

MODULE 3-SYLLUBUS

Classification, measurement of low, medium and high resistance Ammeter voltmeter method (for low and medium resistance measurements)- Kelvin's double bridge- Wheatstones bridge- loss of charge method, measurement of earth resistance.

Measurement of self inductance-Maxwell's Inductance bridge, Measurement of capacitance –Schering's,

Measurement of frequency - Wien's bridge.

Calibration of Ammeter, Voltmeter and Wattmeter using DC potentiometers.

High voltage and high current in DC measurements- voltmeters, Sphere gaps, DC Hall effect sensors.

MODULE 3

Classification of resistances

- (i) Low resistances :- All resistances of the order of $1\text{-}2$ ohm and under may be classified as low resistance.
- (ii) Medium resistances :- This class includes resistances from $1\text{-}2$ upwards to about $0.1\text{M}\Omega$.
- (iii) High resistances :- Resistances of the order $0.1\text{M}\Omega$ and upwards are classified as high resistances.

Measurement of medium resistances

Different methods used for the measurement of medium resistances are,

(1) Ammeter - voltmeter method

(2) substitution method

(3) wheatstone bridge method

(4) Ohmmeter method.

Ammeter - voltmeter method

- This method is very popular since it can be done using the instruments available in a laboratory.
- The two types of connections employed for ammeter - voltmeter method are shown below.

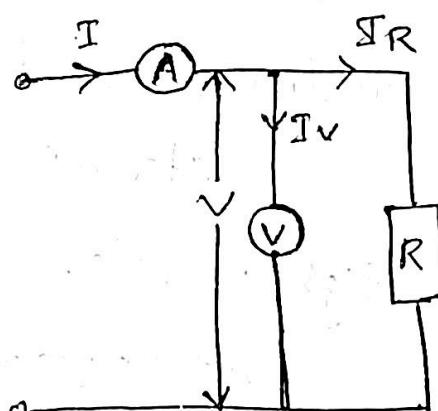
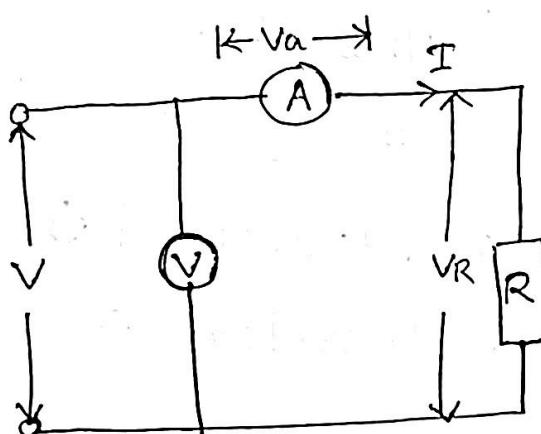


Fig (a)

- In both these cases, if readings of the voltmeters and ammeters are taken, then measured value of resistance is given by,

$$R_m = \frac{\text{voltmeter reading}}{\text{ammeter reading}} = \frac{V}{I}$$

- The measured value of resistance R_m would be equal to true value if the ammeter resistance is zero and voltmeter resistance is infinite.
- However in practice it is not possible and hence both methods give inaccurate

results

- In fig a, ammeter measures the true value of current but voltmeter does not measure the true voltage across the resistance. The voltmeter indicates sum of voltages across the ammeter and resistance.

$$R_{m1} = \frac{V}{I} = \frac{V_R + V_a}{I} = IR + I R_a$$

∴rops in voltmeter to read R_a - internal resistance of ammeter

$$= \frac{IR + I R_a}{I} = R + R_a$$

True value of resistance $R = R_{m1} - R_a$

$$R = R_{m1} \left(1 - \frac{R_a}{R_{m1}} \right)$$

Relative error, $\epsilon_o = \frac{R_{m1} - R}{R} = \frac{R_a}{R}$

- Errors in measurement would be small if the value of resistance under measurement is large as compared to the internal resistance of the ammeter. So this ckt is used when measuring high resistance values
- In fig b, voltmeter measures true value of voltage, but ammeter measures the sum of currents through the resistance and voltmeter

Measured value of resistance, $R_{m2} = \frac{V}{I}$

$$R_{m2} = \frac{V}{I_v + IR} = \frac{V}{\frac{V}{R_v} + \frac{V}{R}}$$
$$= \frac{R}{1 + R/R_v}$$

$$R = \frac{R_{m2} R_v}{R_v - R_{m2}} = R_{m2} \frac{\frac{1}{(1 - R_{m2}/R_v)}}{1 - R_{m2}/R_v}$$

→ True value of resistance is equal to measured value only if the resistance of voltmeter R_v is infinite.

→ $R \approx R_{m2}$,

Relative error $\epsilon_r = \frac{R_{m2} - R}{R}$

$$= \frac{1}{R} \left[R_{m2} - \frac{R_{m2}}{1 - \frac{R_{m2}}{R_v}} \right]$$
$$= \frac{1}{R} \left[\frac{R_{m2} \left(1 - \frac{R_{m2}}{R_v} \right) - R_{m2}}{1 - \frac{R_{m2}}{R_v}} \right]$$
$$= \frac{1}{R} \left[\frac{-R_{m2}^2/R_v}{1 - \frac{R_{m2}}{R_v}} \right]$$

If $R_v \gg R_{m2}$,

$$\epsilon_r = -\frac{R_{m2}^2}{RR_v}$$

$$R_{m2} \approx R \quad \therefore \epsilon_r = -\frac{R}{R_v}$$

- Errors in measurement would be small if the value of resistance under measurement is very small compared to the resistance of voltmeters. Hence should be used when measuring low resistance values.
- The accuracy of these two methods being limited by accuracy of ammeters and voltmeters used even if corrections are made for the voltage drop across the ammeter and for shunting effect of voltmeter.

