TRIVENI PAGE NO. DATE / /

Module 1

Network Theorems.

(DC & AC Steady State Analysis).

Linearity and Superposition Principle.

A linear circuit is the circuit whose parameters are constant; ie they donot change with the application of voltage or current

Eg:- R, L, C.

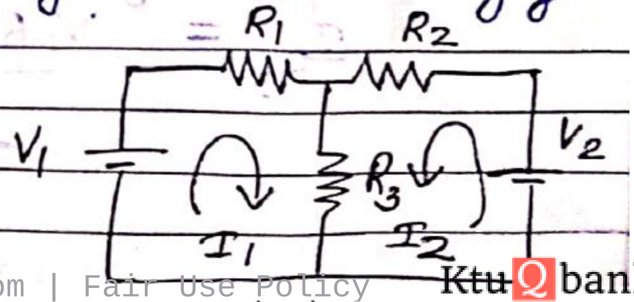
1 Superposition Theorem.

In any linear network, containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non-operative.

ie while considering the effect of individual sources, the other voltage sources and current sources in the network are replaced by the short circuit and open circuit across their terminals.

Proof

Consider a n/w having two energy sources V_1 & V_2.



Suppose we have to find current through R_3.

By applying mesh analysis:

$I_{R_3} = I_1 + I_2$.

For mesh I $V_1 - I_1 R_1 - (I_1 + I_2) R_3 = 0 \Rightarrow V_1 = I_1 (R_1 + R_3) + I_2 R_3$

For mesh II $V_2 - I_2 R_2 - (I_2 + I_1) R_3 = 0 \Rightarrow V_2 = I_1 R_3 + I_2 (R_2 + R_3)$

In matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow I_1 = \frac{\begin{vmatrix} V_1 & R_3 \\ V_2 & R_2 + R_3 \end{vmatrix}}{\begin{vmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{vmatrix}} = \frac{V_1 (R_2 + R_3) - V_2 R_3}{\Delta}$$

$$I_2 = \frac{\begin{vmatrix} R_1 + R_3 & V_1 \\ R_3 & V_2 \end{vmatrix}}{\Delta} = \frac{V_2 (R_1 + R_3) - V_1 R_3}{\Delta}$$

$$I_{R_3} = I_1 + I_2 = \frac{V_1 (R_2 + R_3) - V_2 R_3 + V_2 (R_1 + R_3) - V_1 R_3}{\Delta}$$

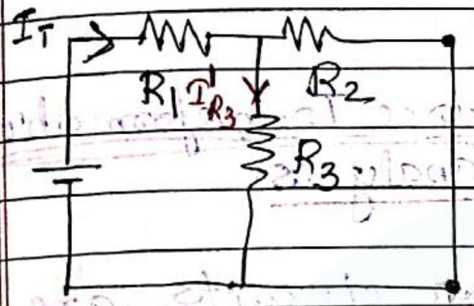
Expanding

$$I_{R_3} = \frac{V_1 R_2 + V_2 R_1}{\Delta}$$

By Superposition theorem

$$I_{R_3} = I_{R_3} \text{ due to } V_1 \Big|_{V_2=0} + I_{R_3} \text{ due to } V_2 \Big|_{V_1=0}$$

To find I_{R_3} due to V_1



$$I'_{R_3} = I_T \times \frac{R_2}{R_2+R_3}$$

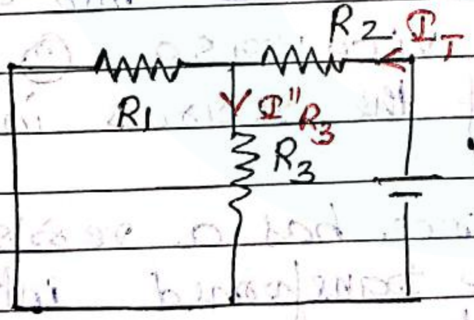
$$I_T = \frac{V_1}{R_{eq}}$$

$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2+R_3}$$

$$\therefore I'_{R_3} = \frac{V_1}{\left(R_1 + \frac{R_2R_3}{R_2+R_3}\right)} \times \frac{R_2}{R_2+R_3}$$

$$= \frac{V_1 R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$

To find I_{R_3} due to V_2



$$I''_{R_3} = I_T \cdot \frac{R_1}{R_1+R_3}$$

$$I_T = \frac{V_2}{R_{eq}}$$

$$R_{eq} = R_2 + \frac{R_1R_3}{R_1+R_3}$$

$$\therefore I''_{R_3} = \frac{V_2}{\left(R_2 + \frac{R_1R_3}{R_1+R_3}\right)} \cdot \frac{R_1}{R_1+R_3}$$

$$= \frac{V_2 R_1}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$I_{R_3} = I'_{R_3} + I''_{R_3}$$

$$= \frac{V_1 R_2 + V_2 R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

It is same as obtained from mesh analysis. This is the proof.

Application of Source Transformation in electric circuit analysis

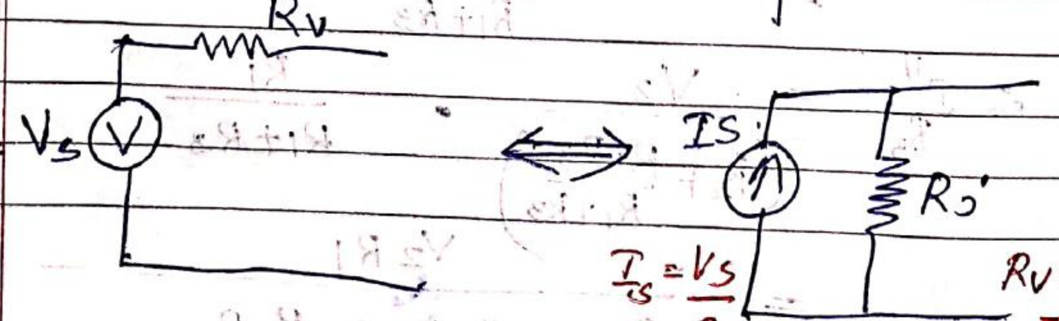
Normally, electric circuits are complicated consisting of many ckt elements. All such circuits can be solved easily by bringing them into the simplest form. This is called network reduction. It includes series ckt concept, parallel ckt concept, Y/Δ transformation, source transformation.

Source transformation means (1) transformation of voltage source into a current source and vice versa (2). Shifting the positions of the sources in a n/w.

If a voltage source has a resistance in series, it can be transformed into an equivalent current source with the resistance in parallel.

$R_V = R_i$

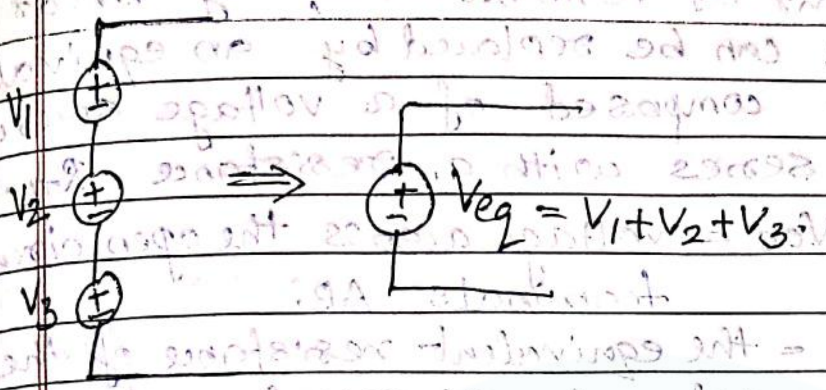
$I_S R_i = V_s$



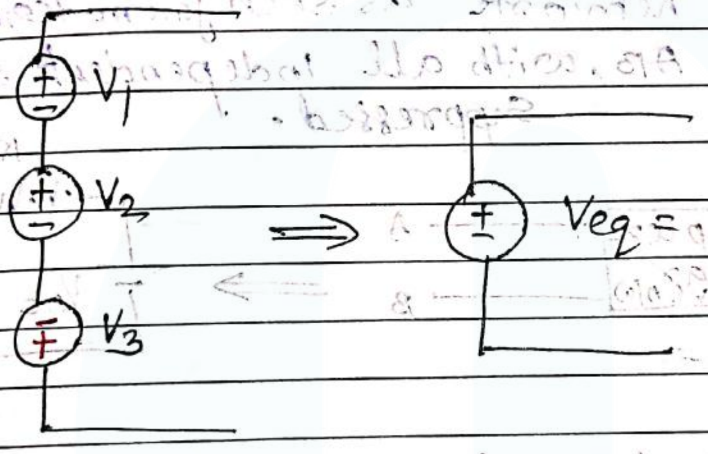
$I_S = \frac{V_S}{R_i}$

$R_V = R_i$

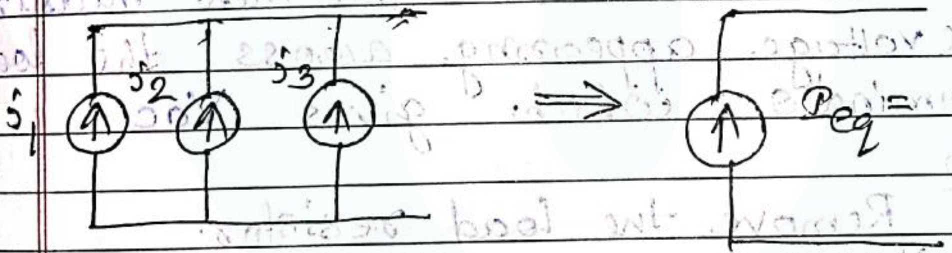
Applications of Source Transformation.



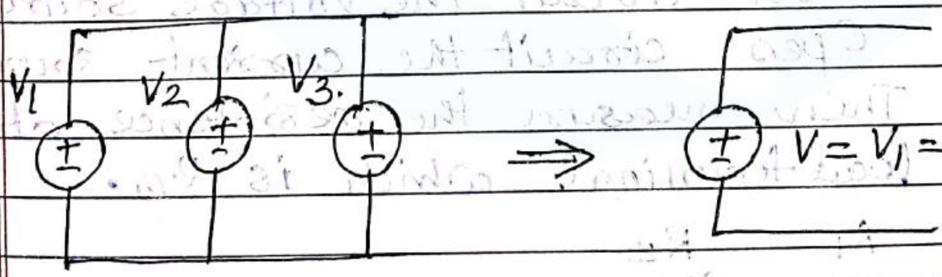
$V_{eq} = V_1 + V_2 + V_3$



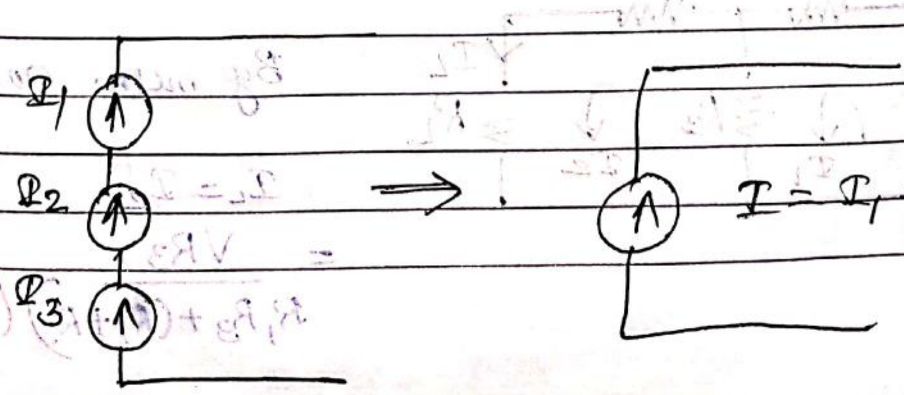
$V_{eq} = V_1 + V_2 - V_3$



$I_{eq} = i_1 + i_2 + i_3$.



$V = V_1 = V_2 = V_3$.



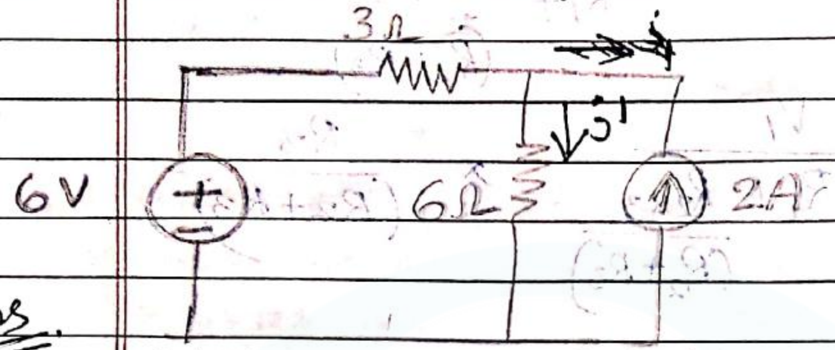
$I = I_1 = I_2 = I_3$

DC analysis.

Problems.

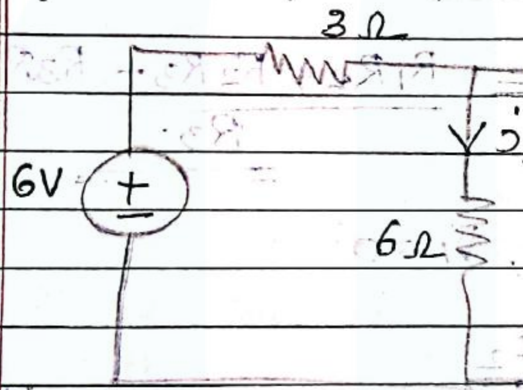
Superposition theorem.

1) Find the current in the 6Ω resistor using the principle of SP.



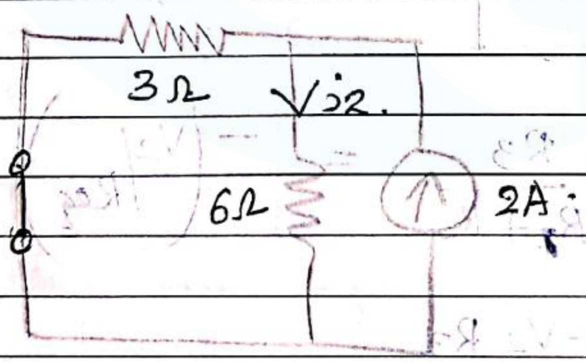
Ans.

Set the current source to zero (2A) ie open ckt., ie find the current due to 6V



$$i_1 = \frac{6}{3+6} = \frac{6}{9}$$

To find current due to 2A (Short ckt 6V)



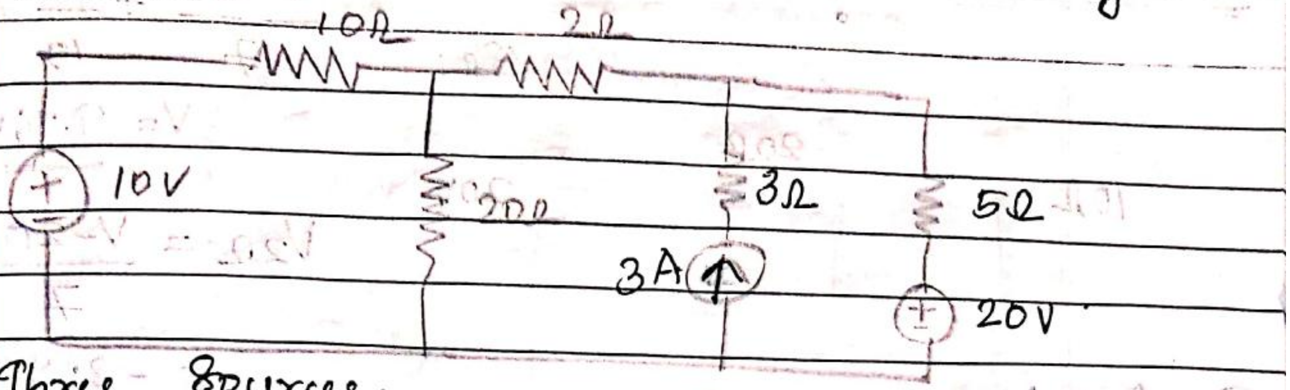
$$i_2 = \frac{2\times 3}{3+6} = \frac{6}{9}$$

$$i' = i_1 + i_2$$

$$= \frac{6}{9} + \frac{6}{9}$$

$$= \frac{12}{9} = \frac{4}{3}\,A.$$

2. Find the voltage across 2Ω resistor by SP theorem.



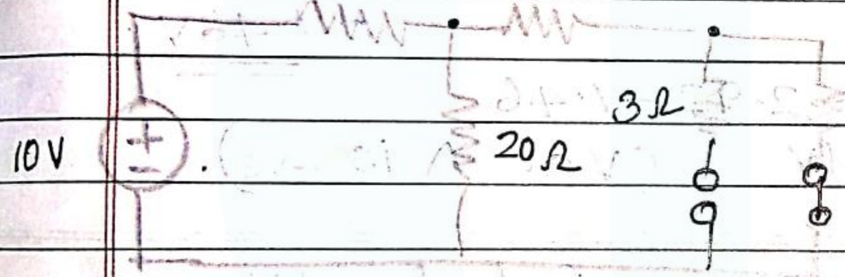
Ans: Three sources.

(10V, 3A, 20V)

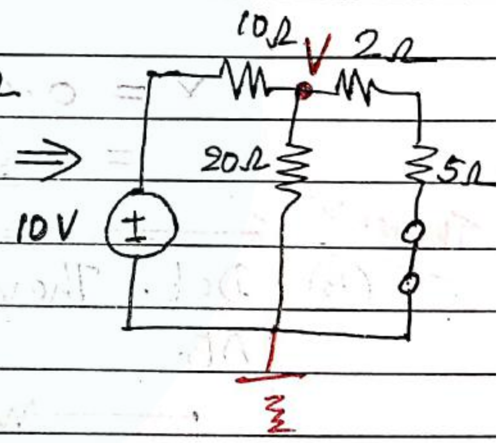
Voltage across 2Ω resistor

= Voltage due to 10V + Due to 3A + Due to 20V

Due to 10V



$\Rightarrow$



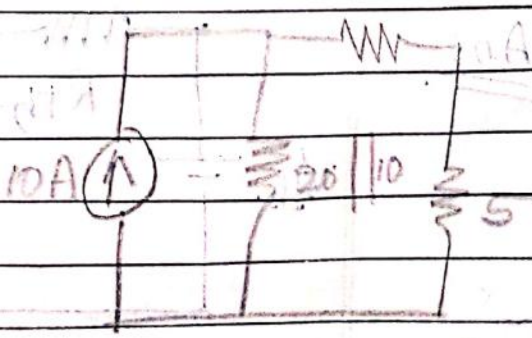
Applying nodal analysis,

$$\frac{V-10}{10} + \frac{V}{20} + \frac{V}{7} = 0 \Rightarrow V = 3.41V$$

$$V_{2\Omega} = \frac{V \times 2}{2+5} = 0.97V$$

OR

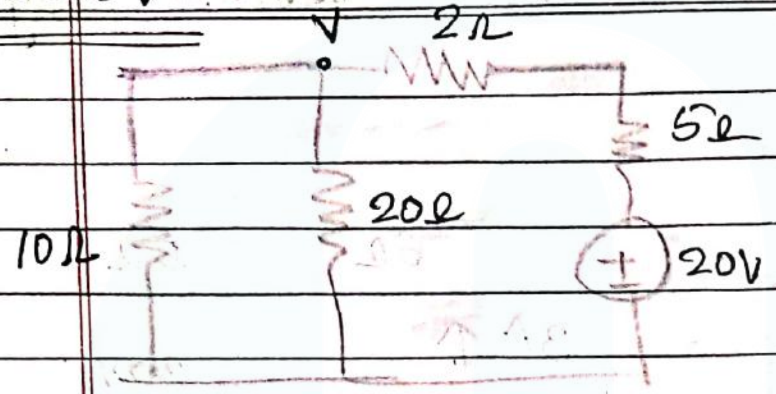
$$I_{2\Omega} = \frac{10 \times 20/3}{\frac{20}{3}+7} = 4.87A$$



15

TRIVENI
PAGE NO.
DATE / /

Due to 20V



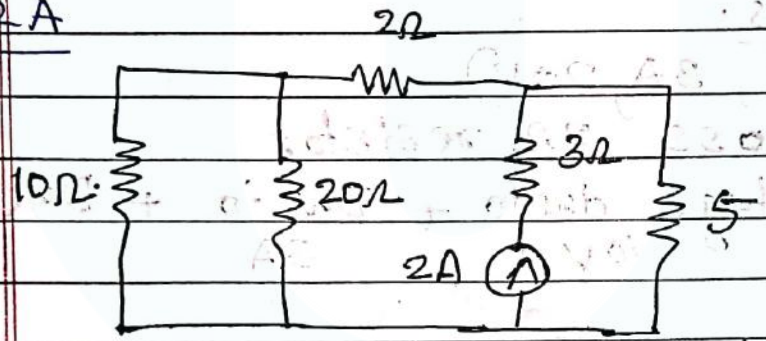
$$\frac{V-20}{7} + \frac{V}{10} + \frac{V}{20} = 0$$

$$V = 9.76V$$

$$V_{2\Omega} = \frac{V-20}{7} \times 2$$

$$= -2.92V$$

Due to 2A



$$I_{2\Omega} = 2 \times \frac{5}{5+8.67}$$

$$= 0.73A$$

$$V_{2\Omega} = 0.73 \times 2$$

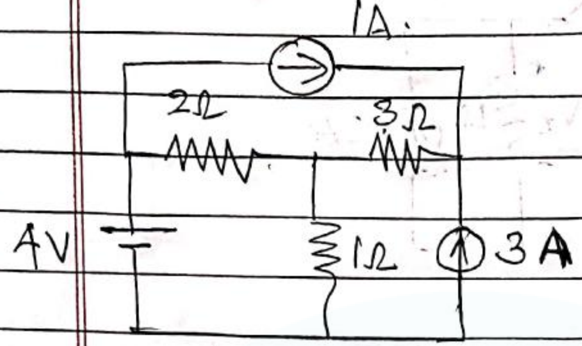
$$= 1.46V$$

$\therefore V = 0.97 - 2.92 - 1.46$

$= -3.41V$ (V at A is -ve).

