

Problems :-

1) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$.

Answer:-

Given $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

The characteristic equation of A is given

by $|A - \lambda I| = 0$

$$\Rightarrow \lambda^2 - B_1\lambda + B_2 = 0, \text{ where } B_1 = \text{Sum of the diagonal elements of } A$$

$$\Rightarrow \lambda^2 - 10\lambda + 24 = 0 \quad \rightarrow (1) \quad = 10$$

By solving (1),

$$B_2 = |A| = 24$$

We get the Eigen values of A .

$$\text{Now, } \lambda^2 - 10\lambda + 24 = 0 \Rightarrow (\lambda - 4)(\lambda - 6) = 0$$

$$\Rightarrow \lambda = 4, 6.$$

Thus, the Eigen values of A are ~~are~~

$$\lambda = 4, 6.$$

Now, to find the Eigen Vectors corresponding to each λ , we ~~will~~ consider the homogeneous system $(A - \lambda I)x = 0$ & solve it for non-trivial solutions.

$$\text{Now, } (A - \lambda I)x = 0 \Rightarrow \begin{bmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow (2)$$

~~Case 1: $\lambda = 4$~~ Case 1: $\lambda = 4$

Put $\lambda = 4$ in (2), we get

$$\begin{bmatrix} 8-4 & -4 \\ 2 & 2-4 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -4 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Now

~~Consider the Coeff matrix with~~

$$\begin{bmatrix} 4 & -4 \\ 2 & -2 \end{bmatrix} \sim \begin{bmatrix} 4 & -4 \\ 0 & 0 \end{bmatrix} R_2 \rightarrow R_2 - \frac{1}{2}R_1$$

→ Echelon form.

Consider the equivalent systems

$$\begin{bmatrix} 4 & -4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 4m_1 - 4m_2 = 0 \rightarrow (E_1)$$

Thus, m_1 is leading & m_2 is free.

$$\text{Let } m_2 = a.$$

$$\text{From } (E_1), 4m_1 = 4m_2 \Rightarrow m_1 = m_2 = a$$

$$\text{Hence } \boxed{m_1 = a, m_2 = a}$$

$$\boxed{m_1 = a}$$

$$\text{Let } X = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} a \\ a \end{bmatrix}$$

Choose $a=1$, then $X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigen vector of A corresponding to the eigen value $\lambda = 4$.

Case 2: $\lambda = b$

Put $\lambda = 6$ in (2), we get

$$\begin{bmatrix} 2 & -4 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

~~Now consider the matrix~~ Now consider the coefficient matrix $\begin{bmatrix} 2 & -4 \\ 2 & -4 \end{bmatrix} \sim \begin{bmatrix} 2 & -4 \\ 0 & 0 \end{bmatrix} R_2 \rightarrow R_2 - R_1$ Echelon form

Consider the equivalent system

$$\begin{bmatrix} 2 & -4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 2m_1 - 4m_2 = 0 \rightarrow (E_1)$$

Now, m_1 is leading & m_2 is free.

$$\text{Let } m_2 = a$$

$$\text{From } (E_1), m_1 = 2m_2 \Rightarrow m_1 = 2a$$

$$= 2a.$$

$$\boxed{m_1 = 2a}$$

$$\text{Hence } \boxed{m_1 = 2a, m_2 = a}$$

$$\text{Let } X = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 2a \\ a \end{bmatrix}$$

Choose $a = 1$, then $X = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigenvector of A corresponding to the eigen value $\lambda = 6$.

2) Find the Eigen value & Eigen Vectors of the matrix $A = \begin{bmatrix} 7 & -1 \\ 4 & 3 \end{bmatrix}$

Answer:-

$$\text{Given } A = \begin{bmatrix} 7 & -1 \\ 4 & 3 \end{bmatrix}$$

Characteristic Eqn. of A is $|A - \lambda I| = 0$

$$\Rightarrow \lambda^2 - B_1\lambda + B_2 = 0 \quad \text{--- (4)}$$

$$\Rightarrow \lambda^2 - 10\lambda + 25 = 0 \quad \left[\begin{array}{l} B_1 = 7+3=10 \\ B_2 = |A| = 21+4=25 \end{array} \right] \quad \rightarrow (1)$$

By solving (1), we get the eigen values of A .

