Time response and stability of system

Time domain response from pole zero plot

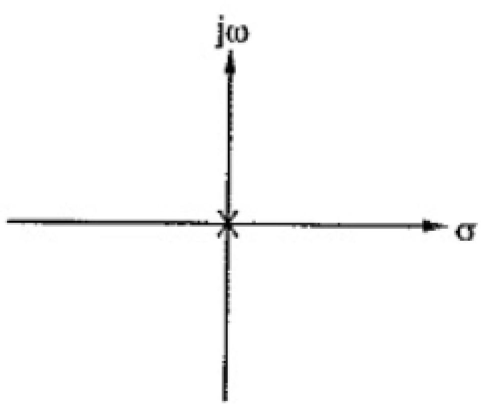
Case i) single pole at origin

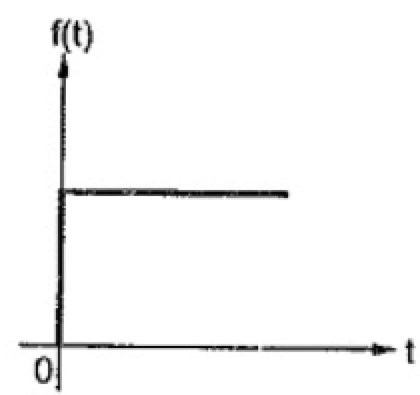
$$F(s) = \frac{K}{s}$$

$$f(t) = L^{-1}\left[\frac{K}{s}\right] = K$$

Thus it is a step type of time response corresponding to the pole at the origin

The pole location and the corresponding time response is shown in the Fig. (a) and





Case ii) multiple pole at origin

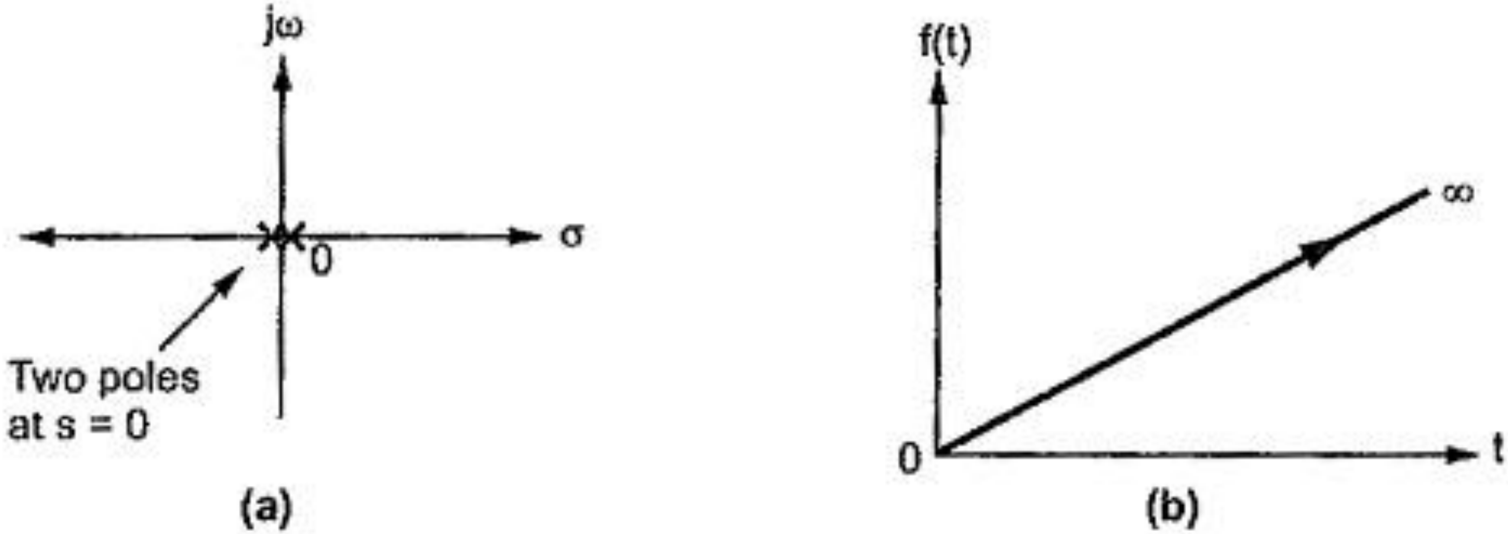
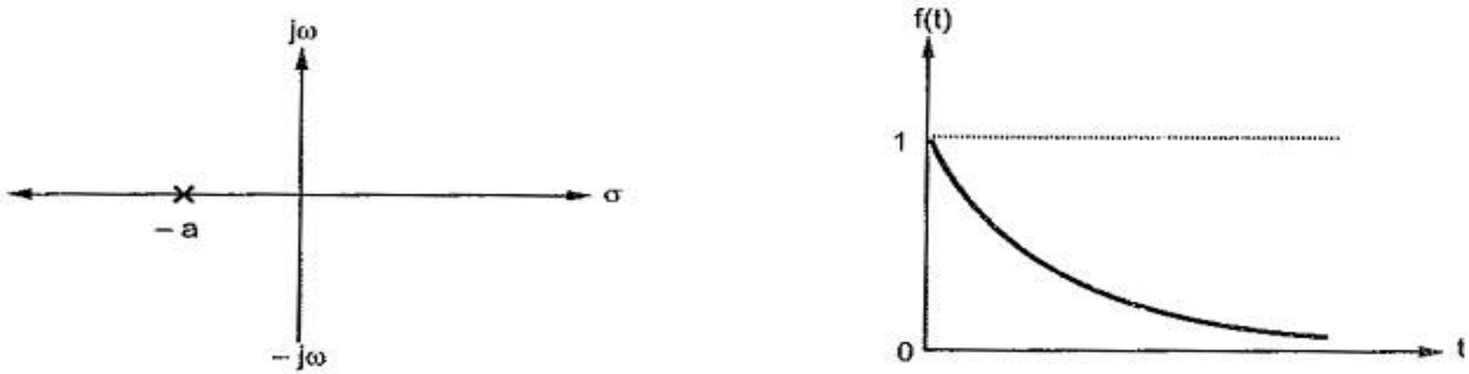


