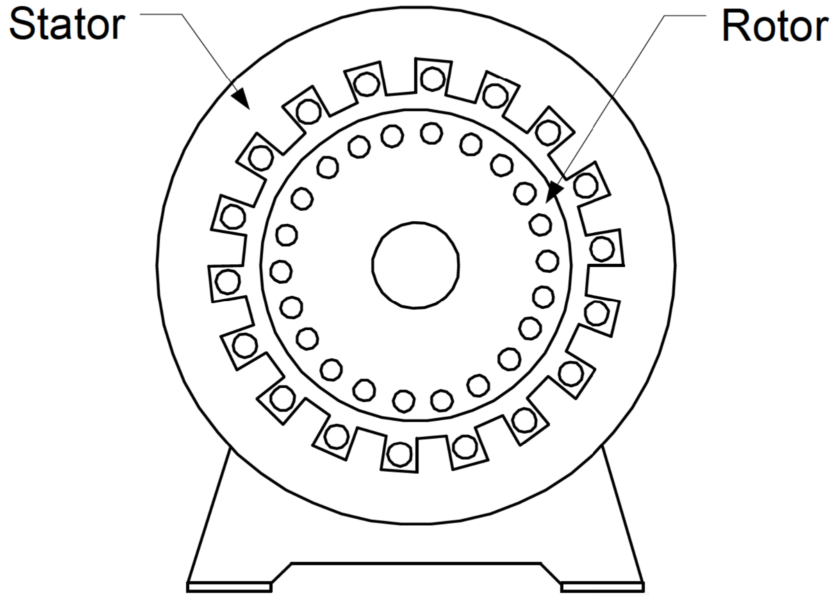
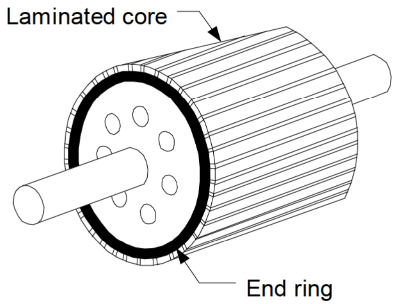
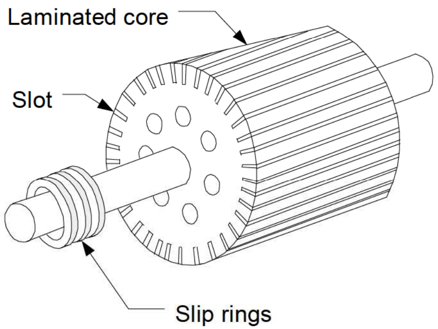
Induction Motor – Construction



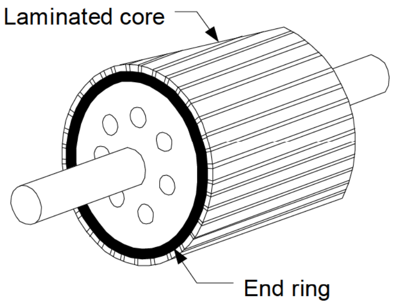


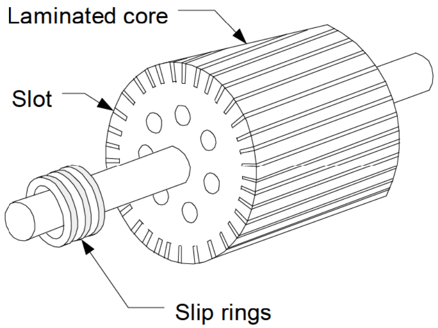
Squirrel cage rotor
Solid rotor bars connected together by the end ring



Slip ring rotor
Three phase star connected winding placed in rotor slots and terminated in the slip rings

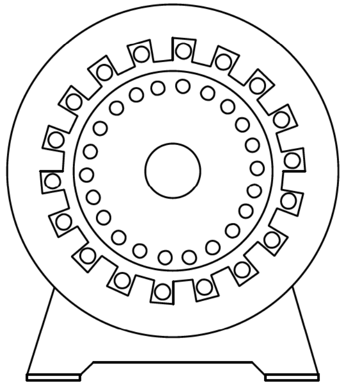
Rotor types





Induction Motor – Basic Facts

A rotating magnetic field is created by the stator winding when three phase balanced supply is applied to it



$$\text{Speed of the rotating magnetic field, } N_s = \frac{120\, f}{P} \text{ rpm}$$

where,

f = frequency of the supply in hertz

P = number of poles of the stator winding.

