Module 2

Performance Analysis of Control Systems

Module 2: **Performance Analysis of Control Systems (9 hours)**

- Time domain analysis of control systems
- Time domain specifications of transient and steady state responses-
- Impulse and Step responses of first and second order systems
- Pole dominance for higher order systems. Error analysis: Steady state error analysis and error constants -Dynamic error coefficients.
- Stability Analysis: Concept of BIBO stability and Asymptotic stability- Time response for various pole locations
- stability of feedback systems - Routh's stability criterionRelative stability

2	**Performance Analysis of Control Systems (9 hours)**	
2.1	Time domain analysis of control systems: Time domain specifications of transient and steady state responses-Impulse and Step responses of First order systems- Impulse and Step responses of Second order systems- Pole dominance for higher order systems	3
2.2	Error analysis: Steady state error analysis - static error coefficient of Type 0, 1, 2 systems. Dynamic error coefficients	2
2.3	Stability Analysis: Concept of stability-BIBO stability and Asymptotic stability- Time response for various pole locations- stability of feedback systems	2
2.4	Application of Routh's stability criterion to control system analysis-Relative stability	2
3	Root Locus Analysis and Compensator Design (11 hours)	

2.1 TIME RESPONSE

The time response of the system is the output of the closed loop system as a function of time. It is denoted by c(t). The time response can be obtained by solving the differential equation governing the system. Alternatively, the response c(t) can be obtained from the transfer function of the system and the input to the system.

The closed loop transfer function, $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)} = M(s) \qquad .....(2.1)$$

The Output or Response in s-domain, C(s) is given by the product of the transfer function and the input, R(s). On taking inverse Laplace transform of this product the time domain response, c(t) can be obtained.

Response in s-domain, $$C(s) = R(s)\,M(s) \qquad .....(2.2)$$

Response in time domain, $$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\{R(s) \times M(s)\} \qquad .....(2.3)$$

where, $$M(s) = \frac{G(s)}{1+G(s)H(s)}$$

Test signals

The standard test signals are,

1. a) Step signal
   b) Unit step signal

2. a) Ramp signal
   b) Unit ramp signal

3. a) Parabolic signal
   b) Unit parabolic signal

4. Impulse signal

5. Sinusoidal signal.

STEP SIGNAL

The step signal is a signal whose value changes from zero to A at t = 0 and remains constant at A for t > 0. The step signal resembles an actual steady input to a system. A special case of step signal is unit step in which **A** is unity.

The mathematical representation of the step signal is,

$$\begin{aligned} r(t) &= 1 \ ; \ t \geq 0 \\ &= 0 \ ; \ t < 0 \end{aligned} \qquad .....(2.4)$$

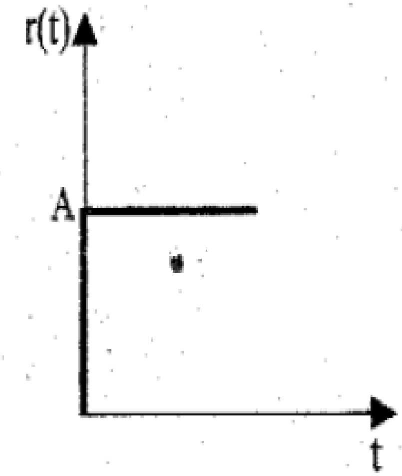


Fig 2.2 : Step signal.

RAMP SIGNAL

The ramp signal is a signal whose value increases linearly with time from an initial value of zero at t = 0. The ramp signal resembles a constant velocity input to the system. A special case of ramp signal is unit ramp signal in which the value of **A** is unity.

The mathematical representation of the ramp signal is,

$$\begin{aligned} r(t) &= A\,t \ ; \ t \geq 0 \\ &= 0 \quad ; \ t < 0 \end{aligned} \qquad .....(2.5)$$

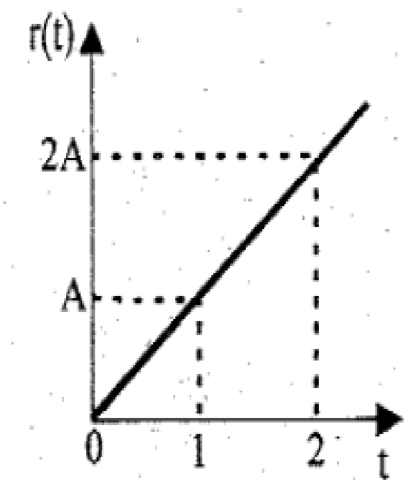


Fig 2.3 : Ramp signal.

PARABOLIC SIGNAL

In parabolic signal, the instantaneous value varies as square of the time from an initial value of zero at t = 0. The sketch of the signal with respect to time resembles a parabola. The parabolic signal resembles a constant acceleration input to the system. A special case of parabolic signal is unit parabolic signal in which A is unity.

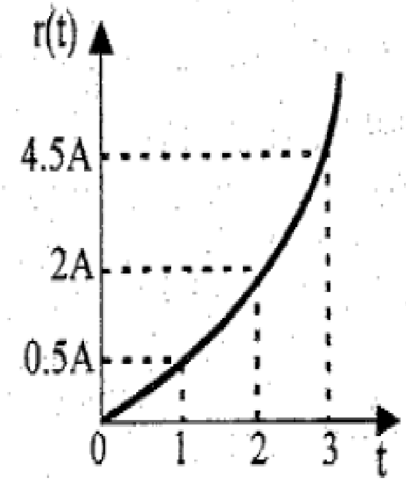


Fig 2.4 : Parabolic signal.

