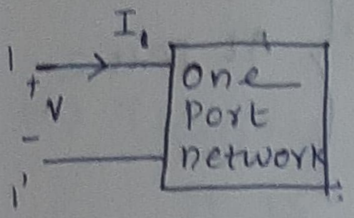
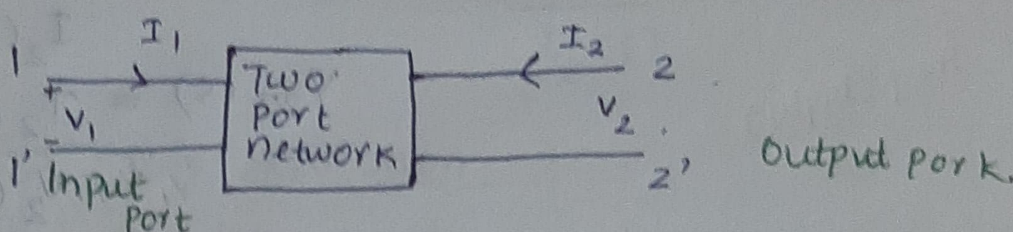
Module- 05

Port in Network

Any network can be represented schematically by a rectangular box. Terminals are needed to connect any network to any other network or for taking some measurement. A network containing one pair of terminals is called one port network and a network containing two pair of terminals is called two port network.





A network containing n pair of terminals is n port network and the port to which the energy source is connected is called input port and the port to which load is connected is known as output port.

Network function is the relation b/w current or voltage and the different parts of a network. It is classified into driving point function and transfer function.

Driving point function (impedance function)

It is defined as the ratio of the voltage transform at one port to the current transform at the same port:

$$Z(s) = \frac{V(s)}{I(s)}$$

Here; excitation and response are measured from same port. [eg:- one port network]

Driving point admittance function

It is defined as the ratio of the current transform at one port to the voltage transform at the same port

$$Y(s) = \frac{I(s)}{V(s)}$$

For two port network; $Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$, $Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$ } At port 1

$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$, $Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$ } At port 2

Transfer function

It relates a voltage or current at one port to the voltage or current at another port. It is also defined as ratio of a response transform to an excitation transform. There are 4 possible transfer functions. Transfer function is described for the network having atleast two ports.

(1) Voltage transfer function

It is defined as the ratio of voltage transform at one port to the voltage transform at another port. It is denoted by $G(s)$

$$G_{12}(s) = \frac{V_2(s)}{V_1(s)} \qquad G_{21}(s) = \frac{V_1(s)}{V_2(s)}.$$

(2) Current transfer function

It is defined as the ratio of the current transform at one port to the current transform at another port. It is denoted as $\alpha(s)$

$$\alpha_{12}(s) = \frac{I_2(s)}{I_1(s)} \qquad \alpha_{21}(s) = \frac{I_1(s)}{I_2(s)}$$

(3) Transfer impedance function

It is defined as the ratio of voltage transform at one port to the current transform at another port. It is denoted by $Z(s)$.

$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)} \qquad Z_{21}(s) = \frac{V_1(s)}{I_2(s)}$$

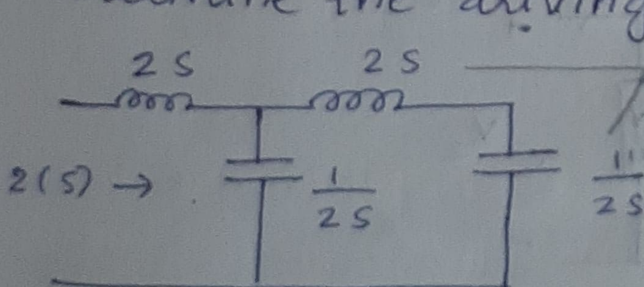
(4) Transfer admittance function.

It is defined as the ratio of current transform at one port to the voltage transform at another port. It is denoted by $Y(s)$.

$$Y_{12}(s) = \frac{I_2(s)}{V_1(s)} \qquad Y_{21}(s) = \frac{I_1(s)}{V_2(s)}$$

Problem:-

1) Determine the driving point impedance of the network.

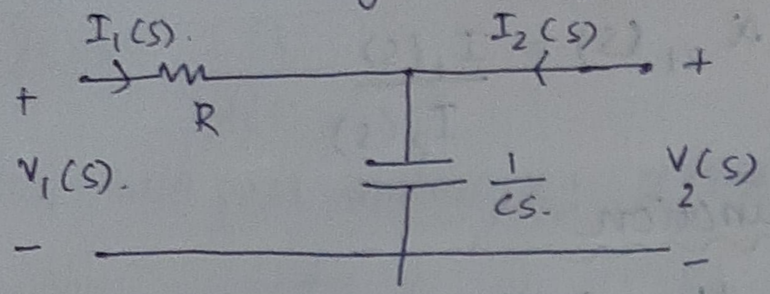


രണ്ട് സമാന്തരം, series. Its equvint parallel it $\frac{1}{2s}$. And that will be Series with 2s

A:- $Z(s) = 2s + \dfrac{\frac{1}{2s}\left(2s + \frac{1}{2s}\right)}{\frac{1}{2s} + 2s + \frac{1}{2s}} = 2s + \dfrac{\frac{1}{2s}\left(2s + \frac{1}{2s}\right)}{\frac{2+4s^2}{2s}}$

$= \dfrac{4s + 8s^3 + 2s + \frac{1}{2s}}{2+4s^2} = \dfrac{16s^4 + 12s^2 + 1}{8s^3 + 4s}$

2] Find the voltage transfer function of the two port network



we need $\frac{V_2(s)}{V_1(s)}$

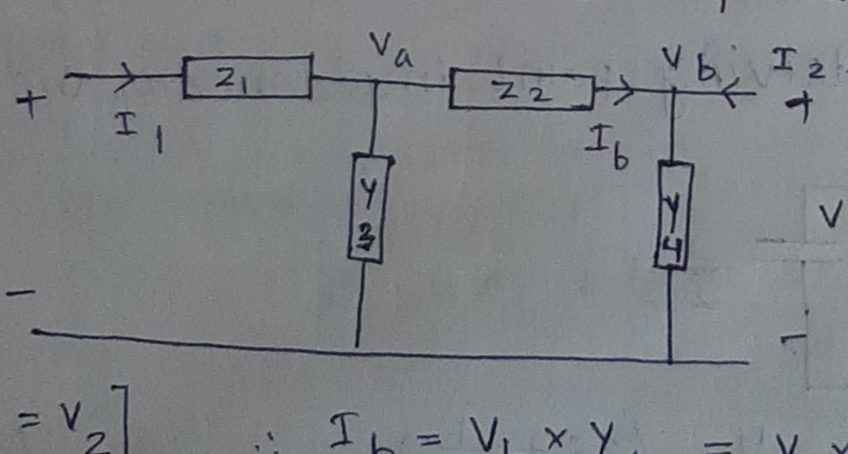
A:- Apply voltage division rule;

$$V_2(s) = V_1(s) \times \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}}$$

Analysis of Ladder Network

Consider the network shown in figure where all the impedance are connected in series branches and all the admittances are connected in parallel branches.



$[V_b = V_2]$ $\therefore I_b = V_b \times Y_4 = V_2 \times Y_4$

$V_a = Z_3 I_b + V_2 = (1 + Z_3 Y_4) V_2$.

$I_1 = Y_2 V_a + I_b$.

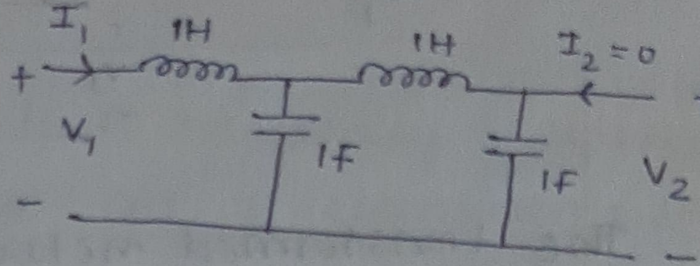
$= [Y_2 (1 + Z_3 Y_4) + Y_4] V_2$

$V_1 = Z_1 I_1 + V_a$

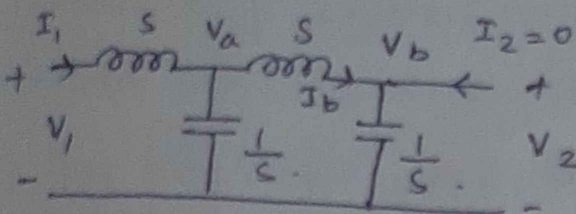
$$V_1 = \left[z_1 \{ y_2 (1 + z_3 y_4) \} + y_4) + (1 + z_3 y_4) \right] V_2$$

<u>Problems:-</u>

1) Find the network functions $\frac{V_1}{I_1}$, $\frac{V_2}{V_1}$ and $\frac{V_2}{I_1}$ for the network



A:- The transferred n/w is shown below



$V_b = V_2$

$$I_b = \frac{V_b}{1/s} = \frac{V_2}{1/s} = sV_2$$

$$V_a = sI_b + V_2 = s(sV_2) + V_2 = (s^2+1)V_2$$

$$I_1 = \frac{V_a}{1/s} + I_b = sV_a + I_b$$

$$= s(s^2+1)V_2 + sV_2 = (s^3+2s)V_2$$

$$V_1 = sI_1 + V_a$$

$$= s(s^3+2s)V_2 + (s^2+1)V_2$$

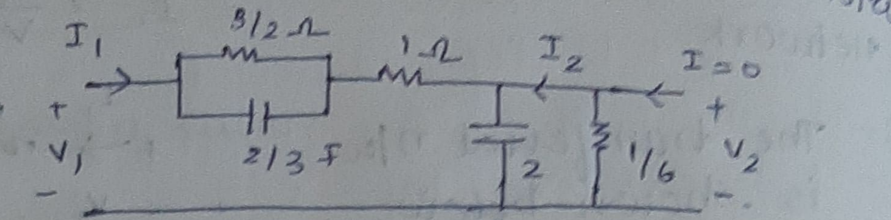
$$= (s^4 + 2s^2 + s^2 + 1)V_2 = (s^4 + 3s^2 + 1)V_2$$

$$\frac{V_1}{I_1} = \frac{(s^4+3s^2+1)V_2}{(s^3+2s)V_2} = \underline{\underline{\frac{s^4+3s^2+1}{s^3+2s}}}$$

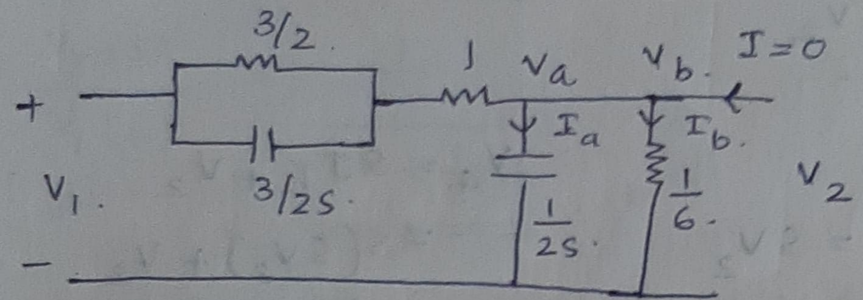
$$\frac{V_2}{V_1} = \frac{V_2}{(s^4+3s^2+1)V_2} = \underline{\underline{\frac{1}{s^4+3s^2+1}}}$$

$$\frac{V_2}{I_1} = \frac{V_2}{(s^3+2s)V_2} = \frac{1}{s^3+2s} = \frac{1}{s(s^2+2)}$$

2] For the network shown in fig; determine the transfer function $\frac{I_2}{V_1}$



