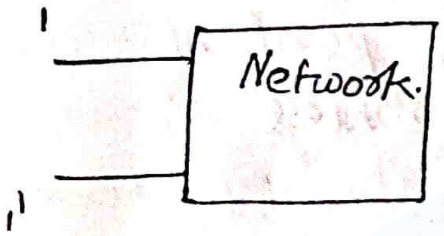
# Module 5

## Two port network

Any n/w may be represented by a rectangular box.

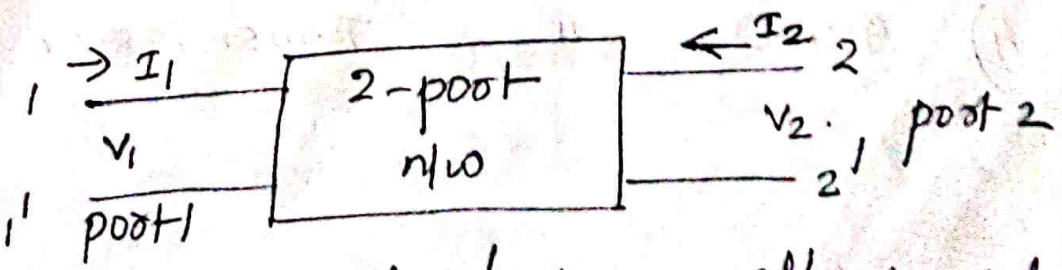



A pair of terminals at which a signal may enter or leave a n/w is called a port.

A port is defined as any pair of terminals into which energy is supplied or from which energy is withdrawn.

N/ws are classified into single port and two port networks.

A single port n/w has a pair of terminals to which the source is connected ie If the n/w consists of only one port then the n/w is called one port n/w. and if the n/w consists of two port, then it is a two port n/w.




The port 11' is normally connected to the driving force or input and is called port 2-2' connected to load is called output port.

This building blocks are common in electronic systems, communication systems, transmission & distribution s/ms.

Each and every two port n/w are represented by 4 variables where $V_1$ and $I_1$ are the i/p port voltage and current and $V_2$ & $I_2$ are the o/p port voltage & current. Two of the 4 variables are independent and another two are dependent.

There are 4 sets of parameters commonly used in the analysis of these networks. The parameters are.

1) z parameter (open ckt impedance parameter)

2) Y " (short ckt admittance parameter)

3) h " (Hybrid ")

4) ABCD " (Transmission ")

# Driving point functions.

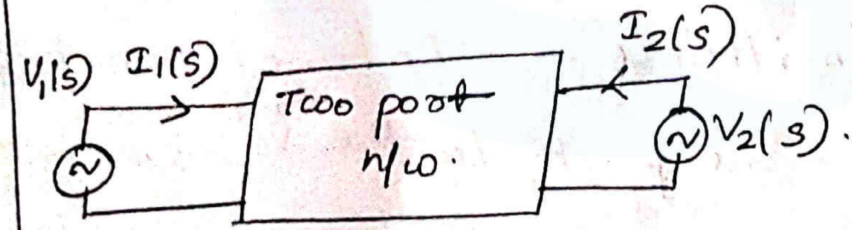
- Driving point impedance
- " admittance.

For a 2 port n/w, driving point impedance is defined as the ratio of transform voltage at the of a port to the transform current at the same port. Denoted as $z(s)$.

$$z(s) = \frac{V(s)}{I(s)}.$$

Driving point admittance is defined as the ratio of transform ~~voltage~~ current of a port to the transform ~~curr~~ voltage at the same port. Denoted as $Y(s)$

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)}.$$




The driving point impedance at port 1

$$z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

At port 2, $z_{22}(s) = \frac{V_2(s)}{I_2(s)}$.

Driving point admittance at port 1

$$Y(s) = Y_{11}(s) = \frac{I_1(s)}{V_1(s)}.$$

at port 2, $Y_{22}(s) = \frac{I_2(s)}{V_2(s)}.$

Transfer function.

= Ratio of transform n/w response to transform n/w excitation.

Transfer impedance — Ratio of transform voltage at one port to the transform current at another port.

$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)}.$$

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)}.$$

Transfer admittance — Ratio of transform current at one port to the transform voltage at another port. $Y(s)$.

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)}.$$

$$Y_{21}(s) = \frac{I_2(s)}{V_1(s)}.$$

**Voltage Transfer ratio** – Ratio of transform voltage at one port to the transform voltage at another port.

$$G_{12}(s) = \frac{V_1(s)}{V_2(s)}$$

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}.$$

**Current Transfer ratio** – ratio of transform current at one port to transform current at another port.

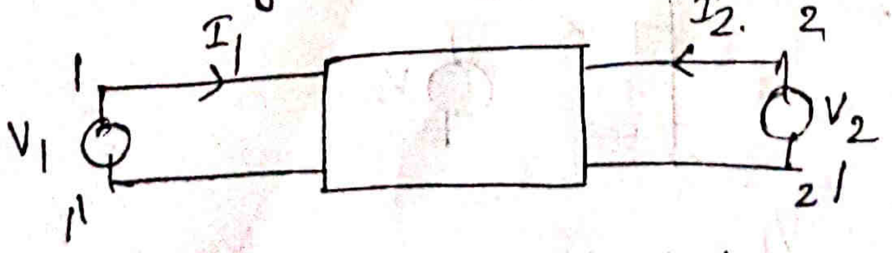
$\alpha(s)$.

$$\alpha_{12}(s) = \frac{I_1(s)}{I_2(s)}.$$

$$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}.$$

**Impedance Parameters.**

4 variables are specified for a 2 port n/w. If any two are specified, the other two can be determined. The 4 variables are related by the eqns.




