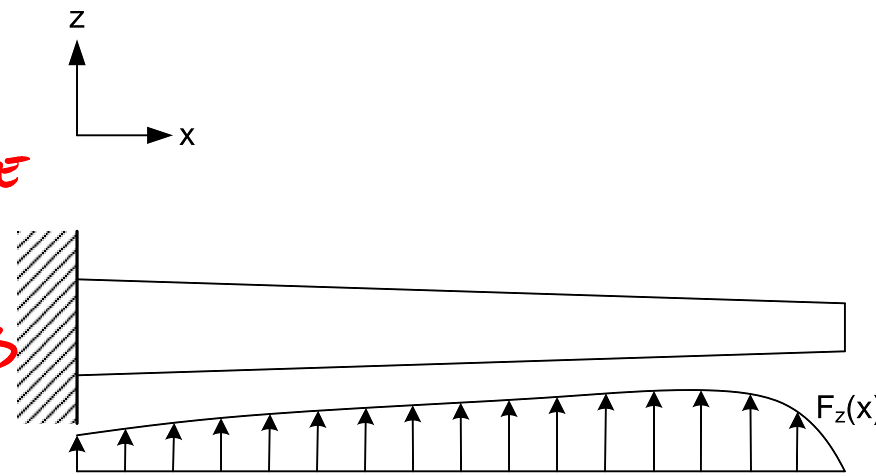
C. Static and Dynamic Modeling of Wind Turbines

WE'VE DISCUSSED FORCES & MOMENTS ACTING ON WIND TURBINES

T, M_y, T_x, M_β

WE'VE DISCUSSED WHERE THEY COME FROM

GRAVITY
INERTIA
AERODYNAMIC } FORCES



WANT TO CONSIDER RESPONSE OF WIND TURBINE TO THESE LOADS

STATICALLY
DYNAMICALLY

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CONSIDER STATIC MODELLING FIRST → BLADE IS IN EQUILIBRIUM

RECALL THAT TURBINE WILL RESPOND STATICALLY TO
→ STEADY LOADS
→ "LOW FREQUENCY" UNSTEADY LOADS

OUR INTEREST HERE IS IN
BLADE FORCES & MOMENTS
BLADE DISPLACEMENT (ANGULAR DEFORMATION & DEFLECTION)

EVEN FOR STATIC LOADS, AERODYNAMICS & STRUCTURAL MECHANICS ARE TIGHTLY COUPLED

AERODYNAMIC MODEL → BLADE LOADS → DETERMINE DEFLECTION → AERODYNAMIC MODEL

REPEAT FOR SEVERAL LOADINGS

HOW DO WE MODEL STRUCTURAL MECHANICS?
BEAM THEORY
→ FINITE ELEMENT ANALYSIS

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NOW CONSIDER DYNAMIC MODELING

