Dynamics is that branch of mechanics which deals with the motion of bodies under the action of forces. Dynamics has two distinct parts, kinematics and kinetics. Kinematics is the study of motion of bodies without any reference to the cause of motion. When a car moves along a straight path with uniform velocity, uniform acceleration or with variable acceleration, determination of the distance travelled by the car in a given time is a problem of kinematics.

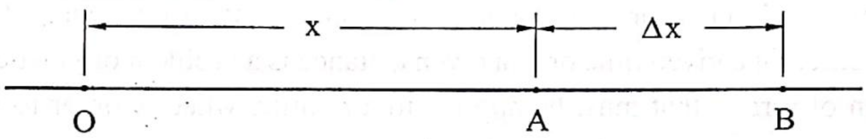
Kinetics deals with the relationship between forces and the resulting motion of the bodies on which they act. When a car moves along a straight path, the force required to bring the car to rest position in a given time or at a given distance is a problem of kinetics.

Displacement

The change of position of a particle with respect to a certain fixed reference point is termed as displacement.

Velocity

The rate of change of position of a particle with respect to time is called velocity.



$$V = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

Acceleration

The rate of change of velocity of a particle with respect to time is called acceleration.

$$a = \lim_{\Delta t \to 0}\left(\frac{\Delta V}{\Delta t}\right) = \frac{dV}{dt} = \frac{d^2x}{dt^2}$$

Q: A particle moves from A to B along a straight line AB at a constant speed of 12 m/s and then returns from B to A along BA at a constant speed of 6 m/s.

Find, (i) The average speed during the entire motion.

(ii) the average velocity during the entire motion.

Solution:

Let t_1 be the time taken by the particle to move from A to B and t_2 be the time taken by the particle to move from B to A.

$$\text{Then } t_1 = \frac{AB}{12} \text{ and } t_2 = \frac{AB}{6}$$

$$\text{Total time of travel} \qquad = t_1 + t_2 = \frac{AB}{12} + \frac{AB}{6}$$

$$= \frac{AB}{4}$$

$$\text{Total distance travelled} \qquad = 2\,AB$$

$$\therefore \text{Average speed} \qquad = \frac{2AB}{\left(\frac{AB}{4}\right)} = 8 \text{ m/s}$$

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{0}{\left(\frac{AB}{4}\right)} = 0$$

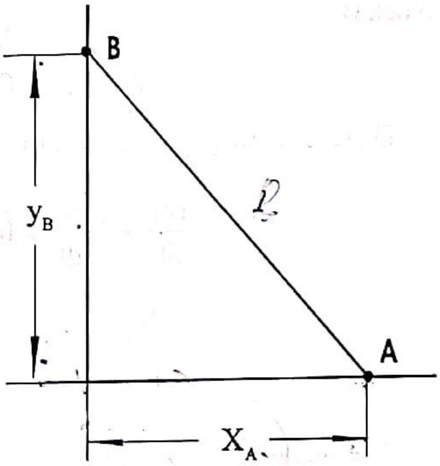
Q: A slender bar AB of length l has its ends A and B constrained A move in contact with a horizontal floor and a vertical wall. The bar starts from a vertical position and the end A is moved along the floor with constant velocity. Obtain the displacement,velocity and acceleration for the vertical motion of the end B.

Solution:

Since the end A is moving with uniform velocity, the distance moved in time t second is $v\times t$.

The displacement y of the end B is given by $y_B=\sqrt{l^2-x_A^2}$

$y_B=\sqrt{l^2-(vt)^2}$. This is the displacement-time equation for the motion of end B of the bar.



Differentiating with respect to time, we will get the expression for vertical velocity of y.

$$\frac{dy_B}{dt}=\frac{d}{dt}\left[l^2-\left(v^2t^2\right)\right]^{\frac{1}{2}}$$

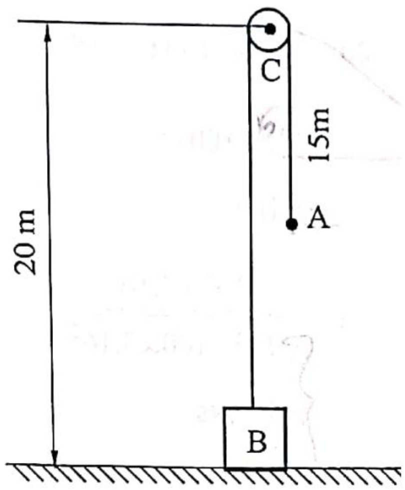
$$v_B=-\frac{v^2t}{\sqrt{l^2-v^2t^2}}$$

Differentiating this expression for velocity of end B of the bar with respect to time,

$$\frac{dv_B}{dt}=\frac{d}{dt}\left[-\frac{v^2t}{\sqrt{l^2-v^2t^2}}\right]$$

$$a_B=-\frac{v^2t^2}{\left[l^2-v^2t^2\right]^{\frac{3}{2}}}$$

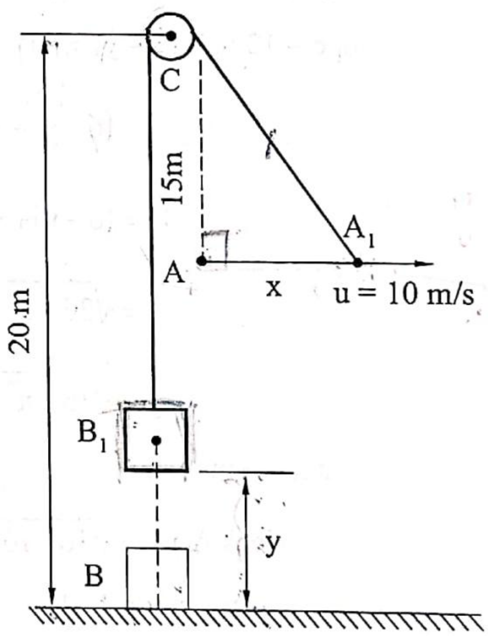
Q: A rope AB, 35 m long passes over a very small pulley as shown in fig 9.5 and a block is attached at B and the end A is moved horizontally with a velocity 10 m/s. Find the time required for the block B to reach the pulley. Also find the velocity of block B when it reaches pulley C.



Solution:

Length of rope = 35 m

Let y be the vertical displacement of block B in t seconds, then the horizontal displacement of end A, $x = AA_1 = u \times t = 10 \times t$



Length of rope $= (20 - y) + A_1C$

$$35 = (20-y)+\sqrt{15^2+x^2}$$

$$35 = (20-y)+\sqrt{15^2+(10\,t)^2}$$

$$35 = (20-y)+\sqrt{225+100\,t^2}$$

$$y = \sqrt{225+100\,t^2}+20-35$$

$$y = \sqrt{225+100\,t^2}-15$$

Differentiating with respect to time,

$$\frac{dy}{dt} = \frac{1}{2}\left[\left(225+100\,t^2\right)\right]^{\frac{1}{2}-1} \times 200\,t$$

$$= \frac{100\,t}{\sqrt{225+100\,t^2}}$$

$$v_B = \frac{100t}{\sqrt{225+100t^2}}$$

When the block is at C, y = 20 m

$$20 = \sqrt{225+100\,t^2} - 15$$

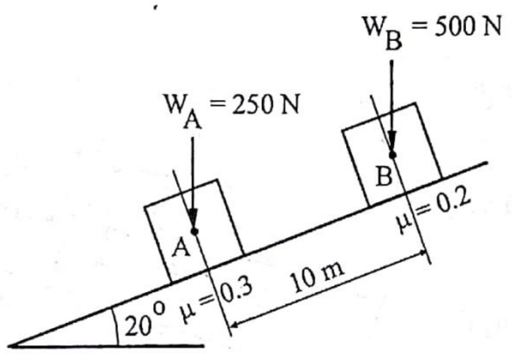
$$35^2 = 225+100\,t^2$$

$$t = 3.16 \text{ s}$$

$$v_B = \frac{100 \times 3.16}{\sqrt{225+100\times 3.16^2}}$$

$$= 9.03 \text{ m/s}$$

Q: Two blocks A and B are held stationary 10 m apart on a 20° incline as shown in fig.9.13. The coefficient of friction between the plane and block A is 0.3 while it is 0.2 between the plane and block B. If the blocks are released simultaneously, calculate the time taken and the distance travelled by each block before they are at the verge of collision.



Solution.

Consider the motion of block A, Net force = mass × acceleration.(Refer fig 9.14)

$$m_A g \sin\theta - \mu R_{NA} = m_A \times a_A$$

$$m_A g \sin\theta - \mu m_A g \cos\theta = m_A \times a_A$$

$$250 \sin 20 - 0.3\, R_{NA} = \frac{250}{9.81} \times a_A$$

$$250 \sin 20 - 0.3 \times 250 \times \cos 20 = \frac{250}{9.81} \times a_A$$

$$a_A = 0.59 \text{ m/s}^2$$

Consider the motion of block B.

