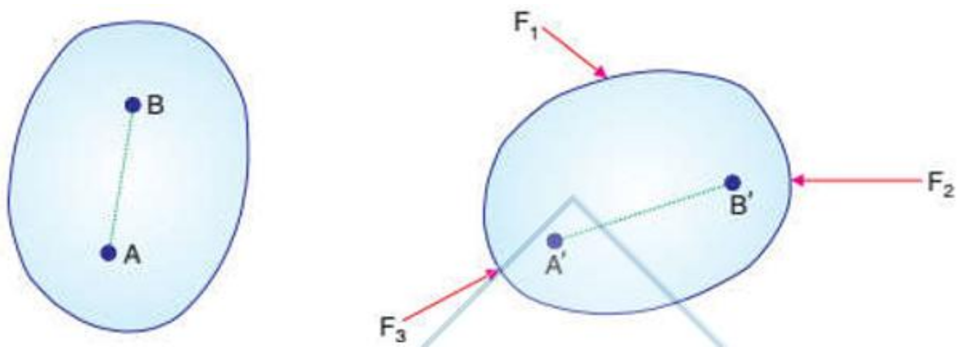
Mechanics can be defined as the branch of physics concerned with the state of rest or motion of bodies that subjected to the action of forces.

❖ RIGID BODY

A rigid body may be defined as a body in which the relative positions of any two particles do not change under the action of forces means the distance between two points/particles remain same before and after applying external forces.

OR

A body which does not deform under the influence of forces is known as a rigid body. For a rigid body, relative positions of A'B' and AB remains same before and after the application of forces



❖ Rigid Body Mechanics

Rigid Body Mechanics can be divided into two branches.

1) **Statics:** It is the branch of mechanics that deals with the study of forces acting on a body in equilibrium. Either the body at rest or in uniform motion is called statics
2) **Dynamics:** It is the branch of mechanics that deals with the study of forces on body in motion is called dynamics. It is further divided into two branches.

❖ Force

Force may be defined as any action that tends to change the state of rest or motion of a body to which it is applied. The three quantities required to completely define force are called its specification or characteristics.

1. Magnitude

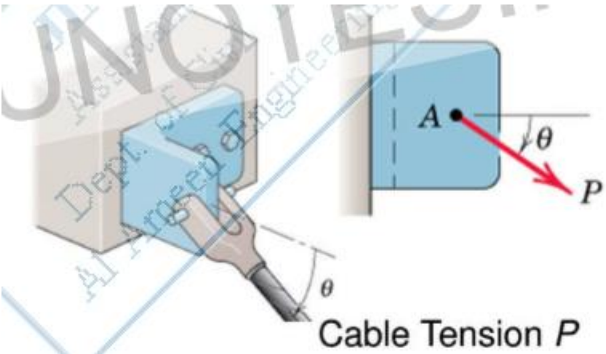
2. Point of application

3. Direction of application/Line of action

Force is a vector quantity and its unit is Newton (N) in S.I. systems and dyne in C.G.S. system.

- **Line of action of force**

The direction of a force is the direction, along a straight-line through its point of application in which the force tends to move a body when it is applied. This line is called line of action of force.



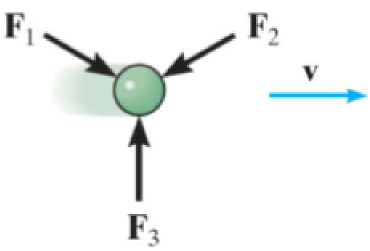
- **LAWS OF MECHANICS**

The following are the fundamental laws of mechanics:

1. Newton's Laws of Motion
2. Newton's Law of Gravitation
3. Law of transmissibility of forces
4. Parallelogram law of forces

- **Newton's Laws of Motion Law**

Law1 : A particle remains at rest or continues to move with uniform velocity if there is no unbalanced force acting on it.



Law 2 : The second law states that the rate of change of momentum of a body, is directly proportional to the force applied and this change in momentum takes place in the direction of the applied force.

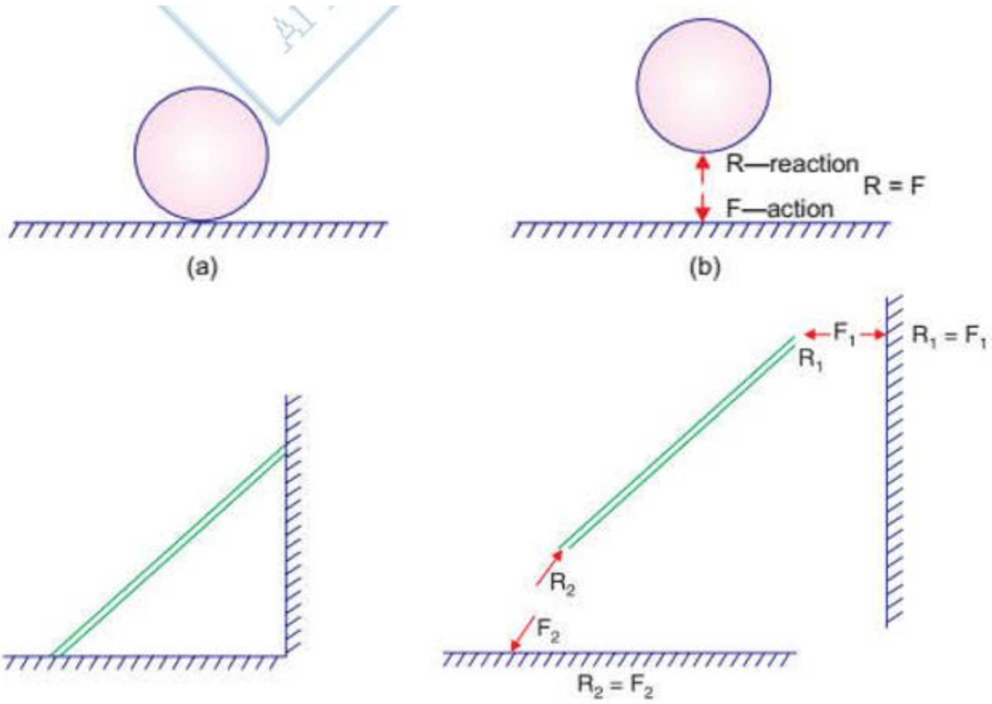
Newton's second law of motion explains how an object will change velocity if acted by a force. For a body with constant mass, the second law can also be stated in terms of an object's acceleration. The acceleration of a particle is proportional to the vector sum of forces acting on it and occurs along a straight line in which the force acts.

$$\sum F = \frac{d(mv)}{dt}$$

$$\sum F = m\frac{d(v)}{dt}$$

$$\sum F = ma$$

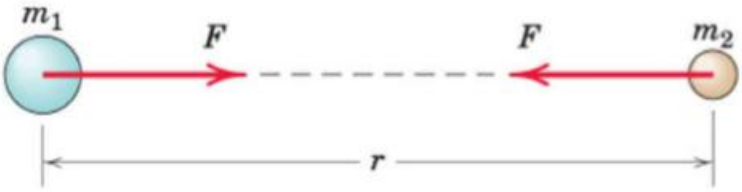
Law 3 : To every action there is always an equal reaction: or the mutual interactions of two bodies are always equal but directed in opposite direction



❖ **Law of Gravitation**

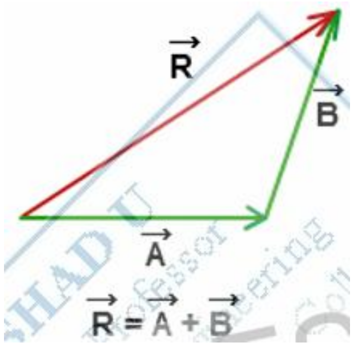
Two particles will be attracted towards each other along their connecting line with a force whose magnitude is directly proportional to the product of the masses and inversely proportional to the distance squared between the particles.

$$F = G\frac{m_1 m_2}{r^2}$$



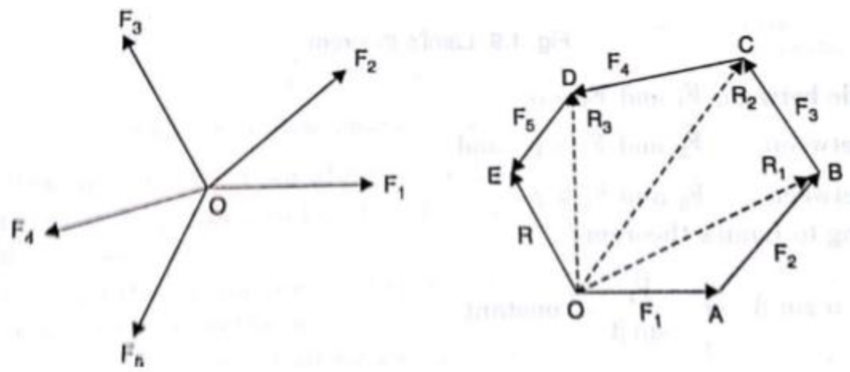
❖ Triangular law of forces

"If the two forces acting on a body are represented in magnitude and direction as two sides of a triangle in order then the third side or the closing side of the triangle would be the resultant in opposite order."