WE ARE INTERESTED IN SAME QUANTITIES { FORCES & MOMENTS, DEFLECTIONS }

BUT UNDER UNSTEADY LOADING

YIELDS THE EFFECTS OF DYNAMIC LOADING AS WELL AS COUPLING BETWEEN THE UNSTEADY LOADS & UNSTEADY RESPONSE

→ FORCED VIBRATIONS → TURBINE RESPONSE

HOW DO WE MODEL THE DYNAMICS

- SIMPLE HINGE/BLADE MODEL
- MDOF MODELS BASED ON STRUCTURAL MODES
- FULLY COUPLED AERO/FEM

COMPLETE MODEL OF TURBINE WILL INCLUDE ALL COMPONENTS

→ SHAFTS, GEARBOX, GENERATOR, TOWER

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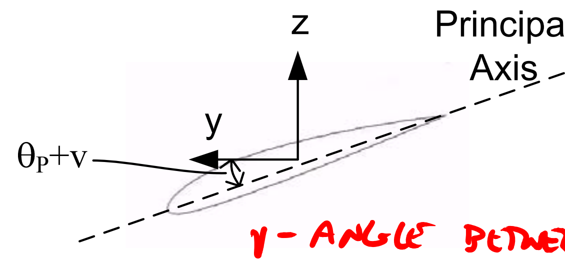
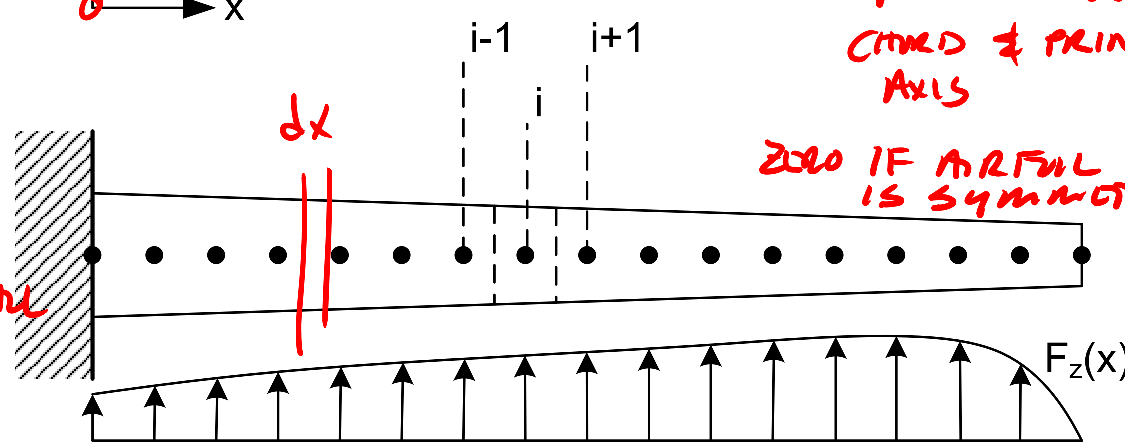
1. Beam Theory



APPROACH IS SAME AS THAT IN BASIC MECHANICS COURSE

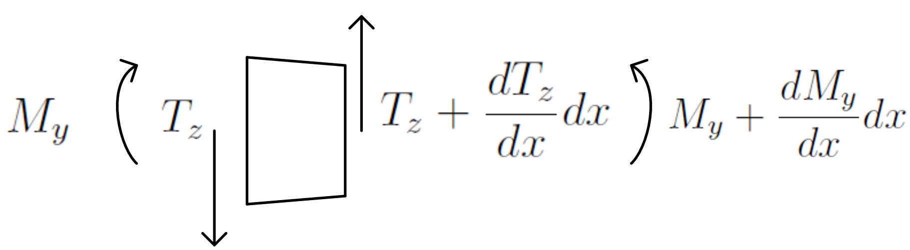
CONSIDER THE BLADE AS A CANTILEVERED BEAM

CONSIDER FORCES ON A SMALL ELEMENT FROM THE BEAM



v - ANGLE BETWEEN CHORD & PRINCIPAL AXIS

ZERO IF AIRFOIL IS SYMMETRIC



$$\frac{dT_z}{dx} = -F_z(x)$$

$$\frac{dM_y}{dx} = T_z$$

T_z DECREASES WITH x

F_z COMES FROM F_N & F_T FORCE / LENGTH

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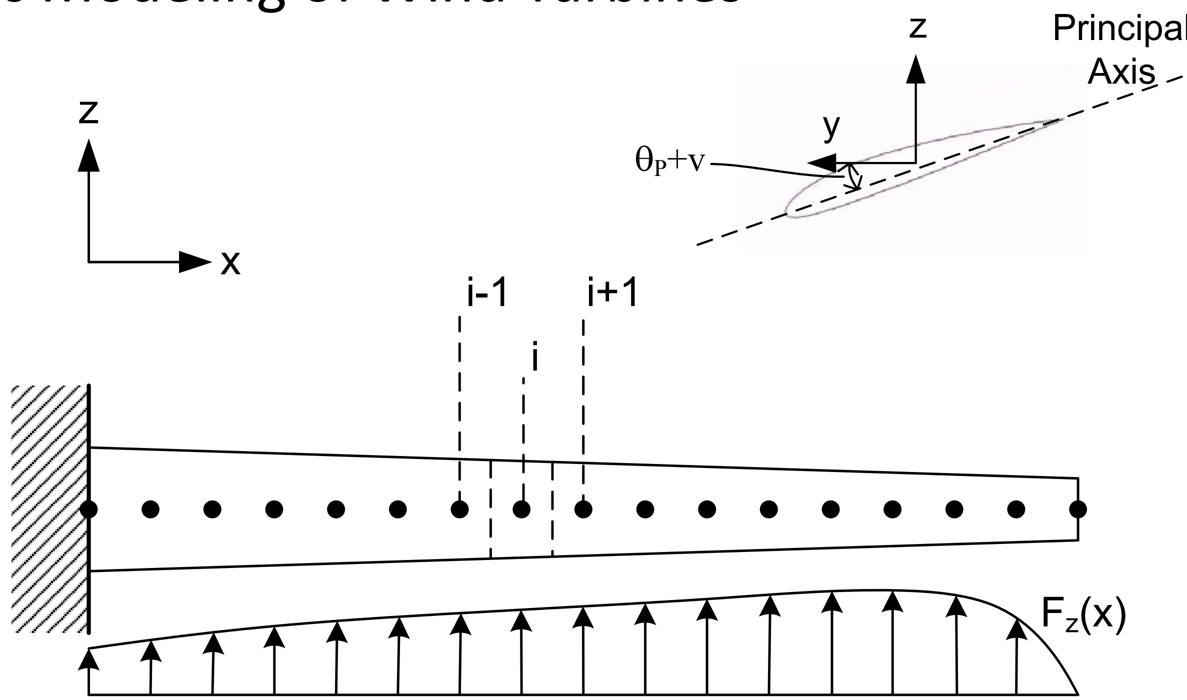