Thevenin's

HW 2 (A) Find the current in the 1Ω resistors.



$I_{4V} = 1.33$

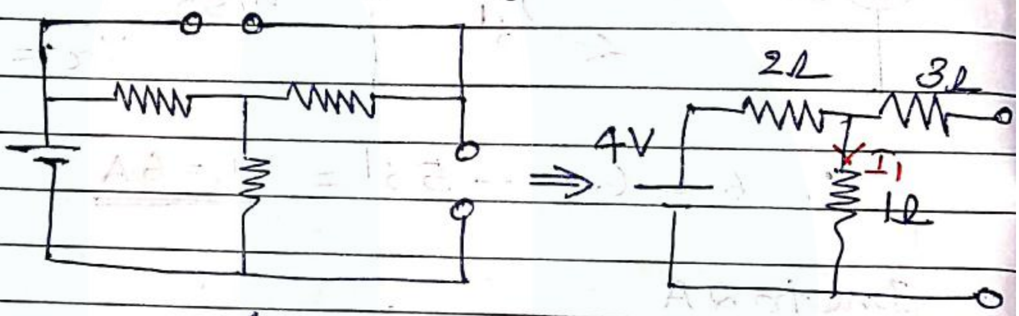
$2A$

$I_{3A} =$

$I_{1A} = 2/3$

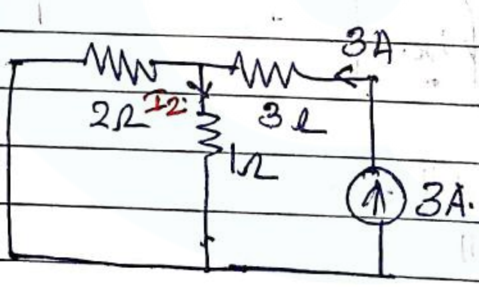
$I = 4A$

When the 4V source is acting alone.



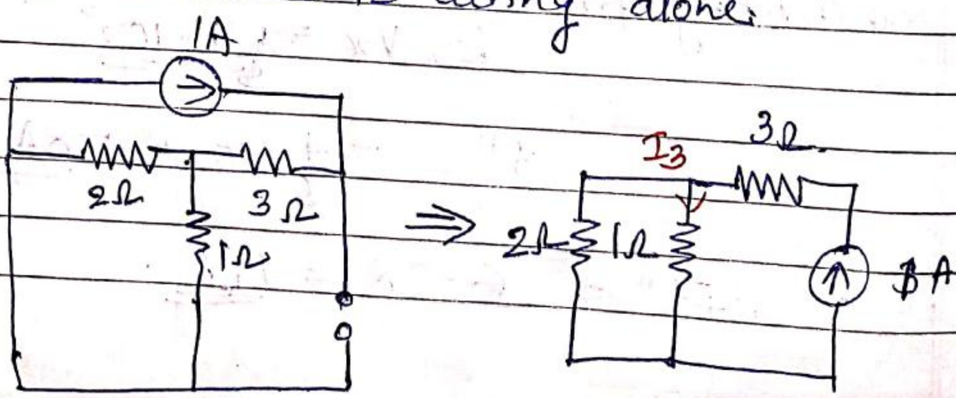
$$I_1 = \frac{4}{2+1} = 1.33A \,(\downarrow)$$

When the 3A source is acting alone.



$$I_2 = \frac{3 \times 2}{2+1} = 2A \,(\downarrow)$$

When the 1A source is acting alone.



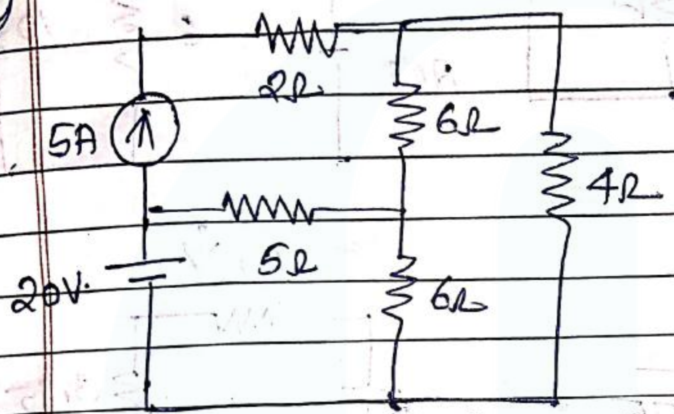
$$I_3 = \frac{1\times 2}{2+1} = 2/3 \text{ A } (\downarrow)$$

Current in the 1Ω resistor = $1.33 + 2 + 2/3$

$= \underline{4A} \downarrow$

HW 3

(5) Find the current through the 4Ω resistor

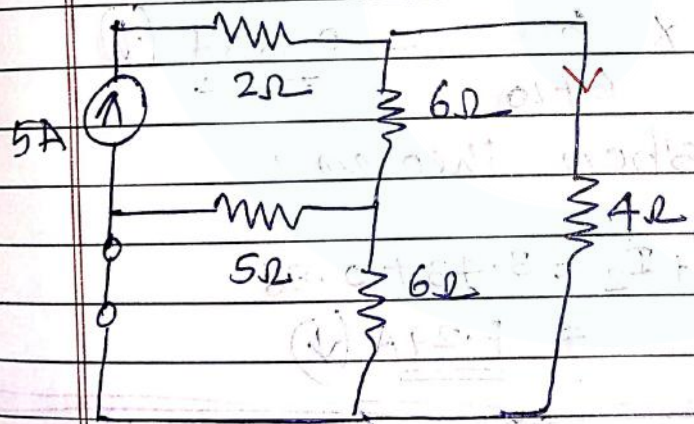


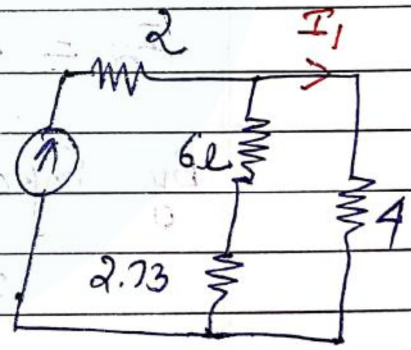
$I_{5A} = 3.43$

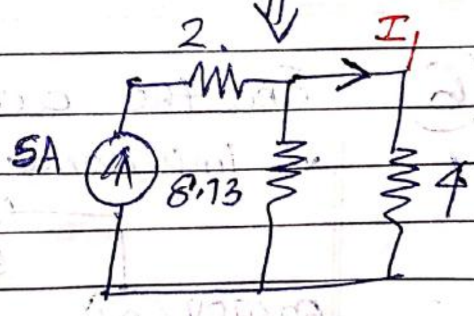
$I_{20V} = .86$

$I = 4.29$

When the 5A source is acting alone.



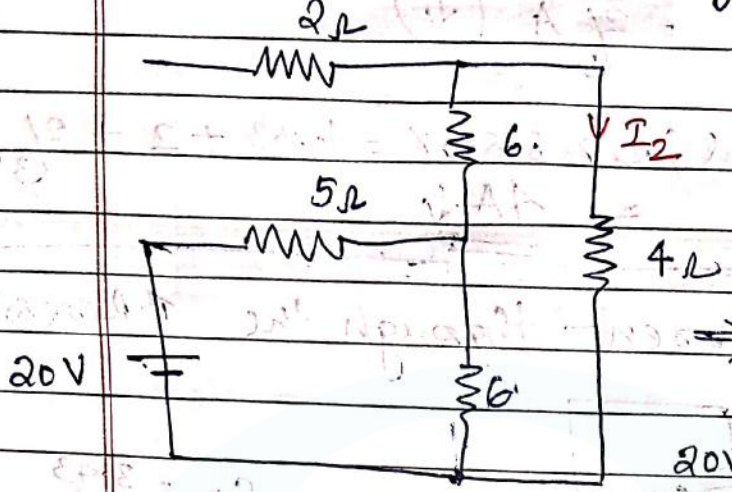


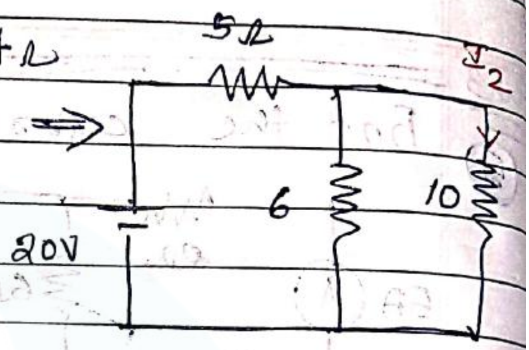


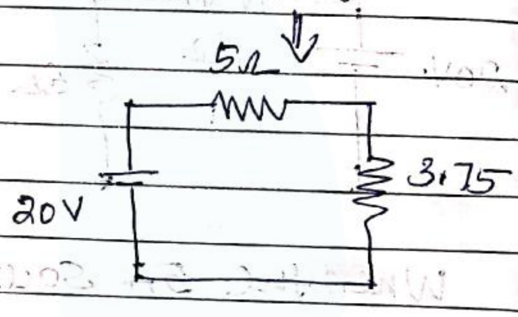
$$I_1 = \frac{5\times 8.73}{8.73+4}$$

$= \underline{3.43A}$ (↓)

When the 20V source is acting alone







$$I = \frac{20}{5+3.75}$$

$$= 2.29A$$

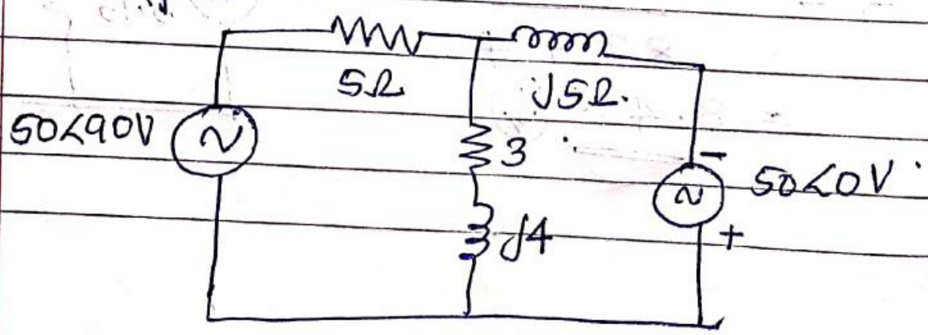
$$I_2 = 2.29 \times \frac{6}{6+10} = 0.86A \ (\downarrow)$$

By Superposition theorem,

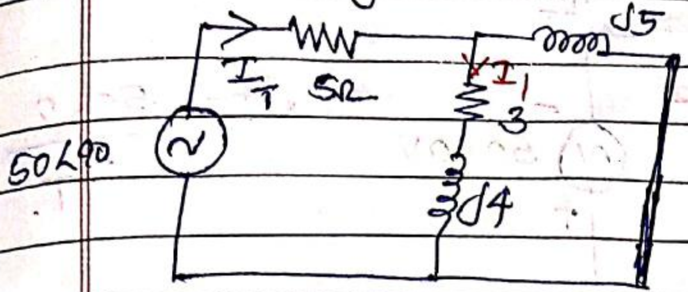
$$I = I_1 + I_2 = 3.43 + 0.86$$

$$= 4.29A \ (\downarrow)$$

(6) Find the current through the (3+j4) Ω impedance.



When the 50<90 V source is acting alone,



$$I_T = \frac{50\angle 90}{Z_T}$$

$$Z_T = 5 + \frac{(3+j4)(j5)}{(3+j4+j5)}$$

$$Z_T = 5 + \frac{j15 - 20}{3+j9}$$

$$= 5 +$$

$$= \underline{\underline{6.35\angle 23.2\ \Omega.}}$$

$$I_T = \frac{50\angle 90}{6.35\angle 23.2} = \underline{\underline{7.87\angle 66.8\ A}}$$

By current division rule,

$$I_1 = \frac{I_T \times j5}{(j5+3+j4)} = I_T \times \frac{5\angle 90}{(3+j9)}$$

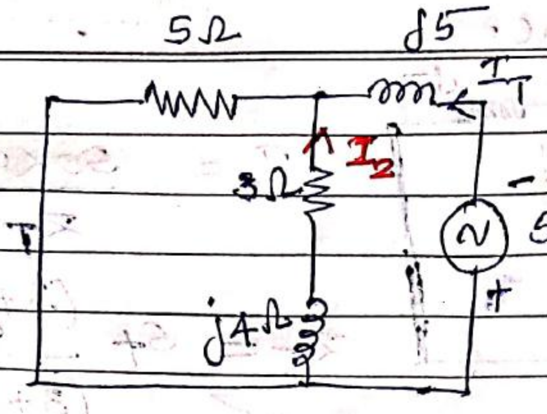
$$= (7.87\angle 66.8) \times \frac{5\angle 90}{(\quad)}$$

$$=$$

When the 50<0V is acting alone.

Ravish

p. 5.16



$I_T = \frac{50\angle 0}{Z_T}$

$$Z_T = j5 + \frac{(3+j4)(5)}{3+j4+5} = j5 + \frac{15+j20}{(8+j4)}$$

$$=$$

$$I_T = \frac{50\angle 0}{6.74\angle 68.2} = 7.42\angle -68.2$$

$$I_2 = I_T \times \frac{5}{(5+3+j4)} = I_T \times \frac{5}{(8+j4)}$$

$$= 4.15\angle -94.77 (\uparrow)$$

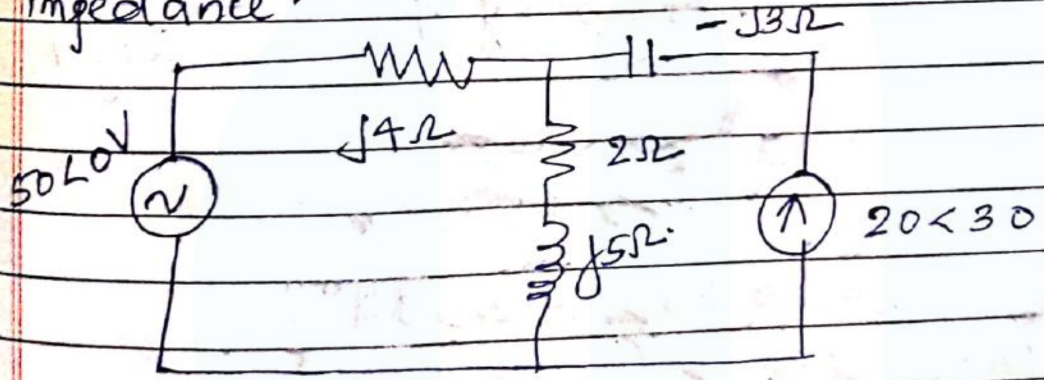
By Superposition theorem:

$$I = I_1 + I_2 = 4.15\angle 85.3 + 4.15\angle -94.77$$

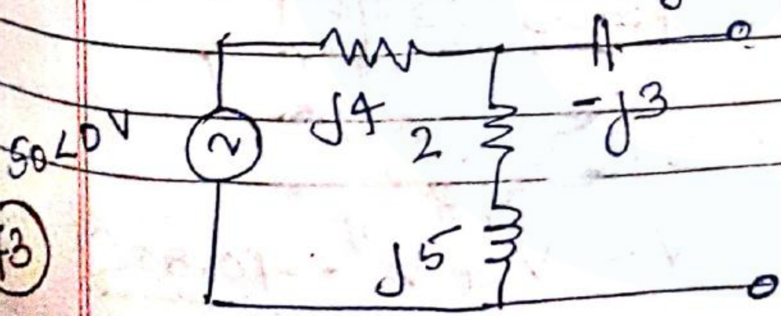
$$= 8.31\angle 85.3 (\downarrow)$$

Assignment 1 Questions

1. Determine the voltage across the $(2+j5)\Omega$ impedance:



Ans. Due to $50\angle0$ voltage source.



43

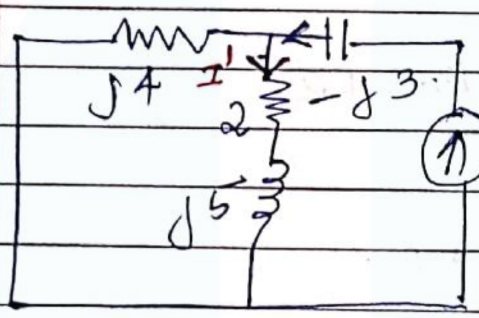
Voltage across $(2+j5)\,\Omega$.

$$= \frac{50\angle 0}{(2+j5)+j4} \times (2+j5)$$

$$V' = \frac{50\angle 0}{2+j9}(2+j5) =$$

$$= \underline{29.16\angle -9.28\,V}$$

Due to $20\angle 30$ A source is acting alone.



By current division rule,

$$I' = \frac{(20\angle 30)(j4)}{2+j5+j4} = \frac{(20\angle 30)(j4)}{2+j9}$$

$$= 8.68\angle 42.53\,A.$$

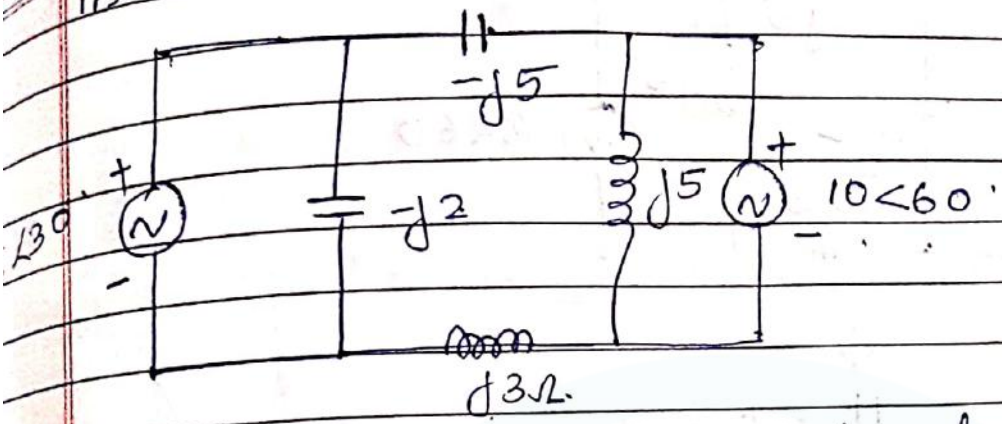
Voltage across $(2+j5)\,\Omega$

$$V'' = 8.68\angle 42.53\,(2+j5)$$

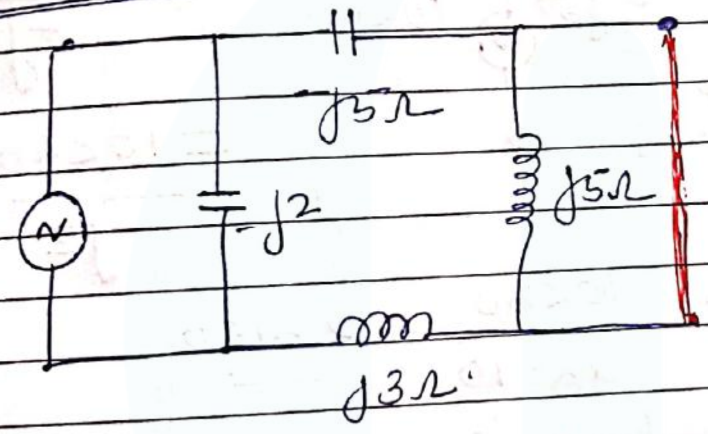
$$= \underline{46.69\angle 110.72\,A}$$

Total voltage $V = V' + V'' \angle 0.$ ... $\angle 42.53$

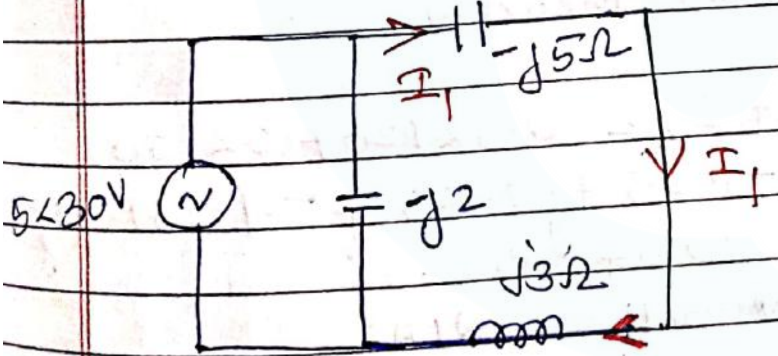
Q2. Find the current through the $j3\Omega$ reactance in the network.



