

Hidden Markov Model

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Oct. 2018

1 Basic Concepts & Examples

1.1 Basic Concepts

Definition 1.1 Assume $X = \{X_n; n \geq 1\}$ is a Markov chain over finite state space $\Phi = \{1, 2, \dots, K\}$, if states of X are unobservable, $Y = \{Y_n; n \geq 1\}$ is an observable random variable series over finite set $V = \{v_1, v_2, \dots, v_L\}$ that is correlated with X , then $(X, Y) = \{(X_n, Y_n); n \geq 1\}$ is a *Hidden Markov Chain*.

Denote $\pi = \{\pi_1, \pi_2, \dots, \pi_k\}$ is the initial distribution of X , $A = [a_{ij}]$ is the transition matrix of X

$$a_{ij} = P\{X_{n+1} = j | X_n = i\}, i, j \in \Phi \quad (1)$$

is the one-step transition probability of X . Denote

$$b_{ij} = P\{Y_n = v_j | X_n = i\}, i \in \Phi, v_j \in V \quad (2)$$

represents the probability that Y equals to v_j given X is at state i at time n . If X is homogeneous, then Y is also irrelevant with time n . Denote $B = [b_{ij}]$ as the observation probability matrix. Due to the unobservability of X , π, A, B can not be directly measured. Generally, we call parameter set $\lambda = \{\pi, A, B\}$ the math model of Hidden Markov chain (X, Y) , as well as *Hidden Markov Model* (HMM).