Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)

APPROACH – USE STREAM TUBES TO ANALYZE BLADE FORCE

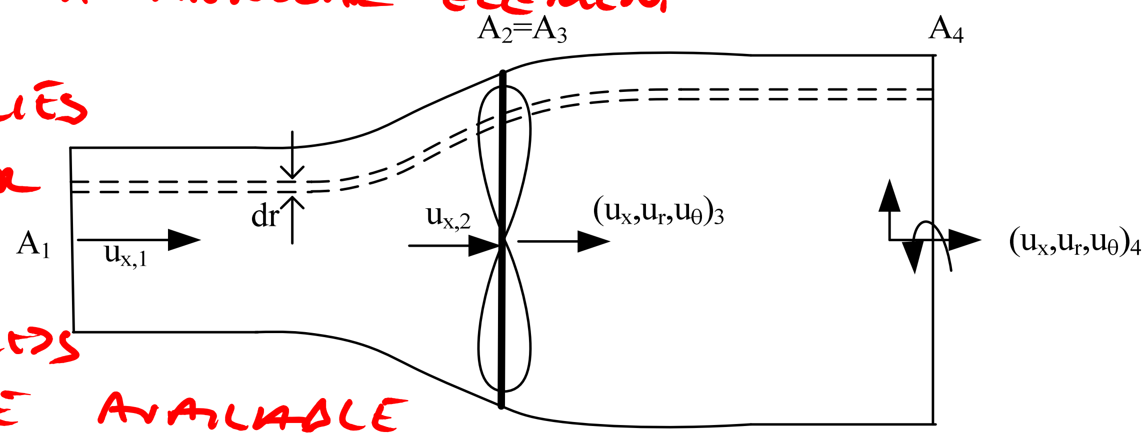
- N ANNULAR ELEMENTS OF RADIAL EXTENT dr
- BOUNDARIES OF ELEMENTS ARE STREAMLINES

FOR EACH ANNULAR ELEMENT, ASSUME

LARGE ASSUMPTION:

1. ANNULAR ELEMENTS ARE INDEPENDENT
2. FORCES FROM BLADES ON FLOW IS CONSTANT IN EACH ANNULAR ELEMENT

ASSUMPTION 2 IMPLIES AN INFINITE NUMBER OF BLADES



CORRECTIONS TO ADDRESS ASSUMPTIONS ARE AVAILABLE

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USED TO CALCULATE LOADS – THRUST, TORQUE
POWER

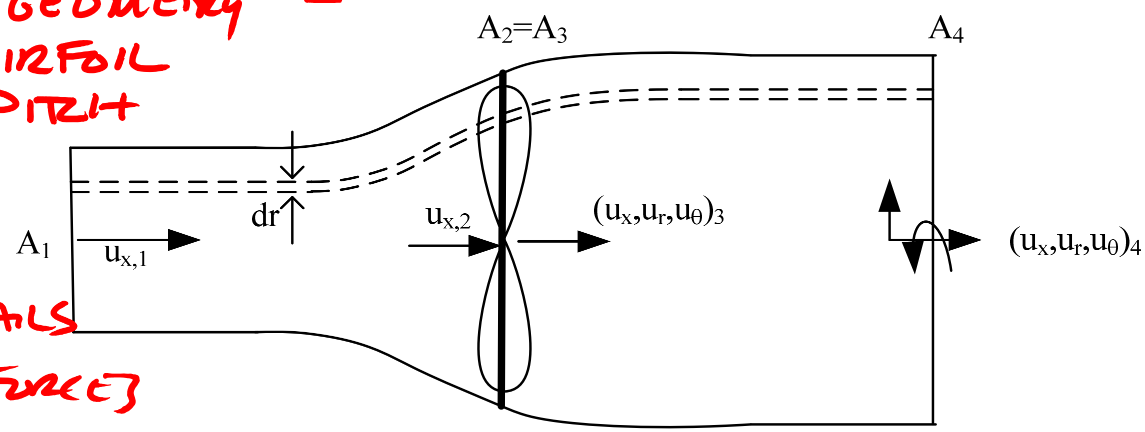
HERE, WE CONSIDER STEADY VERSION

UNSTEADY VERSION ARE A RELATIVELY STRAIGHT-FORWARD EXTENSION

<u>INPUT</u>
- WIND SPEED
- ROTATION SPEED
- BLADE GEOMETRY –
  - AIRFOIL
  - PITCH

BEM

1. MOMENTUM THEORY
   – NO BLADE DETAILS
2. LOCAL BLADE FORCES



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1. Equations

EQUATIONS USED FOR THIS ANALYSIS ALREADY DERIVED

$$dT = \frac{1}{2}\rho(u_{x,1}^2 - u_{x,4}^2)dA$$

X-MOMENTUM BETWEEN ① & ④

$$dT = \frac{1}{2}\rho u_{x,1}^2 4a(1-a)2\pi r dr$$

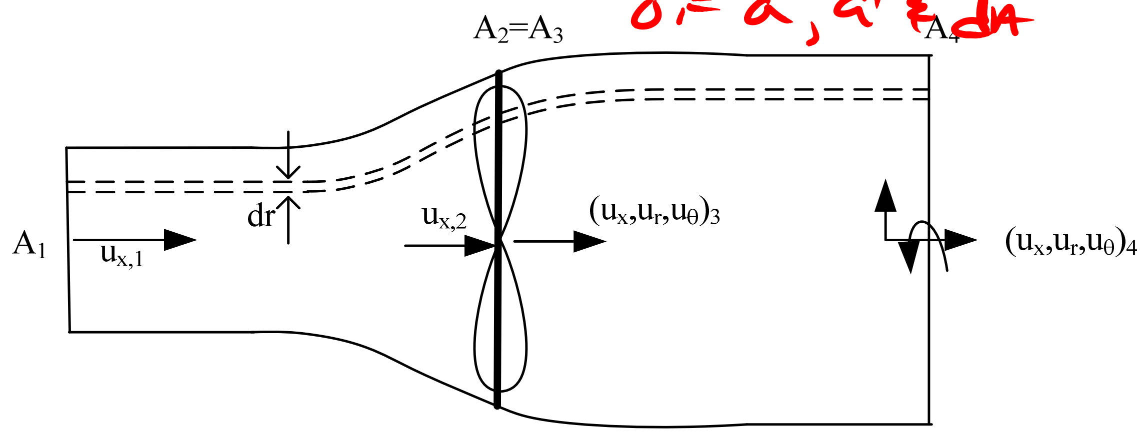
USE DEFINITION OF $dA = 2\pi r dr$ INDUCTION FACTOR

$$dT_x = \rho r u_{\theta,3} u_{x,3} dA$$

ANGULAR MOMENTUM BETWEEN ② & ③

$$dT_x = \frac{1}{2}\rho u_{x,1}\Omega r^2 4a'(1-a)2\pi r dr$$

USE DEFINITION OF a, a' & dA



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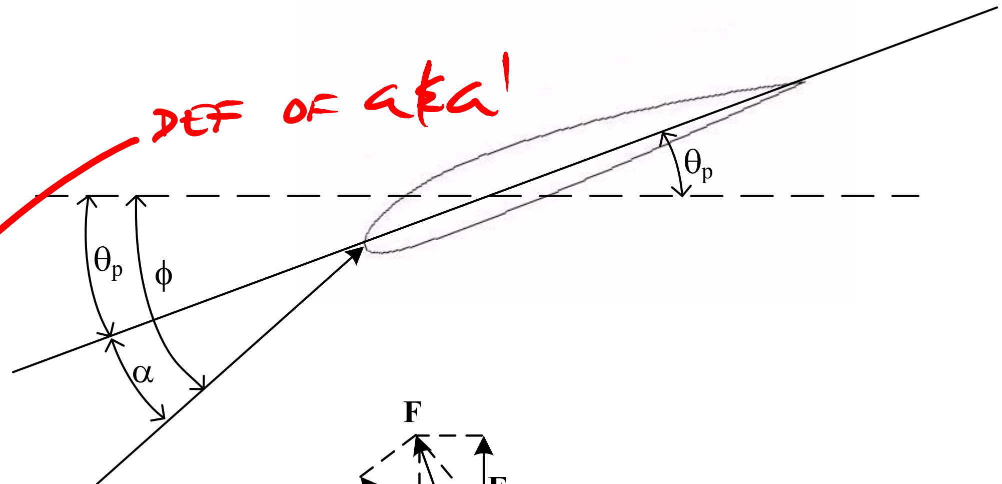
F. Blade Element Momentum Theory (BEM)

1. Equations

BLADE GEOMETRY

DEF OF a & a'



