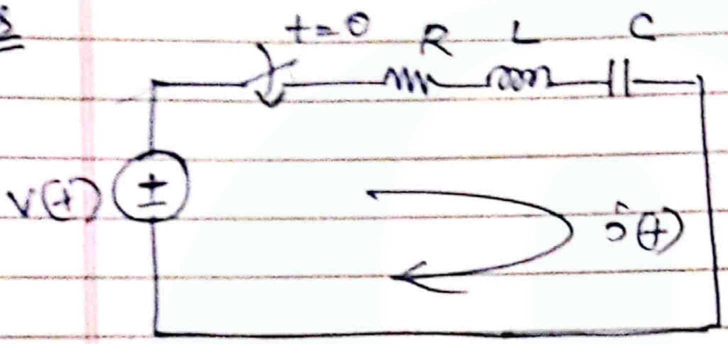
Module 3

Transformed circuits in s-domain

1. A time dependent voltage v(t) is applied to a series RLC circuit. Find s-domain impedance & current. Draw the t-domain & s-domain circuits.

Ans



Applying KVL at t=0

$$v(t) = Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int i(t)\,dt + V(0)$$

Taking LT,

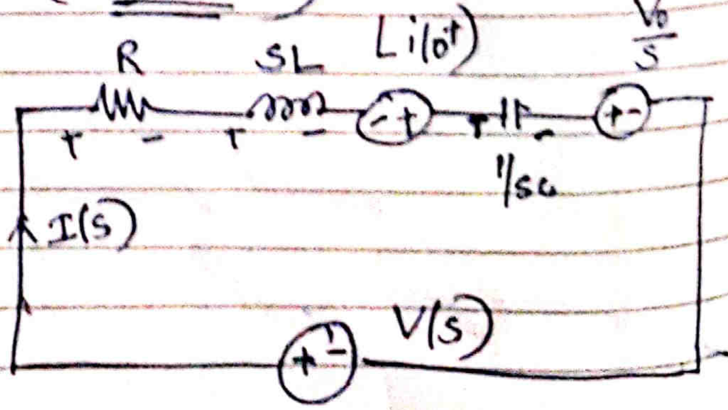
$$V(s) = RI(s) + sLI(s) - Li(0^+) + \frac{1}{C}\frac{1}{s}I(s) + \frac{V(0)}{s}$$

$$= I(s)\left[R + sL + \frac{1}{sC}\right] - Li(0^+) + \frac{V(0)}{s}$$

$$\therefore I(s) = \frac{V(s) + Li(0^+) - \frac{V(0)}{s}}{\left(R + sL + 1/sC\right)}$$

Impedance $Z(s) = \left(R + sL + 1/sC\right)$

S-domain ckt



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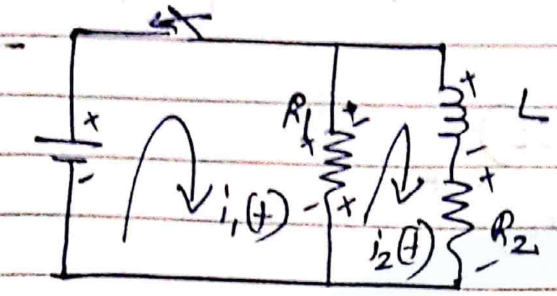
(2) A two-mesh n/w is given. Obtain expression for $I_1(s)$ & $I_2(s)$, when the switch is closed.



Applying KVL,

$$V - R_1(i_1(t) - i_2(t)) = 0$$

$$-R_1(i_2(t) - i_1(t)) - L\frac{di_2(t)}{dt} - R_2 i_2(t) = 0$$

Taking LT,

$$\frac{V}{s} = R_1 I_1(s) - R_1 I_2(s) \quad \text{---} (1)$$

$$-R_1 I_2(s) + R_1 I_1(s) - (LsI_2(s) - Li_2(0^+)) - R_2 I_2(s) = 0$$

$$-R_1 I_1(s) + I_2(s)\begin{bmatrix} R_1 + sL \\ + R_2 \end{bmatrix} = +Li_2(0^+) \quad \text{---} (2)$$

Write these two equations in matrix form,

$$\begin{bmatrix} V/s \\ -Li_2(0^+) \end{bmatrix} = \begin{bmatrix} R_1 & -R_1 \\ -R_1 & +(R_1+R_2+sL) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$i_2(0^+) = 0$

$$I_1(s) = \frac{\begin{vmatrix} V/s & -R_1 \\ 0 & +(R_1+R_2+sL) \end{vmatrix}}{\begin{vmatrix} R_1 & -R_1 \\ -R_1 & +(R_1+R_2+sL) \end{vmatrix}}$$

$$= \frac{+V/s\,(R_1+R_2+SL)}{+R_1(R_1+R_2+SL) - R_1^2} = \frac{+V(R_1+R_2+SL)}{S(+R_1^2+R_1R_2+R_1SL - R_1^2)}$$

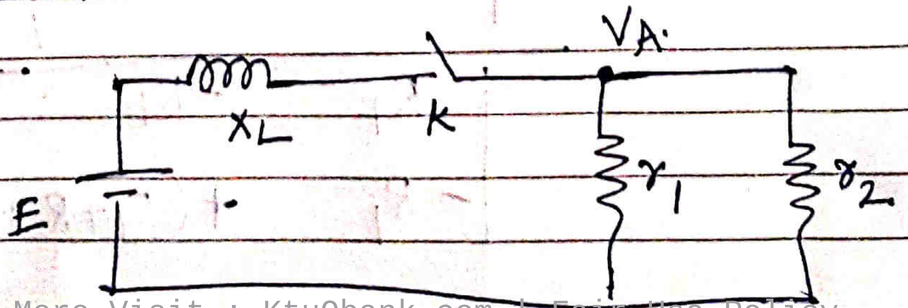
$$= \frac{+V(R_1+R_2+SL)}{S(R_1R_2+R_1SL)}$$

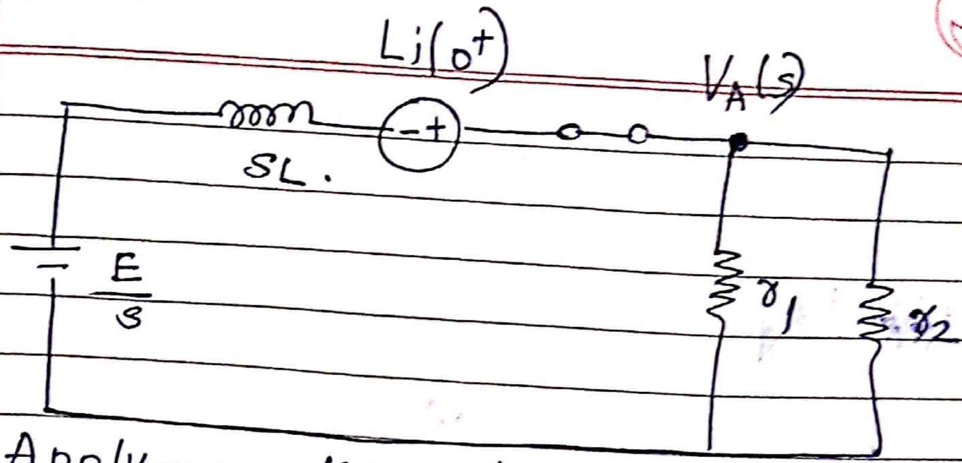
$$I_2(s) = \frac{\begin{vmatrix} R_1 & V/s \\ -R_1 & 0 \end{vmatrix}}{\begin{vmatrix} R_1 & -R_1 \\ -R_1 & R_1+R_2+SL \end{vmatrix}}$$

$$= \frac{R_1 \times 0 - -R_1 V/s}{R_1(R_1+R_2+SL) - R_1^2} = \frac{R_1 V/s}{R_1^2+R_1R_2+R_1SL-R_1^2}$$

$$= \frac{R_1 V}{S(R_1R_2+R_1SL)} = \frac{V}{s(R_2+SL)}$$

3. Apply nodal analysis to find the voltage at node A in the ckt. in S-domain. Assume initial current through the inductor as zero.





Applying KCL at node A,

$$\frac{V_A(s)}{r_1} + \frac{V_A(s)}{r_2} + \frac{V_A(s) - Li(0^+) - E/s}{SL} = 0.$$

$$V_A(s)\left[\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{SL}\right] - \frac{Li(0^+) - E/s}{SL} = 0.$$

$i(0^+) = 0$

$$V_A(s) = \frac{E/s}{SL\left[\frac{1}{r_1} + \frac{1}{r_2} + 1/SL\right]}.$$

$$= \frac{E/s}{\cancel{SL}\left[\frac{r_2 SL + r_1 SL + r_1 r_2}{r_1 r_2 \cancel{SL}}\right]} = \frac{E\, r_1 r_2}{S(r_1 SL + r_2 SL + r_1 r_2)}.$$

$M = k\sqrt{L_1 L_2}$, $\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$

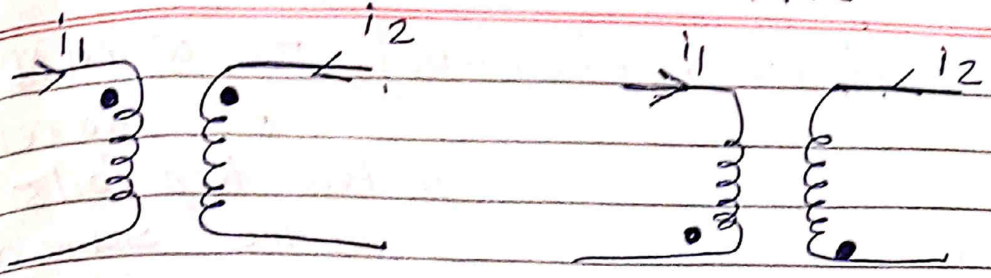
Dot - Convention

* Self induced Voltages are assigned with +ve sign. Mutually induced voltages may be either +ve / -ve depending upon the direction of the winding of the coil. and can be decided by the presence of dots placed at one end of each of the two coils.

