WAVES

WHAT IS A WAVE?

- A wave is a disturbance that propogates and transports energy and momentum without the transport of matter.
- Eg: sound waves, light waves, seismic waves etc

TWO BASIC TYPES OF WAVES

- Mechanical waves

  Waves which require a material medium for its propogation.

Eg: Sound waves

- Non mechanical waves

  Waves which do no require a material medium for its propogation.

Eg: matter wave and electromagneticc waves

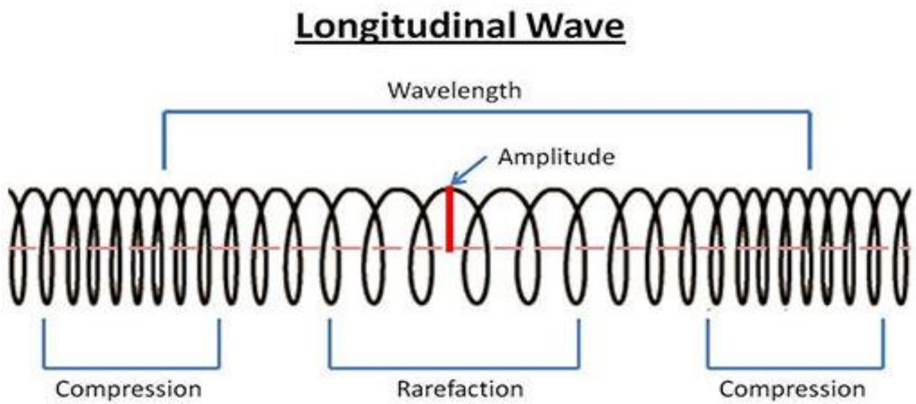
CLASSIFICATION BASED ON DIRECTION OF VIBRATION

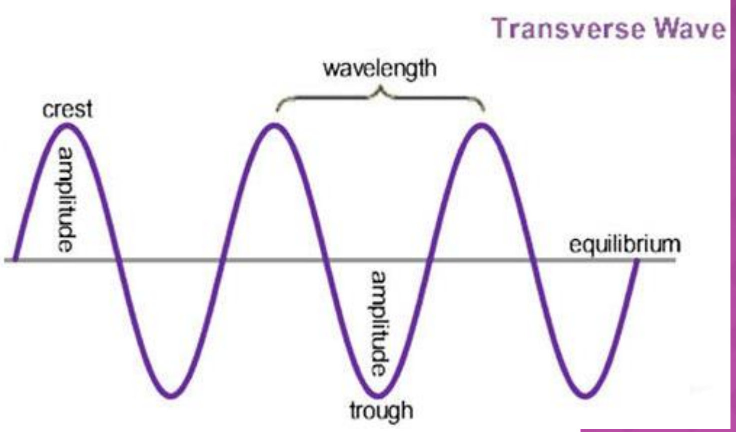
Longitudinal

- The particles of the medium vibrates parallel to the direction of propagation of waves
- Eg: Sound waves
- It cannot be polarised
- It consist of compressions and rarefactions.

Transverse

- The particles of the medium vibrates perpendicular to the direction of propogation of wave
- Eg: Light
- It can be polarised.
- Crests and troughs





CLASSIFICATION BASED ON DIMENSION OF PROPOGATION

- One dimensional wave motion: wave propogates along a straight line.

Eg: wave along a string.

- Two dimensional wave: The disturbances advances along a plane.

Eg: Ripples on liquid surface.

- Three dimensional wave: The disturbances advances in three dimensions.

Eg: Propogation of sound through air.

CLASSIFICATION BASED ON WAVEFRONT

- Depending on the shape of the wave front we have plane waves, spherical waves and cylindrical waves.
- The locus of points having same state of vibration.

CLASSIFICATION BASED ON DURATION OF VIBRATION

- Wave pulse: Particles of the medium disturbed for a short period of time.
- Wave train: Particles of the medium vibrates continuously with time.
- Sinusoidal wave: Particles of the medium are executing simple harmonic motion.

ONE DIMENSIONAL WAVE

- Consider an one dimensional wave pulse moving with a velocity v in the positive x- direction. Its wave function is
- $\Psi(x,t) = f(x-vt)$…….(1)
- Differentiate (1) w.r.t.x twice we get

$\frac{\partial^2\Psi}{\partial x^2}$ = f''(x-vt)……..(2)

- Differentiate (1) w.r.to t twice we get,

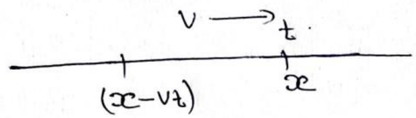
$\frac{\partial^2\Psi}{\partial t^2}$ = v^2 f''(x-vt)…….(3)

From equation (2) and (3) $\frac{\partial^2\Psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2\Psi}{\partial t^2}$

This is the differential equation of a one dimensional wave.