A:- The transformed network is



$$V_a = V_b = V_2$$

$$I_1 = I_a + I_b = \frac{V_2}{1/2s} + \frac{V_2}{1/6} = 2sV_2 + 6V_2 = (2s+6)V_2$$

$$V_1 = \left[\frac{\frac{3}{2} \times \frac{3}{2s}}{\frac{3}{2} + \frac{3}{2s}} + 1\right] I_1 + V_2$$

$$= \left[\frac{9}{6s+6} + 1\right] I_1 + V_2 = \frac{(6s+15)}{6s+6}(2s+6)V_2 + V_2$$

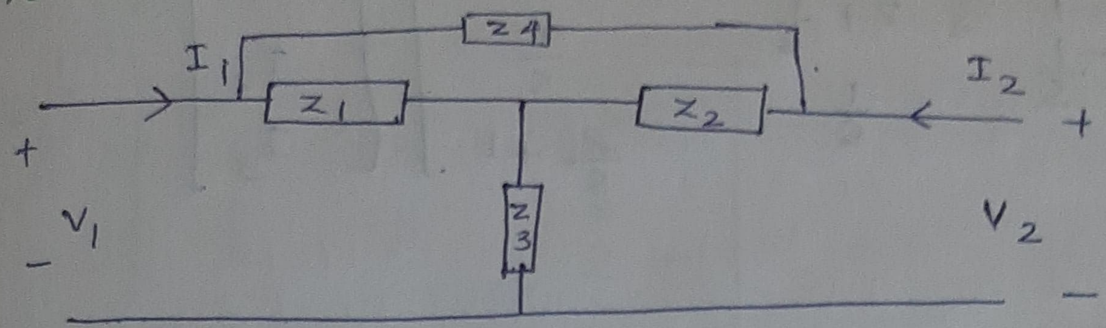
$$= \left[\frac{(2s+5)(s+3)}{s+1} + 1\right] V_2 = \left[\frac{(2s+5)(s+3)+(s+1)}{(s+1)}\right] V_2$$

$$= \frac{2(s^2+16s+8)V_2}{s+1} = \frac{2(s+4)(s+2)V_2}{(s+1)}$$

$$I_2 = -I_b = -6V_2$$

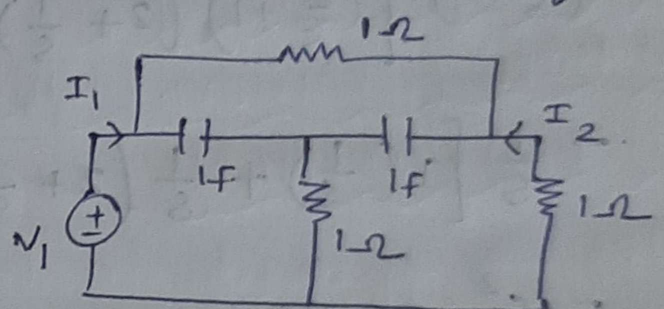
$$\frac{I_2}{V_1} = \frac{-3(s+1)}{(s+4)(s+2)}$$

Analysis of non-ladder Network

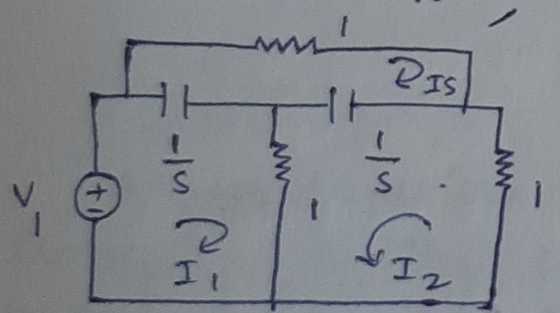


For these type of network; it is necessary to express the network functions as a quotient of determinants formulated on KVL and KCL basis.

1] For the network shown in figure; find the driving point admittance Y_{11} and transfer admittance Y_{12}.



Ans:- The transformed network is:



Apply KVL to mesh 1;

$$V_1 - \frac{1}{s}(I_1 - I_3) - 1(I_1 + I_2) = 0$$

$$\left(\frac{1}{s} + 1\right) I_1 + I_2 - \frac{1}{s} I_3 = V_1 \quad \text{—(1)}$$

Apply KVL to mesh 2;

$$-1 I_2 - \frac{1}{s}(I_2 + I_3) - 1(I_2 + I_1) = 0$$

$$I_1 + \left(2 + \frac{1}{s}\right) I_2 + \frac{1}{s} I_3 = 0 \quad \text{—(2)}$$

Apply KVL to mesh 3;

$$-I_3 - \frac{1}{s}(I_3 + I_2) - \frac{1}{s}(I_3 - I_1) = 0$$

$$-\frac{1}{s} I_1 + \frac{1}{s} I_2 + \left(\frac{2}{s} + 1\right) I_3 = 0.$$

$$\begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s}+1 & 1 & -\frac{1}{s} \\ 1 & 2+\frac{1}{s} & \frac{1}{s} \\ -\frac{1}{s} & \frac{1}{s} & \frac{2}{s}+1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta = \begin{vmatrix} \frac{1}{s}+1 & 1 & -\frac{1}{s} \\ 1 & 2+\frac{1}{s} & \frac{1}{s} \\ -\frac{1}{s} & \frac{1}{s} & \frac{2}{s}+1 \end{vmatrix}$$

$$= \left(\frac{1}{s}+1\right)\left[\left(\frac{2}{s}+1\right)\left(2+\frac{1}{s}\right)-\frac{1}{s^2}\right] - 1\left[1+\left(1+\frac{2}{s}\right)+\frac{1}{s^2}\right]$$

$$-\frac{1}{s}\left[1\left(\frac{1}{s}\right)+\frac{1}{s}\left(2+\frac{1}{s}\right)\right]$$

$$= \frac{s^2+5s+2}{s^2}$$

$$\Delta_1 = \begin{vmatrix} V_1 & 1 & -1/s \\ 0 & 2+1/s & 1/s \\ 0 & 1/s & 2/s+1 \end{vmatrix}$$

$$= V_1\left[(2+1/s)(1+2/s)-\frac{1}{s^2}\right]$$

$$= V_1\left(\frac{2s^2+5s+1}{s^2}\right)$$

$$I_1 = \frac{\Delta_1}{\Delta} = V_1\left(\frac{2s^2+5s+1}{s^2+5s+2}\right)$$

$$I_2 = \frac{\Delta 2}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} \frac{1}{s}+1 & V_1 & -\frac{1}{s} \\ 1 & 0 & \frac{1}{s} \\ -\frac{1}{s} & 0 & \frac{2}{s}+1 \end{vmatrix}$$

$$= -V_1\left[\frac{2}{s}+1+\frac{1}{s^2}\right] = -V_1\left(\frac{s^2+2s+1}{s^2}\right)$$

$$I_2 = -V_1\left(\frac{s^2+2s+1}{s^2+5s+2}\right)$$

$\therefore$ Transfer admittance $= Y_{12} = \frac{I_2}{V_1} = \frac{-s^2+2s+1}{s^2+5s+2}$

$$Y_{11} = \frac{I_1}{V_1} = \frac{2s^2+5s+1}{s^2+5s+2}$$

Poles and zeros of transfer function

Transfer function is the ratio of the laplace transform of the output to the laplace transform of the input considering all initial conditions to zero.

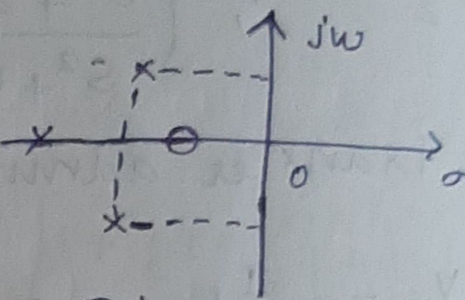
$$u(s) \longrightarrow \boxed{G(s)} \longrightarrow y(s)$$

The transfer function $G(s)$ of the plant $G(s) = \frac{y(s)}{u(s)}$

$$\frac{Y(s)}{u(s)} = \frac{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n}$$

Roots of denominator polynomial of a transfer function are called <u>poles</u>. Roots of numerator polynomial of a transfer function are called <u>zeros</u>.

poles of the system are represented by 'x' and zeros of the system are represented by 'o'. System order is always equal to number of poles of transfer function.



Pole zero plot

Restrictions on pole and zero locations for driving point function (common factors in N(s) and D(s) cancelled)

1) The co-efficients in the polynomials N(s) & D(s) must be real and positive.
2) The poles and zeros, if complex or imaginary must occur in conjugate pairs.
3) The real part of all poles and zeros must be negative or zero, ie; the poles and zeros must lie in left half of s plane.
4) If the real part of pole or zero is zero, then that pole or zero must be simple.
5) The polynomial N(s) and D(s) may not have missing terms between those of highest and

lowest degree unless all even or all odd terms are missing

6) The degree of N(S) & D(S) may differ by either zero or one only. This condition prevents multiple poles and zeros at $S = \infty$.

7) The terms of lowest degree in N(S) & D(S) may differ in degree by one at most. This condition prevents multiple poles and zeros at $S = 0$.

Restriction on poles and zero locations for transfer functions (common factors in N(S) & D(S) cancelled.)

1) The co-efficients in the polynomials N(S) & D(S) must be real and those for D(S) must be positive

2) The poles and zeros if complex or imaginary must occur in conjugate pairs.

3) The real part of poles must be negative or zero. If the real part is zero; then that pole must be simple.

4) The polynomial D(S) may not have any missing terms between that of highest and lowest degree, unless all even or all odd terms are missing.

5) The polynomial N(S) may have terms missing between the terms of lowest and highest degree and some of the co-efficients may be negative.

6) The degree of N(S) may be as small as zero independent of the degree of D(S).

7) For voltage and current transfer functions, the maximum degree of N(S) is the degree of D(S)

8) For transfer impedance and admittance functions, the maximum degree of N(S) is the degree of D(S) plus one.

Question :-

1) Test whether the following represent driving point immittances.

(a) $\dfrac{5s^4 + 3s^2 - 2s + 1}{s^3 + 6s + 20}$ (b) $\dfrac{s^3 + s^2 + 5s + 2}{s^4 + 6s^3 + 9s^2}$

A:- (a) Numerator and denominator polynomial have missing term between those of highest and lowest degree and one of the co-efficient is negative in numerator polynomial. Hence; the function doesnot represent driving point immittance.

(b) $\dfrac{s^3 + s^2 + 5s + 2}{s^4 + 6s^3 + 9s^2}$

The term of lowest degree in numerator and denominator polynomial differ in degree by 2, which is not permitted. Hence, the function doesnot represent driving point immitance.

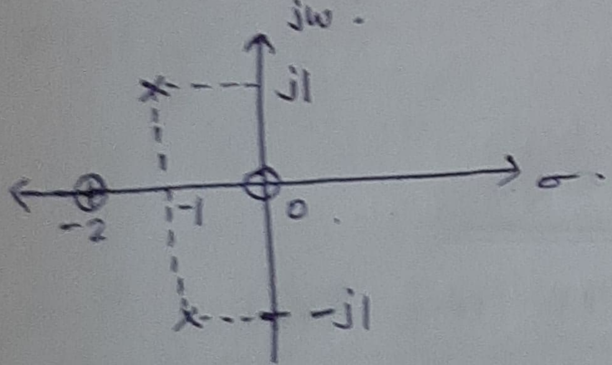
2) obtain the pole zero plot of the following functions

$$F(s) = \frac{s(s+2)}{s^2+2s+2}$$

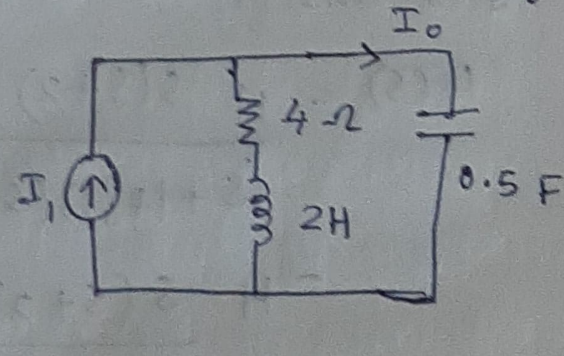
A:- $$F(s) = \frac{s(s+2)}{(s+1+j1)(s+1-j1)}$$

F(s) has zeros at $s=0$ & $s=-2$ and poles at $s=-1-j1$ and $s=-1+j1$

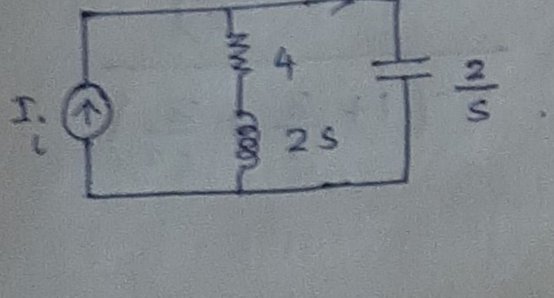
The pole zero plot is shown in figure.