Ans. Due to $5\angle 30\,V$ source is acting alone.



When a short ckt is placed across $j5\Omega$, it gets shorted.



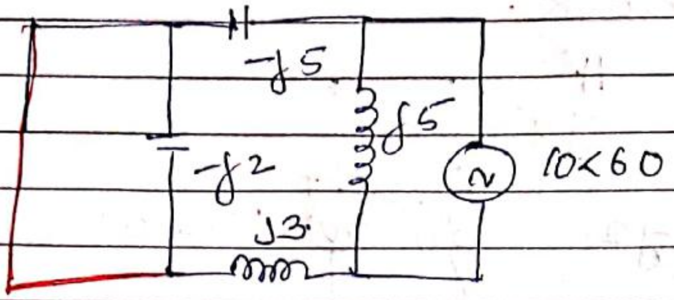
$$I_1 = \frac{5\angle 30}{(-j5+j3)} = \frac{5\angle 30}{-j2}$$

$$= \frac{5\angle 30}{2\angle -90} = 2.5\angle 120\ A.$$

$-j5$ & $j3$ in series
$-j2$ & $(-j5+j3)$ in parallel

Same voltage across parallel branches.

When the $10\angle 60V$ source is acting alone.



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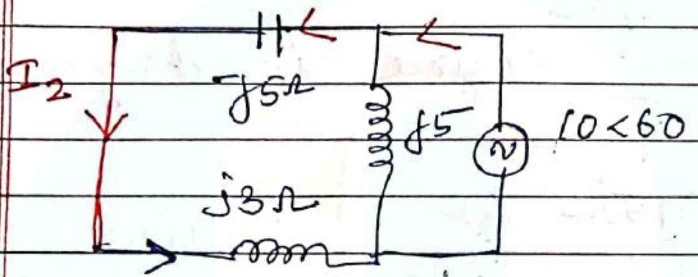


$$I_2 = \frac{10\angle 60}{-j5+j3}$$

$$= \frac{10\angle 60}{-j2}$$

$$= \frac{10\angle 60}{2\angle -90} = 5\angle 150$$

By superposition theorem,

$$I = I_1 + I_2 = 2.5\angle 120 + 5\angle 150$$

$$= -1.25 + j\,2.165 + -4.33 + 0.5j$$

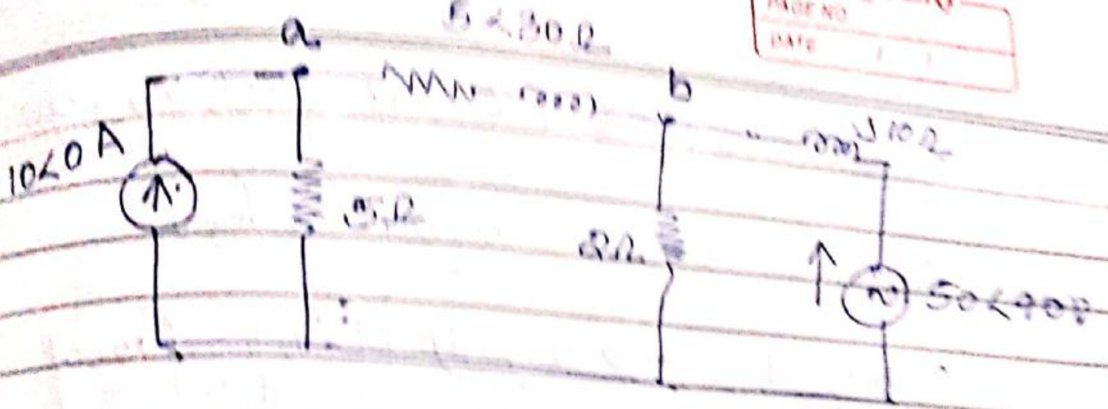
$$= -5.58 + 2.665j$$

$$= 3.1\angle -6.21\,A.$$

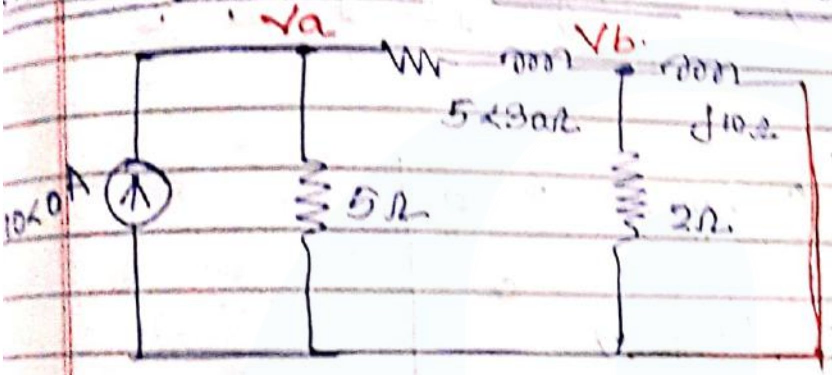
KTU Sample QP

(3) With respect to the following ckt
a) Find the voltages at a and b using superposition theorem.
b) Obtain the active power dissipated in $5\angle 30\Omega$ impedance.

46



When the 10∠0 A source is acting alone.



Applying KCL at node a

$$-10\angle 0 + \frac{V_a - V_b}{5\angle 30} + \frac{V_a}{5} = 0$$

$$V_a\left[\frac{1}{5\angle 30} + \frac{1}{5}\right] - V_b\left[\frac{1}{5\angle 30}\right] = 10\angle 0.$$

$$V_a\left[0.37 - j0.1\right] - V_b\left[0.17 - j0.1\right] = 10\angle 0.$$

Applying KCL at node b.

$$\frac{V_b - V_a}{5\angle 30} + \frac{V_b}{2} + \frac{V_b}{j10} = 0$$

$$(7)\quad V_a\left[\frac{-1}{5\angle 30}\right] + V_b\left[\frac{1}{5\angle 30} + \frac{1}{2} + \frac{1}{j10}\right] = 0$$

TRIVENI PAGE NO. DATE / /

$$-(0.17-j0.1)V_a + (0.67-j0.2)V_b = 0.$$

Express these equations in matrix form,

$$\begin{bmatrix} 0.37-j0.1 & -(0.17-j0.1) \\ -(0.17-j0.1) & 0.67-j0.2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ 0 \end{bmatrix}$$

By Cramer's rule.

$$V_b = \frac{\begin{vmatrix} (0.37-j0.1) & 10\angle 0 \\ -(0.17-j0.1) & 0 \end{vmatrix}}{\begin{vmatrix} (0.37-j0.1) & -(0.17-j0.1) \\ -(0.17-j0.1) & 0.67-j0.2 \end{vmatrix}}$$

$$= \frac{\cancel{(0.37-j0.1)\times 0} + 10\angle 0(0.17-j0.1)}{(0.37-j0.1)(0.67-j0.2) - (0.17-j0.1)(0.17-j0.1)}$$

$$= \frac{1.7-j1}{(0.383\angle -15.12)(0.699\angle 16.6) - [0.1972\angle -30.47 \; 0.1972\angle -30.47]}$$

48

TRIVENI PAGE NO DATE / /

$$= \frac{1.97\angle -30}{(0.383\angle -15.12+16.6 \times .699) - (0.039\angle -67.94)}$$

$$= \frac{1.97\angle -30}{0.2677\angle 1.48 - 0.039\angle -67.94}$$

$$= \frac{1.97\angle -30}{0.2676 - 0.0069j - 0.0146 + j0.036}$$

$$= \frac{1.97\angle -30}{0.253 + 0.0291j} = \frac{1.97\angle -30}{0.255\angle 6.56}$$

$$= \underline{\underline{7.73\angle -36.56}}$$

$$V_a = \frac{\begin{vmatrix} 10\angle 0 & -(0.17 - j0.1) \\ 0 & 0.67 - j0.2 \end{vmatrix}}{\Delta}$$ (Δ same as above)

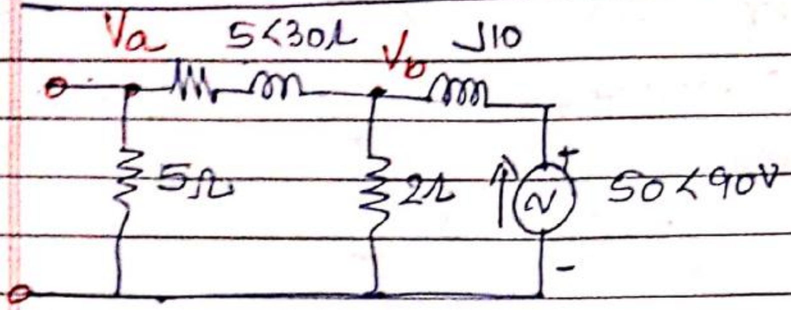
$$= \frac{10\angle 0 (0.67 - j0.2)}{0.255\angle 6.56} = \frac{6.7 - j2}{.255\angle 6.56}$$

$$= \frac{6.99\angle -16.6}{0.255\angle 6.56} = \underline{\underline{27.41\angle -23.16}}$$

V_a & V_b due to $10\angle 0$ A source ↑

TRIVENI
PAGE NO.
DATE / /

V_a & V_b due to $50\angle 90V$.



$$\frac{V_a}{5}+\frac{V_a-V_b}{5\angle 30}=0. \Rightarrow V_a\left[\frac{1}{5}+\frac{1}{5\angle 30}\right]-\frac{V_b}{5\angle 30}=0$$

$$\frac{V_b-V_a}{5\angle 30}+\frac{V_b}{2}+\frac{V_b-50\angle 90}{j10}=0$$

$$V_a\left[-\frac{1}{5\angle 30}\right]+V_b\left[\frac{1}{5\angle 30}+\frac{1}{2}+\frac{1}{j10}\right]=\frac{50\angle 90}{j10}\rightarrow 10\angle 90$$

In matrix form

$$\begin{bmatrix} \left(\frac{1}{5}+\frac{1}{5\angle 30}\right) & -\frac{1}{5\angle 30} \\ -\frac{1}{5\angle 30} & \frac{1}{5\angle 30}+\frac{1}{2}+\frac{1}{j10} \end{bmatrix}\begin{bmatrix} V_a \\ V_b \end{bmatrix}=\begin{bmatrix} 0 \\ 5\angle 0 \end{bmatrix}$$

$$V_a=\frac{\begin{vmatrix} 0 & -1/5\angle 30 \\ 5 & \frac{1}{5\angle 30}+\frac{1}{2}+\frac{1}{j10} \end{vmatrix}}{\Delta}$$

50

TRIVENI PAGE NO. DATE / /

$$\Delta = \begin{vmatrix} 0.2 + 0.2\angle{-30} & -(0.2\angle{-30}) \\ -(0.2\angle{-30}) & 0.2\angle{-30} + 0.5 - j0.1 \end{vmatrix}$$

$$= \begin{vmatrix} 0.2 + 0.173 - 0.1j & -(0.2\angle{-30}) \\ -(0.2\angle{-30}) & 0.173 - 0.1j + 0.5 - j0.1 \end{vmatrix}$$

$$= \begin{vmatrix} 0.373 - 0.1j & -(0.2\angle{-30}) \\ -0.2\angle{-30} & 0.673 - j0.2 \end{vmatrix}$$

Cross-Multiplying

$$= (0.385\angle{-15})(0.702\angle{-16.6}) - (0.2\angle{-30})(0.2\angle{-30})$$

$0.385\angle{-15}$

$0.702\angle{-16.6}$

$$= 0.246\angle{-31.6} - 0.04\angle{-60}$$

$$= 0.204 - 0.129j - (7.02 - 0.006j)$$

$$= -6.916 - 0.123j = \underline{\underline{6.92\angle{-178}}}$$

$$\therefore V_a = \frac{\frac{1}{5\angle{30}} \times 5}{\Delta} = \frac{1\angle{-30}}{6.92\angle{-178}}$$

$$= \underline{0.145\angle{+175}}$$

51

$$V_b = \frac{\begin{vmatrix} \frac{1}{5} + \frac{1}{5\angle 30} & 0 \\ -\frac{1}{5\angle 30} & 5 \end{vmatrix}}{\Delta} = \frac{5\left[\frac{1}{5} + \frac{1}{5\angle 30}\right]}{\Delta}$$

$$= \frac{1 + \frac{5\angle 0}{5\angle 30}}{\Delta} = \frac{1 + 1\angle -30}{6.92\angle -178}$$

$$= \frac{0.865 - 0.5j + 1}{6.92\angle -178} = \frac{1.865 - 0.5j}{6.92\angle -178}$$

$$= \frac{178.13\angle -87.8}{6.92\angle -178} = 25.74\angle 90.23\ V$$

By Superposition theorem

$V_a = 27.41\angle -23.16 + 0.145\angle 175$

$= 25.2 - 10.78j + -0.144 + 0.0126$

$= 25.056 - 10.77j$

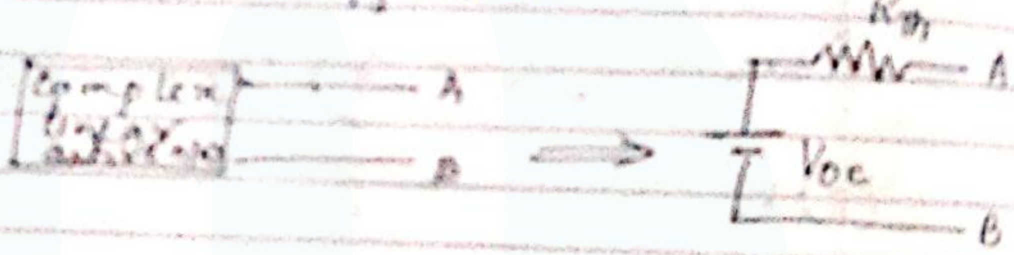
$V_b = 7.73\angle -26.56 + 25.74\angle 90.23$

$= 6.91 - 4.6j - 0.103 + 25.74j$

$= 6.807 + 21.14j$

2. Thevenin's theorem.

At a pair of terminals AB, any linear network can be replaced by an equivalent circuit composed of a voltage source V_{oc} in series with a resistance R_{Th}.
where V_{oc} = voltage across the open circuited terminals AB.
R_{Th} = the equivalent resistance of the network as seen from terminals AB, with all independent sources suppressed.



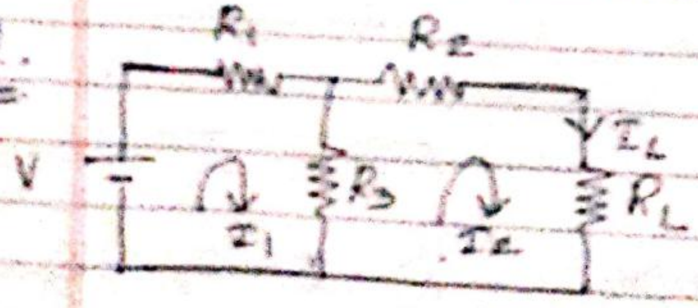
To find V_{oc}

Remove load resistor and then measure the voltage appearing across the load terminals, which gives V_{oc}.

To find R_{Th}

1. Remove the load resistor.
2. Short circuit the voltage source.
3. Open circuit the current source.
4. Then measure the resistance at the load terminal which is R_{Th}.

Proof.

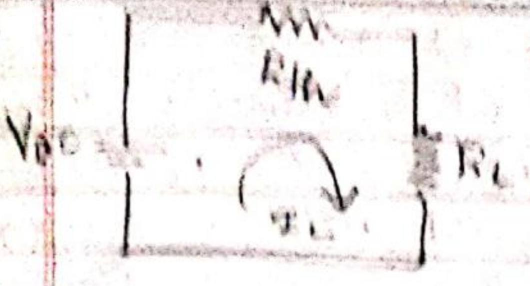


By mesh analysis,

$$I_L = I_2$$

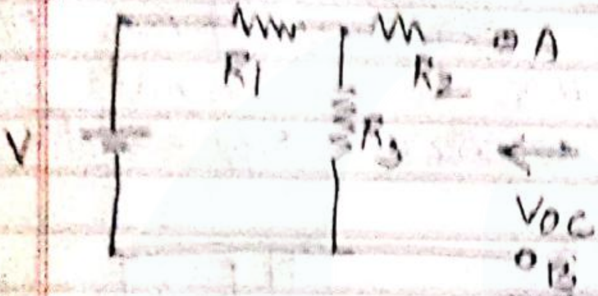
$$= \frac{VR_3}{R_1R_3 + (R_1+R_3)(R_2+R_L)}$$

By theorem



$$I_L = \frac{V_{oc}}{R_{Th}+R_L}$$

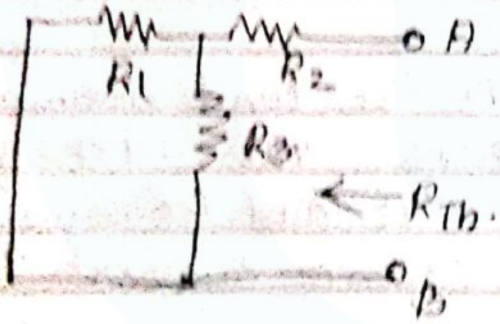
To find V_{oc}



$$V_{oc} = V_{AB} = V_{R_3}$$

$$= V \times \frac{R_3}{R_1+R_3}$$

To find R_{Th}



$$R_{Th} = R_1 \| R_3 + R_2$$

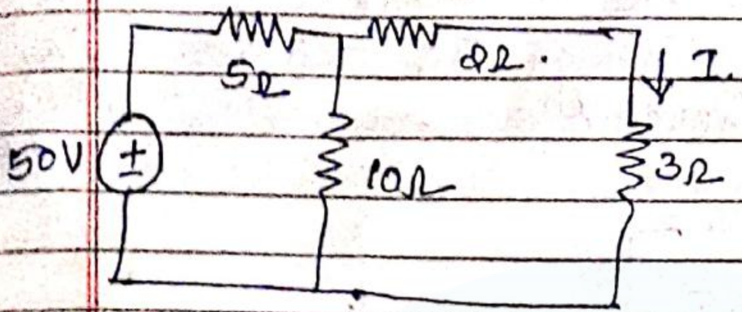
$$= \frac{R_1R_3}{R_1+R_3} + R_2$$

$$I_L = \frac{V_{oc}}{R_{Th}+R_L} = \frac{V\, R_3 / (R_1+R_3)}{\frac{R_1R_3}{R_1+R_3} + R_2 + R_L}$$

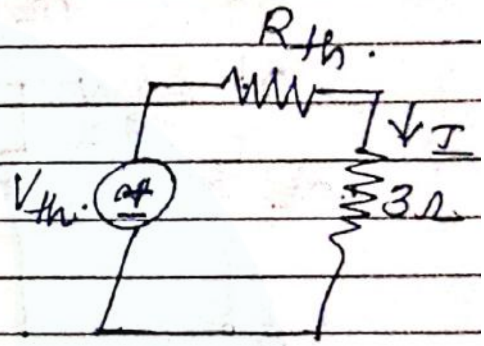
$$= \frac{\frac{VR_3}{(R_1+R_3)}}{\frac{R_L(R_1+R_3) + R_1R_3 + R_2R_1 + R_2R_3}{(R_1+R_3)}} = \frac{VR_3}{R_1R_2 + R_2R_3 + R_3R_1 + R_L(R_1+R_3)}$$

(1) This is the proof

Using Thevenin's theorem, find the current through 3Ω resistor.