The mathematical representation of the parabolic signal is,

$$r(t) = \frac{At^2}{2} \quad ; \quad t \geq 0$$
$$= 0 \quad ; \quad t < 0 \qquad .....(2.6)$$

> ***Note :** Integral of step signal is ramp signal. Integral of ramp signal is parabolic signal.*

IMPULSE SIGNAL

A signal of very large magnitude which is available for very short duration is called ***impulse signal.*** Ideal impulse signal is a signal with infinite magnitude and zero duration but with an area of A. The unit impulse signal is a special case, in which A is unity.

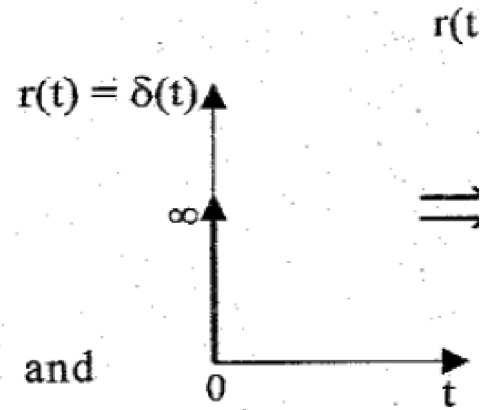


Fig 2.5 :

The impulse signal is denoted by $\delta(t)$ and mathematically it is expressed as,

$$\begin{aligned} \delta(t) &= \infty;\ t = 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\,dt = A \\ &= 0;\ t \neq 0 \qquad\qquad\qquad \text{.....(2.7)} \end{aligned}$$

Response of first order system with step input

The closed loop order system with unity feedback is shown in fig 2.6.

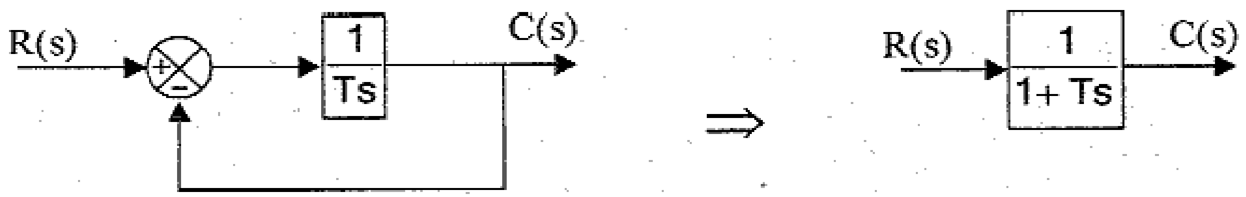


Fig 2.6 : *Closed loop for first order system.*

The closed loop transfer function of first order system, $\dfrac{C(s)}{R(s)} = \dfrac{1}{1+Ts}$

If the input is unit step then, $r(t) = 1$ and $R(s) = \dfrac{1}{s}$.

$$\therefore \text{The response in s-domain, } C(s) = R(s)\frac{1}{(1+Ts)} = \frac{1}{s}\frac{1}{(1+Ts)} = \frac{1}{sT\left(\frac{1}{T}+s\right)} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)}$$

By partial fraction expansion,

$$C(s) = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)} = \frac{A}{s} + \frac{B}{\left(s+\frac{1}{T}\right)}$$

A is obtained by multiplying C(s) by s and letting s = 0.

$$A = C(s) \times s\Big|_{s=0} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)} \times s\Bigg|_{s=0} = \frac{\frac{1}{T}}{s+\frac{1}{T}}\Bigg|_{s=0} = \frac{\frac{1}{T}}{\frac{1}{T}} = 1$$

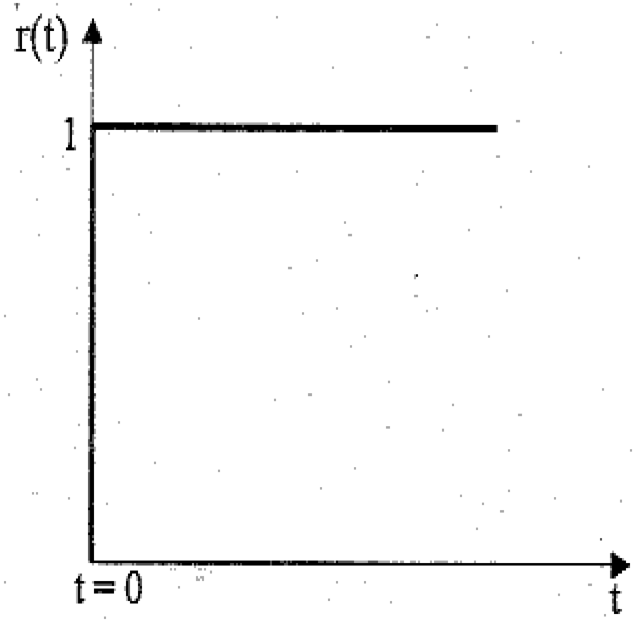
B is obtained by multiplying C(s) by (s+1/T) and letting s = –1/T.

$$B = C(s) \times \left(s+\frac{1}{T}\right)\Bigg|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)} \times \left(s+\frac{1}{T}\right)\Bigg|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{s}\Bigg|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{\frac{-1}{T}} = -1$$

