EE 302
Linear Control Systems

Preamble: This course aims to provide a strong foundation on classical control theory. Modelling, time domain analysis, frequency domain analysis and stability analysis of linear systems based on transfer function approach will be discussed. The compensator design of linear systems is also introduced.

Prerequisite : Basics of Circuits and Networks,
Signals and Systems

Course Outcomes : After the completion of the course the student will be able to:

- CO1: Describe the role of various control blocks and components in feedback systems.
- CO2 :Analyse the time domain responses of the linear systems.
- CO3: Apply Root locus technique to assess the performance of linear systems.
- CO4: Analyse the stability of the given LTI systems.
- CO5:Analyse the frequency domain response of the given LTI systems.
- CO6: Design compensators using time domain and frequency domain techniques.

Module 1 : Feedback Control Systems (9 hours)

• Open loop and closed loop control systems- Examples of automatic control systems

•Transfer function approach to feed back control systems – Effect of feedback Control system components

• Control applications of DC and AC servo motors, Tacho generator, Synchro, Gyroscope and Stepper motor Controllers- Types of controllers & Compensators - Transfer function and basic characteristics of lag, lead and lag-lead phase compensators

Module 2: Performance Analysis of Control Systems (9 hours)

- Time domain analysis of control systems
- Time domain specifications of transient and steady state responses-
- Impulse and Step responses of first and second order systems
- Pole dominance for higher order systems. Error analysis: Steady state error analysis and error constants -Dynamic error coefficients.
- Stability Analysis: Concept of BIBO stability and Asymptotic stability- Time response for various pole locations
- stability of feedback systems - Routh's stability criterionRelative stability

Module 3: Root Locus Analysis and Compensator Design (11 hours)

•Root locus technique: Construction of Root locus-stability analysis- effect of addition of poles and zeroes-Effect of positive feedback systems on Root locus

•Design of Compensators: Design of lag, lead and lag-lead compensators using Root locus technique.

•PID controllers: PID tuning using Ziegler-Nichols methods.

•Simulation based analysis: Introduction to simulation tools like MATLAB/ SCILAB or equivalent for Root locus based analysis (Demo/Assignment only)

Module 4: Frequency domain analysis (9 hours)

- Frequency domain specifications-
- correlation between time domain and frequency domain responses
- Polar plot: Concepts of gain margin and phase margin- stability analysis
- Bode Plot: Construction- Concepts of gain margin and phase margin- stability analysis, Effect of Transportation lag and Non-minimum phase systems

Module 5 : Nyquist stability criterion and Compensator Design using Bode Plot (9 hours)

- Nyquist criterion: Nyquist plot-
- Stability criterion- Analysis Introduction to Log magnitude vs. phase plot and Nichols chart (concepts only)
- Compensator design using Bode plot: Design of lag, lead and lag-lead compensator using Bode plot.
- Simulation based analysis: Introduction to simulation tools like MATLAB/ SCILAB or equivalent for various frequency domain plots and analysis (Demo/Assignment only).

Text books

1. Nagarath I. J. and Gopal M., Control System Engineering, 5/e, New Age Publishers
2. Ogata K, Modern Control Engineering, 5/e, Prentice Hall of India.
3. Nise N. S, Control Systems Engineering, 6/e, Wiley Eastern
4. Dorf R. C. and Bishop R. H, Modern Control Systems, 12/e, Pearson Education

Reference Books

1. Kuo B. C, Automatic Control Systems, 7/e, Prentice Hall of India
2. Desai M. D., Control System Components, Prentice Hall of India, 2008
3. Gopal M., Control Systems Principles and Design, 4/e, Tata McGraw Hill. 4. Imthias Ahamed T. P, Control Systems, Phasor Books, 2016

Module 1
Feedback Control Systems

Module 1 : Feedback Control Systems (9 hours)

• Open loop and closed loop control systems- Examples of automatic control systems

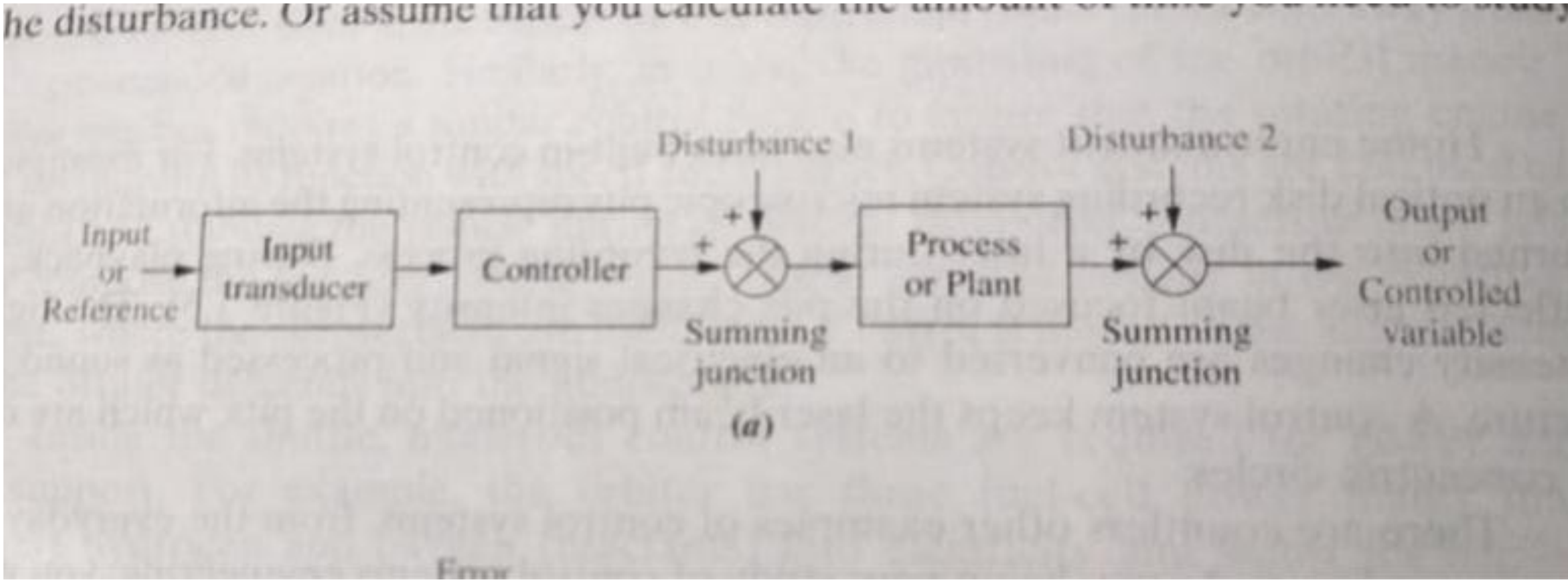
•Transfer function approach to feed back control systems – Effect of feedback

• Control system components

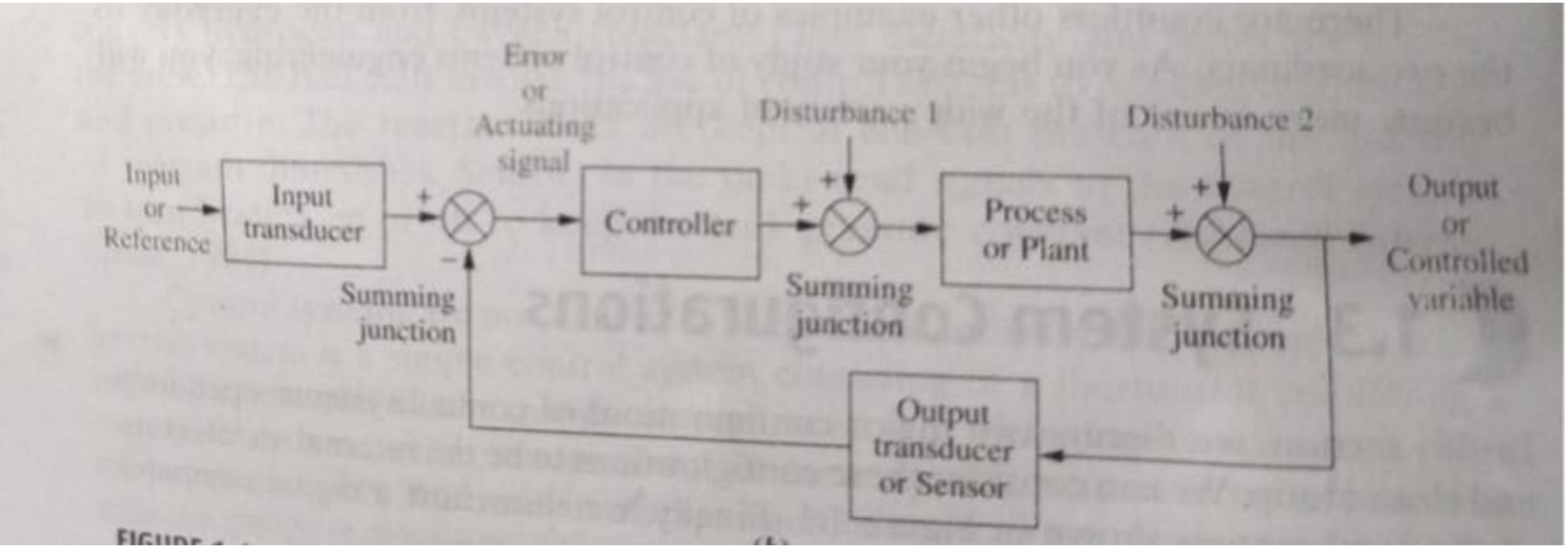
• Control applications of DC and AC servo motors, Tacho generator, Synchro, Gyroscope and Stepper motor Controllers- Types of controllers & Compensators - Transfer function and basic characteristics of lag, lead and lag-lead phase compensators

Module	Topic coverage	No. of Lectures
1	**Feedback Control Systems (9 hours)**	
1.1	Terminology and basic structure of Open loop and Closed loop control systems- Examples of Automatic control systems (block diagram representations only)	2
1.2	Transfer function approach to feed back control systems- Effect of feedback- Characteristic equation- poles and zeroes- type and order.	2
1.3	Control system components: Transfer functions of DC and AC servo motors –Control applications of Tacho generator, Synchro, Gyroscope and Stepper motor	3
1.4	Need for controllers: Types of controllers – Feedback, Cascade and Feed forward controllers Compensators: Transfer function and basics characteristics of lag, lead, and lag-lead phase compensators	2