3) For the network shown, plot poles and zeros of function $\frac{I_o}{I_1}$.



A:- Transformed network is shown below;

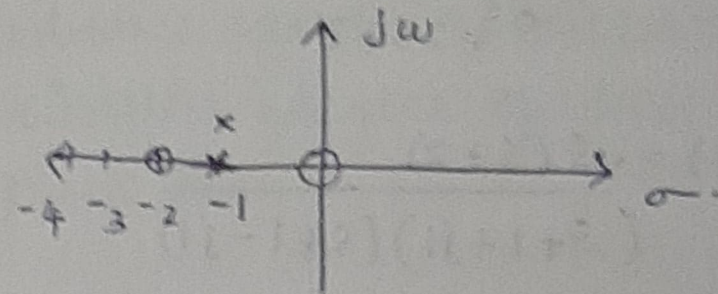


$$I_o = \frac{I_i(4+2s)}{4+2s+\frac{2}{s}}$$

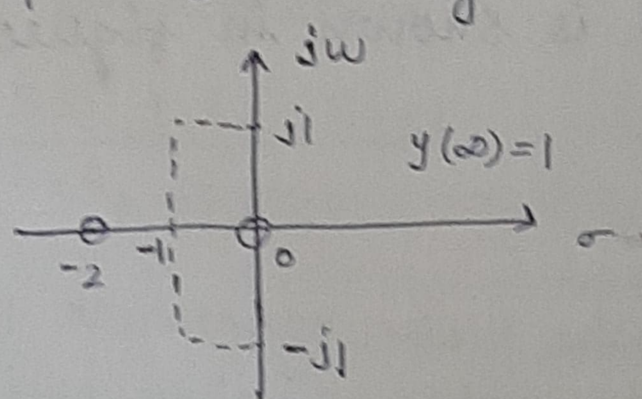
$$\frac{I_o}{I_1} = \frac{s(4+2s)}{4s+2s^2+2}$$

$$\frac{I_o}{I_1} = \frac{s(s+2)}{s^2+2s+1} = \frac{s(s+2)}{(s+1)(s+1)}$$

There is double poles at $s=-1$ and zeros at $s=0$ and $s=-2$



4] obtain the admittance function $y(s)$ for which the pole-zero diagram is shown in fig.



$y(s)$ has poles at $s=-1 \pm j1$ and zeros at $s=0$ & $s=-2$

$$y(s) = H\frac{s(s+2)}{(s+1+j1)(s+1-j1)} = H\left(\frac{s(s+2)}{(s+1)^2+1^2}\right)$$

$$= H\frac{s(s+2)}{s^2+2s+2} = H\frac{s^2\left(1+\frac{2}{s}\right)}{s^2\left(1+\frac{2}{s}+\frac{2}{s^2}\right)}$$

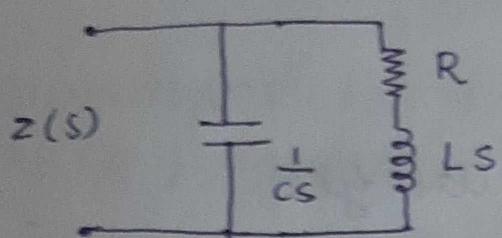
for $s=\infty$

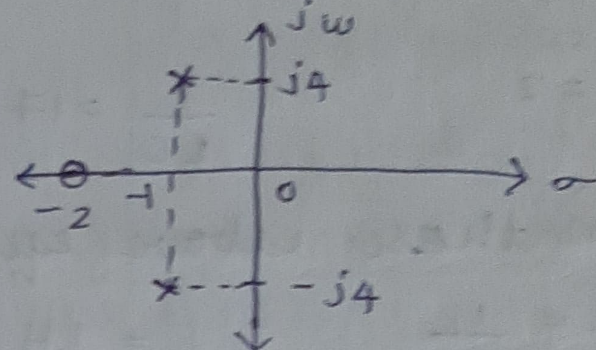
$$y(\infty) = \frac{H(1)}{(1)} = H$$

$y(\infty)=1$ $\therefore H=1$

$$y(s) = \frac{s(s+2)}{s^2+2s+2}$$

5] The pole-zero diagram of the driving point impedance function of the network is shown below. At dc, the input impedance is resistive and equal to 2Ω. Determine the value of R, L, & C.





$$Z(s) = \frac{(Ls+R)\frac{1}{Cs}}{Ls+R+\frac{1}{Cs}} = \frac{Ls+R}{LCs^2+RCs+1}$$

$$= \frac{L\left(s+\frac{R}{L}\right)}{LC\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)} = \frac{\frac{1}{C}\left(s+\frac{R}{L}\right)}{s^2+\frac{R}{L}s+\frac{1}{LC}} \quad \text{——— (1)}$$

From pole-zero diagram, zero is at $s=-2$ and poles are at $s=-1+j4$ and $s=-1-j4$

$$Z(s) = H\frac{s+2}{(s+1+j4)(s+1-j4)}$$

$$= H\frac{(s+2)}{(s+1)^2+4^2} = H\frac{(s+2)}{s^2+2s+17}$$

At dc; ie; $w_1=0$; $z(j0)=2$

$$2 = H \frac{2}{17}$$

$$H = 17$$

$$\therefore Z(s) = 17 \frac{s+2}{s^2+2s+17} \quad \text{— (2)}$$

comparing (1) & (2);

$$\frac{1}{C} = 17$$

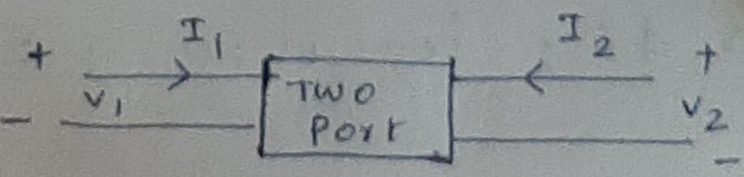
$$\frac{R}{L} = 2 \qquad \frac{1}{LC} = 17$$

From these above equation, we get

$$\underline{\underline{C = \frac{1}{17}}} \qquad \underline{\underline{L = 1\,H}} \qquad \underline{\underline{R = 2\,\Omega}}$$

TWO - PORT NETWORKS

A two port network has two pair of terminals one pair at the input (input port) and one pair at the output (output port). There are four variables associated with a two-port network. They are V_1, V_2, I_1, I_2. Two of these variables can be expressed in terms of other two variables. Thus, there will be two dependent variables and two independent variables. ∴ No: of possible combination is $^4C_2$ (six). There will 6 two port parameters.



TWO PORT PARAMETERS

Parameter	Variables: Express	Variables: In terms of	Equation
open circuit impedance	V_1 V_2	I_1 I_2	$V_1 = Z_{11} I_1 + Z_{12} I_{12}$ $V_2 = Z_{21} I_1 + Z_{22} I_2$
short circuit admittance	I_1, I_2	V_1 V_2.	$I_1 = Y_{11} V_1 + Y_{12} V_2$ $I_2 = Y_{21} V_1 + Y_{22} V_2$
Transmission/ ABCD parameter	V_1 I_1	V_2 I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
Inverse transmission/ $A'B'C'D'$	V_2 I_2	V_1 I_1	$V_2 = A'V_1 - B'I_1$ $I_2 = C'V_1 - D'I_1$
Hybrid / H	V_1, I_2	I_1 V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$
Inverse hybrid/ G	I_1 V_2	V_1 I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$

Open circuit Impedance parameter (Z-parameter)

Z parameter of a two port network may be defined by expressing two port voltages V_1 and V_2 in terms of two port currents I_1 and I_2.

$$(V_1, V_2) = f(I_1, I_2)$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In matrix form;

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V = [Z][I]$$

The individual Z parameter for a given network can be defined by setting each of the port currents equal to zero.

Case I

when output port is open circuited; $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2 = 0}.$$

where Z_{11} is the driving point impedance with output port OC. It is called open circuit input impedence.

Similarily; $Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2 = 0}$:-

Z_{21} = transfer impedance with ~~input~~ output port open circuited. It is called open circuit forward transfer impedance.

Case 2:-

when input port is open circulated; $I_1 = 0$

$$Z_{12} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

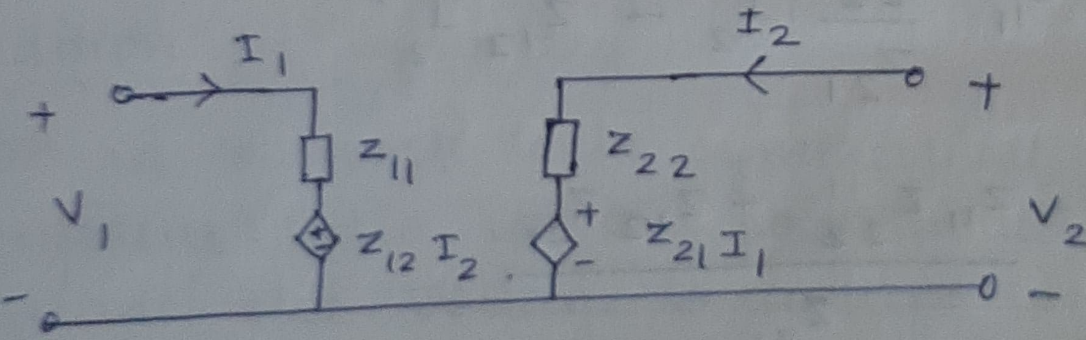
Z_{12} = transfer impedance with the input port OC. It is also called open circuit reverse transfer impedance

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

Z_{22} = open circuit driving point impedance with the input port OC. It is also called open circuit output impedance.

All these impedance parameters are measured with either the input or output port open circuited. ∴ this is called open circuit impedance parameters.

The equivalent circuit of two port network in terms of Z-parameters is



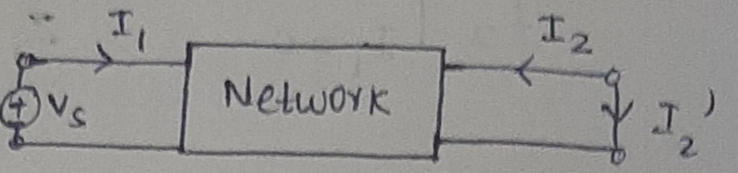
$$V_1 - Z_{11} I_1 - Z_{12} I_2 = 0$$

$$V_2 - Z_{21} I_1 - Z_{22} I_2 = 0.$$

Condition for reciprocity

A network is said to be reciprocal if the ratio of excitation at one port to the response at the port is same if excitation and response are interchanged.