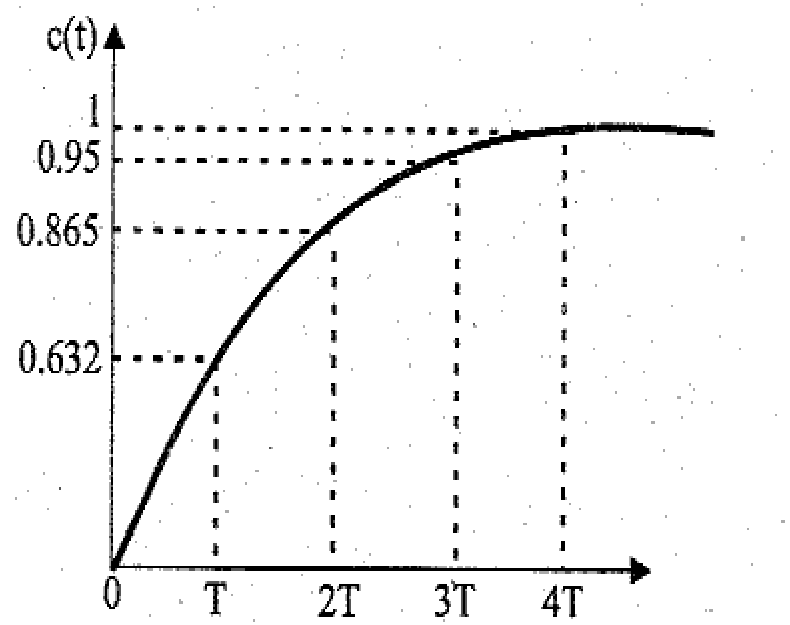
$$\therefore C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

The response in time domain is given by,

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right\} = 1 - e^{-\frac{t}{T}}$$



r(t)
1
t = 0
t



c(t)
1
0.95
0.865
0.632
0
T
2T
3T
4T
t

Second order system

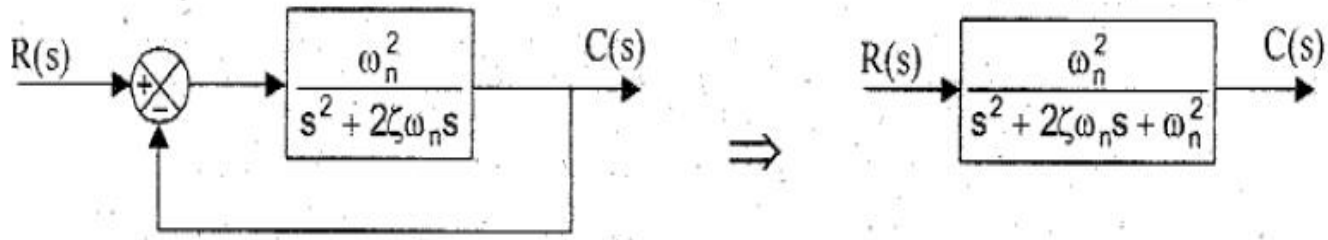


***Fig 2.8 :** Closed loop for second order system.*

The standard form of closed loop transfer function of second order system is given by,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{.....(2.14)}$$

where, ω_n = Undamped natural frequency, rad/sec.

ζ = Damping ratio.

*The **damping ratio*** is defined as the ratio of the actual damping to the critical damping. The response c(t) of second order system depends on the value of damping ratio. Depending on the value of ζ, the system can be classified into the following four cases, The characteristics equation of the second order system is,

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

It is a quadratic equation and the roots of this equation is given by,

$$s_1, s_2 = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} = \frac{-2\zeta\omega_n \pm \sqrt{4\omega_n^2(\zeta^2 - 1)}}{2}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

e system can be classified into the following four cases,

Case 1 : Undamped system, $\zeta = 0$

Case 2 : Under damped system, $0 < \zeta < 1$

Case 3 : Critically damped system, $\zeta = 1$

Case 4 : Over damped system, $\zeta > 1$

$$\text{When } \zeta = 0,\ \ s_1, s_2 = \pm j\omega_n ;\ \begin{cases} \text{roots are purely imaginary} \\ \text{and the system is undamped} \end{cases}$$

$$\text{When } \zeta = 1,\ \ s_1, s_2 = -\omega_n ;\ \begin{cases} \text{roots are real and equal and} \\ \text{the system is critically damped} \end{cases}$$

$$\text{When } \zeta > 1,\ \ s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} ;\begin{cases} \text{roots are real and unequal and} \\ \text{the system is overdamped} \end{cases}$$

$$\text{When } 0 < \zeta < 1,\ \ s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\zeta\omega_n \pm \omega_n\sqrt{(-1)\,(1-\zeta^2)}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{-1}\ \sqrt{1-\zeta^2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$= -\zeta\omega_n \pm j\omega_d ;\ \begin{cases} \text{roots are complex conjugate} \\ \text{the system is underdamped} \end{cases}$$

$$\text{where, } \ \omega_d = \omega_n\sqrt{1-\zeta^2}$$

RESPONSE OF UNDAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For undamped system, z = 0.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

When the input is unit step, $r(t) = 1$ and $R(s) = \frac{1}{s}$.

$$\therefore \text{The response in s-domain, } C(s) = R(s)\,\frac{\omega_n^2}{s^2 + \omega_n^2} = \frac{1}{s}\,\frac{\omega_n^2}{s^2 + \omega_n^2}$$

By partial fraction expansion,

$$C(s) = \frac{\omega_n^2}{s\,(s^2 + \omega_n^2)} = \frac{A}{s} + \frac{B}{s^2 + \omega_n^2}$$

A is obtained by multiplying C(s) by s and letting s = 0.

$$A = C(s) \times s\Big|_{s=0} = \frac{\omega_n^2}{s\,(s^2+\omega_n^2)} \times s\Bigg|_{s=0} = \frac{\omega_n^2}{s^2+\omega_n^2}\Bigg|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

B is obtained by multiplying C(s) by $(s^2 + \omega_n^2)$ and letting $s^2 = -\omega_n^{\ 2}$ or $s = j\omega_n$.