Open loop System

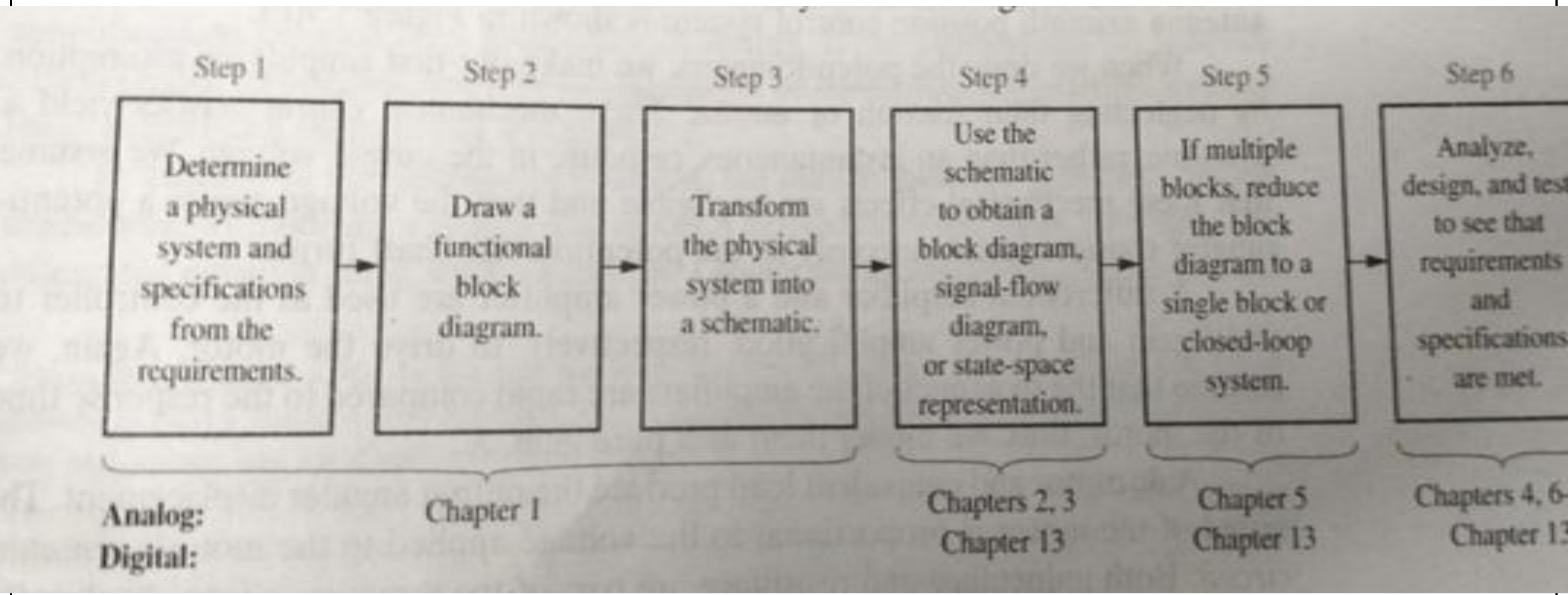


Closed loop System



Basis For Comparison	Open Loop System	Closed Loop System
Definition	The system whose control action is free from the output is known as the open loop control system.	In closed loop, the output depends on the control action of the system.
Components	Controller and Controlled Process.	Amplifier, Controller, Controlled Process, Feedback.
Construction	Simple	Complex
Reliability	Non-reliable	Reliable
Accuracy	Depends on calibration	Accurate because of feedback.
Stability	Stable	Less Stable
Response	Fast	Slow
System Disturbance	Affected	Not-affected
Examples	Traffic light, automatic washing machine, immersion rod, TV remote etc.	Air conditioner, temperature control system, speed and pressure control system, refrigerator, toaster.

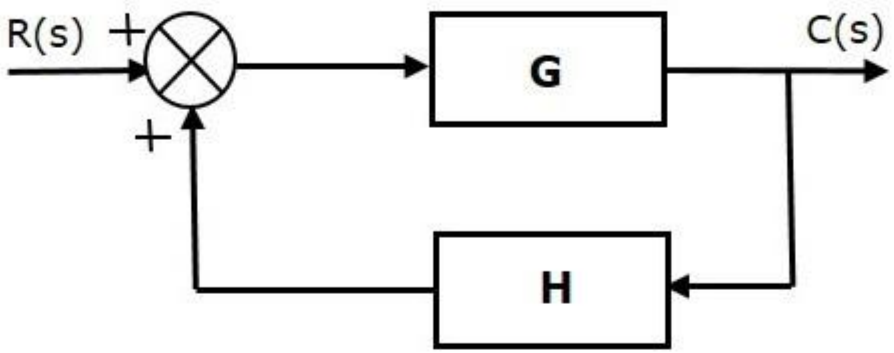
Design Process

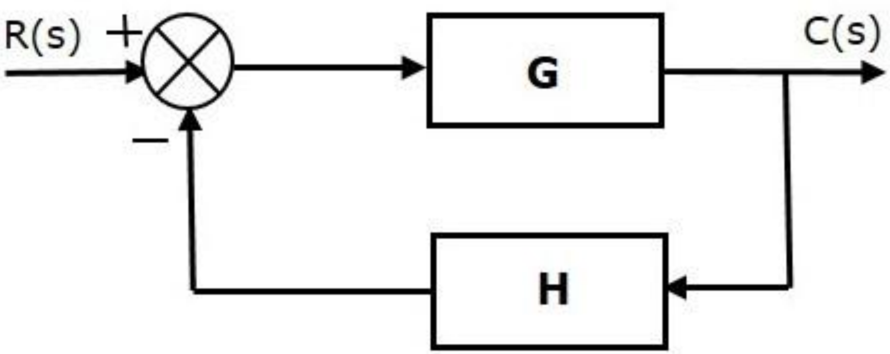


Types of Feedback

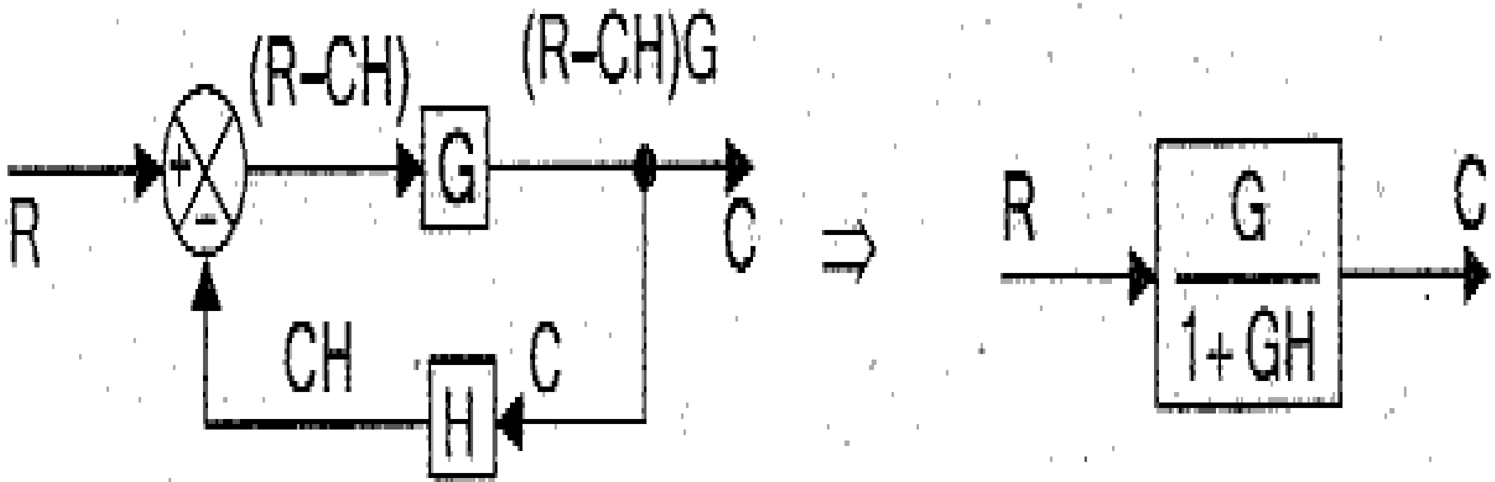
There are two types of feedback

- Positive feedback   Negative feedback





Negative Feed back system

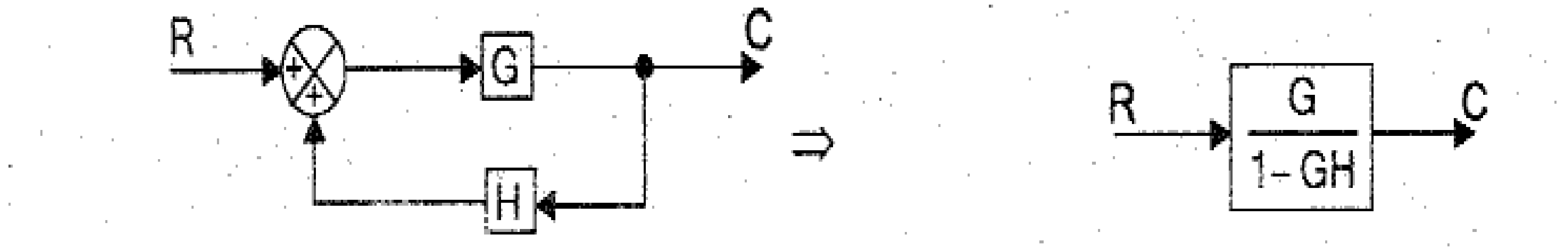


Proof:

$$C=(R-CH)G \Rightarrow C=RG-CHG \Rightarrow C+CHG=RG$$

$$\therefore C(1+HG)=RG \Rightarrow \frac{C}{R}=\frac{G}{1+GH}$$

Positive Feed back system



Effects of Feedback

1) Effect of Feedback on Overall Gain

• Overall gain may increase or decrease depending on the value of (1+GH).

•If the value of (1+GH) is less than 1, then the overall gain increases. In this case, 'GH' value is negative because the gain of the feedback path is negative.

•If the value of (1+GH) is greater than 1, then the overall gain decreases. In this case, 'GH' value is positive because the gain of the feedback path is positive.

•In general, 'G' and 'H' are functions of frequency. So, the feedback will increase the overall gain of the system in one frequency range and decrease in the other frequency range.

2) Effect of Feedback on Stability

A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable.

