9/11/2020

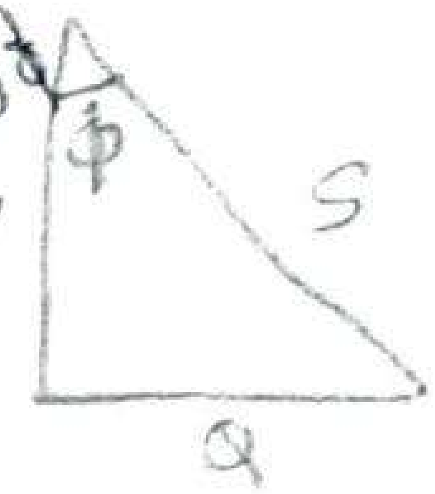
Module IV

$S = P + jQ$

$S = VI$

$P = VI\cos\phi$

$Q = VI\sin\phi$



P → Active → VI Cos φ → W

Q → Reactive → VI Sin φ → VAR Volt Ampere Reactive

S → Apparent → VI → VA

P+jQ

Q. An impedence of $(3+j4)\ \Omega$ is connected in parallel with a resistance 10Ω. Find the ratio of power loss in these parallel circuits.

$$Z_1 = 3+j4 = 5\angle 53^\circ\ \Omega$$

$$Z_2 = 10\ \Omega$$

$$I_1 = \frac{V}{Z_1} = \frac{V}{5\angle 53^\circ}\ \Omega$$

$$I_2 = \frac{V}{Z_2} = \frac{V}{10}\ \Omega$$

Thus $$\frac{I_1}{I_2} = \frac{V/5\angle 53}{V/10} = \frac{10}{5} = \frac{2}{1}$$

$$\frac{P_1}{P_2} = \frac{I_1^2 R_1}{I_2^2 R_1} = \left(\frac{I_1}{I_2}\right)^2 \frac{R_1}{R_2} = 2^2 \times \frac{3}{10} = \frac{6}{5}$$

The ratio of power loss $= 6/5$.

Q. A $\frac{1}{2}$ HP induction motor runs at an efficiency of 85%. If the operating power factor is 0.8 lag find the reactive power taken by motor.

$\frac{1}{5}$ HP $= \frac{746}{5} = 149.2\,W$ active power

$P_{input} = \frac{P_{output}}{\eta}$ η = efficiency

85% = 0.85 $= \frac{149.2}{0.85} = 175.529\,W$

$= 175.53\,W = VI\cos\phi$

VI cos φ - active
VI sin φ - react
VI - apparent

Power factor = 0.8.

$VI\cos\phi = 175.53$.

$VI = \frac{175.53}{0.8}$ $\sin\phi = 0.6$.

=

$VI\sin\phi = 131.65$ VAR. $\rightarrow \frac{149.2}{0.85} \times \frac{1}{0.8} \times 0.6$

Q. On a power destruction system three loads run in parallel.

Load A = 100 VA , 0.5 pf lag
Load B = 150 W , 0.8 pf lead
Load C = 200 VA , 100 VAR lag Find power post

Load A

$S_A = 100$ VA $\cos\phi = 0.5$

$P_A = S_A \times pf = 100 \times 0.5 = 50\,W$

$\sin\phi = 0.866$

$Q_A = 100 \times 0.866$

$= 86.6$ VAR

Load B.

$P_B = 150\,W \quad \cos\phi = 0.8$ lead.

$\sin\phi = 0.6$

$$S_B = \frac{P_B}{\cos\phi} = \frac{150}{0.8} = \underline{185.5\ VA}$$

$$Q_B = \overset{lead}{-} S_B \sin\phi$$

$$= -187.5 \times 0.6 = \underline{-112.5\ VAR}$$

For load C.

$S_C = 200 \qquad Q_C = 100\ VAR$

$$\sin\phi = \frac{Q_C}{S_C} = \frac{100}{200} = \underline{0.5}$$

$\phi = 30^\circ$ and $\cos\phi = 0.866$ (lag)

$$P_C = S_C \times pf = 200 \times 0.866$$

$$= \underline{\underline{173.2\ W}}$$

Hence, Net $P = P_A + P_B + P_C$

$$= 50 + 150 + 173.2$$

$$= \underline{373.2\ W}$$

$$\text{Net } Q = Q_A + Q_B + Q_C$$

$$= 86.6 - 112.5 + 100$$

$$= \underline{74.1\ VAR}$$

$$\text{Net } S = \sqrt{P^2 + Q^2} = \sqrt{373.2^2 + 74.1^2}$$

$$= \underline{\underline{380.46\ VA}}$$

Q. Two parallel branches z_1 and z_2 takes current I_1 and I_2

$I_1 = 3.15\angle 68^\circ A$ $\quad$ $I_2 = 12\angle -45^\circ A$.

Find the complex power drawn from the supply voltage is $17\angle 0^\circ$.

Soln :- $I_1 = 3.15\angle 68^\circ$ $\quad$ $I_2 = 12\angle -45^\circ A$

$$I = I_1 + I_2$$

$$= 3.15\angle 68 + 12\angle -45^\circ$$

$$= \underline{11.1\angle -29.8^\circ}.$$

Complex power

$$S = VI = 17\angle 0 \times 11.1\angle +29.8$$

$$= 189\angle 29.8$$

$$= \underline{164 + j94 \text{ VA}}$$

ie complex power $= 164\text{ W} + j94 \text{ VAR}$.

Q. The current in the circuit lag the voltage by 30°. If the input power be 400 W and the supply voltage be $V = 100\sin(377t + 10^\circ)$. Find complex power.

$\phi = 30^\circ$ lag $\quad$ $\cos\phi = \cos 30 = 0.866$ lag.

$$I = \frac{P}{V\cos\phi} = \frac{400}{\frac{100}{\sqrt{2}} \times 0.866}$$

$$= 6.53 \text{ A at } \angle -30^\circ.$$

Again S

$$V1 = \frac{100}{\sqrt{2}} \times 6.53 \angle +30^\circ$$

$$= 461.74 (\cos 30^\circ + j \sin 30)$$

$$= 10(40 + j23.1) \text{ VA.}$$

10/11/2020

Q. Voltage across 2 series connected circuit elements $V_1 = 100 \sin \omega t$ V and $V_2 = 50 \sin(\omega t - 30^\circ)$ V. If the circuit current is $(4+j2)$ A, find the complex power circuit.

rms value

$$V_1 = \frac{100}{\sqrt{2}} \sin \omega t = \frac{100}{\sqrt{2}} \angle 0.$$

$\angle +30^\circ = \cos 30 + j \sin$

$$= 70.72 + j0 \text{ V.}$$

$$V_2 = \frac{50}{\sqrt{2}} \angle -30^\circ = 35.36(\cos 30^\circ - j \sin 30^\circ)$$

$$= (30.6 - j17.68) \text{ V.}$$

Net voltage $= V_1 + V_2$

$$= (70.72) + (30.62 - j17.68)$$

$$= (101.34 - j17.68) \text{ V}$$

$$= 102.87 \angle -9.9^\circ \text{ V}$$

$$S = V \times \bar{I}$$

$$= 102.87 \angle -9.9^\circ \times 4 + j2$$

$$= 102.87 \angle -9.9 \times 4.47 \angle -26.56$$

$$= 459.8^\circ \angle -36.46^\circ \text{ VA}$$

$$= (370 - j273) \text{ VA}$$

2. A 10KW single phase motor is supplied by an ~~an~~ ac source of 50 Hz. If the terminal voltage of motor is $230\angle 0^\circ$ volts, the pf of the motor be 0.5 lags, the inter connecting cable of the voltage source and the motor be having $0.1\,\Omega$ resistance. find source power.

$$I_{source} = \frac{\text{Active Power}}{\text{Voltage} \times p.f} = \frac{10 \times 10^3}{230 \times .5} = 87\,A$$

$V = 87 \times 0.1\,\Omega$ ie $8.7\angle \cos^{-1} 0.5 = 60^\circ = 8.7\angle -60$

Supply voltage = Vector sum of [terminal V of motor (output end) + drop in cable

$$= 230\angle 0^\circ + 8.7\angle -60^\circ$$

$$= 237.65\angle -1.8^\circ\,V.$$

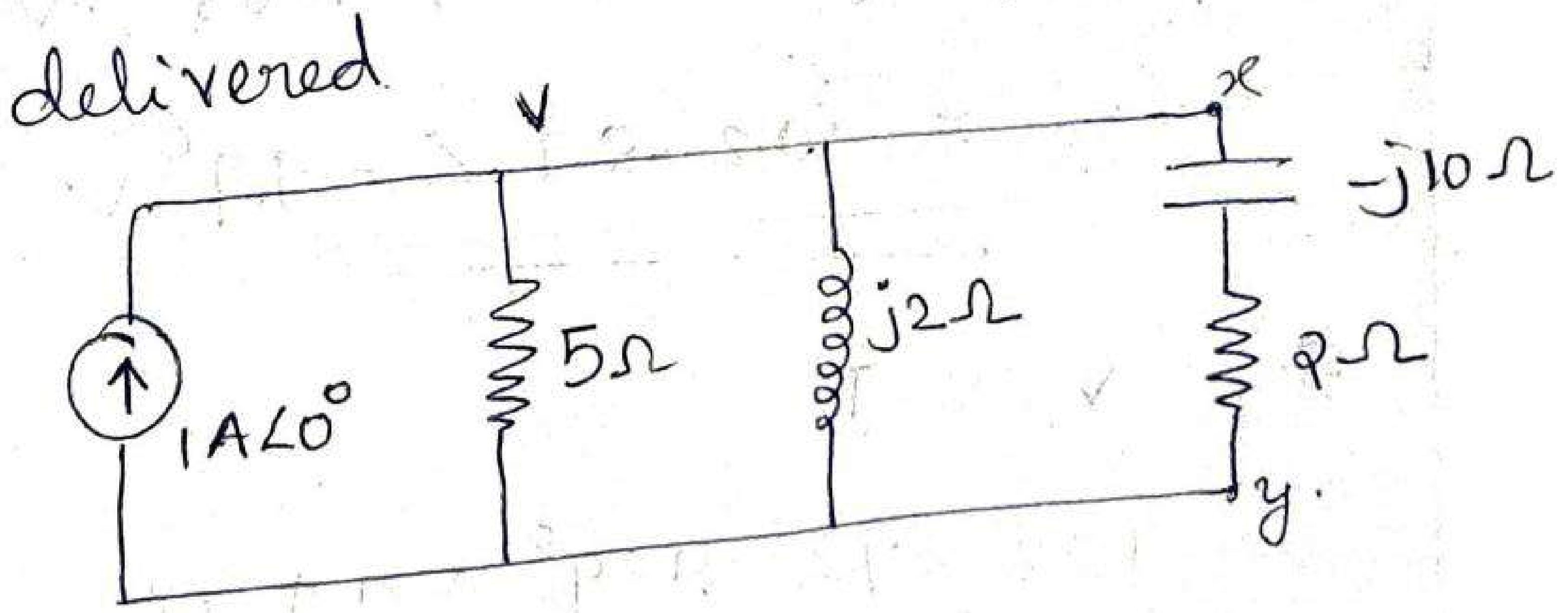
Thus source power $= VI\cos\theta$

$$= 237.65 \times 87 \times \cos(60 - 1.8)$$

$$= 10895\,W.$$

Power source $= 10.89\,KW$

Q3, Find power delivered



$$\frac{V}{5} + \frac{V}{j2} + \frac{V}{2 - j10} = 1 + j0$$

KtuQbank

$$\frac{V}{5}+\frac{V}{j2}+\frac{V}{2}-\frac{V}{j10}=1 \qquad \frac{V}{5}-j\frac{2V}{2^2}+\frac{V(2+j10)}{2^2+10^2}$$

$$\frac{V}{5}-j\frac{V}{2}+\frac{V}{52}+j\frac{5V}{52}=1$$

$$\frac{V}{5}+\frac{V}{2}+\frac{V}{j2}-\frac{V}{j10}=1$$

$$V=(0.2+0.0192)+j(-0.5-0.096)=1$$

$$=\frac{1}{0.2192-j\,0.404}=1.03+1.91i$$

$$=\underline{\underline{2.176\angle 61.52\ V}} \qquad \underline{\underline{|V|=\approx 2.176\,V}}$$

here voltage leads the current by 61.52^0.

Active power $= VI\cos\phi$

$$=2.17\times 1\times\cos 61.52^0$$

$$P=\underline{\underline{1.04\ W}}$$

Reactive power $= VI\sin\phi$

$$=2.17\times 1\times\sin 61.52$$

$$Q=\underline{\underline{1.913\ W}}$$

Total power $=\sqrt{P^2+Q^2}=\sqrt{1.04^2+1.913^2}$

$$=\underline{\underline{2.177\ VA}}$$

How much power is discipated in 5Ω and 2Ω R.

$V=2.176$.

$$P_5=I^2\times R=\left(\frac{V}{5}\right)^2\times 5$$

$$=\frac{2.176^2}{5^2}\times 5=\underline{\underline{0.947\,W}}$$

$$P_2 = I^2 R = \left|\frac{V}{2-j10}\right|^2 \times 2$$

$$= \cancel{\frac{2.176^2}{(2-j10)^2} \times 2} = \left|\frac{1.04-j1.91}{2-j10}\right|^2 \times 2$$

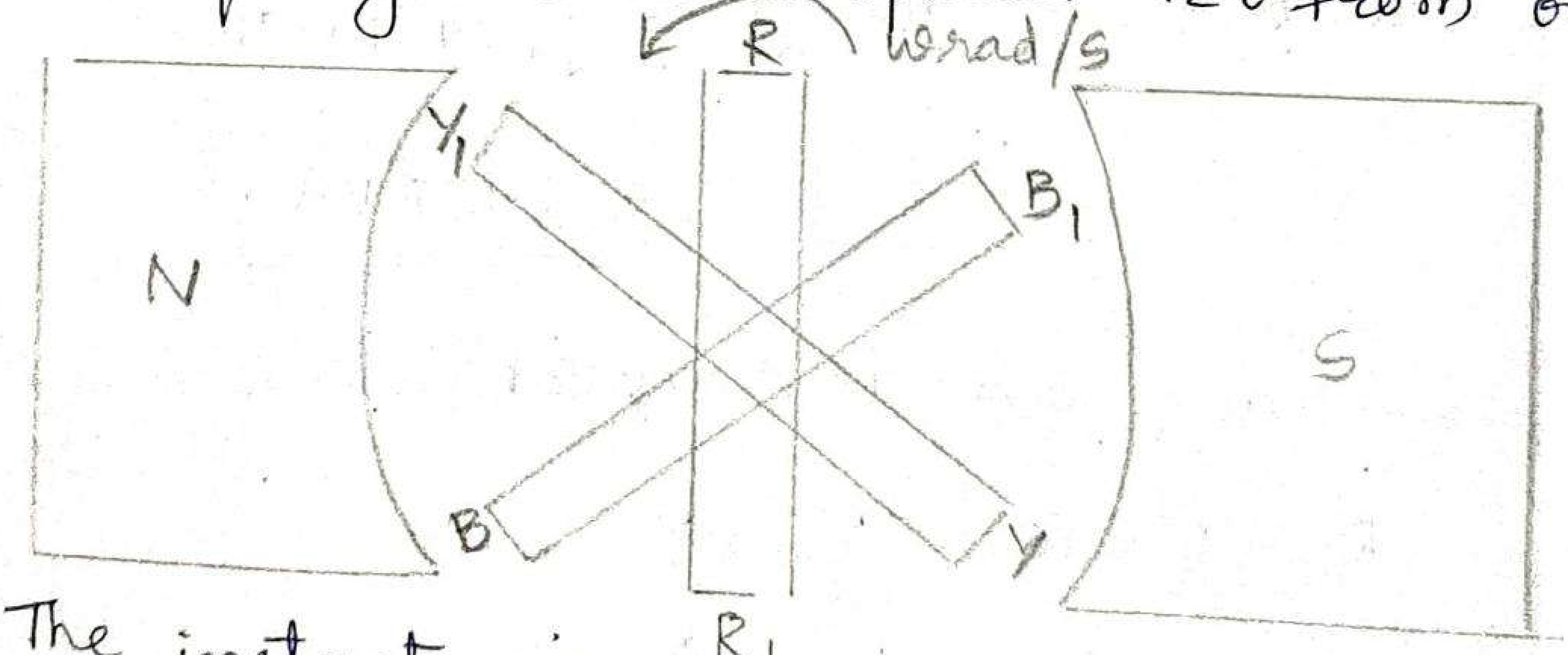
$$= \underline{0.09\ W}$$

12/11/2020

3 phase system

A system which generates a single alternating voltage and current is termed as a single-phase system. It utilises only one winding. A polyphase system utilises more than one winding. It will produce as many induced voltages as the number of windings.

A three phase system consists of three seperate but identical windings displayed by 120° from each other. When these 3 windings are rotated in an anticlock wise direction with constant angular velocity in the uniform magnetic field, the emf's induced in each winding have the same magnitude and frequency but are displaced 120° from one another.



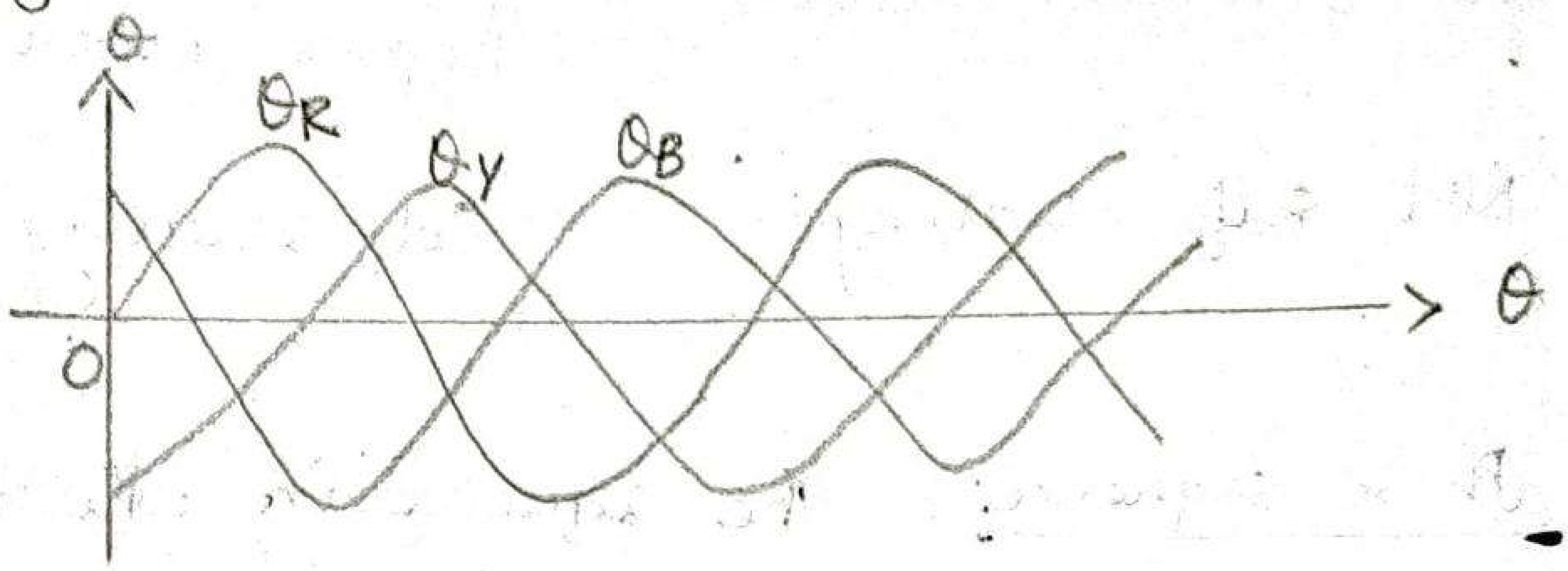
The instantanious values of generated voltage in winding RR_1, YY_1 and BB_1 are

$e_R = E_m \sin\theta$

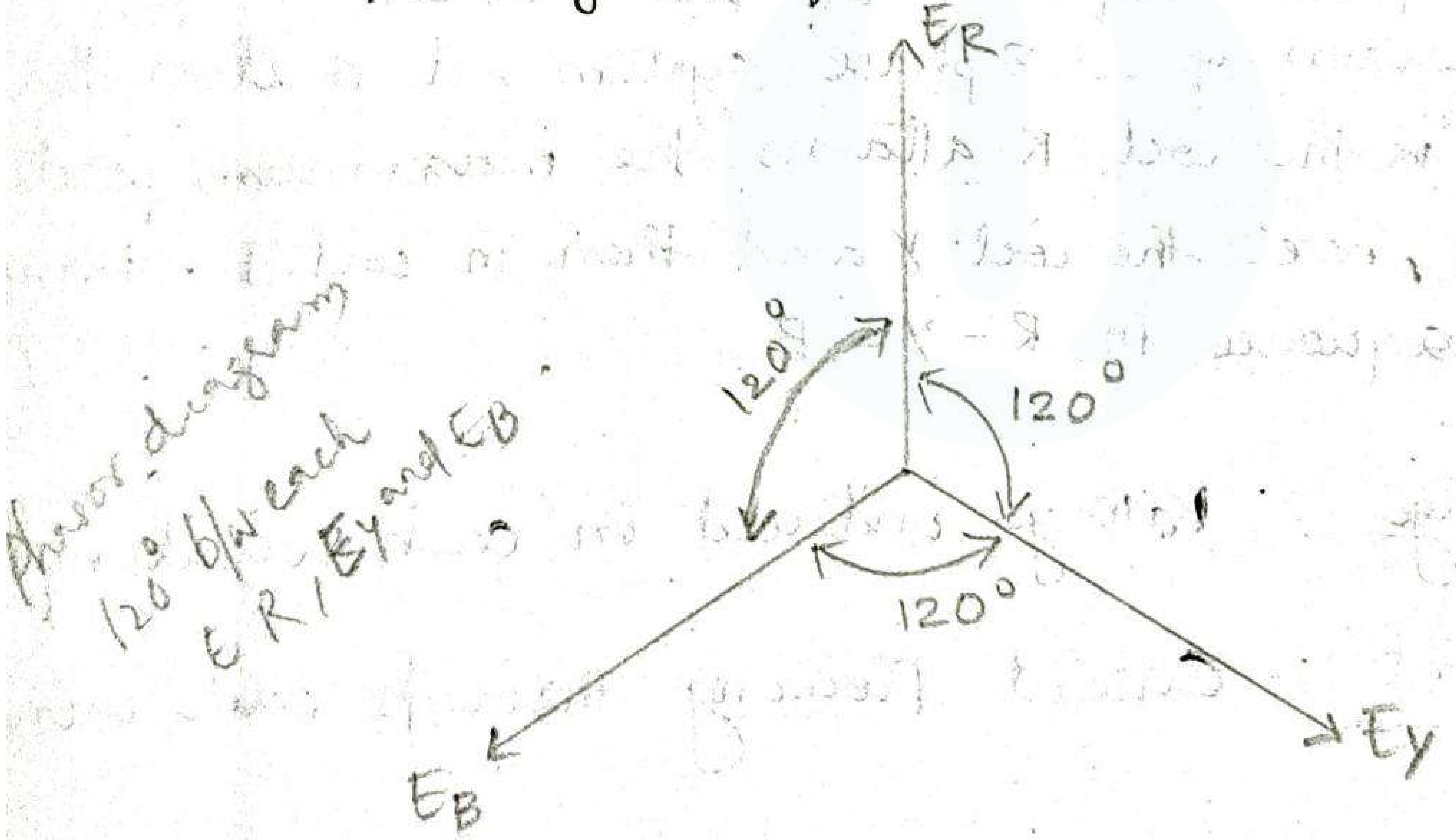
$E_y = E_m \sin(\theta - 120^\circ)$

$E_B = E_m \sin(\theta - 240^\circ)$ / $E_m \sin(\theta + 120^\circ)$

where E_m is the maximum value of the induced voltage in each winding. The waveform of these 3 voltages are :



Phasor diagram of these 3 induced voltages.



