Carnot Cycle

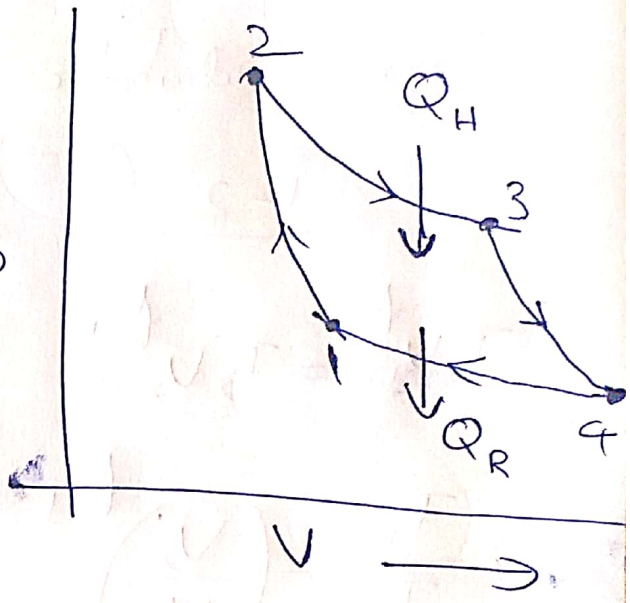
Efficiency Derivation

1-2 ⇒ Adiabatic/Isentropic Compression

2-3 ⇒ Isothermal Heat Addition

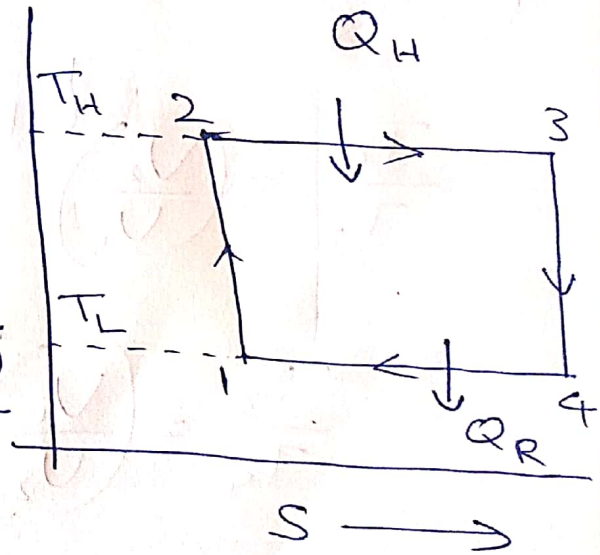
3-4 ⇒ Adiabatic/Isentropic expansion

4-1 ⇒ Isothermal Heat Rejection



$$\eta = \frac{Q_H - Q_R}{Q_H}$$

$$= \frac{P_2V_2 \ln\left\{\frac{V_3}{V_2}\right\} - P_1V_1 \ln\left\{\frac{V_4}{V_1}\right\}}{P_2V_2 \ln\left\{\frac{V_3}{V_2}\right\}}$$



$$= 1 - \frac{mRT_1 \ln\left\{\frac{V_4}{V_1}\right\}}{mRT_2 \ln\left\{\frac{V_3}{V_2}\right\}}$$

$T_H = T_{Higher}$

$T_L = T_{Lower}$

$$\eta = 1 - \frac{T_1}{T_2} \frac{\ln\left\{\frac{V_4}{V_1}\right\}}{\ln\left\{\frac{V_3}{V_2}\right\}} \quad \longrightarrow (1)$$

From adiabatic processes

1-2 & 3-4

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \qquad T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \qquad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \longrightarrow (3)$$

$T_1 = T_4$

$T_2 = T_3$

$$\frac{T_4}{T_3} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\frac{T_3}{T_4} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \longrightarrow (2)$$

Comparing (2) & (3)

$$\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

$$\frac{V_1}{V_2} = \frac{V_4}{V_3}$$

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} \quad \text{——} \quad (4)$$

Sub (4) in (1)

$$\eta = 1 - \frac{T_1}{T_2} \frac{\ln\left\{\frac{V_4}{V_1}\right\}}{\ln\left\{\frac{V_4}{V_1}\right\}}$$

$$\eta = 1 - \frac{T_1}{T_2}$$

$$\boxed{\eta = 1 - \frac{T_L}{T_H}} \quad *$$