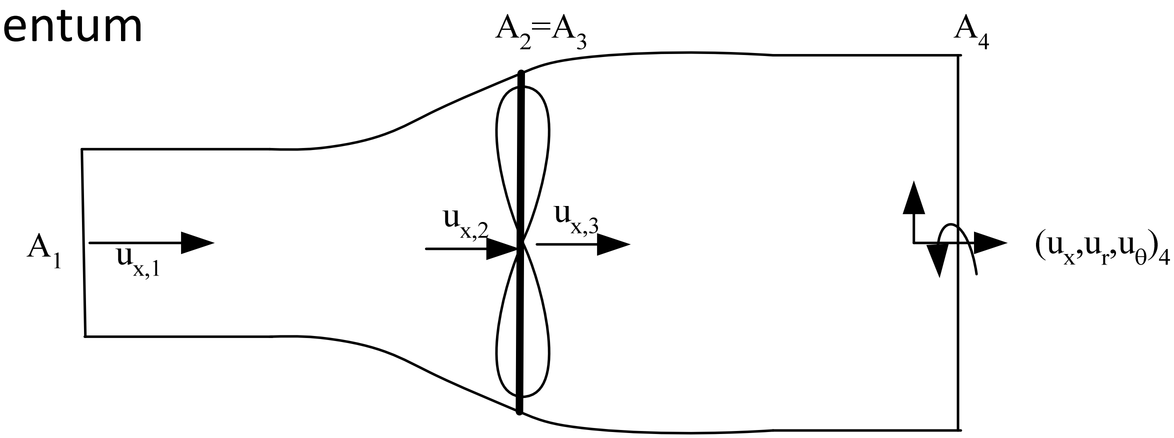
ME/ESE 4470
Wind and Tidal Power
Aerodynamics

Wind Turbine Aerodynamics

A. Introduction

Wind Turbine Aerodynamics

B. One-Dimensional Momentum Theory



Bernoulli Equation Along a Streamline:

$$P + 1/2\rho u^2 + \rho g z = C$$

Continuity Equation:

$$\frac{\partial}{\partial t}\int_{CV}\rho dV + \int_{CS}\rho(\vec{v}\cdot\hat{n})dA = 0$$

x-Momentum Equation:

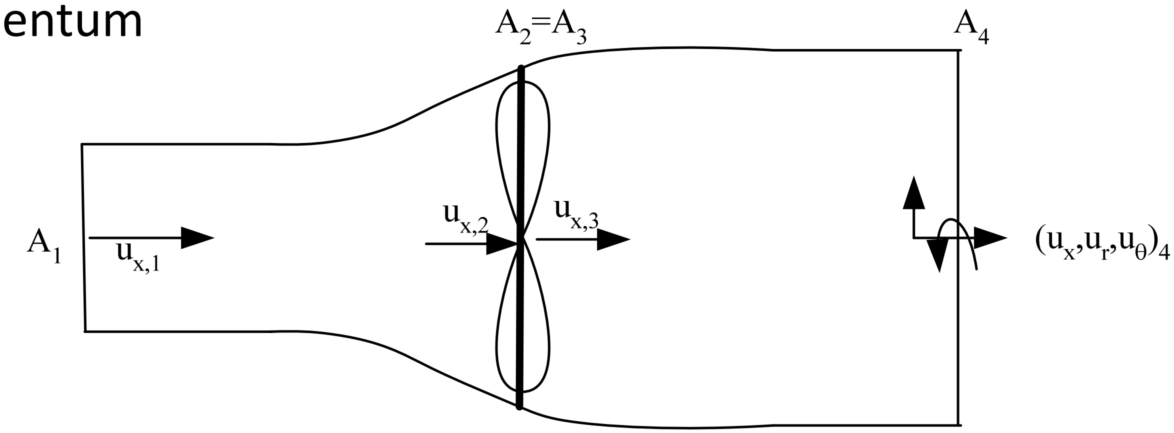
$$\frac{\partial}{\partial t}\int_{CV}\rho u dV + \int_{CS}\rho u(\vec{v}\cdot\hat{n})dA = F_{s,x} + F_{b,x}$$

Energy Equation:

$$\frac{\partial}{\partial t}\int_{CV}(P+1/2\rho u^2+\rho g z+\rho u_i)dV+\int_{CS}(P+1/2\rho u^2+\rho g z+\rho u_i)(\vec{v}\cdot\hat{n})dA = \dot{Q}+\dot{W}$$