Advantages of 3 phase :-

Single	3 phase.
→ Instantanious power is fluctuating in nature	Instantanious power is constant all the time.

Single	3 phase.
→ Output is less compared to 3 phase	Output of 3 phase is greater than single phase.
→ Transmission and distribution is costlier	Transmission and distribution is cheaper.
→ Less efficient and have a lower power factor	More efficient and have higher power factor.
→ Not self starting.	Self starting

Phase sequence :- The sequence in which the voltages in the three phases reach the maximum positive value is called phase sequence or phase order. From phasor diagram of a 3 phase system, it is clear that the voltage in the coil R attains the maximum positive value first, next the coil Y and then in coil B. Hence the phase sequence in R – Y – B

Phase Voltage :- Voltage induced in each winding (230 V)

Phase current :- Current flowing through each winding

Line Voltage :- Voltage available between any pair of terminals or lines (415 V)

Line current :- The current flowing through each line

Symmetrical or Balanced System :- A three phase system is said to be balanced if
(a) Voltages in the three phase are equal in magnitude and differ in phase from one another.
(b) Currents in the three phases are equal in magnitude and differ in phase.

Balanced Load :- If loads connected across the three phases are identical
ie all the loads have the same magnitude and power factor.

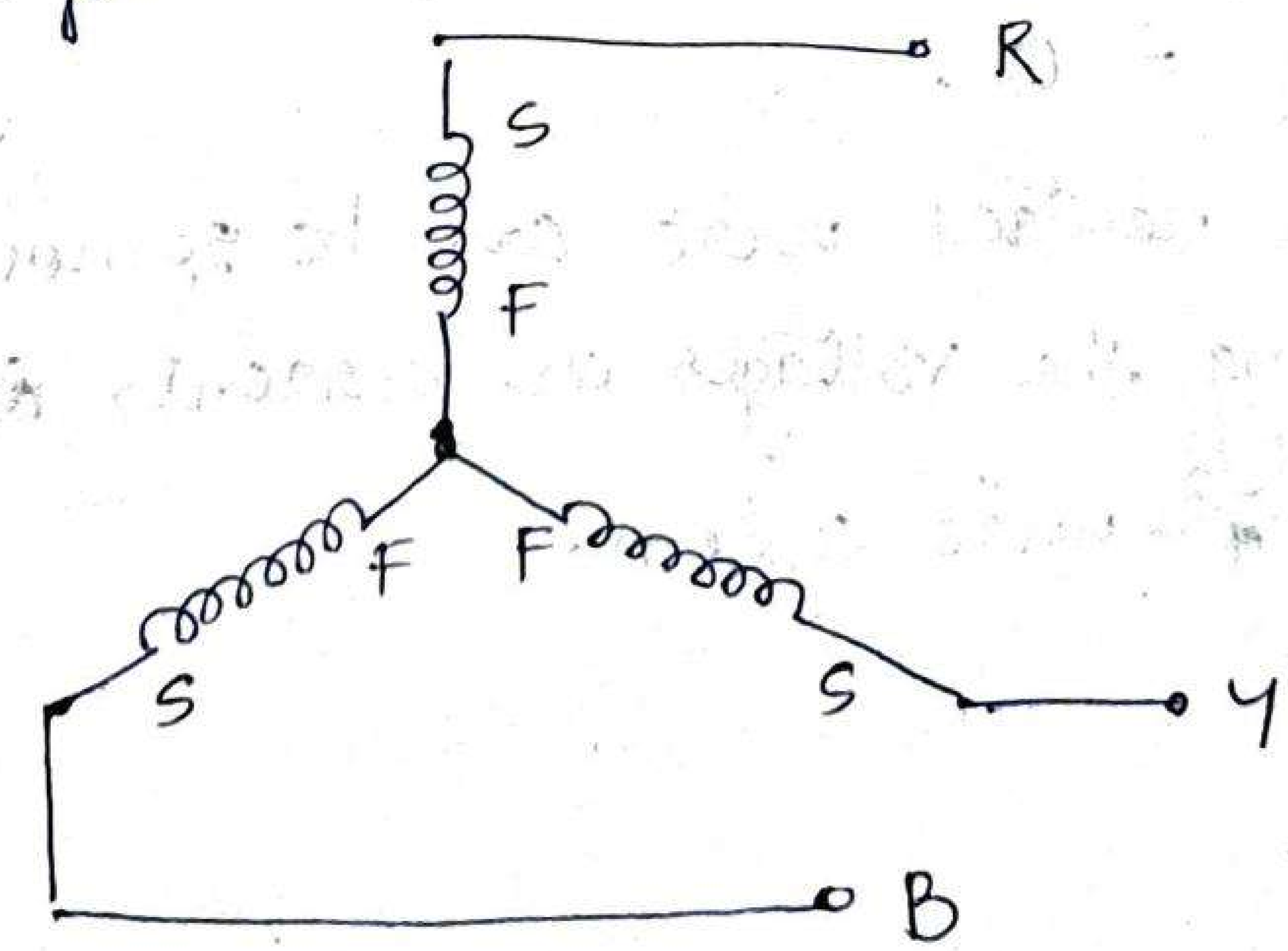
13/11/2020

Interconnection of 3 phases

① Star or Y connection
② Delta or Mesh connection.

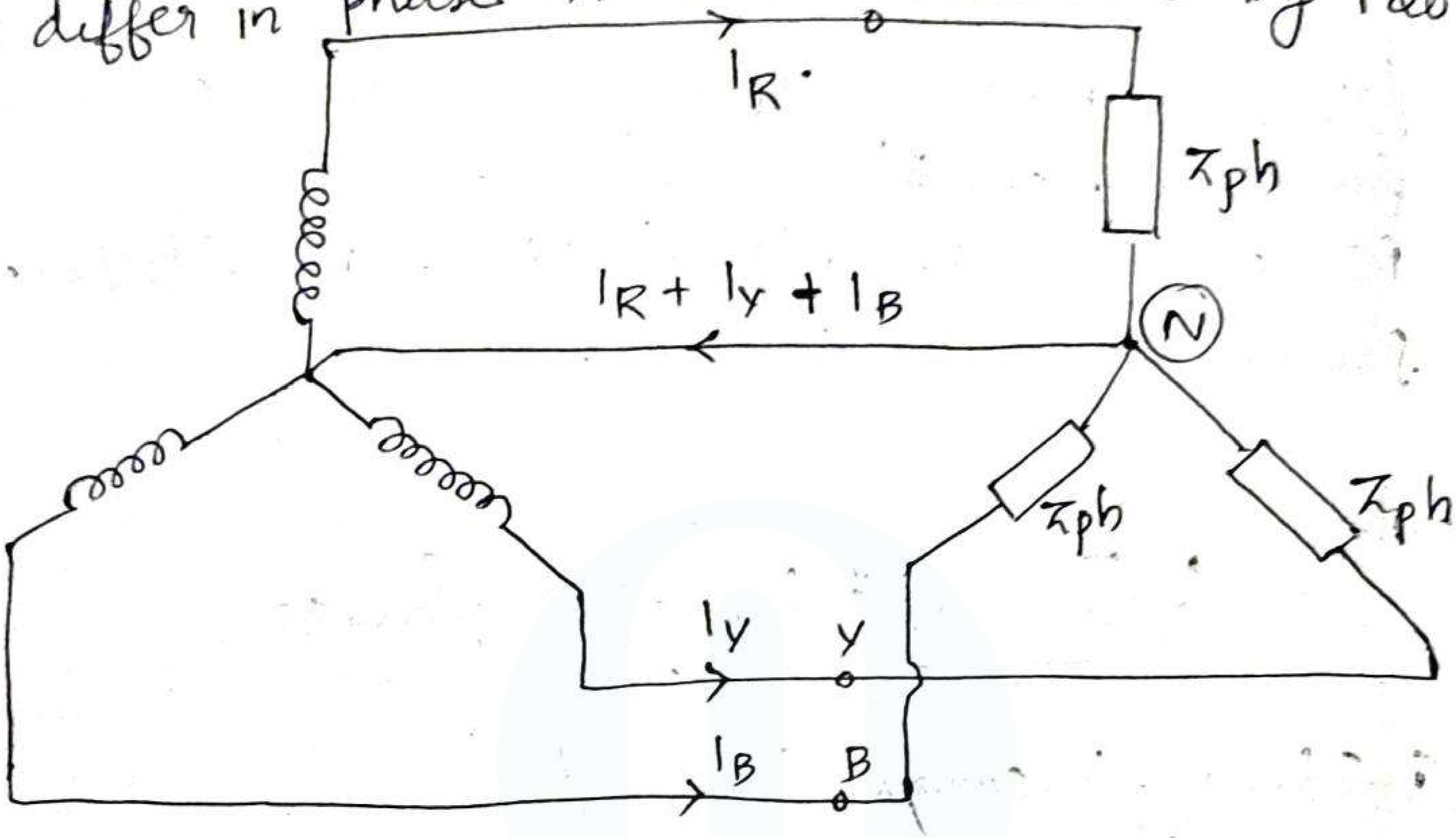
Star Connection :-

In this method, the similiar terminals of the three windings are joined together. The common point is called star or neutral point.



3-phase, 4-wire system :-

The system is called 3 phase 4-wire system. If the three identical loads are connected to each phase, the current flowing through the neutral wire is the sum of 3 currents I_R, I_Y and I_B. Since the impedances are identical, the three currents are equal in magnitude but differ in phase from one another by 120°.



$$I_R = I_m \sin\theta$$

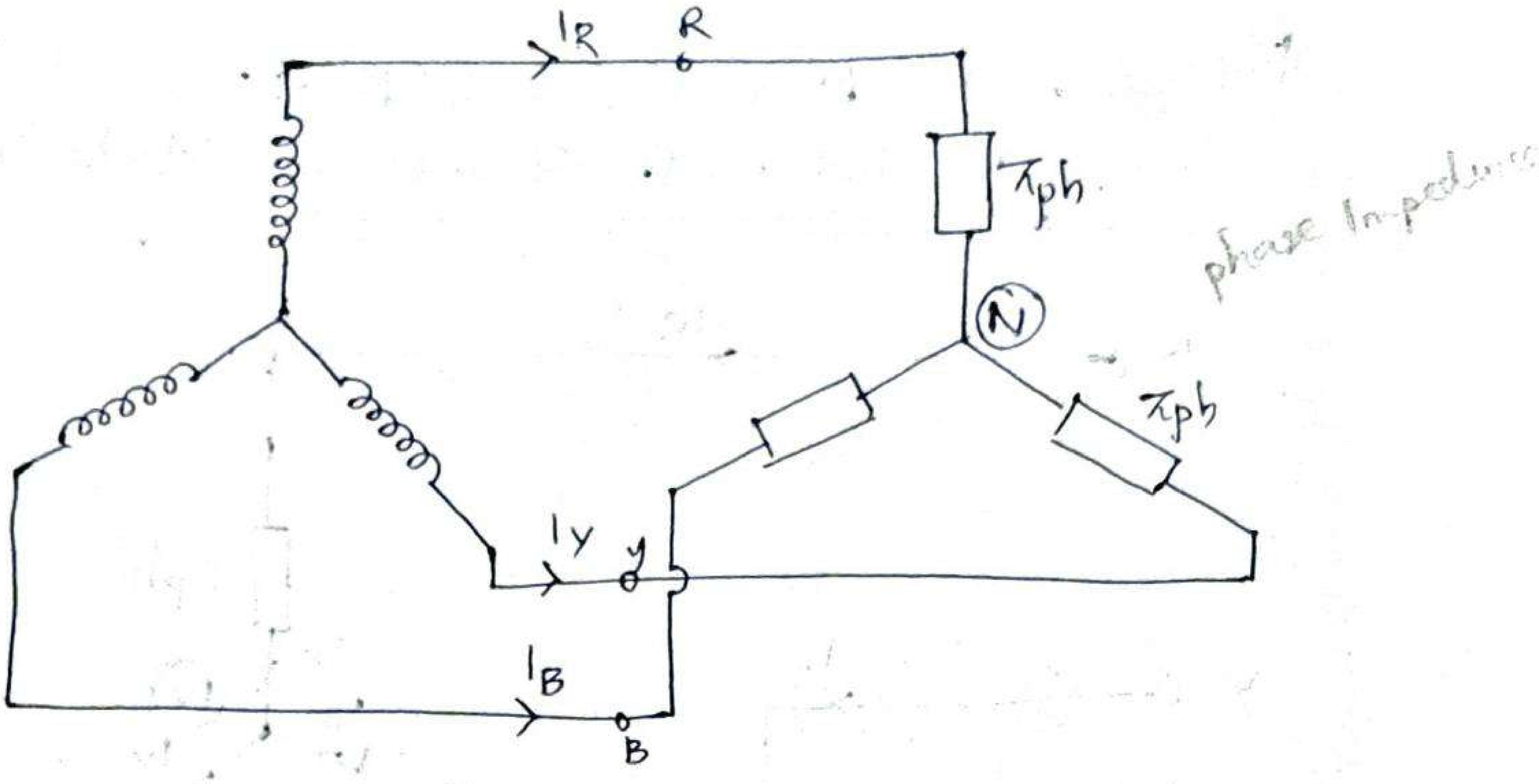
$$I_Y = I_m \sin(\theta - 120^\circ)$$

$$I_B = I_m \sin(\theta - 240^\circ)$$

$$I_R + I_Y + I_B = I_m \sin\theta + I_m \sin(\theta - 120) + I_m \sin(\theta - 240)$$

$$= 0.$$

Hence the neutral wire can be removed without any way affecting the voltages or currents. This constitute a 3-phase, 3-wire system.



Delta or Mesh Connection

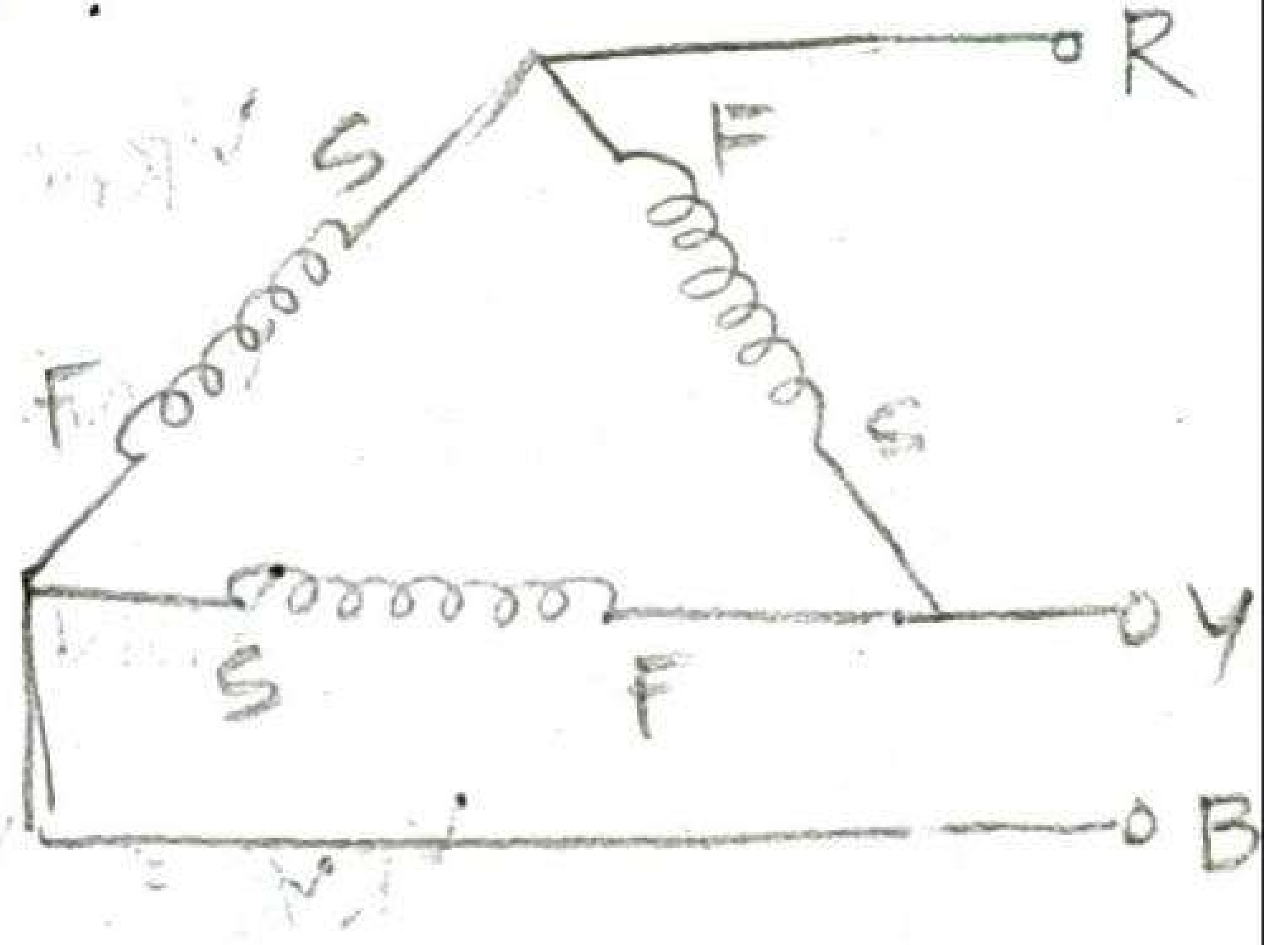
In this method, dissimiliar terminals of the three windings are joined together.
ie, the 'finish' terminal of one winding is connected to 'start' terminal of other winding. This system is also called 3 phase 3 wire system.