Net force = mass × acceleration

Net force = mass × acceleration

$$m_B g \sin\theta - \mu R_{NB} = m_B \times a_B$$

$$m_B g \sin\theta - \mu m_B g \cos\theta = m_B \times a_B$$

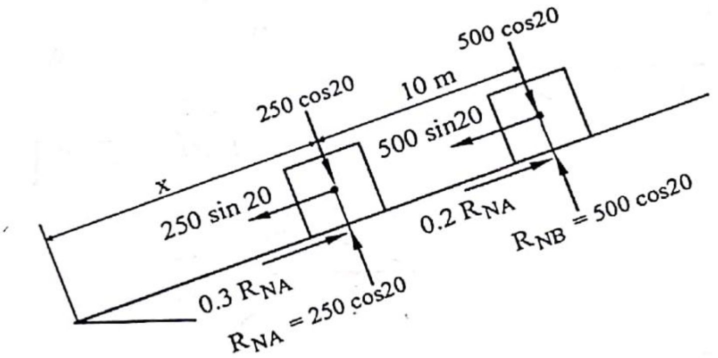
$$500 \sin 20 - 0.2 \times 500 \cos 20 = \frac{500}{9.81} \times a_B$$

$$a_B = 1.51 \text{ m/s}^2$$

Let x be the distance travelled by block A in t seconds, then the distance travelled by block B in the same t second will be (10+x)

$$S_A = x$$

$$= u_A t + \frac{1}{2} a_A t^2$$



$$x = 0 + \frac{1}{2} \times 0.59 \times$$

$$10 + x = 0 + \frac{1}{2} \times a_B t^2$$

$$= \frac{1}{2} \times 1.51 \times t^2$$

$$10 + x - x = \frac{1}{2} \times 1.51 \times t^2 - \frac{1}{2} \times 0.59 \times t^2$$

$$10 = 0.46\ t^2$$

$$t = 4.66\ s.$$

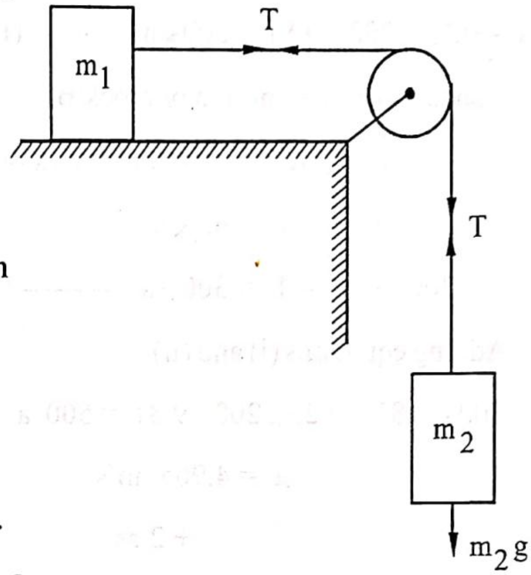
$$x = \frac{1}{2} \times 0.59 \times 4.66^2$$

$$= \frac{1}{2} \times 0.59 \times 4.66^2$$

$$= 6.41\ m.$$

Q: A mass of 60 kg lies on a smooth horizontal table. It is connected to a fine string passing over a smooth guide pulley on the edge of the table to a mass 50 kg. hanging freely. Find the tension in the string and the acceleration of the system.

Solution.



$m_1 = 60$ kg $m_2 = 50$ kg.

Let T be the tension in the string.

Consider the horizontal motion of mass m_1

Net force = mass $\times$ acceleration

$$T = m_1 \times a_1 = m_1 \times a$$

$$T = 60 \times a \text{ ----------(i)}$$

Consider the vertical motion of mass m_2.

Net force = mass $\times$ acceleration.

$$m_2 g - T = m_2 \times a_2 = m_2\ a$$

$$50 \times 9.81 - T = 50 \times a \text{ --------------(ii)}$$

Substituting for T from equation (i),

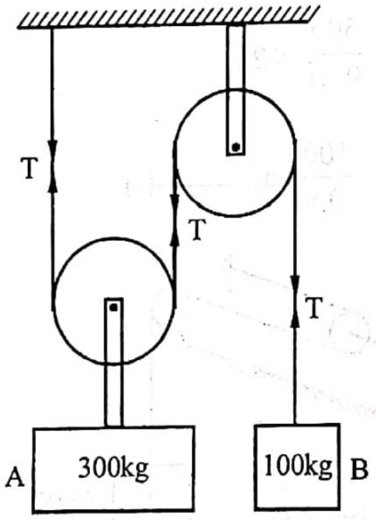
$$50 \times 9.81 - 60 \times a = 50\ a$$

$$110\ a = 50 \times 9.81$$

Acceleration, $a = 4.46\ m/s^2$

Tension, $T = 60 \times a = 60 \times 4.46 = 267.6$ N

Q: Determine the tension in the string and acceleration of the two bodies of mass 300 kg and 100 kg connected by a string and frictionless and weightless pulley as shown in fig 9.21.



Solution.

The downward displacement of 300 kg mass will be only half of the upward displacement of 100 kg mass.

The acceleration, $a_A = \frac{1}{2} a_B$

Consider the downward motion of body A

Net force = mass × acceleration

$$m_A g - 2T = m_A \times a_A$$

$$300 \times 9.81 - 2T = 300 \times a_A$$

$$150 \times 9.81 - T = 150\, a_A \text{-------------(i)}$$

Consider the upward motion of body B

$$\text{Net force} = \text{mass} \times \text{acceleration}$$

$$T - m_B g = m_B a_B$$

$$= m_B \times 2\, a_A$$

$$T - 100 \times 9.81 = 100 \times 2 \times a_A$$

$$T - 100 \times 9.81 = 200\, a_A \text{ -----------(ii)}$$

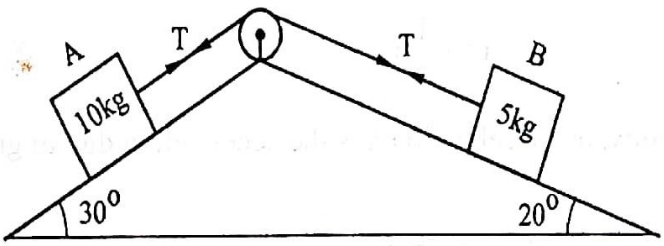
From equations (i) and (ii)

$$50 \times 9.81 = 350\, a_A$$

$$a_A = 1.4 \text{ m/s}^2$$

$$a_B = 2.8 \text{ m/s}^2$$

Q: Two smooth inclined planes whose inclinations with horizontal are 30^0 and 20^0 are placed back to back. Two bodies of mass 10 kg and 5 kg are placed on them and are connected by a string as shown in fig.9.22. Calculate the tension in the string and the acceleration of the bodies



Solution.

The downward displacement of body A will be equal to the upward displacement of body B, along the inclined planes.

$$a_A = a_B = a$$

Consider the motion of A,

$$\text{Net force} = m_A \times a$$

$$m_A g \sin\theta - T = m_A a_A$$

$$10 \times 9.81 \times 0.5 - T = 10 \times a$$

Consider the motion of body B

$$T - m_B\, g \sin 20 = m_B \times a$$

$$T - 5 \times 9.81 \times 0.34 = 5 \times a \text{ --------(ii)}$$

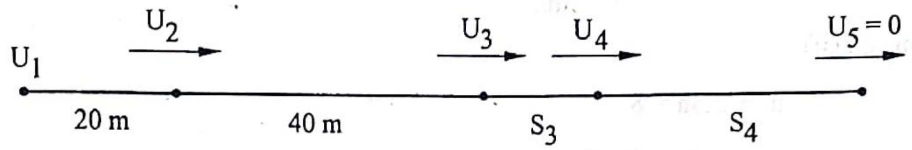
From equations (i) and (ii)

$$32.37 = 15\ a$$

$$a = 2.16\ m/s^2$$

Q: A particle under constant deceleration moves along a straight line and covers a distance of 20 m in first 2 seconds, 40 m in the next 5 seconds. Find the distance it covers in the next 3 seconds and the total distance covered before it comes to rest.

Solution.



$$S_1 = u_1 t_1 + \frac{1}{2} a t_1^2$$

$S_1 = 20$ m, $t_1 = 2$ s

$S_2 = 40$ m, $t_2 = 5$ s

$t_3 = 3$ s

$$20 = 2\,u_1 + \frac{1}{2} \times a \times 2^2$$

$$u_1 + a = 10 \text{ .... (i).}$$

$$S_2 = u_2 t_2 + \frac{1}{2} a\, t_2^2$$

$$40 = u_2 \times 5 + \frac{1}{2} a \times 5^2$$

$$u_2 + 2.5a = 8 \text{ .... (ii).}$$

$$u_2 = u_1 + a t_1$$

$$= u_1 + a \times 2.$$

Substituting this value of u_2 in eqn (ii)

$$u_1 + 2a + 2.5a = 8$$

$$u_1 + 4.5a = 8 \text{ .... (iii)}$$

Subtracting eqn.(i) from eqn.(iii).

$$3.5a = -2$$

$$a = \frac{-2}{3.5}$$

$$= -0.57\ m/s^2$$

$$u_1 + a = 10$$

$$u_1 = 10 - a = 10 - (-0.57)$$

$$= 10.57\ m/s$$

From eqn.(ii).

$$u_2 + 2.5a = 8$$

$$u_2 = 8 - 2.5a$$

$$= 8 - 2.5\,(-0.57)$$

$$= 9.43 \text{ m/s.}$$

$$u_3 = u_2 + at_2 = 9.43 + (-0.57) \times 5$$

$$= 6.58$$

$$u_3 = 6.58 \text{ m/s.}$$

$$s_3 = u_3\,t_3 + \frac{1}{2}\,at_3^2$$

$$= 6.58 \times 3 + \frac{1}{2}\,(-0.57) \times 3^2$$

$$= 17.18 \text{ m}$$

To find the total distance covered.

$$V^2 = u^2 + 2a\,S$$

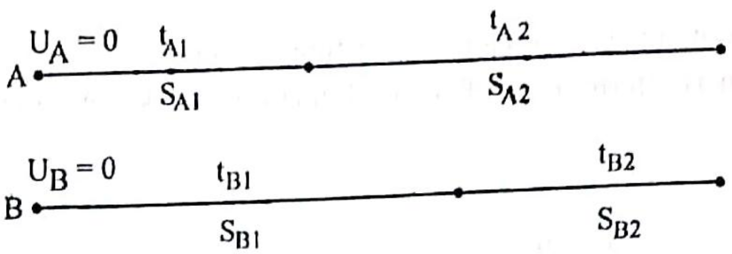
$$0 = u_1^2 + 2 \times (-0.57)\,S_{total}$$

$$0 = 10.57^2 - 2 \times 0.57 \times S_{total}$$

$$S_{total} = 98 \text{ m.}$$

Q: Two trains A and B leave the same station on parallel tracks. The train A starts with a uniform acceleration of $0.2 m/s^2$ and attains a maximum speed of 45 kmph. The train B leaves 60 seconds after with a uniform acceleration of $0.4 m/s^2$ to attain a maximum speed of 72 kmph When and where the train B will overtakes the train A.