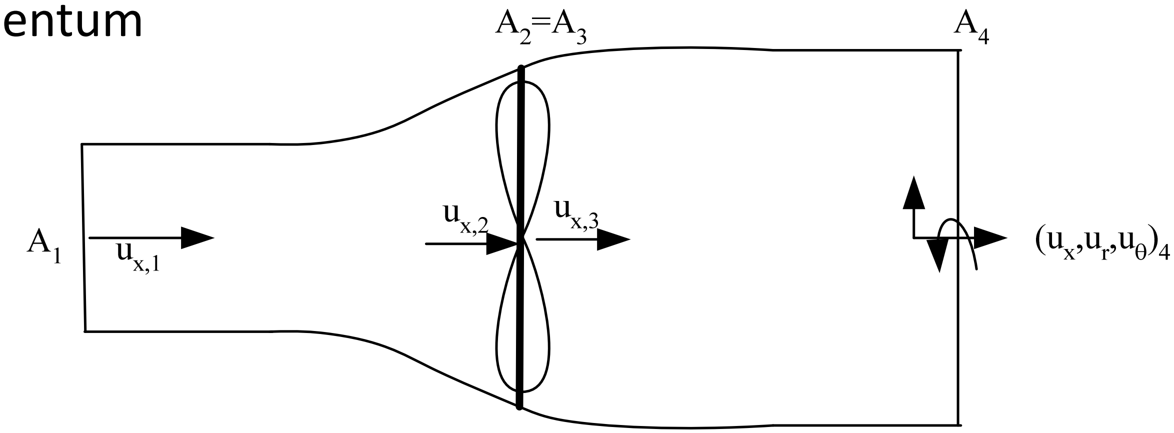
Wind Turbine Aerodynamics

B. One-Dimensional Momentum Theory



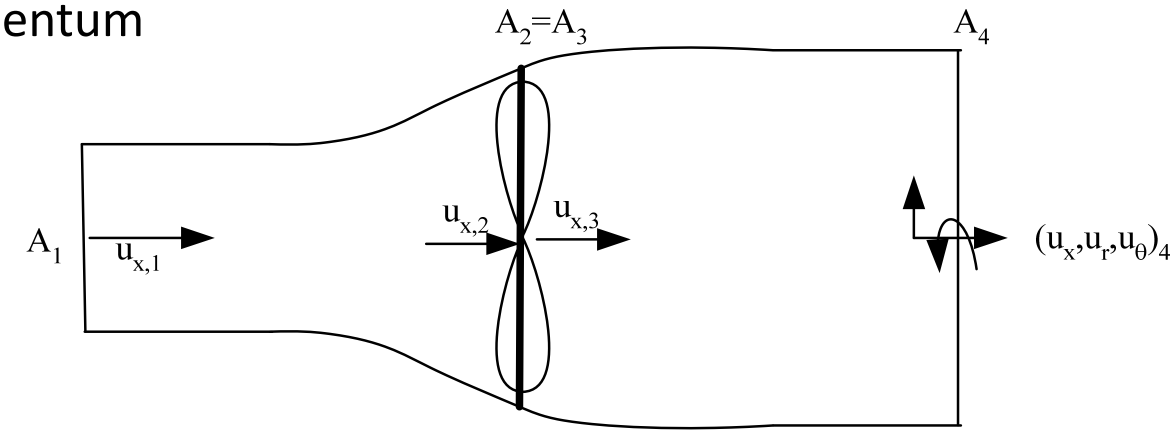
Wind Turbine Aerodynamics

B. One-Dimensional Momentum Theory



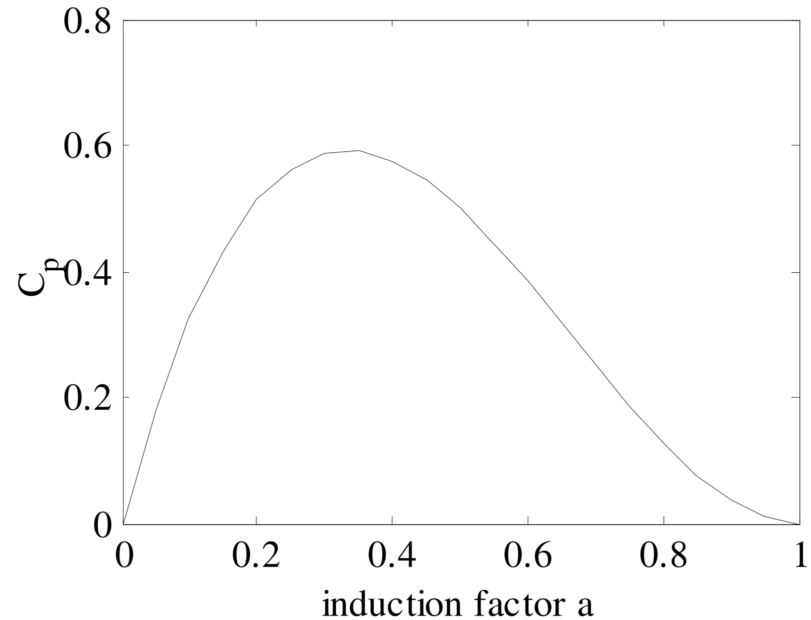
Wind Turbine Aerodynamics

B. One-Dimensional Momentum Theory



Wind Turbine Aerodynamics

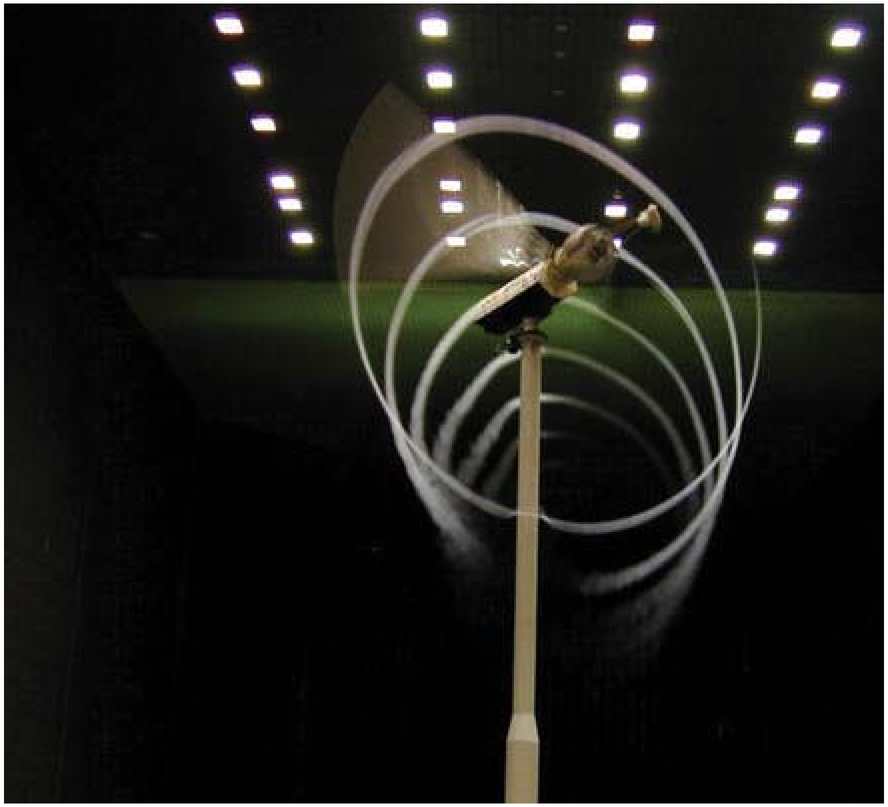
B. One-Dimensional Momentum Theory



$$C_p = \frac{P}{1/2 \rho u_{x,1}^3 A_2} = 4a(1-a)^2$$

Wind Turbine Aerodynamics

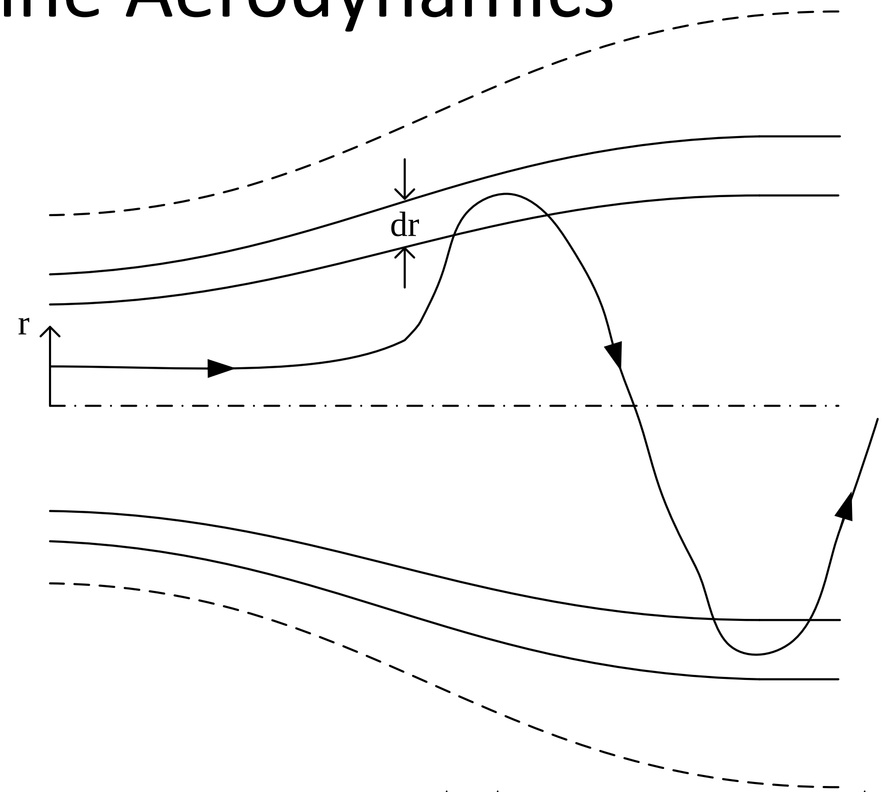
C. Wake Rotation Effects

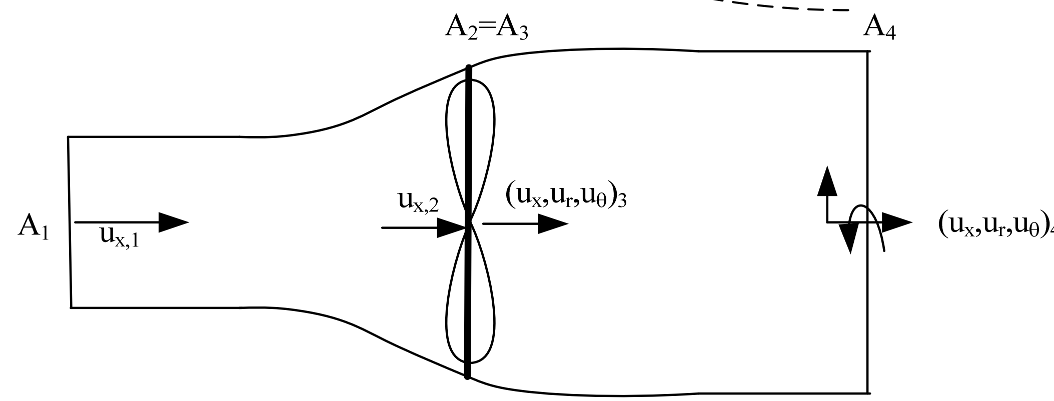


Wind Turbine Aerodynamics

C. Wake Rotation Effects

1. Theory





Wind Turbine Aerodynamics

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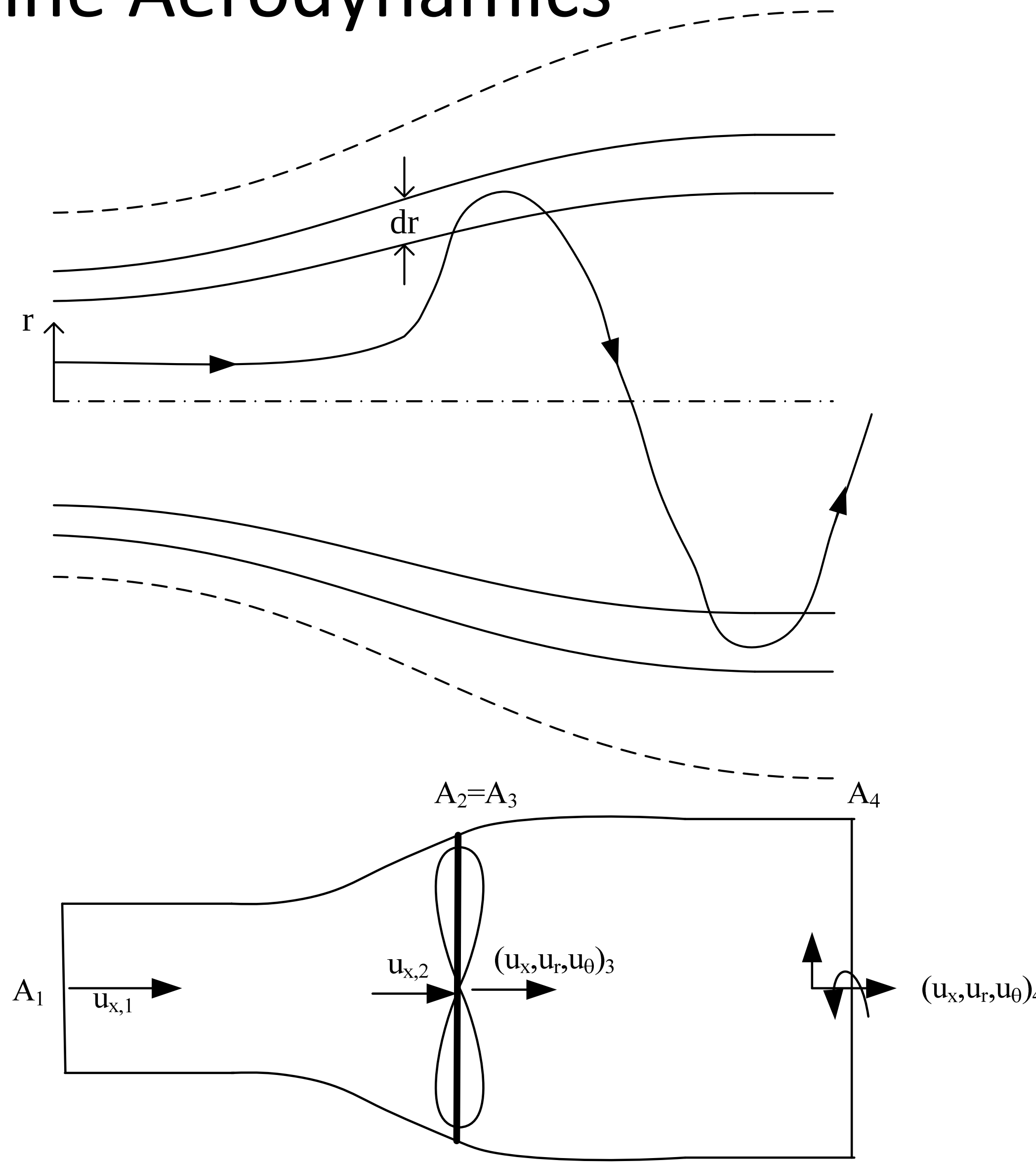

Wind Turbine Aerodynamics

C. Wake Rotation Effects

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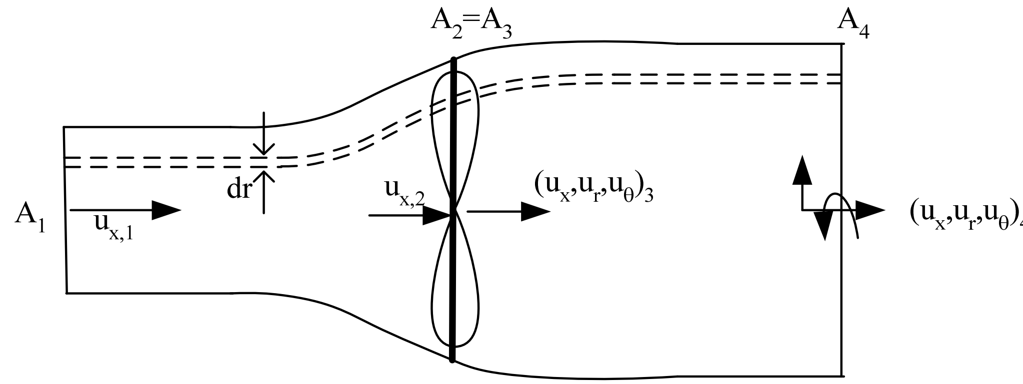

Wind Turbine Aerodynamics

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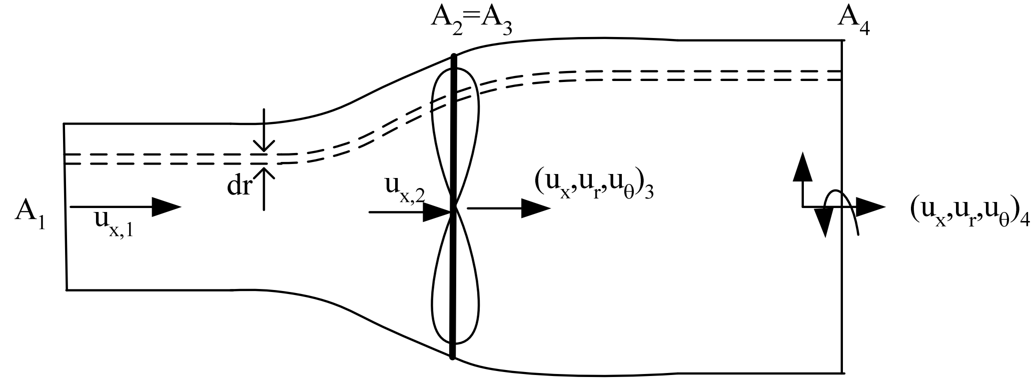

