

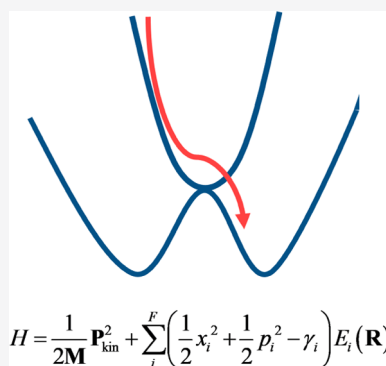
On-the-Fly Symmetrical Quasi-Classical Dynamics with Meyer–Miller Mapping Hamiltonian for the Treatment of Nonadiabatic Dynamics at Conical Intersections

Deping Hu,[§] Yu Xie,[§] Jiawei Peng, and Zhenggang Lan* Cite This: *J. Chem. Theory Comput.* 2021, 17, 3267–3279 Read Online

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ABSTRACT: The on-the-fly version of the symmetrical quasi-classical dynamics method based on the Meyer–Miller mapping Hamiltonian (SQC/MM) is implemented to study the nonadiabatic dynamics at conical intersections of polyatomic systems. The current on-the-fly implementation of the SQC/MM method is based on the adiabatic representation and the dressed momentum. To include the zero-point energy (ZPE) correction of the electronic mapping variables, we employ both the γ -adjusted and γ -fixed approaches. Nonadiabatic dynamics of the methaniminium cation (CH_2NH_2^+) and azomethane are simulated using the on-the-fly SQC/MM method. For CH_2NH_2^+ , both ZPE correction approaches give reasonable and consistent results. However, for azomethane, the γ -adjusted version of the SQC/MM dynamics behaves much better than the γ -fixed version. Further analysis indicates that it is always recommended to use the γ -adjusted SQC/MM dynamics in the on-the-fly simulation of photoinduced dynamics of polyatomic systems, particularly when the excited state is well separated from the ground state in the Franck–Condon region. This work indicates that the on-the-fly SQC/MM method is a powerful simulation protocol to deal with the nonadiabatic dynamics of realistic polyatomic systems.



1. INTRODUCTION

The theoretical description of the nonadiabatic dynamics at conical intersections (CIs) in complex systems has always aroused great research interest due to the breaking down of the Born–Oppenheimer (BO) approximation and the involvement of a large number of degrees of freedom (DoFs).^{1–3} Various theoretical approaches were developed to simulate the nonadiabatic dynamics,^{1–10} which, for example, include the full quantum dynamics^{2,11–15} and different versions of mixed quantum-classical/semiclassical dynamics.^{1,2,5,6,8–10,16–26} The complex potential energy functions of the polyatomic molecules with many DoFs impose some additional challenges in the simulation of the nonadiabatic dynamics. To solve such problems, the combination of on-the-fly simulation with various dynamics methods becomes popular in the simulation of the real-time nonadiabatic dynamics of polyatomic systems at all atomic levels.^{5,6,9,10,21,27}

Because the electronic structure calculations are generally performed at each nuclear time step in the on-the-fly dynamics, the total cost of on-the-fly simulation is rather high. For this reason, on-the-fly simulation of the nonadiabatic dynamics should normally employ practical dynamical methods that can provide a good balance between computational cost and efficiency. Thus, only a few dynamic approaches are possible choices. For example, trajectory surface hopping (TSH) approaches, such as Tully's fewest switches surface hopping (FSSH) approach and its extension^{18,20} and Landau–Zener/

Zhu–Nakamura surface hopping approaches,^{24,28} were employed in the on-the-fly simulation due to their computational efficiency. In recent years, great progress has been made in using Tully's on-the-fly FSSH (and its variations) dynamics to treat the nonadiabatic dynamics of realistic polyatomic systems.^{2,9,21,27} However, the improper treatment of electronic coherence and frustrated hops in Tully's FSSH approach has been widely discussed.^{8,19,20,29} A recent study has also tried to combine a more rigorous surface hopping approach³⁰ with on-the-fly simulation, which is derived from the exact factorization of the electronic-nuclear wavefunction.³¹ Considerable efforts have been made to combine the on-the-fly simulation with various Gaussian wavepacket-based approaches,^{5,10,32,33} such as ab initio multiple spawning (AIMS),^{5,10} variational multi-configurational Gaussians (vMCGs),³² and multiconfigurational Ehrenfest.³⁴ Some numerical details should be considered in the implementation of these approaches. For instance, new Gaussian wavepackets¹⁰ should be properly generated in AIMS, and reasonable basis-function sampling/cloning should be considered in the multiconfigurational

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Ehrenfest dynamics.³⁵ These approaches are highly promising because these theoretical frameworks are more rigorous and the computational cost is reasonable.

As expected, only the dynamical approaches with reasonable computational cost may be suitable for on-the-fly dynamics. Particularly, the methods based on independent trajectories, instead of entangled trajectories, are preferred in the implementation of on-the-fly simulation, if both require a similar number of trajectories. According to this idea, it is clear that the symmetrical quasi-classical dynamics based on the Meyer–Miller mapping Hamiltonian (SQC/MM)^{36–38} may be an alternative and suitable approach for on-the-fly simulation. In the Meyer–Miller (MM) mapping model,³⁶ a Hamiltonian with N discrete quantum states is mapped to an effective Hamiltonian (or a mapping Hamiltonian) with N coupled harmonic oscillators.^{8,36,39,40} It is possible to combine the MM mapping Hamiltonian with different dynamics approaches, such as various quasi-classical/semiclassical dynamics approaches,^{41–49} quantum-classical Liouville equation (QCLE),^{50–53} path integral and extension,^{54–59} surface hopping,^{60,61} centroid molecular dynamics (CMD),⁶² and ring-polymer molecular dynamics (RPMD).^{63–69} In the quasi-classical dynamics, the inclusion of the zero-point energy (ZPE) in the electronic mapping variables in principle provides better dynamical results than the Ehrenfest dynamics,³⁶ while the partial instead of full ZPE should be included in practice.^{8,70–72} When the “bin” technique is taken to perform the initial sampling and the final assignment of the quantum states, SQC/MM^{38,73} is formulated.