❖ Polygon Law of Forces

If a number of concurrent forces acting on a rigid body are represented in magnitude and direction by the sides of a polygon, taken in order, then their resultant is represented in magnitude and direction by the closing side of polygon taken in reverse order.



Q: Find the resultant of the forces shown in fig 2.33.

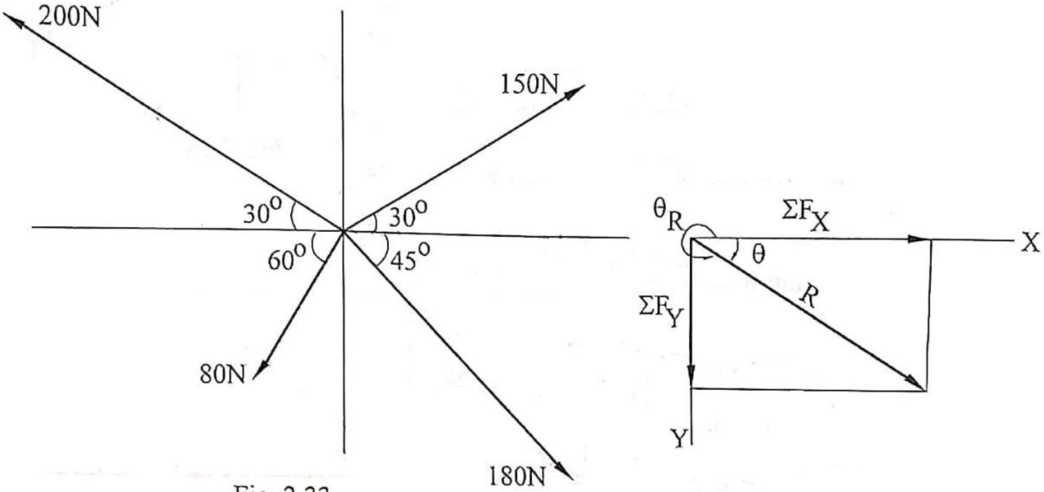


Fig. 2.33

Solution

Resolving the forces along X-axis,

$$\sum F_x = 150 \cos 30 + 180 \cos 45 - 200 \cos 30 - 80 \cos 60$$
$$= 43.98 \text{ N}$$

Resolving the forces along theY -axis

$$\sum F_y = 150 \sin 30 + 200 \sin 30 - 80 \sin 60 - 180 \sin 45$$
$$= -21.56 \text{ N}$$

$$\text{Resultant } R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(43.98)^2 + (-21.56)^2}$$

$$= 48.98 \text{ N}$$

$$\text{Inclination of resultant with horizontal, } \theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\frac{21.56}{43.98}$$

$$= 26.12^0$$

Since the resultant is in the fourth quadrant, the inclination of resultant,

$$\theta_R = 360 - \theta$$

$$= 360 - 26.12$$

$$= 333.88^0$$

Q: Forces of 15 N, 20 N, 25 N, 35 N and 45 N act at an angular point of a regular hexagon towards the other angular points as shown in fig.2.39. Calculate the magnitude and direction of the resultant force.

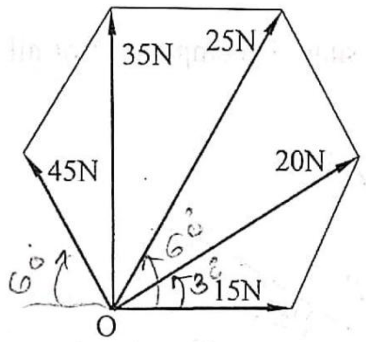


Fig. 2.39

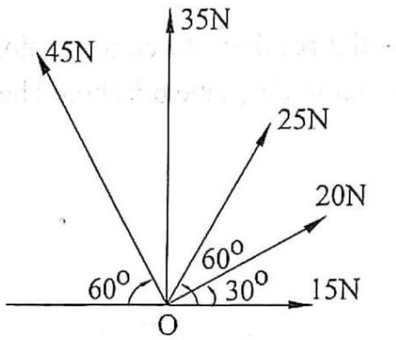


Fig.2.40

Solution.

Resolving the forces along X axis,

$$\Sigma F_x = 15 + 20 \cos 30 + 25 \cos 60 + 0 - 45 \cos 60 = 22.32 \text{ N}$$

Resolving the forces along y axis,

$$\Sigma F_y = 0 + 20 \sin 30 + 25 \sin 60 + 35 + 45 \sin 60 = 105.62 \text{ N}$$

$$\text{Resultant, } R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$= \sqrt{22.32^2 + 105.62^2}$$

$$= 107.95 \text{ N}$$

Inclination of resultant with horizontal,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\frac{105.62}{22.32}$$

$$= 78.07^0$$

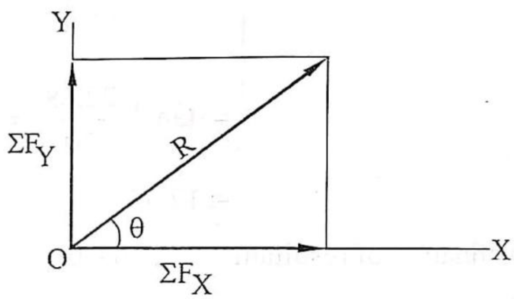
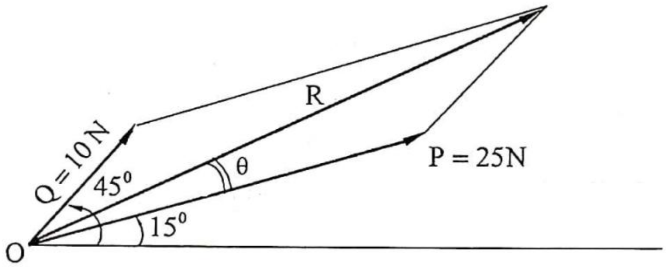


Fig. 2.41

Inclination of resultant, $\theta_R = \theta = 78.07^0$

Q: Two forces P and Q of magnitude 25 N and 10 N are acting at a point. The forces P and Q make angle 15^0 and 45^0, measured counter clockwise with the horizontal. Determine the resultant magnitude and direction.

Solution:



$P = 25$ N

$Q = 10$ N

$\alpha = 45 - 15 = 30^0$

Resultant, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

$= \sqrt{25^2 + 10^2 + 2 \times 25 \times 10 \times \cos 30}$

$R = 34.03$ N.

The inclination of resultant force with the direction of force P,

$$\theta = \tan^{-1}\frac{\sin\alpha}{\cos\alpha + \frac{P}{Q}} = \tan^{-1}\frac{\sin 30}{\cos 30 + \frac{25}{10}} = 8.45^0$$

Inclination of resultant with horizontal is $15^0 + \theta$

$= 15^0 + 8{\cdot}45^0 = 23.45^0$

Q: A boat is moved uniformly by pulling with forces $P = 240$ N and $Q = 200$ N What must be the inclination of the resultant force with P and Q to have the resultant $R = 400$ N as shown

Solution

Given

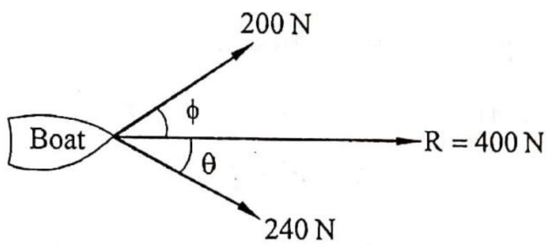


Fig. 2.11

$P = 240$ N

$Q = 200$ N

$R = 400$ N

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

$400 = \sqrt{240^2 + 200^2 + 2 \times 240 \times 200\cos(\theta + \phi)}$

$\cos(\theta + \phi) = 0.65$

$\theta + \phi = 49.46^0$

Inclination of resultant force with P,

$$\theta = \tan^{-1} \frac{\sin(\theta + \phi)}{\cos(\theta + \phi) + \frac{P}{Q}}$$

$$= \tan^{-1} \frac{\sin 49.46}{\cos 49.46 + \frac{240}{200}}$$

$$= 22.33^0.$$

Inclination of resultant with Q,

$$\phi = 49.46^0 - 22.33 = 27.13.$$

$$= 27.13^0$$

Q: What force P, combined with a vertical force Q = 12 N, will give a horizontal resultant, R=16 N ?

