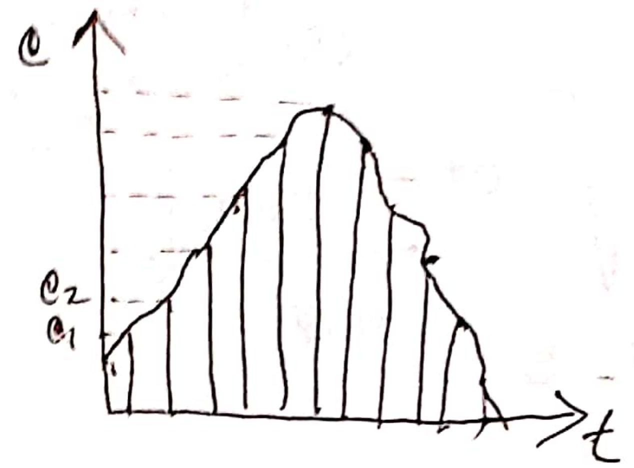
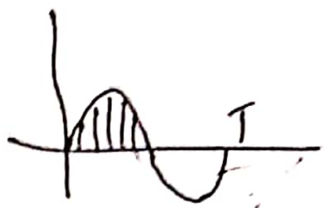
24/10/19 Average value & rms value

* 

$$E_{av} = \frac{E_1 + E_2 + \cdots E_n}{n}$$

$$E_{av} = \frac{\int_0^t e\,dt}{T} \quad \text{(for contin}$$

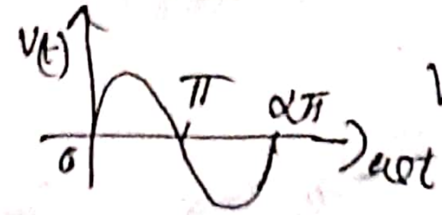
* 

$$E_{ave} = \frac{\int_0^{T/2} e\,dt}{T/2}$$

Average Value = $\frac{\text{Area}}{\text{Base}}$

* $$E_{rms} = \sqrt{\frac{e_1^2 + e_2^2 + \cdots e_n^2}{n}} \rightarrow \text{discrete.}$$

$$E_{rms} = \sqrt{\frac{\int_0 e^2 dt}{T}} \longrightarrow \text{continuous.}$$

? * 

$$V(t) = V_m \sin\omega t$$

$$V_{ave} = \frac{1}{T}\int_0^T v(t)\,dt = 0$$

$$V_{ave} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t$$

$$V_{ave} = \frac{1}{\pi} V_m \left(-\cos\omega t\right)_0^{\pi}$$

$$= \frac{V_m}{\pi}(-2) = \frac{2V_m}{\pi}$$

$$V_{rms} = \sqrt{\frac{\int_0^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t}{2\pi}}$$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

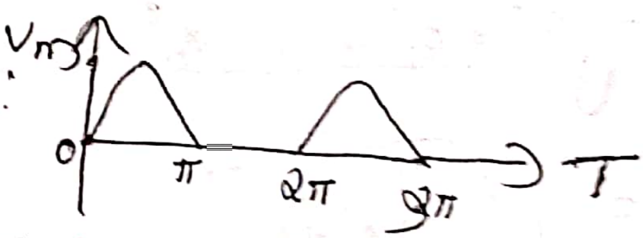
$$= \sqrt{\frac{\int_0^{2\pi} (1 - \cos 2x)\, dx}{4\pi}} = \sqrt{\frac{\left[\omega t - \frac{\sin 2x}{2}\right]_0^{2\pi}}{4\pi}}$$

$$= \frac{V_m}{2}\sqrt{\frac{2\pi - \frac{\sin 2\pi}{2} - 0}{\pi}}$$

$$= \frac{V_m}{\sqrt{2}}$$

(rms value is equivalent to dc)