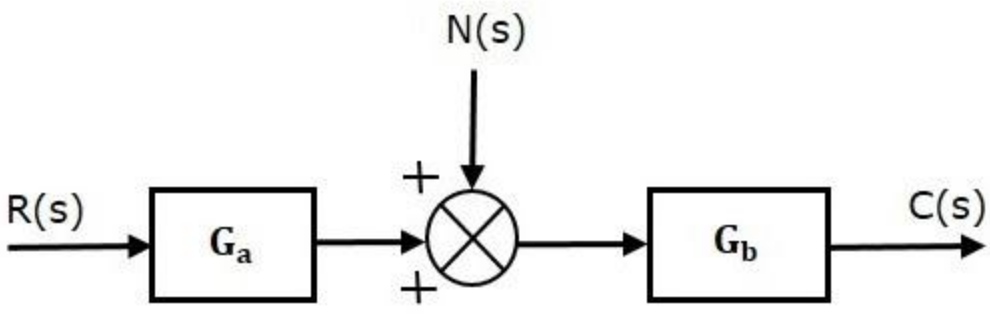
In gain Equation, if the denominator value is zero (i.e., $GH = -1$), then the output of the control system will be infinite. So, the control system becomes unstable.

Therefore, we have to properly choose the feedback in order to make the control system stable.

3) Effect of Feedback on Noise

To know the effect of feedback on noise, let us compare the transfer function relations with and without feedback due to noise signal alone.
Consider an **open loop control system** with noise signal as shown below.

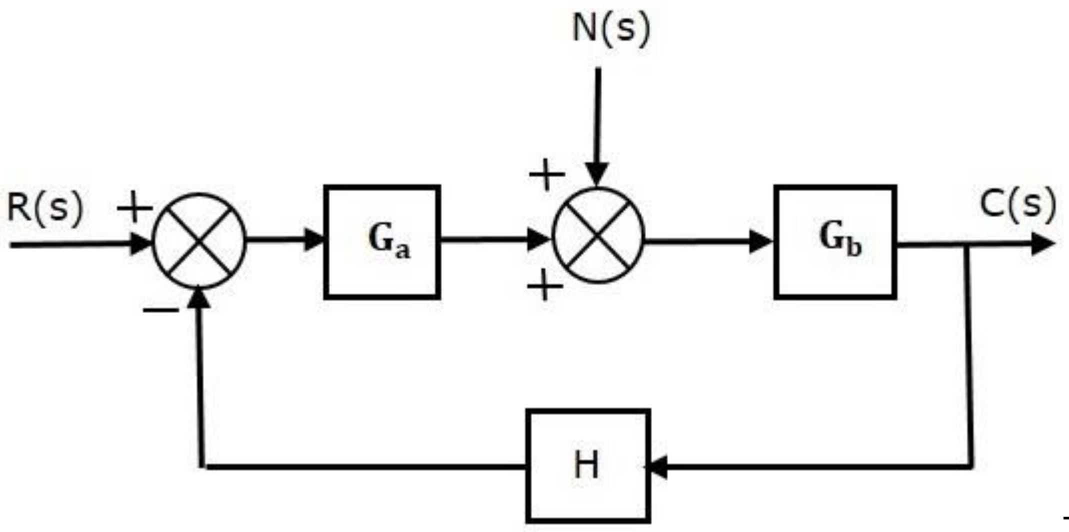
The **open loop transfer function** due to noise signal alone is



The **closed loop transfer function** due to noise signal alone is
C(s)/N(s)=Gb/1+GaGbH

It is obtained by making the other input R(s) equal to zero.

In the closed loop control system, the gain due to noise signal is decreased by a factor of (1+GaGbH) provided that the term (1+GaGbH) is greater than one.



4)Effect of Feedback on sensitivity

Sensitivity of the overall gain of negative feedback closed loop control system (**T**) to the variation in open loop gain (**G**) is defined as

Sensitivity = Percentage change in T/Percentage change i nG

$$= (\partial T/T) \ / \ (\partial G/G)$$

Where, ∂**T** is the incremental change in T.

$$S_G^T = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{Percentage\ change\ in\ T}{Percentage\ change\ in\ G} \quad ($$

Where, ∂T is the incremental change in T due to incremental change in G.

We can rewrite Equation 3 as

$$S_G^T = \frac{\partial T}{\partial G}\frac{G}{T}$$

Do partial differentiation with respect to G on both sides of Equation 2.

$$\frac{\partial T}{\partial G} = \frac{\partial}{\partial G}\left(\frac{G}{1+GH}\right) = \frac{(1+GH).1-G(H)}{(1+GH)^2} = \frac{1}{(1+GH)^2} \quad \text{(Ec}$$

From Equation 2, you will get

$$\frac{G}{T} = 1 + GH$$

Substitute Equation 5 and Equation 6 in Equation 4.

$$S_G^T = \frac{1}{(1+GH)^2}(1+GH) = \frac{1}{1+GH}$$

sensitivity of the overall gain of closed loop control system as the reciprocal of (1+GH).

- **sensitivity** of the overall gain of closed loop control system as the reciprocal of (1+GH).
- Sensitivity may increase or decrease depending on the value of (1+GH).

❖If the value of (1+GH) is less than 1, then sensitivity increases. In this case, 'GH' value is negative because the gain of feedback path is negative.

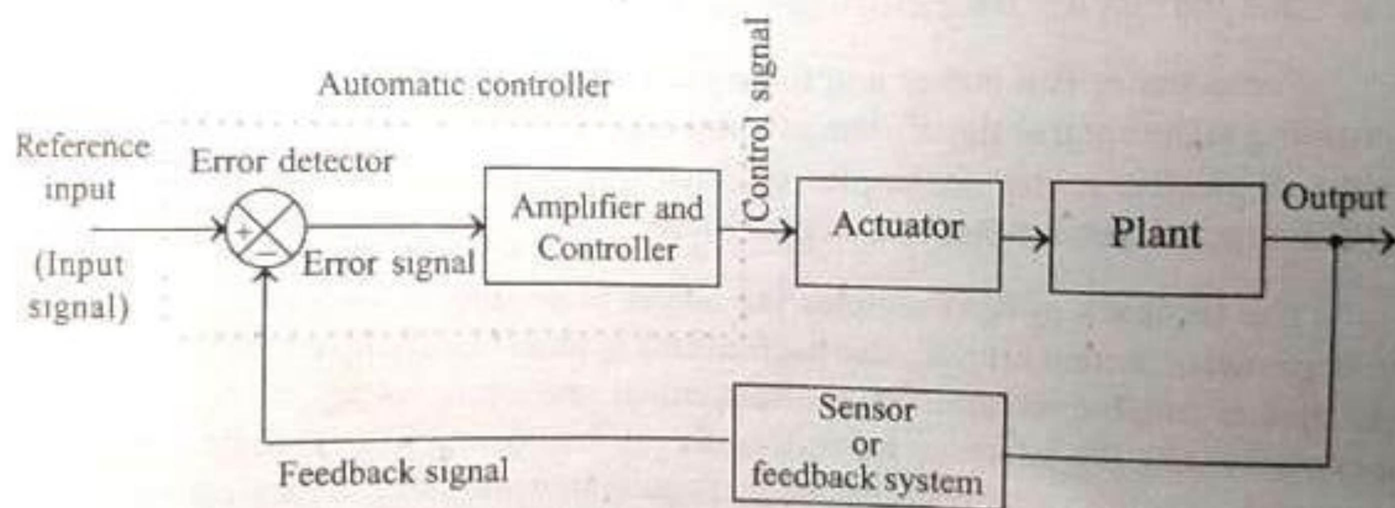
❖If the value of (1+GH) is greater than 1, then sensitivity decreases. In this case, 'GH' value is positive because the gain of feedback path is positive.

❖In general, 'G' and 'H' are functions of frequency. So, feedback will increase the sensitivity of the system gain in one frequency range and decrease in the other frequency range.

Therefore, we have to choose the values of 'GH' in such a way that the system is insensitive or less sensitive to parameter variations.

2.1 COMPONENTS OF AUTOMATIC CONTROL SYSTEM

The basic components of an automatic control system are Error detector, Amplifier and Controller, Actuator (Power actuator), Plant and Sensor or Feedback system. The block diagram of an automatic control system is shown in fig 2.1.



Control System Components

- DC Servo motor
- AC Servo motor
- Stepper motor
- Synchro
- Gyroscope
- Controllers-
  - Types of controllers
  - Compensators

DC servomotor

DC servomotor

- Some Applications require rotation for some specific angle only.
- Some special type of motor with special arrangement is required.
- For this purpose servo motor is used.
- This is normally a simple DC motor which is controlled for specific angular rotation with help of additional **servomechanism**
- Servomechanism- Any system which controls motion automatically using feedback

SERVOMECHANISM

- Servo system consists of three basic components

➢ Controlled device
➢ Output sensor
➢ Feedback system

- Device controlled by feedback signal generated by comparing output signal and reference input signal.
- Automatic closed loop control system

SO WHAT IS A SERVO MOTOR ?

A servo motor is nothing but a special type of motor which is automatically operated up to certain limit for a given command with help of error-sensing feedback to correct the performance.

