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Assignment 1

Instructions:

- This is an individual assignment. You are not allowed to discuss the problems with other students.
- Part of this assignment will be autograded by gradescope. You can use it as immediate feedback to improve your answers. You can resubmit as many times as you want.
- All your solution, code, analysis, graphs, explanations should be done in this same notebook.
- Please make sure to execute all the cells before you submit the notebook to the gradescope. You will not get points for the plots if they are not generated already.
- If you have questions regarding the assignment, you can ask for clarifications in Piazza. You should use the corresponding tag for this assignment.

When Submitting to GradeScope: Be sure to 1) Submit a `.ipynb` notebook to the `Assignment 1 - Code` section on Gradescope. 2) Submit a `pdf` version of the notebook to the `Assignment 1 - Report` entry and tag the answers.

Note: You can choose to submit responses in either English or French.

Before starting the assignment, make sure that you have downloaded all the tests related for the assignment and put them in the appropriate locations. If you run the next cell, we will set this all up automatically for you in a dataset called public, which will contain both the data and tests you use.

This assignment has only one question. In this question, you will learn:

1. To understand how to formalize a dose finding study as a multi-arm bandit problem.
2. To implement ϵ**-greedy**, **UCB**, **Boltzmann**, and **Gradient bandit** algorithms.
3. Understand the role of different hyper-parameters.

In [1]:
```
!pip install -q otter-grader
!git clone https://github.com/chandar-lab/INF8250ae-assignments-2023.git public
```

```
fatal: destination path 'public' already exists and is not an empty directory.
```

In [2]:
```
import otter
grader = otter.Notebook(colab=True, tests_dir='./public/a1/tests')
```

In [3]:
```
import numpy as np
from random import choice, randint
from scipy.stats import bernoulli
from typing import Sequence, Tuple
import matplotlib.pyplot as plt
%matplotlib inline
np.random.seed(8953)
import warnings
warnings.filterwarnings('ignore')
```

Q1: Dose Finding Study (90 points)

In the context of clinical trials, Phase I trials are the first stage of testing in human subjects. Their goal is to evaluate the safety (and feasibility) of the treatment and identify its side effects. The aim of a phase I dose-finding study is to determine the most appropriate dose level that should be used in further phases of the clinical trials. Traditionally, the focus is on determining the highest dose with acceptable toxicity called the Maximum Tolerated Dose (MTD).

A dose-finding study involves a number K of dose levels that have been chosen by physicians based on preliminary experiments (K is usually a number between 3 and 10). Denoting by p_k the (unknown) toxicity probability of dose k, the Maximum Tolerated Dose (MTD) is defined as the dose with a toxicity probability closest to a target:

$$k^* \in \underset{k \in \{1,...,K\}}{\operatorname{argmin}} |\theta - p_k| \tag{1}$$

where θ is the pre-specified targeted toxicity probability (typically between 0.2 and 0.35). A MTD identification algorithm proceeds sequentially: at round t a dose $D_t \in \{1, \ldots, K\}$ is selected and administered to a patient for whom a toxicity response is observed. A binary outcome X_t is revealed where $X_t = 1$ indicates that a harmful side-effect occurred and $X_t = 0$ indicates than no harmful side-effect occurred. We assume that X_t is drawn from a Bernoulli distribution with mean p_{D_t} and is independent from previous observations.

Hint: In this example, the reward definition is a bit different from the usual case. We would like to take the arm with minimum $|\theta - \hat{p}_k|$ where $\hat{p}_k$ is the estimated toxicity probability.

Q1.a: Define your Bandit class (5 points):

Most of the class has been written. Complete the pull method in such a way that:

1. Update both `num_dose_selected` and `num_toxic` arrays,

2. Compute and return the reward $-|\theta - \hat{p}_k|$ where $\hat{p}_k$ is the estimated toxicity probability of arm k.

In [4]:
```
class Bandit(object):

  def __init__(self,
               n_arm: int = 2,
               n_pulls: int = 2000,
               actual_toxicity_prob: list = [0.4, 0.6],
               theta: float = 0.3,
               ):
    self.n_arm = n_arm
    self.n_pulls = n_pulls
    self.actual_toxicity_prob = actual_toxicity_prob
    self.theta = theta
    # ------------------------------------------------
    self.a_star = np.argmax([-abs(theta - p) for p in actual_toxicity_prob])
    self.q_a_star = max([-abs(theta - p) for p in actual_toxicity_prob])
    # ------------------------------------------------
    self.init_bandit()

  def init_bandit(self):
    """
        Initialize the bandit
    """
    self.num_dose_selected = np.array([0]*self.n_arm) # number of times a dose is select
    self.num_toxic = np.array([0]*self.n_arm) # number of times a does found to be toxic
```

```python
    def pull(self, a_idx: int):
      """
      .inputs:
        a_idx: Index of action.
      .outputs:
        rew: reward value.
      """
      assert a_idx < self.n_arm, "invalid action index"
      # ----------------------------------------------
      self.num_dose_selected[a_idx] = self.num_dose_selected[a_idx] + 1
      is_toxic = bernoulli.rvs(self.actual_toxicity_prob[a_idx])
      self.num_toxic[a_idx] = self.num_toxic[a_idx] + is_toxic
      p_hat = self.num_toxic[a_idx]/self.num_dose_selected[a_idx]
      rew = -abs(self.theta - p_hat)
      # ----------------------------------------------
      return rew
```

In [5]:
```python
grader.check("q1a")
```

Out[5]:

`q1.a` passed! □

Dose finding study with three doses

Let's define a dose finding study with three doses ($K = 3$) where you need to choose from with `actual_toxicity_prob=[0.1, 0.35, 0.8]` and targeted toxicity probability is $\theta = 0.3$.

In [6]:
```python
#@title Problem definition
bandit = Bandit(n_arm=3, n_pulls=2000, actual_toxicity_prob=[0.1, 0.35, 0.8], theta=0.3)
```

Q1.b: ϵ-greedy for k-armed bandit and Optimistic initial values (25 points)

Q1.b1: ϵ-greedy algorithm implementation (5 points)

Implement the ϵ-greedy method.

In [7]:
```python
def eps_greedy(
    bandit: Bandit,
    eps: float,
    init_q: float = .0
    ) -> Tuple[list, list, list]:
  """
  .inputs:
    bandit: A bandit problem, instantiated from the above class.
    eps: The epsilon value.
    init_q: Initial estimation of each arm's value.
  .outputs:
    rew_record: The record of rewards at each timestep.
    avg_ret_record: The average of rewards up to step t, where t goes from 0 to n_pulls.
    we define `ret_T` = \sum^T_{t=0}{r_t}, `avg_ret_record` = ret_T / (1+T).
    tot_reg_record: The  regret up to step t, where t goes from 0 to n_pulls.
    opt_action_perc_record: Percentage of optimal arm selected.
  """
  # initialize q values
  q = np.array([init_q]*bandit.n_arm, dtype=float)

  ret = .0
  rew_record = []
  avg_ret_record = []
  tot_reg_record = []
  opt_action_perc_record = []
```

```python
  for t in range(bandit.n_pulls):
    # ---------------------------------------------
    explore = np.random.binomial(1, eps)
    a:int = None
    if explore:
      a = randint(0, len(q)-1)
    else:
      a = np.argmax(q)

    rew = bandit.pull(a)
    q[a] = q[a] + (rew - q[a]) / (bandit.num_dose_selected[a])
    rew_record.append(rew)
    ret += rew
    avg_ret_record.append(ret / (t+1))

    tot_reg_record.append((t+1) * bandit.q_a_star - ret)

    opt_action_perc = np.sum(bandit.num_dose_selected[bandit.a_star])/np.sum(bandit.num_
    opt_action_perc_record.append(opt_action_perc)
    # ---------------------------------------------

  return rew_record, avg_ret_record, tot_reg_record, opt_action_perc_record
```

In [8]:
```python
grader.check("q1b1")
```

Out[8]:

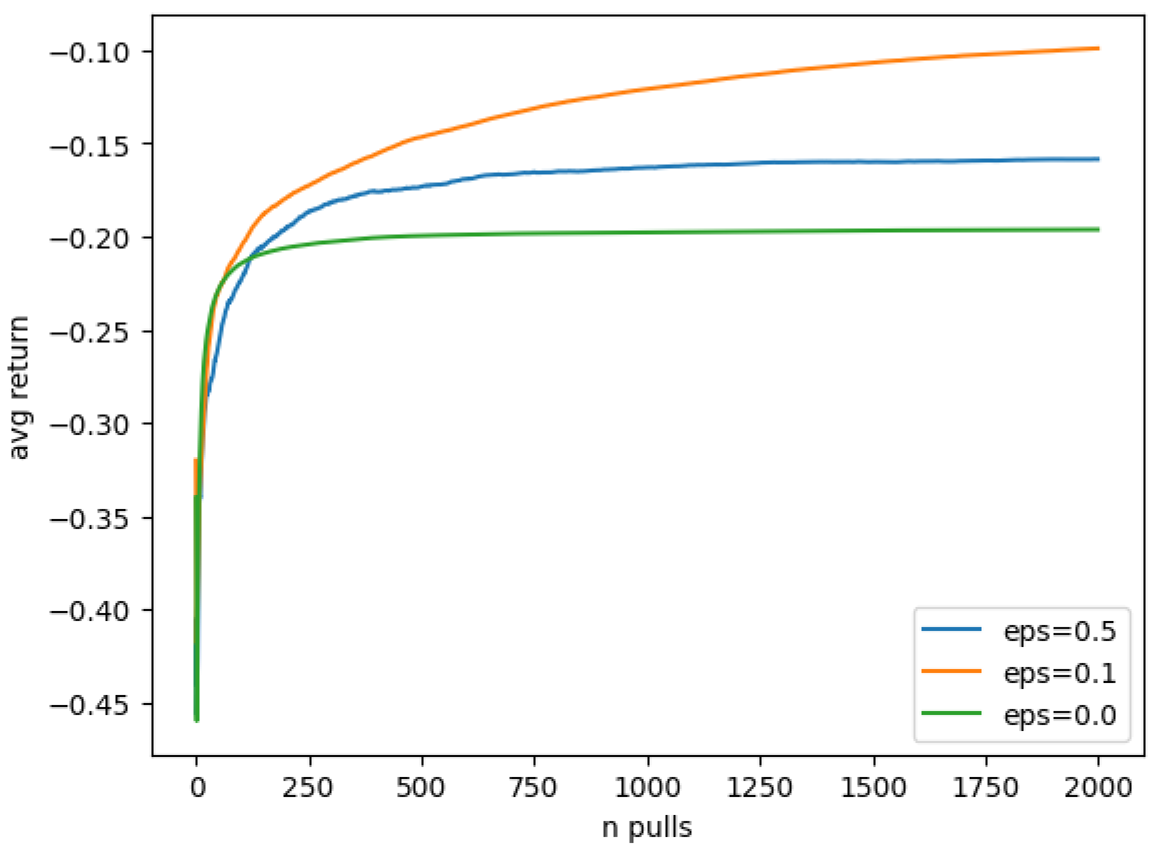
q1.b1 passed! ◻

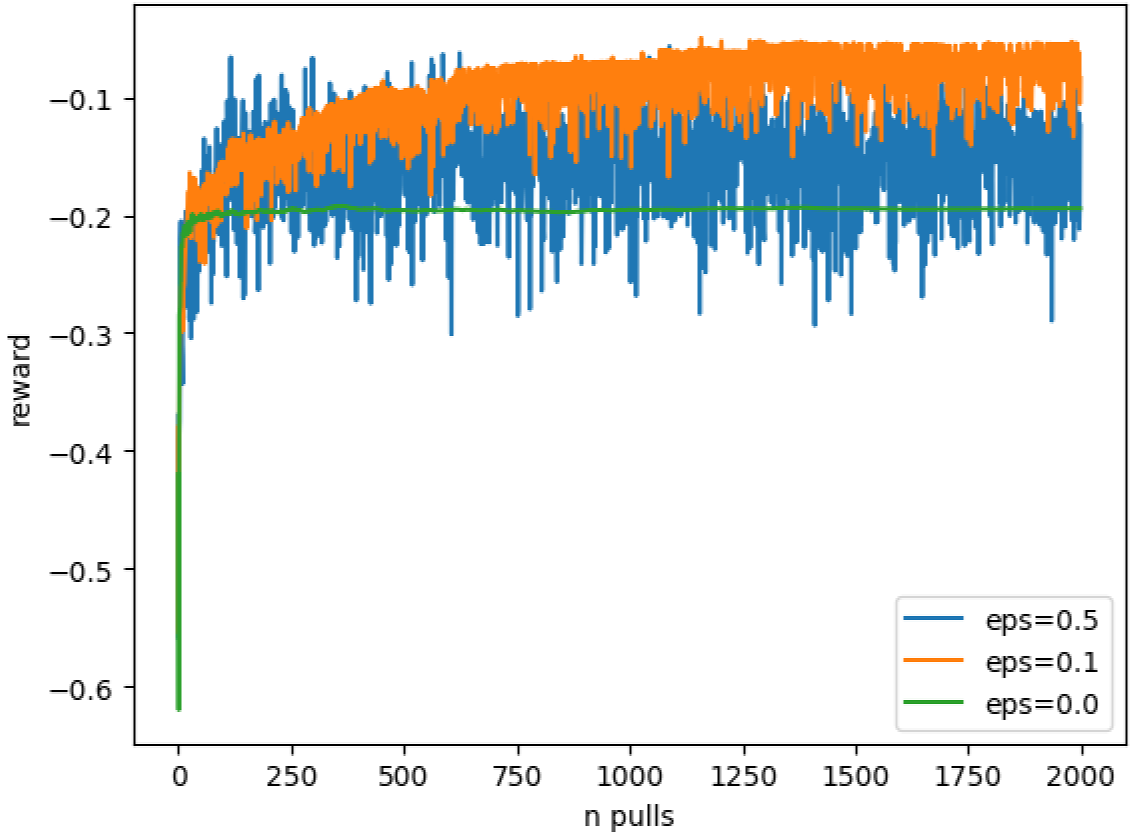
Q1.b2: Plotting the results (5 points)

Use the driver code provided to plot: (1) The average return, (2) The reward, (3) the total regret, and (4) the percentage of optimal action across the N=20 runs as a function of the number of pulls (2000 pulls for each run) for all three ϵ values of 0.5, 0.1, and 0.