Consider $\lambda^2 - 10\lambda + 25 = 0$

$$\Rightarrow (\lambda-5)^2 = 0$$

$$\Rightarrow (\lambda-5)(\lambda-5) = 0$$

$$\Rightarrow \lambda = 5, 5 \quad (\text{i.e. } \lambda = 5 \text{ is a repeated root})$$

\therefore The eigen values of A are

$\lambda = 5, 5$ ($\lambda = 5$ is a repeated root of

the characteristic equation)
or $\lambda = 5$ is a repeated eigen value of A)

Now, to find the eigen vectors corresponding

to $\lambda = 5$, we consider the homogeneous

system $(A - \lambda I)x = 0$ & solve it for

a non-trivial solution

Consider $(A - \lambda I)x = 0$

$$\Rightarrow \begin{bmatrix} 7-\lambda & 3-\lambda \\ 4 & 3-\lambda \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

~~Case I: $\lambda = 5$~~

Here we have to consider only one case, i.e., for $\lambda = 5$, since $\lambda = 5$

is a repeated root eigen value of A .

Now, put $\lambda = 5$ in (2), we get

(5)

$$\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Now, consider the coefficient matrix

$$\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} R_2 \rightarrow R_2 - 2R_1$$

which is in Echelon form

So consider the equivalent system

$$\begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 2m_1 - m_2 = 0 \rightarrow (E_1)$$

Now, m_1 is leading & m_2 is free.

$$\text{Let } m_2 = a.$$

$$\text{From } (E_1), 2m_1 = m_2 \\ = a$$

$$\Rightarrow m_1 = \frac{a}{2}$$

$$m_1 = \frac{a}{2}$$

$$\text{Hence, } \boxed{m_1 = \frac{a}{2}, m_2 = a}$$

$$\text{Let } x = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} a/2 \\ a \end{bmatrix}$$

$$\text{Now, choose } a = 2, \text{ then } x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

An eigen vector of A corresponding to the eigen value $\lambda = 5$.

Now we can observe that $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ has

only one independent Eigen Vector corresponding

to the eigen value, $\lambda = 5$.

- 3) Find the Eigen Values & Eigen Vectors of -
the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

Answer:-

Given $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$

$$\Rightarrow \lambda^3 - 9\lambda^2 + 11\lambda - 6 = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0 \quad \rightarrow (1)$$

By solving (1), we get the
Eigen values of A .

Now, to solve (1), we look
for integer valued solutions
and let's try with $\pm 1, \pm 2, \pm 3$

& ± 6 [Consider only the factors
of the constant term of
(1), i.e., -6]

$$\lambda = -1 \Rightarrow (-1)^3 - 6(-1)^2 + 11(-1) - 6 = 0$$
$$= -1 - 6 + 11 - 6$$
$$= 0$$

$$\lambda = 1 \Rightarrow 1 - 6 + 11 - 6 = 0$$

Thus $\lambda = 1$ is a root of (1).

Now, to obtain the remaining
two roots of (1), let's divide (1)
by $\lambda - 1$ or by ~~its factors~~.

$$C_1 = 1+2+3 -$$

$$m = 6$$

$$C_2 = \left| \begin{array}{cc} 2 & 1 \\ 2 & 3 \end{array} \right| + \left| \begin{array}{cc} 1 & -1 \\ 2 & 3 \end{array} \right|$$

$$= 4 + 5 + 2$$

$$= 11$$

$$C_3 = \left| \begin{array}{ccc} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{array} \right|$$

$$= 1(4) - 0(1) +$$
$$-1(-2)$$

$$= 4 + 2$$

$$= 6$$

by applying the Synthetic division as follows:

The root
we obtained
from trial

$$\begin{array}{c} \xrightarrow{\text{Coefficients of } x^3, x^2, x} \\ \begin{array}{c} | \\ 1 & -6 & 11 & -6 \\ | \\ 0 & 1 & -5 & 6 \\ \hline 1 & -5 & 6 & 0 \end{array} \end{array}$$