Solution.



Given:

$U_A = 0, \quad a_A = 0.2 \text{ m/s}^2,$ $\qquad U_B = 0, \quad a_B = 0.4 \text{ m/s}^2$

$V_A = 45\text{kmph} = 12.5 \text{ m/s.}$ $\qquad V_B = 72 \text{ kmph} = 20 \text{ m/s.}$

$t_B = (t_A - 60)$ s

Let S_{A1} and S_{A2} be the distance travelled by train A during acceleration and during uniform velocity periods of t_{A1} and t_{A2} respectively.

Let S_{B1} and S_{B2} be the distance travelled by train B during acceleration and during uniform velocity periods of t_{B1} and t_{B2} respectively.

When the train B overtakes the train A, the total distance travelled by the trains A and B will be the same.

Therefore $\quad S_{A1} + S_{A2} = S_{B1} + S_{B2}$

The maximum velocity of train A,

$$V_A = u_A + a_A t_{A1}$$

$$12.5 = 0 + 0.2\, t_{A1}$$

$$t_{A1} = 62.5 \text{ s}$$

The maximum velocity of train B,

$$V_B = u_B + a_B t_{B1}$$

$$20 = 0 + 0.4\, t_{B1}$$

$$t_{B1} = 50 \text{ s}$$

$$S_{A1} + S_{A2} = S_{B1} + S_{B2}$$

$$u_A t_{A1} + \frac{1}{2} a_A t_{A1}^2 + V_A \times t_{A2} = u_B t_{B1} + \frac{1}{2} a_B t_{B1}^2 + V_B \times t_{B2}$$

$$0 + \frac{1}{2} \times 0.2 \times 62.5^2 + 12.5 \times t_{A2} = 0 + \frac{1}{2} \times 0.4 \times 50^2 + 20 \times t_{B2}$$

$$390.625 + 12.5 \times t_{A2} = 500 + 20\, t_{B2} \quad \text{............(i)}$$

$$t_B = t_A - 60 \text{ (given)}$$

$$t_{B1} + t_{B2} = t_{A1} + t_{A2} - 60$$

$$50 + t_{B2} = 62.5 + t_{A2} - 60$$

Therefore $t_{A2} = t_{B2} + 50 - 62.5 + 60$

$$= t_{B2} + 47.5.$$

Substituting this value of t_{A2} in eqn.(i).

$$390.625 + 12.5\,(t_{B2} + 47.5) = 500 + 20\, t_{B2}.$$

$$t_{B2} = 64.58 \text{ s}.$$

Time taken by train B to overtake train A,

$$t_B = t_{B1} + t_{B2}$$
$$= 50 + 64.58$$
$$= 114.58 \text{ s.}$$

When the train B overtakes train A, the time of travel of train A,

$$t_A = t_B + 60 = 114.58 + 60 = 174.58 \text{ s}$$
$$S_A = S_B = S_{A1} + S_{A2}$$
$$= 390.625 + 12.5\, t_{A2} \text{ [from eqn. (i)]}$$
$$= 390.625 + 12.5 \times (64.58 + 47.5)$$
$$= 1791.625 \text{ m.}$$

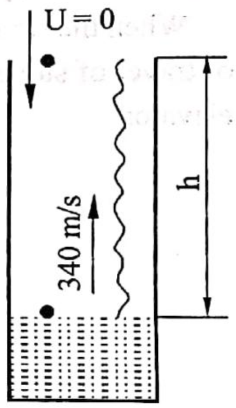
The train B will overtake train A at a distance 1791.625m from the station after 174.58 s from the start of train A or after 114.58 s from the start of train B.

Q: A stone, dropped into a well, is heared to strike the water after 2 seconds. Find the depth of the well, if the velocity of sound is 340 m/s.

Solution:

Velocity of sound, $V_s = 340$ m/s.

Let h be the depth of well and t_1 is the time taken by the stone to reach the bottom of well and t_2 is the time taken by the sound to reach the top of well.



$$t_1 + t_2 = 2\text{s}$$
$$h = \text{velocity of sound} \times \text{time.}$$
$$= 340 \times t_2 = 340\,(2 - t_1) \text{.......(i)}$$
$$h = u\,t + \frac{1}{2}\,g\,t^2$$
$$h = 0 + \frac{1}{2} \times 9.81 \times t_1^2 \text{.............(ii)}$$

From eqns. (i) and (ii)

$$340\,(2 - t_1) = \frac{1}{2} \times 9.81 \times t_1^2$$

$$69.32\,(2 - t_1) = t_1^2$$

$$t_1^2 + 69.32\,t_1 - 138.64 = 0$$

$$t_1 = 1.95\ s$$

$$t_2 = 2 - t_1 = 2 - 1.95 = 0.05\ s$$

Therefore $h = 340 \times t_2$

$$= 340 \times 0.05$$

$$= 17\ m$$

◆ D'Alembert's principle in rectilinear motion.

D' Alembert's principle is an application of Newton's second law to a moving body. A problem in dynamics can be converted into a static equilibrium problem using D'Alembert's principle. Newton's law of motion $F = ma$ can be written as $F - ma = 0$ the term $(-ma)$ is called inertia force. According to Newton's first law of motion, a body continues to be in the state of rest or of uniform motion in a straight line unless acted by an external force. Thus every body has a tendency to continue in its state of rest or of uniform motion. This tendency is called inertia. The magnitude of inertia force is equal to the product of the mass and acceleration and it acts in a direction opposite to the direction of acceleration. $F = ma$ can be written as $F - ma = 0$, or $F + (-ma) = 0$. The statement of the above equation is known D'Alembert's principle which states that the resultant of a system of force acting on a body in motion is in dynamic equilibrium with the inertia force.

Q: A system of weights connected by strings passing over pulleys A and B is shown in fig.9.44. Find the acceleration of the three weights P, Q and R. Using D'Alembert's principle.

Solution.

Let the downward acceleration of weight P be a. Then the upward acceleration of pulley B is a. Let the downward acceleration of weight Q be a_1 with respect to pulley B. Then upward acceleration of weight R is a_1

Absolute acceleration of weight Q is $a_1 - a$

Absolute acceleration of weight R is $a_1 + a$

Consider the downward motion of weight P.

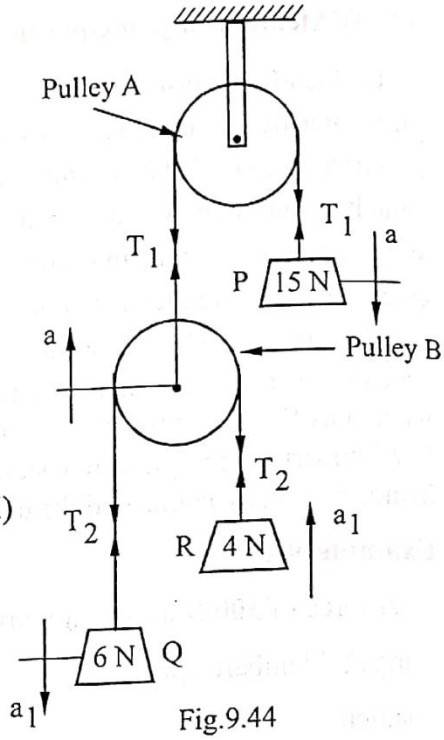


Fig.9.44

$$F + (-ma) = 0$$

$$15 - T_1 - \frac{15}{9.81} \times a = 0$$

$$T_1 = 15 - 1.53\ a \text{ -----(i)}$$

Consider the downward motion of weight Q.

$$F + (-ma) = 0$$

$$6 - T_2 - \frac{6}{9.81} \times (a_1 - a) = 0$$

$$T_2 = 6 - 0.61(a_1 - a) \text{ ---(ii)}$$

Consider the upward motion of weight R

$$F + (-ma) = 0$$

$$T_2 - 4 - \frac{4}{9.81}\ (a_1 + a) = 0$$

$$T_2 = 4 + 0.41\ (a_1 + a) \text{ -----(iii)}$$

Equating eqns (ii) and (iii)

$$T_2 = 6 - 0.61\ (a_1 - a) = 4 + 0.41\ (a_1 + a)$$

$$1.02\ a_1 - 0.2a = 2$$

$$a_1 - 0.196\ a = 1.96 \text{ -----(iv)}$$

Consider the motion of the weightless pulley B.

$$F + (-ma) = 0$$

$$F = 0$$

$$2\ T_2 - T_1 = 0$$

$$T_1 = 2\ T_2$$

Form eqn (i)

$$2\ T_2 = 15 - 1.53\ a$$

$$T_2 = 7.5 - 0.765\ a \text{ ----------------(v)}$$

equating eqns (iii) and (v)

$$6 - 0.61\ (a_1 - a) = 7.5 - 0.765\ a$$

$$-a_1 + 2.25\ a = 2.46 \text{ -----------------(vi)}$$

adding equations (iv) and (vi),

$$2.054\,a = 4.42$$

$$a = 2.15\ m/s^2$$

From equn (iv)

$$a_1 - 0.196\,a = 1.96$$

$$a_1 = 1.96 + 0.196 \times 2.15$$

$$= 2.38\ m/s^2$$

$$\text{Acceleration of } P = a = 2.15\ m/s^2$$

$$\text{Acceleration of } Q = a_1 - a = 2.38 - 2.15$$

$$= 0.23\ m/s^2$$

$$\text{Acceleration of weight } R = a_1 + a = 2.38 + 2.15$$

$$= 4.53\ m/s^2$$

Motion of lift.

