Module - 2

Laplace Transform Basics

Laplace Transform → solving linear differential equations
mainly used to solve the transient behaviour of electrical circuits

LT

$$\mathcal{L}(f(t)) = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$s = \underset{\text{real}}{\sigma} + j\underset{\text{imaginary}}{\omega}$$

Fourier Transform.

$$\rightarrow X(f(t)) = X(j\omega) = \int_0^\infty f(t)\, e^{-j\omega t}\, dt$$

Every signals having FT → LT, but every signals having LT don't have FT.

FT → steady state signal only.

LT → steady state and also transient

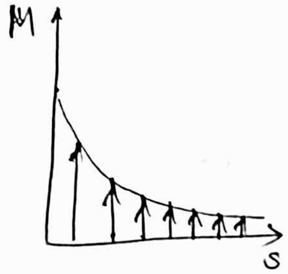
Unit - step function

$u(t) = 0, \quad t \leq 0$

$u(t) = 1, \quad t > 0$

$$\mathcal{L}(f(t)) = \mathcal{L}[u(t)] = \int_0^\infty u(t)\, e^{-st}\, dt$$

$$= \int_0^\infty 1\, e^{-st}\, dt$$



$$= \frac{\left[e^{-st}\right]_0^\infty}{-s}$$

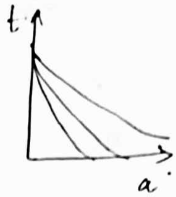
$$= \frac{-1}{s}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{-1}{s}\left[0 - 1\right]$$

$$= \frac{1}{s}$$

Exponential function

$f(t) = e^{-at}$



$F(s) = L(f(t))$

$= \int_0^\infty e^{-at} e^{-st} dt$

$= \int_0^\infty e^{-(s+a)t} dt$

$= \dfrac{e^{-(s+a)t}}{-(s+a)}$

$= \dfrac{-1}{s+a} [0-1]$

$= \dfrac{1}{s+a}$

$$\boxed{L(e^{-at}) = \frac{1}{s+a}}$$

$f(t) = e^{at}$

$$\boxed{L(e^{at}) = \frac{1}{s-a}}$$

Sinusodial function

$f(t) = \sin wt$

$F(s) = L(f(t))$

$= \int_0^\infty \sin wt \, e^{-st} dt$

$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{jwt} = \cos wt + j\sin wt$

$e^{-jwt} = \cos wt - j\sin wt$

$\sin wt = \dfrac{e^{jwt} - e^{-jwt}}{2j}$

$F(s) = L(\sin wt) = \int_0^\infty \dfrac{e^{jwt} - e^{-jwt}}{2j} e^{-st} dt$

$= \dfrac{1}{2j} \int_0^\infty \left[e^{-(s-jw)t} - e^{-(s+jw)t} \right] dt$

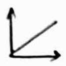
$= \dfrac{1}{2j} \left(-\dfrac{1}{s-jw} \left[e^{-(s-jw)t} \right]_0^\infty + \dfrac{1}{s+jw} \left[e^{-(s+jw)t} \right]_0^\infty \right.$

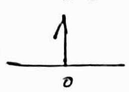
$= \dfrac{1}{2j} \left[\dfrac{1}{s-jw} - \dfrac{1}{s+jw} \right]$

$= \dfrac{1}{2j} \dfrac{s+jw - s+jw}{s^2+w^2}$

$$= \frac{w}{s^2+w^2}.$$

$$\boxed{\mathcal{L}(\sin wt) = \frac{w}{s^2+w^2}}.$$

function	Laplace
$u(t)$	$1/s$.
e^{-at}	$\frac{1}{s+a}$.
e^{at}	$\frac{1}{s-a}$
$\sin wt$	$\frac{w}{s^2+w^2}$
$\cos wt$	$\frac{s}{s^2+w^2}$
 t	$\frac{1}{s^2}$
 t^2	$\frac{2}{s^3}$.
t^n	$\frac{n!}{s^{n+1}}$
$e^{at}\, t^n$	$\frac{n!}{(s+a)^{n+1}}$
$e^{-at} \sin wt$	$\frac{w}{(s+a)^2+w^2}$.
$e^{-at} \cos wt$	$\frac{s+a}{(s+a)^2+w^2}$
Impulse function $\delta(t)$.	1
(hyperbolic function) $\sin hwt$	$\frac{w}{s^2-w^2}$.
$\cos hwt$	$\frac{s}{s^2-w^2}$



At a particular time, it has very large value.

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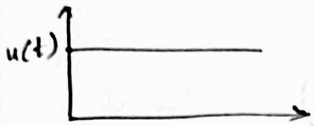
$F(s) = \mathcal{L}\{\frac{d f(t)}{dt}\} = \mathcal{L}\{f'(t)\} = sF(s) - f(0^+)$

f at 0^+

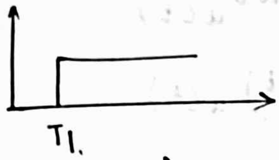
$F(s) = \mathcal{L}\{\int f(t)\,dt\} = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)\right]_{0^+}$

value of integral of $f(t)$ at 0^+

Displacement Theorem



$\mathcal{L}(u(t)) = \frac{1}{s}$.



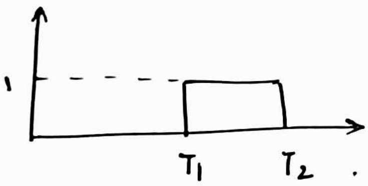
T_1.