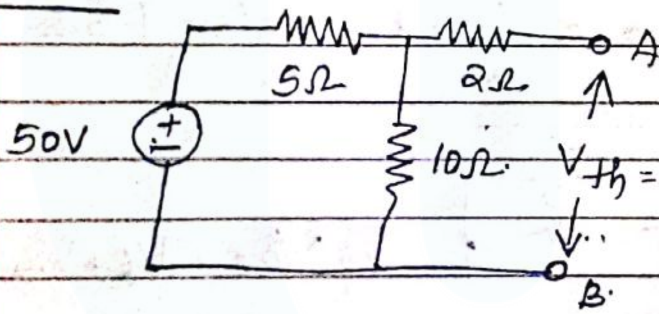
Ans



Thevenin's equivalent Ckt

$$I = \frac{V_{Th}}{R_{Th}+R_L}$$

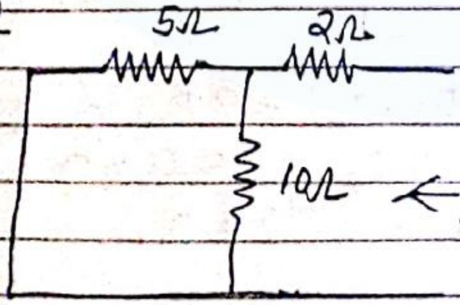
To find V_{Th}



$$V_{Th} = V_{OC} = V_{10\Omega} = \frac{50 \times 10}{10+5}$$

$$= \underline{\underline{33.3\,V}}$$

To find R_{Th}

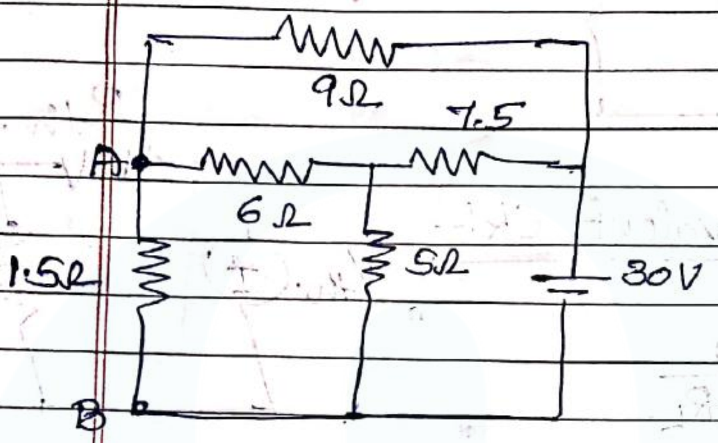


$$\leftarrow R_{Th} = 2 + 5 \| 10$$

$$= \underline{\underline{5.33\Omega}}$$

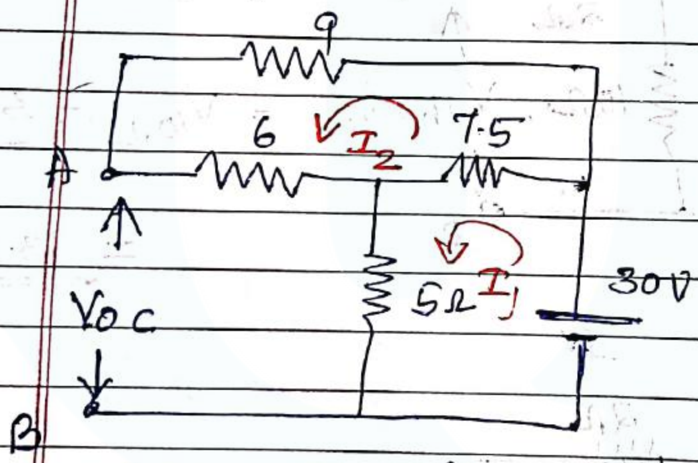
$$I_L = \frac{33.3}{5.33+3} = \underline{\underline{4\,A}}$$

For the ckt, find ① the V_{oc} at AB.
② the looking back resistance at AB.
③ the current through the resistance AB
Draw the Thevenin's equivalent ckt



$V_{oc} = V_{AB}$

To find V_{oc}, Remove load resistor 1.5Ω



$V_{oc} = V_6 + V_5$

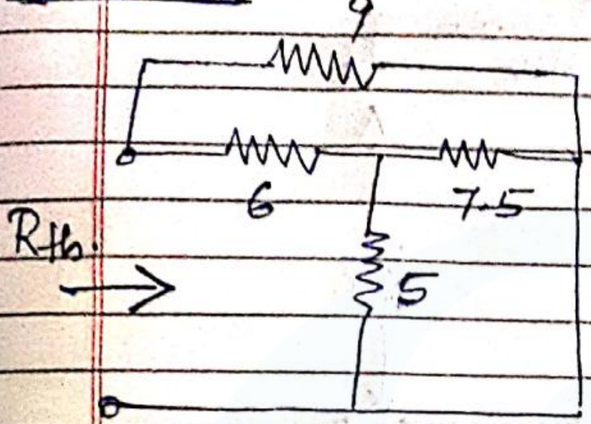
$$6I_2 + 7.5(I_2 - I_1) + 9I_2 = 0$$
$$15I_2 + 7.5I_2 - 7.5I_1 = 0$$
$$22.5I_2 - 7.5I_1 = 0$$

$$30 - 7.5(I_1 - I_2) - 5I_1 = 0$$
$$30 = 12.5I_1 - 7.5I_2$$

$$\Rightarrow I_1 = 3A, \quad I_2 = 1A$$

$$V_{oc} = I_1 \times 5 + I_2 \times 6 = 3 \times 5 + 1 \times 6 = 15 + 6$$
$$= \underline{\underline{21V}}.$$

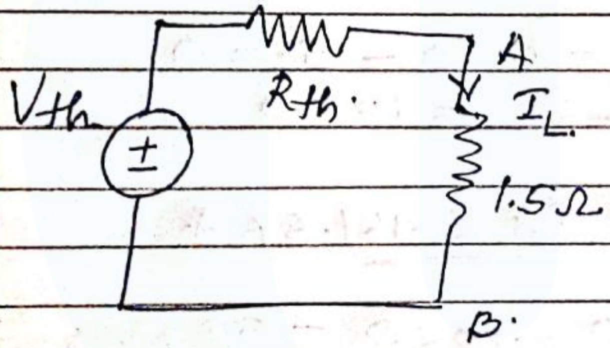
To find R_{th}



$$\left(7.5 \| 5 + 6\right) \| 9$$

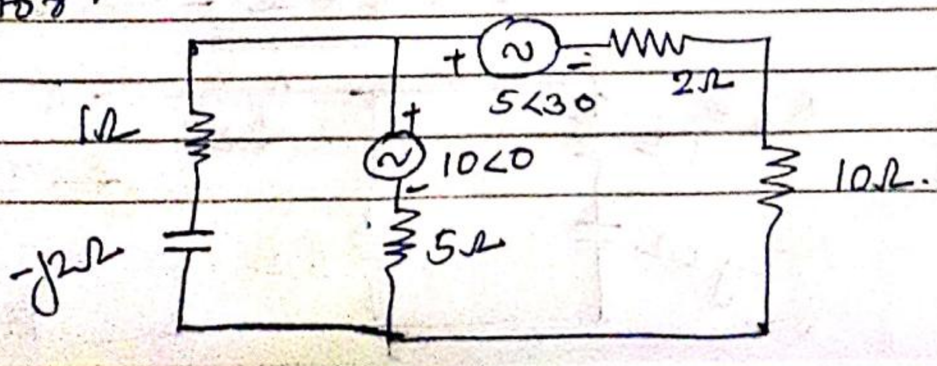
$$= \underline{\underline{4.5\Omega}}.$$

Thevenin's eqt ckt

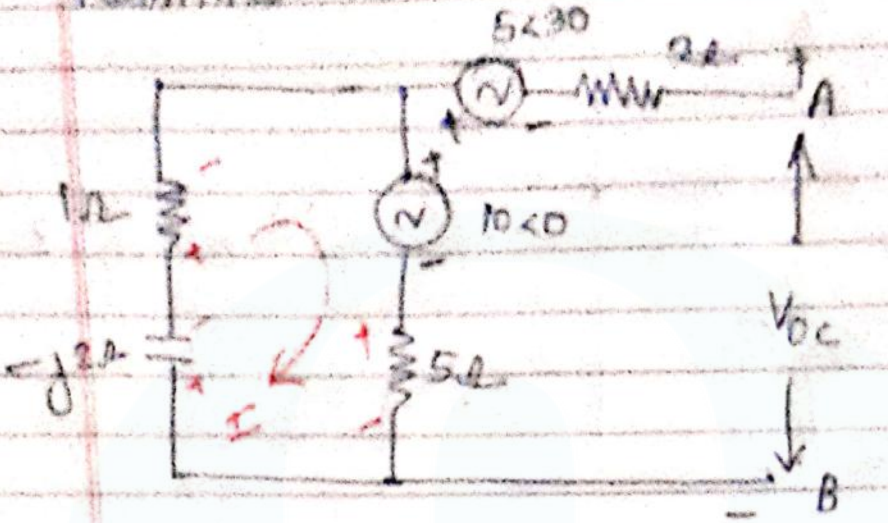


$$I_L = \frac{V_{th}}{R_{th} + 1.5}$$
$$= \frac{21}{4.5 + 1.5}$$
$$= \frac{21}{6} = \frac{7}{2}$$
$$= \underline{\underline{3.5A}}.$$

Draw the Thevenin's eqt ckt. Find the current through the 10Ω resistor.



To find V_{th} – Remove 10Ω resistor, then measure voltage across open circuited terminal.



Applying KVL to the mesh:

$$-j2I - 1I - 10\angle 0 - 5I = 0$$
$$I(+j2-6) = 10\angle 0$$

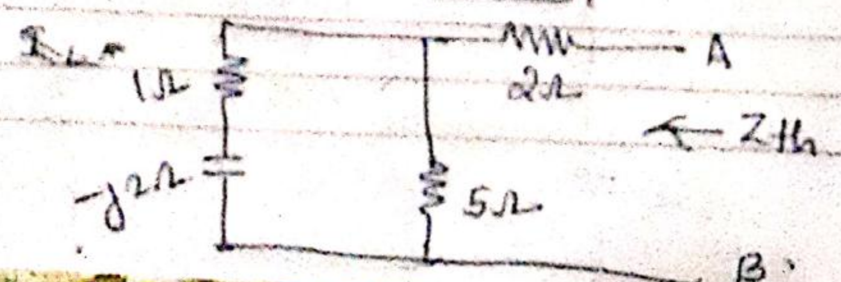
$$I = 1.58\angle -161.57\ A$$

$$-2I' + 5\angle 30 - 10\angle 0 - 5I + V_{th} = 0$$
$$V_{th} = 10\angle 0 - 5\angle 30 + 5I$$

$$= 5.32\angle -110.06\ V$$

To find Z_{th}

$$Z_{th} = 2 + \frac{5(1-j2)}{5+1-j2}$$
$$= 3.48\angle -21.04$$



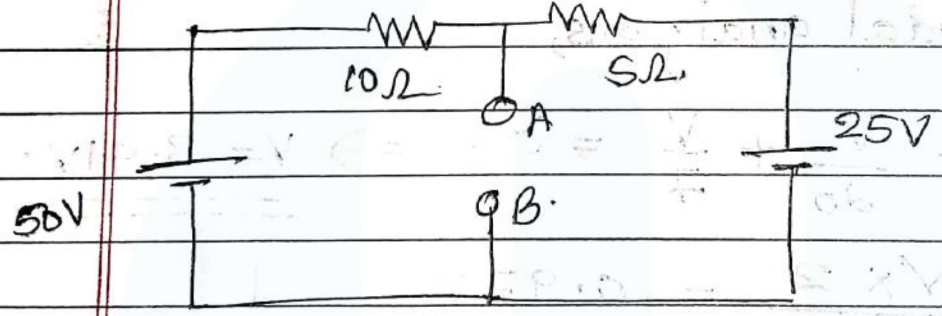
$$I_L = \frac{V_{th}}{Z_{th} + Z_L}$$

$$= \frac{5.32\angle -110.06}{3.48\angle -21.04 + 10}$$

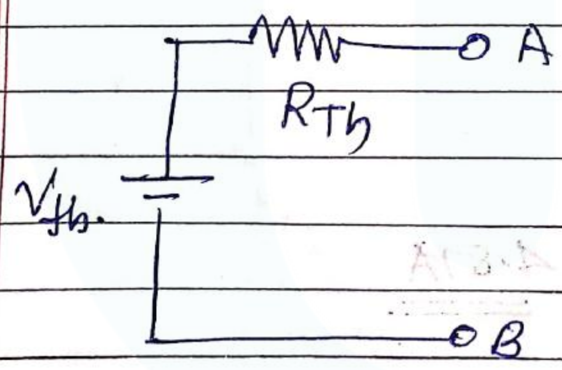
$$= 0.4\angle -104.67$$

Thevenin's

(3) Det. Thevenin's equivalent ckt across AB.



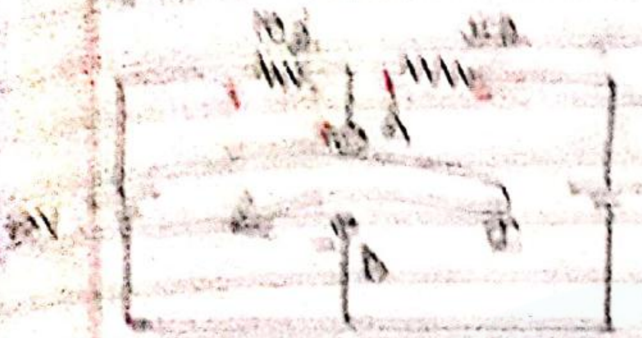
Ans.



(16)

Find V_{th}

Remove the load resistor voltage across open circuited load terminals is V_{th}.



$50 - 10I - 5I - 25 = 0$

$25 = 15I$

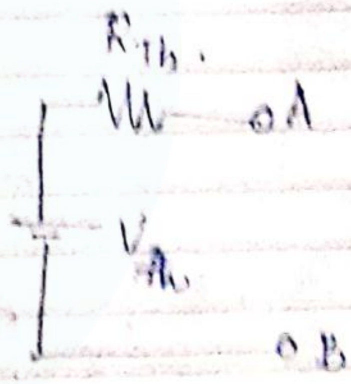
$I = \frac{25}{15} = \frac{5}{3}$ A

$V_{AB} = 50 - 10I = 50 - 10 \times 5/3 = 33.3\,V = V_{Th}$

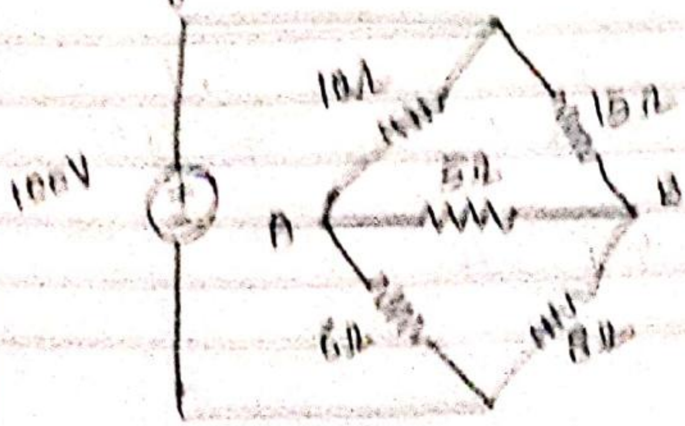
Find R_{th}

$R_{th} = 10 \| 5 = 3.33\,\Omega$

∴ Thevenin's equivalent ckt is

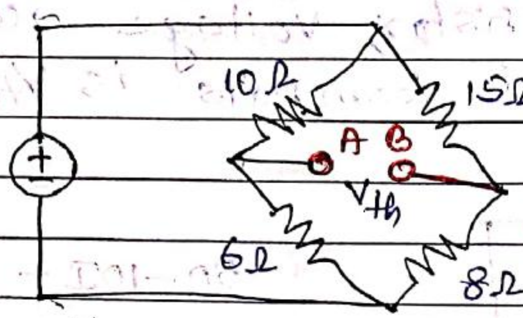


(3) Using Thevenin's theorem, find the current through the 5Ω resistor.

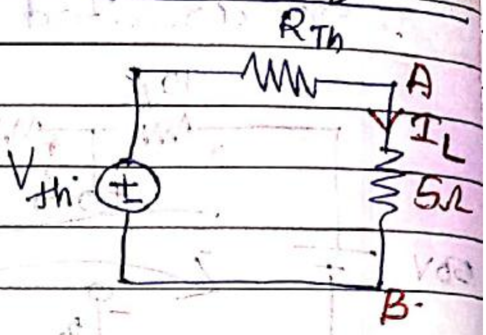


Ans

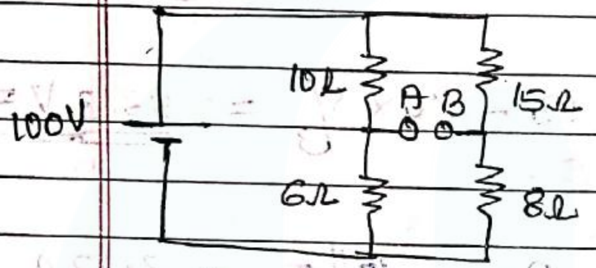
Remove 5Ω resistor



Thevenin's eqt ckt



To find V_{th}



$V_{th} = V_{AB}$

$= V_6 - V_8$

$I_6 = \frac{100}{16}$; $V_6 = I_6 \times 6$

$= 37.5V$

$I_8 = \frac{100}{23}$

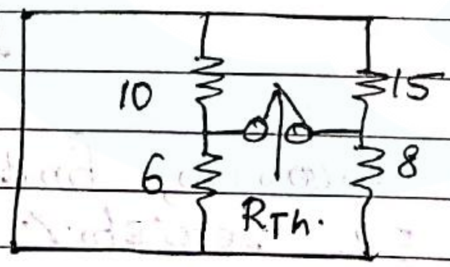
$V_8 = I_8 \times 8 = 4.35 \times 8$

$= 34.8V$

$V_{AB} = 37.5 - 34.8$

$= 2.7V$

To find R_{Th}



$\frac{6 \times 10}{6+10} + \frac{15 \times 8}{15+8} = R_{Th}$

$= 8.97\,\Omega$

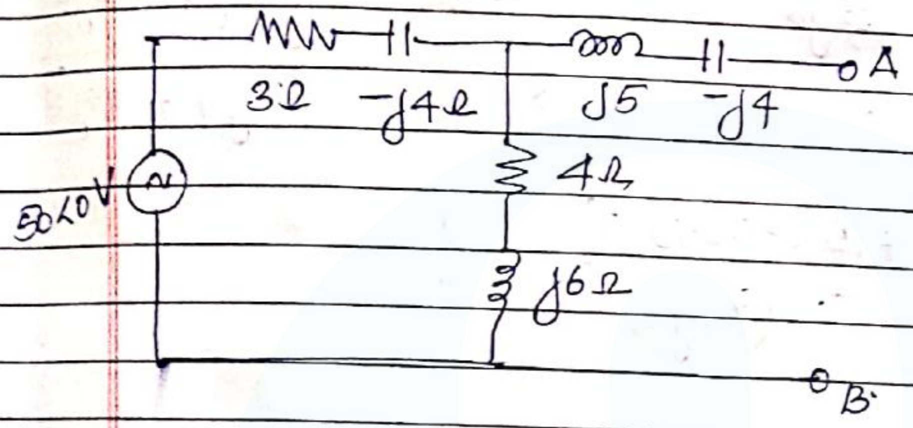
$\therefore I_L = \frac{V_{OC}}{R_{Th}+R_L} = \frac{2.7}{8.97+5}$

$= 0.193\,A$

Thevenin's theorems.

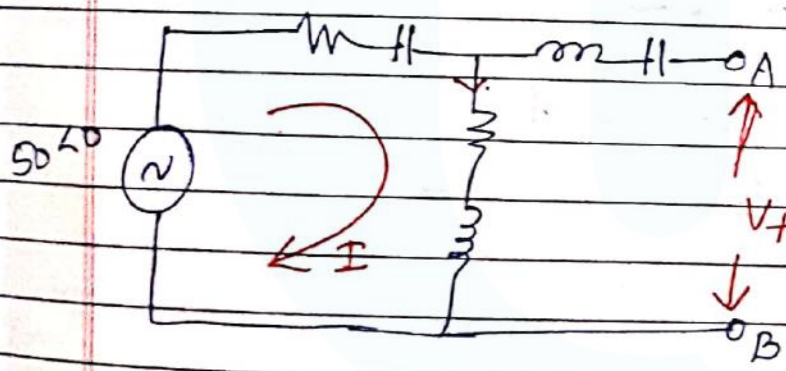
Assignment Qn 3

1) Obtain Thevenin's equivalent network for the terminals A & B.