Solution

Given:

$$Q = 12\text{ N, vertical}$$

$$R = 16\text{ N, horizontal}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$16 = \sqrt{P^2 + 12^2 + 2 \times P \times 12\cos(90 + \theta)}$$

$$= \sqrt{P^2 + 12^2 - 24P.\sin\theta} \text{ -------(i)}$$

Inclination of resultant with force Q is 90^0

$$\therefore \tan 90 = \frac{\sin(90 + \theta)}{\cos(90 + \theta) + (\frac{12}{P})}$$

Since tan 90 is infinite, the denominator of RHS should be zero

$$\cos(90 + \theta) + \frac{12}{P} = 0$$

$$-\sin\theta + \frac{12}{P} = 0$$

$\sin\theta = \frac{12}{P}$, substituting this value of $\sin\theta$ in eqn (i),

$$16 = \sqrt{P^2 + 12^2 - 24P \times \frac{12}{P}}$$

$$= \sqrt{P^2 + 12^2 - 24 \times 12}$$

$$16 = \sqrt{P^2 + 144 - 288}$$

$$P = 20\text{ N}$$

$$\sin\theta = \frac{12}{P} = \frac{12}{20} = 0.6$$

$$\theta = \sin^{-1} 0.6 = 36.87^0$$

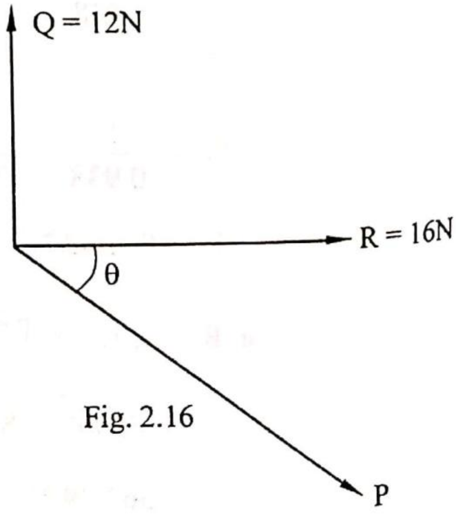
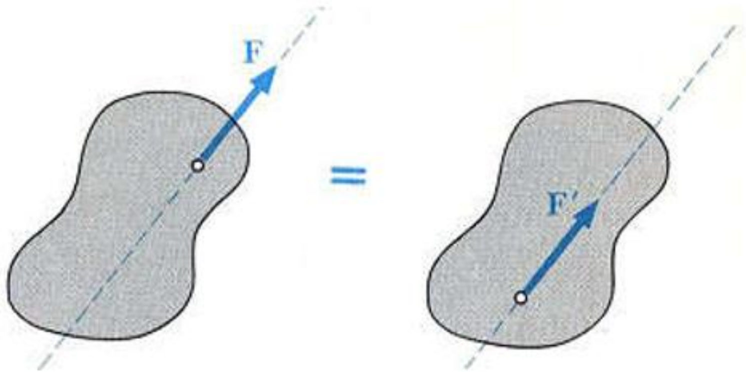


Fig. 2.16

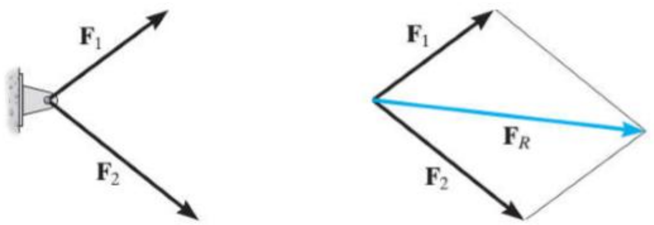
❖ **Law of Transmissibility of Force**

Principle of transmissibility states that a force may be applied at any point on a rigid body along its given line of action without altering the effects of the force on which it acts.

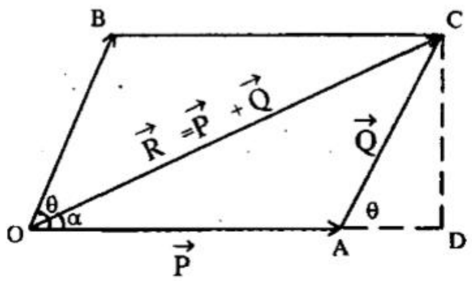


❖ **Parallelogram Law of Forces**

If two forces acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by lite diagonal of the parallelogram passing through that point.



<u>Analytical Proof</u>



Given OA = P force and OB = Q force. Construct parallelogram OBCA as shown and drop perpendicular CD on extension of OA.

Thus, AD= Q cos θ,CD=Q sin θ

OCD is a right angled triangle,

:. $OC^2 = OD^2 + CD^2$

$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$

$= p^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2PQ\cos\theta.$

$= P^2 + Q^2 + 2PQ\cos\theta$ [since $\cos^2 a + \sin^2 a = 1$]

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

<u>Direction of R :</u>

Tanα = CD/OD =Q sin a/ P+Q cos θ

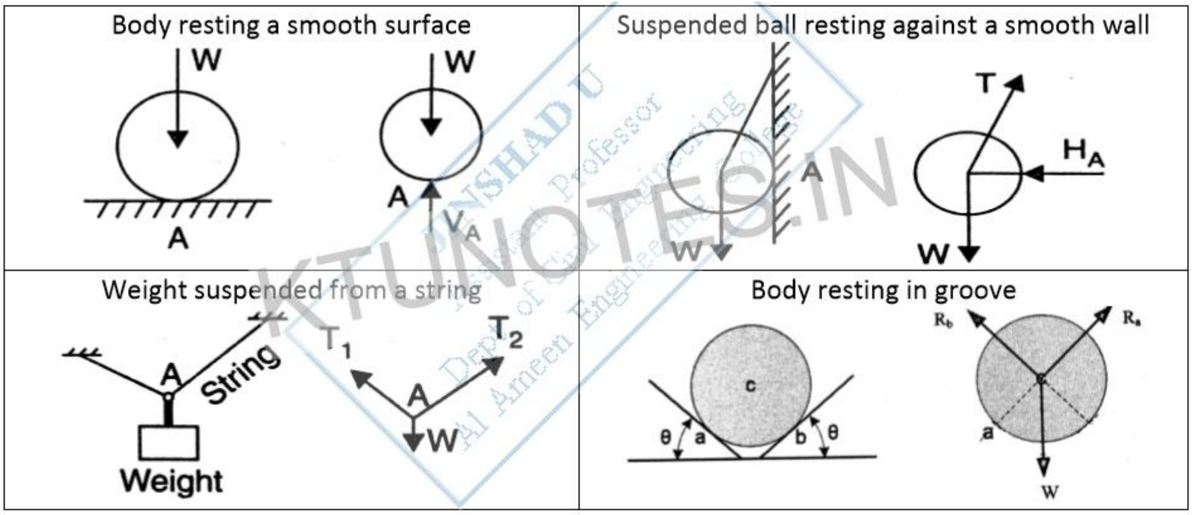
$$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right]$$

❖ Principle of Superposition

According to the *principle of superposition* of forces, when a Rigid body is acted upon by some external forces on various points, then the resultant effect on the body is the sum of the effects caused by each of the loads acting independently on the respective points of the body.

❖ **Free body diagram :**

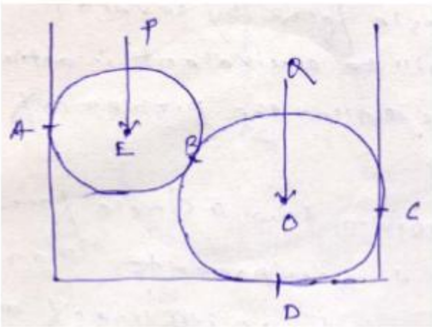
Free body diagram is a diagram in which a rigid body is isolated from the system and all active forces applied to the body and reactive forces as a result of mechanical contact are represented.

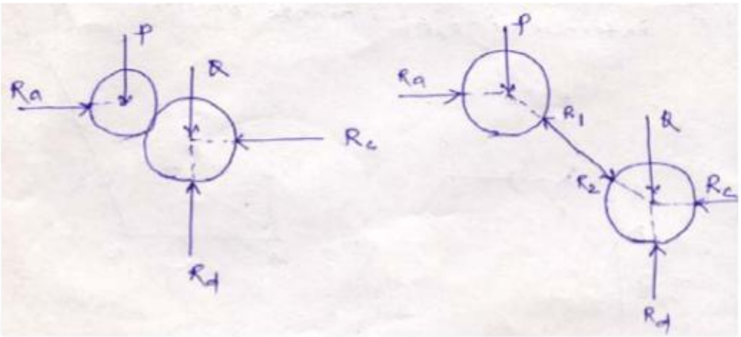


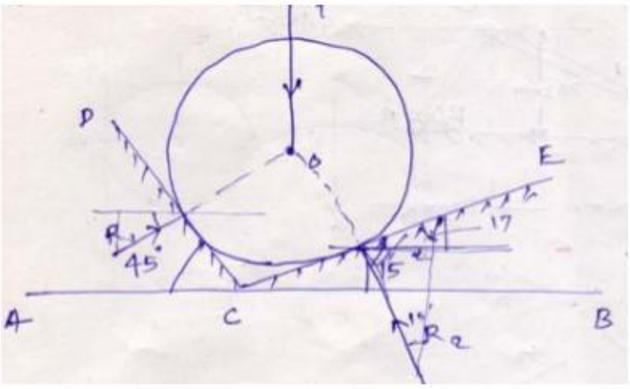
Steps for Drawing Free Body Diagram :

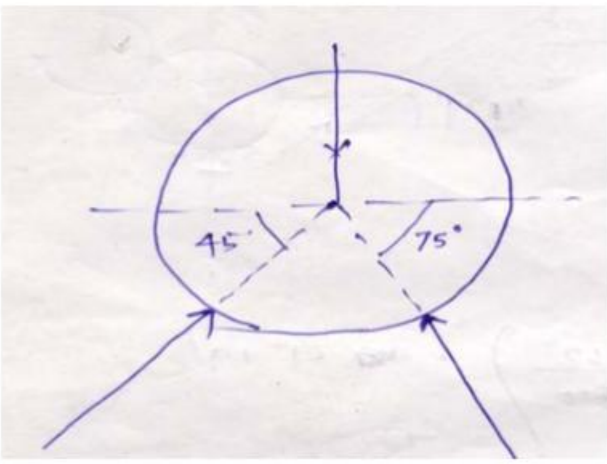
1. A sketch of the body is drawn assuming that all supports (surfaces of contact, supporting cables, etc.) have been removed.
2. All applied forces (including weight) and support reactions are drawn and labeled on the sketch.
3. Apply the weight of the body to its center of gravity (if it is uniform, then apply it to the centroid). If the sense of a reaction is unknown, it should be assumed

Problem 3: Two spheres of weight P and Q rest inside a hollow cylinder which is resting on a horizontal force. Draw the free body diagram of both the spheres, together and separately.

