Exercise 1

1

(a)

F

(b)

T

(c)

F

(d)

F

(e)

T

(f)

T

(g)

F

2

(a)

$\neg A$.

(b)

$A \wedge B$.

(c)

$A \to \neg B$

(d)

$A \vee (\neg A \to B)$.

(e)

$(A \wedge B) \vee (\neg A \wedge \neg B)$.

3

(a)

$p \oplus q = (p \wedge \neg q) \vee (\neg p \wedge q)$.

(b)

$p \to q = \neg(p \wedge \neg q) = \neg p \vee q$

(c)

$p \odot q = (\neg p \vee q) \wedge (p \vee \neg q) = (p \wedge q) \vee (\neg p \wedge \neg q)$.

(d)

$\neg(p \to q) = \neg\neg(p \wedge \neg q) = p \wedge \neg q$.

4

(a)

$$\begin{aligned} f(p,q,r) &= (p \wedge q \wedge r) \vee ((\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)) \\ &= (p \wedge q \wedge r) \vee ((\neg p \wedge \neg q) \wedge (r \vee \neg r)) \\ &= (p \wedge q \wedge r) \vee (\neg p \wedge \neg q). \end{aligned}$$

(b)

$$\begin{aligned} f(p,q,r) &= \neg((p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)) \\ &= (\neg p \vee \neg q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r) \\ &= (\neg q \wedge (p \vee q)) \vee r \\ &= (p \wedge \neg q) \vee r. \end{aligned}$$

5

(a)

Not equivalent.

If $(p, q, r) = (F, F, T)$, then

$$\begin{aligned} pq + r &= FF + T = F + T = T \\ &\neq F = FT = F(F + T) = p(q + r). \end{aligned}$$

(b)

Equivalent.

$$\begin{aligned} pq\bar{r} + p\bar{q} + r &= p(q\bar{r} + \bar{q}) + r \\ &= \bar{r}p(q\bar{r} + \bar{q}) + r \\ &= p(q\bar{r} + \bar{q}\bar{r}) + r \\ &= p\bar{r} + r = p + r. \end{aligned}$$

(c)

Equivalent.

$$\begin{aligned} \neg(p + q + r) &= \neg((p + q) + r) \\ &= \neg(p + q)\neg r \\ &= (\neg p \neg q)\neg r \\ &= \neg p \neg q \neg r. \end{aligned}$$

(d)

Equivalent.

$$p(p + q) = (p + p)(p + q) = p + (pq). \tag{1}$$

(e)

Not equivalent.

If $(p, q, r) = (T, F, T)$, then

$$\begin{aligned} (pq) + (qr) &= (p + r)q = (T + T)F = F \\ &\neq T = F + (TT) = q + (pr). \end{aligned}$$