Homework – maxlik and Poisson

Paulina Reus

1 Function

$$P(X = x, \lambda_i) = \frac{\lambda^x e^{-\lambda}}{x!} \tag{1}$$

For $\lambda_i = \theta_0 + \theta_1 \times z_i$

$$P(X = x, \lambda_i) = \frac{(\theta_0 + \theta_1 \times z_i)^x e^{-(\theta_0 + \theta_1 \times z_i)}}{x!} \tag{2}$$

2 Task 1-generate zi

```
z<-rbern(10000, prob = 0.7)
```

3 Task 2-generate λ_i according to $\theta_0 + \theta_1 \times z_i$

```
theta0 <-0.5
theta1 <-0.5
i<-theta0+theta1*z
i
```

4 Task 3-generate $X \sim Poisson(\lambda_i)$

```
x<-rpois(10000, i)
```

5 Task 4-derive log-likelihood, gradient and hessian

- The likelihood function for Poisson distribution

$$L = \prod_i \frac{(\theta_0 + \theta_1 \times z_i)_i^x}{(e^(\theta_0 + \theta_1 \times z_i) - 1)x_i!} \tag{3}$$

- The log-likelihood function for Poisson distribution

$$\log L = \sum_i x_i \log(\theta_0 + \theta_1 \times z_i) - n(\theta_0 + \theta_1 \times z_i) - \sum_i \ln(x_i!) \quad (4)$$

- Gradient

$$\frac{\partial \log L}{\partial \theta_0} = -\frac{\sum_i x_i}{\theta_0 + \theta_1 \times z_i} - n \quad (5)$$

$$\frac{\partial \log L}{\partial \theta_1} = -\frac{\sum_i x_i z}{\theta_0 + \theta_1 \times z_i} - nz \quad (6)$$

- Hessian

$$H = \begin{bmatrix} -\frac{\sum_i x_i}{(\theta_0+\theta_1\times z_i)^2} & -\frac{\sum_i x_i z}{(\theta_0+\theta_1\times z_i)^2} \\ -\frac{\sum_i x_i z}{(\theta_0+\theta_1\times z_i)^2} & -\frac{\sum_i x_i z^2}{(\theta_0+\theta_1\times z_i)^2} \end{bmatrix}$$

6 Task 5-obtain MLE of $\theta = (\theta_0, \theta_1)$ using Newton-Raphson method

```
    ll <- function(theta, x, z) {

  theta0<- theta[1]
  theta1<- theta[2]
m <- length(x)*(theta0+theta1*z)-log(theta0+theta1*z)*sum(x)
m
}
## gradient
grad <- function(theta,x,z) {
g <- sum(x)/(theta0+theta1*z)-length(x)
g
}
## hessian
hess <- function(theta, x, z) {
  h <- -sum(x)/(theta0+theta1*z)^2
  h
}


res <- maxLik(logLik = ll, grad = grad, hess = hess, start=c(theta0=1, theta1=1) ,x = x,
summary(res)
```

7 Complete code to be uploaded

````
    ---
title: "R Notebook"
output: html_notebook
---
```{r}
library(Rlab)
library(maxLik)
```
```{r}

#generate zi
z<-rbern(10000, prob = 0.7)

```

```{r}
#generate lambda according to 0 + 1 ×zi
theta0 <-0.5
theta1 <-0.5
lambda<-theta0+theta1*z
lambda
```

```{r}

#generate X  Poisson(lambda )
x<-rpois(10000,lambda)
x
```



```{r}

ll <- function(theta, x, z) {

  theta0<- theta[1]
  theta1<- theta[2]
m <- length(x)*(theta0+theta1*z)-log(theta0+theta1*z)*sum(x)
m
}
## gradient
grad <- function(theta,x,z) {
g <- sum(x)/(theta0+theta1*z)-length(x)
````
````

```
g
}
## hessian
hess <- function(theta, x, z) {
  h <- -sum(x)/(theta0+theta1*z)^2
  h
}


res <- maxLik(logLik = ll, grad = grad, hess = hess, start=c(theta0=1, theta1=1) ,x = x,
summary(res)
```

‘‘‘