- The relative errors for the two cases are equal, when

$$\frac{R_a}{R} = \frac{R}{R_v}$$

$$R = \sqrt{R_a R_v}$$

- Q) In the measurement of resistance R by ammeters - voltmeters methods connections shown in fig a. & fig b is used. The resistance of ammeter is 0.01Ω and that of voltmeter 2000Ω . In case of (b), the current measured is $2A$ and the voltage $180V$. Find the % error in calculating resistance R .

as a quotient of the readings and true value of R. Also find the reading of the voltmeters in case (a) if the current indicated by the ammeters is 2A.

Case b (fig b)

$$R_{m2} = \frac{V}{I} = \frac{180}{2} = 90\Omega$$

current through voltmeter $I_V = \frac{V}{R_V} = \frac{180}{2000}$
 $= 0.09 A$

Current through resistance $I_R = I - I_V = (2 - 0.09)$
 $= 1.91 A$

True value of resistance $R = \frac{V}{I_R} = \frac{180}{1.91}$
 $= 94.24 \Omega$

$$\% \text{ error} = \frac{R_{m2} - R}{R} \times 100$$

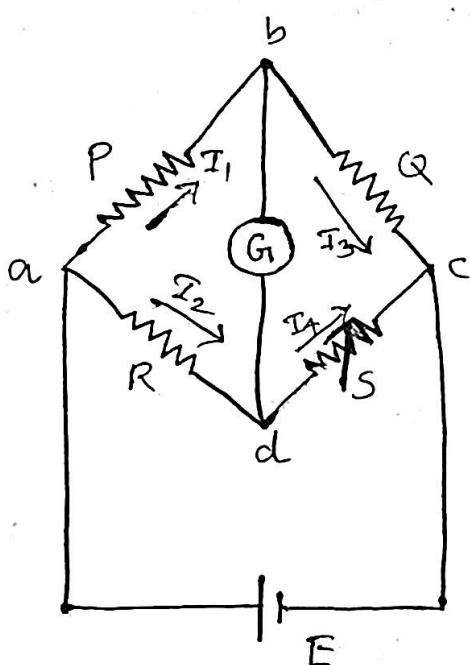
$$= \frac{90 - 94.24}{94.24} = -4.5\%$$

Case a fig (a)

Reading of voltmeter $V = V_a + V_R$
 $= 2(0.01 + 94.24)$
 $= \underline{\underline{188.50 V}}$

Wheatstone bridge

- Important device used for the measurement of medium resistances.
- It is accurate & reliable.
- Accuracy of 0.1% can be achieved.
- Operates upon ^{null} indication principle.



- A wheatstone bridge has four resistive arms, consisting of resistances P, Q, R & S along with a source of emf and a null detector, usually a galvanometer (G) or other sensitive current meters.
- The current through the galvanometer depends upon the potential diff. between points b and d.

- The bridge is said to be balanced if no current flows through the galvanometers or when the P.d across the galvanometers is zero.
- This occurs when voltage from point 'b' to point 'a' equals voltage from point 'd' to point 'b'. Or when voltage from 'd' to 'c' equals voltage from 'b' to 'c'.

i.e., $I_1 R = I_2 S$ under balanced condition

$$\text{and } I_1 = I_3 = \frac{E}{P+Q}$$

$$I_2 = I_4 = \frac{E}{R+S}$$

where E - emf of battery

$$\therefore \frac{P}{P+Q} = \frac{R}{R+S}$$

$$PR + PS = PR + RQ$$

$$PS = RQ$$

If three resistances are known, the fourth one can be determined by

$$R = S \cdot \frac{P}{Q}$$

R = unknown resistance

S = standard arm of bridge

P, Q = ratio arms.

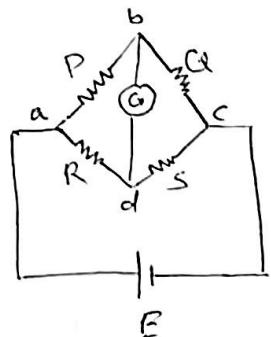
Sensitivity of Wheatstone bridge

The sensitivity to unbalance can be computed by solving the bridge circuit for a small unbalance. The solution can be obtained by converting the wheatstone bridge to its Thevenin equivalent circuit.

Assume that the bridge is balanced when the branch resistances are P, Q, R, S , so that $P/Q = R/S$. Suppose the resistance R is changed to $R + \Delta R$ creating an unbalance. This will cause an emf e to appear across the galvanometer branch. With galvanometer branch open, the voltage drop b/w points a and b is :

$$E_{ab} = I_1 P = \frac{EP}{P+Q}$$

$$E_{ad} = I_2 (R + \Delta R) = \frac{E(R + \Delta R)}{R + \Delta R + S}$$



\therefore voltage difference b/w 'd' and 'b' is ,

$$e = E_{ad} - E_{ab} = E \left[\frac{R + \Delta R}{R + \Delta R + S} - \frac{P}{P+Q} \right]$$

$$\text{Since } \frac{P}{P+Q} = \frac{R}{R+S}$$

$$\begin{aligned}
 e &= E \left[\frac{\frac{R+\Delta R}{R+\Delta R+S}}{} - \frac{R}{R+S} \right] \\
 &= E \left[\frac{R^2 + RS + R\Delta R + S\Delta R - R^2 - R\Delta R - SR}{R^2 + R\Delta R + RS + RS + S\Delta R + S^2} \right] \\
 &= \frac{E S \Delta R}{(R+S)^2 + AR(R+S)} \\
 &\approx \frac{E S \Delta R}{(R+S)^2} \quad \Delta R (R+S) \ll (R+S)
 \end{aligned}$$

Let S_v be the voltage sensitivity of galvanometer

$$\therefore \text{deflection of galvanometer } \Theta = S_v e$$

$$\Theta = \frac{S_v E S \Delta R}{(R+S)^2}$$

Bridge sensitivity S_B is defined as the deflection of the galvanometer per unit fractional change in unknown resistance.

$$\text{Bridge sensitivity } S_B = \frac{\Theta}{\Delta R/R}$$

$$= \frac{S_v E S \Delta R}{(R+S)^2 \Delta R/R}$$

$$= \frac{S_v E S R}{(R+S)^2}$$

Sensitivity of the bridge is dependent upon bridge voltage, bridge parameters and the voltage sensitivity of the galvanometer.