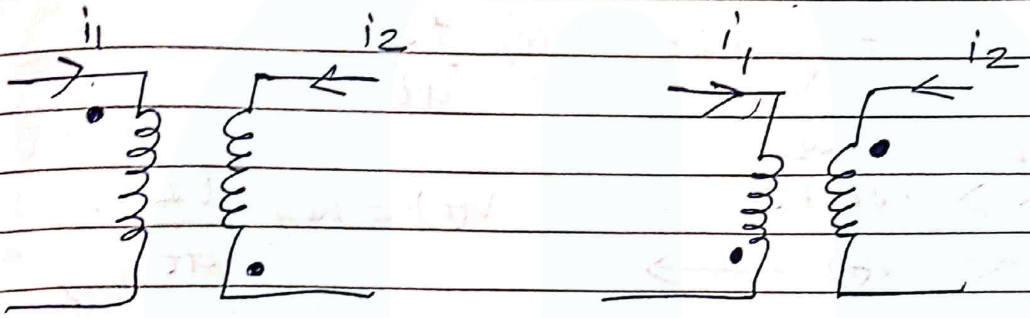
* To determine the relative polarity of the induced voltage in the coupled coil, the coils are marked with dots. This is known as Dot Convention or Dot Rule.

If the currents in two mutually coupled coils enter the dotted ends or leaves the dotted ends, M terms will have the same sign as L terms.

M +ve M +ve.

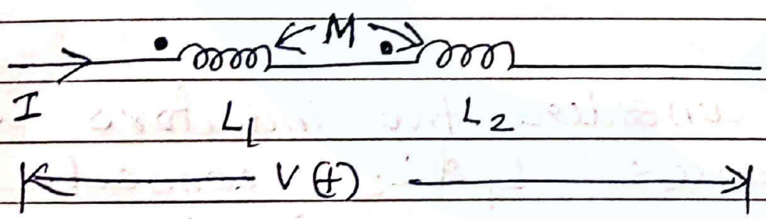


If the current in one coil enters the dotted ends and in the other current leaves the dotted ends, M terms will have the opposite sign as L terms.



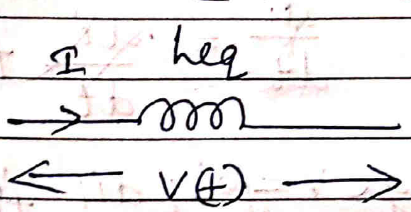
M -ve M -ve

Coupled circuits in Series.



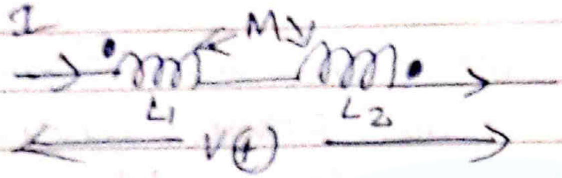
$$V(t) = L_1 \frac{dI}{dt} + M\frac{dI}{dt} + L_2\frac{dI}{dt} + M\frac{dI}{dt}$$

$$= (L_1 + L_2 + 2M)\frac{dI}{dt}$$



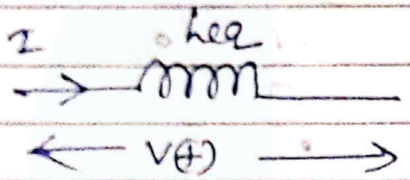
$$V(t) = L_{eq}\frac{dI}{dt}.$$

$\boxed{L_{eq} = L_1 + L_2 + 2M}$ — when M +ve or currents in the two coils are in the same direction or aiding.



$$V(t) = L_1 \frac{dI}{dt} - M\frac{dI}{dt} + L_2 \frac{dI}{dt} - M\frac{dI}{dt}$$

$$= (L_1 + L_2 - 2M)\frac{dI}{dt}$$

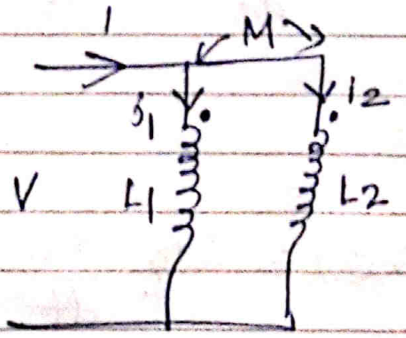


$$V(t) = L_{eq}\frac{dI}{dt}$$

$\boxed{\therefore\ L_{eq} = L_1 + L_2 - 2M}$ when M is -ve. or the currents are opposing.

Parallel

Consider two inductors with self inductances L_1 & L_2 connected parallel which are mutually coupled with mutual inductance M.



$i = i_1 + i_2$

$\cancel{\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}}$ -(1)

Voltage $V = j\omega L_1 \frac{di_1}{dt} + j\omega M \frac{di_2}{dt}$

Ktu Q bank

$V = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$

$\cancel{L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}}$

$\frac{di_1}{dt}$

$V = j\omega L_1 I_1 + j\omega M I_2$ — (1)

Similarly,

$V = j\omega L_2 \frac{di_2}{dt} + j\omega M \frac{di_1}{dt}$

$V = j\omega L_2 I_2 + j\omega M I_1$ — (2)

writing eqns (1) & (2) in matrix form,

$$\begin{bmatrix} V \\ V \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} V & j\omega M \\ V & j\omega L_2 \end{vmatrix}}{\begin{vmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{vmatrix}} = \frac{V j\omega L_2 - V j\omega M}{(j\omega L_1)(j\omega L_2) - (j\omega M)^2}$$

$$= \frac{j\omega V (L_2 - M)}{-\omega^2 L_1 L_2 + \omega^2 M^2} = \frac{j\omega V (L_2 - M)}{\omega^2 (M^2 - L_1 L_2)}$$

Similarly, $I_2 = \frac{j\omega V (L_1 - M)}{\omega^2 (M^2 - L_1 L_2)}$

$$I = I_1 + I_2.$$

$$\left(\begin{aligned} i &= i_1 + i_2 \\ \frac{di}{dt} &= \frac{di_1}{dt} + \frac{di_2}{dt} \\ I &= I_1 + I_2 \end{aligned} \right)$$

$$= \frac{j\omega(L_2 - M)V}{\omega^2(M^2 - L_1L_2)} + \frac{j\omega(L_1 - M)V}{\omega^2(M^2 - L_1L_2)}$$

$$= \frac{j\omega V(L_2 - M + L_1 - M)}{\omega^2(M^2 - L_1L_2)} = \frac{j\omega V(L_1 + L_2 - 2M)}{\omega^2(M^2 - L_1L_2)}$$

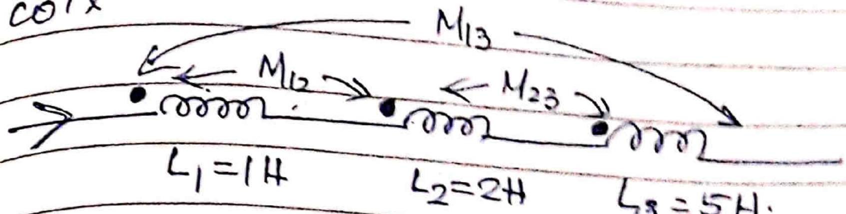
$$Z = \frac{V}{I} = \frac{V}{\dfrac{j\omega V(L_1 + L_2 - 2M)}{\omega^2(M^2 - L_1L_2)}}$$

$$= \cancel{j\omega \frac{(L_1 + L_2 - 2M)}{L_1L_2 - M^2}} = j\omega\left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)$$

$$\boxed{L_{eq} = \frac{L_1 * L_2 - M^2}{L_1 + L_2 - 2M}} \text{ – parallel aiding}$$

$$\boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}}, \text{ parallel opposing.}$$

1. Find out the equivalent inductance of the coil.

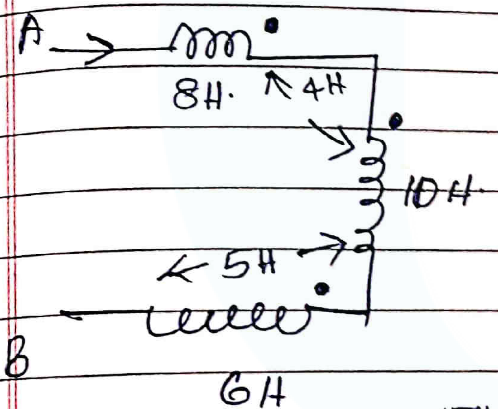


$M_{12} = 0.5H$.
$M_{23} = 1H$.
$M_{13} = 1H$.

