Exercise 1.5

A 3 phase, 8 pole, 750 rpm, star connected alternator has 72 slots on the armature. Each slot has 12 conductors and the winding is short pitched by two slots. Find the induced emf between the lines. Flux per pole is 0.06 Wb

$$\text{Slots/pole/phase, } m = \frac{72}{8\times 3} = 3 \qquad f = \frac{NP}{120} = \frac{750\times 8}{120} = 50\ \text{Hz}$$

$$\text{Slot angle, } \beta = \frac{180}{\text{Slots/pole}} = \frac{180}{72/8} = 20^\circ \qquad \text{Chording angle, } \alpha = 2\times 20 = 40^\circ$$

$$\text{Number of turns, } T = \frac{8\times 3\times 12}{2} = 144$$

$$K_c = \cos\frac{\alpha}{2} = \cos\frac{40}{2} = 0.9397 \qquad K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin\frac{3\times 20}{2}}{3\times\sin\frac{20}{2}} = 0.9598$$

$$\text{Per phase voltage, } V_{ph} = 4.44\, K_c\, K_d\, \Phi\, f\, T$$

$$= 4.44 \times 0.9397 \times 0.9598 \times 0.06 \times 50 \times 144$$

$$= 1730 \text{ volts}$$

$$\text{Line voltage, } V_L = \sqrt{3} \times V_{ph}$$

$$= \sqrt{3} \times 1730$$

$$= 2996 \text{ volts}$$

Exercise 1.6

Find the armature conductors in series per phase required for a 3 phase, 50 cycles/sec, 10 pole alternator with 90 slots. The winding is to be star connected to give a line voltage of 11 kV. The flux per pole is 0.16 Wb.

$$\text{Slots/pole/phase, } m = \frac{90}{10 \times 3} = 3$$

$$\text{Short chording angle,} \alpha = 0 \qquad K_c = \cos\frac{\alpha}{2} = 1$$

$$\text{Slot angle, } \beta = \frac{180}{\text{Slots/pole}} = \frac{180}{9} = 20°$$

$$K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin\frac{3\times 20}{2}}{3\times\sin\frac{20}{2}} = 0.9598$$

$$\text{Per phase voltage} = \frac{\text{line voltage}}{\sqrt{3}} = \frac{11000}{\sqrt{3}} = 6351 \text{ V}$$

$$V_{Ph} = 4.44\, K_c\, K_d\, \Phi\, f\, T$$

$$\text{Number of turns, } T = \frac{V_{Ph}}{4.44\, K_c\, K_d\, \Phi\, f} = \frac{6351}{4.44\times 1\times 0.9598\times 0.16\times 50} = 186$$

Check for consistency

$$\text{Number of conductor in 3 phases} = 186 \times 2 \times 3 = 1116$$

$$\text{Number of conductor in one slot} = \frac{1116}{90} = 12.4$$

This number should be an integer; select 12

$$\text{Number of turns per phase, } T = \frac{12 \times 90}{2 \times 3} = 180$$

$$\text{Number of conductors in series, } Z = 180 \times 2 = 360$$

Exercise 1.7

A 3 phase 4 pole 24 slot alternator has 4 conductors per slot. Flux per pole is 0.2 Wb for the fundamental component and has 30% third harmonic flux. Find the induced rms voltage per phase if the phase spread is a) 60° and b) 120°. Winding is short chorded by 30° in both cases.

$$\text{Slot angle, } \beta = \frac{180}{\text{Slots/pole}} = \frac{180}{24/4} = 30°$$

$$K_c = \cos\frac{\alpha}{2} = \cos\frac{30}{2} = 0.966$$

$$T = \frac{24 \times 4}{2 \times 3} = 16$$

$$\Phi_3 = \frac{0.3 \times 0.2}{3} = 0.02\ \text{Wb}$$

a)

$$\text{Slots/pole/phase, } m = \frac{24}{4 \times 3} = 2$$

$$K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin\frac{2 \times 30}{2}}{2 \times \sin\frac{30}{2}} = 0.966$$

$$\text{Per phase fundamental voltage} = 4.44\, K_c\, K_d\, \Phi\, f\, T$$
$$= 4.44 \times 0.966 \times 0.966 \times 0.2 \times 50 \times 16 = 663 \text{ volts}$$

$$K_{c3} = \cos\frac{3\alpha}{2} = \cos\frac{3\times 30}{2} = 0.707 \qquad K_{d3} = \frac{\sin\dfrac{mn\beta}{2}}{m\sin\dfrac{n\beta}{2}} = \frac{\sin\dfrac{2\times 3\times 30}{2}}{2\times\sin\dfrac{3\times 30}{2}} = 0.707$$

$$\text{Per phase third harmonic voltage} = 4.44\, K_{c3}\, K_{d3}\, \Phi_3\, f_3\, T$$
$$= 4.44 \times 0.707 \times 0.707 \times 0.02 \times 150 \times 16 = 106.5 \text{ volts}$$

$$\text{Per phase voltage} = \sqrt{E_1^{\,2} + E_3^{\,2}}$$
$$= \sqrt{663^2 + 106.5^2} = 671.5 \text{ volts}$$

b)

Slots in a phase group, $m = 4$

$$K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin\frac{4\times 30}{2}}{4\times\sin\frac{30}{2}} = 0.8365$$

$$\text{Per phase fundamental voltage} = 4.44\,K_c\,K_d\,\Phi\,f\,T$$
$$= 4.44\times 0.966\times 0.8365\times 0.2\times 50\times 16 = 574 \text{ volts}$$

$$K_{c3} = \cos\frac{3\alpha}{2} = \cos\frac{3\times 30}{2} = 0.707$$

$$K_{d3} = \frac{\sin\frac{mn\beta}{2}}{m\sin\frac{n\beta}{2}} = \frac{\sin\frac{4\times 3\times 30}{2}}{4\times\sin\frac{3\times 30}{2}} = 0$$

Since $K_d = 0$, the third harmonic voltage $= 0$

Per phase voltage $= 574$ volts