Expressing $V_1, V_2$ in terms of currents $I_1$ & $I_2$. $V_1, V_2 \rightarrow$ dependent variables, $I_1, I_2 \rightarrow$ ind.

$V_1$ is the response produced by 2 currents $I_1$ & $I_2$.

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$

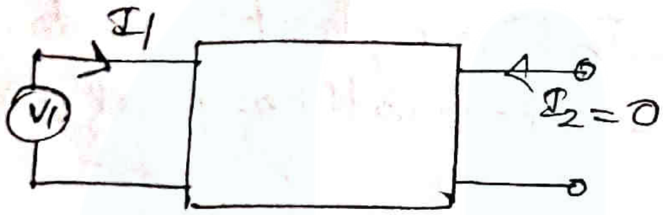
$$V_2 = I_1 Z_{21} + I_2 Z_{22}.$$

$Z_{11}, Z_{12}, Z_{21}, Z_{22}$ are called impedance parameters. Unit is $\Omega$. $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

<u>To find z parameters.</u>

<u>Open ckt either port 1 or port 2.</u>

If port 2 is open ckted, $I_2 = 0$.



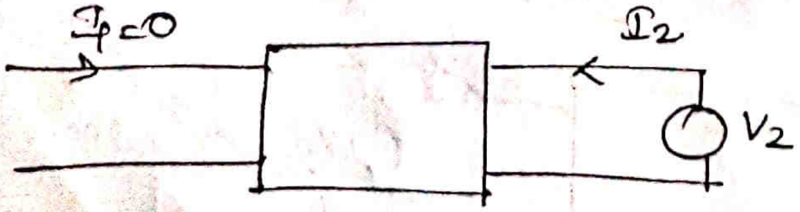

$V_1 = I_1 Z_{11} \Rightarrow Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ → Open ckt i/p impedance

$V_2 = I_1 Z_{21}$

$\Rightarrow Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$.

→ Open ckt transfer impedance.

If port 1 is ockted, $I_1 = 0$




$Z_{12} = \frac{V_1}{I_2}$, $Z_{22} = \frac{V_2}{I_2}$

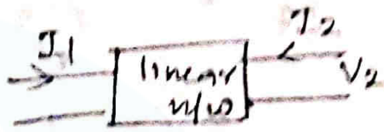
= OC transfer Z      = open ckt o/p Z.

Since these eqns involve oc at one port, these parameters are called Open ckt Impedance parameters. or Z parameters.

$Z_{11}, Z_{22}$ = Driving point impedance

$Z_{12}, Z_{21}$ = oc transfer impedance.

## Admittance Parameters.




The equations relating the four variables can be written as,

dependent variables

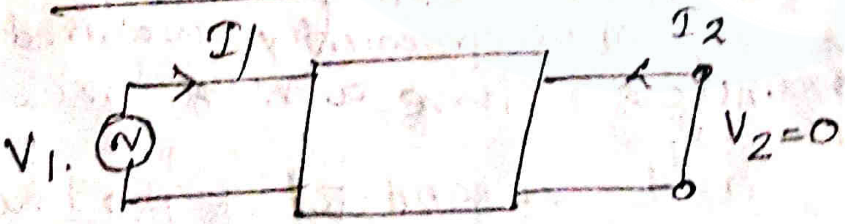
$$\boxed{\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2. \end{aligned}}$$

Unit is ℧.

## To find Y parameters.

Impose condition like short ckting either port 1 or 2.

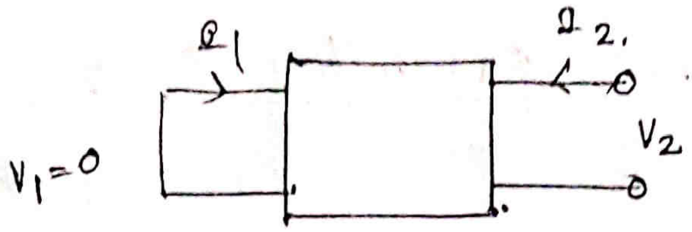
Port 2 is shorted, $V_2 = 0$.




$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \text{SC i/p admittance}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \text{SC transfer admittance}$$

If port 1 is shorted, $V_1 = 0$.




$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = \text{SC transfer admittance}$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \text{SC o/p admittance.}$$

Also called SC Admittance parameters as it involves a SC at one port.

$Y_{11}, Y_{22}$ = SC Driving point admittance

$Y_{12}, Y_{21}$ = " transfer ".

## Hybrid Parameters. (h)

- used in the analysis of transistor networks, ie because these parameters can be conveniently measured.
- the parameters have a mixture of units as $\Omega$ and $℧$. So the name hybrid. (h- represent dimensionally Z, Y, V, G, I, (G)).

If the voltage at 1-1' and I at 2-2' are taken as dependent variables.

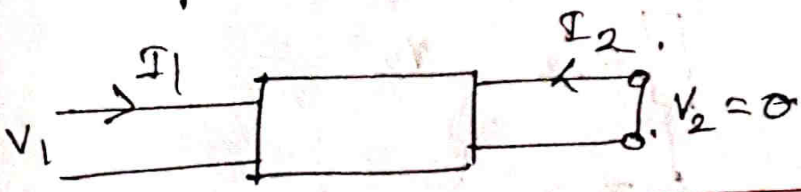
$$\boxed{\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}}$$

- here the voltage of one port and current of other port are taken as the independent variables.

# To find h parameters.

Impose both short ckt and open ckt condition.

