24/09/2020

Module II

Laplace Transform.

Time domain analysis is the conventional method of analysing network for higher order differential equation of network variable, frequency domain analysis using laplace transform is very convenient.

① Solution of differential equation in a system procedure

② Initial conditions are automatically incoporated

③ It gives the complete solution ie, both complementary and particular solution in one step.

Laplace transformation :-

$$\boxed{F(s) = L\{f(t)\} = \int_0^{\infty} f(t)\, e^{-st}\, dt.}$$

where s is the complex frequency variable

$$s = \sigma \pm j\omega.$$

The function f(t) must satisfy following conditions.

$$\int_0^{\infty} |f(t)| e^{-\sigma t}\, dt < \infty.$$ where σ is real & +ve.

Inverse laplace transform. $L^{-1}\{F(s)\}$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s)\, e^{st}\, ds.$$

$F(t)$	$F(s)$. Laplace form.
k	k/s .
1	$1/s$
t^n	$\frac{n!}{s^{n+1}}$
t^2	$\frac{2!}{s^{2+1}} = \frac{2!}{s^3}$
$\delta(t)$	1
t	$1/s^2$
e^{at}	$1/s-a$
e^{-at}	$1/s+a$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$

$e^{-at}\cos\omega t$ $\qquad \dfrac{s+a}{(s+a)^2+\omega^2}$

$e^{at}\sin\omega t$ $\qquad \dfrac{\omega}{(s-a)^2+\omega^2}$.

$e^{at}\cos\omega t$ $\qquad \dfrac{s-a}{(s-a)^2+\omega^2}$.

$t\,e^{at}$ $\qquad \dfrac{1}{(s-a)^2}$

$t\cdot e^{-at}$ $\qquad \dfrac{1}{(s+a)^2}$

Q1 Find the laplace transform of $4t^2+\sin 3t+e^{2t}$.

$$F(s) = L\left[4\times\frac{2!}{s^3}+\frac{3}{s^2+3^2}+\frac{1}{s-2}\right]$$

$$= \frac{8}{s^3}+\frac{3}{s^2+9}+\frac{1}{s-2}$$

Initial value theorem.

If $L[f(t)] = F(s)$ then $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$

Final value theorem

If $L[f(t)] = F(s)$ then $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Q. Find the Inverse Laplace transform of $\frac{s^2-3s+4}{s^3}$.

Standard Results

$$F(s) = \frac{s^2}{s^3} - \frac{3s}{s^3} + \frac{4}{s^3}$$

$$= \frac{1}{s} - \frac{3}{s^2} + \frac{4}{s^3}$$

$$F(t) = 1 - 3t + 2t^2.$$

$t^2 = \frac{2}{s^3}$

$\frac{4}{s^3} = 2 \times \frac{2}{s^3}$

$= 2t^2$

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Q. $$\frac{s+2}{s(s+3)(s+1)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+1}$$

$$\frac{s+2}{\cancel{s(s+3)(s+1)}} = \frac{A(s+3)(s+1) + B(s+1)(s) + C(s(s+3))}{\cancel{s(s+3)(s+1)}}$$

$$s+2 = A(s^2+4s+3) + B(s^2+s) + C(s^2+3s)$$

$$s+2 = As^2 + 4As + 3A + Bs^2 + Bs + Cs^2 + 3Cs.$$

equating coefficients

$s^2 \rightarrow \quad 0 = A+B+C.$

$s \rightarrow \quad 4A+B+3C = 1$

$3A = 2.$

$A = 2/3.$

$\frac{2}{3} + B + C = 0$

$B + C = -\frac{2}{3}$

Sub: A.

$4 \times \frac{2}{3} + B + 3C = 1$

$\frac{8}{3} + B + 3C = 1$

$B + 3C = 1 - \frac{8}{3}$

$B + 3C = -5/3$

$B + C = -2/3$

$(-)\ B + 3C = -5/3$

$-2C = 1$

$C = -1/2$

$B + C = -2/3$
$B = -2/3 + 1/2$
$B = -1/6.$

$A = 2/3$
$B = -1/6$
$C = -1/2.$

$$\frac{s+2}{s(s+3)(s+1)} = \frac{2/3}{s} + \frac{-1/6}{s+3} + \frac{-1/2}{s+1}$$

$$f(t) = \frac{2}{3} + \left(\frac{-1}{6}\right)e^{-3t} + \left(\frac{-1}{2}\right)e^{-t}.$$

Q. Find inverse transform of $\frac{s+2}{s^2(s+3)}$?

$$F(s) = \frac{s+2}{s^2(s+3)}$$

$$\frac{s+2}{s^2(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3}$$

LCM $= s^2(s+3)$

$$s+2 = A(s)(s+3) + B(s+3) + C(s^2)$$

$$= As^2 + 3As + Bs + 3B + Cs^2.$$

coefficients $A + C = 0$

$3A + B = 1$

$3B = 2$

$B = 2/3$

$3A + \frac{2}{3} = 1$

$3A = 1 - 2/3$

$3A = 1/3$

$A = 1/9$

$A + C = 0$
$1/9 + C = 0$
$C = -1/9$

$A = 1/9$
$B = 2/3$
$C = -1/9$

$$\frac{s+2}{s^2(s+3)} = \frac{1/9}{s} + \frac{2/3}{s^2} + \frac{-1/9}{s+3}$$

$$= \frac{1}{9} + \frac{2}{3}t + \left(-\frac{1}{9}\right)e^{-3t}$$

Q3. $\dfrac{5s^2 - 15s - 11}{(s+1)(s-2)^2}$

$$\frac{5s^2 - 15s - 11}{(s+1)(s-2)^2} = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{(s-2)^2}$$

$$5s^2 - 15s - 11 = A(s-2)^2 + B(s+1)(s-2) + C(s+1)$$

$$= A(s^2 + 4s \cancel{\mp} 4) + B(s^2 - s - 2) + Cs + C$$

$$5s^2 - 15s - 11 = As^2 \cancel{-} 4As \cancel{\mp} 4A + Bs^2 - Bs - 2B + Cs + C.$$

$$A + B = 5$$

$$-4A - B + C = -15$$

$$4A - 2B + C = -11$$

$$\underline{B = 4}$$

$$-4A - B + C = -15$$

$$(-)4A \; (+)2B \; (-)C = (+)11$$

$$B = \cancel{26}\; 4$$

$$A + B = 5$$

$$A + 4 = 5$$

$$A = \cancel{3}\; 1$$

$$\cancel{4 \times 1 - 4 + C = -15}$$

$$\cancel{-4 + C = -15}$$

$$\cancel{-16 + C = -15}$$

$$\cancel{C = 1}$$

$$4 - 2 \times 4 + C = -11$$

$$4 - 8 + C = -11$$

$$\underline{C = -7}$$

$$\cancel{A = 1}$$
$$\cancel{B = 4}$$
$$\cancel{C = -7}$$

$A = 1, B = 4, C = -7.$

$$F(s) = \frac{1}{s+1} + \frac{4}{s-2} - \frac{7}{(s-2)^2}$$

$$= e^{-t} + 4e^{2t} - 7te^{2t}$$

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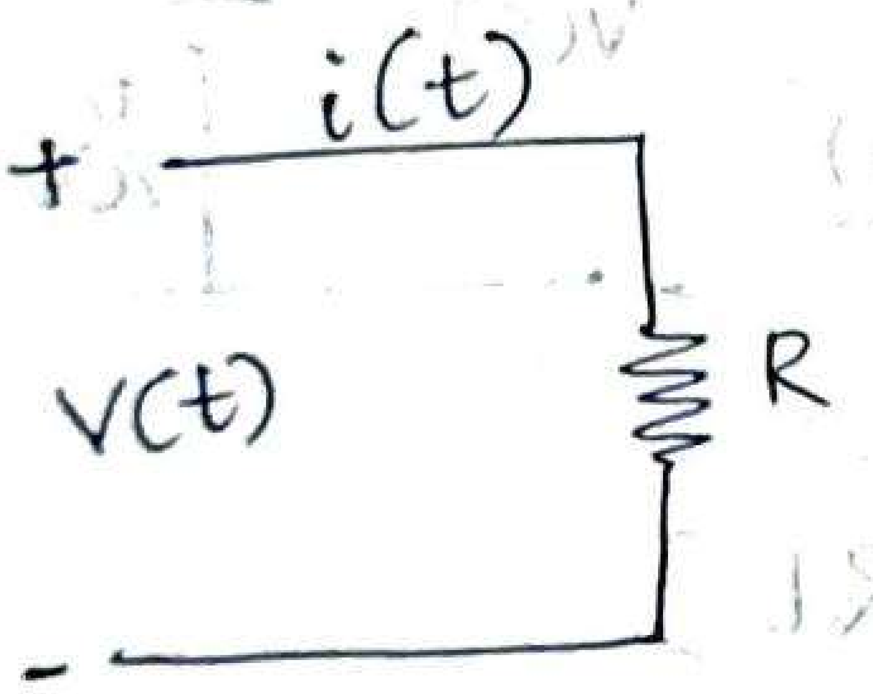
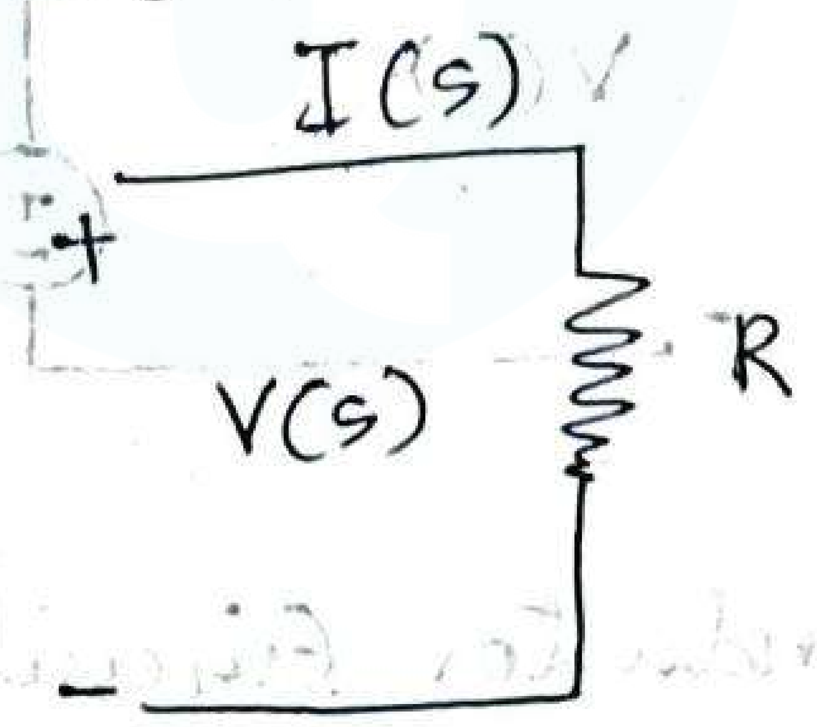
The transformed Circuit

1, Resistor

$$\left.\begin{aligned} V(t) &= R\,i(t) \\ V(s) &= R\,I(s) \end{aligned}\right\} \text{laplace transform}$$





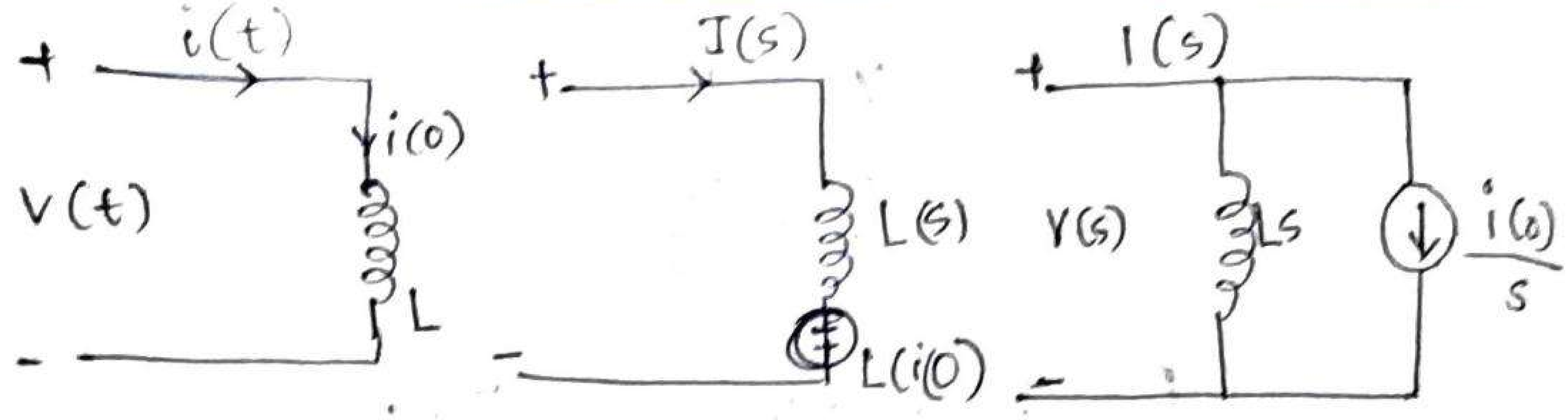
2, Inductor

$$V(t) = L \cdot \frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int_0^t V(t)\,dt + i(0)$$

$$V(s) = L(s) \cdot I(s) - L\,i(0)$$

$$I(s) = \frac{1}{Ls} V(s) + \frac{i(0)}{s}$$



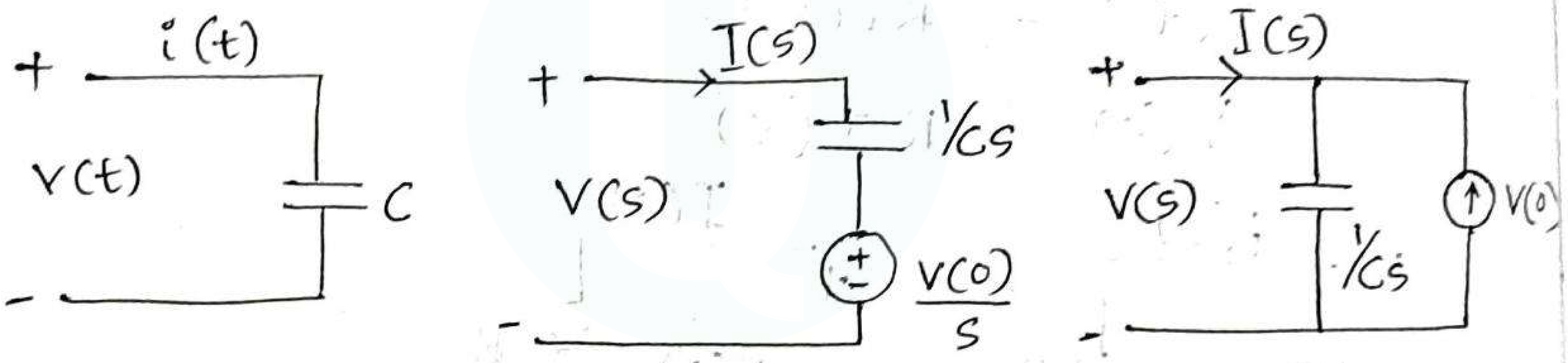
3, Capacitor

$$V(t) = \frac{1}{C}\int_0^t i(t)\,dt + V(0).$$

$$i(t) = C\cdot\frac{dv}{dt}$$

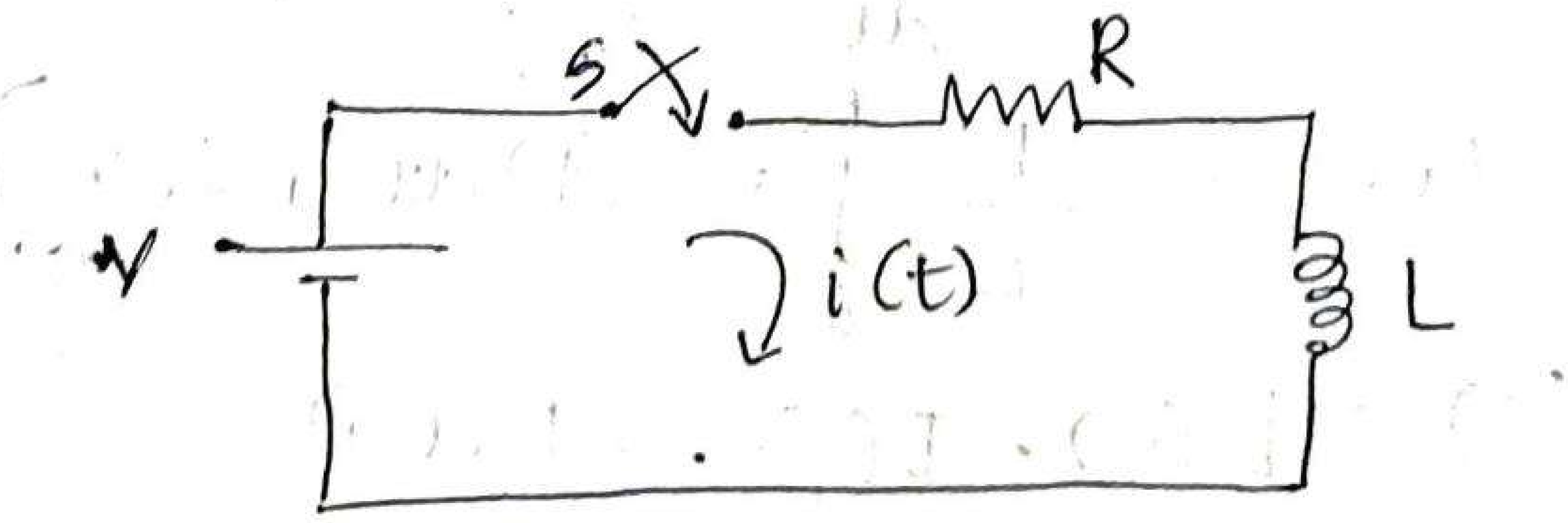
$$V(s) = \frac{1}{Cs} I(s) + \frac{V(0)}{s}$$

$$I(s) = Cs\,V(s) - C\,V(0).$$

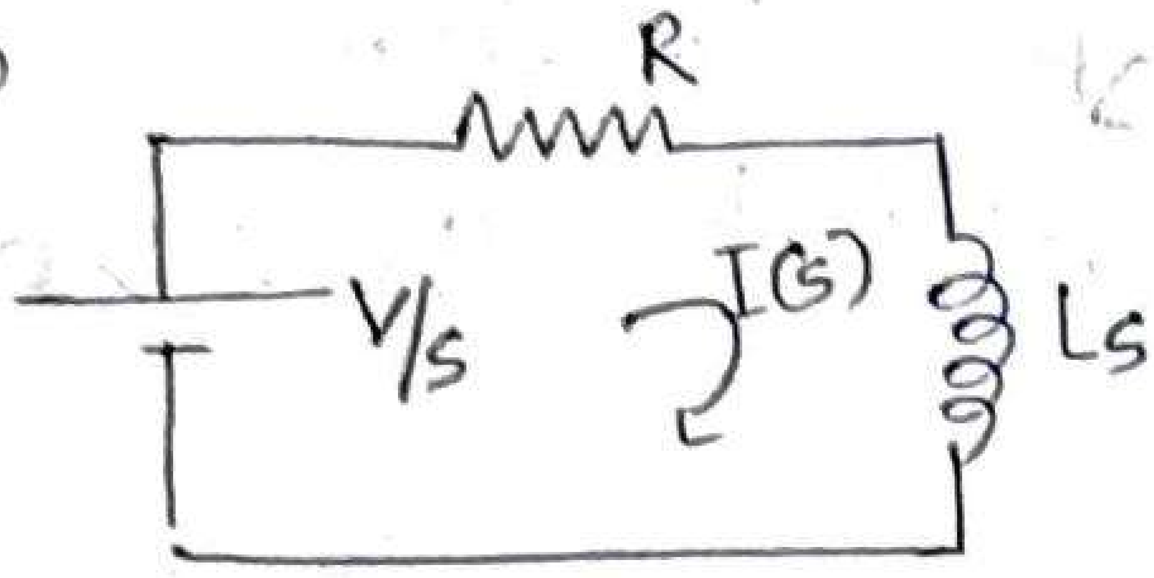


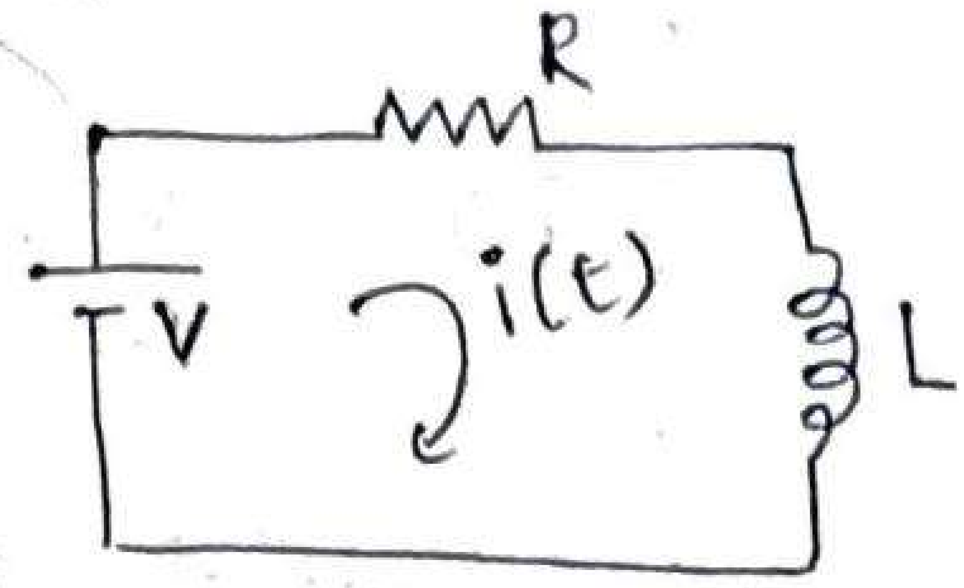
Resistor - Inductor Circuit [RL]

Consider a series RL circuit as shown in fig. The switch is closed at time t=0.



For $t > 0$





$$V = R\,i(t) + L\frac{di(t)}{dt}$$

$$\frac{V}{s} = R\,I(s) + L\cdot s\,I(s).$$

$$= (R + Ls)\,I(s).$$

$$I(s) = \frac{V/s}{R+Ls} = \frac{V}{sL\left[R/L + s\right]}$$

$$I(s) = \frac{V/L}{s\left[s + R/L\right]}$$

$$I(s) = \frac{A}{s} + \frac{B}{s+R/L}$$

$s + \frac{R}{L} = 0$

$s = -R/L$

$$A = s\,I(s)\Big|_{s=0}$$

$$= s \times \frac{V/L}{s(s+R/L)}\Bigg|_{s=0}$$

$$= \frac{V/L}{R/L} = \frac{V}{R}.$$

$$B = (s + R/L) \times \frac{V/L}{s(s+R/L)}\Bigg|_{s=-R/L}$$

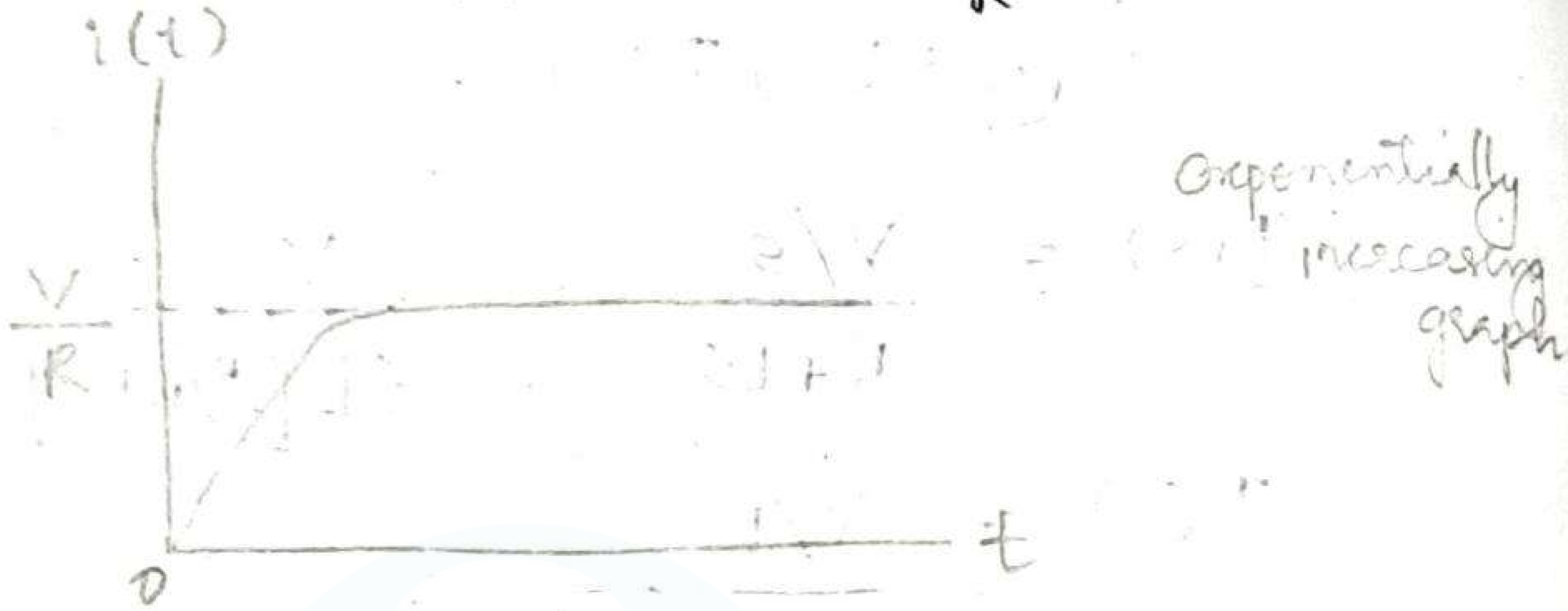
$$= \frac{V/L}{-R/L} = -\frac{V}{R}$$

$$I(s) = \frac{V/R}{s} + \frac{-V/R}{s+R/L}$$

Taking inverse laplace transform

$$i(t) = \frac{V}{R} - \frac{V}{R} e^{-R/L\, t} \quad \text{for } t > 0.$$

The complete response is composed of 2 parts, the steady state response or force response or zero state response. $\frac{V}{R}$ and transient response or natural response or zero i/p response $\frac{V}{R} e^{-R/L\, t}$.



current response of R-L circuit

The transient period is defined as the time taken for current to reach its final or steady state value from its initial value.

The term $\frac{L}{R}$ is called time constant and is denoted by T.

$$T = L/R.$$

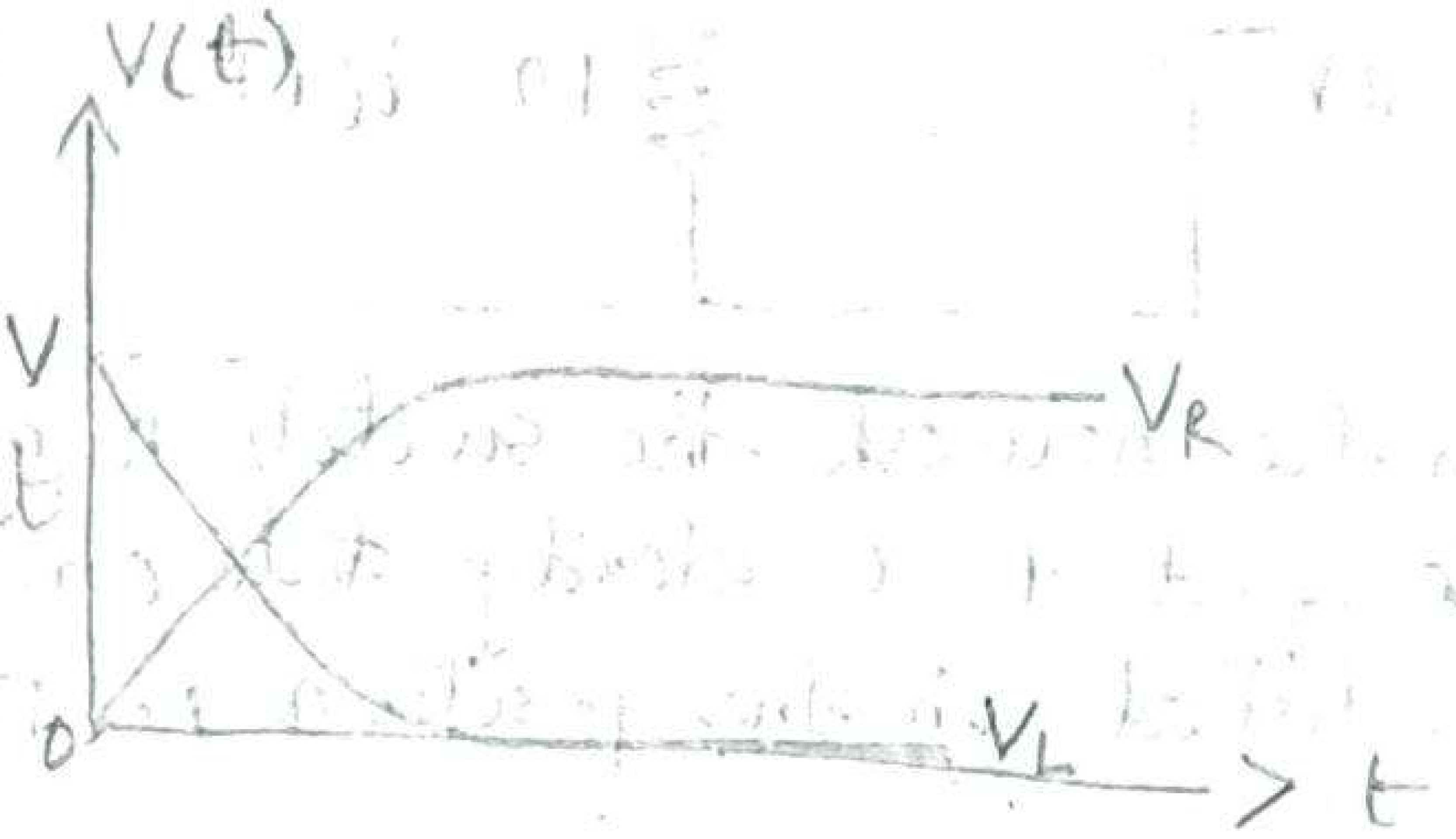
$$V_R = Ri = R \times \frac{V}{R}\left[1 - e^{-R/L\, t}\right]$$

$$= V\left[1 - e^{-R/L\, t}\right]$$

$$V_L = L\frac{di}{dt} = L\frac{V}{R}\frac{d}{dt}\left[1 - e^{-R/L\, t}\right]$$

$$= L\,\frac{V}{R}\left[0 - \left(\frac{-R}{L}\right)e^{-R/L\,t}\right] \quad \frac{LV}{R} \times \frac{R}{L}\,e^{-R/L\,t}.$$

$$= V \cdot e^{-R/L\,t}.$$



Voltage Response of R-L circuit

Q.
$$\frac{s+2}{s(s+3)(s+1)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+1}$$

$$\frac{s+2}{s(s+3)(s+1)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+1}$$

$$A = \left.\frac{s+2}{(s+1)(s+3)}\right|_{s=0} = \frac{2}{3}.$$

$$B = \left.\frac{s+2}{s(s+1)}\right|_{s=-3} = \frac{-3+2}{-3\times -2} = \frac{-1}{6}$$

$$C = \left.\frac{s+2}{s(s+3)}\right|_{s=-1} = \frac{-1+2}{-1(+2)} = -1/2$$

$$\frac{s+2}{s(s+3)(s+1)} = \frac{2/3}{s} + \frac{-1/6}{s+3} + \frac{-1/2}{s+1}$$

$$f(t) = \frac{2}{3} - \frac{1}{6}e^{-3t} - \frac{1}{2}e^{-t}$$

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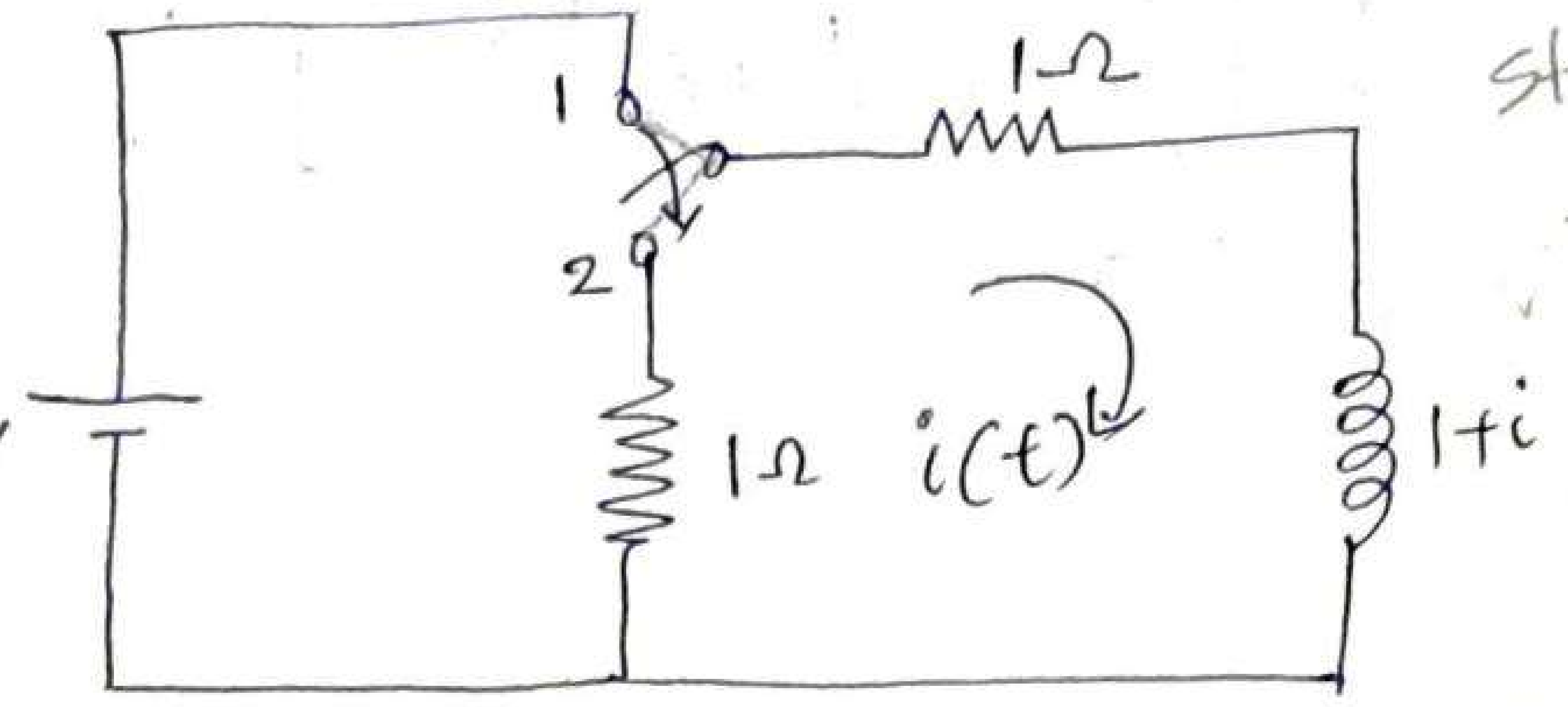
Q1,

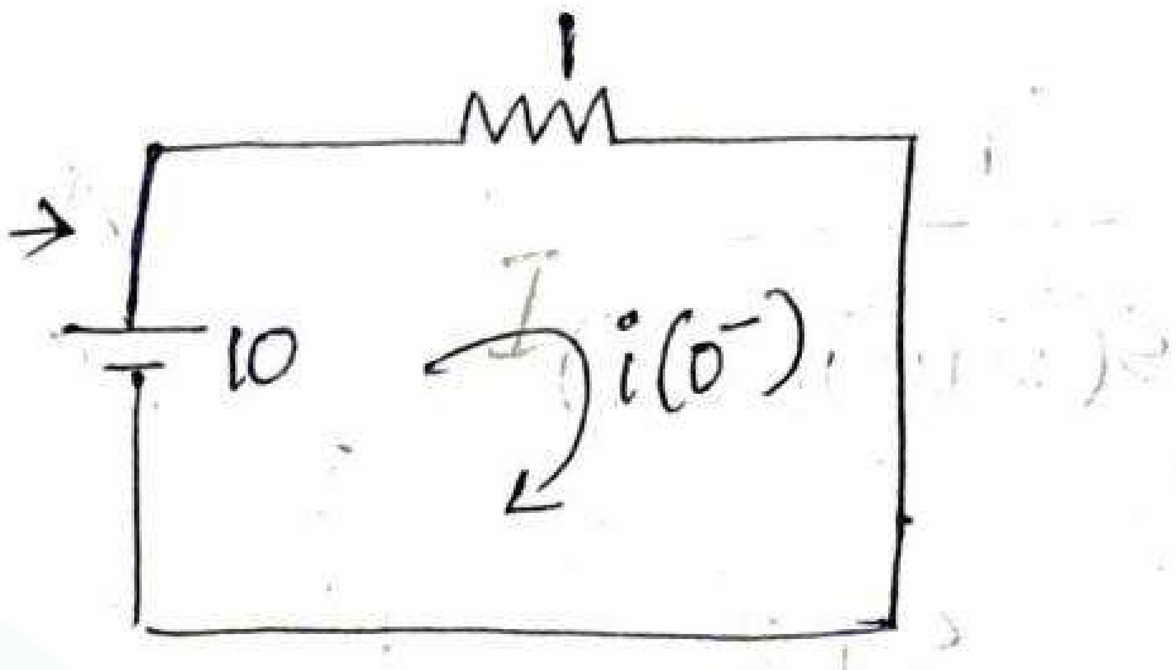


steady state → inductor short

In the network the switch n moved from the position 1 to 2 at $t=0$ steady state condition having been established in the position 1. Determine $i(t)$ for $t>0$

Soln: At $t = 0^-$

the network is shown as below →

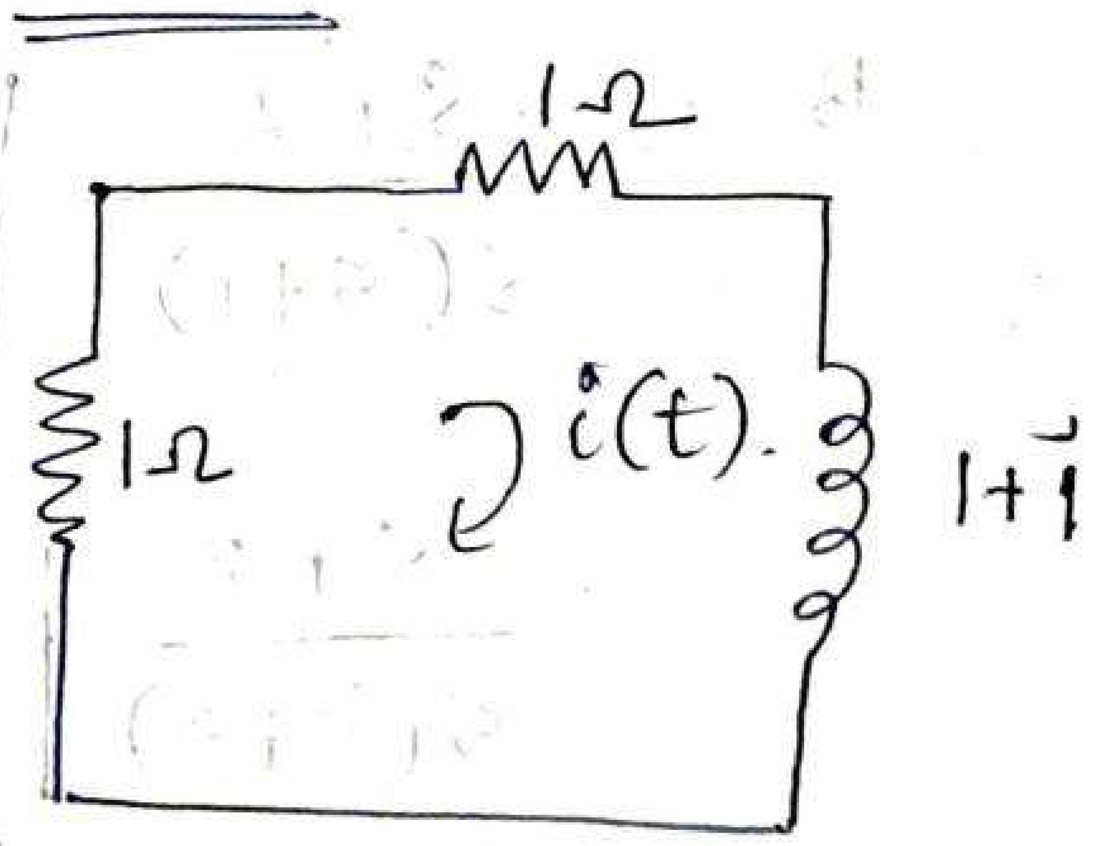


the network has attained steady state condition, hence the inductor act as short circuit.