Rearranging the eqn for bridge sensitivity,

$$S_B = \frac{S_v E}{(R+S)^2 / R_S} = \frac{S_v E}{\frac{R}{S} + 2 + \frac{S}{R}} = \frac{S_v E}{\frac{P}{Q} + 2 + \frac{Q}{P}}$$

Max sensitivity occurs when $R/S = 1$

For a bridge with equal arms, $R = S = P = Q$

Bridge sensitivity $S_B = \frac{S_v E}{4}$

- * sensitivity decreases with $\frac{P}{Q} = \frac{R}{S}$ is greater than or smaller than unity.
- * The reduction in sensitivity is accompanied by a reduction in accuracy with which a bridge can be balanced.

Limitations of wheatstone bridge

- * The use of wheatstone bridge is limited to the measurement of resistances ranging from a few ohm to several Mega ohm.
- * The upper limit is set by the reduction in sensitivity to unbalance caused by high resistance values. The upper limit can be

Example 14.6. Each of the ratio arms of a laboratory type Wheatstone bridge has guaranteed accuracy of $\pm 0.05\%$, while the standard arm has a guaranteed accuracy of $\pm 0.1\%$. The ratio arms are both set at 1000Ω and the bridge is balanced with standard arm adjusted to $3,154 \Omega$. Determine the upper and the lower limits of the unknown resistance, based upon the guaranteed accuracies of the known bridge arms.

Solution. Value of unknown resistance $R = (P/Q) \times S = (1000/1000) \times 3154 = 3154 \Omega$.

\therefore Percentage error in determination of R .

$$\frac{\delta R}{R} = \pm \frac{\delta P}{P} \pm \frac{\delta Q}{Q} \pm \frac{\delta S}{S} = \pm 0.05 \pm 0.05 \pm 0.1 = \pm 0.2\%$$

\therefore Limiting values of $R = 3154 \pm 0.2\% = 3091$ to 3217Ω .

Example 14.7. In the Wheatstone bridge of Fig. 14.3, the values of resistances of various arms are $P = 1000 \Omega$, $Q = 100 \Omega$, $R = 2,005 \Omega$ and $S = 200 \Omega$. The battery has an emf of 5 V and negligible internal resistance. The galvanometer has a current sensitivity of $10 \text{ mm}/\mu\text{A}$ and an internal resistance of 100Ω . Calculate the deflection of galvanometer and the sensitivity of the bridge in terms of deflection per unit change in resistance.

Solution. Resistance of unknown resistor required for balance

$$R = (P/Q) S = (1000/100) \times 200 = 2000 \Omega.$$

In the actual bridge the unknown resistor has a value of 2005Ω or the deviation from the balance conditions is $\Delta R = 2005 - 2000 = 5 \Omega$.

$$\begin{aligned} \text{Thevenin source generator emf } E_0 &= E \left[\frac{R}{R+S} - \frac{P}{P+Q} \right] \\ &= 5 \left[\frac{2005}{2005+200} - \frac{1000}{1000+100} \right] = 1.0307 \times 10^{-3} \text{ V.} \end{aligned}$$

Internal resistance of bridge looking into terminals b and d .

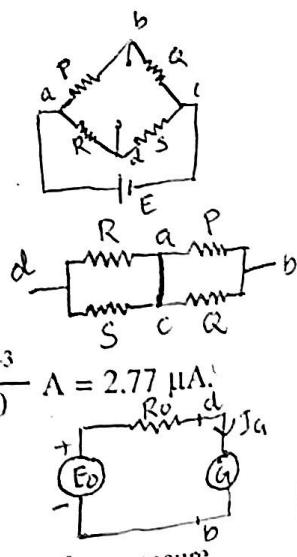
$$R_0 = \frac{RS}{R+S} + \frac{PQ}{P+Q} = \frac{2005 \times 200}{2005+200} + \frac{1000 \times 100}{1000+100} = 272.8 \Omega$$

Hence the current through the galvanometer

$$I_g = \frac{E_0}{R_0+G} = \frac{1.0307 \times 10^{-3}}{272.8 + 100} \text{ A} = 2.77 \mu\text{A}$$

Deflection of the galvanometer $\theta = S_i I_g = 10 \times 2.77 = 27.7 \text{ mm}$.

$$\text{Sensitivity of bridge } S_B = \frac{\theta}{\Delta R} = \frac{27.7}{5} = 5.54 \text{ mm}/\Omega$$

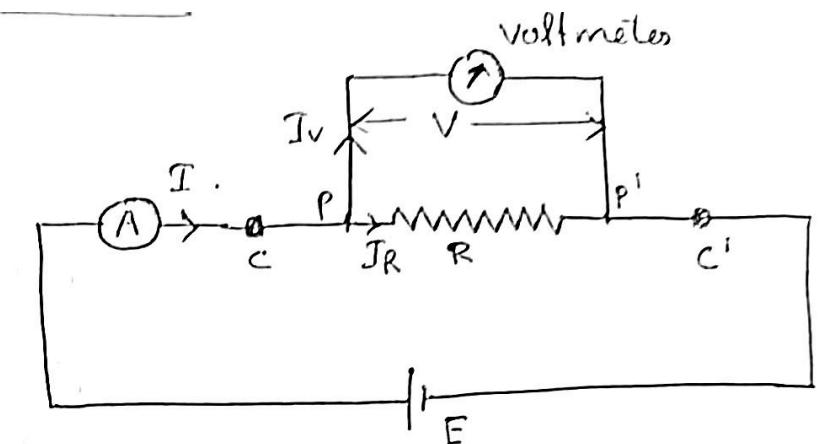


Measurement of low resistance

- The method used for measurement of medium resistances are unsuitable for measurement of low resistance ($R < 1\Omega$) .
- The reason is the resistance leads and contacts , though small are appreciable in the case of low resistances.
- For example, a contact resistance of 0.02Ω causes a negligible error when a resistance of 100Ω is being measured , but the same contact resistance would cause 10% error if a low resistance of the value 0.02Ω is measured .
- So special type of construction and techniques are used for the measurement of low resistance .