Similarly

$$\frac{dT_y}{dx} = -F_y(x)$$

$$\frac{dM_z}{dx} = -T_y$$

From moments we get bending (curvature)

Transfer moments to principal axis

$$M_1 = M_y \cos(\theta_p + v) - M_z \sin(\theta_p + v)$$

$$M_2 = M_y \sin(\theta_p + v) + M_z \cos(\theta_p + v)$$

Curvatures K_1 & K_2

$$K_1 = \frac{M_1}{E I_1} \qquad K_2 = \frac{M_2}{E I_2}$$

E – modulus of elasticity

I_1, I_2 – moment area of inertia

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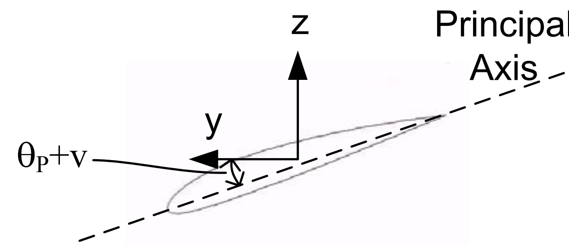
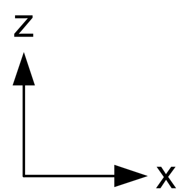


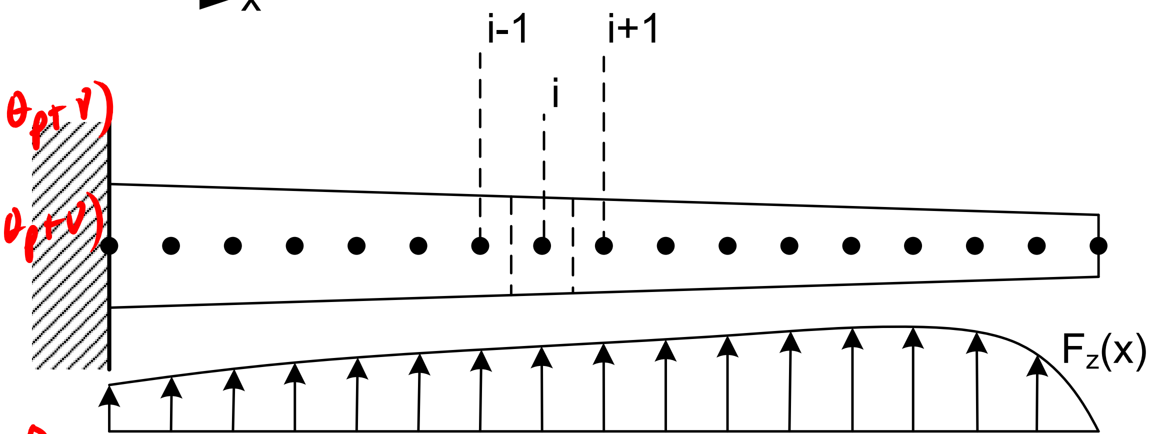
ROTATE CURVATURES BACK TO

X–Z, Y

$$K_z = -K_1 \sin(\theta_p + v) + K_2 \cos(\theta_p + v)$$

$$K_y = K_1 \cos(\theta_p + v) + K_2 \sin(\theta_p + v)$$





WITH THIS INFORMATION, THE ANGULAR DEFORMATION θ DEFLECTIONS u CAN BE CALCULATED

$$\frac{d\theta_y}{dx} = K_y \qquad \frac{d\theta_z}{dx} = K_z$$

$$\frac{du_z}{dx} = -\theta_y \qquad \frac{du_y}{dx} = \theta_z$$

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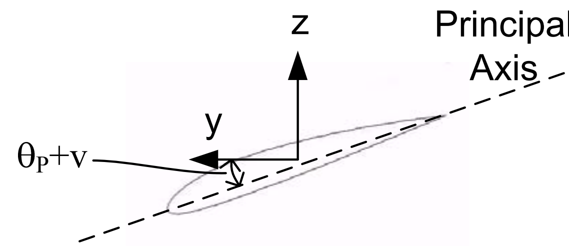