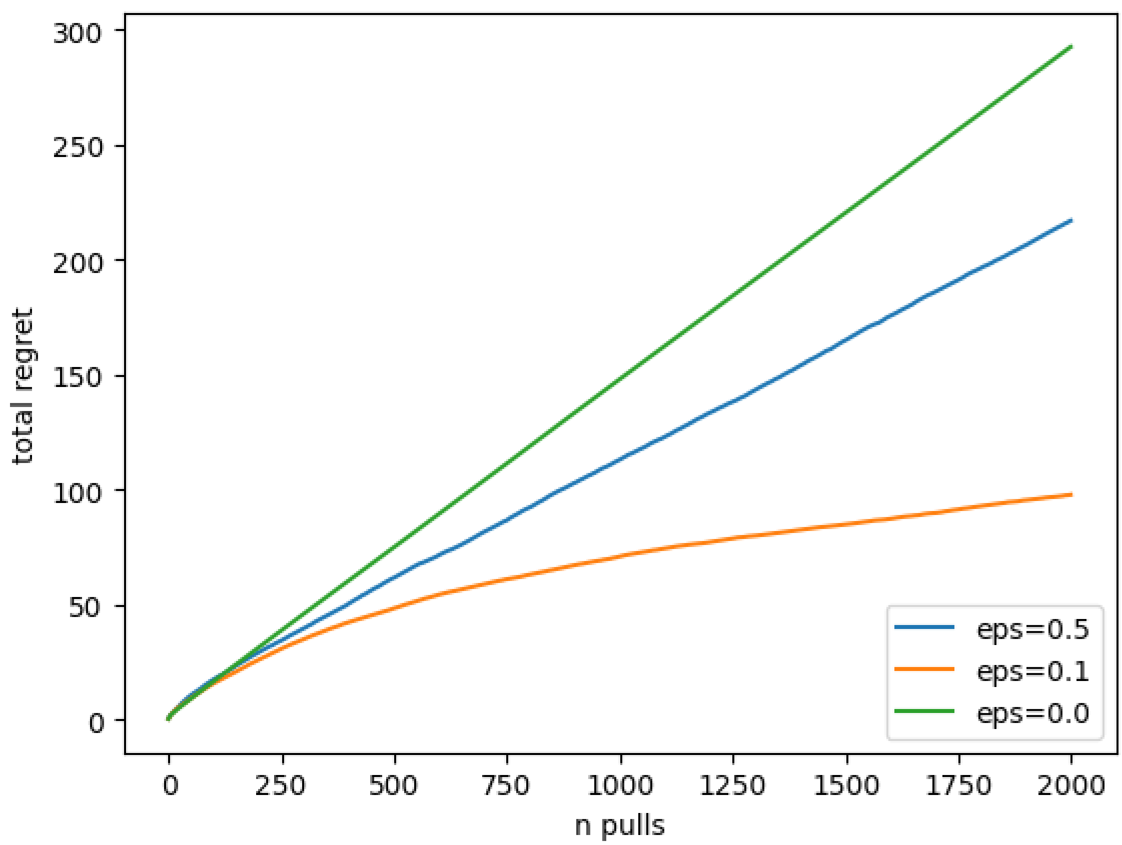
In [9]:
```python
import time
plt.figure(0)
plt.xlabel("n pulls")
plt.ylabel("avg return")
plt.figure(1)
plt.xlabel("n pulls")
plt.ylabel("reward")
plt.figure(2)
plt.xlabel("n pulls")
plt.ylabel("total regret")
plt.figure(3)
plt.xlabel("n pulls")
plt.ylabel("% optimal action")

N = 20
tot_reg_rec_best = 1e8

for eps in [0.5, 0.1, .0]:
  rew_rec = np.zeros(bandit.n_pulls)
  avg_ret_rec = np.zeros(bandit.n_pulls)
  tot_reg_rec = np.zeros(bandit.n_pulls)
  opt_act_rec = np.zeros(bandit.n_pulls)
  start_time = time.time()
  for n in range(N):
    bandit.init_bandit()
    rew_rec_n, avg_ret_rec_n, tot_reg_rec_n, opt_act_rec_n = eps_greedy(bandit, eps)
    rew_rec += np.array(rew_rec_n)
    avg_ret_rec += np.array(avg_ret_rec_n)
```

```
    tot_reg_rec += np.array(tot_reg_rec_n)
    opt_act_rec += np.array(opt_act_rec_n)

  end_time = time.time()
  # take the mean
  rew_rec /= N
  avg_ret_rec /= N
  tot_reg_rec /= N
  opt_act_rec /= N

  plt.figure(0)
  plt.plot(avg_ret_rec, label="eps={}".format(eps))
  plt.legend(loc="lower right")

  plt.figure(1)
  plt.plot(rew_rec[1:], label="eps={}".format(eps))
  plt.legend(loc="lower right")

  plt.figure(2)
  plt.plot(tot_reg_rec, label="eps={}".format(eps))
  plt.legend(loc="lower right")

  plt.figure(3)
  plt.plot(opt_act_rec, label="eps={}".format(eps))
  plt.legend(loc="lower right")


  if tot_reg_rec[-1] < tot_reg_rec_best:
        ep_greedy_dict = {
        'opt_act':opt_act_rec,
        'regret_list':tot_reg_rec,}
        tot_reg_rec_best = tot_reg_rec[-1]
```

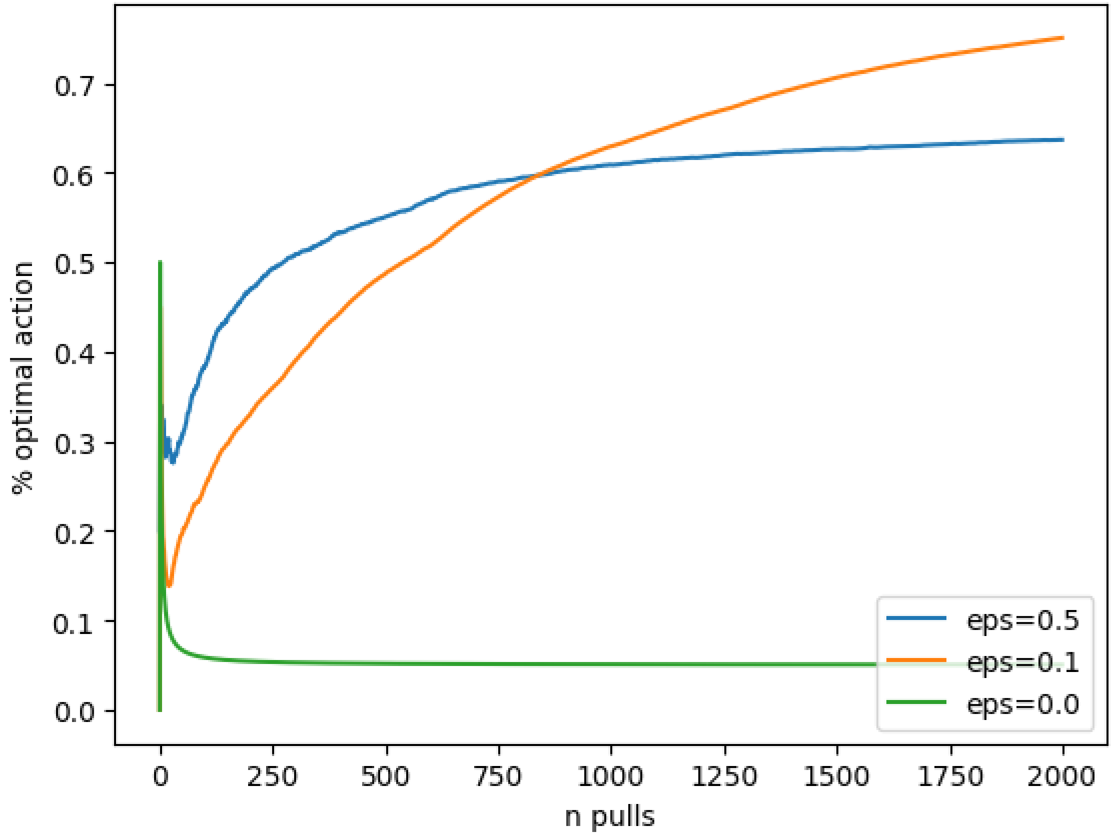




−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
reward
0
250
500
750
1000
1250
1500
1750
2000
n pulls
eps=0.5
eps=0.1
eps=0.0



300
250
200
150
100
50
0
total regret
0
250
500
750
1000
1250
1500
1750
2000
n pulls
eps=0.5
eps=0.1
eps=0.0



Q1.b3: Analysis (5 points)

Explain the results from the perspective of exploration and how different ϵ values affect the results.

With probability ϵ, the agent chooses to explore by taking a random action from its action space. This encourages the agent to try out new actions to discover potentially better strategies.

With probability 1-ϵ, the agent chooses to exploit by selecting the action that is currently estimated to be the best according to its learned value function. This exploits the agent's current knowledge to maximize its expected rewards.

When ϵ is 0.5, the agent is doing a fifty fifty chance on exploration over exploitation. Thus, it frequently takes random actions, which may help it discover new and potentially better strategies. However, it leads to inefficient and erratic behavior, especially when the agent already has a good estimate of the optimal policy. The agent may waste time exploring suboptimal actions when it should be exploiting its current knowledge. It performs less good then the ϵ = 0.1.

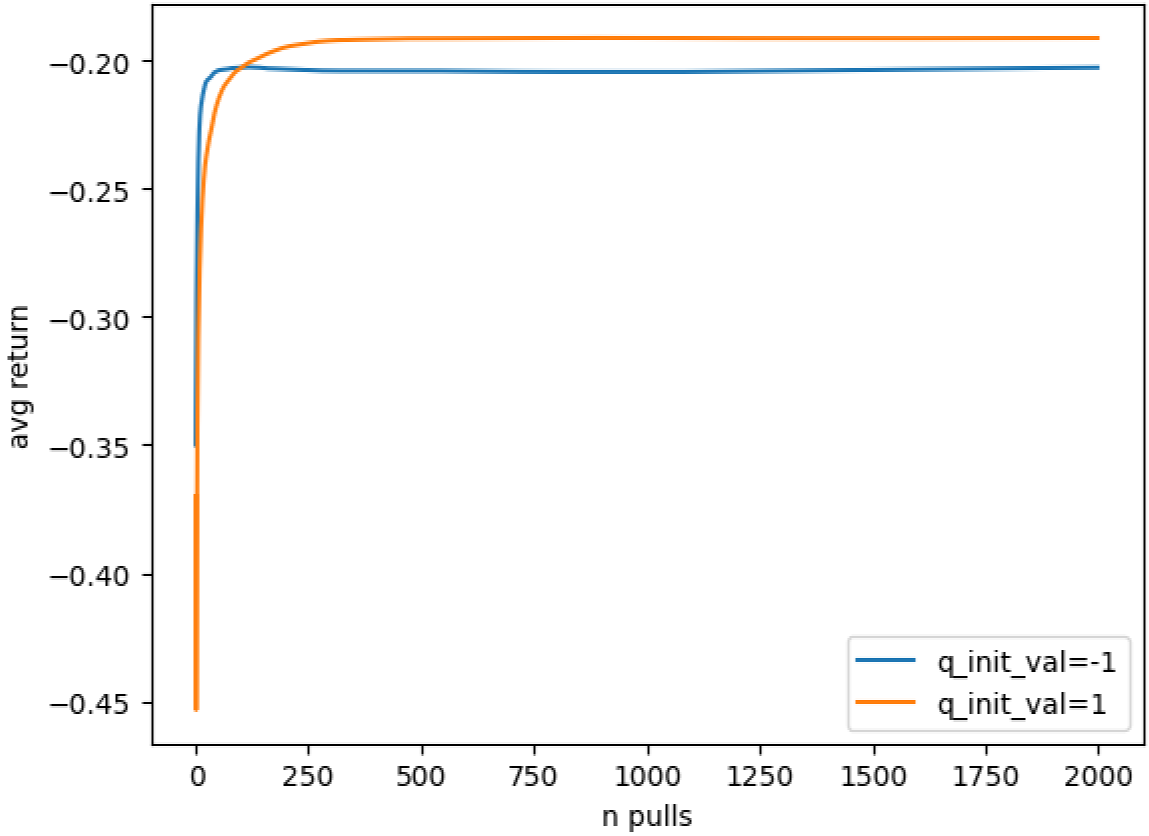
When ϵ is set to 0, the agent strictly follows the policy that it currently believes to be the best. This means it exploits its current knowledge without exploring any further. While ϵ=0.1 ensures that the agent consistently chooses actions it believes to be optimal, it leads to suboptimal results in the long run if the agent's initial estimates are inaccurate like our case.

When, ϵ is set to 0.1, striking a balance between exploration and exploitation. This allows the agent to learn while also making decisions based on its current best estimate. This is the best choice for our case.