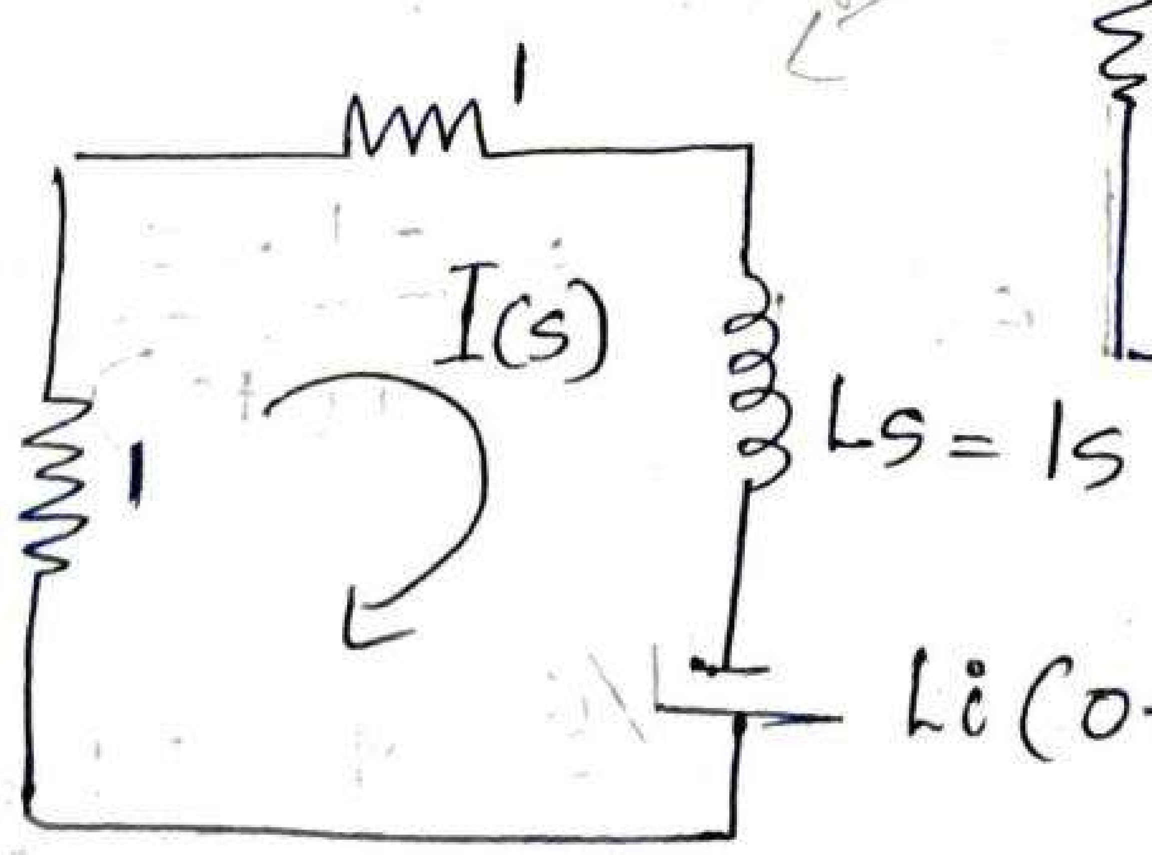
$$i(0^-) = \frac{10}{1} = \underline{10\ A}$$

Since current in the inductor cannot change simultaniously $i(0^+) = \underline{10\,A}$.

— for $t > 0$.



Laplace



$Li(0+) = 1 \times 10 = \underline{10V}$

$$+10 = I(s) + I(s) + Is\,I(s).$$

$$10 = I(s)\,(1+1+1s)$$

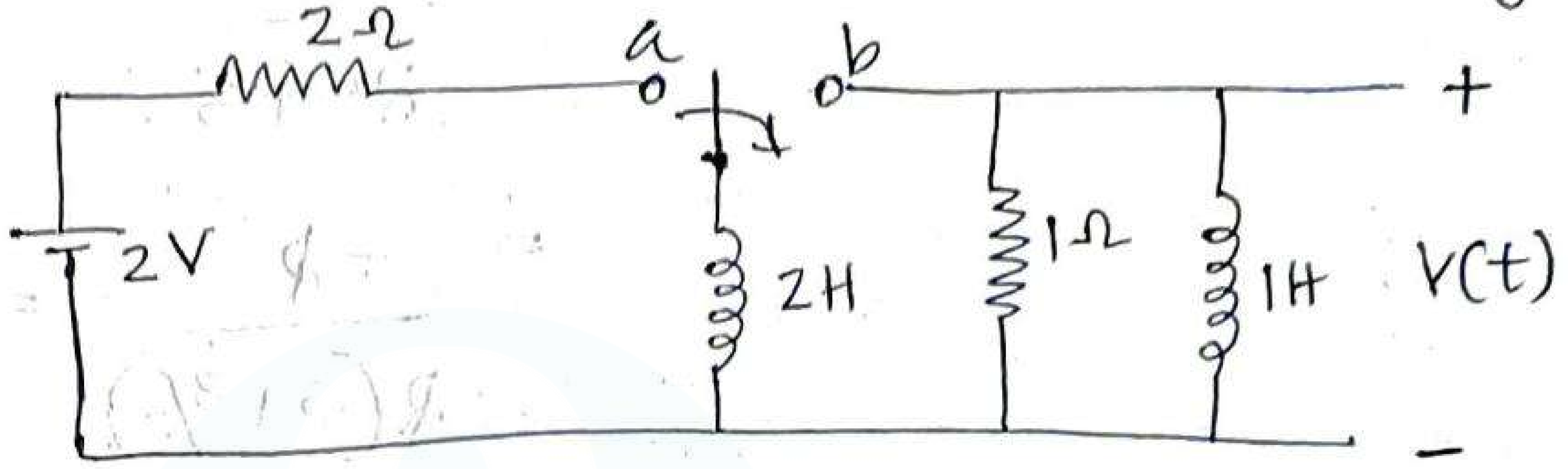
$$10 = I(s)(s+2).$$

$$I(s) = \frac{10}{s+2}.$$

Laplace

$$\underline{\underline{I(t) = 10 \times e^{-2t}}} \quad \text{for } t > 0.$$

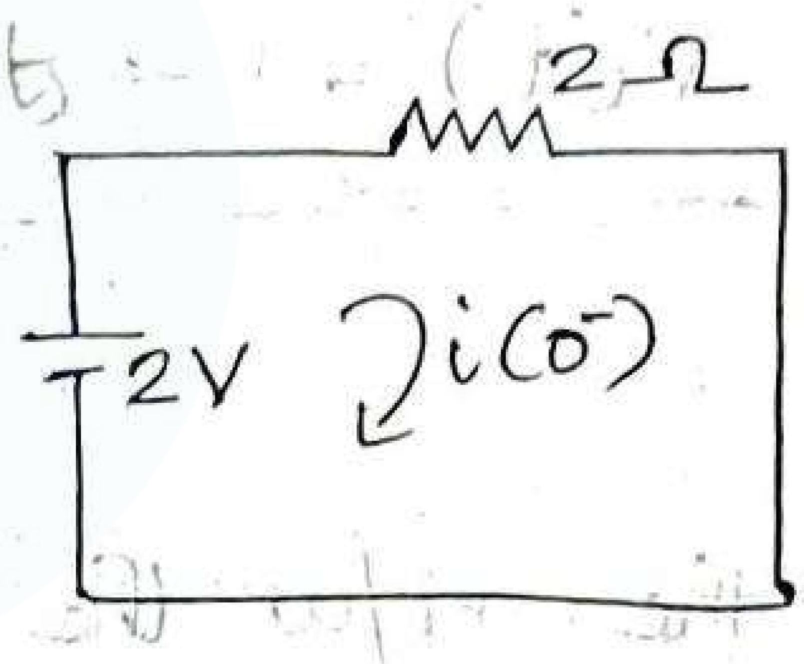
Q2, The network was initially in the steady state with the switch in the position a. At t = 0, the switch goes from a to b. Find the expression for voltage V(t).



At $t = 0^-$

inductor shorts

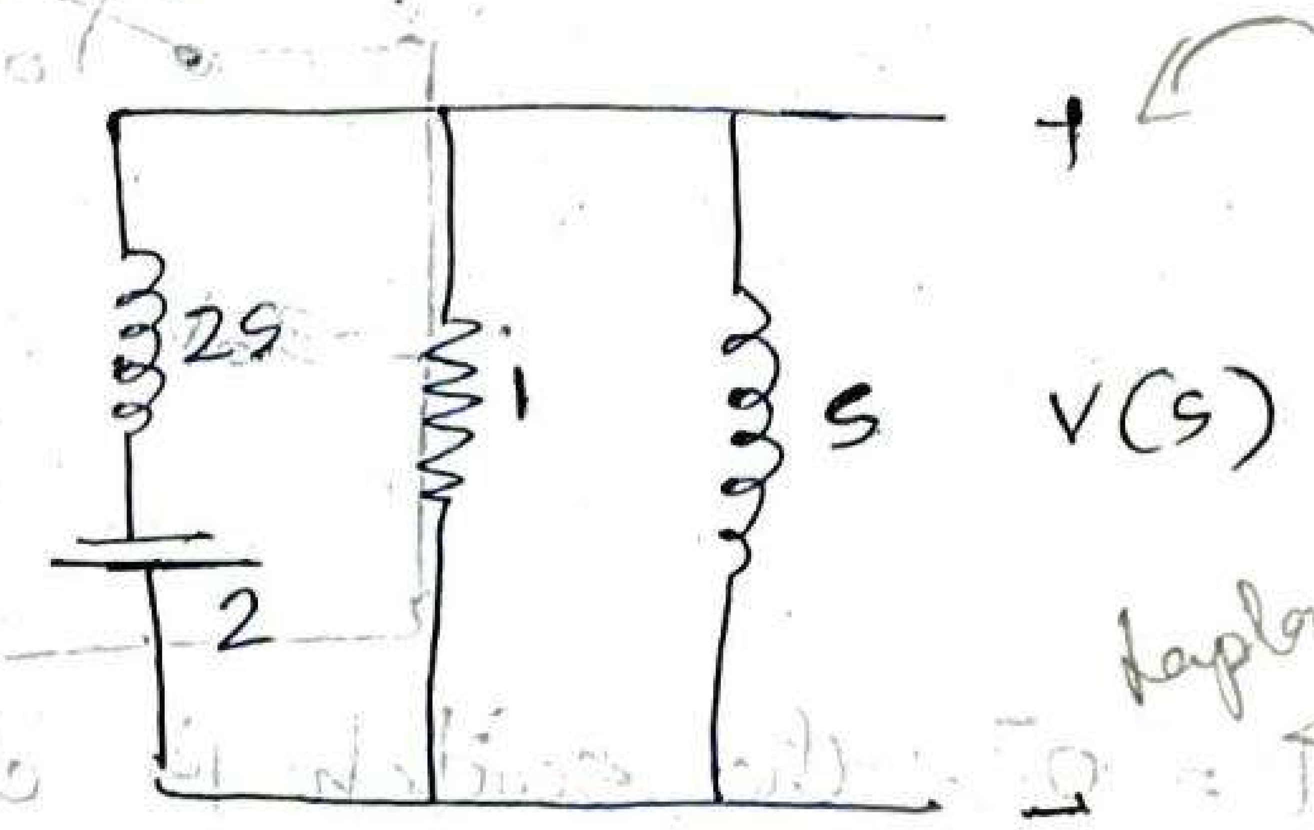
$$i(0^-) = \frac{2}{2} = \underline{\underline{1\,A}}$$

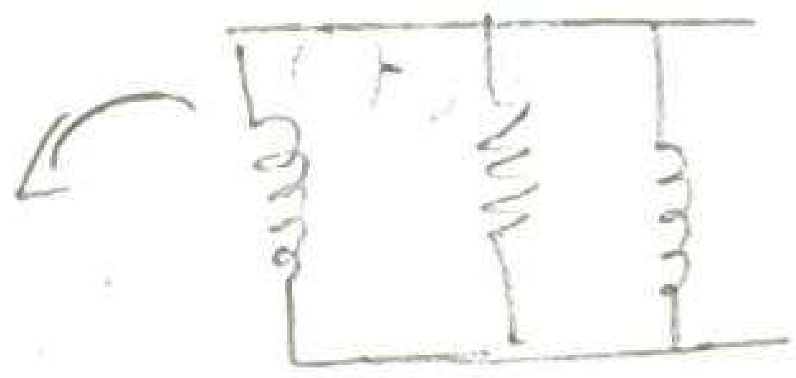


∴ here $i(0^+) = \underline{\underline{1\,A}}.$

At $t = 0^+$

Nodal Analysis





Laplace

$$\frac{V(s)+2}{2s} + \frac{V(s)}{1} + \frac{V(s)}{s} = 0$$

$$\frac{V(s)}{2s} + \frac{\cancel{2}}{\cancel{2}s} + V(s) + \frac{V(s)}{s} = 0.$$

$$V(s)\left[\frac{1}{2s} + 1 + \frac{1}{s}\right] = -\frac{1}{s}$$

$$V(s)\left[\frac{1}{2s}+\frac{s+1}{s}\right] = \frac{-1}{s}$$

$$V(s)\left[\frac{s+2s^2+2s}{2s^2}\right] = \frac{-1}{s}.$$

$$V(s)\left[\frac{\cancel{s}(1+2s+2)}{2s^{\cancel{2}}}\right] = \frac{-1}{s}$$

$$V(s)\,(1+2s+2) = \frac{-1}{\cancel{s}}\times 2\cancel{s}$$

$$V(s) = \frac{-2}{2s+3}.$$

$$= \frac{-\cancel{2}}{\cancel{2}(s+3/2)} = \frac{-1}{s+3/2}.$$

$$v(t) = -e^{-3/2\,t}.$$

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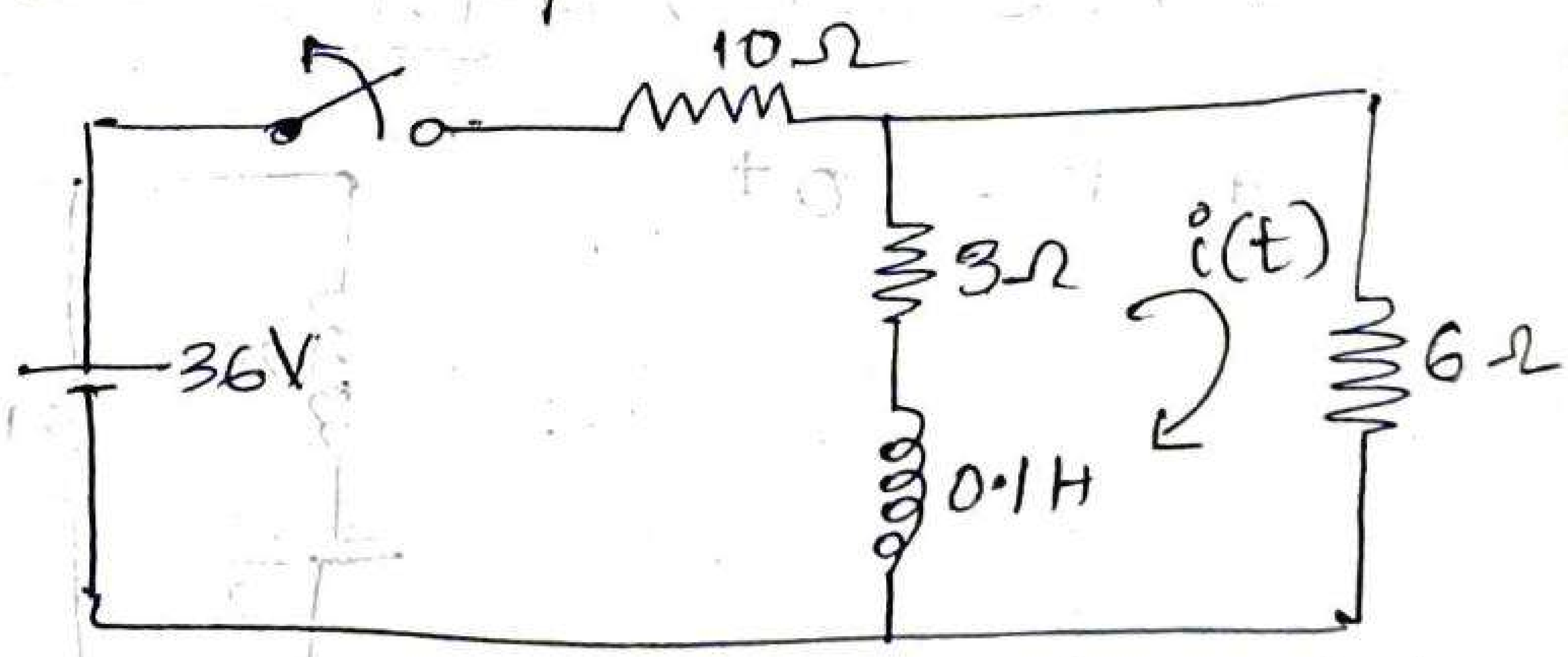
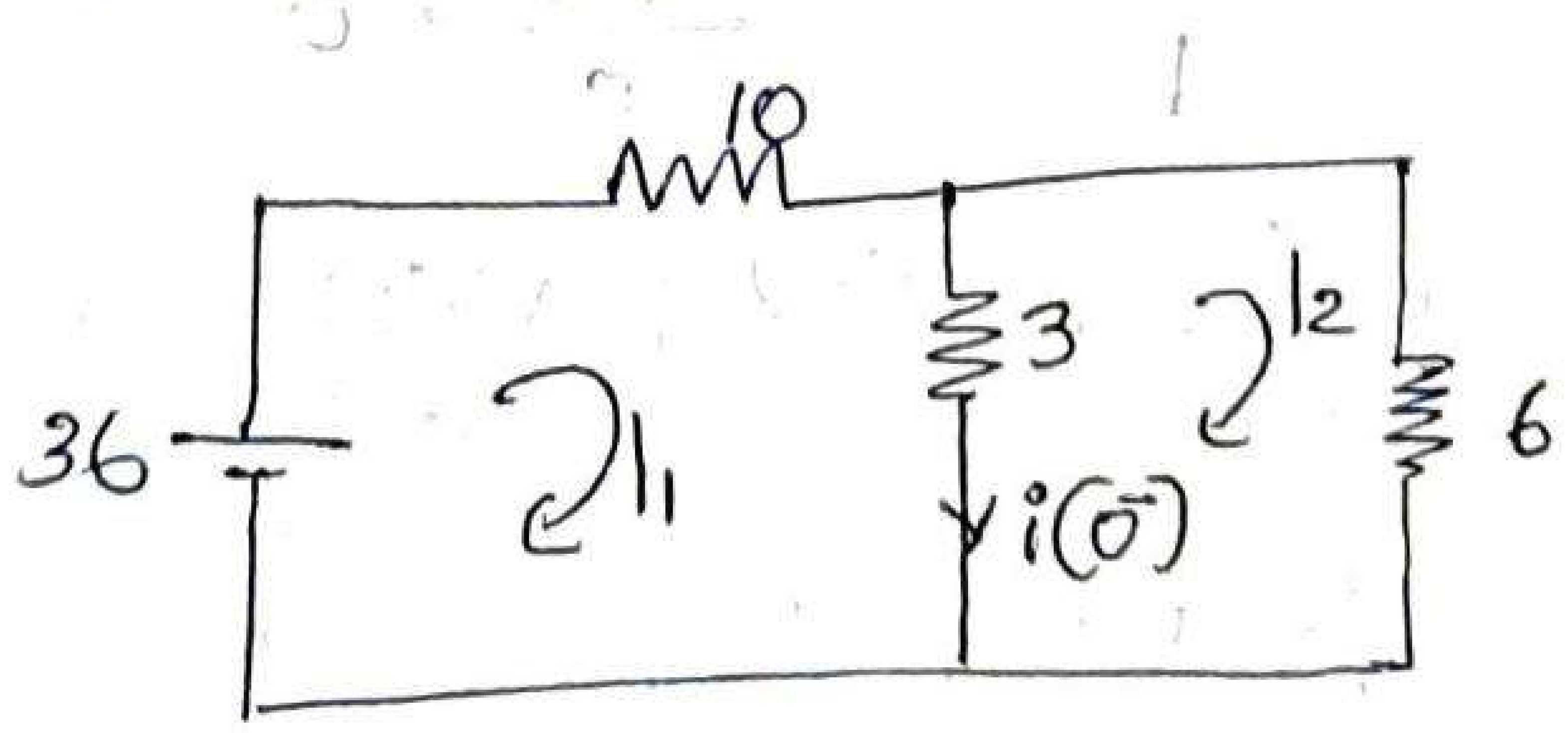
Q. In the n/w the switch is opened at $t=0$, find $i(t)$



At $t=0^-$, the switch is closed and steady state is reached, hence inductor will be short



$36 = \cancel{10}\ 13 I_1 - 3 I_2$

$0 = -3 I_1 + 9 I_2$

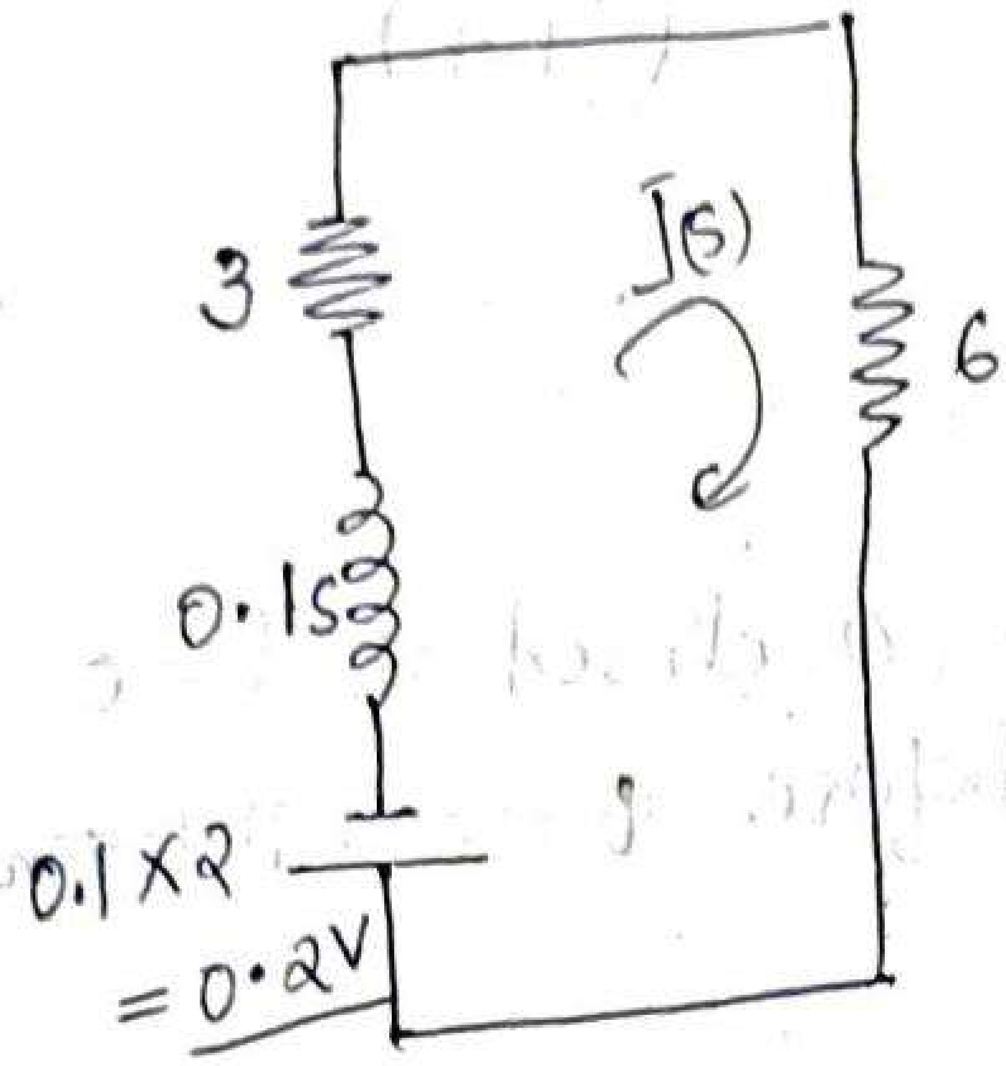
$I_1 = 3A$

$I_2 = 1A$

$i_L(0^-) = I_1 - I_2$

$i(0^-) = 2A$

For $t > 0$



$-0.2 = 9 I(s) + 0.1 s I(s)$

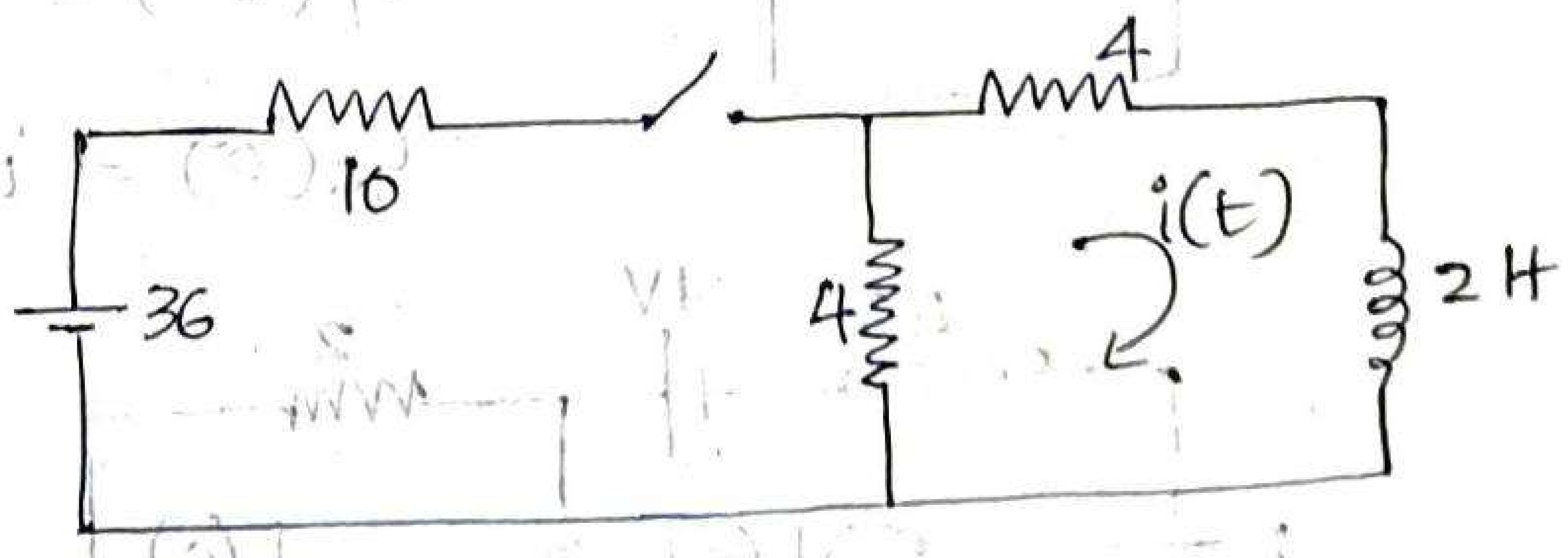
$-0.2 = I(s)[0.1s + 9]$

$$I(s) = \frac{-0.2}{(0.1s + 9)}$$

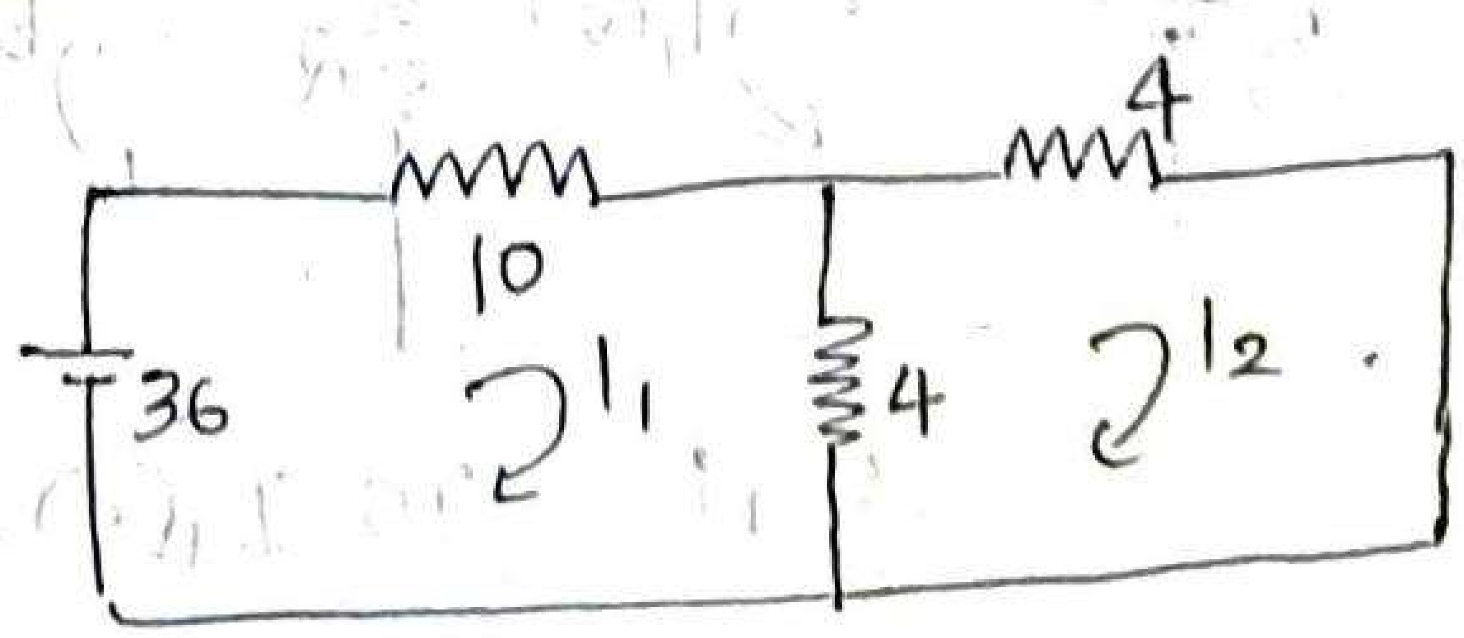
$$I(s) = \frac{-0.2}{0.1[s + 9/0.1]} = \frac{2}{s + 90}$$

$I(t) = 2 \cdot e^{-90t}$ for $t > 0$.

Q. The network shown has acquired steady state with the switch closed for $t < 0$. At $t = 0$, the switch opened. Obtain $i(t)$ for $t > 0$.



At $t = 0^-$



$36 = 14 I_1 - 4 I_2$

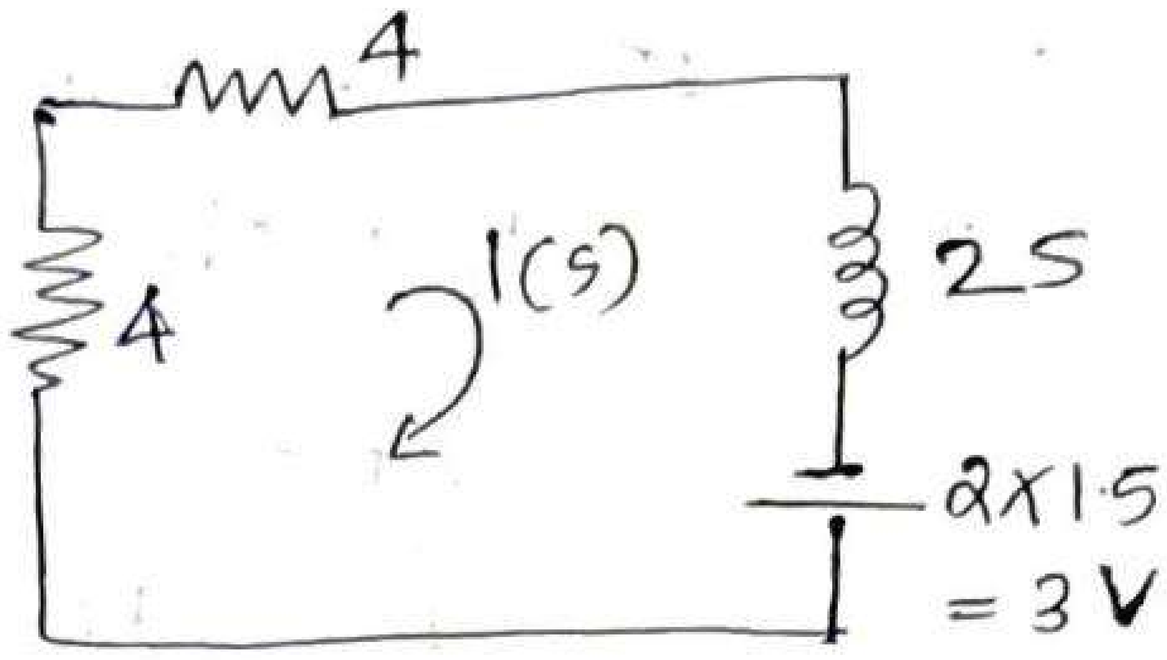
$0 = \cancel{8} - 4 I_1 + 8 I_2$

$I_1 = 3A$

$I_2 = 3/2 = 1.5A$

$i(0^-) = 1.5A$

For $t > 0$,



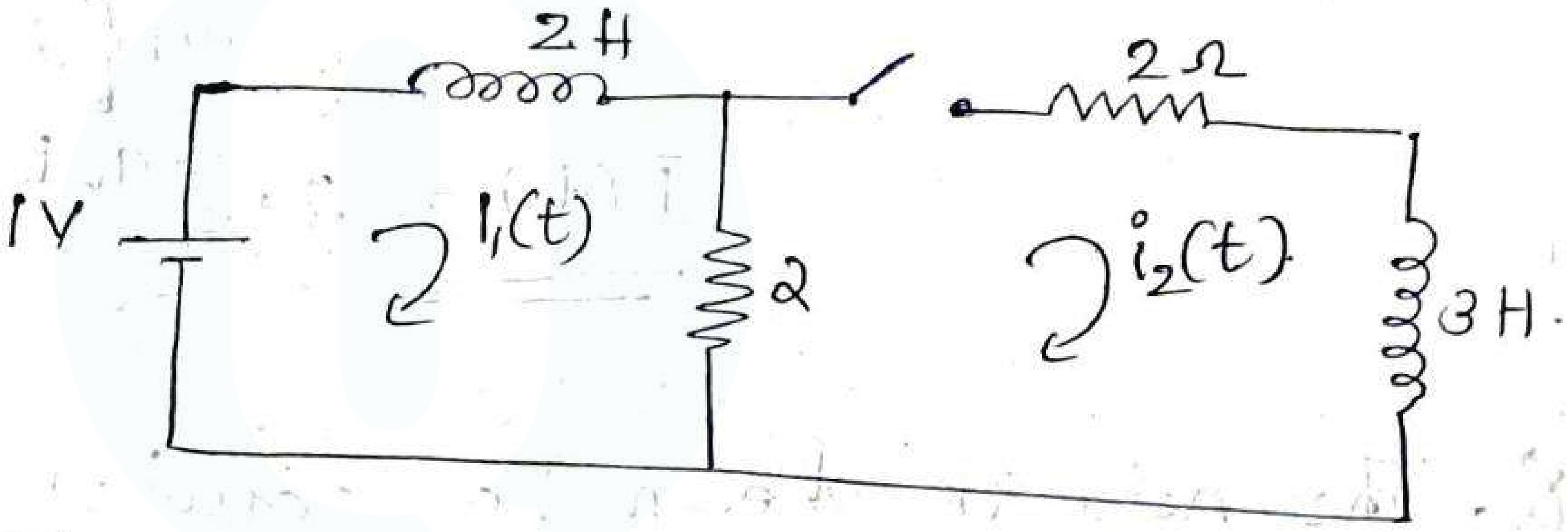
$$+3 = 8I(s) + 2sI(s)$$

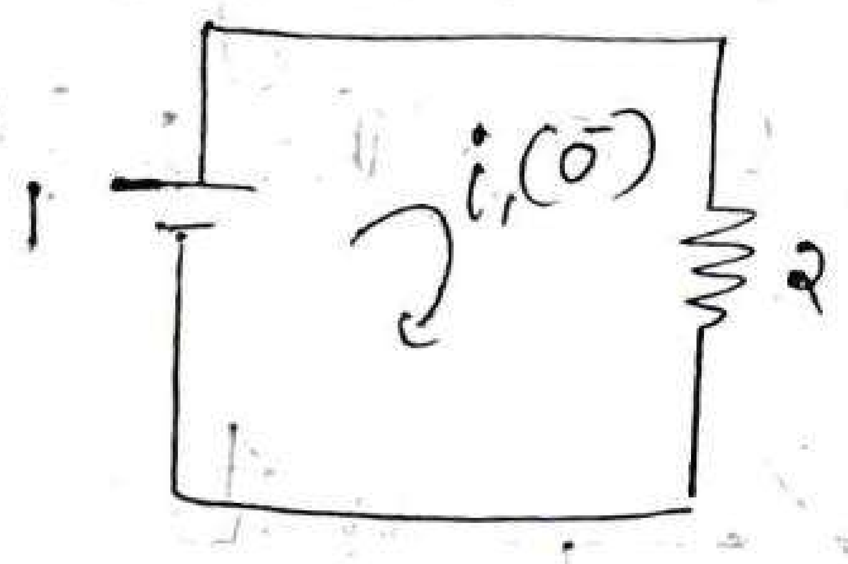
$$= I(s)(2s+8)$$

$$I(s) = \frac{+3}{(2s+8)} = \frac{+3}{2(s+4)} = \frac{+1.5}{(s+4)}$$

$$\underline{\underline{I(t) = +1.5\, e^{-4t}}}$$

Q. In the n/w shown, the switch 0 closed at $t=0$, the steady state being reached before $t=0$. Determine current through Inductor 3H.