$L' = L_1 + L_2 \pm 2M$
$= 1 + 2 + 2 \times 0.5 = 4H$

$L'' = 4 + 5 + 2(M_{13} + M_{23}) = 4 + 5 + 2 \times (1+1)$
$= \underline{\underline{13H}}$.

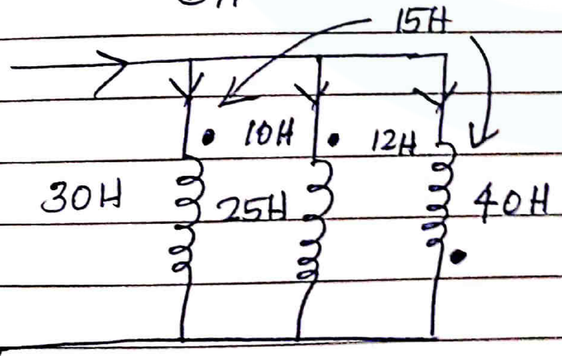
2. Calculate the effective inductance of the ckt across AB.



$L' = 8 + 10 - 4 \times 2$
$= 18 - 8 = \underline{\underline{10H}}$

$L'' = 10 + 2 \times 5 + 6$
$= 16 + 10 = \underline{\underline{26H}}$

3.



$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$.

$L_1 = 30 + 10 - 15 = 25H$
$L_2 = 25 + 10 - 12$
$= \underline{23H}$

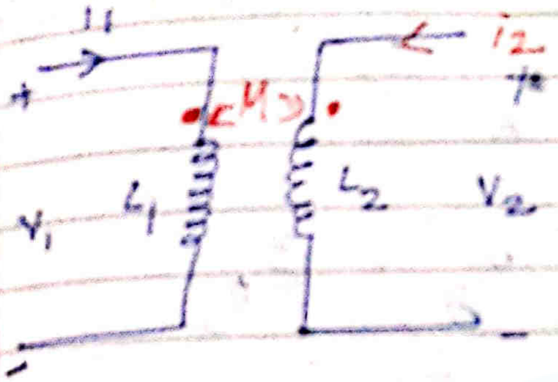
$L_3 = 40 - 15 - 12 = \underline{\underline{13H}}$.

$L_T = \frac{1}{25} + \frac{1}{23} + \frac{1}{13} = \underline{\underline{6.24H}}$.

Analysis of Coupled Ckts Using Dot Rule

M – +ve

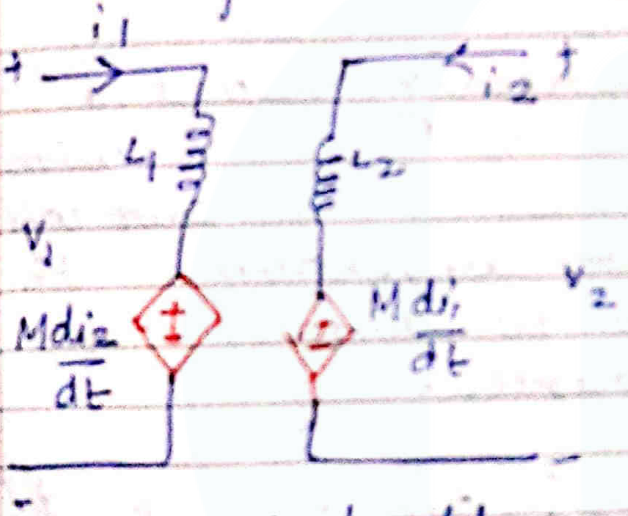
(1)



$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

We can represent using dependent sources.

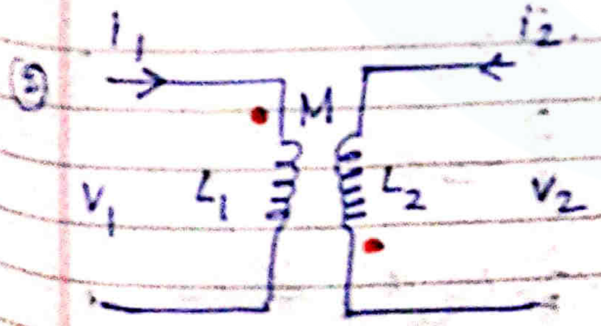


Time domain equivalent ckt

Equivalent ckt

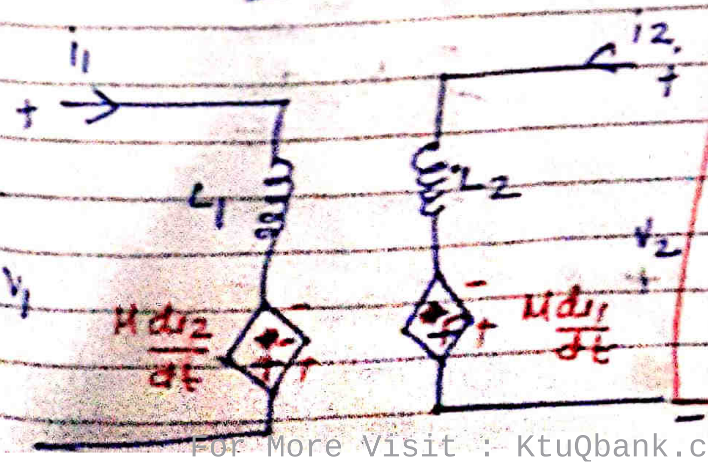
$\boxed{M\text{ –ve}}$

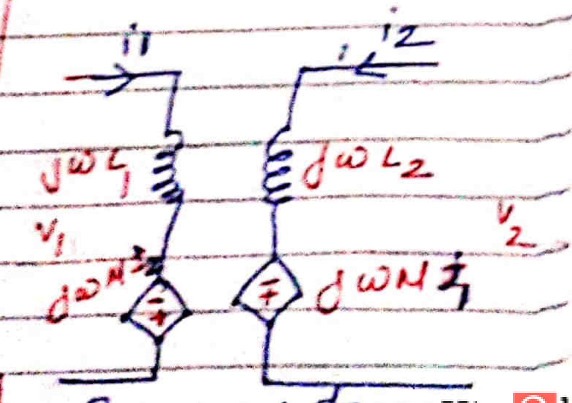
(2)



$$V_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

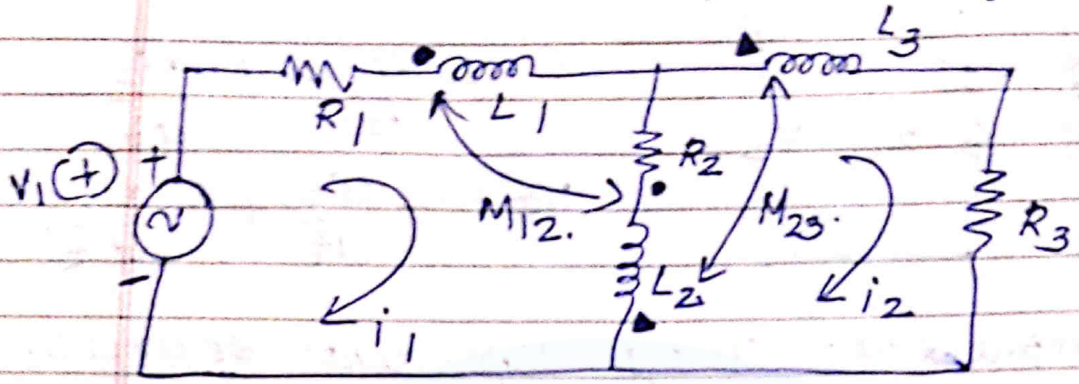
$$V_2 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$$