Consider the motion of a lift with acceleration in the downward direction. Let W be the weight of a man and R be the reaction of force applied by the man on the floor of the lift.

Net force is in the downward direction and hence the inertia force acts in the upward direction.

For dynamic equilibrium,

$$W - R - \frac{W}{g}a = 0$$

$$R = W - \frac{W}{g}a$$

$$= W\left[1 - \frac{a}{g}\right]$$

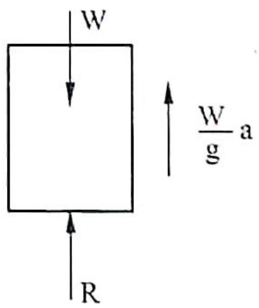


Fig.9.45

When the lift moves upward, the net force is upwards and hence the direction of inertia force is downward.

For dynamic equilibrium,

$$W + \frac{W}{g}a - R = 0$$

$$R = W\left[1 + \frac{a}{g}\right]$$

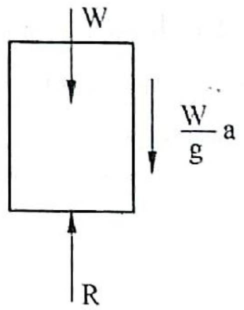


Fig.9.46

Q: A lift has an upward acceleration of 1.2 m/s². What force will a man weighing 750 N exerts on the floor of the lift ? What force would he exert if the lift had an acceleration of 1.2 m/s² downwards. What upward acceleration would cause his weight to exert a force of 900 N on the floor.

Solution.

Case (i), When the lift moves upward.

$$a = 1.2 \text{ m/s}^2$$

$$W = 750 \text{ N}$$

$$R = W\left[1 + \frac{a}{g}\right] = 750\left[1 + \frac{1.2}{9.81}\right]$$

$$= 841.74 \text{ N}$$

Case (ii) When the lift moves downward.

$$a = 1.2 \text{ m/s}^2$$

$$W = 750 \text{ N}$$

$$R = W\left[1 - \frac{a}{g}\right]$$

$$= 750\left[1 - \frac{1.2}{9.81}\right]$$

$$= 658.26 \text{ N}$$

Case (ii) the required acceleration for R = 900 N

When lift moves up

$W = 750$ N

$$R = 900 \text{ N}$$

$$R = W\left[1 + \frac{a}{g}\right]$$

$$900 = 750\left[1 + \frac{a}{9.8}\right]$$

$$a = 1.96 \text{ m/s}^2$$

Q: An elevator of total weight 5000 N starts to move upwards with a constant acceleration of 1 m/s². Find the force in the cable during the accelerated motion.

Also find the force at the floor of the elevator under the feet of a man weighing 600 N when the elevator moves up with a uniform retardation of 1 m/s².

Solution.

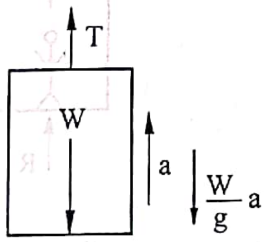
$$W = 5000\ N$$

$$a = 1\ m/s^2$$

Case (i) Elevator moves upwards with acceleration.

In this case the accelerating force is upwards and inertia force is downwards.

For dynamic equilibrium,



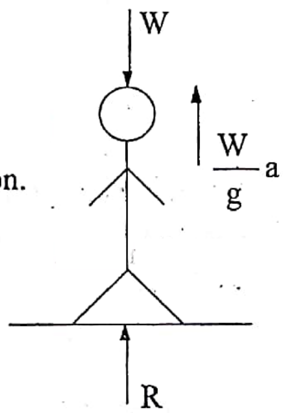
$$F + (-ma) = 0$$

$$T - W - \frac{W}{g}a = 0$$

$$T = W\left[1 + \frac{a}{g}\right]$$

$$= 5000\left[1 + \frac{1}{9.81}\right]$$

$$= 5509.68\ N$$



Case (ii) When the elevator moves up with uniform deceleration.

Consider the dynamic equilibrium of the man

Weight of man, $W = 600\ N$.

$$F + (-ma) = 0$$

Inertia force $\frac{W}{g} \times a$ acts upwards.

$$R - W + \frac{W}{g}a = 0$$

$$R = W\left[1 - \frac{a}{g}\right]$$

$$= 600\left[1 - \frac{1}{9.81}\right]$$

$$= 538.84\ N$$

Q: An elevator has an upward acceleration of $1\ m/s^2$. What pressure will be transmitted to the floor of the elevator by a man weighing 600 N travelling in the elevator ? What pressure will be transmitted if the elevator has a downward acceleration of $2\ m/s^2$?

Solution:

Upward motion.

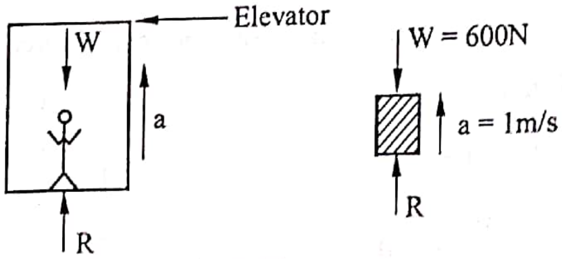


Fig.9.49

Let R be the reaction of pressure exerted by the man on the floor and W be the weight of man.

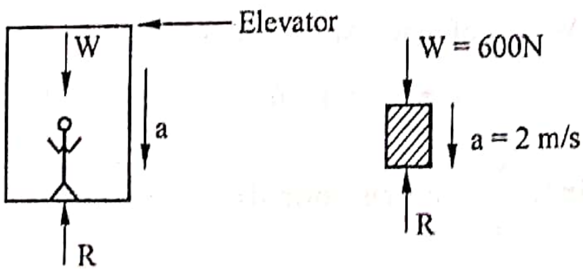
$$\text{Net force} = \text{mass} \times \text{acceleration}$$

$$R - W = m \times a$$

$$R - 600 = \frac{600}{9.81} \times 1$$

$$R = 661.16\ N.$$

Downward motion.



Q: A cylinder of radius 1 m rolls without slipping along a horizontal plane. Its centre has a uniform velocity of 20 m/s. Find the velocities of the points D and E on the circumference of the cylinder as shown in Fig. 5.13.

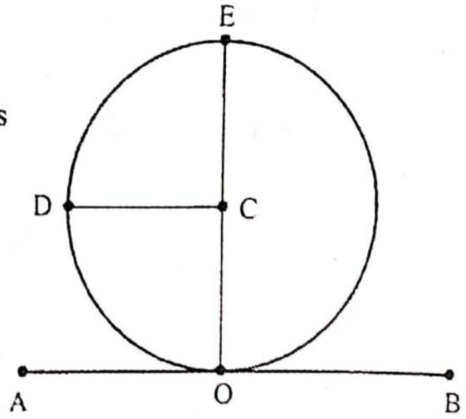
Solution.

$$r = 1\text{ m}$$

$$V_C = 20\text{ m/s}$$

$$V_C = r\,\omega$$

$$\therefore \omega = \frac{V_C}{r} = \frac{20}{1} = 20\text{ rad/s}$$



Velocity of point D

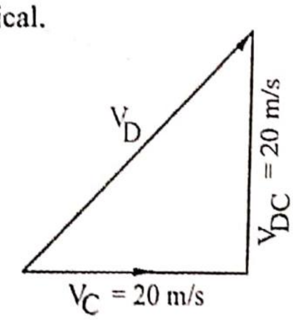
$$V_D = V_C + V_{DC}$$

$$V_C = 20\text{ m/s horizontal}$$

$$V_{DC} = \omega \times CD = 20 \times 1 = 20\text{ m/s, vertical.}$$

$$V_D = \sqrt{20^2 + 20^2}$$

$$= 28.28\text{ m/s}$$



Velocity of point E,

$$V_E = V_C + V_{EC}$$

$$V_C = 20\text{ m/s horizontal}$$

$$V_{EC} = \omega \times CE = 20 \times 1 = 20\text{ m/s horizontal}$$

$$V_E = 20 + 20 = 40\text{ m/s.}$$

Q: A compound wheel rolls without slipping as shown in Fig. 5.18. The velocity of centre is 2 m/s. Find the velocities of points A and B