$$B = C(s) \times (s^2+\omega_n^2)\Big|_{s=j\omega} = \frac{\omega_n^2}{s\,(s^2+\omega_n^2)} \times (s^2+\omega_n^2)\Bigg|_{s=j\omega} = \frac{\omega_n^2}{s}\Bigg|_{s=j\omega_n} = \frac{\omega_n^2}{j\omega_n} = -j\omega_n = -s$$

$$\therefore\ C(s) = \frac{A}{s} + \frac{B}{s^2+\omega_n^2} = \frac{1}{s} - \frac{s}{s^2+\omega_n^2}$$

$\mathcal{L}\{1\} = \dfrac{1}{s}$	$\mathcal{L}\{\cos \omega t\} = \dfrac{s}{s^2+\omega^2}$

Time domain response, $c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\dfrac{1}{s} - \dfrac{s}{s^2+\omega_n^2}\right\} = 1 - \cos \omega_n t$ .....(2.24)

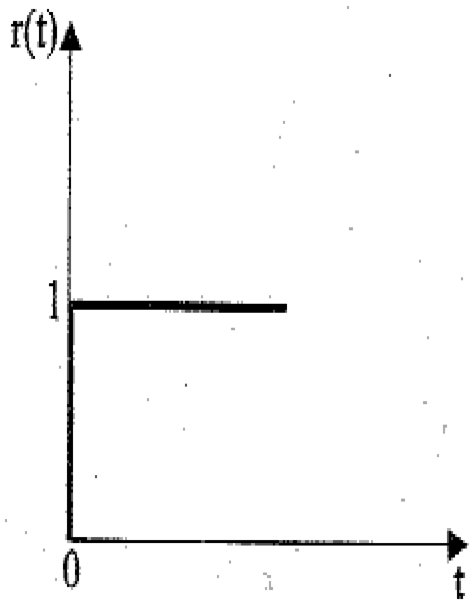


Fig 2.9.a : Input.

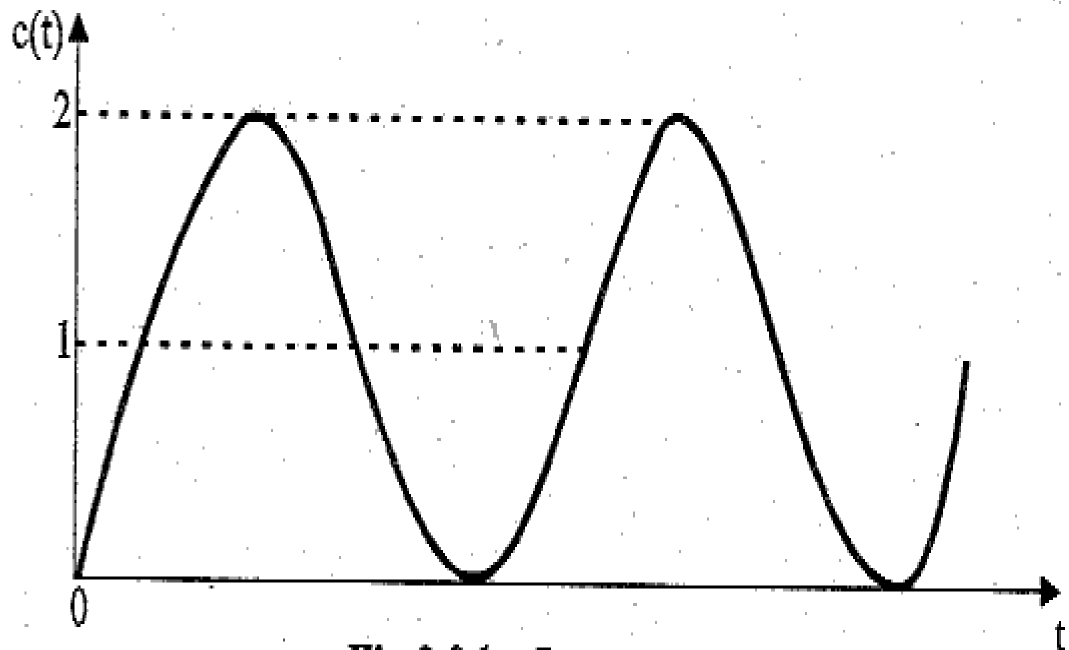


Fig 2.9.b : Response.

***Fig 2.9 :** Response of undamped second order system for unit step input.*

RESPONSE OF UNDERDAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For underdamped system, $0 < \zeta < 1$ and roots of the denominator (characteristic equation) are lex conjugate.

The roots of the denominator are, $s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

Since $\zeta < 1$, ζ^2 is also less then 1, and so $1 - \zeta^2$ is always positive.

$$\therefore s = -\zeta\omega_n \pm \omega_n\sqrt{(-1)(1-\zeta^2)} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

The damped frequency of oscillation, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\therefore s = -\zeta\omega_n \pm j\omega_d$$

The response in s-domain, $C(s) = R(s)\ \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

For unit step input, r(t) = 1 and R(s) = 1/s.

$$\therefore C(s) = \frac{\omega_n^2}{s\,(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

By partial fraction expansion, $C(s) = \dfrac{\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)} = \dfrac{A}{s} + \dfrac{Bs + C}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

A is obtained by multiplying C(s) by s and letting s = 0.

$$\therefore\ A = s \times C(s)\Big|_{s=0} = s \times \frac{\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}\Bigg|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

To solve for B and C, cross multiply equation (2.25) and equate like power of s.