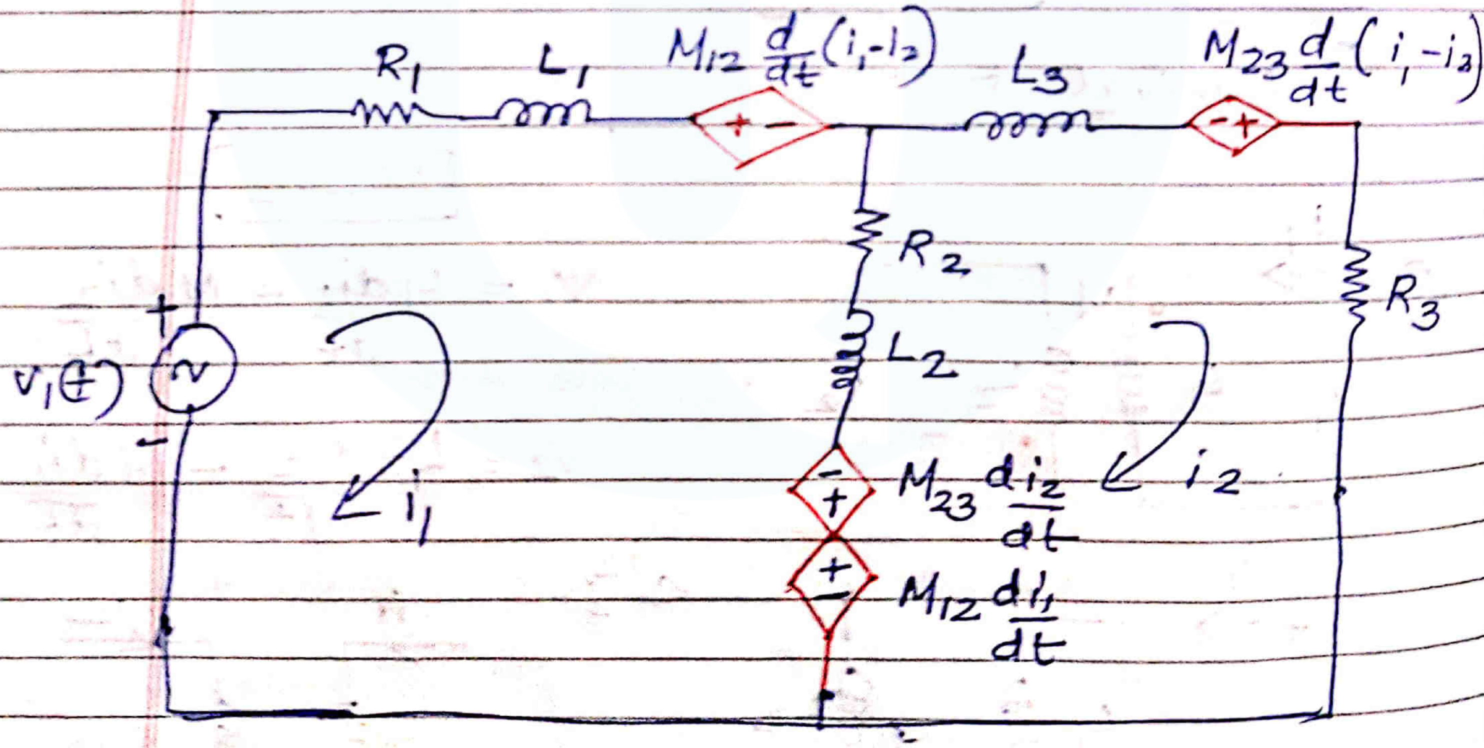
Frequency domain

1. Write the mesh equations for the n/w.



coil 1 is magnetically coupled to coil 2.
coil 2 is " with coil 1 and coil 3.

By applying dot convention, the equivalent ckt is drawn with the dependent sources.



$M_{12} = +ve$, $M_{23} = -ve$

In coil L_1 - mutually induced emf due to $(i_1 - i_2)$ in coil 2

The polarity of mutually induced emf is the same as that of self induced emf because i_1 & $(i_1 - i_2)$ enter in respective coils from the dotted ends.

In coil L_2 — 2 mutually induced emfs. one due to current i_1 in coil 1 & i_2 in coil 3.

The Polarity of mutually induced emf is the same as that of self induced emf.

The polarity of mutually induced emf in coil 2 is due to the current i_2 is opposite to that of self induced emf because current $(i_1 - i_2)$ leaves from the dotted end in coil 2 and the current i_2 enters from the dotted end in coil 3.

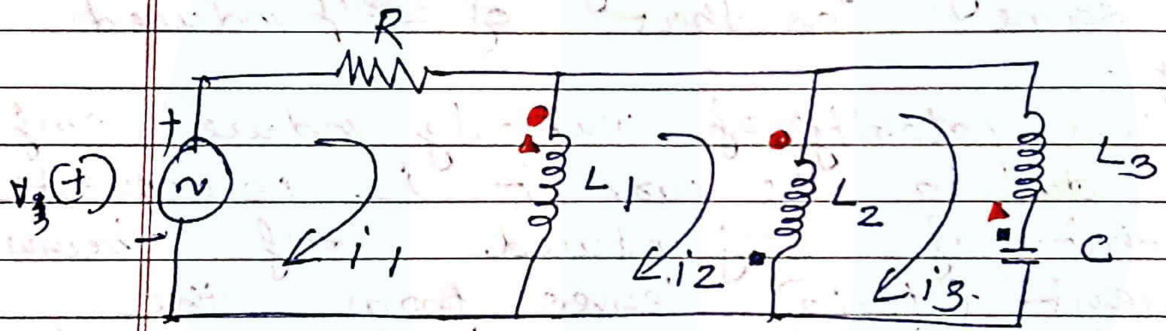
In coil L_3 - the polarity of the mutually induced emf is opposite to that of self induced emf because the current $(i_1 - i_2)$ leaves from the dotted end in coil 2, and the current i_2 enters from the dotted end in coil L_3.

Applying KVL,

$$v_1(t) - R_1 i_1 - L_1 \frac{di_1}{dt} - M_{12}\frac{d}{dt}(i_1 - i_2)$$
$$- R_2(i_1 - i_2) - L_2\left(\frac{d}{dt}(i_1 - i_2)\right) + M_{23}\frac{di_2}{dt} - M\frac{di_1}{dt} = 0$$

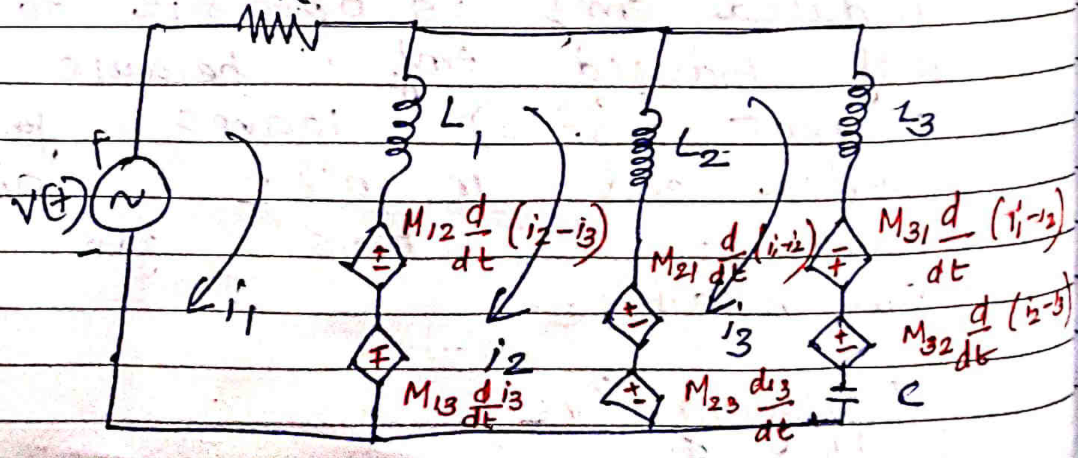
$$M_{12}\frac{di_1}{dt} - M_{23}\frac{di_2}{dt} - L_2\frac{d}{dt}(i_2 - i_1) - R(i_2 - i_1)$$
$$- L_3\frac{di_2}{dt} + M_{23}\frac{d}{dt}(i_1 - i_2) - R_3 i_2 = 0.$$

2) Write mesh equations for the n/w.



B/n L_1 & L_2 = $+M_{12}$ ●
B/n L_1 & L_3 = $-M_{13}$ ▲
B/n L_2 & L_3 = $+M_{23}$ ■

The eq ckt in terms of dependent sources.

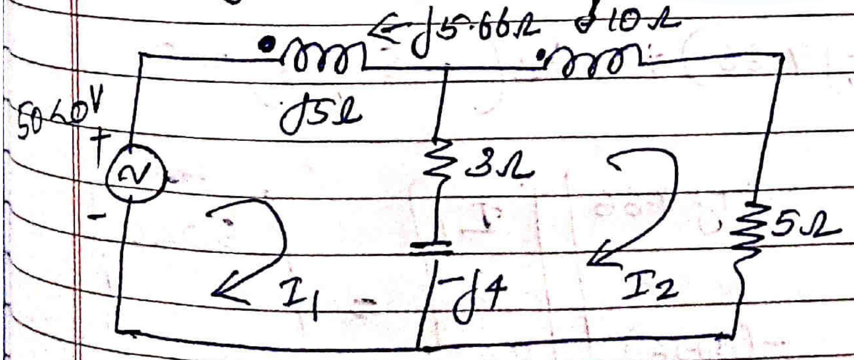


$$v(t) - Ri_1 - L_1\frac{d}{dt}(i_1-i_2) - M_{12}\frac{d}{dt}(i_2-i_3) + M_{13}\frac{di_3}{dt} = 0.$$

