

Problems

Qn) A cycle works between the two temperatures 327°C and 27°C . Find out the thermal efficiency of the cycle. Also find the heat supplied during the heat addition process, if the net work done is 120 kJ .

Solution:

$$\text{Given } T_H = 327^\circ\text{C} = 327 + 273$$

$$= \underline{\underline{600 \text{ K}}}$$

$$T_L = 27^\circ\text{C} = 27 + 273 \\ = 300 \text{ K.}$$

$$w = 120 \text{ kJ}$$

$$\gamma_{\text{cannot}} = 1 - \frac{T_L}{T_H}$$

$$= 1 - \frac{300}{600} = 1 - 0.5 \\ = 0.5 \\ = \underline{\underline{50 \%}}$$

$$\eta = \frac{w}{Q_{in}}$$

$$0.5 = \frac{120}{Q_{in}}$$

$$Q_{in} = \frac{120}{0.5} = \underline{\underline{240 \text{ kJ}}}$$

Q) An engine working in Otto cycle has a compression ratio 6. Find the air standard efficiency of the engine. Take $\gamma = 1.4$. what is the percentage change in efficiency if the compression ratio is reduced to 5.

Solution :

Given : \Rightarrow Compression ratio, $\gamma = 6$

$$\gamma = 1.4$$

$$\eta_{Otto} = 1 - \frac{1}{\gamma^{1-1}}$$

$$= 1 - \frac{1}{6^{1.4-1}}$$

$$\underline{\underline{= 0.51164}}$$

$$\underline{\underline{= 51.16 \%}}$$

Efficiency when $\gamma = 5$

$$\eta_{\text{Otto}} = 1 - \frac{1}{S^{(1-\gamma)}}$$

$$= 0.47469$$

$$= \underline{\underline{47.47\%}}$$

Percentage change in efficiency

$$= \frac{51.16 - 47.47}{51.16} = 0.0721$$

$$= \underline{\underline{7.21\%}}$$

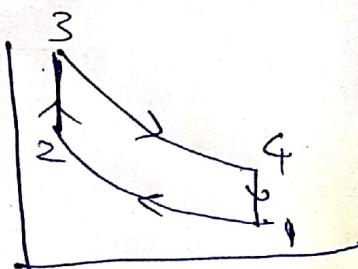
Qn) The pressure ratio between the end & beginning of compression in an engine working on Otto Cycle is 14. Calculate the air standard efficiency of the engine. Take $\gamma = 1.4$.

Solution:

$$\text{Given, } \frac{P_2}{P_1} = 14$$

$$\gamma = 1.4$$

1-2 is an adiabatic process.



$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\frac{V_1}{V_2} = \frac{P_2}{P_1}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\frac{1}{\gamma}}$$

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} = (14)^{\frac{1}{1.4}}$$

$$= \underline{\underline{6.5866}}$$

Compression ratio, $\delta = \frac{V_1}{V_2} = \underline{\underline{6.5866}}$

$$\eta_{\text{Otto}} = 1 - \frac{1}{\delta^{\frac{1}{\gamma-1}}}$$

$$= 1 - \frac{(1-1.4)^{1.4-1}}{6.5866^{1.4-1}}$$

$$= 0.8295$$

$$= \underline{\underline{82.95\%}}$$

Q) The compression ratio of an engine working on Diesel cycle is 16. Cut off takes place at 6% of the stroke. Find the air standard efficiency of the cycle. Take $\gamma = 1.4$.

Given:

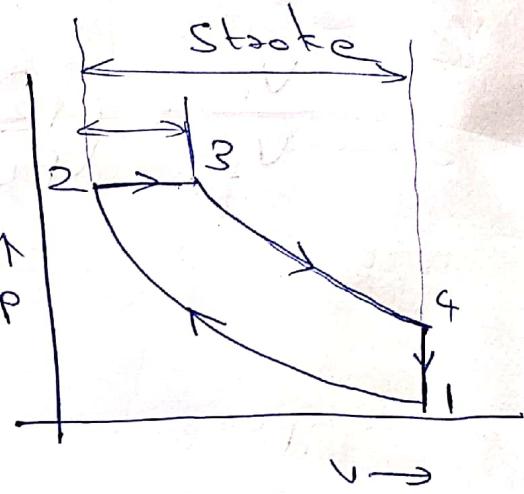
* Compression ratio, γ

$$= \frac{V_1}{V_2} = 16$$

~~$$\therefore V_3 - V_2 = 6\% (V_1 - V_2)$$~~

$$= \frac{6}{100} (V_1 - V_2)$$

$$= 0.06 (V_1 - V_2)$$



Stroke = Swept Volume

$$= V_1 - V_2$$

We need to find $\eta_{std} = \frac{V_3}{V_2}$

$$V_3 - V_2 = 0.06 (V_1 - V_2)$$

$$\frac{V_3 - V_2}{V_2} = 0.06 \frac{(V_1 - V_2)}{V_2}$$

$$\frac{V_3}{V_2} - 1 = 0.06 \left[\frac{V_1}{V_2} - 1 \right]$$

$$\frac{V_3}{V_2} = 1 + \left\{ 0.06 \left[\frac{V_1}{V_2} - 1 \right] \right\}$$

$$= 1 + \left\{ 0.06 [16 - 1] \right\}$$

$$= 1.9$$

$$\eta_{Diesel} = 1 - \frac{V_3 \gamma^{\gamma-1}}{V_1 \gamma^{\gamma-1} - 1}$$

$$= 1 - \left[\frac{1.9^{1.9-1}}{16^{(1.9-1)} \cdot 1.9^{1.9-1}} \right]$$

$$= 0.6188$$

$$= \underline{61.88 \%}$$

Q. An engine working on Otto cycle takes in air at a pressure & temperature of 100 kPa & 300 K. Find out the air standard efficiency of the engine if the clearance volume of the engine is 16% of the cylinder volume. Also, find the maximum pressure of the cycle.

If the maximum temperature is limited to 600°C

Solution:

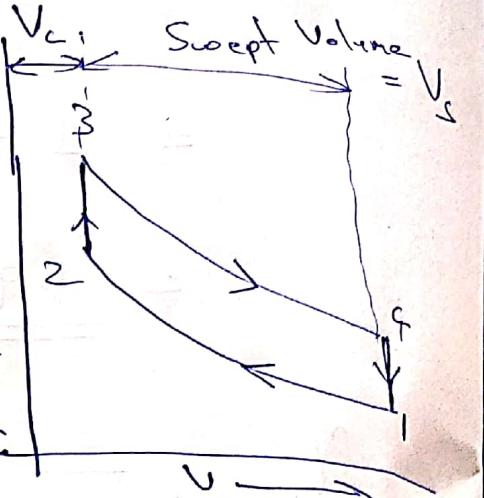
Given: $P_1 = 100 \text{ kPa}$, $T_1 = 300 \text{ K}$; $T_3 = 600^\circ\text{C} = 873 \text{ K}$

$$V_c = V_2 = V_3 = 16\% (V_{cylinder})$$

$$\frac{V_1}{V_2} = \frac{16}{100} (V_1)$$

$$V_2 = \frac{16}{100} V_1$$

$$\gamma = \frac{V_1}{V_2} = \frac{100}{16} = \underline{\underline{6.25}}$$



$V_c = \text{Clearance Volume}$

$V_s = \text{Swept Volume}$

$V_c + V_s = \text{Total cylinder volume}$

$$= V_1$$

$$\gamma = 1 - \frac{1}{6.25(1.4-1)}$$

$$= 1 - \frac{1}{6.25(1.4-1)}$$

$$= 0.51955$$

$$= \underline{\underline{51.96\%}}$$

In the adiabatic process 1-2

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$P_2 = P_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma} = 100 \times (6.25)^{1.4}$$
$$= \underline{\underline{1300.86 \text{ kPa}}}$$

Also

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = T_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 \times (6.25)^{(1.4-1)}$$
$$= \underline{\underline{624.4 \text{ K}}}$$

Process 2-3 is constant volume

$$\therefore P \propto T$$

$$\frac{P_3}{P_2} = \frac{T_3}{T_2}$$

$$P_3 = P_2 \times \frac{T_3}{T_2} = 1300.86 \times \frac{873}{624.4}$$

$$= \underline{\underline{1818.79 \text{ kPa}}}$$

Q) An engine having a swept volume of 0.065 m^3 operates on Otto Cycle. The conditions of air at beginning of compression are 100 kPa and 57°C . The compression ratio of engine is 6. & the heat supplied / cycle is 25 kJ . Find out the values of pressure, Volume & temperature at salient points. Also calculate the work done, thermal efficiency & mean effective pressure of the cycle. Take $C_v = 0.718 \text{ kJ/kgK}$

