DDA4250 Assignment 5

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Notation

- $r : \mathbb{R} \to \mathbb{R}$ is the rectifier function $r(x) = \max\{0, x\}$

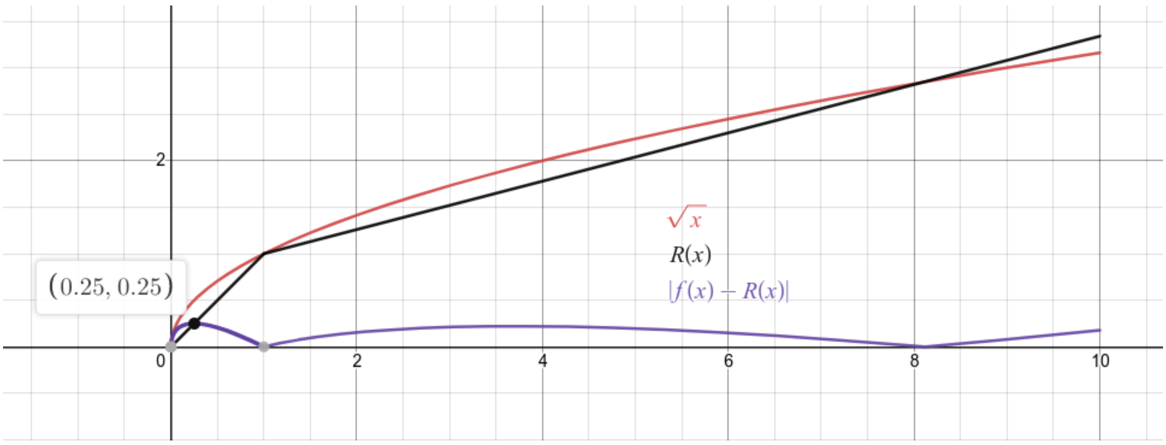
3.1.4

Consider the realization

$$\mathcal{R}(x) := \mathcal{R}_r(\Phi)(x) = r(x) - 0.74 \cdot r(x-1) = \begin{cases} x & : x \in [0,1] \\ 0.26(x-1)+1 & : x \in (1,10] \end{cases}$$

We have $\mathcal{D}(\Phi) = (1,2,1), \mathcal{P}(\Phi) = 1(2+1) + 2(1+1) = 7 < 10$, and

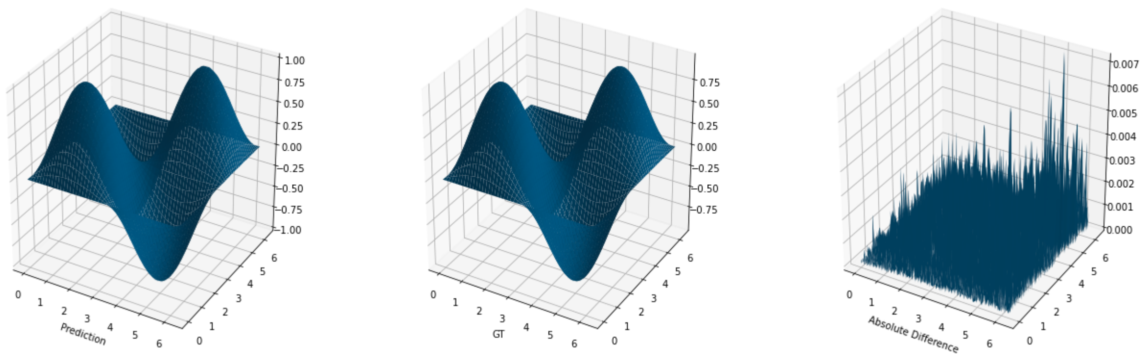
$$\sup_{x\in[0,10]} |\mathcal{R}(x) - \sqrt{x}| = \sup_{x\in[0,1]} \{\sqrt{x} - x\} = \frac{1}{4}$$



