Practice Problems:-

1) Reduce the following matrix to echelon form and hence find its rank:

$$A = \begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

2) Reduce the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$ into echelon form and find the rank.

3) Reduce the matrix $A = \begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix}$ into echelon form and find the rank.

4) Find the value of 'k' such that the rank of the matrix A is 3, where

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & k \end{bmatrix}$$

5) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 0 & 5 \end{bmatrix}$

Practice Problems:

1) Test the consistency & hence solve

$$x+2y+z=3$$
$$2x+3y+2z=5$$
$$3x-5y+5z=2$$
$$3x+9y-z=4.$$

2) Test the consistency & solve

$$-x+2y+3z=-2$$
$$2x-5y+z=2$$
$$3x-8y+5z=2$$
$$5x-12y-z=6$$

3) Test the consistency & solve

$$2x-2z=6$$
$$y+z=1$$
$$2x+y-z=7$$
$$3y+3z=0.$$

4) Find the values of λ and μ for which the system of equations

$$2x+3y+5z=9$$
$$7x+3y-2z=8$$
$$2x+3y+\lambda z=\mu$$

has (i) no solution (ii) a unique solution (iii) a one-parameter family of solutions

5) Show that the equations $x+y+z=a$, $3x+4y+5z=b$, $2x+3y+4z=c$
(i) have no solutions if $a=b=c=1$

(ii) have many solutions if $a = \frac{b}{2} = c = 1$.

Practice Problems.

1) Check whether the following system of equations possesses non-trivial solution.

$$x - 3y - 8z = 0$$
$$3x + y = 0$$
$$2x + 5y + 6z = 0.$$

2) Show that the equations $x + 2y - z = 0$, $3x + y - z = 0$, $2x - y = 0$ have non-trivial solution & find them.

3) Solve completely the system of equations

$$x + y - 2z + 3w = 0$$
$$x - 2y + z - w = 0$$
$$4x + y - 5z + 8w = 0$$
$$5x - 7y + 2z - w = 0$$

4) Find the value of k so that the equations $x + y + 3z = 0$, $4x + 3y + kz = 0$, $2x + y + 2z = 0$ have non-trivial solution.

5) Determine the values of λ for which the following set of equations may possess non-trivial solution:

$$3x_1 + x_2 - \lambda x_3 = 0$$
$$4x_1 - 2x_2 - 3x_3 = 0$$
$$2\lambda x_1 + 4x_2 + \lambda x_3 = 0$$

For each permissible value of λ, determine the general solution.

(3) is a false statement (0 = 12), which shows that the given system has no solution.

Practice Problems

1) Using Gauss elimination method, solve the linear system of equations:

$$x + 2y + z = 3$$
$$2x + 3y + 2z = 5$$
$$3x - 5y + 5z = 2$$
$$3x + 9y - z = 4.$$

2) Using Gauss elimination method, find the solution of the system of equations

$$4y + 3z = 8$$
$$2x - z = 2$$
$$3x + 2y = 5.$$

3) Using Gauss elimination method, solve the linear system:

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 32.$$

4) Using Gauss elimination method, solve the linear system:

$$10x + 4y - 2z = 14$$
$$-3w - 15x + y + 2z = 0$$
$$w + x + y = 6$$
$$8w - 5x + 5y - 10z = 26$$