Ans

To find V_{th}



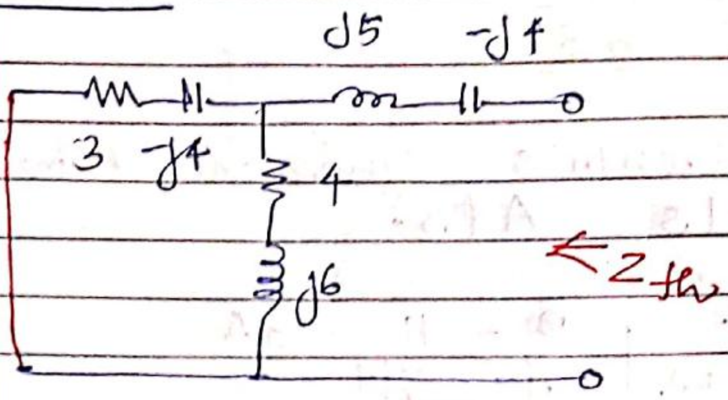
$V_{th} = I(4+j6).$

$$I = \frac{50\angle 0}{3-j4+4+j6} = \frac{50\angle 0}{7+j2}$$

$$= 6.87\angle -15.95\,A.$$

$$V_{th} = 6.87\angle -15.95 \times (4+j6)$$
$$= 6.87\angle -15.95 \times 7.21\angle 56.31$$
$$= 49.53\angle 40.36$$

To find Z_{th}



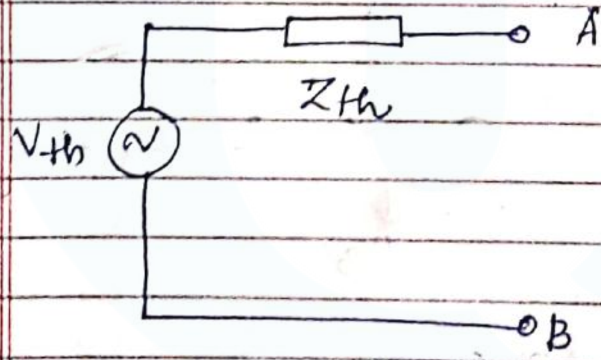
$$Z_{th} = (j5 - j4) + \frac{(3-j4)(4+j6)}{(3-j4+4+j6)}$$

$$= j1 + \frac{(5\angle -53.13)(7.2\angle 56.3)}{(7+j2)} \quad \rightarrow 7.28\angle 15.44$$

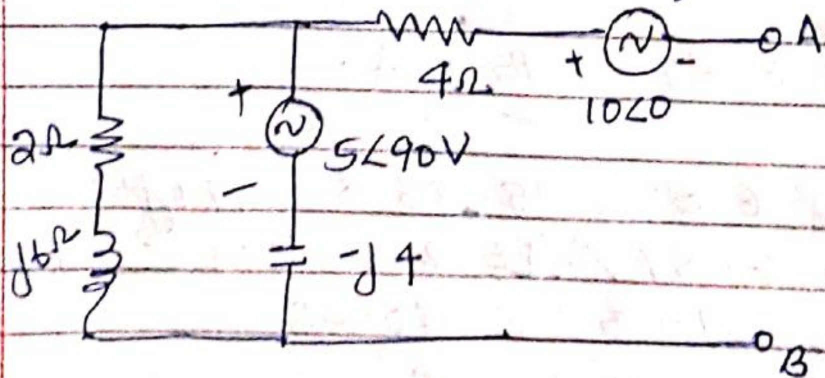
$$= j1 + 4.95\angle -12.27 \qquad = 4.83\angle -1.13$$

$$= j1 + 4.837 - 1.05j$$

Thevenin's equivalent ckt



② Obtain Thevenin's equivalent ckt.



5A.

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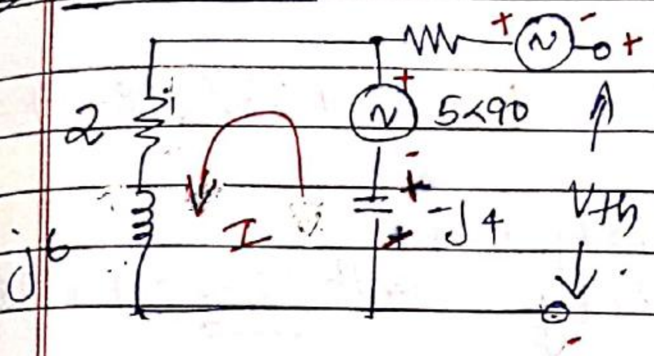
Ans. Find V_{th}



$$-j4I + 5\angle 90 - 2I - j6I = 0$$

$$5\angle 90 - 2I - j2I = 0$$

$$I = \frac{5\angle 90}{2+j2}$$

$$= 1.77\angle 45 \text{ A}$$

$$V_{th} - 5\angle 90 - j4I + 10\angle 0 = 0$$

$$V_{th} = 10\angle 0 + 5\angle 90 - 4jI$$

$$= -10 + 0j + 5j - 4j\,[1.77\angle 45]$$

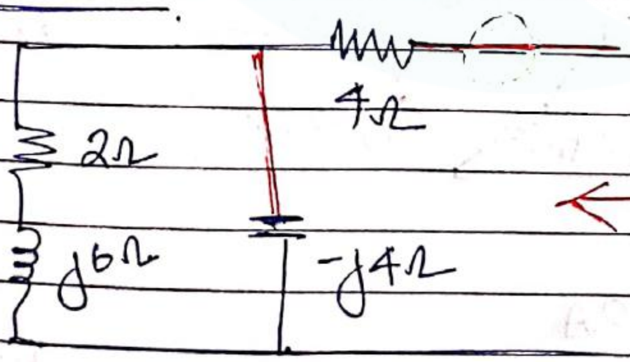
$$= -10 + 5j + 4\angle 90\,(1.77\angle 45)$$

$$= -10 + 5j + 7.08\angle 135$$

$$= -10 + 5j - 5.01 + 5.01j$$

$$= -15.01 + 10.01j = 18.04\angle 146.31$$

To find Z_{th}



$$Z_{th} = \left[(2+j6) \parallel -j4\right] + 4$$

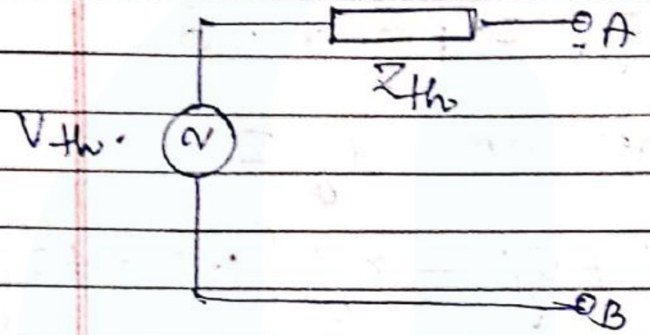
$$= \frac{(2+j6)(-j4)}{2+j6-j4} + 4 = \frac{-8j + 24}{2+j2} + 4$$

55

$$= \frac{25.29 \angle 93.9}{2.83 \angle 45} + 4 = 8.936 \angle 48.9 + 4$$

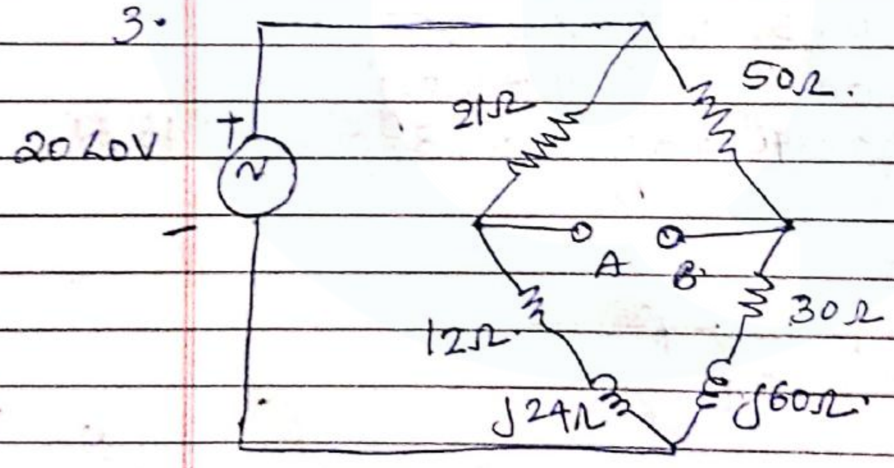
$$= 5.874 + 6.734j + 4$$
$$= 9.874 + 6.734j = \underline{\underline{11.95 \angle -44.93}}$$

Thevenin's equivalent ckt :



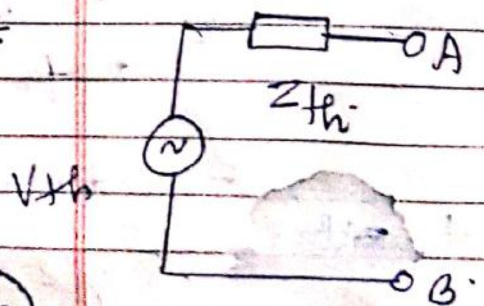
Assignment Qn 4

3.



Obtain Thevenin's equivalent circuit.

Ans.



So find V_{th} & Z_{th}.

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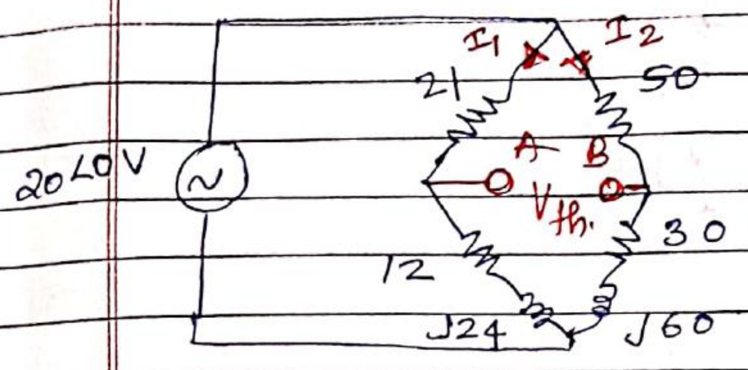
To find Vth.



21, (12+j24) - In series.
50, (30+j60) - "

These 2 branches in parallel.

So $I_1, I_2 \rightarrow$ 2 currents.

$$I_1 = \frac{20\angle 0}{21+12+j24} = \frac{20\angle 0}{33+j24} = \frac{20\angle 0}{40.8\angle 36.02}$$

$$= \underline{0.49\angle -36.02}\ A$$

$$I_2 = \frac{20\angle 0}{50+30+j60} = \frac{20\angle 0}{80+j60} = \frac{20\angle 0}{100\angle 36.86}$$

$$= \underline{\underline{0.2\angle -36.86}}$$

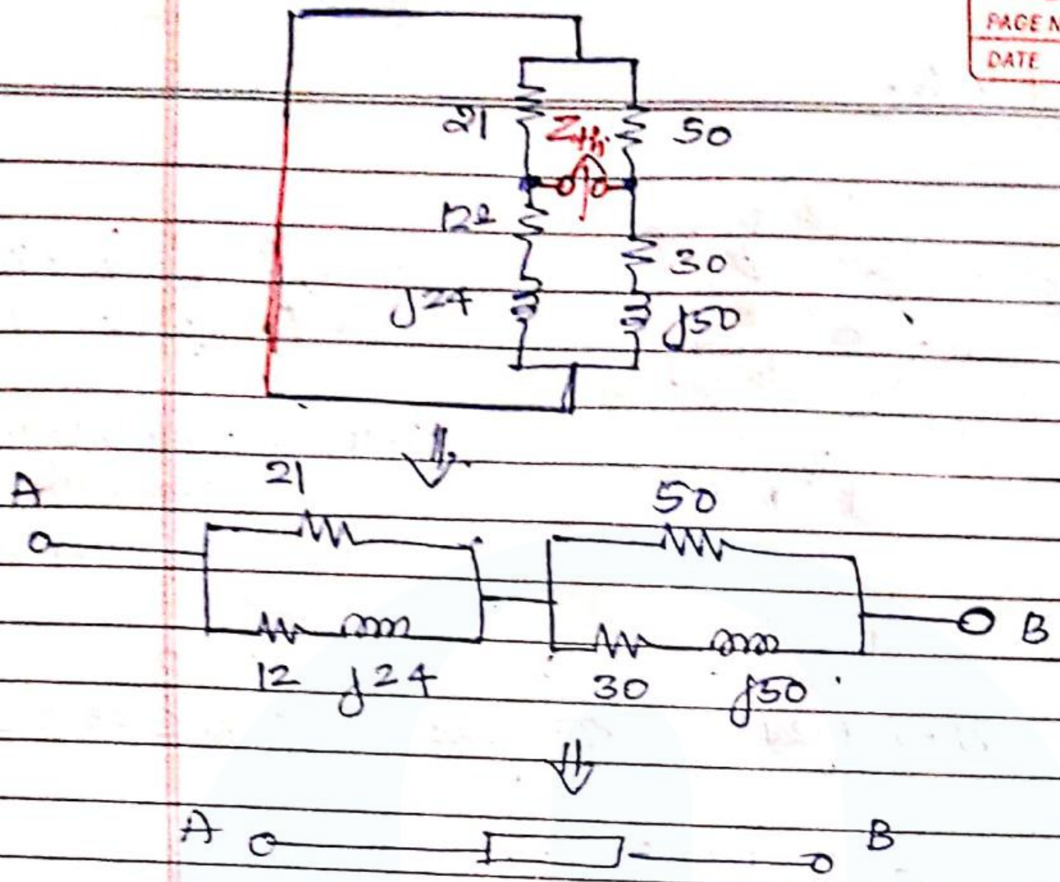
$$V_{th} = 21 I_1 - 50 I_2$$
$$= 21 \times 0.49\angle -36.02 - 50 \times 0.2\angle -36.86$$
$$= 10.29\angle -36.02 - 10\angle -36.86$$
$$= 8.322 - 6.05j - 8+6j$$
$$= 0.322 - 0.05j = \underline{\underline{0.33\angle 171.12}}\ V$$

OR

$$V_{th} = (12+j24) I_1 - (30+j60) I_2$$

$$= \underline{\underline{0.33\angle 171.12\ V}}.$$

To find Zth. Short Circuit 20∠0V



$Z_{th} = 21 \| (12 + j24) + 50 \| (30 + j50)$

$= \dfrac{21(12 + j24)}{21 + 12 + j24} + \dfrac{50 \times (30 + j50)}{50 + 30 + j50}$

$= \dfrac{252 + j504}{33 + j24} + \dfrac{1500 + j2500}{80 + j50}$

$= \dfrac{563.49 \angle 61.1}{40.80 \angle 36.03} + \dfrac{2915.48 \angle 59.04}{94.34 \angle 32.01}$

$= 13.811 \angle 25.07 + 30.9 \angle 27.03$

$= 12.51 + 5.852j + 27.524 + 14.043j$

$= 40.034 + 19.895j$

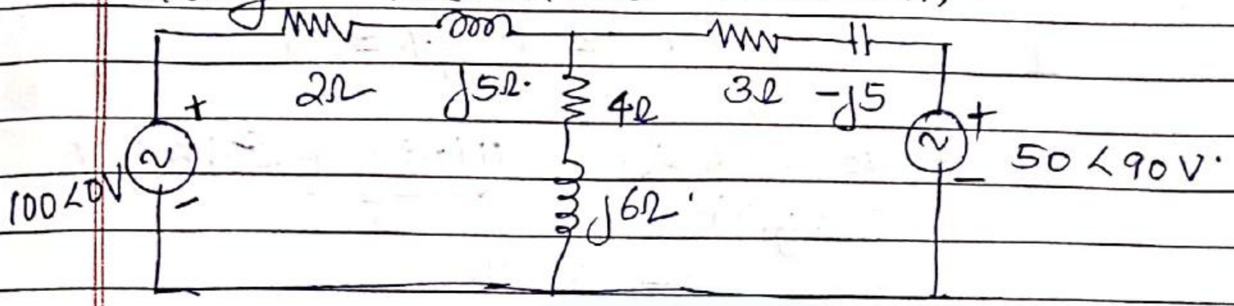
$= \cancel{44.20}\ 47.4 \angle 26.8\ \Omega$

58

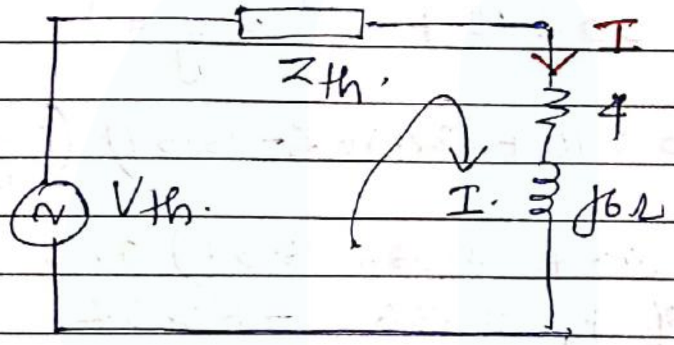
Assignment Qn. 5

TRIVENI PAGE NO. DATE / /

Find the current through $(4+j6)\Omega$ impedance in the network using Thevenin's theorem.



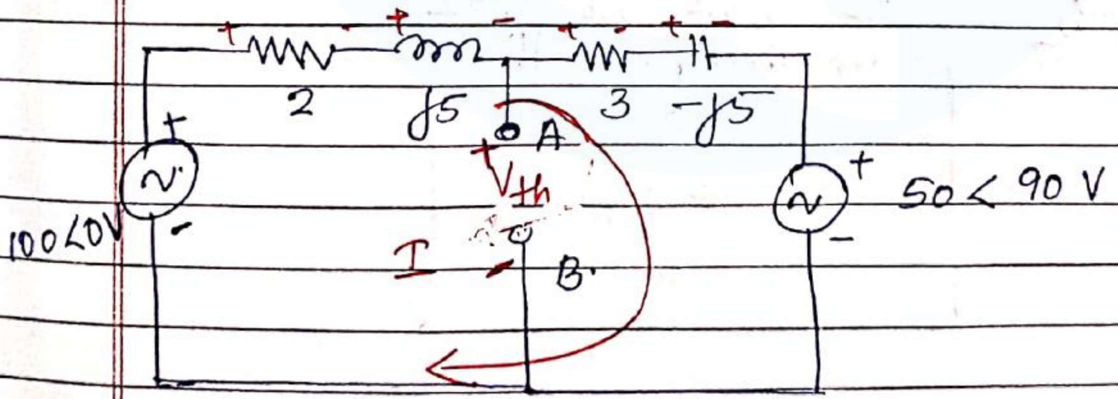
Ans Thevenin's equivalent circuit.



$$I = \frac{V_{th}}{Z_{th} + (4+j6)}$$

So find out V_{th} & Z_{th}.

To find Vth.