The SQC/MM method gained significant attention due to its numerical simplicity and physical insight.^{38,73–90} The reasonable performance of the SQC/MM dynamics was examined by different benchmark studies.^{47,81,82,86,87,89} Within the framework of the SQC/MM dynamics, Cotton and Miller suggested different useful approaches in the implementation, such as the triangle windowing technique^{80,91} and the employment of the trajectory-adjusted electronic ZPE correction⁹² in the SQC/MM approach. The SQC/MM method was employed to treat different types of nonadiabatic dynamics,^{38,73,81–86,93} for example, excited-state energy/electronic transfer dynamics,^{73,81,86,90,93} singlet fission^{83–85} and scattering dynamics,⁸² and photodissociation dynamics.⁹² As shown by several benchmark studies,^{47,81,82,86,87} the SQC/MM dynamics gives a proper description of the nonadiabatic dynamics of various model systems (such as the spin-boson model, etc.) when the bath motion behaves classically. Previous work also proved that the SQC/MM dynamics in principle satisfied the detailed balance in the asymptotic limit.⁷⁴ These studies provide us the baseline of the reasonable behaviors of the SQC/MM dynamics. In addition, it is also shown that the electronic coherence of exciton dynamics in photoharvesting and organic photovoltaics systems is captured by the SQC/MM dynamics.^{81,86} Overall, the SQC/MM method is very promising; thus, it is highly interesting to introduce it into on-the-fly dynamics for all-atomic simulation of the nonadiabatic dynamics of polyatomic molecules in real time.

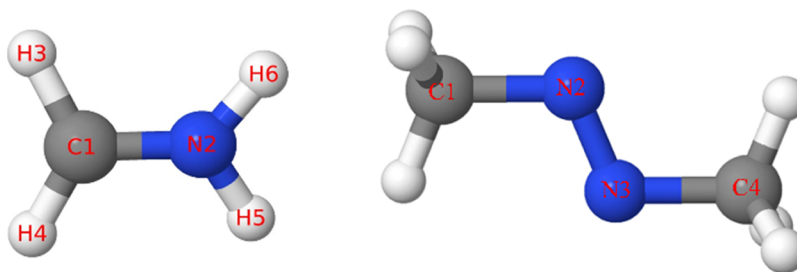
Recently, Huo and co-workers proposed to employ the quasi-diabatization procedure in dynamics propagation.^{88,94–97} They used the quasi-diabatic basis to propagate the MM mapping Hamiltonian-based dynamics, such as the partial-linearized density matrix (PLDM) path-integral approach and the SQC/MM approach. Particularly, they made initial efforts

to combine the PLDM path-integral approach, the classical path approximation, and density functional tight-binding (DFTB) calculations to achieve the on-the-fly simulation of nonadiabatic dynamics of photoinduced charge transfer in organic photovoltaic systems.⁹⁵ Lately, they have extended their approach to study the photoinduced dynamics at CIs.⁹⁷ The quasi-diabatic approach works quite well and improves numerical stability in several cases. When the electronic states involved in the nonadiabatic dynamics have non-neglectable couplings with the higher electronic states, the curl condition is not satisfied.^{1,98–101} In this case, it is not trivial to construct the quasi-diabatic electronic basis within the subspace spanned by a few electronic states unless a much larger number of electronic states are considered in the adiabatic-to-diabatic transformation. The inclusion of many electronic states may introduce additional challenges in the high-level electronic structure calculations. Alternatively, it is also possible to formulate the MM mapping Hamiltonian in the adiabatic representation,^{36,56,75,102–104} providing a natural interface to combine the electronic structure calculations. Along this road, Miller and co-workers⁷⁵ recently reformulated the SQC/MM dynamics in the adiabatic representation using the dressed momentum, namely, kinematic momentum. This formalism provides us an important starting point for the implementation of on-the-fly SQC/MM dynamics. We noticed that Cui and co-workers tried to combine this approach with on-the-fly simulation using the semiempirical electronic structure calculations.¹⁰⁵ This work also showed that the on-the-fly SQC/MM dynamics results are sensitive to the choice of the window width and the ZPE correction parameter γ when γ is fixed in the SQC/MM dynamics. In fact, the possible influence of the window width on the dynamics has also been discussed in several previous studies.^{77,80} This problem may be partially remedied by the trajectory-adjusted electronic ZPE approach proposed by Cotton and Miller.⁹² Thus, in principle, this γ -adjusted approach is highly recommended in the on-the-fly SQC/MM dynamics to study the photochemistry of realistic systems.

In this work, we try to combine the SQC/MM method with the on-the-fly dynamics. To know more about the performance of the SQC/MM method in the on-the-fly nonadiabatic dynamics compared with other methods, such as FSSH and AIMS, two commonly used molecular systems, methaniminium cation (CH_2NH_2^+) and azomethane, were chosen in this work. Since we noticed that the choice of the ZPE correction in the SQC/MM dynamics is not a trivial task, we implemented both the γ -fixed and γ -adjusted approaches. In the latter case, the γ is determined by the initial condition of the electronic DoFs for each trajectory. This combination provides a novel simulation tool to describe the full-dimensional nonadiabatic dynamics at CIs of polyatomic systems. All implementation is performed based on the nonadiabatic dynamics simulation package JADE developed by our group.^{106,107} In this sense, we also largely extend the JADE package to allow for on-the-fly dynamics simulation of nonadiabatic dynamics at CIs with various dynamics approaches.

2. THEORY AND METHODS

2.1. SQC/MM Hamiltonian on Adiabatic Representation. The adiabatic representation of MM Hamiltonian can be written as⁷⁵

Scheme 1. Chemical Structure of CH₂NH₂⁺ (Left) and Azomethane (Right) with Atom Labels

$$H = \frac{1}{2\mathbf{M}}\mathbf{P}_{\text{kin}}^2 + \sum_i^F \left(\frac{1}{2}x_i^2 + \frac{1}{2}p_i^2 - \gamma \right) E_i(\mathbf{R}) \quad (1)$$

with the kinematic momentum \mathbf{P}_{kin} defined as

$$\mathbf{P}_{\text{kin}} = \mathbf{P} + \sum_{i \neq j} x_i p_j \mathbf{d}_{ij}(\mathbf{R}) \quad (2)$$

where (\mathbf{R}, \mathbf{P}) are the coordinates and momenta of the nuclear DoFs, F is the number of electronic states, (x_i, p_i) is the coordinate and momentum of the harmonic oscillator that maps to the i th electronic state, $E_i(\mathbf{R})$ denotes the energy of the i th electronic state, γ represents the ZPE correction parameter, and $\mathbf{d}_{ij}(\mathbf{R})$ is the first-order nonadiabatic coupling (NAC) vector.