A voltage V_s is applied at input port with the output port short circuited.



$$V_1 = V_s \qquad V_2 = 0 \qquad I_2 = -I_2'$$

From z-parameter equations;

$$V_s = Z_{11} I_1 - Z_{12} I_2'$$

$$0 = Z_{21} I_1 - Z_{22} I_2'$$

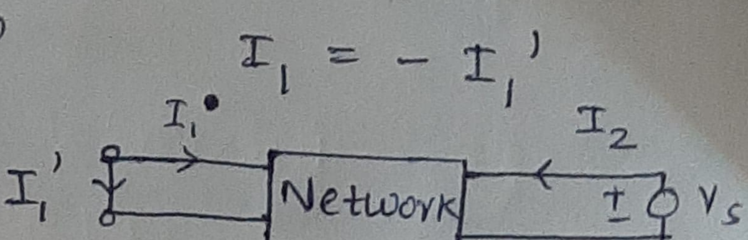
$$I_1 = \frac{Z_{22}}{Z_{21}} I_2'$$

$$V_s = Z_{11} \frac{Z_{22}}{Z_{21}} I_2' - Z_{12} I_2'$$

$$\frac{V_s}{I_2'} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}$$

When voltage V_s is applied at output port with input port short circuited.

$$V_2 = V_s. \qquad V_1 = 0 \qquad I_1 = -I_1'$$



From z-parameter equations;

$$0 = -Z_{11} I_1' + Z_{12} I_2$$

$$V_S = -Z_{21} I_1' + Z_{22} I_2$$

$$I_2 = \frac{Z_{11}}{Z_{12}} I_1'$$

$$V_S = -Z_{21} I_1' + Z_{22} \frac{Z_{11}}{Z_{12}} I_1'$$

$$\frac{V_S}{I_1'} = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{12}}.$$

Hence; the network to be reciprocal;

$$\boxed{\begin{array}{c} \dfrac{V_S}{I_1'} = \dfrac{V_S}{I_2'} \\ Z_{12} = Z_{21} \end{array}}$$

Condition for Symmetry

For a network to be symmetrical, the voltage to current ratio at one port should be the same as the voltage to current ratio at the other port with one of the ports open circuited.

(a) when output port is open circuited;

ie; $I_2 = 0$.

From z-parameter equations.

$$V_S = Z_{11} I_1$$

$$\frac{V_s}{I_1} = Z_{11}$$

(b) when the input port is open circulated, ie;
$I_1 = 0$.

From the z-parameter equation;

$$V_s = Z_{22} I_2$$

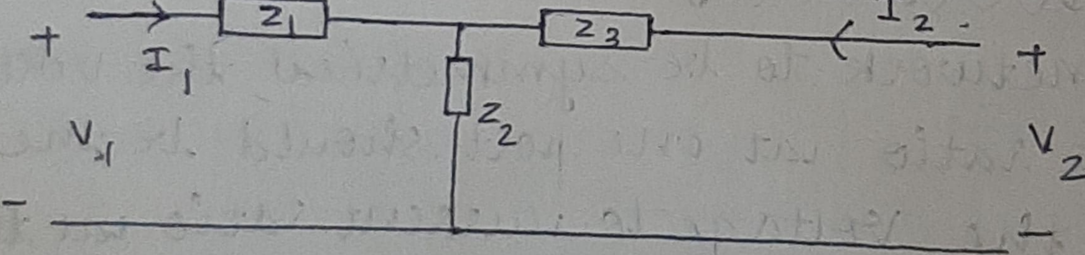
$$\frac{V_s}{I_2} = Z_{22}.$$

Hence; for the network to be symmetrical;

$$\boxed{\begin{gathered} \frac{V_s}{I_1} = \frac{V_s}{I_2} \\ Z_{11} = Z_{22}. \end{gathered}}$$

PROBLEMS :-

1) Find the z-parameters for the network.



First method

case 1 : When the output port is open circulated
$I_2 = 0$

Apply KVL to mesh 1;

$$V_1 = (Z_1 + Z_2) I_1$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = z_1 + z_2$$

$$V_2 = Z_2 I_1$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_2$$

<u>case II:</u> When the input port is open circuited.
ie; $I_1 = 0$

Apply KVL to mesh 2;

$$V_2 = (Z_2 + Z_3) I_2$$

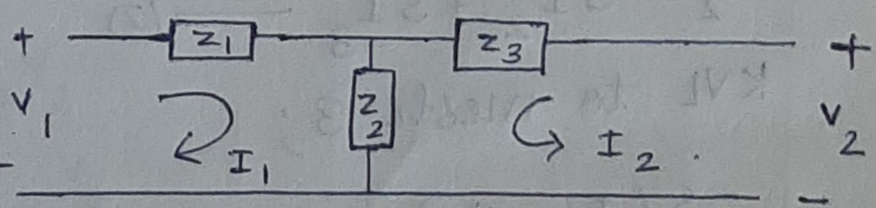
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_2 + Z_3$$

Also; $V_1 = Z_2 I_2$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_2$$

Second method

Apply KVL to mesh 1;



$$V_1 = Z_1 I_1 + Z_2 (I_1 + I_2)$$
$$= (Z_1 + Z_2) I_1 + Z_2 I_2 \quad \text{—— (1)}$$

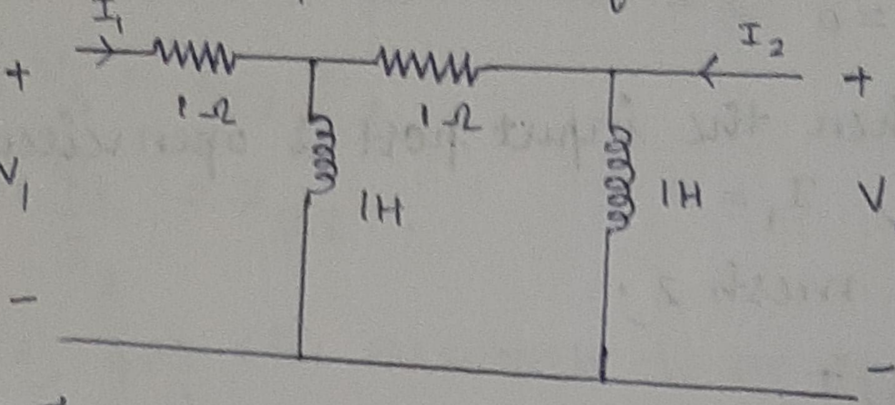
Apply KVL to mesh 2;

$$V_2 = Z_3 I_2 + Z_2 (I_1 + I_2)$$
$$= Z_2 I_1 + (Z_2 + Z_3) I_2 \quad \text{—— (2)}$$

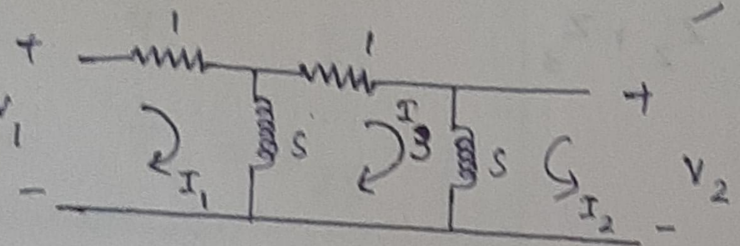
By comparing eqn (1) & (2);

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 + Z_3 \end{bmatrix}$$

2] Find the z-parameter for the network shown.



Ans:- Transformed network is.



KVL to mesh 1;

$V_1 = (s+1) I_1 - sI_3$ —— (1)

KVL to mesh 2;

$V_2 = sI_2 + sI_3$ —— (2)

KVL to mesh 3;

$-s(I_3 - I_1) - I_3 - s(I_2 + I_3) = 0$

$-sI_1 + sI_2 + (2s+1) I_3 = 0$ —— (3)

Substitute (3) in (1)

$V_1 = (s+1) I_1 - s\left(\frac{s}{2s+1} I_1 - \frac{s}{2s+1} I_2\right)$

$$= \left(\frac{s^2+3s+1}{2s+1}\right) I_1 + \frac{s^2}{2s+1} I_2 \quad \text{—— (4)}$$

Substitute (3) in (2);

$$V_2 = sI_2 + s\left(\frac{s}{2s+1} I_1 - \frac{s}{2s+1} I_2\right)$$

$$V_2 = \left(\frac{s^2}{2s+1}\right) I_1 + \left(\frac{s^2+s}{2s+1}\right) I_2 \quad \text{—— (5)}$$

comparing eqn (4) & (5);

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{s^2+3s+1}{2s+1} & \frac{s^2}{2s+1} \\ \frac{s^2}{2s+1} & \frac{s^2+s}{2s+1} \end{bmatrix}$$

SHORT CIRCUIT ADMITTANCE PARAMETERS (Y)

The Y parameters of a two port network may be defined by expressing the two port currents I_1 and I_2 in terms of the two-port voltage V_1 and V_2.

$$(I_1, I_2) = f(V_1, V_2)$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

The individual Y parameters for a given

network can be defined by setting each of port currents equal to zero.

Case I: When the output port is short circuited ie; $V_2 = 0$.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0}.$$

Y_{11} = Short circuit input admittance

Similarily; $I_2 = Y_{21} V_1 + Y_{22} V_2$

$V_2 = 0$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$$

Y_{21} = short circuit forward transfer admittance

Case II: When the input port is short circuited. (ie; $V_1 = 0$)

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}$$

Y_{22} = short circuit output admittance

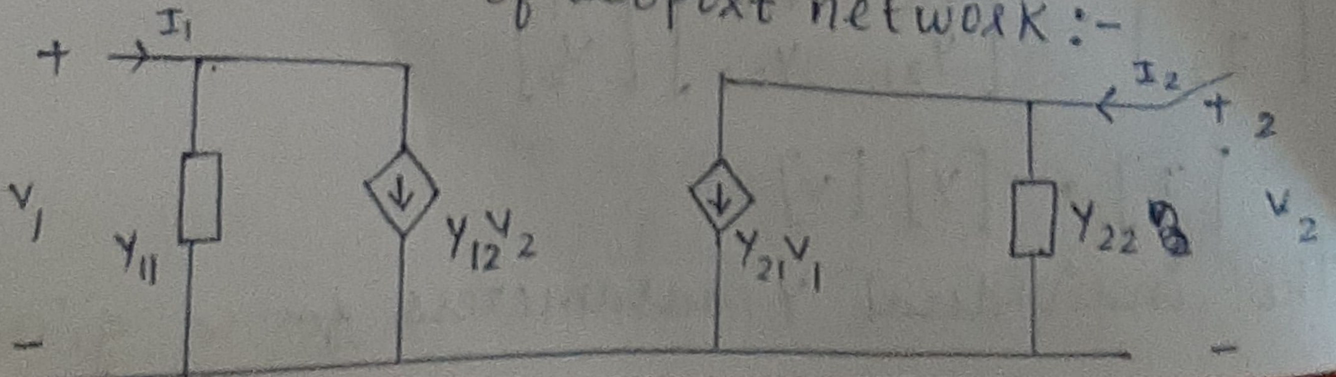
Similarily; $I_1 = Y_{11} V_1 + Y_{12} V_2$

$V_1 = 0$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$

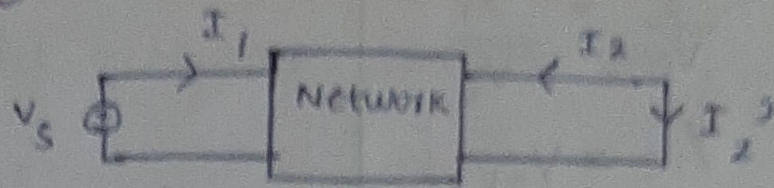
Y_{12} = Short circuit reverse transfer admittance

Equivalent circuit of twoport network :-



Condition for reciprocity

A network is said to be reciprocal if the ratio of excitation



at one port to response at the other port is same if excitation and response are interchanged.