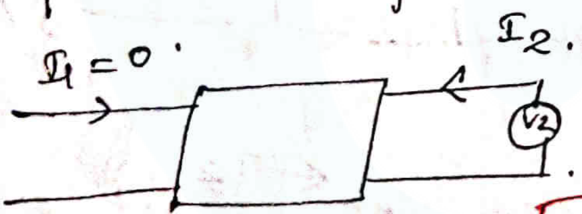
If port 2 is short ckted, $V_2 = 0$




$V_1 = h_{11} I_1$ , $\boxed{h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}}$ → SC i/p Z ohm.

$I_2 = h_{21} I_1$ , $\boxed{h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}}$ → SC forward current ratio.

If port 1 is openckted, $I_1 = 0$



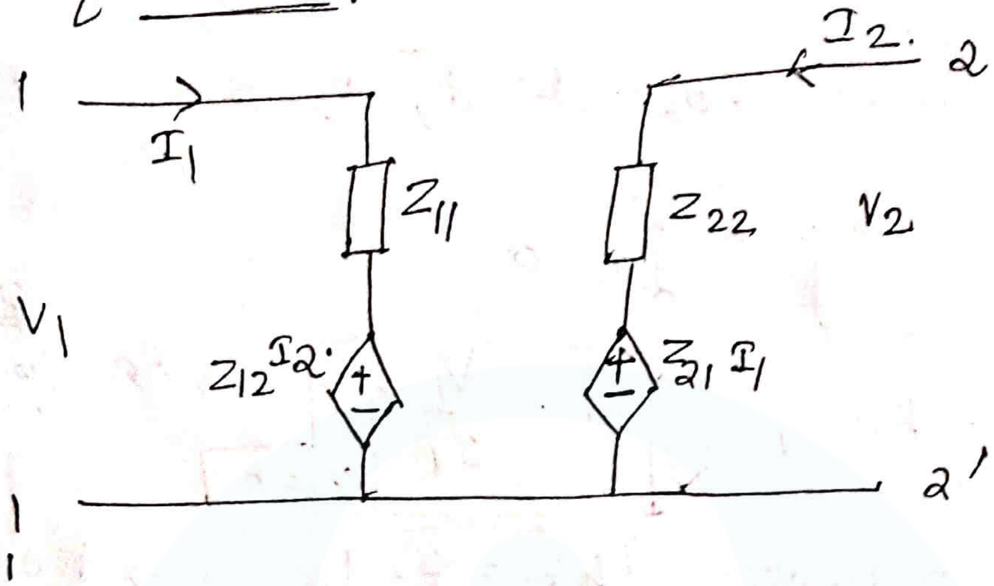

$V_1 = h_{12} V_2$ , $\boxed{h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}}$ OC reverse voltage ratio.

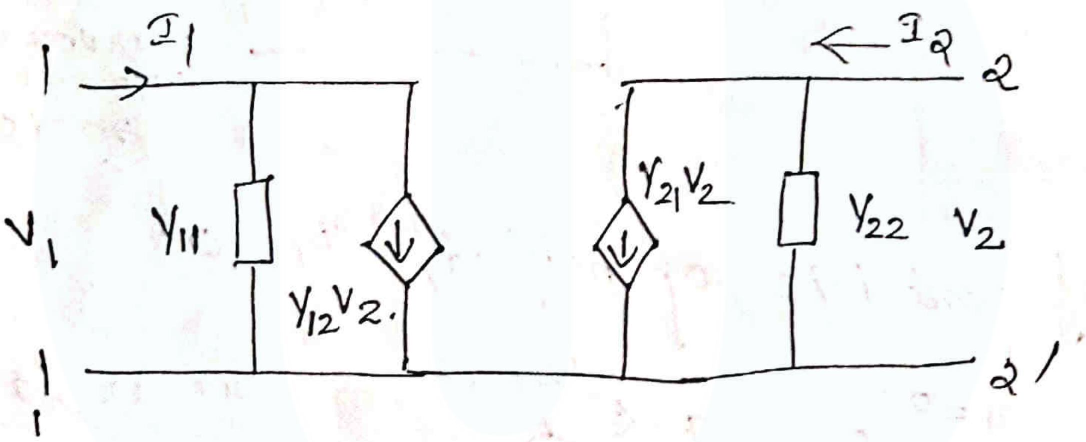
$I_2 = h_{22} V_2$ , $\boxed{h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}}$ — OC o/p admittance → mho.

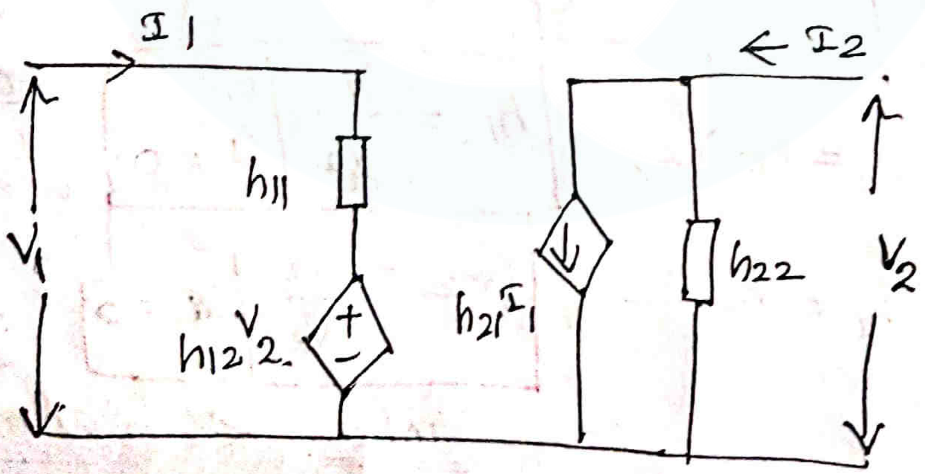
$h_{12}, h_{21}$ = voltage transfer ratio
current transfer ratio

$h_{12}, h_{21} \rightarrow$ dimensionless.