& Constant
term of A

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow (x-2)(x-3) = 0$$

$$\Rightarrow x = 2, 3. \quad (\text{the remaining two roots of } A)$$

[Note: - 1, -5, 6 are
respectively to be the
coefficients of the
2nd Degree equation
to be derived for
the remaining two roots]

Hence the eigen values of A are

$$\lambda = 1, 2, 3.$$

of A

Next, to find the Eigen Vectors corresponding to each λ , we consider the homogeneous system $(A - \lambda I)x = 0$ & solve it for non-trivial solutions.

$$\text{Now, } (A - \lambda I)x = 0$$

$$\Rightarrow \begin{bmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \rightarrow (2)$$

Case 1: $\lambda = 1$

Put $\lambda = 1$ in (2), we get

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now for solving the system, consider the Coefficient

matrix $\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$ ⑧

reduce it into
Echelon form

Now

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \xrightarrow{\text{R}_2 - R_1, \text{R}_3 - 2R_1} \begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 2 & 2 & 2 \end{bmatrix} \xrightarrow{\text{R}_1 \leftrightarrow \text{R}_2} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{R}_2 \rightarrow \text{R}_2 - 2\text{R}_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

which is in Echelon form

Now, consider the equivalent systems

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow m_1 + m_2 + m_3 = 0 \rightarrow (E_1)$$

$$-m_3 = 0 \rightarrow (E_2)$$

Here m_1 & m_3 are leading variable &

m_2 is free variable.

Let $m_2 = a$

From (E_2) , we have $m_3 = 0$.

From (E_1) , $m_1 = -m_2 - m_3$

$$= -\alpha - 0$$

$$= -\alpha$$

$$\Rightarrow m_1 = -\alpha$$

Now, we have

$$m_1 = -\alpha, m_2 = \alpha, m_3 = 0$$

$$\text{Let } X = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -\alpha \\ \alpha \\ 0 \end{bmatrix}$$

Now, choose $\alpha = 1$, thus $X_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ is

an eigen vector of A corresponding to the eigen value $\lambda = 1$.

Case 2 : $\lambda = 2$

Put $\lambda = 2$ in (2), we get

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, for solving this system, consider the

Coefficient matrix

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

& reduce it into Echelon form

Now,

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & -1 \\ 2 & 2 & 1 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$2 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix} \quad R_2 \rightarrow R_2 + R_1 \\ R_3 \rightarrow R_3 - 2R_1$$

$$2 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

↳ which is in Echelon form.
Now, consider the equivalent system

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow m_1 + m_3 = 0 \rightarrow (E_1)$$

$$2m_2 - m_3 = 0 \rightarrow (E_2)$$

Then, the leading variables are m_1 & m_2
& m_3 is ~~a free variable.~~

$$\text{Let } m_3 = a$$

$$\text{From } (E_2), 2m_2 = m_3$$

$$\Rightarrow m_2 = \frac{m_3}{2}$$

$$= \underline{\underline{\frac{a}{2}}}$$

$$\text{From } (E_1), m_1 = -m_3$$

$$= -\underline{\underline{\frac{a}{2}}}$$

Thus, we have $m_1 = -a$, $m_2 = \frac{-a}{2}$, $m_3 = a$ (11)

$$\text{Let } X_2 = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -a \\ a/2 \\ a \end{bmatrix}$$

Now, choosing $a = 2$, thus $X_2 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$

an eigenvector of A corresponding to the eigen value $\lambda = 2$.

Case 3: $\lambda = 3$

Put $\lambda = 3$ in (2), we get

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, for solving the system, consider the Coefficient matrix $\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}$ & reduce it into Echelon form.

$$\text{Now, } \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 \\ -2 & 0 & -1 \\ 2 & 2 & 0 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 4 & -2 \end{bmatrix} R_2 \rightarrow R_2 + 2R_1 \\ R_3 \rightarrow R_3 - 2R_1$$

$$N \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} R_3 \rightarrow R_3 + 2R_2$$

which is in Echelon form.