Fig. 3.20

Case iii) pole is located on negative real axis

$$F(s) = \frac{1}{s+a} \text{ and hence } f(t) = L^{-1}[F(s)] = e^{-at}$$



(a) Pole location

(b) Exponential response

Fig. 3.17

$$F_1(s) = \frac{1}{s+2} \text{ and } F_2(s) = \frac{1}{s+5}$$

$$f_1(t) = e^{-2t} \text{ and } f_2(t) = e^{-5t}$$

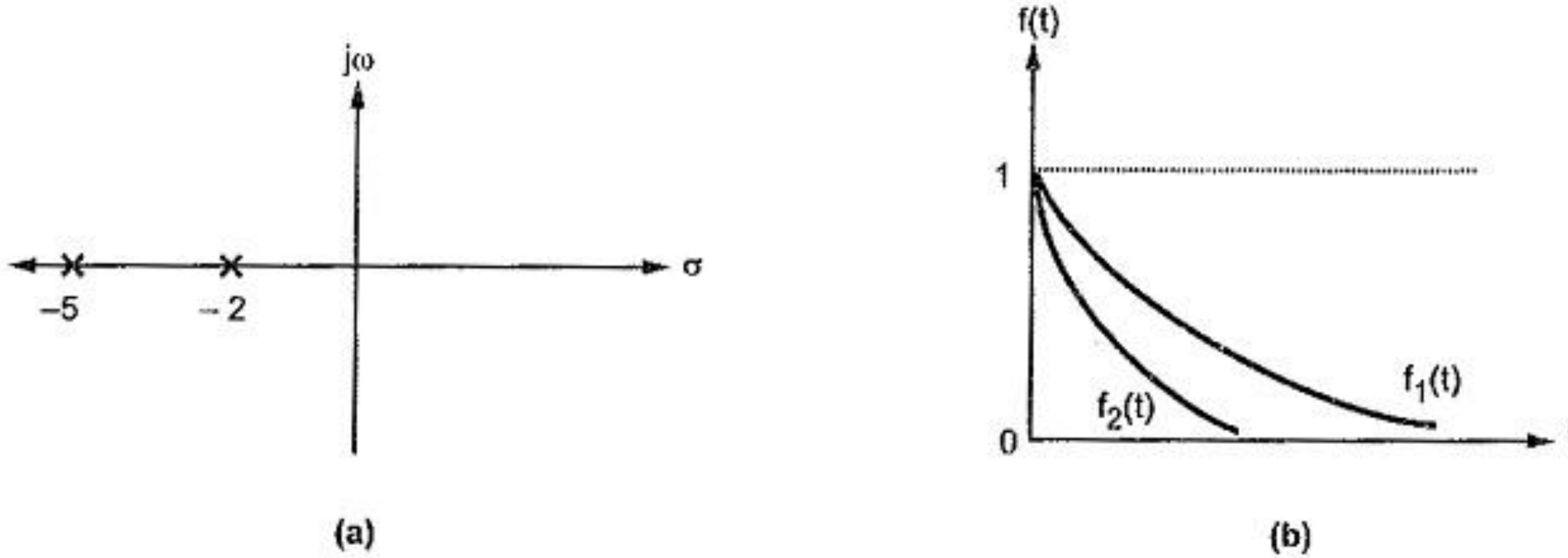
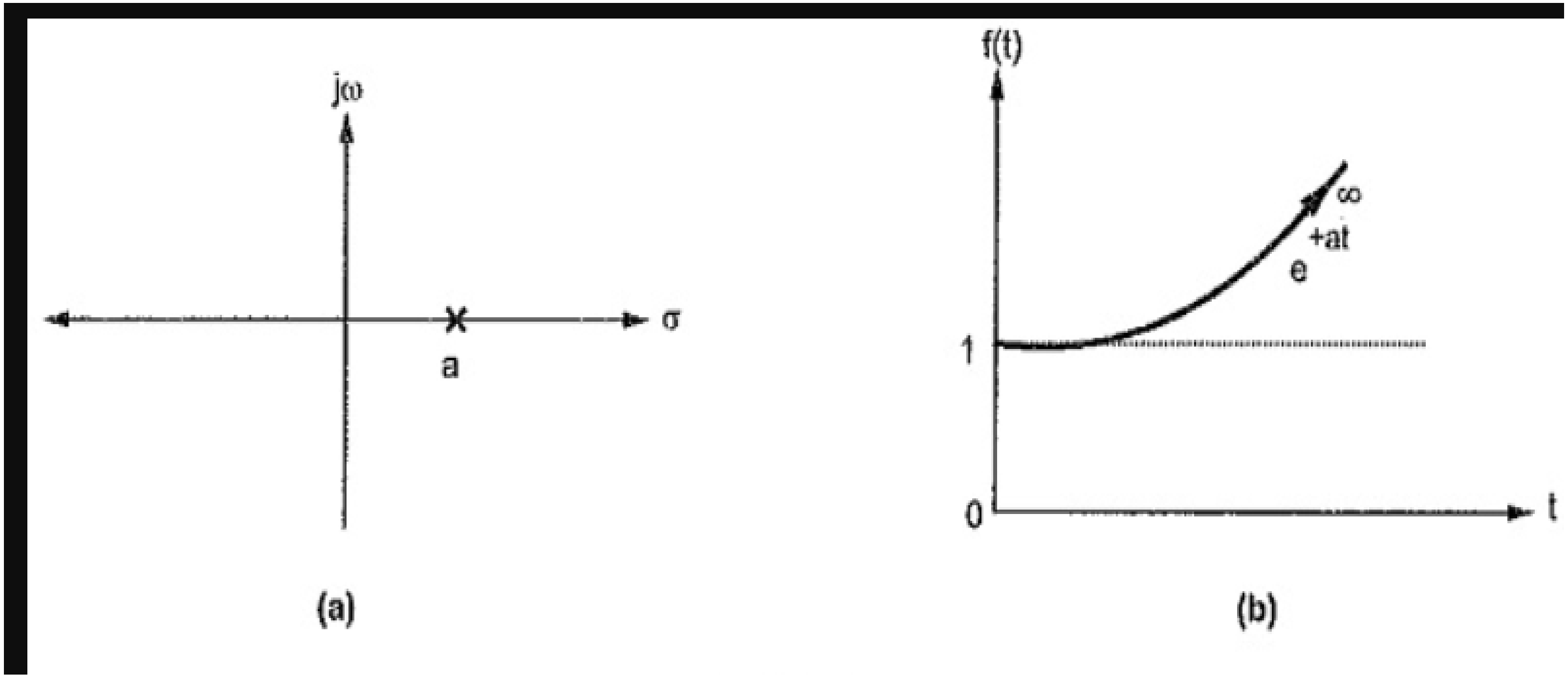


Fig. 3.18

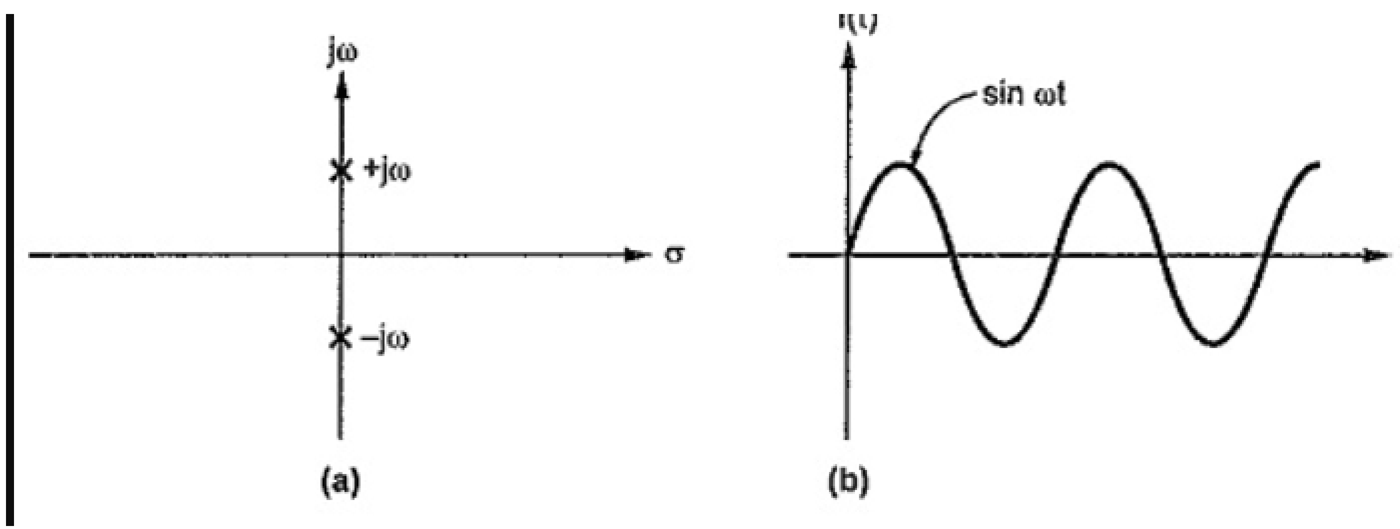
Case iv) Real and positive pole



(a) (b)

Case V) Complex poles on imaginary axis

- The poles are located at $s = \pm j\omega$ from $s^2 + \omega^2 = 0$.