$V_1 = V_s \qquad V_2 = 0 \qquad I_2 = -I_2'$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

From Y- parameter equation;

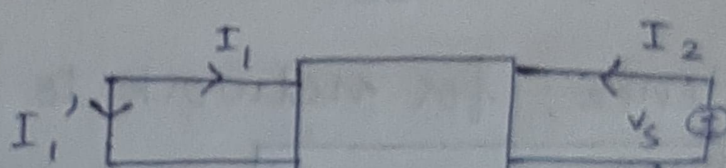
$\therefore \; -I_2' = Y_{21} V_s$

$$\frac{I_2'}{V_s} = -Y_{21}$$

(b) Now when voltage V_s is applied at output port with the input port short circuited.

$V_2 = V_s$

$V_1 = 0$



$I_1 = -I_1'$

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$-I_1' = Y_{12} V_s$

$$\frac{I_1'}{V_s} = -Y_{12}$$

For network to be reciprocal;

$$\boxed{\begin{gathered}\frac{I_2'}{V_s} = \frac{I_1'}{V_s} \\ Y_{12} = Y_{21}\end{gathered}}$$

Condition for Symmetry:

(a) when o/p port is short circuited, $V_2 = 0$

From Y parameter equation:

$$I_1 = Y_{11} V_s$$

$$\frac{V_s}{I_1} = \frac{1}{Y_{11}}$$

(b) when the input port is short circuited, ie, $V_1 = 0$

From Y-parameter equation,

$$I_2 = Y_{22} V_s$$

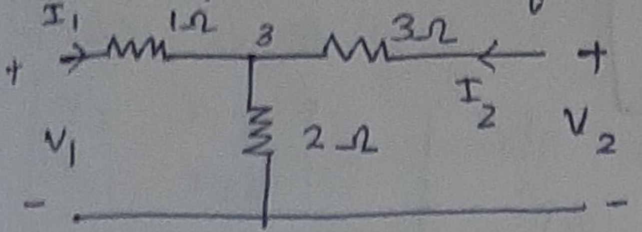
$$\frac{V_s}{I_2} = \frac{1}{Y_{22}}$$

Hence, for network to be symmetrical,

$$\boxed{\begin{gathered}\frac{V_s}{I_1} = \frac{V_s}{I_2} \\ Y_{11} = Y_{22}\end{gathered}}$$

Problems:-

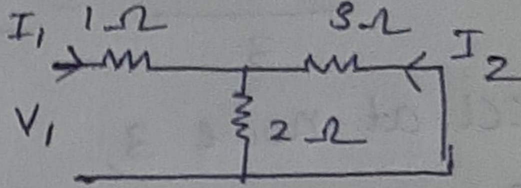
1) Find Y parameters for the network shown in fig.



Ans:- First method

case I: when output port is short circuited

$V_2 = 0$.



$$R_{eq} = \frac{3 \times 2}{3+2} + 1 = \frac{11}{5}\ \Omega$$

$$V_1 = \frac{11}{5} \times I_1$$

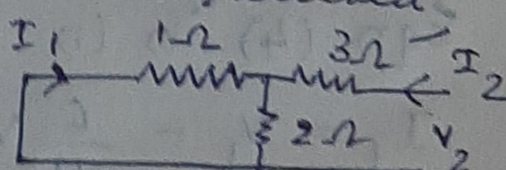
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{5}{11}\ \mho$$

$$I_2 = \frac{2}{5}(-I_1) = -\frac{2}{5} \times \frac{5}{11} V_1 = -\frac{2}{11} V_1$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-2}{11}\ \mho$$

case II: when input port is short circuited.

$V_1 = 0$.



$$R_{eq} = 3 + \frac{1 \times 2}{1+2} = 3 + \frac{2}{3} = \frac{11}{3}\ \Omega$$

$$V_2 = \frac{11}{3} I_2$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3}{11}\ \mho$$

$$I_1 = \frac{2}{3}(-I_2) = \frac{-2}{3} \times \frac{3}{11} V_2 = \frac{-2}{11} V_2$$

Y parameters are;

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 5/11 & -2/11 \\ -2/11 & 3/11 \end{bmatrix}$$

Second method

$$I_1 = \frac{V_1 - V_S}{1} = V_1 - V_S \quad \text{——(1)}$$

$$I_2 = \frac{V_2 - V_S}{3} = \frac{V_2}{3} - \frac{V_3}{3} \quad \text{——(2)}$$

KCL at node 3;

$$I_1 + I_2 = \frac{V_3}{2} \quad \text{——(3)}$$

Substitute (1) & (2) in (3)

$$V_1 - V_3 + \frac{V_2}{3} - \frac{V_3}{3} = \frac{V_3}{2}$$

$$V_1 + \frac{V_2}{3} = V_3\left(\frac{1}{2} + \frac{1}{3} + 1\right) = \frac{11}{6} V_3$$

$$V_3 = \frac{6}{11} V_1 + \frac{2}{11} V_2 \quad \text{——(4)}$$

Eqn (4) in (1)

$$I_1 = V_1 - \frac{6}{11} V_1 - \frac{2}{11} V_2$$

$$= \frac{5}{11} V_1 - \frac{2}{11} V_2 \quad \text{——(5)}$$

Eqn (4) in (2)

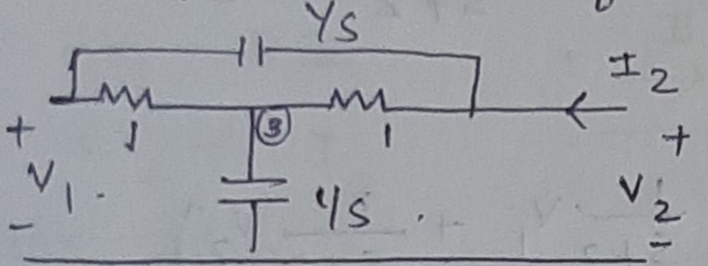
$$I_2 = \frac{V_2}{3} - \frac{1}{3}\left(\frac{6}{11} V_1 + \frac{2}{11} V_2\right)$$

$$= -\frac{2}{11} V_1 + \frac{3}{11} V_2 \quad \text{--- (6)}$$

From (5) & (6) ;

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 5/11 & -2/11 \\ -2/11 & 3/11 \end{bmatrix}$$

2] Obtain Y parameter of the network.



A:- Apply KCL at node 1 ;

$$I_1 = \frac{V_1 - V_3}{1} + \frac{V_1 - V_2}{1/s}$$

$$I_1 = (s+1) V_1 - s V_2 - V_3 \quad \text{--- (1)}$$

Apply KCL at node 2.

$$I_2 = \frac{V_2 - V_3}{1} + \frac{V_2 - V_1}{1/s} = (s+1) V_2 - s V_1 - V_3 \quad \text{--- (2)}$$

Apply KCL at node 3.

$$\frac{V_3}{1/s} + \frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{1} = 0$$

$$(s+2) V_3 - V_1 - V_2 = 0$$

$$V_3 = \frac{1}{s+2} V_1 + \frac{1}{s+2} V_2 \quad \text{--- (3)}$$

Eqn (3) in (1) gives.

$$I_1 = (s+1)V_1 - sV_2 - \left(\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right)$$

$$= \left[\frac{(s+1)(s+2)-1}{(s+2)}\right]V_1 - \left[\frac{s(s+2)+1}{(s+2)}\right]V_2$$

$$= \left[\frac{s^2+3s+1}{s+1}\right]V_1 - \left(\frac{s^2+2s+1}{s+2}\right)V_2 \quad \text{--- (4)}$$

Eqn (3) in (2).

$$I_2 = (s+1)V_2 - sV_1 - \left(\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right)$$

$$= -\left[\frac{s(s+2)+1}{s+2}\right]V_1 + \left[\frac{(s+1)(s+2)-1}{(s+2)}\right]V_2$$

$$= -\left(\frac{s^2+2s+1}{s+2}\right)V_1 + \left(\frac{s^2+3s+1}{s+2}\right)V_2 \quad \text{--- (5)}$$

comparing (4) & (5)

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{s^2+3s+1}{s+2} & -\frac{(s^2+2s+1)}{s+2} \\ -\frac{(s^2+2s+1)}{s+2} & \frac{s^2+3s+1}{s+2} \end{bmatrix}$$

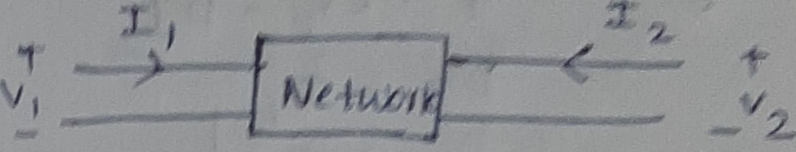
TRANSMISSION PARAMETERS

Transmission parameters/chain parameters/ABCD parameters serve to relate the voltage and current at the input port to voltage and current at the output port.

$(V_1, I_1) = f(V_2, -I_2)$

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$



Negative sign is used with I_2 and not for parameters B & D.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{\text{transmission matrix}} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The parameters are determined as follows:—

Case I: when output port is open circuited,

$V_1 = AV_2 - BI_2$

$I_2 = 0$

$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0}$

A = reverse voltage gain with o/p port open circuited.

Similarly; $I_1 = CV_2 - DI_2$

when $I_2 = 0$

$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0}$

C = transfer admittance with the port open circuited.

Case II: When o/p port is short circuited:

$V_1 = AV_2 - BI_2$

ie; $V_2 = 0$

$B = -\frac{V_1}{I_2}\Big|_{V_2=0}$

B = transfer impedance with o/p short circuited.

||rly;

$I_1 = CV_2 - DI_2$

$D = -\frac{I_1}{I_2}\Big|_{V_2=0}$

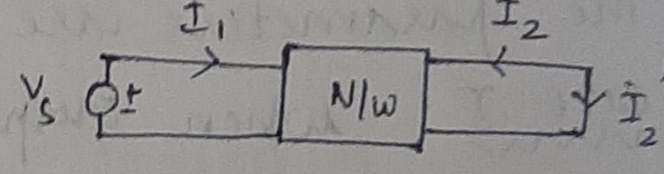
D = reverse current gain with o/p port short circuited

Condition for reciprocity

(a) When the voltage V_s is applied at the input port and o/p port is short circuited.

$V_1 = V_s \quad V_2 = 0 \quad I_2' = -I_2$

From parameter eqn:

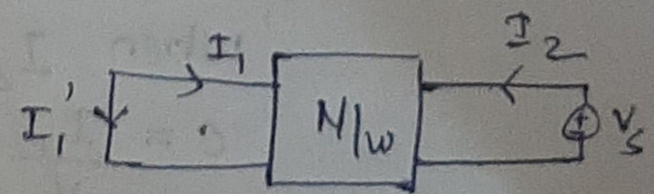


$V_s = BI_2'$

$\frac{V_s}{I_2'} = B$

(b) when voltage V_s is applied at the o/p port with input port short circuited.

$V_2 = V_s \quad V_1 = 0 \quad I_1' = -I_1$



$0 = AV_s - BI_2$

$-I_1' = CV_s - DI_2$

$I_2 = \frac{A}{B}V_s$

$= I_1' = CV_s - \frac{DA}{B} V_s$

$\frac{V_s}{I_1'} = \frac{A}{AD - BC}$

Hence, for the network to be reciprocal;

$\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$

$B = \frac{B}{AD - BC}$

ie; $\boxed{AD - BC = 1}$

Condition for Symmetry

When the o/p port is open circulated, $I_2 = 0$

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

when $I_2 = 0$

$V_s = AV_2$

$I_1 = CV_2$

$\frac{V_s}{I_1} = \frac{A}{C}$

When the I/p port is open circulated, $I_1 = 0$

from parameter equation; $CV_s = DI_2$

$\frac{V_s}{I_2} = \frac{D}{C}$

Hence, for network to be symmetrical.

$$\boxed{\begin{gathered}\frac{V_s}{I_1} = \frac{V_s}{I_2} \\ A = D\end{gathered}}$$

HYBRID PARAMETERS (h parameter)

Hybrid parameter of a two port network may be defined by expressing the voltage of input port V_1 & current of o/p port I_2 in terms of current of input I_1 and voltage of o/p port V_2.

$$(V_1, I_2) = f(I_1, V_2)$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

In matrix form; $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$

The individual h parameter can be determined by setting $I_1 = 0$ & $V_2 = 0$

Case I: When o/p port is short circuited.

$$V_2 = 0$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0}$$

h_{11} = short circuit impedance.