Applying KVL to the mesh,

$$100\angle 0 - 2I - j5I - 3I - (-j5I) - 50\angle 90 = 0$$

$$100\angle 0 - 50\angle 90 = 5I$$

$$\frac{100\angle 0 - 50\angle 90}{5} = I$$

$$I = \frac{100 - j50}{5} = \cancel{20 - j10 = 20}$$

$$= \cancel{\frac{100\angle 25.4}{5}} \quad \frac{111.8034\angle -26.57}{5}$$ 10

$$= 22.36\angle -26.57$$ Ans

$$V_{th} - 3I + j5I - 50\angle 90 = 0$$

$$V_{th} = 50\angle 90 + I(3 - j5)$$

$$= 50\angle 90 + (22.36\angle -26.57)(3 - j5)$$

$$= 50\angle 90 + (22.36\angle -26.57)\, 5.83\angle -59$$

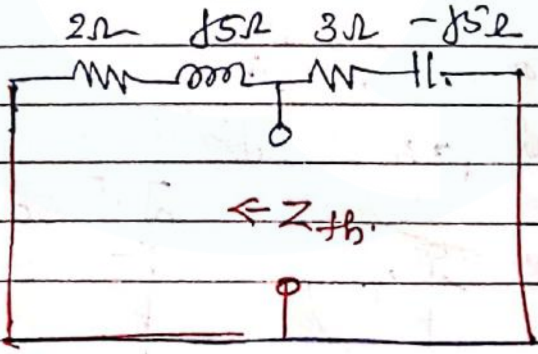
$$= +50j + 130.36\angle -85.57$$ To

$$= 50j + 10.069 - 129.97j$$

$$= 10.069 - 79.97j$$

$$= 79.33\angle -82.88 \text{ V}$$

To find Z_{th}



$$Z_{th} = \frac{(2+j5)(3-j5)}{2+j5+3-j5} = 6.28\angle 9.16\,\Omega$$

(60)

$$I_{\text{through }(4+j6)} = \frac{V_{th}}{Z_{th} + (4+j6)} = 6.52\angle -117$$ (61)

TRIVENI
PAGE NO.
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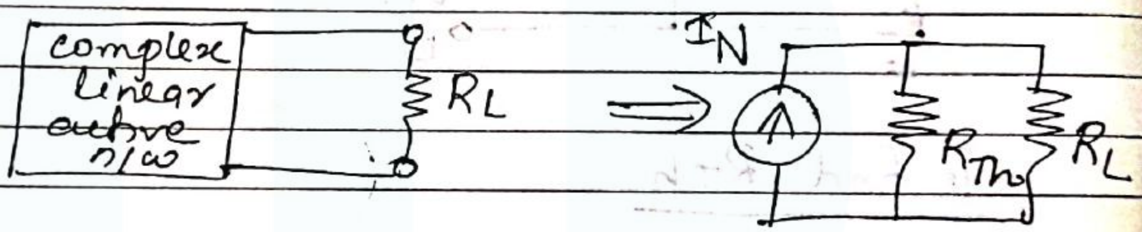
3. Norton's theorem.

Any two terminals of a n/w containing linear, passive and active elements maybe replaced by an equivalent current source I_N in parallel with a resistance R_{Th}, where I_N is the current flowing through a short circuit placed across the terminals AB.

R_{Th} = equivalent resistance of the n/w as seen from the two terminals with all independent sources suppressed.



To find I_N

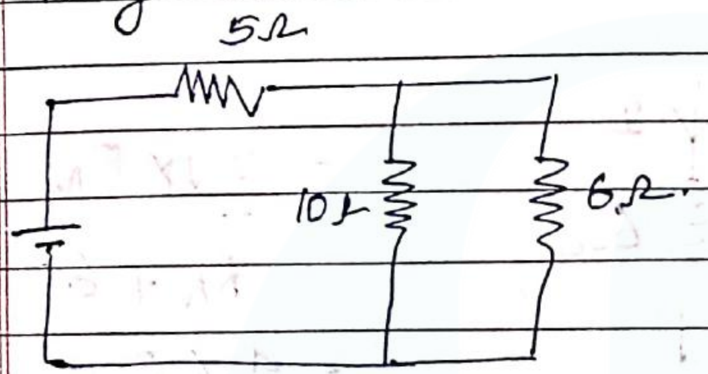
Short circuit the load terminal and then measure the current passing through the load terminal that gives short circuit current I_N at AB.

To find R_{Th}

① Remove R_L ② Short circuit the voltage source ③ Open circuit the current source ④ Then measure the resistance at the load terminal that gives R_{Th}.

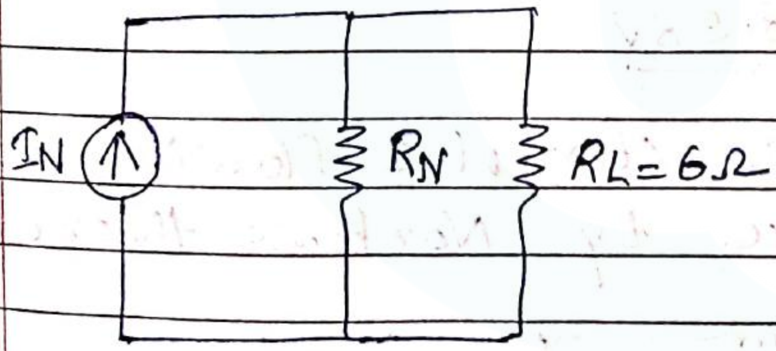
Norton's theorem

1) Find the voltage across the 6Ω resistor using Norton's theorem.



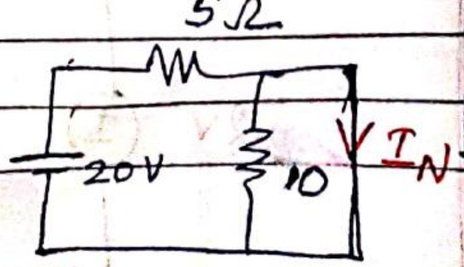
Ans

Norton's equivalent ckt



To find I_N.

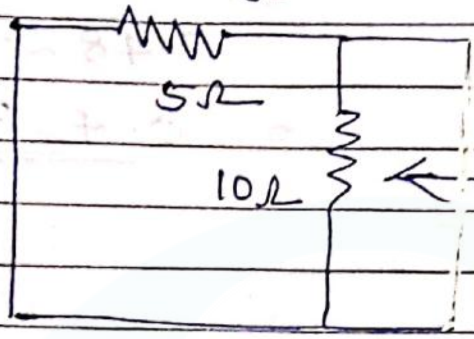
Short Circuit R_L



$$I_N = \frac{20}{5} = 4\,A$$

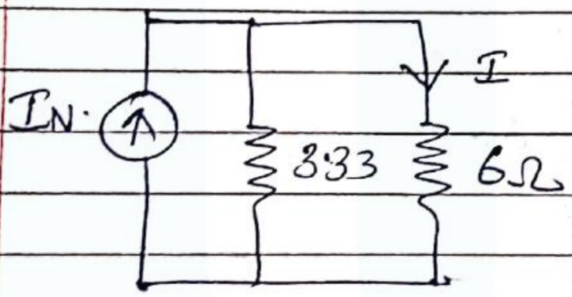
To find R_N

Remove R_L, ~~short ckt~~.
Voltage source shorted.



$$R_N = 5 \| 10 = \frac{5 \times 10}{5 + 10} = 3.33\,\Omega$$

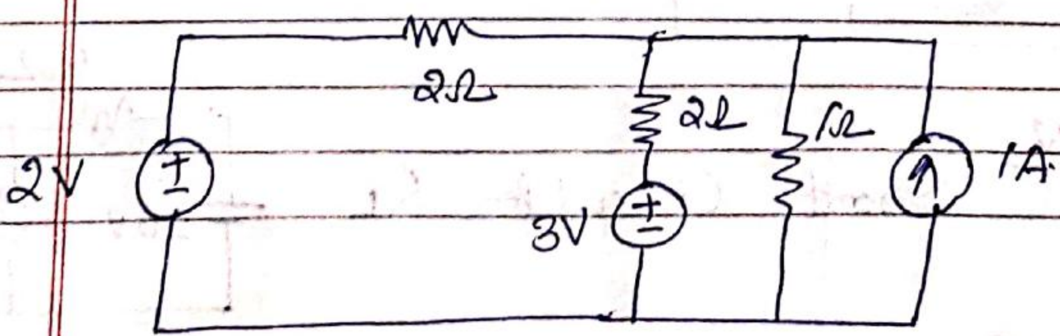
Norton's equivalent ckt



$$I = \frac{I_N \times R_N}{R_N + 6} = \frac{4 \times 3.33}{3.33 + 6} = 1.43\,A$$

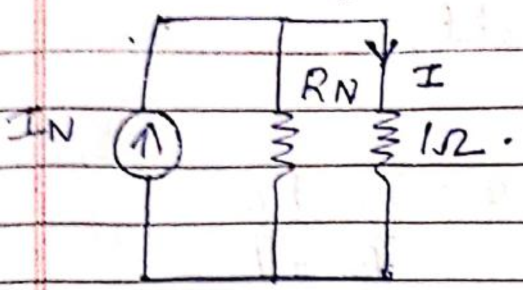
$$V_{6\Omega} = 1.43 \times 6 = 8.58\,V$$

② Evaluate the current I flowing through 1Ω resistance by Norton's theorem



TRIVENI
PAGE NO.
DATE / /

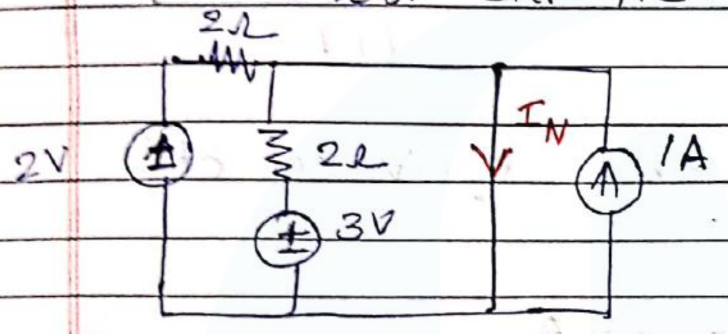
Norton's eqt ckt



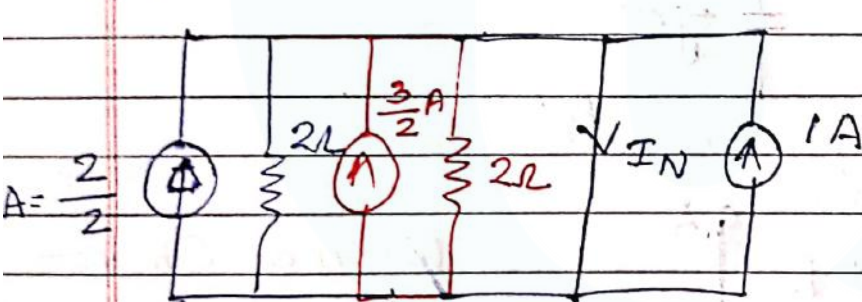
$$I = \frac{I_N \times R_N}{R_N + R_L}$$

To find I_N

Short ckt R_L.



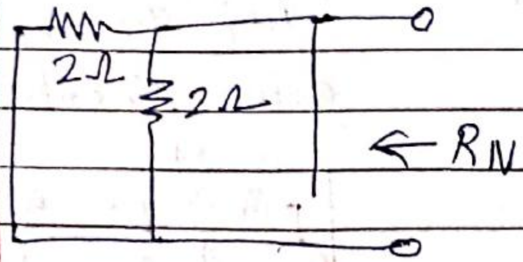
We can convert 2 voltage sources to current source.



$$I_N = 1 + \frac{3}{2} + 1$$

$$= \underline{3.5A}.$$

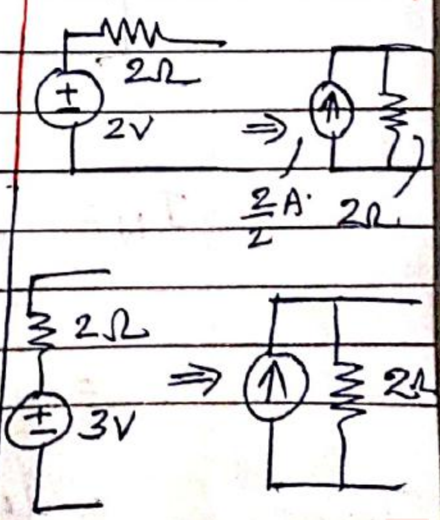
To find R_N



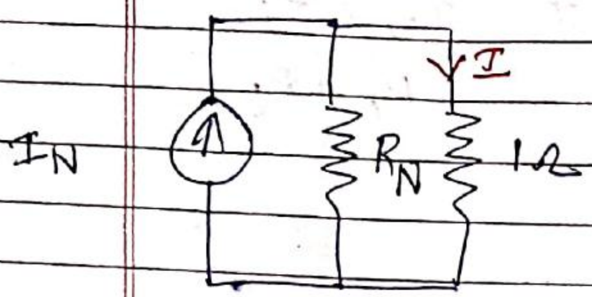
$$R_N = 2 \| 2 = \frac{2 \times 2}{2+2}$$

$$= \underline{1}\Omega.$$

Using Source transformation



From Norton's eqt ckt,

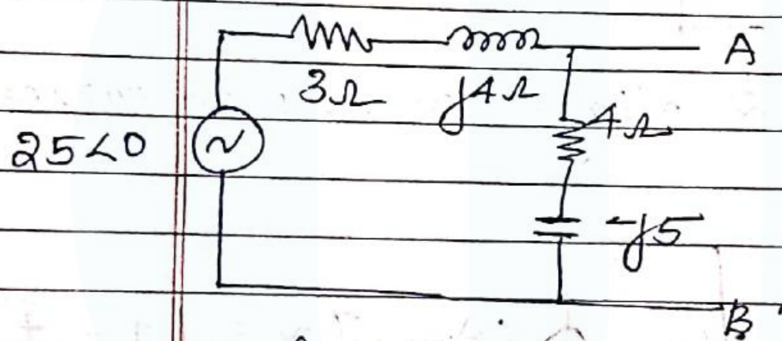


$$I = \frac{I_N R_N}{R_N + 1}$$

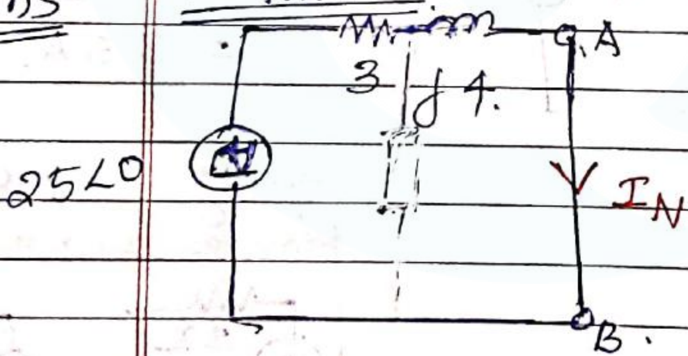
$$= \frac{3.5 \times 1}{1+1} = \underline{1.75A}$$

AC Analysis

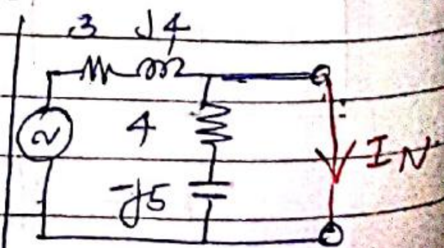
(3) Obtain Norton's equivalent ckt across AB.



Ans To find I_N



When a short circuit is placed across $(4-j5)\Omega$ impedance, it gets shorted



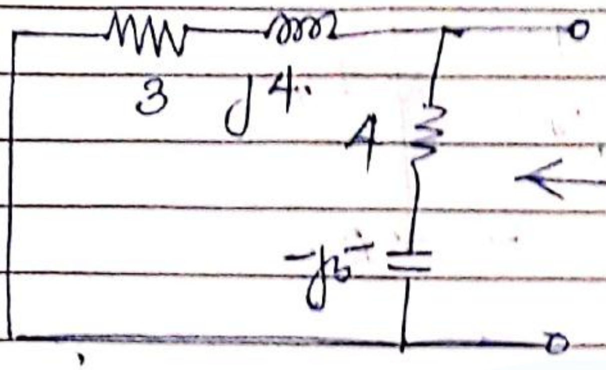
$$I_N = \frac{25\angle 0}{3+j4}$$

$$= \frac{25\angle 0}{5\angle 53.13}$$

$$= 5\angle -53.13\ A.$$

To find Z_N

Remove load.



$$Z_N = \frac{(3+j4)(4-j5)}{(3+j4)+(4-j5)}$$

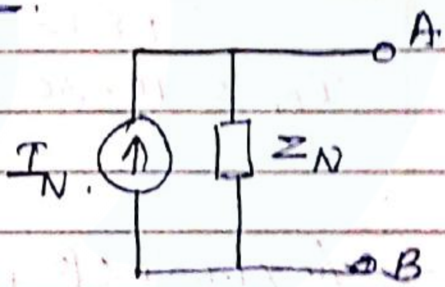
$$Z_N = \frac{(5\angle 53.13)(6.4\angle -51.1)}{(7-j1)}$$

$(7-j1) \rightarrow 7.07\angle -8.13$

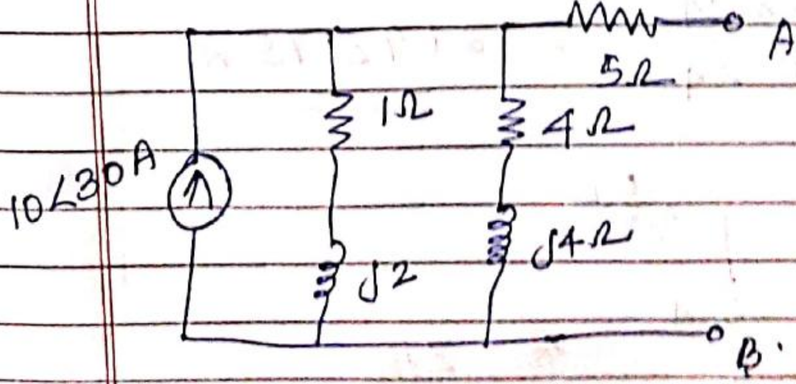
$$= \frac{5\times 6.4}{7.07}\angle(53.13 + -51.34 + 8.13)$$

$$= \underline{4.53\angle 9.92}$$

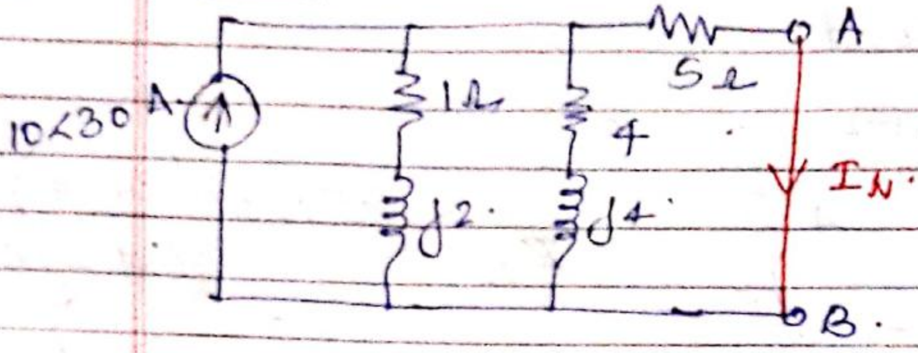
Norton's eqt ckt



④ Obtain Norton's eqt ckt. across AB.



Ans Find I_N – Short ckt AB.