Case vi) complex poles with negative real part

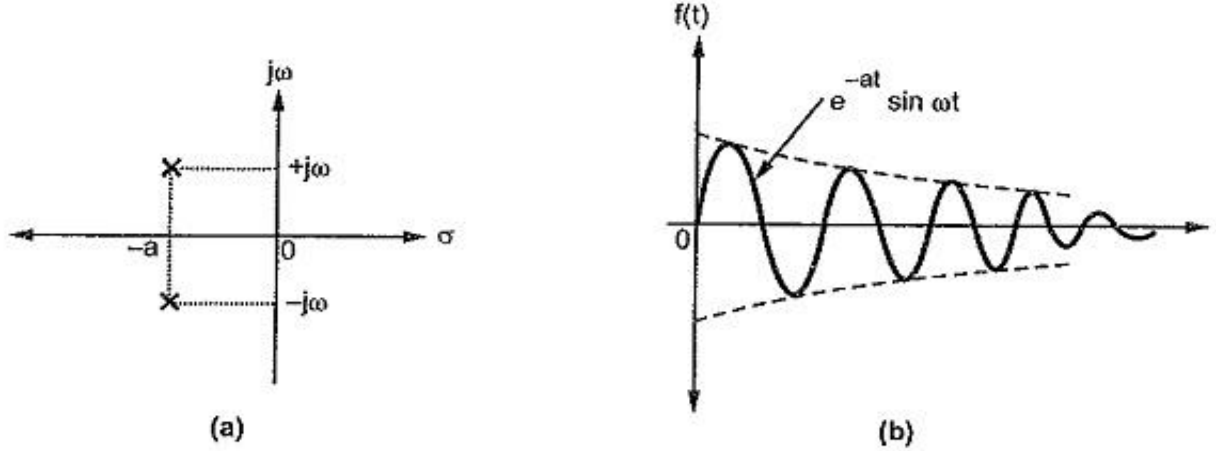


Fig. 3.24

$$F(s) = \frac{K}{s^2 + \alpha s + \beta} = \frac{K}{(s+a)^2 + \omega^2}$$

$$(s+a)^2 = -\omega^2 \text{ i.e. } (s+a) = \pm j\omega \text{ i.e. } s = -a \pm j\omega$$

$$f(t) = L^{-1}\left[\frac{K}{(s+a)^2 + \omega^2}\right] = \frac{K}{\omega} e^{-at} \sin \omega t$$

Case vii) complex poles with positive real part

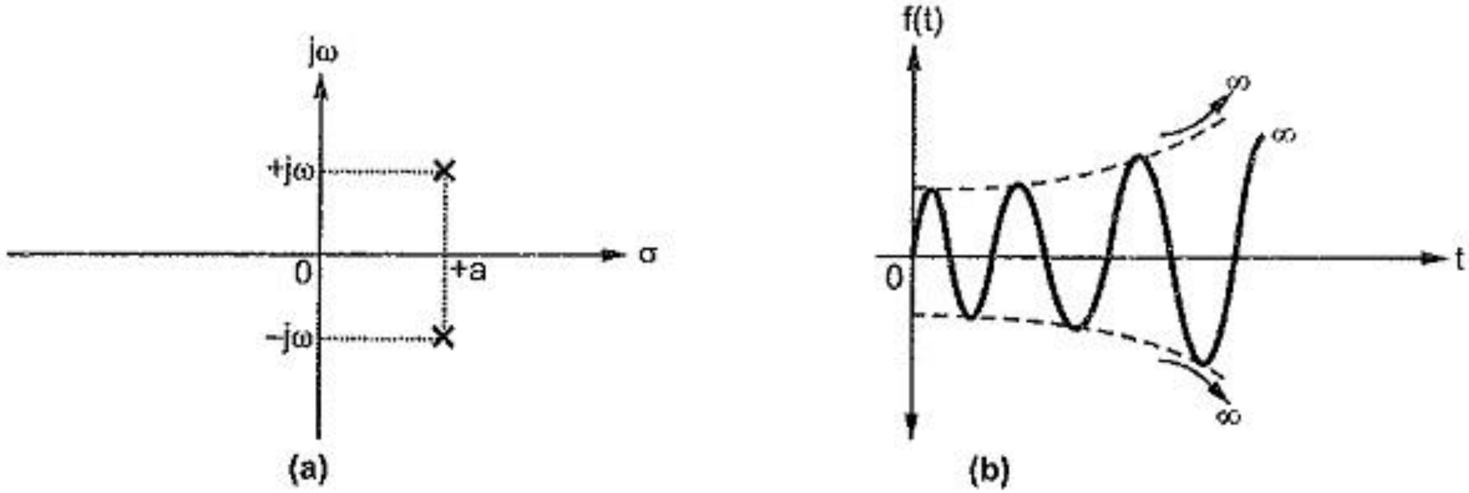


Fig. 3.25

Case viii) multiple pair of poles on imaginary axis

$$F(s) = \frac{s}{(s^2+\omega^2)^2}$$

The corresponding time response is,

$$f(t) = L^{-1}\left[\frac{s}{(s^2+\omega^2)^2}\right] = \frac{t}{2\omega}\sin\omega t$$

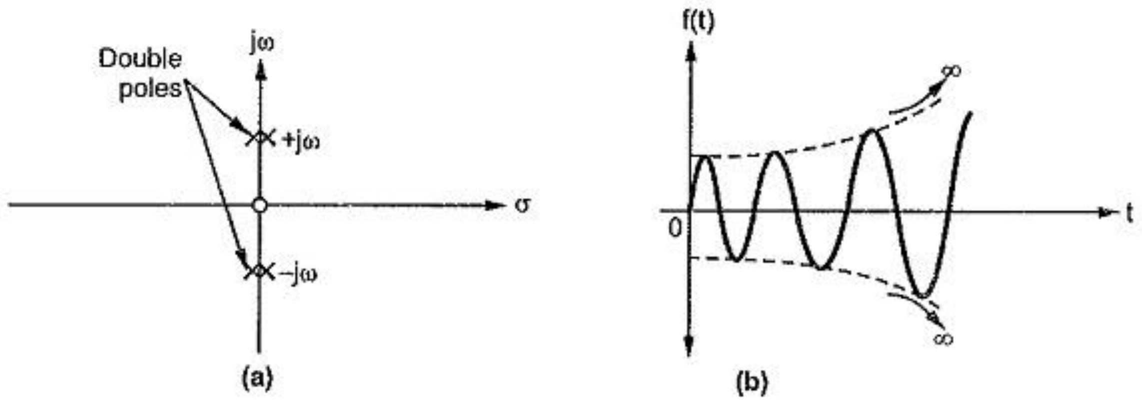


Fig. 3.26

Example.1

Let, $T(s) = \dfrac{2}{s(s+1)(s+2)}$

By partial fraction expansion, T(s) can be expressed as,

$$T(s) = \frac{2}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

A is obtained by multiplying T(s) by s and letting s = 0.