$u(t-T_1)$ $\qquad \mathcal{L}(u(t-T_1)) = e^{-sT_1}\frac{1}{s}$

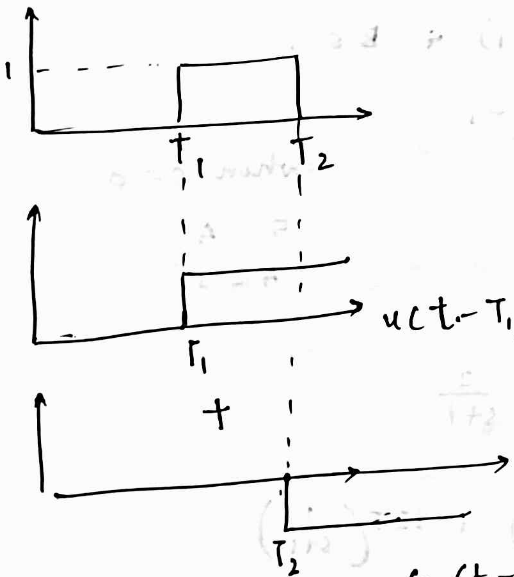
$\mathcal{L}(f(t)) = F(s)$

$\mathcal{L}(f(t-T)) = e^{-sT}F(s)$

Qn)



obtain the Laplace Transform of the pulse.
This pulse is known as the Gate function



$u(t-T_1)$

$+$

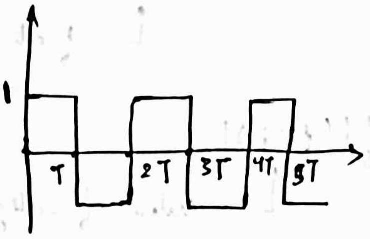
$-(u(t-T_2))$

$f(t) = f_1(t) + f_2(t)$

$= u(t-T_1) - u(t-T_2)$

$= e^{-sT_1}\frac{1}{s} - \frac{e^{-sT_2}}{s} = \frac{e^{-sT_1} - e^{-sT_2}}{s}$

Homework



Find the Laplace Transform of square wave train.

Ans:- $\left(\frac{1}{s} \left[1 - 2e^{-Ts} \left[1 - e^{-Ts} + e^{-2Ts} - e^{-3Ts} + \dots\right]\right]\right)$

Qn Find the inverse Laplace transform of

(i) $F(s) = \frac{7}{s} - \frac{31}{s+17}$.

$$ILT(F(s)) = ILT(7/s) - ILT\left(\frac{31}{s+17}\right)$$

$$= 7u(t) - 31e^{-17t}u(t)$$

$$= \left(7 - 31e^{-17t}\right)u(t)$$

(ii) $F(s) = \frac{7s+5}{s^2+s}$

$$ILT\left(\frac{7s+5}{s^2+s}\right) = ILT\left(\frac{7s+5}{s^2+s}\right).$$

$$\cancel{\frac{7s+5}{s^2+s} = \frac{A}{s} + \frac{B}{s^2}}$$

$$\frac{7s+5}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$7s+5 = A(s+1) + Bs.$$

when $s = -1$

$$-B = -7+5$$

$$B = 2$$

when $s = 0$.

$$5 = A$$

$$A = 5$$

$$\frac{7s+5}{s(s+1)} = \frac{5}{s} + \frac{2}{s+1}$$

$$ILT\left(\frac{7s+5}{s^2+s}\right) = ILT\left(\frac{5}{s}\right) + ILT\left(\frac{2}{s+1}\right)$$

$$= \cancel{7}\, 5u(t) + 2e^{-t}u(t)$$

$$= \left(5 + 2e^{-t}\right)u(t)$$

(iii) $F(s) = \frac{2}{s^3+12s^2+36s}$

81

12.

$\frac{2}{s^3+12s^2+36s} = \frac{2}{s(s^2+12s+36)}$

$s^2+6s+6s+36$

$(s+6)s+6(s+6)$

$\cancel{\frac{2}{s(s^2+12s+36)}} = \frac{A_2}{s} +$

$F(s) = \frac{2}{s(s+6)^2}$

$\frac{2}{s(s+6)^2} = \frac{A}{s} + \frac{B}{s+6} + \frac{C}{(s+6)^2}$

$2 = A(s+6)^2 + Bs(s+6) + Cs$

Put $s=0$

$36A = 2$

$A = \frac{1}{18}$

Put $s=-6$

$-6C = 2$

$C = -\frac{2}{6} = -1/3$

$B + A = 0$

$B = -A$

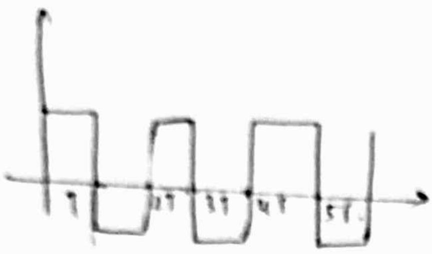
$= -\frac{1}{18}$

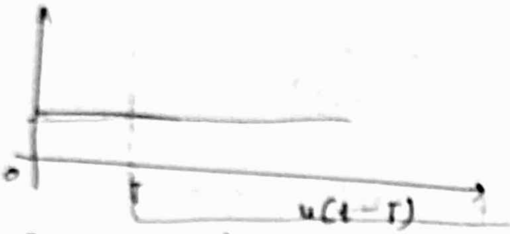
$\frac{2}{s(s+6)^2} = \cancel{-\frac{1/18}{s}} + \frac{1/18}{s} + \frac{-1/18}{s+6} + \frac{-1/3}{(s+6)^2}$

$\text{ILT}(F(s)) = \frac{1}{18}u(t) - \frac{1}{18}e^{-6t}u(t) - \frac{1}{3}te^{-6t}u(t)$

$f(t) = \left(\frac{1}{18} - \frac{1}{18}e^{-6t} - \frac{1}{3}te^{-6t}\right)u(t)$