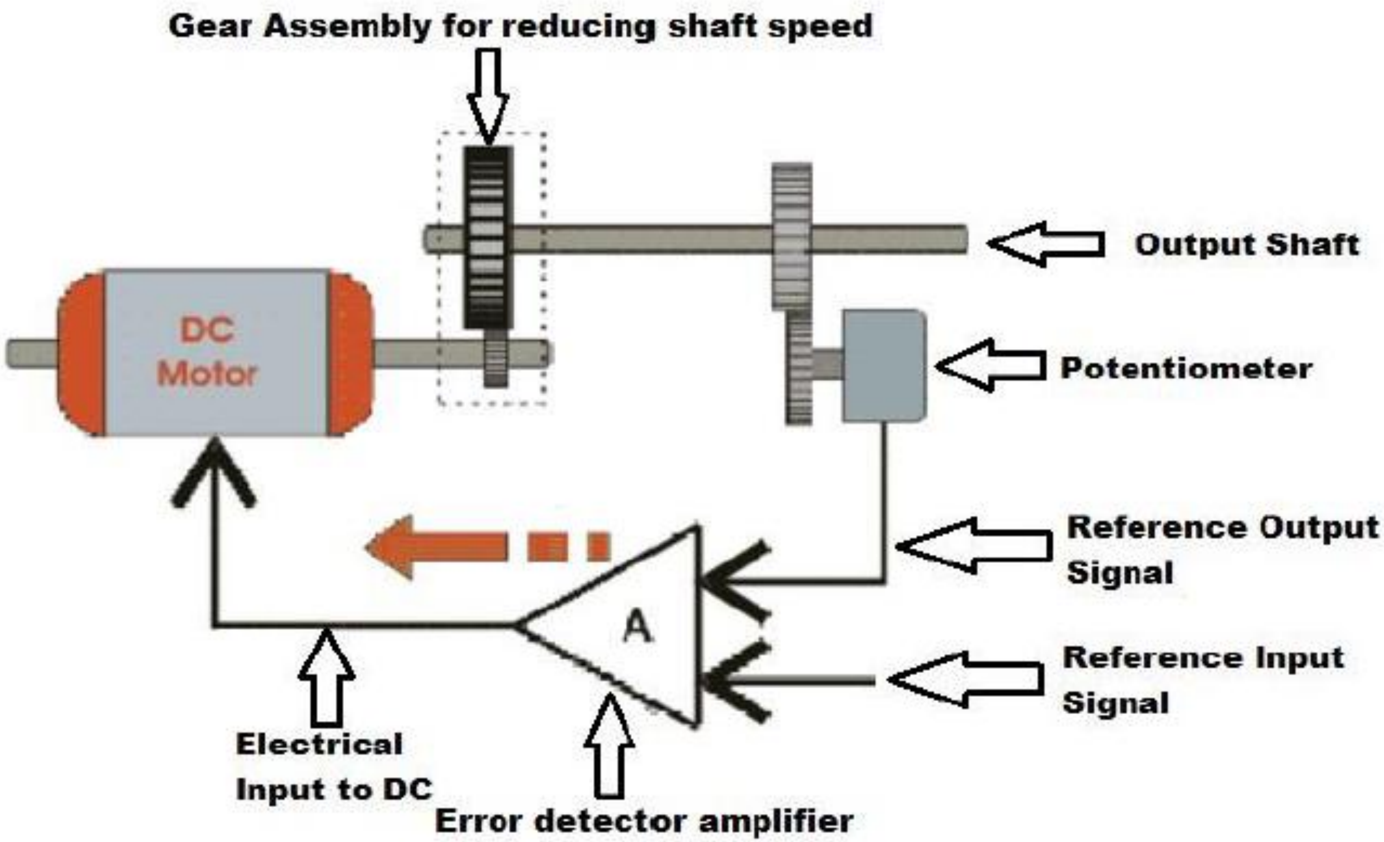
Primary Task is to maintain the output of a system at the desired value in the presence of disturbances.

presence of disturbances.

WORKING PRINCIPLE OF A SERVO MOTOR

- Consist of a DC motor with some special purpose Components:
  - A Gear arrangement
  - A potentiometer
  - An intelligent circuitry
- The intelligent circuitry along with the potentiometer makes the servo to rotate according to our wishes.

WORKING PRINCIPLE OF A SERVO MOTOR



WORKING PRINCIPLE OF A SERVO MOTOR

- Hence the working can be concluded as:
  - The shaft of the servo is connected to a potentiometer
  - The circuitry inside the servo, to which the potentiometer is connected, knows the position of the servo.
  - The Current position will be compared with the desired position continuously with the help of an Error Detection Amplifier.
  - If a mismatch is found, then an error signal is provided at the output of the error amplifier and the shaft will rotate to go the exact location required.
  - Once the desired location is reached, it stops and waits.

FEATURES OF SERVO MOTOR/REQUIREMENTS OF A GOOD SERVOMOTOR

- Linear relationship between the speed and electric control signal over a wide range
- Steady state stability- Its operation should be stable without any oscillations or overshoot
- Wide range of speed control
- Low mechanical and electrical inertia- must stop running without any time delay if control signal to it is removed

FEATURES CONTD…

- Should be quickly reversible
- Linearity of mechanical characteristics throughout the entire speed range (T Vs N)
- Fast response –with quickly changing error signals, it must react with good response. Achieved by keeping torque to weight ratio high.

Transfer function of armature controlled dc motor

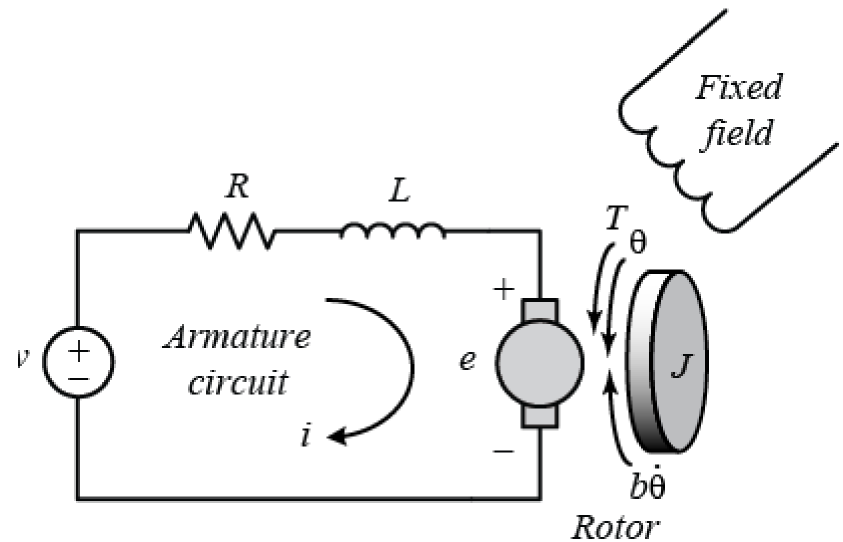
$$V_a = i_a R_a + L_a \frac{di_a}{dt} + e_b \quad .....(1)$$

$$T = K_t i_a \quad .....(2)$$

$$T = J \frac{d\omega}{dt} + B\omega \quad .....(3)$$

$$e_b = K_b\, \omega \quad .....(4)$$

$$\omega = \frac{d\theta}{dt} \quad .....(5)$$

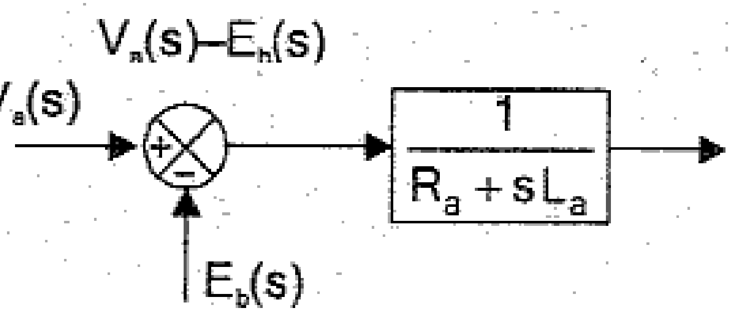


On taking Laplace transform of equation (1) we get,

$$V_a(s) = I_a(s)\, R_a + L_a\, s\, I_a(s) + E_b(s)$$

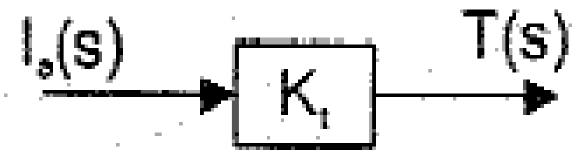
$$V_a(s) - E_b(s) = I_a(s)\left[R_a + s\, L_a\right]$$

$$\therefore I_a(s) = \frac{1}{R_a + s\, L_a}\left[V_a(s) - E_b(s)\right]$$



On taking Laplace transform of equation (2) we get,

$$T(s) = K_t \, I_a(s)$$

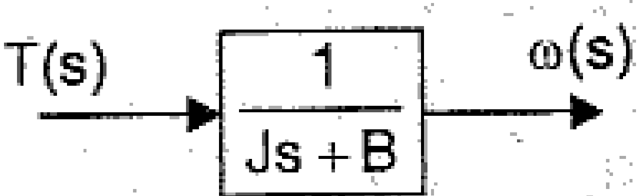


On taking Laplace transform of equation (3) we get,

$$T(s) = Js\,\omega(s) + B\,\omega(s)$$

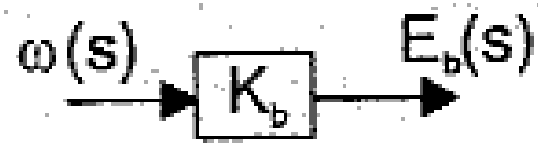
$$T(s) = (Js + B)\,\omega(s)$$

$$\therefore \omega(s) = \frac{1}{Js + B}\,T(s)$$



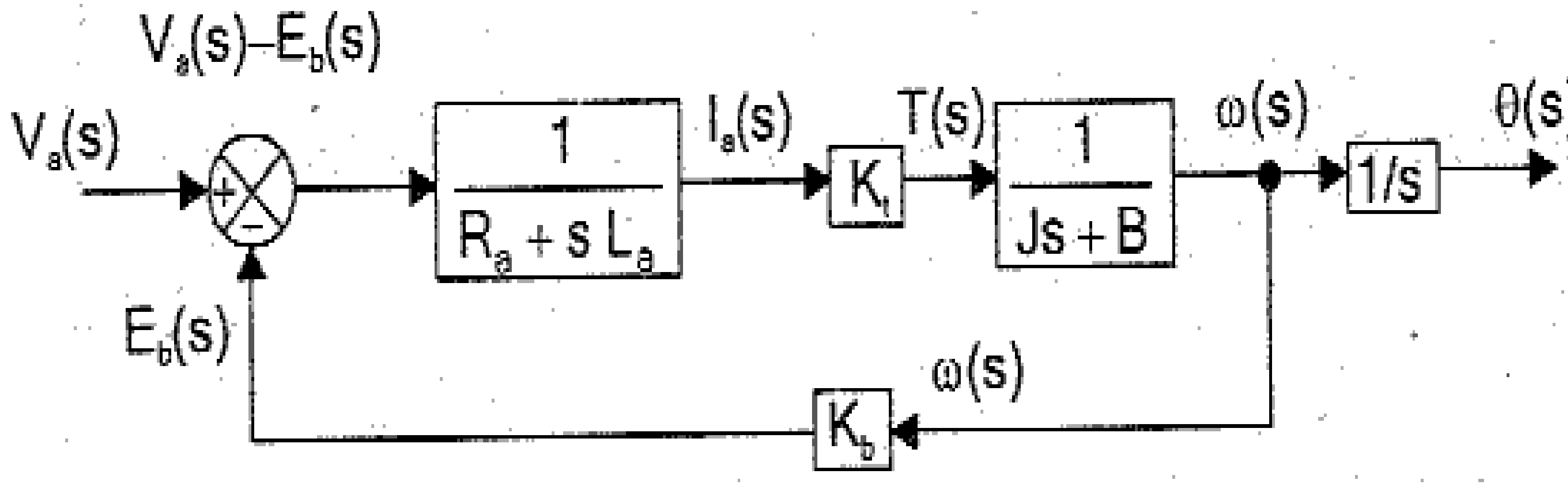
On taking Laplace transform of equation (4) we get,

