Linear System, Coefficient matrix, Augmented Matrix

A linear system of m equations in n unknowns $x_1, x_2, \ldots, x_n$ is a set of equations of the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \quad \longrightarrow (1) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

The system is called linear because each variable x_j appears in the first power only.

$a_{11}, a_{12}, \ldots, a_{mn}$ are given numbers, called the coefficients of the system.

$b_1, b_2, \ldots, b_m$ on the right are also given numbers.

If all the b_j are zero, then (1) is called a homogeneous system.

If atleast one b_j is not zero, then (1) is called a non-homogeneous system.

A solution of (1) is a set of numbers $x_1, x_2, \ldots, x_n$ that satisfies all the m equations.

A solution vector of (1) is a vector

x, (say $x = (x_1, x_2, \ldots, x_n)$) whose components form a solution of (1).

If the system (1) is homogeneous, it always has atleast the trivial solution $x_1 = 0, x_2 = 0, \ldots, x_n = 0$.

Matrix form of the Linear System (1)

From the definition of matrix multiplication we see that the m equations of (1) may be written as a single vector equation

$$Ax = b \longrightarrow (2)$$

where the coefficient matrix $A = [a_{jk}]$ is the $m \times n$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \text{ and}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ and } b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$ are column vectors.

We assume that the coefficients a_{jk} are not all zero, so that A is not a zero matrix. Note that x has n components, whereas b has m components.

The matrix

$$\tilde{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{bmatrix}$$

is called the augmented matrix of the system (1).

Note:-

The last column of $\tilde{A}$ did not come from matrix A but came from vector b. Thus we augmented the matrix A.

Note:-

The augmented matrix $\tilde{A}$ determines the system (1) completely because it contains all the given numbers appearing in (1).

Elementary Row operations for Matrices.

Three types of elementary row operations are associated with a matrix. They are

(i) Interchange of two rows

$(R_i \longleftrightarrow R_j)$ (Here R_i means i^{th} row of a matrix)

(ii) Multiplication of a row by a nonzero constant c.

$(R_i \longrightarrow cR_i)$

(iii) Addition of a constant multiple of one row to another row.

$(R_i \longrightarrow R_i + cR_j)$

for Example:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 3 & 0 & 2 \\ -1 & 0 & 4 & 3 & 1 \\ 2 & 1 & 0 & -4 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 & 2 \\ 2 & 1 & 0 & -4 & 6 \\ -1 & 0 & 4 & 3 & 1 \end{bmatrix} \quad R_2 \longleftrightarrow R_3$$

(We say that A is equivalent to the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 & 2 \\ 2 & 1 & 0 & -4 & 6 \\ -1 & 0 & 4 & 3 & 1 \end{bmatrix}$ by the elementary row operation $R_2 \longleftrightarrow R_3$)

(5)

Let $A = \begin{bmatrix} 1 & 2 & 1 & -1 & 5 & 2 & 4 \\ 3 & -2 & -4 & 3 & 2 & -1 & 0 \\ 0 & 0 & 2 & 1 & 1 & -2 & 5 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 2 & 1 & -1 & 5 & 2 & 4 \\ 12 & -8 & -16 & 12 & 8 & -4 & 0 \\ 0 & 0 & 2 & 1 & 1 & -2 & 5 \end{bmatrix} R_2 \to 4R_2$$

$$\sim \begin{bmatrix} 12 & -8 & -16 & 12 & 8 & -4 & 0 \\ 1 & 2 & 1 & -1 & 5 & 2 & 4 \\ 0 & 0 & 2 & 1 & 1 & -2 & 5 \end{bmatrix} R_1 \Leftrightarrow R_2$$

$= C$ [A is equivalent to C by a sequence of elementary row operations]

Let $B = \begin{bmatrix} 0 & -1 & 2 & 3 \\ -2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}$

$$\sim \begin{bmatrix} 0 & -1 & 2 & 3 \\ -2 & 1 & 0 & 0 \\ 0 & \frac{5}{2} & 3 & 4 \end{bmatrix} R_3 \to R_3 + \frac{1}{2}R_2$$

$$\sim \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -1 & 2 & 3 \\ 0 & \frac{5}{2} & 3 & 4 \end{bmatrix} R_1 \Leftrightarrow R_2$$

$$\sim \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 8 & \frac{23}{2} \end{bmatrix} R_3 \to R_3 + \frac{5}{2}R_2$$

$= D$ (where $D = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 8 & 23/2 \end{bmatrix}$)

so that B is equivalent to D by a sequence of elementary row operations.