Solution

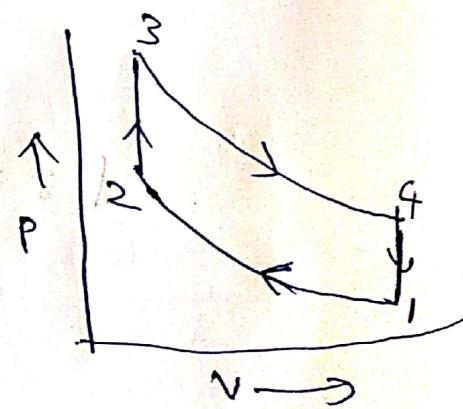
Given \rightarrow Swept Volume, $V_s = 0.065 \text{ m}^3$

Otto Cycle

$$P_1 = 100 \text{ kPa}$$

$$T_1 = 57^\circ\text{C} =$$

$$57 + 273 = \underline{\underline{330 \text{ K}}}$$



$$\alpha = 6 \left\{ \frac{V_1}{V_2} \right\}$$

$$Q_{in} = 75 \text{ kJ}$$

Swept volume, $V_s = V_1 - V_g$

$$V_1 - V_g = 0.065 \quad \textcircled{1}$$

$$\frac{V_1}{V_g} = 6$$

$$V_1 = 6V_g \quad \textcircled{2}$$

Sub in $\textcircled{1}$

$$6V_g - V_g = 0.065$$

$$\cancel{V_g (6)} - 0.065 V_g = 0.065$$

$$\underline{\underline{V_g = 0.013 \text{ m}^3}}$$

From $\textcircled{1}$

$$\underline{\underline{= V_3}}$$

$$V_1 = 0.065 + V_g$$

$$= 0.065 + 0.013$$

$$= \underline{\underline{0.078 \text{ m}^3}} \quad \underline{\underline{= V_4}}$$

From process 1-2 adiabatic

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$P_2 = P_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$= 100 \times 6^{1.4}$$

$$= \underline{1228.6 \text{ kPa}}$$

Also

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = T_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 330 \times 6^{(1.4-1)}$$

$$V = \underline{675.73}$$

From ideal gas relation,

$$PV = mRT$$

$$m = \frac{PV}{RT}$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{(100 \times 10^3) \times (0.078)}{(287) \times (330)}$$

J/kg K K

$$= 0.082357 \text{ kg}$$

In process 2-3

$$Q_{23} = m C_V (T_3 - T_2)$$

$$75000 = 0.082357 \times 718 (T_3 - 675.73)$$

$$T_3 = 675.73 + \left(\frac{75000}{718 \times 0.082357} \right)$$

$$T_3 = \underline{\underline{1944.08 \text{ K}}}$$

Process 2-3 is Constant Volume

$$\therefore P \propto T$$

$$\frac{P_3}{P_2} = \frac{T_3}{T_2}$$

$$P_3 = P_2 \times \frac{T_3}{T_2}$$

$$= \frac{1228.6 \times 1944.08}{675.75}$$

$$= \underline{\underline{3534.64 \text{ kPa}}}$$

For process 3-4 (adiabatic)

$$\frac{P_4}{P_3} = \left(\frac{V_3}{V_4} \right)^{\gamma} = \left(\frac{V_2}{V_1} \right)^{\gamma} = \left(\frac{1}{6} \right)^n$$

$$= 0.08139$$

$$P_4 = P_3 \times 0.08139 = \underline{\underline{282.68 \text{ kPa}}}$$

Similarly

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4} \right) \left(\frac{V_3}{V_4} \right)^{n-1} = \left(\frac{V_2}{V_1} \right)^{n-1}$$

$$T_4 = T_3 \times \left(\frac{V_2}{V_1} \right)^{n-1}$$

$$= 1944.08 \times \left(\frac{1}{6} \right)^{(4-1)}$$

$$= \underline{\underline{949.41 \text{ K}}}$$

Heat Rejected

$$\begin{aligned} Q_R &= m C_v (T_1 - T_2) \\ &= m C_v (T_4 - T_1) \\ &= 0.082357 \times 0.718 \times (949.41 - 330) \\ &= \underline{\underline{36.627 \text{ kJ}}} \end{aligned}$$