For a balanced system, the sum of the three phase voltages round the closed mesh is zero. The three emfs are equal in magnitude but differ in phase from one another by $120°$.

$$e_R = E_m \sin\theta$$

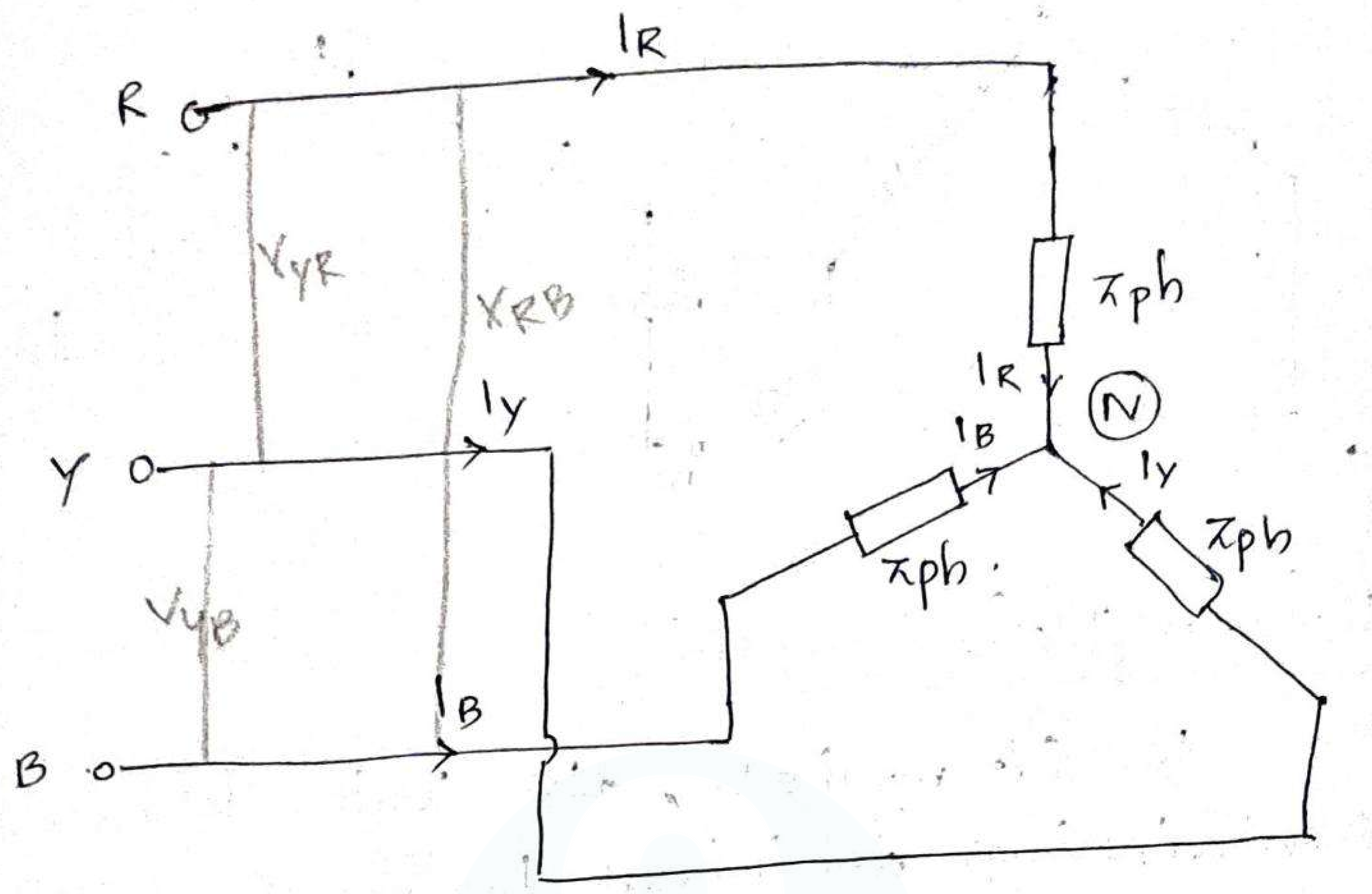
$$e_Y = E_m \sin(\theta - 120°)$$

$$e_B = E_m \sin(\theta - 240)$$



$$e_R + e_Y + e_B = 0$$

Voltage, Current and Power Relation in balanced Star Connected Load.



The figure shows a balanced star connected load. Since the system is balanced V_{RN}, V_{YN}, V_{BN}, the 3 phase voltages are equal in magnitude and differ in phase from one another by 120°.

$$V_{RN} = V_{YN} = V_{BN} = V_{ph}.$$

where V_{ph} indicates the rms value of phase Voltage

$$V_{RN} = V_{ph} \angle 0^\circ$$

$$V_{YN} = V_{ph} \angle -120^\circ$$

$$V_{BN} = V_{ph} \angle -240^\circ$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$ Line voltage.

where $V_L \rightarrow$ rms value of line Voltage

Applying kirchoff's voltage law,

$$V_{RY} = V_{RN} + V_{NY}$$

$$= V_{RN} - V_{YN}$$

$$= V_{ph} \angle 0^\circ - V_{ph} \angle -120^\circ \rightarrow 1 \angle 120^\circ$$

$$= (V_{ph} + j0) - (-0.5 V_{ph} - j\,0.866 V_{ph})$$

$$= 1.5 V_{ph} + j\,0.866 V_{ph}.$$

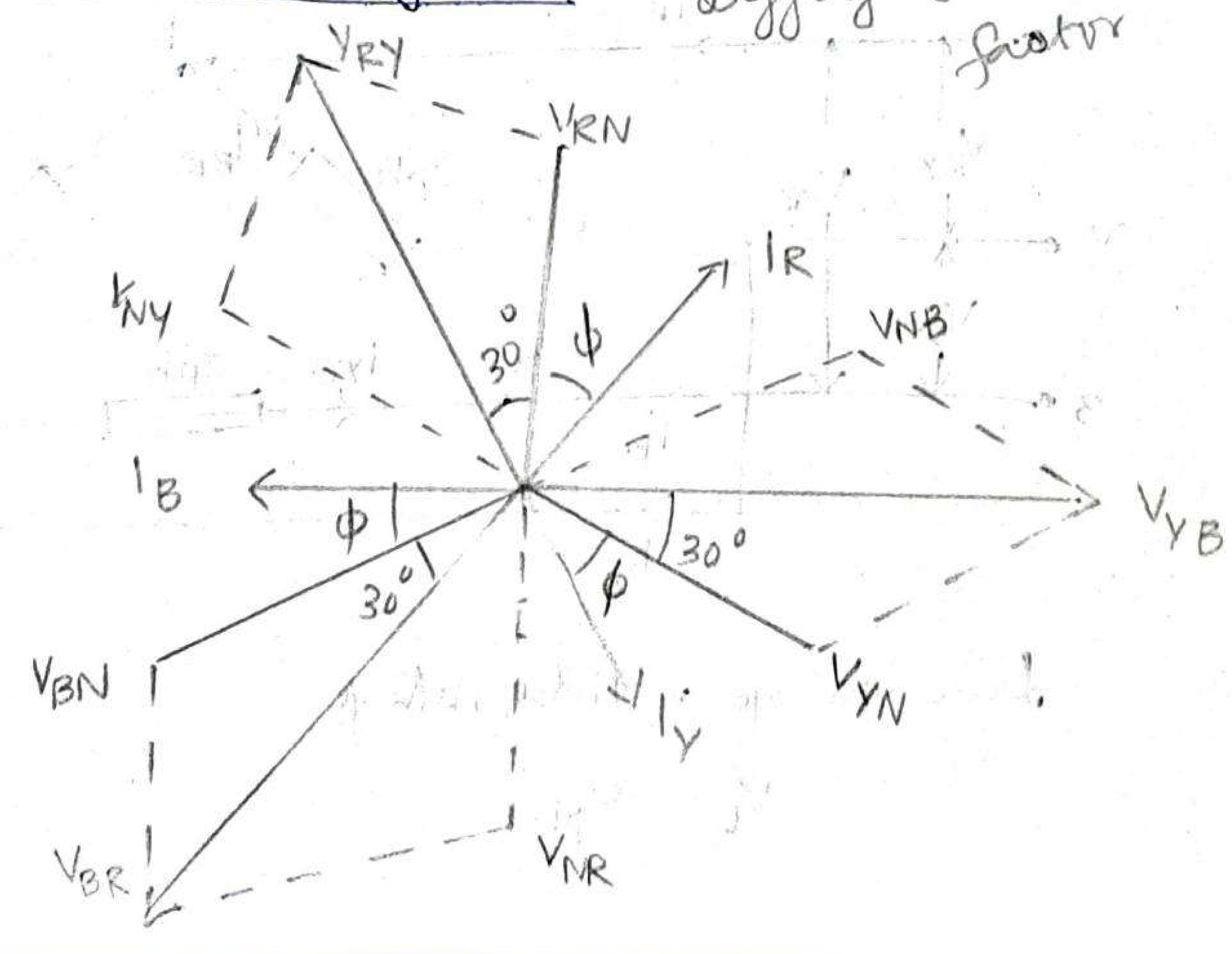
$$= \sqrt{3}\, V_{ph} \angle 30^\circ$$

Similarly $V_{YB} = V_{YN} + V_{NB} = \sqrt{3}\, V_{ph} \angle 30^\circ$

$V_{BR} = V_{BN} + V_{NR} = \sqrt{3}\, V_{ph} \angle 30^\circ$.

Thus in a star-connected, three phase system, $V_L = \sqrt{3}\, V_{ph}$ and line voltages lead respective phase voltages by 30°.

<u>Phasor Diagram</u> :- lagging power factor



17/11/2020

Active power $P = 3 \times$ power in each phase

$$= 3 \times V_{ph} I_{ph} \cos\phi$$

In star connected

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$P = 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi = \underline{\sqrt{3} V_L I_L \cos\phi}$$

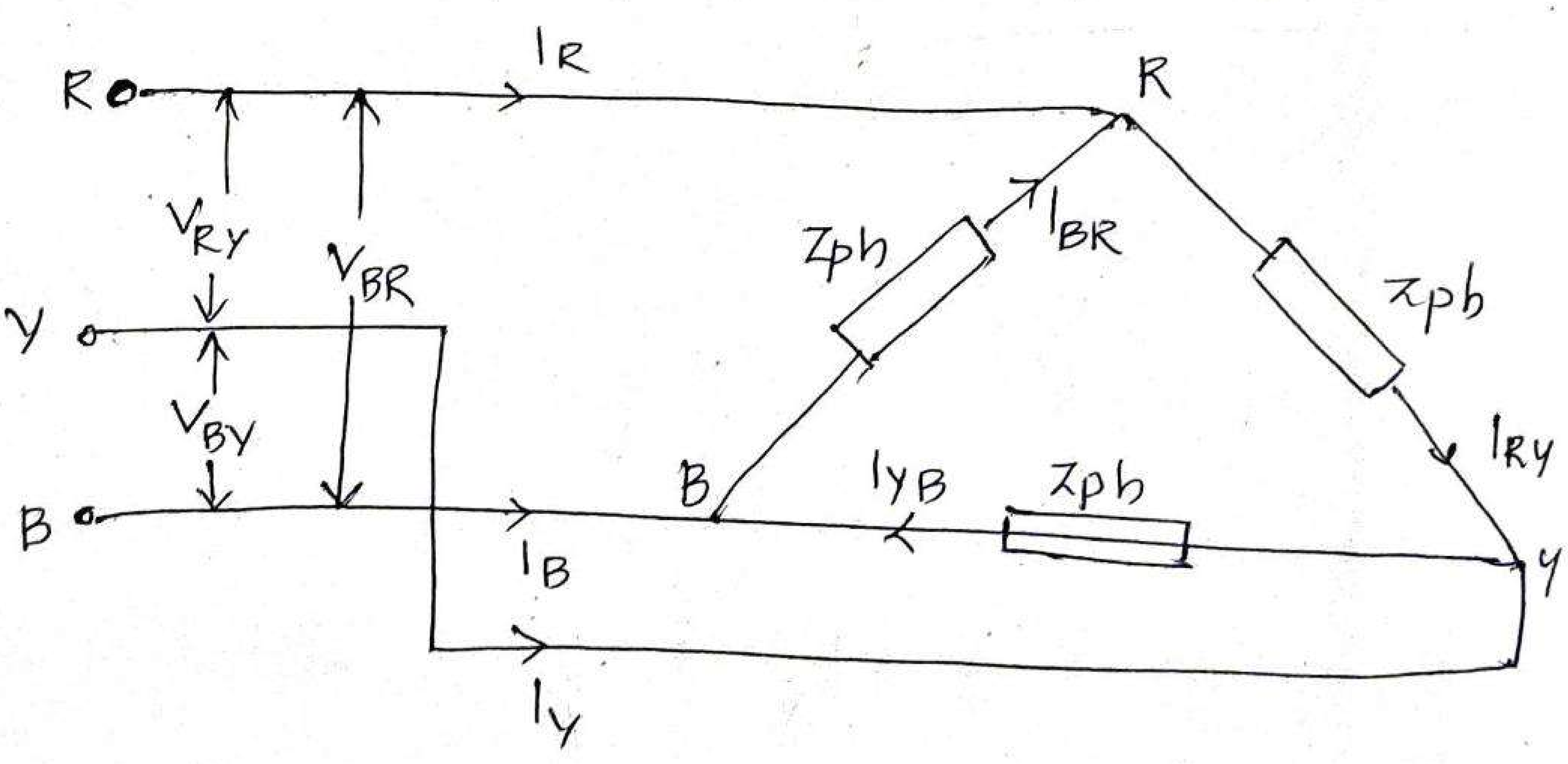
Reactive power $Q = 3 V_{ph} I_{ph} \sin\phi$

$$= \underline{\sqrt{3} V_L I_L \sin\phi}$$

Apparent power $S = 3 V_{ph} I_{ph}$

$$= \underline{\sqrt{3} V_L I_L}$$

Relation between Line Voltage and Phase Voltage



Line voltage = Phase voltage

$$V_L = V_{ph}$$

Line current and Phase current

$$I_{RY} = I_{YB} = I_{BR} = I_{ph}.$$

$I_{ph} \rightarrow$ rms value of phase current

$$I_{RY} = I_{ph} \angle 0^\circ$$

$$I_{YB} = I_{ph} \angle -120^\circ$$

$$I_{BR} = I_{ph} \angle -240^\circ$$

$$I_R = I_Y = I_B = I_L$$

$I_L \rightarrow$ RMS value of line current.

Kirchoff's current law

$$I_R + I_{BR} = I_{RY}$$

$$I_R = I_{RY} - I_{BR}$$

$$= I_{ph} \angle 0 - I_{ph} \angle -120^\circ$$

$$= (I_{ph} + j0) - (-0.5 I_{ph} + j0.866 I_{ph})$$

$$= 1.51 I_{ph} - j0.866 I_{ph}$$

$$I_R = \sqrt{3} I_{ph} \angle -30^\circ$$

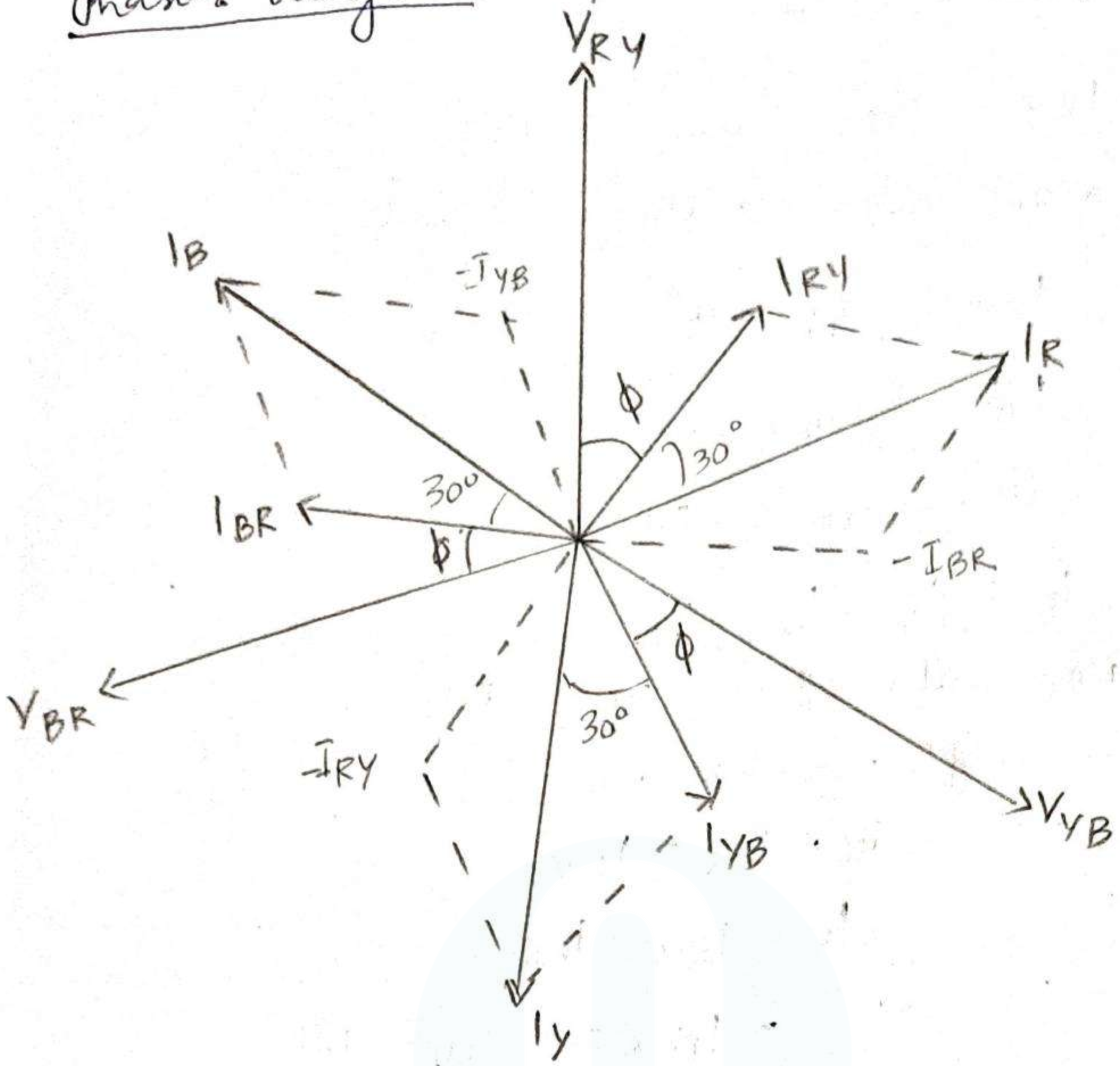
Similiarly

$$I_Y = I_{YB} - I_{RY} = \sqrt{3} I_{ph} \angle -30^\circ$$

$$I_B = I_{BR} - I_{YB} = \sqrt{3} I_{ph} \angle -30^\circ$$

Thus in a delta connected, three phase system $I_L = \sqrt{3} I_{ph}$ and line current lag behind the phase current by 30°.

Phasor diagram



$$\text{Power} = 3\, V_{ph} \cdot I_{ph} \cos\phi$$

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$P = 3\, V_L \times \frac{I_L}{\sqrt{3}} \cos\phi = \underline{\sqrt{3}\, V_L I_L \cos\phi}$$

$$\text{Reactive power } Q = 3\, V_{ph} I_{ph} \sin\phi$$

$$\underline{Q = \sqrt{3}\, V_L I_L \sin\phi}$$

$$\text{Apparent power } S = 3\, V_{ph} I_{ph}$$

$$\underline{S = \sqrt{3}\, V_L I_L}$$

Balanced Y/Δ and Δ/Y conversions

Any balanced star-connected system can be converted into the equivalent delta connected system and vice versa

For balanced star connected load

Line voltage $= V_L$

Line current $= I_L$

Impedence / phase $= Z_Y$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$Z_Y = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{\sqrt{3}\, I_L} \qquad \frac{V_L}{I_L} = \sqrt{3}\, Z_Y$$

For equivalent delta connected system, the line voltages and currents may have the same value as the star connected system

Line voltage $= V_L$

Line current $= V_I$

Impedence / phase $= Z_\Delta$

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$\sqrt{3} \times \sqrt{3}\, Z_Y = 3 Z_Y$$

$$Z_\Delta = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{I_L/\sqrt{3}} = \frac{\sqrt{3}\, V_L}{I_L} = 3 Z_Y$$

$$Z_Y = \frac{1}{3} Z_\Delta$$

When three equal phases impedences are connected in delta, the equivalent star impedence is of delta impedence

Relation b/w power in Delta and star systems

Balanced load be connected in star having impedence per phase Z_{ph}.

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$V_{ph} = \frac{V_L}{\sqrt{3}}$

$I_{ph} = I_L$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{\sqrt{3}\, Z_{ph}}$$

$$I_L = I_{ph} = \frac{V_L}{\sqrt{3}\, Z_{ph}}$$

$$P_Y = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3}\, V_L \times \frac{V_L}{\sqrt{3}\, Z_{ph}} \cos\phi$$

$$P_Y = \frac{V_L^2}{Z_{ph}} \cos\phi$$

Delta connected

$$V_{ph} = V_L$$

$V_{ph} = V_L$

$I_{ph} = \frac{I_L}{\sqrt{3}}$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{Z_{ph}}$$

$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3}\, \frac{V_L}{Z_{ph}}$$

$$P_\Delta = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3}\, V_L \times \sqrt{3}\, \frac{V_L}{Z_{ph}} \cos\phi$$

$$P_\Delta = \frac{3 V_L^2}{Z_{ph}} \cos\phi$$

$$P_Y = \frac{1}{3} P_\Delta$$

Q. 3 equal impedence, each of $(8+j10)\,\Omega$ are connected in star. This is further connected to a 440 V, 50 Hz 3ϕ supply. Calculate a) phase voltage b) phase angle c) phase current d) line current e) active power f) reactive phower.