$$E_b(s) = K_b\,\omega(s)$$



$$\omega(s) = s\,\theta(s)$$

$$\theta(s) = \frac{1}{s}\omega(s)$$



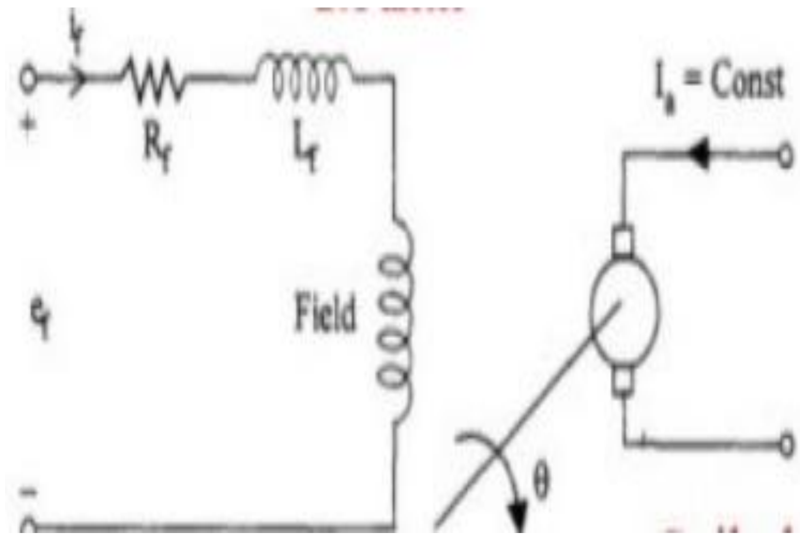
$V_a(s)-E_b(s)$
$V_a(s)$
$\frac{1}{R_a + s L_a}$
$I_a(s)$
K_t
$T(s)$
$\frac{1}{Js+B}$
$\omega(s)$
$1/s$
$\theta(s)$
$E_b(s)$
$\omega(s)$
K_b

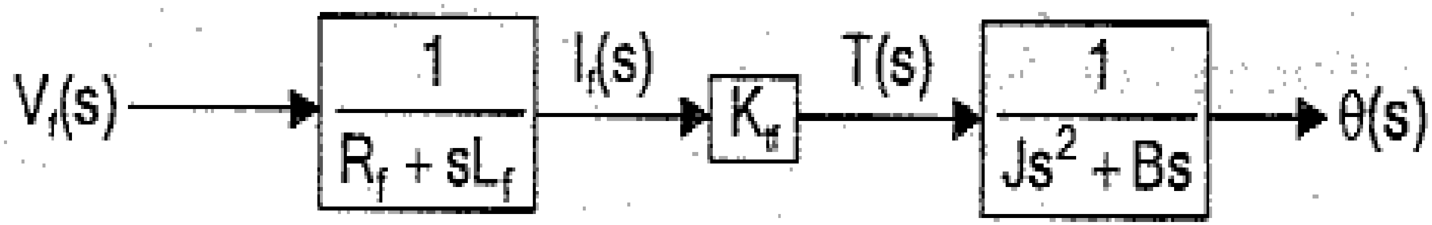
Block diagram representation of Field controlled dc motor

$$v_f = R_f' i_f + L_f \frac{di_f}{dt} \qquad .....(1)$$

$$T = K_{tf}\, i_f \qquad .....(2)$$

$$T = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} \qquad .....(3)$$





$V_f(s)$
$\frac{1}{R_f + sL_f}$
$I_f(s)$
K_{tf}
$T(s)$
$\frac{1}{Js^2 + Bs}$
$\theta(s)$

AC Servomotor

WHY AC SERVOMOTORS?

- DC motors have greater rotor inertia than AC motors for the same horse power rating because of the heavier winding of DC armatures.
- Drag due to brush friction also discourages the use of DC motors in extremely small and sensitive instrument servo systems.
- Commutation is also a problem with DC servomotors, although interpoles and compensating windings help the situation considerably.
- At high altitudes, because of lack of oxygen, the oxide film may wear away by friction from the commutator, causing commutation difficulties.

WHY AC SERVOMOTORS?

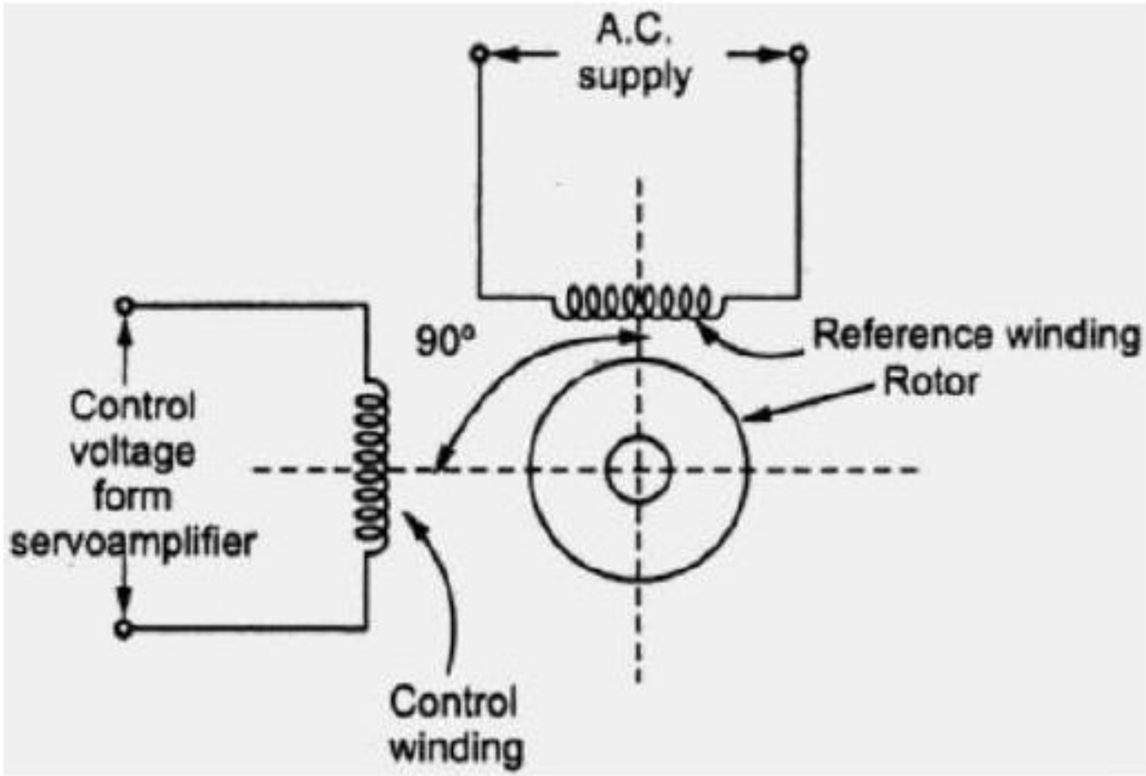
- **Hermetically sealed** small DC servomotors have been developed to overcome this particular problem.
- Motors operate most of the time from rest position, and large currents flow to the slowly moving bars, producing **high arcing of the commutator**.
- The arcing of any commutator motor produces **radiation and radio interference.**
- Brushes require periodic **maintenance**.

AC SERVOMOTORS

- Most of servomotors used for low power servomechanism are AC servomotors.
- AC servomotors are basically **two-phase induction motor type**
- Output power varies from few fraction of watts to few hundred of watts
- Operating frequency is 50hz to 400 hz

CONSTRUCTION

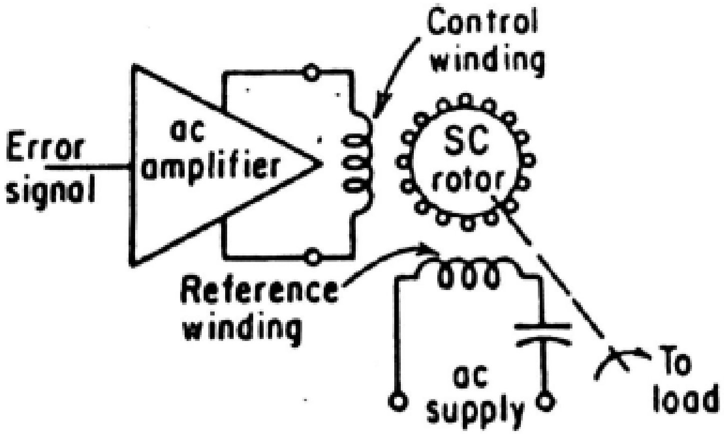
- Basically consist of stator and rotor
- Stator carries two windings, uniformly distributed and displaced by 90 degree
- One winding is called main or reference or fixed winding and is excited by constant ac voltage
- Other winding is control winding, excited by variable control voltage from servoamplifier
- Obtain rotating magnetic field (90 degree displacement)
- Also called as two-phase induction motor type



Stator of a.c. servomotor

TWO-PHASE SERVOMOTOR

- This motor is a true two-phase motor having two stator windings displaced 90° in space on the stator.
- The reference winding is constantly and usually excited through a capacitor by the fixed ac supply.



2.37 b illustrates this effect.

TRANSFER FUNCTION OF AC SERVOMOTOR

Let, T_m = Torque developed by servomotor

q = Angular displacement of rotor

$$w = \frac{d\theta}{dt} = \text{Angular speed}$$

T_l = Torque required by the load

J = Moment of inertia of load and the rotor

B = Viscous-frictional coefficient of load and the rotor.

K_1 = Slope of control-phase voltage Vs Torque characteristic

K_2 = Slope of speed-torque characteristic.

With ref

Torque developed by motor, $T_m = K_1 e_e - K_2 \frac{d\theta}{dt}$ ....

The rotating part of motor and the load can be modelled by the equatior given below.

Load torque , $T_l = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt}$ ....