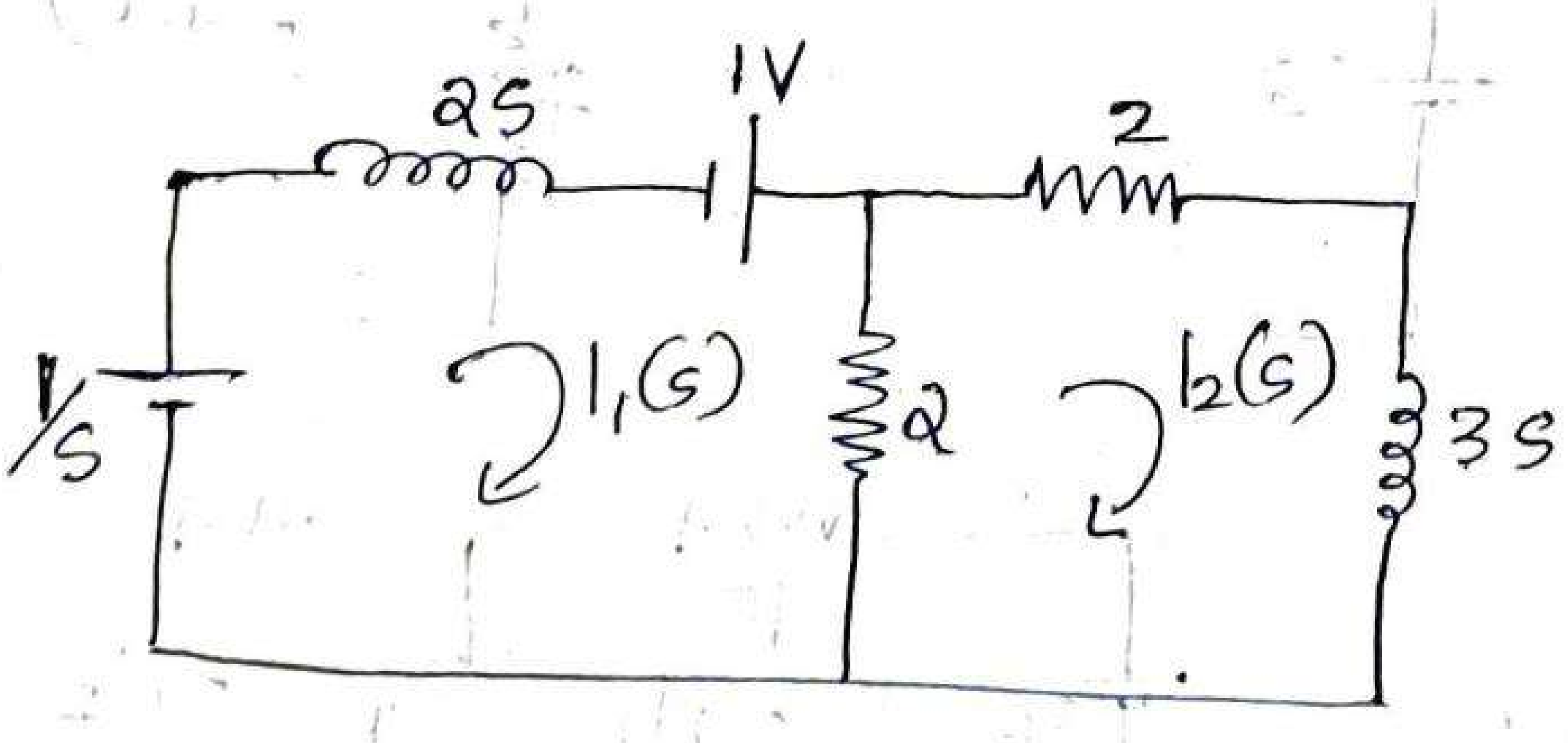




$$i_1(0^-) = 1/2\ A$$

$$i_1(0^+) = 1/2\ A$$

$$i_2(0^-) = i_2(0^+) = 0\ A$$



$$\frac{1}{s} + 1 = 2s\, I_1(s) + 2[I_1(s) - I_2(s)]$$

$$1 + 1/s = I_1(s)[2 + 2s] - 2I_2(s)$$

$$0 = 2\left(I_2(s) - I_1(s)\right) + 2I_2(s) + 3sI_2(s).$$

$$= -2I_1(s) + 4I_2(s) + 3sI_2(s)$$

$$0 = -2I_1(s) + I_2(s)[4+3s]$$

$$\begin{bmatrix} 2s+2 & -2 \\ -2 & 3s+4 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} 1+1/s \\ 0 \end{bmatrix}$$

$$I(s) = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 2s+2 & 1+1/s \\ -2 & 0 \end{vmatrix}}{\begin{vmatrix} 2s+2 & -2 \\ -2 & 3s+4 \end{vmatrix}}$$

$$= \frac{(2s+2)\times 0 - (-2)(1+1/s)}{(2s+2)(3s+4) - (-2\times -2)}$$

$$= \frac{2/s\,[s+1]}{6s^2+8s+6s+8+4} = \frac{2[s+1]}{2s[3s^2+7s+2]}$$

$$= \frac{s+1}{s(3s^2+7s+2)} = \frac{s+1}{3s(s^2+7/3\,s+2/3)}$$

$a+b = \frac{7}{3}$

$a \times b = \frac{2}{3}$

$a = \frac{1}{3}$

$b = 2$

$$I_2(s) = \frac{s+1}{3s[s+1/3][s+2]} = \frac{1/3\,(s+1)}{s(s+1/3)(s+2)}$$

$$\frac{1/3\,(s+1)}{s(s+1/3)(s+2)} = \frac{A}{s} + \frac{B}{s+1/3} + \frac{C}{s+2}$$

$$A = sI_2(s)\Big|_{s=0}$$

$$= s \times \frac{(s+1)}{s(s+1/3)(s+2)} = \frac{1/3}{1/3 \times 2} = \frac{1}{2}$$

$$B = \left(s+\tfrac{1}{3}\right) I_2(s)\Big|_{s=-1/3}$$

$$= \left(s+\tfrac{1}{3}\right) \times \frac{(s+1)\left(\tfrac{1}{3}\right)}{s\left[\left(s+\tfrac{1}{3}\right)(s+2)\right]}\Bigg|_{s=-1/3}$$

$$= \frac{2/3 \times 1/3}{-1/3 \times +5/3} = \frac{2/9}{-5/9} = \frac{-2}{5}$$

$$C = s+2\ I(s)\Big|_{s=-2}$$

$$= \frac{(s+2)\left(1/3(s+1)\right)}{s(s+2)\left(s+1/3\right)}\Bigg|_{s=-2}$$

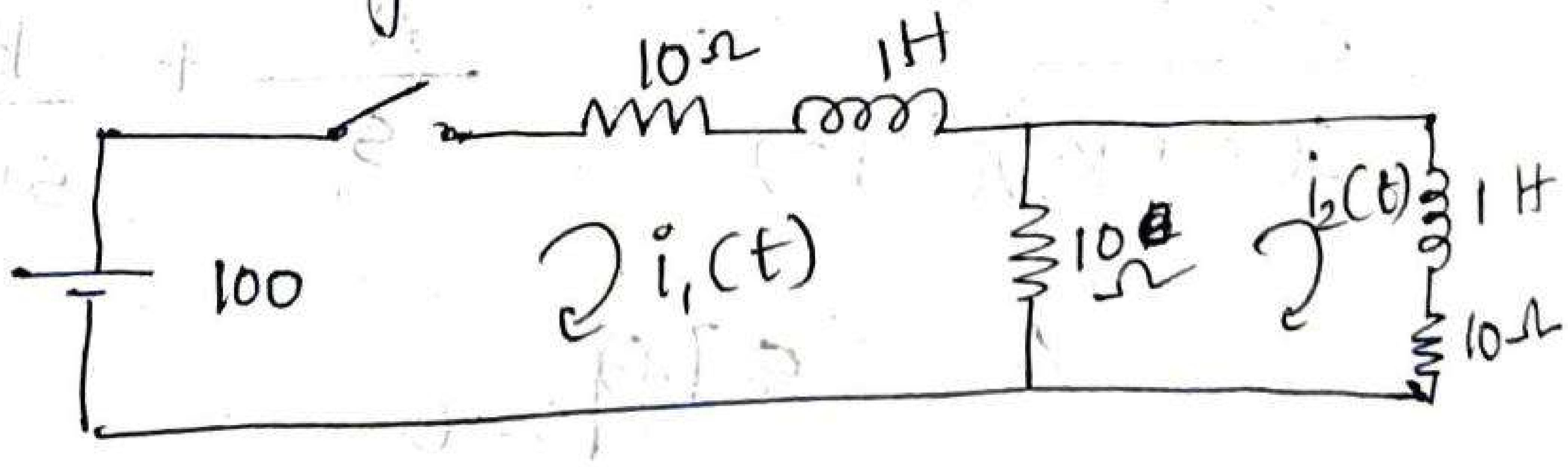
$$= \frac{1/3 \times -1}{-2\left(-5/3\right)} = \frac{-1/10}{}$$

$$I_2(s) = \frac{1/2}{s} + \frac{-2/5}{s+1/3} + \frac{-1/10}{s+2}$$

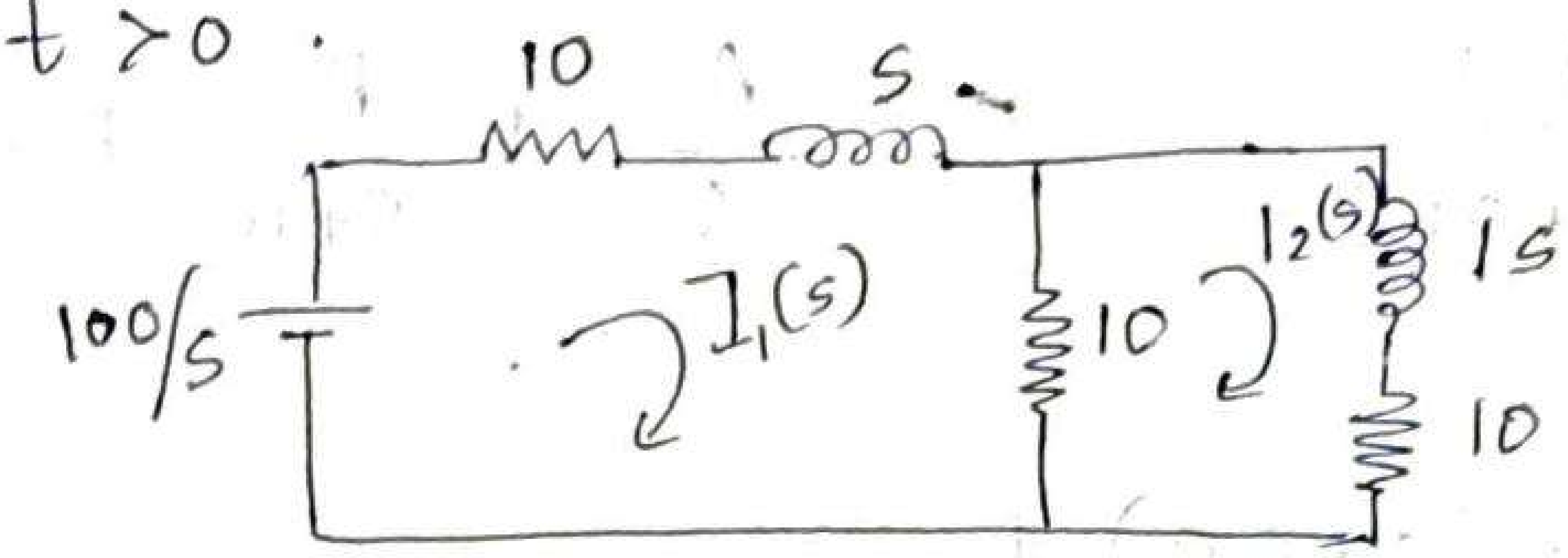
$$I_2(s) = \frac{1}{2}\cancel{e} - \frac{2}{5}e^{-1/3\, t} - \frac{1}{10}e^{-2t}$$

1/10/2020

Q. In the n/w the switch is closed at $t=0$, the n/w is previously unenergised. Determine current $i_1(t)$.



For $t > 0$.



$$\frac{100}{s} = 20\, I_1(s) + s\, I_1(s) - 10 I_2(s)$$

$$\frac{100}{s} = (20+s)\, I_1(s) - 10\, I_2(s) \quad \text{--- (1)}$$

$$0 = 20\, I_2(s) + s\, I_2(s) - 10\, I_1(s)$$

$$0 = -10\, I_1(s) + (20+s)\, I_2(s) \quad \text{--- (2)}$$

$$\begin{bmatrix} 20+s & -10 \\ -10 & (20+s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} 100/s \\ 0 \end{bmatrix}$$

$$I_1(s) = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 100/s & -10 \\ 0 & 20+s \end{vmatrix}}{\begin{vmatrix} 20+s & -10 \\ -10 & 20+s \end{vmatrix}}$$

$$= \frac{100/s\,(20+s) - 0}{(20+s)(20+s) - 100}$$

$$= \frac{100\,(20+s)}{s\,(s^2 + 40s + 400) - 100}$$

$$= \frac{100\,(20+s)}{s\,(s^2 + 40s + 300)}$$

30, 10

$$I_1(s) = \frac{100\,(20+s)}{s\,(s+10)(s+30)}$$

$$\frac{100(s+20)}{s(s+10)(s+30)} = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+30}.$$

$$A = \left.\frac{100(s+20)}{(s+10)(s+30)}\right|_{s=0}$$

$$= \frac{10\cancel{0} \times 2\cancel{0}}{1\cancel{0} \times 3\cancel{0}} = \frac{20}{3}$$

$$B = \left.\frac{100(s+20)}{s(s+30)}\right|_{s=-10}$$

$$= \frac{10\cancel{0}(\cancel{10})}{-\cancel{10} \times 2\cancel{0}} = -5.$$

$$C = \left.\frac{100(s+20)}{s(s+10)}\right|_{s=-30}$$

$$= \frac{\overset{5}{\cancel{100}} \times -1\cancel{0}}{-3\cancel{0} \times -\cancel{20}} = \frac{-5}{3}$$

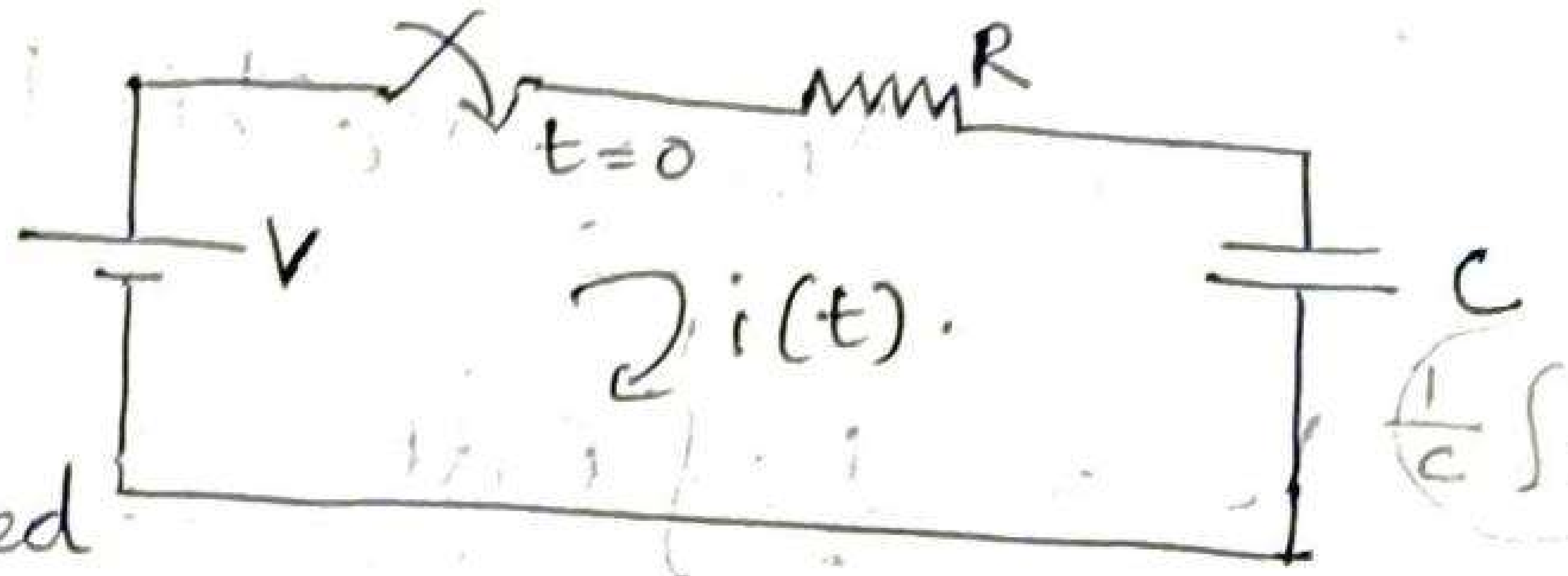
$$I(s) = \frac{20/3}{s} + \frac{-5}{s+10} + \frac{-5/3}{s+30}.$$

$$I_1(t) = \frac{20}{3} + -5e^{-10t} - \frac{5}{3}e^{-30t}$$

8/10/2020

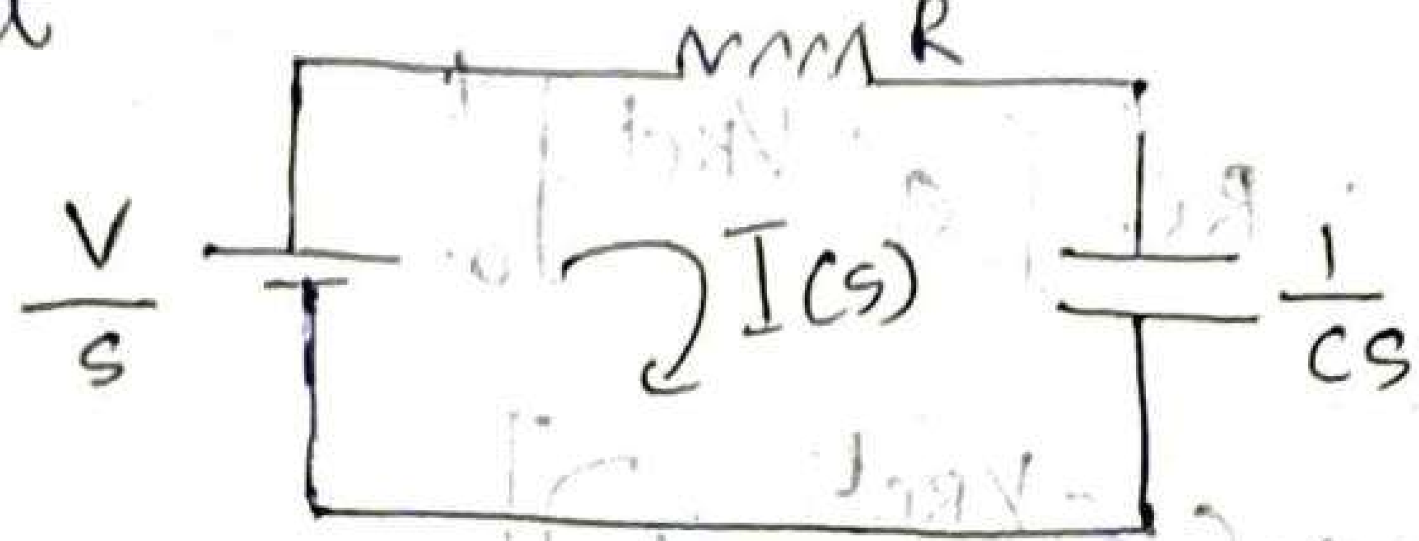
RC circuits

Consider a series RC circuit. The switch is closed at time $t = 0$.



$\frac{1}{C}\int \frac{dq}{dt}$

For $t > 0$ the transformed circuit



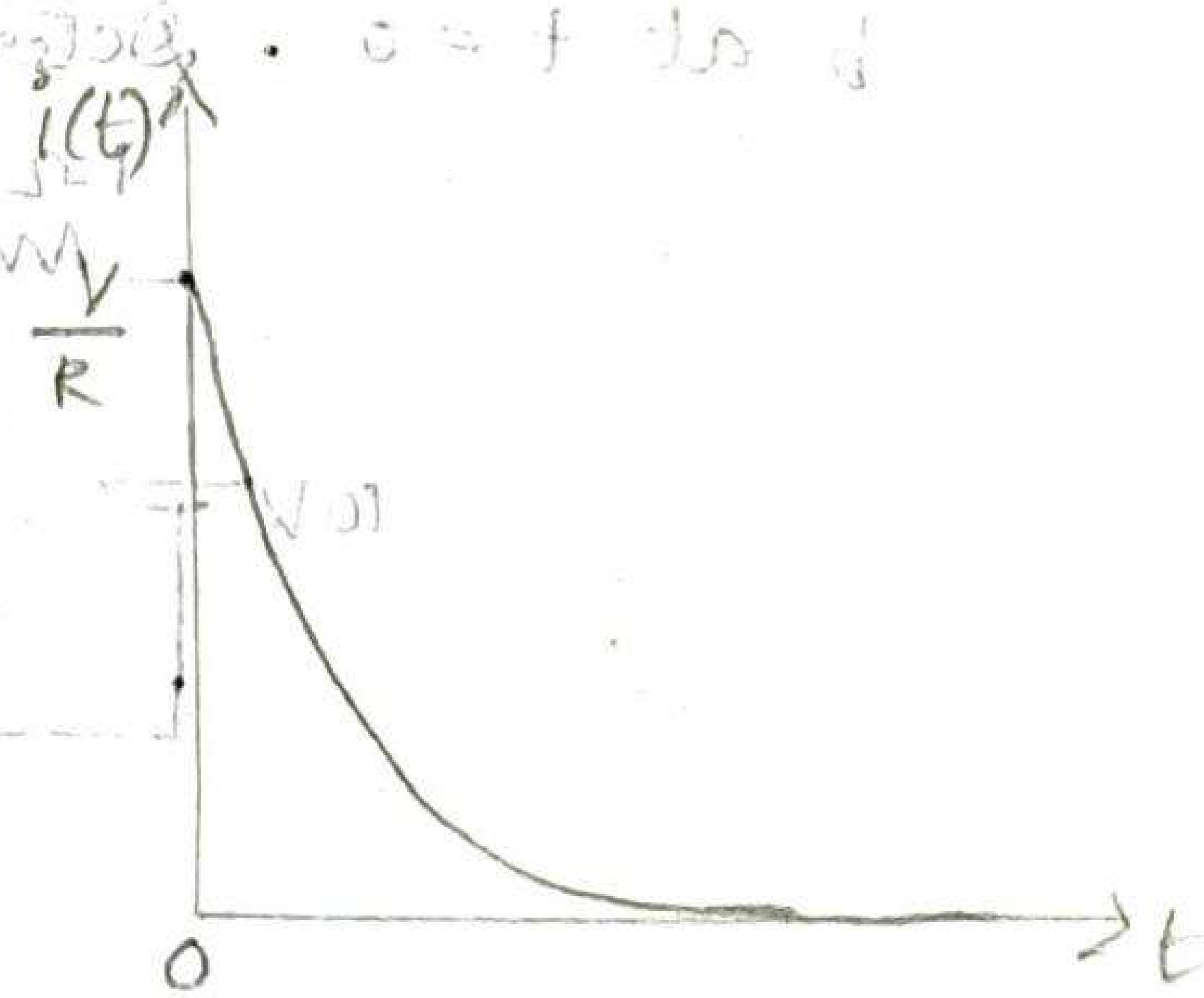
$$R\,I(s) + \frac{1}{Cs} I(s) = \frac{V}{s}$$

$$\left[R + \frac{1}{Cs}\right] I(s) = \frac{V}{s}$$

$$I(s) = \frac{V/s}{\left[R + \frac{1}{Cs}\right]} = \frac{V/s}{\left[\frac{RCs+1}{Cs}\right]} = \frac{\frac{V}{\cancel{s}} \times C\cancel{s}}{\left[RCs+1\right]}$$

$$= \frac{VC}{RC\left[s + 1/RC\right]} = \frac{V/R}{s + 1/RC}$$

$$i(t) = \frac{V}{R} e^{-1/RC\, t}$$



The term RC is the time constant.

$$T = RC$$

$$T = RC$$

$$V_R = Ri = \cancel{R} \times \frac{V}{\cancel{R}} e^{-1/RC\, t}$$

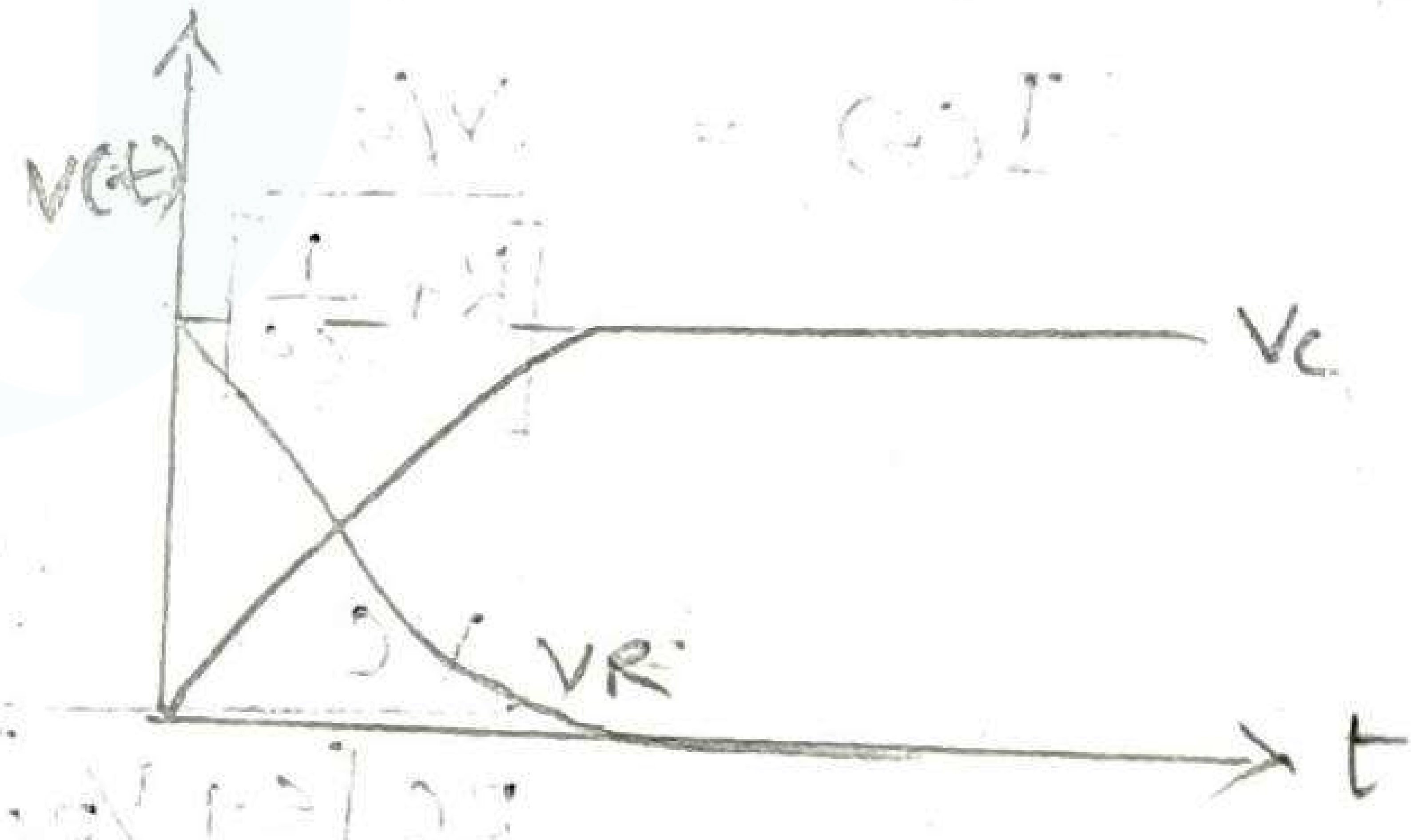
$$V_R = Ve^{-1/RC\, t}$$

$$V_c = \frac{1}{C}\int_0^t i\, dt = \frac{1}{C}\int \frac{V}{R} e^{-1/RC\, t}\, dt$$

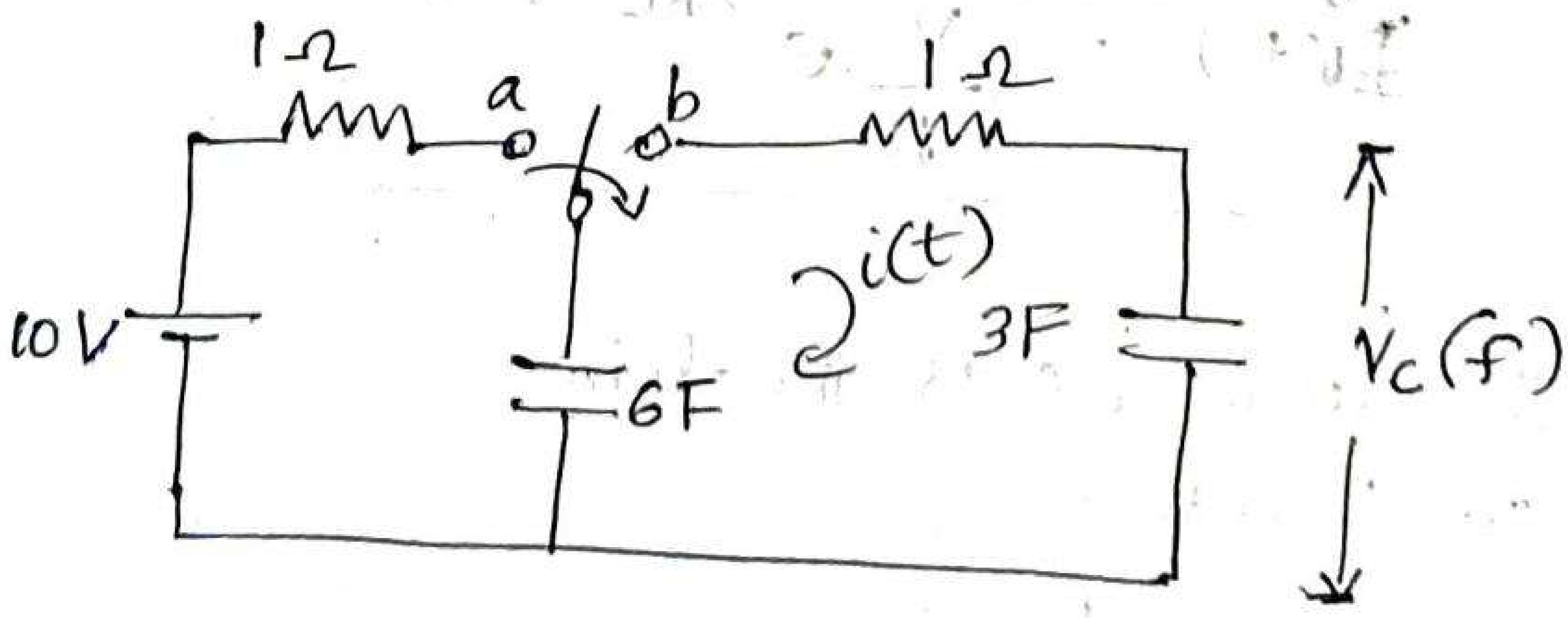
$$= \frac{1}{C} \cdot \frac{V}{R}\left[-RC\left[e^{-1/RCt}\right]_0^t\right]$$

$$= \frac{V}{CR}\left[-RC\left(e^{-1/RCt} - 1\right)\right]$$

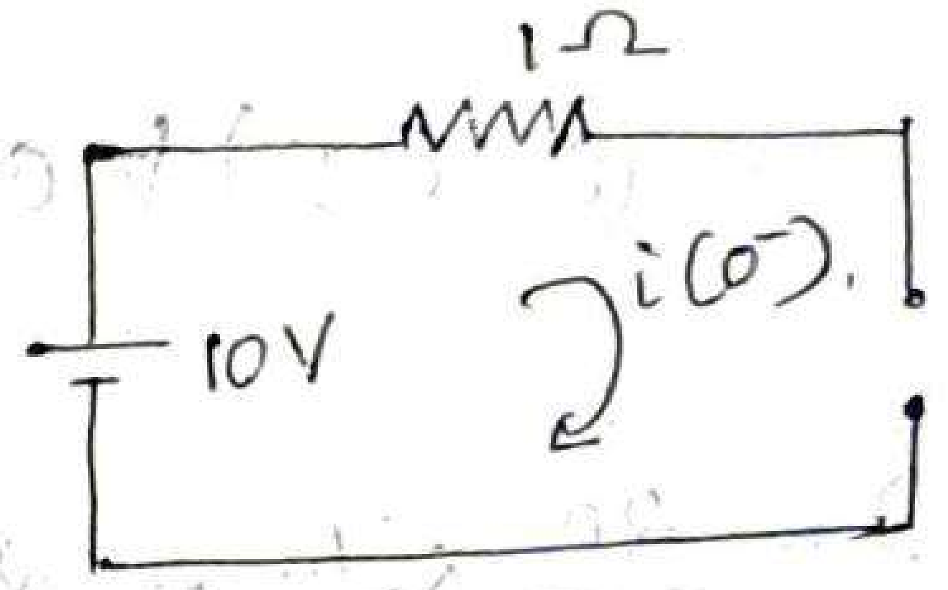
$$= -Ve^{-1/RCt} + V = V\left[1 - e^{-1/RCt}\right]$$



Q: In the network, the switch is moved from a to b at $t = 0$. Determine $i(t)$ and $v(t)$.



At $t = 0^-$, the network has ~~assigned~~ attained steady state condition, Hence 6 F capacitor act as an open circuit.



$V_{6F}(0^-) = 10\,V$

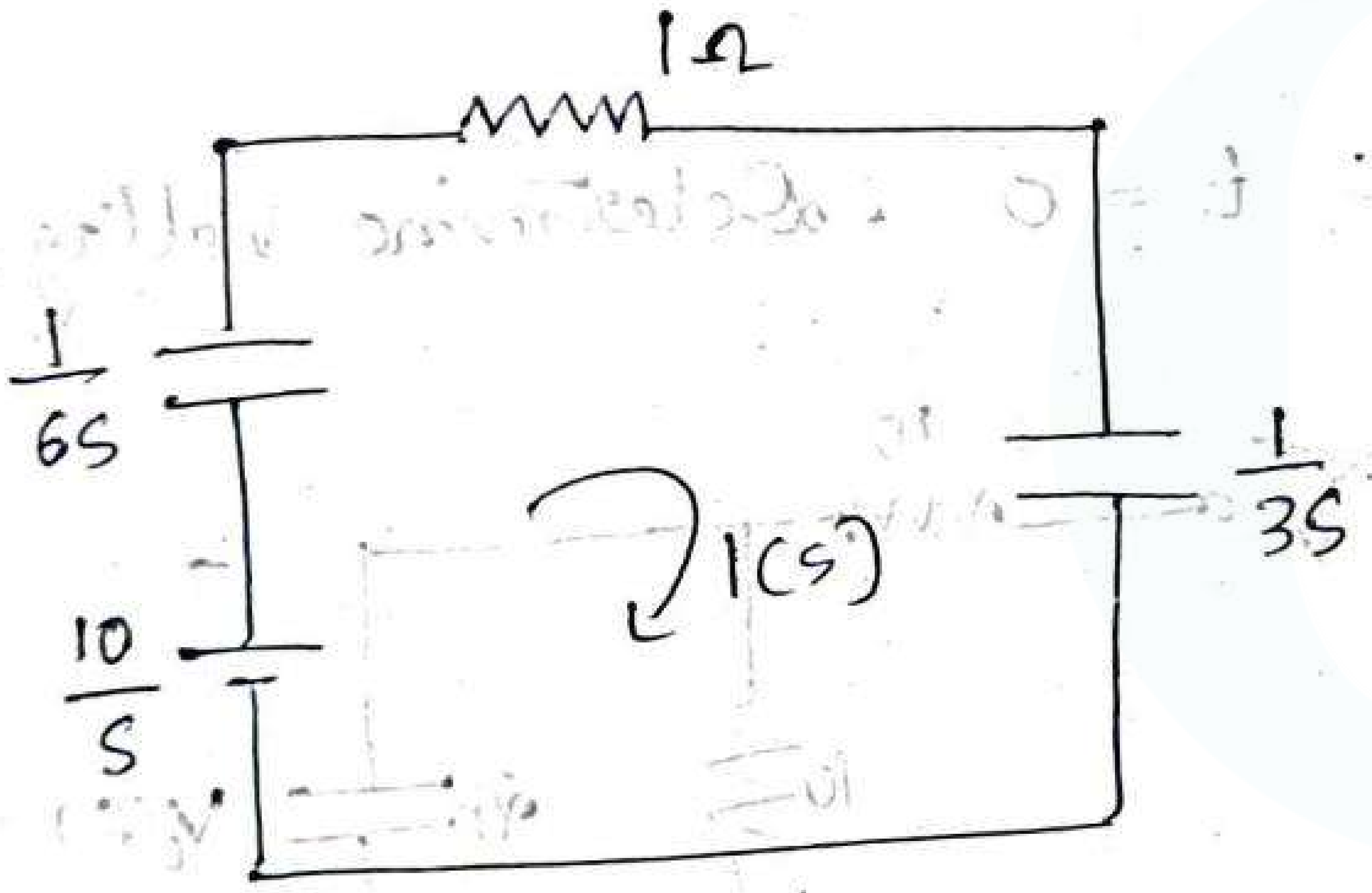
$i(0^-) = 0$

$V_{3F}(0^-) = 0$

Since voltage across the capacitor cannot change simultaniously $V_{6F}(0^-) = 10\,V$

$V_{3F}(0^+) = 0$

For $t > 0$



Mesh

$$\frac{10}{s} = \frac{1}{6s} I(s) + I(s) + \frac{1}{3s} I(s)$$

$$\frac{10}{s} = I(s)\left[\frac{1}{6s} + 1 + \frac{1}{3s}\right]$$

$$I(s) = \frac{10}{s\left(1 + \frac{1}{6s} + \frac{1}{3s}\right)}$$

$$= \frac{60}{6s+3} = \frac{10}{s+0.5}$$

9/10/2020

$$I(s) = \frac{10}{s+0.5}$$

$$\underline{\underline{i(t) = 10\,e^{-0.5t}}}$$

Voltage across 3 F,

$$V_c(s) = \frac{1}{3s} I(s) = \frac{10}{3s(s+0.5)}$$

$$\frac{10/3}{s(s+0.5)} = \frac{A}{s} + \frac{B}{s+0.5}$$

$$A = sV_c(s)\Big|_{s=0} = \frac{10/3}{s+0.5} = \frac{20}{3}$$

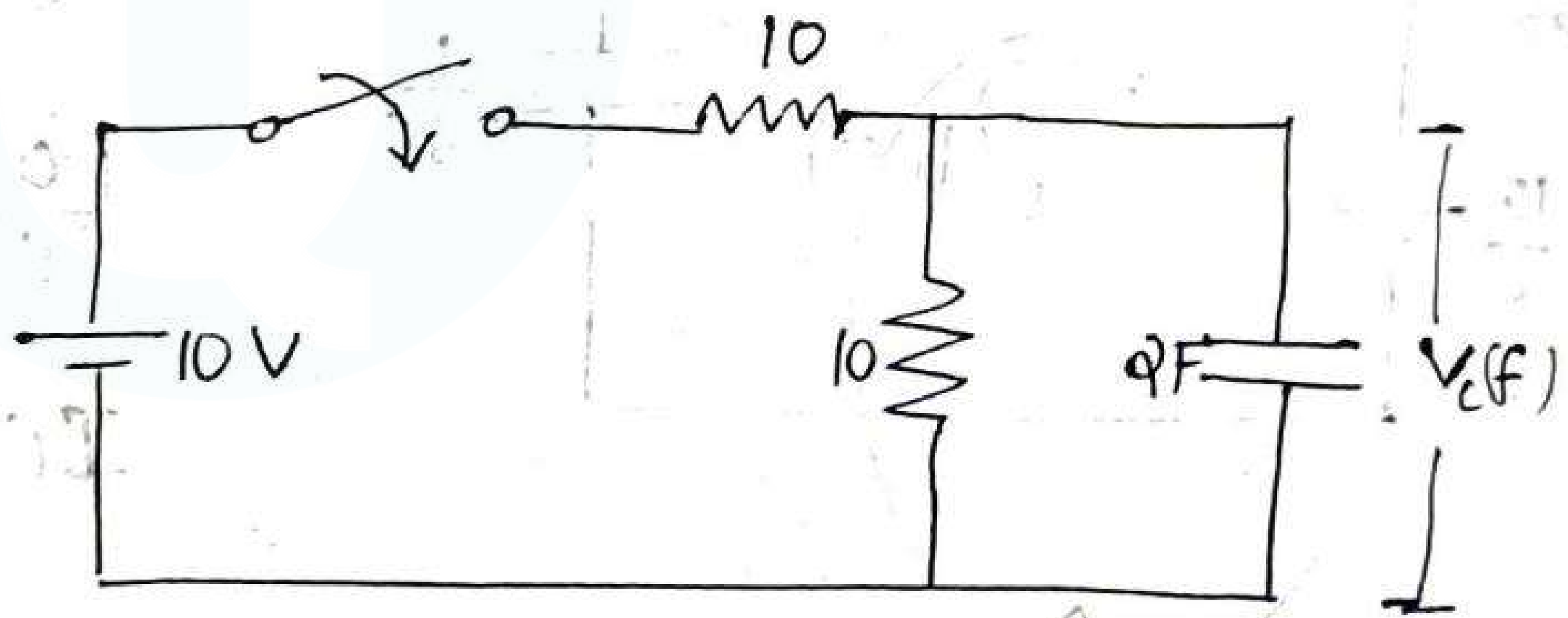
$$B = (s+0.5)V_c(s)\Big|_{s=0.5} = \frac{10}{3s}\Big|_{s=0.5} = -\frac{20}{3}$$

$$V_C(s) = \frac{20}{3} \times \frac{1}{s} - \frac{20}{3} \times \frac{1}{s+0.5}$$

$$= \frac{20}{3} - \frac{20}{3} e^{-0.5t}$$

$$V_C(s) = \frac{20}{3}\left(1 - e^{-0.5t}\right)$$

Q. The switch is closed at $t = 0$. Determine voltage across the capacitor

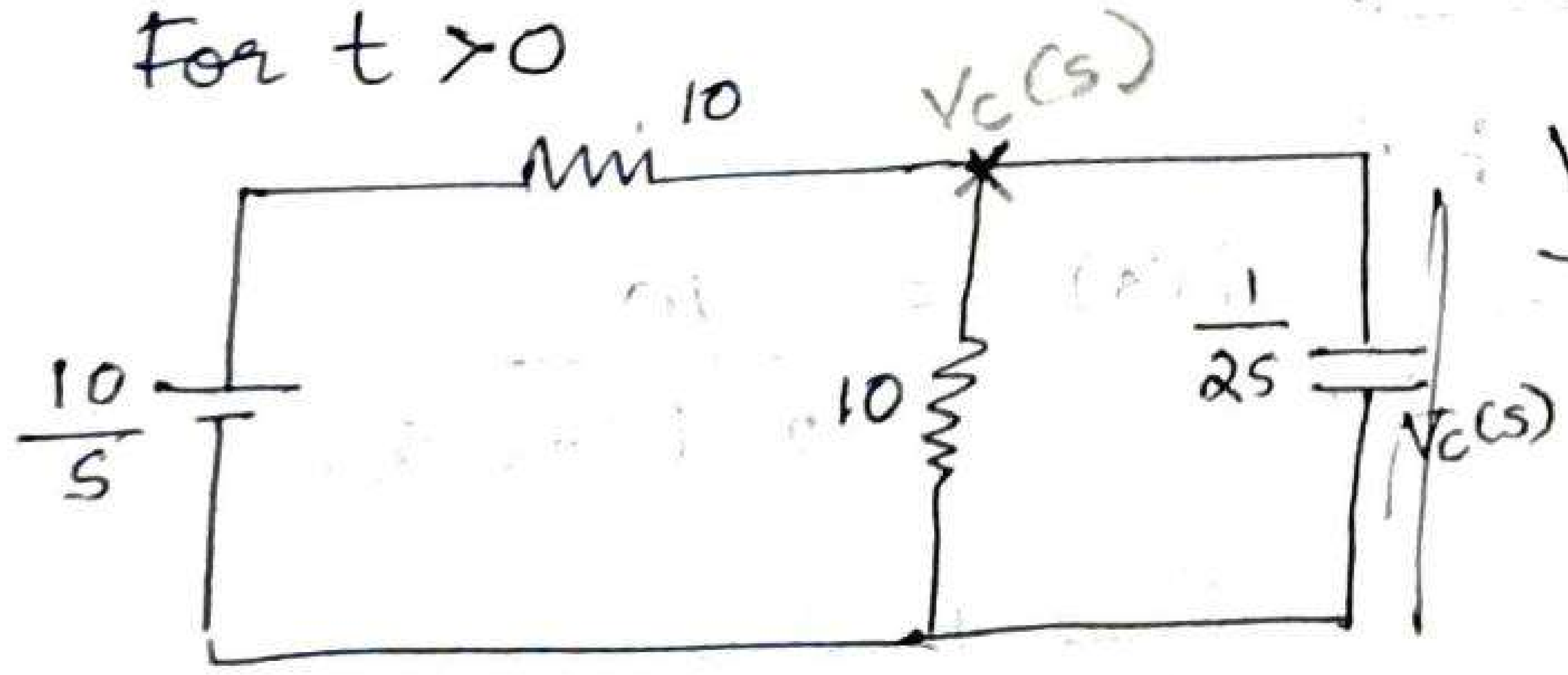


At $t = 0^-$ capacitor is uncharged.

$$V_c(0^-) = 0.$$

$$V_c(0^+) = 0.$$

For $t > 0$



$$\frac{V_c(s) - \frac{10}{s}}{10} + \frac{V_c(s)}{10} + \frac{V_c(s)}{1/2s} = 0$$

$$2s\,V_c(s) + 0.2\,V_c(s) = \frac{1}{s}$$

$$V_C(s) = \frac{1}{s(2s+0.2)} = \frac{0.5}{s(s+0.1)}$$

$$\frac{0.5}{s(s+0.1)} = \frac{A}{s} + \frac{B}{s+0.1}$$

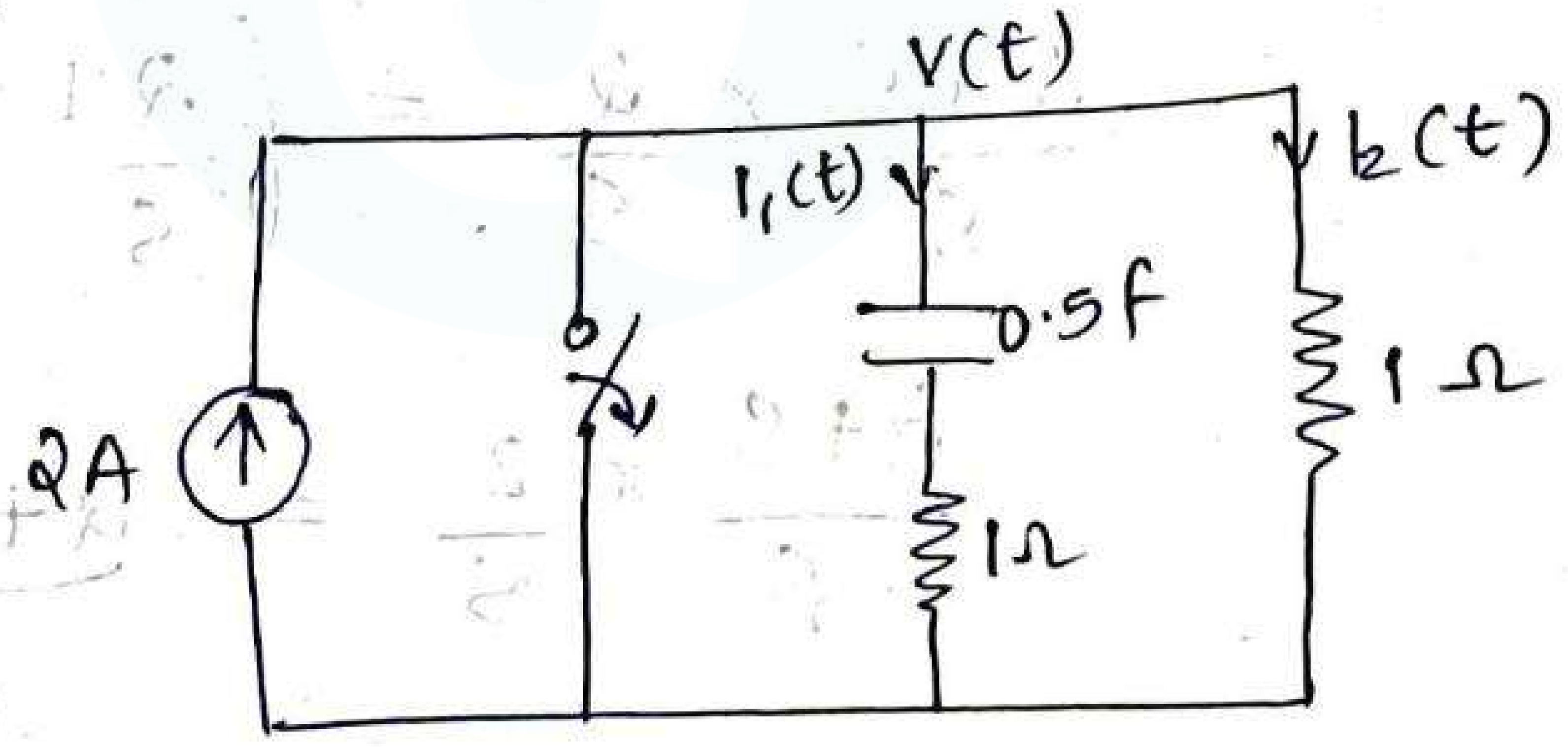
$$A = s\,V_C(s)\Big|_{s=0} = \frac{0.5}{s+0.1} = 5$$

$$B = (s+0.1)\,V_C(s)\Big|_{s=-0.1} = \frac{0.5}{s} = -5.$$

$$V_C(s) = \frac{5}{s} - \frac{5}{s+0.1} = 5 - 5e^{-0.1t}$$

$$= \underline{5\left(1 - e^{-0.1t}\right)}$$

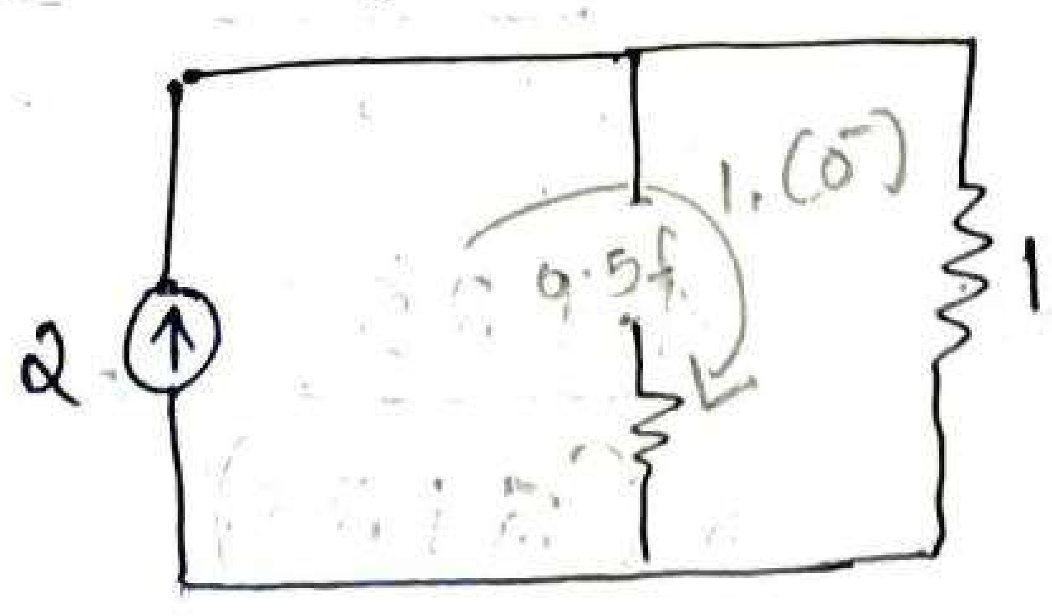
Q. The switch is closed for a long time and at $t=0$ the switch is opened. Determine current through capacitor.

