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```
procedure Kruskal(edges: Array of Edge, V: Integer, mode: Integer);
var
  graph: Graph;
  currentTime: LongInt;
  result: Array of Edge;
  1. subsets: Array of Subset;
  2. v, e, i, x, y, mstCost: Integer;
  3. execTime: Float;
  4. next_edge: Edge;
begin
  5. graph := DrawGraph(edges, 'MST');
  6. currentTime := GetCurrentTime();
  7. result := EmptyArray();

  8. edges := SortArrayList(edges);

  9. SetLength(subsets, V);
  10. for v := 0 to V-1 do
  11. subsets[v] := Subset(v, 0);
    end;

  12. e := 0;
  13. i := 0;

  14. while (e < V - 1) and (i < Length(edges)) do
    begin
  15. next_edge := edges[i];
  16. i := i + 1;

  17. x := Find(subsets, next_edge.from.index);
  18. y := Find(subsets, next_edge.to.index);

  19. if x <> y then
    begin
  20. AddToArray(result, next_edge);
  21. Union(subsets, x, y);
  22. e := e + 1;
    end;
    end;

  23. execTime := (GetCurrentTime() - currentTime) / 1000000.0;
  24. mstCost := 0;

  25. for edge in result do
    begin
```

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26. Print(edge.from.name + ' - ' + edge.to.name + ': ' + edge.weight);
27. mstCost := mstCost + edge.weight;
    end;

28. Print('MST cost: ' + mstCost);
29. Print('Time: ' + execTime + 'us');

30. GenerateGraphVizSVG(edges, result, 'Minimum Spanning Tree', mstCost);

31. graph.SetAttribute('ui.title', 'Minimum Spanning Tree');
32. graph.SetAttribute('ui.quality');
33. graph.SetAttribute('ui.antlralias');

34. OutputGraphCSV(result, mstCost);

35. highlightedGraph := HighlightGraph(graph, result);
36. AddTextToGraph(highlightedGraph, 'Cost: ' + mstCost + ' Time: ' + execTime + 'ms');
end;

```

Worst-case time complexity

Linha Operações Primitivas

1	1A
2	1A
3	1A
4	1A
5	1A
6	1A
7	1A
8	-
9	1A
10-11	1A + 1I + 1C + 1A
12	1A
13	1A
14	2C
15	1A
16	1I
17	1A
18	1A
19	1C
20	1A

21 | 1A
22 | 1I
23 | 1A + 1Op
24 | 1A
25-27 | 1C + 1A
28 | 1A
29 | 1A
30 | 1A
31 | 1A
32 | 1A
33 | 1A
34 | 1A
35 | 1A
36 | 1A

Estimativa de O: $O(E \log V)$, onde E é o número de arestas e V é o número de vértices.

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Pseudo code:

```
procedure dijkstra(pointNames: array of String; distancesMatrix: array of array of Integer;  
source: String);  
var  
  1. n, sourceIndex, i, u, v: Integer;  
  2. distance: array of Integer;  
  3. visited: array of Boolean;  
  4. predecessor: array of Integer;  
begin  
  5. n := Length(pointNames);  
  6. SetLength(distance, n);  
  7. SetLength(visited, n);  
  8. SetLength(predecessor, n);  
  
  9. for i := 0 to n - 1 do  
    begin  
      10. distance[i] := MAXINT;
```

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11. visited[i] := False;
12. predecessor[i] := -1;
end;

13. sourceIndex := IndexOf(pointNames, source);
14. distance[sourceIndex] := 0;

15. for i := 0 to n - 2 do
begin
16. u := minDistance(distance, visited);
17. visited[u] := True;
18. for v := 0 to n - 1 do
begin
    if (not visited[v]) and (distancesMatrix[u][v] <> 0) and (distance[u] <> MAXINT) and
(distance[u] + distancesMatrix[u][v] < distance[v]) then
    begin
19. distance[v] := distance[u] + distancesMatrix[u][v];
20. predecessor[v] := u;
    end;
end;
end;