A previous study demonstrated that replacement of the effective potential term in eq 1

$$V_{\text{eff}} = \sum_i^F \left(\frac{1}{2}x_i^2 + \frac{1}{2}p_i^2 - \gamma \right) E_i(\mathbf{R}) \quad (3)$$

by a symmetrized one

$$V_{\text{eff}} = \frac{1}{F} \sum_i^F E_i(\mathbf{R}) + \frac{1}{F} \sum_{ij} \frac{1}{2}(n_i - n_j)(E_i(\mathbf{R}) - E_j(\mathbf{R})) \quad (4)$$

in the propagation generally gives better dynamics results,⁸² where

$$n_i = \frac{1}{2}x_i^2 + \frac{1}{2}p_i^2 - \gamma \quad (5)$$

is the action variable for the i th electronic state. Therefore, our current on-the-fly implementation used the following equations of motions (EOMs)

$$\begin{aligned} \frac{dx_i}{dt} &= p_i \frac{1}{F} \sum_j^F (E_i(\mathbf{R}) - E_j(\mathbf{R})) + \sum_j x_j \mathbf{d}_{ji}(\mathbf{R}) \cdot \frac{\mathbf{P}_{\text{kin}}}{\mathbf{M}} \\ \frac{dp_i}{dt} &= -x_i \frac{1}{F} \sum_j^F (E_i(\mathbf{R}) - E_j(\mathbf{R})) + \sum_j p_j \mathbf{d}_{ji}(\mathbf{R}) \cdot \frac{\mathbf{P}_{\text{kin}}}{\mathbf{M}} \\ \frac{d\mathbf{R}}{dt} &= \frac{\mathbf{P}_{\text{kin}}}{\mathbf{M}} \\ \frac{d\mathbf{P}_{\text{kin}}}{dt} &= -\frac{\partial V_{\text{eff}}}{\partial \mathbf{R}} - \frac{1}{2} \sum_{ij} (x_i x_j + p_i p_j)(E_j(\mathbf{R}) - E_i(\mathbf{R})) \mathbf{d}_{ij}(\mathbf{R}) \end{aligned} \quad (6)$$

2.2. Symmetrical Triangle Window and Initial Sampling. In the SQC/MM dynamics, the “window function” was used for both the initial sampling of the electronic DoFs and final assignment for the quantum states. The triangle

window function⁹¹ for multistate used in this work is defined as shown below

$$W_i(\mathbf{n} = n_1, \dots, n_i, \dots, n_f) = w_i(n_i) \cdot \prod_{i \neq j}^F w_0(n_i, n_j) \quad (7)$$

with

$$w_i(n_i) = \begin{cases} (2 - \gamma - n_i)^{2-F} & \text{for } 1 - \gamma < n_i < 2 - \gamma \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

and

$$w_0(n_i, n_j) = \begin{cases} 1 & \text{for } n_j < 2 - 2\gamma - n_i \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

The system is regarded as staying at the i th electronic state if $W_i = 1$ at any moment along the trajectory.

The action-angle sampling method was used to generate the initial x_i^0 and p_i^0 for the i th electronic state, which is

$$\begin{aligned} x_i^0 &= \sqrt{2(n_i^0 + \gamma)} \cos \theta \\ p_i^0 &= \sqrt{2(n_i^0 + \gamma)} \sin \theta \end{aligned} \quad (10)$$

where n_i^0 is sampled according to the triangle function and $\theta \in [-\pi, \pi]$. The final assignment is performed by binning the action variables into the corresponding windows. In the γ -fixed approach, this γ value is determined by the triangle window function.⁹¹

2.3. Trajectory-Adjusted Electronic ZPE. Recently, Cotton and Miller proposed a ZPE adjustment protocol,⁹² which modifies the ZPE correction parameter γ for each trajectory. After the initial sampling according to the triangle window, the coordinate and momentum of the electronic DoFs are x_i^0 and p_i^0 , respectively. This in fact defines the ZPE for each trajectory, namely

$$\gamma_i = \frac{1}{2}(x_i^0)^2 + \frac{1}{2}(p_i^0)^2 - \delta_{ij} \quad (11)$$

where j is the initial adiabatic electronic state used in the sampling process. This indicates that the corresponding ZPE correction parameter γ_i of each trajectory is different. Under this trajectory-adjusted γ_i , the action of electronic DoFs becomes

$$n_i = \frac{1}{2}x_i^2 + \frac{1}{2}p_i^2 - \gamma_i \quad (12)$$

during the trajectory propagation. At the same time, the expression of the MM Hamiltonian in the adiabatic

representation should also take the trajectory-adjusted ZPE into account, namely

$$H = \frac{1}{2\mathbf{M}}\mathbf{P}_{\text{kin}}^2 + \sum_i^F \left(\frac{1}{2}x_i^2 + \frac{1}{2}p_i^2 - \gamma_i \right) E_i(\mathbf{R}) \quad (13)$$

In the trajectory propagation, the time-dependent action of the electronic DoFs is calculated according to eq 12. Next, the final assignment of the quantum state is performed according to eqs 7–9 in which the γ value is replaced by the trajectory-adjusted γ_i of each trajectory. Detailed discussion on the technical details of this trajectory-adjusted electronic ZPE is given in a previous reference.⁹²

2.4. Computational Details. Both CH_2NH_2^+ and azomethane (Scheme 1) have often been used as typical examples to test the direct nonadiabatic dynamics approaches.^{30,106,108–121} Therefore, we also examined their excited-state dynamics using the on-the-fly SQC/MM dynamics. In addition, the FSSH dynamics was also performed.

The ground-state (S_0) minima were optimized at the DFT/B3LYP/6-31G* level using the Gaussian 16 package.¹²² The initial sampling of the nuclear coordinates and momenta was performed by the Wigner sampling of the lowest vibrational level in the electronic ground state. Here, the anharmonic effect is neglected for simplicity, and the current Wigner sampling is a practical and standard approach in on-the-fly nonadiabatic dynamics simulation. To directly compare the dynamics results based on different approaches, the same initial conditions of nuclear coordinates and momenta were used for FSSH, γ -adjusted, and γ -fixed SQC/MM dynamics.

For CH_2NH_2^+ , we considered the dynamics starting from both the first (S_1) and the second (S_2) excited state individually, while for azomethane, we explored the dynamics starting from S_1 .

In the SQC/MM dynamics, the initial sampling and final assignment of the quantum states for electronic DoFs were performed using symmetrical triangle window tricks. In the SQC/MM dynamics, both γ -adjusted and γ -fixed approaches were implemented.

In the FSSH dynamics, the trajectories were first directly put in the adiabatic electronic state where the dynamics started. The hop probabilities were calculated according to Tully's fewest switches algorithm.¹⁸ We employed the decoherence correction proposed by Granucci et al. and set the parameter $\alpha = 0.1$.²⁹

In the on-the-fly simulation, electronic structure calculations are performed in each nuclear time step. The electronic motion is generally faster than the nuclear motion. If we use the same time step to integrate the electronic and nuclear motions, the computational cost becomes extremely high. To reduce it, a standard implementation of the on-the-fly TSH dynamics often takes different time steps to propagate the nuclear and electronic motions. For each time step of nuclear motion, a much smaller time step is employed to propagate the electronic motion, and the effective Hamiltonian at each electronic time step is built by linear interpolation. This approach has been clearly discussed in previous studies.^{108,111,123} Thus, in the SQC/MM and FSSH dynamics, after several test calculations [see the Supporting Information (SI)], the nuclear equations of motion were integrated with time steps, 0.2 and 0.5 fs, respectively. For each step of nuclear motion, 100 steps of electronic motion were performed. The final results were obtained by averaging over 200 trajectories

for all dynamics. The convergence tests with respect to the number of trajectories and nuclear and electronic time steps are given in Figures S1–S3 (SI), respectively.