On cross multiplication equation (2.25) after substituting A = 1, we get,

$$\omega_n^2 = s^2 + 2\zeta\omega_n s + \omega_n^2 + (Bs + C)s$$
$$\omega_n^2 = s^2 + 2\zeta\omega_n s + \omega_n^2 + Bs^2 + Cs$$

Equating coefficients of s^2 we get, $0 = 1 + B$ $\qquad \therefore B = -1$

Equating coefficient of s we get, $0 = 2\zeta\omega_n + C$ $\qquad \therefore C = -2\zeta\omega_n$

$$\therefore\ C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Let us add and subtract $\zeta^2\omega_n^{\,2}$ to the denominator of second term in the equation (2.26).

$$\therefore C(s)=\frac{1}{s}-\frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2+\zeta^2\omega_n^2-\zeta^2\omega_n^2}=\frac{1}{s}-\frac{s+2\zeta\omega_n}{(s^2+2\zeta\omega_n s+\zeta^2\omega_n^2)+(\omega_n^2-\zeta^2\omega_n^2)}$$

$$=\frac{1}{s}-\frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}=\frac{1}{s}-\frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$$\boxed{\omega_d=\omega_n\sqrt{1-\zeta^2}}$$

$$=\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} \qquad .....(2.27)$$

Let us multiply and divide by ω_d in the third term of the equation (2.27).

$$\therefore\ C(s) = \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}}$$

$$\boxed{\mathcal{L}\{e^{-at}\sin\omega t\} = \frac{\omega}{(s+a)^2+\omega^2}}$$

$$\boxed{\mathcal{L}\{e^{-at}\cos\omega t\} = \frac{s+a}{(s+a)^2+\omega^2}}$$

The response in time domain is given by,

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}\right\}$$

$$= 1 - e^{-\zeta\omega_n t}\cos\omega_d t - \frac{\zeta\omega_n}{\omega_d}e^{-\zeta\omega_n t}\sin\omega_d t = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta\omega_n}{\omega_n\sqrt{1-\zeta^2}}\sin\omega_d t\right)$$

$$\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2}}$$

$$= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sqrt{1-\zeta^2}\,\cos\omega_d t + \zeta\,\sin\omega_d t\right) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sin\omega_d t \times \zeta + \cos\omega_d t \times \sqrt{1-\zeta^2}\right)$$

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sin\omega_d t \times \cos\theta + \cos\omega_d t \times \sin\theta\right)$$

$$= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \theta) \qquad .....(2.28)$$

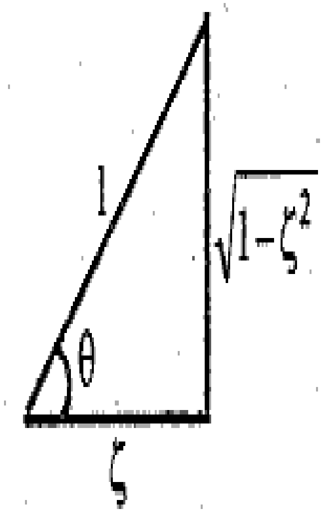
$$\text{where, } \left(\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

***Note :** On constructing right angle triangle with ζ and $\sqrt{1-\zeta^2}$, we get*

$\sin\theta = \sqrt{1-\zeta^2}$

$\cos\theta = \zeta$

$\tan\theta = \dfrac{\sqrt{1-\zeta^2}}{\zeta}$



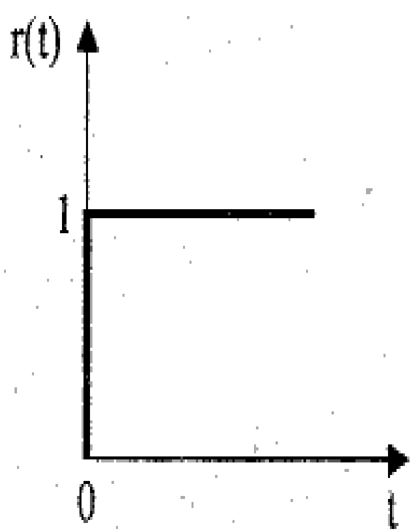


Fig 2.10.a : *Input.*

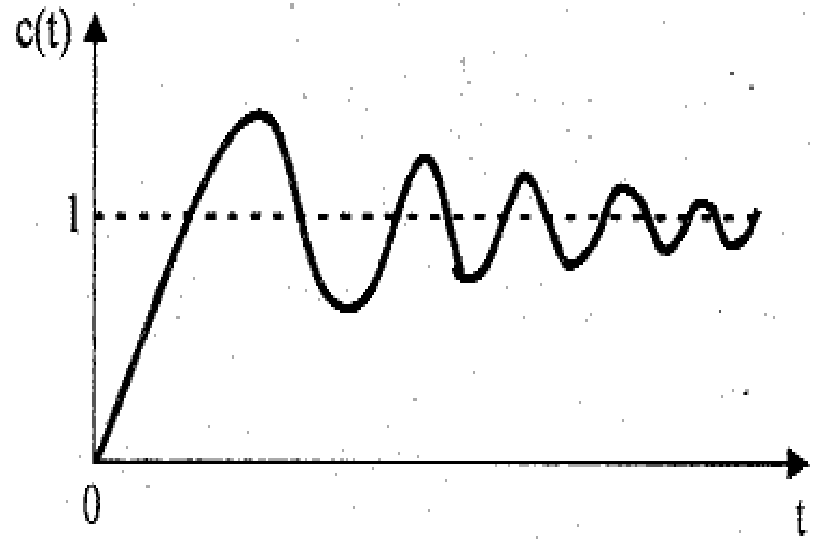


Fig 2.10.b : *Response.*

Fig 2.10 : *Response of under damped second order system for unit step input.*

RESPONSE OF CRITICALLY DAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For critical damping $\zeta = 1$.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

When input is unit step, $r(t) = 1$ and $R(s) = 1/s$.