→ Low resistances are constructed with four terminals as shown in fig.

- One pair of terminals cc' called as current terminals is used to lead current to and from the resistor.
- The voltage drop is measured between the other two terminals pp' , called the potential terminals.
- Ammeter - voltmeter method for measuring R is shown above. The voltage indicated by voltmeter is I_R times the resistance R b/w terminals pp' and it does not include any contact resistance drop that may be present at the current terminals cc' .
- Contact resistance



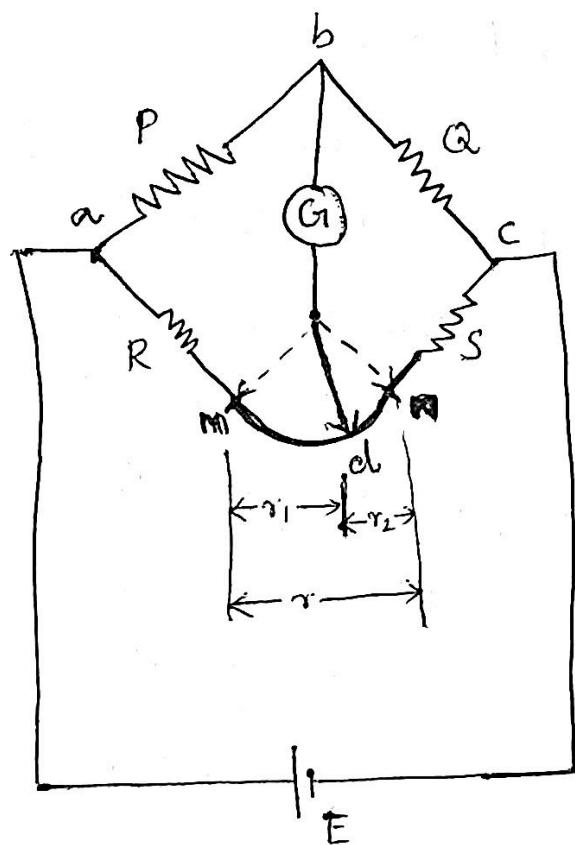
Methods for measurement of low resistance

Usually used methods are,

- 1) Ammeter - voltmeter method (same as medium R measurement)
- 2) Kelvin's double bridge method
- 3) Potentiometer method

Kelvin Double Bridge Method

- It is a modification of Wheatstone bridge.
- Provides good accuracy in measurement of low value resistances.
- The principle of Kelvin bridge can be illustrated by the fig. given below.



- Here 'r' represents the resistance of the lead that connects the unknown resistance R to standard resistance S .
- Two galvanometer connections indicated by dotted lines are possible. The connection may be either to point 'm' or to point 'n'.

→ When galvanometer is connected to point 'm', the resistance r of the connecting leads is added to the standard resistance s , resulting in indication of too low an indication for unknown resistance R .

→ When galvanometer is connected to point 'n', r is added to unknown resistance resulting in indication of too high a value for R .

→ Suppose that instead of using point m or n , we take the galvanometer connection is made at any intermediate point d as shown by the solid line in the fig above. Now if at point d , the resistance r is divided into two parts, r_1 and r_2 such that

$$\frac{r_1}{r_2} = \frac{P}{Q}$$

Then the presence of r the resistance of connecting leads causes no error in the result.

$$R + r_1 = \frac{P}{Q} (s + r_2) \quad \text{--- (1)} \quad \text{but } \frac{r_1}{r_2} = \frac{P}{Q}$$

$$\text{or } \frac{r_1}{r_1 + r_2} = \frac{P}{P+Q}$$

$$\text{or } r_1 = \left(\frac{P}{P+Q} \right) (r_1 + r_2) = \frac{P}{P+Q} r$$

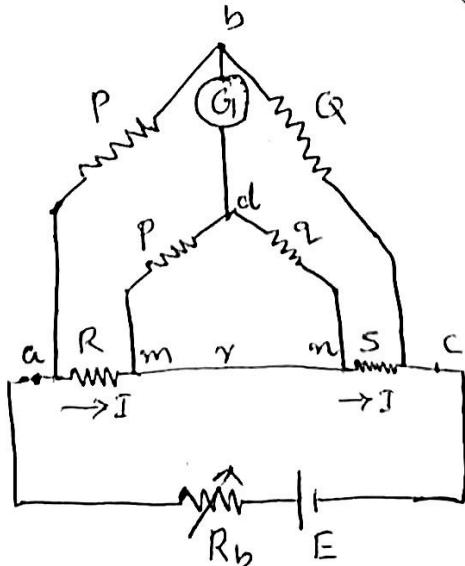
$$\text{& } r_2 = \frac{Q}{P+Q} r$$

Now eqn ① can be written as,

$$\left(R + \frac{P}{P+Q} r \right) = \left(\frac{P}{Q} \left(S + \frac{Q}{P+Q} r \right) \right)$$

$$R = \frac{P}{Q} \cdot S$$

- The above eqn for R indicates that making the galvanometer connection as at d , the resistance of leads does not affect the result.
- But this process is not a practical way since it is not possible to determine the point d .
- Thus some modifications are done in the above circuit that two actual resistance units of correct ratio be connected b/w points m and n , the galvanometer be connected to the junction of the resistors.
- Actual Kelvin bridge arrangement is shown below



- The Kelvin double bridge incorporates a second set of ratio arms (hence the name double bridge) and a 4 terminal resistor for the low resistance arms. The first ratio arms arm is P and Q and second is P & Q.
- The ratio P/Q is made equal to P/Q .
- Under balance conditions there is no current through the galvanometer. Then so $E_{ab} = E_{amid}$.