Equivalent Ckt



## Transmission Parameters (ABCD) (Chain) (General ckt)

Express voltage & current at one port 1 interms of voltage & current at port 2. They are defined by

$$\boxed{V_1 = AV_2 - BI_2}, \quad \boxed{I_1 = CV_2 - DI_2}$$

A, B, C, D are transmission parameters.

Negative sign for $I_2$ is that $I_2$ is in a dirn opposite to that of conventional current dirn

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Transmission Matrix.

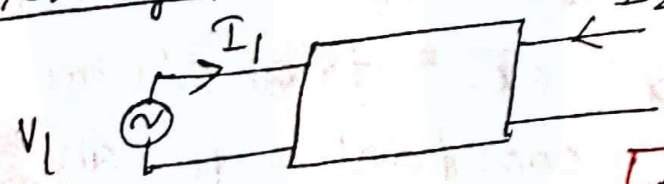
- widely used in transmission line & cascade networks.

$V_1$, $I_1$ at port 1-1' as sending end are expressed interms of $V_2$, $I_2$ - receiving end.

**To find ABCD.**

Both short ckt & open ckt on port 2.

open ckting port 2, applying a $V_1$ at 1-1', $I_2 = 0$



$V_1 = AV_2$

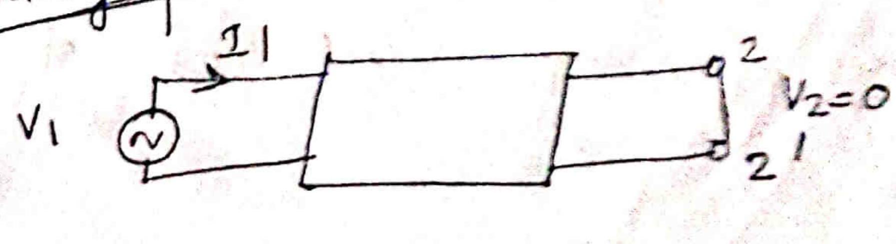
$I_1 = CV_2$

$$\Rightarrow \boxed{A = \frac{V_1}{V_2}\bigg|_{I_2=0}}$$

= OC reverse voltage ratio (voltage transfer ratio)

$$\boxed{C = \frac{I_1}{V_2}\bigg|_{I_2=0}}$$ = OC transfer admittance.

shortckting port 2, $V_2 = 0$ applying $V_1$ at 1-1'.



$$\boxed{B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}}$$ = sc transfer impedance

$$\boxed{D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}}$$ = sc reverse current ratio.

= current transfer ratio.

Dimensionless — A & D.

C — oc admittance parameter — ℧

B — sc impedance " — Ω

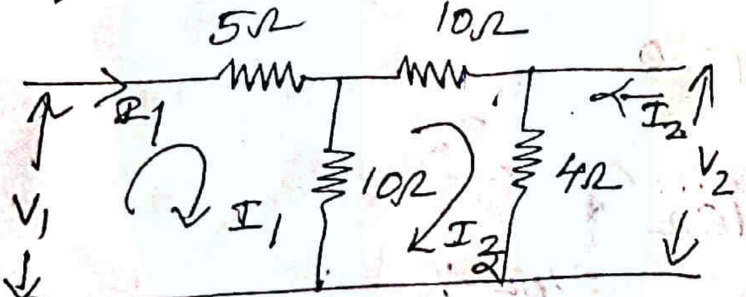
Condition for Reciprocity & Symmetry.

A 2port n/w is said to be reciprocal, if the ratio of the transform n/w response to the transform n/w excitation is constant to an Interchange of the positions of the excitation and the response in the n/w.

A 2port n/w is said to be symmetric, if the ports can be interchanged without changing the port voltages & currents.

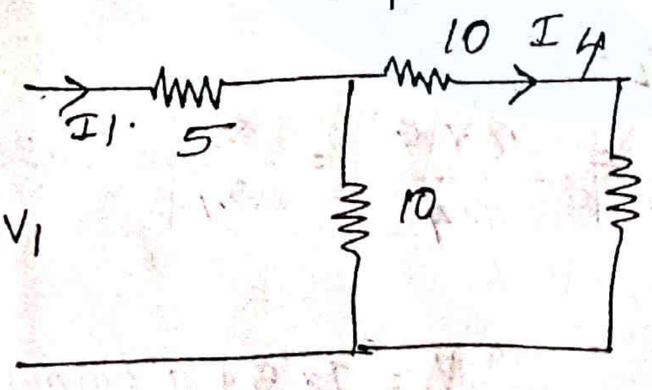
| Parameter | Condition for Reciprocity | Condition for Symmetry |
|---|---|---|
| Z | $Z_{12}=Z_{21}$ | $Z_{11}=Z_{22}$ |
| Y | $Y_{12}=Y_{21}$ | $Y_{11}=Y_{22}$. |
| h | $h_{12}=h_{21}$ | $h_{11}h_{22}-h_{12}h_{21}=1$ |
| ABCD | $AD-BC=1$. | $A=D$ |

① Find the z parameter of the following n/w.