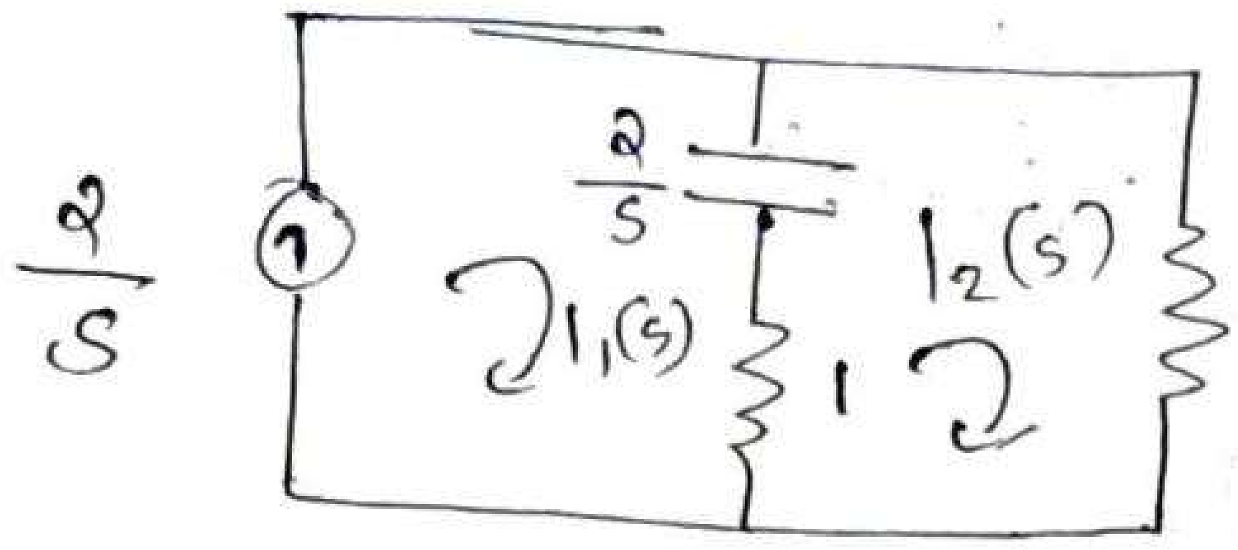


at $t(0^-)$ switch is closed

$0.5f(0^-) = 2$

$i(0^-) = 0.$





$$\frac{2}{s} = \frac{2}{s} I_1(s) + I_1(s) - I_2(s) - \frac{2}{s} I_2(s)$$

$$\frac{2}{s} = \left(\frac{2}{s}+1\right) I_1(s) - \left(\frac{2}{s}+1\right) I_2(s)$$

$$0 = \frac{2}{s}\left(I_2(s) - I_1(s)\right) + 2 I_2(s) - 1 I_1(s)$$

$$= \left(\frac{2}{s}-1\right) I_1(s) + \left(\frac{2}{s}+2\right) I_2(s)$$

$$\cancel{\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} \begin{bmatrix} \frac{2}{s}+1 & -\frac{2}{s}+1 \\ \frac{2}{s}-1 & \frac{2}{s}+2 \end{bmatrix} = \begin{bmatrix} \frac{2}{s} \\ 0 \end{bmatrix}}$$

$$\left(\frac{2}{s}+1\right) I_1(s) = \left[\frac{2}{s}+2\right] I_2(s)$$

$$I_1(s) = 2/s$$

$$\frac{2+s}{s} \times \frac{2}{s} = \left(\frac{2}{s}+2\right) I_2(s)$$

$$\frac{2+s}{\cancel{s}} \times \frac{2}{s} = \frac{2+2s}{\cancel{s}} I_2(s)$$

$$\frac{4+2s}{s} = (2+2s) I_2(s)$$

$$I_2(s) = \frac{4+2s}{s(2+2s)} = \frac{\cancel{2}(2+s)}{\cancel{2}s(1+s)}$$

$$I_2(s) = \frac{2+s}{s(1+s)} = \frac{A}{s} + \frac{B}{s+1}$$

$$A = \left.\frac{2+s}{s+1}\right|_{s=0} = \frac{2}{1} = \underline{\underline{2}}$$

$$B = \left.\frac{2+s}{s}\right|_{s=-1} = \frac{1}{-1} = \underline{\underline{-1}}$$

$$I_2(s) = \frac{2}{s} - \frac{1}{s+1}$$

$$I_2(t) = \underline{\underline{2 - e^{-t}}}$$

$$I_1(s) = \frac{2}{s}$$

$$\underline{\underline{I_1(t) = 2}}.$$

Current through capacitor

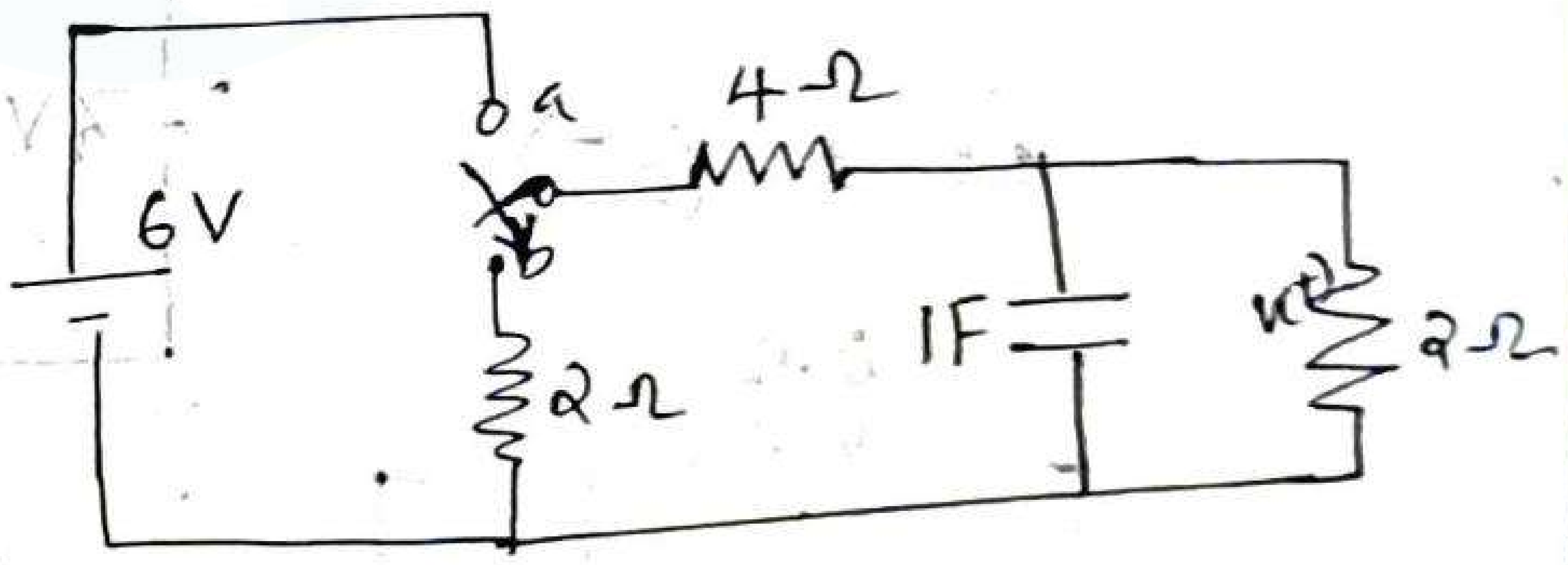
$$= I_1(t) - I_2(t)$$

$$= 2 - (2 - e^{-t})$$

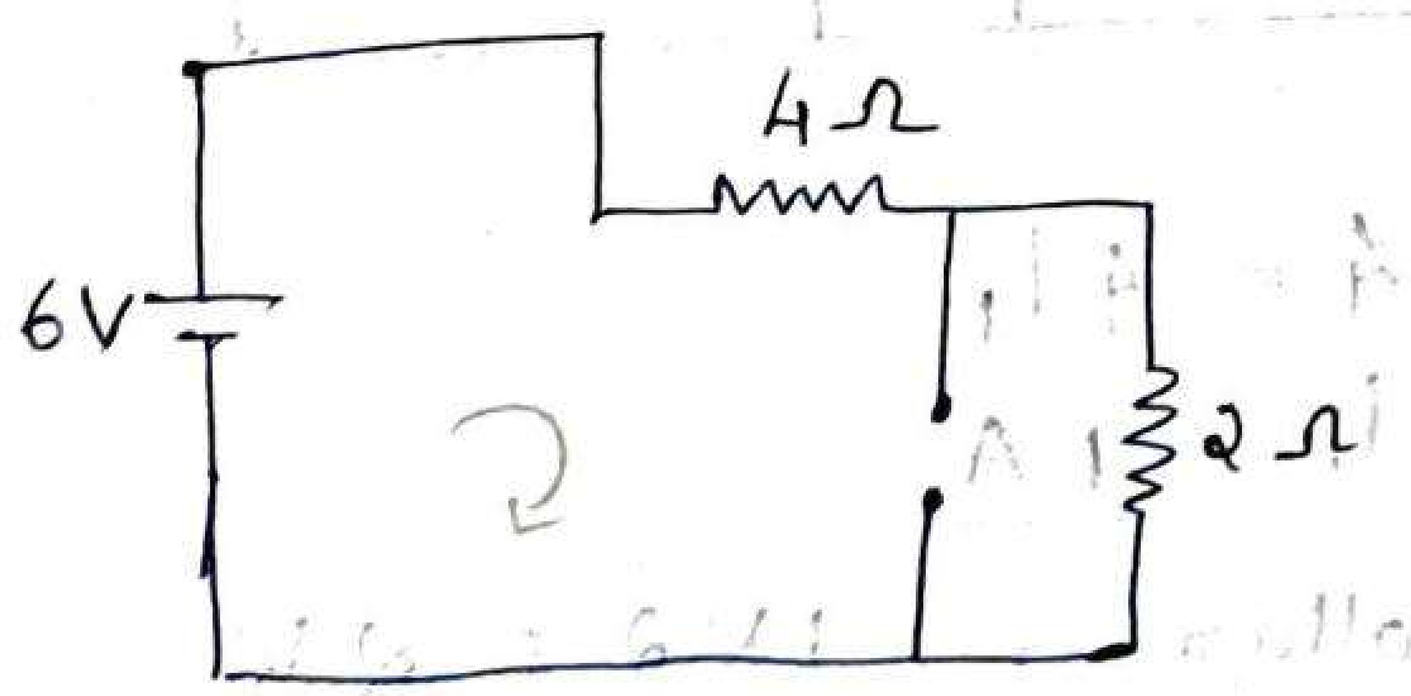
$$= \underline{\underline{e^{-t}}}.$$

12/10/2020

Q. In this network, the switch is moved from a to b at $t = 0$, find $v(t)$



at $t = 0$



Voltage = Total V × $\dfrac{\text{Resi of branch}}{\text{Sum of resistor}}$

$$= 6 \times \frac{2}{6}$$

$$= \underline{\underline{2\,V}}.$$

mesh $6 = 6I_1$

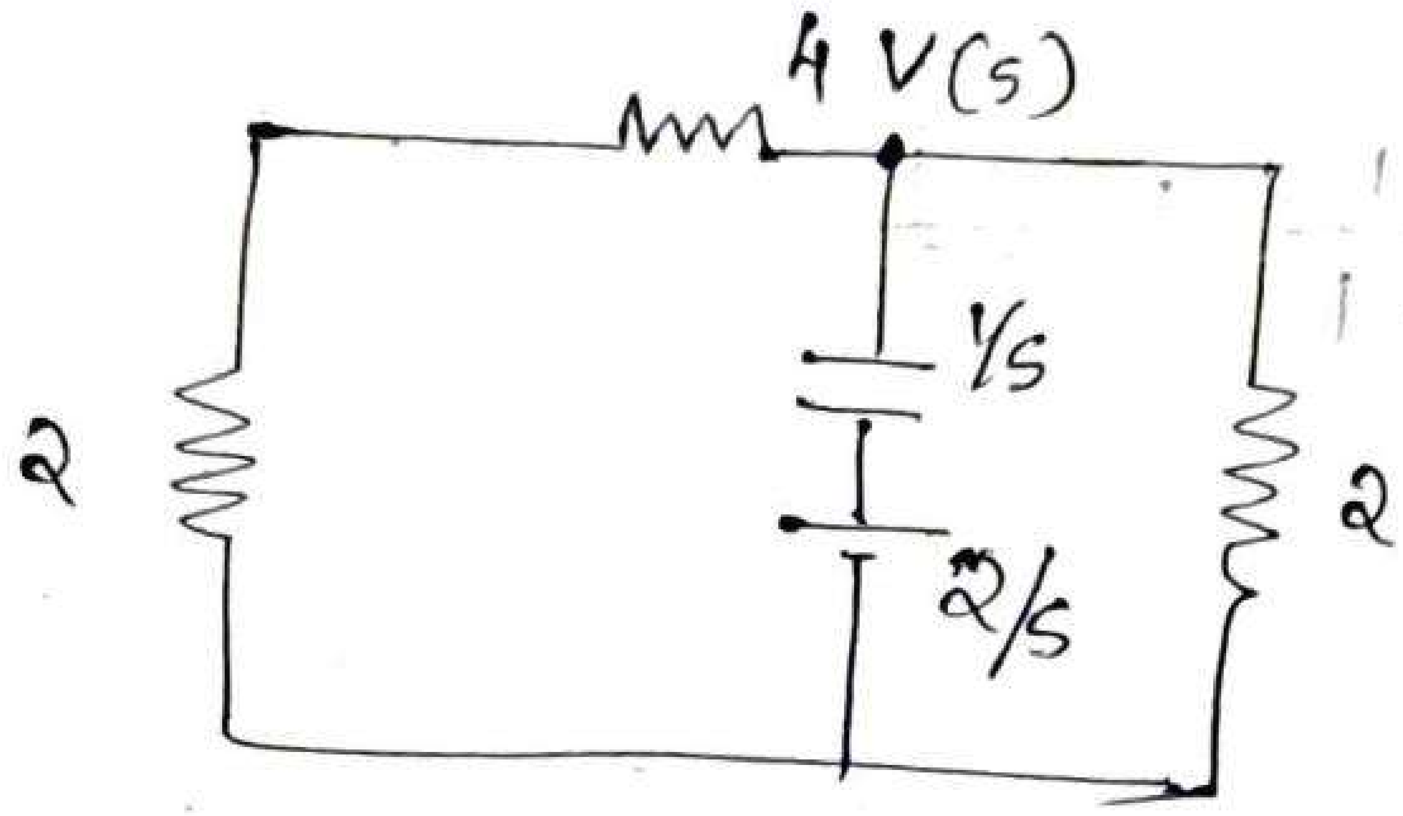
$I_1 = \underline{\underline{1\,A}}$.

V across 2 Ω

$= 2 \times 1 = \underline{\underline{2\,V}}$

Since the voltage across capacitor doesnot change

$V(0^+) = 2V.$



$$\frac{V(s)}{6} + \frac{V(s) - 2/s}{1/s} + \frac{V(s)}{2} = $$

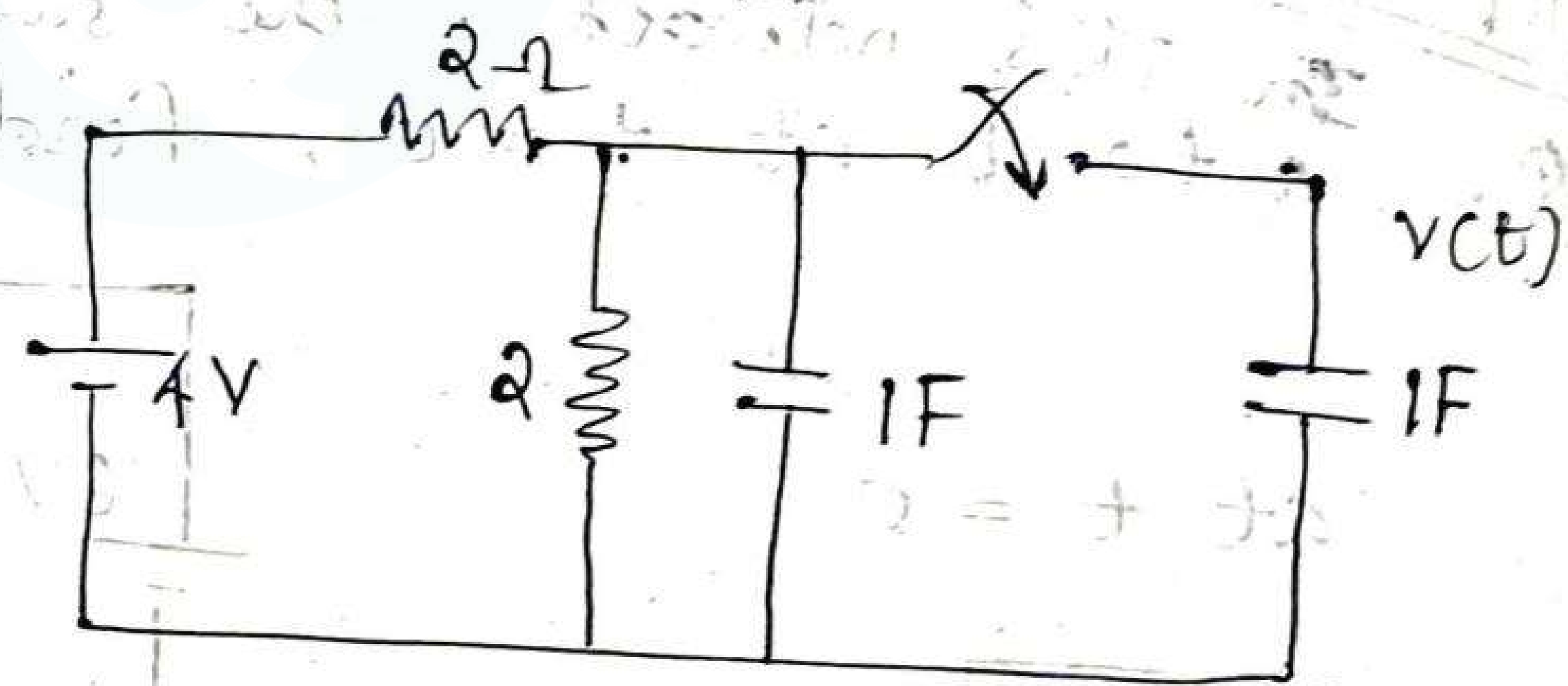
$$V(s)\left[\frac{1}{6} + \frac{1}{2}\right] + sV(s) = 2$$

$$V(s)\left[\frac{2}{3} + s\right] = 2.$$

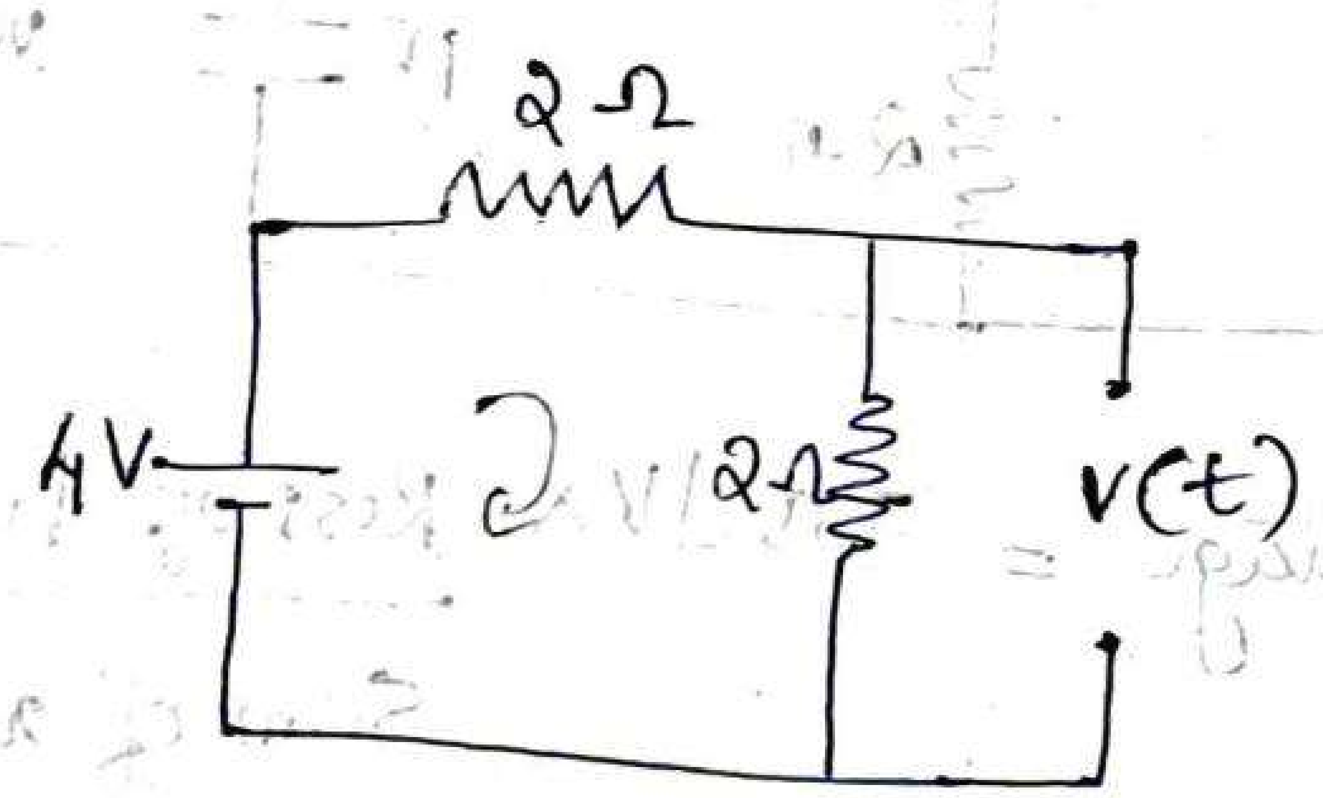
$$V(s) = \frac{2}{s + 2/3}$$

$$V(t) = 2e^{-2/3\, t}.$$

Q. The network shown has acquired steady state at $t < 0$ with the switch is open. The switch is closed at $t = 0$. Determine $V(t)$



at $t = (0^-)$



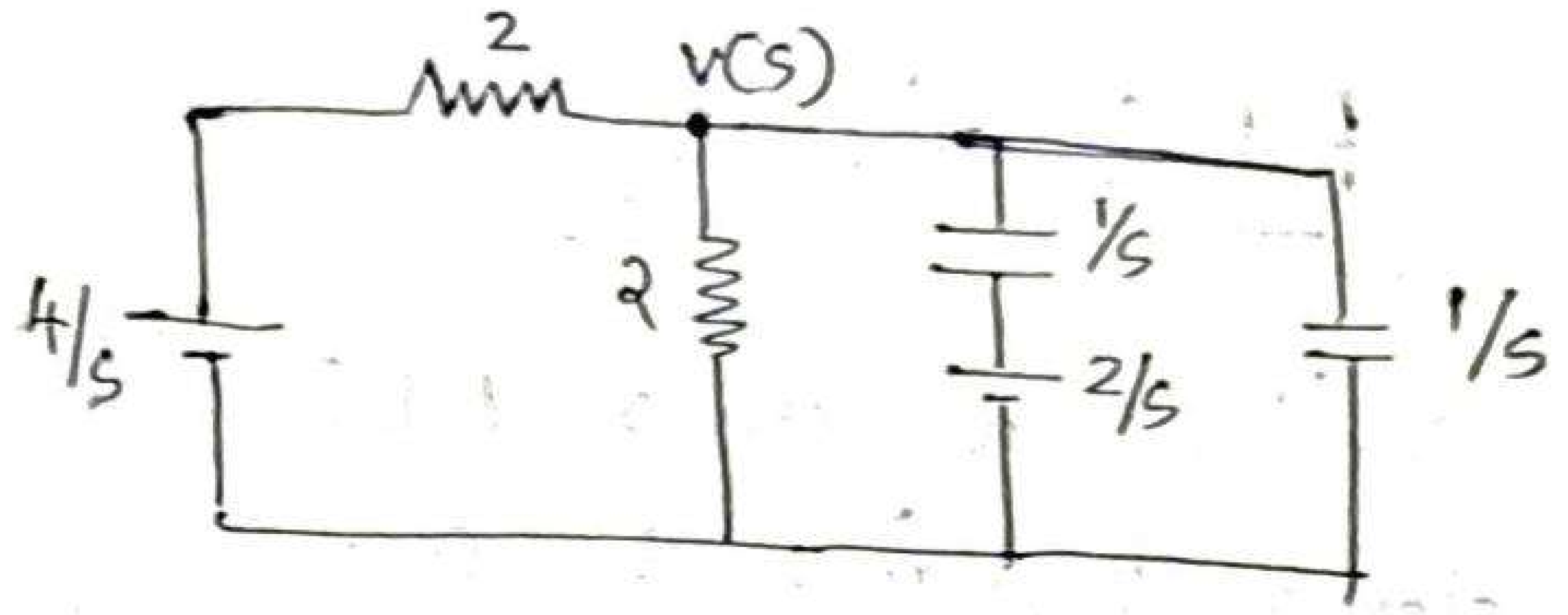
$4 = 4I_1$

$I_1 = 1\,A$

Voltage $= 1 \times 2 = 2V$

$V(0^-) = 2V.$

at $t = 0$



$$\frac{V(s) - 4/s}{2} + \frac{V(s)}{2} + \frac{V(s) - 2/s}{1/s} + \frac{V(s)}{1/s} = 0.$$

$$\frac{V(s)}{2} - \frac{4}{2s} + \frac{V(s)}{2} + sV(s) - 2 + sV(s) = 0.$$

$$\frac{2V(s)}{\cancel{2}} - \frac{\cancel{4}2}{\cancel{2}s} + 2sV(s) - 2 = 0.$$

$$2sV(s) + V(s) = \frac{2}{s} + 2.$$

$$V(s)\left[2s+1\right] = \frac{2(s+1)}{s}. \qquad \frac{2+2s}{s} \qquad \frac{2(1+s)}{s}$$

$$V(s) = \frac{2(s+1)}{s(2s+1)}$$

$$= \frac{2(s+1)}{2s(s+1/2)}$$

$$V(s) = \frac{s+1}{s(s+1/2)} = \frac{A}{s} + \frac{B}{(s+1/2)}$$

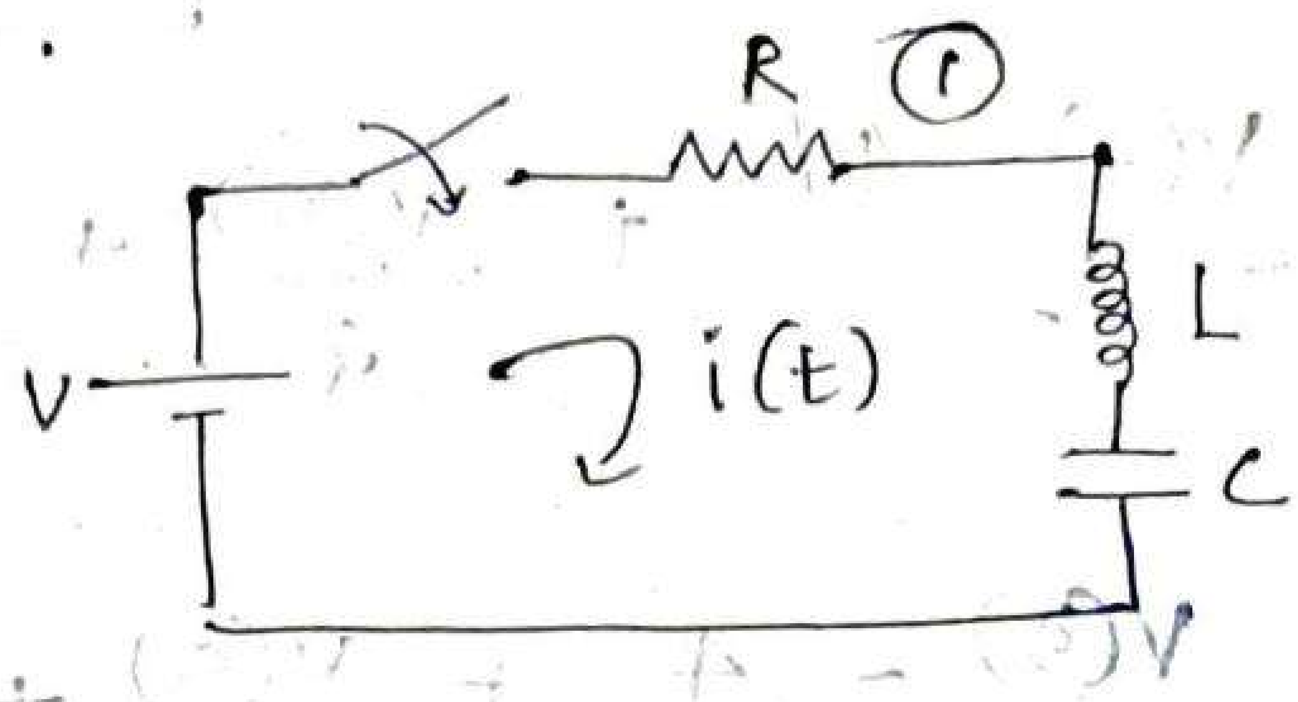
$$A = \left.\frac{s+1}{s+1/2}\right|_{s=0} = \frac{1}{1/2} = \underline{\underline{2}}$$

$$B = \left.\frac{s+1}{s}\right|_{s=-1/2} = \frac{1/2}{-1/2} = \underline{\underline{-1}}$$

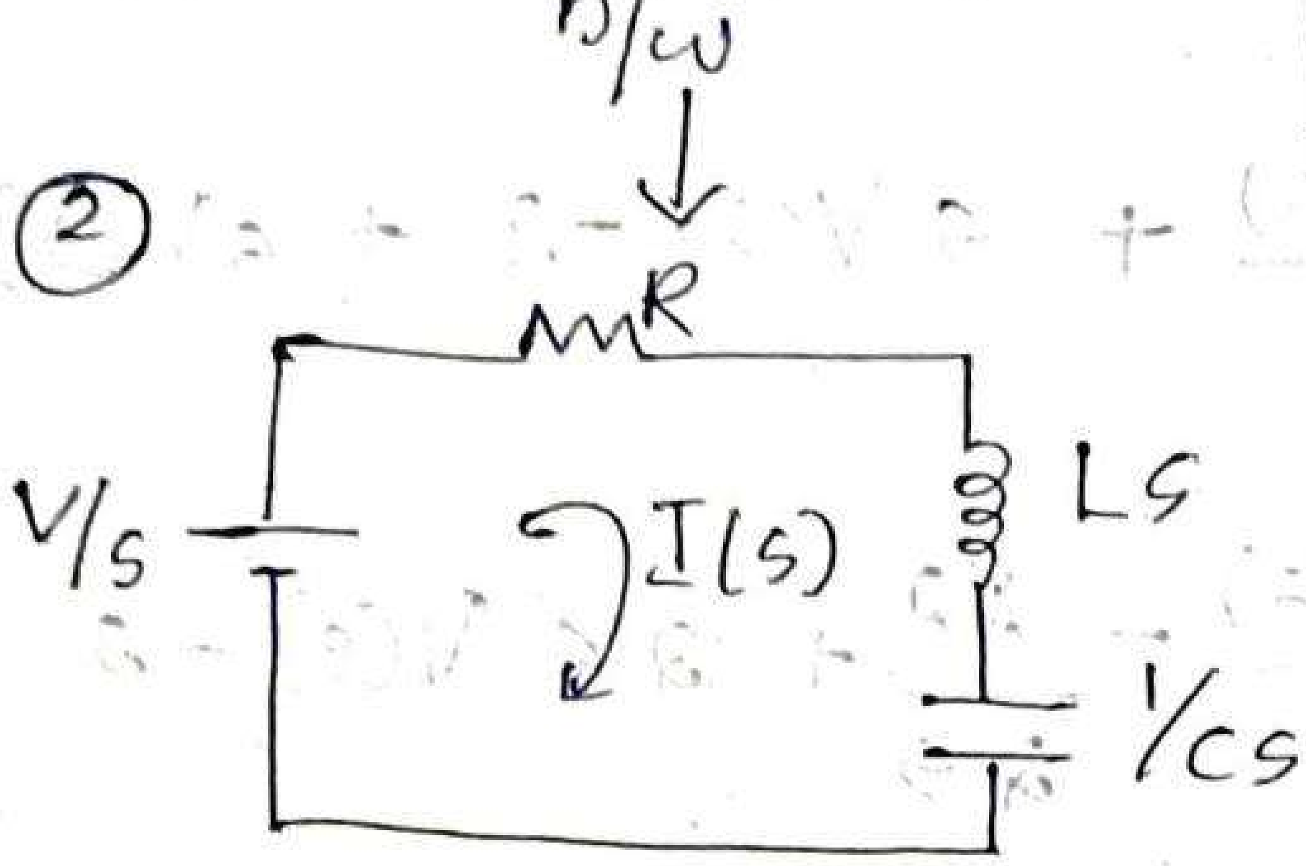
$$V(s) = \frac{2}{s} - \frac{1}{s+1/2} \qquad \underline{\underline{v(t) = 2 - e^{-1/2\,t}}}$$

RLC circuit

Consider a series RLC circuit. The switches is closed at time $t = 0$.



For $t > 0$, the transfor n/w



Applying KVL

$$\frac{V}{S} = R\,I(S) + LS\,I(S) + \frac{1}{CS} I(S)$$

$$\frac{V}{S} = I(S)\left[R + LS + \frac{1}{CS}\right]$$

$$\frac{V}{\cancel{S}} = I(S)\left[\frac{RCS + LCS^2 + 1}{C\cancel{S}}\right]$$

$$I(S) = \frac{VC}{RCS + LCS^2 + 1}$$

$$= \frac{VC}{LCS^2 + RCS + 1}$$

$$I(S) = \frac{VC}{LC\left[S^2 + \frac{R}{L}S + \frac{1}{LC}\right]}$$

$$= \frac{V/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{V/L}{(s-a)(s-b)}$$

a and b are roots of $s^2 + \frac{R}{L} + \frac{1}{LC}$

$$a,b = \frac{-R/L \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4 \times \frac{1}{LC}}}{2}$$

$$= -R/2L \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$a = -R/2L + \sqrt{(R/2L)^2 - \frac{1}{LC}}$$

$$= -\alpha + \sqrt{\alpha^2 - \omega_0^2} = \underline{\underline{-\alpha + \beta}}$$

$$b = -R/2L - \sqrt{(R/2L)^2 - \frac{1}{LC}}$$

$$= -\alpha - \sqrt{\alpha^2 - \omega_0^2} = \underline{\underline{-\alpha - \beta}}$$

$\alpha = \frac{R}{2L}$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$ $\quad \beta = \sqrt{\alpha^2 - \omega_0^2}$

$$I(s) = \frac{A}{s-a} + \frac{B}{s-b}.$$

$$A = (s-a)\, I(s)\Big|_{s=a} = \frac{V/L}{(s-a)(s-b)}\Big|_{s=a}$$

$$= \frac{V/L}{a-b}.$$

$$B = (s-b)\, I(s)\Big|_{s=b} = \frac{V/L}{(s-a)(s-b)}\Big|_{s=b}.$$

$$= \frac{V/L}{b-a}.$$

$$= \frac{-V/L}{a-b}$$

$$I(s) = \frac{V/L/a-b}{s-a} - \frac{V/L/a-b}{s-b}.$$

$$= \frac{V}{L(a-b)}\left[\frac{1}{s-a} - \frac{1}{s-b}\right]$$

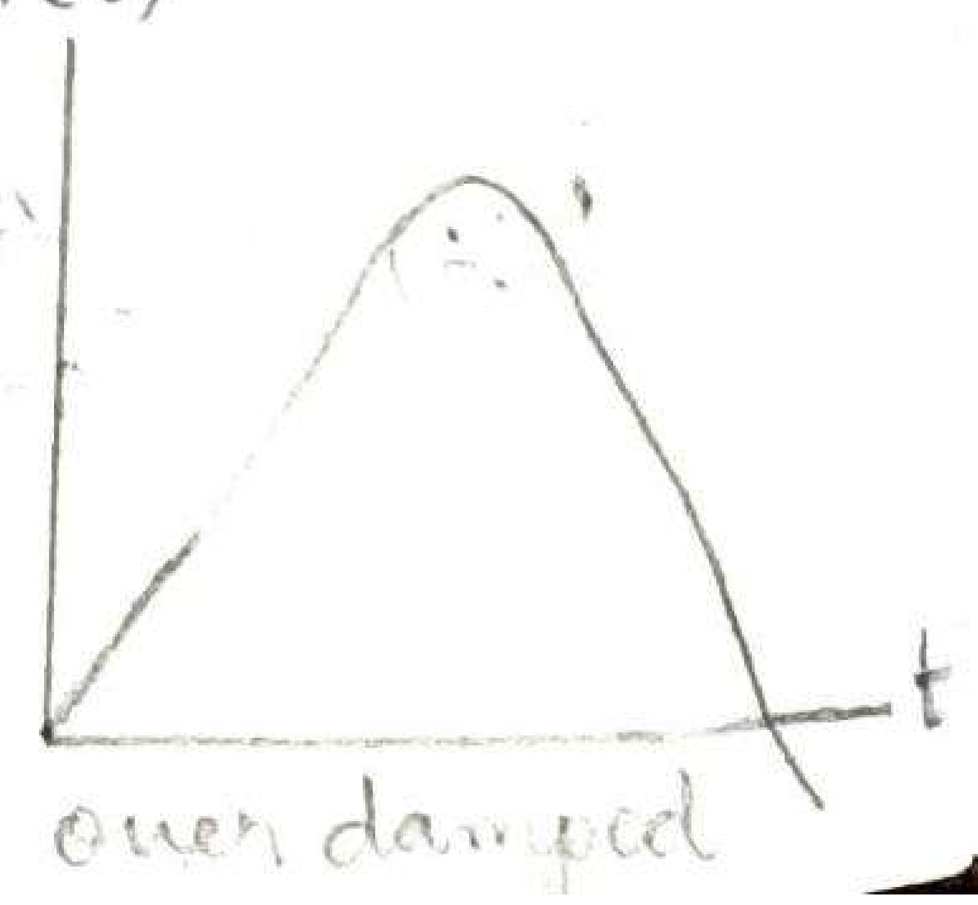
$$i(t) = \frac{V}{L(a-b)}\left[e^{at} - e^{bt}\right]$$

$$= \underline{\underline{k_1 e^{at} - k_2 e^{bt}}}.$$

13/10/2020

Now depending upon the values of a and b.