Now, consider the equivalent system

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow m_1 - m_2 + m_3 = 0 \rightarrow (E_1)$$

$$-2m_2 + m_3 = 0 \rightarrow (E_2)$$

Here m_1 & m_2 are leading variable
& m_3 is free variable.

$$\text{Let } m_3 = a$$

$$\text{From } (E_2), -2m_2 = m_3$$

$$\Rightarrow m_2 = \frac{m_3}{2}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \frac{a}{2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{From } (E_1), m_1 = m_2 - m_3$$

$$= \frac{a}{2} - a$$

$$= -\frac{a}{2}$$

Thus, we have

$$m_1 = -\frac{a}{2}, m_2 = \frac{a}{2}, m_3 = a$$

$$\text{Let } X_3 = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} -a/2 \\ a/2 \\ a \end{bmatrix}$$

$$\text{Now choose } a=2, \text{ then } X_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

is an eigen vector of A corresponding to the eigen value $\lambda=3$.

$$\text{Hence the Eigen values of } A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

But $\lambda = 1, 2, 3$ & the corresponding Eigen Vectors of A are $X_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, X_2 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$

$$\text{and } X_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

Q) Find the Eigen Values and Eigen Vectors of the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

Solution:-

$$\text{Given } A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

The characteristic equation of A is $(A - \lambda I)^0$

$$\Rightarrow \lambda^3 - 9\lambda^2 + 27\lambda - 54 = 0$$

$$(14) \Rightarrow \lambda^3 - 0\lambda^2 + (-12)\lambda - 16 = 0$$

$$\Rightarrow \lambda^3 - 12\lambda - 16 = 0$$

$$\Rightarrow \lambda^3 - 12\lambda - 16 = 0 \quad \text{--- (1)}$$

By solving (1), we get the Eigen values of A.

Now, for solving, we consider

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$ as the factors of the constant term $-(-16)$ in (1).

$$\begin{aligned} \lambda &= -1, (-1)^3 - 12(-1) - 16 \\ &= -1 + 12 - 16 \neq 0 \end{aligned}$$

$$\lambda = 1, 1 - 12 - 16 \neq 0$$

$$\begin{aligned} \lambda &= -2, (-2)^3 - 12(-2) - 16 \\ &= -8 + 24 - 16 \end{aligned}$$

$$\Rightarrow \lambda = -2 \text{ is a root of (1)}$$

Let us find the other two roots of (1) as follows:

$$\begin{array}{r} -2 \\ \hline 1 & 0 & -12 & -16 \\ 0 & -2 & 4 & 16 \\ \hline 1 & -2 & -8 & 0 \end{array}$$

$$\Rightarrow \lambda^2 - 2\lambda - 8 = 0$$

$$\Rightarrow (\lambda + 2)(\lambda - 4) = 0$$

$$\Rightarrow \lambda = -2, 4$$

$$C_1 = 0 (1+(-5)+4=0)$$

$$C_2 = \begin{vmatrix} -5 & 3 \\ -6 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 6 & 4 \end{vmatrix}$$

$$+ \begin{vmatrix} 1 & -3 \\ 3 & -5 \end{vmatrix}$$

$$= -2 + (-14) + 4$$

$$= -12$$

$$C_3 = \begin{vmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{vmatrix}$$

$$= 1(-2) - (-3)(-6)$$

$$+ 3(12)$$

$$= -2 - 18 + 36$$

$$= 16$$

$$= 2 \times 16$$

for method copy own value copy into part (4)

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Hence the Eigen values of A are (15)
 $\lambda = 4, -2, -2$ ($\lambda = -2$ is a repeated eigen value of A)

Next, to find the Eigen Vectors of A corresponding to each λ , consider the homogeneous system $(A - \lambda I)x = 0$ & solve it for a non-trivial solution.

Now, $(A - \lambda I)x = 0$

$$\Rightarrow \begin{bmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \rightarrow (2)$$

Case 1 : $\lambda = 4$

Put $\lambda = 4$ in (2), we get

$$\begin{bmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, for solving this system, consider the coefficient matrix & reduce it into the echelon form.

i.e. $\begin{bmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix}$

$\xrightarrow{R_2 \rightarrow R_2 + R_3}$
 $\xrightarrow{R_3 \rightarrow R_3 + 2R_1}$

$$\begin{bmatrix} -3 & -3 & 3 \\ 0 & -12 & 6 \\ 0 & -12 & 6 \end{bmatrix}$$

(16)

$$\sim \begin{bmatrix} -3 & -3 & 3 \\ 0 & -12 & 6 \\ 0 & 0 & 0 \end{bmatrix} R_3 \rightarrow R_3 - R_2$$

for which it is in Echelon form.