Opencircuiting port 2.

$$Z_{11}=\left.\frac{V_1}{I_1}\right|_{I_2=0}$$




$V_1 = I_1 Z_{eq}$

$Z_{eq} = 14 \| 10 + 5$

$= \frac{14\times10}{24}+5$

$= \underline{10.8}$.

$$\frac{V_1}{I_1}=Z_{11}=\underline{10.8\Omega}$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$
$$V_2 = I_1 Z_{21} + I_2 Z_{22}$$

$$V_2 = V_{4\Omega} = 4 \times I_4$$

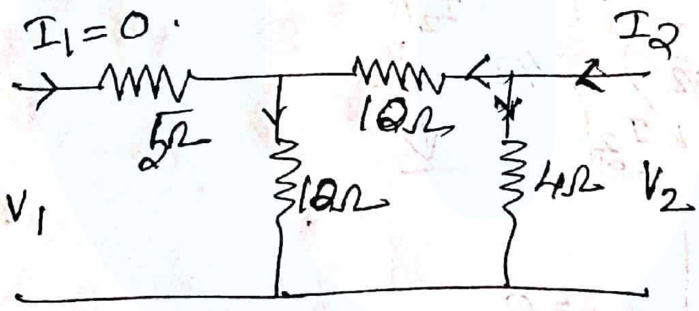
$$I_4 = \frac{I_1 \times 10}{10+10+4}$$

$$= I_1 \times \frac{10}{24}$$

$$V_2 = I_1 \times \frac{10}{\cancel{24}\,6} \times \cancel{4} = \frac{10}{6} I_1$$

$$\frac{V_2}{I_1} = Z_{21} = \frac{10}{6} = \frac{5}{3}\Omega = \underline{\underline{1.67\Omega}}$$

<u>O.C port 1</u>, $I_1 = 0$




$$V_2 = I_2 \, Z_{eq}$$

$$Z_{eq} = 20 \parallel 4 = \frac{20 \times 4}{24} = \frac{80}{24} = 3.33\Omega$$

$$Z_{22} = \frac{V_2}{I_2}$$

$$Z_{22} = \frac{V_2}{I_2}$$

$$V_1 = I_5 \times 5 + I_{10} \times 10\Omega$$
$$= 10 \times I_{10}$$

$$I_{10} = \frac{I_2 \times 4}{10+10+4}$$

$$V_1 = 10 \times \frac{I_2 \times 4}{10+10+4} = \frac{40}{24} I_2$$

$$\frac{V_1}{I_2} = \frac{40}{24} = \underline{1.67\,\Omega}.$$

# Relationship b/n Parameter

## ① Z interms of Y

Y parameter eqns are

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{— (b)}$$

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$
$$V_2 = I_1 Z_{21} + I_2 Z_{22}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{—— (a)}$$

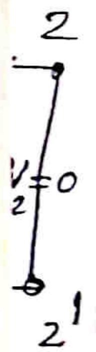
Solving for $V_1$ & $V_2$, using Cramer's rule.

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22}I_1 - Y_{12}I_2}{Y_{11}Y_{22} - Y_{21}Y_{12}}$$

$$\cancel{V_1 = \frac{I_1 - Y_{12}V_2}{Y_{11}}}$$

$$= \frac{Y_{22}}{\Delta Y} I_1 - \frac{Y_{12}}{\Delta Y} I_2$$

$$\cancel{V_1 = I_1\left(\frac{1}{Y_{11}}\right) - V_2 \frac{Y_{12}}{Y_{11}}}$$




$$V_2 = \frac{Y_{11}}{\Delta Y} I_2 - \frac{Y_{21}}{\Delta y} I_1$$

Comparing these eqns with eqn (a).

$$Z_{11} = \frac{Y_{22}}{\Delta y}, \quad Z_{12} = \frac{-Y_{12}}{\Delta y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta y}, \quad Z_{22} = \frac{Y_{11}}{\Delta y}.$$

$$\Delta y = Y_{11}Y_{22} - Y_{12}Y_{21}$$

(2) Interms of ABCD.

$I_1 = CV_2 - DI_2$ , $V_1 = AV_2 - BI_2$.

$$V_2 = \frac{I_1 + DI_2}{C} = \frac{1}{C} I_1 + \frac{D}{C} I_2.$$

$$V_1 = A\left(\frac{1}{C} I_1 + \frac{D}{C} I_2\right) - BI_2.$$

$$= \frac{A}{C} I_1 + \frac{AD}{C} I_2 - BI_2.$$

$$= \frac{A}{C} I_1 + \frac{AD - BC}{C} I_2.$$

Comparing with std eqn of Z parameters,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\boxed{Z_{11} = \frac{A}{C}, \quad Z_{12} = \frac{AD - BC}{C} \\ Z_{21} = \frac{1}{C}, \quad Z_{22} = \frac{D}{C}.}$$

(3) Interms of h parameters.

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2. \quad \text{--- (2)}$$

$$\Rightarrow V_2 = \frac{I_2 - h_{21} I_1}{h_{22}} = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1.$$

$$= -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2.$$

$$V_1 = h_{11} I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right]$$

$$= h_{11} I_1 + \frac{h_{12}(-h_{21})}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

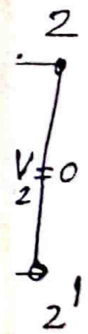
$$= \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$= \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 .$$

Comparing with std eqns.

$$\boxed{\begin{array}{ll} Z_{11} = \dfrac{\Delta h}{h_{22}}, & Z_{12} = \dfrac{h_{12}}{h_{22}} \\[2ex] Z_{21} = \dfrac{-h_{21}}{h_{22}}, & Z_{22} = \dfrac{1}{h_{22}}. \end{array}}$$

(4) Y parameters interms of Z parameter




$$Y = \frac{1}{Z}$$

$$[I] = [Y][V] = [Y][I][Z]$$

$$[Y] = [Z]^{-1}.$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

$$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}} \qquad Y_{12} = \frac{-Z_{12}}{\Delta Z}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z}, \qquad Y_{22} = \frac{Z_{11}}{\Delta Z}.$$