STEP 3

$$\alpha = \phi - \theta_p$$

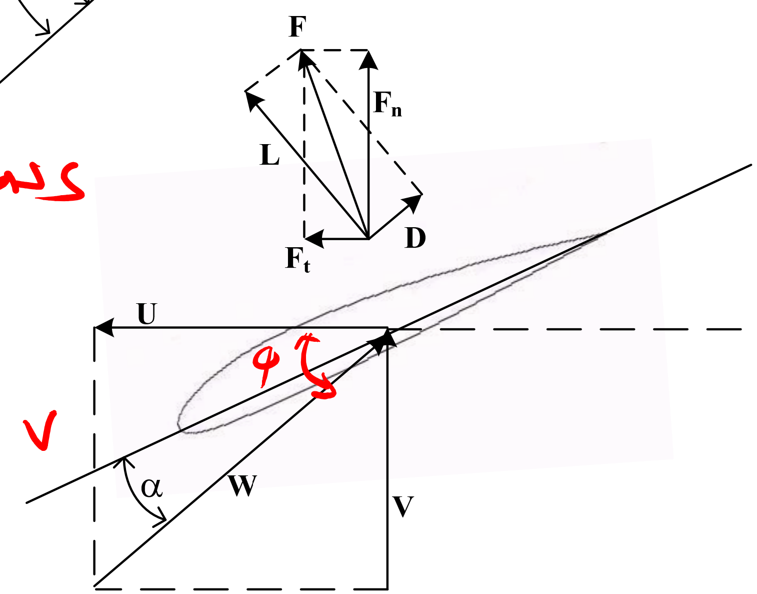
STEP 2

$$\tan\phi = \frac{V}{U} = \frac{(1-a)u_{1,x}}{(1+a')\Omega r}$$

LIFT & DRAG DEFINITIONS

$$L = \frac{1}{2}\rho W^2 c C_L$$

$$D = \frac{1}{2}\rho W^2 c C_D$$



Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)

1. Equations

RELATIONSHIPS BETWEEN LIFT & DRAG →

$$F_N = L\cos(\phi) + D\sin(\phi) \qquad C_N = C_L\cos(\phi) + C_D\sin(\phi)$$

$$F_T = L\sin(\phi) - D\cos(\phi) \qquad C_T = C_L\sin(\phi) - C_D\cos(\phi)$$

NORMAL & TANGENTIAL

STEP 3

$$C_N = \frac{F_N}{\frac{1}{2}\rho W^2 c} \qquad C_T = \frac{F_T}{\frac{1}{2}\rho W^2 c}$$

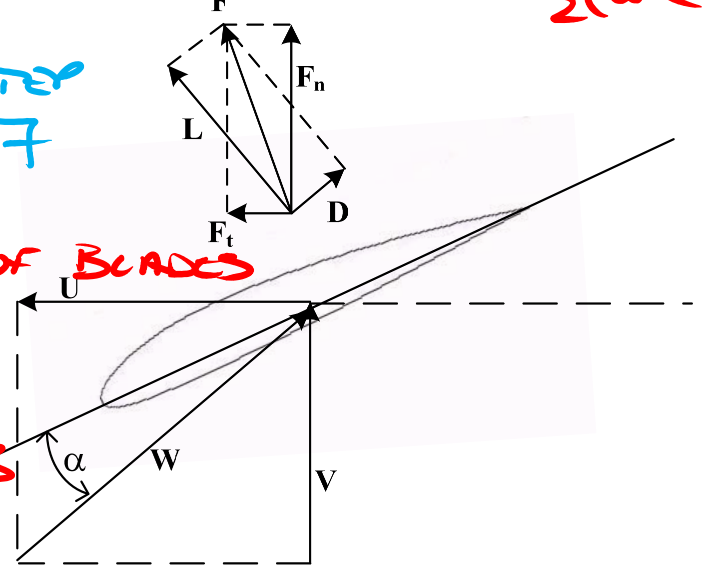
STEP 7

DEFINE SOLIDITY OF BLADE AT LOCATION r

$$\sigma(r) = \frac{c(r)B}{2\pi r}$$

B = NO OF BLADES

FRACTION OF ANNULAR AREA COVERED BY BLADES



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F. Blade Element Momentum Theory (BEM)

