Exercise 2

First order Logic

1

$$\exists s(s \in S \wedge [\forall c(c \in C \implies \exists t[t \in T \wedge Q(t,c) \wedge P(s,t)])]). \tag{1}$$

3

(a)

$$\forall p[\text{oddprime}(p) \implies \text{sumofsquare}(p)]. \tag{2}$$

(b)

$$\forall n[n \in \mathbb{Z}^+ \implies \exists a \exists b \exists c \exists d(n = a^2 + b^2 + c^2 + d^2)]. \tag{3}$$

(c)

$$\forall a \forall b \forall c[\text{abc}(a,b,c) \implies \text{cf}(a,b,c)]. \tag{4}$$

(d)

$$[\exists p \; P(p)] \wedge [P(p) \wedge P(q) \implies p = q]. \tag{5}$$

4

$$\text{TTFFTFTFF} \tag{6}$$

Set Theory

2

$$\text{TFFTTTT} \tag{7}$$

4

If $x \in A \cup C$, then either $x \in A$ or $x \in C$. If $x \in C$, then $x \in B \cup C$; If $x \in A$, then since $A \subset B$ we have $x \in B$. So $x \in B \cup C$. In either case, $x \in B \cup C$. Therefore $A \cup C \subset B \cup C$. $\square$