The values of electronic energies, first-order derivative NACs, and gradients can be obtained from the on-the-fly quantum chemistry calculation. In this work, the complete active space self-consistent field (CASSCF) method^{124,125} with the 6-31G* basis set was used in the electronic structure calculation process. For CH_2NH_2^+ , three state average with an active space of six electrons in orbitals [SA-3-CAS(6,4)] was used in the CASSCF calculation. For azomethane, to directly compare with previous AIMS dynamics,¹²⁰ the state average and active space [SA-2-CAS(6,4)] consistent with that in a previous study¹²⁰ were used in the SQC/MM and FSSH dynamics.

All of the dynamics calculations were performed with a development version of the JADE package.^{106,107} The CASSCF calculations were performed with the MOLPRO software.¹²⁶

3. RESULTS AND DISCUSSION

3.1. CH_2NH_2^+ . **3.1.1. Nonadiabatic Dynamics from S_2 .** For CH_2NH_2^+ , all selected dynamics approaches basically give consistent results on the time-dependent electronic populations regardless of whether the dynamics starts from S_2 or S_1 ; see Figure 1a,b. Here, we take γ -adjusted SQC/MM dynamics

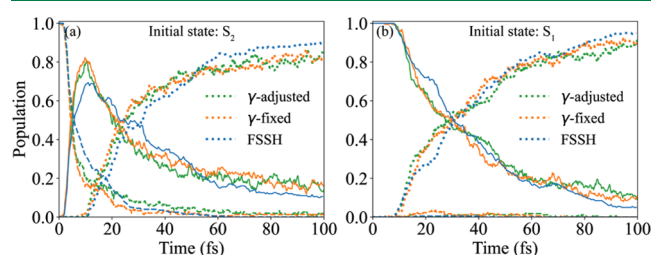


Figure 1. Time-dependent electronic populations of S_0 (dotted line), S_1 (solid line), and S_2 (dashed line) of dynamics starting from (a) S_2 and (b) S_1 for the CH_2NH_2^+ molecule. A total of 200 trajectories were used for all dynamics.

as an example. When the dynamics starts from S_2 , the system switches from S_2 to S_1 rapidly (<10 fs), as shown in Figure 1a. At 10 fs, the population in S_0 starts to increase. The lifetimes of S_2 and S_1 are approximately 9 and 30 fs, respectively, consistent with many previous theoretical studies.^{2,30,106,108–112,115,118,119}

It is well known that the CN stretching motion plays an important role in the nonadiabatic decay dynamics of CH_2NH_2^+ ,^{106,108,112,127} when the dynamics starts from S_2 . To properly show the CN bond stretching motion in the dynamics starting from S_2 , we adopted the contour presentation in Figure 2. It is obvious that two types of trajectories (labeled as I and II) exist during the dynamical run. The first group (I) of trajectories displays significant CN elongation, while for the second group (II) of trajectories, the CN bond only experiences weak oscillation patterns. Consistent with previous FSSH results,¹¹¹ we also found that the first group (I) of trajectories shows bipyramidalization at both C and N sites in the CN elongation process (discussion given below), while the second group (II) of trajectories displays highly mixed motions of the bipyramidalization motions at both C and N sites and the torsional motion of the CN bond.

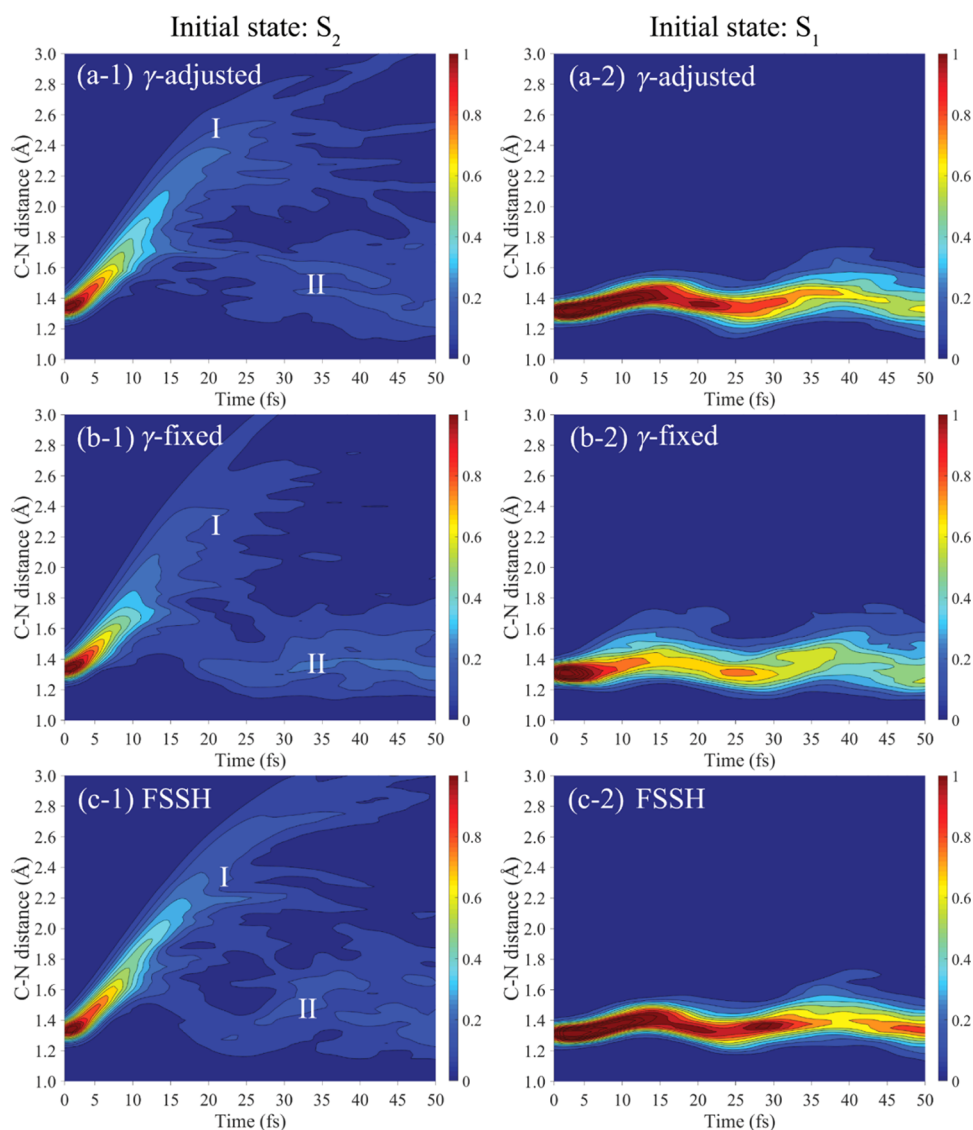


Figure 2. Time-dependent CN distance distribution of CH_2NH_2^+ in the (a) γ -adjusted, (b) γ -fixed SQC/MM, and (c) FSSH dynamics. The initial states are (1) S_2 and (2) S_1 , respectively. A total of 200 trajectories were used for all dynamics. For the dynamics starting from S_2 , two types of trajectory I and II are given in the subfigures a-1, b-1, and c-1.