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$$

h_{21} = short ckt frwd current gain.

Case 2: When i/p port is open circuited;

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_1 = 0$$

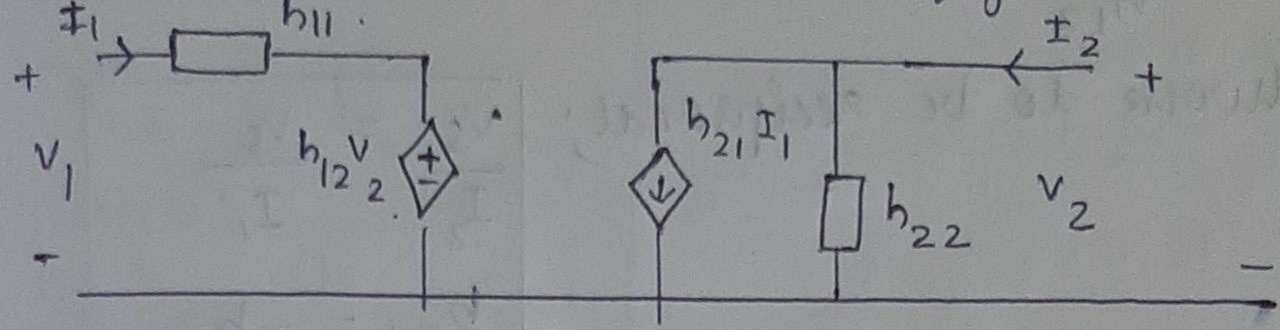
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$$

h_{12} = open ckt reverse voltage gain.

$I_2 = h_{21} I_1 + h_{22} V_2$ $\qquad$ h_{22} = open ckt o/p admittance

$I_1 = 0$ $\qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0}$

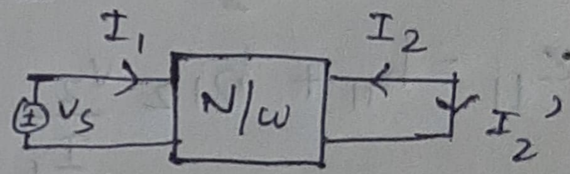
Equivalent circuit of two port n/w in terms of h parameters is shown in fig.



Condition for reciprocity

(a) When voltage V_s is applied at the input port & the o/p port is short circuited.

$V_1 = V_s \qquad V_2 = 0 \qquad I_2' = -I_2$



From h-parameter eqns.

$V_1 = h_{11} I_1 + h_{12} V_2$

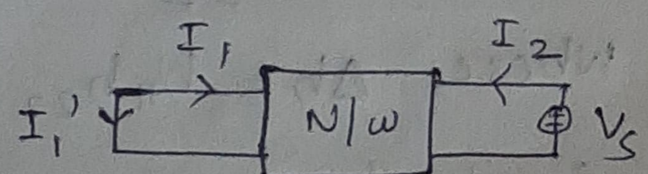
$I_2 = h_{21} I_1 + h_{22} V_2$

$-I_2' = h_{21} I_1$

$$\frac{V_s}{I_2'} = \frac{-h_{11}}{h_{21}}$$

(b) when voltage V_s is applied at the o/p port with input short circuited.

$V_1 = 0 \qquad V_2 = V_s \qquad I_1 = -I_1'$



From h parameter equations.

$V_1 = h_{11} I_1 + h_{12} V_2$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$0 = h_{11} I_1 + h_{12} V_s$$

$$h_{12} V_s = -h_{11} I_1 + h_{11} I_1'$$

$$\frac{V_s}{I_1'} = \frac{h_{11}}{h_{12}}$$

$\therefore$ Network to be reciprocal; $\boxed{\frac{V_s}{I_2'} = \frac{V_s}{I_1'} \quad h_{21} = -h_{12}}$

Condition for Symmetry

Cond^n for Symmetry is obtained from z parameters

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{h_{11} I_1 + h_{12} V_2}{I_1}\right|_{I_2=0}$$

$$z_{11} = h_{11} + h_{12} \frac{V_2}{I_1}$$

But, with $I_2 = 0$

$$0 = h_{21} I_1 + h_{22} V_2$$

$$\frac{V_2}{I_1} = \frac{-h_{21}}{h_{22}}$$

$$z_{11} = h_{11} - \frac{h_{12} h_{21}}{h_{22}} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

Where; $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

Similarily; $z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

With $I_1 = 0$

$I_2 = h_{22} V_2$

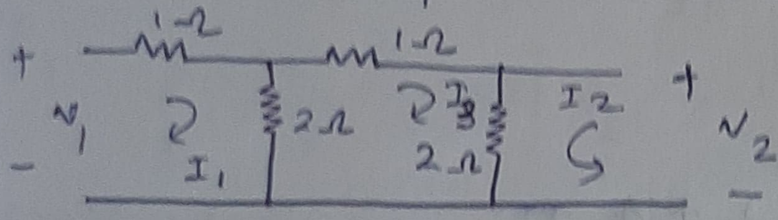
$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1}{h_{22}}$

for symmetrical n/w $Z_{11} = Z_{22}$

$$\boxed{\begin{gathered}\frac{1}{h_{22}} = \frac{\Delta h}{h_{22}} \\ \Delta h = 1 \\ h_{11}h_{22} - h_{12}h_{21} = 1\end{gathered}}$$

Problems :-

1) obtain ABCD parameters for the n/w shown



Apply KVL to mesh 1;

$V_1 - I_1 - 2(I_1 - I_3)$

$V_1 = 3I_1 - 2I_3$ —— (1)

Apply KVL to mesh 2;

$V_2 - 2(I_2 + I_3) = 0$

$V_2 = 2I_2 + 2I_3$ —— (2)

In mesh 3;

$-2(I_3 - I_1) - I_3 - 2(I_3 + I_2) = 0$

$5I_3 = 2I_1 - 2I_2$

$$I_3 = \frac{2}{5} I_1 - \frac{2}{5} I_2 \quad \text{—— (3)}$$

Eqn (3) in (1)

$$V_1 = 3I_1 - 2\left(\frac{2}{5} I_1 - \frac{2}{5} I_2\right)$$

$$= \frac{11}{5} I_1 + \frac{4}{5} I_2 \quad \text{—— (4)}$$

Eqn (3) in (2)

$$V_2 = 2I_2 + 2\left(\frac{2}{5} I_1 - \frac{2}{5} I_2\right)$$

$$= \frac{4}{5} I_1 + \frac{6}{5} I_2$$

$$\frac{4}{5} I_1 = V_2 - \frac{6}{5} I_2$$

$$I_1 = \frac{5}{4} V_2 - \frac{6}{4} I_2 \quad \text{—— (5)}$$

Eqn (3) in (4)

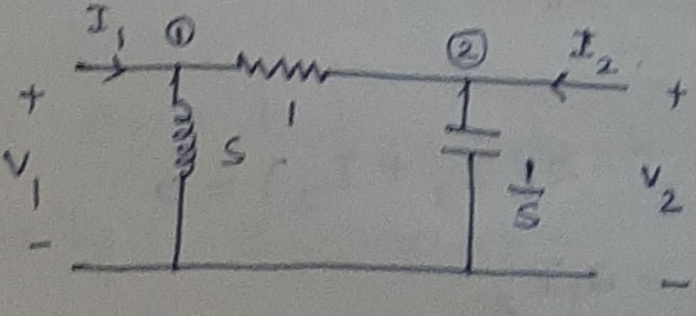
$$V_1 = \frac{11}{5}\left(\frac{5}{4} V_2 - \frac{3}{2} I_2\right) + \frac{4}{5} I_2$$

$$= \frac{11}{4} V_2 - \frac{5}{2} I_2 \quad \text{—— (6)}$$

comparing (5) & (6)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 11/4 & 5/2 \\ 5/4 & 3/2 \end{bmatrix}$$

→ Determine the transmission parameters



Apply KCL at node 1;

$$I_1 = \frac{V_1}{S} + (V_1 - V_2)$$

$$= \frac{S+1}{S} V_1 - V_2 \quad \text{—— (1)}$$

Apply KCL at node 2;

$$I_2 = \frac{V_2}{1/S} + (V_2 - V_1)$$

$$I_2 = (S+1) V_2 - V_1$$

$$V_1 = (S+1) V_2 - I_2 \quad \text{—— (2)}$$

Substitute (2) in (1)

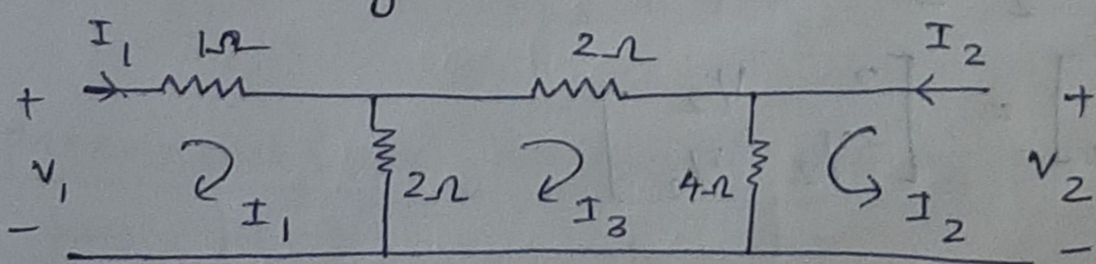
$$I_1 = \frac{S+1}{S} \left[(S+1) V_2 - I_2 \right] - V_2$$

$$I_1 = \left[\frac{(S+1)^2}{S} - 1 \right] V_2 - \frac{S+1}{S} I_2$$

comparing eqn (2) & (3) with ABCD parameter.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} S+1 & -1 \\ \frac{S^2+S+1}{S} & \frac{S+1}{S} \end{bmatrix}$$

→ Determine hybrid parameters.