Due to the effect of induction, a current is induced in rotor conductors; and a torque is generated

The rotor rotates at a speed *N*, which is slightly less than the synchronous speed, N_s

A factor called **slip** denotes the difference between N_s *and* *N*, which is expressed by

$$\text{Slip, } s = \frac{N_S - N}{N_S} \qquad \%\ \text{Slip} = \frac{N_S - N}{N_S} \times 100\ \%$$

Speed of Rotor MMF

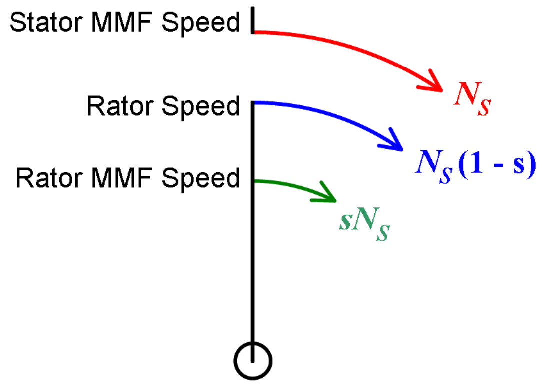
Slip, $s = \frac{N_s - N}{N_s}$ ⇒ $N = N_s(1 - s)$

N_s – Speed of RMF or stator mmf

N – Speed of rotor

Frequency of rotor current $= sf$ (slip x frequency)

Speed of rotor mmf w.r.t. rotor $= sN_s$



Actual speed of rotor mmf $= N + sN_s$

$$= N_s(1 - s) + sN_s$$

$$= N_s - sN_s + sN_s$$

$$= N_s$$

∴ Stator mmf and rotor mmf are stationary w.r.t. each other

Induced Voltage

EMF induced in the stator phase, $E_1 = 4.44\, K_{w1}\, \Phi_m\, f\, T_1$ volts

EMF induced in the rotor phase, $E_2 = 4.44\, K_{w2}\, \Phi_m\, sf\, T_2$ volts

Winding factor, $K_w = K_c \times K_d\ (pitch\ \ factor \times Distribution\ \ factor)$