Wind Turbine Aerodynamics

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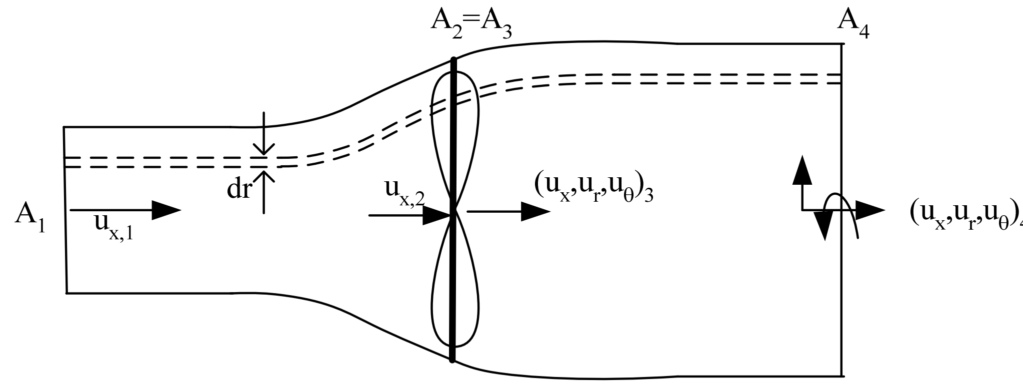

Wind Turbine Aerodynamics

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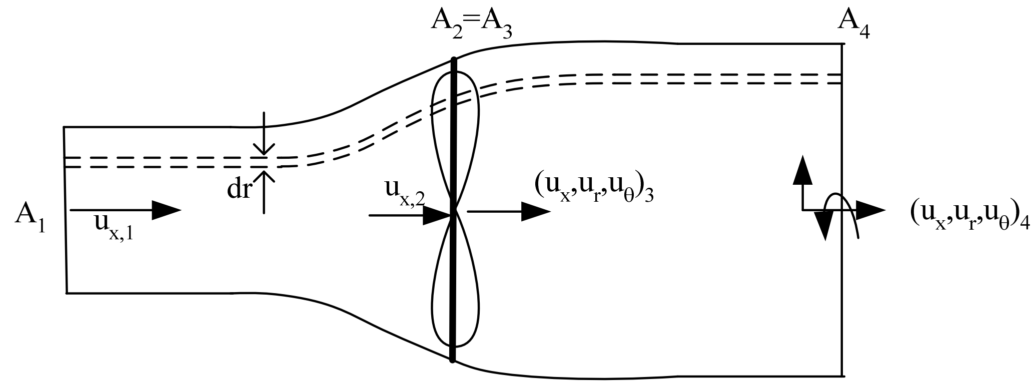

Wind Turbine Aerodynamics

C. Wake Rotation Effects

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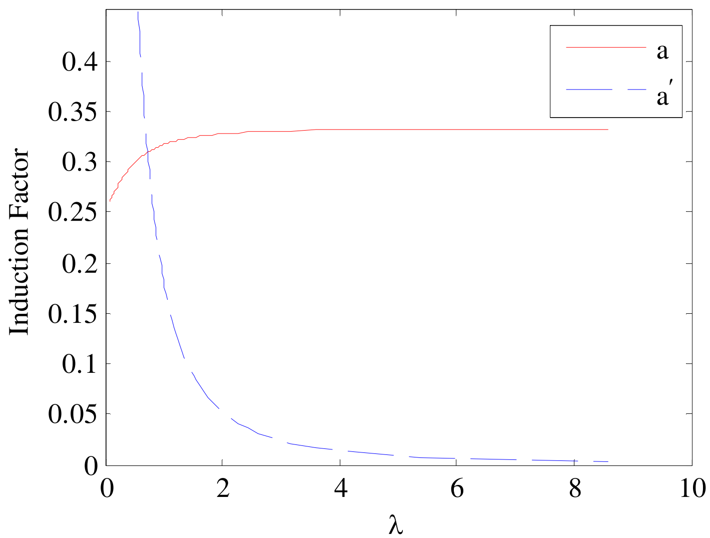
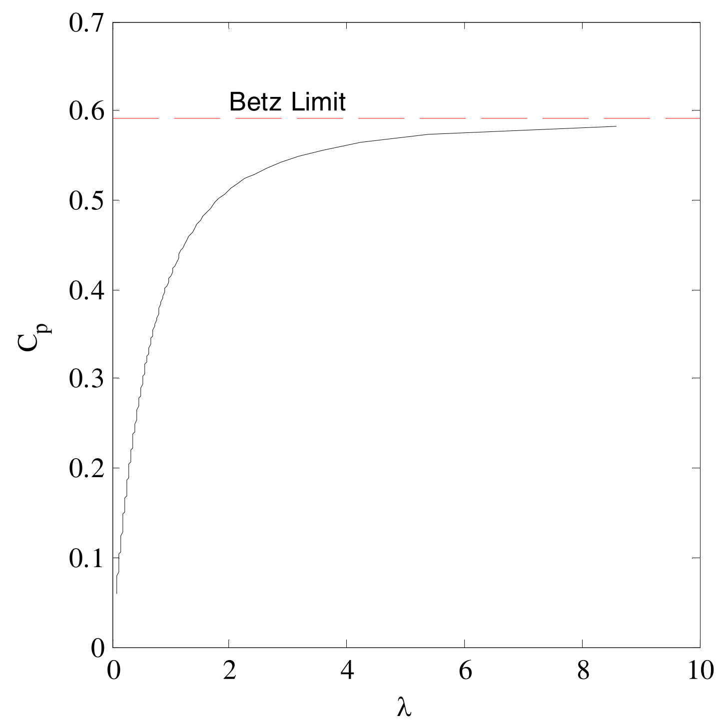
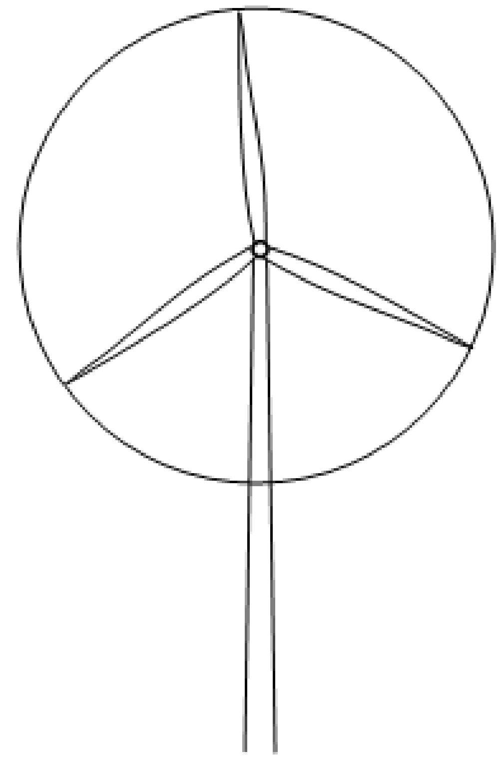




Wind Turbine Aerodynamics

C. Wake Rotation Effects

2. Practical Consideration



Wind Turbine Aerodynamics

C. Wake Rotation Effects

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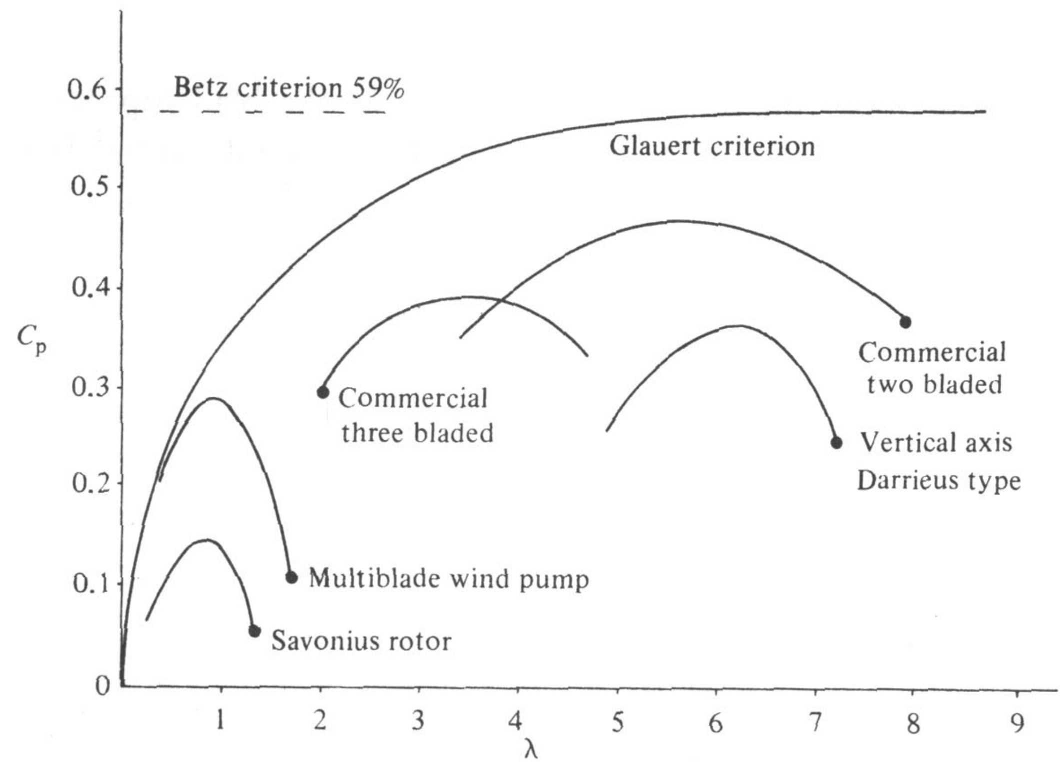
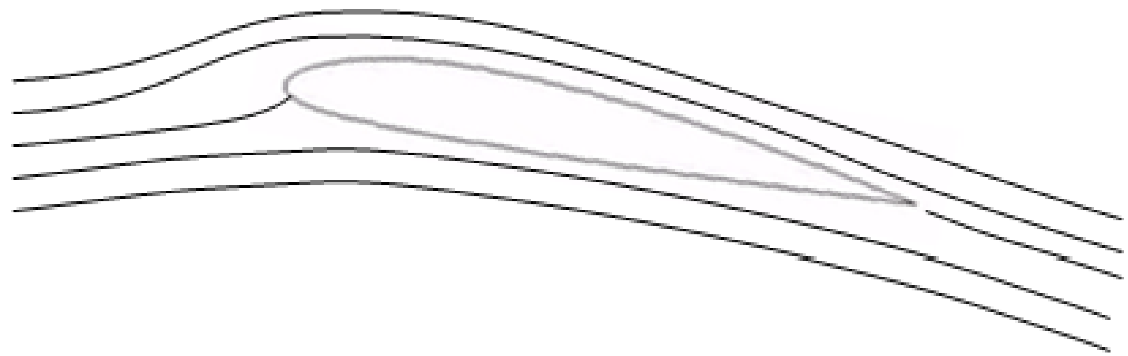