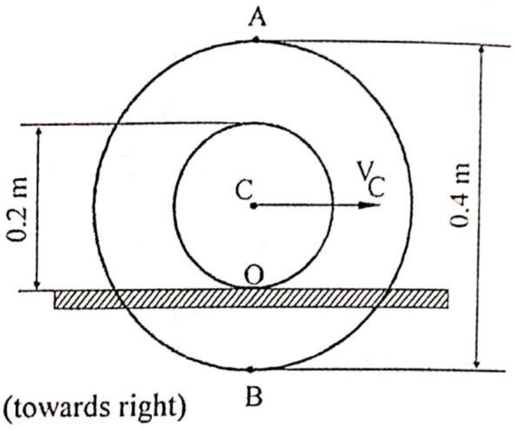
Solution

$$V_C = 2\text{ m/s horizontal}$$

$$V_C = \omega \times OC$$

$$2 = \omega \times 0.1$$

$$\omega = 20\text{ rad/s}$$



$$V_A = V_C + V_{AC}$$

$$V_{AC} = \omega \times AC\text{ horizontal (towards right)}$$

$= 20 \times 0.2 = 4$ m/s

$V_A = 2 + 4 = 6$ m/s

$V_B = V_C + V_{BC}$

$V_{BC} = \omega \times BC = 20 \times 0.2$

$= 4$ m/s, towards left

$V_B = V_C + V_{BC}$

$= 2 - 4$

$= -2$ m/s

$= 2$ m/s, towards left

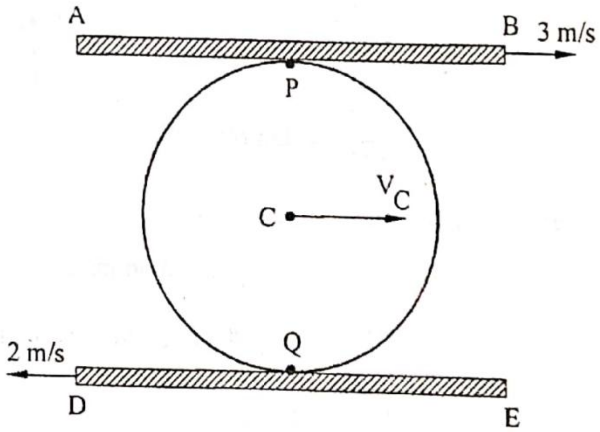
Q: A cylindrical roller is in contact at its at its top and bottom, with two conveyor belts AB and DE as shown in Fig.5.19. If the belts run at uniform speeds of 3 m/s and 2 m/s respectively, find the linear velocity and the angular velocity of the roller, when,

(i) the velocities are in the same direction and

(ii) the direction of velocities are opposite. The diameter of the roller is 40 cm.

Solution.

Case (i)



$r = 20$ cm $= 0.2$ m

$V_P = 3$ m/s

$V_P = V_C + V_{PC}$

$V_{PC} = r\omega$

$= 0.2\,\omega$ towards right

$3 = V_C + 0.2\,\omega$ ------(i)

$V_Q = 2$ m/s

$V_Q = V_C + V_{QC}$

$V_{QC} = r\omega$

$= 0.2\,\omega$ towards left

$2 = V_C - 0.2\,\omega$ ----(ii)

adding (i) and (ii)

$$5 = 2V_c$$
$$V_c = 2.5 \text{ m/s}$$
$$3 = V_c + 0.2\,\omega$$
$$3 = 2.5 + 0.2\,\omega$$

$$\omega = 2.5 \text{ rad/s}$$

Case (ii) $V_P = V_C + V_{PC}$

$$= V_c + r\,\omega$$
$$3 = V_c + 0.2\,\omega \text{ ------------------(i)}$$

$$V_Q = V_C + V_{QC}$$
$$-2 = V_c - 0.2\,\omega \text{ ------------------(ii)}$$

From eqns (i) and (ii)

$$1 = 2V_c$$
$$V_c = 0.5 \text{ m/s}$$
$$3 = V_c + 0.2\,\omega$$
$$3 = 0.5 + 0.2\,\omega$$
$$\omega = 12.5 \text{ rad/s}$$

Q: A bar PQ of length 1 m has its ends P and Q constrained to move horizontally and vertically as shown in Fig. 5.20. The end P moves with constant velocity of 5 m/s horizontally.

Find,

(i) the angular velocity of the bar

(ii) the velocity of the end Q and

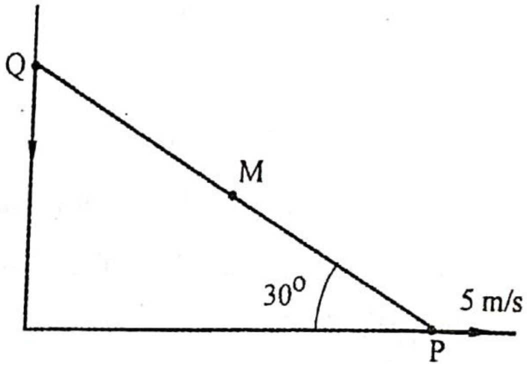
(iii) the velocity of the mid point M of the bar at the instant when the bar makes an angle of 30^0 with the horizontal .

$V_P = 5$ m/s horizontal

$$V_Q = V_P + V_{QP}$$

$$V_{QP} = V_Q - V_P$$

$$\frac{V_Q}{V_P} = \tan 60$$



Velocity of end Q, $V_Q = V_P \tan 60$

$$= 5 \tan 60 = 8.66 \text{ m/s.}$$

$V_{QP} = \omega \times PQ$, is $\perp^r$ to PQ, i.e, inclined 60° with horizontal.

$$\sqrt{5^2 + 8.66^2} = \omega \times 1$$

Angular velocity of bar, $\omega = 10$ rad/s

$$V_M = V_P + V_{MP}$$

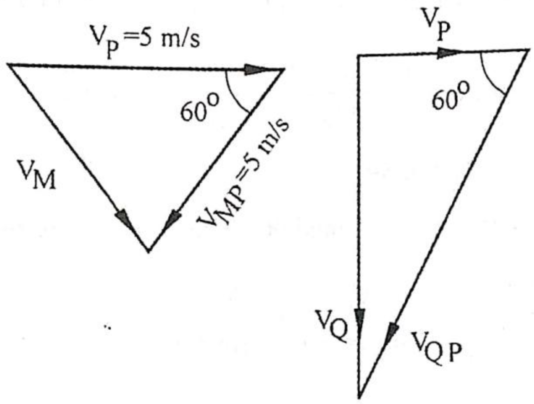
$$= 5 + \omega \times MP$$

$$= 5 + 10 \times 0.5$$

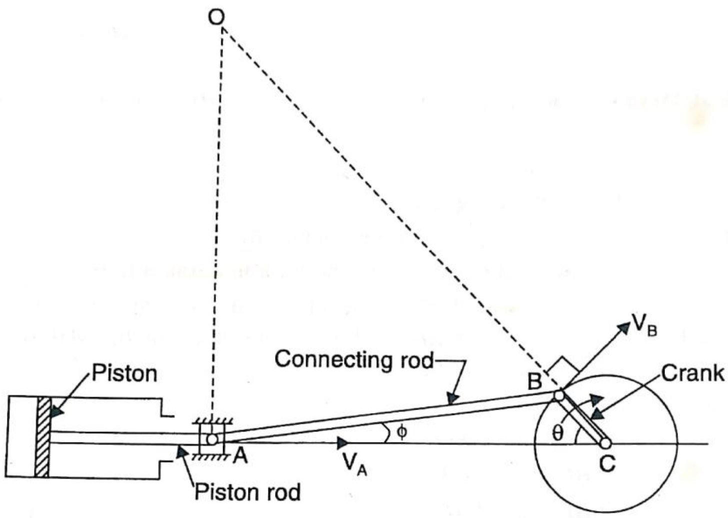
$= 5 + 5$ (vector sum) Velocity of mid point,

$$V_M = \sqrt{5^2 + 5^2 - 2 \times 5 \times 5 \cos 60}$$

$= 5$ m/s



◆ Connecting Rod Mechanism of Piston



$$V_A = \omega\,[L \sin\phi + r\cos\theta\tan\phi]$$

Q: *The crank of a reciprocating engine is rotating at 210 r.p.m. The lengths of the crank and connecting rod are 20 cm and 100 cm respectively. Find the velocity of the point A (i.e., velocity of piston), when crank has turned through an angle of 45° with the horizontal as shown in Fig. 13.12.*

Sol. Given :

Rotation of crank, $N = 120$ r.p.m.

$\therefore$ Angular velocity of crank, $\omega = \dfrac{2\pi N}{60} = 2 \times \dfrac{22}{7} \times \dfrac{210}{60} = 22$ rad/s.

Crank length or radius, $r = 20\,\text{cm} = 0.20\,\text{m}$

Length of connecting rod, $L = 100\,\text{cm} = 1.0\,\text{m}$

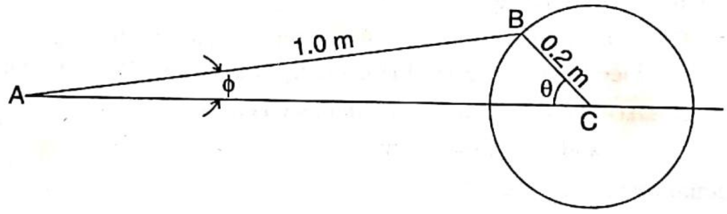
Angle turned by crank, $\theta = 45°$.

1st Method. The velocity of the point A (*i.e.,* V_A) can be calculated analytically by using equation (13.14).

Let us first calculate the angle ϕ.