$$A = T(s) \times s\Big|_{s=0} = \frac{2}{s(s+1)(s+2)} \times s\Big|_{s=0} = \frac{2}{(s+1)(s+2)}\Bigg|_{s=0} = \frac{2}{1\times 2} = 1$$

B is obtained by multiplying T(s) by (s +1) and letting s = −1.

$$B = T(s) \times (s+1)\Big|_{s=-1} = \frac{2}{s(s+1)(s+2)} \times (s+1)\Big|_{s=-1} = \frac{2}{s(s+2)}\Bigg|_{s=-1} = \frac{2}{-1(-1+2)} = -2$$

C is obtained by multiplying T(s) by (s +2) and letting s = −2.

$$C = T(s) \times (s+2)\Big|_{s=-2} = \frac{2}{s(s+1)(s+2)} \times (s+2)\Big|_{s=-2} = \frac{2}{s(s+1)}\Bigg|_{s=-2} = \frac{2}{-2(-2+1)} = +1$$

$$\therefore T(s) = \frac{2}{s(s+1)(s+2)} = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2}$$

Example

$$\text{Let, } T(s)=\frac{2}{s(s+1)\,(s+2)^2}$$

By partial fraction expansion, T(s) can be expressed as,

$$T(s)=\frac{K}{s(s+1)\,(s+2)^2}=\frac{A}{s}+\frac{B}{(s+1)}+\frac{C}{(s+2)^2}+\frac{D}{(s+2)}$$

$$A=T(s)\times s\Big|_{s=0}=\frac{2}{s(s+1)\,(s+2)^2}\times s\Big|_{s=0}=\frac{2}{(s+1)\,(s+2)^2}\Bigg|_{s=0}=\frac{2}{1\times 2^2}=0.5$$

B is obtained by multiplying T(s) by (s +1) and letting s = −1.

$$B=T(s)\times(s+1)\Big|_{s=-1}=\frac{2}{s(s+1)\,(s+2)^2}\times(s+1)\Bigg|_{s=-1}=\frac{2}{s\,(s+2)^2}\Bigg|_{s=-1}=\frac{2}{-1(-1+2)^2}=-2$$

C is obtained by multiplying T(s) by $(s+2)^2$ and letting s = −2.

$$C=T(s)\times(s+2)^2\Big|_{s=-2}=\frac{2}{s(s+1)\,(s+2)^2}\times(s+2)^2\Bigg|_{s=-2}=\frac{2}{s\,(s+1)}\Bigg|_{s=-2}=\frac{2}{-2(-2+1)}=1$$

D is obtained by differentiating the product T(s) $(s+2)^2$ with respect to s and then letting s = −2.

$$D=\frac{d}{ds}\left[T(s)\times(s+2)^2\right]\Bigg|_{s=-2}=\frac{d}{ds}\left[\frac{2}{s\,(s+1)}\right]\Bigg|_{s=-2}=\frac{-2(2s+1)}{s^2(s+1)^2}\Bigg|_{s=-2}=\frac{-2(2(-2)+1)}{(-2)^2(-2+1)^2}=+1.5$$

$$\therefore\ T(s)=\frac{2}{s\,(s+1)\,(s+2)^2}=\frac{0.5}{s}-\frac{2}{s+1}+\frac{1}{(s+2)^2}+\frac{1.5}{s+2}$$

Response of first order system with step input

The closed loop order system with unity feedback is shown in fig 2.6.

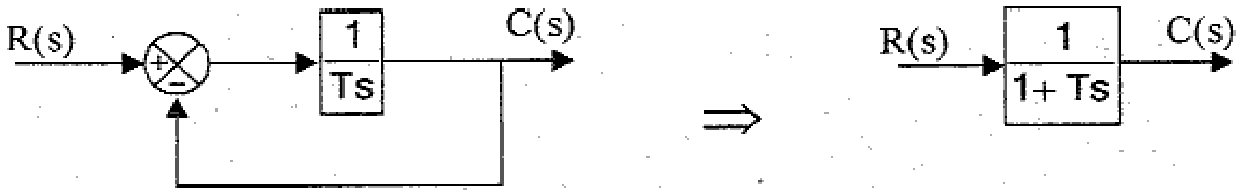


Fig 2.6 : *Closed loop for first order system.*

The closed loop transfer function of first order system, $\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$

If the input is unit step then, $r(t) = 1$ and $R(s) = \frac{1}{s}$.

$\therefore$ The response in s-domain, $C(s) = R(s)\frac{1}{(1+Ts)} = \frac{1}{s}\frac{1}{(1+Ts)} = \frac{1}{sT\left(\frac{1}{T}+s\right)} = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)}$

By partial fraction expansion,

$$C(s) = \frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)} = \frac{A}{s} + \frac{B}{\left(s+\frac{1}{T}\right)}$$

A is obtained by multiplying C(s) by s and letting s = 0.

$$A = C(s)\times s\Big|_{s=0} = \left.\frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)}\times s\right|_{s=0} = \left.\frac{\frac{1}{T}}{s+\frac{1}{T}}\right|_{s=0} = \frac{\frac{1}{T}}{\frac{1}{T}} = 1$$

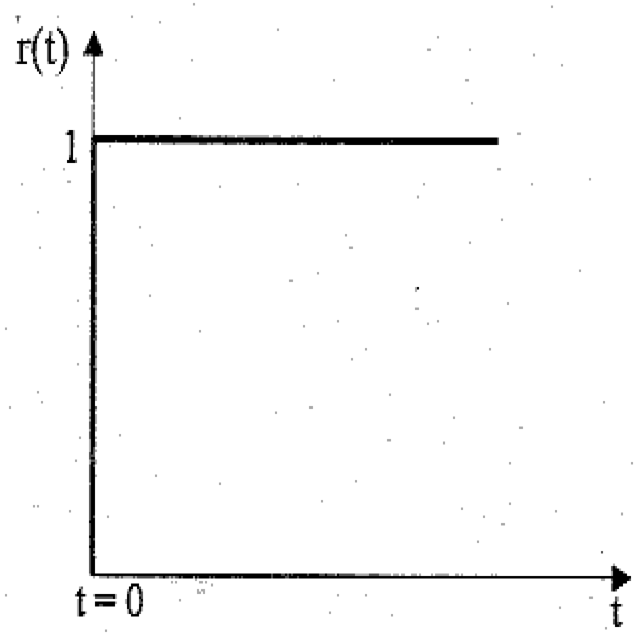
B is obtained by multiplying C(s) by (s+1/T) and letting s = –1/T.

$$B = C(s)\times\left(s+\frac{1}{T}\right)\Big|_{s=-\frac{1}{T}} = \left.\frac{\frac{1}{T}}{s\left(s+\frac{1}{T}\right)}\times\left(s+\frac{1}{T}\right)\right|_{s=-\frac{1}{T}} = \left.\frac{\frac{1}{T}}{s}\right|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{\frac{-1}{T}} = -1$$