One dimensional wave equation

$$\varphi(x,t) = f(x,t)$$
$$= f(x-vt).$$



$$\frac{\partial \varphi}{\partial x} = \frac{\partial f(x-vt)}{\partial x}$$
$$= \frac{\partial f(x-vt)}{\partial (x-vt)} \times \frac{\partial (x-vt)}{\partial x} = f' \times 1$$
$$= f'.$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial f'}{\partial x} = \frac{\partial f'(x-vt)}{\partial x} = \frac{\partial f'(x-vt)}{\partial (x-vt)} \times \frac{\partial (x-vt)}{\partial x}.$$
$$= f'' \times 1$$
$$= f'' \quad \text{——— (1)}$$

$$\frac{\partial \varphi}{\partial t} = \frac{\partial f(x-vt)}{\partial t} = \frac{\partial f(x-vt)}{\partial (x-vt)} \times \frac{\partial (x-vt)}{\partial t}$$
$$= f' \times -v$$

$$\frac{\partial^2 \varphi}{\partial t^2} = -v \times \frac{\partial f'}{\partial t} = -v \times \frac{\partial f'(x-vt)}{\partial t}$$
$$= -v \times \frac{\partial f'(x-vt)}{\partial (x-vt)} \times \frac{\partial (x-vt)}{\partial t}$$
$$= -v \times f'' \times -v$$
$$= v^2 f'' \quad \text{——— (2)}$$

Combine equation (1) and (2)

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2}.$$

SOLUTION OF ONE DIMENSIONAL WAVE

- The one dimensional wave equation is
  $\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}$.
- Some of its solutions are Ψ=a sin (kx-ωt)
- Ψ=acos (kx-ωt)
- Ψ=ae $^{i(kx-\omega t)}$

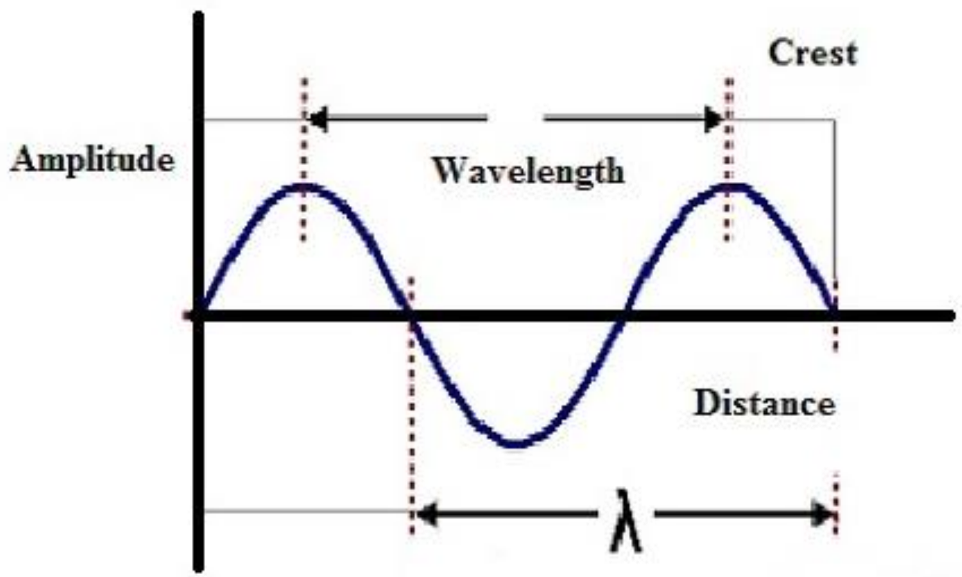
$\frac{2\pi}{\lambda}$

PHASE DIFFERENCE AND PATH DIFFERENCE

- The difference in states of two particles in the medium expressed in an angle , it is called phase difference.
- The same quantity expressed in λ is called path difference.
- The phase difference of 2π radian corresponds to a path difference λ.
- The phase difference corresponds to unit path difference is $2\pi/\lambda$. It is called phase shift constant. It is also known as wave vector 'k'.
- The phase difference(δ) corresponding to path difference x is, $\delta = 2\pi/\lambda \quad x = kx$

WAVELENGTH

- It is the minimum separation between two particles of the medium having the same state of vibration.