$$E_{ab} = \frac{P}{P+Q} E_{ac}$$

$$E_{ac} = I \left[R + S + \frac{(P+Q)\gamma}{P+Q+\gamma} \right]$$

$$E_{ab} = \frac{P}{P+Q} I \left[R + S + \frac{(P+Q)\gamma}{P+Q+\gamma} \right]$$

$$E_{amid} = I \left[R + \frac{P\gamma}{P+Q+\gamma} \right]$$

For zero galvanometer deflection $E_{ab} = E_{amid}$

$$\frac{P}{P+Q} I \left[R + S + \frac{(P+Q)\gamma}{P+Q+\gamma} \right] = I \left[R + \frac{P\gamma}{P+Q+\gamma} \right]$$

$$R = \frac{P}{Q} S + \frac{Q\gamma}{P+Q+\gamma} \left[\frac{P}{Q} - \frac{P}{Q} \right] \quad \text{--- (2)}$$

$$\text{Now if } \frac{P}{Q} = \frac{P}{Q}, \text{ becomes } R = \frac{P}{Q} S \quad \text{--- (3)}$$

- This is ^{eqn(3)} the usual working equation of Kelvin bridge. It indicates that the lead resistance has no effect on the measurement provided that two set of ratio arms have equal ratios.
- If the ratios of ratio arms are not exactly equal, eqn (2) shows the error introduced.
- If there is a difference b/w ratios $\frac{P}{Q} \neq \frac{P}{Q}$; to minimize the error, lead resistance r should be kept as small as possible.
- In a typical Kelvin bridge, the range of resistance covered is $0.1\text{ m}\Omega$ to $1\text{ }\Omega$.

Measurement of high resistance

Different methods used are,

- 1) Direct deflection method
- 2) Loss of charge method
- 3) Megohm bridge
- 4) Meggar

Loss of charge method

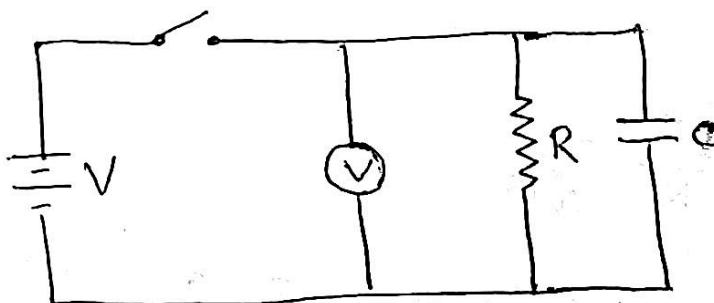


fig 1

* The insulation resistance (high resistance) R to be measured is connected in parallel with a capacitor C and an electrostatic voltmeter.

* The capacitor is charged to some suitable voltage, by means of a battery having voltage V , and is then allowed to discharge through the resistance.

* The terminal voltage is observed

over a considerable period of time during discharge.

* The voltage across the capacitor at any instant t after the application of voltage is,

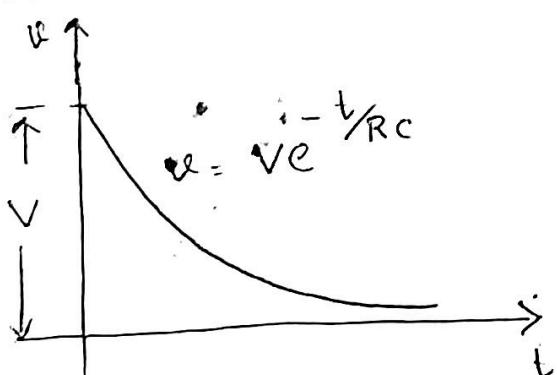
$$v = V e^{(-t/RC)}$$

$$\frac{V}{v} = e^{-t/RC} \Rightarrow \frac{V}{v} = \frac{1}{e^{t/RC}}$$

$$R = \frac{t}{C \log_e \frac{V}{v}} = \frac{0.4343 t}{C \log_{10} \frac{V}{v}} \quad \text{--- (1)}$$

The variation of voltage v with time is shown

below



From eqn (1) if V, v, C & t are known, the value of R can be computed.

* This method is applicable to measure high resistances but requires a capacitor of a very high leakage resistance as high as resistance being measured.

- * So this method is very attractive if the resistance being measured is the leakage resistance of a capacitor as in this case auxiliary R and C units are not required.

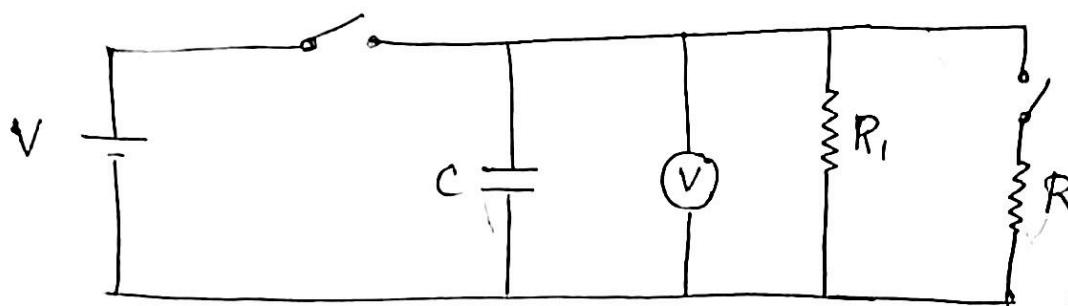


Fig 2

- * In the loss of charge method shown in fig 1, we are not actually getting the true resistance since we assume that the value of resistance of electrostatic voltmeters and the leakage resistance of the capacitor have infinite value.
- * But in practice corrections must be applied for that assumption. The Fig 2 shown above represents the actual circuit of test where R_1 is the leakage resistance of capacitor.
- * R' is the resistance of R_1 and R in parallel.