Homework





Upper cycle

$$u(t) + u(t-T) + u(t-2T) + u(t-2T) + \cdots$$

~~Lower cycle~~

Upper cycle

$$u(t) - \cancel{2}u(t-T) + u(t-2T) - u(t-3T) + u(t-4T) - u(t-5T) + \cdots$$

$$\frac{1}{s} - \frac{e^{-sT}}{s} + \frac{e^{-2sT}}{s} - \frac{e^{-3sT}}{s} + \frac{e^{-4sT}}{s} - \frac{e^{-5sT}}{s}$$

$$= \frac{1}{s}\left[1 - e^{-sT} + e^{-2sT} - e^{-3sT} + e^{-4sT} - e^{-5sT} + \cdots\right]$$

Lower cycle

$$+ u(t-T) - 2\cdot u(t-2T) + u(t-3T) - u(t-4T) + u(t-5T) - \cdots$$

$$= \frac{e^{-sT}}{s} - \frac{e^{-2sT}}{s} + \frac{e^{-3sT}}{s} - \frac{e^{-4sT}}{s} + \frac{e^{-5sT}}{s}$$

~~Upper~~

Total

$$\frac{1}{s}\left[1 - 2e^{-sT} + 2e^{-2sT} - 2e^{-3sT} + 2e^{-4sT} - 2e^{-5sT} + \cdots\right]$$

$$= \frac{1}{s}\left[1 - 2e^{-sT}\left[1 - e^{-sT} + e^{-2sT} - e^{-3sT} + e^{-4sT} - e^{-6sT} + \cdots\right]\right.$$

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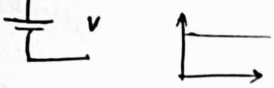
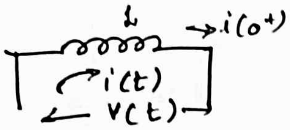
Time domain | s-domain

constant voltage / DC-V | $\frac{V}{s}$ $[Vu(t)]$



Time dependent source | $V(s)$

$v(t)$

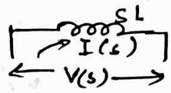


$v(t) = L\frac{di}{dt}$

$V(s) = L[sI(s) - i(0^+)]$

if the initial current is 0

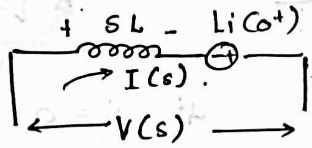
then, $V(s) = L\,s\,I(s)$

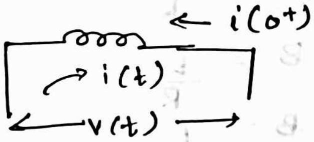


if $i(0^+) \neq 0$

$V(s) = L(sI(s) - i(0^+))$

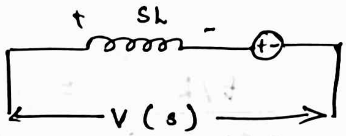
$= sL\,I(s) - L\,i(0^+)$



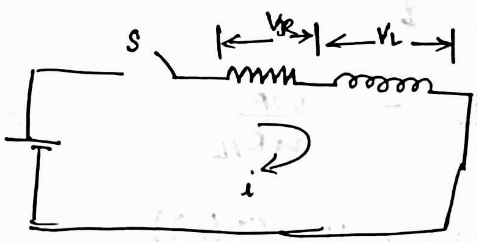


$V(s) = L(sI(s) + i(0^+))$

$V(s) = sLI(s) + L\,i(0^+)$



Qn) consider a dc circuit, find the current



at $t = 0$, s is closed.

at $t = 0$, $i = 0$

$t = 0^+$, $i(0^+) = 0$

since, the current through inductor can't change instanteneously

$E = iR + L\frac{di}{dt}$

converting to s-domain

$$\frac{E}{s} = I(s)R + L(sI(s) - 0)$$

$$\frac{E}{s} = RI(s) + LsI(s)$$

$$\frac{E}{s} = I(s)[R + Ls]$$

$$I(s) = \frac{E}{s(R+Ls)}$$

$$I(s) = \frac{E}{sL(\frac{R}{L}+s)}$$

$$I(s) = \frac{(E/L)}{s(s+\frac{R}{L})}$$

$$\frac{E/L}{s(s+R/L)} = \frac{A}{s} + \frac{B}{s+\frac{R}{L}}$$

$$\frac{E}{L} = A\left[s + \frac{R}{L}\right] + Bs$$

At $s = 0$

$$\frac{AR}{L} = \frac{E}{L}$$

$$A = \frac{E}{R}$$

At $s = -\frac{R}{L}$

$$-\frac{BR}{L} = \frac{E}{L}$$

$$-B = \frac{E}{R}$$

$$B = -\frac{E}{R}$$

$$\frac{E/L}{s(s+R/L)} = \frac{E}{sR} - \frac{E}{Rs + \frac{R^2}{L}}$$

$$= \frac{\cancel{E}}{Rs}$$

$$I(s) = \frac{E/R}{s} - \frac{E/R}{s+R/L}$$

~~I(t)~~ $i(t) = \text{ILT}(I(s))$

$$i(t) = \frac{E}{R} - \frac{E}{R}e^{-\frac{R}{L}t}$$

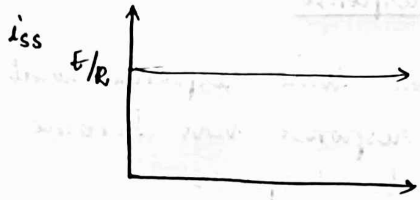
$$i(t) = \frac{E}{R}\left(1 - e^{-R/L\,t}\right)$$

$$i(t) = \underbrace{\frac{E}{R}}_{\text{steady state}} - \underbrace{\frac{E}{R} e^{-R/L\, t}}_{\text{transient state}}$$

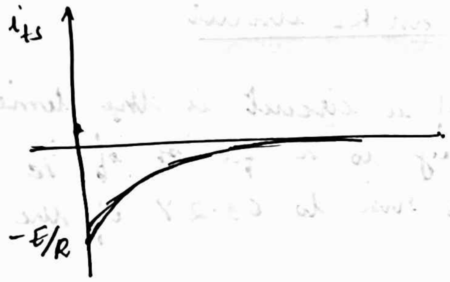
at $t = \infty$

$$i = \frac{E}{R}$$

i_{ss} - steady state current.