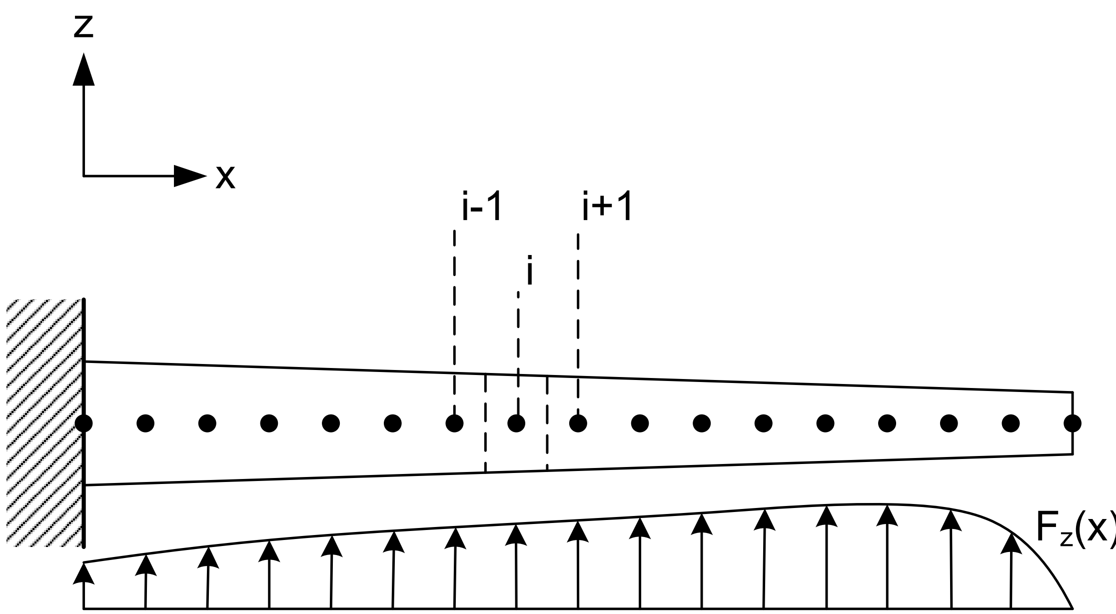
TO SOLVE THESE EQUATIONS, BOUNDARY CONDITIONS ARE REQUIRED

$\theta_y(x=0)=0$

$\theta_z(x=0)=0$

$u_y(x=0)=0$

$u_z(x=0)=0$



$T_y(x=R)=0$

$T_z(x=R)=0$

$M_y(x=R)=0$

$M_z(x=R)=0$

WITH EQUATIONS & BOUNDARY CONDITIONS

→ NOW CAN DETERMINE FORCES, MOMENTS, DEFLECTIONS

Wind Turbine Mechanics

Why numeric?
→ Discrete load
→ Profiles & principal axes change with r

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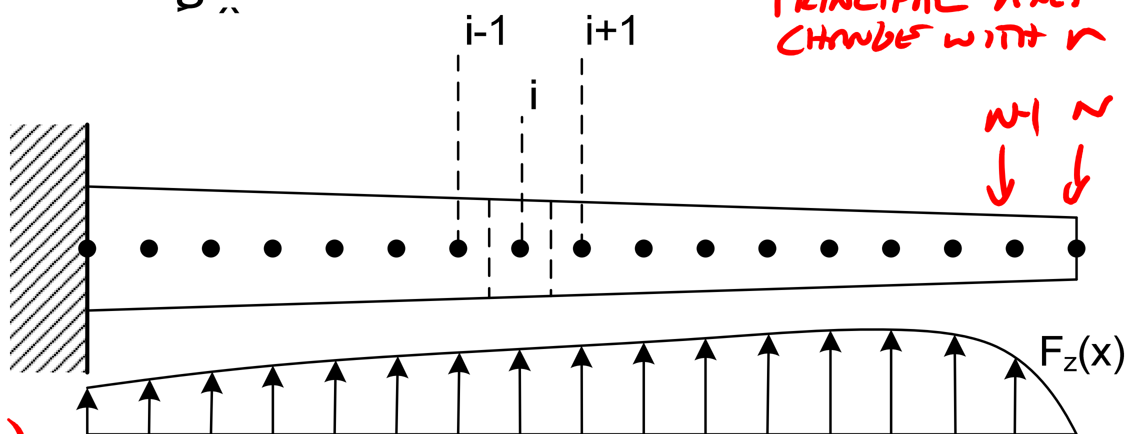
1. Beam Theory