$\therefore$ The response in s-domain,

$$C(s) = R(s)\frac{\omega_n^2}{(s+\omega_n)^2} = \frac{1}{s}\,\frac{\omega_n^2}{(s+\omega_n)^2} = \frac{\omega_n^2}{s\,(s+\omega_n)^2}$$

By partial fraction expansion, we can write,

$$C(s) = \frac{\omega_n^2}{s\,(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{s+\omega_n}$$

$$A = s \times C(s)\Big|_{s=0} = \left.\frac{\omega_n^2}{(s+\omega_n)^2}\right|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

$$B = (s+\omega_n)^2 \times C(s)\Big|_{s=-\omega_n} = \left.\frac{\omega_n^2}{s}\right|_{s=-\omega_n} = -\omega_n$$

$$C = \frac{d}{ds}\Big[(s+\omega_n)^2 \times C(s)\Big]\Big|_{s=-\omega_n} = \left.\frac{d}{ds}\left(\frac{\omega_n^2}{s}\right)\right|_{s=-\omega_n} = \left.\frac{-\omega_n^2}{s^2}\right|_{s=-\omega_n} = -1$$

$$\therefore C(s) = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{s+\omega_n} = \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}$$

$\mathcal{L}\{1\} = \frac{1}{s}$
$\mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$
$\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$

The response in time domain,

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{s+\omega_n}\right\}$$

$$c(t) = 1 - \omega_n t\, e^{-\omega_n t} - e^{-\omega_n t}$$

$$c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t) \quad .....(2.3$$

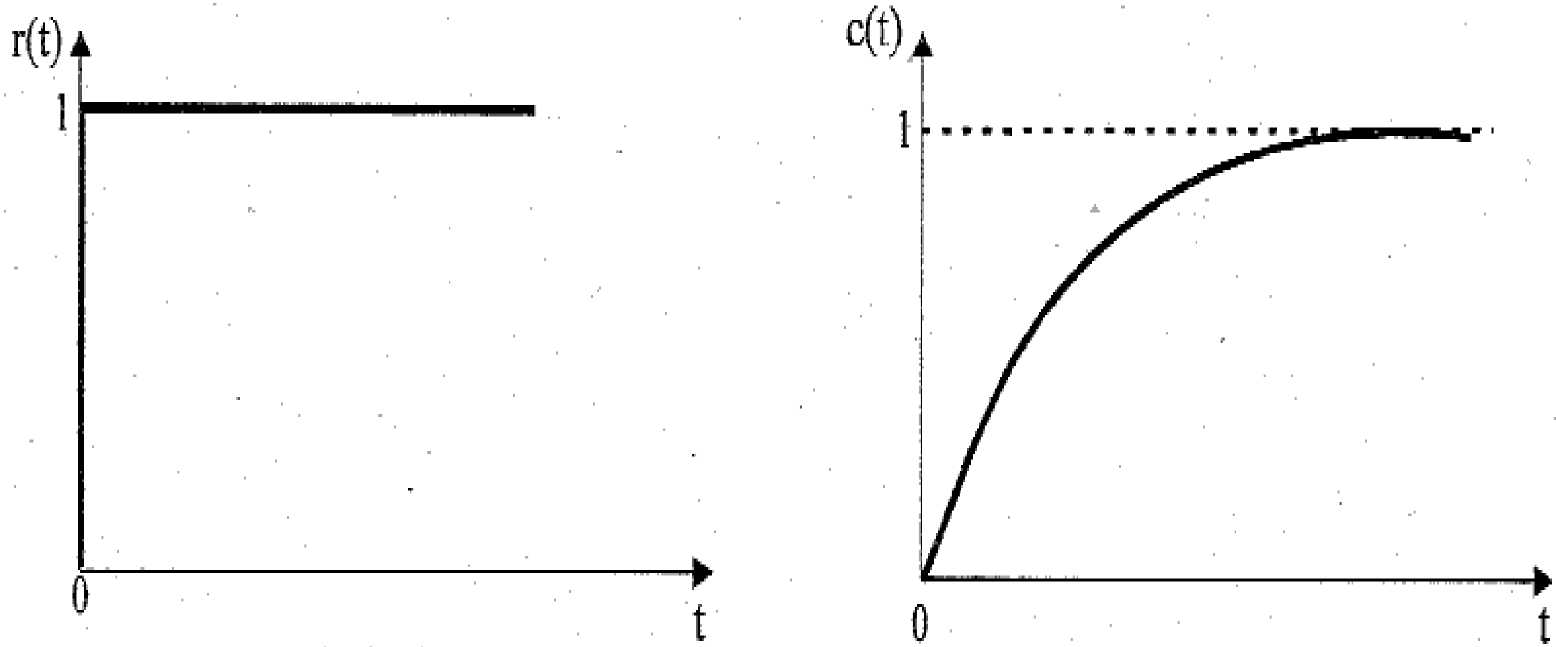


Fig 2.11.a : *Input.*

Fig 2.11.b : *Response.*

Fig 2.11 : *Response of critically damped second order system for unit step input.*

RESPONSE OF OVER DAMPED SECOND ORDER SYSTEM FOR UNIT STEP INPUT

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For overdamped system $\zeta > 1$. The roots of the denominator of transfer function are r
ct. Let the roots of the denominator be s_a, s_b.

$$s_a, s_b = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\left[\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}\right]$$

Let $s_1 = -s_a$ and $s_2 = -s_b$ $\qquad \therefore s_1 = \zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$

$$s_2 = \zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$$

The closed loop transfer function can be written in terms of s_1 and s_2 as shown below.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s + s_1)(s + s_2)}$$

$\therefore$ For closed loop over damped second order system,

$$\text{Unit step response} = 1 - \frac{\omega_n}{2\sqrt{\zeta^2-1}}\,\frac{1}{s_1}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right) \qquad \text{where, } s_1 = \zeta\omega_n - \omega_n\sqrt{\zeta^2-1}$$

$$\text{Step response} = A\left[1 - \frac{\omega_n}{2\sqrt{\zeta^2-1}}\,\frac{1}{s_1}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right)\right] \qquad s_2 = \zeta\omega_n + \omega_n\sqrt{\zeta^2-1}$$