It is necessary to know whether the results are different if we only include the data points in the windows during the trajectory propagation. We plotted the time-dependent CN distance distribution under such a constraint in Figure S4 (SI), and the results are very similar.

Here, we wish to know whether a similar S_0/S_1 CI seam is reached in the SQC/MM and FSSH dynamics. To perform a more “direct” comparison, we first found the geometry with the minimum S_0/S_1 energy gap in the FSSH dynamics, before the trajectory jumped to S_0 . Next, we tried to extract the geometry with the minimum S_0/S_1 energy gap in the SQC/MM dynamics before the trajectory entered the window belonging to S_0 . For all dynamics approaches, we did not consider the trajectory that did not return to S_0 . The geometrical distributions of these minimum-energy-gap structures are plotted in Figure 3. The key role of the CN bond stretching motion is confirmed, while the strong bipyramidalization motions at both C and N sites are also important. For the nonadiabatic dynamics starting from S_2 , all dynamics approaches show broad distributions of the torsional

angles along the CN motion. However, it is clear that the γ -adjusted SQC/MM and FSSH dynamics give rather similar descriptions of the CN stretching motion and the bipyramidalization motion.

To check whether the single minimum-energy-gap structure in each trajectory can represent the key geometries that play important roles in the nonadiabatic decay process of CH_2NH_2^+ , we also extracted the five geometries with the smallest energy gaps for each trajectory before it reached S_0 and plotted the distributions of key DoFs in Figure S5 (SI). The results remain similar.

For the CH_2NH_2^+ nonadiabatic dynamics starting from S_2 , the nonadiabatic evolutions given by the three dynamics approaches are not far from each other, while some minor differences exist. For instance, the time-dependent populations predicted by the two SQC/MM dynamics are more similar, while slightly different results are obtained in the FSSH dynamics. As a contrast, we notice that the distribution of minimum-energy-gap structures from the γ -adjusted SQC/MM dynamics is more similar to the FSSH dynamics, while

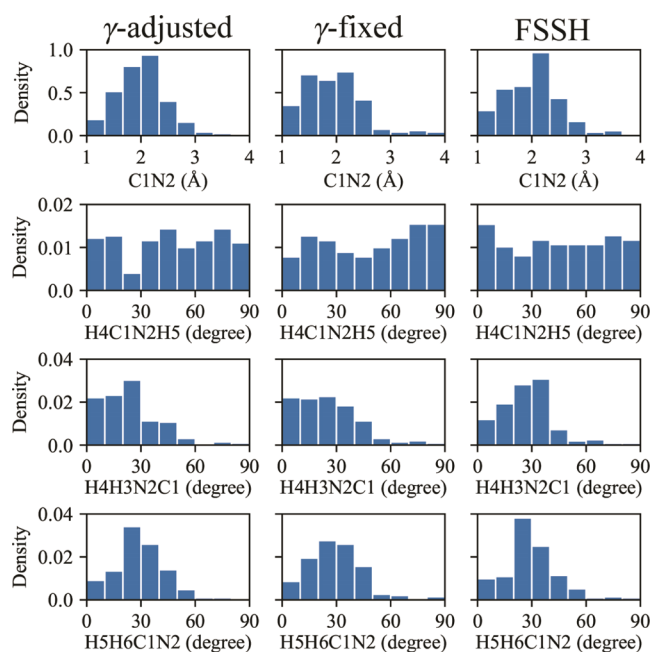


Figure 3. Normalized distribution of key DoFs: bond length C1N2 (CN stretching), dihedral angle H4C1N2H5 (CN torsion angle), dihedral angle H4H3N2C1 (pyramidalization at C atom), and dihedral angle H5H6C1N2 (pyramidalization at N atom) of the CH_2NH_2^+ molecule at S_1/S_0 minimum-energy-gap structures in γ -adjusted SQC/MM, γ -fixed SQC/MM, and FSSH dynamics starting from S_2 .

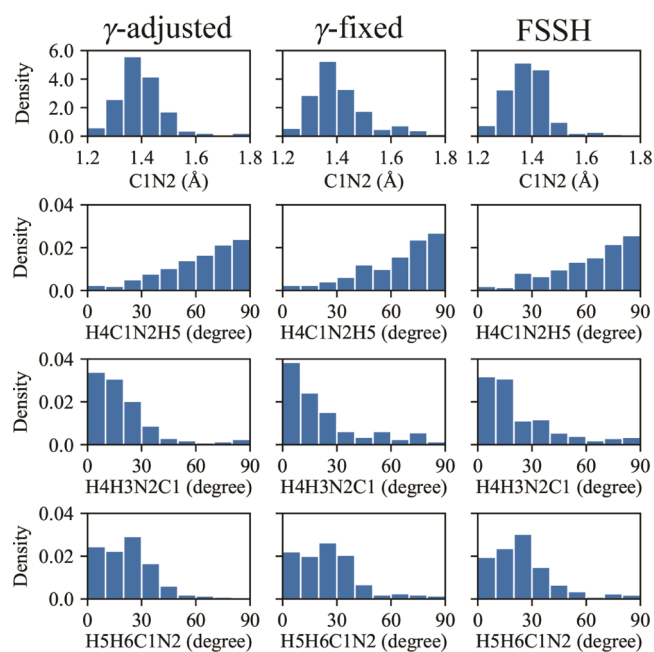


Figure 4. Normalized distributions of key DoFs: bond length C1N2 (CN stretching), dihedral angle H4C1N2H5 (CN torsion angle), dihedral angle H4H3N2C1 (pyramidalization at the C atom), and dihedral angle H5H6C1N2 (pyramidalization at the N atom) of the CH_2NH_2^+ molecule at S_1/S_0 minimum-energy-gap structures in γ -adjusted SQC/MM, γ -fixed SQC/MM, and FSSH dynamics starting from S_1 .

different distribution patterns were found in the γ -fixed SQC/MM dynamics.