Now, consider the equivalent system

$$\begin{bmatrix} -3 & -3 & 3 \\ 0 & -12 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -3m_1 - 3m_2 + 3m_3 = 0 \rightarrow E_1$$

$$-12m_2 + 6m_3 = 0 \rightarrow E_2$$

Here m_1 & m_2 are leading variables
& m_3 is a free variable.

$$\text{Let } m_3 = a$$

$$\text{From } E_2, -12m_2 = -6m_3$$

$$\Rightarrow m_2 = \frac{1}{2}m_3$$

$$= \frac{a}{2}$$

$$\text{From } E_1, -3m_1 = 3m_2 - 3m_3$$

$$\Rightarrow m_1 = m_2 - m_3$$

$$= \frac{a}{2} - a$$

$$= -\frac{a}{2}$$

Thus, we have $m_1 = -\frac{a}{2}$, $m_2 = \frac{a}{2}$, $m_3 = a$

$$\text{Let } x_1 = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -a/2 \\ a/2 \\ a \end{bmatrix}$$

Now, choose $a = 2$, then $x_1 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ is an eigen vector of A corresponding to the eigen value $\lambda = 4$.

Case 2: $\lambda = -2$

Put $\lambda = -2$ in (2), we get

$$\begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, for solving this system, consider the coefficient matrix & reduce it into the Echelon form.

$$\text{i.e., } \begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & -3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - 2R_1$$

which is in Echelon form.

Now consider the equivalent system,

$$\begin{bmatrix} 3 & -3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow 3m_1 - 3m_2 + 3m_3 = 0 \rightarrow E_1$$

Here m_1 is a leading variable & m_2 & m_3 are free variables.

$$\text{Let } m_2 = a, m_3 = b.$$

$$\text{From } E_1, 3m_1 = 3m_2 - 3m_3$$

$$\rightarrow m_1 = m_2 - m_3$$

$$= a - b$$

$$\text{Thus, we have } m_1 = a - b, m_2 = a, m_3 = b$$

$$\text{Let } X = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} a-b \\ a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} a + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} b$$

~~Now choose $a=1, b=0$~~

~~Now choose $a=0, b=1$, we get~~

Now, we can find two independent eigenvectors of A corresponding to the eigen value $\lambda = -2$ by choosing $a=1, b=0$ & $a=0, b=1$.

$$\text{is when } a=1, b=0, \text{ we get } X_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{When } a=0, b=1, \text{ we get } X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Thus the Eigen values of A are $-4, -2, -2$
 & the corresponding Eigen vectors are
 $x_1 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ & $x_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

5) Find the Eigen Value & Eigen vectors
 of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$.

Solution:-

Given $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$

The characteristic equation of A is
 $|A - \lambda I| = 0$.

$$\Rightarrow \lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$$

$$\Rightarrow \lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0 \quad \rightarrow ①$$

Now By solving ①
 We get the Eigen value
 of A .

Now for solving ①, we
 consider the factors of
 -12 as $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6,$
 ± 12

$$\lambda = -1, (-1)^3 - 7(-1)^2 + 16(-1) - 12 \\ = -1 - 7 - 16 - 12 \neq 0$$

$$\lambda = 1, 1 - 7 + 16 - 12 \neq 0$$

$$C_1 = 2 + 1 + 4 = 7$$

$$C_2 = \begin{vmatrix} 1 & -1 \\ 2 & 4 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} \\ = 6 + 8 + 2$$

$$= 16$$

$$C_3 = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{vmatrix}$$

$$= 2(6) - 1(6) + 0(0)$$

$$= 12$$

$$\lambda = -2, -8 - 2\lambda - 32 - 12 \neq 0 \text{ with } \lambda$$

$$\lambda = 2, 8 - 2\lambda + 32 - 12 = 0 \text{ good with } \lambda$$

Thus $\lambda = 2$ is a root of ①.