Wind Turbine Aerodynamics

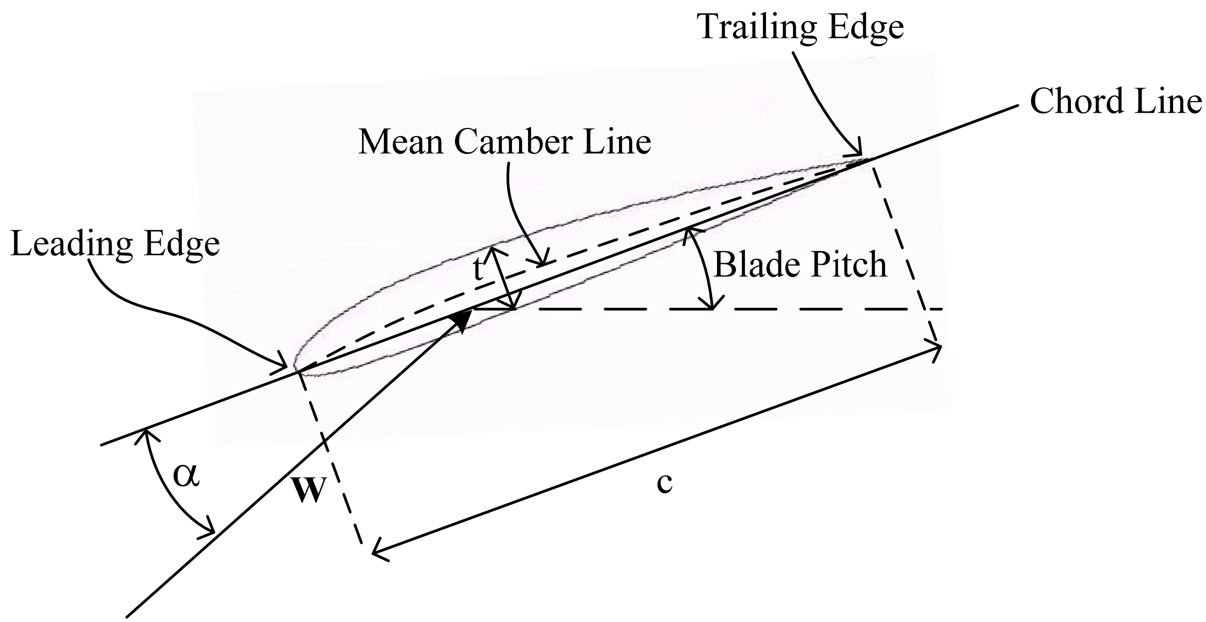
D. Blade Aerodynamics



Wind Turbine Aerodynamics

D. Blade Aerodynamics

1. Terminology



Wind Turbine Aerodynamics

D. Blade Aerodynamics

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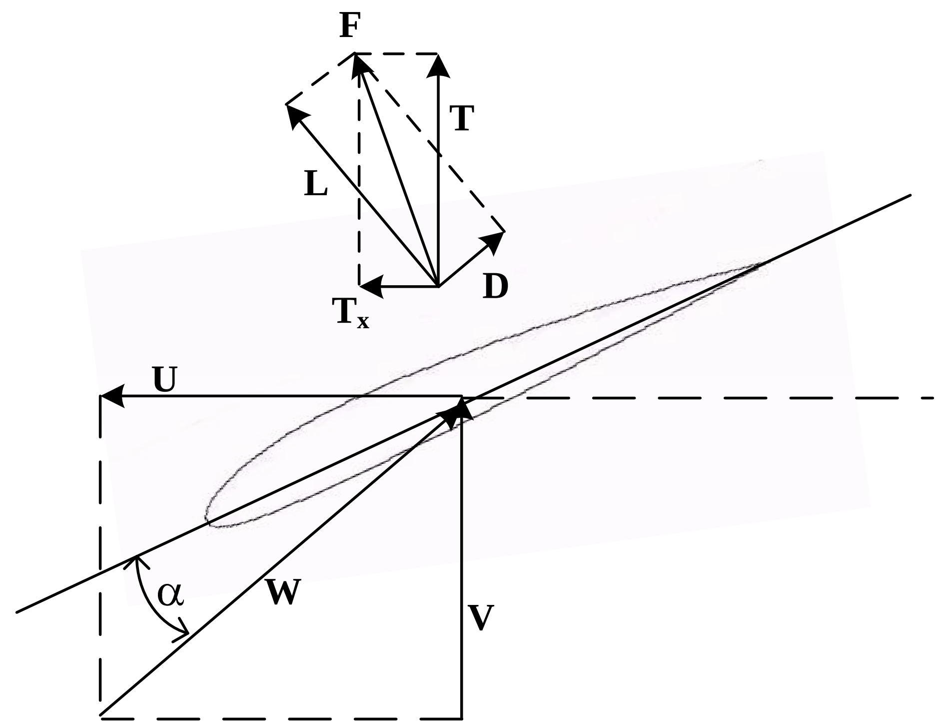
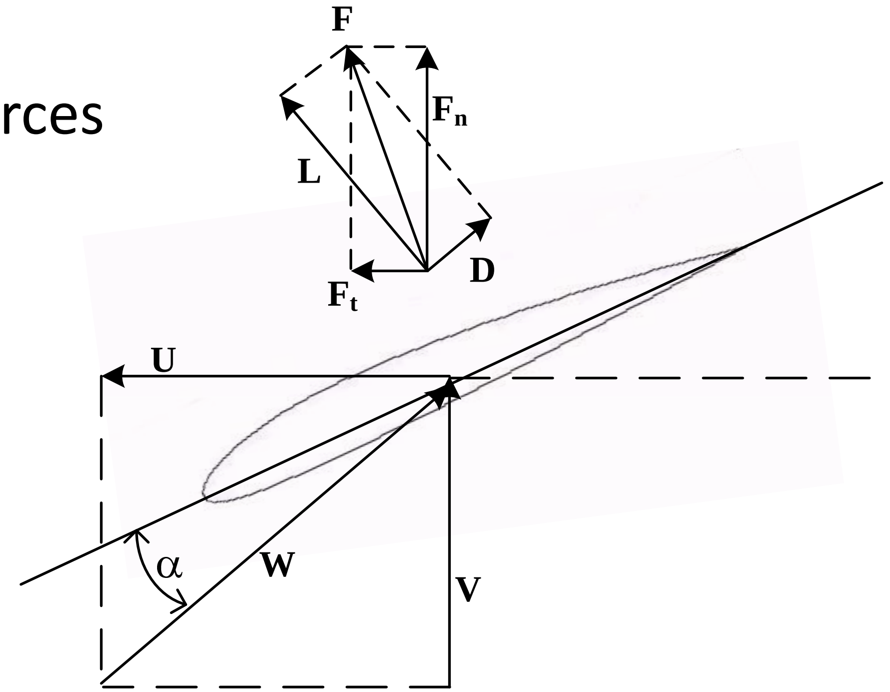


Wind Turbine Aerodynamics

D. Blade Aerodynamics

2. Lift, Drag and Related Forces

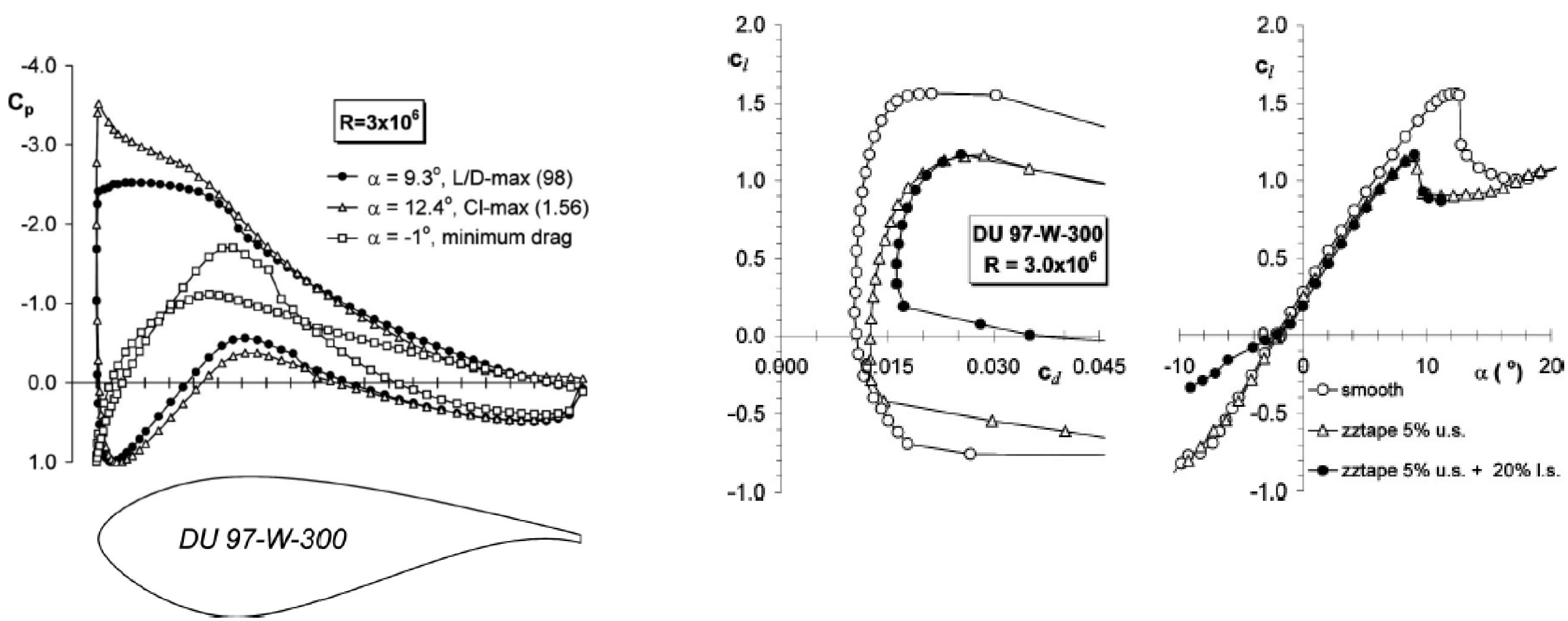


Wind Turbine Aerodynamics

D. Blade Aerodynamics

2. Lift, Drag and Related Forces

Source:
Timmer and Rooij
J. Solar Energy Engineering
Vol. 125, November 2003



Wind Turbine Aerodynamics

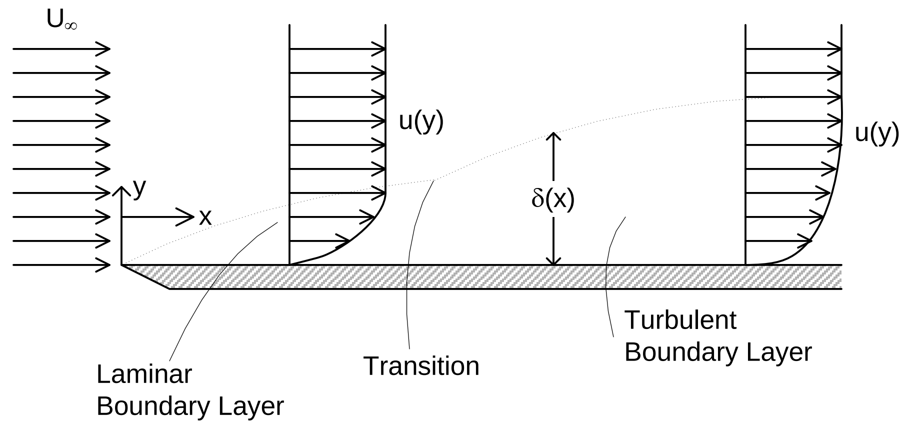
D. Blade Aerodynamics

3. Boundary Layers and Stall

$$Re_x = \frac{\rho U_\infty x}{\mu} = \frac{U_\infty x}{\nu}$$

$$\begin{array}{lll} \text{laminar boundary layer} \longrightarrow & \text{transition} \longrightarrow & \text{turbulent boundary layer} \\ \mathrm{Re}_x < 2 \times 10^4 & & \mathrm{Re}_x > 3 \times 10^6 \end{array}$$