5) Interms of ABCD

$$V_1 = AV_2 - BI_2$$

$$\Rightarrow I_2 = \frac{AV_2 - V_1}{B} = \left(-\frac{1}{B}\right)V_1 + \frac{A}{B}V_2.$$

$$I_1 = CV_2 - DI_2.$$

$$= CV_2 - D\left[\left(-\frac{1}{B}\right)V_1 + \frac{A}{B}V_2\right]$$

$$= CV_2 + \frac{D}{B}V_1 - \frac{AD}{B}V_2.$$

$$= \frac{BC - AD}{B}V_2 + \frac{D}{B}V_1$$

$$I_1 = \frac{D}{B}V_1 - \frac{(AD - BC)}{B}V_2.$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

Comparing,

$$Y_{11} = \frac{D}{B}, \quad Y_{12} = \frac{AD - BC}{B}.$$

$$Y_{21} = -\frac{1}{B}, \quad Y_{22} = A/B.$$

6) Interms of h parameter.

$$V_1 = h_{11}I_1 + h_{12}V_2.$$

$$\Rightarrow I_1 = \frac{V_1 - h_{12}V_2}{h_{11}} = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2.$$

& $I_2 = h_{21}I_1 + h_{22}V_2.$

$$= h_{21}\left[\frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2.$$

$$= \frac{h_{21}}{h_{11}} V_1 - \frac{h_{21}h_{12}}{h_{11}} V_2 + h_{22} V_2$$

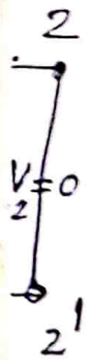
$$= \frac{h_{21}}{h_{11}} V_1 - \frac{h_{21}h_{12} V_2 + h_{11}h_{22} V_2}{h_{11}}$$

$$= \frac{h_{21}}{h_{11}} V_1 + V_2 \left[ \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} \right]$$

Comparing,

$$\boxed{\begin{aligned} Y_{11} &= \frac{1}{Y_{11}}, & Y_{12} &= -\frac{h_{12}}{h_{11}} \\ Y_{21} &= \frac{h_{21}}{h_{11}}, & Y_{22} &= \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} \end{aligned}}$$

<u>ABCD parameters</u> – <u>interms of z parameter</u>




$V_1 = z_{11} I_1 + z_{12} I_2$ $\qquad$ $V_2 = z_{21} I_1 + z_{22} I_2$

$\Downarrow$

$$\left.\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}\right| A$$

$$I_1 = \frac{V_2 - z_{22} I_2}{z_{21}}$$

$$= \frac{1}{z_{21}} V_2 - \frac{z_{22}}{z_{21}} I_2$$

$$V_1 = z_{11}\left[\frac{1}{z_{21}} V_2 - \frac{z_{22}}{z_{21}} I_2\right] + z_{12} I_2.$$

$$= \left(\frac{z_{11}}{z_{21}}\right) V_2 - I_2 \left[\frac{z_{22}z_{11} - z_{12}z_{21}}{z_{21}}\right].$$

Comparing with eqns (A),

$$A = \frac{Z_{11}}{Z_{21}} , \quad B = \frac{Z_{22}Z_{11} - Z_{12}Z_{21}}{Z_{21}} .$$

$$C = \frac{1}{Z_{21}} , \quad D = \frac{Z_{22}}{Z_{21}} ,$$

Interms of Y parameters.

$$I_2 = Y_{21}V_1 + Y_{22}V_2 .$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2 .$$

$$V_1 = \frac{I_2 - Y_{22}V_2}{Y_{21}}$$

$$= \frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2 = V_2\left(\frac{-Y_{22}}{Y_{21}}\right) - I_2\left(\frac{1}{Y_{21}}\right)$$

$$I_1 = Y_{11}\left[\frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2\right] + Y_{12}V_2 .$$

$$= \frac{Y_{11}}{Y_{21}} I_2 - \frac{Y_{11}Y_{22}}{Y_{21}} V_2 + Y_{12}V_2$$

$$= I_2 \frac{Y_{11}}{Y_{21}} - V_2\left[\frac{Y_{11}Y_{22}}{Y_{21}} - \frac{Y_{12}Y_{21}}{Y_{21}}\right]$$

Comparing with the eqns,

$$A = -\frac{Y_{22}}{Y_{21}} , \quad B = -\frac{1}{Y_{21}}$$

$$C = -\left(\frac{Y_{11}Y_{22}}{Y_{21}} - \frac{Y_{12}Y_{21}}{Y_{21}}\right)$$

$$D = -\frac{Y_{11}}{Y_{21}}.$$

## Interms of h parameters.

$$I_2 = h_{21} I_1 + h_{22} V_2.$$

$$I_1 = \frac{I_2 - h_{22} V_2}{h_{21}} = \frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2$$

$$V_1 = h_{11} I_1 + h_{12} V_2 = h_{11}\left[\frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2\right].$$

$$+ h_{12} V_2.$$

$$= \frac{h_{11}}{h_{21}} I_2 - \frac{h_{11} h_{22}}{h_{21}} V_2 + h_{12} V_2.$$

$$= \frac{h_{11} I_2 - h_{11} h_{22} V_2 + h_{12} h_{21} V_2}{h_{21}}$$

$$= I_2\left[\frac{h_{11}}{h_{21}}\right] - V_2\left[\frac{h_{11} h_{22}}{h_{21}} + \frac{h_{12} h_{21}}{h_{21}}\right]$$


$$V_1 = A V_2 - B I_2, \quad I_1 = C V_2 - D I_2.$$

$$V_1 = V_2\left[-\frac{(h_{11} h_{22} - h_{12} h_{21})}{h_{21}}\right] + \left(-\frac{h_{11}}{h_{21}}\right) - I_2.$$

$$A = -\frac{(h_{11} h_{22} - h_{12} h_{21})}{h_{21}}, \quad B = \frac{-h_{11}}{h_{21}}$$

$C = -\frac{h_{22}}{h_{21}}$ , $D = -\frac{1}{h_{21}}$ ,

(2) Find the Y parameter.

h parameter interms of z parameters.