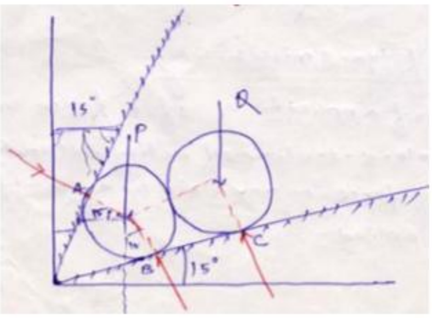


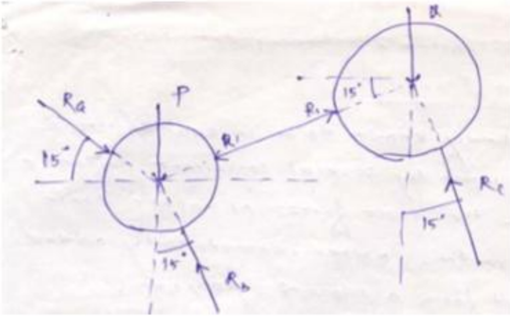


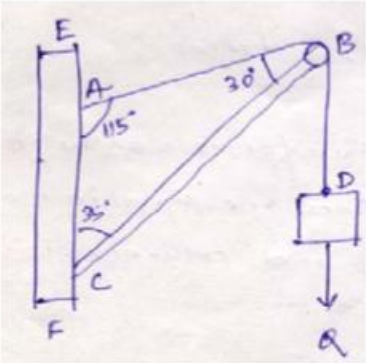


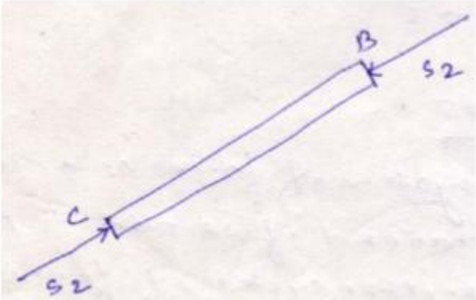


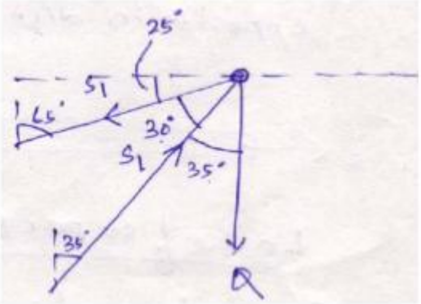
Problem 7: Two rollers of weight P and Q are supported by an inclined plane and vertical walls as shown in the figure. Draw the free body diagram of both the rollers separately.











❖ Reaction

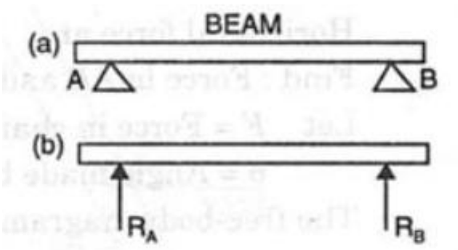
Reaction is the opposing force that a support offers whenever it is acted upon by external or inherent forces.

❖ TYPES OF SUPPORTS AND THEIR REACTIONS

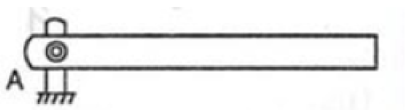
Though there are many types of supports, yet the following are important from the subject point of view:

1. Roller support
2. Pin-joint (or hinged) support
3. Fixed or built in support

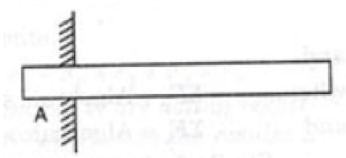
1. **Roller Support.** A beam supported on the rollers at points A and B as shown. The reactions in case of roller supports will be normal to the surface on which rollers are placed.



2. **Pin Joint (or Hinged) Support.** A beam, which is hinged (or pin-joint) at A as. The reaction at the hinged end may be either vertical or inclined depending upon the type of loading.



3. **Fixed or Build-in-Support**. In case of fixed support, the reaction will be inclined. Also the fixed support will provide a couple/moment.



❖ Resultant

Resultant is a single force that will replace a system of forces and produces the same effect on the rigid body as that of the system of forces.

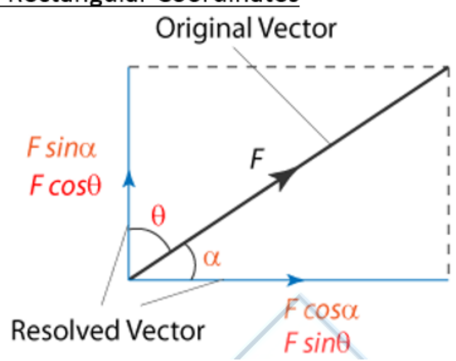
The following sign conventions shall be used.

1. Upward forces are considered as positive, whereas the downwards as negative.
2. Forces acting towards right are considered as positive, whereas those towards left as negative.

❖ Resolution Of Forces

The replacement of a single force by a several components which will be equivalent in action to the given force is called resolution of a force. Forces can be resolved in any 2 directions. However, it is convenient to resolve them into the two orthogonal components (mutually perpendicular directions)

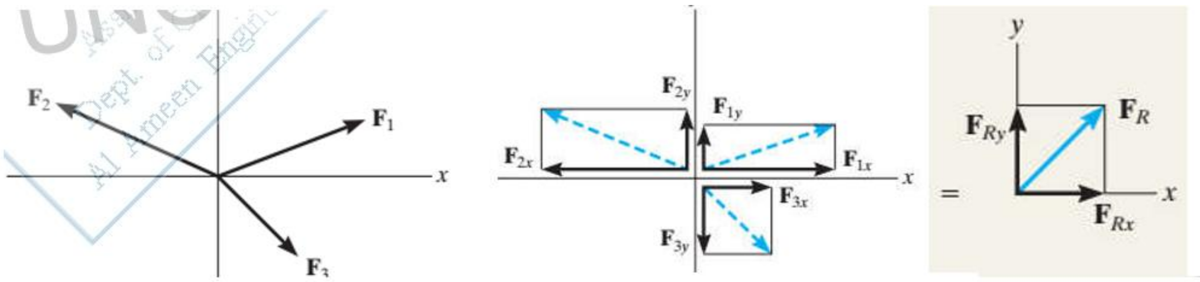
Resolution of Coplanar Forces in Rectangular Coordinates



❖ RESULTANT OF CONCURRENT COPLANAR FORCE SYSTEMS

Procedure:

1. Resolve all the forces into x and y components
2. Add the components of forces along the x and y axes with proper sense of direction.
3. Find the resultant and inclination of the forces



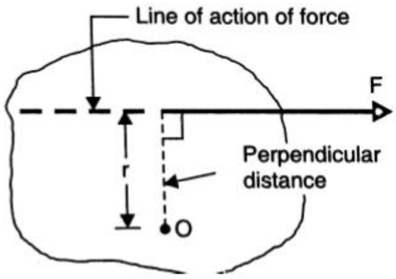
$$F_{Rx} = \Sigma F_x \qquad F_{Ry} = \Sigma F_y$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

❖ Moment

The tendency of a force to rotate the body in the direction of its application a force about a point that is not on the line of action of the force is called Moment of force or simply moment. Moment is also referred to as torque.

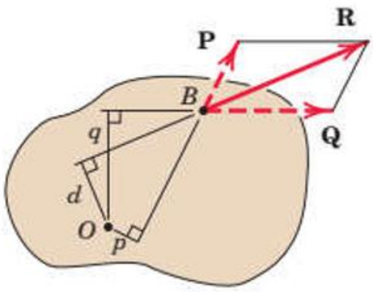


$$M = Fd$$

❖ Varignon's Theorem or Principle of Moments

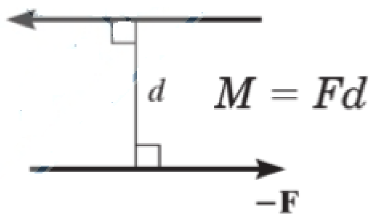
"The moment of a force about any point is equal to the sum of the moments of the components of the force about the same point."

$$M_O = Rd = -pP + qQ$$



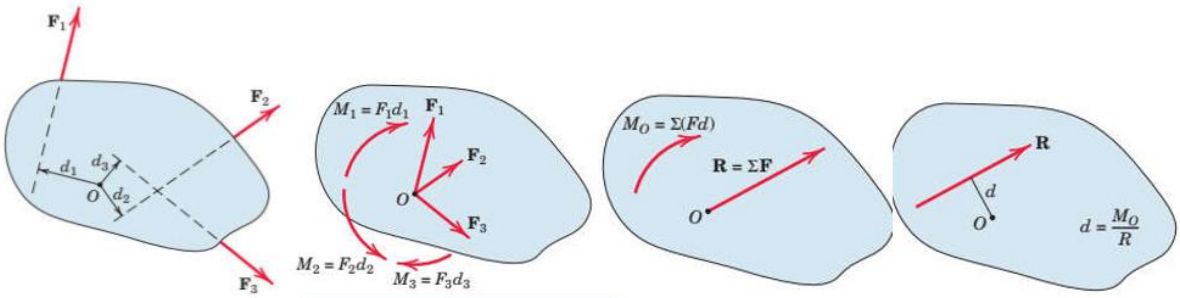
❖ Couple

The moment produced by two equal, opposite, and non-collinear forces is called a couple. The perpendicular distance between the lines of action of the two and opposite parallel forces is known as arm of the couple.