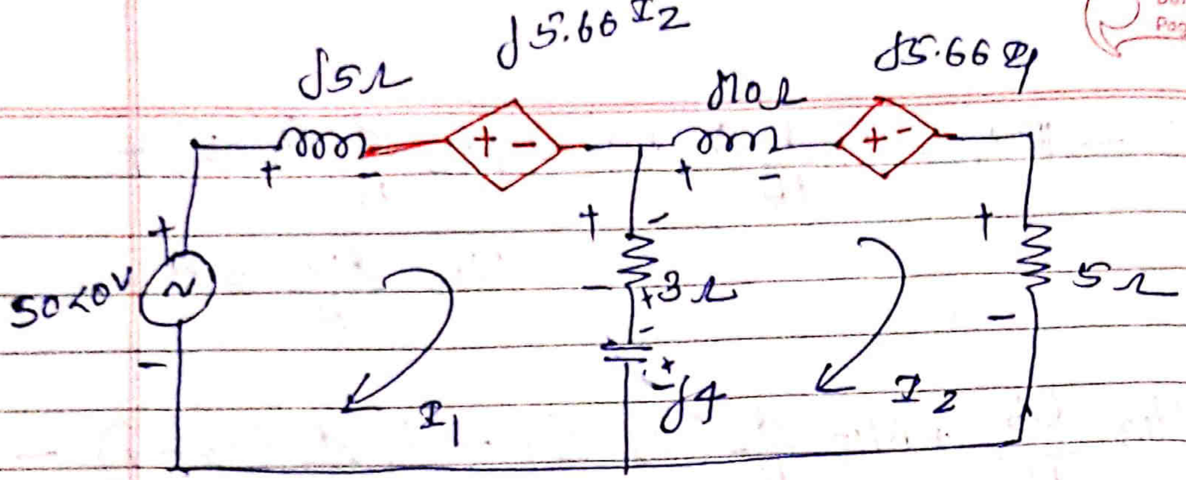
$$-M_{13}\frac{di_3}{dt} + M_{12}\frac{d}{dt}(i_2-i_3) - L_1\frac{d}{dt}(i_2-i_1) - L_2\frac{d}{dt}(i_2-i_3) - M_{21}\frac{d}{dt}(i_1-i_2) - M_{23}\frac{di_3}{dt} = 0$$

$$M_{23}\frac{di_3}{dt} + M_{21}\frac{d}{dt}(i_1-i_2) - L_2\frac{d}{dt}(i_3-i_2) - L_3\frac{di_3}{dt} + M_{31}\frac{d}{dt}(i_1-i_2) - M_{32}\frac{d}{dt}(i_2-i_3) + \frac{1}{C}\int i_3\,dt = 0$$

3 Find the voltage across 5Ω resistor using mesh analysis.



Ans Draw the equivalent ckt interms of dependent sources.



Applying KVL to mesh 1

$$50\angle 0 - j5 I_1 - j5.66 I_2 - 3(I_1 - I_2) + j4(I_1 - I_2) = 0$$

$$50\angle 0 = I_1(j5 + 3 - j4) + I_2(j5.66 - 3 + j4)$$

Applying KVL to mesh 2

$$\cancel{I} j4(I_2 - I_1) - 3(I_2 - I_1) - j10 I_2 - j5.66 I_1 - 5I_2 = 0$$

$$I_1(-j4 + 3 - j5.66) + I_2(j4 \underset{-5}{-3} - j10) = 0$$

$$I_1(3 - j9.66) + I_2(-8 - j6) = 0$$

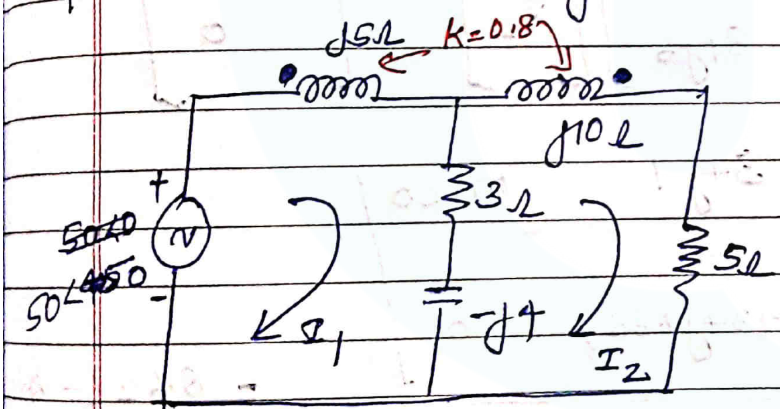
$$\begin{bmatrix} 3+j1 & -3+j9.66 \\ 3-j9.66 & -(8+j6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0 \\ 0 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 3+j1 & 50\angle 0 \\ 3-j9.66 & 0 \end{vmatrix}}{\begin{vmatrix} 3+j1 & -3+j9.66 \\ 3-j9.66 & -(8+j6) \end{vmatrix}}$$

$$= 3.82\angle -112.14$$

$$V_{5\Omega} = 5I_2 = 19.1\angle -112.14\ V$$

4 Find the voltage across 5Ω resistor



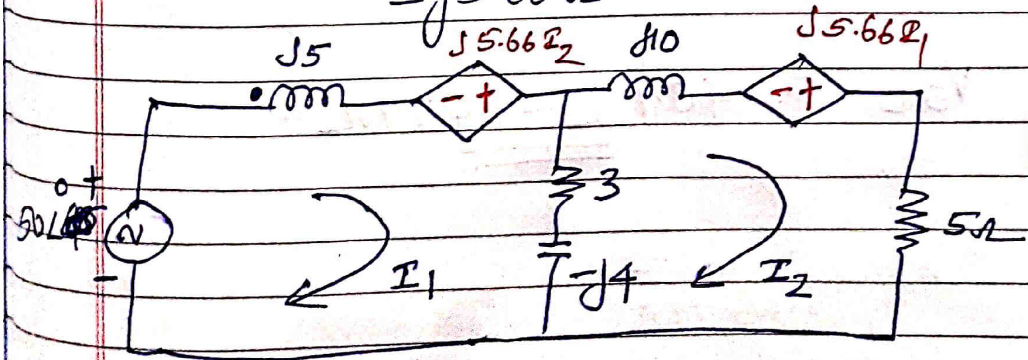
$$M = k\sqrt{L_1L_2} = j0.8\sqrt{5\times 10} = j0.8\times\sqrt{50}$$

$$= j5.66\ \Omega.$$

$\angle 0$

63.43

$\boxed{M\ -ve}$



$$\cancel{50\angle 45 - (3+j4)I_1 - (I_1 - I_2)j3 - j5(I_1 - I_2) - j3I_1 = 0}$$

$$\cancel{(3+j15)I_1 - j8I_2 = 50\angle 45}$$

Mesh 1 $50\angle 0 - j5I_1 + j5.66I_2 - (3-j4)(I_1 - I_2) = 0$

$$(3+j1)I_1 - (3+j1.66)I_2 = 50\angle 0$$

Mesh 2 $-(3-j4)(I_2 - I_1) - j10I_2 + j5.66I_1 - 5I_2 = 0$

$$-(3+j1.66)I_1 + (8+j6)I_2 = 0$$

$$\begin{bmatrix} 3+j1 & -(3+j1.66) \\ -(3+j1.66) & 8+j6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0 \\ 0 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 3+j1 & 50\angle 0 \\ -(3+j1.66) & 0 \end{vmatrix}}{\begin{vmatrix} 3+j1 & -(3+j1.66) \\ -(3+j1.66) & 8+j6 \end{vmatrix}} = 8.62\angle -24.7$$

$$V_{5\Omega} = \underline{\underline{43.1\angle -24.79\Omega}}$$

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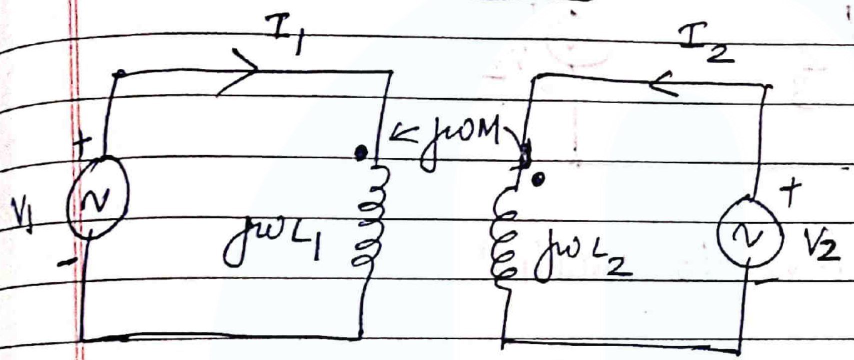
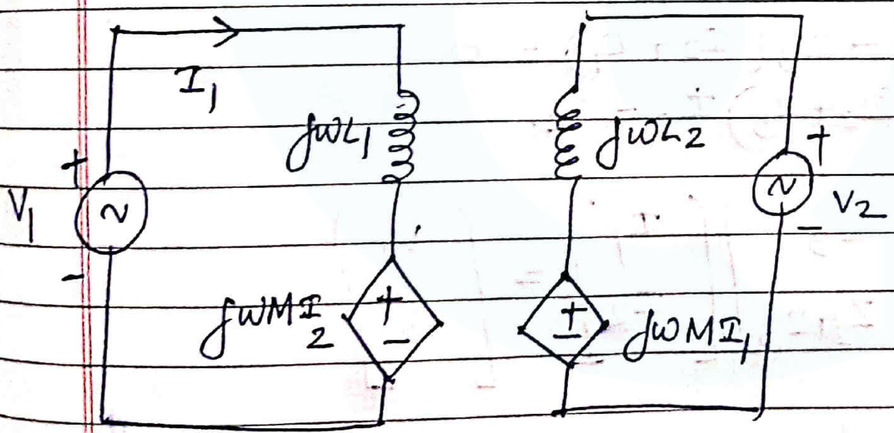
Conductively Coupled Equivalent Ckts.