$$\delta = y \quad \text{where} \quad u = 0.99 U_\infty$$



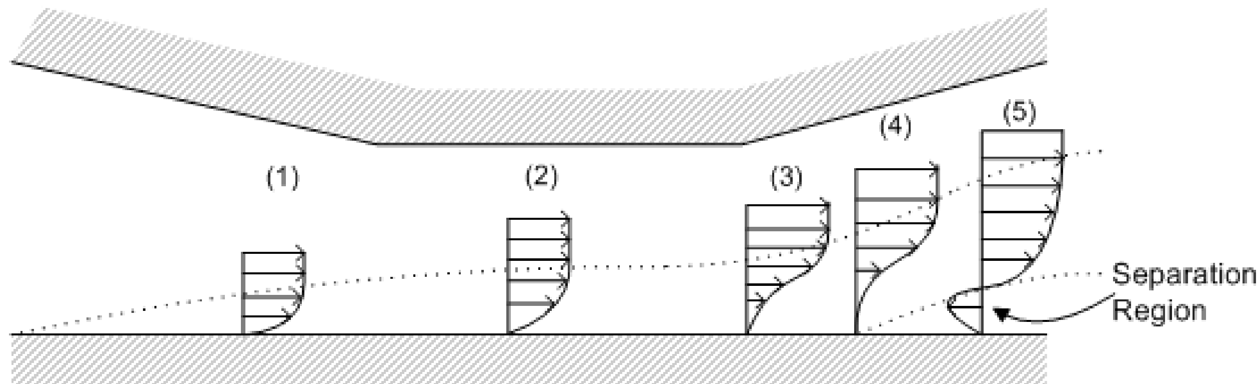
$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy$$