Per

$$Z_{ph} = 8 + j10 \qquad V_L = 440\,V \qquad f = 50\,Hz$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = \underline{254.03\,V.} \quad (a)$$

$$Z_{ph} = 8 + j10 = \underline{12.81 \angle 51.34^\circ\,\Omega.}$$

$$Z_{ph} = 12.81\,\Omega$$

$$\phi = \underline{51.34^\circ} \quad (b)$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{12.81} = \underline{19.83\,A} \quad (c)$$

$$I_L = I_{ph} = 19.83\,A \quad (d)$$

$$P = \sqrt{3}\,V_L I_L \cos\phi = \sqrt{3} \times 440 \times 19.83 \times \cos(51.34)$$

$$= \underline{9.44\,kW} \quad (e)$$

$$Q = \sqrt{3}\,V_L I_L \sin\phi = \sqrt{3} \times 440 \times 19.83 \times \sin(51.34)$$

$$= \underline{11.81\,kVA} \quad (f)$$

Apparent Power $= V \times I$

Q2 A balanced delta connected load of impedence $(8 - j6)\,\Omega$ per phase is connected to a 3ϕ 230 V 50 Hz supply. Calculate a) pf, (b) line current c) reactive power

$Z_{ph} = (8 - j6)\,\Omega \qquad V_L = 230V \qquad f = 50\,Hz$

$Z_{ph} = (8 - j6) = \underline{10 \angle -36.87}$ leading

$\phi = 36.87$

T → lag

P.f

$\cos\phi = \cos(36.87) = \underline{0.8 \text{ leading}}$

$V_{ph} = V_L = 230V$

$I_p = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{230}{10} = \underline{\underline{23A}}$

1 question

$I_L = \sqrt{3}\, I_p$

$= \sqrt{3} \times 23$

$= \underline{39.84\ A}$

$Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3} \times 230 \times 39.84 \sin(36.87)$

$= \underline{9.53\ KVAR}$

18/11/2020

3 phase unbalanced circuits

If the loads connected across the three phases are not identical to each other ie; the loads have different magnitude and power factor, load are said to be unbalanced. The phase current in delta and phase or line currents in star connection differ in unbalanced loading giving rise to flow of current in a neutral wire.

$$I_N = I_Y + I_R + I_B$$

There may be three cases of unbalanced loads.

(1) Unbalanced delta connected load
(2) Unbalanced 3 wire star connected load
(3) Unbalanced 4 wire star connected load.

(1) Unbalanced delta connected load.

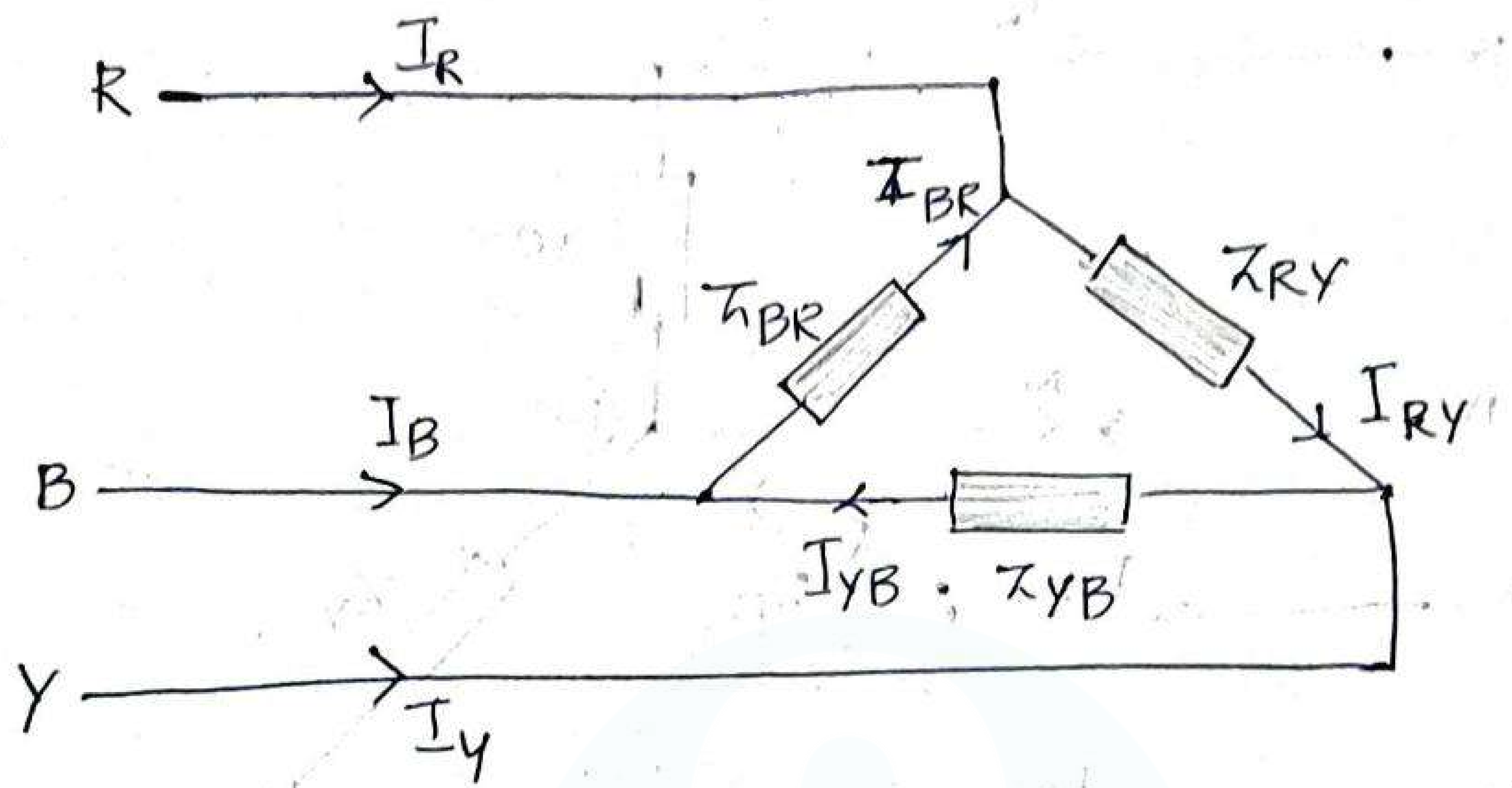


Figure shows an unbalanced delta connected load connected to a balanced 3 phase supply.
For a delta connected load

$$V_L = V_{ph}$$

$$V_{RY} = V_{ph} \angle 0$$

$$V_{YB} = V_{ph} \angle -120^\circ$$

$$V_{BR} = V_{ph} \angle -240^\circ$$

Phase current

$$I_{RY} = \frac{V_{RY}}{Z_{RY}}$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}}$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}}$$

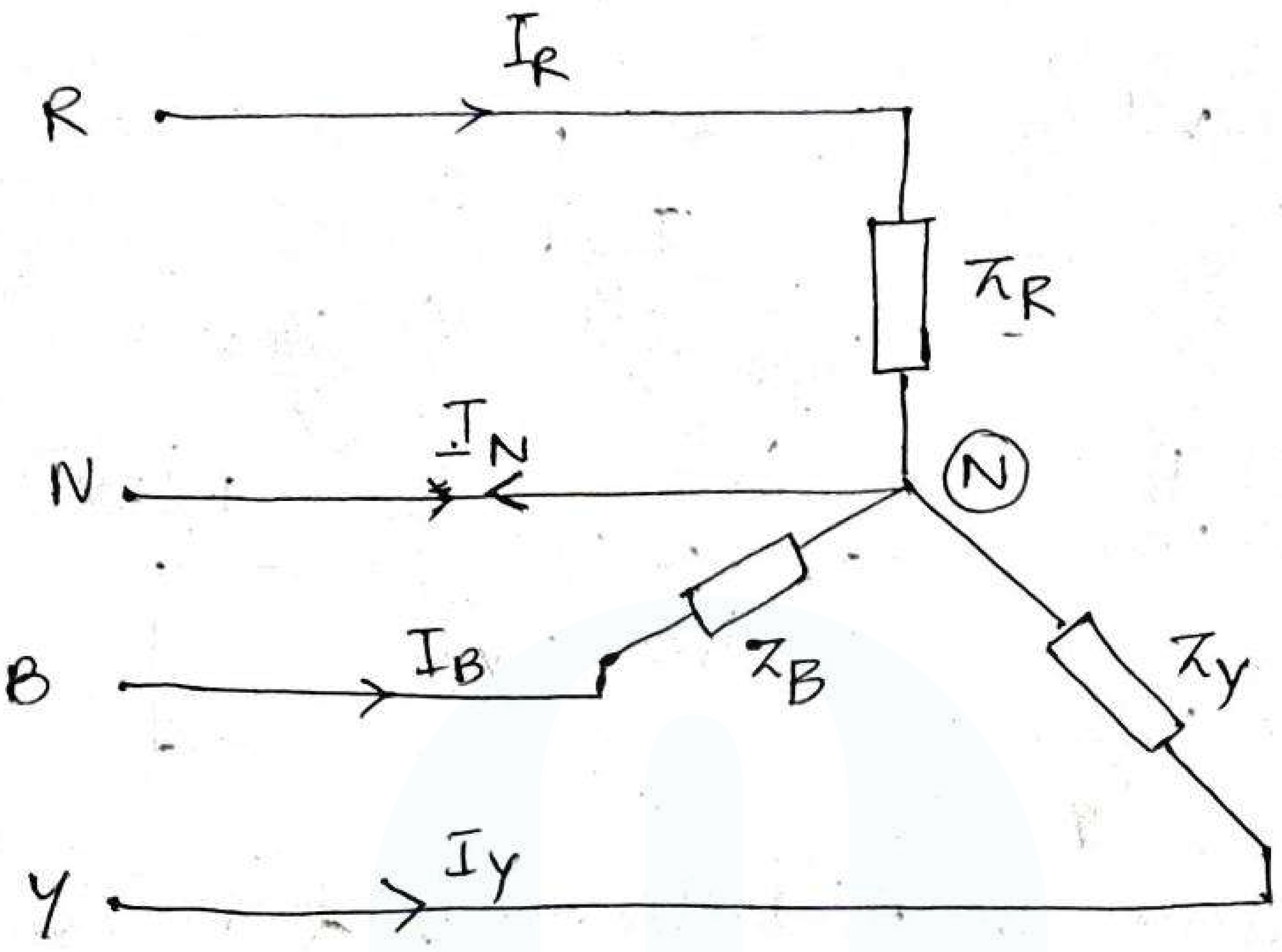
Line current is given by

$$I_R = I_{RY} - I_{BR} \qquad I_R + I_{BR} = I_{RY}$$

$$I_Y = I_{YB} - I_{RY}$$

$$I_B = I_{BR} - I_{YB}$$

(2) Unbalanced 4 wire star connected load.



The fig: shows unbalanced star connected load connected in a balanced 3 ϕ, 4 wire supply. The neutral point N of the load is connected to the neutral part of the supply. Hence voltage across the 3 load impedence is equal to the phase voltage of supply

$$V_L = \sqrt{3}\, V_{ph}$$

$$V_{RN} = V_p \angle 0$$

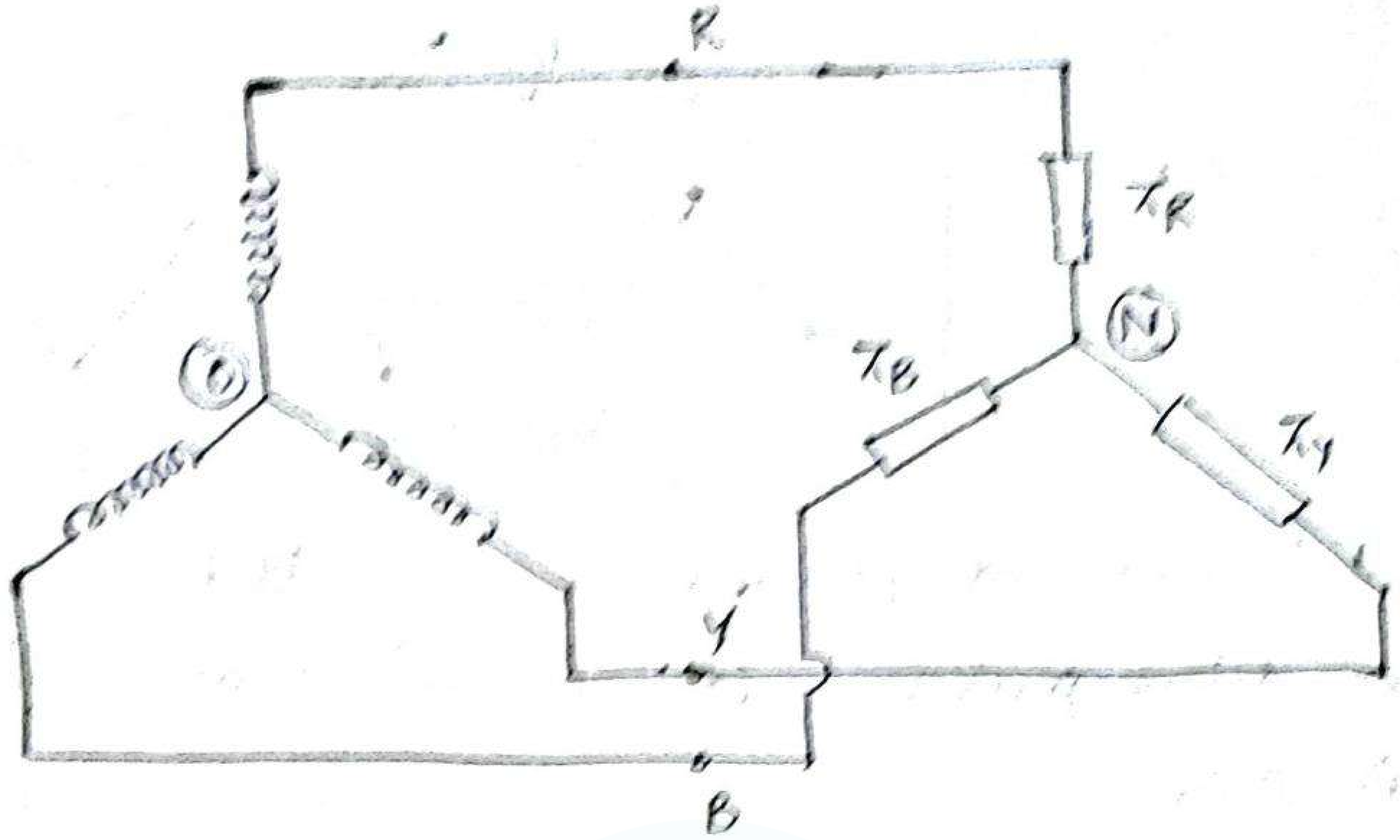
$$V_{YN} = V_p \angle -120^\circ$$

$$V_{BN} = V_p \angle -240^\circ$$

$$I_R = \frac{V_{RN}}{Z_R} \qquad I_Y = \frac{V_{YN}}{Z_Y} \qquad I_B = \frac{V_{BN}}{Z_B}$$

$I_N = I_R + I_Y + I_B$

(3) Unbalanced 3 wire star connected load.



If the neutral point N of the load is not connected to the neutral point O of the supply, there exist a voltage b/w supply neutral point and load neutral point. The load phase voltage is not same as that of the supply phase voltage.

There are many methods to solve such unbalanced star connected load.

(1) Star-delta transformation.

$$Z_{RY} = Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}$$

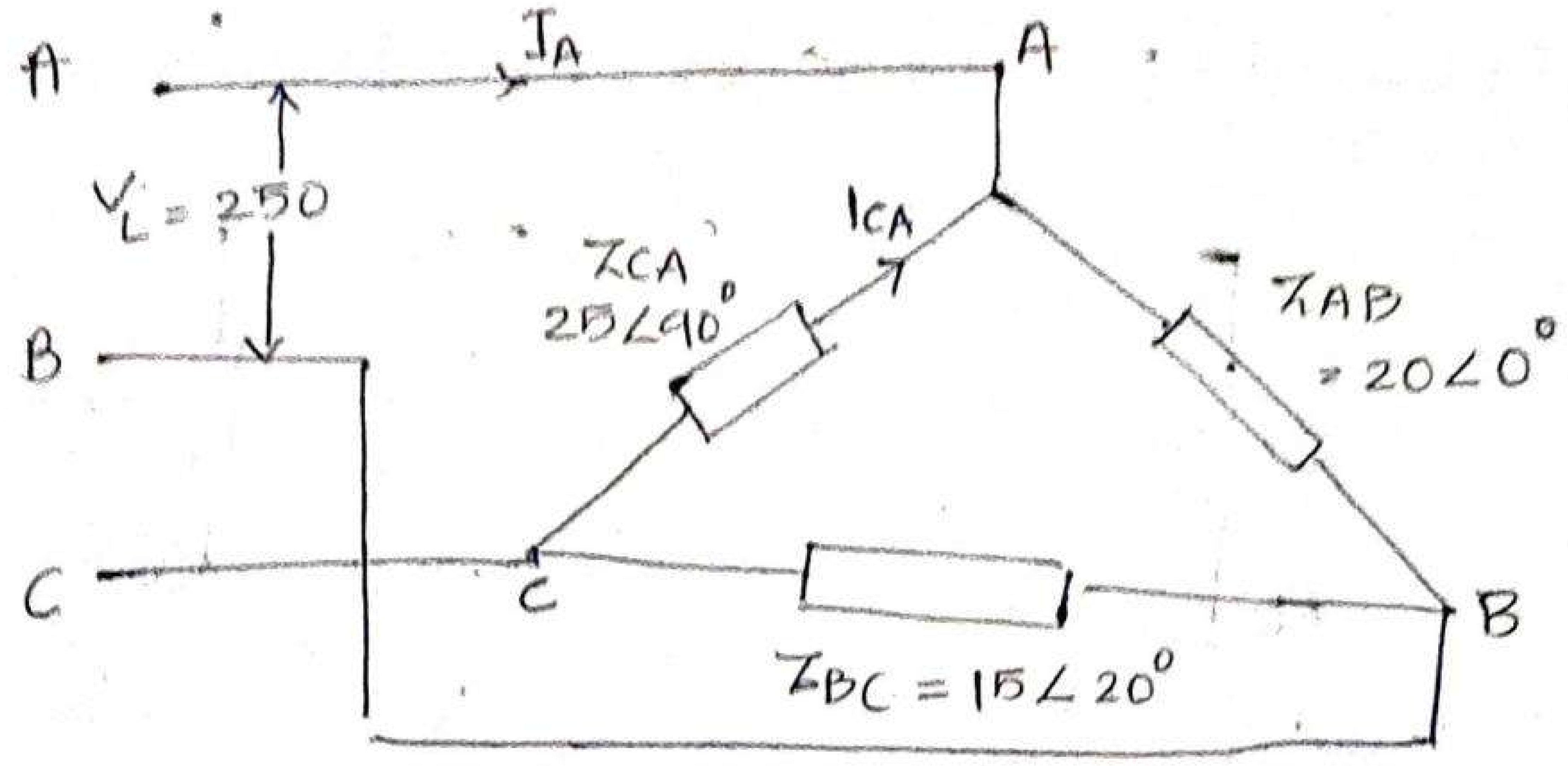
$$Z_{YB} = Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}$$

$$Z_{BR} = Z_B + Z_R + \frac{Z_B \cdot Z_R}{Z_Y}$$

The problem is solved by as an unbalanced delta connected load.

12/11/2020 Tutorial - 15

Q. A 3 ϕ supply with line voltage of 250V has an unb. Δ connected load as in fig.



Determine (a) phase current (b) line current (c) Active power (d) T Reactive power if phase se is ABC.