- For simplifying circuit analysis, we can replace a magnetically coupled ckt with an equivalent ckt called conductively coupled circuit. In this no magnetic coupling is involved. The dot convention is also not needed.



Equivalent ckt



Applying KVL to mesh 1,

$$V_1 - j\omega L_1 I_1 - j\omega M I_2 = 0$$

$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

Mesh 2

$$V_2 - j\omega L_2 I_2 - j\omega M I_1 = 0$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2$$

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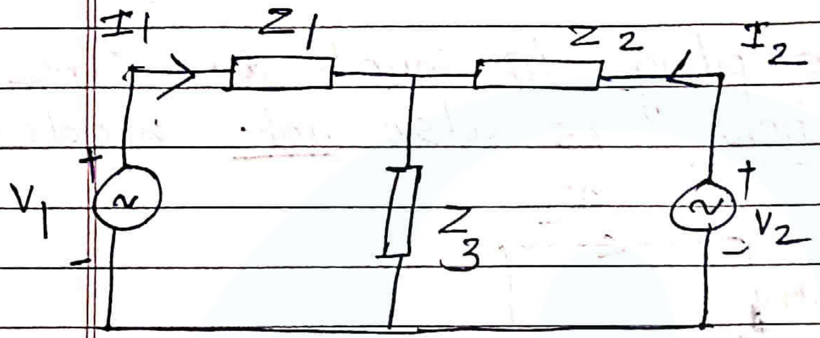
$$\begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Consider a T network



Applying KVL to mesh 1

$$V_1 - I_1 Z_1 - Z_3(I_1 + I_2) = 0$$
$$(Z_1 + Z_3) I_1 + Z_3 I_2 = 0.$$

Mesh 2

$$V_2 - I_2 Z_2 - Z_3(I_2 + I_1) = 0$$
$$Z_3 I_1 + (Z_2 + Z_3) I_2 = V_2.$$

$$\begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Comparing two matrix eqns.

$$\left.\begin{aligned} Z_1 + Z_3 &= j\omega L_1 \\ Z_3 &= j\omega M \\ Z_2 + Z_3 &= j\omega L_2. \end{aligned}\right\}$$

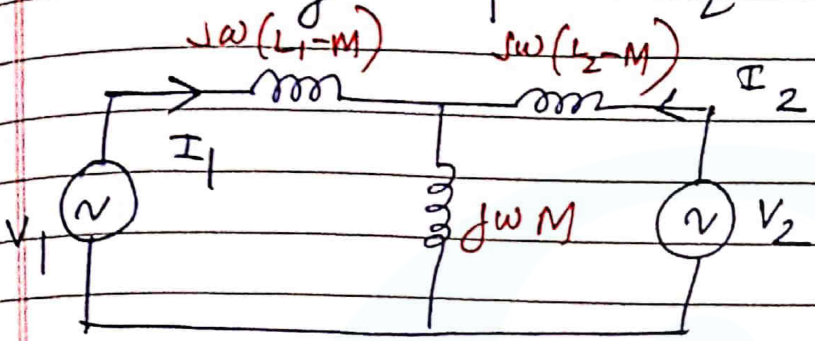
Solving

$$Z_1 = j\omega L_1 - j\omega M.$$
$$= j\omega(L_1 - M)$$

$Z_2 = j\omega L_2 - j\omega M = j\omega(L_2 - M)$

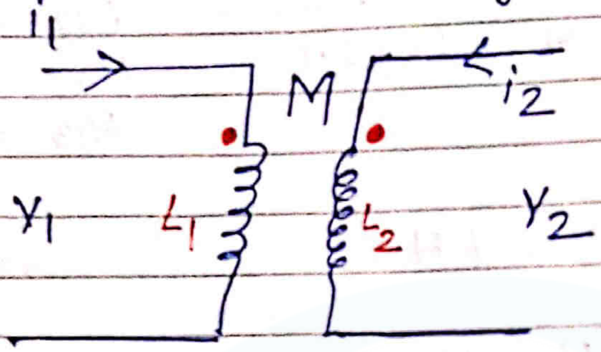
$Z_3 = j\omega M.$

Conductively coupled equivalent ckt.



Conductively Coupled Equivalent Circuit

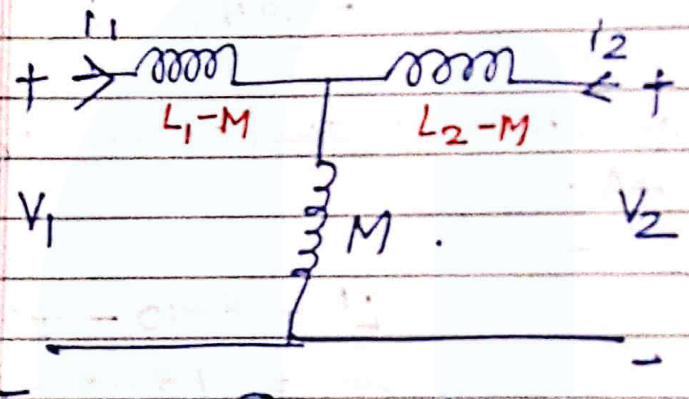
- Energy transfer takes place b/n two circuits through a conductor.



$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad (1)$$

M +ve



← conductively equivalent of the magnetic ckt

$$V_1 = (L_1 - M)\frac{di_1}{dt} + M\frac{d}{dt}(i_1 + i_2)$$

$$V_2 = (L_2 - M)\frac{di_2}{dt} + M\frac{d}{dt}(i_1 + i_2)$$

If we simplify these 2 eqns, we get eqns (1), ie the two circuits are equivalent.

$$Z_A = L_1 - M, \quad Z_B = L_2 - M, \quad Z_C = M.$$

Case 2: M is +ve, both the currents are additive.

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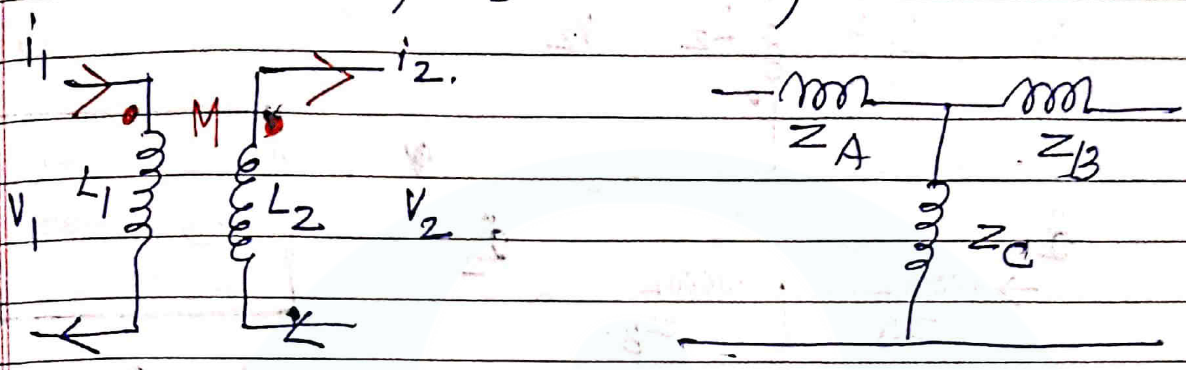
II. If M is −ve, & currents in the common branch are opposite to each other.

$Z_A = L_1 - M$, $Z_B = L_2 - M$, $Z_C = M$

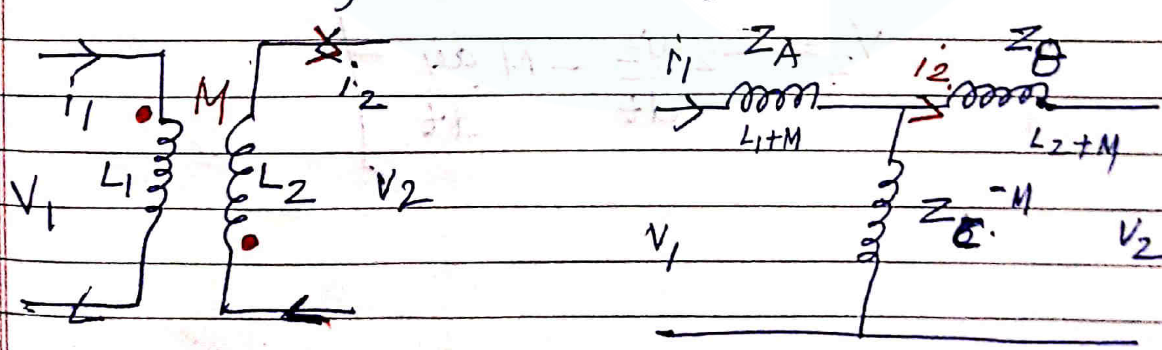


$$V_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

III. If M is +ve, two currents are opposite to each other.

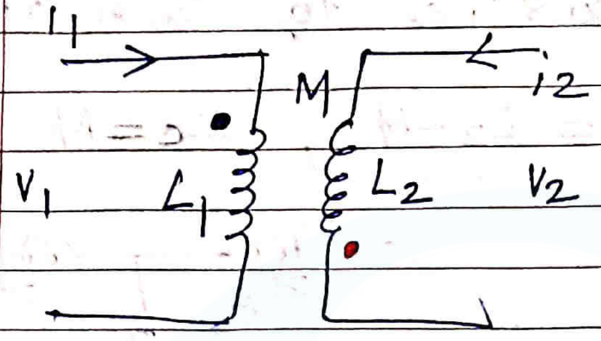
$Z_A = L_1 + M$, $Z_B = L_2 + M$, $Z_C = -M$