$$\theta = \int_0^\infty \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

Wind Turbine Aerodynamics

D. Blade Aerodynamics

3. Boundary Layers and Stall

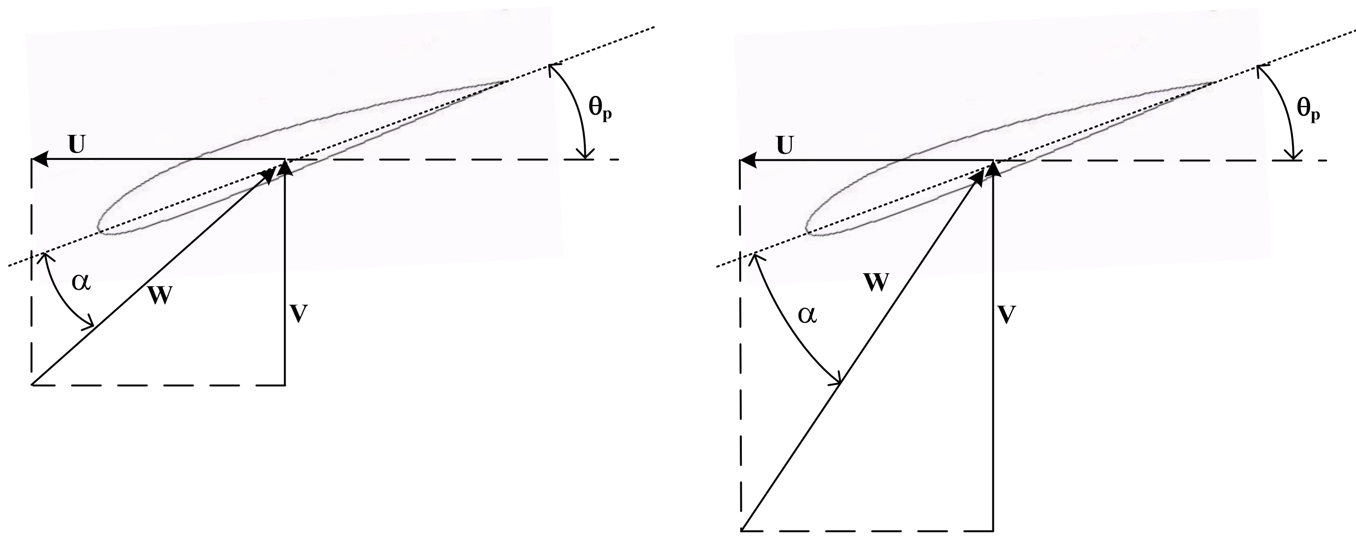


Wind Turbine Aerodynamics

D. Blade Aerodynamics

4. Various Effects on Angle of Attack

a. Variations in Wind Speed

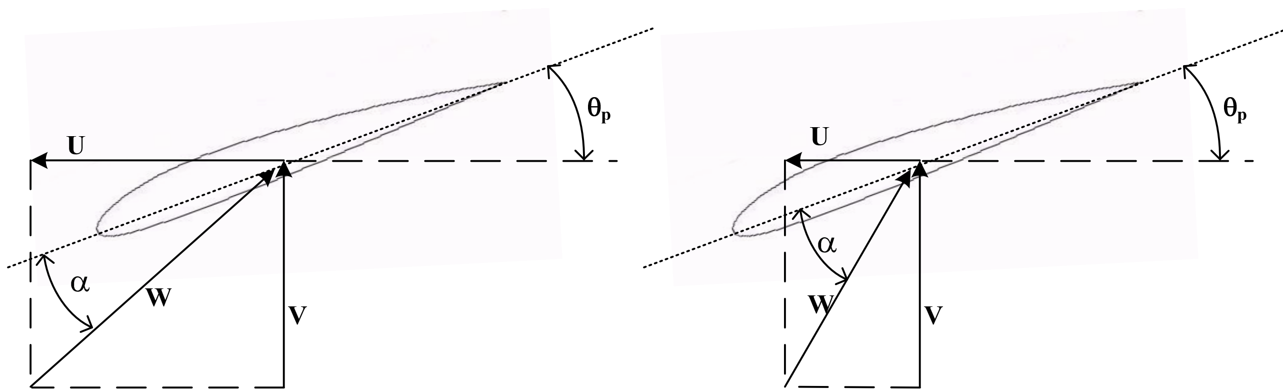


Wind Turbine Aerodynamics

D. Blade Aerodynamics

4. Various Effects on Angle of Attack

b. Variations in Rotational Velocity

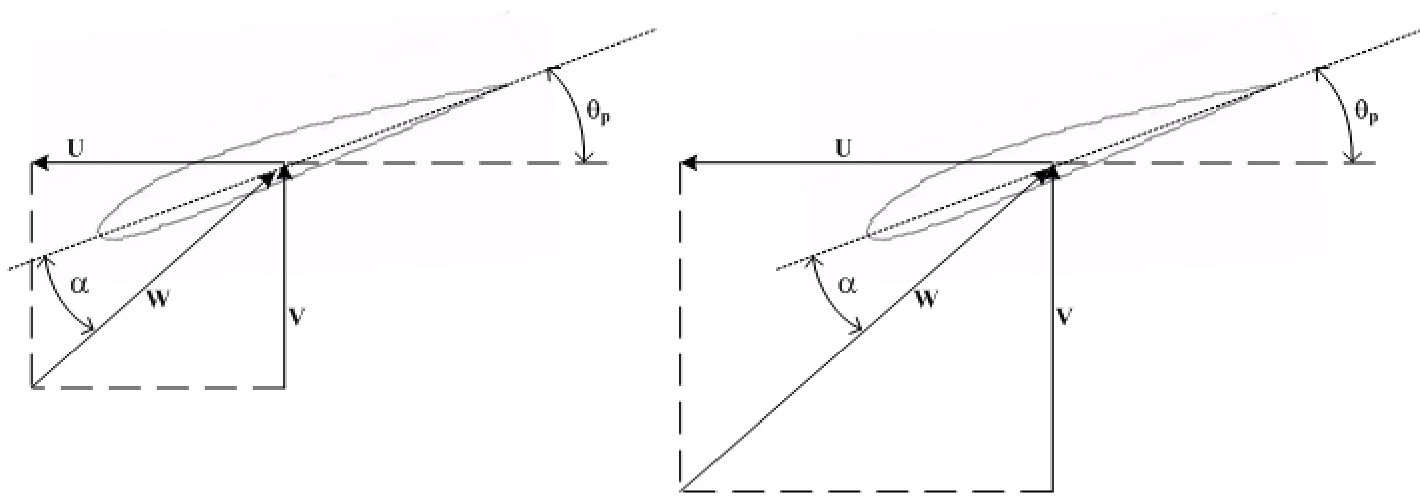


Wind Turbine Aerodynamics

D. Blade Aerodynamics

5. Design Features to Overcome Varying α

a. Vary Rotation Speed to Overcome Variations in Wind Speed

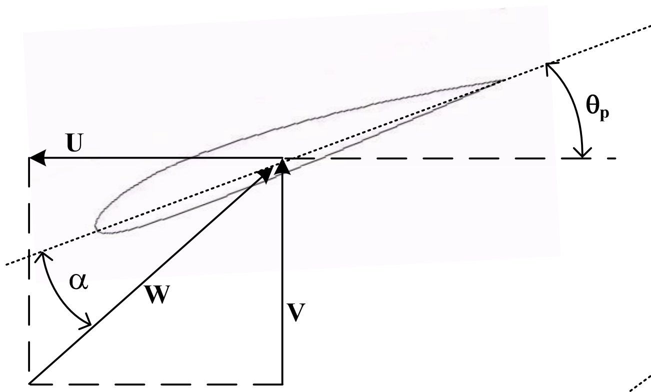


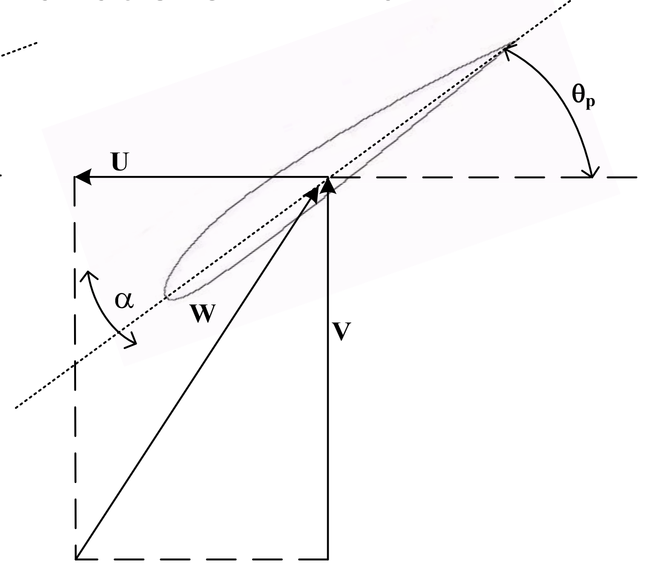
Wind Turbine Aerodynamics

D. Blade Aerodynamics

5. Design Features to Overcome Varying α

b. Vary Pitch to Overcome Variations in Wind Sp



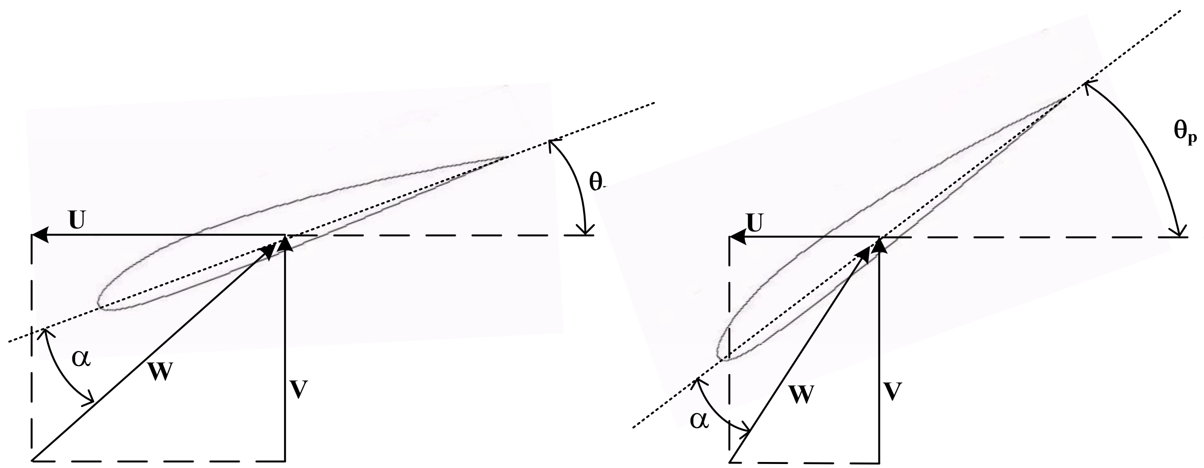


Wind Turbine Aerodynamics

D. Blade Aerodynamics

5. Design Features to Overcome Varying α

c. Vary Pitch to Overcome Variations in Rotation Rate



Wind Turbine Aerodynamics

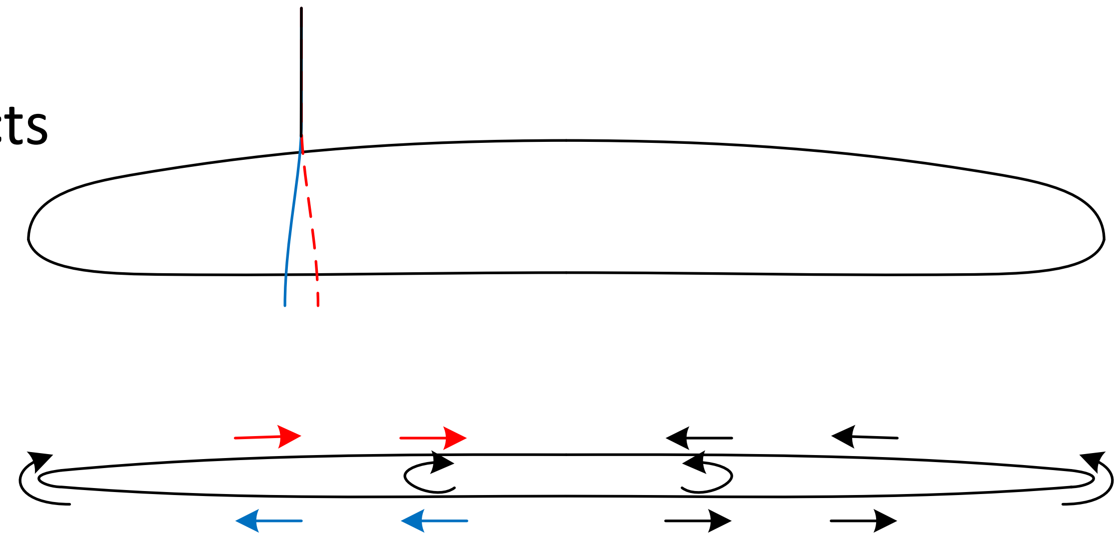
D. Blade Aerodynamics

    4. Effects of Blade Rotation

Wind Turbine Aerodynamics

E. 3-D Blade Aerodynamics

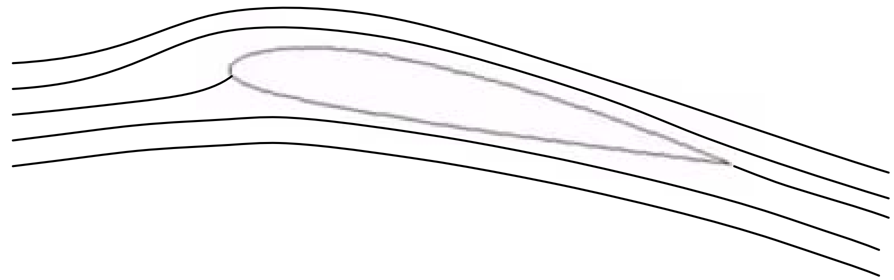
1. Finite Wing Effects

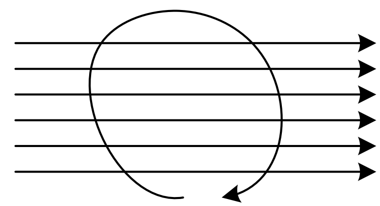


Wind Turbine Aerodynamics

E. 3-D Blade Aerodynamics

1. Finite Wing Effects

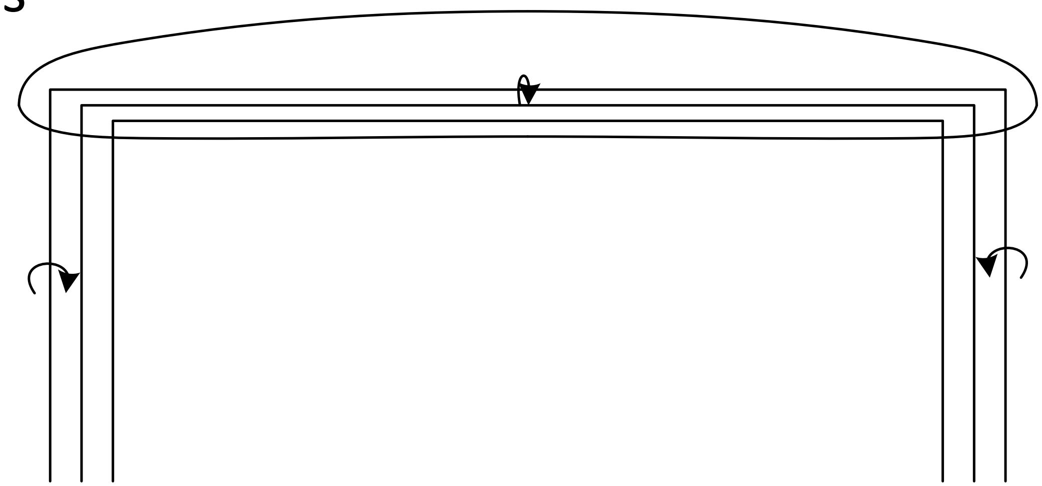




Wind Turbine Aerodynamics

E. 3-D Blade Aerodynamics

2. Lifting Line Theory



Wind Turbine Aerodynamics

E. 3-D Blade Aerodynamics

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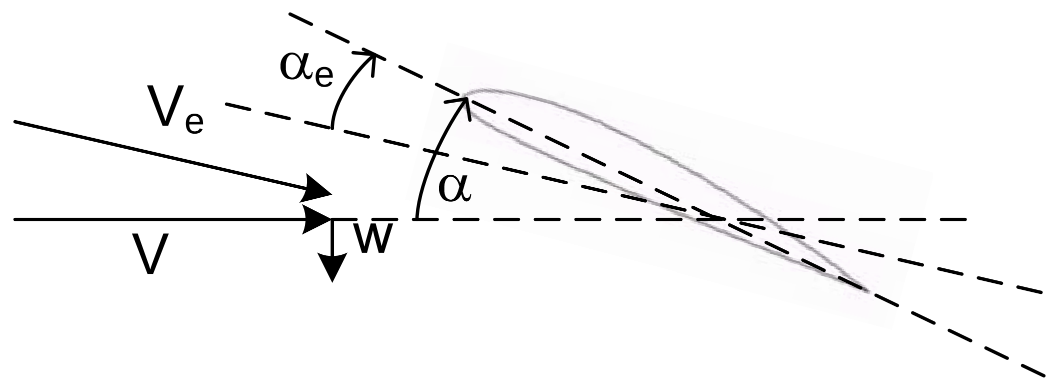
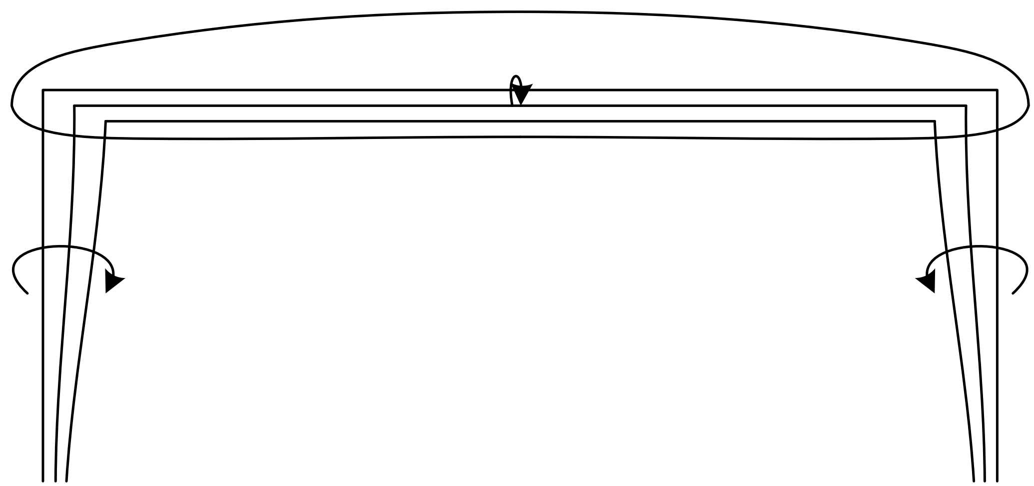