The other two roots of ① can be

obtained as follows:

$$2 \left| \begin{array}{ccc|c} 1 & -4 & 16 & -12 \\ 0 & 2 & -10 & 12 \\ \hline 1 & -5 & 6 & 0 \end{array} \right.$$

$$\Rightarrow \lambda^2 - 5\lambda + 6 = 0$$

$$\Rightarrow (\lambda - 2)(\lambda - 3) = 0$$

$$\Rightarrow \lambda = 2, 3.$$

Hence the eigen values of A are

$$\lambda = 3, 2, 2 \quad (\lambda = 2 \text{ is a repeated eigen value of } A)$$

Next, to find the eigen vectors of A corresponding to each λ , consider the homogeneous system $(A - \lambda I)x = 0$ & solve it for a non-trivial solution.

$$\text{Now, } (A - \lambda I)x = 0$$

$$\Rightarrow \begin{bmatrix} 2-\lambda & 1 & 0 \\ 0 & 1-\lambda & -1 \\ 0 & 2 & 4-\lambda \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

→ ②

Case 1: $\lambda = 3$

Put $\lambda = 3$ in ②, we get

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, for solving this system, consider the coefficient matrix & reduce it into the Echelon form

is

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$\sim \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $R_3 \rightarrow R_3 + R_2$

which is in Echelon form
 Now, consider the equivalent system

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -m_1 + m_2 = 0 \rightarrow \textcircled{E}_1$$

$$\Rightarrow -2m_2 - m_3 = 0 \rightarrow \textcircled{E}_2$$

here m_1 & m_2 are leading variables &
 m_3 is a free variable.

Let $m_3 = a$

From \textcircled{E}_2 , $-2m_2 = m_3$

$$\Rightarrow m_2 = -\frac{1}{2}m_3$$

$$= -\frac{a}{2}$$

From (E), $-m_1 = -m_2 \Rightarrow m_1 = m_2$

$$\Rightarrow m_1 = m_2$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus, we have, $m_1 = -\frac{a}{2}$, $m_2 = -\frac{a}{2}$, $m_3 = a$.

$$\text{Let } X = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -a/2 \\ -a/2 \\ a \end{bmatrix}$$

$$\text{Now, choose } a=2, \text{ then } X_1 = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

is an eigenvector of A corresponding to the eigen value $\lambda = 3$.

Case 2: $\lambda = 2$

Put $\lambda = 2$ in ②, we get

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, for solving this system, consider the coefficient matrix & reduce it into the Echelon form.

$$\text{ie } \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\xrightarrow[R_2 \rightarrow R_2 + R_1]{R_3 \rightarrow R_3 - 2R_1} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\sim \left[\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{array} \right] R_3 \rightarrow R_3 + 2R_2$$

which is in Echelon form.

Now, consider the equivalent system

$$\left[\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{array} \right] \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow m_2 = 0 \rightarrow \textcircled{E}_1$$

$$-m_3 = 0 \rightarrow \textcircled{E}_2$$

Here m_2 & m_3 are leading variables
& m_1 is a free variable.

$$\text{Let } m_1 = a$$

$$\text{From } \textcircled{E}_1, m_2 = 0$$

$$\text{From } \textcircled{E}_2, m_3 = 0.$$

Thus, we have $m_1 = a, m_2 = 0, m_3 = 0$.

$$\text{Let } x = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

Now, choose $a = 1$, then $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is
an eigen vector of A corresponding
to the eigen value $\lambda = 2$.

Here, we noticed that there is only one
independent eigen vector of A corresponding
to the repeated eigen value $\lambda = 2$.

Thus, we found that $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$ (24)

only

two independent eigenvectors

corresponding to the eigen values $\lambda = 3, 2, 2$

one is $x_1 = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$ corresponding to the

eigen value $\lambda = 3$ & the other is

$x_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ corresponding to the eigen

value $\lambda = 2$ ($\lambda = 2$ is a repeated eigen value)

Q6) Find the Eigenvalues & Eigenvectors of the matrix $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$ (25)

Solution:-

Given $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\Rightarrow \lambda^3 - 9\lambda^2 + 27\lambda - 1 = 0$$

$$\Rightarrow \lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0 \quad \rightarrow ①$$

By solving ①, we get the eigen values of A .