Soln :- $V_{AB} = 250\angle 0$ $\quad V_{BC} = 250\angle -120$ $\quad V_{CA} = 250\angle$

$Z_{AB} = 20\angle 0$ $\quad Z_{BC} = 15\angle 20$ $\quad Z_{CA} = 25\angle 9$

(a) Phase current

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{250\angle 0}{20\angle 0} = 12.5\angle 0^\circ$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{250\angle -120}{15\angle 20} = 16.67\angle -140^\circ \text{ A}$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{250\angle -240}{25\angle 90} = 10\angle 30^\circ \text{ A}$$

(b) Line currents

$$I_A = I_{AB} - I_{CA} = \underline{6.3\angle -52.48 \text{ A}}$$

$$I_B = I_{BC} - I_{AB} = \underline{27.45 \angle -157.02^\circ \text{ A}}$$

$$I_C = I_{CA} - I_{BC} = \underline{26.57 \angle 36.25^\circ \text{ A}}$$

(c) Total active power

$\phi = Z$ ang

$$P_{AB} = V_{AB} I_{AB} \cos \phi_{AB}$$

$$= 250 \times 12.5 \times \cos(0^\circ) = \underline{3.13 \text{ KW}}$$

$$P_{BC} = V_{BC} I_{BC} \cos \phi_{BC}$$

$$= 250 \times 16.67 \times \cos(20^\circ) = \underline{3.92 \text{ KW}}$$

$$P_{CA} = V_{CA} \times I_{CA} \cos \phi_{CA}$$

$$= 250 \times 25 \times \cos(90^\circ) = \underline{0}$$

$$P = P_{AB} + P_{BC} + P_{CA}$$

$$= 3.13 + 3.92 + 0 = \underline{\underline{7.05 \text{ KW}}}$$

(d) Total reactive power

$$Q_{AB} = V_{AB} I_{AB} \sin \phi$$

$$= 250 \times 12.07 \times \sin(0) = \underline{0}$$

$$Q_{BC} = V_{BC} I_{BC} \sin \phi$$

$$= 250 \times 16.67 \times \sin(20^\circ) = \underline{1.43 \text{ kVAR}}$$

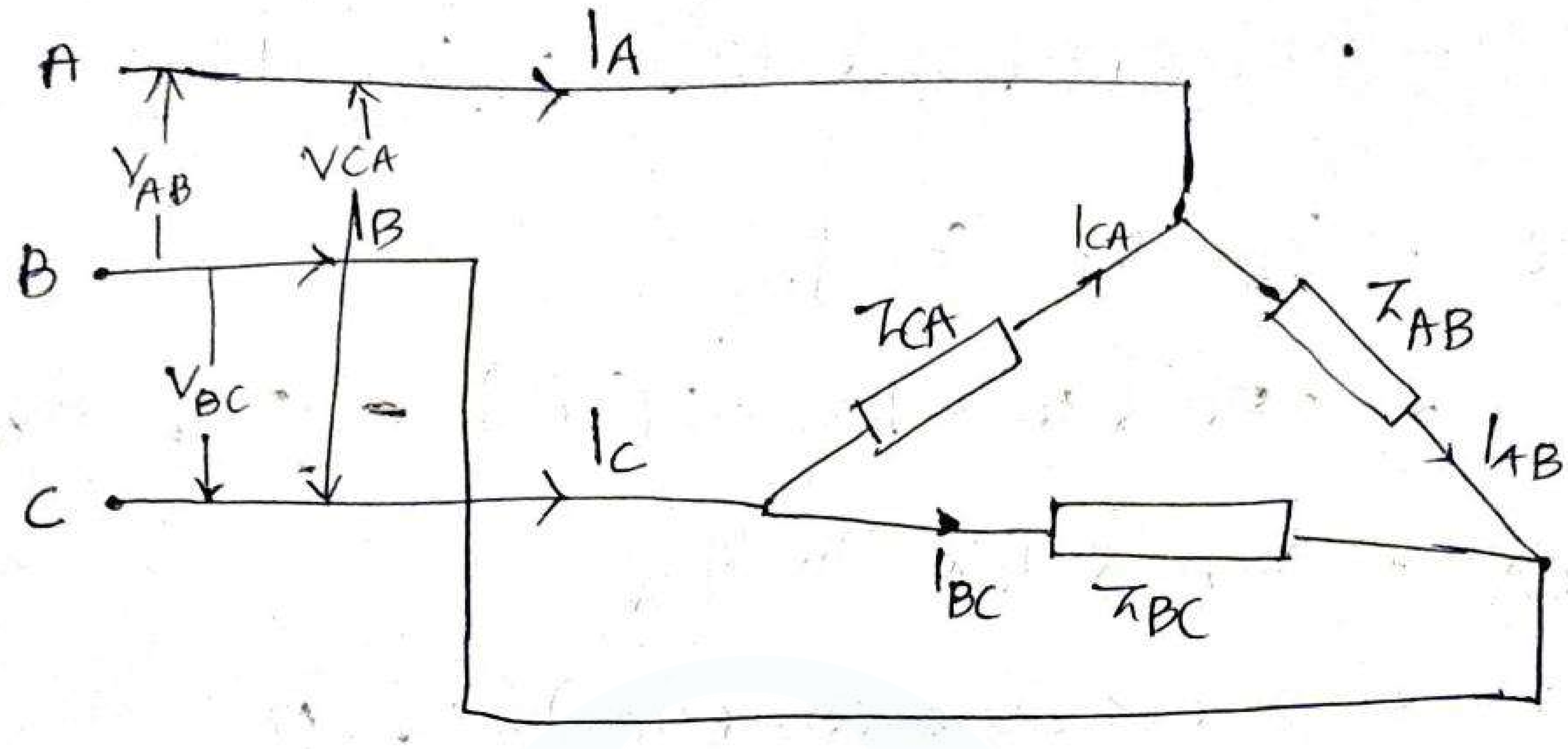
$$Q_{CA} = V_{CA} I_{CA} \sin \phi$$

$$= 250 \times 25 \times \cos(90^\circ) = \underline{6.25 \text{ kVAR}}$$

$$Q = Q_{AB} + Q_{BC} + Q_{CA}$$

$$= 0 + 1.43 + 6.25 = \underline{7.68 \text{ kVAR}}$$

Q. A 400 V, 50 Hz, 3 phase supply of phase sequence ABC is applied to a delta connected load consisting of a 100 Ω resistor b/w lines A and B, a 378 mH inductor b/w B and C and 37.8 μF capacitor b/w lines C and A. Determine phase and line current.



Soln:

$V_L = 400\,V \qquad f = 50\,Hz \qquad R = 100\,\Omega$

$L = 318\,mH \qquad C = 31.8\,\mu F$

$V_{AB} = 400\angle 0°\,V$

$V_{BC} = 400\angle -120°\,V$

$V_{CA} = 400\angle -240°\,V$

$X_L = 2\pi f \times L = 2\pi \times 50 \times 318 \times 10^{-3}$

$= 99.90\,\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times 31.8 \times 10^{-6}}$

$= 100\,\Omega$

$Z_{AB} = R = 100\angle 0^\circ\ \Omega$

$Z_{BC} = jX_L = j99.9 = 99.9\angle 90^\circ\ \Omega$

$Z_{CA} = -jX_C = -j100 = 100\angle -90^\circ\ \Omega$.

(a) Phase currents

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{400\angle 0}{100\angle 0} = \underline{\underline{4\angle 0\ A}}$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{400\angle -120}{99.9\angle 90} = \underline{\underline{4\angle 150\ A}}$$

$3.99\angle 149.9$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{400\angle -240}{100\angle -90^\circ} = \underline{\underline{4\angle -150\ A}}$$

(b) Line currents

$I_A = I_{AB} - I_{CA} = 4\angle 0 - 4\angle -150^\circ = 7.46 + 2i$

$= \underline{\underline{7.72\angle 15^\circ A}}$

$I_B = I_{BC} - I_{AB} = 4\angle 150 - 4\angle 0 = -7.46 + 2i$

$= \underline{\underline{7.72\angle 165^\circ A}}$

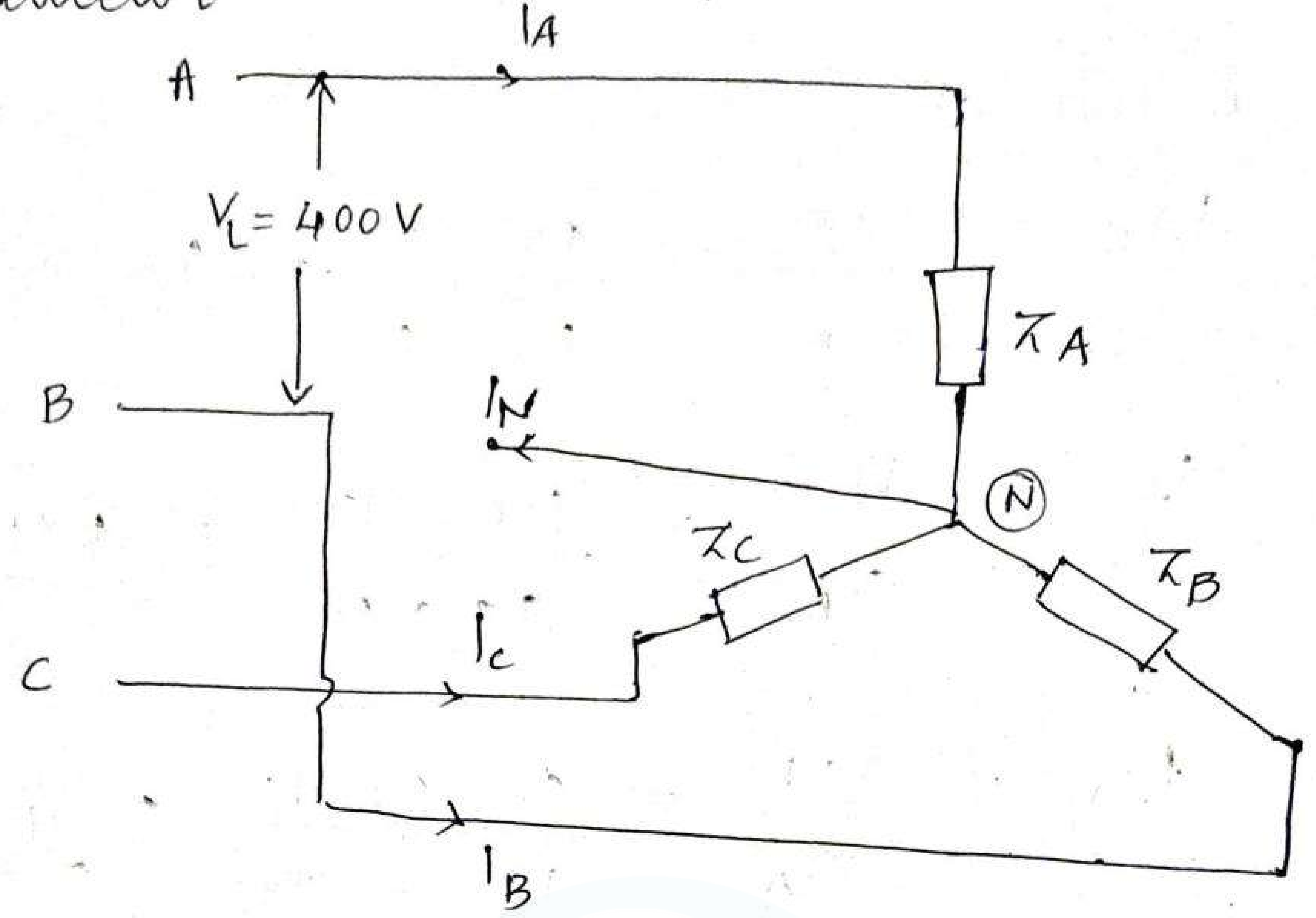
$I_C = I_{CA} - I_{BC} = 4\angle -150 - 4\angle 150 = -4i$

$= \underline{\underline{4\angle -90^\circ A}}$

Tutorial - 16

Q. A 3 ϕ, 400V, 4-wire system has a star-connected load $Z_A = (10 + j0\ \Omega)$, $Z_B = (15 + j10)\ \Omega$, $Z_C = (0 + j5)\ \Omega$. Find the line currents and current through the conductor.

6/11/2020



$V_L = 400$ V

$Z_A = (10 + j0)\ \Omega = 10\angle 0\ \Omega$.

$Z_B = (15 + j10)\ \Omega = 18.03\angle 33.6\ \Omega$.

$Z_C = (0 + j5)\ \Omega = 5\angle 90°\ \Omega$.

In star connected load

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = \underline{230.94\ V}$$

$V_{AN} = 230.94\angle 0°$ V

$V_{BN} = 230.94\angle -120°$ V

$V_{CN} = 230.94\angle -240°$ V

Phase current

$$I_A = \frac{V_{AN}}{Z_A} = \frac{230.94\angle 0}{10\angle 0} = \underline{23.09\angle 0^\circ A}$$

$-7.10 - 10.65i$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{230.94\angle -120^\circ}{18.03\angle 33.69} = \underline{12.81\angle -123.69^\circ A}$$

$39.99 + 23.09i$

$$I_C = \frac{V_{CN}}{Z_C} = \frac{230.94\angle -240^\circ}{5\angle 90^\circ} = \underline{46.19\angle 30^\circ A}$$

Line current are equal to phase current in star connected

$I_{L1} = I_A = \underline{23.09\angle 0^\circ}\ A$

$I_{L2} = I_B = \underline{12.81\angle -123.69^\circ}\ A$

$I_{L3} = I_C = \underline{46.19\angle 30^\circ A}$

Current through neutral

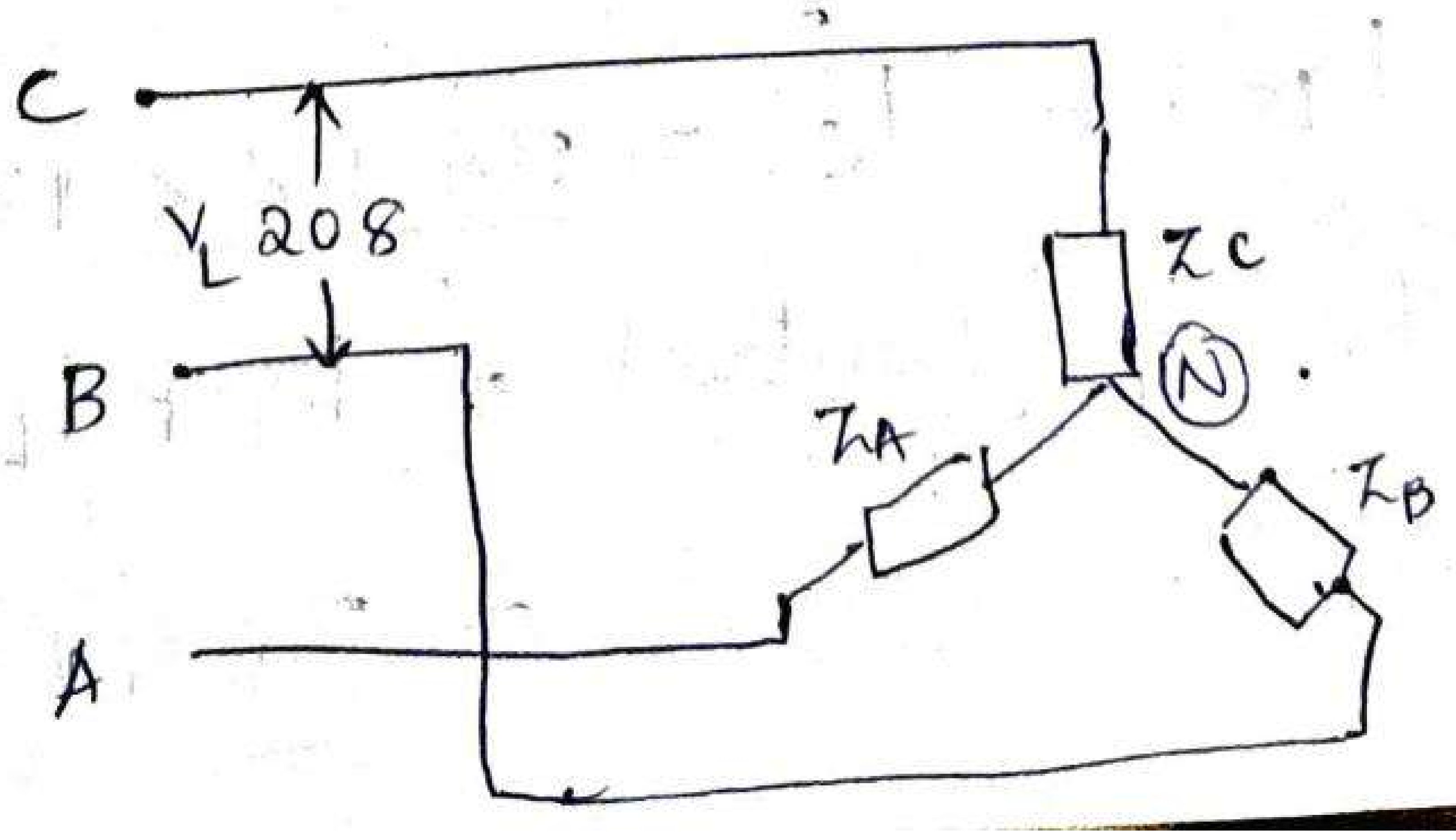
$I_N = I_A + I_B + I_C$

$= 23.09\angle 0 + 12.81\angle -123.69 + 46.19\angle 30^\circ$

$= \underline{\underline{54.47\angle 18.65^\circ A}}$ $55.98 + 12.43i$

$= \underline{57.34}\angle 12.51$

Q. A 3ϕ, 4 wire or 208 V, CBA system has a star connected load with $Z_A = 5\angle 0^\circ\ \Omega$, $Z_B = 5\angle 30^\circ$, $Z_C = 10\angle -60^\circ$. Obtain phase current, line current, current through neutral wire.



$V_L = 208\ V$

$Z_A = 5\angle 0$

$Z_B = 5\angle 30$

$Z_C = 10\angle -60^\circ$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = \underline{120.09\ V}$$

$V_{AN} = 120.09\angle 0\ V$

$V_{BN} = 12.09\angle -120\ V$

$V_{AN} = 12.09\angle -240\ V$

phase currents

$$I_A = \frac{V_{AN}}{Z_A} = \frac{12.09\angle -240}{5\angle 0} = \underline{24.02\angle 120^\circ A}$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{12.09\angle -120}{5\angle 30} = \underline{24.02\angle -150\ A}$$

$$I_C = \frac{V_{CN}}{Z_C} = \frac{12.09\angle 0}{10\angle -60} = \underline{12\angle 60^\circ A}$$

Line current

$I_{L1} = I_C = \underline{12\angle 60^\circ}$

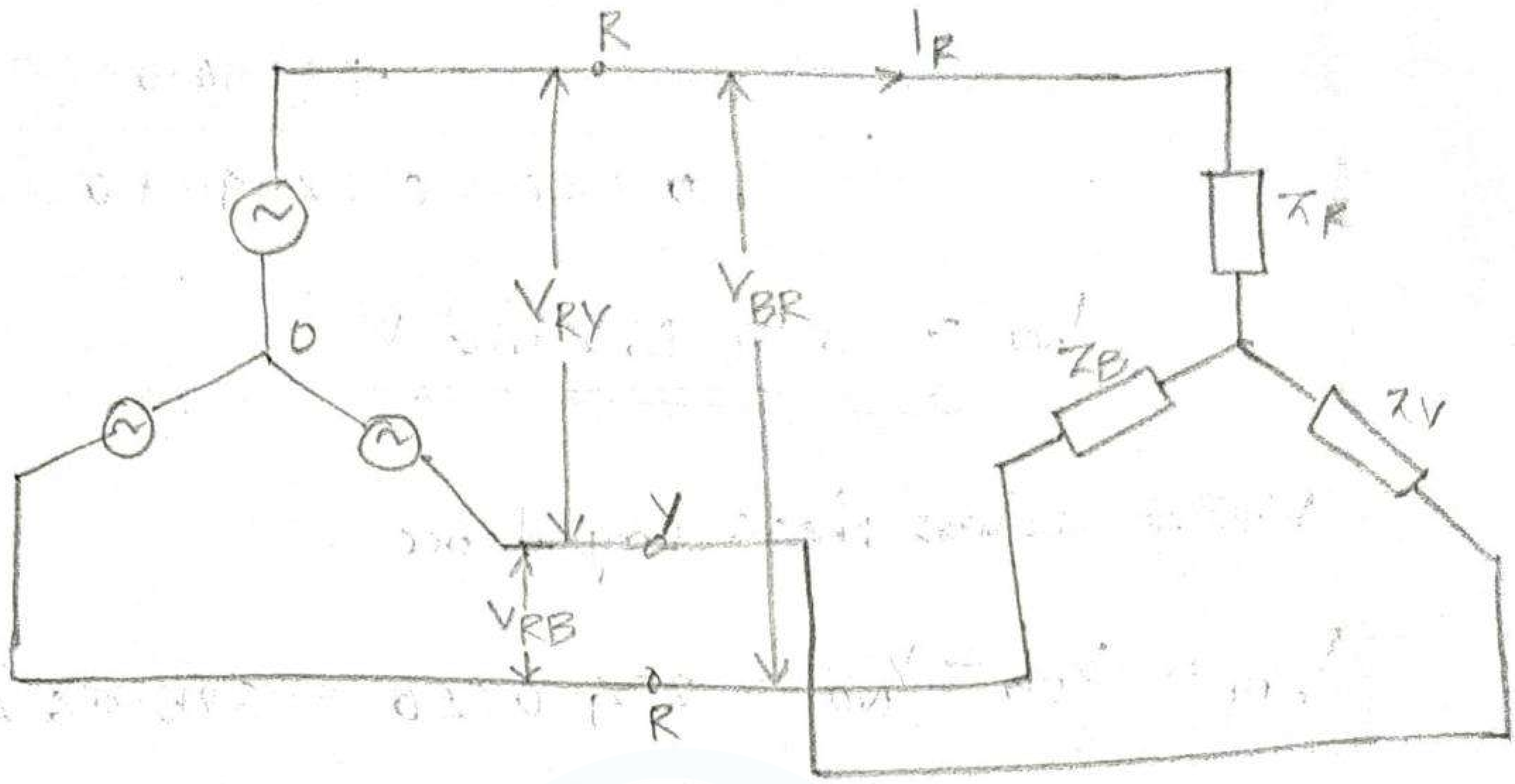
$I_{L2} = I_B = \underline{24.02\angle -150}$

$I_{L3} = I_C = \underline{24.02\angle 120}$

Current through Neutral $= I_{L1} + I_{L2} + I_{L3}$

$= \underline{\underline{32.97\angle 144.47\ A}}$

Q. A symmetrical 440 V, 3 phase system supplied to a star connected load with the branch impedence $Z_R = 10\,\Omega$, $Z_Y = 5j\,\Omega$, $Z_B = -j5\,\Omega$. Calculate the ~~b~~ line current, and voltage across each phase impedence by milman's theorum. RYB sequence.