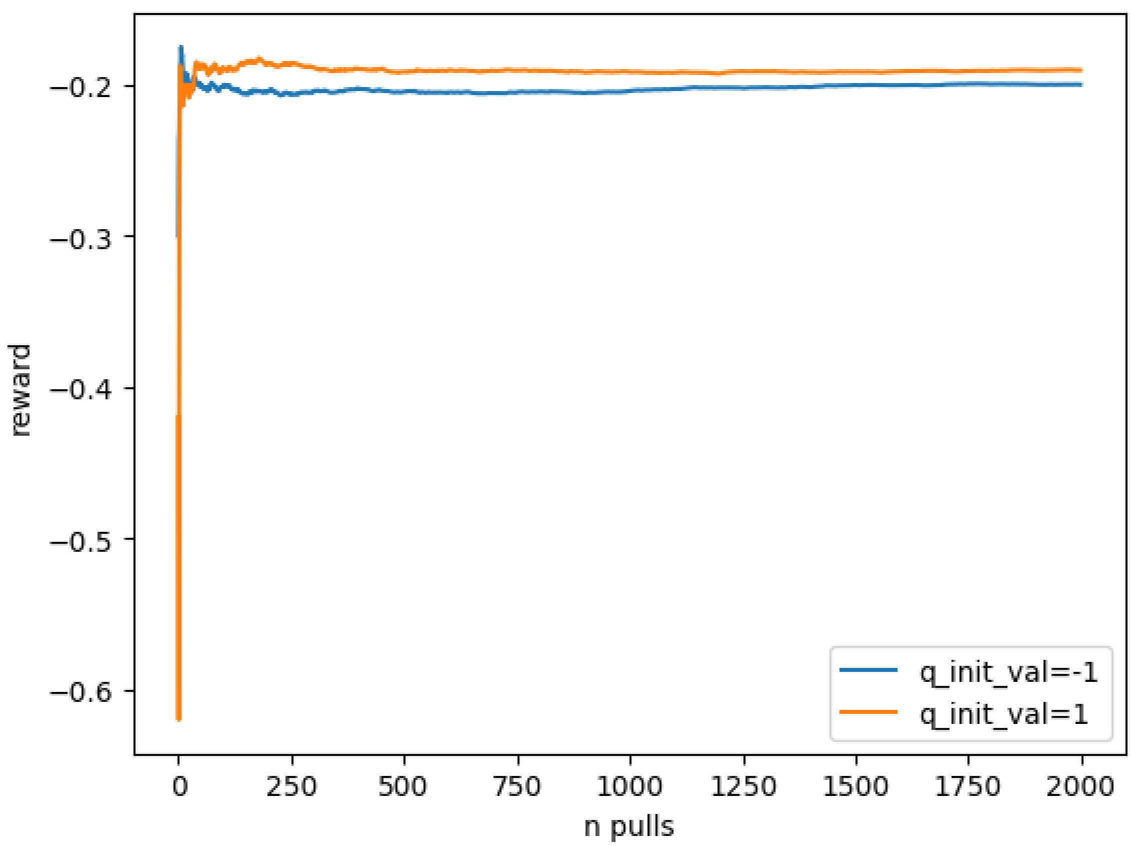
Q1.b4: Optimistic Initial Values (5 points)

We want to run the optimistic initial value method on the same problem described above for the initial q values of -1 and +1 for all arms. Compare its performance, measured by the average reward across N=20 runs as a function of the number of pulls, with the non-optimistic setting with initial q values of 0 for all arms. For both optimistic and non-optimistic settings, ϵ=0.

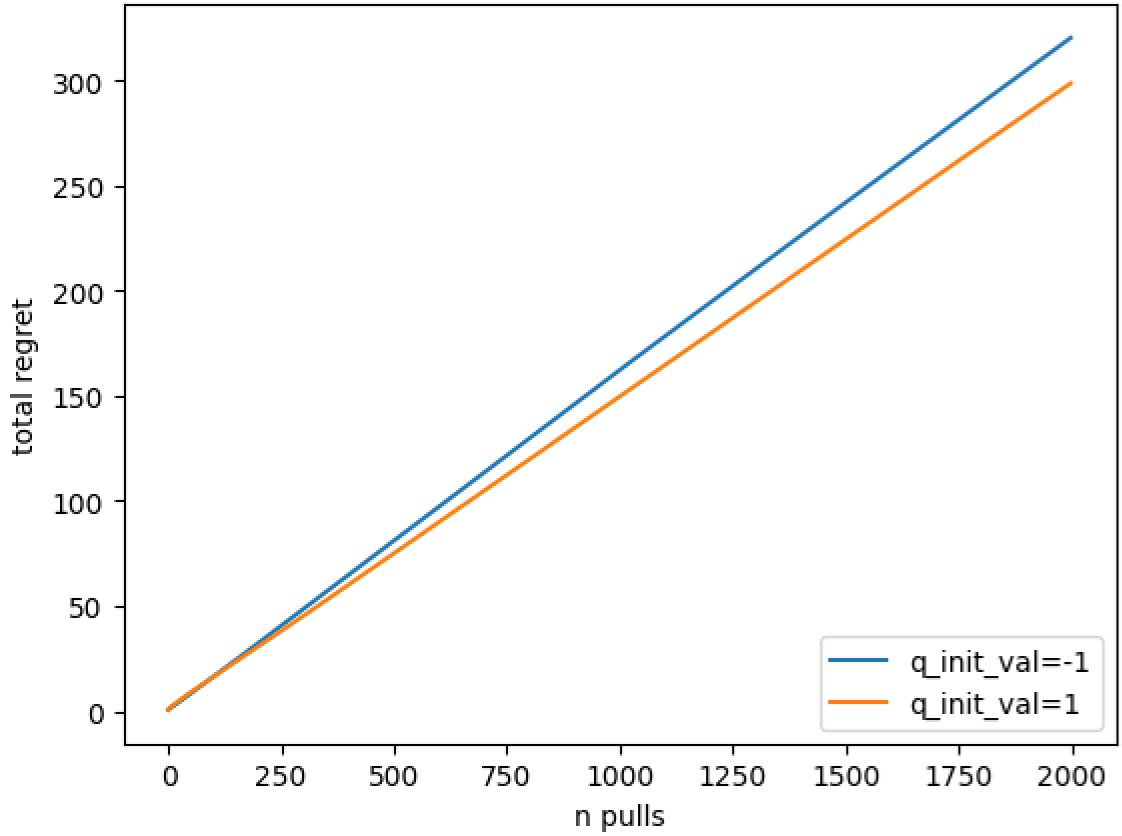
In [10]:
```
plt.figure(4)
plt.xlabel("n pulls")
plt.ylabel("avg return")

plt.figure(5)
plt.xlabel("n pulls")
plt.ylabel("reward")

plt.figure(6)
plt.xlabel("n pulls")
plt.ylabel("total regret")

N = 20
for init_q in [-1, 1]:
  rew_rec = np.zeros(bandit.n_pulls)
  avg_ret_rec = np.zeros(bandit.n_pulls)
  for n in range(N):
    bandit.init_bandit()
    rew_rec_n, avg_ret_rec_n, tot_reg_rec_n, opt_act_rec_n = eps_greedy(bandit, eps=0.0,

    rew_rec += np.array(rew_rec_n)
    avg_ret_rec += np.array(avg_ret_rec_n)
    tot_reg_rec += np.array(tot_reg_rec_n)

  avg_ret_rec /= N
  rew_rec /= N
  tot_reg_rec /= N
  plt.figure(4)
  plt.plot(avg_ret_rec[1:], label="q_init_val={}".format(init_q))
  plt.legend(loc="lower right")

  plt.figure(5)
  plt.plot(rew_rec[1:], label="q_init_val={}".format(init_q))
  plt.legend(loc="lower right")

  plt.figure(6)
  plt.plot(tot_reg_rec[1:], label="q_init_val={}".format(init_q))
  plt.legend(loc="lower right")
```



avg return
n pulls
q_init_val=-1
q_init_val=1



reward
n pulls
q_init_val=-1
q_init_val=1



Q1.b5: Analysis (5 points)

Explain how initial q values affect the exploration and the performance.

When the initial Q-values is set to Q=1, all initial Q-values are set to a high positive value, the agent starts with an optimistic view of all actions. This means it initially believes that every action can lead to high rewards. The impact on exploration is that high initial Q-values encourage exploration, as the agent is inclined to try various actions in different states in pursuit of the assumed high rewards. The impact on performance is that initially, the agent may perform better than if it had pessimistic initial Q-values, as it is more willing to explore and try different actions. However, if the environment does not align with the initial optimistic estimates like in this case, the agent may experience initial episodes of poor performance as it tries suboptimal actions.

When the initial Q-values set to Q=-1 the agent starts with a pessimistic view of all actions. It assumes that taking any action will result in low rewards. The impact on exploration is that low initial Q-values discourage exploration, as the agent is less motivated to try actions that it believes will yield poor outcomes. The impact on performance is that initially, the agent may be risk-averse and less likely to try new actions. This can result in slower learning and potentially suboptimal performance which is the case observed.

Q1.c: Upper-Confidence-Bound action selection (15 points)

Q1.c1: UCB algorithm implementation (5 points)

Implement the UCB algorithm on the same MAB problem as above.

In [11]:
```
def ucb(
    bandit: Bandit,
    c: float,
    init_q: float = .0
    ) -> Tuple[list, list, list]:
```

```
    """
    .inputs:
      bandit: A bandit problem, instantiated from the above class.
      c: The additional term coefficient.
      init_q: Initial estimation of each arm's value.
    .outputs:
      rew_record: The record of rewards at each timestep.
      avg_ret_record: The average summation of rewards up to step t, where t goes from 0 t
      we define `ret_T` = \sum^T_{t=0}{r_t}, `avg_ret_record` = ret_T / (1+T).
      tot_reg_record: The  regret up to step t, where t goes from 0 to n_pulls.
      opt_action_perc_record: Percentage of optimal arm selected.
    """
    # init q values (the estimates)
    q = np.array([init_q]*bandit.n_arm, dtype=float)

    ret = .0
    rew_record = []
    avg_ret_record = []
    tot_reg_record = []
    opt_action_perc_record = []

    for t in range(bandit.n_pulls):
      # Assuming to take the first arm always when there is no exploration
      # ----------------------------------------------
      ucb = []
      for a in range(bandit.n_arm):
          if bandit.num_dose_selected[a] == 0:
              ucb.append(float('inf'))
          else:
              ucb.append(q[a] + c * np.sqrt(np.log(t+1) / (bandit.num_dose_selected[a])))

      a = np.argmax(ucb)

      rew = bandit.pull(a)
      q[a] = q[a] + (rew - q[a]) / (bandit.num_dose_selected[a])
      rew_record.append(rew)
      ret += rew
      avg_ret_record.append(ret / (t+1))

      tot_reg_record.append((t+1) * bandit.q_a_star - ret)

      opt_action_perc = np.sum(bandit.num_dose_selected[bandit.a_star])/np.sum(bandit.num_
      opt_action_perc_record.append(opt_action_perc)
      # ----------------------------------------------

    return rew_record, avg_ret_record, tot_reg_record, opt_action_perc_record
```

In [12]:
```
grader.check("q1c1")
```

Out[12]:

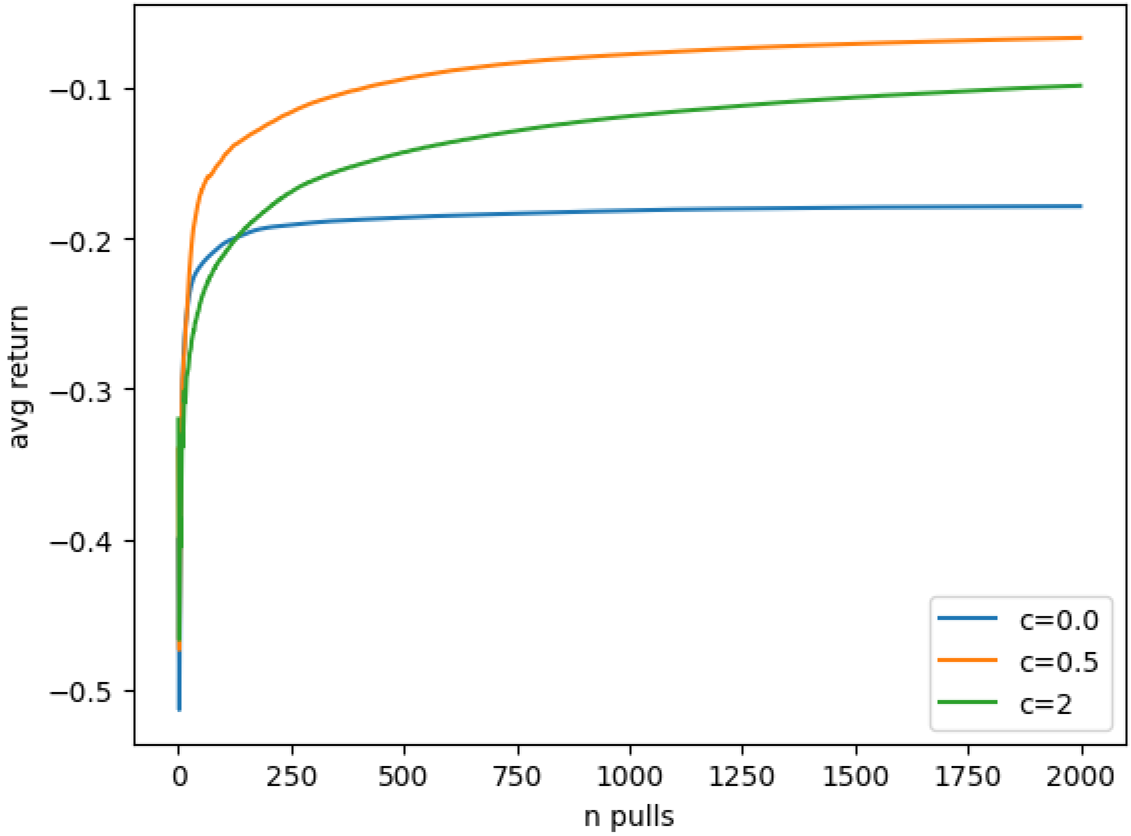
q1.c1 passed! ▯

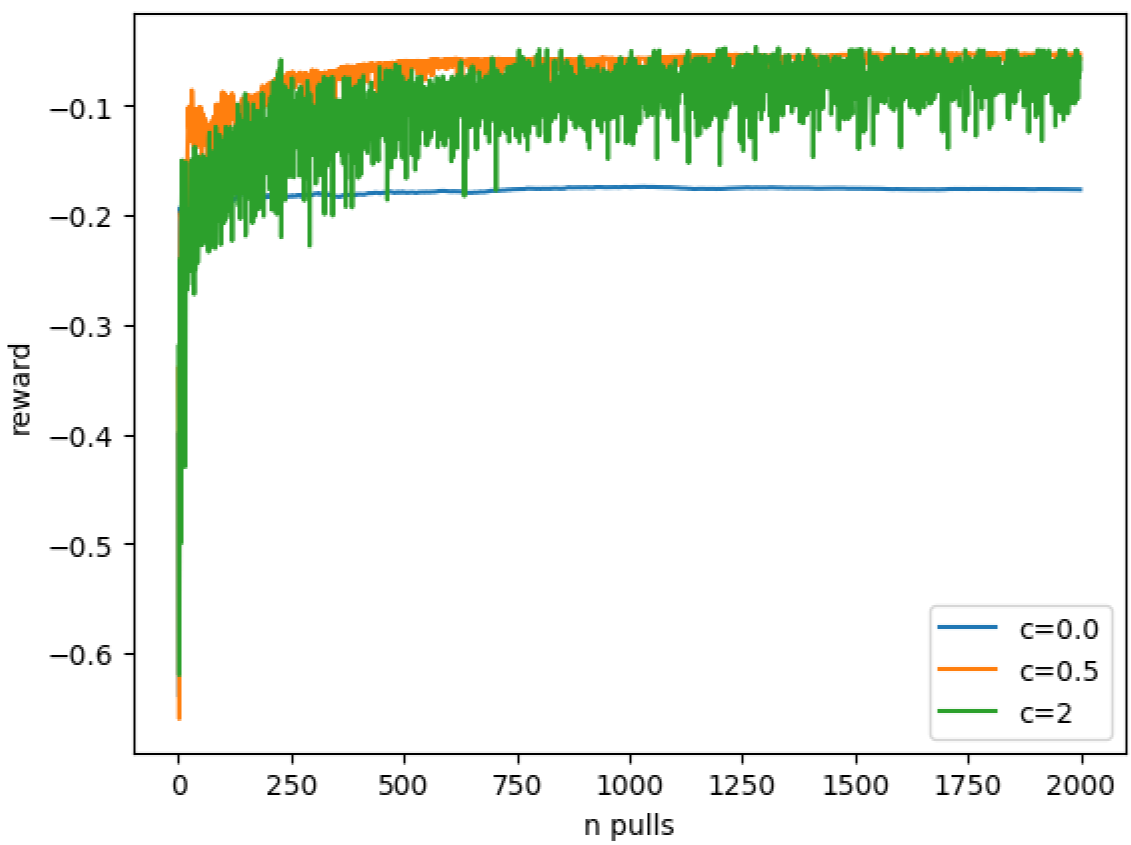
Q1.c2: Plotting the results (5 points)