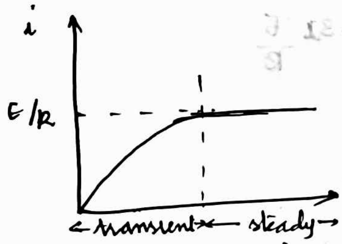
$$-\frac{E}{R} e^{-R/L\, t}$$



at initial state $t = 0$

$$-\frac{E}{R} e^{-R/L\, t} = -\frac{E}{R}$$

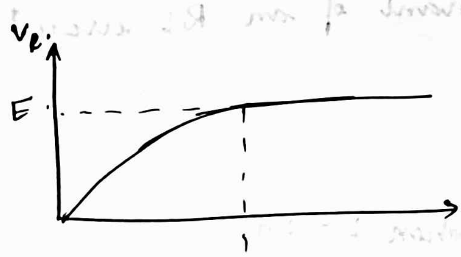
$\therefore$ The graph of $\frac{E}{R} - \frac{E}{R} e^{-R/L\, t}$



Voltage across resistor $\quad V_R = IR$

$$= \frac{E}{R} R$$

$$= E$$



Transient state response

After applying an input, the output takes certain time to reach steady state. ∴ The response of the system during transient state is known as transient state response.

Steady state response

The part of the time response that remains even after the transient response has become zero is known as the steady state response.

The time constant of an RL circuit

The time constant of a circuit is the time required for the response to decay to a factor of $1/e$ [36.8%] of the initial value or rise to 63.2% of the final value.

$$i = \frac{E}{R}\left(1 - e^{-tR/L}\right)$$

$$\frac{E}{R}\left(1 - e^{-tR/L}\right) = 0.632\,\frac{E}{R}$$

$$1 - e^{-tR/L} = 0.632$$

$$e^{-tR/L} = 0.368$$

$$t = \frac{L}{R}$$

The time constant of an RL circuit is $\frac{L}{R}$

$$t = \frac{L}{R} = 1\tau$$

$$\tau = \frac{L}{R} \qquad \text{when } L = 1\,H, \; R = 1\,\Omega$$

$$\tau = \frac{L}{R} = 1\,\text{sec}$$

The transient response time is 4τ.

If the time constant $\tau\uparrow$, it takes more time to reach steady state.

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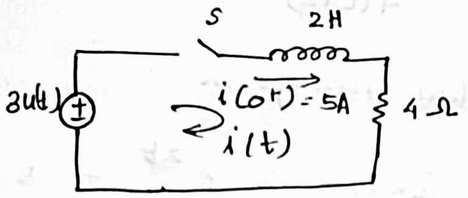


The initial current through inductor $i(0^+) = 5A$, at time $t = 0$, switch is closed.

Find $i(t)$.

$$3u(t) = L\frac{di}{dt} + 4i$$

Taking Laplace Transform

$$\frac{3}{s} = 2\left[sI(s) - i(0^+)\right] + 4\,\cancel{\$(s)}\,I(s)$$

$$= 2\left[sI(s) - 5\right] + 4I(s)$$

$$\frac{3}{s} = 2sI(s) - 10 + 4I(s)$$

$$\frac{3}{s} = I(s)\left[2s + 4\right] - 10$$

$$\therefore \quad I(s)(2s+4) = \frac{3}{s} + 10$$

$$I(s)(2s+4) = \frac{3 + 10s}{s}$$

$$I(s) = \frac{3 + 10s}{s(2s+4)}$$

$$I(s) = \cancel{\frac{3+10}{}}\ \frac{3}{s(2s+4)} + \frac{10s}{s(2s+4)}$$

$$I(s) = \frac{3}{2s(s+2)} + \frac{5}{s+2}.$$

$$\frac{\cancel{3}\ 1.5}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$1.5 = A(s+2) + Bs$$

$s = 0$. $\qquad s = -2$

$2A = 1.5 \qquad -2B = 1.5$

$A = \frac{3}{4} \qquad B = \frac{-3}{4}$

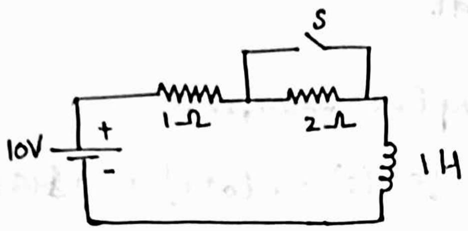
$$I(s) = \frac{3}{4s} - \frac{3}{4(s+2)} + \frac{5}{s+2}$$

Inverse Laplace transform.

$$i(t) = \frac{3}{4}u(t) - \frac{3}{4}e^{-2t} + 5e^{-2t}$$

$$= \underbrace{\frac{3}{4}u(t)}_{\text{steady state}} + \underbrace{4.25e^{-2t}}_{\text{transient state}}$$

Qn 2)



The battery voltage is applied for a steady state period. Obtain the complete expression for the current after closing the switch S.

Initially switch is opened.