❖ RESULTANT OF COPLANAR NON-CONCURRENT FORCE SYSTEMS

A system of several forces and couple moments acting on a body can be reduced to equivalent single resultant force acting at a point O and a resultant couple moment.



$$\mathbf{R} = \Sigma \mathbf{F}$$

$$M_O = \Sigma M = \Sigma(Fd)$$

$$Rd = M_O$$

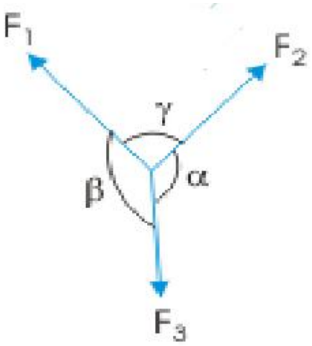
$$d = \frac{M_O}{R}$$

❖ EQUILBRIUM OF RIGID BODIES

A rigid body is said to be in equilibrium if the resultant of all external and reactive forces and moments acting on it is zero.

❖ Lami's Theorem

If three coplanar concurrent forces acting on a body keep it in equilibrium, then each force is proportional to the sine of the angle between the other two.



$$\frac{F_1}{Sin\alpha} = \frac{F_2}{Sin\beta} = \frac{F_3}{Sin\gamma}$$

Note: Lami's theorem is applicable only to 3 coplanar concurrent forces in equilibrium

❖ General Equations of Equilibrium

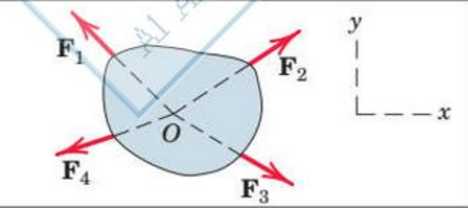
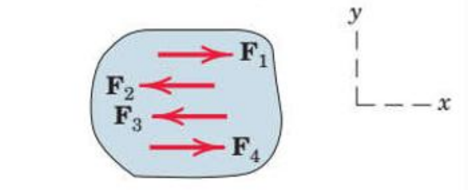
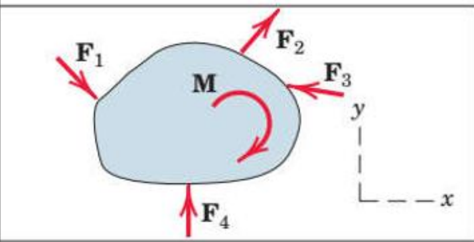
1. The algebraic sum of all forces in a force system is zero.

$$R = \sum F = 0$$

2. The algebraic sum of all moments in a force system is zero.

$$M = \sum M = 0$$

Equations of Equilibrium For Coplanar Systems

Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

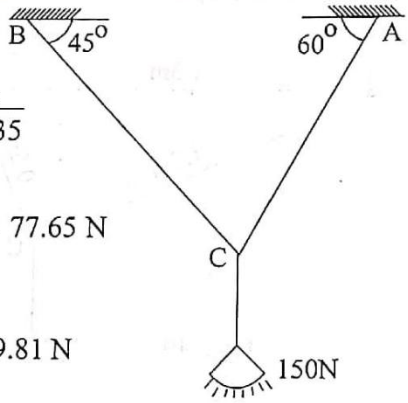
- **Solving Equilibrium Problems**
  1. Draw proper Free- Body Diagram
  2. Resolve all the forces into x and y components
  3. Apply Equilibrium conditions along the x and y directions
  4. Solve the resultant algebraic equations

In case of moments, try to select the point you take moments around such that the line of action of at least one unknown force passes through that point. This will eliminate one unknown from your moment equation and will result in simpler equations to work with.

Q: An electric light fixture weighing 150 N hangs from a point C by two stay wires AC and BC as shown. Determine the tensions in the stay wires AC and BC using Lami's theorem. Verify the answer using any other method.

Solution

Using Lami's theorem

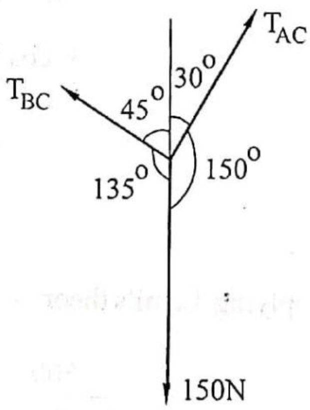


$$\frac{150}{\sin 75} = \frac{T_{BC}}{\sin 150} = \frac{T_{AC}}{\sin 135}$$

$$T_{BC} = \frac{150 \times \sin 150}{\sin 75} = 77.65 \text{ N}$$

$$T_{AC} = \frac{150 \times \sin 135}{\sin 75} = 109.81 \text{ N}$$

Using method of projections.



For $\Sigma F_x = 0$, $\quad T_{AC} \cos 60 - T_{BC} \cos 45 = 0$

$$0.5\, T_{AC} - 0.707\, T_{BC} = 0 \text{ ---------(i)}$$

For $\Sigma F_y = 0$, $\quad T_{AC} \sin 60 + T_{BC} \sin 45 - 150 = 0$

$$0.866\, T_{AC} + 0.707\, T_{BC} = 150 \text{ ------(ii)}$$

Adding equations (i) and (ii)

$$T_{AC}\,(0.5 + 0.866) + 0 = 150$$

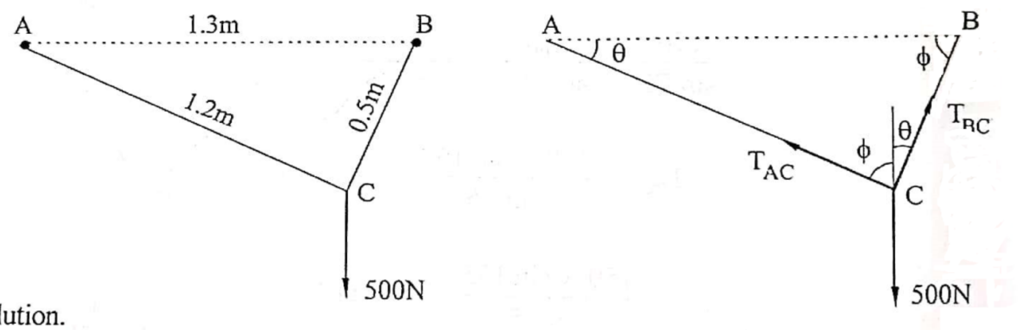
$$T_{AC} = \frac{150}{(0.5 + 0.866)} = 109.81 \text{ N}$$

Substituting this value of T_{AC} in eqn (i)

$$0.5 \times 109.81 - 0.707\, T_{BC} = 0$$

$$T_{BC} = \frac{0.5 \times 109.81}{0.707} = 77.65 \text{ N}$$

Q: Two cables AC and BC are tied together at the point C to support a load of 500 N at C. A and B are at a distance of 1.3 m and are on the same horizontal plane. AC and BC are 1.2 m and 0.5 m respectively. Find the tensions in AC and BC.



Solution.

$AC^2 + BC^2 = 1.2^2 + 0.5^2 = 1.69 = 1.3^2 = AB^2$

Since $AC^2 + BC^2 = AB^2$, angle ACB = 90^0

$$\theta + \phi = 90^0$$

$$1.3 \cos\theta = 1.2$$

$$\theta = \cos^{-1}\frac{1.2}{1.3} = 22.62^0$$

$$\phi = 90 - \theta = 90 - 22.62 = 67.38^0$$

Applying Lami's theorem,

$$\frac{500}{\sin(\theta + \phi)} = \frac{T_{AC}}{\sin(180 - \theta)} = \frac{T_{BC}}{\sin(180 - \phi)}$$

$$\frac{500}{\sin 90} = \frac{T_{AC}}{\sin(180 - 22.62)} = \frac{T_{BC}}{\sin(180 - 67.38)}$$

$$T_{AC} = \frac{500 \sin(180 - 22.62)}{\sin 90} = 192.31 \text{ N}$$

$$T_{BC} = \frac{500 \sin(180 - 67.38)}{\sin 90} = 461.54 \text{ N}$$

Q: A rope 9 m long is connected at A and B, two points on the same level, 8 m apart. A load of 300 N is suspended from a point C on the rope, 3 m from A. What load connected to a point D, on the rope, 2 m from B is necessary to keep portion CD parallel to AB.