$V_1 = I_1 z_{11} + I_2 z_{12}$

$V_2 = I_1 z_{21} + I_2 z_{22}$

$V_1 = h_{11} I_1 + h_{12} V_2$

$\cancel{V}_2 = h_{21} I_1 + h_{22} V_2$

$\Downarrow$

$$I_2 = \frac{V_2 - I_1 z_{21}}{z_{22}}$$

$$= \frac{1}{z_{22}} V_2 - \frac{z_{21}}{z_{22}} I_1$$

k.

$$V_1 = z_{11} I_1 + z_{12}\left[\frac{V_2}{z_{22}} - \frac{z_{21}}{z_{22}} I_1\right]$$

$$V_1 = I_1\left[z_{11} - \frac{z_{12}z_{21}}{z_{22}}\right] + V_2 \frac{z_{12}}{z_{22}}.$$

$$I_2 = \frac{1}{z_{22}} V_2 - \frac{z_{21}}{z_{22}} I_1.$$

$$h_{11} = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}}, \quad h_{12} = \frac{z_{12}}{z_{22}}$$

$=0.$

$$h_{21} = -\frac{z_{21}}{z_{22}}, \quad h_{22} = 1/z_{22}.$$

$\Big|_{V_2=0}$

$\Big|_{V_2=0}$

<u>Interms of Y parameters.</u>

$$I_1 = Y_{11}V_1 + Y_{12}V_2.$$

$$\Rightarrow V_1 = \frac{I_1 - Y_{12}V_2}{Y_{11}} \quad \text{——} ①$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2.$$

$$= Y_{21}\left[\frac{I_1 - Y_{12}V_2}{Y_{11}}\right] + Y_{22}V_2.$$

$$= I_1 \cdot \frac{Y_{21}}{Y_{11}} - \frac{Y_{21}Y_{12}}{Y_{11}} V_2 + Y_{22}V_2$$

Comparing,

$$h_{11} = \frac{1}{Y_{11}}, \quad h_{12} = \frac{-Y_{12}}{Y_{11}}$$

$$h_{21} = \frac{Y_{21}}{Y_{11}}, \quad h_{22} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}} + 1$$

Interms of ABCD

$I_1 = CV_2 - DI_2$, $\Rightarrow I_2 = \frac{CV_2 - I_1}{D}$

$V_1 = AV_2 - BI_2$ $\Rightarrow V_1 = AV_2 - B\left[\frac{CV_2 - I_1}{D}\right]$

$= AV_2 - \frac{BC}{D}V_2 + I_1\frac{B}{D}$

$= V_2\left(\frac{AD-BC}{D}\right) + I_1 \cdot \frac{B}{D}$

Comparing,

$h_{11} = \frac{B}{D}$, $h_{12} = \frac{AD-BC}{D}$,

$h_{21} = -\frac{1}{D}$, $h_{22} = \frac{C}{D}$.

$$\boxed{B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}}$$ = sc transfer impedance

$$\boxed{D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}}$$ = sc reverse current ratio

= current transfer ratio.

Dimensionless — A & D.

C — oc admittance parameter — ℧

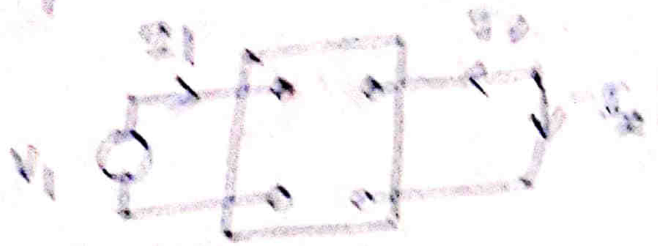
B — sc impedance " — Ω

Condition for Reciprocity & Symmetry.

A 2port n/w is said to be reciprocal, if the ratio of the transform n/w response to the transform n/w excitation is constant to an interchange of the positions of the excitation and the response in the n/w.

A 2port N/w is said to be symmetric, if the ports can be interchanged without changing the port voltages & currents.

Reciprocity in Z parameters:
Z parameter based two port n/ws can be represented as.




$I_2$ is assumed to be in reverse dirn of $I_2$ assumed.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \Rightarrow V_1 = Z_{11} I_1 - Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \qquad 0 = Z_{21} I_1 - Z_{22} I_2$$

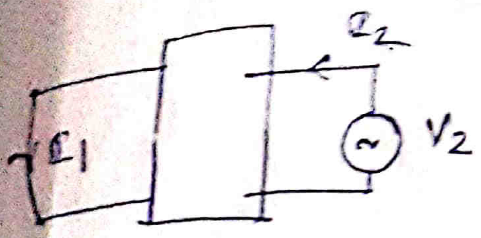
($V_2$ is zero as o/p is shorted).

$$\Rightarrow I_2 = \frac{Z_{11} I_1 - V_1}{Z_{12}}, \qquad Z_{21} I_1 = Z_{22} I_2$$
$$I_1 = \frac{Z_{22} I_2}{Z_{21}}$$

$$= \frac{-V_1 + Z_{11} \frac{Z_{22} I_2}{Z_{21}}}{Z_{12}} = \frac{-V_1 Z_{21} + Z_{11} Z_{22} I_2}{Z_{21} Z_{12}}$$

$$I_2 Z_{21} Z_{12} = -V_1 Z_{21} + I_2 Z_{11} Z_{22}$$
$$I_2 \left[ Z_{12} Z_{21} - Z_{11} Z_{22} \right] = -V_1 Z_{21}$$
$$I_2 = \frac{V_1 Z_{21}}{Z_{11} Z_{22} - Z_{12} Z_{21}}.$$

Interchange the i/p & o/p voltage & currents,




Governing eqns are

$$0 = I_1 z_{11} + I_2 z_{12}$$

$$V_2 = -z_{21} I_1 + z_{22} I_2 .$$

Solving, $I_1 = \dfrac{V_2 z_{12}}{z_{11} z_{22} - z_{12} z_{21}}$.