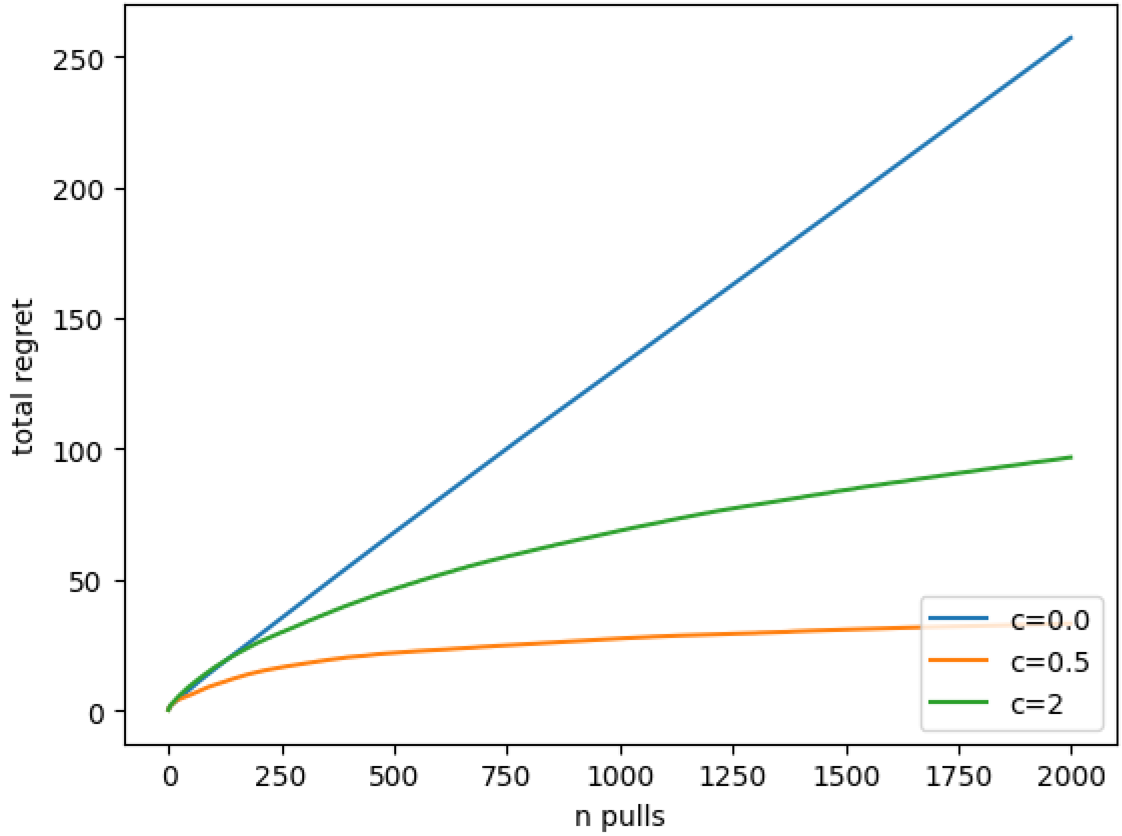
Use the driver code provided to plot: (1) The average return, (2) The reward, (3) the total regret, and (4) the percentage of optimal action across the N=20 runs as a function of the number of pulls (2000 pulls for each run) for three values of c=0, 0.5, and 2.0.

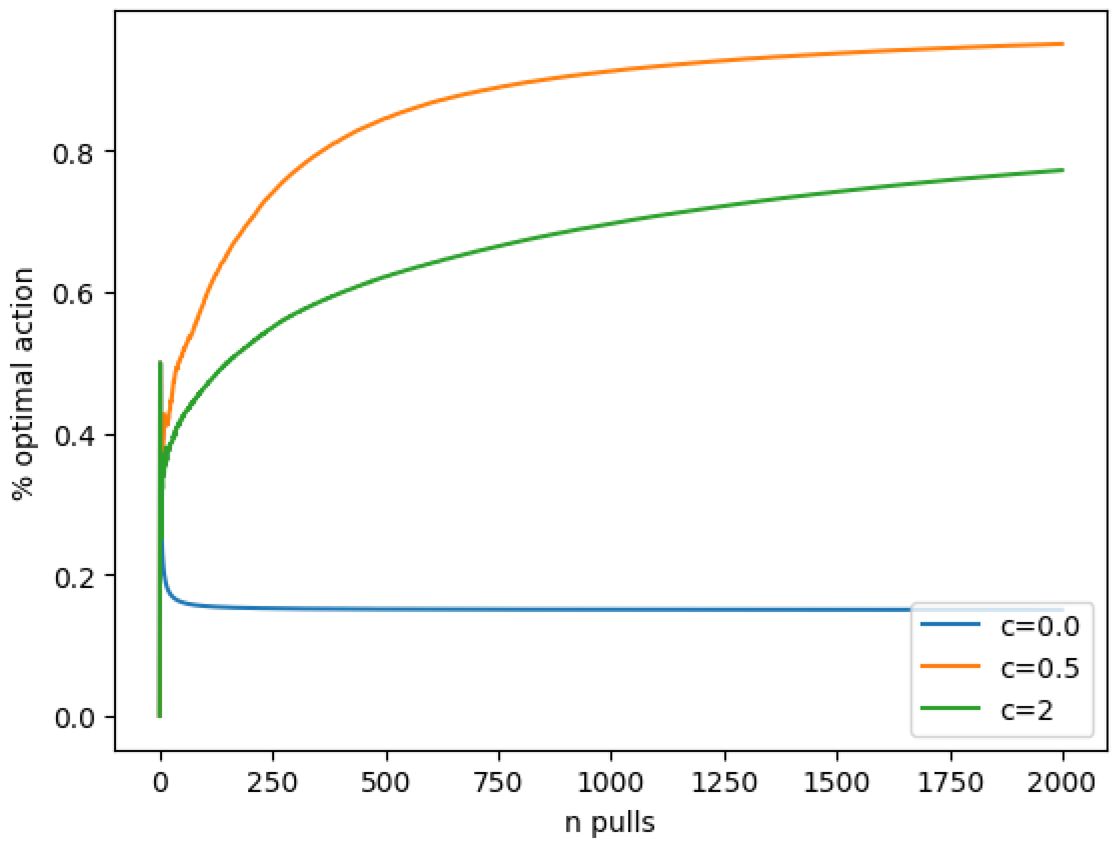
In [13]:
```
plt.figure(7)
plt.xlabel("n pulls")
plt.ylabel("avg return")
plt.figure(8)
plt.xlabel("n pulls")
```

```
plt.ylabel("reward")
plt.figure(9)
plt.xlabel("n pulls")
plt.ylabel("total regret")
plt.figure(10)
plt.xlabel("n pulls")
plt.ylabel("% optimal action")

N = 20
tot_reg_rec_best = 1e8
for c in [.0, 0.5, 2]:
  rew_rec = np.zeros(bandit.n_pulls)
  avg_ret_rec = np.zeros(bandit.n_pulls)
  tot_reg_rec = np.zeros(bandit.n_pulls)
  opt_act_rec = np.zeros(bandit.n_pulls)

  for n in range(N):
    bandit.init_bandit()
    rew_rec_n, avg_ret_rec_n, tot_reg_rec_n, opt_act_rec_n = ucb(bandit, c)
    rew_rec += np.array(rew_rec_n)
    avg_ret_rec += np.array(avg_ret_rec_n)
    tot_reg_rec += np.array(tot_reg_rec_n)
    opt_act_rec += np.array(opt_act_rec_n)

  # take the mean
  rew_rec /= N
  avg_ret_rec /= N
  tot_reg_rec /= N
  opt_act_rec /= N

  plt.figure(7)
  plt.plot(avg_ret_rec, label="c={}".format(c))
  plt.legend(loc="lower right")

  plt.figure(8)
  plt.plot(rew_rec, label="c={}".format(c))
  plt.legend(loc="lower right")

  plt.figure(9)
  plt.plot(tot_reg_rec, label="c={}".format(c))
  plt.legend(loc="lower right")

  plt.figure(10)
  plt.plot(opt_act_rec, label="c={}".format(c))
  plt.legend(loc="lower right")

  if tot_reg_rec[-1] < tot_reg_rec_best:
        ucb_dict = {
        'opt_act':opt_act_rec,
        'regret_list':tot_reg_rec,}
        tot_reg_rec_best = tot_reg_rec[-1]
```


avg return
n pulls
c=0.0
c=0.5
c=2


reward
n pulls
c=0.0
c=0.5
c=2





Q1.c3: Analysis (5 points)

Explain the results from the perspective of exploration and how different c values affect the results.

When c is set to 0, the UCB algorithm essentially becomes a purely exploitation-driven strategy. As a result, it will not explore any arms that it hasn't already deemed as the best, which leads to suboptimal performance, especially if initial estimates are inaccurate or if there are arms with higher rewards that the algorithm has not yet explored. It explains the suboptimal performance of the algorithm in our case. It is the worst choice for our case.

With *c* set to 0.5, the UCB algorithm achieves a balance between exploration and exploitation. The algorithm assigns exploration bonuses to actions, but they are not overly aggressive. It is willing to explore arms that have some uncertainty in their estimated rewards but still tends to exploit the arms it believes to be better. It is the one that performed the best in our case.

When *c* is set to 2, the UCB algorithm prioritizes exploration significantly. The exploration bonuses assigned to actions are substantial, encouraging the algorithm to explore arms even if they have been sampled less frequently or have higher uncertainty in their reward estimates. It explains the suboptimal performance of the algorithm in our case.

Q1.d: Boltzmann algorithm (20 points)

Q1.d1: Boltzmann policy implementation (5 points)

Implement a Boltzmann policy that gets an array and temprature value (τ) and returns an index sampled from the Boltzmann policy.

In [14]:
```
def boltzmann_policy(x, tau):
    """ Returns softmax probabilities with temperature tau
        Input:  x -- 1-dimensional array
        Output: idx -- chosen index
    """
    # ---------------------------------------------
    x = np.array(x)
    exp_x = np.exp(x/tau)
    p = exp_x / np.sum(exp_x)
    idx = np.random.choice(len(x), p=p)
    # ---------------------------------------------
    return idx
```

In [15]:
```
grader.check("q1d1")
```

Out[15]:

q1.d1 passed! □

Q1.d2: Boltzmann algorithm implementation (5 points)

Evaluate the Boltzmann algorithm on the same MAB problem as above, for three values of the parameters τ: 0.01, 0.1, and 1. Use the driver code provided to plot their performances across N=20 runs as a function of the number of pulls.

Note: You can use action-value estimates for the Boltzmann distribution.