3.2.1

We shall show the existence of such an ANN by directly training one. Using PyTorch, we managed to find a Φ with $\mathcal{D}(\Phi) = [2, 256, 256, 256, 256, 256, 256, 1], \mathcal{P}(\Phi) = 329986 < 60000000$, and

$$\sup_{x,y\in[0,2\pi]} |\mathcal{R}_r(\Phi)(x,y) - \sin(x)\sin(y)| = 0.0071... < \frac{1}{5}$$



Left: *Prediction output from the network;* ***Mid***: *Ground truth;* ***Right***: *Absolute difference between prediction and GT.*

```
diff.max()
0.007159508
```

Please see `params.pt` attached for trained parameters of Φ.

3.2.2

Let $c := 2(\sqrt{2}-1)$. Consider the neural network S with realization $\mathcal{R}_r(S) : \mathbb{R} \to \mathbb{R}$ given by

$$\mathcal{R}_r(S)(x) = -cr(x) + 2cr(x-1) + c =: \mathcal{R}(x)$$

We have $\mathcal{D}(S) = (1,2,1)$, and

$$\sup_{x\in[0,2]} |(x-1)^2 - \mathcal{R}(x)| = 3 - 2\sqrt{2} = 0.1715... < 0.1875 = \frac{3}{16}$$

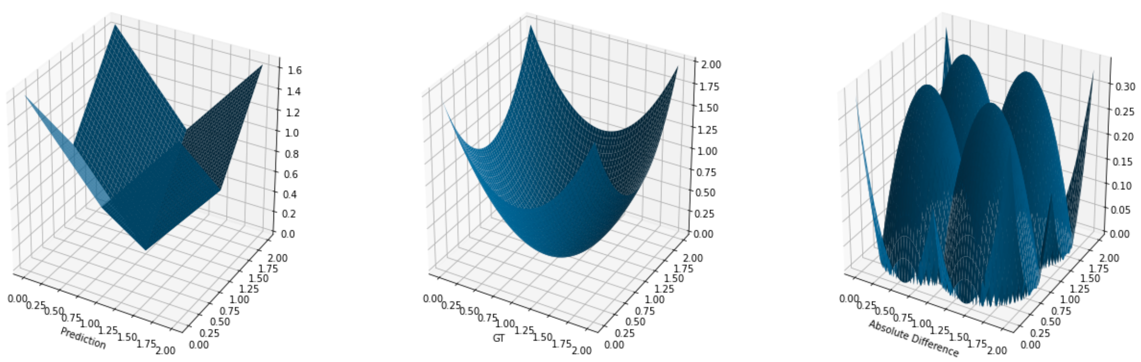
Construct $\Phi := \mathbf{A}_{[1,1],0} \bullet \mathbf{P}(S,S)$. Then

$$\mathcal{R}_r(\Phi)(x,y) = \mathcal{R}_r(S)(x) + \mathcal{R}_r(S)(y) + 0 = \mathcal{R}(x) + \mathcal{R}(y)$$

Thus

$$\begin{aligned}
\sup_{x,y\in[0,2]} |\mathcal{R}_r(\Phi)(x,y) - (x-1)^2 - (y-1)^2| &= \sup_{x,y\in[0,2]} |\mathcal{R}(x) - (x-1)^2 + \mathcal{R}(y) - (y-1)^2| \\
&\le \sup_{x,y\in[0,2]} \{|\mathcal{R}(x) - (x-1)^2| + |\mathcal{R}(y) - (y-1)^2|\} \\
&= 2\sup_{x\in[0,2]} |\mathcal{R}(x) - (x-1)^2| \\
&= 2(3-2\sqrt{2}) \\
&< 2\cdot 3/16 \\
&= 3/8
\end{aligned}$$

Also note that $\mathcal{D}(\Phi) = (1,4,1)$, and so $\mathcal{P}(\Phi) = 1(4+1) + 4(1+1) = 13 < 20$.



Left: *Prediction output from the network;* ***Mid***: *Ground truth;* ***Right***: *Absolute difference between prediction and GT.*