At equilibruim the motor torque is equal to load torque.

$$\therefore J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = K_1 e_e - K_2 \frac{d\theta}{dt}$$ ....

On taking laplace transform of equation (2.86) with zero initial con we get,

$$\therefore J\frac{d^2\theta}{dt^2}+B\frac{d\theta}{dt}=K_1 e_c - K_2\frac{d\theta}{dt} \quad \text{....}$$

On taking laplace transform of equation (2.86) with zero initial con
we get,

$$J\, s^2\theta(s)+B\, s\theta(s)=K_1\, E_c\,(s)-K_2\, s\,\theta(s)$$

$$\left[Js^2+Bs+K_2 s\right]\theta(s)=K_1\, E_c\,(s)$$

$$\therefore \frac{\theta(s)}{E_c(s)}=\frac{K_1}{s\,(Js+B+K_2)}=\frac{K_1/(B+K_2)}{s\left(\frac{J}{B+K_2}s+1\right)}$$

$$=\frac{K_m}{s\,(\tau_m s+1)}$$

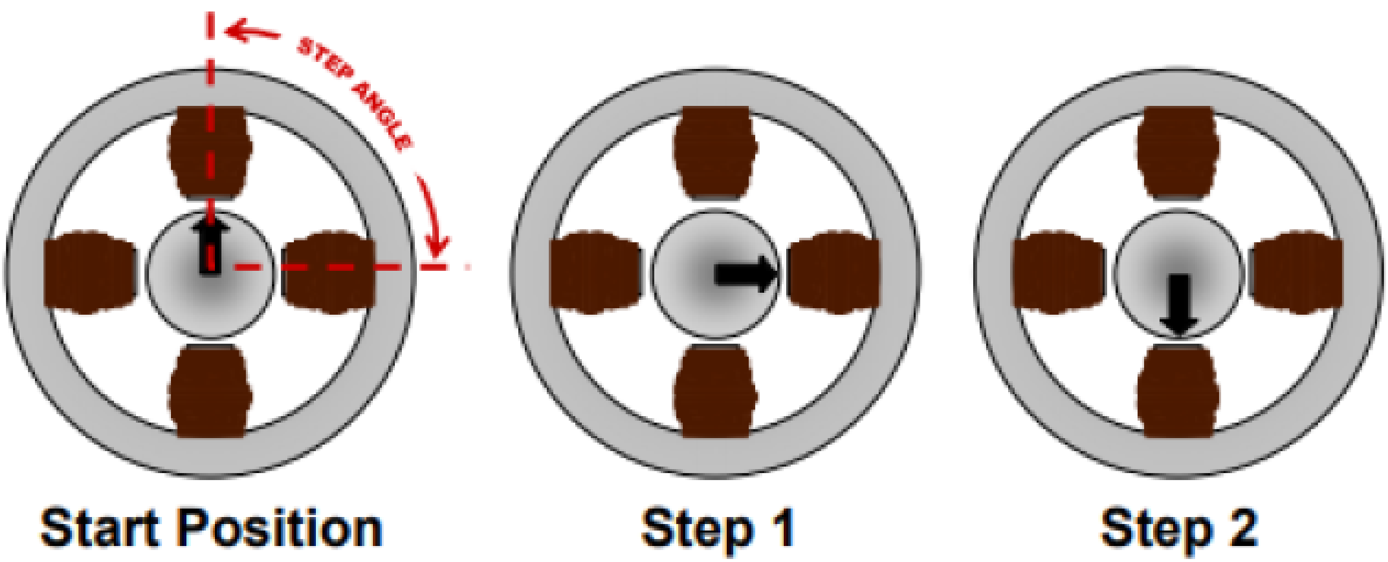
Where, $K_m=\dfrac{K_1}{B+K_2}$ = Motor gain constant

$\tau_m=\dfrac{J}{B+K_2}$ = Motor time constant

The equation (2.87) is the transfer function of ac servomotor.

STEPPER MOTOR

- Motor that moves one <u>step</u> at a time
  - A digital version of an electric motor
  - Each step is defined by a **Step Angle**



STEPPER MOTOR

- Rotate a specific number of degrees as a respond to each input electric pulse.
- Typical types of stepper motors can rotate 2°, 2.5°, 5°, 7.5°, and 15° per input electrical pulse.
- Rotor position sensors or sensor less feedback based techniques can be used
- Stepping motors are categorized as doubly salient machines.
- They have teeth of magnetically permeable material on both stator and the rotor.
- Rotor is that it has a different number of teeth to the stator
- For a machine with '*p*' rotor teeth, the tooth pitch is $(360/p)^\circ$ corresponding to a movement of '*N*' steps, so

$$\text{step length} = (360/Np)^\circ$$

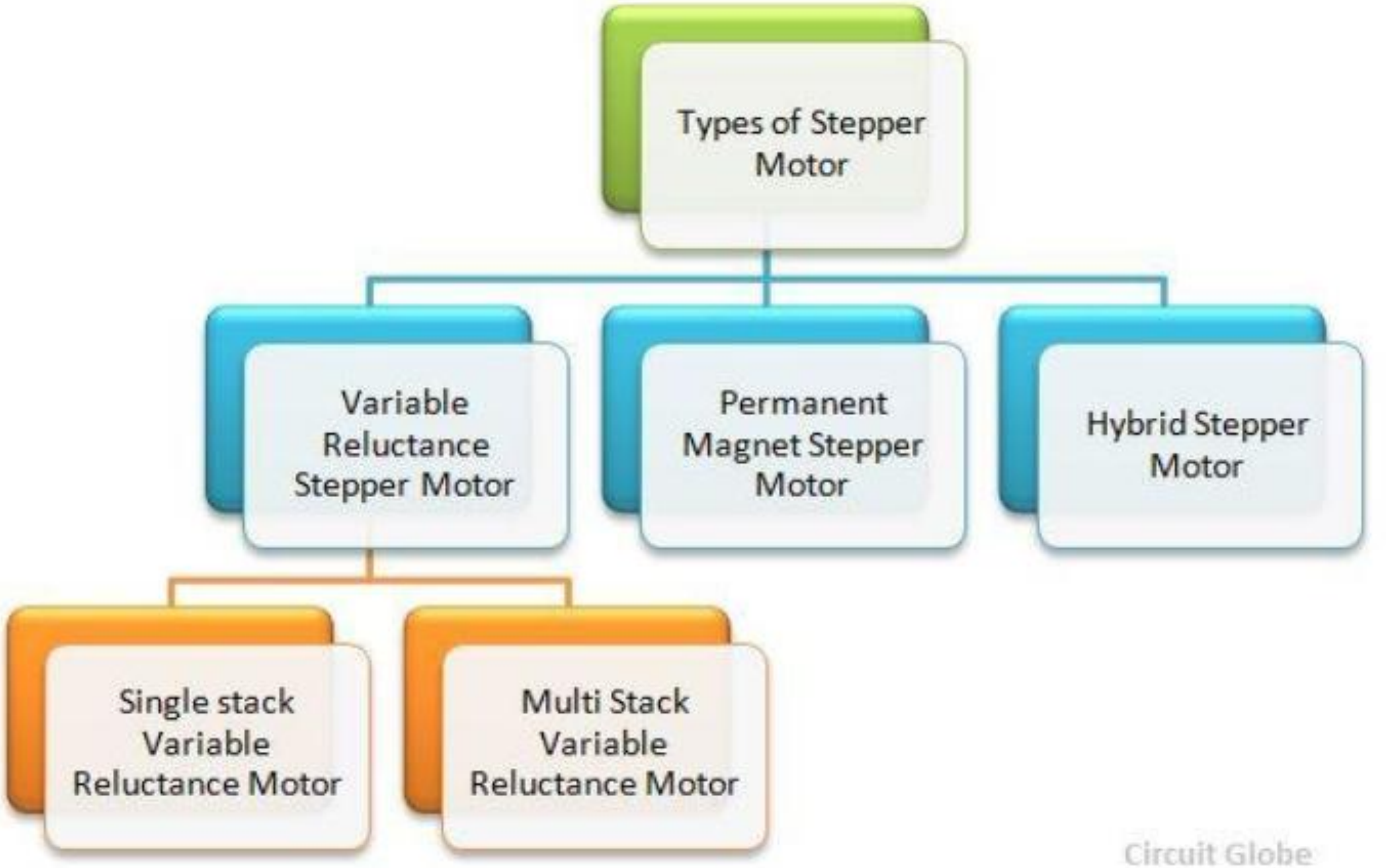
Power range- 1W to 2.5KW

Toque range- 1 micro N-m to 40Nm

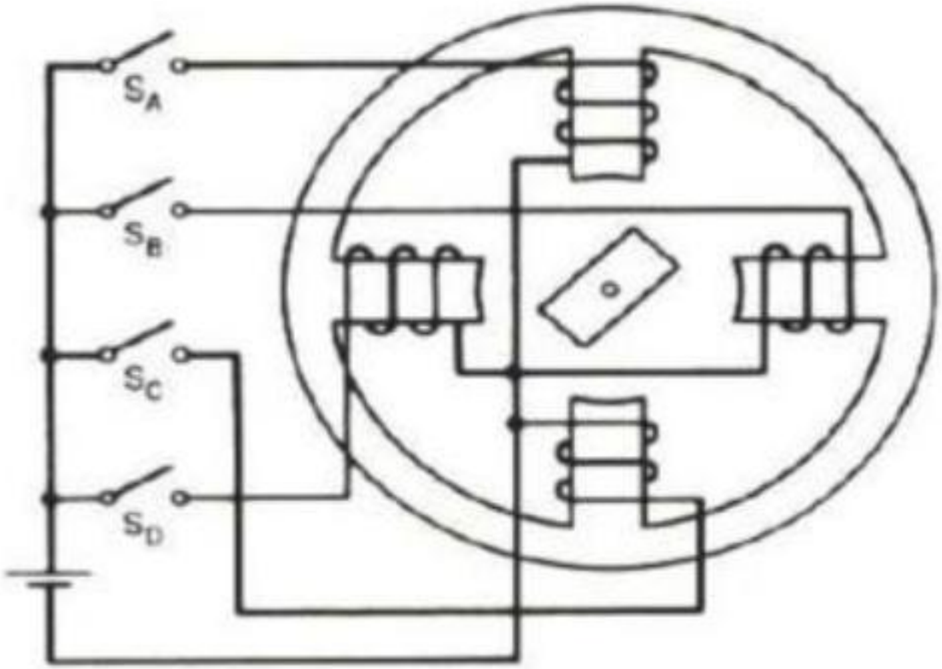
STEPPER MOTOR

- Relatively inexpensive
- Ideal for open loop positioning control
  - Can be implemented without feedback
  - Minimizes sensing devices
  - Just count the steps!
- Torque
  - Holds its position firmly when not turning
  - Eliminates mechanical brakes
  - Produces better torque than DC motors at lower speeds
- Positioning applications