* Half rectified



$$V_{ave} = \frac{1}{T}\int_0^T e\,dt = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin(\omega t)\, d\omega t$$

$$= \frac{-V_m}{2\pi}\left[(\cos \omega t)\right]_0^{2\pi} = \frac{-V_m}{2\pi}(-2) = \frac{V_m}{\pi}$$

$$V_{rms} = \sqrt{\frac{\int_0^T V_m^2 \sin^2 \omega t \, d\omega t}{T}} = \sqrt{\frac{\int_0^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t}{2\pi}}$$

$$\sqrt{\frac{\int_0^{\pi} V_m^2 \sin^2 \omega t \, d\omega t}{2\pi} + \frac{\int_{\pi}^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t}{2\pi}}$$

$$\frac{1}{\sqrt{2\pi}}\sqrt{\frac{\left(\omega t - \frac{\sin\omega t}{2}\right)_0^{\pi} + \left(\omega t - \frac{\sin\omega t}{2}\right)_{\pi}^{2\pi}}{2}}$$

$\frac{2\pi}{2}$

$$= \frac{1}{\sqrt{2\pi}}\sqrt{\frac{\pi + \pi - (0-1)}{2}} = \frac{V_m}{\sqrt{2\pi}}\sqrt{2\pi - 1}$$

$\frac{2\pi}{2} - 0 - 1$

$$= \frac{V_m}{2}$$

$\frac{1}{2}$

* Phases

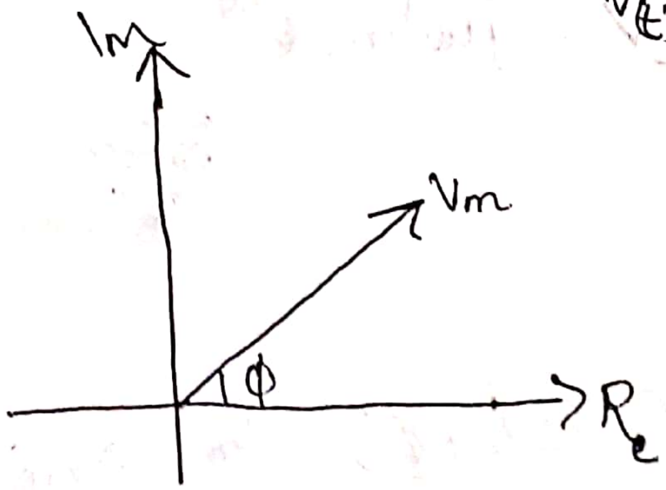
$V(t) =$

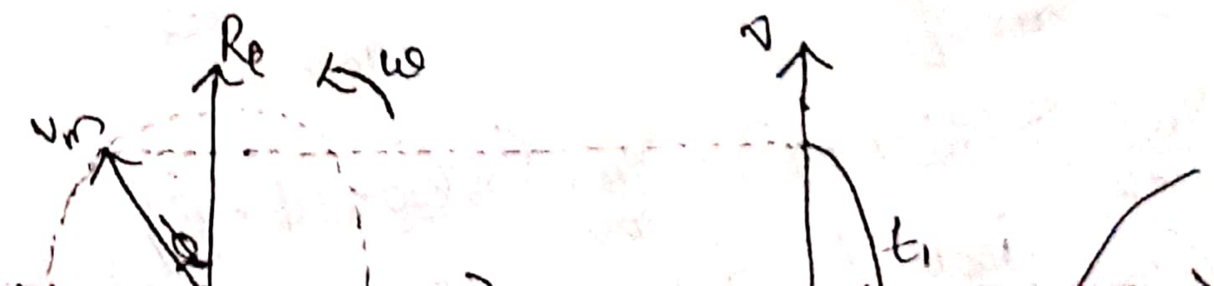
$\rightarrow V_m \cos(\omega t + \phi)$

$= Re\left[V_m e^{j(\omega t + \phi)}\right] = Re\left[V_m e^{j\omega t} e^{j\phi}\right]$

$= Re\left[\vec{V} e^{j\omega t}\right]$

$\vec{V} = V_m e^{j\phi} \rightarrow$ phaser representation of $V(t) = V_m \cos(\omega t + \phi)$

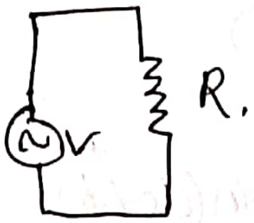




3/10/2019

Phasor Relationships of Circuit Elements

* Resistor :