FREQUENCY PERIOD

- The number of cycles passing through a point in one second is called the frequency of the wave.(f)
- Time taken by a particle in the medium to complete one vibration.(T)
- $f = 1/T$

WAVE VELOCITY

- The displacement of the disturbance in unit time is called the wave velocity.
- Wave velocity= displacement / time

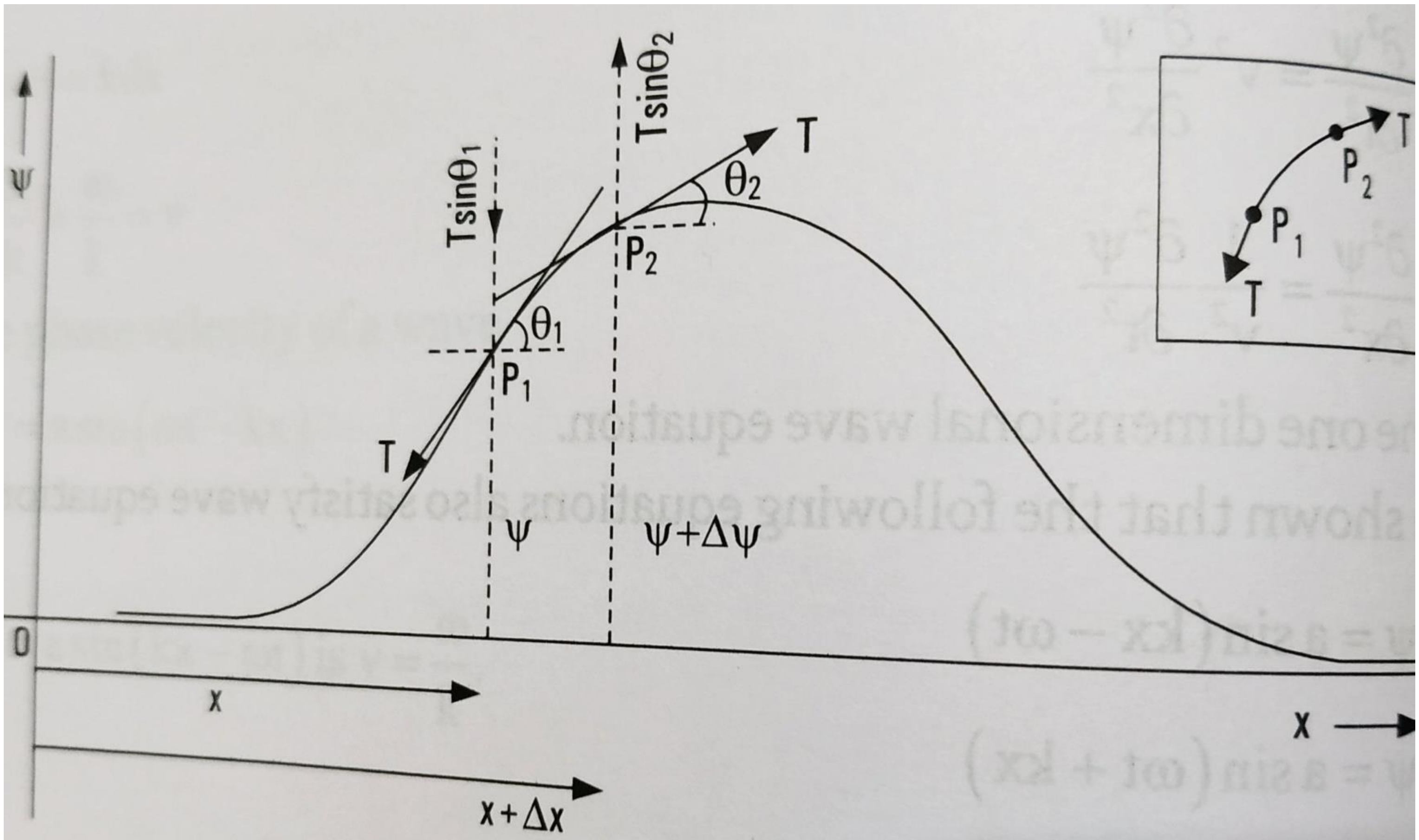
$$= \lambda / T$$

$$= f\lambda$$

TRANSVERSE VIBRATION OF A STRETCHED STRING

- Consider a long uniform stretched string having tension T. On plucking this string at a point produces a pulse that travels along the string. Since the displacement of the particles are perpendicular the direction of propagation the wave is transverse in nature.
- Consider two neighbouring points P_1 and P_2 separated by a small distance Δx.Tension is uniform throughout the string.
- The vertical components of tensions at P1 and P2 are $T \sin\theta_1$ and $T \sin\theta_2$. Where θ1 and θ2 are the angles made by the tangents at the horizontal directions.