Numerically solve equations

1. Divide blade into N sections
2. Determine (or have provided) the load at each point $F_y(x)$, $F_z(x)$
3. Integrate force & moment equations from tip inward → From N-1 inward to N=1

Node N is where we know T, M

$$T_y^{i-1} = T_y^i + \frac{1}{2}(F_y^{i-1} + F_y^i)(x^i - x^{i-1})$$

$$T_z^{i-1} = T_z^i + \frac{1}{2}(F_x^{i-1} + F_x^i)(x^i - x^{i-1})$$

$$M_y^{i-1} = M_y^i - T_z^i(x^i - x^{i-1}) - \left(\frac{1}{6}F_z^{i-1} + \frac{1}{3}F_z^i\right)(x^i - x^{i-1})^2$$

$$M_z^{i-1} = M_z^i + T_y^i(x^i - x^{i-1}) + \left(\frac{1}{6}F_y^{i-1} + \frac{1}{3}F_y^i\right)(x^i - x^{i-1})^2$$

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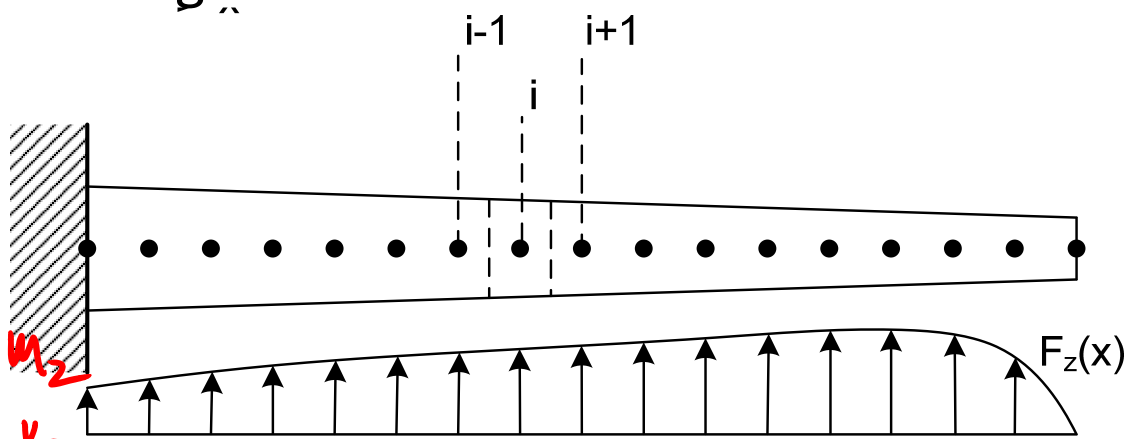

4. DETERMINE CURVATURES

→ AT EACH x_i, TRANSFORM BENDING MOMENTS TO PRINCIPAL AXES → M_1, M_2

→ DETERMINE CURVATURES K_1, K_2

→ TRANSFORM BACK → K_y, K_z

$$\theta_y^{i+1} = \theta_y^i + \frac{1}{2}(\kappa_y^{i+1} + \kappa_y^i)(x^{i+1} - x^i)$$

$$\theta_z^{i+1} = \theta_z^i + \frac{1}{2}(\kappa_z^{i+1} + \kappa_z^i)(x^{i+1} - x^i)$$

5. INTEGRATE TO DETERMINE ANGULAR DEFORMATION & DEFLECTION

$$u_y^{i+1} = u_y^i + \theta_z^i(x^{i+1} - x^i) + \left(\frac{1}{6}\kappa_z^{i+1} + \frac{1}{3}\kappa_z^i\right)(x^{i+1} - x^i)^2$$

START @ ROOT & INTEGRATE OUTWARD

$$u_z^{i+1} = u_z^i - \theta_y^i(x^{i+1} - x^i) - \left(\frac{1}{6}\kappa_y^{i+1} + \frac{1}{3}\kappa_y^i\right)(x^{i+1} - x^i)^2$$