CSC3001 PQ6

1

(a)

$$(12, 8) = (4, 8) = (4, 0) = 4.$$

(b)

$$(36, 84) = (36, 12) = (0, 12) = 12.$$

(c)

$$(120, 98) = (22, 98) = (22, 10) = (2, 10) = (2, 0) = 2.$$

3

(a)

False; $(1, 2) = 1; (1, 4) = 1$. But $(2, 4) = 2$.

(b)

True;

$$(a, b) = (a, c) = 1 \implies ax + by = 1 \land aw + cz = 1 \implies (x + w - xwa)a + (yz)bc = 1 \implies (a, bc) = 1$$

4

Let $d := (x + y, x - y) = (x + y, 2x)$.

$$(x, y) = 1 \implies ax + by = 1 \implies (2b)(x + y) + (a - b)(2x) = 2 \implies d|2 \implies d = 1 \lor d = 2$$

6

If $p|a$ we are done. May assume $p \nmid a$. Write $b = pq + r, r \in [0, p)$. We want to show $r = 0$. Since $p|ab, ab = kp$. Therfore $a(pq + r) = aqp + ar = kp$. Or $r = (\frac{k}{a} - q)p$, whence $p|r$. But $r \in [0, p)$. It follows that $r = 0$.

8

Suppose m even. Then $(n, m) = 2(\frac{n}{2}, \frac{m}{2})$. So $2|(n, m); 2|5$. Contradiction.