K_{w1} - Winding factor of stator

K_{w1} - Winding factor of rotor

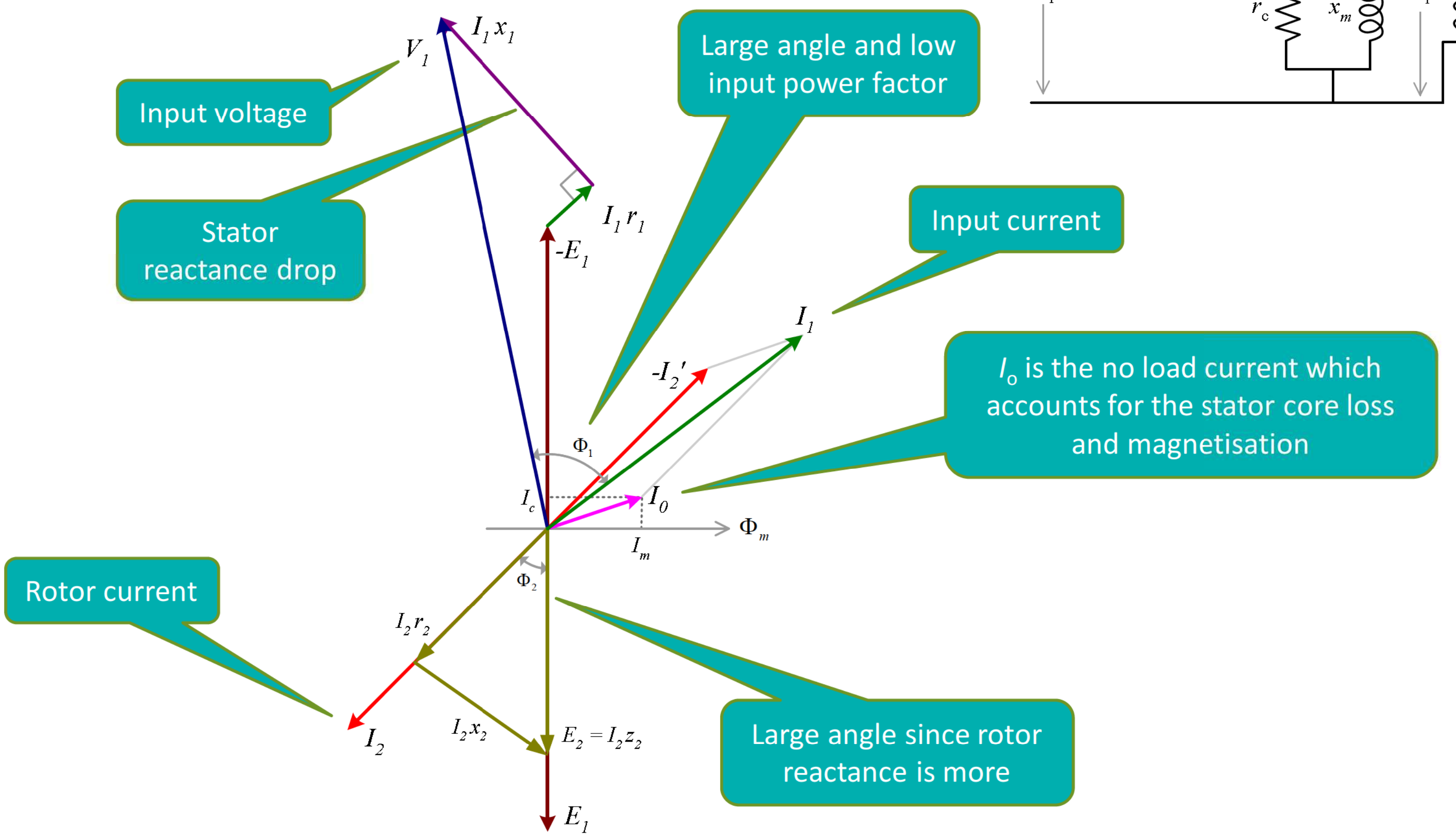
T_1 - Per phase turns in stator

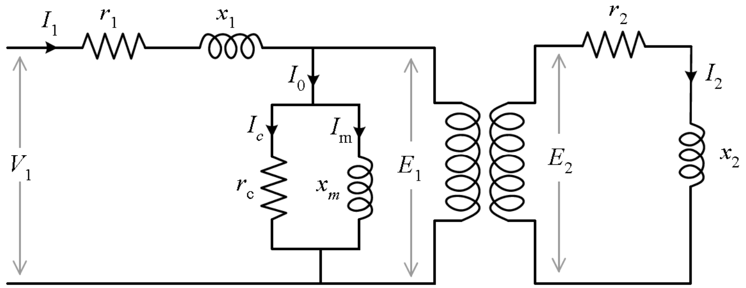
T_2 - Per phase turns in rotor

f - Input frequency

Phasor Diagram

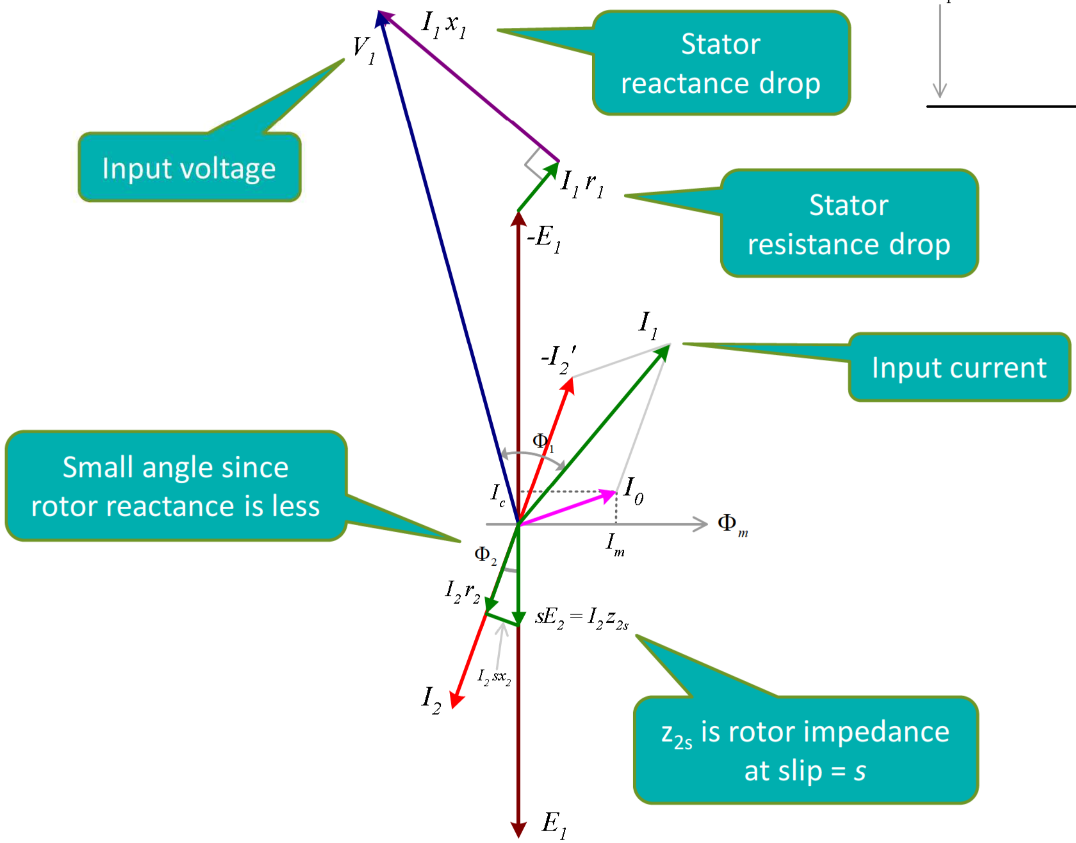
At Standstill

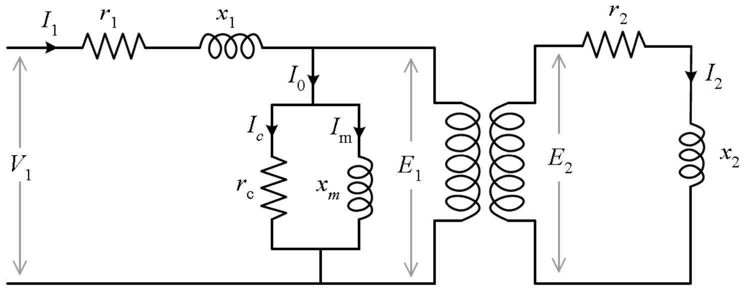




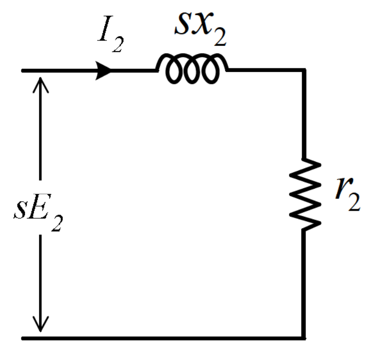
Phasor Diagram

Operating at slip, *s*





Equivalent Circuit



- Primary side (stator side) of equivalent circuit is similar to that of transformer
- The secondary side (rotor side) is short circuited and has resistance and reactance
- The secondary voltage will vary with change in rotor speed
- The secondary side has variable reactance due to change in rotor frequency