21. printSolution(distance, n, pointNames, predecessor);
end;

function minDistance(distance: array of Integer; visited: array of Boolean): Integer;
var
22. min, minIndex, i: Integer;
begin
23. min := MAXINT;
24. minIndex := -1;
25. for i := 0 to Length(distance) - 1 do
begin
    if (not visited[i]) and (distance[i] <= min) then
    begin
26. min := distance[i];
27. minIndex := i;
    end;
end;
28. minDistance := minIndex;
end;

```

Worst-case time complexity

- 1- 1A
- 2- 1A
- 3- 1A
- 4- 1A

- 5- 1A
- 6- 1A
- 7- 1A
- 8- 1A
- 9- $(n+1)C + nI$
- 10- nA
- 11- nA
- 12- nA
- 13- nC
- 14- 1A
- 15- $(n-1)C + (n-1)I$
- 16- $(n-1)(nC + nC) = n(n-1)C$
- 17- $(n-1)A$
- 18- $(n-1)(nC + nI)$
- 19- $(n-1)nA$
- 20- $(n-1)nA$
- 21- 1R
- 22- 1A
- 23- 1A
- 24- 1A
- 25- $(n+1)C + nI$
- 26- nC + nA
- 27- nA
- 28- 1R

Estimativa total da complexidade: $O(n^2)$.

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Pseudocode:

1. function Dijkstra(pointNames: array of String; distancesMatrix: array of array of Integer; source, point: String; finalPrint: Boolean): List;
2. var
3. n, i, u, v, sourceIndex: Integer;
4. distance, predecessor: array of Integer;
5. visited: array of Boolean;
6. output: List;
7. begin
8. $n := \text{Length}(\text{pointNames});$
9. $\text{SetLength}(\text{distance}, n);$
10. $\text{SetLength}(\text{visited}, n);$
11. $\text{SetLength}(\text{predecessor}, n);$
12. $\text{output} := \text{CreateList}();$
13. for $i := 0$ to $n - 1$ do
14. begin
15. $\text{distance}[i] := \text{MaxInt};$
16. $\text{visited}[i] := \text{False};$
17. $\text{predecessor}[i] := -1;$
18. end;

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19. sourceIndex := IndexOf(pointNames, source);
20. distance[sourceIndex] := 0;
21. for i := 0 to n - 2 do
22. begin
23. u := MinDistance(distance, visited);
24. visited[u] := True;
25. if pointNames[u] = point then
26. Break;
27. for v := 0 to n - 1 do
28. begin
29. if (not visited[v]) and (distancesMatrix[u][v] <> 0) and (distance[u] <> MaxInt) and
(distance[u] + distancesMatrix[u][v] < distance[v]) then
30. begin
31. distance[v] := distance[u] + distancesMatrix[u][v];
32. predecessor[v] := u;
33. end;
34. end;
35. end;
36. output.Append(PrintSolution(distance, n, pointNames, predecessor, point));
37. output.Append(distance);
38. Result := output;
39. end;
40. function GetClosestAP(assemblyPoints, pointNames: array of String;
distancesMatrix: array of array of Integer; point: String): String;
41. var
42. minDistance, distanceToPoint: Integer;
43. closestAP: String;
44. distancesFromAP: array of Integer;
45. ap: String;
46. begin
47. minDistance := MaxInt;
48. closestAP := nil;
49. for ap in assemblyPoints do
50. begin
51. distancesFromAP := Dijkstra(pointNames, distancesMatrix, ap, point, False)[1];
52. distanceToPoint := distancesFromAP[IndexOf(pointNames, point)];
53. if distanceToPoint < minDistance then
54. begin
55. minDistance := distanceToPoint;
56. closestAP := ap;
57. end;
58. end;
59. Result := closestAP;
60. end;

```

Complexity analysis:

1 -
 2 -
 3 5A
 4 2A
 5 1A
 6 1A
 7 -
 8 1A
 9 $1A + nA$
 10 $1A + nA$
 11 $1A + nA$
 12 1A
 13 $(n+1)A + (n+1)C$
 14 -
 15 nA
 16 nA
 17 nA
 18 -
 19 $nA + nC$
 20 1A
 21 $nA + nC$
 22 -
 23 $n^2(A + C)$
 24 nA
 25 nC
 26 nC
 27 $n(nA + nC)$
 28 -
 29 $n^2(4C)$
 30 -
 31 $n^2(Op + A)$
 32 $n^2(A)$
 33 -
 34 -
 35 -
 36 $nA + nOp$
 37 1A
 38 1R
 39 -

Estimativa de O: $O(n^2)$