<u>Case 1</u>; when the roots are real and unequal, it gives an overdamped response



over damped

$$\frac{R}{2L} > \frac{1}{\sqrt{LC}}$$

$$\alpha > \omega_0$$

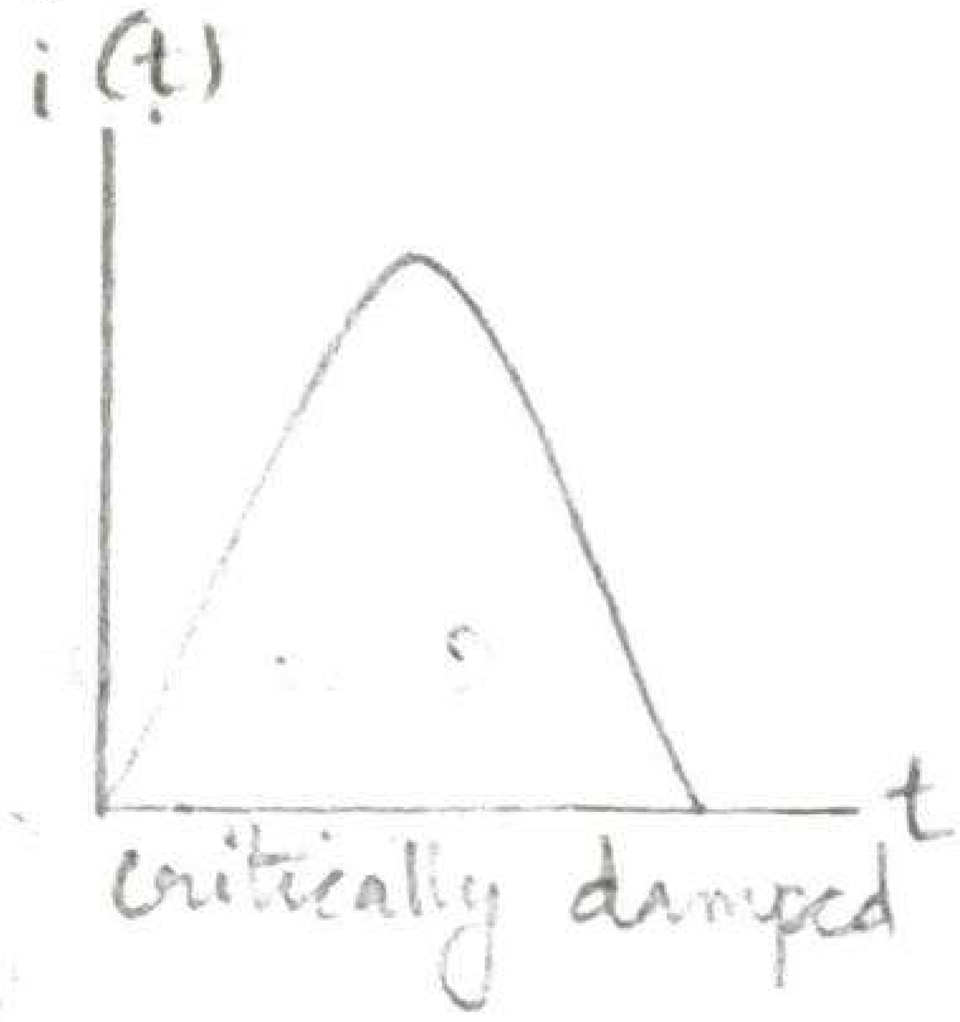
$$i(t) = K_1 e^{at} + K_2 e^{bt}$$

Case II ; when the roots are real and equal, it gives a critically damped response

$$\frac{R}{2L} = \frac{1}{\sqrt{LC}}$$

$$\alpha = \omega_0$$

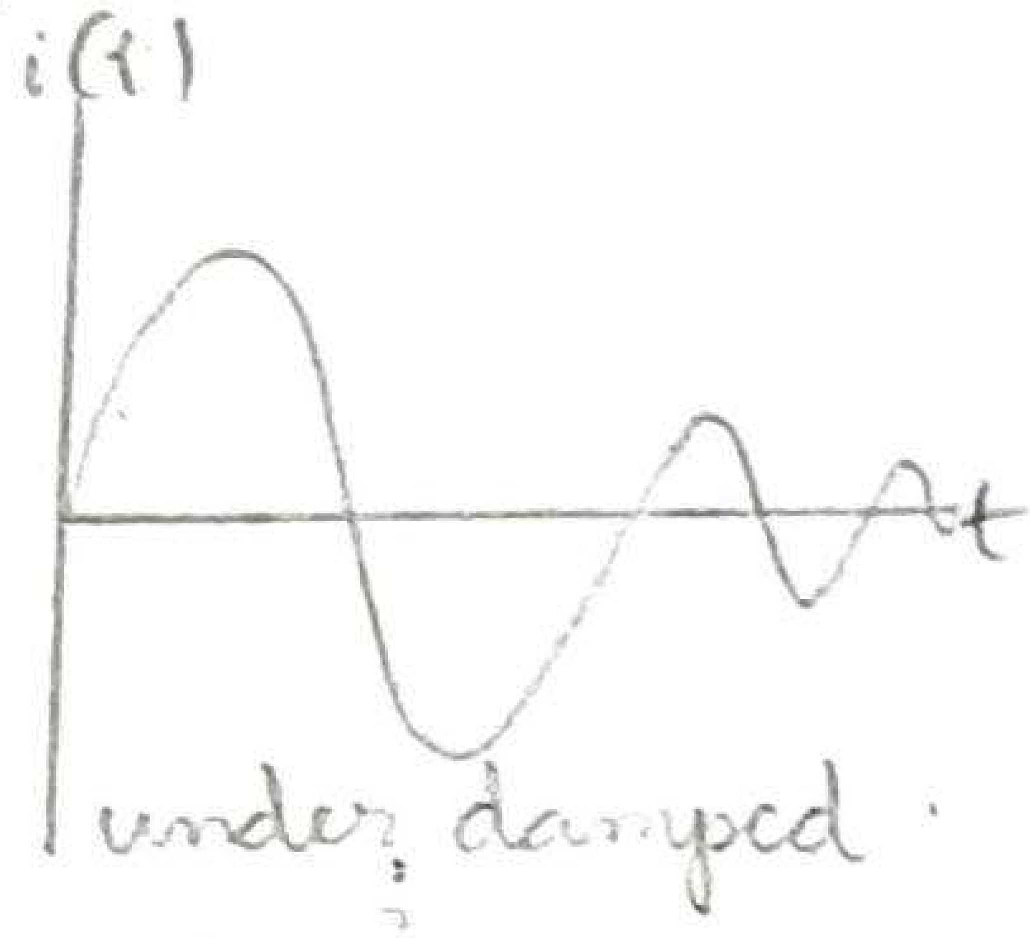
$$i(t) = e^{-\alpha t}\left[K_1 + K_2 t\right]$$



Case III ; when the roots are complex conjugate, it gives an under damped response

$$\frac{R}{2L} < \frac{1}{\sqrt{LC}}$$

$$i(t) = k_1 e^{at} + k_2 e^{bt}$$



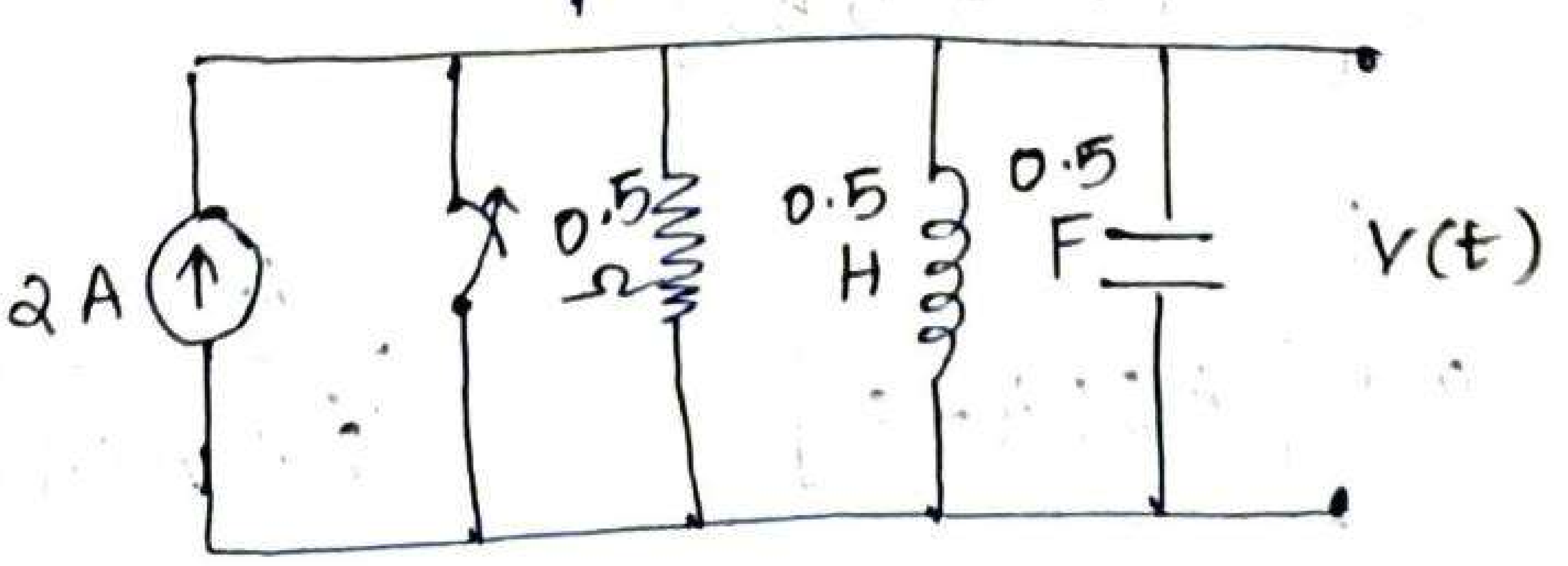
$$a, b = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\sqrt{\alpha^2 - \omega_0^2} = \sqrt{-1}\sqrt{\omega_0^2 - \alpha_2^2} = j\,\omega d.$$

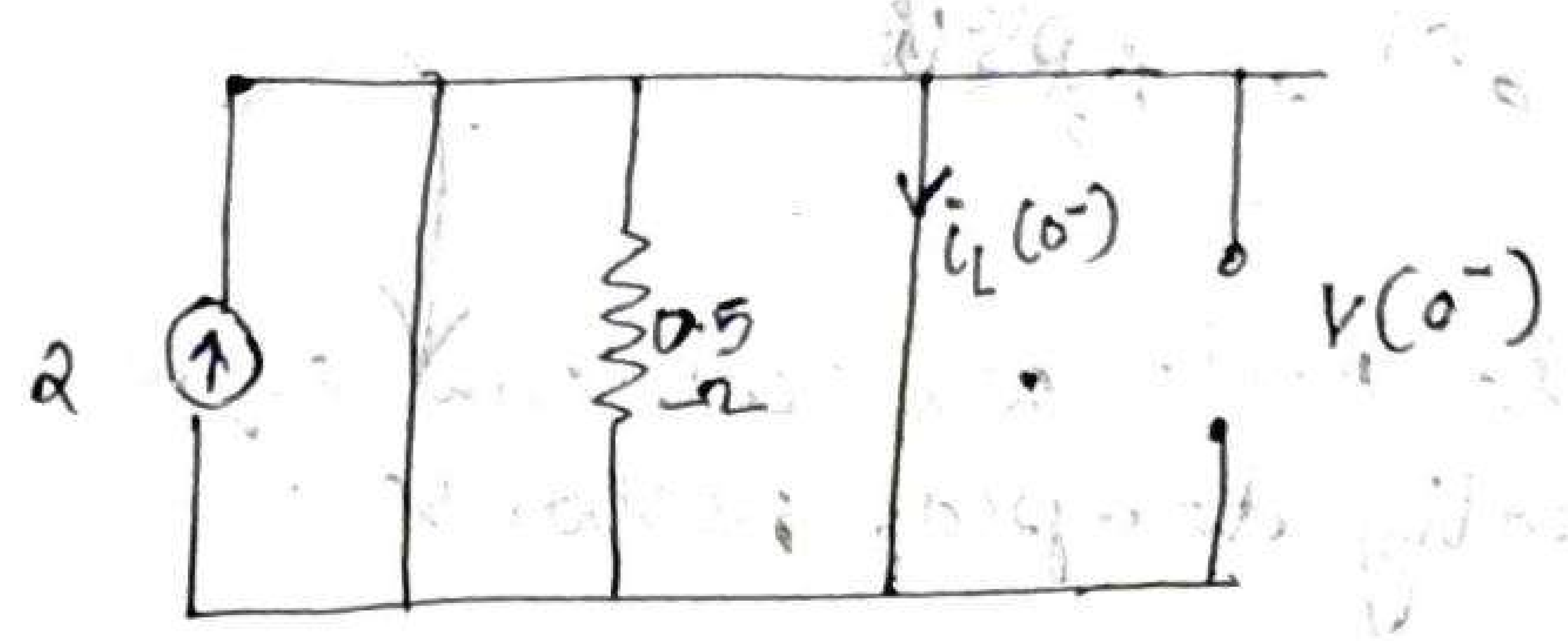
$$\omega d = \sqrt{\omega_0^2 - \alpha^2}$$

$$i(t) = e^{-\alpha t}\left[k_1 e^{j\omega d t} + k_2 e^{-j\omega d t}\right]$$

Q. The switch is open at $t=0$, Determine $V(t)$



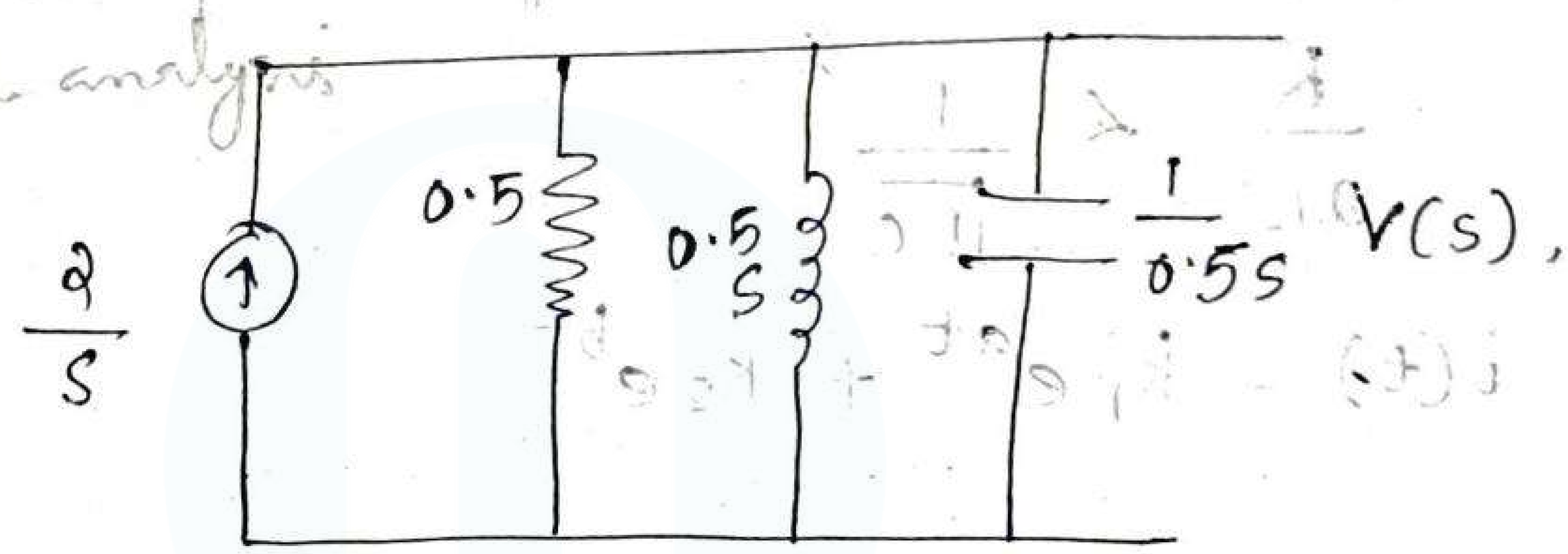
llel n/w



$$i_L(0^-) = 0$$
$$V(0^-) = 0$$

Since the current through inductor and capacitor cannot change instantaniously

$$i_L(0+) = 0$$
$$V(0^+) = 0$$



$$\frac{2}{s} = \frac{V(s)}{0.5} + \frac{V(s)}{0.5s} + \frac{V(s)}{1/0.5s}$$

$$= 2V(s) + \frac{2V(s)}{s} + 0.5sV(s)$$

$$\boxed{\frac{2}{s} = V(s)\left[2 + \frac{2}{s} + 0.5s\right]}$$

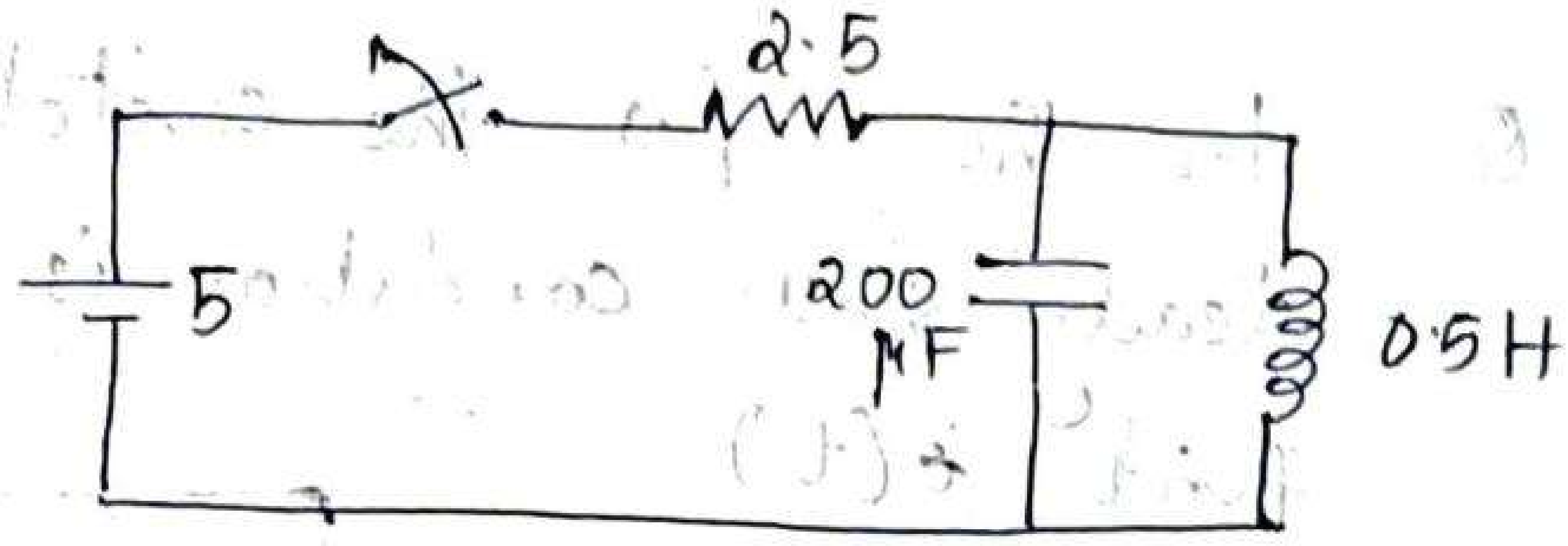
$$V(s) = \frac{2}{s\left[2 + \frac{2}{s} + 0.5s\right]}$$

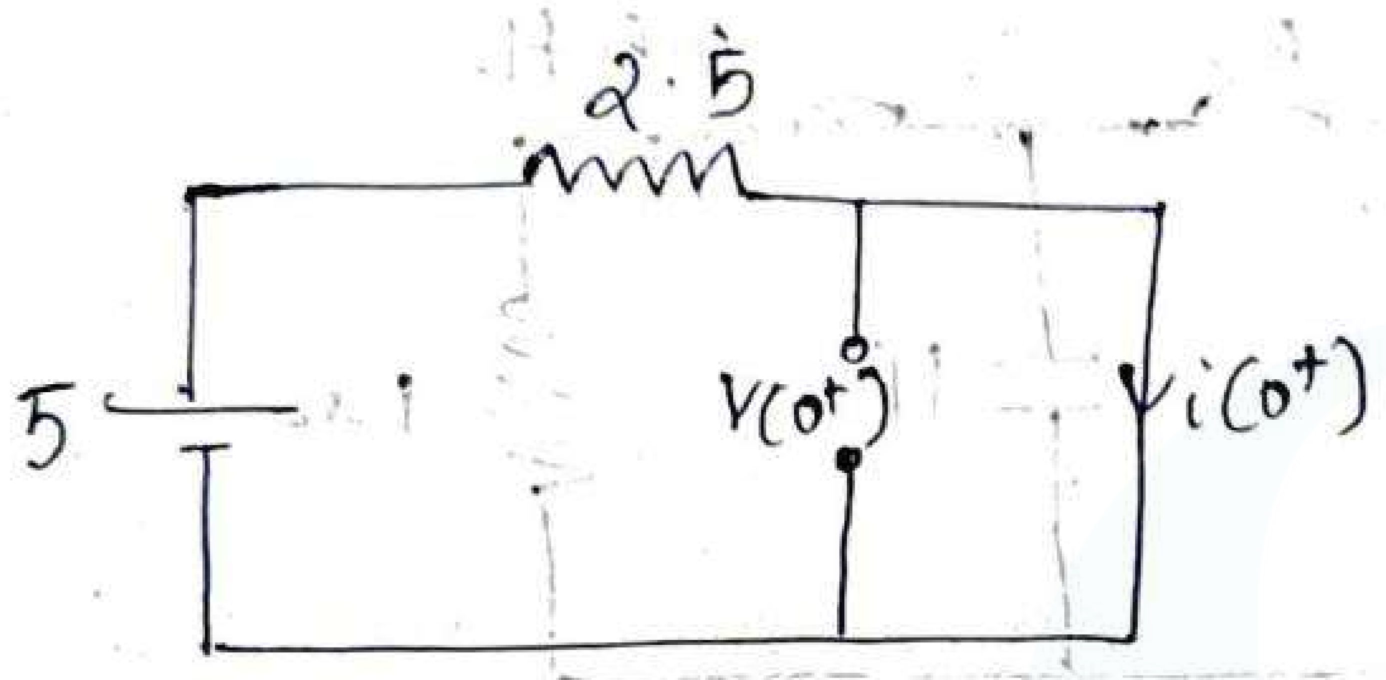
$$= \frac{2}{0.5\left[4s + 4 + s^2\right]} = \frac{4}{s^2 + 4s + 4}$$

$$= \frac{4}{(S+2)^2}$$

$$v(t) = \underline{\underline{4t\,e^{-2t}}}.$$

2) In the n/w, the switch is closed and steady state o is attained. At $t=0$, switch is opened. Determine current through inductor.





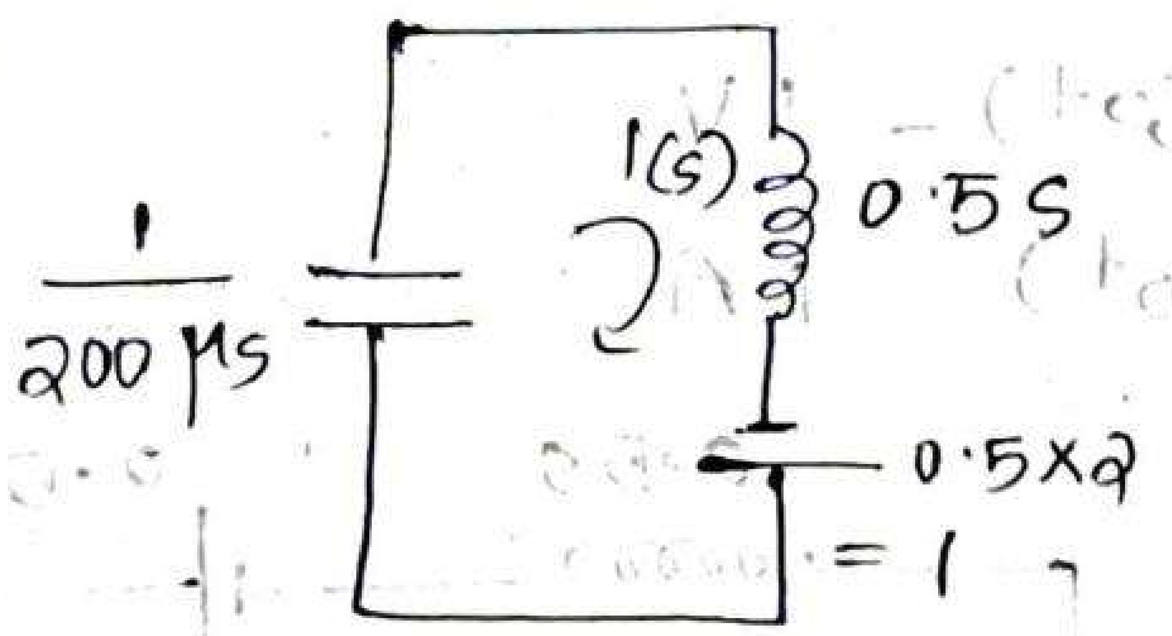
$$i_L(0^-) = \frac{5}{2.5} = 2A$$

$V_C(0^-) = 0$ (it is connected || to short)

$i_L(0^+) = 2.$

$V_C(0^+) = 0$

At $t > 0$



$$1 = \frac{1}{200\mu s} I(s) + 0.5\,s\,I(s)$$

$$= I(s)\left[\frac{1}{200\times10^{-6}} + 0.5s\right]$$

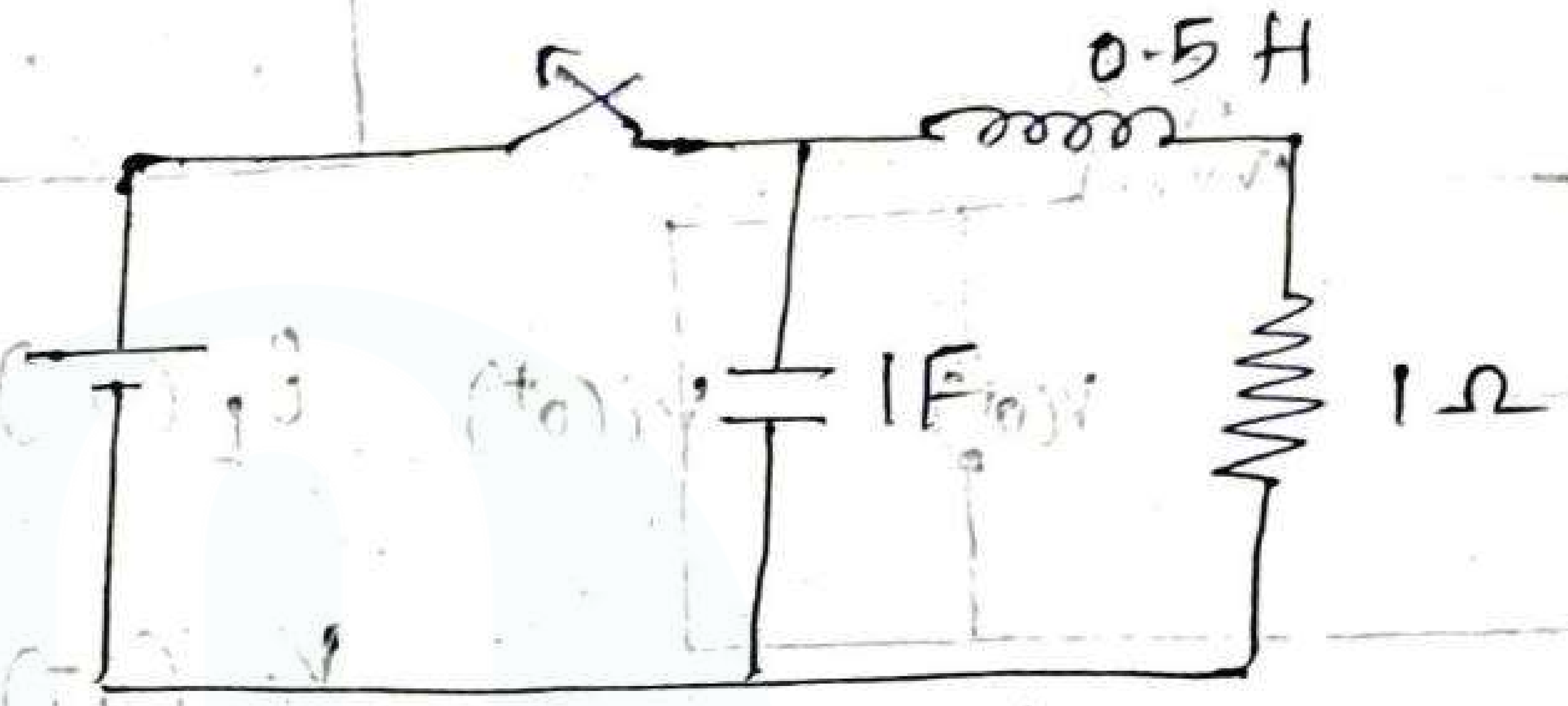
$$= I(s)\left[\frac{5000}{s} + 0.5s\right]$$

$$I(s) = \frac{1}{\frac{5000+0.5s^2}{s}} = \frac{s}{0.5\left[s^2+10000\right]}$$

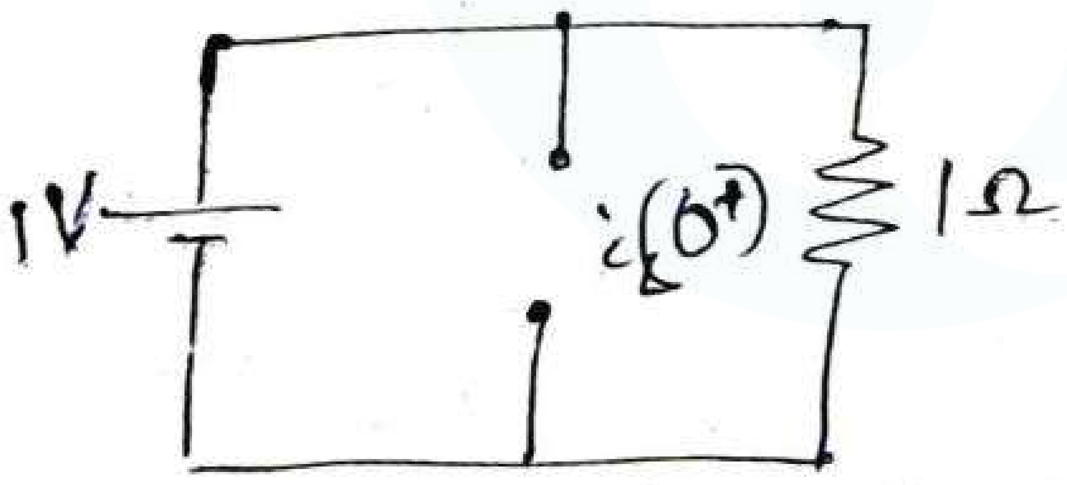
$$= \frac{2s}{s^2+10000}$$

$$i(t) = 2\cos 100t$$

Q. In the n/w the switch is opened at $t=0$, steady state condition is acheived before $t=0$. Find $i(t)$



At $t=0^-$



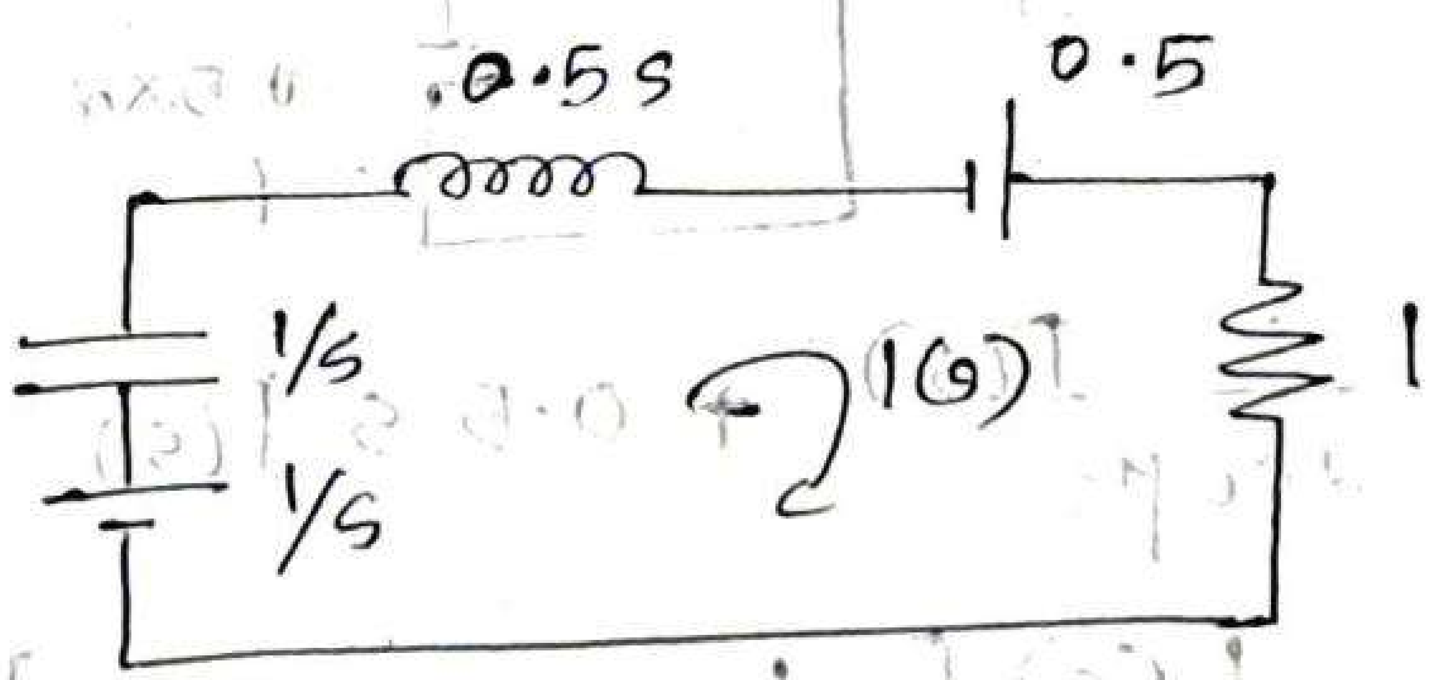
$V_C(0^-) = 1\,V$

$i_L(0^-) = 1\,A$.

$V_C(0+) = 1\,V$

$i_L(0^+) = 1\,A$.

At $t>0$



$$\frac{1}{s} + 0.5 = \frac{1}{s}I(s) + 0.5s\,I(s) + I(s)$$

$$\frac{1+0.5}{s} = I(s)\left[\frac{1+0.5s^2+s}{s}\right]$$

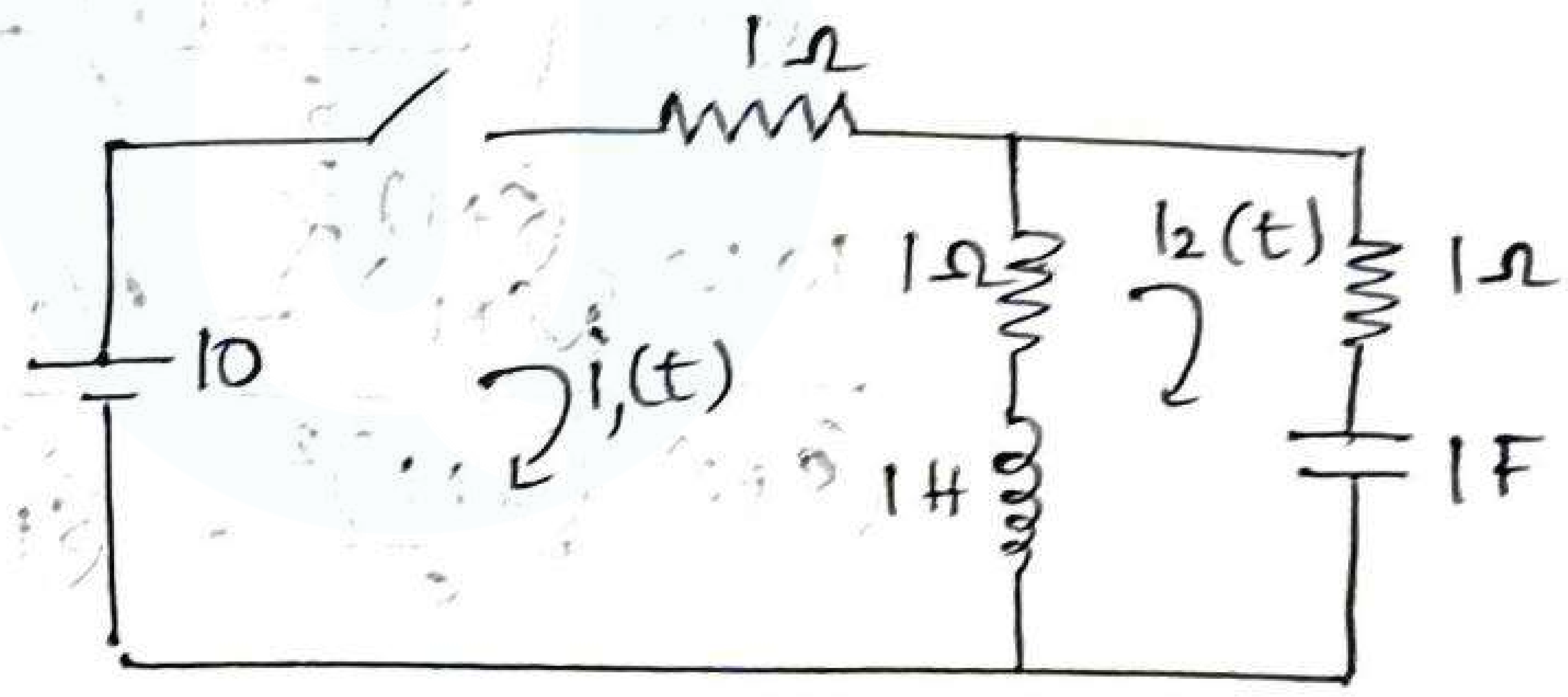
$$I(s) = \frac{0.5s+1}{0.5s^2+s+1} = \frac{\cancel{0.5}(s+2)}{\cancel{0.5}(s^2+2s+2)}$$

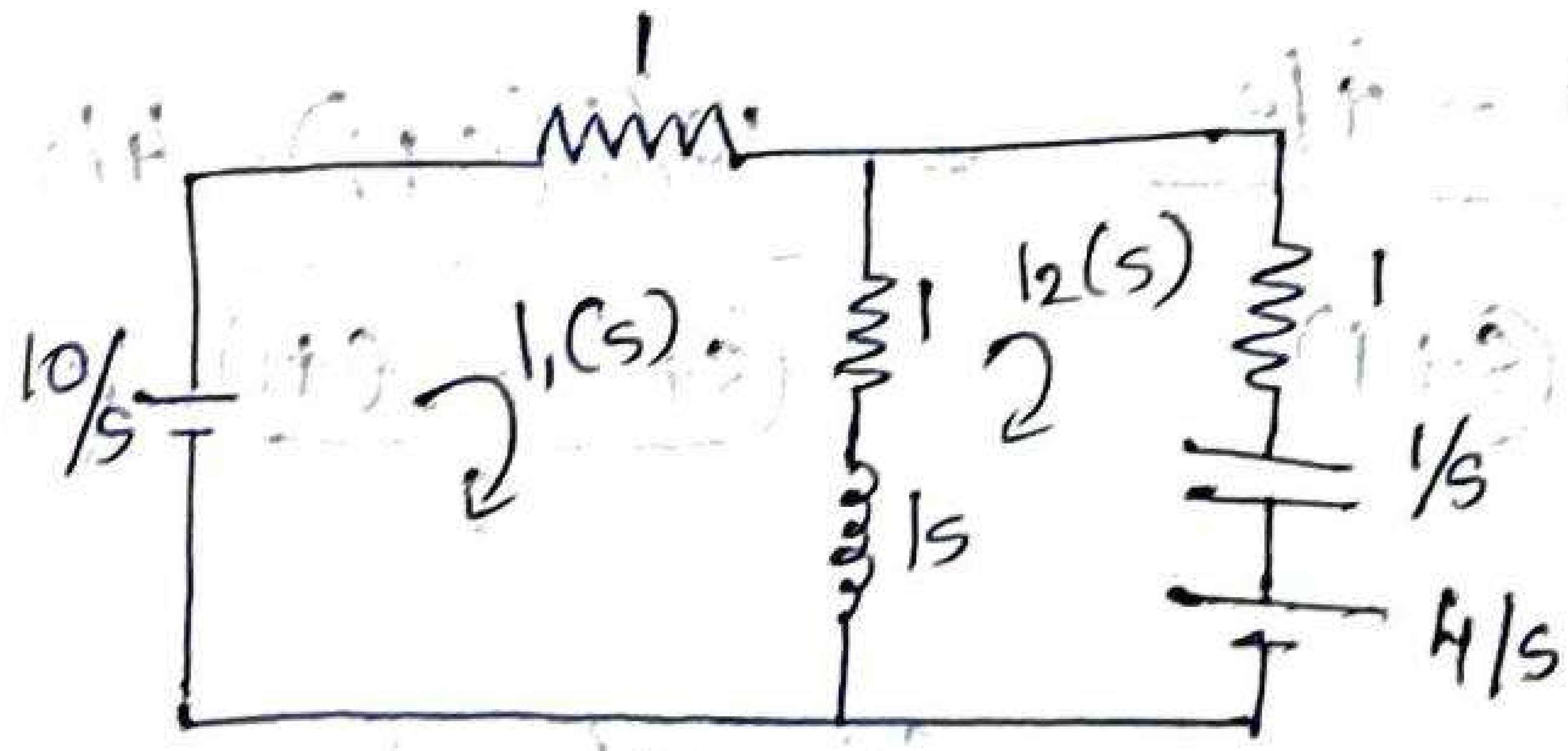
$$= \frac{s+2}{s^2+2s+2} = \frac{s+1+1}{(s+1)^2+1}$$

$$I(s) = \frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}$$

$$I(t) = \underline{e^{-t}\cos t + e^{-t}\sin t}$$

Q4, In the n/w the switch is closed at $t=0$, Find the current $i_1(t)$ when the initial current through inductor is 0 and initial voltage on capacitor is 4 V.