Solution.

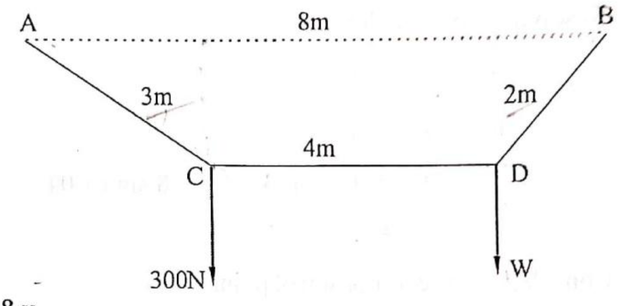


Fig. 2.51

From the triangle ACE,

$$y^2 = 3^2 - x^2$$

From the triangle BDF,

$$y^2 = 2^2 - (4 - x)^2$$

$$3^2 - x^2 = 2^2 - (4 - x)^2$$

$$9 - x^2 = 4 - 16 - x^2 + 8x$$

$$8x = 21$$

$$x = 2.625 \text{ m}$$

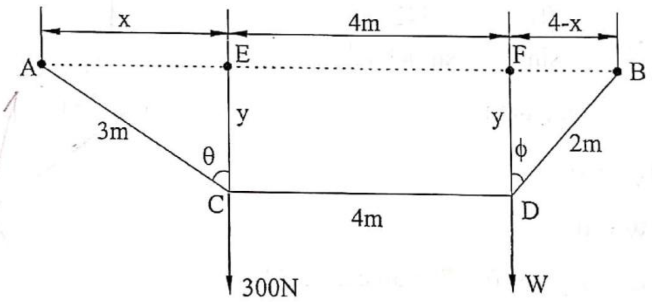


Fig. 2.52

$$\sin\theta = \frac{x}{3} = \frac{2.625}{3} = 0.875$$

$$\theta = 61.04^0$$

$$\sin\phi = \frac{4 - x}{2} = \frac{4 - 2.625}{2} = 0.6875$$

$$\phi = 43.43^0$$

Consider the equilibrium of point C.

Resolving the forces vertically,

For $\sum F_V = 0$

$$T_{AC} \cos\theta - 300 = 0$$

$$T_{AC} = \frac{300}{\cos 61.04} = 619.58 \text{ N}$$

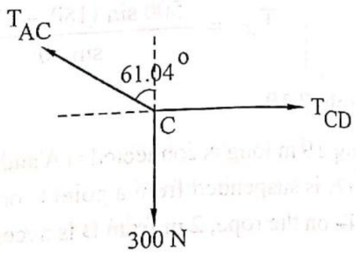


Fig. 2.53

Resolving the forces horizontally,

or $\sum F_H = 0$

$$T_{CD} - T_{AC} \sin\theta = 0$$

$$T_{CD} = T_{AC} \sin\theta = 619.58 \sin 61.04$$

$$= 542.11 \text{ N.}$$

Considering the equilibrium of point D

For $\sum F_H = 0$

$T_{DB} \sin \phi - T_{CD} = 0$

$$T_{DB} = \frac{T_{CD}}{\sin \phi} = \frac{542.11}{\sin 43.43^0}$$

$= 788.56$ N

For $\sum F_V = 0$

$T_{DB} \cos \phi - W = 0$

$W = T_{DB} \cos \phi = 788.56 \cos 43.43$

$W = 572.66$ N

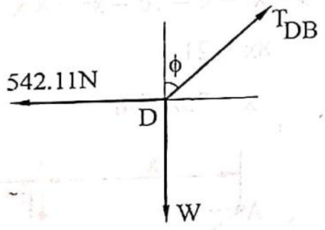


Fig. 2.54

Q: A steel ball rests in a groove, the sides of which are smooth. One side of the groove is vertical while the other side is at 40° to the horizontal. If the ball has a weight of 10 N find the reaction on each wall of the groove.

Solution.

For $\sum F_V = 0$

$R_2 \cos 40 - 10 = 0$

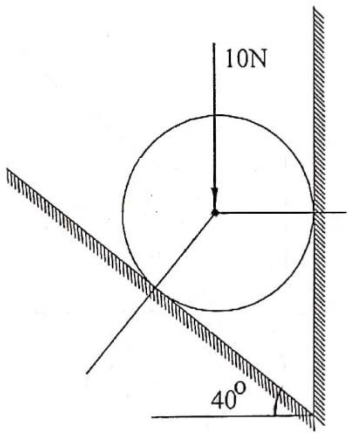
$$R_2 = \frac{10}{\cos 40} = 13.05 \text{ N}$$

For $\sum F_H = 0$

$R_2 \sin 40 - R_1 = 0.$

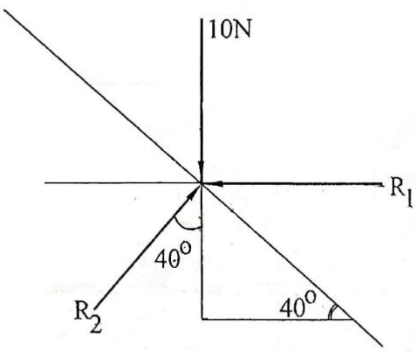
$R_1 = R_2 \sin 40 = 13.05 \sin 40$

$= 8.39$ N



Reaction on vertical wall = 8.39 N.

Reaction on inclined wall = 13.05 N.



Q: A roller of weight W = 1000 N rests on a smooth inclined plane and is kept from rolling down by a string AC as shown in fig.2.66. Using method of projections, find the tension S in the string and the reaction at the point of contact B.

Solution.

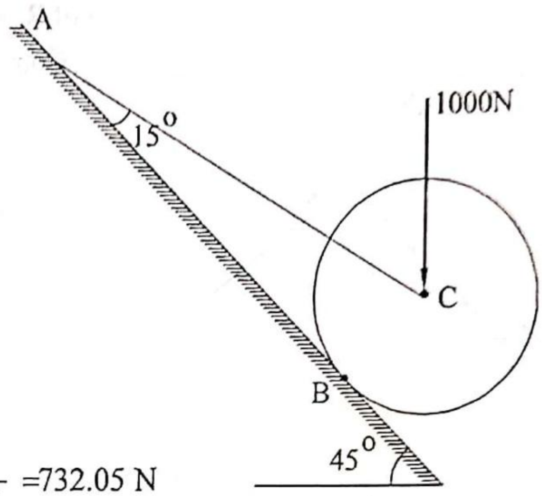
Let S be the tension in the string and R be the reaction at B.

Resolving the forces along X direction, $\sum F_x = 0$

$$R \cos 45 - S \cos 30 = 0 \text{ ---------(i)}$$

Resolving the forces along Y direction,

$$\text{For } \sum F_y = 0$$



$$R \sin 45 + S \sin 30 - 1000 = 0 \text{ ---------(ii)}$$

From eqns (i),

$$R \cos 45 = S. \cos 30$$

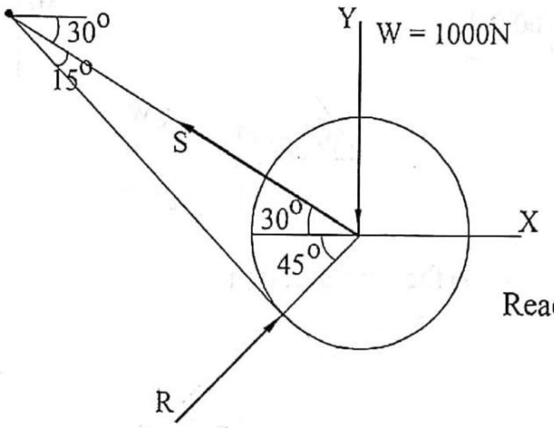
$$R \sin 45 = S \cos 30$$

From eqn (ii),

$$S \cos 30 + S \sin 30 - 1000 = 0$$

$$S (\cos 30 + \sin 30) = 1000.$$

$$\therefore S = \frac{1000}{(\cos 30 + \sin 30)} = 732.05 \text{ N}$$



$$\text{Reaction } R = \frac{S \cos 30}{\cos 45} = \frac{732.05 \times \cos 30}{\cos 45}$$

$$R = 896.57 \text{ N.}$$

Q: Two smooth circular cylinders each of weight 100 N and radius 15 cm are connected at their centres by a string AB of length 40 cm. and rest upon a horizontal plane as shown in fig.2.74. The cylinder above them has a weight 200 N and radius of 15 cm. Find the force in the string AB and the pressure produced in the floor at the points of contact D and E.

Solution.