$V = V\cos(\omega t+\phi)$

$i = \frac{V}{R}\cos(\omega t+\phi)$

$i = i_m\cos(\omega t+\phi)$

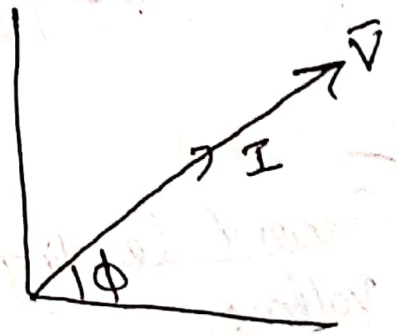
$i_m = \left(\frac{V}{R}\right)$

Phasor : $i = I_m \angle \phi$

$\bar{V} = \cancel{I_m} R I_m \angle \phi$

Both the voltage and current are in same phase.

Purely Resistive.



* Purely Inductive circuit

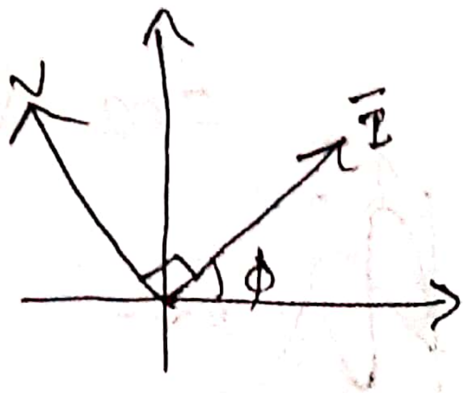
$i = I_m\cos(\omega t+\phi)$

$v = L\frac{di}{dt}$

$\frac{di}{dt} = I_m \omega \sin(\omega t+\phi)$

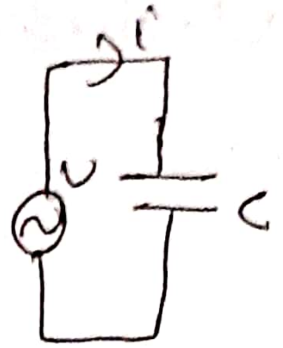
$v = +\omega L I_m \cos(\omega t+\phi+90^\circ)$

$\bar{V} = \omega L I_m \angle(90+\phi)$



$\boxed{\text{Current laggs behind the Voltage by } 90^\circ.}$

* Purely Capacitive Circuit



$Q = CV$

$V = V_m \cos(\omega t + \phi)$

$i = C\frac{dv}{dt} = -C\,\omega V_m \sin(\omega t + \phi)$

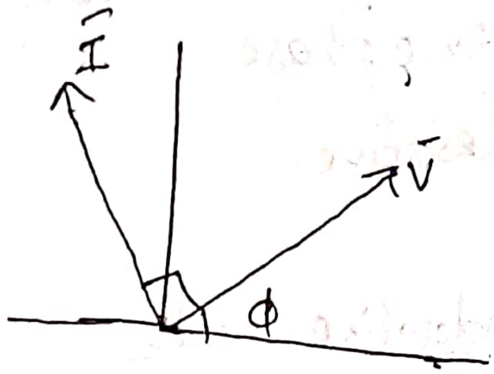
$i = -C\omega V_m \sin(\omega t + \phi)$

$C\omega V_m \cos(\omega t + \phi + 90^\circ)$

$\bar{V}$ and $\bar{I}$

$V = V_m \angle \phi$, $i = \omega C V_m \angle (90 + \phi)$

Current leading the voltage.



* Inductive

$P_{inst} = Vi$

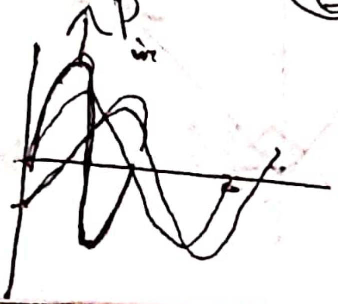
$= V_m \cos(\omega t + \phi)\, \omega C V_m \cos(\omega t + \phi + 90^\circ)$ $\quad \cos A \cdot \cos(A+B)$

$= \omega C V_m^2 \cos(\omega t + \phi)(-\sin(\omega t + \phi))$ $\quad \cos A \cos B - \sin\theta.$

$= \omega C V_m^2 \frac{\sin 2(\omega t + \phi)}{2}$ $\quad = 2\omega C V_m^2 t$

$= -\frac{1}{2} V_m I_m \sin\{2(\omega t + \phi)\}$

$= -\frac{V_m I_m}{2} \sin(2\theta)$, $\theta = \omega t + \phi$



$\boxed{P_{ave} = 0}$

* Capacitive : $P_{inst} = i_m V_m$

$= V_m \cos(\omega t + \phi)$

$= -\frac{V_m I_m}{2} \sin 2\theta$, $\theta = \omega t + \phi$

Angular fergeuncy of Power = 2ω.