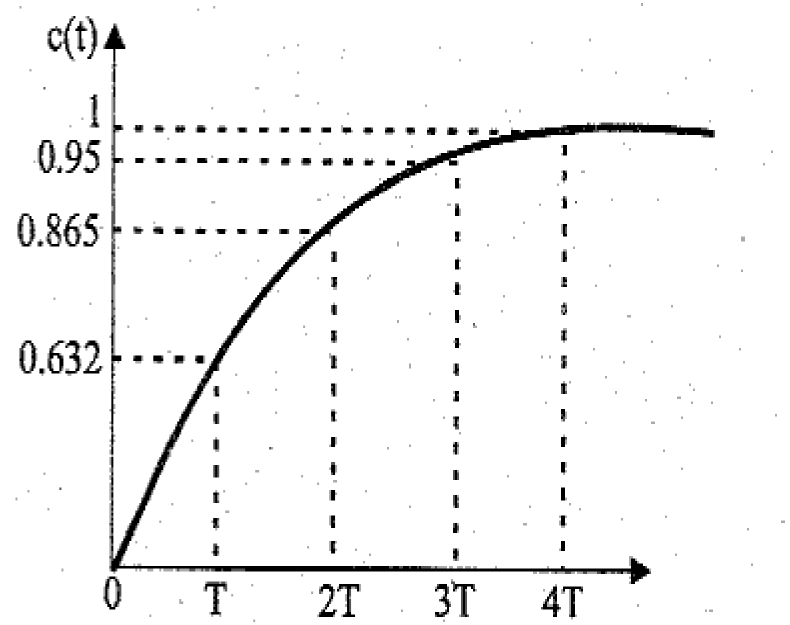
$$\therefore C(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{T}}$$

The response in time domain is given by,

$$c(t) = \mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s + \frac{1}{T}}\right\} = 1 - e^{-\frac{t}{T}}$$



r(t)
1
t = 0
t



c(t)
1
0.95
0.865
0.632
0
T
2T
3T
4T
t

Response of second order system

The closed loop second order system is shown in fig 2.8

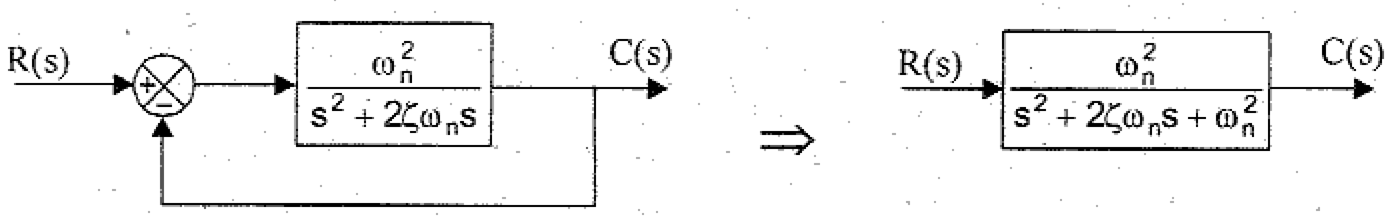
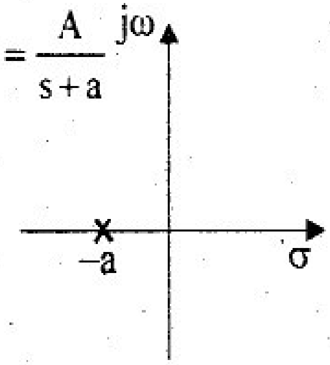
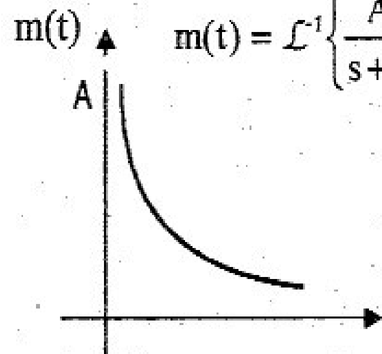
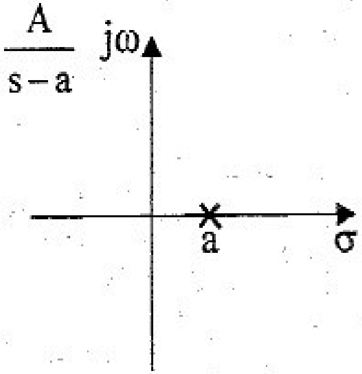
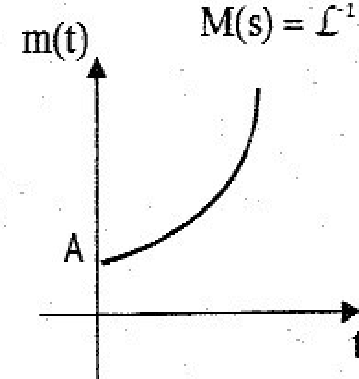


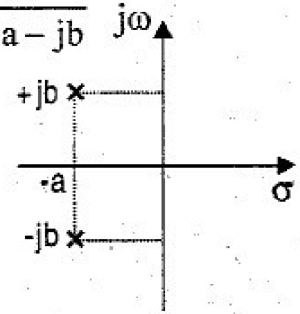
Fig 2.8 : *Closed loop for second order system.*

The standard form of closed loop transfer function of second order system is given by,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Transfer function, M(s) and location of roots on s-plane	**Impulse response, m(t)**
$M(s) = \dfrac{A}{s+a}$  *Root on negative real axis*	$m(t) = \mathcal{L}^{-1}\left\{\dfrac{A}{s+a}\right\} = Ae^{-at}$  *Impulse response is exponentially decaying. Stable system.*
$M(s) = \dfrac{A}{s-a}$  *Root on positive real axis*	$M(s) = \mathcal{L}^{-1}\left\{\dfrac{A}{s-a}\right\} = Ae^{+at}$  *Impulse response is exponentially increasing. Unstable system.*