$V_L = 440\,V$ $\quad Z_R = 10\,\Omega$ $\quad Z_Y = j5\,\Omega$ $\quad Z_B = -j5\,\Omega$

For star connected

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = \underline{254.03\ V}$$

O be the neutral point

$$V_{RO} = 254.03 \angle 0^\circ$$

$$V_{YO} = 254.03 \angle -120$$

$$V_{BO} = 254.03 \angle -240$$

admittance

$$Y_R = \frac{1}{Z_R} = \frac{1}{10\angle 0} = \underline{0.1 \angle 0^\circ}\ ℧$$

$$Y_Y = \frac{1}{Z_Y} = \frac{1}{5\angle 90} = \underline{0.2 \angle -90^\circ}\ ℧$$

$$Z_B = \frac{1}{Z_B} = \frac{1}{5\angle -90} = \underline{0.2 \angle 90^\circ}\ ℧$$

By Millman's theorem

$$V_{NO} = \frac{V_{RO}Y_R + V_{YO}Y_Y + V_{BO}Y_B}{Y_R + Y_Y + Y_B}$$

$$= \frac{(254.03\angle 0)(0.1\angle 0) + (254.03\angle -120)(0.2\angle -90) + (254.03\angle -240)(0.2\angle 90)}{0.1\angle 0 + 0.2\angle -90 + 0.2\angle 90}$$

$$V_{NO} = 625.96\angle 180^\circ \text{ V}$$

Voltage across phase impedence

$$V_{RN} = V_{RO} - V_{NO} = 254.03\angle 0 - 625.96\angle 180$$

$$= 880\angle 0 \text{ V}$$

$$V_{YN} = V_{YO} - V_{NO} = 254.03\angle -120 - 625.96\angle 180$$

$$= 545.29\angle -23.79 \text{ V}$$

$$V_{BN} = V_{BO} - V_{NO} =$$

$$= 545.29\angle 23.79 \text{ V}$$

Phase current = line current =>

$$I_R = \frac{V_{RN}}{Z_R} = \frac{880\angle 0}{10\angle 0} = 88\angle 0^\circ \text{A}$$

$$I_Y = \frac{V_{YN}}{Z_Y} = \frac{545.29\angle -23.79}{5\angle 90} = 109.06\angle -113.79^\circ \text{A}$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{545.29\angle 23.79}{5\angle -90} = 109.06\angle 113.79^\circ \text{A}$$

20/11/2020

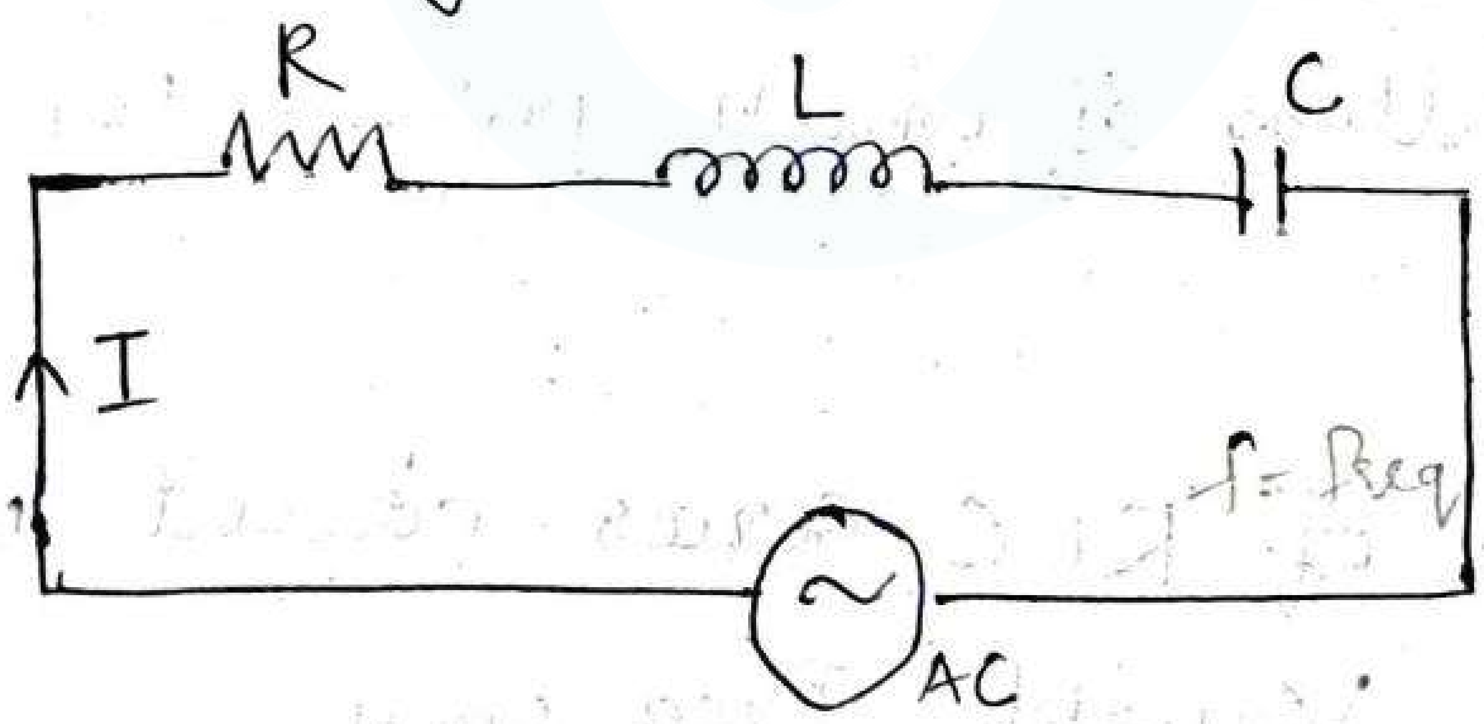
Resonance

Resonance is a particular type of phenomenum inherently found normally in every kind of system Electrical, Mechanical, Optical, Acoustical and even Atomic. Usually resonance occurs in any of these systems, when energy storage elements interchange exactly equal amount of energy.

Series Resonance :- only Resistance

$X_L + X_C = 0$

A circuit is said to be under resonance, when the applied voltage V and the resulting current I are in phase. Thus a series RLC circuit under resonance behaves like a pure resistance network and the net resistance of the circuit should be zero. Since V and I are in phase, $\phi = 0$ the p.f is unity at resonance. P = maximum



Power Factor

$\cos\phi = \frac{R}{Z}$

$\cos\phi = 1$

$\phi = 0$

The complex impedence

$$Z = R + j(X_L - X_C)$$

$$Z = R + j\left(\omega L - \left(\frac{1}{\omega C}\right)\right)$$

The absolute value of impedence is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Under resonance, the circuit should be purely resistive, ie; net reactance should be zero

$$X = X_L - X_C = \omega L - \frac{1}{\omega C} = 0$$

$$\Rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega = \left(\frac{1}{\sqrt{LC}}\right) \quad \text{or} \quad f = \frac{1}{2\pi\sqrt{LC}}$$

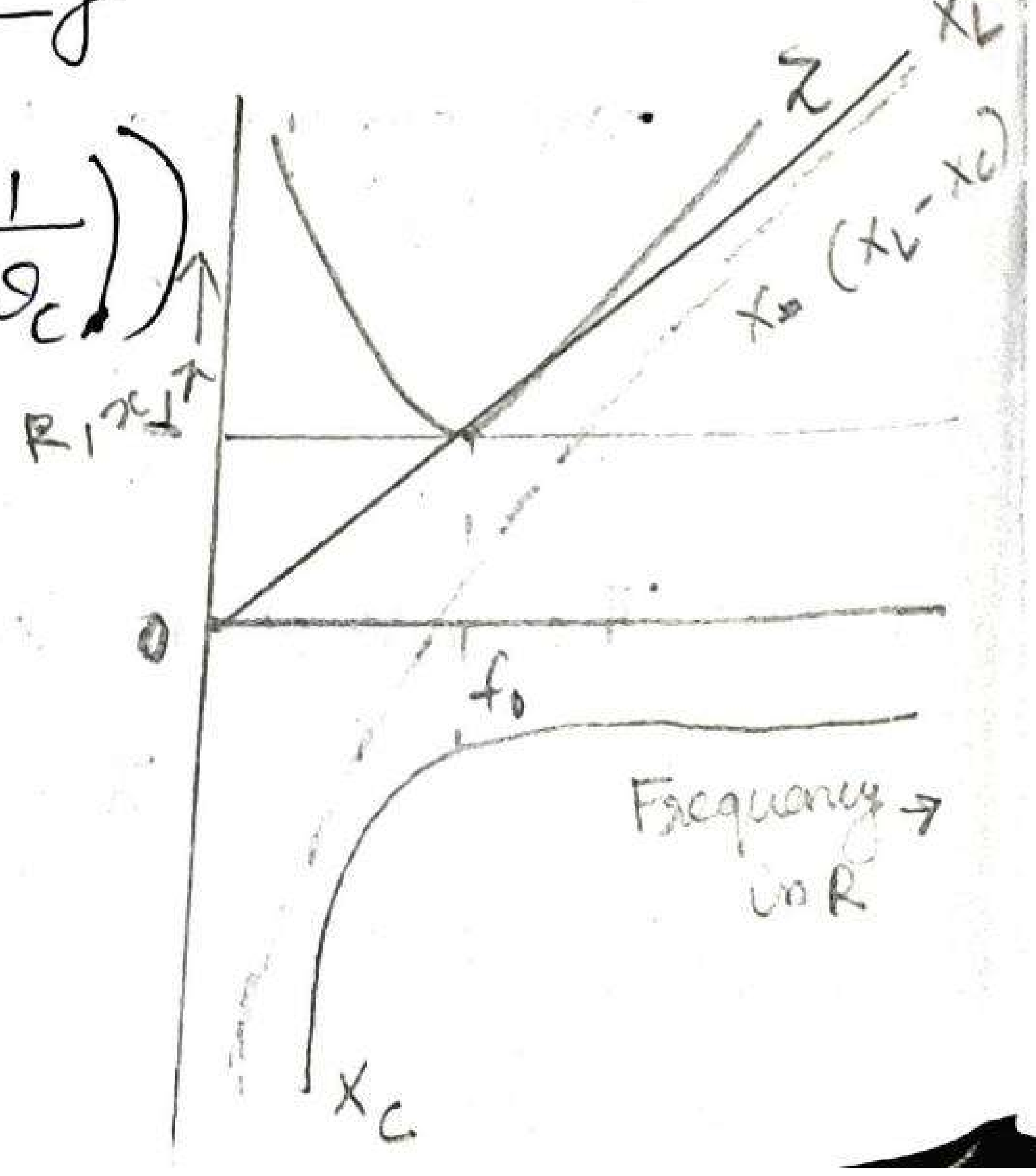
$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

is called the natural frequency of the circuit

When the frequency is equal to $\frac{1}{2\pi\sqrt{LC}}$ ie; the natural frequency of the circuit, it is under resonance and this frequency is called resonant frequency. Resonance can also be obtained by varying L or C. The general consideration at resonance are the same regardless of which parameter is varied

Behavior of RLC series circuit under <u>Variable Frequency</u>

$$Z = R + j\left(\omega L - \left(\frac{1}{\omega C}\right)\right)$$



① Resistance R → independent of frequency

② Inductive Reactance $X_L \propto$ frequency and +ve

③ Capacitive Reactance $X_C \propto \frac{1}{\text{frequency}}$ and is −ve

④ Net reactance $X = X_L - X_C$, initially −ve and becomes zero and then +ve.

ie; for $f < f_0$, it is capacitive p.f → leading

for $f = f_0$, it is zero p.f → unity

for $f > f_0$, it is inductive p.f → lag

when $X_L = X_C$, circuit is under resonance.

⑤ Impedence (Z)

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance $X_L - X_C = 0$, hence $Z = R$. At any other frequency $(X_L - X_C) \neq 0$ and hence $|Z| > R$. Hence impedence at resonance is minimum and it is equal to R. The circuit behaves as capacitive circuit below f_0 and inductive circuit f_0.

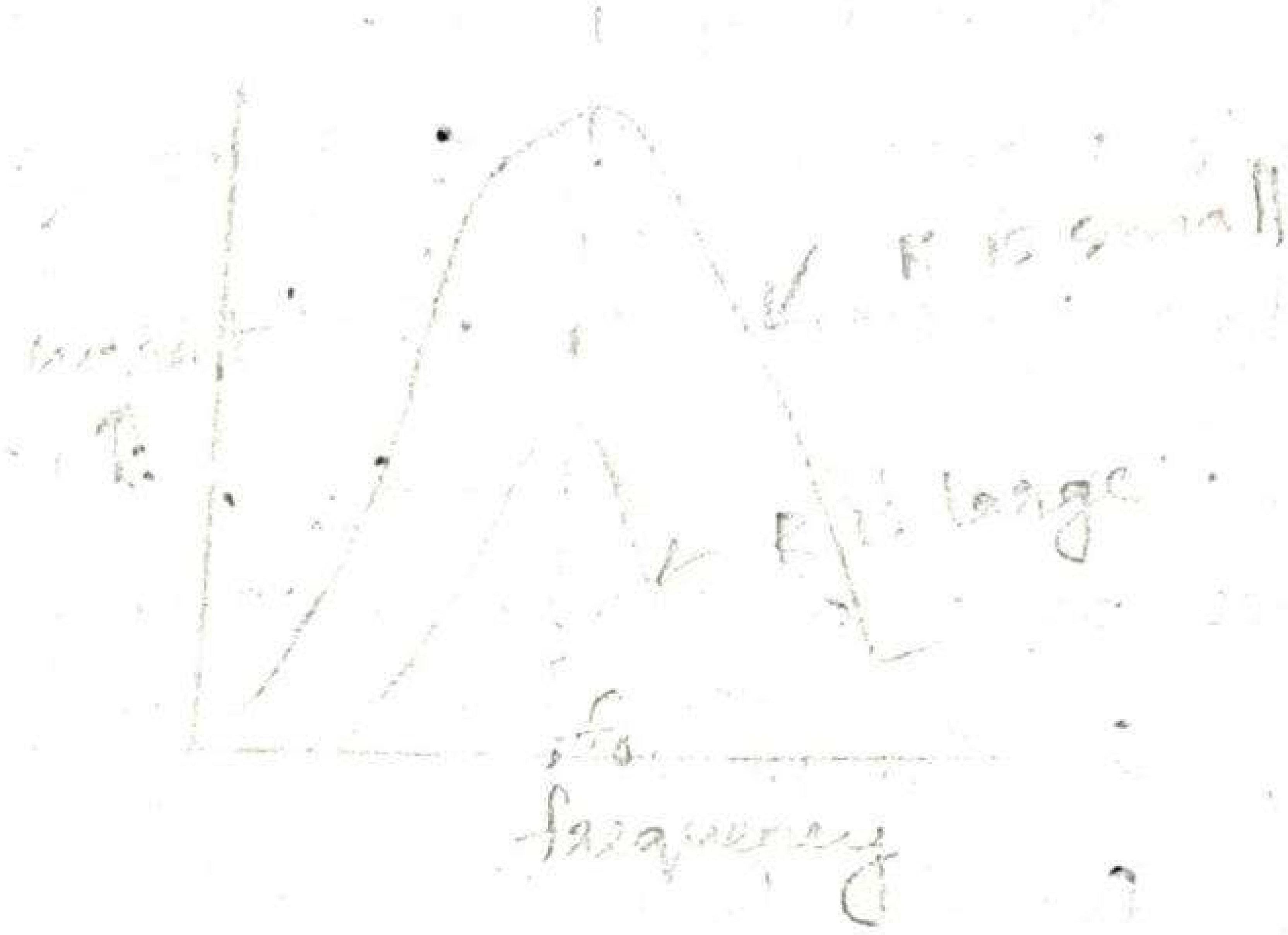
Current at Resonance :-

$$\text{current} = \frac{\text{Voltage}}{\text{Impedence}} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Since the impedence is minimum and equal to R at resonance, the current is maximum and equal to $\frac{V}{R}$ and in phase with V.

It varies inversely at impedence Z

$$I_{max} = V/R.$$

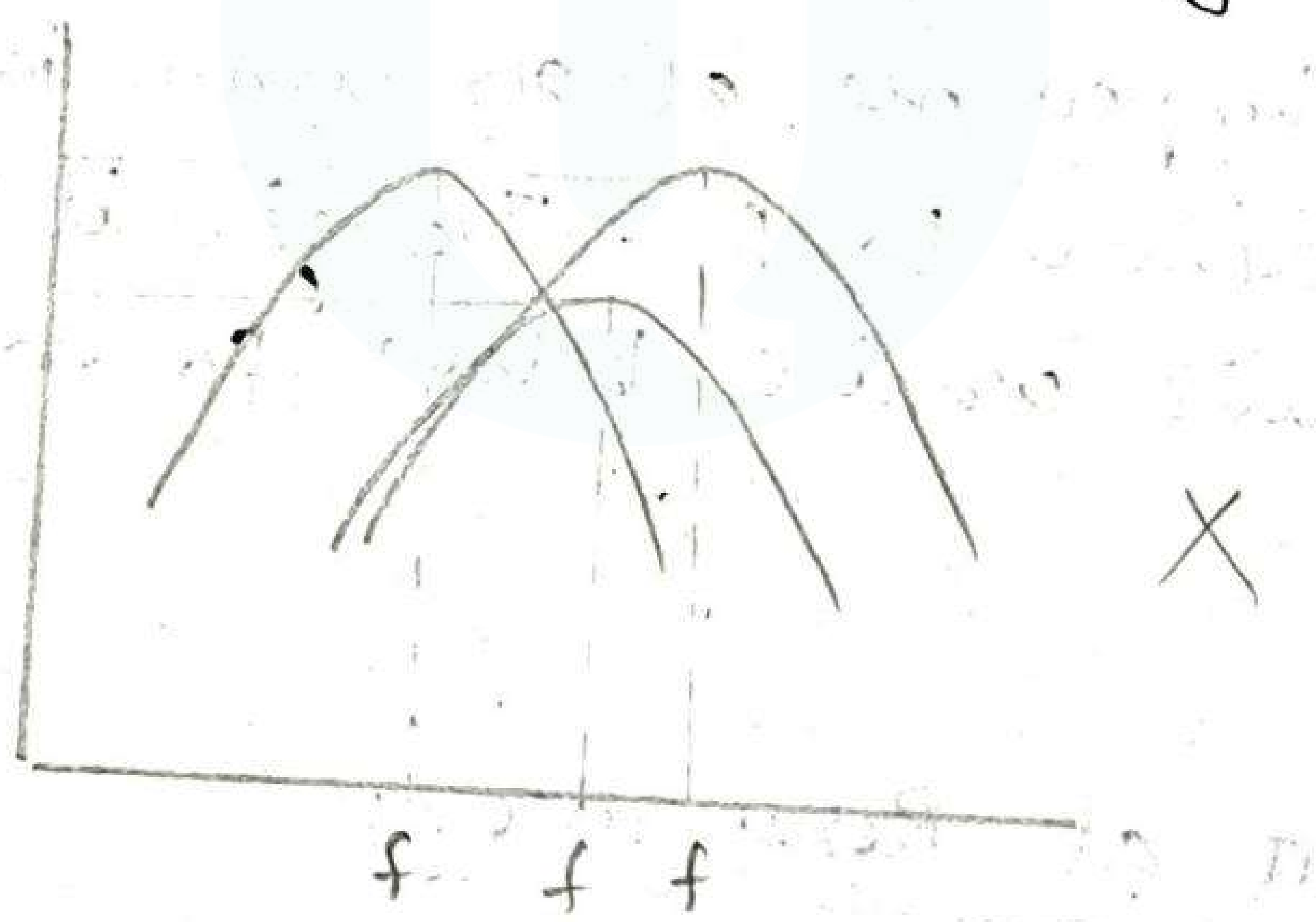


Voltage across R, L and C.