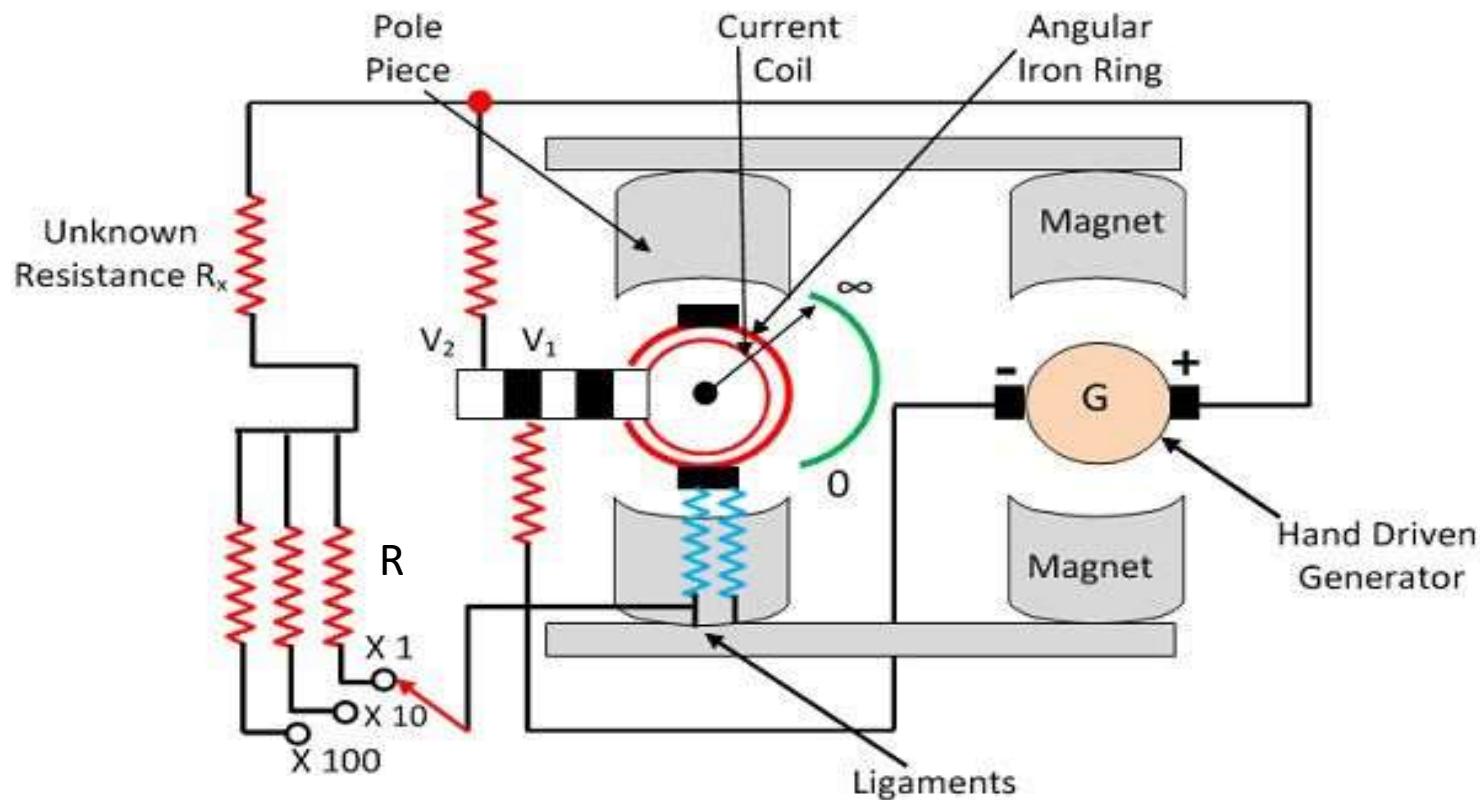
$$R' = \frac{RR_1}{R+R_1} \quad \text{--- (2)}$$

* Then the discharge eqn of capacitor gives

$$R' = \frac{0.4343 t}{C \log_{10} \frac{V_0}{V}} \quad \text{--- (3)}$$

* The test is then repeated with the unknown resistance R_1 disconnected and the capacitor discharging through R_1 . Thus value of R_1 is obtained and substitute in (2) to get the unknown resistor R .

MEGGER



Megger

Circuit Globe

MEGGER

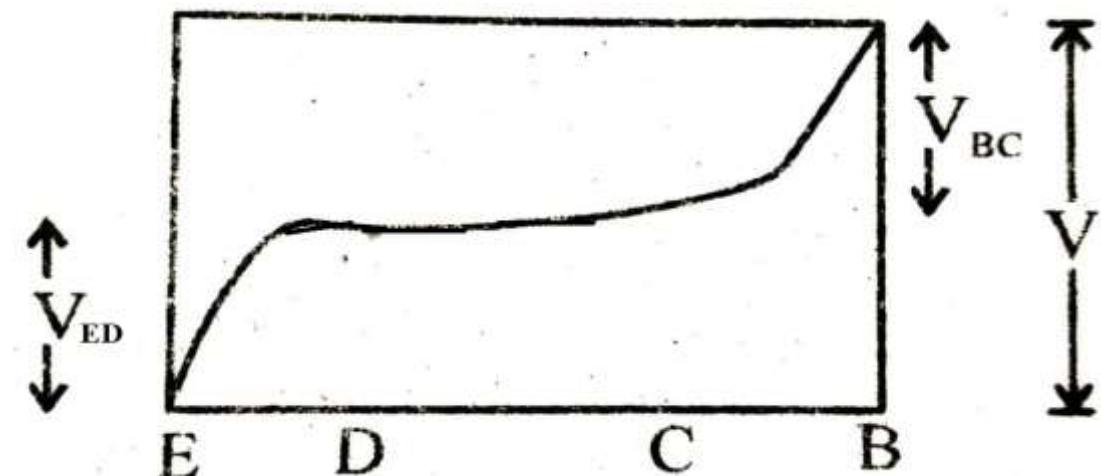
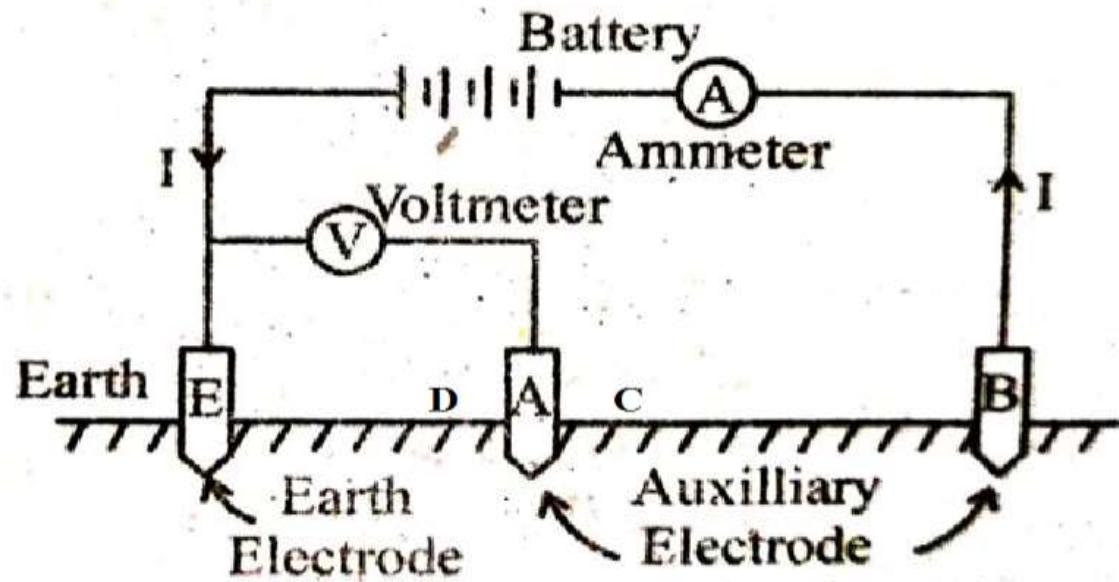
- It is an insulation resistance testing instrument
- It consists of a hand driven dc generator and a ohm meter.
- Permanent magnets provide field for both generator and ohmmeter.
- Ohmmeter consists of 2 coils. Current or deflecting coil, pressure or control coil,. Coils are connected to circuit by means of flexible leads (ligaments).
- CC connected in series with R b/w one generator terminal and test terminal L. R protects CC when L & E are short circuited.
- PC connected across generator terminals.