3.1.2. Nonadiabatic Dynamics from S_1 . When the initial state is S_1 , exponential decay of S_1 is observed with a lifetime of ~ 27 fs; see Figure 1b. During the whole decay dynamics, S_2 is only weakly populated. Including S_2 in the dynamics calculations does not change the population dynamics significantly; see Figure S6 in SI. It is important to notice that the geometrical evolution in this case is quite different with respect to the dynamics starting from S_2 . For most trajectories, the CN distance remains relatively short (about 1.2–1.6 Å) during the evolution; see Figure 2. All dynamics approaches predicted that bipyramidalization at both C and N sites and the torsional motion of the CN bond are responsible for the $S_1 \rightarrow S_0$ decay, as shown in Figure 4. We also noticed that a similar distribution pattern of the minimum-energy-gap structures in the trajectory propagation was predicted by all dynamics methods.

The same conclusion is drawn when five minimum-energy-gap structures are considered for each trajectory; see Figure S7 in SI.

Irrespective of whether the dynamics starts from S_1 or S_2 , the dynamics features, including excited-state lifetimes, the major reactive molecular motion in the excited-state nonadiabatic dynamics, and the geometrical feature in the relevant CI seam, are similar in description by different dynamics methods used in this work, the γ -adjusted and γ -fixed SQC/MM and FSSH; see Figures 1–4. In addition, the dynamics results are highly consistent with previous on-the-fly studies with various methods, such as the TSH dynamics,^{106,108–112,118,128} and the more rigorous TSH approach from the exact factorization of the electronic-nuclear wavefunction.³⁰

Both SQC/MM and FSSH approaches should be rather important in the on-the-fly dynamics community. In the current work, the dynamics results from SQC/MM and FSSH are very similar for CH_2NH_2^+ , while these results are not trivial and contain important meanings. For instance, this work confirms that the on-the-fly SQC/MM dynamics can be used to describe the nonadiabatic dynamics of CH_2NH_2^+ , a prototype system to test the on-the-fly nonadiabatic dynamics methods. This gives us the confidence to use this approach in further studies of realistic systems. At the same time, the current work also gives additional evidence to support the previous understanding of the excited-state process of CH_2NH_2^+ by the TSH method,^{106,108–112,118,128} making sure that proper description of the nonadiabatic dynamics of CH_2NH_2^+ is achieved. In fact, this type of comparison is very important for the general on-the-fly simulation. Only when different dynamics simulations give consistent results we gain confidence that the nonadiabatic dynamics is properly described theoretically.

3.2. Azomethane. Azomethane is often used as a chemical source for radical generation since it can decompose into alkyl radicals and nitrogen after absorption of ultraviolet light in the gas phase.¹²⁹ Previous theoretical studies with the FSSH and AIMS methods showed that azomethane undergoes an ultrafast nonadiabatic decay process after excitation.^{120,121} As a typical system for benchmark, we chose azomethane to examine the SQC/MM dynamics. Here, we used the CASSCF level with SA-2-CAS(6,4) in the SQC/MM and FSSH dynamics, and the same electronic structure level was employed in a previous AIMS study.¹²⁰

For azomethane, only two states, S_1 and S_0 , are involved in the nonadiabatic dynamics process in this work. When the systems are put in S_1 , most of the trajectories decay to S_0

within 350 fs for γ -adjusted SQC/MM dynamics, consistent with our own FSSH dynamics and previous AIMS dynamics.^{120,121} While for the γ -fixed SQC/MM dynamics, about half of the trajectories still stay at S_1 at 350 fs; see Figure 5a. This

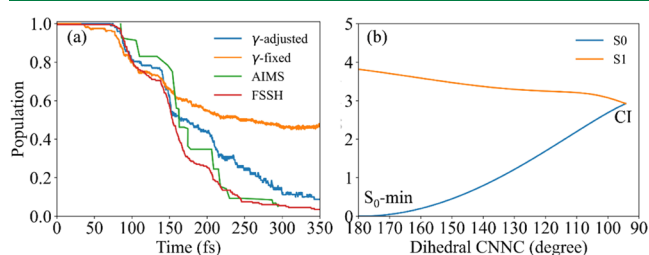


Figure 5. (a) Time-dependent electronic populations of S_1 with different dynamics methods. A total of 200 trajectories are used in the SQC/MM and FSSH dynamics. The results of the AIMS dynamics are taken from a previous study.¹²⁰ (b) Linear interpolated potential energy curve from S_0 minimum to CI.

phenomenon is very unlike that of CH_2NH_2^+ , where the time-dependent electronic populations of S_1 of SQC/MM dynamics with both γ -adjusted and γ -fixed versions are very close (Figure 1).

To understand the general feature of nonadiabatic dynamics of azomethane, we optimized the S_1/S_0 CI and plotted the linear interpolated potential energy curve from S_0 minimum to CI; see Figure 5b. Consistent with a previous study,¹²⁰ there is no barrier along the photoisomerization pathway.

Since the rotation of the central N–N double bond, characterized by the dihedral angle of CNNC, plays an

important role in the nonadiabatic decay of azomethane, the time-dependent evolution of the dihedral angle of CNNC for all trajectories of SQC/MM dynamics with γ -adjusted and γ -fixed is plotted; see Figure 6. The dihedral angles after the minimum-energy-gap structures are not plotted for a better view of the trajectory propagation in the excited state from the Franck–Condon (FC) region to the S_1/S_0 CI.

Clearly, the trajectory must experience a strong twisting motion of the CNNC dihedral angles if it goes to S_0 via the S_0/S_1 CI. Otherwise, the trajectory remains on the excited state, and no S_1 population decays are observed. For γ -adjusted dynamics, most trajectories show the strong rotation of the NN double bond, and the trajectory can access these geometries with a very small energy gap. This means that the majority of the trajectories follow isomerization channels and decay to S_0 via relevant CIs. While for γ -fixed dynamics, a large number of trajectories do not show a significant CNNC twisting motion but display an oscillation of the CNNC angle at around 180 (-180)°. These trajectories simply stay on S_1 and do not decay to S_0 at all.

Figure 5b shows that the energy of the S_0/S_1 CI is lower than that of S_1 at the S_0 -min geometry. No energy barrier exists along the isomerization reaction pathway. Thus, the trajectory should naturally follow the barrierless pathway and access the S_0/S_1 CI without any doubt. However, for γ -fixed dynamics, a large number of trajectories seem to just stay in S_1 . This problematic feature can be explained by the fact that the nuclear motion is propagated by a kind of mean potential energy surface (PES) instead of S_1 PES even before the trajectory reaches the S_0/S_1 CI. In γ -fixed dynamics, both S_1 and S_0 were involved in the nonadiabatic dynamics of