Work Done / cycle

$$\begin{aligned} w &= Q_{in} - Q_R \\ &= 75 - 36.627 \\ &= \underline{\underline{38.373 \text{ kJ}}} \end{aligned}$$

Thermal efficiency, $\eta = \frac{w}{Q_{in}}$

$$= \frac{38.373}{75} = 0.5116$$
$$= \underline{\underline{51.16\%}}$$

Mean effective pressure

$$\begin{aligned} P_m &= \frac{w}{V_s} = \frac{38.373}{0.065} \\ &= \underline{\underline{590.35 \text{ kPa}}} \end{aligned}$$

Q) The bore & stroke of an engine working on Diesel cycle is 200 mm and 300 mm respectively. The clearance volume of the engine is 0.8 litres & cut off takes place at 50% of the stroke. Assuming $\gamma = 1.4$, find the air standard efficiency of the cycle.

Solution:

Given: Bore = Cyl dia = 200 mm
 $= 0.2 \text{ m}$

Stroke = Cyl length = 300 mm
 $= 0.3 \text{ m}$

Clearance Volume, $V_c = 0.8 \text{ litres}$

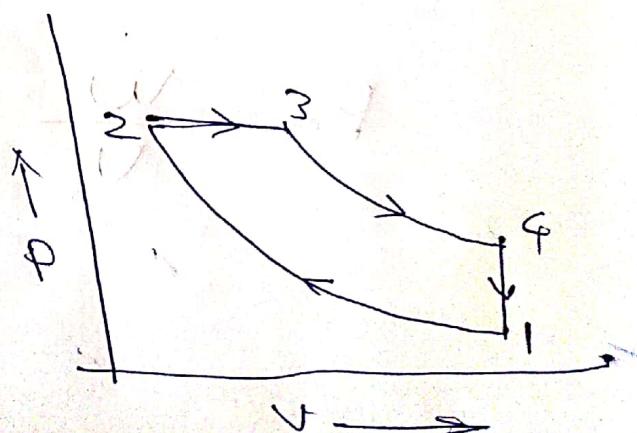
$$\underline{V_c = V_2} \quad \text{i.e. } V_2 = \frac{0.8}{1000} \text{ m}^3$$

$$\begin{aligned} 1 \text{ litre} \\ = \frac{1}{1000} \text{ m}^3 \\ 1000 \text{ litre} \\ = 1 \text{ m}^3 \end{aligned}$$

Cut off $\Rightarrow 5\% \text{ (stroke)}$

$$V_3 - V_2 = 5\% (V_1 - V_2)$$

$$\underline{V_3 - V_2 = \frac{5}{100} (V_1 - V_2)}$$



Swept Volume =

$$(V_1 - V_2) = \text{Piston area} \times \text{Stroke length}$$
$$= \pi r^2 \times L_s$$
$$= \pi (0.1)^2 \times 0.3$$
$$= \underline{\underline{0.009425 \text{ m}^3}}$$

(1)

$$V_1 - \frac{0.8}{1000} = 0.009425$$

$$V_1 = 0.009425 + \frac{0.8}{1000}$$
$$= \underline{\underline{0.010225 \text{ m}^3}}$$

$$\gamma = \frac{V_1}{V_2} = \frac{0.010225}{\left(0.8 / 1000\right)}$$
$$= \underline{\underline{12.78}}$$

$$V_3 - V_2 = \frac{5}{100} (V_1 - V_2)$$

$$V_3 = V_2 + 0.05 (V_1 - V_2)$$

From (1)

$$V_3 = \frac{0.8}{1000} + 0.05(0.009425)$$

$$= \underline{\underline{0.00127125}}$$

$$\gamma_c = \frac{V_3}{V_2} = \frac{0.00127125}{\underline{\underline{0.8/1000}}}$$

$$\gamma_c = \underline{\underline{1.589}}$$

$$\gamma = 1 - \left[\frac{1}{\gamma^{8.1}} \left(\frac{\gamma_c^{1.4} - 1}{\gamma(\gamma_c - 1)} \right) \right]$$

$$= 1 - \left[\frac{1}{12.78^{(1-4-1)}} \frac{(1.589^{1.4} - 1)}{1.4(1.589 - 1)} \right]$$

$$= 0.6007$$

$$= \underline{\underline{60.07 \%}}$$

Qn) The pressure & temperature of air at the beginning of compression stroke in a Diesel engine is 100 kPa & 17°C respectively. The compression ratio of the engine is 15. If the maximum temperature in the cycle is limited to 2300°C, find the required cut off ratio and the achievable air standard efficiency.