For solving ①, we consider the factors of -1.

the constant term, ($\lambda = -1$)

$\lambda \neq 1$.

$$\lambda = -1, (-1)^3 - 3(-1)^2 + 3(-1) - 1$$

$$= -1 - 3 - 3 - 1$$

$\neq 0$

$$\lambda = 1, 1 - 3 + 3 - 1 = 0$$

Then $\lambda = 1$ is a root of ①.

To find the other two roots of ①, we

$$C_1 = -3 + 4 + 2$$

$$= 3$$

$$C_2 = \begin{vmatrix} 4 & 3 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} -3 & -5 \\ 1 & 2 \end{vmatrix}$$

$$+ \begin{vmatrix} -3 & -7 \\ 2 & 4 \end{vmatrix}$$

$$= 2 + (-1) + 2$$

$$= \frac{3}{3}$$

$$C_3 = \begin{vmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{vmatrix}$$

$$= -3(2) - (-7)(1) - 5(0)$$

$$= -6 + 7$$

$$= 1$$

proceeded as follows;

$$\left[\begin{array}{ccc|c} 2 & 1 & -3 & 3 \\ 2 & 0 & 1 & -2 \\ 2 & 0 & 1 & 1 \end{array} \right] \xrightarrow{\text{R}_1 - R_2, \text{R}_3 - R_2} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\Rightarrow \lambda^2 + 2\lambda + 1 = 0$$

$$\Rightarrow (\lambda + 1)^2 = 0$$

$$\Rightarrow (\lambda + 1)(\lambda + 1) = 0$$

$$\Rightarrow \lambda = 1, 1, 1$$

Thus, the eigen values of A are

$\lambda = 1, 1, 1$ ($\lambda = 1$ is a repeated eigen value of A)

Next, to find the eigen vectors of A corresponding to the eigen value λ , consider the homogeneous system

$(A - \lambda I)x = 0$ & solve it for a non-trivial solution.

$$\text{Now, } (A - \lambda I)x = 0$$

$$\Rightarrow \left[\begin{array}{ccc|c} -3 & -1 & -5 & 0 \\ 2 & 4 & 3 & 0 \\ 1 & 2 & 2 & 0 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \quad (2)$$

Here we have only one for

$\lambda = 1$, since $\lambda = 1$ is repeated 3 times.

Now put $\lambda = 1$ in (2), we get

$$\begin{bmatrix} -4 & -7 & -5 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, for solving this system, consider the coefficient matrix & reduce it into the Echelon form.

i.e. $\begin{bmatrix} -4 & -7 & -5 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ -4 & -7 & -5 \end{bmatrix} R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 + 4R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} R_3 \rightarrow R_3 + R_2$$

which is in Echelon form.

Now consider the equivalent system

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow m_1 + 2m_2 + m_3 = 0 \rightarrow E_1 \\ -m_2 + m_3 = 0 \rightarrow E_2$$

Here m_1 & m_2 are leading variables &

m_3 is a free variable.

(28)

Let $m_3 = a$

From $E_2 \rightarrow m_2 = -m_3$ or $m_2 = -a$
the number of wolf
= $m_2 = m_3$ (with
wolf counted twice)
 $= a$.

Thus $m_2 = a$.

From $E_1 \rightarrow m_1 = -2m_2 - m_3$
 $= -2a - a$
 $= -3a$

Thus, we have $m_1 = -3a$, $m_2 = a$, $m_3 = a$

$$\text{Let } X = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -3a \\ a \\ a \end{bmatrix}$$

Now choose $a=1$, then $X = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$

is an eigenvector of A corresponding
to the eigen value $\lambda = 1$.

Here, we noticed that $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 6 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

There is only one independent eigenvector $X = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$ corresponding to
the repeated eigen value $\lambda = 1$.

∴ We can find only one solution.

Note:-

- 1) If A is a 3×3 matrix and A has three distinct eigen values, then we can always find three independent eigen vectors corresponding to the eigen values.
- 2) If A is a 3×3 matrix and A has repeated eigen values, then it is not always to find three independent eigen vectors corresponding to the eigen values.

In some cases, we ~~can't~~ we can find three independent eigen vectors (refer : Problem 4), but in some cases, if we cannot find three independent eigen vectors corresponding to the eigen value (refer : Problem 5) & (refer : Problem 6)