CAUTION:- The operations are for rows only, not for columns!

Row Echelon form of a matrix.

A matrix is in Echelon form (or row echelon form) if ~~it~~ it has the following three properties:

(1) Each leading entry of a row is in a column to the right of the leading entry of the row above it.

(2) All entries in a column below a leading entry are zero.

(3) All non-zero rows are above any rows of all zeros.

Example:

(1) $A = \begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5/2 \end{bmatrix}$ (2) $A = \begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(3) $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ (4) $A = \begin{bmatrix} -1 & 2 & -1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Note:-

~~Property (1) says that the leading~~

(1) The leading entry of a row means the first non-zero entry in a row.

for Ex: let $A = \begin{bmatrix} 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & \boxed{-1} & 2 \\ 0 & 0 & 0 & \boxed{4} \end{bmatrix}$

→ leading entry in 1st row

→ leading entry in 3rd row.

→ leading entry in 2nd row

(2) Property (1) says that the leading ~~entry~~ entries form an echelon ("steplike") pattern that moves down and to the right through the matrix.

for Ex:

$$A = \begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5/2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Examples:

The following matrices are in echelon form. The leading entries ($\square$) may have any non-zero value; the starred entries may have any values (including zero)

$$A = \begin{bmatrix} \square & * & * & * \\ 0 & \square & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \square & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \square & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & \square & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \square & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \square & * \end{bmatrix}$$

Note:- Any matrix can be reduced to Echelon form by a sequence of Elementary row operations.

Note:- The Echelon form of a given matrix need not be unique.

Problem :- 1

Reduce the matrix $A = \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 6 & -6 & 6 & 12 & 36 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix}$

into the Echelon form.

Solution :-

$$A = \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 6 & -6 & 6 & 12 & 36 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 1 & -1 & -5 & -13 \\ 0 & 1 & -3 & 0 & 4 \end{bmatrix} \begin{matrix} R_2 \to R_2 - \left(\frac{6}{2}\right) R_1 \; (R_2 - 3R_1) \\ R_3 \to R_3 - \left(\frac{4}{2}\right) R_1 \; (R_3 - 2R_1) \\ R_4 \to R_4 - \left(\frac{2}{2}\right) R_1 \; (R_4 - R_1) \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 0 & -1 & -4 & -11 \\ 0 & 0 & -3 & 1 & 6 \end{bmatrix} \begin{matrix} R_3 \to R_3 + \left(\frac{1}{9}\right) R_2 \; \left(R_3 - \frac{1}{-9} R_2\right) \\ R_4 \to R_4 + \left(\frac{1}{9}\right) R_2 \; \left(R_4 - \frac{1}{-9} R_2\right) \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 0 & -1 & -4 & -11 \\ 0 & 0 & 0 & 13 & 39 \end{bmatrix} \quad R_4 \to R_4 + 3R_3 \; \left(R_4 - \left(\frac{-3}{-1}\right) R_3\right)$$

which is in Echelon form.

Problem: 2

Reduce the matrix $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$

into the Echelon form.

Solution:-

$$A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & -21 & -14 & -29 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + 2R_1 & \left(R_2 - \left(\frac{-6}{3}\right)R_1\right) \\ R_3 \to R_3 - 7R_1 & \left(R_3 - \left(\frac{21}{3}\right)R_1\right) \end{matrix}$$

$$\sim \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} \left(R_3 - \left(\frac{-21}{42}\right)R_2\right) \\ R_3 \to R_3 + \frac{1}{2}R_2 \end{matrix}$$

which is in Echelon form.

Problem: 3

Reduce the matrix $A = \begin{bmatrix} 0 & 1 & 3 & 9 \\ 2 & 2 & -1 & 8 \\ -1 & 0 & 5 & 8 \end{bmatrix}$

into Echelon form.