1. Equations

USE DEFINITIONS OF C_N & C_T

$V = W\sin\phi = u_{1,x}(1-a)$

$U = W\cos\phi = \Omega r(1+a')$

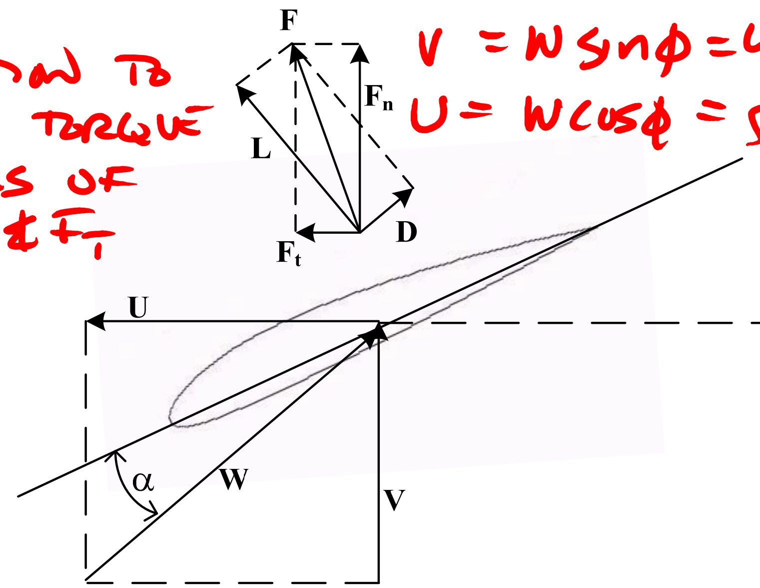
WRITE THE ANNULUS CONTRIBUTION TO THRUST & TORQUE IN TERMS OF F_N & F_T

$$dT = BF_N dr$$

$$dT = \frac{1}{2}\rho\frac{u_{1,x}^2(1-a)^2}{\sin^2(\phi)}cC_N B dr$$

$$dT_x = rBF_T dr$$

$$dT_x = \frac{1}{2}\rho r\frac{u_{1,x}(1-a)\Omega r(1+a')}{\sin(\phi)\cos(\phi)}cC_T B dr$$



SET $dT = dT$
$dT_x = dT_x$

ONE dT FROM MOMENTUM
ONE dT FROM BLADE ELEMENT

STEP 6

$$a = \left(1 + \frac{4\sin^2(\phi)}{\sigma C_N}\right)^{-1} \qquad a' = \left(\frac{4\sin(\phi)\cos(\phi)}{\sigma C_T} - 1\right)^{-1}$$

RELATIONSHIP BETWEEN INDUCTION FACTOR & SOLIDITY & C_N & C_T

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2. Implementation

FIRST, SPLIT BLADE INTO N REGIONS OF RADIAL EXTENT Δr

FOR EACH ANNULAR ELEMENT

ITERATE

1. GUESS THE INDUCTION FACTORS a & a'
2. COMPUTE FLOW ANGLE ϕ
3. COMPUTE LOCAL ANGLE OF ATTACK α
4. DETERMINE $C_L(\alpha)$ & $C_D(\alpha)$ USING AIRFOIL PROPERTIES
5. DETERMINE C_N & C_T FROM C_L & C_D
6. CALCULATE a & a'
7. CALCULATE F_N & F_T FROM C_N & C_T

RESULT $F_N(r_i)$ & $F_t(r_i)$

CAN DETERMINE POWER, THRUST, SHAFT TORQUE, ROOT BEND MOM

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3. Corrections

a. Prandtl's Tip Loss Factor – ACCOUNTS FOR TIP EFFECTS AS WELL AS FINITE # OF BLADES

$$a = \left(\frac{4F\sin^2\phi}{\sigma C_N} + 1\right)^{-1}$$

$$a' = \left(\frac{4F\sin\phi\cos\phi}{\sigma C_T} - 1\right)^{-1}$$

$$F = \frac{2}{\pi}\cos^{-1}\left(e^{-f}\right)$$

$$f = \frac{B}{2}\frac{(R-r)}{r\sin\phi}$$

b. Glauert's Correction for High Values of a

REALIZE $a > 0.5$ IS MEANINGLESS

FOR $a < a_c$ $\quad a_c \sim 0.2$

a CALCULATED AS GIVEN

$a > a_c$

$$a = \frac{1}{2}\left[2 + K(1-2a_c) - \sqrt{\left(K(1-2a_c)+2\right)^2 + 4\left(Ka_c^2 - 1\right)}\right]$$

$$K = \frac{4F\sin^2\phi}{\sigma C_N}$$