In [16]:
```
def boltzmann(
    bandit: Bandit,
    tau: float = 0.1,
    init_q: float = .0
    ) -> Tuple[list, list, list]:
  """
  .inputs:
    bandit: A bandit problem, instantiated from the above class.
    tau: The additional term coefficient.
    init_q: Initial estimation of each arm's value.
  .outputs:
```

```
        rew_record: The record of rewards at each timestep.
        avg_ret_record: The average summation of rewards up to step t, where t goes from 0 t
        we define `ret_T` = \sum^T_{t=0}{r_t}, `avg_ret_record` = ret_T / (1+T).
        tot_reg_record: The  regret up to step t, where t goes from 0 to n_pulls.
        opt_action_perc_record: Percentage of optimal arm selected.
    """
    # init q values (the estimates)
    q = np.array([init_q]*bandit.n_arm, dtype=float)

    ret = .0
    rew_record = []
    avg_ret_record = []
    tot_reg_record = []
    opt_action_perc_record = []

    for t in range(bandit.n_pulls):
        # ----------------------------------------------
        a = boltzmann_policy(q, tau)

        rew = bandit.pull(a)
        q[a] = q[a] + (rew - q[a]) / (bandit.num_dose_selected[a])
        rew_record.append(rew)
        ret += rew
        avg_ret_record.append(ret / (t+1))

        tot_reg_record.append((t+1) * bandit.q_a_star - ret)

        opt_action_perc = np.sum(bandit.num_dose_selected[bandit.a_star])/np.sum(bandit.num_
        opt_action_perc_record.append(opt_action_perc)
        # ----------------------------------------------

    return rew_record, avg_ret_record, tot_reg_record, opt_action_perc_record
```

In [17]:

```
grader.check("q1d2")
```

Out[17]:

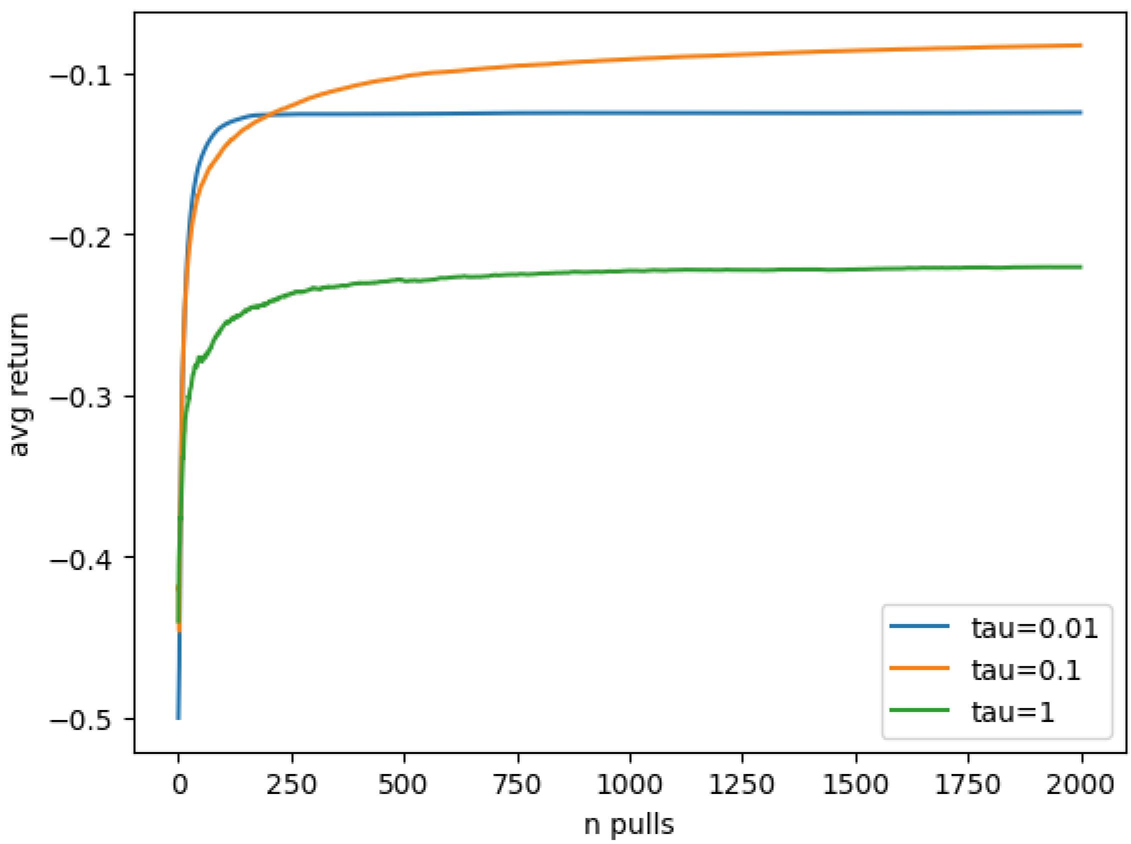
q1.d2 passed!

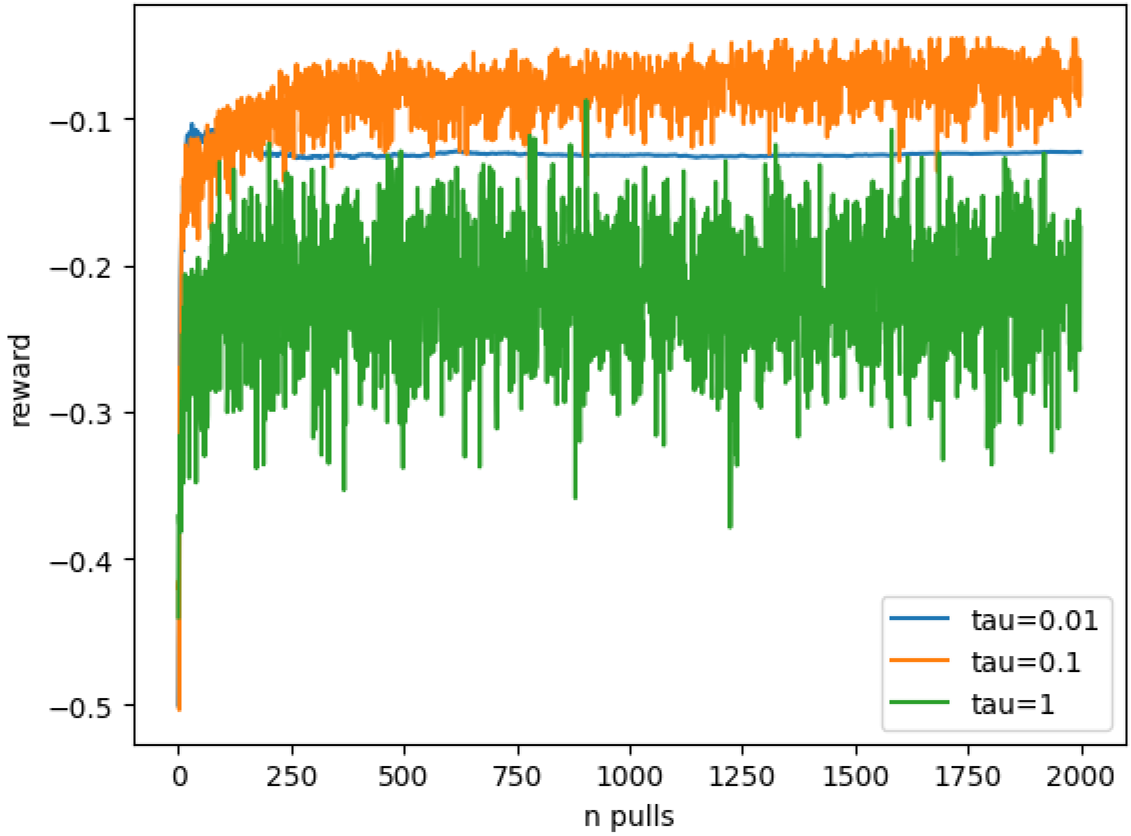
Q1.d3: Plotting the results (5 points)

In [18]:

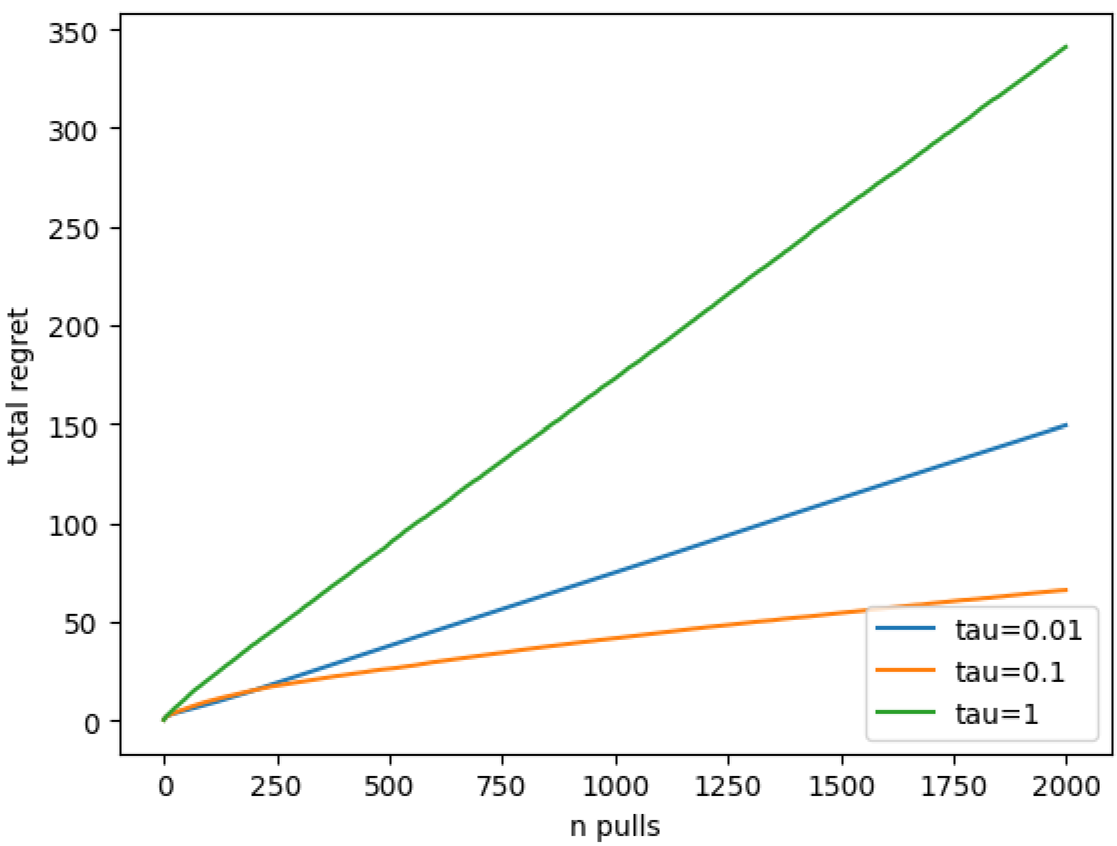
```
plt.figure(11)
plt.xlabel("n pulls")
plt.ylabel("avg return")
plt.figure(12)
plt.xlabel("n pulls")
plt.ylabel("reward")
plt.figure(13)
plt.xlabel("n pulls")
plt.ylabel("total regret")
plt.figure(14)
plt.xlabel("n pulls")
plt.ylabel("% optimal action")

N = 20
tot_reg_rec_best = 1e8
for tau in [0.01, 0.1, 1]:
    rew_rec = np.zeros(bandit.n_pulls)
    avg_ret_rec = np.zeros(bandit.n_pulls)
    tot_reg_rec = np.zeros(bandit.n_pulls)
    opt_act_rec = np.zeros(bandit.n_pulls)

    for n in range(N):
        bandit.init_bandit()
        rew_rec_n, avg_ret_rec_n, tot_reg_rec_n, opt_act_rec_n = boltzmann(bandit, tau=tau)
```

```
    rew_rec += np.array(rew_rec_n)
    avg_ret_rec += np.array(avg_ret_rec_n)
    tot_reg_rec += np.array(tot_reg_rec_n)
    opt_act_rec += np.array(opt_act_rec_n)

  # take the mean
  rew_rec /= N
  avg_ret_rec /= N
  tot_reg_rec /= N
  opt_act_rec /= N

  plt.figure(11)
  plt.plot(avg_ret_rec, label="tau={}".format(tau))
  plt.legend(loc="lower right")

  plt.figure(12)
  plt.plot(rew_rec, label="tau={}".format(tau))
  plt.legend(loc="lower right")

  plt.figure(13)
  plt.plot(tot_reg_rec, label="tau={}".format(tau))
  plt.legend(loc="lower right")

  plt.figure(14)
  plt.plot(opt_act_rec, label="tau={}".format(tau))
  plt.legend(loc="lower right")

  if tot_reg_rec[-1] < tot_reg_rec_best:
        boltzmann_dict = {
        'opt_act':opt_act_rec,
        'regret_list':tot_reg_rec,}
        tot_reg_rec_best = tot_reg_rec[-1]
```

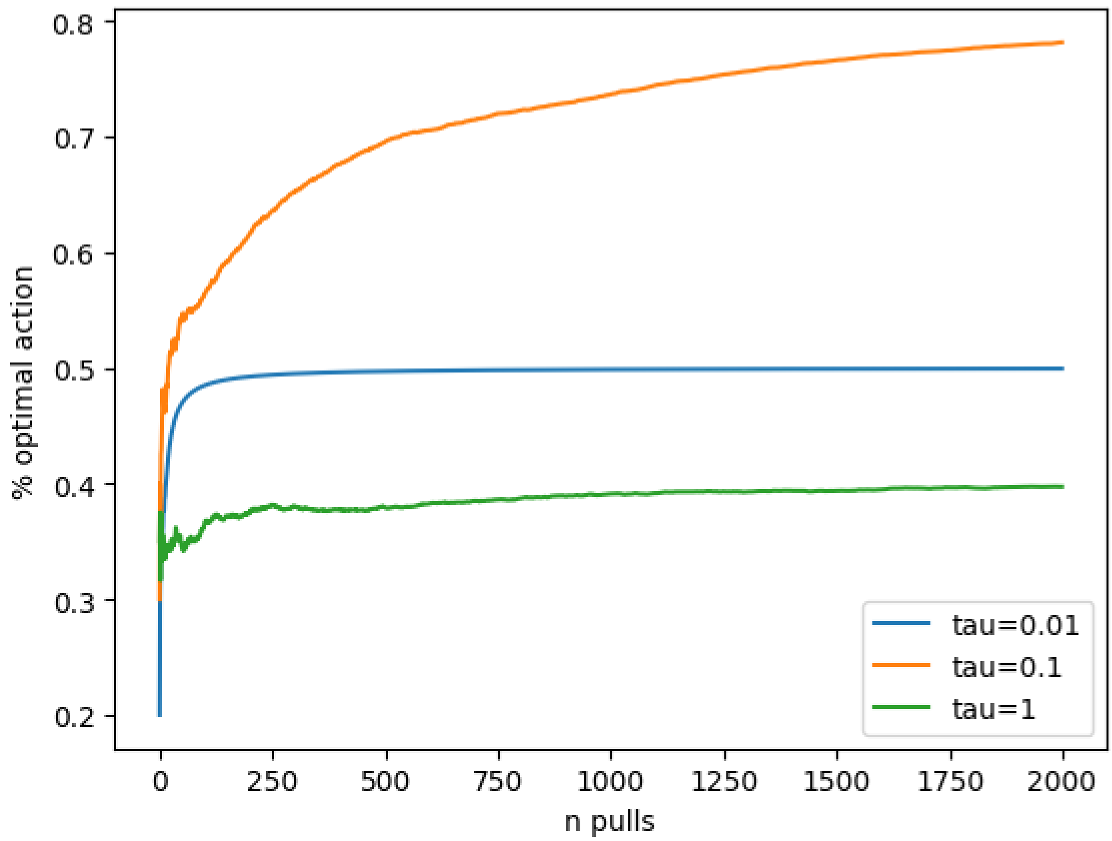




reward
n pulls
−0.1
−0.2
−0.3
−0.4
−0.5
0
250
500
750
1000
1250
1500
1750
2000
tau=0.01
tau=0.1
tau=1



total regret
n pulls
350
300
250
200
150
100
50
0
0
250
500
750
1000
1250
1500
1750
2000
tau=0.01
tau=0.1
tau=1



Q1.d4: Analysis (5 points)

Explain the role of τ paramtere on the results.