$\frac{V}{I} = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

1. $\frac{V}{I} = R$. — Resistive

2. $\frac{V}{I} = j\omega L I$ — Inductive

3. $\frac{V}{I} = \frac{-j}{C\omega}$. — Capacitive

* Impedence Z

~~Ratio~~ of Resistance offered. $Z = \frac{V}{I}$

$Z = R$

$= j\omega L$

$= \frac{-j}{C\omega}$

→ if $\omega = 0$, dc circuit, $Z = 0$ purely ~~inductive~~ Inductive.

$Z = \infty$ purely Capac.

Purely Inductive : $\omega = 0$, $Z = 0$.

$\omega = \infty$, $Z = \infty$

Purely Capacitive : $\omega = 0$, $Z = \infty$

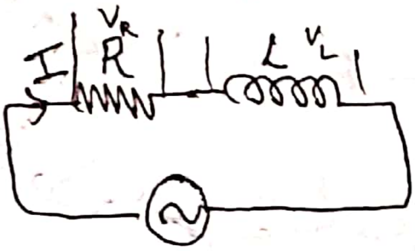
$\omega = \infty$, $Z = 0$

* Admittance : $\left(\frac{1}{Z}\right)$, $Y = \frac{1}{Z} = \underbrace{G}_{\text{Conductance}} + j\underbrace{B}_{\text{Susceptance}}$ $\boxed{G \neq \frac{1}{R}}$

* $Z = R \pm j\underbrace{X}_{\text{reactance}}$

11/2019

RL Circuit



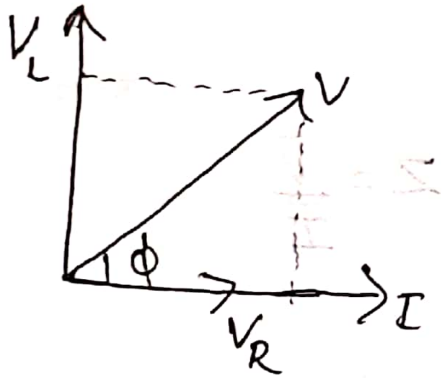
$V = V_m \sin \omega t$

$V_L = I X_L$

$V_R = IR$

X_L - reactance offered by inductance

$X_L = L\omega$



$I = I_m \sin(\omega t - \phi)$

$V = \sqrt{V_R^2 + V_L^2}$

$V = I\sqrt{R^2 + (L\omega)^2}$

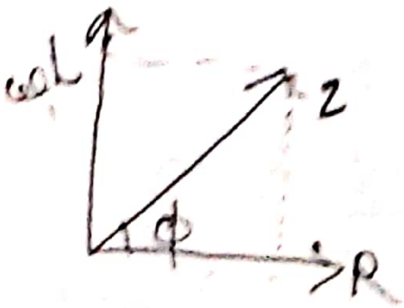
$$\Rightarrow I = \frac{V}{\sqrt{R^2 + (L\omega)^2}}$$

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

* $|Z| = \sqrt{R^2 + (L\omega)^2}$

$Z = R + j\omega L$

$Z\angle\phi$; $\sqrt{R^2 + (L\omega)^2} \angle \tan^{-1}\left(\frac{L\omega}{R}\right)$



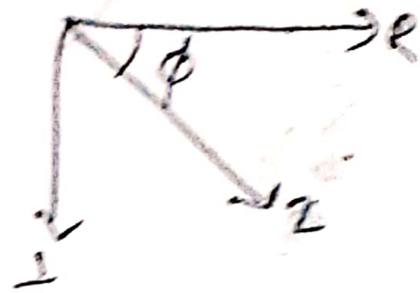
- $V = \sqrt{V_R^2 + V_L^2}$

$V = I\sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}$

* $|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$Z = R - \frac{j}{C\omega}$

$\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$



Power, $P = V_R I$

$= VI\cos\phi$

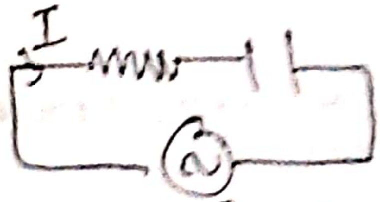
$\cos\phi$ = power factor of ckt.