Solution:-

Given $A = \begin{bmatrix} 0 & 1 & 3 & 9 \\ 2 & 2 & -1 & 8 \\ -1 & 0 & 5 & 8 \end{bmatrix}$

$$\sim \begin{bmatrix} -1 & 0 & 5 & 8 \\ 2 & 2 & -1 & 8 \\ 0 & 1 & 3 & 9 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} -1 & 0 & 5 & 8 \\ 0 & 2 & 9 & 24 \\ 0 & 1 & 3 & 9 \end{bmatrix} \quad R_2 \to R_2 + 2R_1 \quad \left(R_2 - \left(\frac{2}{-1}\right)R_1\right)$$

$$\sim \begin{bmatrix} -1 & 0 & 5 & 8 \\ 0 & 2 & 9 & 24 \\ 0 & 0 & -\frac{3}{2} & -3 \end{bmatrix} \quad R_3 \to R_3 - \frac{1}{2}R_2$$

which is in Echelon form.

Rank of a matrix.

The rank of a matrix A is the maximum number of linearly independent rows of A. It is denoted by rank A or $r(A)$ or $\rho(A)$.

for Ex:

Consider the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \end{bmatrix}$

Since the 2^{nd} row A is twice the 1^{st} row, we then have A has only one linearly independent row & $\therefore \rho(A) = 1$.

Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & -1 & 2 & 1 & 0 \\ 3 & 6 & 9 & 12 & 0 \\ 1 & 0 & 7 & 6 & 0 \end{bmatrix}$

$\rho(A) = 2$, since it is easy to see that 3^{rd} row of A is 3-times of first row & also 4^{th} row of A is the linear combination of 1^{st} & 2^{nd} row (i.e., $R_3 = R_1 + 2R_2$).

Thus, no: of linearly independent rows of $A = 2$ & hence $\rho(A) = 2$.

~~Rank of~~

Determination of rank of a matrix

Let A be a given matrix. Reduce the matrix A to the Echelon form. Identify the no. of non-zero rows in the Echelon form. The no. of non-zero rows in the Echelon form of A gives the rank of A or $\rho(A)$.

Problems.

Find the rank of the following matrices.

1) $A = \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 6 & -6 & 6 & 12 & 36 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix}$

2) $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$

3) $A = \begin{bmatrix} 0 & 1 & 3 & 9 \\ 2 & 2 & -1 & 8 \\ -1 & 0 & 5 & 8 \end{bmatrix}$

4) $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ -2 & -5 & 5 & 6 \\ 0 & -3 & 15 & 18 \\ 0 & -2 & 10 & 8 \\ 3 & 6 & 0 & 6 \end{bmatrix}$

5) $A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$

Solution:-

1) $A = \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 6 & -6 & 6 & 12 & 36 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix}$

$\sim \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 1 & -1 & -5 & -13 \\ 0 & 1 & -3 & 0 & 4 \end{bmatrix}$ $\begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - R_1 \end{matrix}$

$\sim \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 0 & -1 & -4 & -11 \\ 0 & 0 & -3 & 1 & 6 \end{bmatrix}$ $\begin{matrix} R_3 \to R_3 + \frac{1}{9} R_2 \\ R_4 \to R_4 + \frac{1}{9} R_2 \end{matrix}$

$\sim \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 0 & -9 & 0 & 9 & 18 \\ 0 & 0 & -1 & -4 & -11 \\ 0 & 0 & 0 & 13 & 39 \end{bmatrix}$ $R_4 \to R_4 - 3R_3$

which is in Echelon form.

Now $\rho(A)$ = no: of non-zero rows in the Echelon form of A

$= \underline{\underline{4}}$

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2) $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$

$\sim \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & -21 & -14 & -29 \end{bmatrix}$ $\begin{matrix} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 - 7R_1 \end{matrix}$

$\sim \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 + \frac{1}{2} R_2$

which is in Echelon form.