Wind Turbine Aerodynamics

E. 3-D Blade Aerodynamics

2. Lifting Line Theory



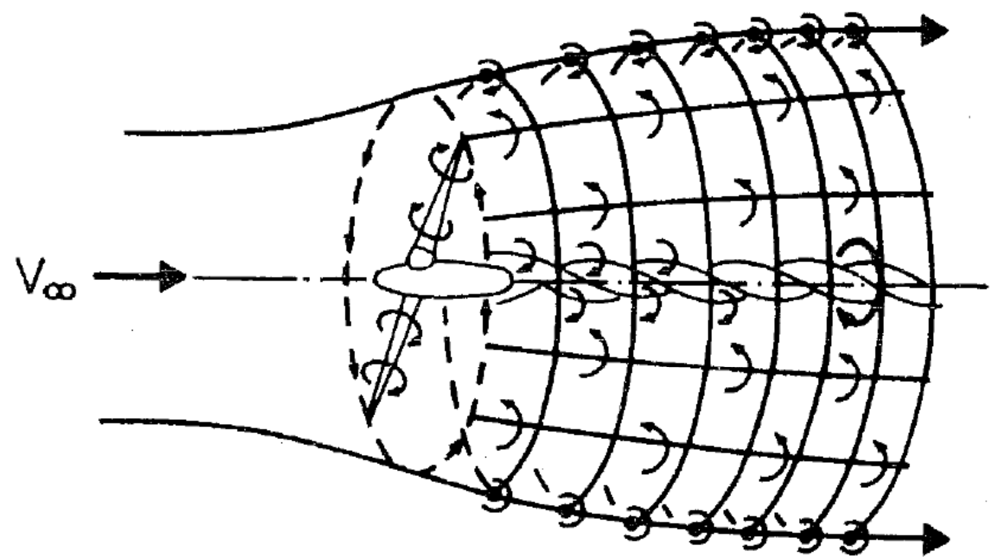
Wind Turbine Aerodynamics

E. 3-D Blade Aerodynamics
   3. Other 3-D Effects

Wind Turbine Aerodynamics

E. 3-D Blade Aerodynamics

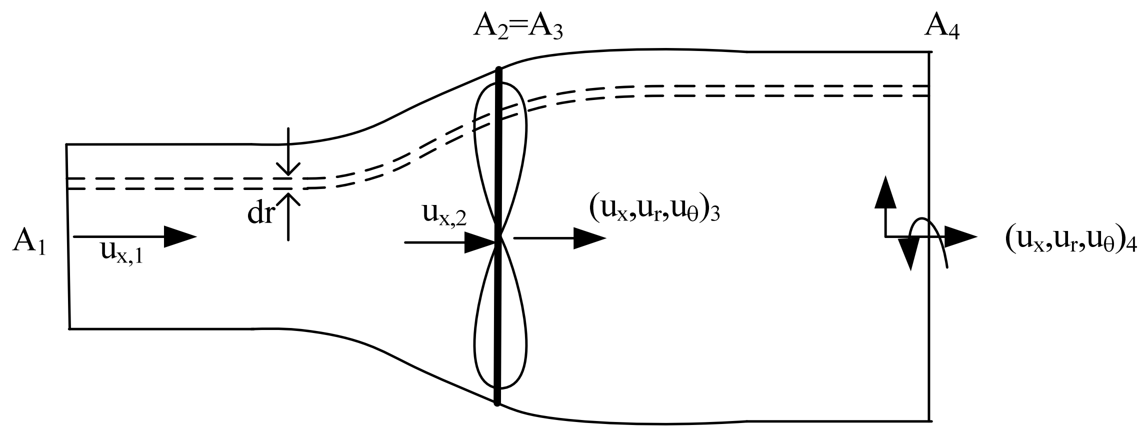
4. Vortex System Behind a Wind Turbine



Wilson and Lissaman
Applied Aerodynamics of Wind Power Machines, 1974

Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)



Wind Turbine Aerodynamics

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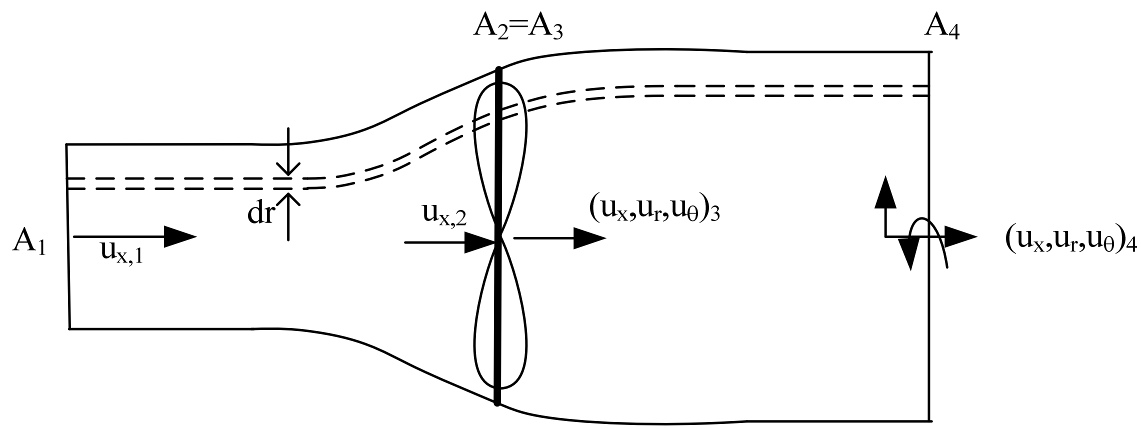


Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)

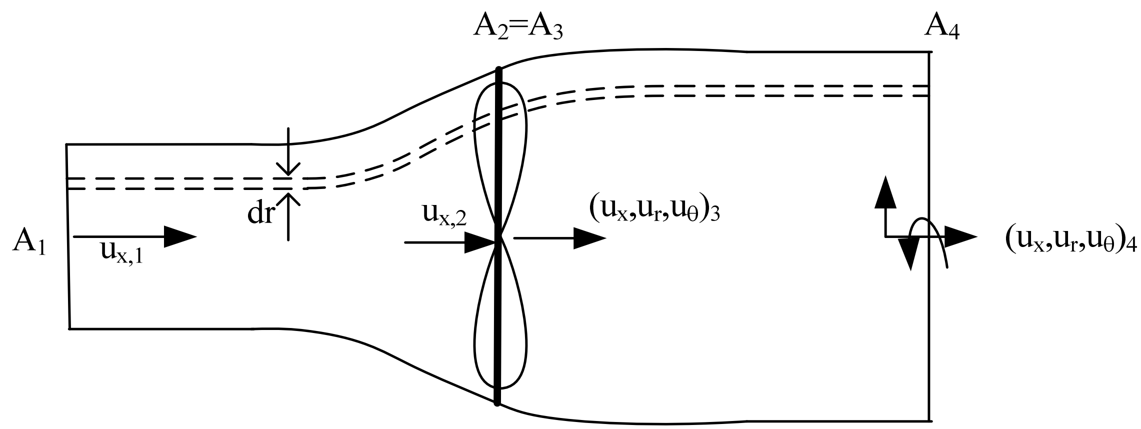
1. Equations

$$dT = \frac{1}{2}\rho(u_{x,1}^2 - u_{x,4}^2)dA$$

$$dT = \frac{1}{2}\rho u_{x,1}^2 4a(1-a)2\pi r dr$$

$$dT_x = \rho r u_{\theta,3} u_{x,3} dA$$

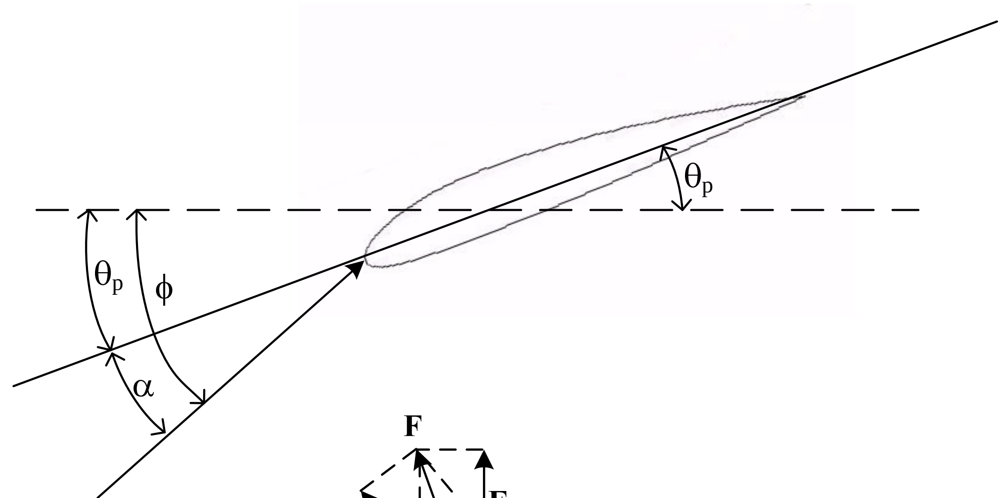
$$dT_x = \frac{1}{2}\rho u_{x,1}\Omega r^2 4a'(1-a)2\pi r dr$$



Wind Turbine Aerodynamics

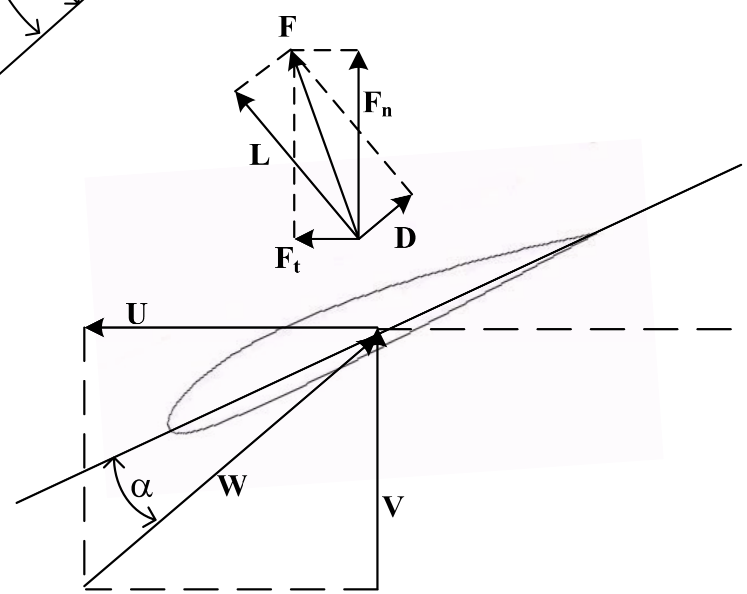
F. Blade Element Momentum Theory (BEM)

1. Equations



$$\alpha = \phi - \theta_p$$

$$\tan\phi = \frac{V}{U} = \frac{(1-a)u_{1,x}}{(1+a')\Omega r}$$



$$L = \frac{1}{2}\rho W^2 c C_L$$

$$D = \frac{1}{2}\rho W^2 c C_D$$

Wind Turbine Aerodynamics

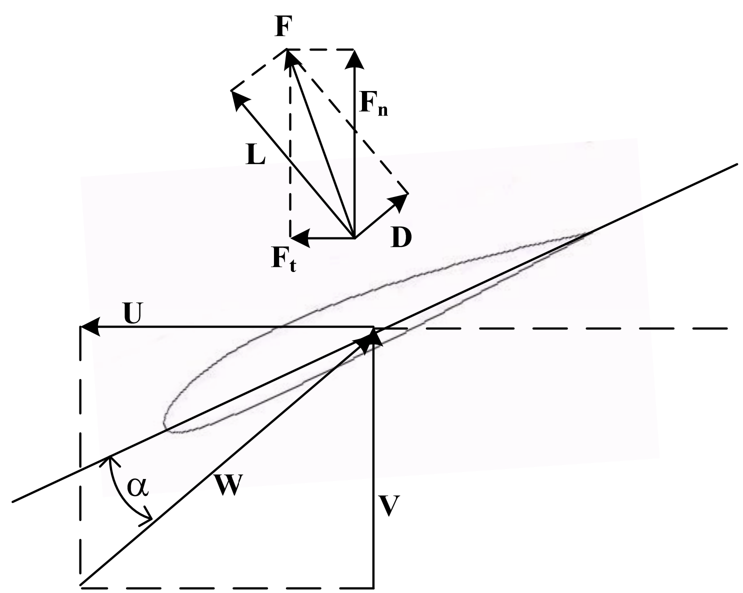
F. Blade Element Momentum Theory (BEM)