Applying the sine rule for the triangle ABC shown in Fig. 13.12

$$\frac{BC}{\sin\phi} = \frac{AB}{\sin\theta}$$



$$\therefore \quad \sin\phi = \frac{BC\sin\theta}{AB} = \frac{0.2}{1.0} \times \sin 45° = \frac{0.2}{1.0} \times 0.7071 = 0.1414$$

$$\therefore \quad \phi = \sin^{-1} 0.1414 = 8.13°.$$

Using equation (13.14), we get

$$V_A = \omega\,(L\sin\phi + r\cos\theta\tan\phi)$$
$$= 22\,(1 \times 0.1414 + 0.2 \times \cos 45° \times \tan 8.13°) \qquad (\because \ \sin\phi = 0.1414)$$
$$= 22\,(0.1414 + 0.2 \times 0.7071 \times 0.1428)$$
$$= 22\,(0.1414 + 0.0202) = \mathbf{3.555\ m/s}. \quad \textbf{Ans.}$$

Projectiles

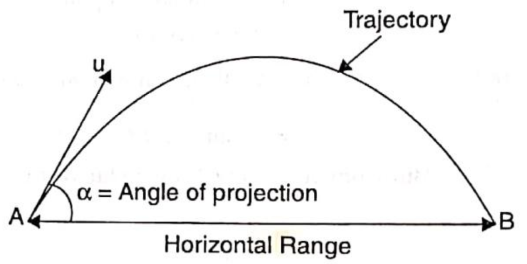
When a particle is projected upwards at a certain angle (but not vertical), the particle traces some path in the air and falls on the ground at a point, other than the point of projection. The path traced by the particle in air is known as *trajectory* of the particle whereas the particle is called a *projectile.* The path traced by the particle is parabolic.*

TERMS USED WITH THE PROJECTILES

The following terms are generally used with the projectiles :

1. Velocity of projection.
2. Angle of projection.
3. Time of flight.
4. Horizontal range.

They are shown in Fig. 14.1.



14.2.1. Velocity of projection. The velocity, with which a projectile is projected into space, is called the velocity of projection. This will be denoted by the symbol u.

14.2.2. Angle of projection. This is the angle, with the horizontal, at which a projectile is projected. This will be denoted by 'α'.

14.2.3. Time of flight. It is total time taken by a projectile for which the projectile remains in space. Or this is the interval of time since the projectile is projected and hits the ground again. This will be denoted by 'T'.

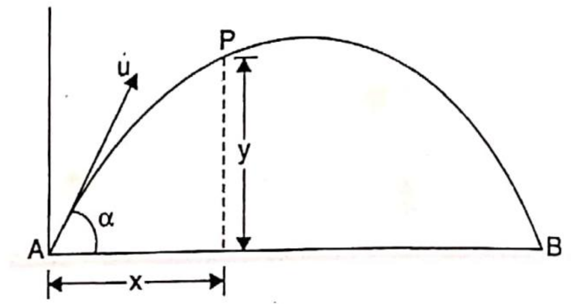
14.2.4. Horizontal range. The horizontal distance, between the point of projection and the point where projectile strikes the ground, is called horizontal range. This will be denoted by 'R'.

EQUATION FOR THE PATH OF A PROJECTILE

Let a particle is projected upwards at an angle α with the horizontal with an initial velocity 'u' m/s from a point A.

$\therefore$ α = Angle of projection and

u = Velocity of projection.



$$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

Maximum height attained by the projectile.

$$h_{max} = \frac{u^2 \sin^2 \alpha}{2g}$$

- **Time of Flight.**

$$T = \frac{2u \sin \alpha}{g}$$

- **Horizontal range of the projectile (R).**

$$R = \frac{u^2}{g} \times \sin 2\alpha$$

Q:

A projectile is fired with an initial velocity of 250 m/s at a target located at a horizontal distance of 4 km and vertical distance of 700 m above the gun. Determine the value of firing angle to hit the target. Neglect air resistance.

Sol. Given :

Initial velocity, $u = 250$ m/s

Horizontal distance, $x = 4$ km $= 4000$ m

Vertical distance, $y = 700$ m

Let α = angle of firing.

Using equation (14.3),

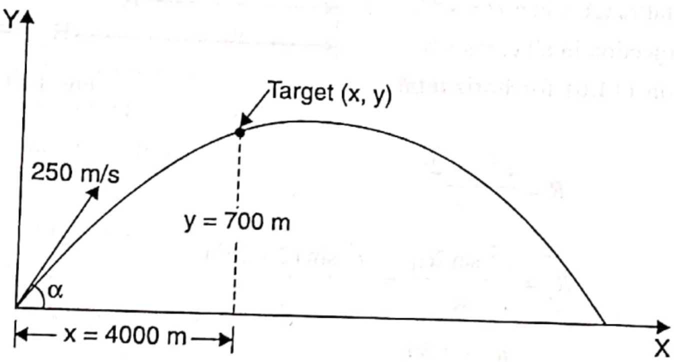


Fig. 14.3(*b*)

$$y = x \tan \alpha - \frac{gx^2}{2\mu^2 \cos^2 \alpha} \quad 700 = 4000 \tan \alpha - \frac{9.81 \times 4000^2}{2 \times 250^2 \times \cos^2 \alpha}$$

$$= 4000 \tan \alpha - \frac{1255.68}{\cos^2 \alpha} = 4000 \tan \alpha - 1255.68 \sec^2 \alpha$$

$$= 4000 \tan \alpha - 1255.68 (1 + \tan^2 \alpha) \qquad (\because \sec^2 \alpha = 1 + \tan^2 \alpha)$$

$$= 4000 \tan \alpha - 1255.68 - 1255.68 \tan^2 \alpha$$

or $$1255.68 \tan^2 \alpha - 4000 \tan \alpha + 1255.68 + 700 = 0$$

or $$1255.68 \tan^2 \alpha - 4000 \tan \alpha + 1955.68 = 0$$

or $$\tan^2 \alpha - \frac{4000}{1255.68} \tan \alpha + \frac{1955.68}{1255.68} = 0 \quad \tan^2 \alpha - 3.185 \tan \alpha + 1.557 = 0$$

The above equation is a quadratic equation in tan α.

Hence its roots are

$$\tan\alpha = \frac{3.185 \pm \sqrt{3.185^2 - 4 \times 1.557}}{2} = \frac{3.185 \pm 1.979}{2}$$

$$= \frac{3.185 + 1.979}{2} \quad \text{and} \quad \frac{3.185 - 1.979}{2} = 2.582 \quad \text{and} \quad 0.603$$

$\therefore$ $\alpha = \tan^{-1} 2.582$ and $\tan^{-1} 0.603 =$ **68.82° and 31.08°. Ans.**

Q: *A bullet is fired upwards at an angle of 30° to the horizontal from a point P on a hill and it strikes a target which is 80 m lower than B. The initial velocity of the bullet is 100 m/s. Calculate :*

(a) The maximum height to which the bullet will rise above the horizontal ;

(b) The actual velocity with which it will strike the target ; and

(c) The total time required for the flight of the bullet.

Neglect the resistance due to air.

Sol. Given :

Velocity of projection, $u = 100$ m/s

Angle of projection, $\alpha = 30°$

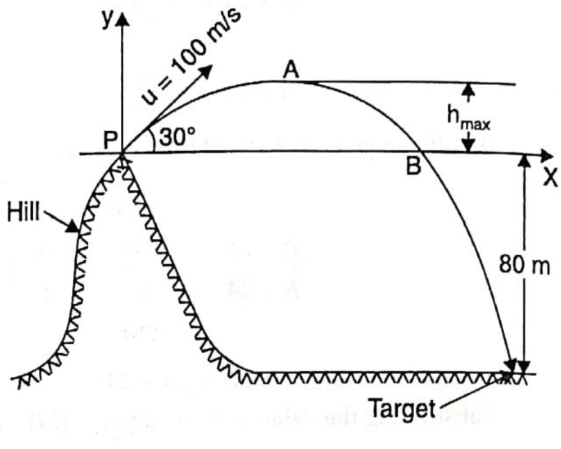
Vertical downward distance of the target from the point of projection = 80 m.

(*a*) The maximum height (h_{max}) attained by the bullet above the point of projection P is given by equation (14.4).

$$\therefore \quad h_{max} = \frac{u^2 \sin^2\alpha}{2g} = \frac{100^2 \times \sin^2 30°}{2 \times 9.81} = \frac{100 \times 100 \times \frac{1}{2} \times \frac{1}{2}}{2 \times 9.81} \quad (\because \sin 30° = \tfrac{1}{2})$$

= **127.42 m. Ans.**

(*b*) *Actual velocity with which the bullet will strike the target.*



In Fig. 14.5, point A is the highest point of the projectile. At the highest point the velocity in the vertical direction is zero. Consider the motion from point A to target.

Let v_1 = Vertical component of velocity at the highest point A = 0 m/s.

v_2 = Vertical component of velocity striking the target

S = Vertical distance between point A and target = 127.42 + 80 = 207.42 m

g = Acceleration due to gravity = 9.81 m/s².

Now using the relation,

(Final velocity)² – (Initial velocity)² = $2gS$

or $$v_2^2 - v_1^2 = 2 \times 9.81 \times 207.42$$

or $$v_2^2 - 0 = 2 \times 9.81 \times 207.42 \quad (\because v_1 = 0)$$

or $$v_2 = \sqrt{2 \times 9.81 \times 207.42} = 63.79 \text{ m/s.}$$

$\therefore$ Vertical component of velocity striking the target = 63.79 m/s.

The horizontal component of the velocity from the point of projection and during its flight remains constant as the resistance due to air is to be neglected.

But horizontal component of velocity at the point of projection = $u \cos \alpha$

$= 100 \times \cos 30° = 86.6$ m/s.

∴ Horizontal component of velocity striking the target = 86.6 m/s.