Take $\gamma = 1.4$.

Solution

$$\text{Given} \Rightarrow P_1 = 100 \text{ kPa}$$

$$T_1 = 17^\circ\text{C}$$

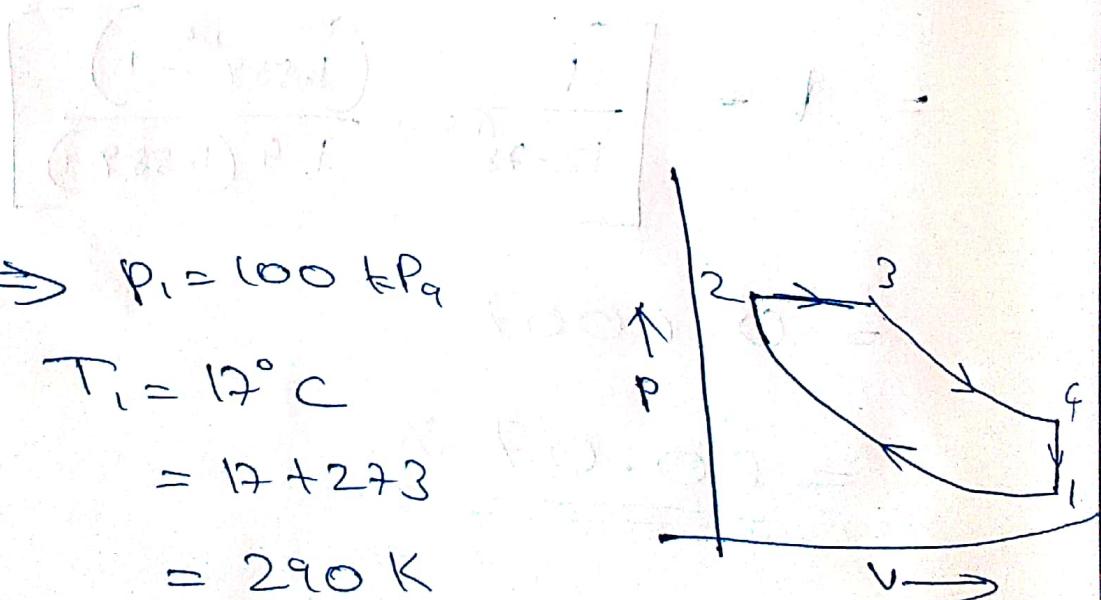
$$= 17 + 273$$

$$= \underline{\underline{290 \text{ K}}}$$

Max temp occurs at 3

$$\begin{aligned}
 \therefore T_3 &= 2300^\circ\text{C} \\
 &= 2300 + 273 \\
 &= \underline{\underline{2573 \text{ K}}}
 \end{aligned}$$

$$\delta = 15 = \frac{V_1}{V_2}$$



In the adiabatic process 1-2

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\begin{aligned} T_2 &= T_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma-1} \\ &= 290 \times (15)^{(1.4-1)} \\ &= \underline{\underline{856.71 \text{ K}}} \end{aligned}$$

In the process 3-2 (constant Pressure)

$V \propto T$

$$\frac{V_3}{V_2} = \frac{T_3}{T_2}$$

$$\frac{v_3}{v_2} = \gamma_c = \frac{2573}{856-71}$$

$$= \underline{\underline{3}}$$

$$\gamma = 1 - \left[\frac{1}{\gamma^{n-1}} \left(\frac{\gamma^n - 1}{\gamma(1-\gamma)} \right) \right]$$

$$= 1 - \left[\frac{1}{15^{(4-1)}} \left(\frac{3^{1.4} - 1}{1.4(3-1)} \right) \right]$$

$$= 0.5581$$

$$= \underline{\underline{55.81\%}}$$