Given: $W_A = W_B = 100$ N

$r_A = r_B = 15$ cm

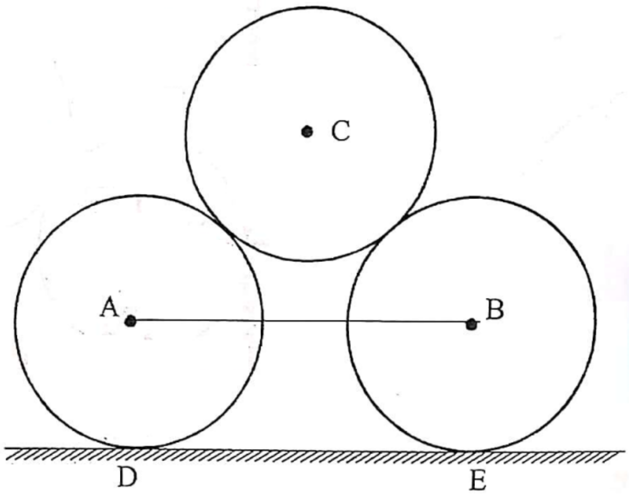
$W_C = 200$ N, $r_C = 15$ cm

$l = AB = 40$ cm.

$$\sin\theta = \frac{20}{30}$$

$$\theta = \sin^{-1}\frac{20}{30}$$

$$= 41.81^0$$



$$\angle CAB = \angle CBA = 90 - \theta$$

$$= 90 - 41.81$$

$$= 48.19^0$$

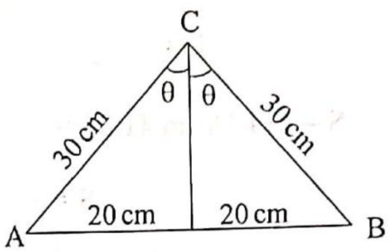
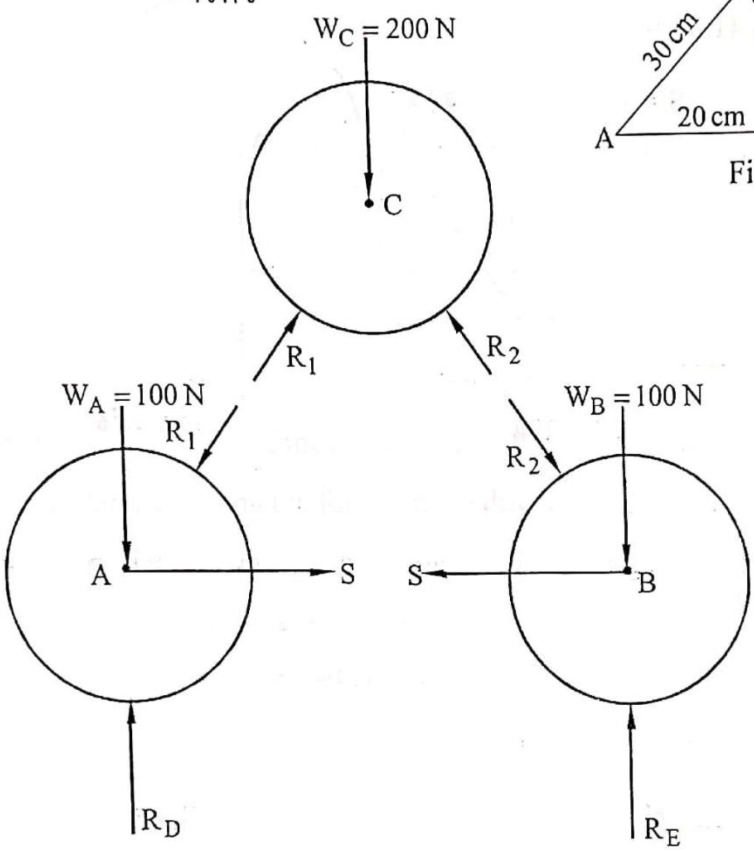
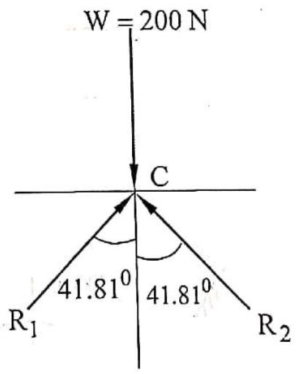


Fig. 2.75



Consider the equilibrium of upper cylinder;



For $\sum F_x = 0$

$$R_1 \sin 41.81 - R_2 \sin 41.81 = 0$$

$$R_1 = R_2$$

For $\sum F_y = 0$

$$R_1 \cos 41.81 + R_2 \cos 41.81 - 200 = 0$$

$$2\,R_1 \cos 41.81 = 200$$

$$R_1 = \frac{200}{2.\cos 41.81} = 134.16N$$

$$R_1 = R_2 = 134.16\text{ N}$$

Consider the equilibrium of lower cylinder,

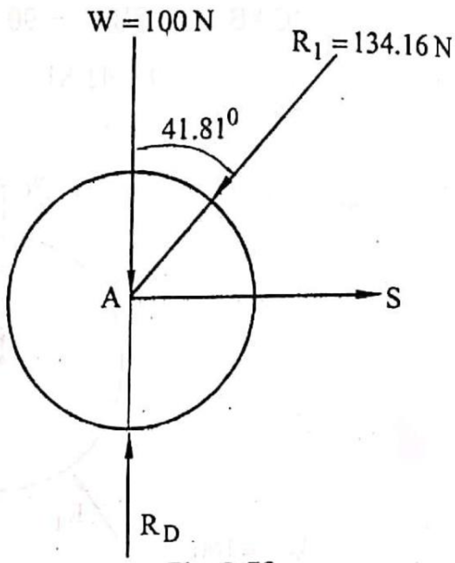


Fig. 2.78

For $\sum F_x = 0$

$$S - 134.16 \sin 41.81 = 0$$

$$S = 134.16 \times \sin 41.81$$

$$= 89.44\text{ N}$$

For $\sum F_y = 0$,

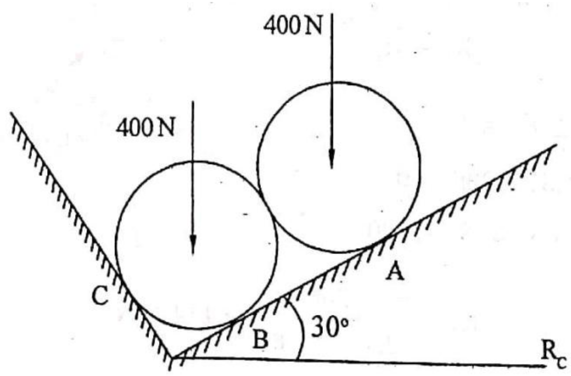
$$R_D - 100 - 134.16 \cos 41.81 = 0$$

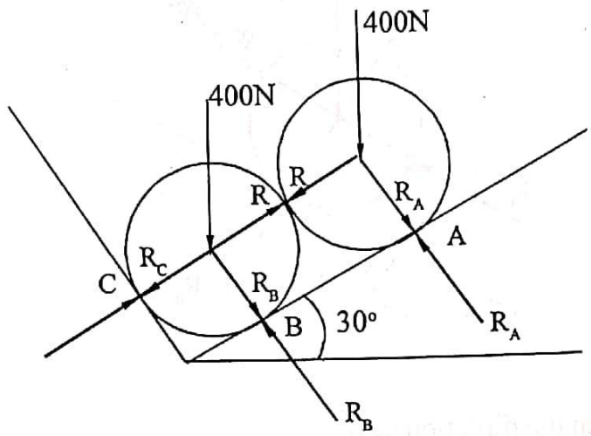
$$R_D = 100 + 134.16 \cos 41.81$$

$$= 200\text{ N}$$

Because of symmetrical arrangement of cylinders, the reaction at E will be same as that at D. Therefore, $R_E = R_D = 200$ N

Q: Two identical rollers each weighing 400N are supported by an inclined plane inclined at 30^0 to the horizontal and a wall, at right angles to the inclined plane as shown in the fig.2.79. Find the reactions at the supports A ,B and C: Neglect friction.



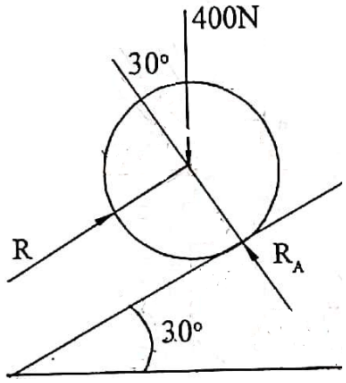


The freebody diagram of the two rollers is shown in fig.2.80.Consider the equilibrium of the upper roller. Resolving the forces in the direction of R_A,

$$R_A - 400\ \cos 30 = 0$$

$$R_A = 400\ \cos 30$$

$$= 346.41\text{N}.$$



Resolving the forces in the direction of R,

$$R - 400 \sin 30 = 0$$

$$R = 200\ \text{N}.$$

Consider the equilibrium of lower roller. Resolving the forces in the direction of R_B,