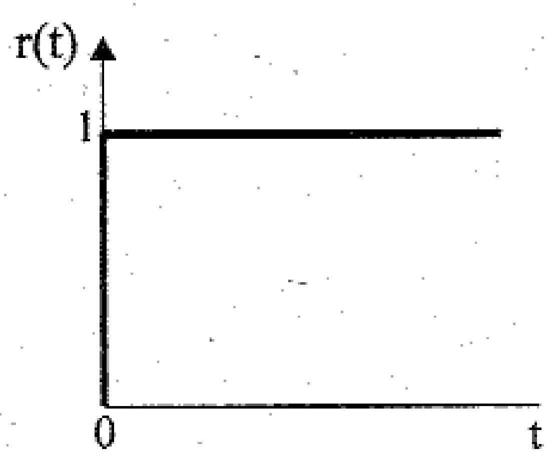


Fig 2.12.a : *Input.*

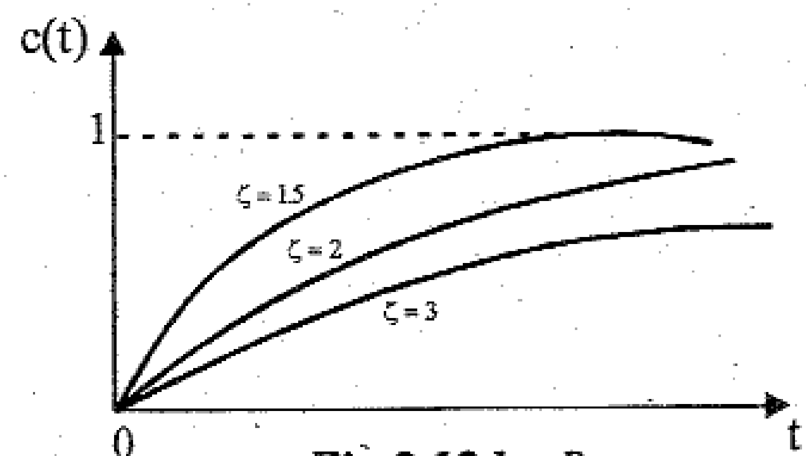


Fig 2.12.b : *Response.*

Fig 2.12 : *Response of over damped second order system for unit step input.*

Time domain specifications

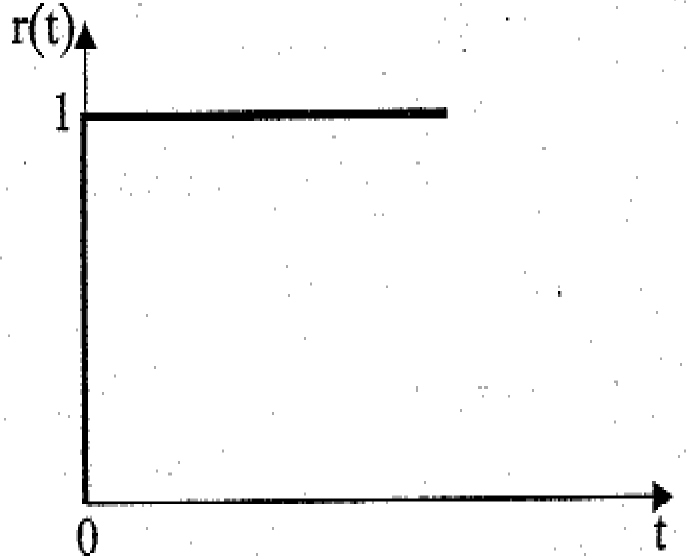


Fig 2.13.a : Input.

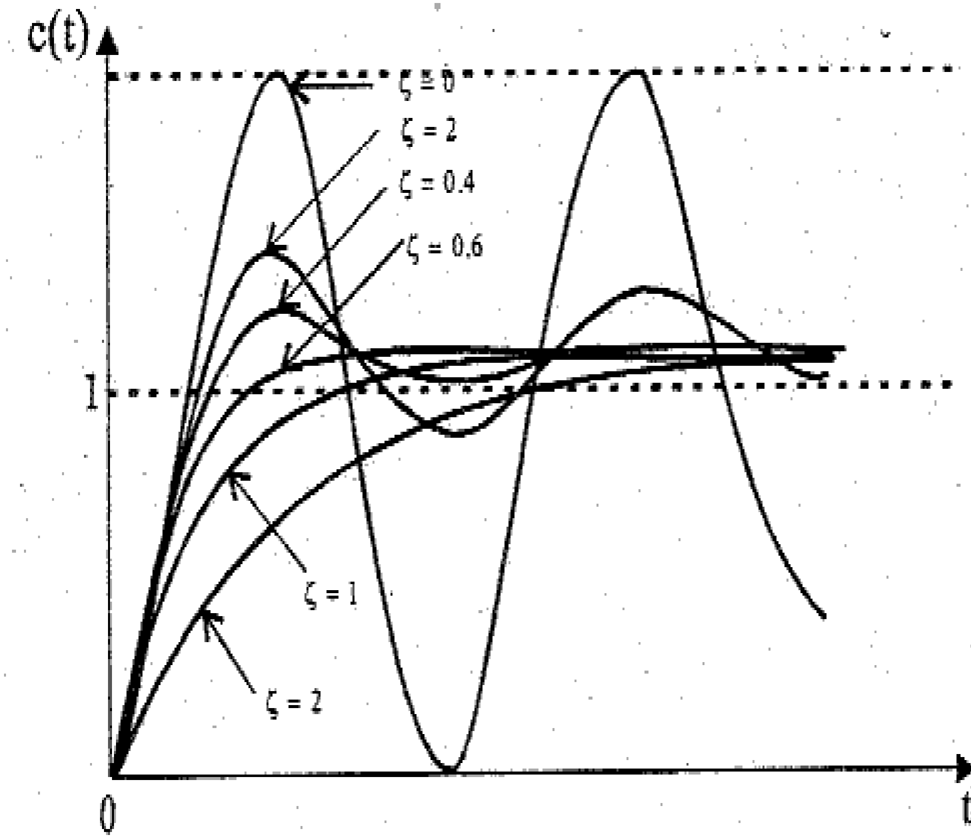


Fig 2.13.b : Response.