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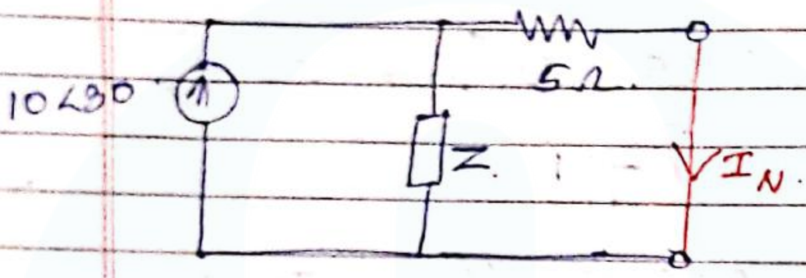


$$Z = \frac{(1+j2)(4+j4)}{(1+j2)+(4+j4)}$$

$$= \frac{(2.236\angle 63.43)(5.657\angle 45)}{5+j6}$$

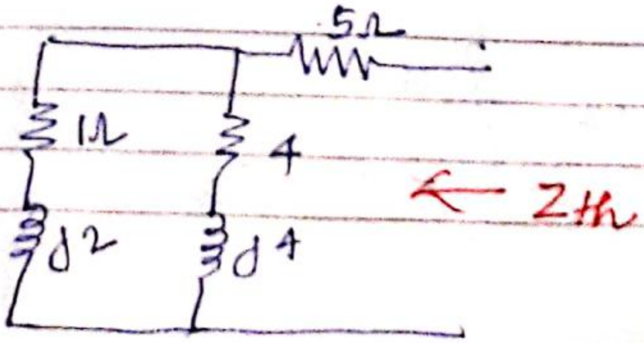
$$Z = \frac{12.649\angle 108.43}{7.81\angle 50.19} = 1.6196\angle 58.24$$

$$I_N = 10\angle 30 \times \frac{1.62\angle 58.24}{1.62\angle 58.24 + 5}$$

$$= \frac{(10\times 1.62)\angle(30+58.24)}{0.8527+1.377j+5} = \frac{16.2\angle 88.24}{5.8527+j1.377}$$

$$= \frac{16.2\angle 88.24}{6.011\angle 13.24} = 2.69\angle 75 \text{ A}$$

To find Z_N

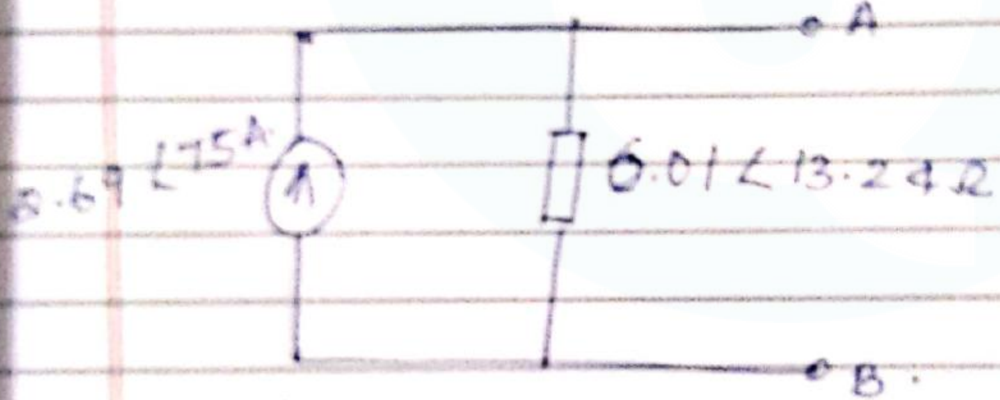


$$Z_N = 5 + \frac{(1+j2)(4+j4)}{(1+j2)+(4+j4)} = 5 + 1.6196\angle 58.24$$

$$= 5 + 0.852 + 1.377j = 5.852 + j1.372.$$

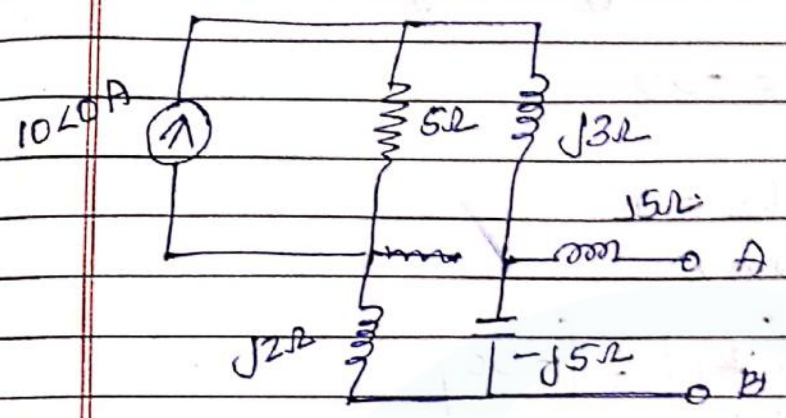
$$= 6.01\angle 13.24\ \Omega.$$

Eqt Ckt.



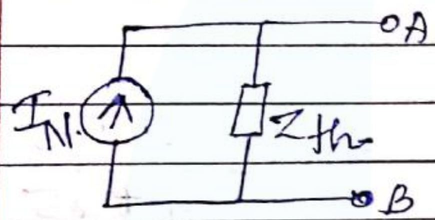
Assignment Qn: 7

Obtain the Norton's equivalent ckt.



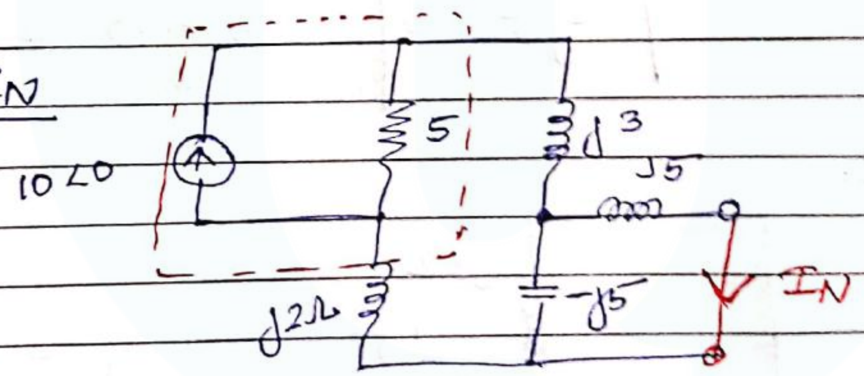
Ans

Norton's equivalent ckt

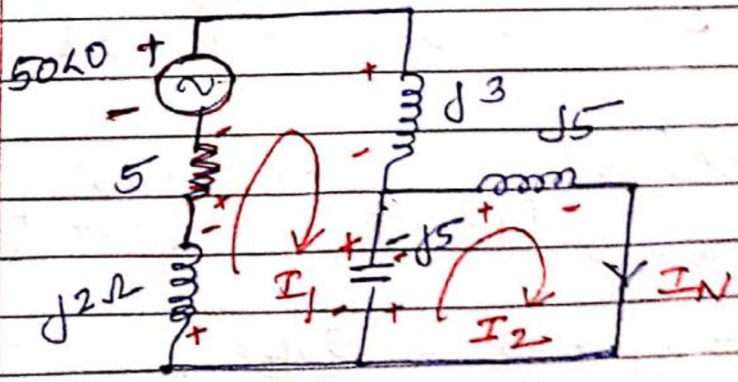


So find out I_N & Z_{th}

To find I_N



By source transformation, the n/w can be redrawn as,



$I_2 = I_N$

61

Applying mesh analysis

$$-j2I_1 - 5I_1 + 50 - j3I_1 - j5(I_1 - I_2) = 0$$

$$-5I_1 - j5I_2 = -50$$

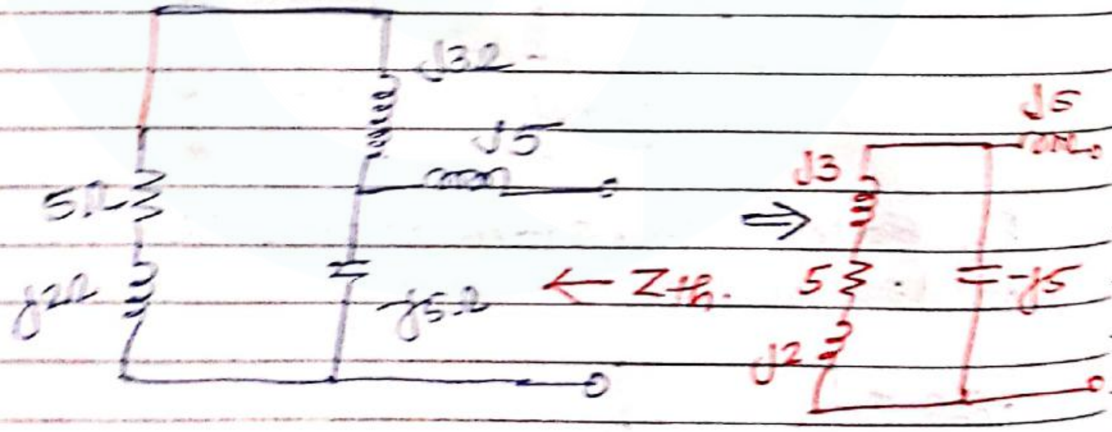
$$-j5(I_2 - I_1) - j5I_2 = 0$$

In matrix form,

$$\begin{bmatrix} -5 & -j5 \\ -j5 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -50 \\ 0 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} -5 & -50 \\ -j5 & 0 \end{vmatrix}}{\begin{vmatrix} -5 & -j5 \\ -j5 & 0 \end{vmatrix}} = \frac{-50 \times j5}{+25} = -j10 = \underline{\underline{10\angle -90^\circ}}$$

To find Z_{th}



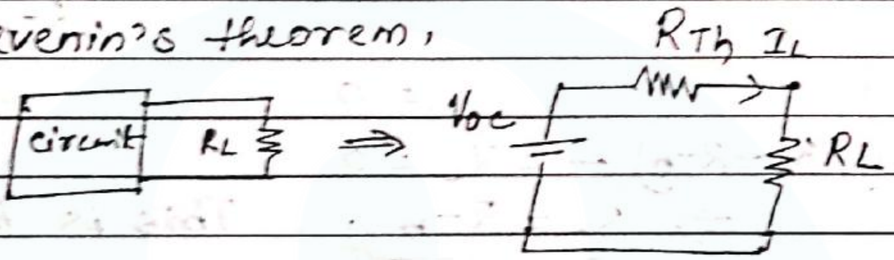
$$Z_{th} = \frac{(5+j5)(-j5)}{5+j5-j5} + j5 = \frac{-j25+25}{5} + j5$$

$$= 5 - 5j + j5 = \underline{\underline{5\Omega}}$$

4. Maximum Power Transfer Theorem

In dc circuits, maximum power is transferred from a source to the load when the load R is made equal to the resistance of the n/w as viewed from the load terminals, with load removed and all the sources replaced by their internal resistances.

By Thevenin's theorem,



Here, V_{oc} = open circuit voltage.

R_{Th} = R measured b/n terminals AB with R_L removed and sources replaced by their internal resistance

Proof

The current supplied to R_L

$$I_L = \frac{V_{oc}}{R_{Th}+R_L}$$

$$\left.\begin{array}{l}\text{Power}\\ \text{delivered to } R_L\end{array}\right] = P_L = I_L^2 R_L$$

$$= \left(\frac{V_{oc}}{(R_{Th}+R_L)}\right)^2 R_L \quad \text{—(1)}$$

For a given ckt, V_{oc} & R_{Th} are constant.

$\therefore$ Power delivered to load depends on R_L.

In order to find the value of R_L for which the value of P_L is maximum, differentiate eqn (1) w.r.t to R_L.

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{d}{dR_L}\left(\frac{V_{oc}}{(R_{Th}+R_L)}\right)^2 R_L = 0$$

$$V_{oc}^2 \frac{d}{dR_L}\left[\frac{R_L}{(R_{Th}+R_L)^2}\right] = V_{oc}^2 \left[\frac{(R_{Th}+R_L)^2 \times 1 - R_L\, 2(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right]$$

$$= 0$$

$$(R_L+R_{Th}) - 2R_L = 0$$

$$\Rightarrow \underline{\underline{R_L = R_{Th}}}$$ This is the proof.

$$\therefore P_{max} = \frac{V_{oc}^2}{(R_{Th}+R_L)^2} R_L = \frac{V_{oc}^2}{(R_{Th}+R_{Th})^2} R_{Th}.$$

$$= \frac{V_{oc}^2}{4R_{Th}^2} R_{Th} = \underline{\underline{\frac{V_{oc}^2}{4R_{Th}}}}.$$

This theorem is used to find the value of load resistance for which there would be maximum amount of power transfer from source to load.

Under the condition of maximum power transfer, the η is only 50%, as only one half of the total power generated is dissipated in the internal resistance R_{Th} of the ckt

ie;

$$\eta = \frac{o/p}{i/p} = \frac{I^2 R_L}{I^2(R_L+R_{Th})}$$

TRIVENI
PAGE NO.
DATE / /

At max, power transfer condition,

$R_L = R_{Th}$,

$$\eta = \frac{I^2 R_L}{2I^2 R_L} = \frac{1}{2} = 50\%$$

Load voltage is one half of the open circuited voltage at the load terminals.

$$\text{Load voltage} = \frac{V_{oc}}{(R_L + R_{Th})} R_L$$ (using voltage division rule).

$$= \frac{V_{oc}}{2R_L} \cdot R_L$$

$$= \frac{V_{oc}}{2}$$

Steps.

(1) Remove the load resistor R_L and find R_{Th} of the source network looking through the open circuited load terminals.

(2) As per max. power transfer theorem, this R_{Th} is the load resistance of the n/w ie $R_L = R_{Th}$ that allows max. power transfer.

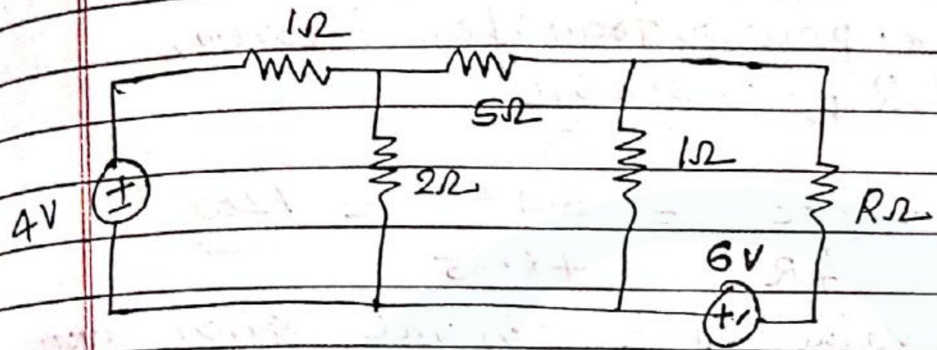
(3) Find the thevenin's voltage V_0 across the open circuited load terminals.

(4) Max. power transfer, $V_0^2 / 4R_{Th}$.

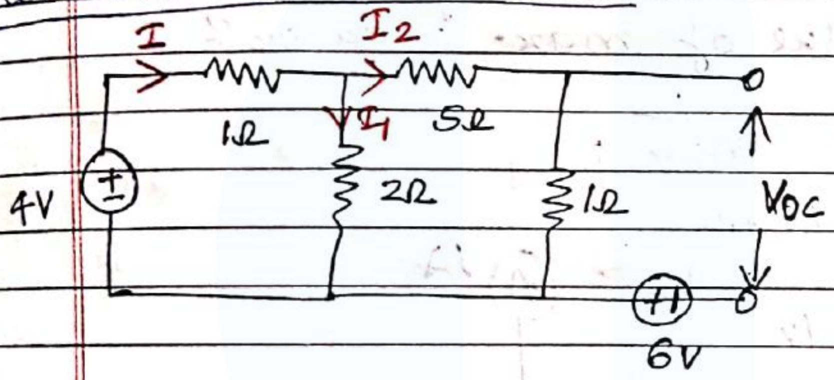
Maximum power transfer

TRIVENI PAGE NO. DATE / /

(1) Find the value of R, such that maximum power transfer takes place. What is the amount of this power?



Remove the load resistor, R



$V_{oc} = V_1 + 6V$

$$R = \frac{6\times 2}{8} + 1 = \frac{12}{8} + 1 = \frac{20}{8}\,\Omega.$$

$$I = \frac{4}{R} = 4 \div \frac{20}{8} = 4 \times \frac{8}{20} = \frac{8}{5}\,A.$$

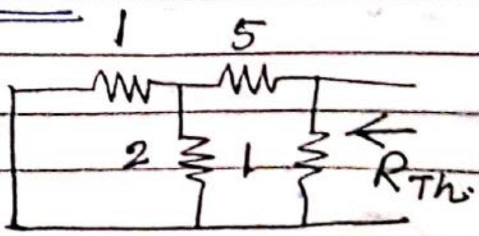
$$I_2 = \frac{\frac{8}{5}\times 2}{2+5+1} = \frac{16}{5} \div 8 = \frac{16}{5}\times\frac{1}{8} = \frac{2}{5}\,A$$

$$V_{1\Omega} = \frac{2}{5}\times 1 = \frac{2}{5}\,V.$$

$$V_{oc} = \frac{2}{5} + 6 = \frac{32}{5} = 6.4\,V$$

To find R_{Th}

$$R_{Th} = (2 \| 1 + 5) \| 1$$



TRIVENI
PAGE NO.
DATE / /

$$= \left(\frac{2\times1}{3}+5\right) \| 1 = \left(\frac{2}{3}+5\right) \| 1 = \frac{\frac{17}{3}\times 1}{\frac{17}{3}+1}$$

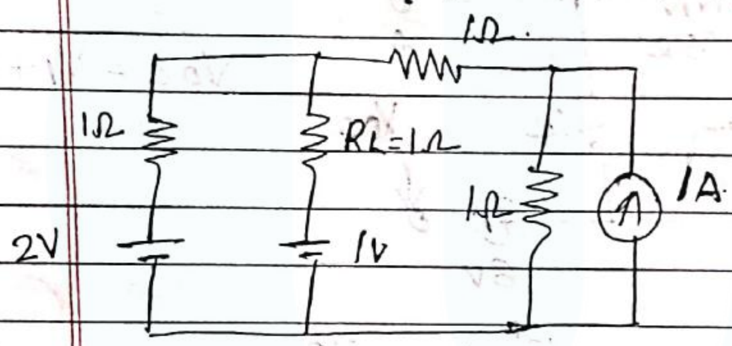
$$= 0.85\,\Omega$$

By max. power transfer theorem,

$R = R_{Th} = 0.85\,\Omega$

$$P_{max} = \frac{V_{oc}^2}{4R} = \frac{6.4^2}{4\times.85} = 12.0$$

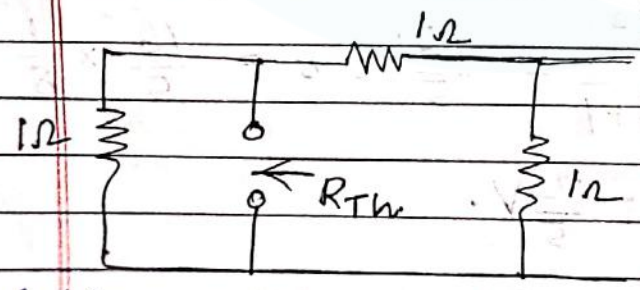
(8) What value of R in the given ckt will receive the max. power? What is the value of max. power?



Ans.

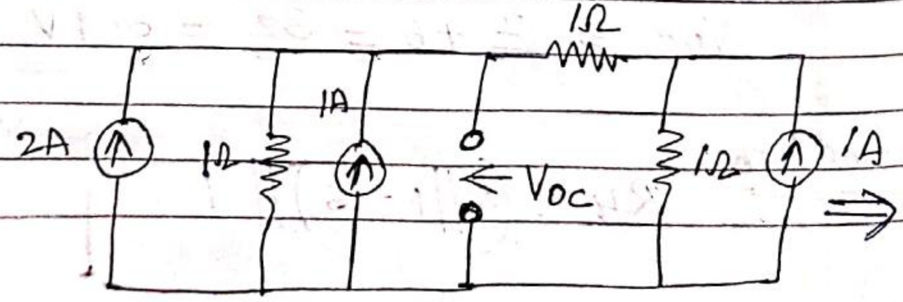
To find R_{Th}

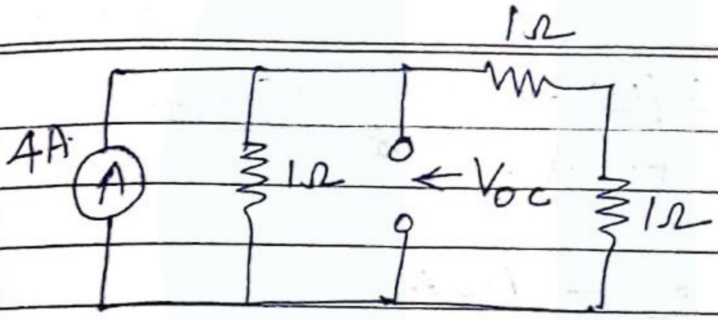
Remove R_L, open ckt current source & short ckt voltage source.



$2 \| 1 = 2/3 = R_{Th} = R$

To find V_{oc}





$$= \frac{4\times 2}{3}\times 1$$

$$V_{oc} = \frac{8}{3}\times 1 \text{ V}$$

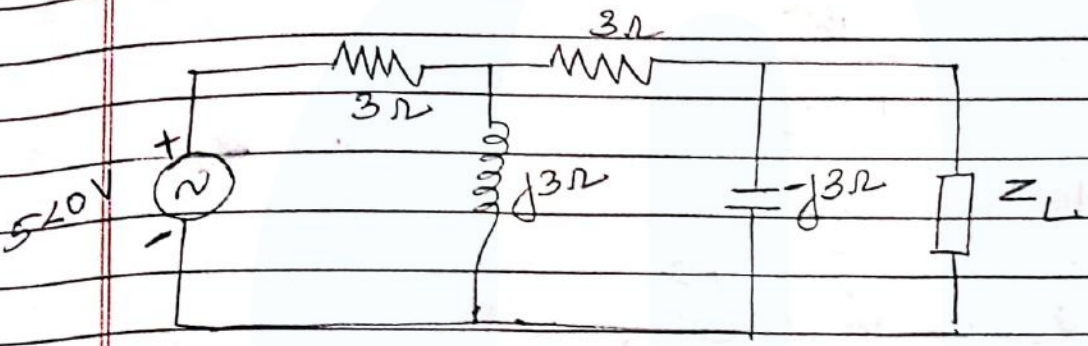
$$P_{max} = \frac{V_{oc}^2}{4R_{Th}} = \frac{(8/3)^2}{4\times 2/3}$$

Reciprocity

Maximum power transfer

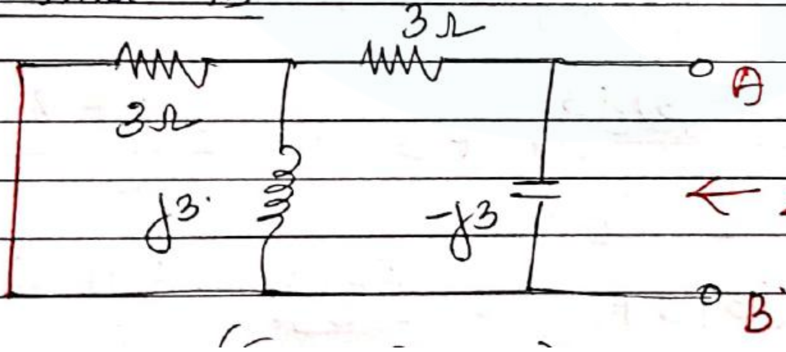
PAGE NO
DATE / /

(3) Find the impedance Z_L so that maximum power can be transferred to it. Find maximum power.