1. Equations

$$F_N = L\cos(\phi) + D\sin(\phi) \qquad C_N = C_L\cos(\phi) + C_D\sin(\phi)$$

$$F_T = L\sin(\phi) - D\cos(\phi) \qquad C_T = C_L\sin(\phi) - C_D\cos(\phi)$$

$$\sigma(r) = \frac{c(r)B}{2\pi r}$$



Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)

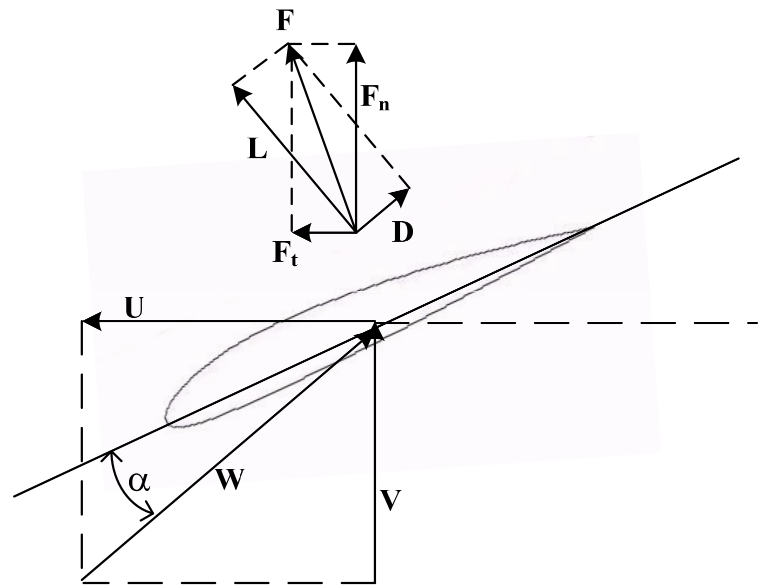
1. Equations

$$dT = BF_N dr$$

$$dT = \frac{1}{2}\rho\frac{u_{1,x}^2(1-a)^2}{\sin^2(\phi)}cC_N B dr$$

$$dT_x = rBF_T dr$$

$$dT_x = \frac{1}{2}\rho r\frac{u_{1,x}(1-a)\Omega r(1+a')}{\sin(\phi)\cos(\phi)}cC_T B dr$$



$$a = \left(1 + \frac{4\sin^2(\phi)}{\sigma C_N}\right)^{-1}$$

$$a' = \left(\frac{4\sin(\phi)\cos(\phi)}{\sigma C_T}\right)^{-1}$$

Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)

 2. Implementation

Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)

3. Corrections

a. Prandtl's Tip Loss Factor

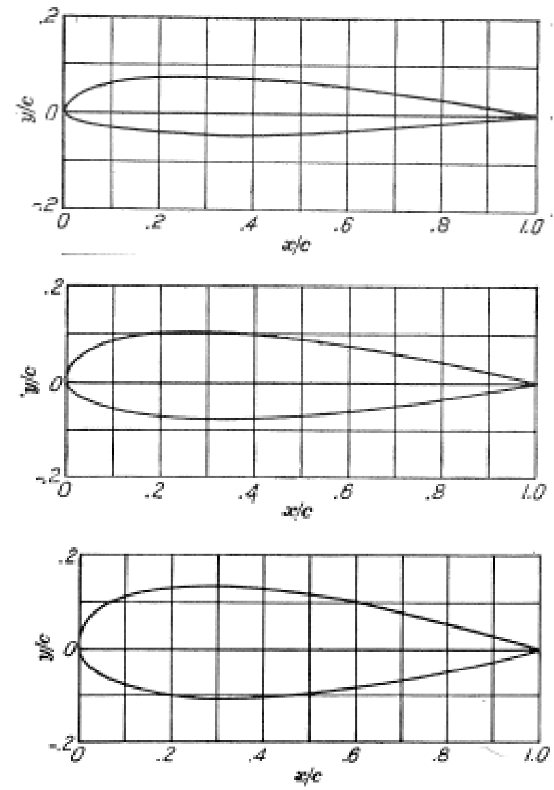
b. Glauert's Correction for High Values of a

Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)

4. Example

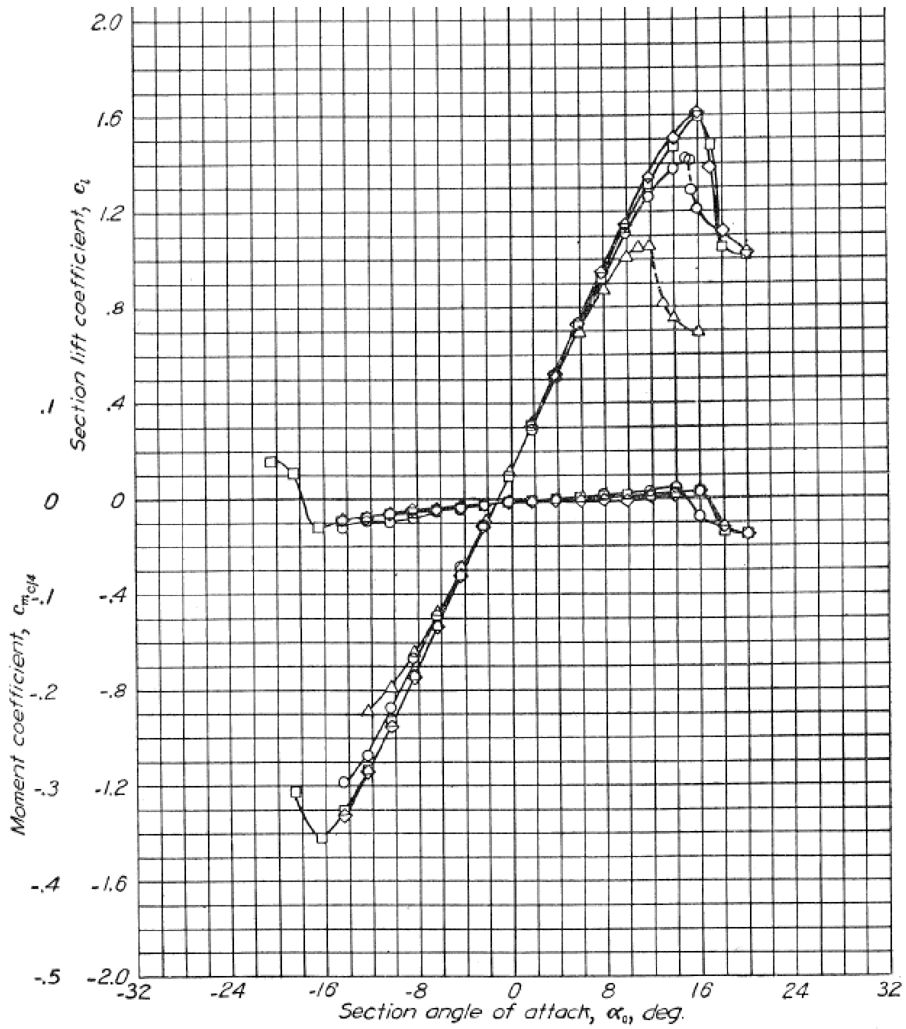
r (m)	c (m)	θ_p(deg)
4	1.5	20
6	1.4	13
8	1.3	8
10	1.2	6
12	1.0	4
14	0.8	3
16	0.6	2
18	0.4	1
20	0.2	0



Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)

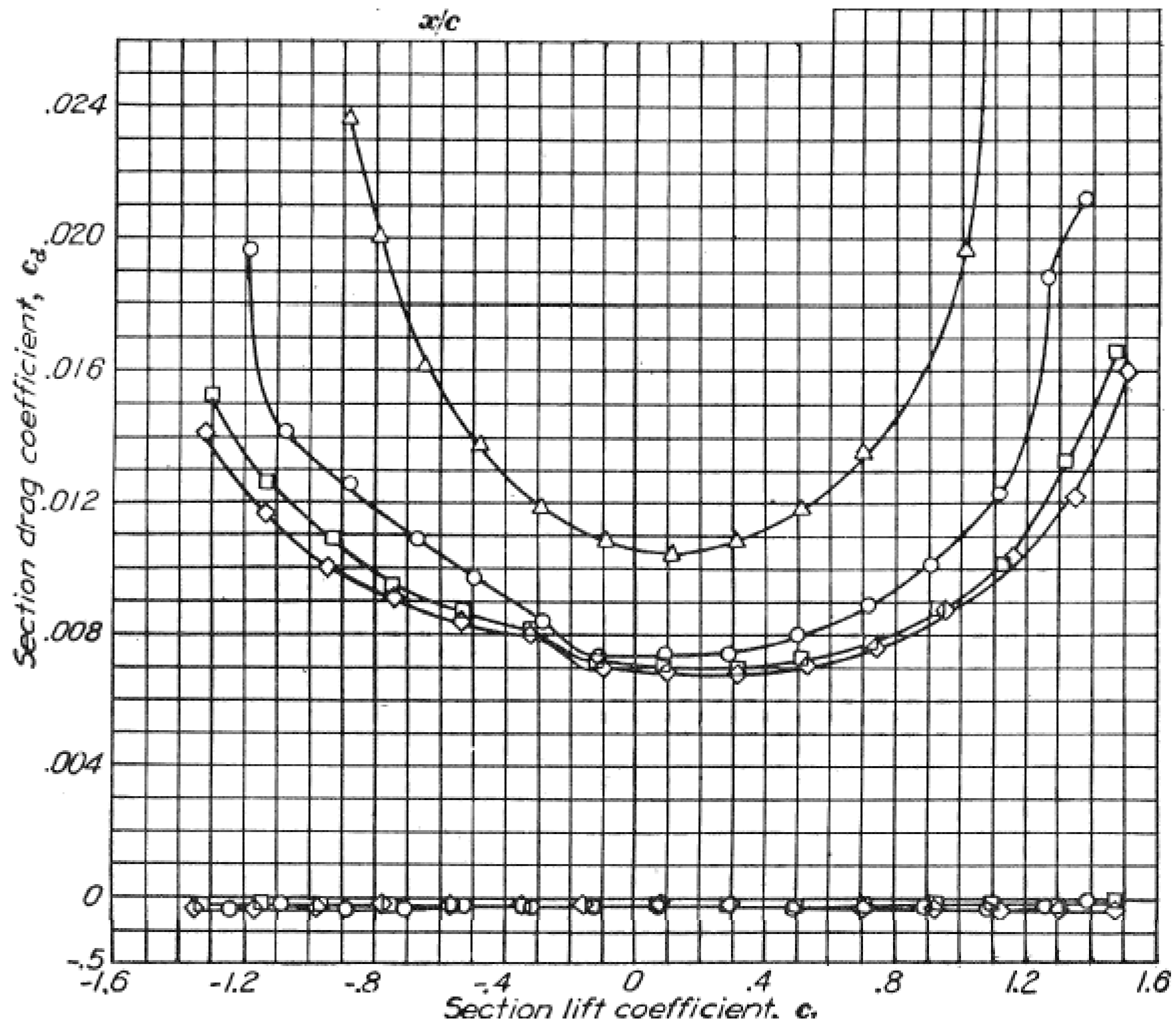
4. Example



Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)

4. Example



Wind Turbine Aerodynamics

F. Blade Element Momentum Theory (BEM)

4. Example

a. Results