$$M(s)=\frac{A}{s+a+jb}+\frac{A^*}{s+a-jb}$$

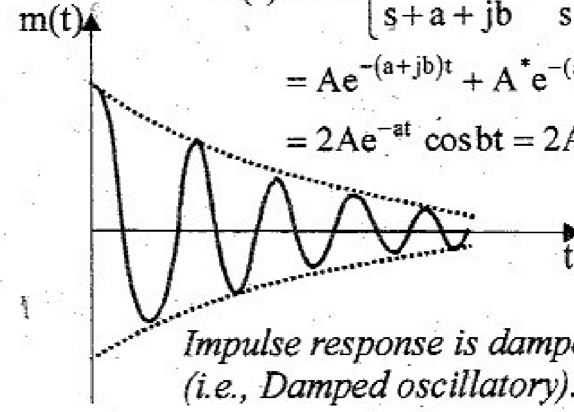


Complex conjugate roots on left half of s-plane

$$m(t)=\mathcal{L}^{-1}\left\{\frac{A}{s+a+jb}+\frac{A^*}{s+a-jb}\right\}$$

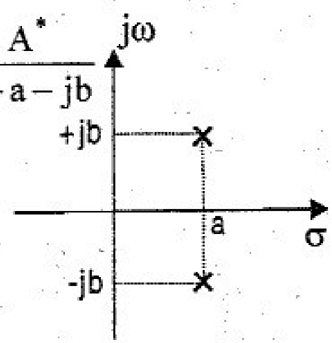
$$=Ae^{-(a+jb)t}+A^*e^{-(a-jb)t}$$

$$=2Ae^{-at}\cos bt=2Ae^{-at}\sin\left(bt+90^\circ\right)$$



Impulse response is damped sinusoidal (i.e., Damped oscillatory). Stable system

$$M(s)=\frac{A}{s-a+jb}+\frac{A^*}{s-a-jb}$$

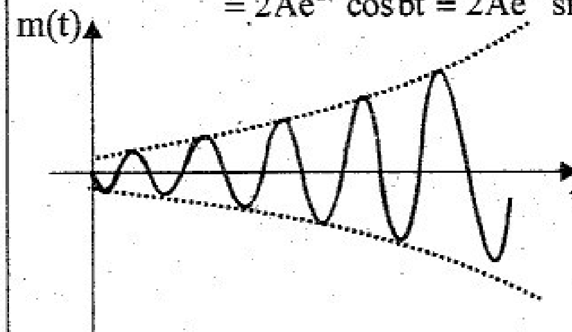


Complex conjugate roots on right half of s-plane

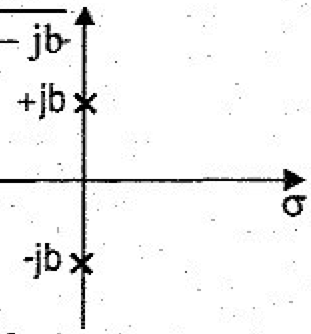
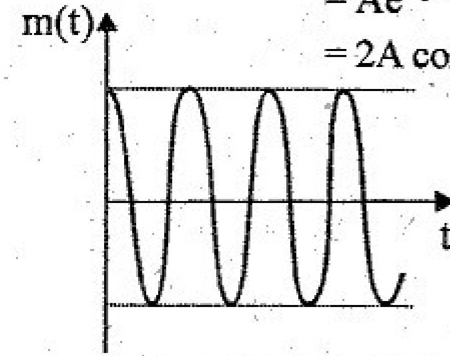
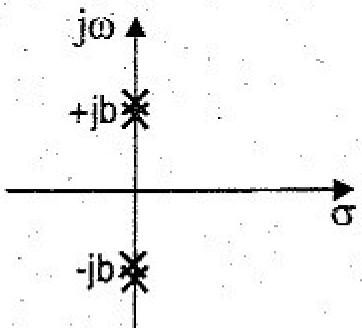
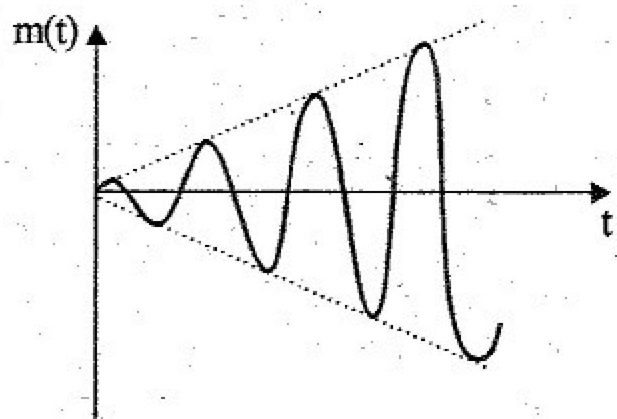
$$m(t)=\mathcal{L}^{-1}\left\{\frac{A}{s-a+jb}+\frac{A^*}{s-a-jb}\right\}$$

$$=Ae^{-(-a+jb)t}+A^*e^{-(-a-jb)t}$$

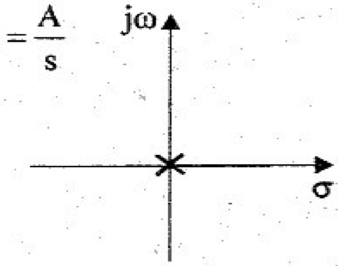
$$=2Ae^{at}\cos bt=2Ae^{at}\sin\left(bt+90^\circ\right)$$



Impulse response is exponentially increasing sinusoidal (i.e., Amplitude of oscillations exponentially increases with time). Unstable system.

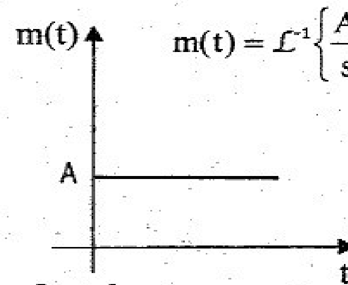
$M(s)=\dfrac{A}{s+jb}+\dfrac{A^*}{s-jb}$  *Single pair of roots on imaginary axis*	$m(t)=\mathcal{L}^{-1}\left\{\dfrac{A}{s+jb}+\dfrac{A^*}{s-jb}\right\}$ $=Ae^{-jbt}+A^*e^{+jbt}$ $=2A\cos bt=2A\sin(bt+90°)$  *Impulse response is oscillatory* *Marginally stable*
$M(s)=\dfrac{A}{(s+jb)^2}+\dfrac{A^*}{(s-jb)^2}$  *Double pair of roots on imaginary axis*	$m(t)=\mathcal{L}^{-1}\left\{\dfrac{A}{(s+jb)^2}+\dfrac{A^*}{(s-jb)^2}\right\}$ $=At\,e^{-jbt}+A^*\,t\,e^{+jbt}$ $=2At\cos bt=2At\sin(bt+90°)$ 

$$M(s) = \frac{A}{s}$$



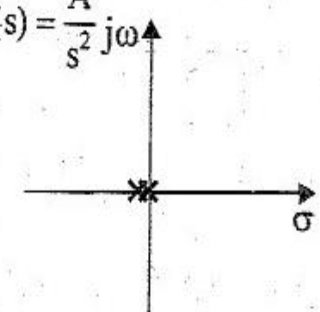
Single root at origin

$$m(t) = \mathcal{L}^{-1}\left\{\frac{A}{s}\right\} = A$$



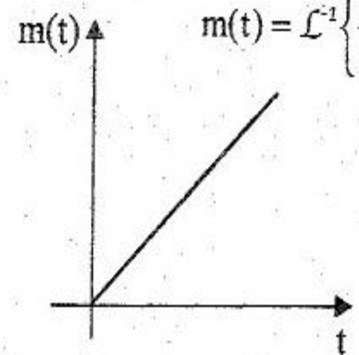
Impulse response is constant.
Marginally stable system.

$$M(s) = \frac{A}{s^2}$$



Double root at origin

$$m(t) = \mathcal{L}^{-1}\left\{\frac{A}{s^2}\right\} = At$$



Impulse reponse linearly increases with time. Unstable system

stability

If all the roots of characteristic equation has negative real parts, then the system is stable.

If any root of the characteristic equation has a positive real part or if there is a repeated root on the imaginary axis then the system is unstable.

If the condition (i) is satisfied except for the presence of one or more non repeated roots on the imaginary axis, then the system is limitedly or marginally stable.