Assuming $V_1 = V_2$, and comparing $I_1$ & $I_2$

$\underline{\underline{z_{12} = z_{21}}}$.

Symmetry.

Applying voltage V at o/p port and keeping the o/p port open.

$V_1 = I_1 z_{11}$ , $z_{11} = \dfrac{V_1}{I_1}\Big|_{I_2=0}$.

Applying voltage V at o/p port and keeping openckt at i/p port

$V_2 = I_2 z_{22}$ , $z_{22} = \dfrac{V_2}{I_2}\Big|_{I_1=0}$.

Symmetry is when $\dfrac{V}{I_1} = \dfrac{V}{I_2}$ leads to the condition,

$\underline{\underline{z_{11} = z_{22}}}$.

In Yparameter

$$\boxed{\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}}$$

Shorting o/p, $V_2 = 0$

$I_1 = Y_{11}V_1$ , $I_2 = -Y_{21}V_1$, $-ve$ sign indicates the dirn is outward of the n/w.

$$Y_{21} = -\frac{I_2}{V_1} \quad \text{--- (a)}$$

With interchanging of response & excitation

$V_1 = 0$, $I_1$ is flowing outwards

$-I_1 = Y_{12}V_2$, $\quad Y_{12} = -\frac{I_1}{V_2}$ --- (b)

If $V_1 = V_2$, LHS of eqns (a) & (b) will be identical, provided the ckt is reciprocal.

$\boxed{Y_{21} = Y_{12}}$, the ckt will be reciprocal.

For symmetrical n/w, $Y_{11} = Y_{22}$,

This is possible if $Y_{11} = Y_{22}$.

Y parameter will be symmetric if the i/p & o/p ports can be interchanged without changing the voltage or current at each port.

In ABCD parameter.

$$\boxed{\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2. \end{aligned}}$$

Shorting the o/p port and $V_1$ at i/p port,

$$V_2 = 0$$

$$V_1 = -BI_2, \quad \frac{I_2}{V_1} = -\frac{1}{B}. \quad \text{— (a)}$$

With interchange of excitation & response, the voltage source $V_2$ is at the o/p port. $I_1$ is the sc current at the i/p port.

$$0 = AV_2 - BI_2, \quad I_1 = CV_2 - DI_2 \text{ —}$$

$$\Rightarrow I_2 = \frac{A}{B}V_2, \quad I_1 = CV_2 - \frac{ADV_2}{B}.$$

$$= V_2\left(\frac{BC-AD}{B}\right).$$

$$\Rightarrow \frac{I_1}{V_2} = \frac{BC-AD}{B} = -\frac{(AD-BC)}{B} \text{ — (b)}$$

When $V_1 = V_2$, LHS of eqns will be identical, provided $AD - BC = 1$.

Condition for reciprocity is,

$$\boxed{AD - BC = 1}.$$

For symmetry, $Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$

$$= \frac{A}{C} \Big| I_2 = 0.$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_P=0} = \underline{\underline{D/C}}$$

$\Big[ I_1 = 0 = CV_2 - DI_2$, $V_2 = \frac{DI_2}{C}$.

$\frac{V_2}{I_2} = D/C \Big]$.

For z parameter, $Z_{11} = Z_{22}$ to be symmetrical.

$\therefore \boxed{Z_{11} = Z_{22}}$, $\boxed{\frac{A}{C} = \frac{D}{C}}$, $\boxed{A = D}$

In h parameter.

$\boxed{V_1 = h_{11} I_1 + h_{12} V_2}$

with $V_2 = 0$, $V_1 = h_{11} I_1$, $h_{11} = \frac{V_1}{I_1}$. — (1)

$\boxed{I_2 = h_{21} I_1 + h_{22} V_2.}$

$V_2 = 0$, $\Rightarrow -I_2 = h_{21} I_1$. — (2)

$\frac{(2)}{(1)} \Rightarrow \frac{h_{21}}{h_{11}} = -\frac{I_2}{V_1}$.

Interchanging the excitation & response $V_2$ is applied at o/p port, i/p terminals are shorted., $I_1$ is reversed

$$0 = -h_{11} I_1 + h_{12} V_2.$$

$$\Rightarrow \frac{I_1}{V_2} = \frac{h_{12}}{h_{11}}.$$

Assume $V_1 = V_2$,

LHS will be same, provided $h_{21} = -h_{12}$

This is the condition for reciprocity.

For symmetry.

O/p is ~~shorted~~ open, $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1} = \frac{h_{11} I_1 + h_{12} V_2}{I_1} = h_{11} + h_{12} \frac{V_2}{I_1}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \qquad = h_{11} - h_{12} \times \frac{h_{21}}{h_{22}}$$

$$\Rightarrow \frac{V_2}{I_1} = -\frac{h_{21}}{h_{22}}$$

$$\boxed{Z_{11} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}} = Z_{11}}$$

Taking $I_1 = 0$,

$$Z_{22} = \frac{V_2}{I_2} = \frac{V_2}{h_{22} V_2} = \frac{1}{h_{22}}$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$I_2 = h_{22} V_2$$

If $\Delta h = 1$, $Z_{11} = Z_{22}$, this leads to condition of symmetry.

ie $\underline{h_{11} h_{22} - h_{12} h_{21} = 1}$.