$$V_1 = (L_1+M)\frac{di_1}{dt} - M\frac{d(i_1 - i_2)}{dt}$$

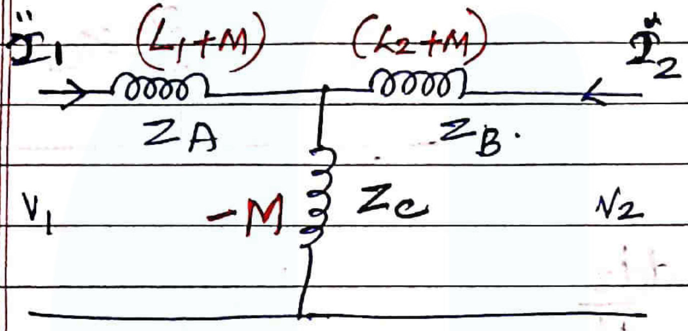
$$= L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

IV If M is −ve, two currents are additive.



−M.
i_1 & i_2 are same dirn.

$$\boxed{\begin{aligned} Z_A &= L_1+M \\ Z_B &= L_2+M \\ Z_C &= -M \end{aligned}}$$



$$V_1 = (L_1+M)\frac{di_1}{dt} - M\frac{d}{dt}(i_1+i_2)$$

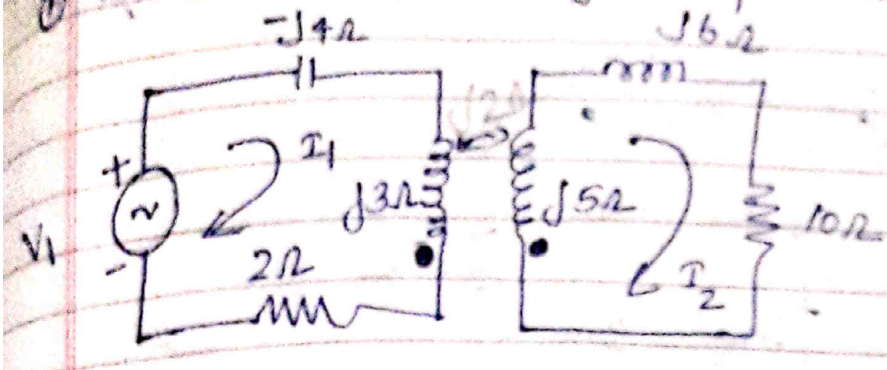
$$V_2 = (L_2+M)\frac{di_2}{dt} - M\frac{d}{dt}(i_1+i_2)$$

For magnetically coupled ckt,

$$V_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$$

1. Find the conductively coupled equivalent ckt.



In both the coils, currents leave the dotted ends.

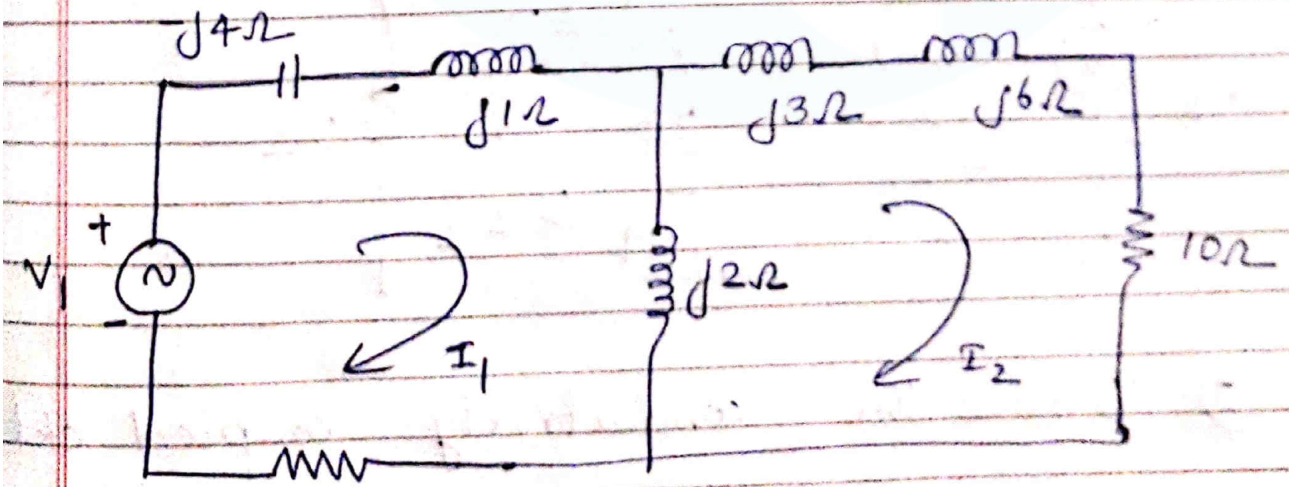
~~In both the coils, currents leave the dotted ends.~~ ~~M is neg~~

In coil 1, current leaves the dotted end. In second coil, current enters the dotted end. ∴ M is negative.

$$Z_A = j\omega(L_1 - M) = j\omega L_1 - j\omega M$$
$$= j3 - j2 = j1\Omega.$$

$$Z_B = j\omega(L_2 - M) = j\omega L_2 - j\omega M$$
$$= j5 - j2 = j3\Omega$$

$$Z_C = j\omega M = j2\Omega$$



Conductively Coupled equivalent ckt

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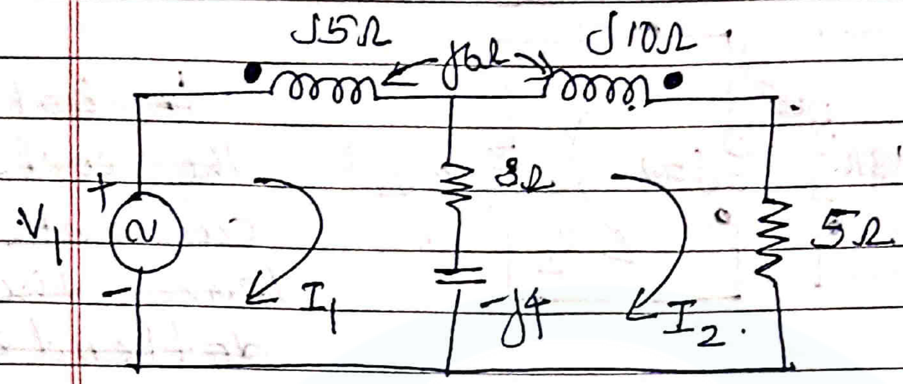
(2) Draw the, conductively coupled equivalent ckt.

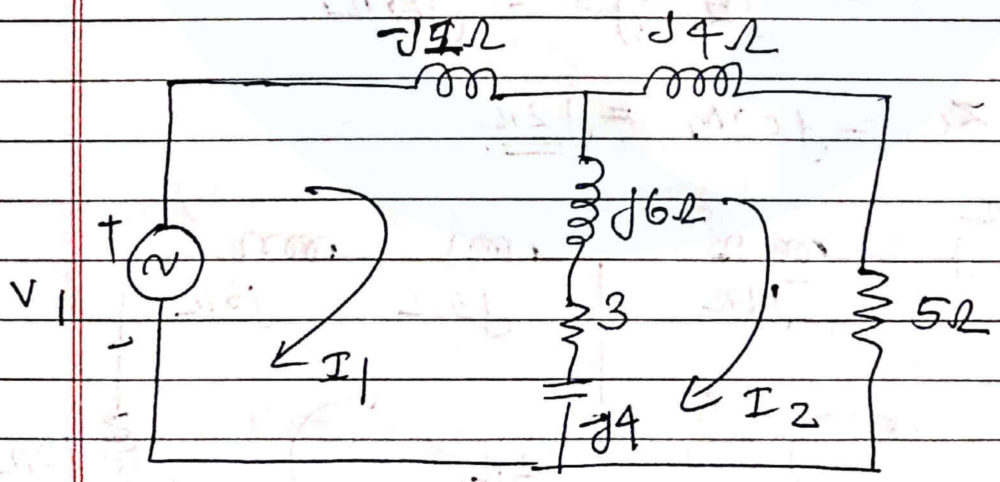


Current I_1 enters at the dotted end
Current I_2 leaves the dotted end.
Hence, M is negative.