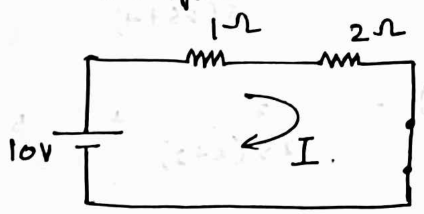
Note :-

An inductor

at $t=0$.

at $t=\infty$, short unless at steady state unless there is no transient

S→ open



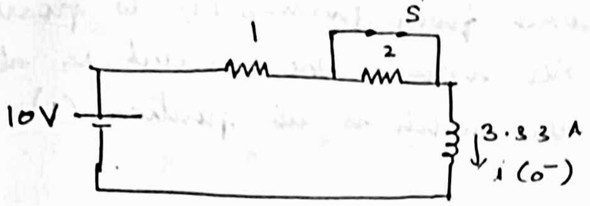
at $t=0^-$

Initially switch s open then when it.

~~switch s is closed.~~

$$I = \frac{10}{1+2} = 3.33A = i(0^-)$$

when switch S is closed, a transient occurs.



when switch is closed 2Ω is shorted.

$$10 = 1 \times i + 1\frac{di}{dt}.$$

$$\frac{10}{s} = I(s) + 1\left(sI(s) - i(0^+)\right)$$

$$\frac{10}{s} = I(s) + sI(s) - 3.33.$$

$$\frac{10}{s} + 3.33 = I(s)(1+s).$$

$$I(s) = \frac{10}{s(1+s)} + \frac{3.33}{s+1}.$$

$$\frac{10}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$10 = A(s+1) + Bs.$$

at $s = -1$ At $s = 0$.

$-B = 10$ $A = 10$.

$B = -10$.

$$I(s) = \frac{10}{s} - \frac{10}{s+1} + \frac{3.33}{s+1}$$

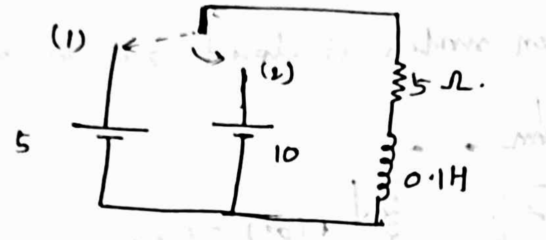
Inverse

$$i(t) = 10 - 10e^{-t} + 3.33e^{-t}$$

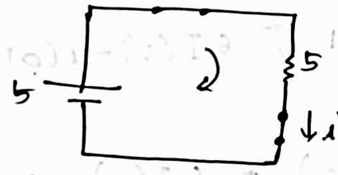
$$i(t) = 10 + (3.33 - 10)e^{-t}.$$

$$i(t) = 10 - 6.667e^{-t}$$

Qn (2) Obtain the expression for the current $i(t)$ when the switch is moved from position (1) to position (2) at $t = 0$. Here assume the circuit is at steady state when the switch is at position (1).



when circuit is at steady state when the switch is in ~~at~~ position - 1



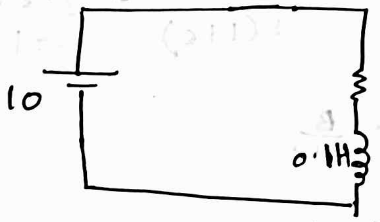
$\downarrow i(0^-) = 1\,A = i(0^+)$

$i(0^-) = 1A$

A transient occurs.

$5 + \frac{1}{10}s$

$\frac{50+1}{10}$



$\downarrow i(0^-) = i(0^+) = 1A$

Ans:- $i(t) = [2u(t) - e^{-50t}u(t)]\,A$

$$10 = 5i + 0.1\frac{di}{dt}$$

$$\frac{10}{s} = 5I(s) + 0.1(sI(s) - i(0^+))$$

$$\frac{10}{s} = 5I(s) + 0.1sI(s) - 0.1$$

$$\frac{10}{s} + \frac{1}{10} = I(s)\,\cancel{[5 \cdot 50 + s]}\,[5 + 0.1s]$$

$$I(s) = \frac{100}{s(50+s)} + \frac{1}{(50+s)}$$

$$\frac{100}{s(50+s)} = \frac{A}{s} + \frac{B}{50+s}$$

$$100 = A(50+s) + Bs$$

$s = 0$ $\qquad\qquad$ $s = -50$

$50A = 100$ $\qquad\qquad$ $-50B = 100$

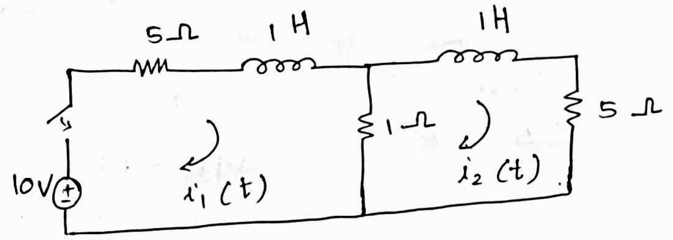
$A = 2$ $\qquad\qquad$ $B = -2$

$$I(s) = \frac{2}{s} - \frac{2}{50+s} + \frac{1}{50+s}$$

$$I(s) = \frac{2}{s} - \frac{1}{50+s}$$

$$i = 2u(t) - e^{-50t}u(t)$$

Qn) Using LT, find $i_2(t)$ at $t = 0^+$ following switching at $t = 0$. Assume the network de-energized ($i_1(0^-) = 0, i_2(0^-) = 0$)