$$\cancel{\frac{10}{s} = I_1(s) + sI_1(s) + I_1(s) - I_2(s) + I_2(s)}$$

$$\frac{10}{s} = [2+s]\, I_1(s) - (1-s)\, I_2(s)$$

$$\frac{-4}{s} = -(1+s)\, I_1(s) + \left(2+s+\frac{1}{s}\right) I_2(s).$$

$$\begin{bmatrix} s+2 & -(s+1) \\ -(s+1) & (2+s+1/s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} 10/s \\ -4/s \end{bmatrix}$$

$$I_1(s) = \frac{\begin{vmatrix} 10/s & -(s+1) \\ -4/s & s+2+1/s \end{vmatrix}}{\begin{vmatrix} (s+2) & -(s+1) \\ -(s+1) & (s+2+1/2) \end{vmatrix}}$$

$$= \frac{10/s\,(s+2+1/s) - -(s+1)(-4/s)}{(s+2)(s+2+1/s) - -(s+1) - (s+1)}$$

$$= \frac{10/s^2\,[s^2+2s+1] - \frac{4}{s}(s+1)}{(s+2)\left(\frac{s^2+2s+1}{s}\right) - (s+1)^2}$$

$$= \frac{10/s^2\,\cancel{(s+1)^2}(s+1)^2 - 4/s\,(s+1)}{(s+2)\frac{(s+1)^2}{s} - \cancel{(s+1)^2}} = (s+1)$$

$$= \frac{10/s^2\,(s+1) - 4/s}{\left[\frac{s+2}{s} - 1\right](s+1)} = \frac{10/s^2(s+1) - 4/s}{\frac{(s+2) - (s+1)}{s} - (s+}$$

$$= \frac{\frac{10s+10-4s}{s^2}}{4s+2/s} = \frac{6s+10/s^2 \times s}{4s+2}$$

$$= \frac{3s+5}{s(s+1)}$$

$$\frac{3s+5}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$A = \left.\frac{3s+5}{s+1}\right|_{s=0} = \underline{\underline{5}}$$

$$B = \left.\frac{3s+5}{s}\right|_{s=-1} = \frac{-3+5}{-1} = \underline{\underline{-2}}$$

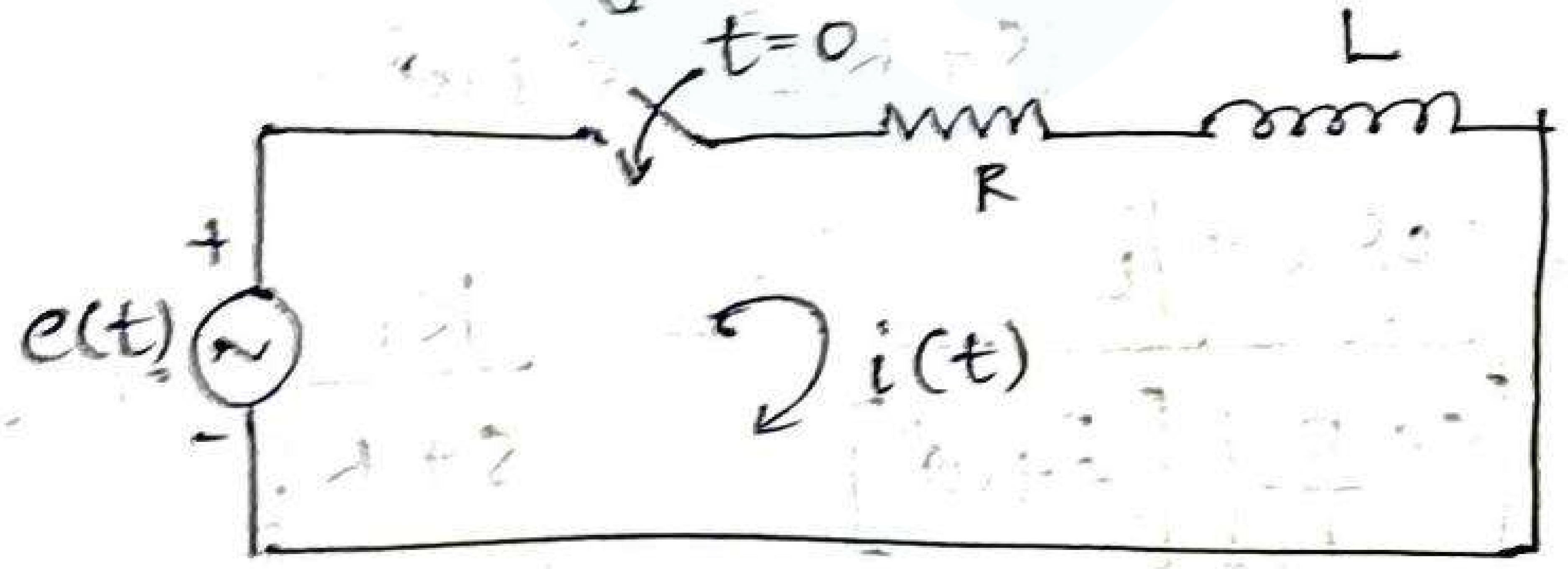
$$I(s) = \frac{5}{s} + \frac{-2}{s+1}$$

$$i(t) = \underline{\underline{5 - 2e^{-t}}}$$

14/10/2020

Transients in circuits excited by AC source.

RL circuit excited by AC source



Consider the RL circuit. Let the circuit is excited by sinusoidal source $e(t) = E_m \sin \omega t$. When the switch is closed at $t=0$, let $i(t)$ be the current through the circuit. Let us assume that there is no stored energy in the circuit.

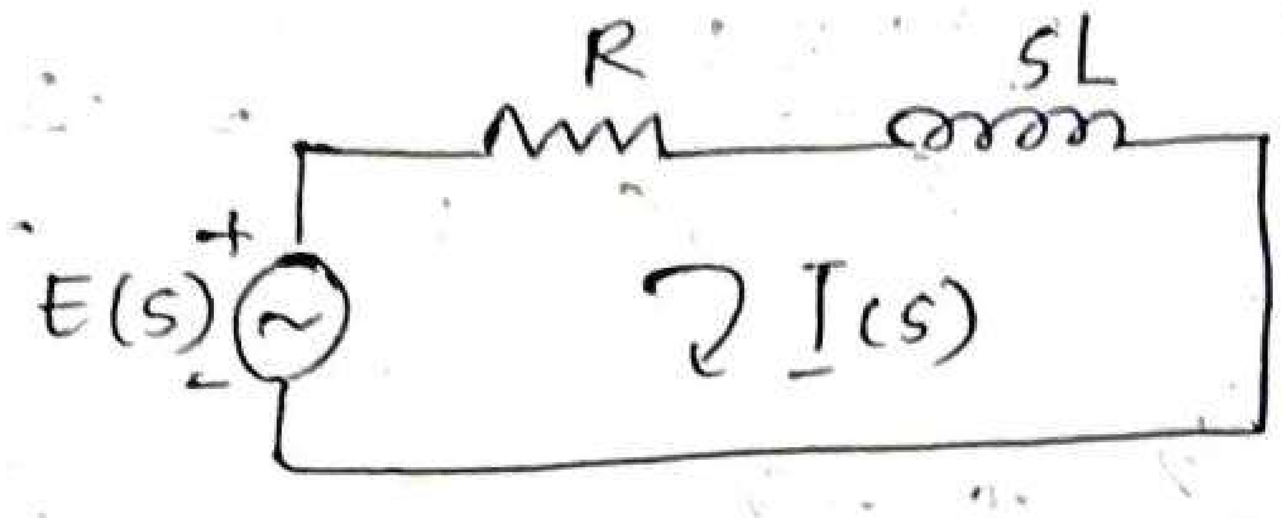
$$L[i(t)] = I(s)$$

$$L[e(t)] = E(s)$$

$$E(s) = L[e(t)] = L[E_m \sin \omega t]$$

$$= E_m \cdot \frac{\omega}{s^2+\omega^2}$$

The s domain RL circuit is shown below



$$[R+Ls]\ I(s) = E(s)$$

$$I(s) = \frac{E(s)}{R+Ls} = E(s) \cdot \frac{1}{L[s+R/L]}$$

$$= \frac{\omega E_m}{s^2+\omega^2} \times \frac{1}{L[s+R/L]}$$

$$= \frac{\omega E_m / L}{(s+R/L)(s^2+\omega^2)}$$

$$I(s) = \frac{\omega E_m / L}{\left[s+\frac{R}{L}\right]\left[s^2+\omega^2\right]} = \frac{K_1}{s+R/L} + \frac{K_2 s + K_3}{s^2+\omega^2}$$

$ax+b$

$$K_1 = \frac{\omega E_m/L}{\cancel{(s+R/L)}(s^2+\omega^2)} \times \cancel{(s+R/L)} \Bigg|_{s=-R/L}$$

$$= \frac{\omega E_m/L}{-R/L^2+\omega^2} = \frac{\omega E_m/L}{R^2/L^2+\omega^2}$$

$$= \frac{\omega E_m / \cancel{L}}{\frac{R^2+\omega^2 L^2}{L^{\cancel{2}}}}$$

$$= \frac{\omega L E_m}{R^2 + \omega^2 L^2}.$$

In frequency domain, magnitude of impedence of RL circuit is given by

$$Z = R + jX_L \qquad X_L = \omega L$$

$$= \sqrt{R^2 + \omega^2 L^2}$$

$$Z^2 = R^2 + \omega^2 L^2$$

$$\boxed{K_1 = \frac{\omega L E_m}{Z^2}}$$

$$\frac{\omega E_m / L}{\left(S + \frac{R}{L}\right)\left(S^2 + \omega^2\right)} = \frac{K_1}{S + R/L} + \frac{K_2 S + K_3}{S^2 + \omega^2}$$

$$\frac{\omega E_m / L}{\cancel{(S + R/L)(S^2 + \omega^2)}} = \frac{K_1 (S^2 + \omega^2) + (K_2 S + K_3)(S + R/L)}{\cancel{(S + R/L)(S^2 + \omega^2)}}$$

$$\frac{\omega E_m}{L} = K_1 S^2 + K_1 \omega^2 + K_2 S^2 + K_2 S\, R/L + K_3 S + K_3 R/L$$

$$= (K_1 + K_2) S^2 + [K_2 R/L + K_3] S + K_1 \omega^2 + K_3 R/L$$

coeff: of s^2

$$K_1 + K_2 = 0$$

$$K_2 = -K_1$$

$$\boxed{K_2 = \frac{-\omega L E_m}{Z^2}}$$

Equating coeff: of s ..

$$K_2 \cdot \frac{R}{L} + k_3 = 0$$

$$-\frac{\omega L E_m}{Z^2} \times \frac{R}{L} = -k_3$$

$$K_3 = \frac{\omega E_m R}{Z^2}$$

$$I(s) = \frac{\frac{\omega L E_m}{Z^2}}{s + R/L} + \frac{-\frac{\omega L E_m}{Z^2} s + \frac{\omega R E_m}{Z^2}}{s^2 + \omega^2}$$

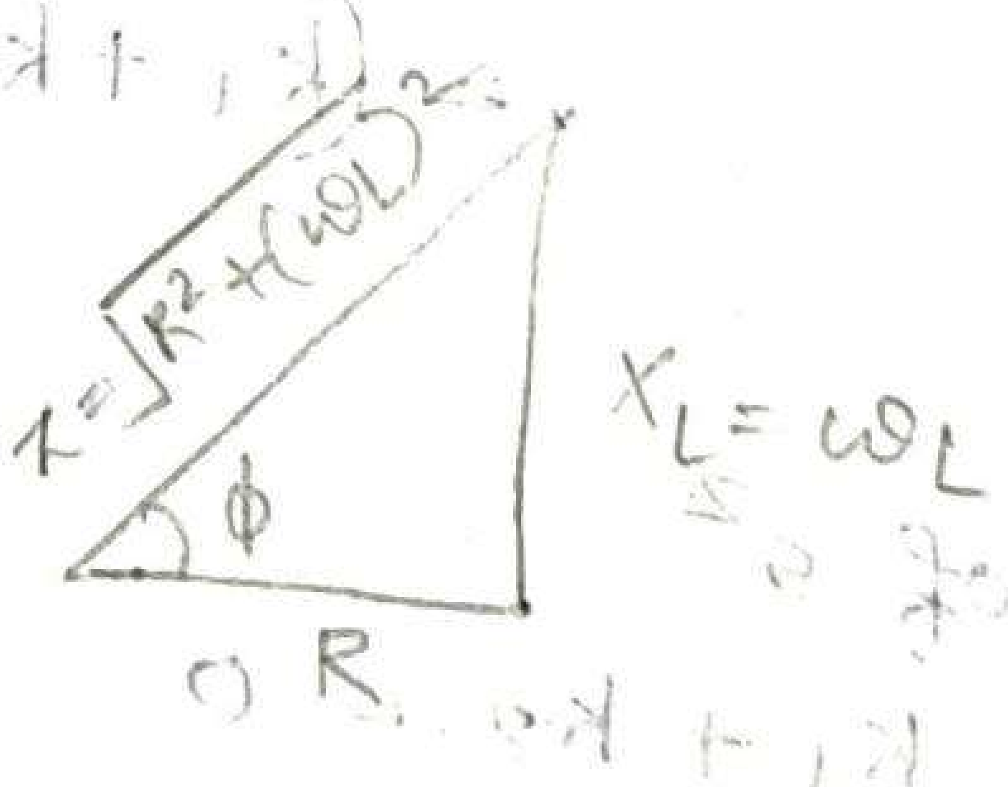
$$= \frac{\omega L E_m}{Z^2} \times \frac{1}{s + R/L} - \frac{\omega L E_m}{Z^2} \cdot \frac{s}{s^2+\omega^2} + \frac{R E_m}{Z^2} \times \frac{\omega}{s^2+\omega^2}$$

$$i(t) = \frac{\omega L E_m}{Z^2} \times e^{-R/L\, t} - \frac{\omega L E_m}{Z^2} \cos\omega t + \frac{R E_m}{Z^2} \sin\omega t$$

$$i(t) = \frac{\omega L E_m}{Z^2} e^{-R/L\, t} + \frac{E_m}{Z^2}\left[R \sin\omega t - \omega \cos\omega t\right]$$

$$\tan\phi = \frac{\omega L}{R}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$



$$\cos\phi = R/Z$$

$$R = Z\cos\phi$$

$$\sin\phi = \frac{\omega L}{Z}$$

$$\omega L = Z \sin\phi$$

$$i(t) = \frac{\omega L\, E_m}{Z^2} e^{-R/L\, t} + \frac{E_m}{Z^2}\left[\sin\omega t \cdot Z\cos\phi - Z\sin\phi \cdot \cos\omega t\right]$$

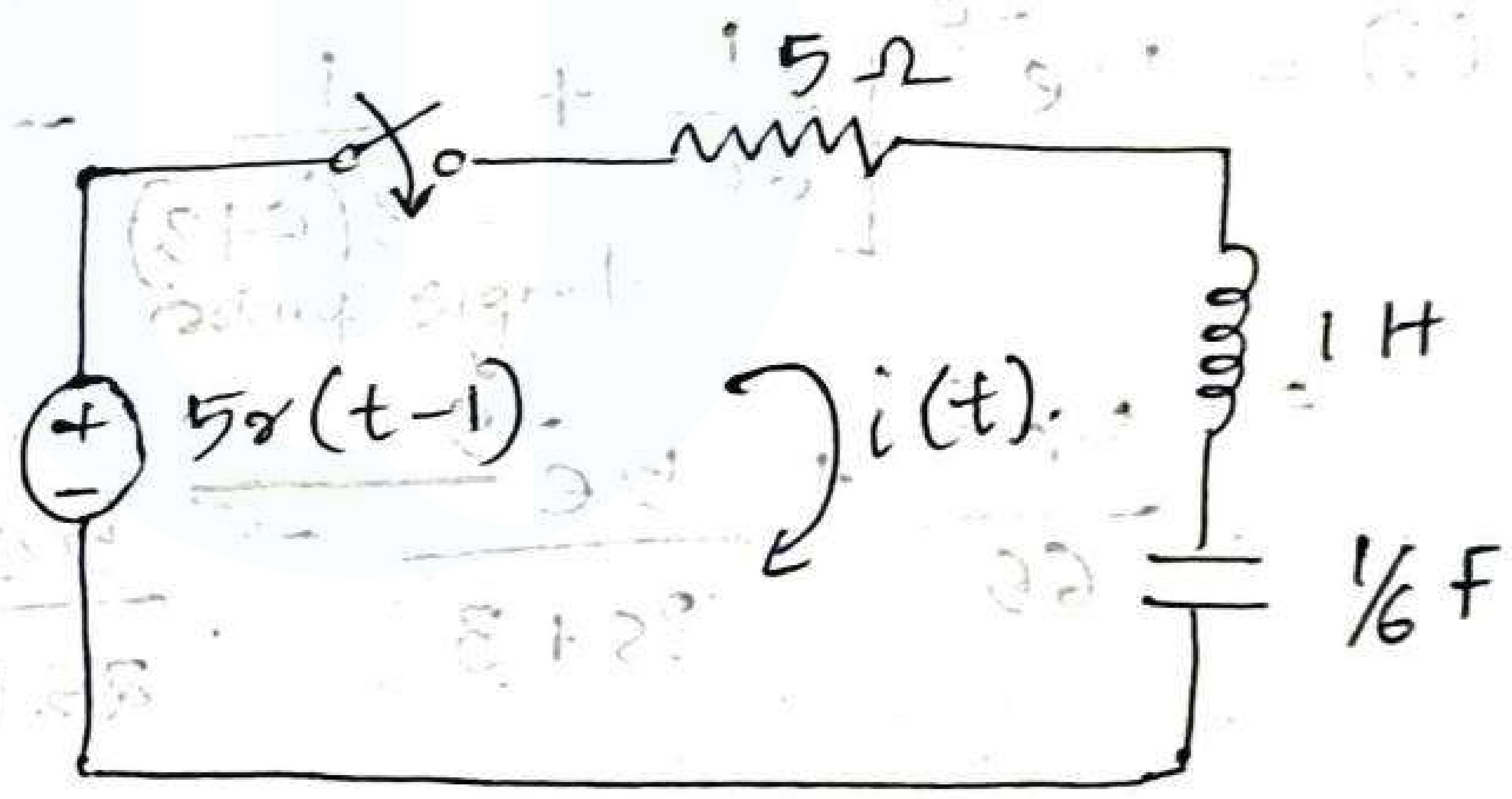
$$= \frac{\omega L\, E_m}{Z^2} e^{-R/L\, t} + \frac{E_m}{Z^{\cancel{2}}} \times \cancel{Z}\left[\sin\omega t\cos\phi - \sin\phi\cos\omega t\right]$$

$$i(t) = \underbrace{\frac{\omega L\, E_m}{Z^2} e^{-R/L\, t}}_{\text{Transient part}} + \underbrace{\frac{E_m}{Z}\sin(\omega t - \phi)}_{\text{Steady state part}}$$

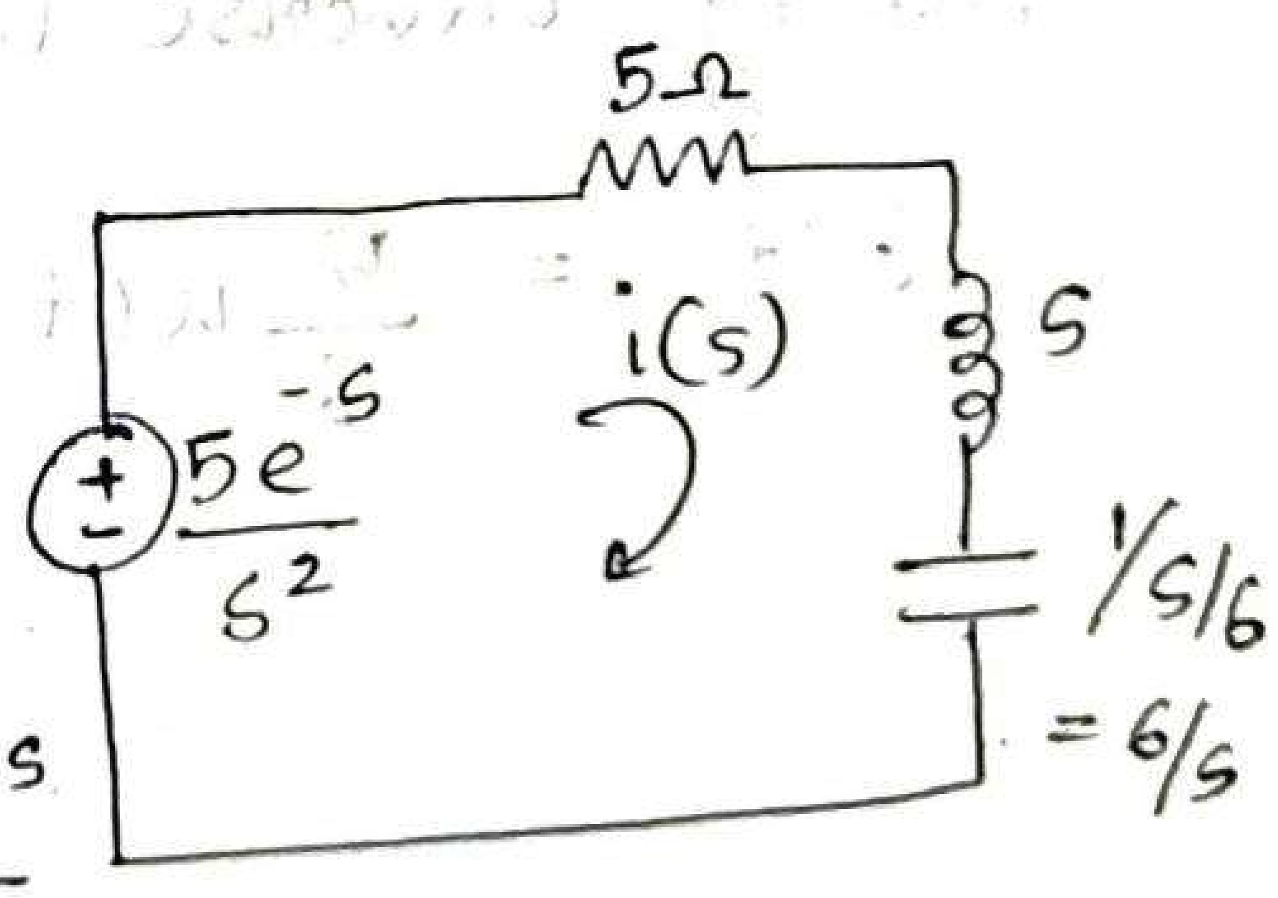
15/10/2020

Q. Determine the current $i(t)$ when the switch is closed at $t=0$ with zero initial conditions.

Laplace	
unit signal	$= \frac{1}{s}$
Ramp "	$= \frac{1}{s^2}$



At $t=0$, KVL applied.



$$\frac{5e^{-s}}{s^2} - 5I(s) - sI(s) - \frac{6}{s}I(s) = 0$$

$$5I(s) + sI(s) + \frac{6}{s}I_s = \frac{5e^{-s}}{s^2}$$

~~5Is+sI(s)~~

$$I(s) = \frac{5e^{-s}}{s(s^2+5s+6)}$$

$$I(s) = \frac{5e^{-s}}{s(s^2+5s+6)} = \frac{5e^{-s}}{s(s+3)(s+2)}$$

By partial fraction expansion

$$\frac{1}{s(s+3)(s+2)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+2}$$

$$A = \left.\frac{1}{(s+3)(s+2)}\right|_{s=0} = \frac{1}{6}.$$

$$B = \left.\frac{1}{s(s+2)}\right|_{s=-3} = \frac{1}{3}$$

$$C = \left.\frac{1}{s(s+3)}\right|_{s=-2} = -\frac{1}{2}$$

$$I(s) = 5e^{-s}\left[\frac{1}{6s} + \frac{1}{3(s+3)} - \frac{1}{2(s+2)}\right]$$

$$= \frac{5e^{-s}}{6s} + \frac{5e^{-s}}{3s+3} - \frac{5e^{-s}}{2s+2}$$

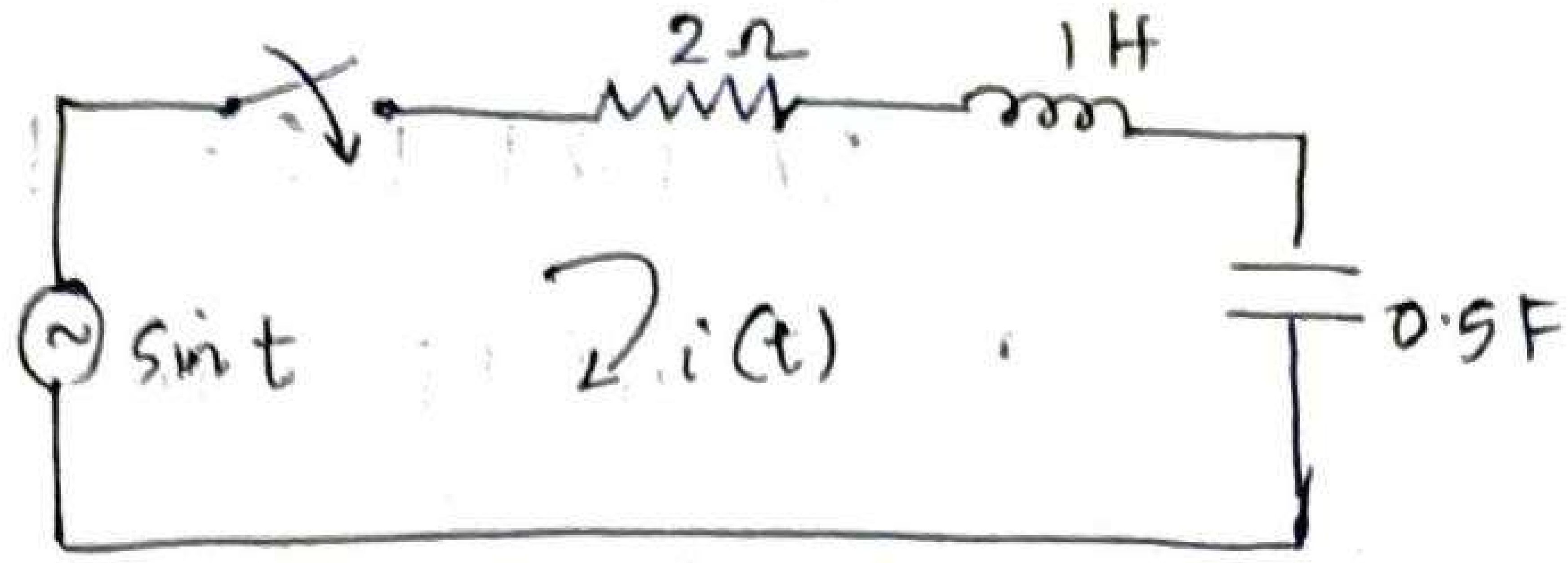
$u(t-1)$

Taking inverse laplace.

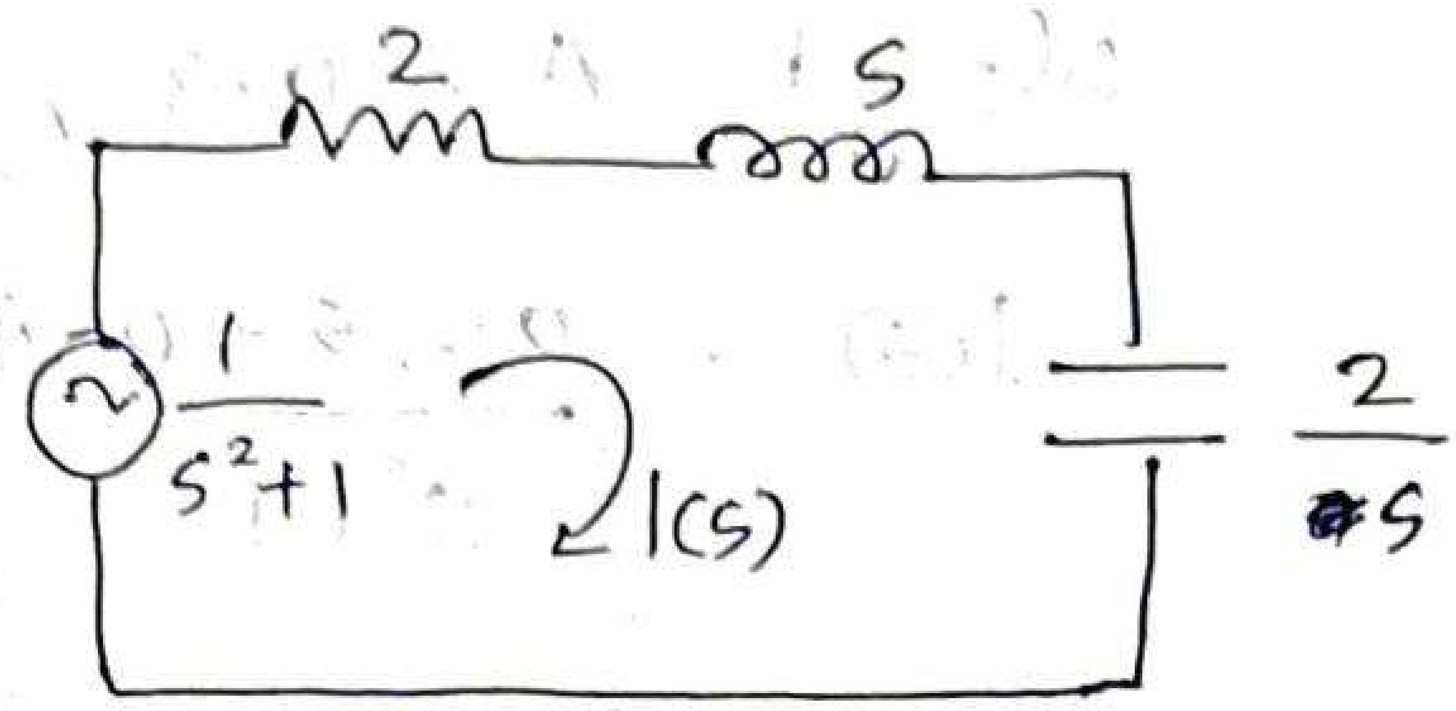
$$i(t) = \frac{5}{6}u(t-1) + \frac{5}{3}e^{-3(t-1)}u(t-1) - \frac{5}{2}e^{-2(t-1)}u(t-1)$$

$$e^{-s} = u(t-1).$$

Q. Switch is closed at $t = 0$. Determine the current $i(t)$ assuming zero initial condition.



Transformed network



$$\frac{1}{s^2+1} - 2\,I(s) - s\,I(s) - \frac{2}{s}I(s) = 0$$

$$\left(2 + s + \frac{2}{s}\right) I(s) = \frac{1}{s^2+1}$$

$$I(s) = \frac{s}{(s^2+1)(2+s^2+2s)}$$

$$= \frac{s}{(s^2+1)(s^2+2s+2)}$$

Partial fraction

$$I(s) = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+2s+2}$$

$$s = (As+B)(s^2+2s+2) + (Cs+D)(s^2+1)$$

$$= As^3 + 2As^2 + 2As + Bs^2 + 2Bs + 2B + Cs^3 + Cs + Ds^2 + D.$$

$$= (A+C)s^3 + (2A+B+D)s^2 + (2A+2B+C)s + 2B + D$$

$$A + C = 0$$

$$2A + B + D = 0$$

$$2A + 2B + C = 1$$

$$2B + D = 0$$

We get $A = 0.2$, $B = 0.4$, $C = -0.2$, $D = -0.8$.

$$I(s) = \frac{0.2s + 0.4}{s^2+1} - \frac{0.2s + 0.8}{s^2 + 2s + 2}$$

$$= \frac{0.2s}{s^2+1} + \frac{0.4}{s^2+1} - \frac{0.2s + 0.2 + 0.6}{(s+1)^2 + (1)^2}$$

$$= \frac{0.2s}{s^2+1} + \frac{0.4}{s^2+1} - \frac{0.2(s+1)}{(s+1)^2+1} - \frac{0.6}{(s+1)^2+1}$$

taking inverse laplace.

$$i(t) = 0.2\cos t + 0.4\sin t - 0.2e^{-t}\cos t - 0.6e^{-t}\sin t$$

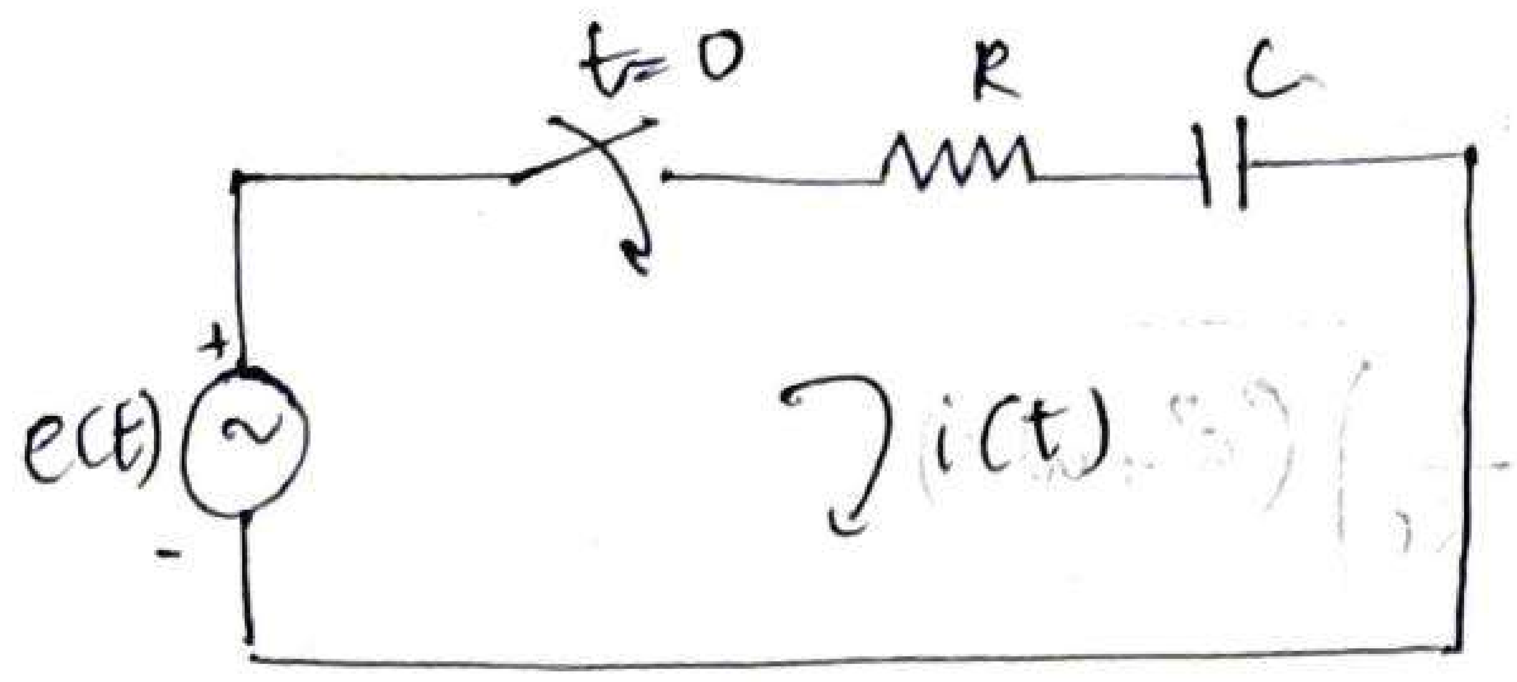
$$= 0.2\cos t + 0.4\sin t - e^{-t}(0.2\cos t + 0.6\sin t)$$

17/10/2020

RC circuit Excited by an AC Source

Consider the RC circuit. Let the circuit be excited by a sinusoidal source.

$$e(t) = E_m \sin \omega t$$



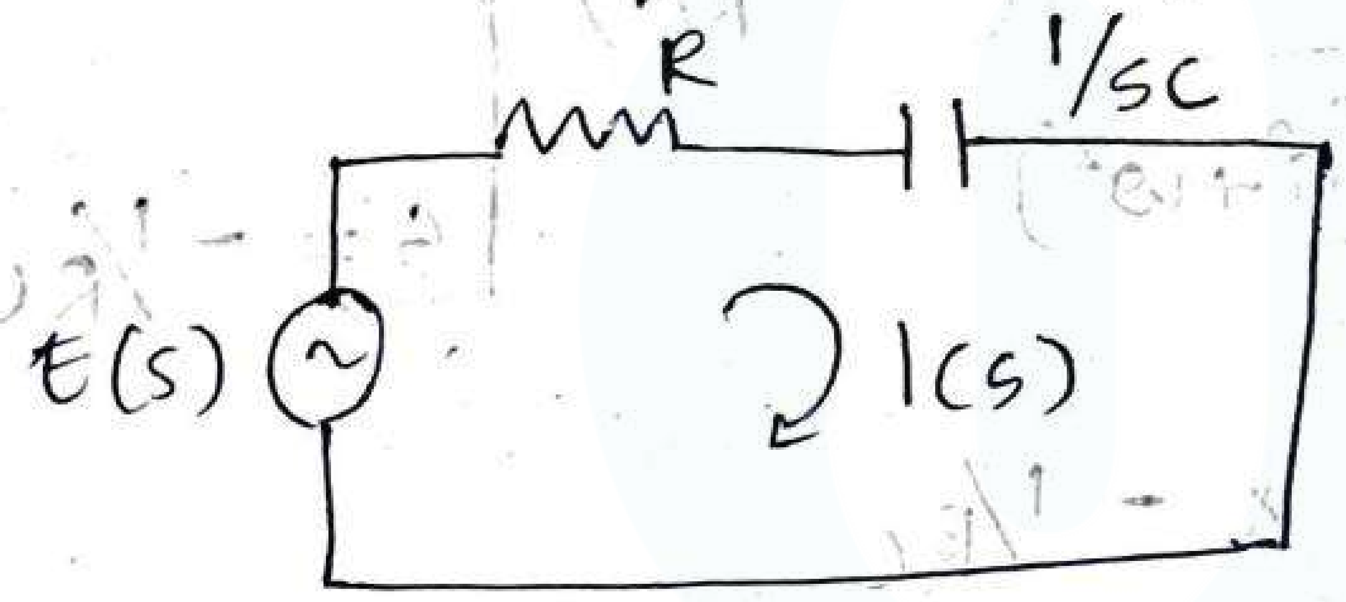
when the switch is closed at $t=0$, let $i(t)$ be the current through the circuit. Let us assume that there is no standard energy in the circuit

$$L[i(t)] = I(s)$$

$$L[e(t)] \cdot E(s) = L[E_m \sin \omega t]$$

$$= E_m \frac{\omega}{s^2+\omega^2}$$

The s domain equilant of RC circuit.