A:- Apply KVL to mesh 1;

$$V_1 = 3I_1 - 2I_3 \quad \text{—— (1)}$$

$$V_2 = 4I_2 + 4I_3 \quad \text{—— (2)}$$

Apply KVL to mesh 3;

$$-2(I_3 - I_1) - 2I_3 - 4(I_3 + I_2) = 0$$

$$8I_3 = 2I_1 - 4I_2$$

$$I_3 = \frac{I_1}{4} - \frac{I_2}{2} \quad \text{---(3)}$$

Substitute (3) in (1);

$$V_1 = 3I_1 - 2\left(\frac{I_1}{4} - \frac{I_2}{2}\right) = \frac{5}{2} I_1 + I_2 \quad \text{---(4)}$$

Substitute (3) in (2);

$$V_2 = 4I_2 + 4\left(\frac{I_1}{4} - \frac{I_2}{2}\right)$$

$$= 4I_2 + I_1 - 2I_2$$

$$= I_1 + 2I_2$$

$$I_2 = -\frac{1}{2} I_1 + \frac{1}{2} V_2 \quad \text{---(5)}$$

Substitute eqn (5) in (4);

$$V_1 = \frac{5}{2} I_1 - \frac{1}{2} I_1 + \frac{1}{2} V_2$$

$$V_1 = 2I_1 + \frac{1}{2} V_2$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

INTERRELATIONSHIP B/W PARAMETERS

Z Parameters in terms of other parameters

1) Z parameters in terms of Y parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

By cramer's rule;

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22}I_1 - Y_{12}I_2}{Y_{11}Y_{22} - Y_{12}Y_{21}} = \frac{Y_{22}}{\Delta Y}I_1 - \frac{Y_{12}}{\Delta Y}I_2$$

comparing with; $V_1 = Z_{11}I_1 + Z_{12}I_2$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \qquad Z_{12} = -\frac{\Delta Y_{12}}{\Delta Y}$$

Also; $$V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\Delta Y} = \frac{Y_{11}}{\Delta Y}I_2 - \frac{Y_{21}}{\Delta Y}I_1$$

Comparing with; $V_2 = Z_{21}I_1 + Z_{22}I_2$

$$Z_{22} = \frac{Y_{11}}{\Delta Y} \qquad Z_{21} = -\frac{Y_{21}}{\Delta Y}$$

2) Z parameters in terms of ABCD parameters

We know that $V_1 = AV_2 - BI_2$

$$I_1 = CV_2 - DI_2$$

Rewriting the second equation;

$$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2$$

Comparing with $V_2 = Z_{21} I_1 + Z_{22} I_2$

$$Z_{22} = \frac{D}{C} \qquad Z_{21} = \frac{1}{C}$$

Also; $V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right] - B I_2$

$$= \frac{A}{C} I_1 + \left[\frac{AD}{C} - B\right] I_2$$

$$= \frac{A}{C} I_1 + \left[\frac{AD - BC}{C}\right] I_2$$

Comparing with; $V_1 = Z_{11} I_1 + Z_{12} I_2$

$$Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{AD - BC}{C}$$

3) Z-parameters in terms of hybrid parameters

We know that; $V_1 = h_{11} I_1 + h_{12} V_2$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Rewriting the second equation;

$$V_2 = \frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

Comparing with; $V_2 = Z_{21} I_1 + Z_{22} I_2$

$$Z_{21} = \frac{-h_{21}}{h_{22}} \qquad Z_{22} = \frac{1}{h_{22}}$$

Also; $V_1 = h_{11} I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right]$

$$= h_{11} I_1 + \frac{h_{12}}{h_{22}} I_2 - \frac{h_{12} h_{21}}{h_{22}} I_1$$

$$= \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

Comparing with $V_1 = Z_{11} I_1 + Z_{12} I_2$

$$Z_{11} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

Y parameters in terms of other parameters

1) Y- parameters in terms of Z - parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

By cramer's rule;

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{Z_{22} V_1 - Z_{12} V_2}{Z_{11} Z_{22} - Z_{12} Z_{21}}$$

$$= \frac{Z_{22}}{\Delta Z} V_1 - \frac{Z_{12}}{\Delta Z} V_2$$

where; $\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$

Comparing with; $I_1 = Y_{11} V_1 + Y_{12} V_2$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

Also;

$$I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\Delta Z} = \frac{Z_{11} V_2 - Z_{12} V_1}{\Delta Z}$$

$$= \frac{-Z_{21}}{\Delta Z} V_1 + \frac{Z_{11}}{\Delta Z} V_2$$

Comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} \qquad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

2) Y- parameters in terms of ABCD parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Rewriting the first equation;

$$I_2 = \frac{-1}{B}V_1 + \frac{A}{B}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Comparing with $Y_{21} = -\frac{1}{B}$ $\quad Y_{22} = \frac{A}{B}$

Also, $I_1 = CV_2 - D\left[-\frac{1}{B}V_1 + \frac{A}{B}V_2\right]$

$$= \frac{D}{B}V_1 + \left[\frac{BC - AD}{B}\right]V_2$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$Y_{11} = \frac{D}{B}$$

$$Y_{12} = \frac{BC - AD}{B} = \frac{AD - BC}{B} = \frac{-\Delta T}{B}$$

3) Y - Parameters in terms of Hybrid parameters

We know that; $V_1 = h_{11}I_1 + h_{12}V_2$
$I_2 = h_{21}I_1 + h_{22}V_2$

Rewriting the first equation;

$$I_1 = \frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2$$

comparing with; $I_1 = Y_{11}V_1 + Y_{12}V_2$

$$Y_{11} = \frac{1}{h_{11}} \qquad Y_{12} = \frac{-h_{12}}{h_{11}}$$

Also; $I_2 = h_{21}\left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2\right] + h_{22}V_2$

$$= \frac{h_{21}}{h_{11}} V_1 + \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right] V_2$$

comparing with; $I_2 = Y_{21}V_1 + Y_{22}V_2$

$$Y_{21} = \frac{h_{21}}{h_{11}} \qquad Y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} = \frac{\Delta h}{h_{11}}$$

ABCD parameters in terms of other parameters

1) ABCD parameters in terms of Z-parameters

We know that; $V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

Rewriting the second eqn;

$$I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2$$ (Comparing with)

$$I_1 = CV_2 - DI_2$$

$$C = \frac{1}{Z_{21}} \qquad D = \frac{Z_{22}}{Z_{21}}$$

Also; $V_1 = Z_{11}\left[\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right] + Z_{12}I_2$

$$= \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{22} Z_{11}}{Z_{21}} I_2 + Z_{12} I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 - \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}\right] I_2$$

comparing with; $V_1 = AV_2 - BI_2$

$$A = \frac{Z_{11}}{Z_{21}} \qquad B = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} = \frac{\Delta Z}{Z_{21}}$$

2) ABCD parameters in terms of Y parameters.

We know that; $I_1 = Y_{11} V_1 + Y_{12} V_2$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Rewriting the second equation;

$$V_1 = -\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2$$

comparing with; $V_1 = AV_2 - BI_2$

$$A = -\frac{Y_{22}}{Y_{21}} \qquad B = -\frac{1}{Y_{21}}$$

Also; $I_1 = Y_{11}\left[-\frac{Y_{22}}{Y_{11}} V_2 + \frac{1}{Y_{11}} I_2\right] + Y_{12} V_2$

$$= \left[\frac{Y_{12} Y_{21} - Y_{11} Y_{22}}{Y_{21}}\right] V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

comparing with; $I_1 = CV_2 - DI_2$

$$C = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}} = -\frac{\Delta Y}{Y_{21}}$$

$$D = \frac{-Y_{11}}{Y_{21}}$$

3) ABCD parameters in terms of hybrid parameters

We know that; $V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

Rewriting the second equation;

$$I_1 = \frac{-h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2 \quad \text{(comparing with)}$$

$$I_1 = CV_2 - DI_2 .$$

$$C = \frac{-h_{22}}{h_{21}} \qquad D = -\frac{1}{h_{21}}$$

Also; $$V_1 = h_{11}\left[\frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2\right] + h_{12} V_2$$

$$= \left[\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right] V_2 + \frac{h_{11}}{h_{21}} I_2$$

Comparing with; $V_1 = AV_2 - BI_2$

$$A = \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}} = \frac{-\Delta h}{h_{21}}$$

$$B = \frac{-h_{11}}{h_{21}} .$$

Hybrid parameters in terms of other parameters

1) Hybrid parameters in terms of Z-parameters

We know that; $V_1 = Z_{11} I_1 + Z_{12} I_2$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Rewriting the second eqn; $I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2$

comparing with; $I_2 = h_{21} I_1 + h_{22} V_2$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} \qquad h_{22} = \frac{1}{Z_{22}}$$

Also; $V_1 = Z_{11} I_1 + Z_{12}\left[-\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2\right]$

$$= \left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}}\right] I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

Comparing with $V_1 = h_{11} I_1 + h_{12} V_2$

$$h_{11} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}} = \frac{\Delta Z}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

2) Hybrid parameters in terms of Y - parameters

We know that; $I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

Rewriting the first eqn;

$$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2$$

comparing with; $h_{11} = \frac{1}{Y_{11}}$ $\quad h_{12} = -\frac{Y_{12}}{Y_{11}}$

Also; $I_2 = Y_{21}\left[\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22}V_2$

$$= \left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}\right] V_2 + \frac{Y_{21}}{Y_{11}} I_1$$

comparing with; $I_2 = h_{21}I_1 + h_{22}V_2$

$$h_{21} = \frac{Y_{22}}{Y_{11}}$$

$$h_{22} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}} = \frac{\Delta Y}{Y_{11}}.$$

3) Hybrid parameters in terms of ABCD parameters

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

Rewriting the second equation;

$$I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2$$

comparing with $I_2 = h_{21} I_1 + h_{22} V_2$

$$h_{21} = -\frac{1}{D} \qquad h_{22} = \frac{C}{D}$$

Also; $V_1 = AV_2 - B\left[-\frac{1}{D} I_1 + \frac{C}{D} V_2\right] = \frac{B}{D} I_1 + \left[\frac{AD - BC}{D}\right] V_2$

comparing with; $V_1 = h_{11} I_1 + h_{12} V_2$

$$h_{11} = \frac{B}{D}$$

$$h_{12} = \frac{AD - BC}{D} = \frac{\Delta T}{D}.$$

Interrelation b/w parameters

$$\Delta X = X_{11} X_{22} - X_{12} X_{21}$$

	In terms of [z]		[y]		[T]		[h]	
[z]	z_{11}	z_{12}	$\frac{y_{22}}{\Delta y}$	$\frac{-y_{12}}{\Delta y}$	$\frac{A}{C}$	$\frac{\Delta T}{C}$	$\frac{\Delta h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$
	z_{21}	z_{22}	$\frac{-y_{21}}{\Delta y}$	$\frac{y_{11}}{\Delta y}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{-h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$
[y]	$\frac{z_{22}}{\Delta z}$	$\frac{-z_{12}}{\Delta z}$	y_{11}	y_{12}	$\frac{D}{B}$	$\frac{-\Delta T}{B}$	$\frac{1}{h_{11}}$	$\frac{-h_{12}}{h_{11}}$
	$\frac{-z_{21}}{\Delta z}$	$\frac{z_{11}}{\Delta z}$	y_{21}	y_{22}	$\frac{-1}{B}$	$\frac{A}{B}$	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta h}{h_{11}}$

[T]	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$	$\begin{matrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{-\Delta Y}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$	$\begin{matrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{matrix}$
[h]	$\begin{matrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta T}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$

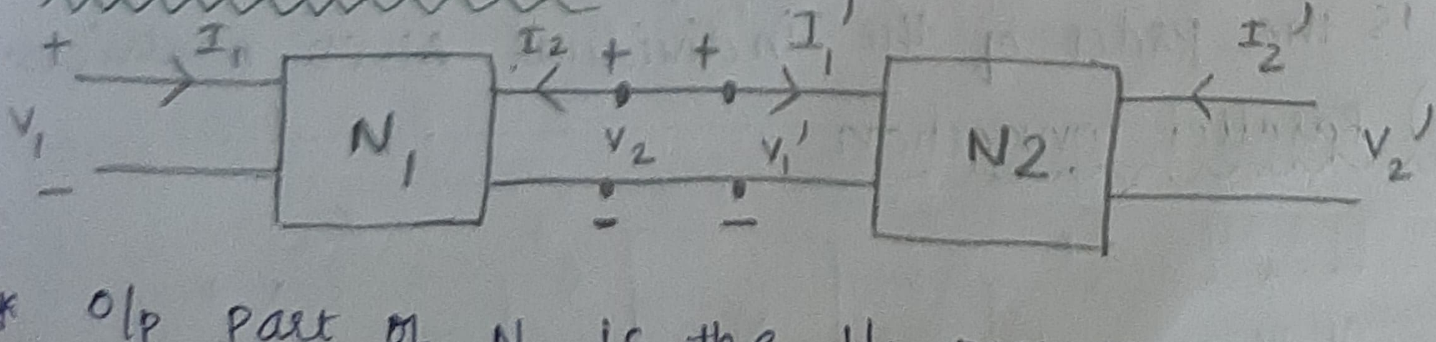
Interconnection of two-port networks

Various types of interconnections:-

1) cascade
2) parallel
3) series
4) series-parallel
5) parallel-series

(4, 5) Not required

1) cascade connection



* o/p port of N_1 is the i/p port of N_2.