Voltage across resistance $R = IR$ is maximum at resonance and is equal to voltage applied in the series circuit.

Voltage across inductive Reactance $= I \times X_L$

Both I and X_L are increasing before resonance and the product must be increasing



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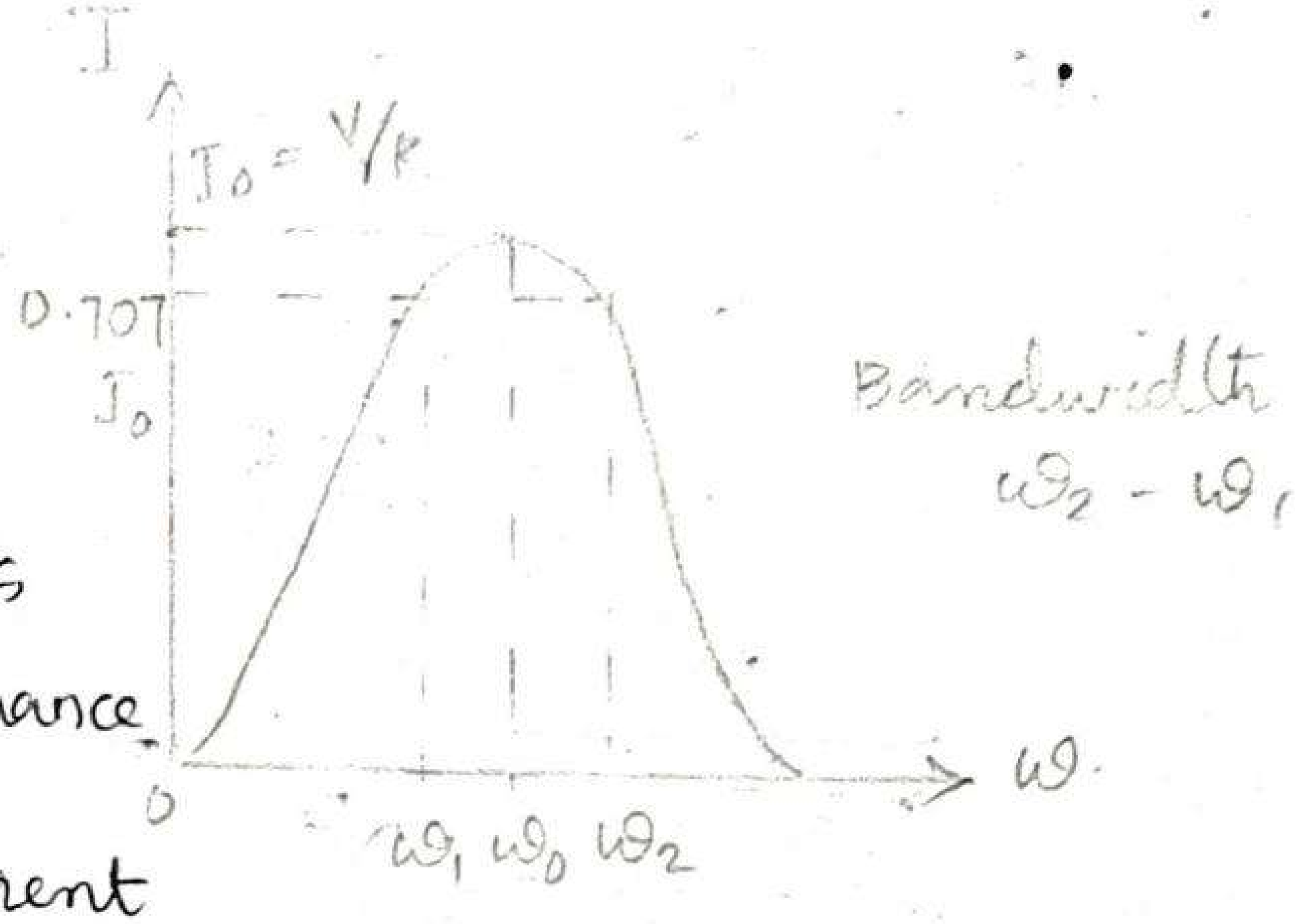
Bandwidth

For a series RLC circuit the bandwidth is defined as the range of frequencies for which the power delivered to R is greater than or equal to $\frac{P_0}{2}$ where P_0 is the power delivered to R at Resonance.

From the shape of resonance curve, it is clear that there are 2 frequency for which the power delivered to R is half the power at resonance



The magnitude of current at each half-power point is the same.

$$I_1^2 R = \frac{1}{2} I_0^2 R = I_2^2 R \qquad P_1 = \frac{P_0}{2} = P_2$$

1 denotes the lower half point and 2 denotes the higher half point.

$$I_1 = I_2 = \frac{I_0}{\sqrt{2}} = .707\, I_0$$

Accordingly the bandwidth may be identified on the resonance curve as the range of frequencies over which the magnitude of current is equal to or greater than .707 of current at resonance.

Bandwidth is $\omega_2 - \omega_1$.

<u>Expression for Bandwidth</u> :-

$$Z = R + j(X_L - X_C)$$

$$I = \frac{V}{Z}$$

$$= \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

At half power points

$$I = \frac{I_0}{\sqrt{2}}.$$

then $I_0 = \frac{V}{R}$

$$I = \frac{V}{\sqrt{2}\,R}.$$

$$\frac{\cancel{V}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{\cancel{V}}{\sqrt{2}\,R}.$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}.R$$

Squaring both sides

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2\,R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} \pm R = 0$$

multiplying with $\frac{\omega}{L}$

$$\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0$$

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} \qquad -b \pm \sqrt{b^2 - 4ac}$$

For low values of R, the term $\frac{R^2}{4L^2}$ can be neglected in comparison to term $\frac{1}{LC}$.

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{1}{LC}} = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$$

The resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \pm \frac{R}{2L} + \omega_0$$

$$\omega_1 = \omega_0 - \frac{R}{2L}$$

$$\omega_2 = \omega_0 + \frac{R}{2L}$$

$$f_1 = f_0 - \frac{R}{4\pi L}$$

$$f_2 = f_0 + \frac{R}{4\pi L}$$

$$\boxed{\begin{array}{l} \text{Bandwidth} = \omega_2 - \omega_1 = \dfrac{R}{L} \\ \text{Bandwidth} = f_2 - f_1 = \dfrac{R}{2\pi L} \end{array}}$$

Quality factor :-

Measure of voltage magnification in the series resonant circuit. Also a mesure of selectivity or sharpness of series R

$$Q_0 = \frac{\text{Voltage across inductor / capacitor}}{\text{Voltage across resonance}}$$

$$= \frac{V_{L0}}{V} = \frac{V_{C0}}{V}$$

Sub: value of V_{L0} and V

$$Q_0 = \frac{I_0 \times X_{L0}}{I_0 R} = \frac{X_{L0}}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}.$$

Sub: value of ω_0

$$Q_0 = \frac{(1/\sqrt{LC})\, L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad Q_0$$

Q. A series RLC circuit has the following parameter values $R = 10\Omega$, $L = 0.01$ H, $C = 100\mu F$. Compute the resonant frequency, bandwidth and lower and upper frequencies of the bandwidth.

$R = 10\Omega$, $L = 0.01$ H, $C = 100\mu F$

Ans: Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 100 \times 10^{-6}}}$$

$$= \underline{159.15 \text{ Hz}}.$$

(b) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi \times 0.01} = \underline{159.15 \text{ Hz}}.$$

(c) Lower frequency of bandwidth

$$f_1 = f_0 - \frac{BW}{2}$$

$$= 159.15 - \frac{159.15}{2}$$

$$= \underline{79.58 \text{ Hz}}$$

(d) Upper frequency

$$f_2 = f_0 + \frac{BW}{2}$$

$$= 159.15 + \frac{159.15}{2}$$

$$= \underline{238.73 \text{ Hz}}$$

24/11/2020

Q. A series RLC circuit is connected to a 200 V ac supply. The current drawn by the circuit at resonance is 20 A. The voltage drop across the capacitor is 5000 V at series resonance. Calculate resistance and ~~unpe~~ inductance of it the capacitance is 4 μF. Also calculate the resonant frequency.

$V = 200\ V \qquad I_0 = 20\ A \qquad V_C = 5000\ V$

$C = 4\ \mu F$

(a) Resistance $= \frac{V}{I_0} = \frac{200}{20} = \underline{10\ \Omega}$

(b) Resonant frequency

$$X_{C_0} = \frac{V_{C_0}}{I_0} = \frac{5000}{20} = \underline{250\ \Omega}$$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$250 = \frac{1}{2\pi \times f_0 \times 4 \times 10^{-6}}$$

$$\underline{f_0 = 159.15\ Hz}$$

(c) Inductance, At resonance

$$X_{C_0} = X_{L_0} = 250\ \Omega$$

$$X_{L_0} = 2\pi f_0 L$$

$$250 = 2\pi \cdot 159.15 \times L$$

$$\underline{L = 0.25\ H}$$

Q A coil of 2Ω resistance and 0.01 H inductance is connected in series with a capacitor across 200 V mains. What must be the capacitance in order that max: current occurs at a frequency of 50 Hz. Find also the current and voltage across the capaci..

$$R = 2\Omega, \quad L = 0.01\ H, \quad V = 200\ V$$

$$f_0 = 50\ Hz$$

Capacitance

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$50 = \frac{1}{2\pi\sqrt{0.01 \times C}}$$

$$\underline{C = 1013.2\ \mu F}$$

Current $I_0 = \frac{V}{R}$

$$= \frac{200}{2} \Rightarrow \underline{100\ A}$$

$$V_{C_0} = I_0 X_{C_0} = I_0 \times \frac{1}{2\pi f_0 C}$$

$$= 100 \times \frac{1}{2\pi\, 50 \times 1013.2 \times 10^{-6}}$$

$$= \underline{\underline{314.16\ V}}$$

Q. A series RLC circuit has $R = 2.5\,\Omega$, $C = 100\ \mu F$ and a variable inductance. The applied voltage is 50 V at 800 rad/sec. The inductance is varied till the voltage

across the resistance is maximum. Find
(a) value of inductance under this condition (b) Q.
(c) current (d) Voltage across R, C and L.

$R = 2.5\,\Omega$, $C = 100\,\mu F$, $V = 50\,V$, $\omega = 800$ rad/s.

(a) Value of inductance

$$\omega = \frac{1}{\sqrt{LC}}$$

$$800 = \frac{1}{\sqrt{L \times 100 \times 10^{-6}}}$$

$$\underline{L = 0.0156\ H}$$

(b) Quality factor

$$Q_0 = \frac{\omega_0 L}{R} = \frac{800 \times 0.0156}{2.5} = \underline{5.}$$

(C) current

$$I_0 = \frac{V}{R} = \frac{50}{2.5} = \underline{20\ A}$$

(d) Voltage across R, L, C

~~Resistance~~ $V_R = I_0 R = 20 \times 2.5 = 50\,V$

$$V_L = Q_0 V = 5 \times 50 = 250\,V$$

$$V_C = Q_0 V = 5 \times 50 = 250\,V$$

Q. A series RLC circuit which resonates at 500 KHz has, $R = 25\,\Omega$, $L = 100\,\mu F$, $C = 1000\,pF$. Determine the quality factor, new value of C required to resonate at 500 KHz when the L is doubled and the new quality factor

$f_0 = 500\,Hz$, $R = 25\,\Omega$, $L = 100\,\mu H$, $C = 100$

(a) Quality factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{25}\sqrt{\frac{100 \times 10^{-6}}{1000 \times 10^{-12}}} = \underline{12.65}$$

(b) New value of C when $L = 200\,\mu H$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$500 \times 10^{3} = \frac{1}{2\pi\sqrt{200 \times 10^{-6} \times C}}$$

$$\underline{C = 506.61\,pF}$$

(c) New Q Factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$$

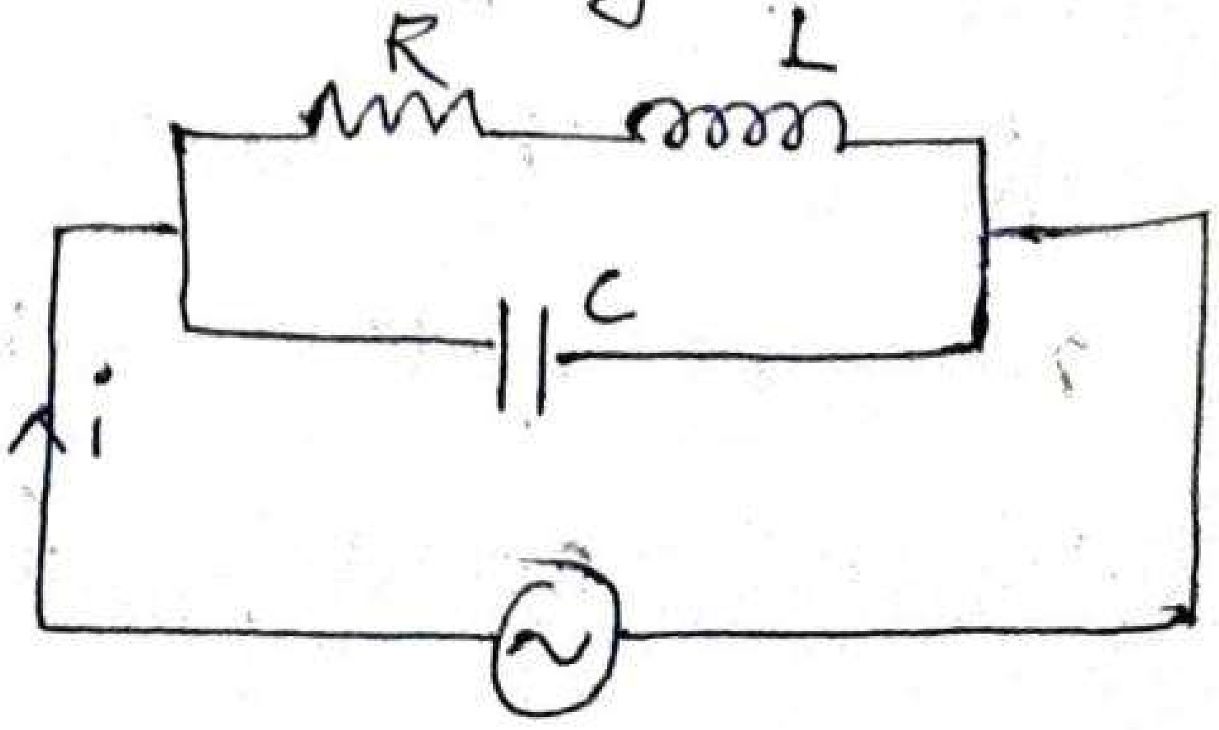
$$= \frac{1}{25}\sqrt{\frac{200 \times 10^{-6}}{506.61 \times 10^{-12}}} = \underline{\underline{25.13\,pF}}$$

Parallel Resonance

Consider a parallel ~~resonant~~ circuit consisting of a coil and circuit as shown in fig

$Z_1 = R + jX_L$

$Z_2 = -jX_C$



$$Y_1 = \frac{1}{Z_1} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{-jX_C} = \frac{j}{X_C}$$

Admittance of circuit

$$Y = Y_1 + Y_2$$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} - j\left[\frac{X_L}{R^2 + X_L} - \frac{1}{X_C}\right]$$

At resonance, the circuit is purely resistive, hence condition of resonance is

$$\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C} = 0$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

$$X_L X_C = R^2 + X_L^2$$

$$\omega_0 L \times \frac{1}{\omega_0 C} = R^2 + X_L^2$$

$$\frac{L}{C} = R^2 + \omega_0^2 L^2$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

f_0 is called resonant frequency of the circuit.

If R is very small compared to L

$R << L$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

→ Dynamic Impedence of Parallel Circuit :-

The real part of admittance is $\frac{R}{R^2 + X_L^2}$.

Hence dynamic impedence at resonance

$$Z_D = \frac{R^2 + X_L^2}{R}$$

At resonance,

$$R^2 + X_L^2 = X_L X_C = \frac{L}{C}$$

$$Z_D = \frac{L}{CR}$$

→ Current

Impedence is max at Resonance, I is minimum at resonance

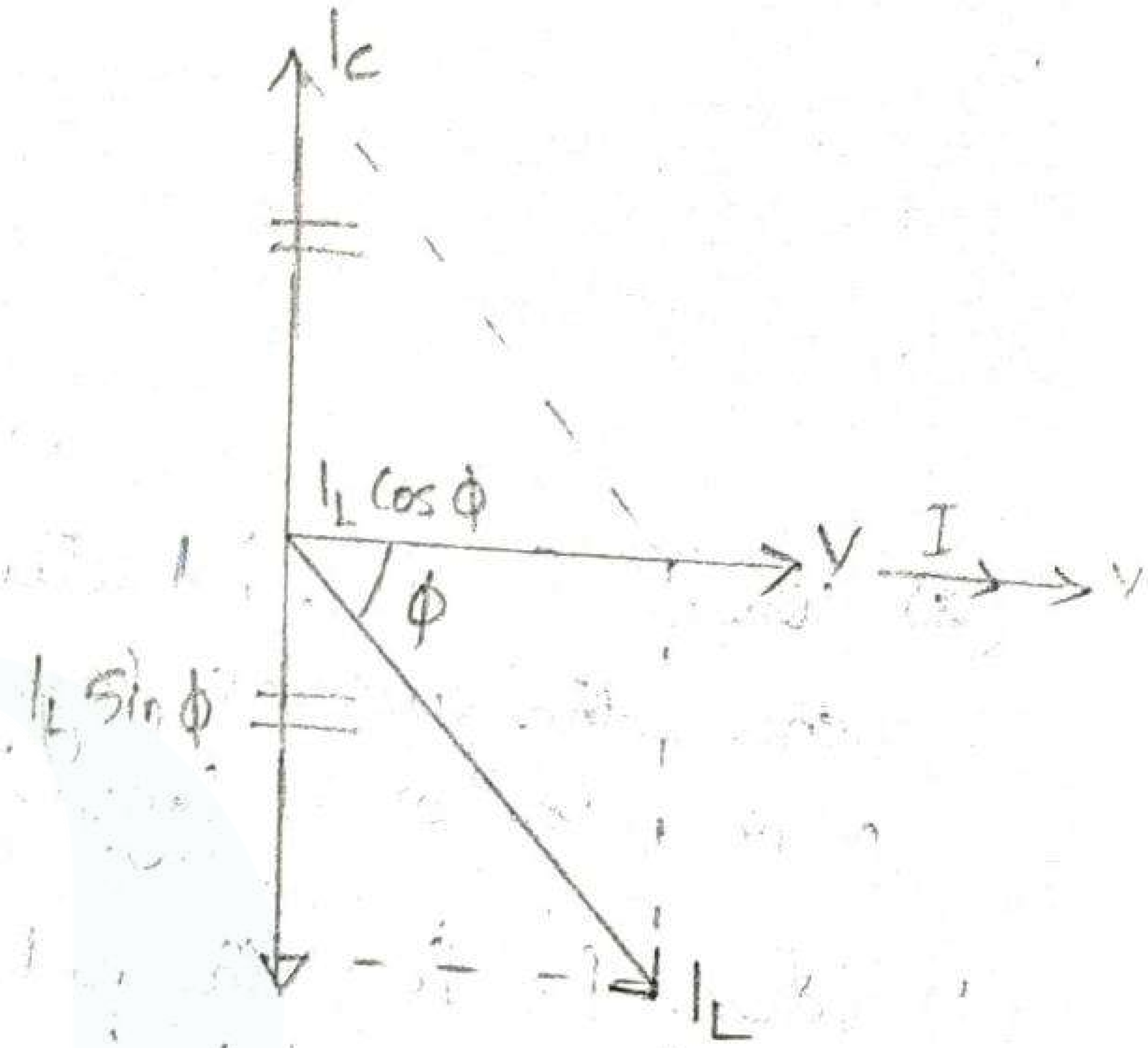
$$I_0 = \frac{V}{Z_D} = \frac{V}{L/CR} = \frac{VCR}{L}$$

25/11/2020

Phasor diagram :-

At resonance power factor is unity. So, the total current drawn by the circuit is the inphase with voltage. This happens only when current I_C is equal to reactive component of the current in the inductive brance.

$$I_C = I_L \sin\phi$$



Hence at resonance

$$I_C = I_L \sin\phi$$

$$I = I_L \cos\phi$$

Behaviour of Conductance G, Inductive susceptance B_L and Capacitive susceptance with change in Frequency.

Conductance remains constant with the change in frequency.