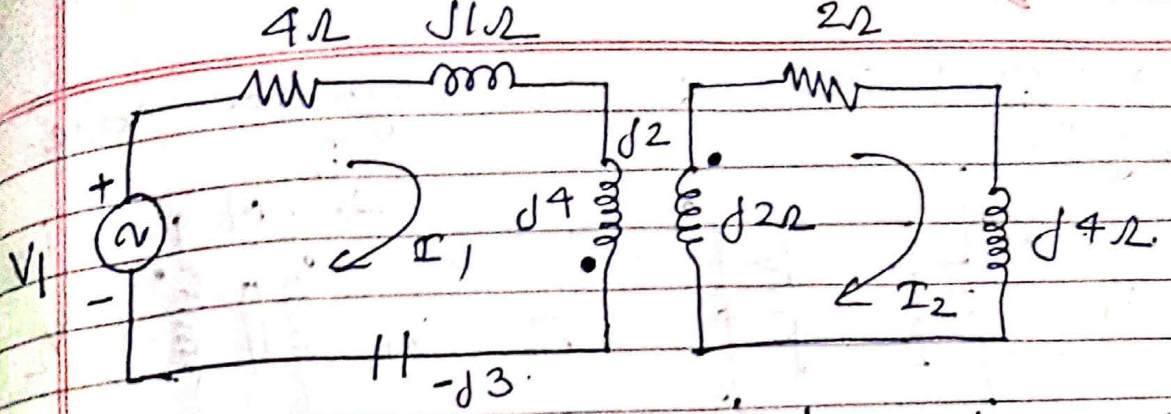
$$Z_A = j\omega(L_1 - M) = j5 - j6 = -j1\Omega$$

$$Z_B = j\omega(L_2 - M) = j10 - j6 = j4\Omega.$$

$$Z_3 = j\omega M = j6\Omega.$$



(3) Draw the conductively coupled ckt.

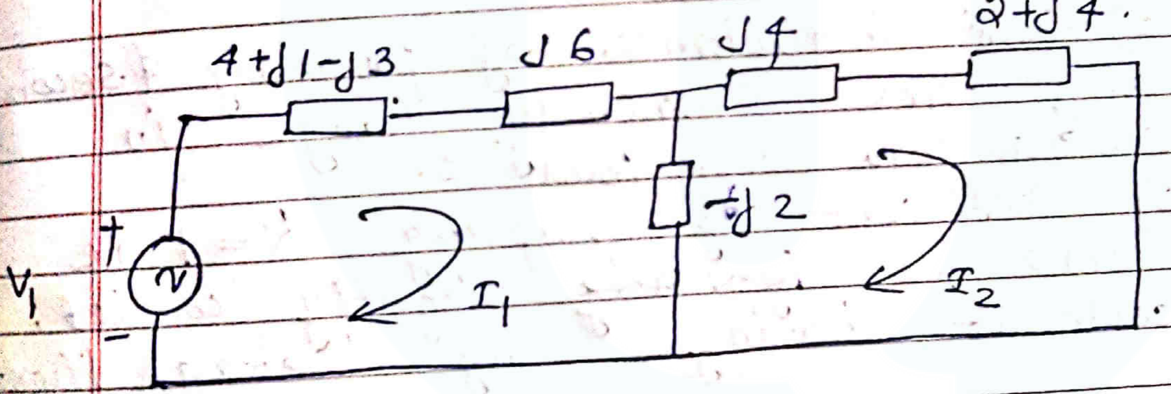


Both currents leave from the dotted ends.
M is +ve.

$$Z_A = j\omega(L_1+M) = j4 + j2 = j6\Omega.$$

$$Z_B = j\omega(L_2+M) = j2 + j2 = j4\Omega$$

$$Z_C = -j\omega M = -j2\,\Omega.$$

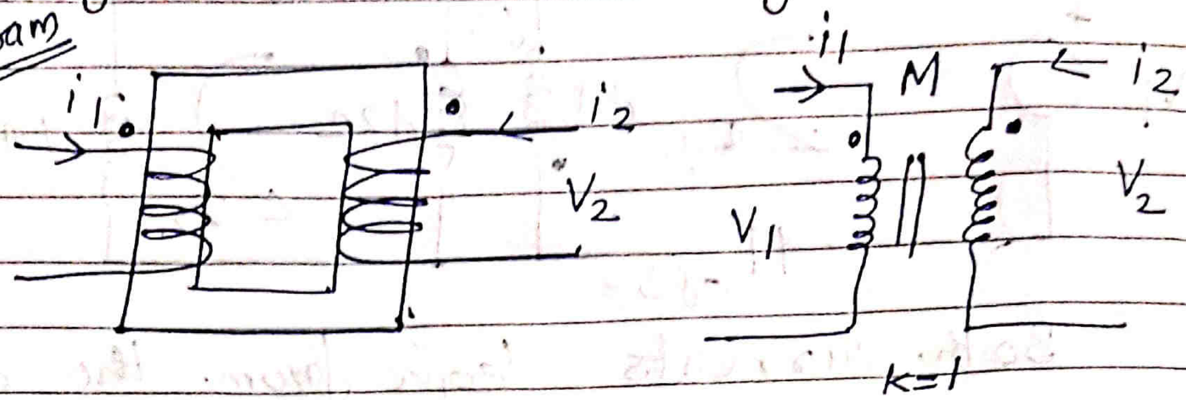


Ideal Transformer

Transfer of energy from one circuit to another circuit through mutual induction is widely utilised in power systems. This purpose is served by transformers.

Transformer is a stationary device having two or more windings arranged in a common magnetic core.

Circuit Diagram



The winding to which supply is connected is primary and winding connected to load is secondary.

For an ideal transformer,

1) Resistance in both the coils are assumed to be 0.
2) Self inductance of primary & secondary are extremely large in comparison with load Z.
3) Coefficient of coupling $k = 1$ ie coils are typically tightly coupled, without having any leakage flux.

The magnitude of self induced emf

$$V = L\frac{di}{dt}.$$

Also, $V = N\frac{d\phi}{dt}$; N = no. of turns

$N\frac{d\phi}{dt}$ = Fluxlinkage of the coil.

$$L\frac{di}{dt} = N\frac{d\phi}{dt} \Rightarrow L = N\frac{d\phi}{di}$$

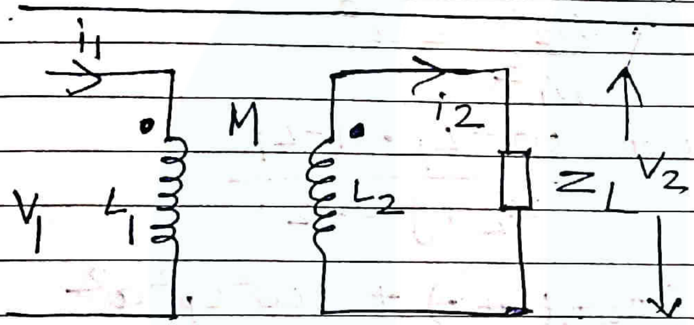
But $\phi = \frac{mmf}{Reluctance} = \frac{Ni}{S}$

$$L = N\frac{d\phi}{di} = N\frac{d}{di}\left(\frac{Ni}{S}\right).$$

$$L = \frac{N^2}{S} \quad , \quad \boxed{L \propto N^2}$$

ie $\frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} = k^2$, k = turns ratio.

Transformer with Sinusoidal Excitations



$$V_1 - I_1 j\omega L_1 \mp j\omega M I_2 = 0$$

$$V_1 = I_1 j\omega L_1 \mp j\omega M I_2 \quad \text{——} \ (1)$$

$$0 = -j\omega M I_1 + (Z_L + j\omega L_2) I_2 \quad \text{——} \ (2)$$

$$\Rightarrow I_2 = \frac{j\omega M \cdot I_1}{(Z_2 + j\omega L_2)}$$

$$V_1 = I_1 j\omega L_1 - j\omega M \times \frac{j\omega M I_1}{Z_2 + j\omega L_2}$$

$$= I_1 j\omega L_1 + \frac{\omega^2 M^2 I_1}{Z_2 + j\omega L_2}.$$

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Z_{in} = Input Impedance $= \frac{V_1}{I_1}$

$$= j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2}$$

When $k=1$, $M = k\sqrt{L_1 L_2} = \sqrt{L_1 L_2}$.

$$Z_{in} = j\omega L_1 + \frac{\omega^2 L_1 L_2}{Z_L + j\omega L_2}$$

Putting $L_2/L_1 = \frac{N_2^2}{N_1^2} = a^2$

$$\therefore Z_{in} = j\omega L_1 + \frac{\omega^2 L_1 \cdot a^2 L_1}{Z_L + j\omega L_2}$$

$$= \frac{j\omega L_1(Z_L + j\omega L_2) + \omega^2 L_1^2 a^2}{Z_L + j\omega L_2}$$

$$= \frac{Z_L j\omega L_1 - \omega^2 L_1 L_2 + \omega^2 L_1^2 a^2}{Z_L + j\omega L_2}$$

$L_2 = a^2 L_1$

$$= \frac{Z_L j\omega L_1 - \omega^2 L_1^2 a^2 + \omega^2 L_1^2 a^2}{Z_L + j\omega L_2}$$

$$= \frac{Z_L j\omega L_1}{Z_L + j\omega L_2}$$

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As L_2 is assumed to be infinitely large compared to Z_L

$$Z_{in} = \frac{Z_L j\omega L_1}{j\omega a^2 L_1} = \frac{Z_L}{a^2} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

ie ideal transformer change the impedance of a load and can be used to match circuits with different impedances inorder to achieve maximum power transfer.