$L \rightarrow sL$

$R \rightarrow R$

$V \rightarrow \frac{V}{s}$

$$\frac{10}{s} - 5I_1 - sI_1 - (I_1 - I_2)\times 1 = 0$$

$$5I_1 + sI_1 + I_1 - I_2 = \frac{10}{s}$$

$$(6+s)I_1 - I_2 = \frac{10}{s} \quad — ①$$

$$-(I_2 - I_1)\times 1 - sI_2 - 5I_2 = 0$$

$$-I_1 + (6+s)I_2 = 0 \quad — ②$$

Using cramer's rule

$$\begin{bmatrix} 6+s & -1 \\ -1 & 6+s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10/s \\ 0 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 6+s & 10/s \\ -1 & 0 \end{vmatrix}}{\begin{vmatrix} 6+s & -1 \\ -1 & 6+s \end{vmatrix}}$$

$$I_2 = \frac{0 - (-1 \times \frac{10}{s})}{(6+s)(6+s) - (-1 \times -1)} = \frac{\frac{10}{s}}{(s+6)^2 - 1}$$

$$I_2 = \frac{10}{s(s+7)(s+5)} = \frac{A}{s} + \frac{B}{s+7} + \frac{C}{s+5}$$

$$10 = A(s+7)(s+5) + Bs(s+5) + Cs(s+7)$$

$s = 0$, $35A = 10$, $A = \frac{2}{7}$

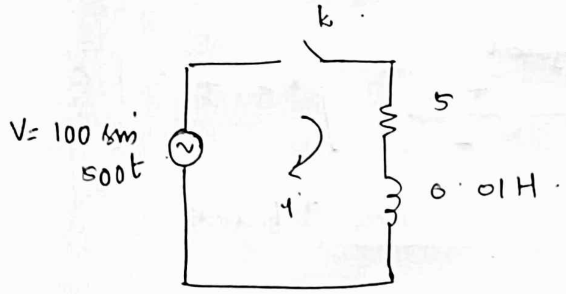
$s = -7$, $14B = 10$, $B = \frac{5}{7}$

$s = -5$, $-10C = 10$, $C = -1$

$$I_2 = \frac{2}{7s} + \frac{5}{7(s+7)} - \frac{1}{s+5}$$

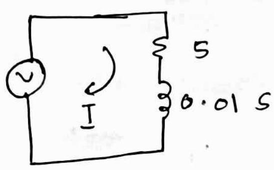
$$i_2(t) = \underbrace{\frac{2}{7}u(t)}_{\text{steady state}} + \underbrace{\frac{5}{7}e^{-7t}u(t) - e^{-5t}u(t)}_{\text{transient state}}$$

Determine the current i, if the switch k is closed at $t = 0$ $\quad i_2(0^-) = 0$



$$LT(\sin \omega t) = \frac{\omega}{s^2 + \omega^2}$$

$$\frac{100 \times 500}{s^2 + 500}$$



$$\frac{5 \times 10^4}{s^2 + 500^2} = 5I + 0.01sI$$

$$I = \frac{5 \times 10^4}{(0.01s + 5)(500^2 + s^2)}$$

$$a^2 + b^2 = (a + jb)(a - jb)$$

$$s^2 + 500^2 = (s + j500)(s - j500)$$

$$I = \frac{5 \times 10^6}{(s+500)(s^2 + 500^2)}$$

$$I = \frac{5 \times 10^6}{(s+500)(s - j500)(s + j500)}$$

$$= \frac{A}{s + j500} + \frac{B}{s - j500} + \frac{C}{s + 500}$$

On solving

$A = 5(-1+j)$, $B = 5(-1-j)$; $C = 10$

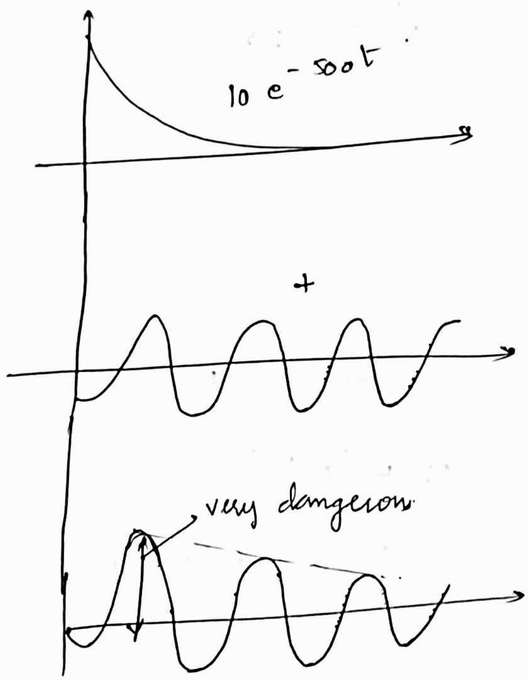
$$I = \frac{5(-1+j)}{s + j500} + \frac{5(-1-j)}{s - j500} + \frac{10}{s + 500}$$

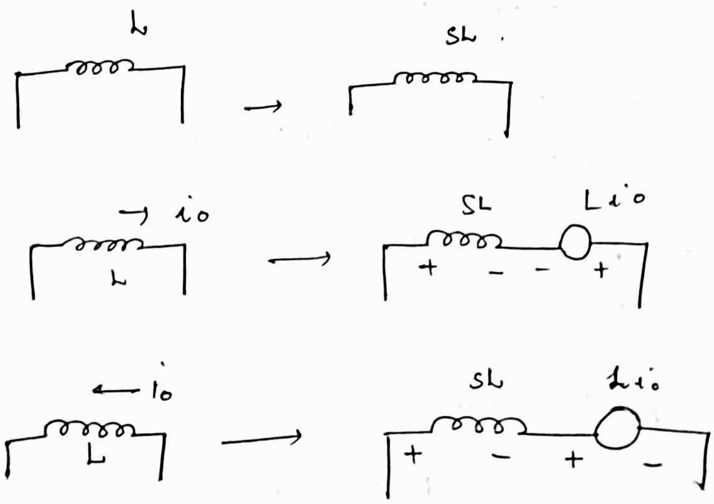
$$I = \frac{-5}{s+j500} - \frac{j5}{s+j500} - \frac{5}{s-j500} - \frac{5j}{s-j500} + \frac{10}{s+500}$$

$$= -5\left[\frac{e^{-j500t} + e^{j500t}}{2}\right] \times 2 + 5\left[\frac{e^{-j500t} + e^{j500t}}{2j}\right] \times 2 + 10e^{-500t}$$