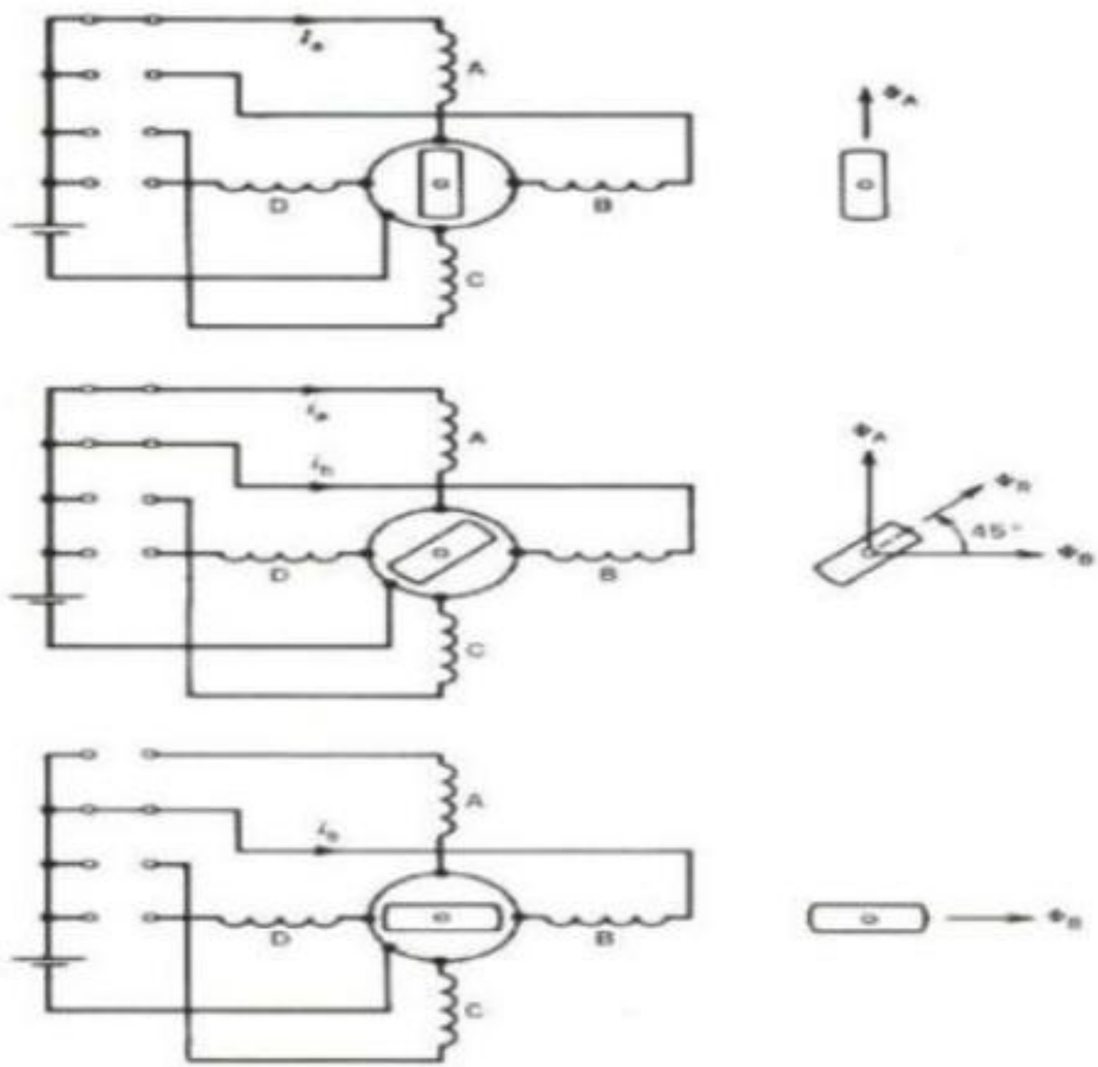
TYPES OF STEPPER MOTORS



Variable reluctance stepper motor



Basic circuit configuration of a typical 4-phase, 2-pole, single-stack, variable reluctance stepper motor



Operation modes of single-stack, 2-poles, and variable reluctance stepper motor with 45° step

1. SINGLE STACK VR STEPPER MOTORS

- Above figure illustrates the different modes of operation of the 4-phase, 2-rotor pole, single-stack, variable reluctance stepper motor for 45° step in the following energizing sequence A, A+B, B, B+C, C, C+D, D, and then D+A. Then this switching sequence is repeated.
- ***Energizing winding A:*** The resultant air-gap flux will be aligned along the axis of pole A windings. Consequently, the rotor aligns itself along the phase A axis as shown in the upper part of Figure.
- ***Energizing windings A and* B:** The resultant air-gap flux will be oriented in the midway between pole A and pole B i.e., the resultant mmf rotated 45° in the clockwise direction.
- Consequently, the rotor aligns itself with the resultant mmf (45°) as shown in the middle part of Figure
- • ***Energizing winding B:*** The resultant air-gap flux will be aligned along the axis of pole B windings. Consequently, the rotor aligns itself along the phase B axis as shown in the lower part of Figure
- The direction of rotation can be reversed by reversing the switching sequence to be A, A+D, D, D+C, C, C+B, B, and then B+A. Then this switching sequence is repeated.

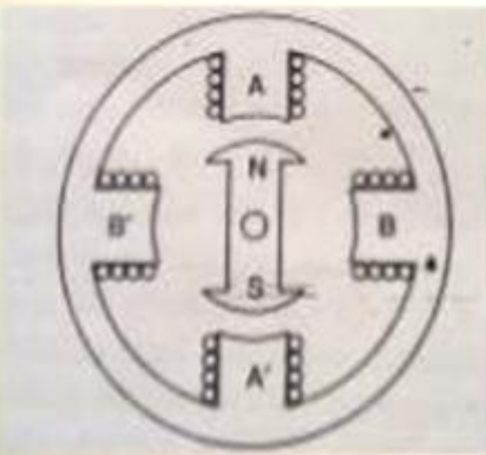
PERMANENT MAGNET STEPPER MOTOR Cont…

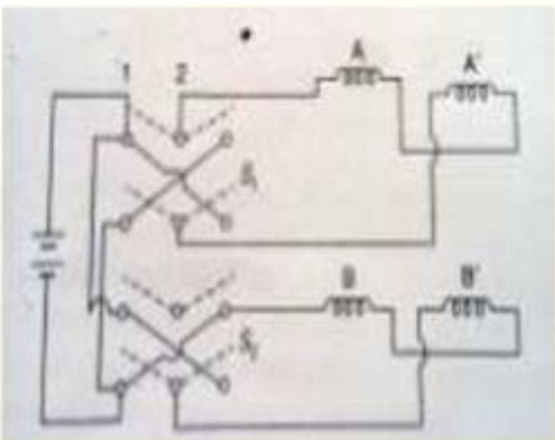
To study the principle of operation of PM stepper motor, a two phase motor is considered.

•It has four stator poles and two rotor poles.

•The stator has winding on its poles.

•The motor structure is shown in fig.





•When a phase is energized, it sets up a magnetic flux and rotor will position to lock its N pole and S pole to stator S pole and N pole respectively.

PERMANENT MAGNET STEPPER MOTOR Cont…

(a) Sigle phase ON mode

The switching sequence table is given below

S-1	S-2	Φ (degree)
+	0	0
0	+	90
_	0	180
0	_	270
+	0	360

+ = position 1

- = position 2

o = open

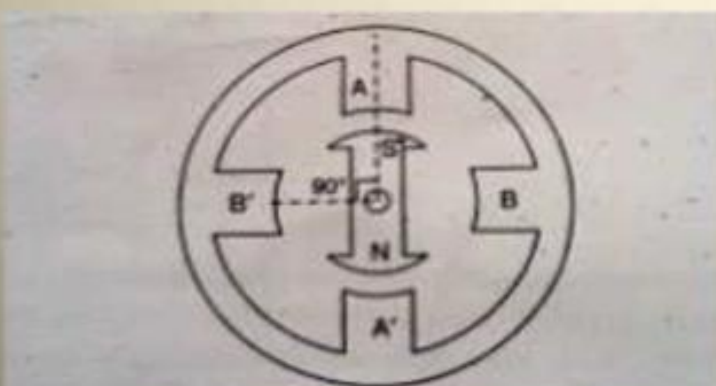


Fig. 1.16 Position of rotor when A is energised

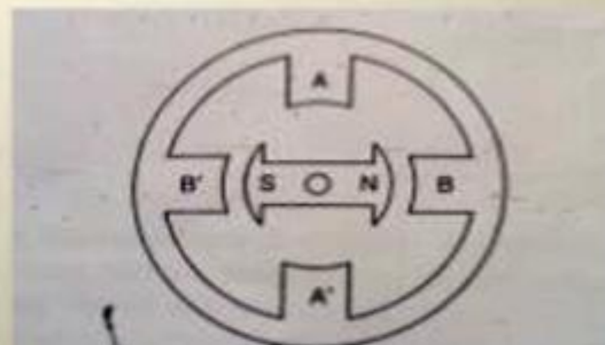


Fig. 1.15 Position of rotor when B is energised

PERMANENT MAGNET STEPPER MOTOR Cont…

(a) Two phase ON mode

The switching sequence table is given below

S-1	S-2	Φ (degree)
+	+	45
-	+	135
-	-	225
+	-	315
+	+	45

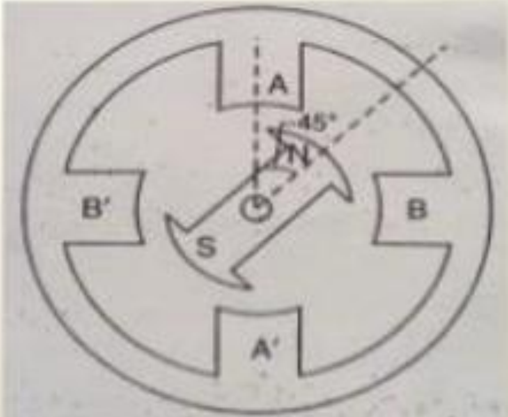


Fig. 1.17 S_1 and S_2 in position 1

Fig. 1.18 S_1 in position 2, S_2 in position 1

Fig. 1.19 S_1 and S_2 in position 2

PERMANENT MAGNET STEPPER MOTOR Cont...