∴ Actual velocity with which the bullet will strike the target

= [(Vertical component striking the target)^2 + (Horizontal component striking the target)^2]$^{1/2}$

$= [63.79^2 + 86.6^2]^{1/2} = \sqrt{4069.164 + 7499.56} = $ **107.55 m/s. Ans.**

And the angle made with the horizontal by actual velocity striking the target is given by

$$\tan \theta = \frac{\text{Vertical component striking the target}}{\text{Horizontal component striking the target}} = \frac{63.79}{86.6} = .7366$$

or $\theta = \tan^{-1} .7366 = $ **36.37°. Ans.**

(c) *Total time required for the flight of the bullet*

Let t_1 = Time of flight from point of projection P to the highest point A

t_2 = Time of flight from point A to target.

Then total time, $t = t_1 + t_2$.

Consider the motion from point P to point A

Initial velocity in vertical direction at P

$= u \sin \alpha = 100 \times \sin 30° = 100 \times \frac{1}{2} = 50$ m/s.

Final velocity in vertical direction at $A = 0$

Acceleration due to gravity, $g = -9.81$ m/s^2

Time, $t = t_1$

Using the equation, final velocity = Initial velocity + gt

$$0 = 50 - 9.81 \times t_1 \qquad t_1 = \frac{50}{9.81} = 5.096 \text{ s.}$$

Now consider the motion from point A to target

Initial velocity in vertical direction at $A = 0$

Final velocity in vertical direction at target = 63.79

Acceleration due to gravity, $g = 9.81$

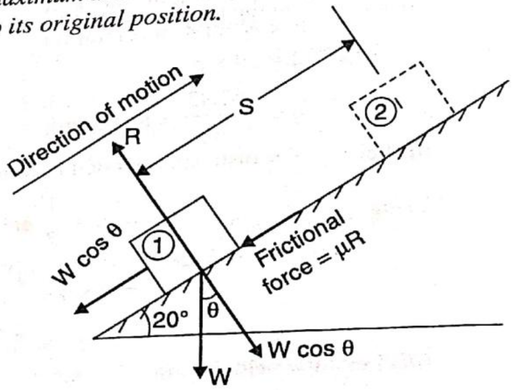
Time, $t = t_2$

∴ Using, final velocity = Initial velocity + $g \times t$

or
$$63.79 = 0 + 9.81 \times t_2 \qquad \therefore \quad t_2 = \frac{63.79}{9.81} = 6.50 \text{ s.}$$

∴ Total time of flight, $t = t_1 + t_2 = 5.096 + 6.50 = $ **11.596 s. Ans.**

Q: *A body weighing 20 N is projected up a 20° inclined plane with a velocity of 12 m/s, co-efficient of friction is $\mu = 0.15$. Find : (i) the maximum distance S, that the body will move up the inclined plane, (ii) velocity of the body when it returns to its original position.*



Sol. Given :

Weight, $W = 20$ N

Angle, $\theta = 20°$

Initial velocity, $u = 12$ m/s

Co-efficient of friction,

$\mu = 0.15$

Mass, $m = \dfrac{W}{g} = \dfrac{20}{9.81}$ kg.

Let S = Maximum distance moved by the body up the inclined plane.

(*i*) **First Case**

Body is moving up the inclined plane from position 1 to position 2 where velocity is zero.

At the maximum distance, the velocity of the body (*i.e.*, final velocity) will become zero.

∴ Initial velocity, $u = 12$ m/s

Final velocity, $v = 0$

Force of friction, $F = \mu R = \mu \times W \cos 20°$ $(\because \; R = W \cos 20°)$

$= 0.15 \times 20 \times \cos 20°$

Fig. 18.15 (*b*) shows the free-body diagram. The net force in the direction of motion is given by,

$$F = -W \sin\theta - \mu R$$

$$= -20 \times \sin 20° - 0.15 \times W \cos\theta \qquad (\because \; R = W\cos\theta)$$

$$= -20 \times 0.342 - 0.15 \times 20 \times \cos 20°$$

$$= -6.84 - 2.819 = -9.659 \text{ N}$$

*(Negative sign means the net force** is acting opposite to the direction of motion.)*

Using, net force = mass × acceleration

or $$-9.659 = \left(\frac{20}{9.81}\right) \times a$$

∴ $$a = \frac{-9.659 \times 9.81}{20} = -4.737 \text{ m/s}^2$$

Now using $v^2 - u^2 = 2aS$

or $0^2 - 12^2 = 2 \times (-4.737) \times S$ or $\dfrac{-144}{-2 \times 4.737} = S$

or $$S = \frac{144}{2 \times 4.737} = \textbf{15.19 m.} \quad \textbf{Ans.}$$

But change of K.E.

or -14

∴ $$S = \frac{-146.78}{9.659} = \textbf{15.196 m.} \quad \textbf{Ans.}$$

Q: *The tractive force, exerted by a railway car weighing 50 kN, is 2000 N. If the frictional resistance is 5 N per kN of the railway car's weight, determine the acceleration when the railway car is moving on a level track.*

Sol. Given :

Tractive force exerted by railway car,

$$F_1 = 2000 \text{ N}$$

Weight of car, $W = 50 \text{ kN} = 50 \times 1000 \text{ N}$

$\therefore$ Mass of car, $m = \dfrac{W}{g} = \dfrac{50 \times 1000}{9.81}$ kg

Frictional resistance, $F_2 = 5$ N per kN of car's weight

$= 5 \text{ N} \times \text{weight of car in kN} = 5 \times 50 \text{ N} = 250 \text{ N}.$

The tractive force is acting in the direction of motion, while frictional resistance is acting in opposite direction of motion.

$\therefore$ Net force in the direction of motion,

$$F = F_1 - F_2 = 2000 - 250 = 1750 \text{ N}.$$

As the net force is acting in the direction of motion, it will produce acceleration.

Let a = acceleration produced

Using equation (15.2), we have

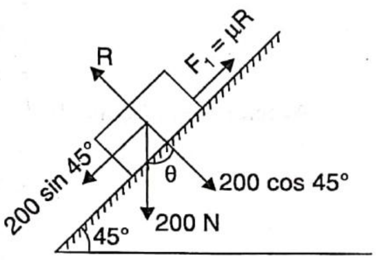
$$F = m \times a$$

or
$$1750 = \frac{50 \times 1000}{9.81} \times a \qquad \left(\because \; m = \frac{50 \times 1000}{9.81}\right)$$

$$\therefore \quad a = \frac{1750 \times 9.81}{50 \times 1000} = \mathbf{0.343 \ m/s^2}. \quad \textbf{Ans.}$$

◆ *A body of weight 200 N is initially stationary on a 45° inclined plane. What distance along the inclined plane must the body slide, before it reaches a speed of 2 m/s. The co-efficient of friction between the body and the plane = 0.1.*

Sol. Given :



Weight of body, $W = 200$ N

$\therefore$ Mass of body, $m = \dfrac{W}{g} = \dfrac{200}{9.81}$ kg

Angle of plane, $\theta = 45°$

Initial velocity, $u = 0$

Final velocity, $v = 2$ m/s

Co-efficient of friction, $\mu = 0.1$.

The acceleration of the body is given by equation (15.4) as

$$a = g[\sin\theta - \mu\cos\theta] = 9.81[\sin 45° - 0.1 \cos 45°]$$
$$= 9.81[0.707 - 0.1 \times .707] = 6.242 \text{ m/s}^2.$$

Now using the relation,

$$v^2 - u^2 = 2as \qquad \text{or} \qquad 2^2 - 0^2 = 2 \times 6.242 \times s$$

$$\therefore \quad s = \frac{2 \times 2}{2 \times 6.242} = 0.32 \text{ m} = \mathbf{32 \ cm.} \quad \textbf{Ans.}$$

Impulse and momentum

Principle of impulse and momentum is derived from Newton's second law, F = ma. This principle relates force, mass, velocity and time and is suitably used for solving problems where large forces act for a very small time.

If F is the resultant force acting on a body of mass m, then from Newton's second law,

$$F = ma$$

$$a = \frac{dV}{dt}$$

$$\therefore F = m \times \frac{dV}{dt}$$

$$= \frac{d}{dt}(mV)$$

The product of mass and velocity is called momentum. ie., the resultant force acting on a body is equal to the rate of change of momentum of the body.

The total change in the momentum of a body during a time interval is equal to the impulse of the force acting during the same interval of time.

$$Ft = m(V_2 - V_1)$$

Impulse = Final momentum – Initial momentum.

Unit of impulse is Ns.

Unit of momentum is

$$kg\ \frac{m}{s} = kg\ \frac{m}{s^2} \times s = N.s$$

Q: A ball of weight 0.6 N falls from a height of 15 m and rebounds to a height of 10 m. Find the impulse and the average force between the ball and the floor if the time during which they are in contact is 0.1 second.

Solution.

The velocity with which the ball strikes the floor,

$$V_1 = \sqrt{2gh} = \sqrt{2\,g \times 15}$$

$$= 17.16 \text{ m/s (downward)}$$

Velocity of rebound, $V_2 = \sqrt{2\ gh} = \sqrt{2\ g \times 10}$

$= 14$ m/s. (upward)

$$\text{Impulse} = m\ (V_2 - V_1)$$

$$= \frac{0.6}{9.81}[14 - (-17.16)]$$

$$= 1.91 \text{ Ns}$$

If F is the average force, then,

$$F \times t = m\ (V_2 - V_1)$$

$$F \times 0.1 = 1.91$$

$$F = 19.1 \text{ N}$$

Q: A body weighing 40 N drops freely from a height of 50 m and penetrates into the ground by 100 cm. Find the average resistance to penetration and the time of penetration.