For example, consider the characteristic polynomial with all positive coefficients,

$$s^3 + s^2 + 2s + 8 = 0.$$

The characteristic polynomial can be written as,

$$(s^3 + s^2 + 2s + 8) = (s+2)\left(s - \frac{1}{2} - j\frac{\sqrt{15}}{2}\right)\left(s - \frac{1}{2} + j\frac{\sqrt{15}}{2}\right) = 0$$

Now the roots are,

$$s = -2, \quad +\frac{1}{2} + j\frac{\sqrt{15}}{2}, \quad +\frac{1}{2} - j\frac{\sqrt{15}}{2}$$

The coefficients of the polynomial are all positive, but two roots have positive real part and so will lie on on right half of s-plane, therefore the system is unstable.

Routh Stability Criterion

- All the coefficients of characteristic equation are to be positive.
- If some of the coefficients are zero or negative then system is not stable
- Routh array

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} s + a_n = 0, \text{ where } a_0 > 0,$$

$$\begin{array}{llllllll}
s^n & : & a_0 & a_2 & a_4 & a_6 & a_8 & \ldots\ldots \\
s^{n-1} & : & a_1 & a_3 & a_5 & a_7 & a_9 & \ldots\ldots \\
s^{n-2} & : & b_0 & b_1 & b_2 & b_3 & b_4 & \ldots\ldots \\
s^{n-3} & : & c_0 & c_1 & c_2 & c_3 & c_4 & \ldots\ldots \\
 & & & & & & & \\
s^1 & : & g_0 & & & & & \\
s_0 & : & h_0 & & & & &
\end{array}$$

- The necessary and sufficient condition for stability is that all the elements of first column of Routh array should be positive for a stable system
- The number of sign changes in the elements of first column of Routh array corresponds to number of roots of characteristic equation in the right half of s plane

Construction of Routh array

Let the characteristic polynomial be,

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \dots a_{n-1} s^1 + a_n s^0$$

The coefficients of the polynomial are arranged in two rows as shown below.

$$\begin{array}{llllll} s^n & : & a_0 & a_2 & a_4 & a_6 \dots\dots \\ s^{n-1} & : & a_1 & a_3 & a_5 & a_7 \dots\dots \end{array}$$

$$s^{n-x} : x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \ldots\ldots$$

$$s^{n-x-1} : y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \ldots\ldots$$

Let the next row be,

$$s^{n-x-2} : z_0 \quad z_1 \quad z_2 \quad z_3 \quad z_4 \ldots\ldots$$

The elements of s^{n-x-2} row are given by,

$$z_0 = \frac{(-1)\begin{vmatrix} x_0 & x_1 \\ y_0 & y_1 \end{vmatrix}}{y_0} = \frac{y_0 x_1 - y_1 x_0}{y_0}$$

$$z_1 = \frac{(-1)\begin{vmatrix} x_0 & x_2 \\ y_0 & y_2 \end{vmatrix}}{y_0} = \frac{y_0 x_2 - y_2 x_0}{y_0}$$

$$z_2 = \frac{(-1)\begin{vmatrix} x_0 & x_3 \\ y_0 & y_3 \end{vmatrix}}{y_0} = \frac{y_0 x_3 - y_3 x_0}{y_0}$$

$$z_3 = \frac{(-1)\begin{vmatrix} x_0 & x_4 \\ y_0 & y_4 \end{vmatrix}}{y_0} = \frac{y_0 x_4 - y_4 x_0}{y_0}$$

$$z_4 = \frac{(-1)\begin{vmatrix} x_0 & x_5 \\ y_0 & y_5 \end{vmatrix}}{y_0} = \frac{y_0 x_5 - y_5 x_0}{y_0} \quad \text{and so on.}$$

In the construction of Routh array one may come across the following three cases.

Case-I : Normal Routh array (Non-zero elements in the first column of routh array).

Case-II : A row of all zeros.

Case-III : First element of a row is zero but some or other elements are not zero.

Ex.1) Using Routh stability criterion, determine the stability of the system represented by characteristic equation, $S^4+8s^3+18s^2+16s+5=0$

s^4	:	1	18	5	.... Row-1
s^3	:	8	16		.... Row-2
s^2					
s^1					
s^0					

$$s^2 : \frac{1\times 18-2\times 1}{1} \quad \frac{1\times 5-0\times 1}{1}$$

$$s^2 : \quad 16 \quad 5$$

$$s^1 : \frac{16\times 2-5\times 1}{16}$$

$$s^1 : 1.6875 \approx 1.7$$

$$s^0 : \frac{1.7\times 5-0\times 16}{17}$$

$$s^0 : 5$$

s^4	:	1	18	5	.... Row-1
s^3	:	1	2		.... Row-2
s^2	:	16	5		.... Row-3
s^1	:	1.7			.... Row-4
s^0	:	5			.... Row-5
		Column-1			

$$s^2 : \frac{1\times 18 - 2\times 1}{1} \quad \frac{1\times 5 - 0\times 1}{1}$$

$$s^2 : 16 \quad 5$$

$$s^1 : \frac{16\times 2 - 5\times 1}{16}$$

$$s^1 : 1.6875 \approx 1.7$$

$$s^0 : \frac{1.7\times 5 - 0\times 16}{17}$$

$$s^0 : 5$$

- System is stable

all four roots lies on left half of s plane

Ex.2) Using Routh stability criterion, determine the stability of the system represented by characteristic equation, $9S^5-20s^4+10s^3-s^2-9s-10=0$. Comment on the location of roots of characteristic equation

$$
\begin{array}{llllll}
s^5 & : & \boxed{9} & 10 & -9 & \text{....Row-1} \\
s^4 & : & -20 & -1 & -10 & \text{....Row-2} \\
s^3 & : & & & & \\
s^2 & : & & & & \\
s^1 & : & & & & \\
s^0 & : & & & &
\end{array}
$$

$$s^3: \frac{-20\times 10-(-1)\times 9}{-20} \quad \frac{-20\times(-9)-(-10)\times 9}{-20}$$

$$s^3: 9.55 \quad -13.5$$

$$s^2: \frac{9.55\times(-1)-(-13.5)\times(-20)}{9.55} \quad \frac{9.55\times(-10)}{9.55}$$

$$s^2: -29.3 \quad -10$$

$$s^1: \frac{-29.3\times(-13.5)-(-10)\times 9.55}{-29.3}$$

$$s^1: -16.8$$

$$s^0: \frac{-16.8\times(-10)}{-16.8}$$

$$s^0: -10$$

s^5	:	9	10	−9	.... Row-1
s^4	:	-20	−1	−10	.... Row-2
s^3	:	9.55	−13.5		.... Row-3
s^2	:	−29.3	−10		.... Row-4
s^1	:	−16.8			.... Row-5
s^0	:	−10			.... Row-6

Column-1

•System is unstable

Two roots lies on left half of s plane

Case ii) Ex. 3:

$$s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0.$$

s^6	:	1	8	20	16	.... Row-1
s^5	:	2	12	16		.... Row-2

s^6	:	1	8	20	16	....Row-1
s^5	:	1	6	8		....Row-2
s^4	:	1	6	8		....Row-4
s^3	:	0	0			....Row-4

$$s^4 : \frac{1\times 8-6\times 1}{1} \quad \frac{1\times 20-8\times 1}{1} \quad \frac{1\times 16-0\times 1}{1}$$

s^4 : 2 12 16

divide by 2

s^4 : 1 6 8

$$s^3 : \frac{1\times 6-6\times 1}{1} \quad \frac{1\times 8-8\times 1}{1}$$

s^3: 0 0

The auxiliary equation is, $A = s^4+6s^2+8$. On differentiating A with respect to s we get,

$$\frac{dA}{ds} = 4s^3 + 12s$$

The coefficients of $\frac{dA}{ds}$ are used to form s^3 row.