$\therefore \rho(A)$ = No: of non-zero rows in the Echelon form of A

$= \underline{\underline{2}}$

3) $A = \begin{bmatrix} 0 & 1 & 3 & 9 \\ 2 & 2 & -1 & 8 \\ -1 & 0 & 5 & 8 \end{bmatrix}$

$\sim \begin{bmatrix} -1 & 0 & 5 & 8 \\ 2 & 2 & -1 & 8 \\ 0 & 1 & 3 & 9 \end{bmatrix}$ $R_1 \leftrightarrow R_3$

$\sim \begin{bmatrix} -1 & 0 & 5 & 8 \\ 0 & 2 & 9 & 24 \\ 0 & 1 & 3 & 9 \end{bmatrix}$ $R_2 \to R_2 + 2R_1$

$\sim \begin{bmatrix} -1 & 0 & 5 & 8 \\ 0 & 2 & 9 & 24 \\ 0 & 0 & -3/2 & -3 \end{bmatrix}$ $R_3 \to R_3 - \frac{1}{2} R_2$

which is in Echelon form.

Hence $\rho(A)$ = no. of non-zero rows in the Echelon form of A

$= \underline{\underline{3}}$

4) $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ -2 & -5 & 5 & 6 \\ 0 & -3 & 15 & 18 \\ 0 & -2 & 10 & 8 \\ 3 & 6 & 0 & 6 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & 5 & 10 \\ 0 & -3 & 15 & 18 \\ 0 & -2 & 10 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 \to R_2 + 2R_1$
$R_5 \to R_5 - 3R_1$

$\sim \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & 5 & 10 \\ 0 & 0 & 0 & -12 \\ 0 & 0 & 0 & -12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - 3R_2$
$R_4 \to R_4 - 2R_2$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & 5 & 10 \\ 0 & 0 & 0 & -12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_4 \to R_4 - R_3$$

which is in Echelon form.

Hence $\rho(A)$ = no. of non-zero rows in the Echelon form of A

$= \underline{\underline{3}}$

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5) $$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & -5 & 1 & 5 \\ -2 & -5 & 8 & 0 & -17 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 2 & -4 & 4 & -14 \\ 0 & 4 & -8 & 4 & -8 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 20 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 - 4R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 0 & 0 & -4 & 20 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \leftrightarrow R_4$$

which is in Echelon form.

$\therefore \rho(A)$ = no. of non-zero rows in the Echelon form of $A = 3$

Homework.

Reduce the following matrices into Echelon form.

1) $A = \begin{bmatrix} 2 & 3 & 1 & 9 \\ 1 & 2 & 3 & 6 \\ 3 & 1 & 2 & 8 \end{bmatrix}$

2) $A = \begin{bmatrix} 2 & 3 & -1 & 5 \\ 4 & 4 & -3 & 3 \\ -2 & 3 & -1 & 1 \end{bmatrix}$

3) $A = \begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}$

4) $A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$

5) $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

Practice Problems.

Reduce the following matrices into Echelon form.

(1) $A = \begin{bmatrix} 5 & 1 & 1 & 1 & 4 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 14 & -5 \\ 1 & 1 & 1 & 4 & -6 \end{bmatrix}$

(2) $A = \begin{bmatrix} 10 & 2 & 1 & 9 \\ 2 & 20 & -2 & -44 \\ -2 & 3 & 10 & 22 \end{bmatrix}$

(3) $A = \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 6 & -6 & 6 & 12 & 36 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix}$

(4) $A = \begin{bmatrix} 4 & -4 & -9 & -9 \\ 6 & -8 & -7 & 5 \\ 5 & -7 & -4 & 9 \\ -9 & 11 & 16 & 7 \end{bmatrix}$

(5) $A = \begin{bmatrix} 2 & -3 & 10 & 9 \\ 7 & -5 & 6 & -7 \\ -5 & 4 & -2 & 2 \\ 8 & -9 & 7 & 15 \end{bmatrix}$

Practice Problems:-

1) Reduce the following matrix to echelon form and hence find its rank:

$$A = \begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

2) Reduce the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$ into echelon form and find the rank.

3) Reduce the matrix $A = \begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix}$ into echelon form and find the rank.

4) Find the value of 'k' such that the rank of the matrix A is 3, where

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & k \end{bmatrix}$$

5) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 0 & 5 \end{bmatrix}$