Solution.

Velocity body when it just strikes the ground,

$$V = \sqrt{2gh} = \sqrt{2g \times 50}$$

$$= 31.32 \text{ m/s.}$$

Velocity of body after penetrating 100 cm $= 0$.

Using work energy principle.

Change in K.E = work done

Let R be the average resistance of penetration

$$\frac{1}{2} m\ (V_2^2 - V_1^2) = mgx - R \times x$$

$$\frac{1}{2} m \left(0^2 - 31.32^2\right) = 40 \times 1 - R \times 1$$

$$R = 40 + \frac{1}{2} \times \frac{40}{9.81} \times 31.32^2$$

$$= 2039.88 \text{ N}$$

Using impulse momentum equation

$$F \times t = m\,(V_2 - V_1)$$

F is the resultant force on the body during penetration.

$$F = mg - R$$

$$= 40 - 2039.88$$

$$= -1999.88 \text{ N}$$

$$-1999.88 \times t = \frac{40}{9.81}(0 - 31.32)$$

$$t = 0.064 \text{ s}$$

Q: A ball of mass 100 gm is bowled towards a batsman. The velocity of ball was 25 m/s horizontally just before the batsman hit it. After hitting, the ball went away with a velocity of 40 m/s at an angle of 40^0 with horizontal. Find the average impulsive force exerted by the bat on the ball if the impact lasts for 0.015 second.

Solution.

$$F \times t = m\,(V_2 - V_1)$$

The component equation in the X direction,

$$F_x\, t = m\,(V_{2x} - V_{1x}) \text{ and}$$

the component equation in the Y direction,

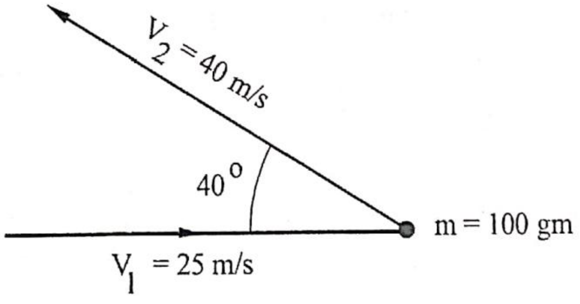
$$F_y\, t = m\,(V_{2y} - V_{1y})$$

$$F_x \times 0.015 = 0.1\ [40 \cos 40 - (-25)]$$

$$F_x = 370.95 \text{ N}$$

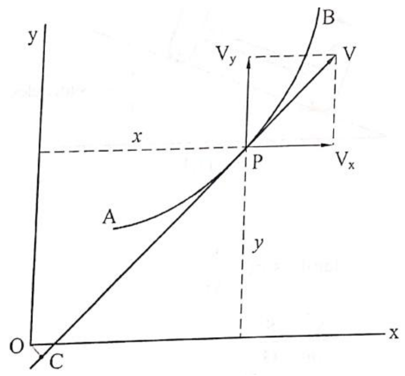
$$F_y \times 0.015 = 0.1\ [40 \sin 40 - 0]$$

$$F_y = 171.41 \text{ N}$$



Moment of momentum

The product of mass and velocity is called momentum. It is a vector in the same direction of velocity.



The moment of momentum,

$$H_0 = m\left[V_x y - V_y x\right]$$

$$= m \times V \times OC$$

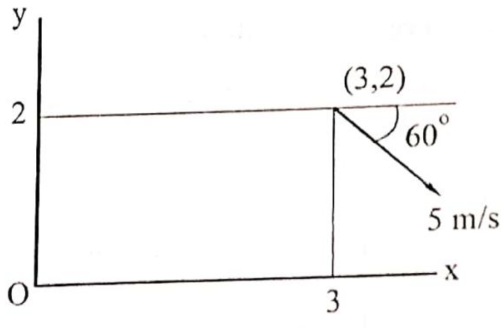
$$= mV_x \times y - n$$

The moment of force about O,

$$M_o = F \times OC = F_x \times y - F_y \times x \;= \frac{d}{dt}\left[m\left(V_x y - V_y x\right)\right] \;= \frac{d}{dt} H_o$$

Q: A particle of mass 1 kg is moving with a velocity of 5m/s as shown. The coordinates of the particle are (3,2). Find the angular momentum about the origin O.

Solution:



mass m = 1 kg

V = 5 m/s

$V_x = 5 \cos 60$

$V_y = 5 \sin 60$

x = 3 m

y = 2 m

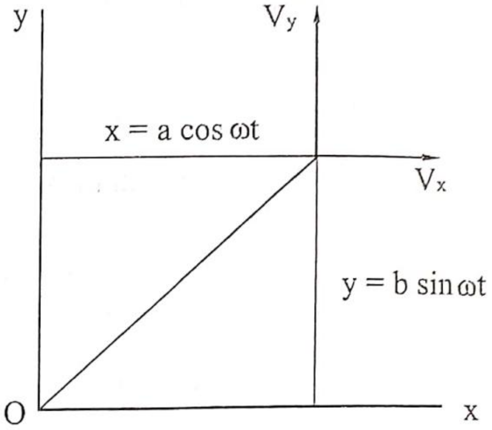
Angular momentum about O,

$$H_O = m\left[V_x \times y + V_y \times x\right] = 1\left[5\cos 60 \times 2 + 5\sin 60 \times 3\right] = 18 \text{ Nms}$$

Q: The motion of a particle of mass m in the x - y plane is defined by the equations $x = a \cos \omega t$ and $y = b \sin \omega t$. Where a, b and ω are constants. Calculate the moment of momentum of the particle with respect to the origin.

Solution:

$$\text{Moment of momentum} = m\left(V_x \times y - V_y \times x\right)$$



$$x = a \cos \omega t$$

$$V_x = -a\omega \sin \omega t$$

$$y = b \sin \omega t$$

$$V_y = b\,\omega \cos \omega t$$

$$\text{Moment of momentum} = m\left[\left(-a\omega \sin \omega t\right) \times b \sin \omega t - b\omega \cos \omega t \times a \cos \omega t\right]$$

$$= -m\,a\,b\omega\left[\sin^2 \omega t + \cos^2 \omega t\right]$$

$$= -m\,ab\,\omega$$

◆ **Work and energy in curvilinear motion**

Work done by the resultant force acting on the body,

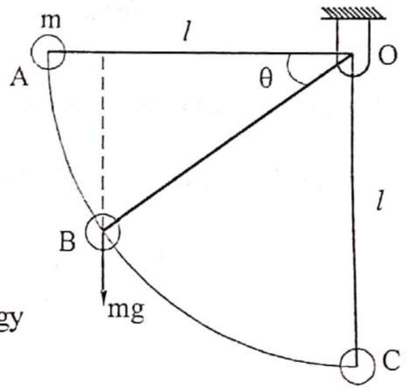
$$\int_1^2 F_t ds = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$$

Q: A simple pendulum is released from rest at A with the string horizontal and swings downward. Express the velocity of the bob as a function of the angle θ. Also obtain the expression for angular velocity of bob when, the string is in the vertical position.

Solution:

Work done when the pendulum swings from OA to OB is mg × vertical distance between A and B

$$= mg \times l \sin \theta$$



$$\text{Change in kinetic energy} = \frac{1}{2}mV_B^2 - \frac{1}{2}mV_A^2$$

$$= \frac{1}{2}mV_B^2$$

Equating the work done and change in kinetic energy

$$mgl \sin \theta = \frac{1}{2}mV_B^2$$

$$V_B = \sqrt{2gl\sin\theta}$$

When the string is vertical, $\theta = 90°$

$$\therefore V_c = \sqrt{2gl\sin 90}$$

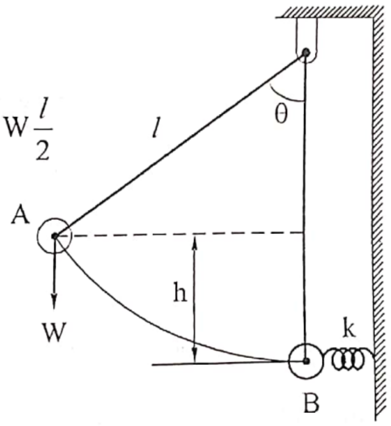
$$= \sqrt{2gl}$$

Angular velocity $\omega = \dfrac{V}{r} = \dfrac{\sqrt{2gl}}{l} = \sqrt{\dfrac{2g}{l}}$

Q: A simple pendulum of weight W and length l is released from rest at A as shown. It strikes a spring of stiffness k at B. Calculate the deflection of the spring.

Solution:

Work done by the simple pendulum when it swing from A to B is W × Vertical distance between A and B.



$$\text{Work done} = W(l - l\cos\theta)$$

$$= Wl(1-\cos 60) = W\frac{l}{2}$$

Let V be the velocity of bob at B.

Work done = Change in K.E

$$\frac{Wl}{2} = \left[\frac{1}{2}mV^2 - 0\right] = \frac{1}{2}\frac{W}{g}V^2$$

$$V^2 = l \times g$$

Let x be the compression of spring.

Work of compression of spring $= -\dfrac{1}{2}kx^2$

After compression, the velocity of bob reduces to zero.

Change in KE of bob $= 0 - \dfrac{1}{2}\dfrac{W}{g}V^2 \quad = -\dfrac{1}{2}\dfrac{W}{g}lg$

Equating the work done and change in KE,

$$-\frac{1}{2}kx^2 = -\frac{1}{2}\frac{W}{g}lg$$

$$x = \sqrt{\frac{Wl}{k}}$$