$P = V_R I$

$= VI\cos\phi$

Power factor is leading
Becoz current is leading

RC Circuit



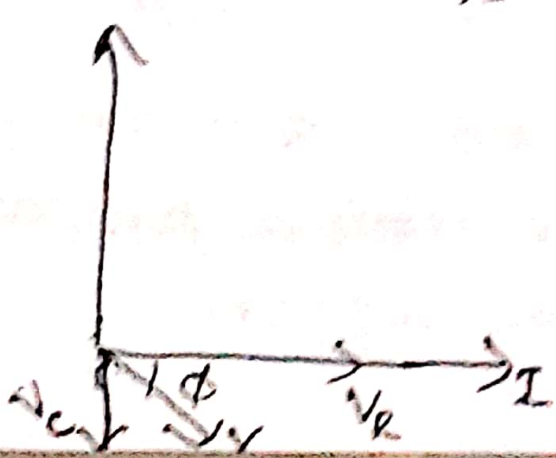
$V_t = V_m \sin\omega t$

$V_R = IR$

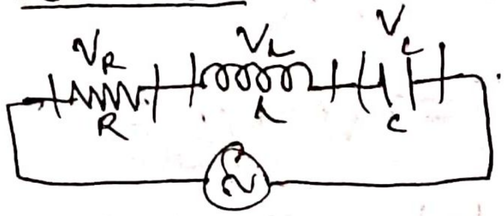
$V_L = I X_C$

$X_C = \frac{1}{C\omega}$

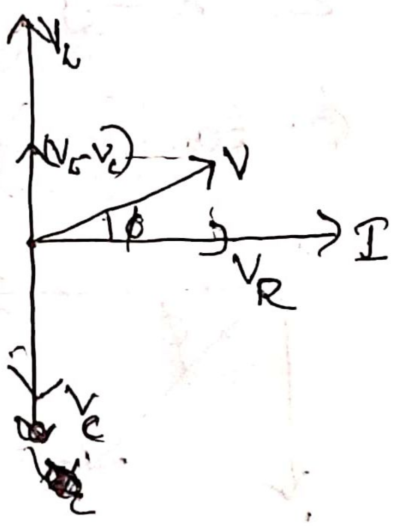
$I = I_m \sin(\omega t + \phi)$



* RLC Circuit



$V = V_m \sin \omega t$



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= I\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$\phi = \tan^{-1} \frac{\left(\omega L - \frac{1}{C\omega}\right)}{R}$$

$Z = |Z| \angle \phi$, $Z = R + j\left(\omega L - \frac{1}{C\omega}\right)$

$Z = |Z| \angle \phi$

$$\bar{I} = \frac{V_m \sin(\omega t + 0)}{Z}$$

$$= \frac{V\angle 0^\circ}{|Z|\angle\phi} = \frac{V}{Z}\angle -\phi$$

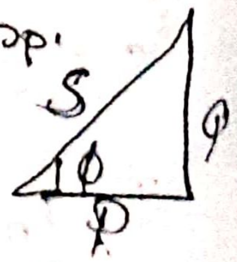
$$\bar{I} = I_m \sin(\omega t - \phi)$$

* if $X_L > X_C$, current laggs behind (V) by ϕ

* $X_C > X_L$, current lead the voltage by ϕ

* $P = VI \cos\phi$ - Active power (P) (Watts) (resistance)
* $P = VI \sin\phi$ - Wattless compnt (Q) Reactive power Volt Amp Reac
* $P = VI$ is Apparent power (S) Volt Amp.

$S = \sqrt{P^2 + Q^2}$



P - ale

A resistor of R=50Ω L=0.1H are connected in series across a 200V 50Hz supply. Find the impedence, powerfactor, current, active power and reactive power