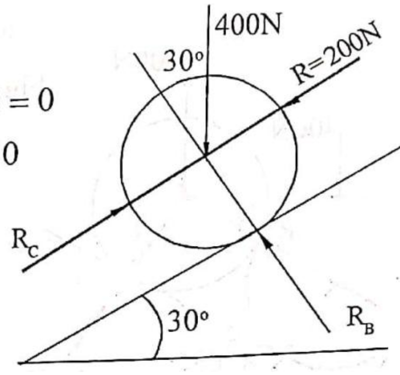
$$R_B - 400\ \cos 30 = 0$$

$$R_B = 400 \cos 30$$

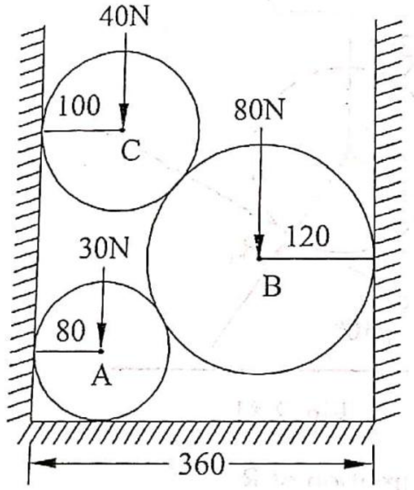
$$= 346.41\text{N}.$$

Resolving the forces in the direction of $R_{C,}$

$$R_C - 200 - 400 \sin 30 = 0$$
$$R_C = 200 + 400 \sin 30$$
$$= 400 \text{ N}$$



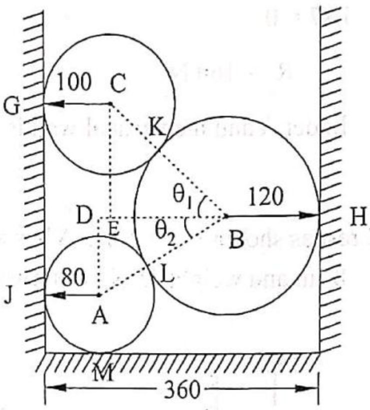
Q: Three cylinders are piled in a rectangular ditch as in fig.2.83. Neglecting friction, determine the reaction between cylinder A and vertical wall

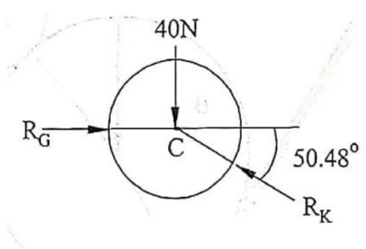


Solution:

$$\text{Cos}\,\theta_1 = \frac{BE}{BC} = \frac{360-120-100}{120+100} = \frac{140}{220}$$
$$\theta_1 = 50.48°$$

$$\text{Cos}\,\theta_2 = \frac{BD}{AB} = \frac{360-120-80}{120+80} = \frac{160}{200}$$





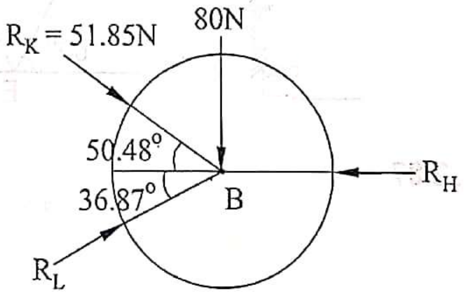
$\theta_2 = 36.87°$

Consider the free body diagram of cylinder C.

For $\sum F_V = 0, R_K \sin 50.48 - 40 = 0$

$$R_K = 51.48 \text{ N}$$

Consider the free body diagram of cylinder B.

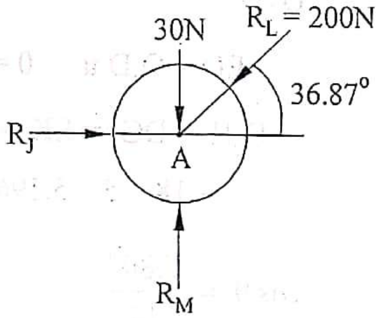


For $\sum F_V = 0,$

$R_L \sin 36.87 - 80 - 51.85 \sin 50.48 = 0$

$$R_L = \frac{51.85 \sin 50.48 + 80}{\sin 36.87}$$

$$= 200 \text{ N}$$



Consider the free body diagram of cylinder A.

For $\sum F_H = 0,$

$$R_J - 200 \cos 36.87 = 0$$

$$R_J = 160 \text{ N}$$

Reaction between the cylinder A and the vertical wall is 160 N.

Q: A uniform wheel 60 cm diameter weighing 1000 N rests against a rectangular obstacle 15 cm height as shown. Find the least force required, which when acting through the centre of the wheel will just turn the wheel over the corner A of the block. Also find the reaction of the block.

Solution

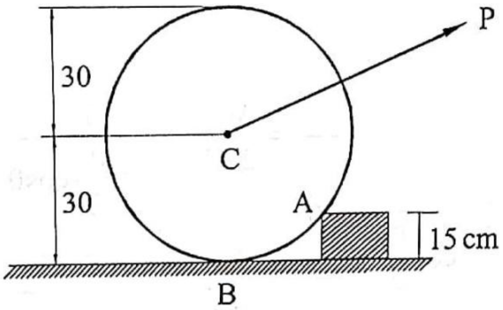
Let the inclination of the force P with AC be α. The inclination of line joining A and C with vertical, $\theta = 60^0$ (refer example 2.21). Taking moments of P and W about A,

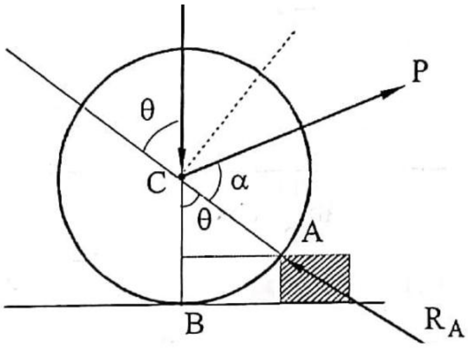
For $\sum M = 0$, $(P \sin \alpha) \times AC - W \times (AC \sin \theta) = 0$

$$P \sin \alpha = W \sin 60 = 1000 \sin 60$$

$$= 866 \text{ N}$$

W = 1000 N





$$P = \frac{866}{\sin\alpha}$$

For P to be minimum, sin α should be maximum . For this α should be 90°

$$P = 866 \text{ N}$$

Resolving the forces along AC, $R_A - P\cos\alpha - W\cos\theta = 0$

$$R_A = 866\cos 90 + 1000\cos 60$$

$$= 500 \text{ N}$$

Example 1.33

Forces of 15N, 20N, 25N, 35N and 45N act at an angular point of a regular hexagon towards the other angular points as shown in Fig.1.121. Calculate the moment of these forces about A, using Varignon's principle.

Solution.

Resolving the forces along X axis,

$$\Sigma F_x = 15 + 20 \cos 30 + 25 \cos 60 + 0 - 45 \cos 60 = 22.32 \text{ N}$$

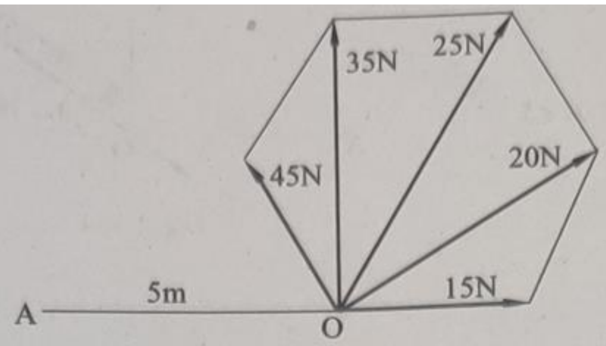


Fig. 1.121

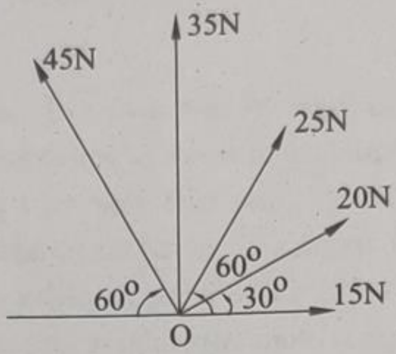


Fig.1.122

Resolving the forces along y axis,

$$\Sigma F_y = 0 + 20 \sin 30 + 25 \sin 60 + 35 + 45 \sin 60 = 105.62 \text{ N}$$

$$\text{Resultant, } R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$= \sqrt{22.32^2 + 105.62^2}$$

$$= 107.95 \text{ N}$$

Inclination of resultant with horizontal,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\frac{105.62}{22.32}$$

$$= 78.07^0$$

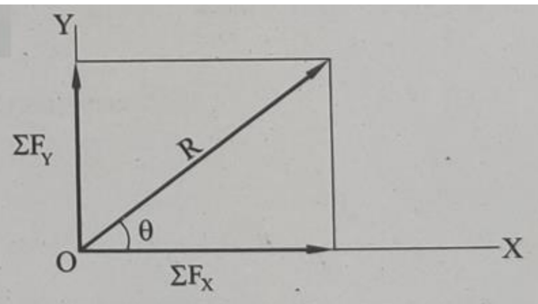


Fig. 1.123

According to Verignon's principle, sum of moment of the forces about A is equal to the moment of their resultant about A.

Moment of resultant R about A,

$$M_A = R\sin\theta \times 5$$

$$= 107.95 \times \sin 78.07 \times 5$$

$$= 528.09\text{Nm}.$$

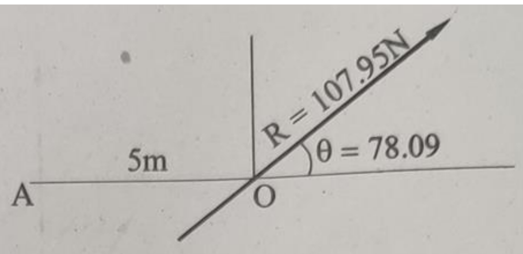


Fig. 1.124