$$I(s)\left[R + \frac{1}{sC}\right] = E(s)$$

$$I(s) = \frac{E(s)}{R + \frac{1}{sC}} = \frac{E(s)}{\frac{RsC+1}{sC}}$$

$$= \frac{E(s)}{\frac{R\cancel{C}}{s\cancel{C}}\left[s + \frac{1}{RC}\right]}$$

$$= \frac{E(s)}{\frac{R}{s}\left[s + \frac{1}{RC}\right]}$$

$$= \frac{\frac{\omega E_m}{R} \cdot s}{\left(s + \frac{1}{RC}\right)(s^2 + \omega^2)}$$

$$I(s) = \frac{k_1}{s + \frac{1}{RC}} + \frac{k_2 s + k_3}{s^2 + \omega^2}$$

$$\frac{\frac{\omega E_m}{R} \cdot s}{\left(s + \frac{1}{RC}\right)(s^2 + \omega^2)} = \frac{k_1}{s + \frac{1}{RC}} + \frac{k_2 s + k_3}{s^2 + \omega^2}$$

$$k_1 = \left. \frac{\frac{\omega E_m}{R} \cdot s}{\cancel{\left(s + \frac{1}{RC}\right)}(s^2 + \omega^2)} \times \cancel{\left(s + \frac{1}{RC}\right)} \right|_{s = -1/RC}$$

$$= \frac{\frac{\omega E_m}{R} \times -1/RC}{\left(\frac{-1}{RC}\right)^2 + \omega^2} = \frac{\frac{-\omega E_m}{R^2 C}}{\left(\frac{1}{R^2 C^2}\right) + \omega^2}$$

$$= \frac{-\frac{\omega E_m}{R^2 C}}{\frac{\omega^2}{R^2}\left[\frac{1}{\omega^2 C^2} + R^2\right]} = \frac{-\frac{\omega E_m}{\cancel{R^2} C} \times \frac{\cancel{R^2}}{\omega^2}}{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$k_1 = \frac{-E_m/\omega C}{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\chi = \sqrt{R^2 + (\omega C)^2} \Rightarrow \chi^2 = R^2 + (\omega C)^2$$

$$= \frac{-E_m}{\omega C \chi^2}$$

$$\frac{\frac{\omega E_m}{R} \times s}{\left(s + \frac{1}{RC}\right)\left(s^2 + \omega^2\right)} = \frac{k_1}{\left(s + \frac{1}{RC}\right)} + \frac{k_2 s + k_3}{s^2 + \omega^2}$$

$$\frac{\omega E_m}{R} s = k_1 (s^2 + \omega^2) + (k_2 s + k_3)\left(s + \frac{1}{RC}\right)$$

$$\frac{\omega E_m}{R} s = (k + k_2) s^2 + k_1 \omega^2 + \frac{k_2}{RC} s + k_3 s + \frac{k_3}{RC}$$

$$\frac{\omega E_m}{R} \times s = (k_1 + k_2) s^2 + \left(\frac{k_2}{RC} + k_3\right) s + k_1 \omega^2 + \frac{k_3}{RC}$$

$$k_1 + k_2 = 0 \Rightarrow k_1 = -k_2.$$

$$k_1 = \frac{-E_m}{\omega C \chi^2}$$

$$k_2 = \frac{E_m}{\omega C \chi^2}$$

$$\frac{k_2}{RC} + k_3 = \frac{\omega E_m}{R}$$

$$\frac{\frac{E_m}{\omega C \chi^2}}{RC} + k_3 = \frac{\omega E_m}{R}$$

$$k_3 = \frac{\omega E_m}{R} - \frac{E_m / \omega C \chi^2}{RC}$$

$$= \frac{\omega E_m}{R} - \frac{E_m}{\omega C^2 \chi^2 R}$$

$$k_3 = \frac{\omega E_m (\omega C^2 Z^2 R) - E_m R}{\omega C^2 Z^2 R^2}$$

$$= \frac{\cancel{R}[\omega E_m (\omega C^2 Z^2) - E_m]}{\omega C^2 Z^2 R^{\cancel{2}}}$$

$$= \frac{\omega E_m (\omega C^2 Z^2) - E_m}{\omega C^2 Z^2 R}$$

$$= \frac{E_m [\omega^2 C^2 Z^2 - 1]}{\omega C^2 Z^2 R.}$$

Sub: Z^2 on numerator

$$= \frac{E_m \left[\omega^2 C^2 \left[R^2 + \frac{1}{\omega^2 C^2}\right] - 1\right]}{\omega C^2 Z^2 R}$$

$$= \frac{E_m [\omega^2 C^2 R^2 + 1 - 1]}{\omega C^2 Z^2 R}$$

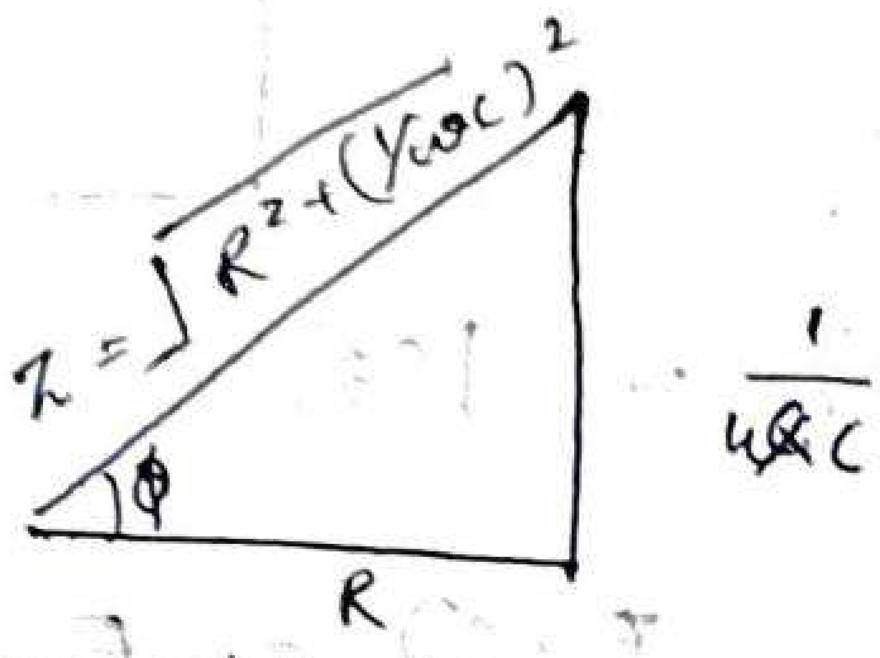
$$= \frac{E_m \omega^{\cancel{2}} \cancel{C^2} R^{\cancel{2}}}{\cancel{\omega} \cancel{C^2} Z^2 \cancel{R}} = \frac{E_m \omega R}{Z^2} \Rightarrow k_3$$

$$I(s) = \frac{-E_m / \omega C Z^2}{s + \frac{1}{RC}} + \frac{E_m / \omega C Z^2 \, s + \omega R E_m / Z^2}{s^2 + \omega^2}$$

$$= \frac{-E_m}{\omega C Z^2} \times \frac{1}{s + \frac{1}{RC}} + \frac{E_m}{\omega C Z^2} \cdot \frac{s}{s^2 + \omega^2} + \frac{\omega R E_m}{Z^2} \frac{\omega}{s^2 + \omega^2}$$

$$i(t) = \frac{-E_m}{\omega C Z^2} e^{-1/RCt} + \frac{E_m}{\omega C Z^2} \cos\omega t + \frac{R E_m}{Z^2} \sin\omega t$$

$$= \frac{-E_m}{\omega C Z^2} e^{-1/RCt} + \frac{E_m}{Z^2}\left[R \sin\omega t + \frac{1}{\omega C}\cos\omega t\right]$$



$$\tan\phi = \frac{1/\omega C}{R} \qquad \phi = \tan^{-1}\frac{1/\omega C}{R}$$

$$\cos\phi = R/Z \qquad R = Z\cos\phi$$

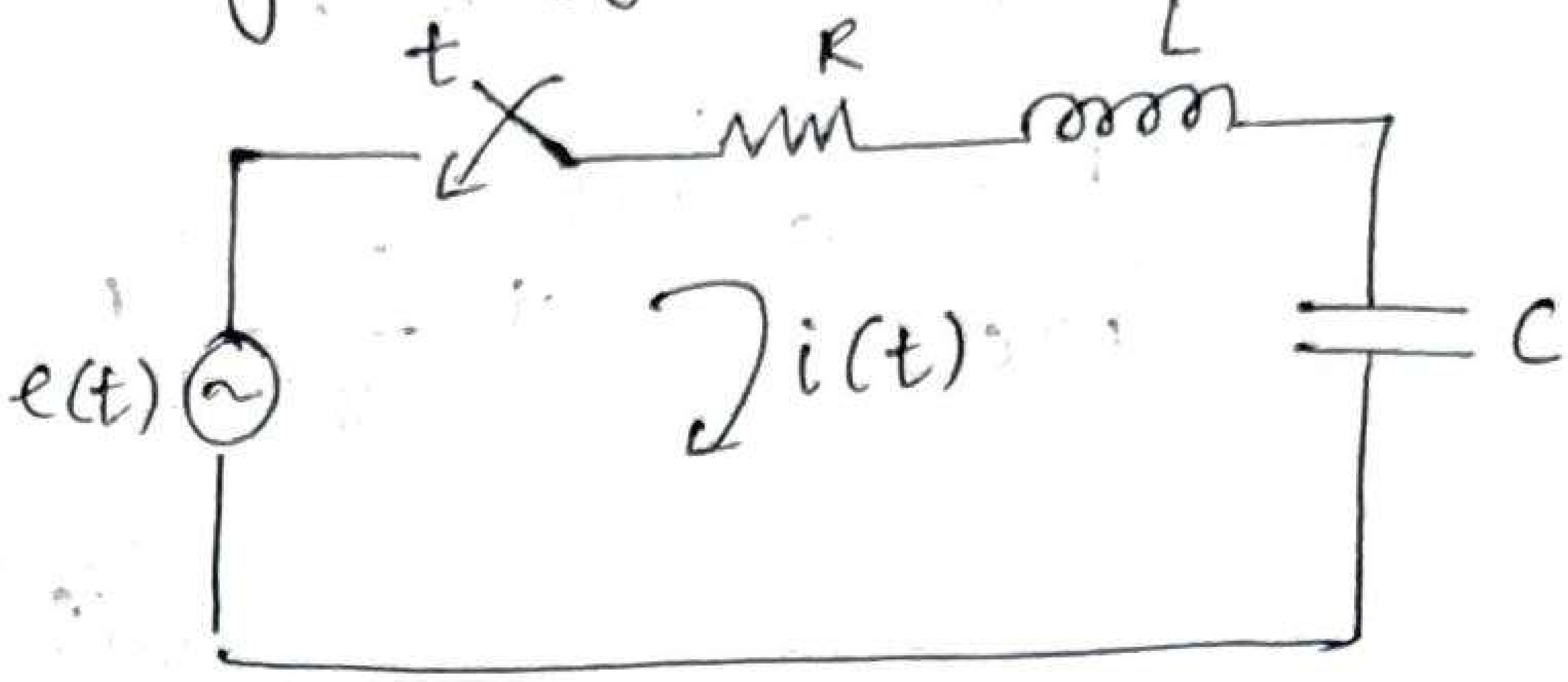
$$\sin\phi = \frac{1/\omega C}{Z} \qquad \frac{1}{\omega C} = Z\sin\phi$$

$$i(t) = \frac{-E_m}{\omega C Z^2} e^{-1/RC\,t} + \frac{E_m}{Z^2}\left[Z\cos\phi \sin\omega t + Z\sin\phi\cos\omega t\right]$$

$$= \underbrace{\frac{-E_m}{\omega C Z^2} e^{-1/RCt}}_{\text{transient}} + \underbrace{\frac{E_m}{Z}\left[\sin(\omega t+\phi)\right]}_{\text{steady state part}}$$

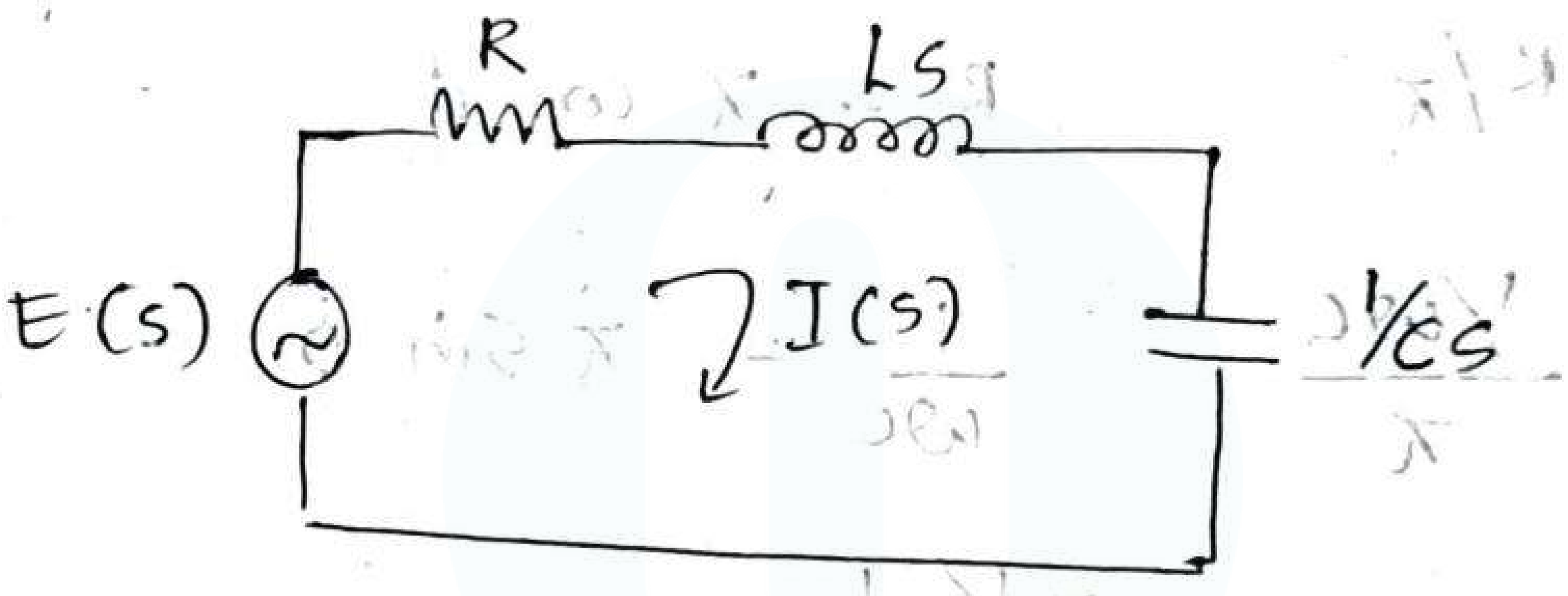
RLC circuit excited by AC source.

Consider the RLC circuit. Let the ckt be excited by a sinusoidal sowre $e(t) = E_m \sin\omega t$ when switch is closed at $t=0$. Let $i(t)$ be the current through the ckt. Let us assume there is no steady energy in the ckt.



$$L\, i(t) = I(s) \qquad L[e(t)] = E(s)$$

$$E(s) = E_m \frac{\omega}{s^2+\omega^2}$$



$$I(s)\left[R + Ls + \frac{1}{Cs}\right] = E(s)$$

$$I(s) = \frac{E(s)}{R + Ls + \frac{1}{Cs}}$$

$$= \frac{\omega E_m}{s^2+\omega^2}\left[\frac{1}{R+Ls+\frac{1}{Cs}}\right]$$

$$= \frac{\omega E_m}{s^2+\omega^2}\left[\frac{1}{\frac{L}{s}\left[\frac{R}{L}s + s^2 + \frac{1}{LC}\right]}\right]$$

$$= \frac{\omega E_m s/L}{(s^2+\omega^2)\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)}$$

Case I $\Rightarrow$ $\dfrac{\omega E_m s/L}{(s^2+\omega^2)(s+a)^2}$

The roots are real and equal

Let $\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right) = (s+a)^2$

$$s = \pm a$$

$$I(s) = \frac{(\omega E_m/L).\, s}{(s^2+\omega^2)(s+a)^2}$$

$$= \frac{K_1}{s+a} + \frac{K_2}{(s+a)^2} + \frac{K_3 s + K_4}{s^2+\omega^2}$$

$$i(t) = K_1 e^{-at} + K_2 t e^{-at} + I_m \sin(\omega t \pm \phi)$$

Case II $\Rightarrow$ roots are real and unequal

Let $\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right) = (s+a)(s+b)$

$$I(s) = \frac{\frac{\omega E_m}{L} s}{(s^2+\omega^2)-(s+a)(s+b)}$$

$$= \frac{K_1}{(s+a)} + \frac{K_2}{(s+b)} + \frac{K_3 s + K_4}{s^2+\omega^2}$$

$$= K_1 e^{-at} + K_2 e^{-bt} + I_m\left[\sin(\omega t \pm \phi)\right]$$

Case III ⇒ The roots are complex conjugate

$$\det\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right) = s^2+\frac{R}{L}s+\frac{1}{LC}+\left(\frac{R}{2L}\right)^2-\left(\frac{R}{2L}\right)^2$$

$$= (s+a)^2+\omega_d^2$$

$$a=\left(\frac{R}{2L}\right) \;;\; \omega_d = \sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$$

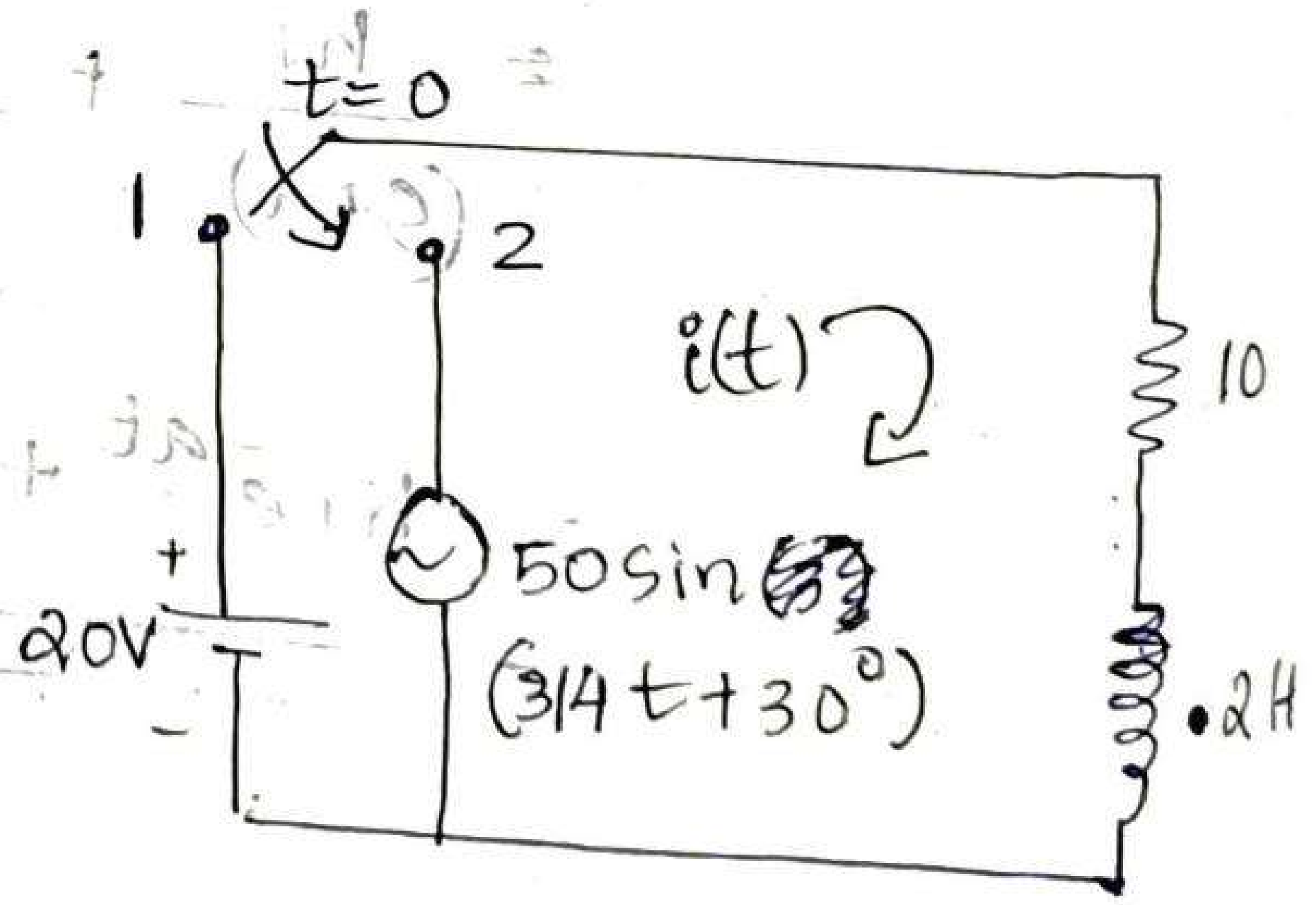
$$I(s) = \frac{\frac{\omega E_m}{L}\cdot s}{(s^2+\omega^2)\left((s+a)^2+\omega_d^2\right)}$$

$$= \frac{k_1 s+k_2}{(s+a)^2+\omega_d^2} + \frac{k_3 s+k_4}{s^2+\omega^2}$$

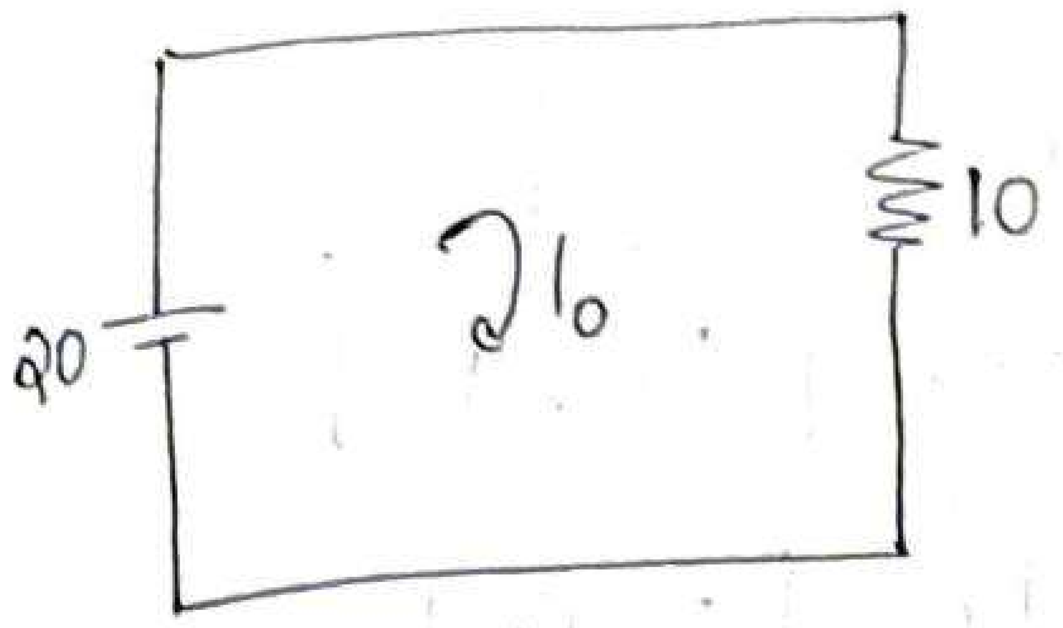
$$i(t) = I_0 e^{-at}\sin(\omega_d t+\theta)+I_m\sin(\omega t\pm\phi)$$

20/10/2020

Q The switch in the circuit is closed at position-1 for a time. At time $t=0$, switch is moved to position 2. Find $i(t)$ for $t\geq 0$



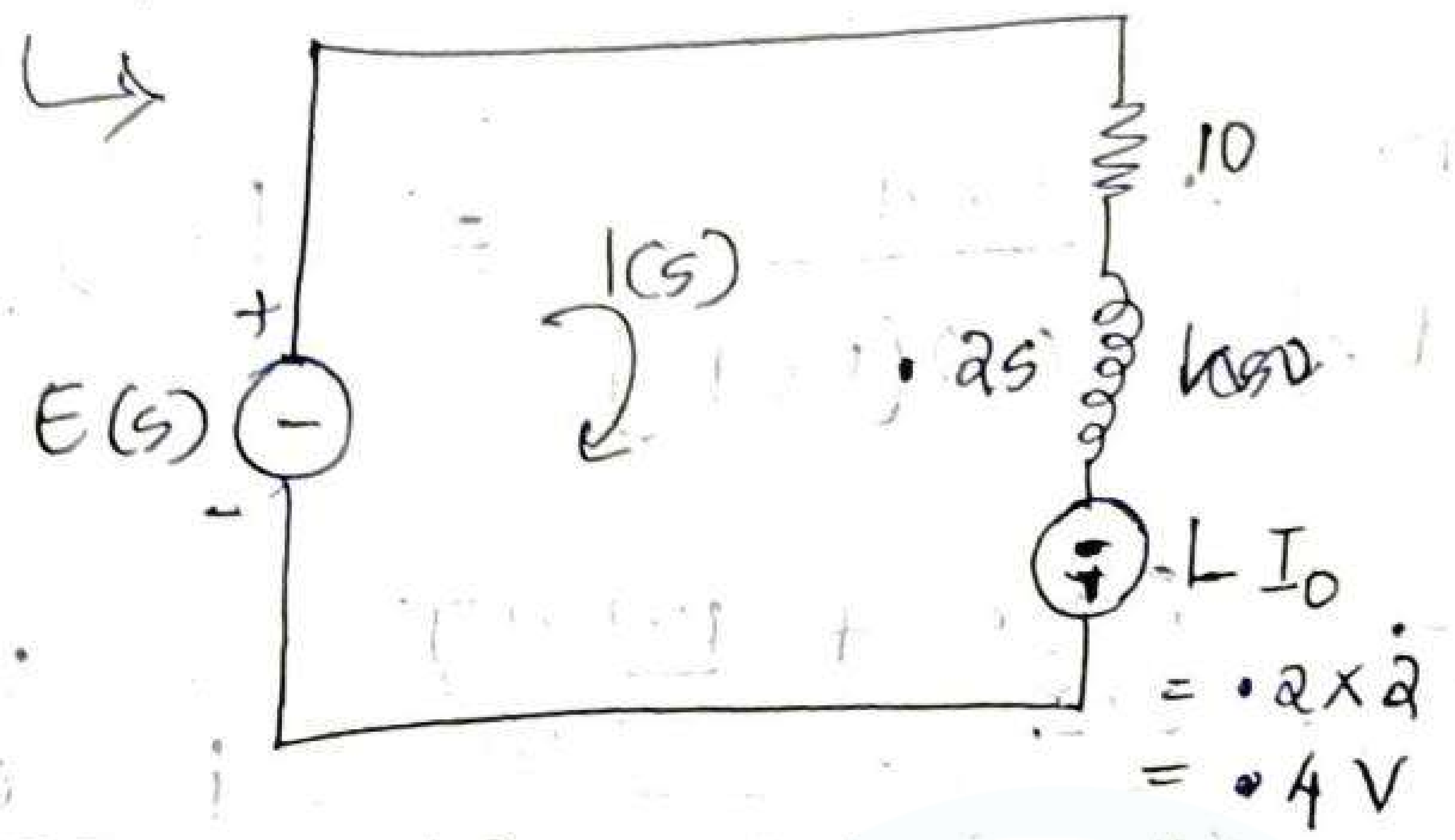
position 1



$20 = 10 I_0$

$\underline{\underline{I_0 = 2A}}$

position : 2



$E(t) = 50 \sin(314t + 30^\circ)$

$E(S) = L\left[50 \sin(314t + 30^\circ)\right]$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= 50 \times L\left[\sin 314t \cos 30 + \cos 314t \sin 30\right]$

$= 50 \times L\left[\sin 314t \times \frac{\sqrt{3}}{2} + \cos 314t \times \frac{1}{2}\right]$

$= 50 \times \frac{\sqrt{3}}{2} L\left[\sin 314t\right] + 50 \times \frac{1}{2} L\left[\cos 314t\right]$

$= \overset{25}{\cancel{50}} \times \frac{\sqrt{3}}{\cancel{2}} \times \frac{314}{s^2 + 314^2} + \overset{25}{\cancel{50}} \times \frac{1}{\cancel{2}} \times \frac{s}{s^2 + 314^2}$

$= \frac{13597}{s^2 + 314^2} + \frac{25s}{s^2 + 314^2}$

$$E(s) = \frac{25s + 13597}{s^2 + 314^2}$$

Mesh $\rightarrow$ $E(s) + .4 = 10I(s) + 0.2s\,I(s)$

$= I(s)\,[10 + 0.2s]$

$= 0.2\,I(s)\,[10/.2 + s]$

$$\frac{25s + 13597}{s^2 + 314^2} + 0.4 = 0.2\,I(s)\,[50 + s]$$

$$\frac{25s + 13597}{(s^2 + 314^2)\,0.2\,[50 + s]} + \frac{0.4}{0.2(50 + s)} = I(s)$$

$$I(s) = \frac{\frac{25}{0.2}s + \frac{13597}{0.2}}{(s+50)(s^2 + 314^2)} + \frac{\frac{0.4}{0.2}}{(s+50)}$$

$$I(s) = \frac{125s + 67985}{(s+50)(s^2 + 314^2)} + \frac{2}{(s+50)}$$

Let $$\frac{125s + 67985}{(s+50)(s^2+314^2)} + \frac{2}{(s+50)} = \frac{k_1}{s+50} + \frac{k_2 s + k_3}{s^2 + 314^2}$$

$$k_1 = \left.\frac{125s + 67985}{\cancel{(s+50)}(s^2 + 314^2)} \times \cancel{(s+50)}\right|_{s=-50}$$

$$= \frac{125 \times -50 + 67985}{(-50^2 + 314^2)} = \underline{\underline{0.6107}}$$

On cross multiplication

$$125s + 67985 = k_1(s^2+314^2) + (k_2 s + k_3)(s+50)$$

$$125s + 67985 = k_1 s^2 + k_1 314^2 + k_2 s^2 + k_2 s \times 50 + sk_3 + 50k_3$$

$$125s + 67985 = (k_1 + k_2)s^2 + (50k_2 + k_3)s + k_1 314^2 + 50k_3$$

$k_1 + k_2 = 0$, $\quad (50k_2 + k_3) = 125$.

$k_1 = -k_2$ $\quad 50 \times -0.6107 + k_3 = 125$

$\underline{k_2 = -0.6107}$ $\quad k_3 = 125 + 30.537$

$\underline{k_3 = 155.535}$

$$I(s) = \frac{0.6107}{s+50} + \frac{-0.6107s + 155.535}{s^2+314^2} + \frac{2}{s+50}$$

$$= \frac{2.6107}{s+50} - \frac{0.6107s}{s^2+314^2} + \frac{155.535}{s^2+314^2}$$

$$= 2.6107 \times \frac{1}{s+50} - 0.6107 \times \frac{s}{s^2+314^2} + \frac{155.535}{314} \times \frac{314}{s^2+314^2}$$

$$= 2.6107 \times \frac{1}{s+50} - 0.6107 \times \frac{s}{s^2+314^2} + 0.4953 \times \frac{314}{s^2+314^2}$$

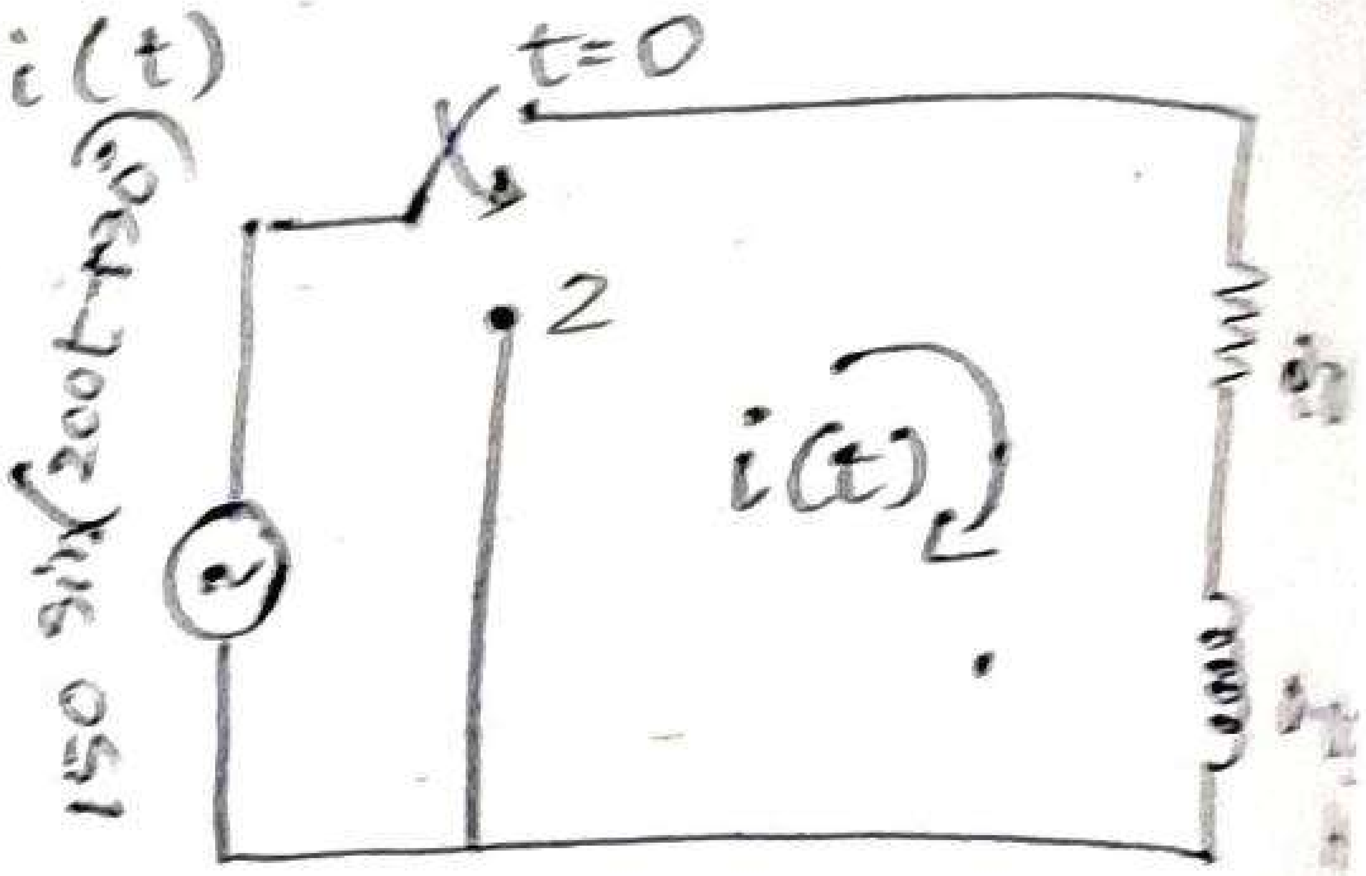
Inverse laplace.

$$= 2.6107 e^{-50t} - 0.6107 \cos 314t + 0.4953 \sin 314t$$

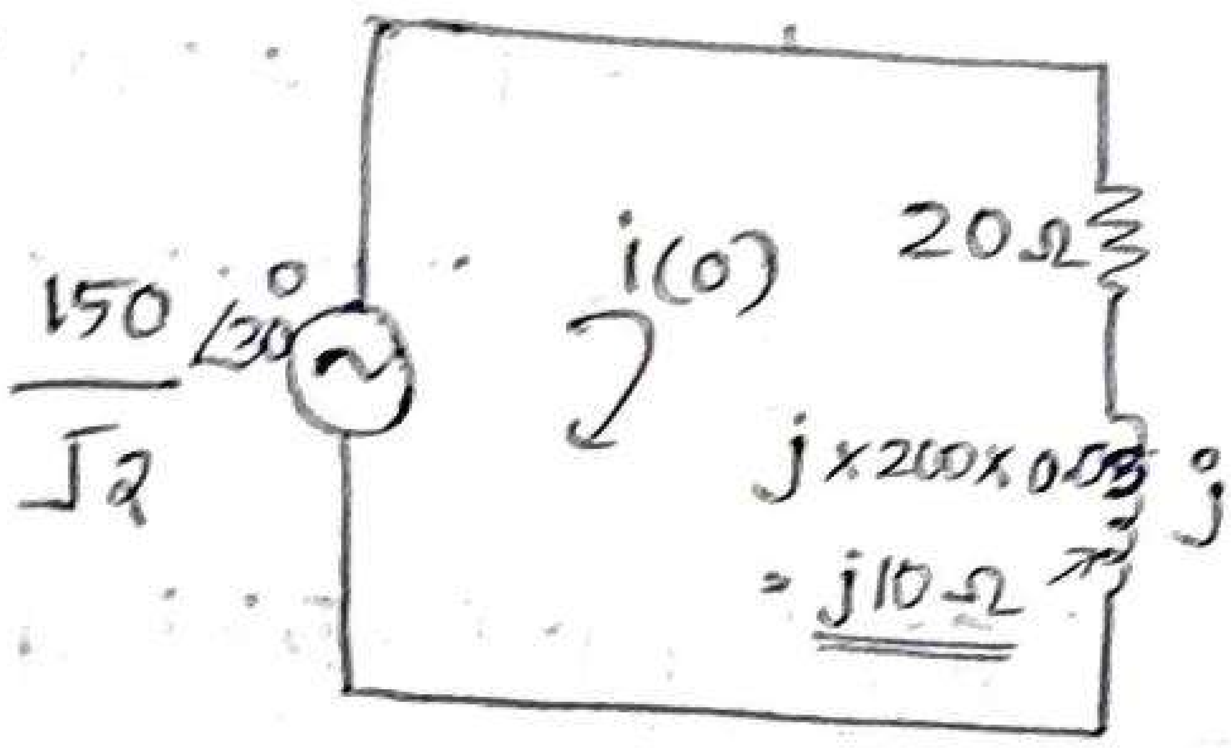
$$2.6107 e^{-50t} + 0.7863 \sin(314t - 51) \text{ A}$$

trans — Steady part

Q. In the circuit the switch remains in position 1 until steady state is reached. At time $t=0$, switch is changed to position 2. Find $i(t)$



At position 1



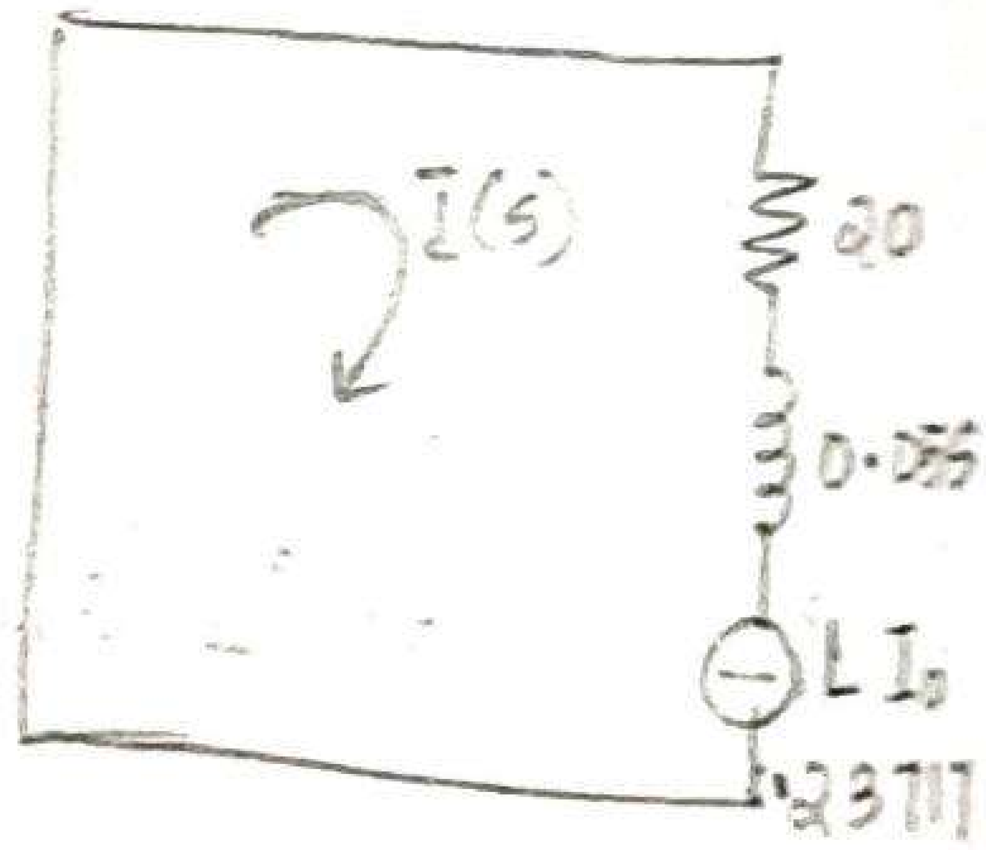
jωL

here circuit is attained steady state, hence we can perform steady state analysis.

$$E_m \sin(\omega t \pm \phi) = 150 \sin(200t + 30)$$

$$I_0 = \frac{E_m}{\sqrt{Z^2}} = \frac{150/\sqrt{2}}{\sqrt{20^2+10^2}} = \underline{\underline{4.7434\,A}}$$

At position 2;



$$20\,I(s) + 0.05s\,I(s) = 0.23717$$

$$I(s) \times 0.05\left(\frac{20}{0.05} + s\right) = 0.23717$$

$$I(s) \times 0.05(400+s) = 0.23717$$

$$I(s) = \frac{0.23717}{0.05(400+s)}$$

$$I(s) = \frac{4.7434}{s+400}$$

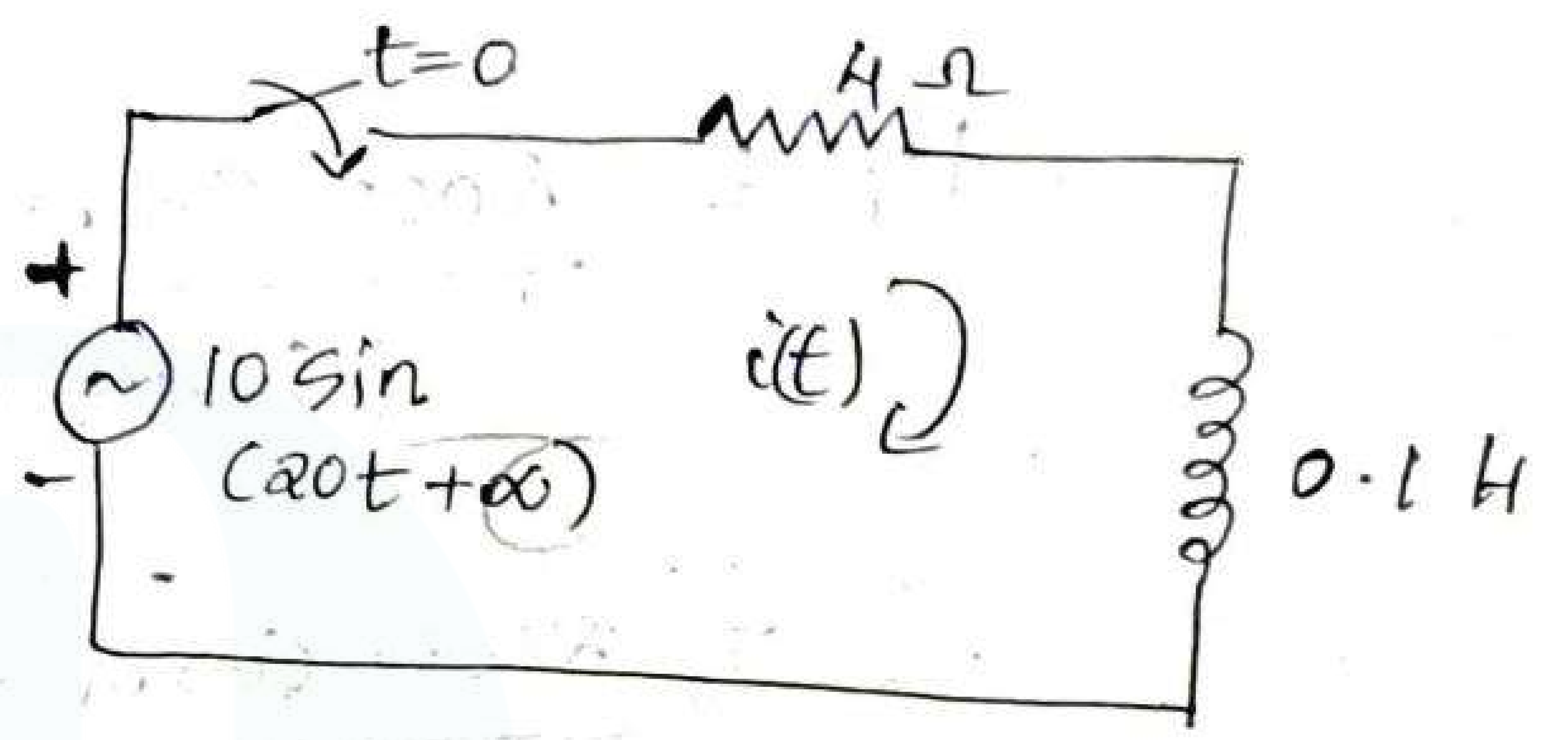
Taking inverse Laplace

$$L^{-1}[I(s)] = L^{-1}\left[\frac{4.7434}{s+400}\right]$$

$$\underline{\underline{i(t) = 4.7434\, e^{-400t}\ A}}$$

21/10/2020

Q. In the circuit, the switch is closed at a particular value of 'α' so that there is no transient in the RL circuit. Find the value of 'α'.