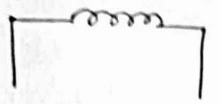
$$= -10\cos 500t + 10\sin 500t + 10e^{-500t}$$

$$= \underbrace{10e^{-500t}}_{\text{transient}} + \underbrace{14.14\sin(500t - 45^\circ)}_{\text{steady}}$$

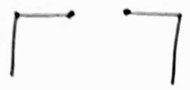




snap shot

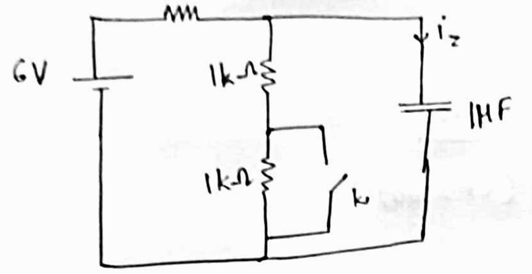


at $t = 0^+$



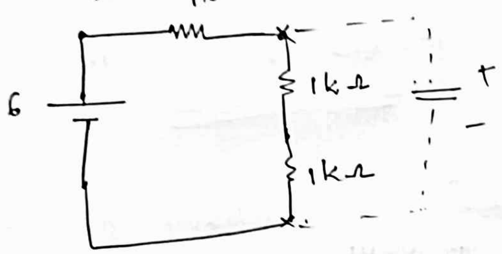
at $t = \infty$

Q n)



Initially k is open & the circuit is in steady state. If the switch k is closed at $t=0$, find the current through capacitor.

At steady state



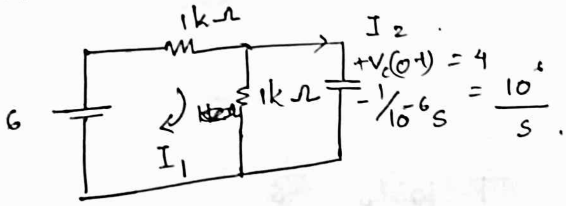
$V_c(0^-) = 4$

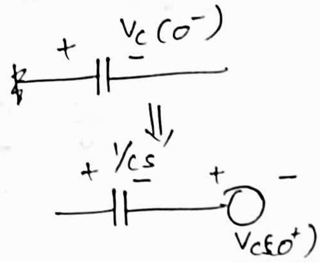
$$V_c(0^-) = \frac{6 \times 2}{3} = 4\,V.$$

At transient

~~The circuit~~

$V_c(0^-) = V_c(0^+) = 4$





$$\frac{6}{s} - I_1 \times 1000 - 1000(I_1 - I_2) = 0$$

$$-2000 I_1 + 1000 I_2 + \frac{6}{s} = 0$$

$$\frac{6}{s} = 2000 I_1 - 1000 I_2 \quad \text{—(1)}$$

$$-(I_2 - I_1)1000 - \frac{10^6}{s} \times I_2 - \frac{4}{s} = 0$$

$$\frac{4}{s} = +1000 I_1 - I_2\left(1000 - \frac{10^6}{s}\right) \quad \text{—(2)}$$

$$\begin{bmatrix}2000 & -1000\\ 1000 & -\left(1000+\frac{10^6}{s}\right)\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix}=\begin{bmatrix}6/s\\ 4/s\end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix}2000 & 6/s\\ 1000 & 4/s\end{vmatrix}}{\begin{vmatrix}2000 & -1000\\ 1000 & -1000+\frac{10^6}{s}\end{vmatrix}}$$

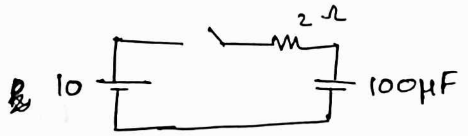
$$I_2 = \frac{-2\times10^3}{10^6(s+2000)}$$

$$I_2 = \frac{-2\times10^{-3}}{s+2000}$$

$$i_2 = -2\times10^{-3}e^{-2000t}\ \text{A}$$

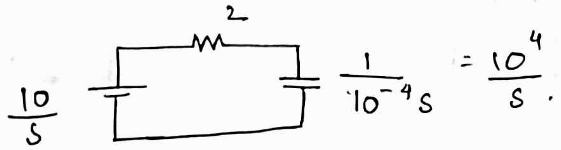
Q8)

Q9)



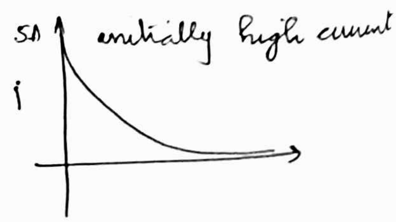
Obtain the current expression through the capacitor c at $t=0^+$, following switching at $t=0$. Assume the capacitor is initially discharged.