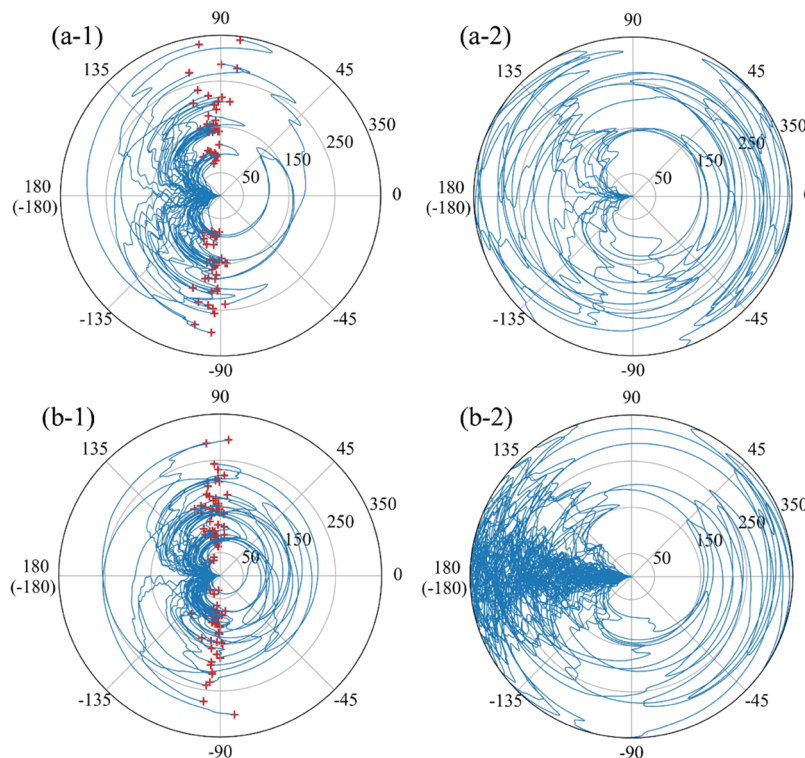


Figure 6. Time-dependent evolutions of the dihedral angle of CNNC for all trajectories before decaying to S_0 of (a) γ -adjusted and (b) γ -fixed SQC/MM dynamics. The trajectories that decay and do not decay to S_0 within 350 fs are shown in (1) and (2), respectively. The red points indicate the S_1/S_0 minimum-energy-gap structures. The radial and angular coordinates in the polar coordinates represent the time (fs) and dihedral angle (degree), respectively.

azomethane, and the nuclear motion is governed by the effective potential

$$V_{\text{eff}} = n_0 E_0(\mathbf{R}) + n_1 E_1(\mathbf{R}) \quad (14)$$

where

$$\begin{aligned} n_0 &= \frac{1}{2}((x_0)^2 + (p_0)^2) - \gamma \\ n_1 &= \frac{1}{2}((x_1)^2 + (p_1)^2) - \gamma \end{aligned} \quad (15)$$

are time-dependent action variables of electronic states for each trajectory. Please note that the effective potential on the nuclear part given in eq 14 is rather approximated when the symmetrized form of the effective potential (eq 4) is employed. For each trajectory, only when $n_0 + n_1 = 1$, eq 4 becomes completely equivalent to eq 14. However, this condition is not always true for each trajectory since n_0 and n_1 are sampled independently in standard γ -fixed SQC dynamics. This feature has been carefully discussed in detail by several previous studies.^{77,78,92} However, the essential ideas behind them should be rather similar, and thus we still can use eqs 14 and 15 for the below discussions.

Certainly, the effective potential given in eq 14 defines the nuclear motion at the very beginning of the dynamics. Because two action variables were obtained stochastically by the sampling procedure, the effective potential is the linear combination of the S_0 and S_1 potentials. As a consequence, the BO dynamics is not recovered for the single trajectory, while in principle the early-stage dynamics should follow the S_1 surface due to the negligible S_0/S_1 NAC in the FC region. In azomethane, S_0 is a strong bounded state near the S_0 minimum geometry. If the contribution of n_0 is large enough, the effective potential may become a bounded potential that prevents the trajectory from escaping from the FC region. To clarify this idea, we plotted the initial sampled action variables and labeled their final assignment in Figure 7.

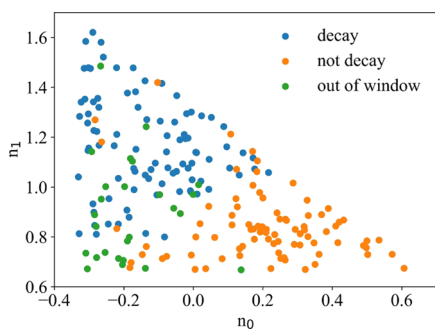


Figure 7. Distribution of the initial sample n_0 and n_1 of all trajectories for γ -fixed dynamics. The labels “decay” and “not decay” imply that the trajectory decays and does not decay to S_0 within 350 fs in the γ -fixed SQC/MM dynamics, respectively. The label “out of window” implies that the action values of the trajectory are not in any triangle windows at the end of the dynamics.

It is clear that the trajectories are more likely to stay in the excited state when n_0 is large. In this condition, the effective PES might become bounded, and the trajectory cannot arrive at the CI region. This explains why many trajectories do not decay in Figure 5a.

As a contrast, in the γ -adjusted situation, the ZPE correction value of each trajectory is determined by its own initial

condition. In such a case, the early-time dynamics almost becomes the pure BO dynamics when the NAC is small enough under the large S_0/S_1 energy gap. In this way, most trajectories may follow the correct potential and access the CI. This explains the fast and completed population decay of the γ -adjusted SQC/MM dynamics in Figure 5a.

Clearly, the γ -adjusted SQC/MM dynamics result is much closer to that of the AIMS dynamics in Figure 5a;¹²⁰ we suggest that this γ -adjusted approach should always be used. In addition, this choice is consistent with physical insight. When the ground state and excited states are well separated in the energy domain in the FC region, the early-time excited-state dynamics should essentially be the BO dynamics due to the extremely weak NAC. Thus, the employment of the γ -adjusted approach allows for the trajectory propagation to recover this limit. This avoids the possible deficiency of the effective potential acting on the nuclear propagation in the γ -fixed approach. This observation is consistent with the ideas by Cotton and Miller,⁹² who strongly suggested using the γ -adjusted approach in the study of the gas-phase photochemistry of the molecular systems without too many DoFs.

3.3. Discussions. Here, we wish to give a few additional comments and discussions on the current work.

The understanding of the excited-state nonadiabatic dynamic is rather challenging for the realistic molecules. Many ultrafast spectroscopic experiments give time-resolved spectra. To understand the nonadiabatic dynamics and analyze such spectra, theoretical simulations are necessary. To describe the realistic systems with many DoFs and arbitrary PESs, on-the-fly nonadiabatic dynamics simulation becomes irreplaceable. Different electronic methods have different advantages and shortcomings, and previous studies have shown that the on-the-fly dynamics with different electronic structure methods may give different results.^{107,130} In principle, the same situations exist for the nonadiabatic dynamics methods.