  $I_1' = -I_2$.

* Let A_1, B_1, C_1, D_1 & A_2, B_2, C_2, D_2 be the

transmission parameters of N_1 & N_2 respectively.

For N_1 :- $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$ —— (1)

For N_2 :- $\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$

$\because V_1' = V_2$ & $I_1' = -I_2$

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

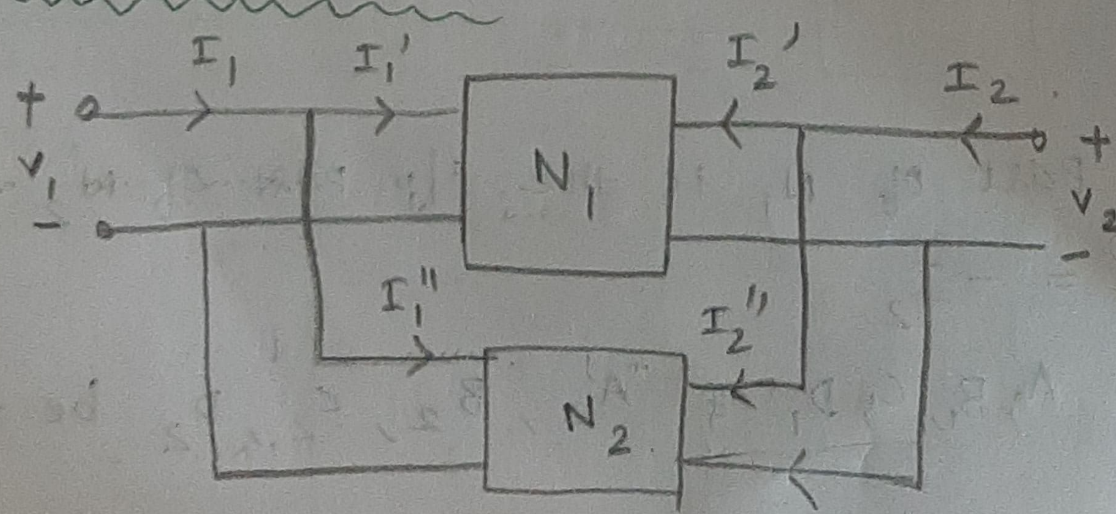
In (1) $\Rightarrow$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

where $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$

Resultant ABCD matrix of the cascade connection is the prdct of the individual ABCD matrices.

2) Parallel connection



* In parallel connection, the two n/ws have the same i/p voltages and the same o/p voltages.
* Let $Y_{11}', Y_{12}', Y_{21}', Y_{22}'$ & $Y_{11}'', Y_{12}'', Y_{21}'', Y_{22}''$ be Y parameters of N_1 & N_2; then

for n/w N_1 :- $\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

for n/w N_2 :- $\begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

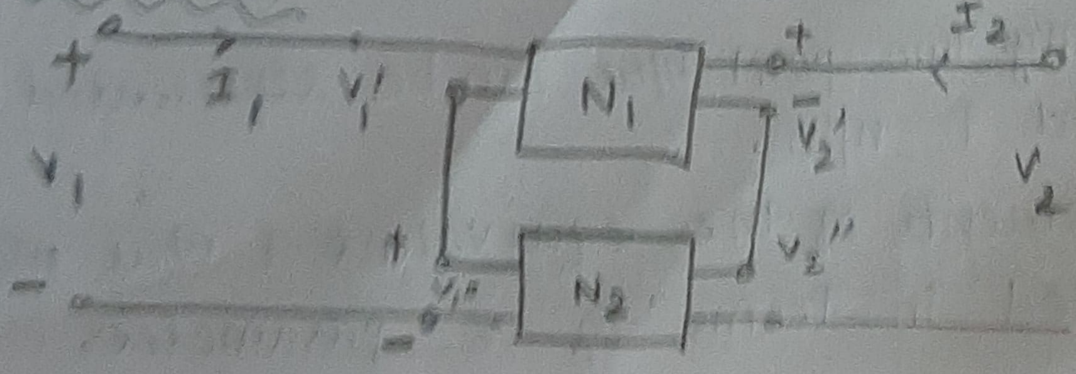
for the combined n/w :-

$$I_1 = I_1' + I_1''$$
$$I_2 = I_2' + I_2''$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1' + I_1'' \\ I_2' + I_2'' \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
$$= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The resultant Y-parameter matrix for parallel connected n/ws is the sum of Y matrices of each individual two-port n/ws.

3) Series connection



* In a Series connection, both the n/ws carry the same i/p current. their o/p currents are also equal.
* Let $Z_{11}', Z_{12}', Z_{21}', Z_{22}'$ & $Z_{11}'' \; Z_{12}'' \; Z_{21}'' \; Z_{22}''$ be the Z-parameters of N_1 & N_2 respt.

for n/w $N_1 \Rightarrow \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}' & Z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

for n/w $N_2 \Rightarrow \begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = \begin{bmatrix} Z_{11}' & Z_{12}'' \\ Z_{21}' & Z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

for the combined n/w:-

$$V_1 = V_1' + V_1'' \qquad V_2 = V_2' + V_2''$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1' + V_1'' \\ V_2' + V_2'' \end{bmatrix} = \begin{bmatrix} Z_{11}' + Z_{11}'' & Z_{12}' + Z_{12}'' \\ Z_{21}' + Z_{21}'' & Z_{22}' + Z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

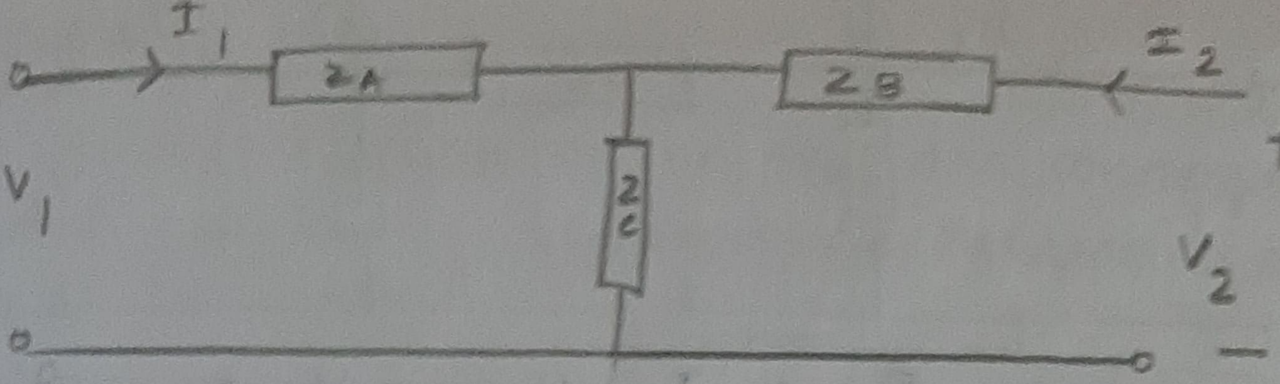
$$= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

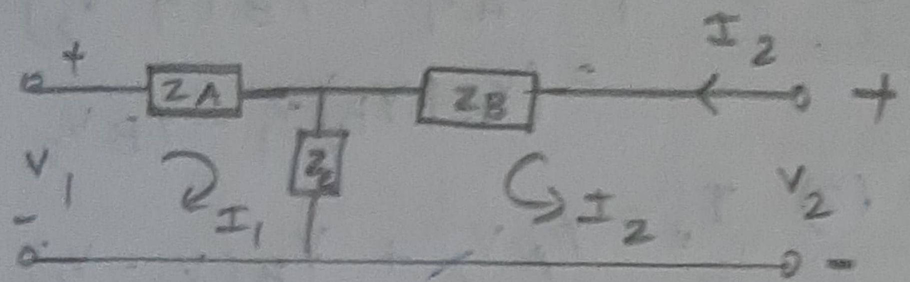
* Resultant Z parameter matrix for series connected n/ws is the sum of Z matrices of each individual 2 port n/w.

T - N/w

Any two port n/w can be represented by an equivalent T- n/w.

·Elements of the equivalent T n/w may be expressed in terms of Z-parameters.





KVL at mesh 1; $V_1 = Z_A I_1 + Z_C(I_1 + I_2)$
$= (Z_A + Z_C) I_1 + Z_C I_2$

KVL at mesh 2; $V_2 = Z_B I_2 + Z_C(I_1 + I_2)$
$= Z_C I_1 + (Z_B + Z_C) I_2$

Comparing with z-parameter eqns;

$Z_{11} = Z_A + Z_C$ $\quad Z_{12} = Z_C$ $\quad Z_{21} = Z_C$

$Z_{22} = Z_B + Z_C$

Solving the above eqns;

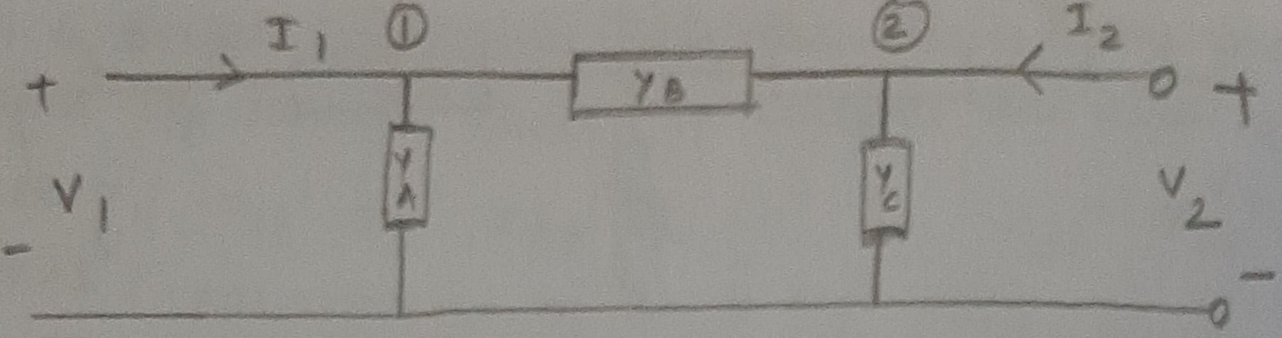
$Z_A = Z_{11} - Z_{12} = Z_{11} - Z_{21}$

$Z_B = Z_{22} - Z_{21} = Z_{22} - Z_{12}$

$Z_C = Z_{12} = Z_{21}$

4) π Network

Any two n/w can be represented by an equivalent (π) pi n/w.



KCL at node ① $\Rightarrow$ $I_1 = Y_A V_1 + Y_B (V_1 - V_2)$

$$= (Y_A + Y_B) V_1 - Y_B V_2$$

KCL at node (2) $I_2 = Y_A V_2 + Y_B (V_2 - V_1)$

$$= -Y_B V_1 + (Y_B + Y_C) V_2$$

Comparing with Y parameter eqns:

$Y_{11} = Y_A + Y_B$

$Y_{12} = -Y_B = Y_{21}$

$Y_{22} = Y_B + Y_C$

Solving the above eqns:

$Y_A = Y_{11} + Y_{12} = Y_{11} + Y_{21}$

$Y_B = -Y_{12} = -Y_{21}$

$Y_C = Y_{22} + Y_{12} = Y_{22} + Y_{21}$