Rotor current at standstill, $I_2 = \dfrac{E_2}{r_2 + jx_2}$

$\because$ At standstill, rotor frequency is equal to stator freuency

Rotor current at slip s, $I_2 = \dfrac{sE_2}{r_2 + jsx_2}$

$\because$ With slip s, rotor induced voltage is reduced by the factor s and rotor frequency $f_2 = sf_1$ where f_1 is the stator frequency

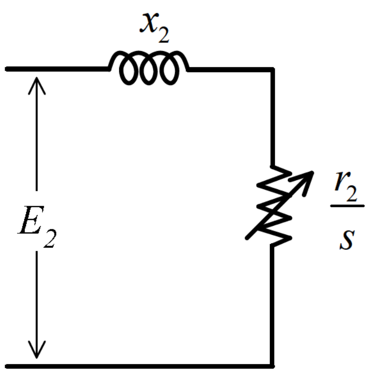
Dividing by s in the numerator and denominator

$$I_2 = \frac{E_2}{\dfrac{r_2}{s} + jx_2}$$

Two elements of rotor side of eqv cct

Equivalent Circuit Contd.....1

Rotor Equivalent Circuit



$$I_2 = \frac{E_2}{\frac{r_2}{s} + jx_2}$$

Turns ratio, $k = \dfrac{T_2}{T_1}$

where T_1 – Effective stator tuns per phase

T_2 – Effective rotor turns per phase

To transfer the quantities to stator side:

Rotor resistnce referred to stator side, $r_2' = \dfrac{r_2}{k^2}$

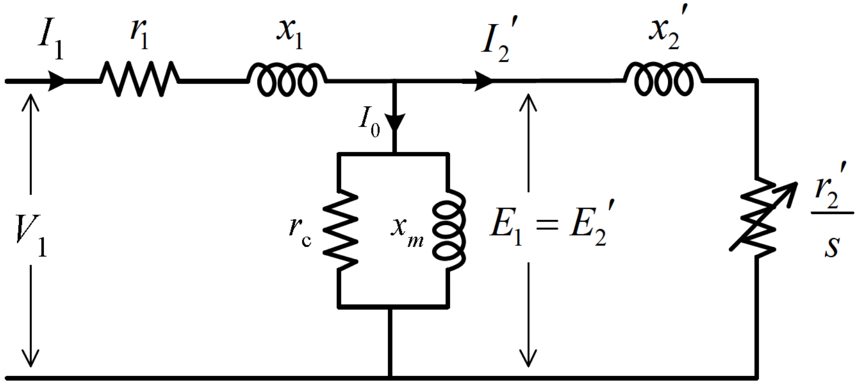
Rotor reactance referred to stator side, $x_2' = \dfrac{x_2}{k^2}$

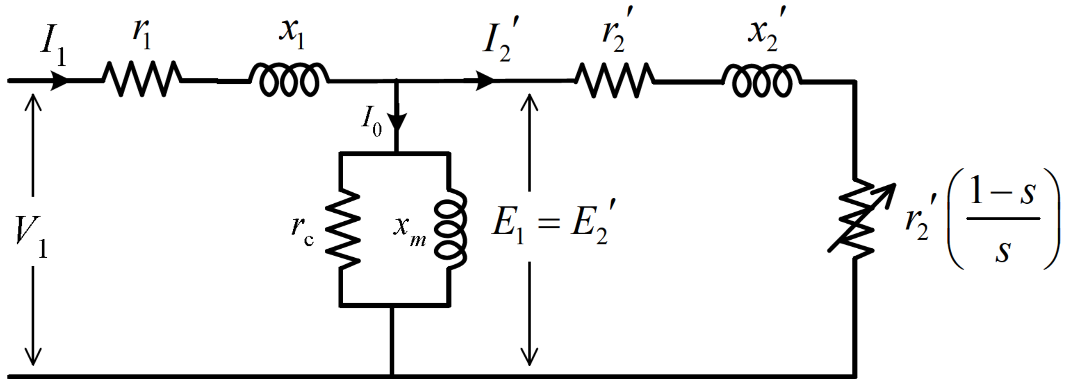
Rotor current referred to stator side, $I_2' = k\, I_2$