Ans: By maximum power transfer theorem, Z_L should be the complex conjugate of the source impedance. (Z_{th})

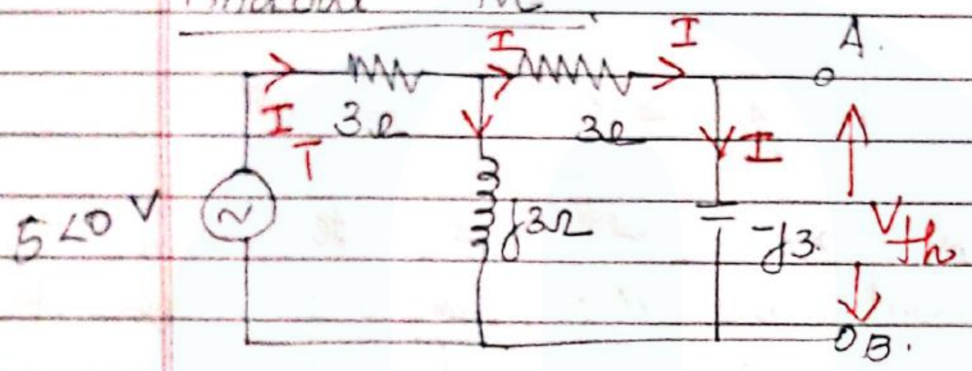
To find Z_{th}



$$= \frac{\frac{(-j9+12)}{(1+j1)}}{\frac{3+j4-j3(1+j1)}{(1+j1)}} = \frac{12-j9}{(6+j1)} = \frac{\cancel{15\angle -36.87}}{6.08\angle 9.46} =$$

To find $P_{max} = \frac{V_{th}^2}{4R_L}$

Findout V_{th}:



$V_{th} = (-j3)\,I$

$$Z_{eq} = (3-j3)\,||\,j3 + 3.$$

$$= \frac{(3-j3)j3}{(3-j3)+j3} + 3 = \frac{j9+9}{3} + 3$$

$$= j3+3+3 = 6+j3 = 6.71\angle 26.57\,\Omega$$

$$I_T = \frac{5\angle 0}{6+j3} = 0.75\angle -26.57$$

$$I = I_T \times \frac{j3}{j3+3-j3} = \frac{(0.75\angle -26.57)(3\angle 90)}{3\angle 0}$$

$$= 0.75\angle \cancel{-26.57}\; 63.43\ A$$

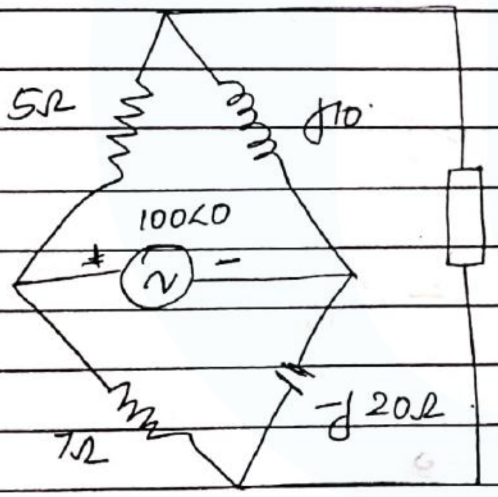
$$V_{th} = (-j3)(0.75\angle 63.43) = 2.24 \angle -26.57\ V$$

$$P_{max} = \frac{|V_{th}|^2}{4R_L} = \frac{2.24^2}{4 \times 1.8} = 0.70$$

By max power transfer, load impedance should be equal to complex conjugate of source impedance.

$$Z_L = Z_{th}^* = 1.8 + j2.4\ \Omega.$$

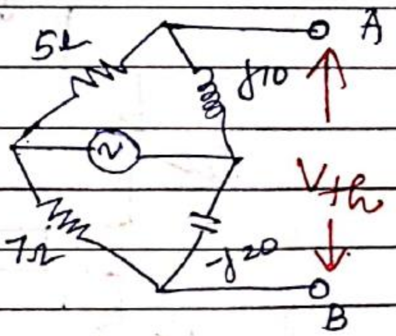
(4) Find the value of Z_L for maximum power transfer and find max. power.

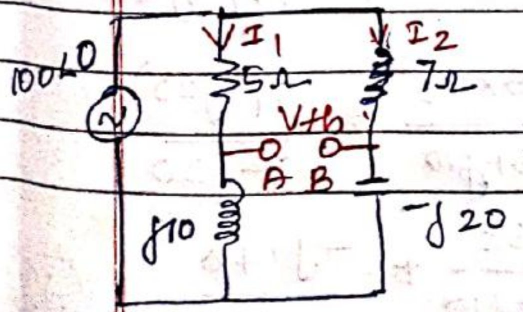


Z_L for max. power transfer will be, complex conjugate of Z_{th}.

Ans. To find V_{th}

Redrawing the circuit





$$I_1 = \frac{100\angle 0}{5+j10} = 8.94\angle -63.43\ A$$

$$I_2 = \frac{100\angle 0}{7-j20} = 4.72\angle 70.7\ A$$

$$V_{th} = (V_A - V_B) = (8.94\angle -63.43)\, j10 - (4.72\angle 70.7)(-j20)$$

$$= (8.94\angle -63.43)(10\angle 90) + (4.72\angle 70.7)(20\angle +90)$$

$$= \cancel{89.4\angle 26.57 + 94.4\angle 19.3} \ 160.7$$

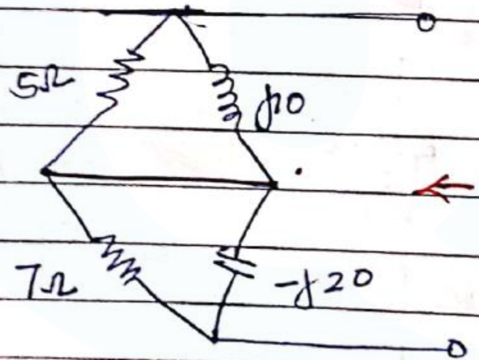
$$= 93.26 + 16.55j - \cancel{96.35 - 11.55j}$$

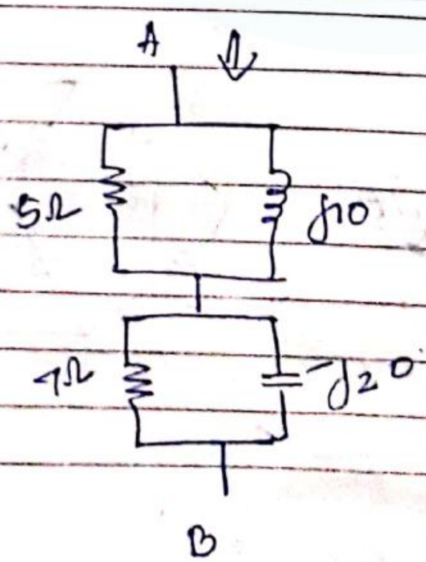
$$= -3.09 + 4.995j =$$

$$= 5.87\angle 121.74$$

$$= 71.76\angle 97.3\ V$$

To find Z_{th}





$$Z_{th} = 5 \parallel j10 + 7 \parallel -j20$$

$$= \frac{5 \times j10}{5+j10} + \frac{7(-j20)}{7-j20}$$

$$= \frac{j50}{5+j10} + \frac{-j140}{7-j20}$$

TRIVENI
PAGE NO.
DATE / /

$$= \frac{50\angle 90}{11.18\angle 63.43} + \frac{140\angle -90}{21.19\angle -70.7} = (10.23 - j0.18)$$

For maximum power transfer the Z_L should be complex conjugate of the Source impedance.

$$Z_L = 10.23 + j0.18\ \Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{71.76^2}{4 \times 10.23} = 125.84\,V$$

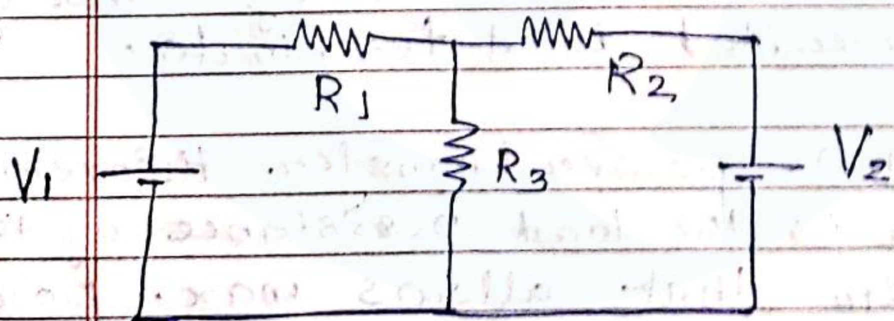
5. Reciprocity Theorem

In any linear network containing linear bilateral resistances & energy sources the ratio of voltage 'V' introduced in one mesh to the current 'I' in any second mesh is the same as ratio obtained if the positions of V & I are interchanged other voltage sources are removed.

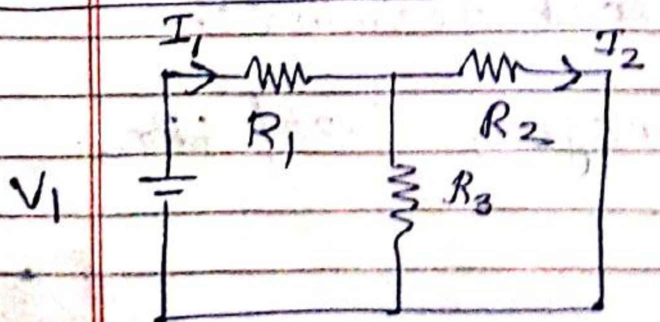
This theorem leads to a transfer reactance R_T where V is the voltage in one mesh ~~to~~ and I is the current in any second mesh.

$$\boxed{R_{T12} = R_T\ V_1 / I_2}$$

Proof Consider a network consists of resistors R_1, R_2 & R_3.



To calculate R_{T12}, $V_2 = 0$



$R_{T12} = V_1 / I_2$

$R_{eq} = R_1 + R_2 || R_3$

(12)

$$I_2 = I_1 \times \frac{R_3}{R_2+R_3}$$

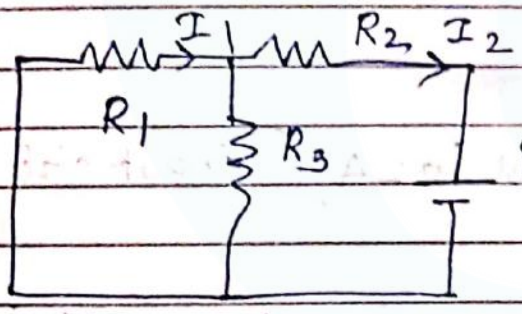
$$I_1 = \frac{V_1}{R_{eq}} = \frac{V_1}{R_1 + \frac{R_2R_3}{(R_2+R_3)}}$$

$$I_2 = \frac{V_1}{R_1 + \frac{R_2R_3}{(R_2+R_3)}} \times \frac{R_3}{(R_2+R_3)}$$

$$= \frac{V_1R_3}{R_1R_2 + R_2R_3 + R_1R_3}.$$

$$R_{T1,2} = \frac{V_1}{I_2} = \underline{\underline{\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}.}}$$

<u>To calculate R_{T21}</u>, $V_1 = 0$.



$R_{eq} = R_1 || R_3 + R_2$

$R_{T21} = V_2 / I_1$

$$I_1 = -I_2 \times \frac{R_3}{R_1+R_3} = -\left(V_2/R_{eq}\right)\frac{R_3}{R_1+R_3}$$

$$= \frac{-V_2R_3}{\frac{R_1R_3}{R_1+R_3} + R_2} = \frac{-V_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

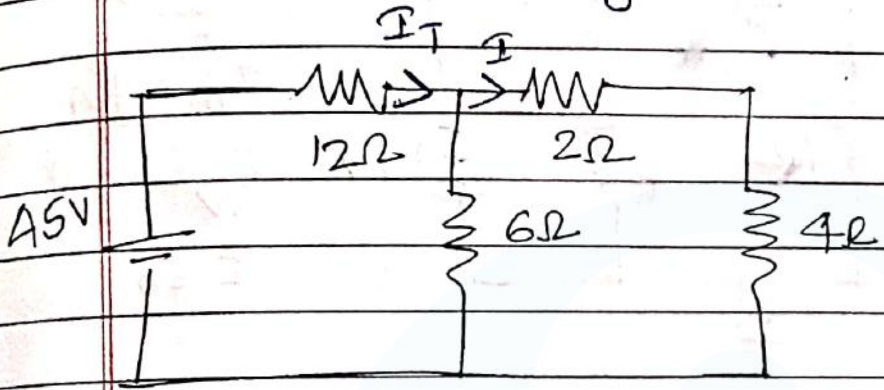
$$R_{T21} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

i.e. $R_{T12} = R_{T21}$

This is the proof

Reciprocity

Q. For the n/w, find the value of current I and also verify the reciprocity theorem.



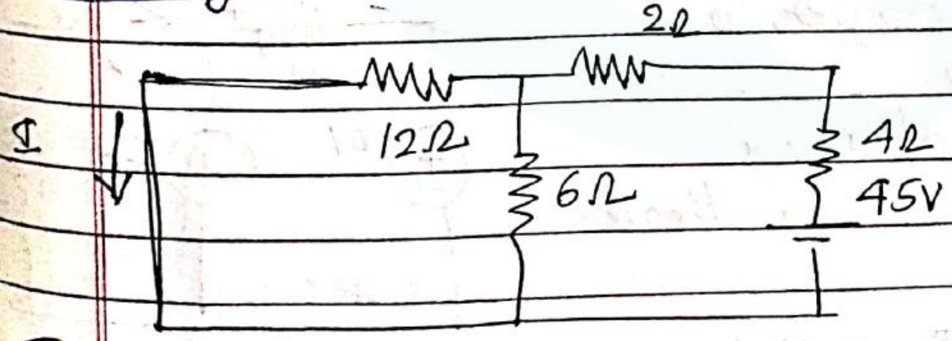
$$I = I_T \times \frac{6}{6+2+4} \quad , \quad R_{eq} = \left[(2+4) \parallel 6\right] + 12$$

$$I_T = \frac{V}{R_{eq}} = \frac{45}{15} \qquad = 15\Omega$$

$$= 3A$$

$$I = 3 \times \frac{6}{12} = 1.5A$$

Interchange V & I



TRIVENI

PAGE NO.

DATE / /

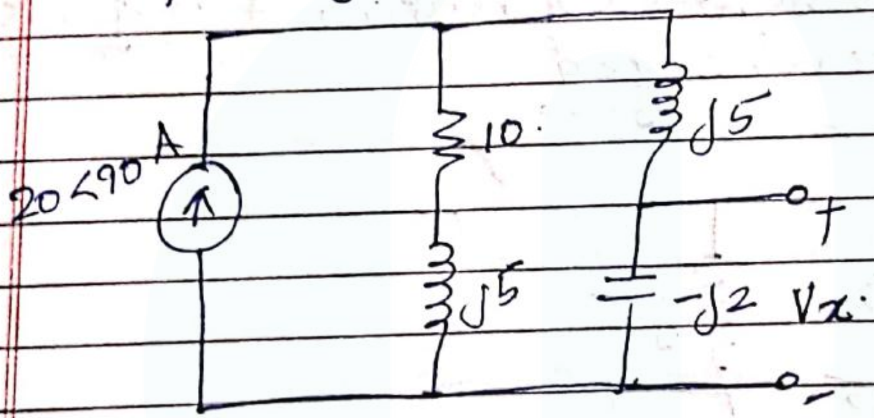
$$R_T = 6 + (6 \| 12) = 6 + \frac{72}{18} = \underline{\underline{10\Omega}}.$$

$$I_T = \frac{45}{10} = \underline{\underline{4.5A}}.$$

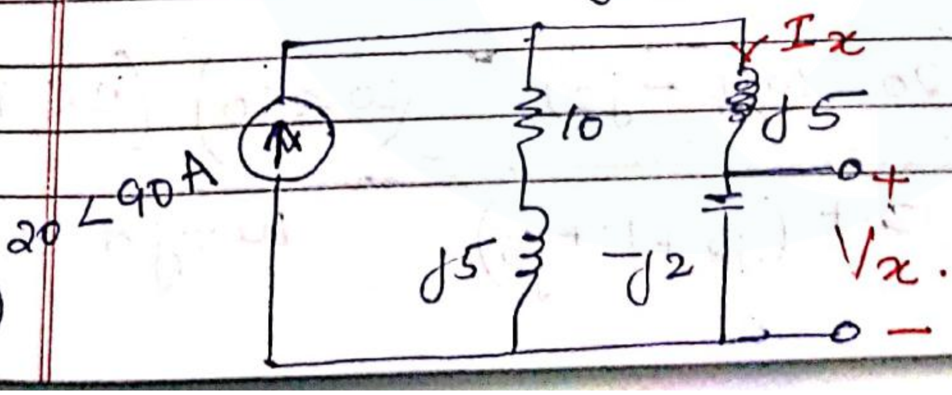
$$I = \frac{4.5 \times 6}{6+12} = \underline{\underline{1.5A}}.$$

Reciprocity theorem:

Find the voltage V_x & then verify reciprocity theorem.



Ans Calculate V_x when excitation & response are interchanged.



Find I_x, by current division rule:

$$I_x = 20\angle 90 \times \frac{(10+j5)}{(10+j5)+(j5-j2)}$$

$$= \frac{(20\angle 90)(11.18\angle 26.57)}{10+j8}$$

$10+j8 \longrightarrow 12.81\angle 38.66$

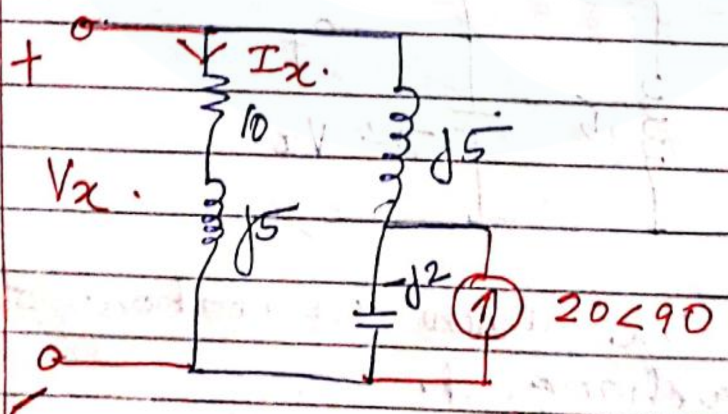
$$= \frac{20\times 11.18}{12.81}\angle(90+26.57-38.66)$$

$$= \underline{\underline{17.46\angle 77.91 \text{ A.}}}$$

$$V_x = (-j2)\, I_x = (2\angle -90)(17.46\angle 77.91)$$

$$= \underline{\underline{34.92\angle -12.09 \text{ V.}}}$$

Find V_x when excitation & response are interchanged



$$I_x = \frac{(20\angle 90)(-j2)}{-j2+(j5+10+j5)} = \frac{(20\angle 90)(2\angle -90)}{10+j8}$$

$$= \frac{40\angle 0}{12.81\angle 38.66} = 3.12\angle -38.66 \text{ A.}$$

$$V_x = I_x \times (10 + j5)$$

$$= (3.12\angle -38.66)(11.18\angle 26.57)$$

$$= 34.8816\angle -12.09 \text{ V.}$$

Since, V_x is same in both cases, reciprocity theorem is verified.