s^3: 4 12

divide by 4

s^3: 1 3

s^6	:	1	8	20	16	.... Row-1
s^5	:	1	6	8		....Row-2
s^4	:	1	6	8		....Row-4
s^3	:	0	0			....Row-4
s^3	:	1	3			....Row-4
s^2	:	3	8			....Row-5
s^1	:	0.33				....Row-6
s^0	:	8				....Row-7

Column-1

$$s^2 : \frac{1\times 6 - 3\times 1}{1} \quad \frac{1\times 8 - 0\times 1}{1}$$

$$s^2 : \quad 3 \qquad 8$$

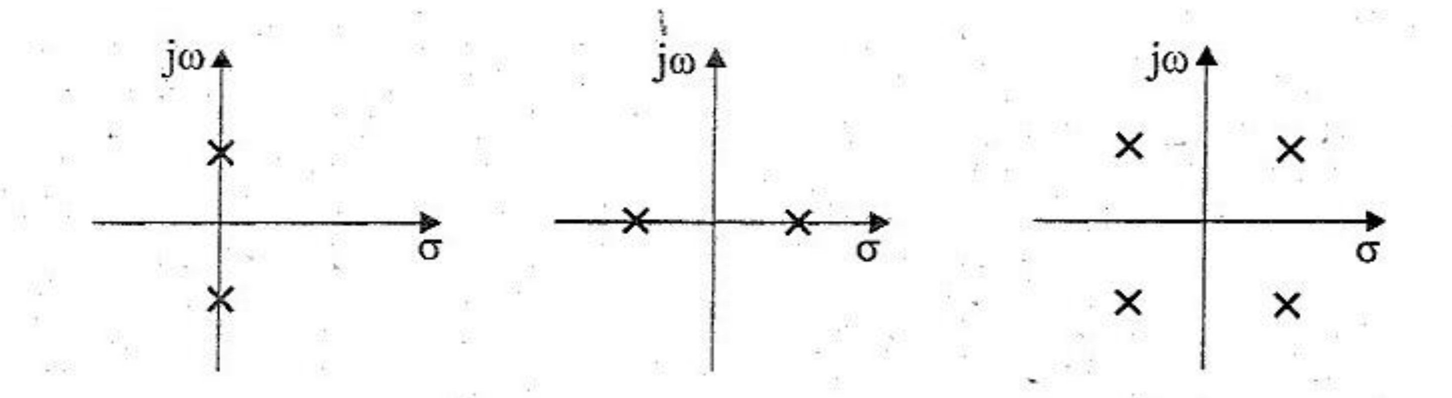
$$s^1 : \frac{3\times 3 - 8\times 1}{3}$$

$$s^1 : 0.33$$

$$s^0 : \frac{0.33\times 8 - 0\times 3}{0.33}$$

$$s^0 : 8$$

Even polynomial - roots



The auxiliary polynomial is,

$$s^4 + 6s^2 + 8 = 0$$

Let, $s^2 = x$

$$\therefore x^2 + 6x + 8 = 0$$

The roots of quadratic are, $x = \dfrac{-6 \pm \sqrt{6^2 - 4 \times 8}}{2}$

$$= -3 \pm 1 = -2 \text{ or } -4$$

The roots of auxiliary polynomial is,

$$s = \pm\sqrt{x} = \pm\sqrt{-2} \text{ and } \pm\sqrt{-4}$$

$$= +j\sqrt{2}, -j\sqrt{2}, +j2 \text{ and } -j2$$

- System is marginaly stable

Two roots lie on left half of s plane and 4 roots lie on imaginary axis

Case iii) Ex. 4: $s^5+s^4+2s^3+2s^2+3s+5=0.$

s^5	:	1	2	3	.... Row-1
s^4	:	1	2	5	.... Row-2

$$
\begin{array}{llllll}
s^5 & : & 1 & 2 & 3 & \text{.... Row-1} \\
s^4 & : & 1 & 2 & 5 & \text{.... Row-2} \\
s^3 & : & \in & -2 & & \text{.... Row-3} \\
s^2 & : & \dfrac{2\in+2}{\in} & 5 & & \text{.... Row-4} \\
s^1 & : & \dfrac{-\left(5\in^2+4\in+4\right)}{2\in+2} & & & \text{.... Row-5} \\
s^0 & : & 5 & & & \text{.... Row-6}
\end{array}
$$

s^5	:	1	2	3	.... Row-1
s^4	:	1	2	5	.... Row-2
s^3	:	0	-2		.... Row-3
s^2	:	∞	5		.... Row-4
s^1	:	-2			.... Row-5
s^0	:	5			.... Row-6

Column-1

Ex.5: $s^7+9s^6+24s^5+24s^4+24s^3+24s^2+23s+15=0.$

s^7	:	1	24	24	23	.... Row-1
s^6	:	3	8	8	5	.... Row-2
s^5	:	1	1	1		.... Row-3
s^4	:	1	1	1		.... Row-4
s^3	:	0	0			.... Row-5
s^3	:	2	1			.... Row-5
s^2	:	0.5	1			.... Row-6
s^1	:	-3				.... Row-7
s^0	:	1				.... Row-8

Column-1

$$s^4 + s^2 + 1 = x^2 + x + 1 = 0$$

The roots of quadratic are, $x = \dfrac{-1 \pm \sqrt{1-4}}{2} = -\dfrac{1}{2} \pm j\dfrac{\sqrt{3}}{2}$

$= 1\angle 120^\circ$ or $1\angle -120^\circ$

But $s^2 = x$, $\therefore s = \pm\sqrt{x} = \pm\sqrt{1\angle 120^\circ}$ or $\pm\sqrt{1\angle -120^\circ}$

$= \pm\sqrt{1}\angle 120^\circ/2$ or $\pm\sqrt{1}\angle -120^\circ/2$

$= \pm 1\angle 60^\circ$ or $\pm 1\angle -60^\circ$

$= \pm(0.5 + j0.866)$ or $\pm(0.5 - j0.866)$

Ex.6: $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0.$

Ex.6: $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0.$

s^5	:	1	8	7	.... Row-1
s^4	:	4	8	4	.... Row-2

Divide s^4 row by 4 to simplify the calculations.

s^5	:	1	8	7	.... Row-1
s^4	:	1	2	1	.... Row-2
s^3	:	1	1		.... Row-3
s^2	:	1	1		.... Row-4
s^1	:	$\in$			.... Row-5
s^0	:	1			.... Row-6

Column-1

The auxiliary polynomial is, $s^2 + 1 = 0$; $\therefore s^2 = -1$ or $s = \pm\sqrt{-1} = \pm j1$

•System is marginally stable
Three roots lie on left half of s plane and 2 roots lie on imaginary axis

Ex.7 : $s^7 + 5s^6 + 9s^5 + 9s^4 + 4s^3 + 20s^2 + 36s + 36 = 0$.

Ex.8 : $s^6 + s^5 + 3s^4 + 3s^3 + 3s^2 + 2s + 1 = 0$.