When $\tau = 0.01$, a very low value, the Boltzmann exploration strategy becomes highly deterministic. Action selection is primarily driven by the estimated action values, with very little randomness. The action with the highest estimated value is almost always chosen. This leads to strong exploitation, where the agent focuses almost exclusively on the action it currently believes to have the highest expected reward. It performs as the middle one in our case.

When $\tau = 0.1$, an intermediate value, the Boltzmann strategy achieves a balance between exploration and exploitation. Action selection is influenced by both estimated action values and randomness, with actions having higher estimated values being more likely to be chosen, but there is still some variability. This level of exploration allows the agent to occasionally explore other actions, even if it believes one action is superior. It strikes a balance between refining its estimates and maximizing. It performs as the best one in our case.

When $\tau = 1$, a high value, the Boltzmann strategy encourages extensive exploration. Action selection probabilities are much more evenly distributed, and actions with lower estimated values have a higher chance of being selected. This leads to significant exploration, as the agent is willing to try out different actions, even if it has a good estimate of the optimal action. It performs as the worst one in our case.

Q1.f: Gradient Bandits Algorithm (15 points)

Q1.f1: GB implementation (5 points)

Follow the lecture notes to implement the Gradient Bandits algorithm with and without the baseline.

In [19]:

```
def softmax(x):

    return np.exp(x) / np.sum(np.exp(x), axis=0)
```

In [20]:
```python
def gradient_bandit(
    bandit: Bandit,
    alpha: float,
    use_baseline: bool = True,
    ) -> Tuple[list, list, list]:
  """
  .inputs:
    bandit: A bandit problem, instantiated from the above class.
    alpha: The learning rate.
    use_baseline: Whether or not use avg return as baseline.
  .outputs:
    rew_record: The record of rewards at each timestep.
    avg_ret_record: The average summation of rewards up to step t, where t goes from 0 t
    we define `ret_T` = \sum^T_{t=0}{r_t}, `avg_ret_record` = ret_T / (1+T).
    tot_reg_record: The  regret up to step t, where t goes from 0 to n_pulls.
    opt_action_perc_record: Percentage of optimal arm selected.
  """
  # init h (the logits)
  h = np.array([0]*bandit.n_arm, dtype=float)

  ret = .0
  r_bar_t = 0
  rew_record = []
  avg_ret_record = []
  tot_reg_record = []
  opt_action_perc_record = []


  for t in range(bandit.n_pulls):
    # ----------------------------------------------
    p = softmax(h)
    a = np.random.choice(bandit.n_arm, p=p)

    rew = bandit.pull(a)

    if use_baseline:
        r_bar_t = r_bar_t + (rew - r_bar_t) / (t+1)
        for i in range(len(h)):
            if i == a:
                h[a] = h[a] + alpha * (rew - r_bar_t) * p[a] * (1 - p[a])
            else:
                h[i] = h[i] - alpha * (rew - r_bar_t) * p[a] * p[i]
    else:
        for i in range(len(h)):
            if i == a:
                h[a] = h[a] + alpha * rew * p[a] * (1 - p[a])
            else:
                h[i] = h[i] - alpha * rew * p[a] * p[i]

    rew_record.append(rew)
    ret += rew
    avg_ret_record.append(ret / (t+1))

    tot_reg_record.append((t+1) * bandit.q_a_star - ret)

    opt_action_perc = np.sum(bandit.num_dose_selected[bandit.a_star])/np.sum(bandit.num_
    opt_action_perc_record.append(opt_action_perc)
    # ----------------------------------------------

  return rew_record, avg_ret_record, tot_reg_record, opt_action_perc_record
```

In [21]:
```python
grader.check("q1f1")
```

Out[21]:

q1.f1 passed! □

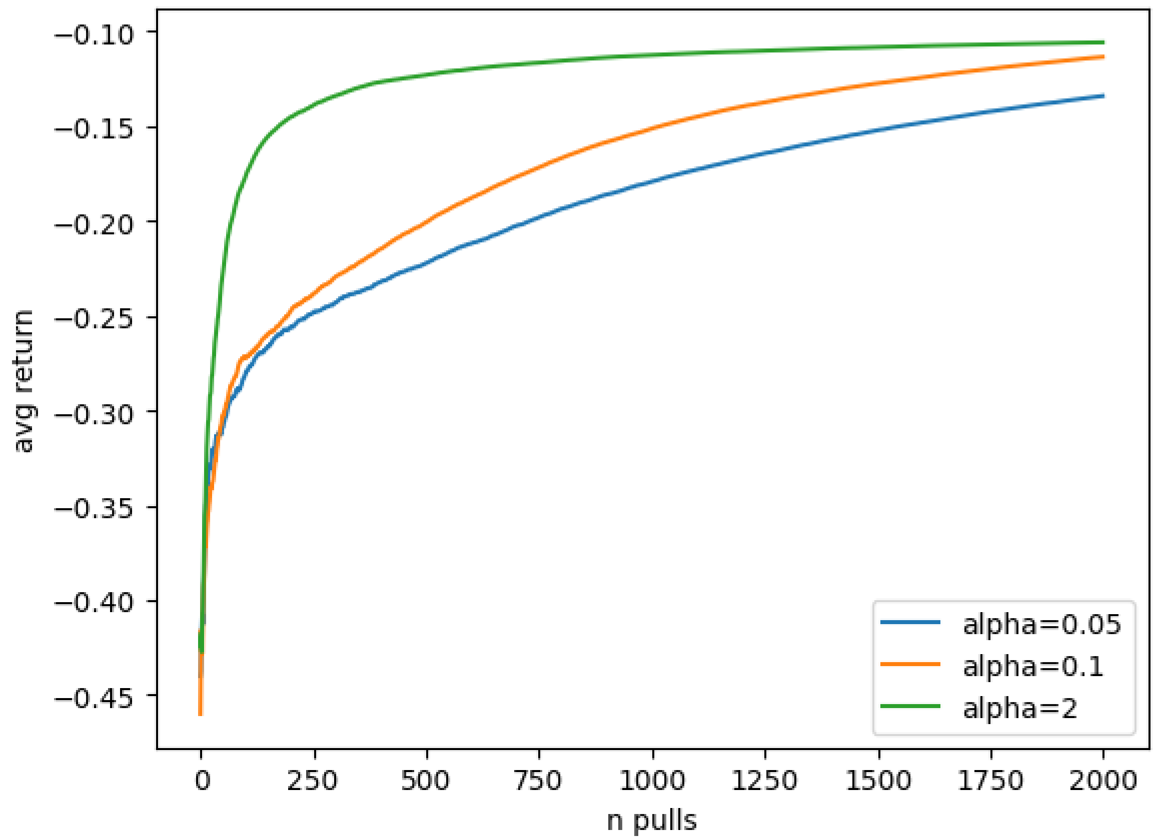
Q1.f2: Plotting the results (5 points)

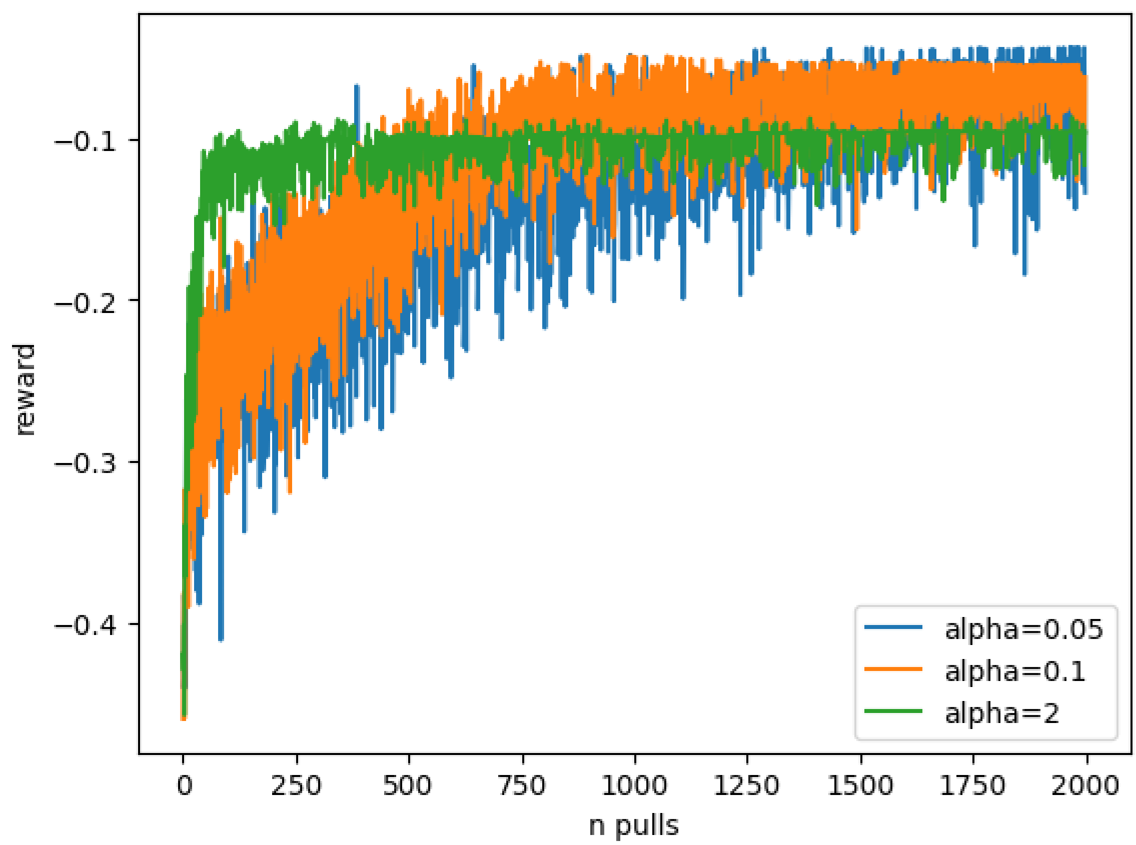
Evaluate the GB algorithm on the same MAB problem as above, for three values of the parameters α: 0.05, 0.1, and 2. Use the driver code provided to plot their performances.

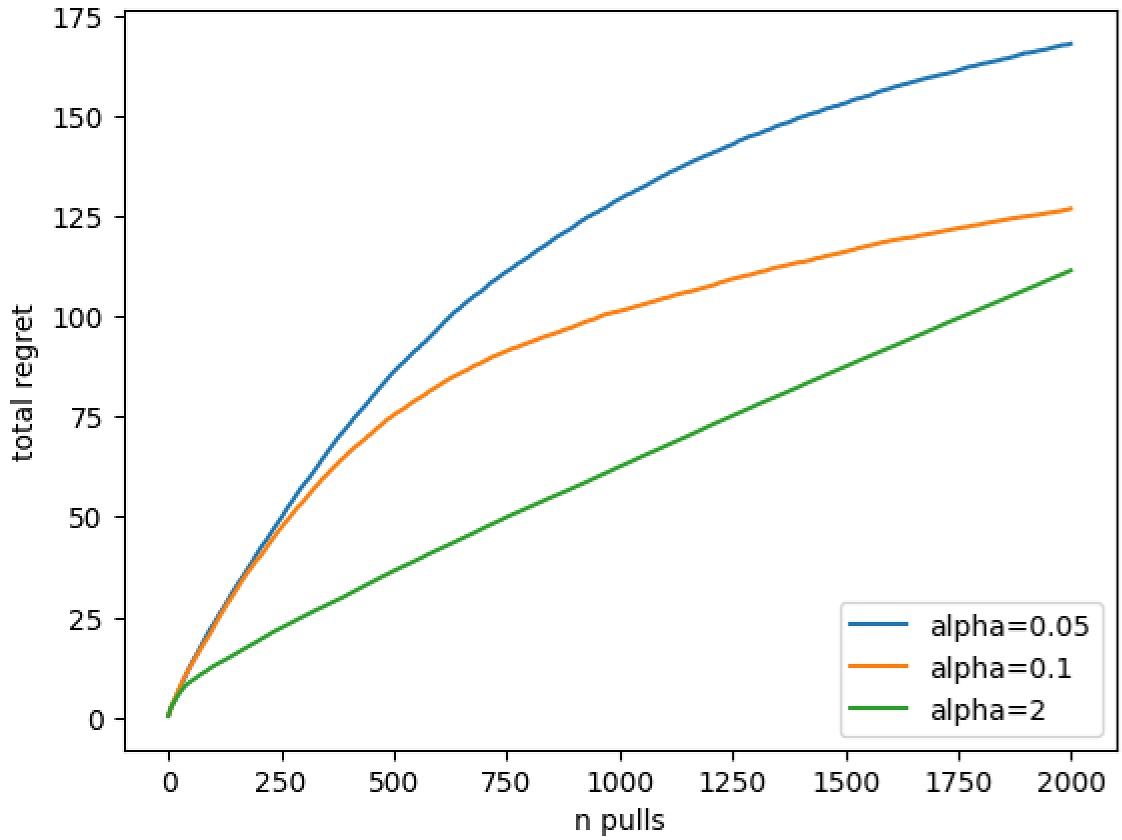
With baseline:

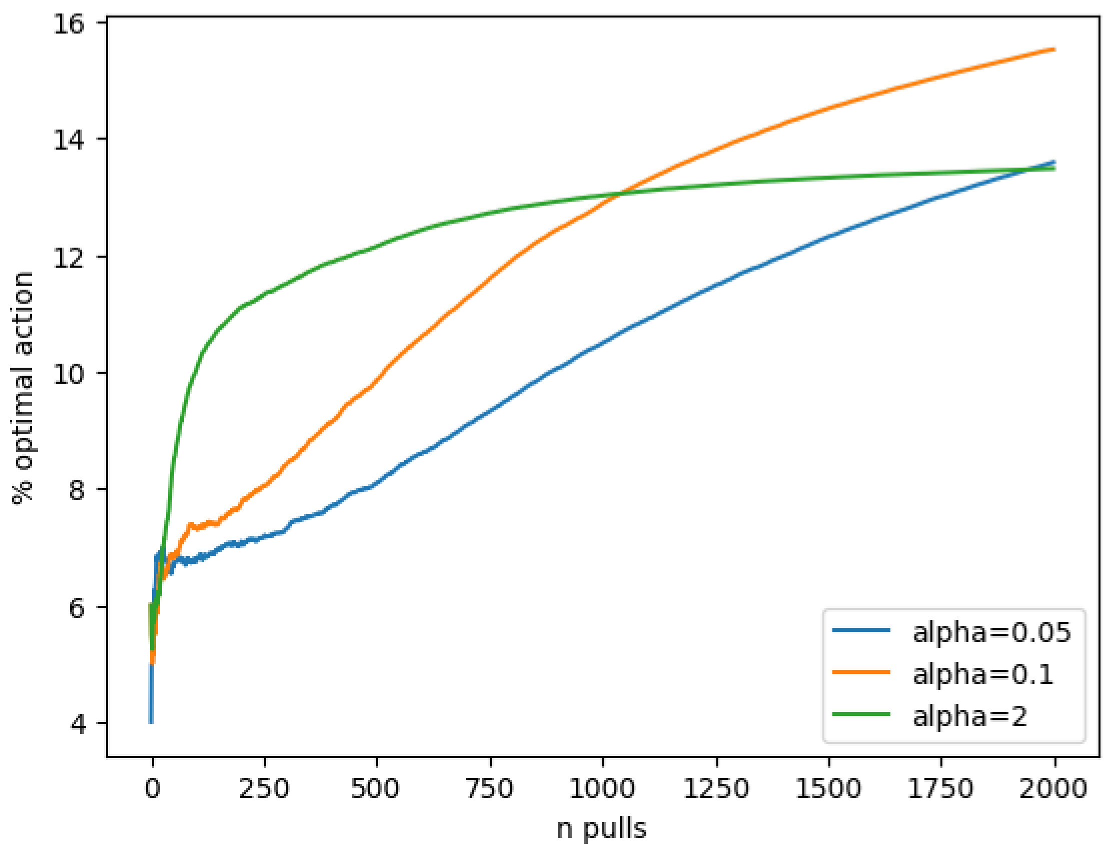
In [22]:
```
plt.figure(15)
plt.xlabel("n pulls")
plt.ylabel("avg return")
plt.figure(16)
plt.xlabel("n pulls")
plt.ylabel("reward")
plt.figure(17)
plt.xlabel("n pulls")
plt.ylabel("total regret")
plt.figure(18)
plt.xlabel("n pulls")
plt.ylabel("% optimal action")

N = 20
tot_reg_rec_best = 1e8
for alpha in [0.05, 0.1, 2]:
  rew_rec = np.zeros(bandit.n_pulls)
  avg_ret_rec = np.zeros(bandit.n_pulls)
  tot_reg_rec = np.zeros(bandit.n_pulls)
  opt_act_rec = np.zeros(bandit.n_pulls)

  for n in range(N):
    bandit.init_bandit()
    rew_rec_n, avg_ret_rec_n, tot_reg_rec_n, opt_act_rec_n = gradient_bandit(bandit, alp
    rew_rec += np.array(rew_rec_n)
    avg_ret_rec += np.array(avg_ret_rec_n)
    tot_reg_rec += np.array(tot_reg_rec_n)
    opt_act_rec += np.array(opt_act_rec_n)


  # take the mean
  rew_rec /= N
  avg_ret_rec /= N
  tot_reg_rec /= N

  plt.figure(15)
  plt.plot(avg_ret_rec, label="alpha={}".format(alpha))
  plt.legend(loc="lower right")

  plt.figure(16)
  plt.plot(rew_rec, label="alpha={}".format(alpha))
  plt.legend(loc="lower right")

  plt.figure(17)
  plt.plot(tot_reg_rec, label="alpha={}".format(alpha))
  plt.legend(loc="lower right")

  plt.figure(18)
  plt.plot(opt_act_rec, label="alpha={}".format(alpha))
  plt.legend(loc="lower right")

  if tot_reg_rec[-1] < tot_reg_rec_best:
        gradient_bandit_dict = {
        'opt_act':opt_act_rec,
```

```
        'regret_list':tot_reg_rec,}
        tot_reg_rec_best = tot_reg_rec[-1]
```





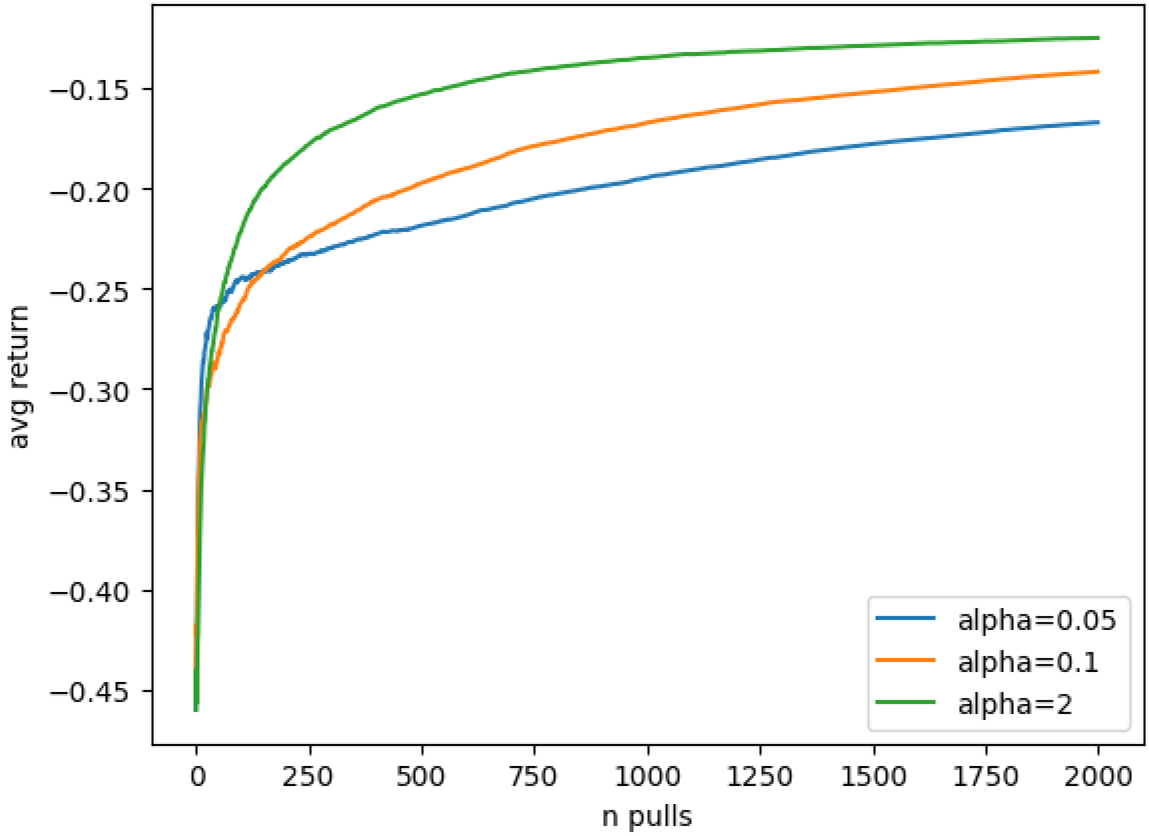


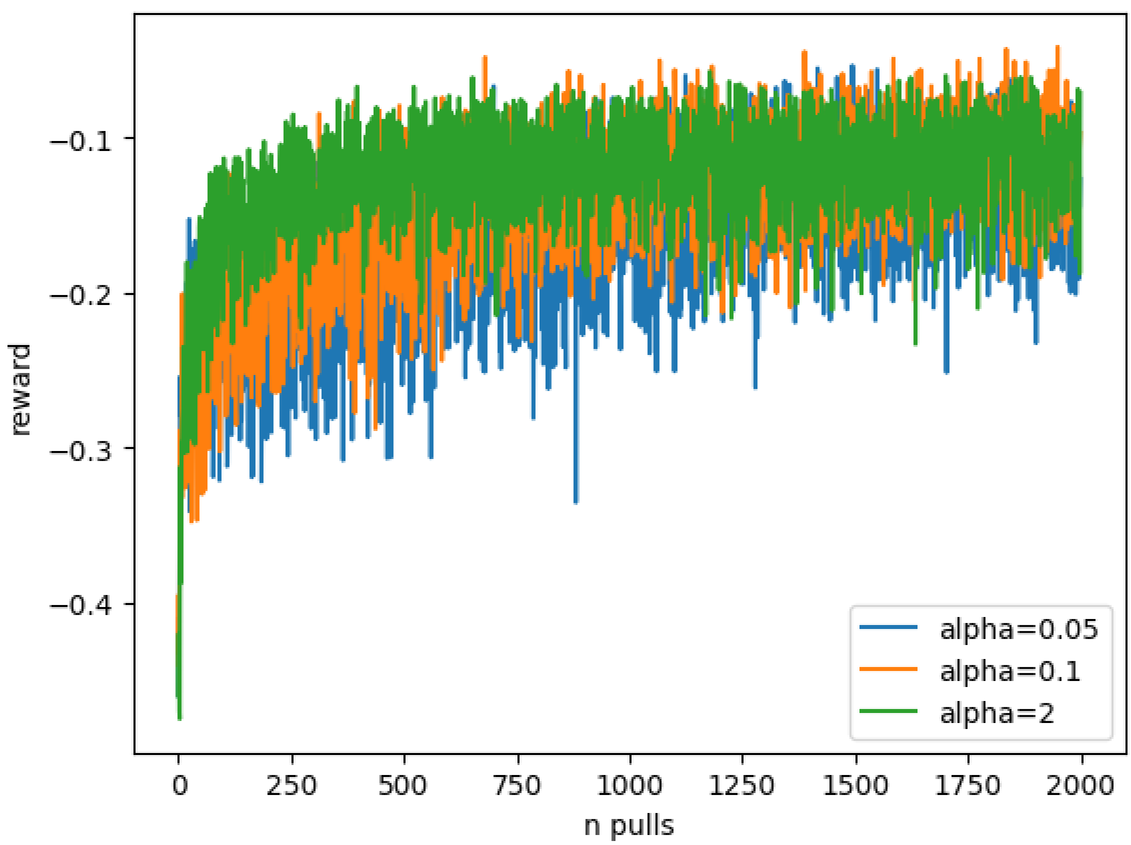


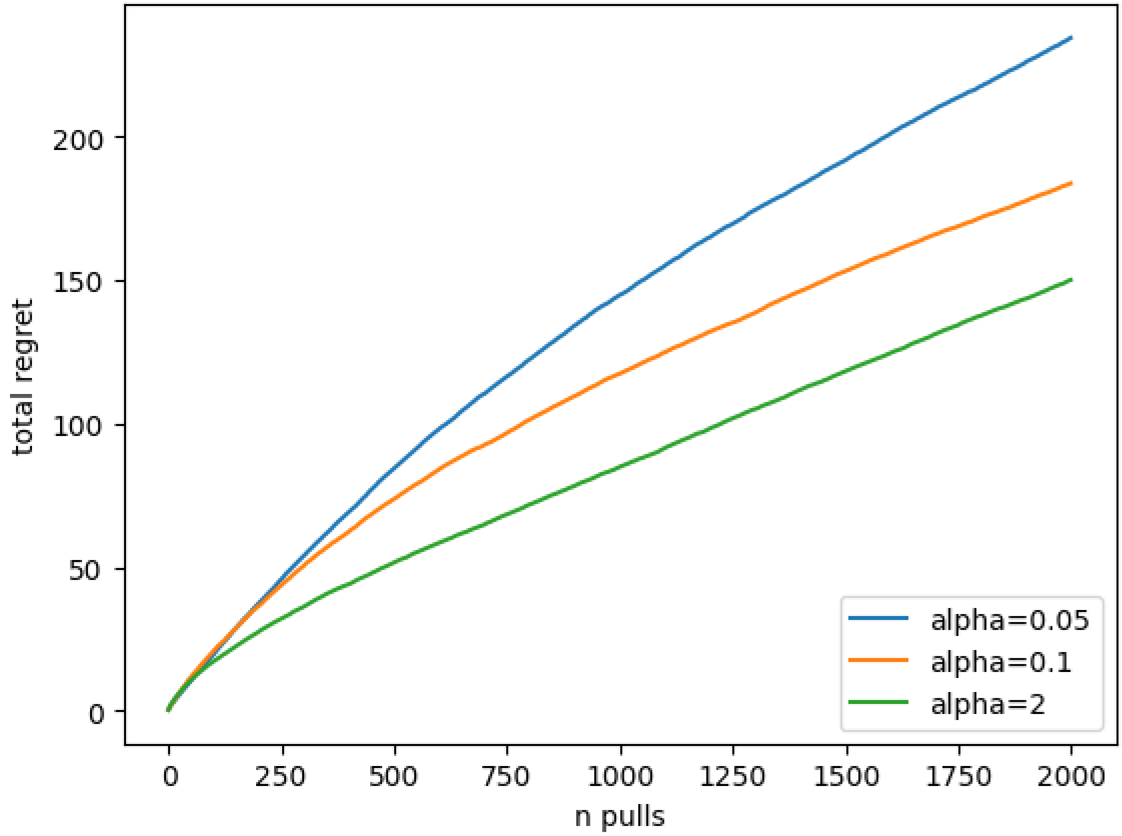
Without baseline:

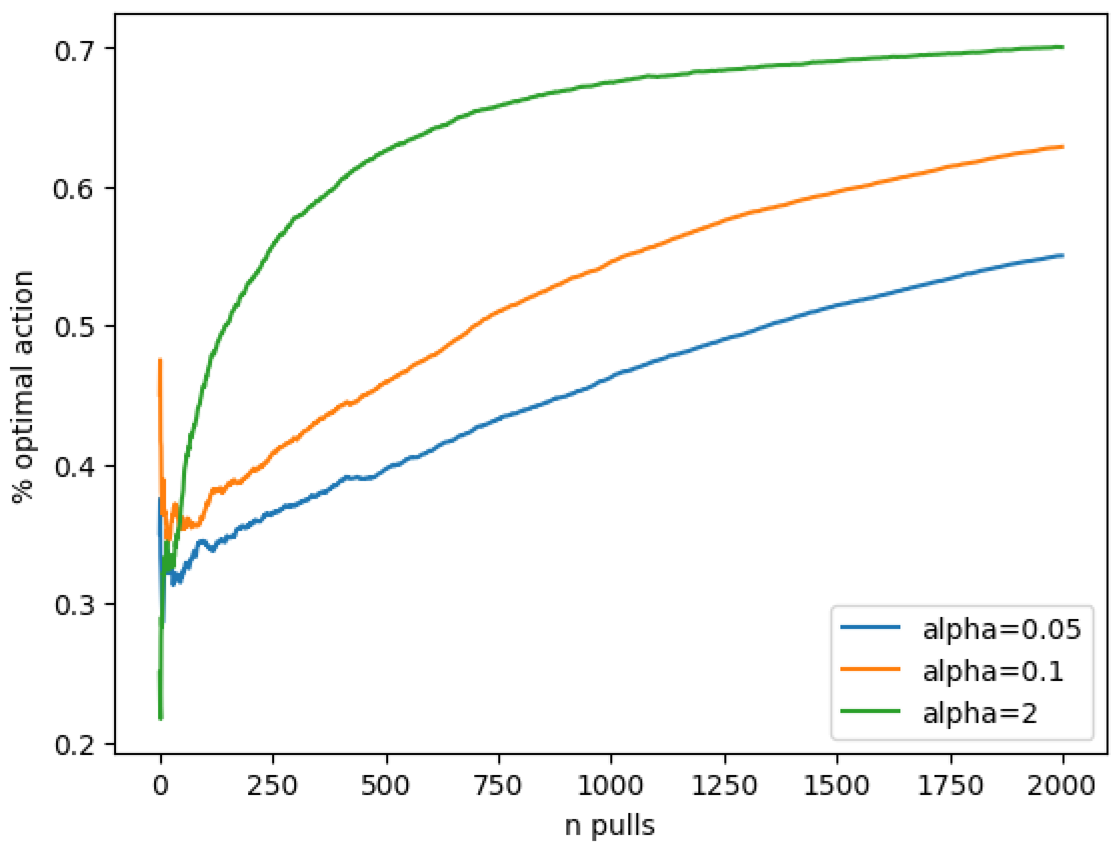
In [23]:
```
plt.figure(19)
plt.xlabel("n pulls")
plt.ylabel("avg return")
plt.figure(20)
plt.xlabel("n pulls")
plt.ylabel("reward")
plt.figure(21)
```

```
plt.xlabel("n pulls")
plt.ylabel("total regret")
plt.figure(22)
plt.xlabel("n pulls")
plt.ylabel("% optimal action")

N = 20
tot_reg_rec_best = 1e8
for alpha in [0.05, 0.1, 2]:
  rew_rec = np.zeros(bandit.n_pulls)
  avg_ret_rec = np.zeros(bandit.n_pulls)
  tot_reg_rec = np.zeros(bandit.n_pulls)
  opt_act_rec = np.zeros(bandit.n_pulls)

  for n in range(N):
    bandit.init_bandit()
    rew_rec_n, avg_ret_rec_n, tot_reg_rec_n, opt_act_rec_n = gradient_bandit(bandit, alp
    rew_rec += np.array(rew_rec_n)
    avg_ret_rec += np.array(avg_ret_rec_n)
    tot_reg_rec += np.array(tot_reg_rec_n)
    opt_act_rec += np.array(opt_act_rec_n)


  # take the mean
  rew_rec /= N
  avg_ret_rec /= N
  tot_reg_rec /= N
  opt_act_rec /= N

  plt.figure(19)
  plt.plot(avg_ret_rec, label="alpha={}".format(alpha))
  plt.legend(loc="lower right")

  plt.figure(20)
  plt.plot(rew_rec, label="alpha={}".format(alpha))
  plt.legend(loc="lower right")

  plt.figure(21)
  plt.plot(tot_reg_rec, label="alpha={}".format(alpha))
  plt.legend(loc="lower right")

  plt.figure(22)
  plt.plot(opt_act_rec, label="alpha={}".format(alpha))
  plt.legend(loc="lower right")

  if tot_reg_rec[-1] < tot_reg_rec_best:
        gradient_bandit_dict = {
        'opt_act':opt_act_rec,
        'regret_list':tot_reg_rec,}
        tot_reg_rec_best = tot_reg_rec[-1]
```


avg return
n pulls
alpha=0.05
alpha=0.1
alpha=2
−0.15
−0.20
−0.25
−0.30
−0.35
−0.40
−0.45
0
250
500
750
1000
1250
1500
1750
2000


reward
n pulls
alpha=0.05
alpha=0.1
alpha=2
−0.1
−0.2
−0.3
−0.4
0
250
500
750
1000
1250
1500
1750
2000





Q1.f3: Analysis (5 points)

Explain the role of α and the baseline on the results.

When $\alpha = 0.05$, a low learning rate, the parameter updates during the learning process are relatively small. Low learning rates are cautious and result in more stable learning, but they can be slow to converge. A positive aspect is that it is stable. The learning process is less likely to diverge or exhibit significant oscillations. Another positive aspect is that is explore safely. Smaller updates ensure that the agent's policy changes gradually However, it is slow to converge.

When $\alpha = 0.1$, a moderate learning rate, it balances the trade-off between learning speed and stability. It allows for reasonably sized updates, promoting relatively fast convergence while maintaining some degree of stability.

When $\alpha = 2$, a high learning rate, it leads to large parameter updates, which can result in faster learning but at the cost of stability. High learning rates leads to oscillations, divergence, and difficulty converging to a good policy.

With a Baseline, there is a reduction in variance of the gradient estimates. This results in less noisy updates and more stable learning. With a lower variance, the learning process is less likely to exhibit large fluctuations and is more predictable. It also leads to more efficient learning. Smoother gradient estimates allow the agent to learn more reliably and converge to a better policy faster.

Without Baseline, the gradient estimates tend to have higher variance. This results in noisier updates, which can lead to more erratic learning behavior. High variance may make it challenging to predict the direction and magnitude of policy updates accurately. The increased variance in gradient estimates can slow down the learning process. The agent may require more episodes or experience to converge to a good policy, as the updates are less stable and reliable.

Q1.g: Final comaprison (10 points)

Q1.g1: plots (5 points)

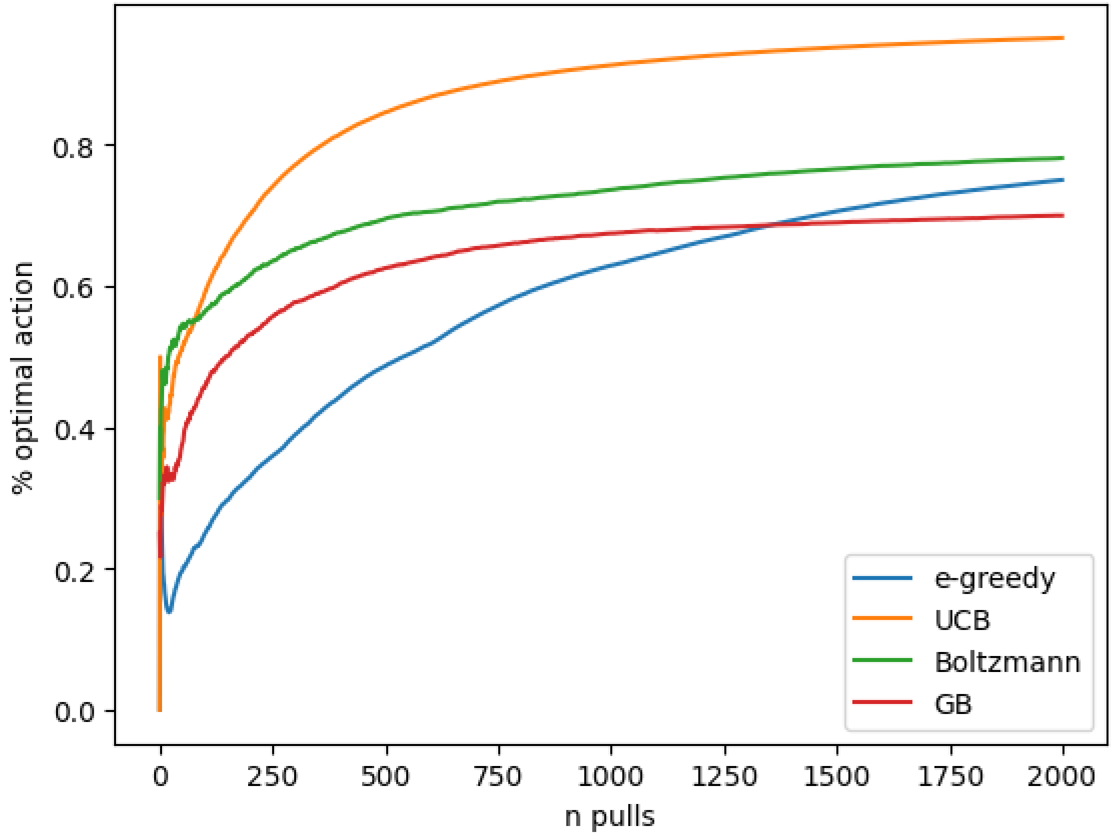
Compare the performance of ϵ-greedy, UCB, Boltzmann algorithm, and Gradient Bandit algorithm in a single plot as measured by the average reward and total regret.

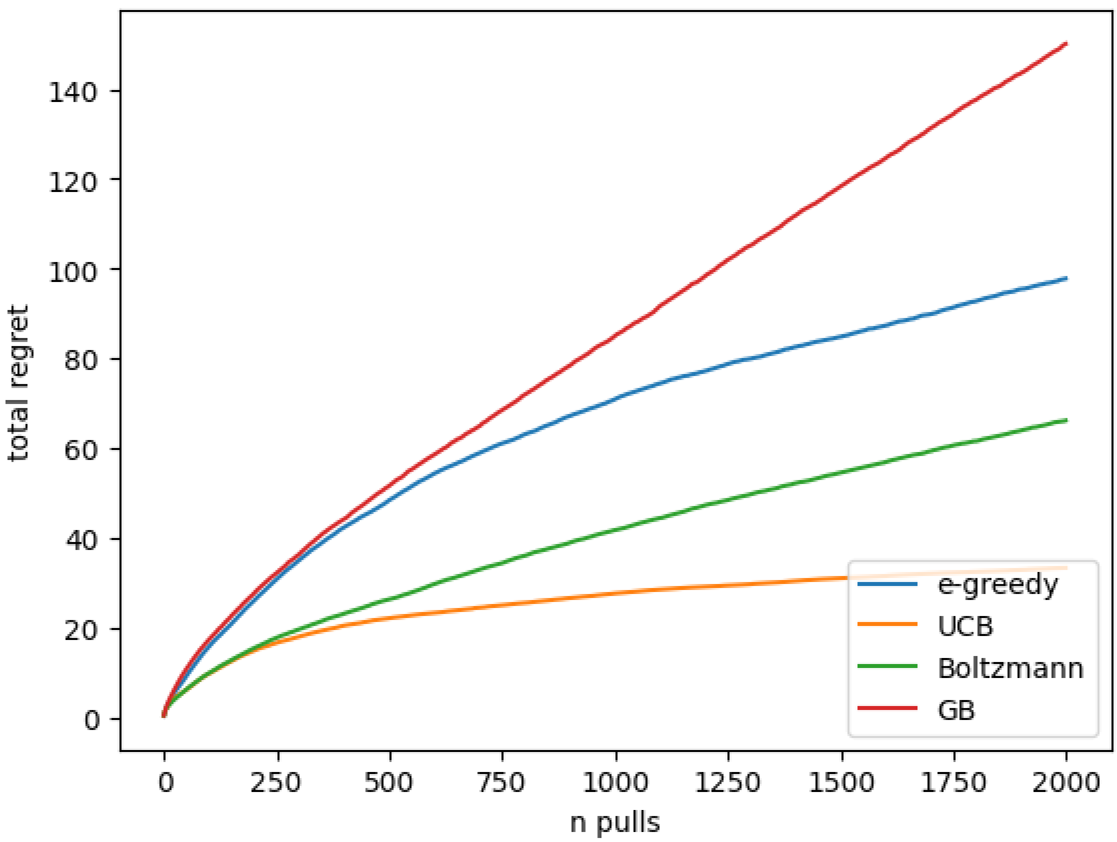
In [24]:

```
plt.figure(23)
plt.plot(ep_greedy_dict["opt_act"], label="e-greedy")
plt.legend(loc="lower right")
plt.plot(ucb_dict["opt_act"], label="UCB")
plt.legend(loc="lower right")
plt.plot(boltzmann_dict["opt_act"], label="Boltzmann")
plt.legend(loc="lower right")
plt.plot(gradient_bandit_dict["opt_act"], label="GB")
plt.legend(loc="lower right")
plt.xlabel("n pulls")
plt.ylabel("% optimal action")

plt.figure(24)
plt.plot(ep_greedy_dict["regret_list"], label="e-greedy")
plt.legend(loc="lower right")
plt.plot(ucb_dict["regret_list"], label="UCB")
plt.legend(loc="lower right")
plt.plot(boltzmann_dict["regret_list"], label="Boltzmann")
plt.legend(loc="lower right")
plt.plot(gradient_bandit_dict["regret_list"], label="GB")
plt.legend(loc="lower right")
plt.xlabel("n pulls")
plt.ylabel("total regret")
```

Out[24]:

```
Text(0, 0.5, 'total regret')
```





Q1.g2: Analysis (5 points)

Compare all the algorithms in terms of their performance.

The ϵ-greedy algorithm performs really good compared to Boltzmann algorithm, and UCB algorithm. It is simpler to implement and achieved better results. However, it is not as good as the Gradient Bandit algorithm. The Gradient Bandit algorithm is the best one in our case. There are also hyperparameter to tune for the Boltzmann algorithm, UCB algorithm and Gradient Bandit that could lead to better results that the one observed. However, fine tuning theses hyperparameter is dependent on the problem and it is not always easy to do. In summary, the choice of exploration strategy depends on the specific problem's characteristics and requirements. Epsilon-greedy is simple but may not be the best choice in highly uncertain environments. UCB is effective in uncertain environments but may be more computationally demanding. Boltzmann exploration provides adaptability and smooth transitions between exploration and exploitation but requires tuning of the temperature parameter. Experimentation and empirical evaluation are often necessary to determine the best exploration strategy for a given RL problem. In the Multi-Armed Bandit problem, the Gradient Bandit algorithm is the best one.

In [25]:
```
plt.close('all')
```