MEGGER

- With L & E open(α resistance) no current flows through CC. The PC thus governs the motion of moving element causing it to move extreme counter clock position
- With L& E short circuited (0 resistance) current through CC is large enough to produce torque to overcome counter clock torque of PC, pointer moves extreme clockwise position. CC is wound to produce clock wise torque on moving element.
- When resistance under test is connected b/w L & E opposing torques balance & pointer comes to rest at intermediate position.

MEASUREMENT OF EARTH RESISTANCE

1. Fall of potential method

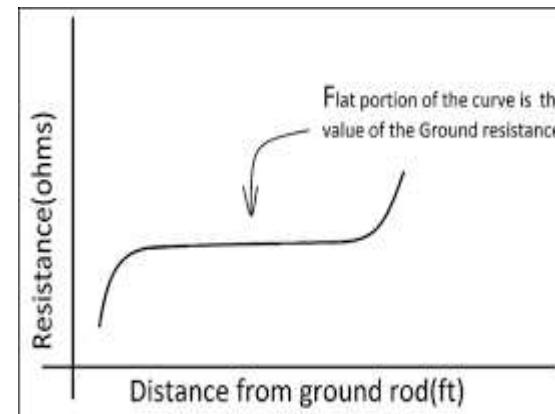


MEASUREMENT OF EARTH RESISTANCE

- A current is passed through earth electrode E to an auxiliary electrode inserted in earth at a distance away from the earth electrode.
- Between E and B a second auxiliary electrode A is inserted in earth.
- The potential difference V, between A and E is measured for a given current I.
- Potential rises in proximity of E & B and constant along middle portion.
- The resistance of earth therefore is: $R_E = V/I$ or V_{EA}/I .