At $t=0$



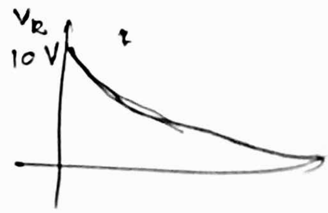
$$\frac{1}{10^{-4}s} = \frac{10^4}{s}$$

$$\frac{10}{s} = 2I + \frac{10^4 I}{s}$$



$$\frac{10}{s} = I\left(2+\frac{10^4}{s}\right)$$

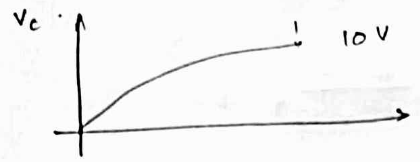
$$I = \frac{10}{s}\times\frac{s}{2s+10^4}$$



$$I = \frac{10}{2s+10^4}$$

$$I = \frac{\cancel{10}\,5}{s+5000}$$

$\cancel{I(s)} = i(t) = \underline{5e^{-5000t} A}$



Time constant of this circuit:

$V = 10 \quad R = 2$

$$i_c = \frac{V}{R} e^{-t/Rc}$$

when current reduces to 1/e of the initial value (time constant)

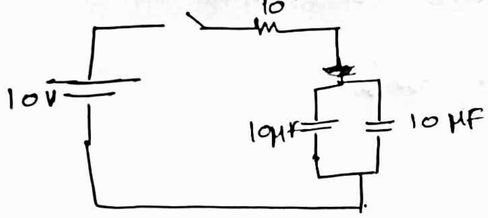
Here time constant

$t = RC$

$= 2 \times 100 \times 10^{-6}$

$= \underline{200 \times 10^{-6} s}$

After 4τ current will become 0. After $4 \times 200 \times 10^{-6} s = 800 \mu s$, i will be zero



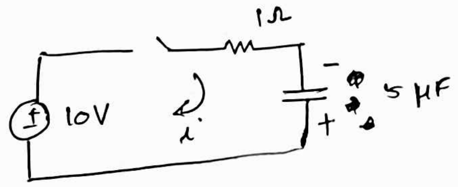
$\tau = (10 + 10) \times 10$

$= \underline{200 \mu s}$.

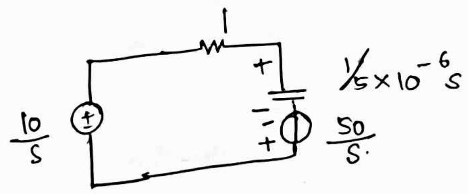
after $800 \mu s$, circuit will be in steady state

$\boxed{\text{For R-L circuit } \tau = L/R}$

Qn) Find $i(t)$ following switching at $t = 0$. Assume initial charge on capacitor $250 \mu C$ as shown in fig



$$V_0 = \frac{250 \times 10^{-6}}{5 \times 10^{-6}} = 50 V$$



$$\frac{10}{s} + \frac{50}{s} = I + \frac{I}{5 \times 10^{-6} s}$$

$$\frac{60}{s} = I\left[1 + \frac{10^6}{5s}\right]$$

$$\frac{60}{s} = I\left[\frac{5s + 10^6}{5s}\right]$$

$$I(s) = \frac{60}{s} \times \frac{5s}{5s + 10^6}$$

$$I(s) = \frac{60 \times 5}{5s + 10^6}$$

$$I(s) = \frac{60}{s + 2 \times 10^5}$$

$$i(t) = \underline{\underline{60\, e^{-200000t}}}$$

Time constant $\tau = RC$

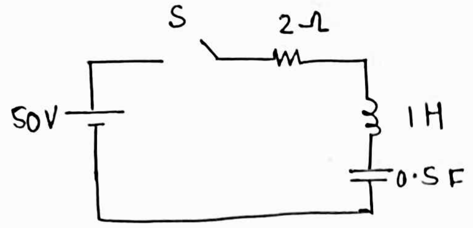
$$= 1 \times 5 \times 10^{-6}$$

$$= \underline{\underline{5\mu s}}$$

After $20\mu s$, the circuit will be in steady state

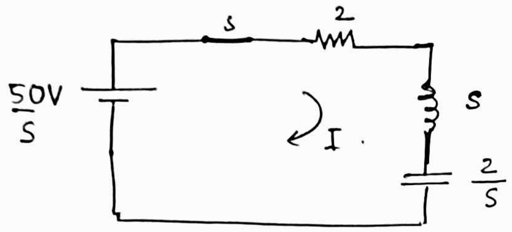
R-L-C circuits

(x)



In the series R-L-C circuit $i_L(0^-) = 0$ $v_c(0^-) = 0$, switch is closed at $t = 0$. Determine the resulting current i.

s-domain



$$\frac{50}{s} - 2I - sI - \frac{2}{s}I = 0$$

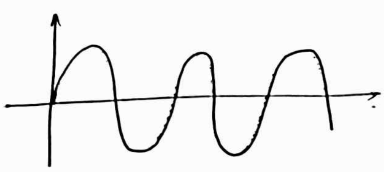
$$I = \frac{\frac{50}{s}}{2 + s + \frac{2}{s}}$$

$$I = \frac{50}{s^2 + 2s + 2}$$

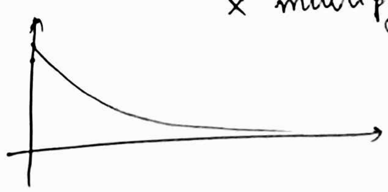
$$= \frac{50}{(s+1)^2 + 1^2}$$

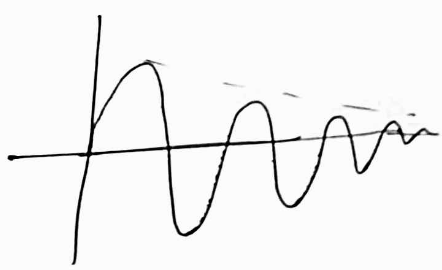
$$\text{ILT}(I) = 50 \ \text{ILT}\left(\frac{1}{(s+1)^2 + 1^2}\right)$$

$$= 50 \, e^{-t} \sin t$$



× multiplying





Exponentially decaying sine wave.