1. Delay time, t_d
2. Rise time, t_r
3. Peak time, t_p
4. Maximum overshoot, M_p
5. Settling time, t_s

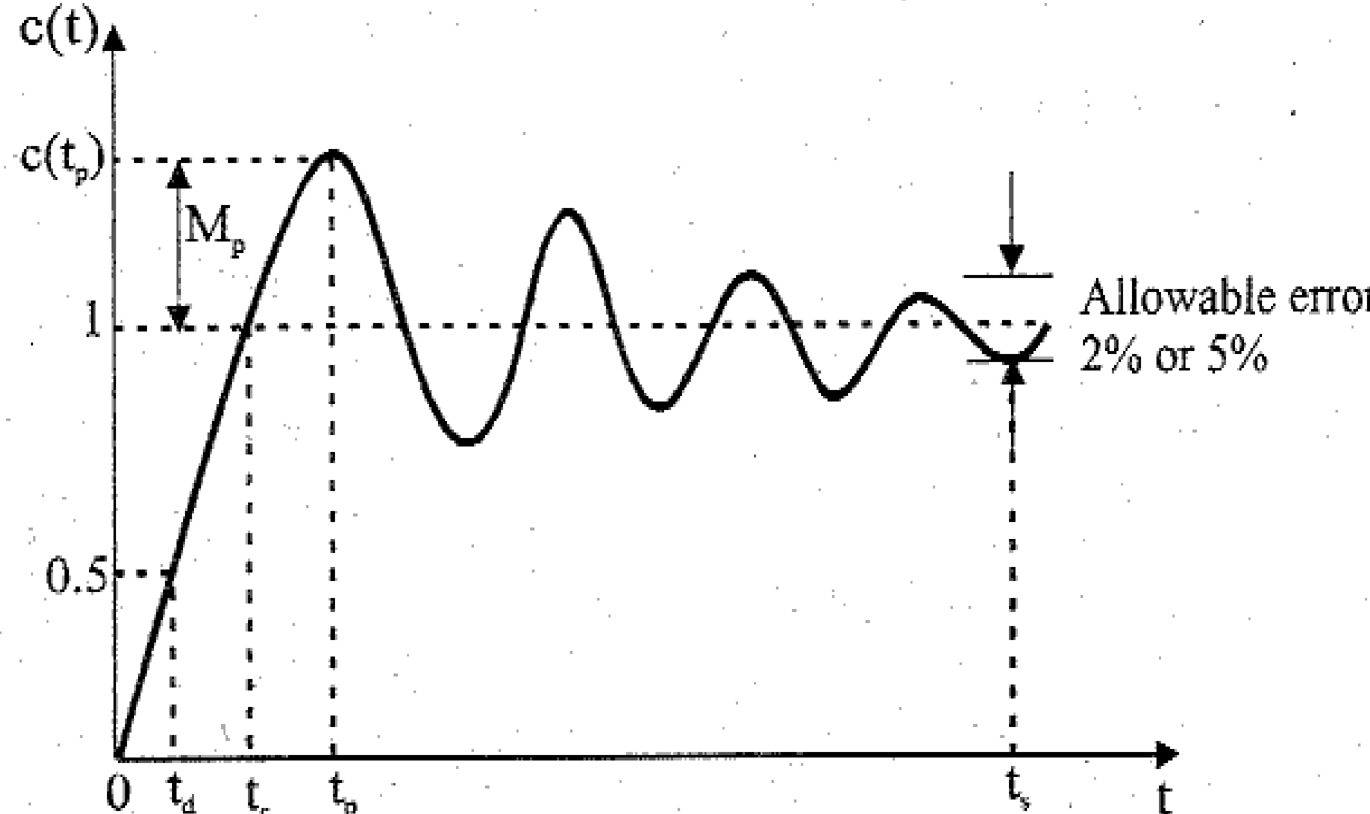


Fig 2.14 : Damped oscillatory response of second order system for unit step input.

1. DELAY TIME (t_d) : It is the time taken for response to reach 50% of the final value, for the very first time.

2. RISE TIME (t_r) : It is the time taken for response to raise from 0 to 100% for the very first time. For underdamped system, the rise time is calculated from 0 to 100%. But for overdamped system it is the time taken by the response to raise from 10% to 90%. For critically damped system, it is the time taken for response to raise from 5% to 95%.

3. PEAK TIME (t_p) : It is the time taken for the response to reach the peak value the very first time. (or) It is the time taken for the response to reach the peak overshoot, M_p.

4. PEAK OVERSHOOT (M_p) : It is defined as the ratio of the maximum peak value to the final value, where the maximum peak value is measured from final value.

Let, $c(\infty)$ = Final value of c(t).

$c(t_p)$ = Maximum value of c(t).

$$\text{Now, Peak overshoot, } M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \qquad \text{.....(2.37)}$$

$$\text{\% Peak overshoot, } \%M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100 \qquad \text{.....(2.38)}$$

5. SETTLING TIME (t_s) : It is defined as the time taken by the response to reach and stay within a specified error. It is usually expressed as % of final value. The usual tolerable error is 2 % or 5% of the final value.