(a) Half step mode

The switching sequence table is given below

S-1	S-2	Φ (degree)
+	0	0
+	+	45
0	+	90
-	+	135
-	0	180
-	-	225
0	-	270
+	-	315
+	0	360

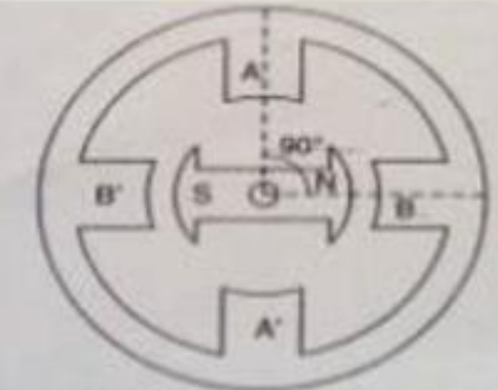


Fig. 1.20 S_2 in position 1 and S_1 opened.

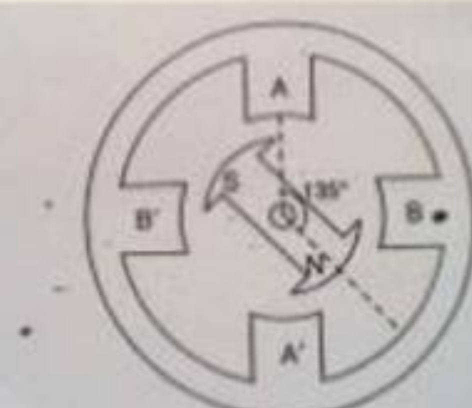


Fig. 1.21 S_2 in position 1 and S_1 in position 2.

Synchro

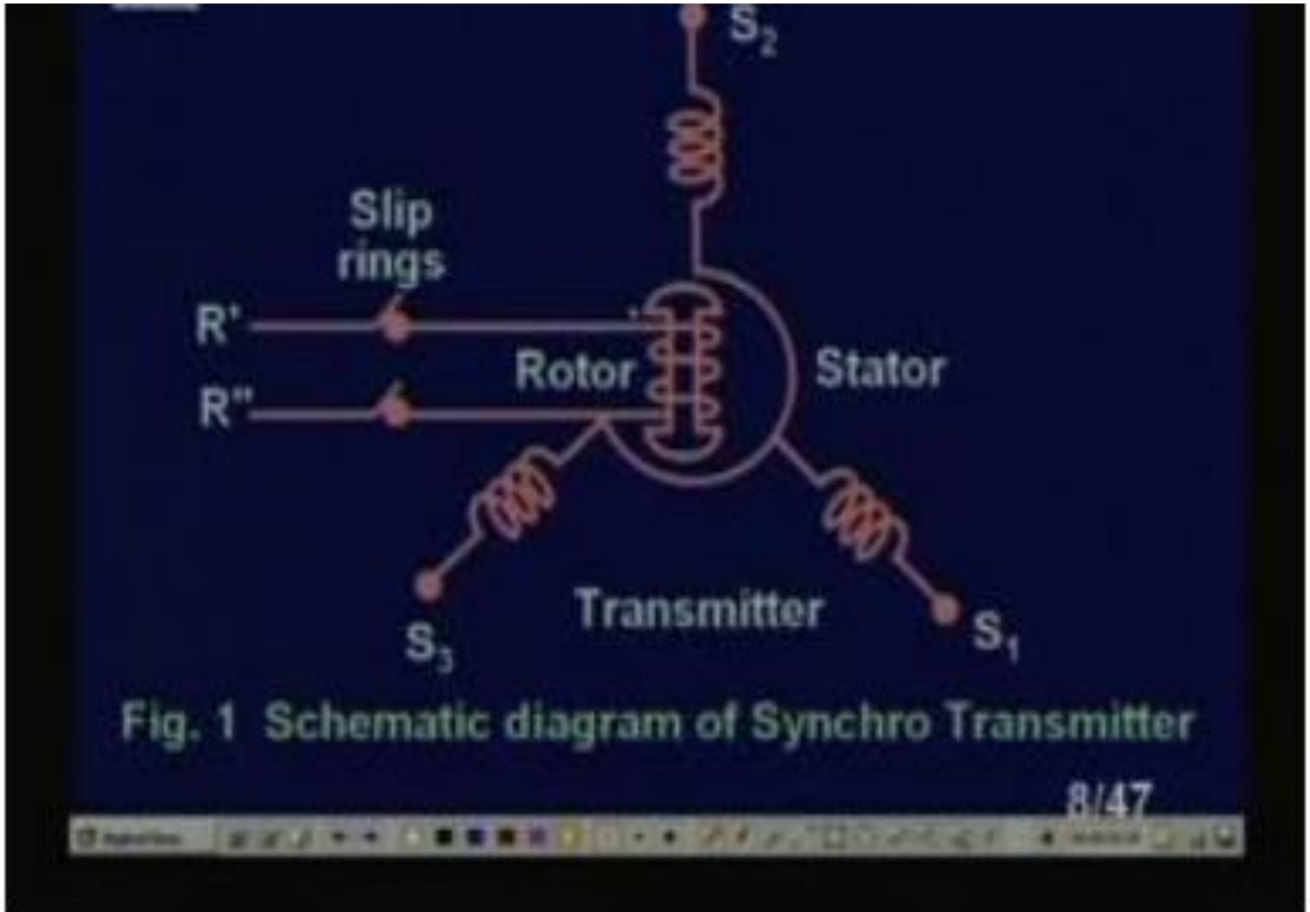
Introduction

- Synchro is an AC electromechanical system which basically makes the angle measurement.
- Synchros for angle measurement are most widely used as components of servo-mechanism where they are used to measure and compare the actual rotational position of a load with its commanded position.

Theory

Synchro is a rotating device that operates on the same principle as a transformer and produces a set of voltages, correlated to angular position.

Synchro transmitter



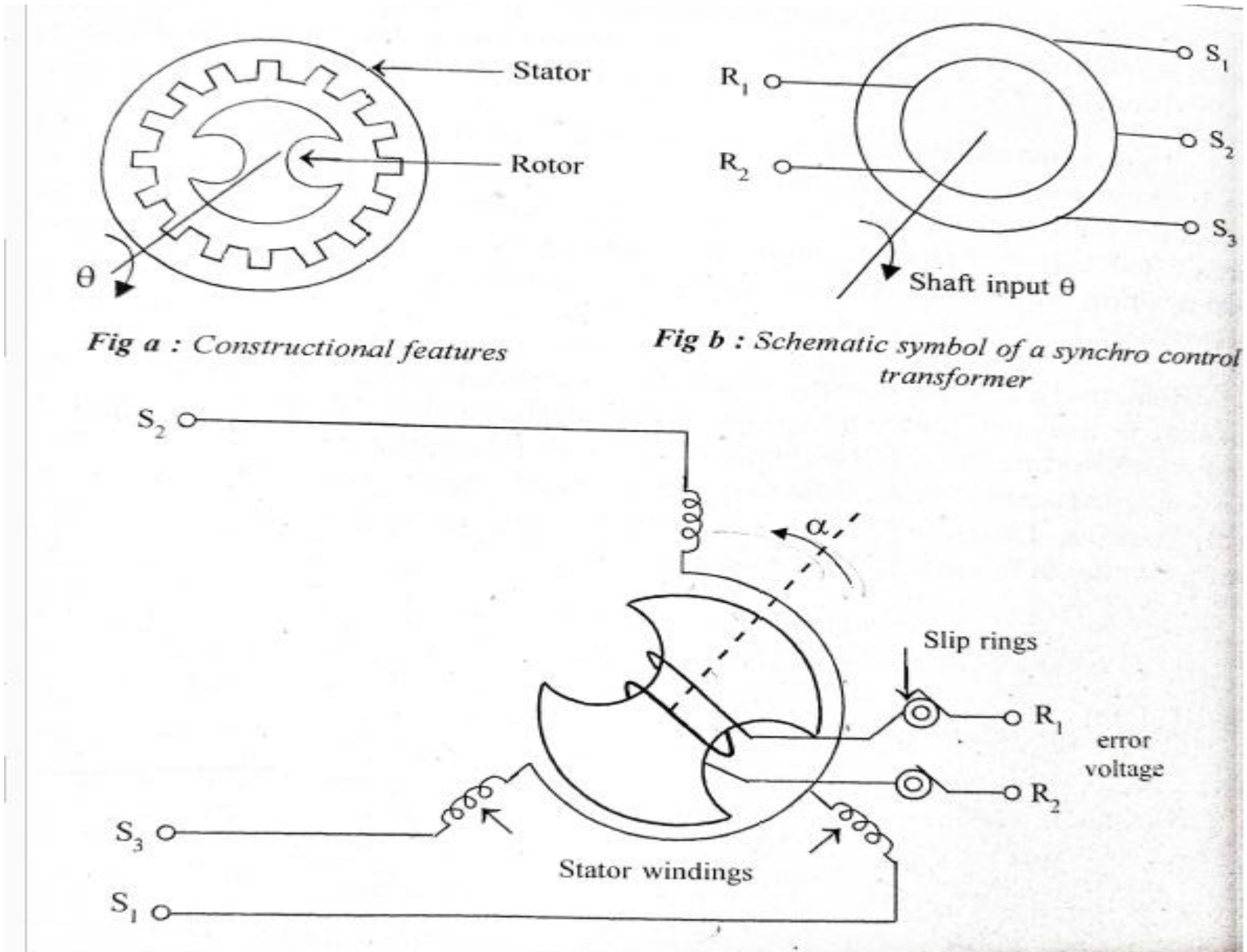
When the rotor is in position shown in Fig. 1 which is defined as the electric zero, the voltage induced across the stator winding between s_2 and the neutral n is maximum and is written as

$$e_{s_{2n}}(t) = KE_{r} \sin \omega t$$

where K is the cbnstant of proportionality and two other stator voltages can be written as

Fig. 2 Rotor is at angle ϕ from electric zero

Synchro control transformer



Synchro as error detector

is that position of its rotor for which the ...
the transmitter in its electrical zero position.