Inductive susceptane B_L

$$B_L = \frac{1}{jX_L} = -j\frac{1}{2\pi fL} = -j\frac{1}{2\pi fL}$$

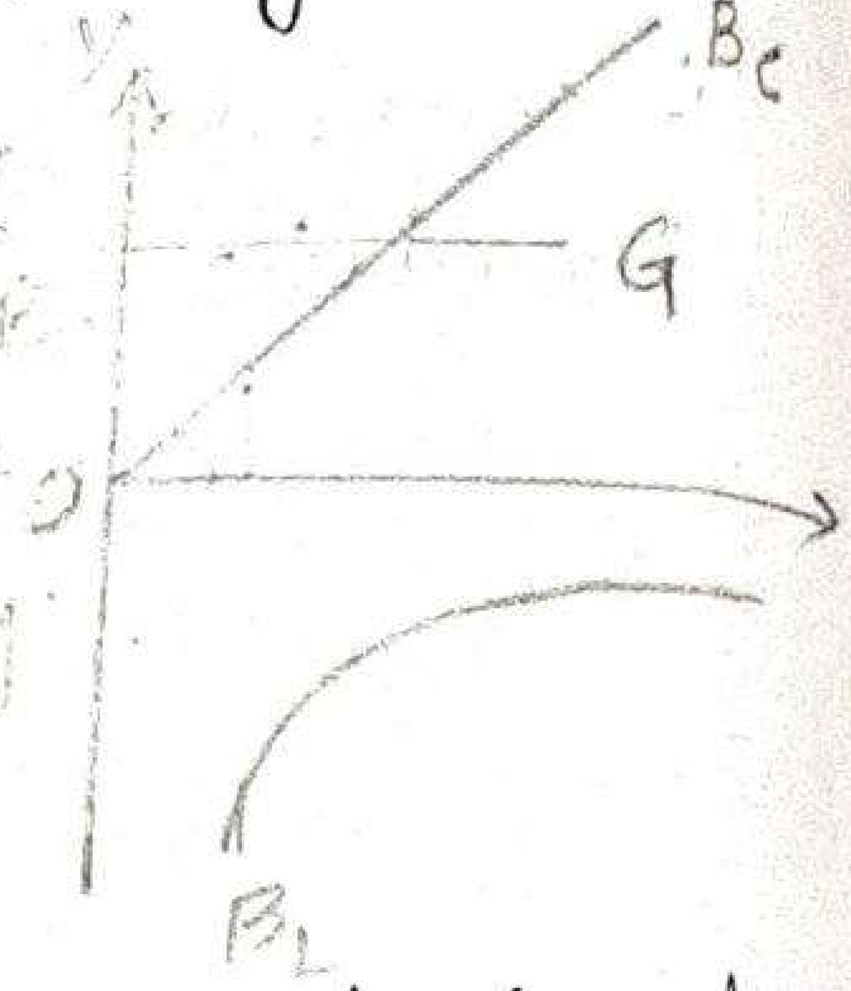
It is inversely propotional to the frequency thus, it decreases with the increase in frequency.

drawn as a rectangular hyperbola in the 4th quadrant.

Capacitive susceptance B_C

$$B_C = \frac{1}{-jX_C} = j\frac{1}{X_C} = \underline{\underline{j2\pi fC}}$$

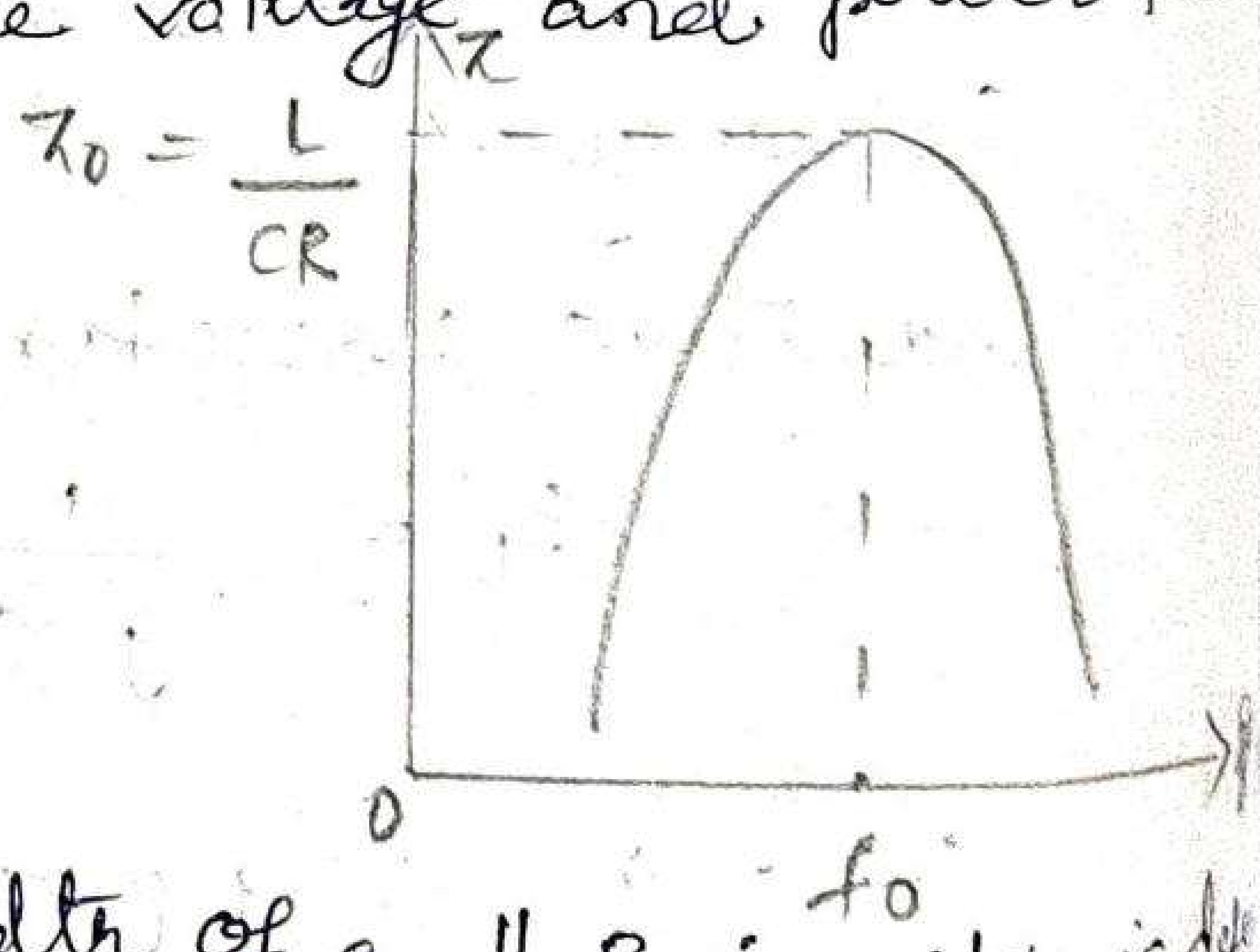
It is directly propotional to frequency. It can be a straight line passing through the origin.



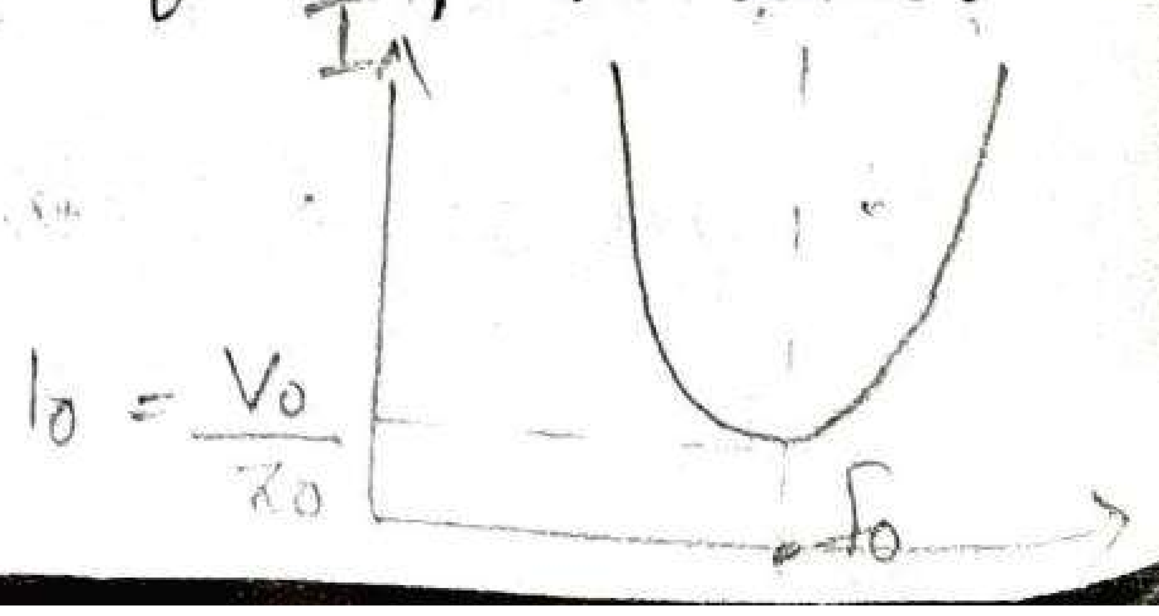
(a) When $f < f_0$, inductive susceptance predominates. Hence, the current lags behind the voltage and the power factor is lagging in nature.

(b) When $f = f_0$, the net susceptance is zero. Hence the admitance is minimum and impedence is maximum. At f_0, current is in phase with voltage and the power factor is unity.

(c) When $f > f_0$, capacitive susceptance predominates. Hence the current leads the voltage and power factor is leading in nature.



→ Bandwidth :- The bandwidth of a ∥ R circuit is defined in the same way as that of series resonance circuit



→ Quality factor :- Measure of current magnification in a parallel resonant circuit.

$$Q_0 = \frac{\text{Current through inductance or Capacitor}}{\text{Current at resonance}}$$

$$Q_0 = \frac{I_{C_0}}{I_0}$$

$$= \frac{V/X_{C_0}}{VCR/L} = \frac{\frac{1}{X_{C_0}}}{\frac{CR}{L}}$$

$$= \frac{\omega_0 C}{CR/L} = \frac{\omega_0 L}{R}.$$

Neglecting Resistance R.

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right) L}{R}$$

$$\boxed{Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Parameter	Series Circuit	Parallel Ci
Current at resonance	$I = V/R$, maximum	$I = VCR/L$ min
Impedence at resonance	$Z = R$, minimum	$Z = \frac{L}{CR}$ maxim
Power factor at resonance	unity	unity
Resonant frequency	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Q - factor	$Q = \frac{2\pi f_0 L}{R}$	$Q = \frac{2\pi f_0 L}{R}$
it magnifies	Voltages across L and C	Current across L and C

Q. A coil having an inductance of L henry and resistance of 12 Ω is connected in parallel with a variable capacitor. At $\omega = 2.3 \times 10^6$ rad/s, resonance is acheived and at this instant, capacitance $C = 0.021\,\mu F$. Find inductance.

$R = 12\,\Omega$ $\quad \omega_0 = 2.3 \times 10^6$ rad/s $\quad C = 0.021 \times 10^{-6}$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$2.3 \times 10^6 = \sqrt{\frac{1}{L \times 0.021 \times 10^{-6}} - \frac{12^2}{L^2}}$$

$$L = 89.7\,\mu H$$

Q. A coil of 20 Ω Resistance has an inductance of ·2 H and is connected in parallel with a condensor of 100 μF capacitor. Calculate the frequency at which this circuit will have a non inductive resistance. Also find dynamic resistance.

$R = 20\,\Omega$, $L = 0.2\,H$, $C = 100\,\mu F$.

(a) Resonant frequency :-

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 100 \times 10^{-6}} - \frac{20^2}{(0.2)^2}}$$

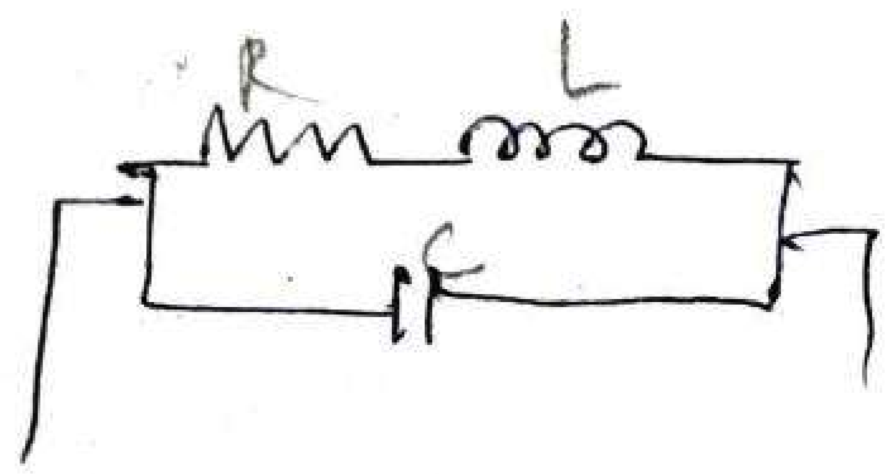
$$= 31.83\ Hz.$$

(b) Dynamic Resistance

$$Z_D = \frac{L}{CR} = \frac{0.2}{100 \times 10^{-6} \times 20}$$

$$= 100\,\Omega$$

Q, An inductance at 0.1 H having a Q_0 of 5 is in parallel with capacitor. Determine the value of capacitance and coil resistance at a resonant frequency of 500 rad/s.



$L = 0.1\ H \quad Q_0 = 5 \quad \omega_0 = 500\ rad/s$.

$$Q_0 = \frac{\omega_0 L}{R}$$

$$5 = \frac{500 \times 0.1}{R}$$

$$\underline{R = 10\ \Omega}.$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$500 = \sqrt{\frac{1}{0.1 \times C} - \frac{10^2}{0.1^2}}$$

$$\underline{\underline{C = 38.46\ \mu F}}.$$

Q. A coil of $10\,\Omega$ R and 0.2 H inductance is connected in parallel with a variable capacitor across of 220 V, 50 Hz supply. Calculate

(a) capacitance of capacitor for resonance.

(b) Dynamic impedence of circuit

(c) Supply current $I = \frac{V}{Z_C}$

$R = 10\,\Omega \quad L = 0.2\ H \quad f_0 = 50\ Hz$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$50 = \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times C} - \frac{10^2}{0.2^2}}$$

$$C = 49.41\ \mu F$$

(b) Dynamic impedence

$$Z_D = \frac{L}{CR} = \frac{0.2}{49.41\times10^{-6}\times10}$$

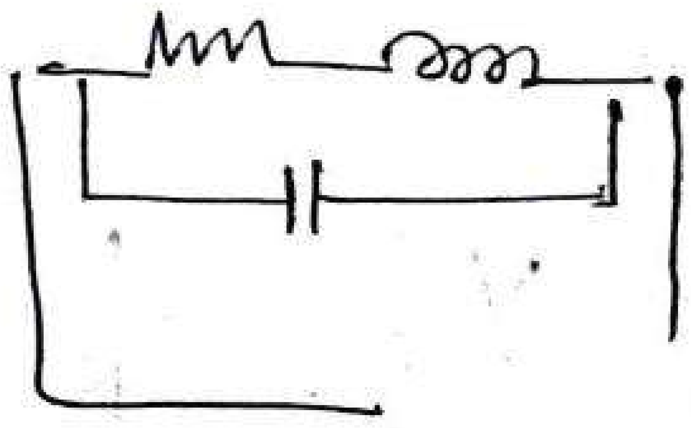
$$= 404.78\,\Omega$$

(c) Supply current

$$I = \frac{V}{Z_D} = \frac{220}{404.78}$$

$$= 0.543\ A$$

Q. Find the value of dynamic impedence of circuit at frequency of 500 KHz and bandwidth of operation equal to 20KHz. The resistance of the coil is 5Ω.



$f_0 = 500\ kHz \qquad BW = 20\ kHz$

$R = 5\Omega$.

$$B.W = \frac{R}{2\pi L}$$

$$20\times10^{3} = \frac{5}{2\pi L} \qquad L = 39.79\ \mu H$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

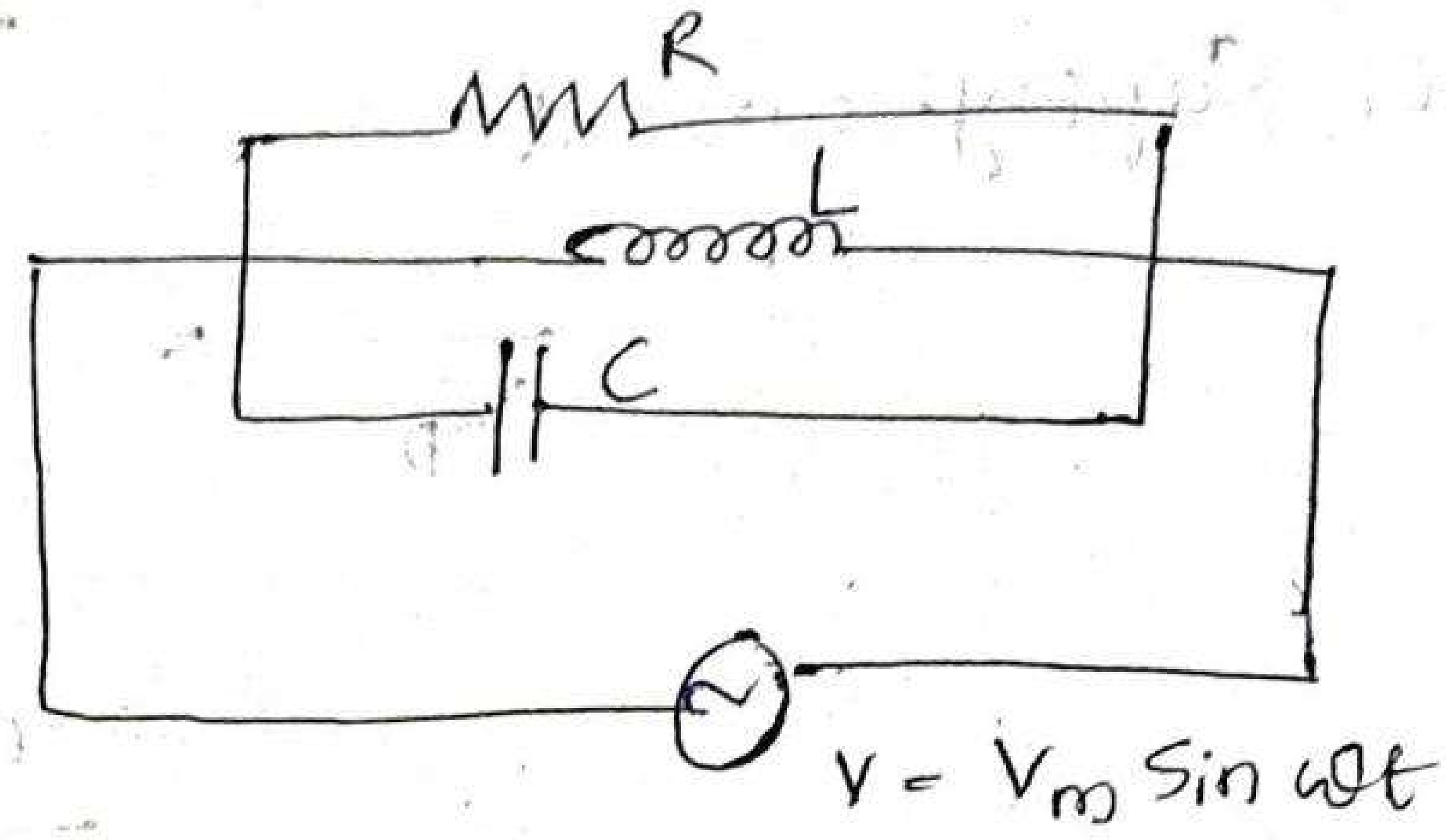
$$500\times10^{3} = \frac{1}{2\pi}\sqrt{\frac{1}{39.79\times10^{-6}\times C} - \frac{5^2}{(39.79\times10^{-6})^2}}$$

$$C = 2.54\ nF$$

$$Z_D = \frac{L}{CR} = \frac{39.79 \times 10^{-6}}{2.54 \times 10^{-9} \times 5}$$

$$= \underline{\underline{3.13\ k\Omega}}.$$

Q. Derive expression for resonant frequency for the parallel circuit. Calculate impedence and current at resonance.



$Z_1 = R$

$Z_2 = jX_L$

$Z_3 = -jX_C$.

Resonant frequency,

For parallel circuit

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$Y = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R} - j\frac{1}{X_L} + j\frac{1}{X_C}$$

$$= \frac{1}{R} - j\left(\frac{1}{X_L} - \frac{1}{X_C}\right)$$

At resonance, the circuit is purely resistive, Hence the condition for resonance is

$$\frac{1}{X_L} - \frac{1}{X_C} = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$f_0 \Rightarrow$ resonant frequency.

Impedence at resonance

At resonance, circuit is purely resistive

$$Y_D = \frac{1}{R}$$

$$Z_D = R$$

Current at resonance

$$I_0 = \frac{V}{Z_D} = \frac{V}{R}$$