For surface hopping dynamics, the over-decoherence problem is well known.^{19,20,29} Although many theoretical approaches were proposed to add the decoherence corrections and many on-the-fly implementation studies used them for practical reasons,^{19–21,29,106,127,131} many corrections are rather phenomenological and the empirical parameters are often added. The underlying reason is that researchers have still not found a formal way to derive the surface hopping dynamics from rigorous semiclassical approximations. Thus, the improvement of the performance of surface hopping dynamics is rather challenging. As a contrast, the SQC/MM dynamics can more or less capture the electronic coherence without the inclusion of empirical corrections.^{81,86} More importantly, the SQC/MM approach can be improved systematically by adding more quantum corrections based on more rigorous semiclassical dynamics. In the SQC/MM dynamics, the trajectories are independently propagated, and thus the computational cost is in principle smaller than that of the wavepacket-based methods, such as AIMS and vMCG.

Only when we combine various dynamics approaches and different electronic structure methods in the on-the-fly simulation is it possible to construct correct and comprehensive understandings of the nonadiabatic procedures of polyatomic systems. For example, when the on-the-fly simulations with different dynamics and electronic structure methods give consistent results, we more or less believe that the proper description of the nonadiabatic dynamics is achieved. This situation is observed in our simulation of

CH_2NH_2^+ . Certainly, when the results are strongly dependent on the selected dynamics methods, more additional theoretical and experimental studies should be performed. In addition, when different problems are concerned, it may be suitable to choose proper methods. Thus, it is always worthwhile to implement different dynamics approaches in on-the-fly simulation. Previous studies have already demonstrated that the SQC/MM dynamics is a rather promising approach;^{38,73–90} therefore, it is highly preferable to introduce it in on-the-fly simulation.

In the γ -fixed SQC/MM dynamics, we employed the triangle window proposed by Miller and Cotton.^{80,91} They recommended the triangle window because it gives a similar description for normal nonadiabatic dynamics compared with the SQC/MM dynamics with the rectangle window, while it gives a better description in the weak diabatic coupling situation. In such studies, they also strongly suggest using $\gamma = 1/3$ in the application of the triangle window due to the geometrical reason.^{80,91} Here, we preferred to use this ZPE correction instead of modifying its value due to the following reasons.

For the γ -fixed SQC/MM dynamics with the square window or its extensions, several studies have clearly demonstrated that the results are strongly dependent on the selection of the γ value.^{77,105} In principle, a similar conclusion should be reached in the on-the-fly implementation of this approach. However, in on-the-fly simulation, it is not possible to choose the γ value by running many preliminary calculations because of the huge computational costs and the lack of accurate results. In addition, a very narrow window may give better results for anharmonic systems,⁷⁷ while in this setup, a large number of trajectories go out of windows and it is not easy to converge calculations. In this sense, it is more important to select a reasonable γ value within a proper window before the treatment of realistic systems. Since the γ value can be derived by the geometry feature for the triangle window,^{80,91} we think that it is suitable to simply use it instead of adjusting its value.

At the same time, we also realize that the γ -adjusted SQC/MM dynamics always results in much better dynamics, particularly for the second test system. This again confirms that we should use the γ -adjusted SQC/MM dynamics in future on-the-fly simulations. Thus, the examination of the influence of the γ value for the γ -fixed SQC/MM dynamics is not essential in the current work.

4. CONCLUSIONS

In this work, we tried to implement the on-the-fly SQC/MM dynamics. The SQC/MM dynamics formalism in the adiabatic representation provides a rather clear idea for direct combination with the electronic structure calculations. We chose CH_2NH_2^+ and azomethane as typical examples to examine the performance of the on-the-fly SQC/MM dynamics. The results show that all dynamical features of the nonadiabatic dynamics of CH_2NH_2^+ are well captured by the on-the-fly SQC/MM dynamics, irrespective of using the γ -adjusted or γ -fixed approach, while for azomethane, the γ -adjusted SQC/MM dynamics performs much better than the γ -fixed SQC/MM dynamics. The reason for this is that the former can recover the BO dynamics features at the beginning of the trajectory when the electronic states are far from each other, while the latter always uses a kind of mean PES. As a result, the γ -adjusted approach is strongly recommended to be used in all on-the-fly SQC/MM nonadiabatic dynamics.

This work demonstrates that the current on-the-fly SQC/MM dynamics provides a promising approach to simulate the nonadiabatic dynamics of polyatomic molecules. When the MM Hamiltonian is considered, the SQC/MM dynamics is a simple dynamics approach, which serves as a zero-order approximation of more advanced semiclassical dynamics methods with the inclusion of quantum effects. Starting from the SQC/MM dynamics, it is possible to obtain other sophisticated semiclassical methods with different accurate levels by introducing more quantum correction terms. Thus, the current implementation work provides a good starting point to incorporate more rigorous semiclassical dynamics in on-the-fly dynamics simulation.

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at <https://pubs.acs.org/doi/10.1021/acs.jctc.0c01249>.

Convergence tests of the number of trajectories; time steps of nuclear and electronic motions; time-dependent CN distance distribution spanned by trajectories in the window; and key DoF distributions of five S_1/S_0 minimum-energy-gap structures and time-dependent electronic populations with and without S_2 in the dynamics starting from S_1 (PDF)

■ AUTHOR INFORMATION

Corresponding Author

Zhenggang Lan – SCNU Environmental Research Institute, Guangdong Provincial Key Laboratory of Chemical Pollution and Environmental Safety & MOE Key Laboratory of Environmental Theoretical Chemistry, South China Normal University, Guangzhou 510006, China; School of Environment, South China Normal University, Guangzhou 510006, China; Email: zhenggang.lan@m.scnu.edu.cn, zhenggang.lan@gmail.com

Authors

Deping Hu – SCNU Environmental Research Institute, Guangdong Provincial Key Laboratory of Chemical Pollution and Environmental Safety & MOE Key Laboratory of Environmental Theoretical Chemistry, South China Normal University, Guangzhou 510006, China; School of Environment, South China Normal University, Guangzhou 510006, China; orcid.org/0000-0001-7161-1253

Yu Xie – SCNU Environmental Research Institute, Guangdong Provincial Key Laboratory of Chemical Pollution and Environmental Safety & MOE Key Laboratory of Environmental Theoretical Chemistry, South China Normal University, Guangzhou 510006, China; School of Environment, South China Normal University, Guangzhou 510006, China

Jiawei Peng – SCNU Environmental Research Institute, Guangdong Provincial Key Laboratory of Chemical Pollution and Environmental Safety & MOE Key Laboratory of Environmental Theoretical Chemistry, South China Normal University, Guangzhou 510006, China; School of Environment, South China Normal University, Guangzhou 510006, China

Complete contact information is available at: <https://pubs.acs.org/doi/10.1021/acs.jctc.0c01249>

Author Contributions

[§]D.H. and Y.X. contributed equally to this work.

Notes

The authors declare no competing financial interest.

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