TRANSVERSE VIBRATION OF A STRETCHED STRING



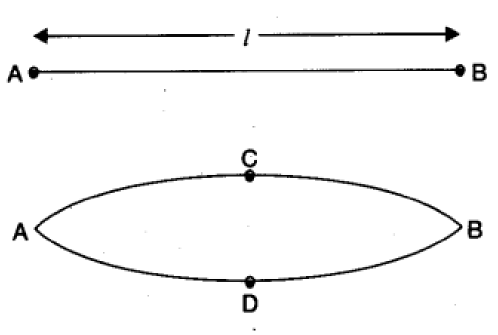
TRANSVERSE VIBRATION OF A STRETCHED STRING

- The resultant vertical force on the small element P1P2 is, F= T sinθ_2 – T sin θ_1
- Since θ_1 and θ_2 small, sin $\theta_1 \approx \theta_1$ =tan θ_1 and sin $\theta_2 \approx \theta_2$ =tan θ_2 .
- So F= T tanθ_2 – T tan θ_1 ……….(1)
- Slope at a point is $\frac{d\Psi}{dx}$ = tan θ. But here Ψ is a function of position and time , So use $\frac{\partial\Psi}{\partial x}$ for the slope.
- tan θ_1 = $(\frac{\partial\Psi}{\partial x})_x$ and tan θ_2 = $(\frac{\partial\Psi}{\partial x})_{x+\Delta x}$
- Equation (1) becomes, F= T $(\frac{\partial\Psi}{\partial x})_{x+\Delta x}$ -T $(\frac{\partial\Psi}{\partial x})_x$

- Force on the element P1P2 of length Δx is, F= mass X acceleration= m Δx X $\frac{\partial^2 \Psi}{\partial t^2}$ where 'm' is the mass per unit length of the wire.
- So m Δx X $\frac{\partial^2 \Psi}{\partial t^2}$ =T[$\left(\frac{\partial \Psi}{\partial x}\right)_{x+\Delta x}$ - $\left(\frac{\partial \Psi}{\partial x}\right)_x$]
- $\frac{m}{T} \frac{\partial^2 \Psi}{\partial t^2} = \frac{\left(\frac{\partial \Psi}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial \Psi}{\partial x}\right)_x}{\Delta x}$ RHS of this equation represents the rate of change of $\frac{\partial \Psi}{\partial x}$ w.r.t. x. ie:- $\frac{\partial^2 \Psi}{\partial x^2}$ as $\Delta x \longrightarrow 0$.
- Therefore $\frac{m}{T} \frac{\partial^2 \Psi}{\partial t^2} = \frac{\partial^2 \Psi}{\partial x^2}$..........(3) . This equation is similar to one dimensional wave equation $\frac{m}{T} \frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}$ ....(4). Equation (3) is known as wave equation of the string. Comparing equation (3) and (4) v= $\sqrt{\frac{T}{m}}$, velocity of propagation of wave.

LAWS OF VIBRATION OF A STRETCHED STRING

- Velocity of a wave that propagates along a stretched string, v= $\sqrt{\frac{T}{m}}$
- If l is the length of the vibrating segment, then l=λ/2 or λ=2l
- Velocity=f λ or f=$\frac{1}{2l}\sqrt{\frac{T}{m}}$



- I Law:- fl=constant, provided the tension and linear density are constants.
- II Law:- $\frac{n}{\sqrt{T}}$=constant, provided l and m are constants.
- III Law:-n$\sqrt{m}$ =constant, provided T and l are constants

THREE DIMENSIONAL WAVE EQUATION

- $\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{v2} \frac{\partial^2 \Psi}{\partial t^2}$ or $\boldsymbol{\nabla}^2 \Psi = \frac{1}{v2} \frac{\partial^2 \Psi}{\partial t^2}$ where
- $\boldsymbol{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is called Laplacian operator.

Solution

Ψ =ae i($k_x x+k_y y+k_z z-\omega t$)

Or $\Psi = ae^{i(\mathbf{k}.\mathbf{r}-\omega t)}$

$\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$

k= $k_x\mathbf{i}+k_y\mathbf{j}+k_z\mathbf{k}$

QUESTIONS

- Derive one dimensional wave equation.(3)
- Discuss the propagation of transverse wave along a stretched string and derive the expression for frequency.(10)
- A uniform steel wire has length 10m and mass 2kg. Find the tension in the string if the speed of transverse wave on the wire is 340m/s. (2.31*104 N/m) (4)
- What is the relation between path difference and phase difference in wave motion? (3)

- Derive the differential equation for transverse wave in a stretched string and hence obtain the expression for fundamental frequency.
- Calculate the fundamental frequency of a string of 1m long and mass 2g when stretched by a weight of 4kg.(70Hz) (4)
- List two differences between longitudinal and transverse wave . Give an example for each.
- A string of mass 0.65kg is stretched between two supports 30m apart.If the tension in the string is 160N, find the velocity of the wave in the string. How long will a pulse take to travel from one support to the other?