Also, $E_2' = \dfrac{E_2}{k}$

Equivalent Circuit Contd.....2

Complete Equivalent Circuit

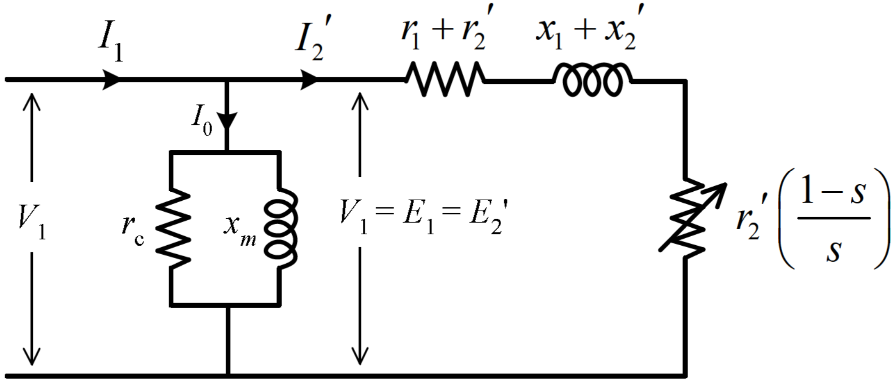




Modifying $\frac{r_2'}{s}$ as

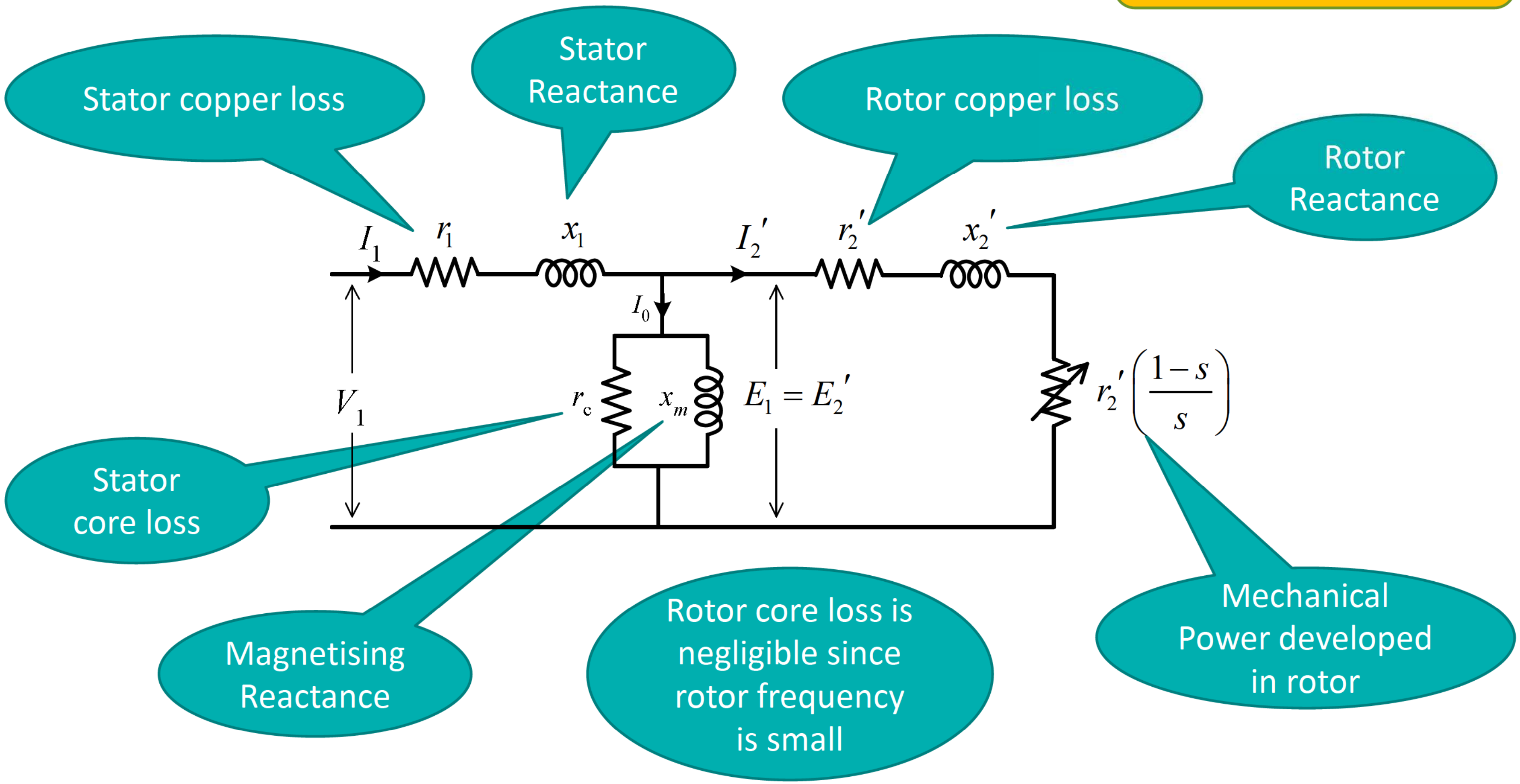
$$\left[r_2' + r_2'\left(\frac{1-s}{s}\right)\right]$$

Approximate eqv. cct. combining stator and rotor parameters



Parameter representation

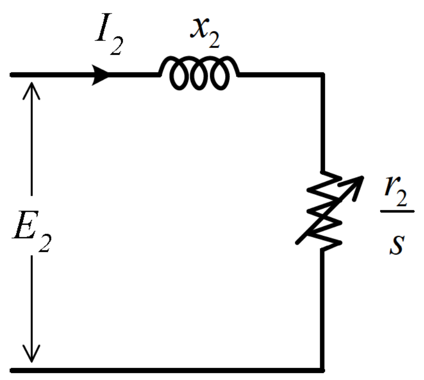
All parameters are per phase values



Power in Rotor Circuit

All parameters are per phase values

Rotor Equivalent Circuit



$$\text{Power transfered to rotor, } P_2 = \frac{I_2^{\,2}\, r_2}{s}$$

$$\text{Power loss in rotor circuit, } P_2 = I_2^{\,2}\, r_2 = sP_2$$

$$\text{Mechanical power developed, } P_m = \frac{I_2^{\,2}\, r_2}{s} - I_2^{\,2}\, r_2 = (1-s)\frac{I_2^{\,2}\, r_2}{s}$$

