(1)

Properties of Eigen Values & Eigen Vectors

1) Eigen values of A and A^T are same.

2) If λ is an eigen value of A, then λ^n is an eigen value of A^n, where n is a +ve integer.

3) If λ is an eigen value of A, then $k\lambda$ is an eigen value of kA.

4) If λ is an eigen value of A, then $\lambda - k$ is an eigen value of $A - kI$

5) If λ is an eigen value of ~~A~~ a non-singular matrix A, then $\frac{1}{\lambda}$ is an eigen value of A^{-1}.

6) If λ is an eigen value of A, then $\frac{|A|}{\lambda}$ is an eigen value of adj A.
($|A|$ denotes the determinant of A)

7) Eigen values of triangular matrices (upper or lower) and diagonal matrices are its diagonal elements.

8) The sum of the eigen values of a matrix is equal to the sum of ~~the~~

its diagonal elements.

9) The product of the eigen values of a matrix is equal to its determinant value.

10) The eigen values of a symmetric matrix are real.

11) The eigen values of a skew-symmetric matrix are purely imaginary or zero.

12) The eigen values of an orthogonal matrix are real or complex conjugates in pairs and have absolute value 1.

13) Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be distinct ~~of~~ eigen values of an $n \times n$ matrix, then the corresponding eigen vectors $X_1, X_2, \ldots, X_n$ are independent to each other.

Problems:-

If 2 is an eigen value of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, without using its characteristic equation, find the other eigen values. Also find the eigen values of A^3, A^T, A^{-1}, $5A$, $A-3I$ and adj A.

Solution:-

Let $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. Now, given that 2 is an eigen value of A.

Let $\lambda_1 = 2$ & λ_2, λ_3 be the other two eigen values of A.

Now, we know that sum of the eigen values of A is equal to the sum of the diagonal elements of A.

Thus, we have $\lambda_1 + \lambda_2 + \lambda_3 = 11$

$$\Rightarrow 2 + \lambda_2 + \lambda_3 = 11$$

$$\Rightarrow \lambda_2 + \lambda_3 = 9. \longrightarrow (1)$$

Also, we know that the product of the eigen values of A is equal to its determinant value.

Thus, we have $\lambda_1 \lambda_2 \lambda_3 = |A| = 36$

$$\Rightarrow 2\lambda_2\lambda_3 = 36$$

$$\Rightarrow \lambda_2\lambda_3 = 18 \longrightarrow (2)$$

We have $(\lambda_2-\lambda_3)^2 = (\lambda_2+\lambda_3)^2 - 4\lambda_2\lambda_3$ (4)

$$= 9^2 - 4\times 18$$

$$= 81 - 72$$

$$= 9$$

$$\Rightarrow \lambda_2-\lambda_3 = \sqrt{9} = 3.$$

ie, $\lambda_2 - \lambda_3 = 3 \longrightarrow$ (3)

(1) + (3) $\Rightarrow$

$$\begin{array}{r} \lambda_2 + \cancel{\lambda_3} = 9 \\ \lambda_2 - \cancel{\lambda_3} = 3 \\ \hline 2\lambda_2 = 12 \end{array}$$

$$\Rightarrow \boxed{\lambda_2 = 6}$$

Now, $\lambda_2 + \lambda_3 = 9$

$$\Rightarrow 6 + \lambda_3 = 9$$

$$\Rightarrow \lambda_3 = 9-6 = 3$$

$$\boxed{\lambda_3 = 3}$$

Thus, the other eigen values of A are 3, 6.

Hence, the eigen values of A are 2, 3, 6.

i) Eigen values of A^3.

It is given by $2^3, 3^3, 6^3$

ie, 8, 27, 216.

(5) 2) Eigen values of A^T are 2, 3, 6.
($\because$ Eigen values of A & A^T are same).

3) Eigen values of A^{-1} are $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$.

4) Eigen values of $5A$ are $5\times2, 5\times3, 5\times6$

i.e. 10, 15, 30.

5) Eigen values of $A-3I$ are $2-3, 3-3, 6-3$

i.e. $-1, 0, 3$.

6) Eigen values of adj A are $\frac{|A|}{2}, \frac{|A|}{3}, \frac{|A|}{6}$

$\because |A| = 36$, then we have

$\frac{36}{2}, \frac{36}{3}, \frac{36}{6}$

$\Rightarrow 18, 12, 6$