Ans Given that

$$e(t) = 10\sin(20t+\alpha)$$

Let $L[e(t)] = E(s)$

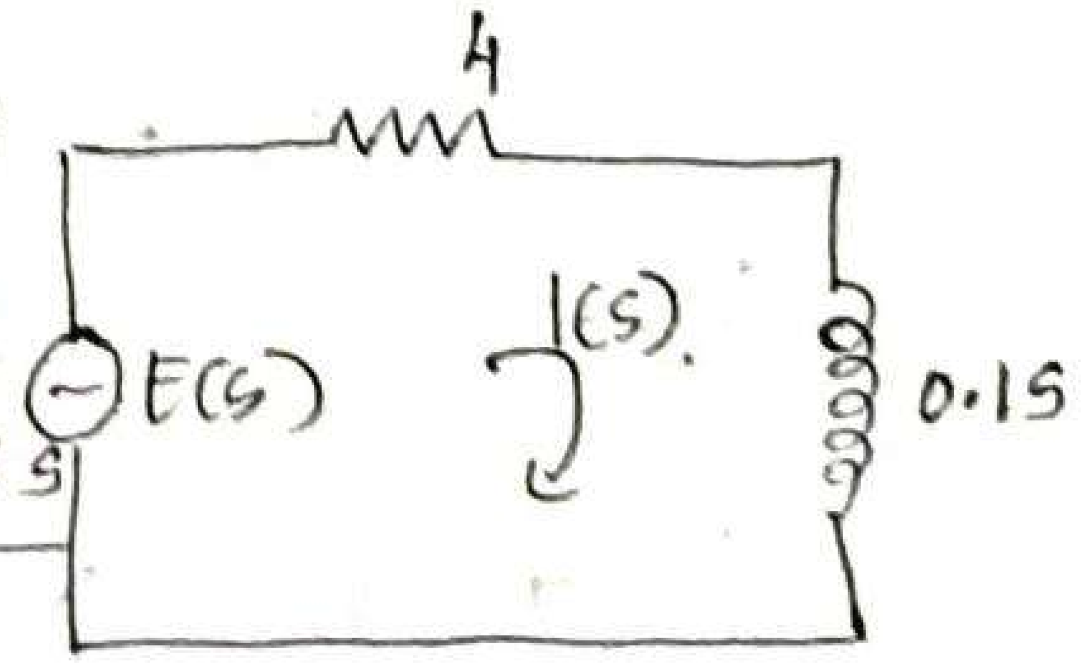
$$E(s) = L[10\sin(20t+\alpha)]$$

$$= 10 \times L[\sin 20t \cos\alpha + \cos 20t \sin\alpha]$$

$$= 10\cos\alpha\, L(\sin 20t) + 10\sin\alpha\, L(\cos 20t)$$

$$= 10\cos\alpha \frac{20}{s^2+20^2} + 10\sin\alpha \frac{s}{s^2+20^2}$$

$$= \frac{200\cos\alpha + 10\sin\alpha\, s}{s^2+20^2}$$



$$4 I(s) + 0.1s\, I(s) = E(s)$$

$$I(s) \times 0.1\left(\frac{4}{0.1} + s\right) = \frac{200\cos\alpha + 10\sin\alpha\, s}{s^2+20^2}$$

$$I(s) \times 0.1(s+40) = \frac{200\cos\alpha + 10\sin\alpha s}{s^2+20^2}$$

$$I(s) = \frac{200\cos\alpha + 10\sin\alpha s}{0.1(s+40)(s^2+20^2)}$$

$$I(s) = \frac{2000\cos\alpha + 100\sin\alpha s}{(s+40)(s^2+20^2)}$$

$$\frac{2000\cos\alpha + 100\sin\alpha s}{(s+40)(s^2+20^2)} = \frac{k_1}{s+40} + \frac{k_2 s + k_3}{s^2+20^2}$$

$$k_1 = \frac{2000\cos\alpha + 100\sin\alpha s}{\cancel{(s+40)}(s^2+20^2)} \times \cancel{(s+40)} \Big|_{s=-40}$$

$$= \frac{2000\cos\alpha + 100\sin\alpha \times -40}{-40^2+20^2}$$

$$= \frac{2000\cos\alpha - 4000\sin\alpha}{2000}$$

$$= \frac{\cancel{2000}(\cos\alpha - 2\sin\alpha)}{\cancel{2000}}$$

$$\underline{k_1 = \cos\alpha - 2\sin\alpha.}$$

$$2000\cos\alpha + 100\sin\alpha s = k_1(s^2+20^2) + (k_2 s + k_3)(s+40)$$

$$= k_1 s^2 + k_1 20^2 + k_2 s^2 + k_3 s + 40 k_2 s + 40 k_3$$

$$= (k_1+k_2)s^2 + (k_3 + 40k_2)s + 40k_3 + k_1 20^2$$

$$k_1 + k_2 = 0$$

$$k_1 = -k_2 \qquad \underline{k_2 = -\cos\alpha + 2\sin\alpha.}$$

$k_3 + 40k_2 = 100\sin\alpha.$

$k_3 = 100\sin\alpha - 40(-(\cos\alpha + 2\sin\alpha))$

$= 100\sin\alpha + 40(\cos\alpha - 2\sin\alpha.)$

$= 100\sin\alpha + 40\cos\alpha - 80\sin\alpha$

$\underline{k_3 = 40\cos\alpha + 20\sin\alpha}.$

$$I(s) = \frac{k_1}{s+40} + \frac{k_2 s + k_3}{s^2+20^2}$$

$$= \frac{k_1}{s+40} + k_2\frac{s}{s^2+20^2} + \frac{k_3}{20}\frac{20}{s^2+20^2}$$

taking inverse laplace

$$= \underbrace{k_1 e^{-40t}}_{\text{transient}} + \underbrace{k_2\cos 20t + \frac{k_3}{20}\sin 20t}_{\text{steady}}$$

For no transient part $i(t)$

$k_1 e^{-40t} = 0$

At $t=0$ $[e^0 = 1$, so $t \neq 0]$.

therefore putting $k_1 = 0$

Put $k_1 = \cos\alpha - 2\sin\alpha = 0$

$\cos\alpha = 2\sin\alpha$

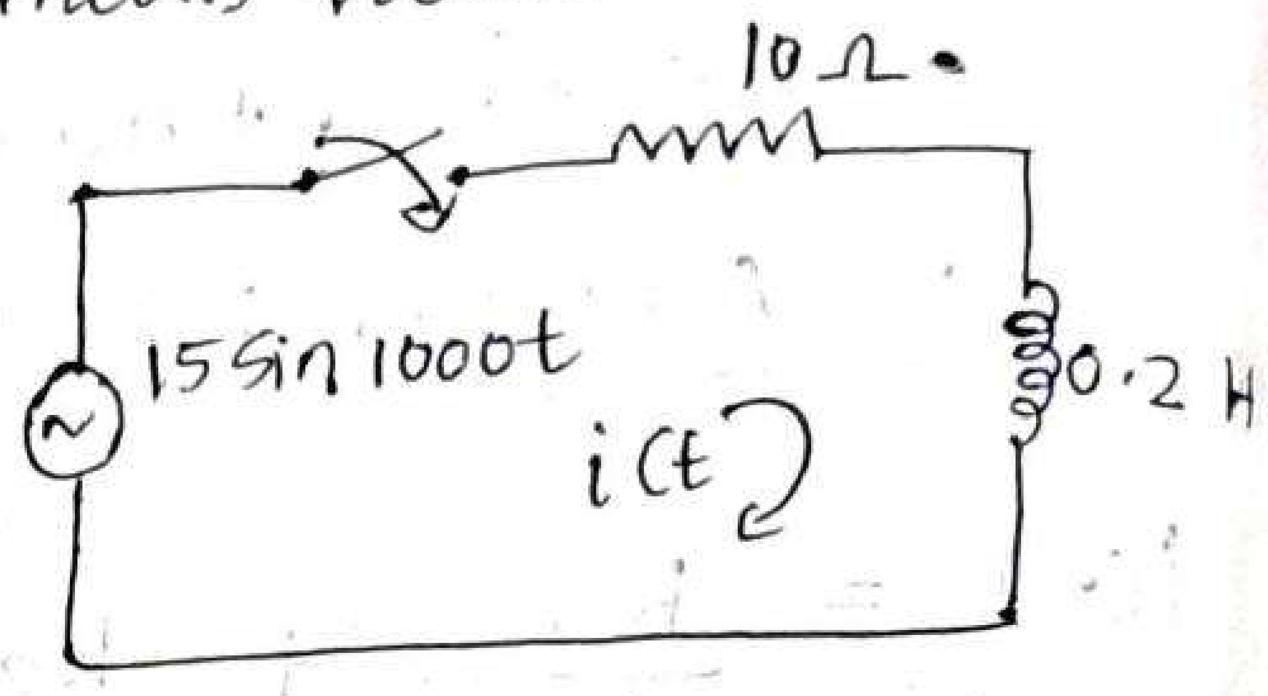
$$\frac{\sin\alpha}{\cos\alpha} = \frac{1}{2}$$

$\tan\alpha = .5$

$\alpha = \tan^{-1}(0.5) = 26.6$

For no transient in $i(t)$ $\underline{\alpha = 26.6^\circ}$

Q. In the RL circuit excited by sinusoidal source, the switch is closed at t = 0.001 second. Find the ratio of max. value to which the current may rise to steady state maximum value.



Given $E_m \sin \omega t$

$= 15 \sin 1000\, t$

$\omega = 1000$ rad/s

At t = 0.001 second

$\omega t = 1000 \times \cancel{10}\, 0.001 = 1$ radian

$= 1 \times \frac{180}{\pi}$ 0. $= \underline{57.3^\circ}$

Let us assume that the excitation source is 15 Sin $(1000t + 57.3^\circ)$ and switch is closed at t=0

$e(t) = 15 \sin(1000\,t + 57.3^\circ)$ V.

Let, $L(e(t)) = E(s)$

$E(s) = L(e(t)) = L[15 \sin(1000t + 57.3)]$

$= 15 \times L[\sin(1000t + 57.3)]$

$= 15 \times L(\sin 1000t \cos 57.3 + \cos 1000t \cdot \sin 57.3)$

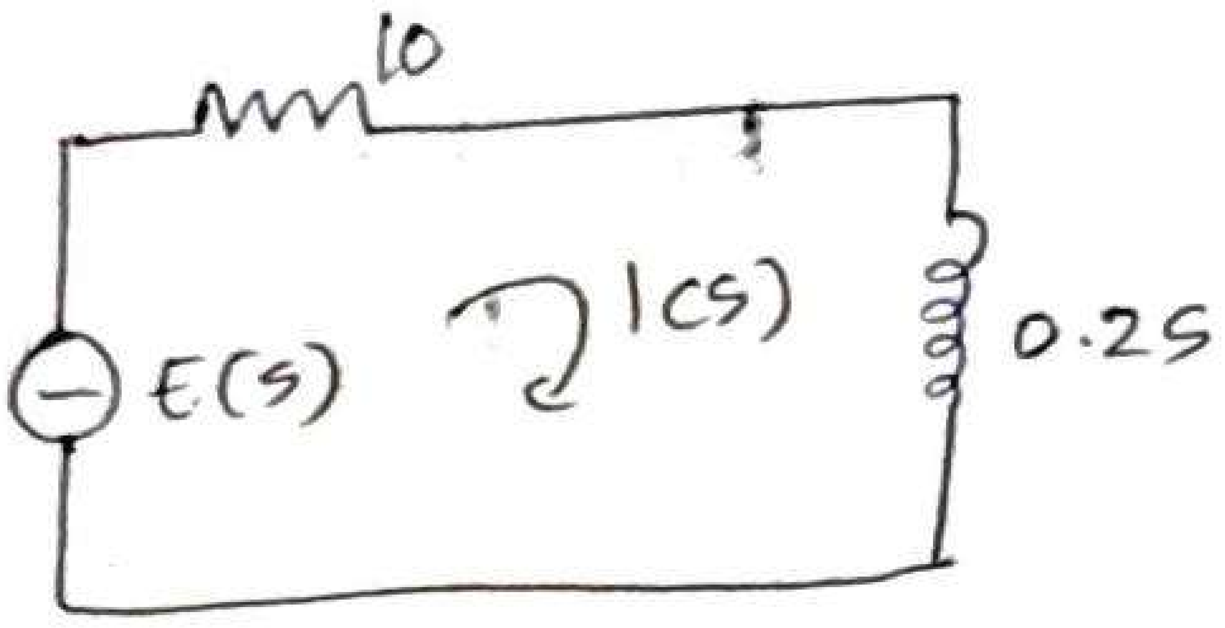
$= 15 \times L(\sin 1000t) \times 0.5402 + \cos 1000t \times 0.8415$

$= 15 \times 0.5402 \times L(\sin 1000t) + 15 \times 0.8415 \times \cos 1000t$

$= 8.103 \times \frac{1000}{s^2 + 1000^2} + 12.6225 \times \frac{s}{s^2 + 1000^2}$

$= \frac{12.6225s + \cancel{8103}\, 8103}{s^2 + 1000^2}$

$$10\,I(s) + 0.25\,I(s) = E(s)$$



$$I(s) \times 0.2\left(\frac{10}{0.2} + s\right) = E(s)$$

$$I(s) \times 0.2\,(s+50) = \frac{12.6225\,s + 8103}{s^2 + 10^6}$$

$$I(s) = \frac{12.6225\,s + 8103}{0.2\,(s+50)\,(s^2+10^6)}$$

$$= \frac{63.1125\,s + 40515}{(s+50)\,(s^2+10^6)}$$

By partial expansion $I(s) =$

$$\frac{63.1125\,s + 40515}{(s+50)\,(s^2+10^6)} = \frac{k_1}{(s+50)} + \frac{k_2 s + k_3}{s^2+10^6}$$

$$k_1 = \left.\frac{63.1125\,s + 40515}{\cancel{(s+50)}\,(s^2+10^6)} \times \cancel{(s+50)}\right|_{s=-50}$$

$$= \frac{63.1125 \times -50 + 40515}{-50^2 + 10^6}$$

$$= \underline{0.0375}$$

$$63.1125\,s + 40515 = k_1(s^2+10^6) + (k_2 s + k_3)(s+50)$$

$$= k_1 s^2 + k_1 10^6 + k_2 s^2 + k_3 s + 50 k_2 s + 50 k_3$$

$$= (k_1 + k_2)\,s^2 + (k_3 + 50 k_2)\,s + 50 k_3 + k_1 10^6$$

$$k_1 + k_2 = 0$$

$$k_2 = -k_1$$

$$\underline{k_2 = -0.037}$$

$$50k_2 + k_3 = 63.1125$$

$$k_3 = 63.1125 - 50 \times -0.037$$

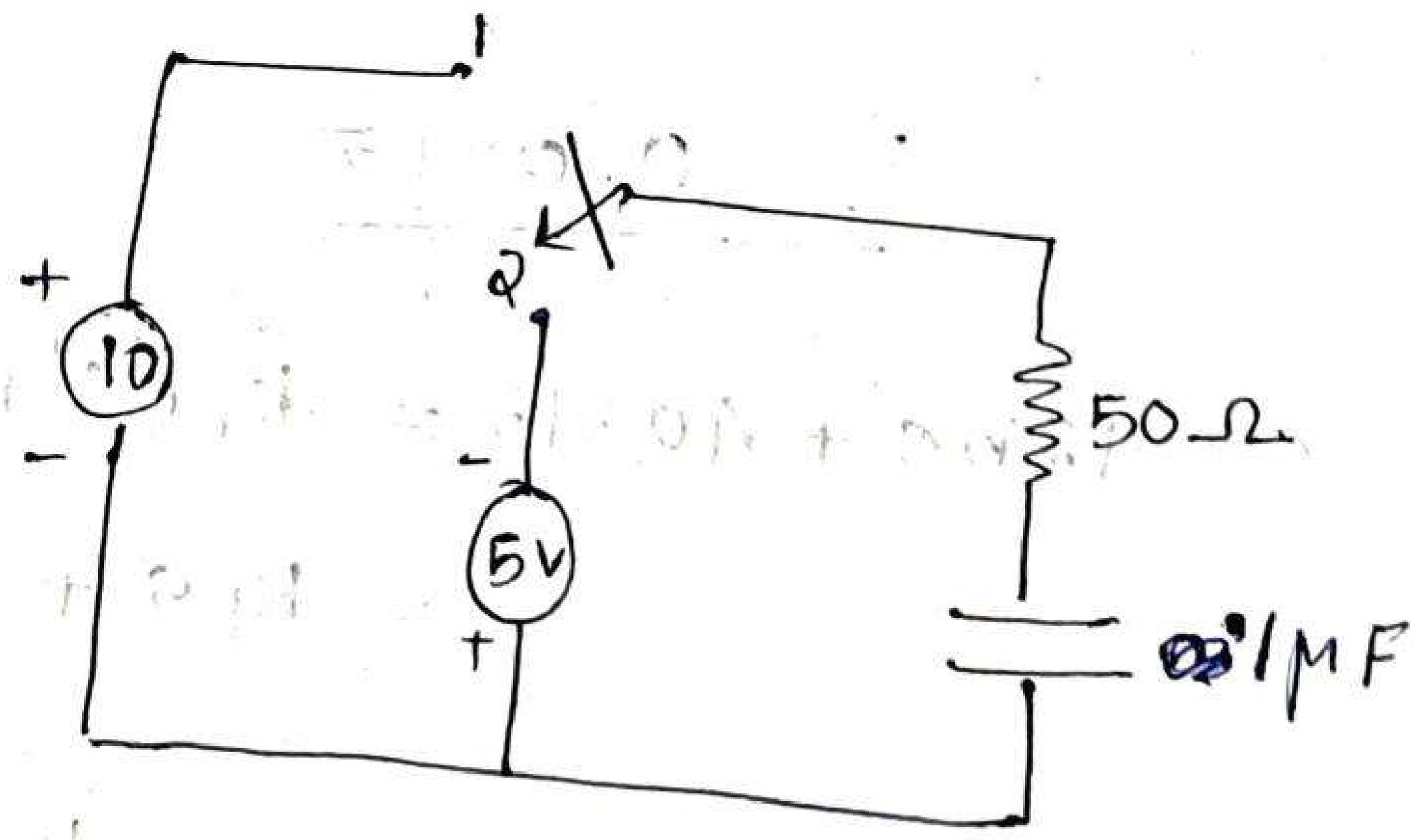
$$= \underline{64.9775}$$

$$I(s) = 0.0373 \times \frac{1}{s+50} - 0.0373 \times \frac{s}{s^2+10^6} + \frac{64.997}{10^3} \times \frac{10^3}{s^2+10^6}$$

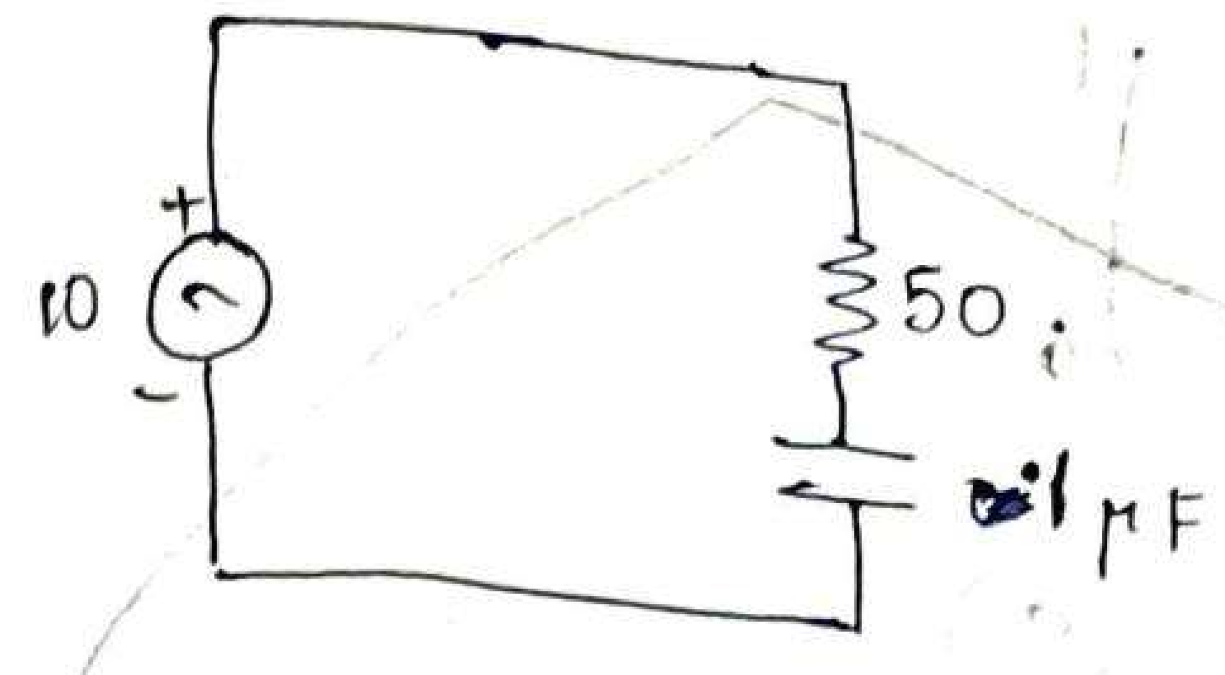
$$\underline{i(t) = 0.0373\,e^{-50t} - 0.0373 \cos 10^3 t + 64.997 \sin 10^3 t}$$

22/10/2020

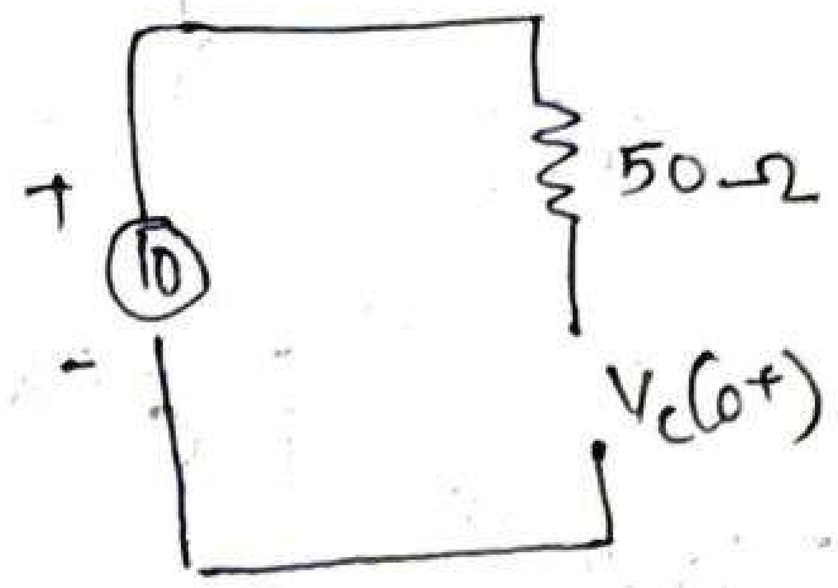
Q. In the circuit, steady state is reached, while the switch is in position 1. At $t = 0$, the switch is moved in position 2. Determine the energy stored in capacitor at $t = 0.1$ ms.



Ans

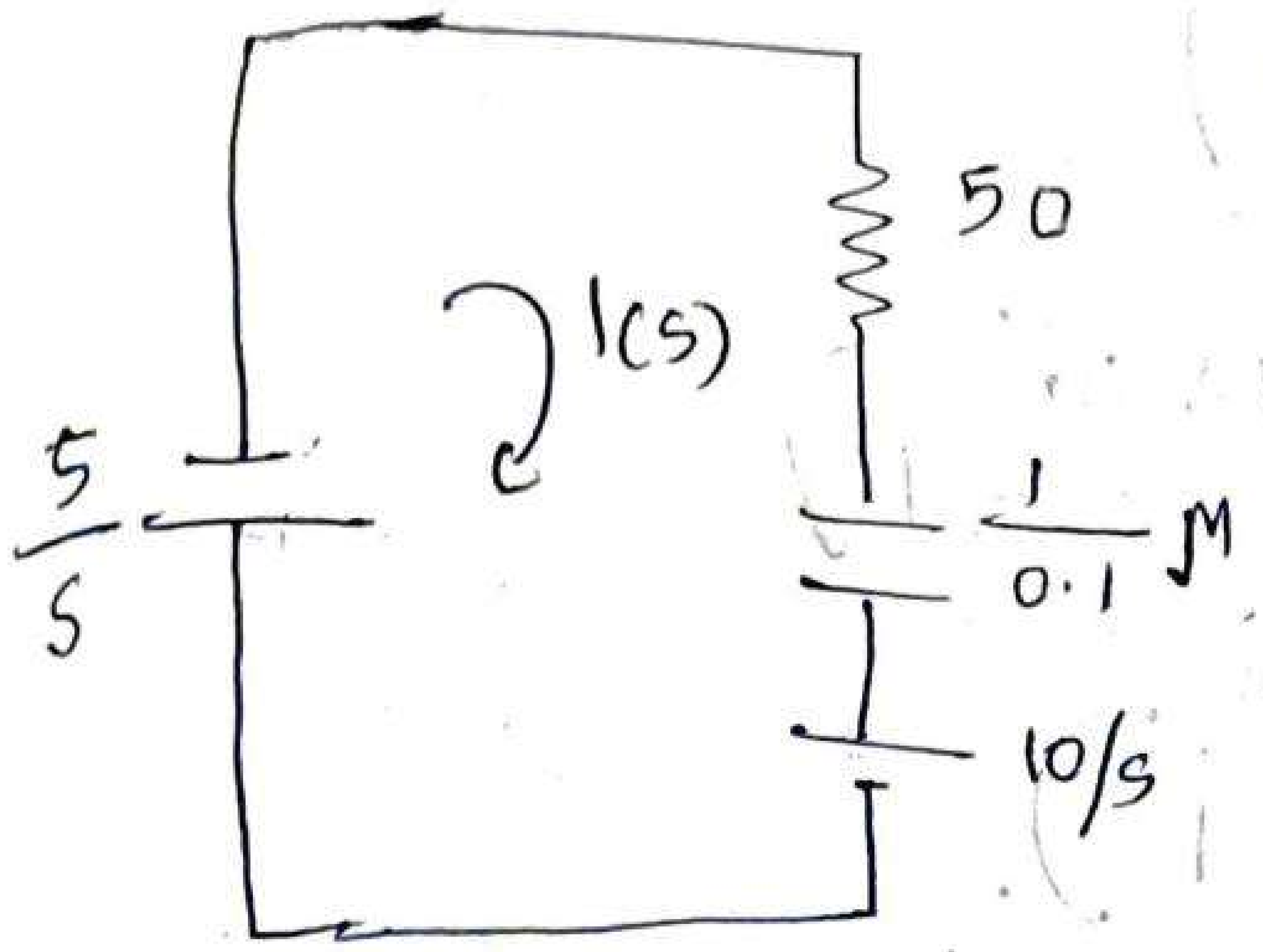


At $t = (0^-)$



$V_c(0^+) = 10\,V$.

At $t = 0$.



$$-\frac{10}{s} - \frac{5}{s} = 50\, I(s) + \frac{10^6}{0.1 s} I(s)$$

$$\frac{-15}{s} = I(s)\left[50 + \frac{10^7}{s}\right]$$

$$I(s) = \frac{-15/s}{\left[50 + \frac{10^7}{s}\right]}$$

$$= \frac{-15/\cancel{s}}{\frac{50s+10^7}{\cancel{s}}}$$

$$= \frac{-15}{50s+10^7}$$

Q = CV

$$I(s) = \frac{-15}{50\left(s + \frac{10^7}{50}\right)} = \frac{-15/50}{s + .2\times10^6} = \frac{-0.3}{s+.2\times10^6}$$

$$i(t) = -0.3\, e^{-.2\times10^6 t}$$

$$V_c(t) = \frac{1}{C}\int i(t)\, dt$$

$$= \frac{1}{1\times10^{-6}} \int -0.3\, e^{-.2\times10^6 t}$$

$$= \frac{-0.3}{10^{-6}} \left[\frac{e^{-2\times10^{6}t}}{-.2\times10^{6}}\right]_0^t$$

$$= \frac{+0.3}{+.2\,\cancel{10^{-6}}} \left[e^{-2\times10^{6}t} - e^{0}\right)$$

$$= \frac{0.3}{.2}\left(e^{-2\times10^{6}t} - 1\right)$$

$$= \frac{0.3}{.2}\left(e^{-2\times10^{6}\times 0.1\times10^{-3}} - 1\right)$$

$$= 1.5\left(e^{-2\times10^{6}\times0.1\times10^{-3}} - 1\right).$$

$$= -1.5.$$

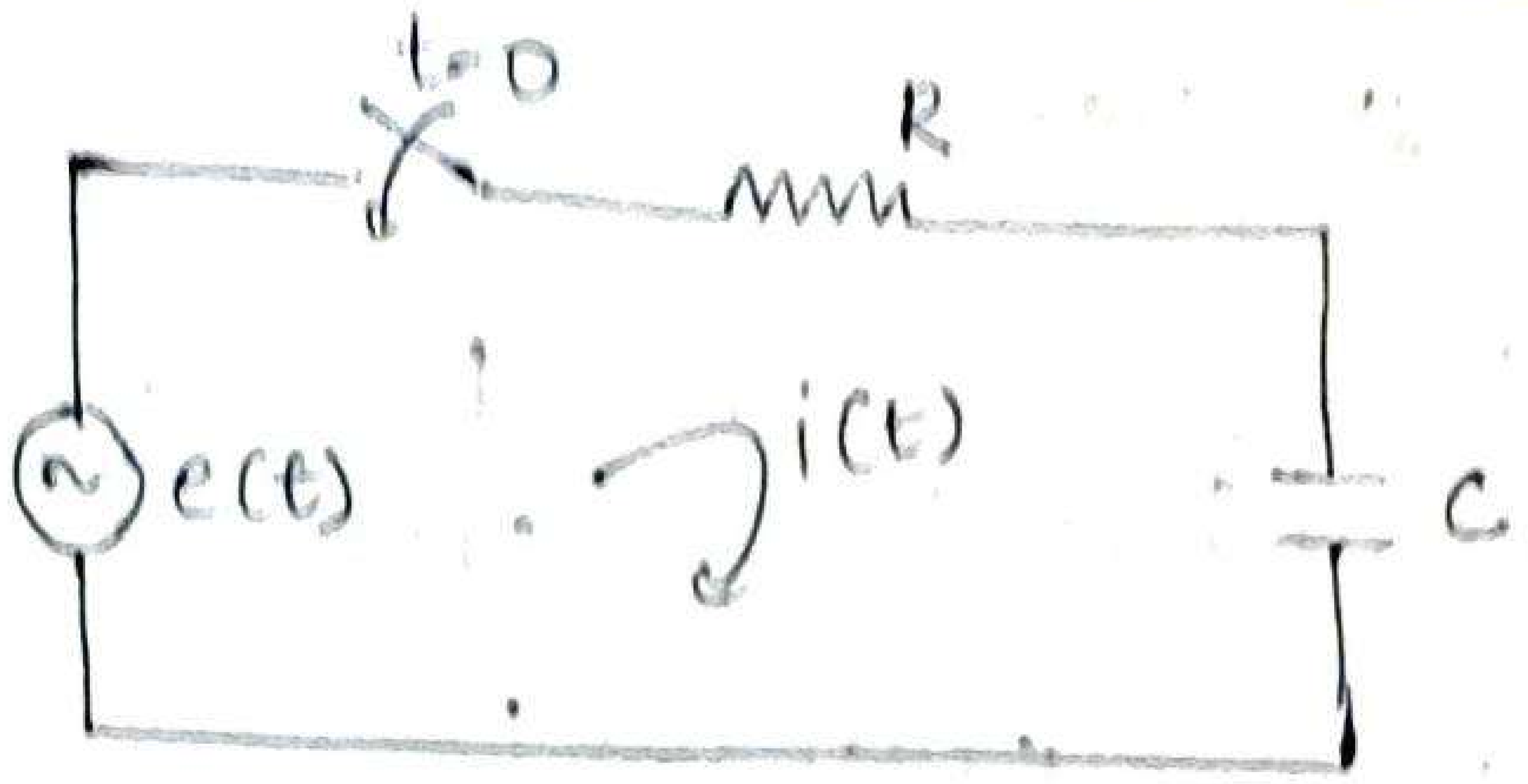
Energy $Q = C \times V$

$= 0.1 \times 10^{-6} \times +1.5$

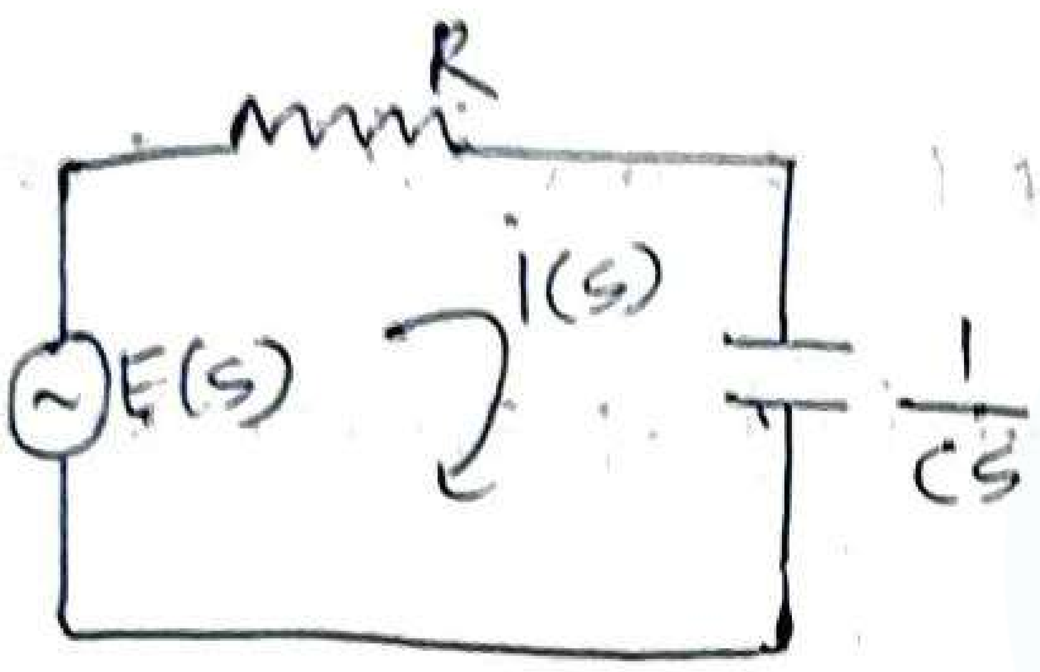
$= 1.5 \times 10^{-7}$.

Q. An RC circuit is exited by a sinusoidal source of voltage 50 sin 314t volts. Find the voltage and current in capacitor. Take $R = 100\,\Omega$ and $C = 20\,\mu F$.

Given that $e(t) = 50 \sin 314t$ volts.



$$E(s) = L(e(t))$$
$$= L[50 \sin 314t]$$
$$= 50 \times L(\sin 314t)$$
$$= 50 \times \frac{314}{s^2+314^2} = \frac{15700}{s^2+314^2}$$



$$I(s) = \frac{E(s)}{R + \frac{1}{Cs}}$$
$$= \frac{E(s)}{\frac{RCs+1}{Cs}}$$

$$I(s) = E(s) \times \frac{\cancel{C}s}{R\cancel{C}\left(s+\frac{1}{RC}\right)}$$
$$= \frac{15700}{s^2+314^2} \times \frac{s}{R\left(s+\frac{1}{RC}\right)}$$

here $RC = 100 \times 20 \times 10^{-4} = 0.002\, s$.

$$I(s) = \frac{15700}{s^2+314^2} \times \frac{s}{100\left(s+\frac{1}{0.002}\right)}$$
$$= \frac{157\, s}{(s+500)(s^2+314^2)}$$

By partial fraction:

$$I(s) = \frac{157s}{(s+500)(s^2+314^2)} = \frac{k_1}{s+500} + \frac{k_2 s + k_3}{s^2+314^2}$$

$$k_1 = \left.\frac{157s}{\cancel{(s+500)}(s^2+314^2)} \times \cancel{(s+500)}\right|_{s=-500}$$

$$= \frac{157 \times -500}{((-500)^2 + 314^2)} = \underline{\underline{-0.2252}} = k_1.$$

$$157s = k_1(s^2+314^2) + k_2 s + k_3 (s+500)$$

$$= k_1 s^2 + k_1 314^2 + k_2 s^2 + k_2 500 s + k_3 s + 500 k_3$$

$$= (k_1+k_2)s^2 + (k_2 500 + k_3)s + 500k_3 + 314^2 k_1$$

$$k_1 + k_2 = 0$$

$$k_2 = -k_1$$

$$\underline{k_2 = 0.2252}$$

$$500k_2 + k_3 = 157$$

$$k_3 = 153 - 500 \times 0.2252$$

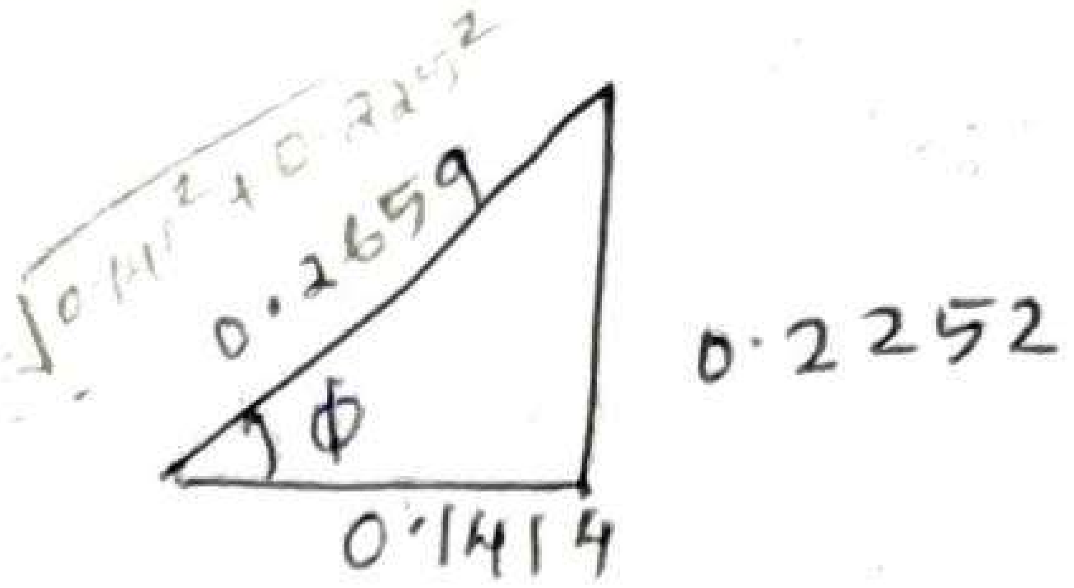
$$\underline{k_3 = 44.4}$$

$$I(s) = \frac{-0.2252}{s+500} + \frac{0.2252 s + 44.4}{s^2+314^2}$$

$$= \frac{-0.2252}{s+500} + 0.2252 \times \frac{s}{s^2+314^2} + \frac{44.4}{314} \times \frac{314}{s^2+314^2}$$

$$i(t) = -0.2252 e^{-500t} + 0.2252 \cos 314t + 0.141 \sin 314t$$

$$i(t) = -0.2252 e^{-500t} + (\sin 314t \times 0.141 + \cos 314t \times 0.225)$$



$$\tan\phi = \frac{0.2252}{0.1414} = 1.5926$$

$$\phi = \tan^{-1}(1.5926)$$

$$= 57.9^{\circ}$$

$$= 58^{\circ}$$

$$\cos\phi = \frac{.1414}{0.2659}$$

$$0.1414 = 0.2659 \cos\phi$$

$$= 0.2659 \cos 58$$

$$\sin\phi = \frac{0.2252}{0.2659}$$

$$0.2252 = \sin\phi \; 0.2659$$

$$= .2659 \sin 58^{\circ}$$

$$i(t) = -0.2252\, e^{-500t} + \left[\sin 314t \times 0.2659 \cos 58^{\circ} + \cos 314t \times 0.2659 \sin 58^{\circ}\right]$$

$$= -0.2252\, e^{-500t} + 0.2659\left(\sin 314t \cos 58 + \cos 314t \sin 58^{\circ}\right)$$

$$i(t) = -0.2252\, e^{-500t} + 0.2659 \sin(314t + 58^{\circ}) \text{ Ampere}$$

$$V_c(t) = \frac{1}{C}\int i(t)\, dt$$

$$= \frac{1}{C}\int \left[-0.2252\, e^{-500t} + 0.2659 \sin(314t + 58^{\circ})\right] dt$$

$$= \frac{1}{20\times10^{6}}\left[\frac{-0.2252\, e^{-500t}}{-500} - \frac{0.2659 \cos(314t + 58^{\circ})}{314}\right]$$

$$= \frac{-0.2252\, e^{-500t}}{20\times10^{6} \times (-500)} + \frac{0.2659}{20\times10^{6}\times 314} \cos(314t + 58 - 90)$$

$$= 22.52\, e^{-500t} + 42.34 \sin(314t - 32^{\circ})\ V$$