$$= (1-s)\,P_2$$

$$\text{Torque, } T = \frac{P_m}{2\pi n}$$

$$= \frac{(1-s)\,P_2}{2\pi n_s\,(1-s)}$$

$$= \frac{P_2}{2\pi n_s} \text{ newton metre}$$

$$= P_2 \text{ synchronous watts}$$

n – Speed in rps

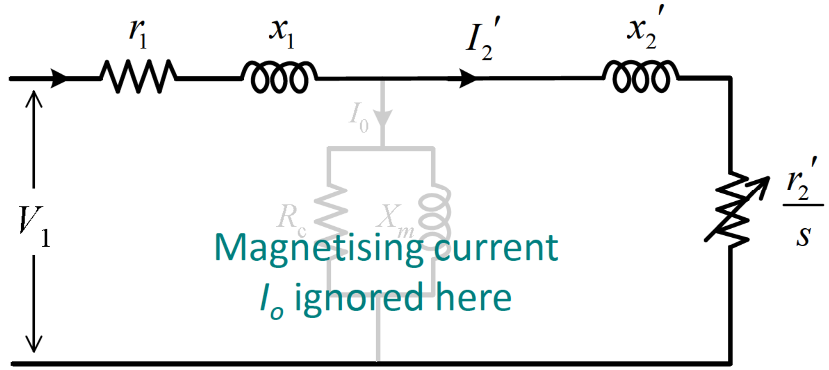
n_s – Syn. speed in rps

Synchronous watts is the torque which, at synchronous speed, would develop one watt of power

Developed Torque

Per phase torque, $T = \dfrac{P_2}{2\pi n_s} = \dfrac{I_2^{\,2}\, r_2}{s\omega_s}$

Total electromagnetic torque, $T_e = 3\dfrac{I_2^{\,2}\, r_2}{s\omega_s}$



From the equivalent circuit,

Total electromagnetic power, $P_e = 3\, I_2'^2\, r_2' \dfrac{(1-s)}{s}$

$$I_2' = \frac{V_1}{\sqrt{\left(r_1 + \dfrac{r_2'}{s}\right)^2 + \left(x_1 + x_2'\right)^2}}$$

$$P_e = \frac{(1-s)}{s} \times \underbrace{\frac{3\, V_1^2\, r_2'}{\left\{\left(r_1 + \dfrac{r_2'}{s}\right)^2 + \left(x_1 + x_2'\right)^2\right\}}}_{I_2'^2}$$

Developed Torque Contd.....

To get torque in terms of voltage:

$$\text{Total electromagnetic torque, } T_e = \frac{P_e}{\omega} = \frac{(1-s)}{s\omega} \times \frac{3\,V_1^2\, r_2'}{\left\{\left(r_1 + \frac{r_2'}{s}\right)^2 + \left(x_1 + x_2'\right)^2\right\}}$$

$$= \frac{3\,V_1^2\, r_2'}{s\,\omega_s \left\{\left(r_1 + \frac{r_2'}{s}\right)^2 + \left(x_1 + x_2'\right)^2\right\}} \text{ Nm} \qquad \because \omega = (1-s)\,\omega_s$$

$$\text{Starting torque, } T_s = \frac{3\,V_1^2\, r_2'}{\omega_s \left\{\left(r_1 + r_2'\right)^2 + \left(x_1 + x_2'\right)^2\right\}} \text{ Nm} \qquad \because \text{At starting, } s = 1$$

Slip at Maximum Torque

$$\text{Total electromagnetic torque, } T_e = \frac{3\,V_1^2\, r_2'}{s\,\omega_s \left\{ \left(r_1 + \frac{r_2'}{s} \right)^2 + \left(x_1 + x_2' \right)^2 \right\}} \text{ Nm}$$

- Torque varies with change in slip. To get condition for maximum torque, the above expression should be differentiated with respect to *s*.
- Since The numerator is independent of *s*, and ω_s is constant, we may differentiate the denominator part alone as below:

$$\frac{d}{ds}\, s \left\{ \left(r_1 + \frac{r_2'}{s} \right)^2 + \left(x_1 + x_2' \right)^2 \right\} = 0$$

On solving

$$r_1^2 - \frac{r_2'^2}{s^2} + \left(x_1 + x_2' \right)^2 = 0$$

$$\text{Slip at maximum torque, } s_{t_{max}} = \frac{r_2'}{\sqrt{r_1^2 + \left(x_1 + x_2' \right)^2}}$$

If stator impedance is neglected

$$s_{t_{max}} = \frac{r_2'}{x_2'} = \frac{r_2}{x_2}$$