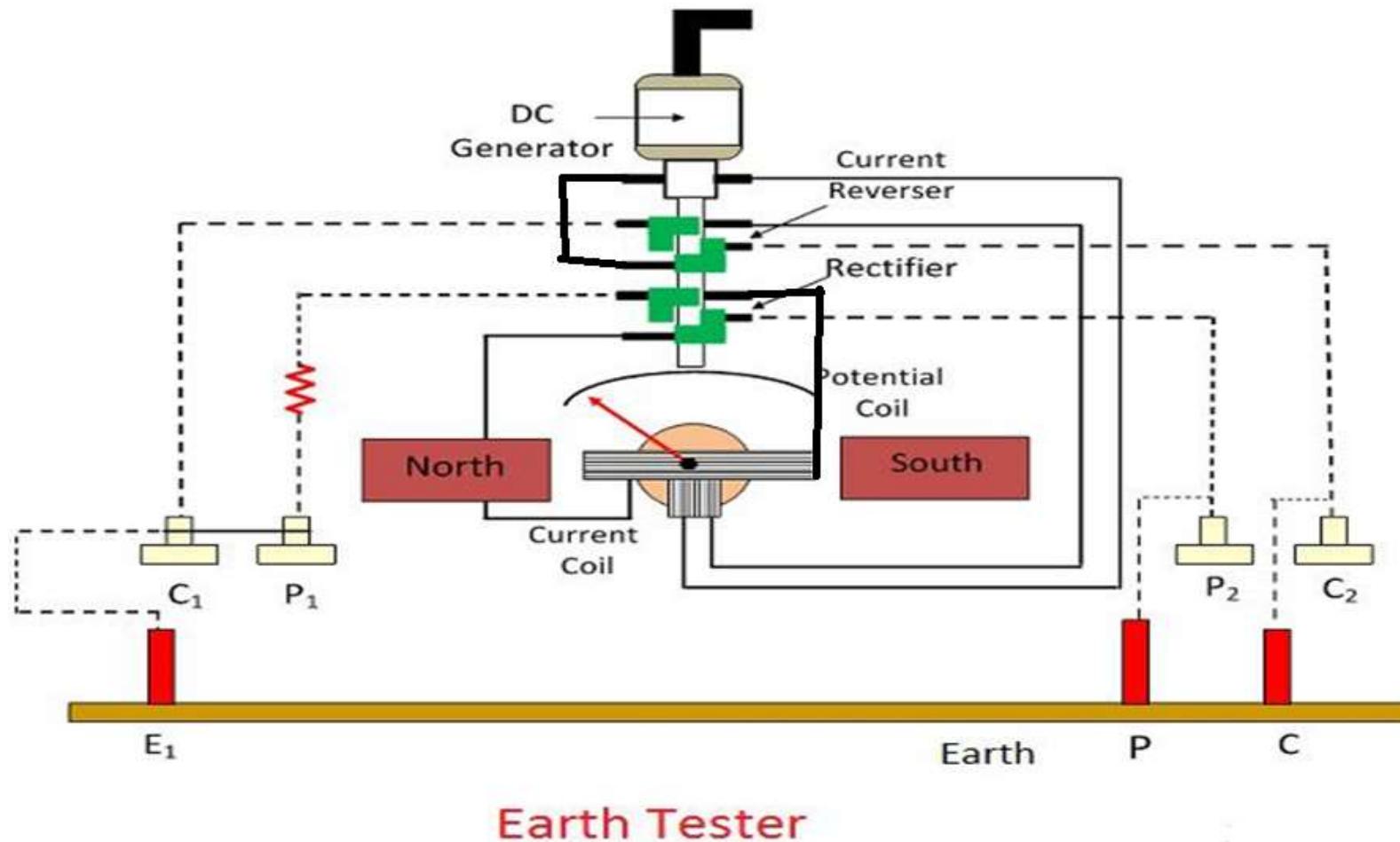
MEASUREMENT OF EARTH RESISTANCE

- The position of electrodes B and E is fixed and the position of electrode A is changed and resistance is measured for various positions of electrode A.
- It is clear that the measured value of earth resistance depends upon the position of the auxiliary electrode A. The earth resistance rises rapidly initially, when the distance between earth electrode E and auxiliary electrode A is increased, it then becomes constant and when the auxiliary electrode A approaches the auxiliary electrode B, the resistance rises again.
- The placing of electrodes is thus very important and serious error may be cause by incorrect placing of the electrodes. The correct value of resistance of earth, $R.$, is when the auxiliary electrode A is at such a distance that the resistance lies on the flat part of curve.



MEASUREMENT OF EARTH RESISTANCE

2.EARTH TESTER



MEASUREMENT OF EARTH RESISTANCE

- Special type of megger with additional features like rotating current reverser and rectifier
- Both these additional features consists of simple commutators made of L shaped segments mounted on shaft of generator.
- Tester sends ac through earth & dc through measuring instrument.
- It has 4 terminals P1, C1, P2 ,C2. P1 & C1 are shorted to form a common point which is connected to earth electrode. Other two terminals P2 &C2 are connected to auxiliary electrodes P & C.
- The indication of earth tester depends upon ratio of voltage across pressure coil and current through current coil.

MEASUREMENT OF EARTH RESISTANCE

- The deflection of pointer indicates resistance of earth. Earth tester is PMMC instrument can operate on dc only .By the inclusion of current reverser and rectifying device it is possible to make measurements with ac flowing through soil.
- Testing voltage produced by hand operated generator for manual operation and battery for automatic type.

AC Bridges

- Alternating current bridge methods are used for the measurement of inductance, capacitance etc.
- It consists of four arms, a source of excitation and a balance detector. The detector is sensitive to small alternating potential differences.

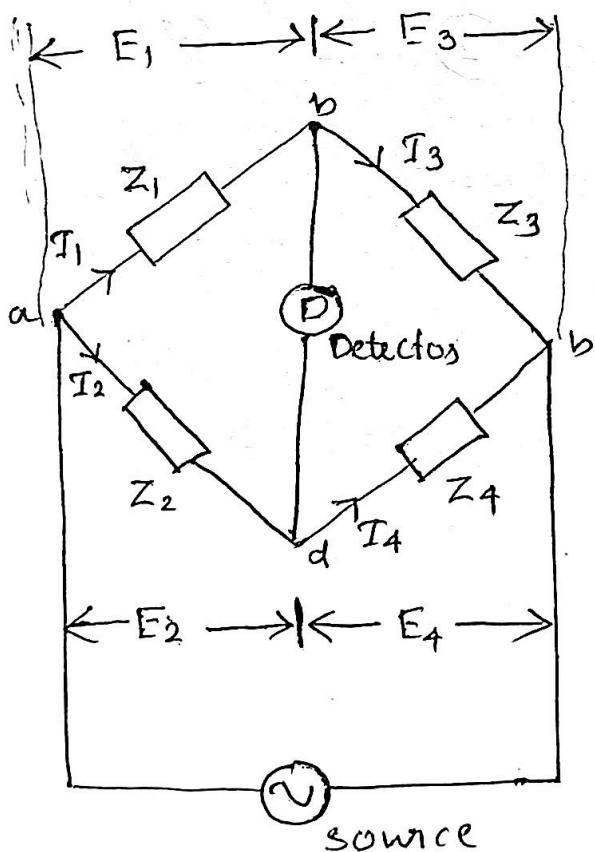


Fig 1

- The four arms of the bridge are impedances Z_1, Z_2, Z_3 and Z_4 .
- Under balanced condition there will not be any current through the detector. That means the potential difference b/w points b and d is zero.

→ So the voltage drop from a to b equals the voltage drop from a to d both in magnitude and phase.

$$E_1 = E_2$$

$$I_1 Z_1 = I_2 Z_2$$

$$I_1 = I_3 = \frac{E}{Z_1 + Z_3} \quad \text{--- (1)}$$

$$I_2 = I_4 = \frac{E}{Z_2 + Z_4} \quad \text{--- (2)}$$

$$\frac{Z_1}{Z_1 + Z_3} = \frac{Z_2}{Z_2 + Z_4}$$

$$\boxed{Z_1 Z_4 = Z_2 Z_3} \quad \text{--- (3)}$$

→ In polar form, the admittances can be written as $Z\angle\theta$, where Z is the magnitude and θ is the phase angle of complex impedance.

$$\therefore \text{eqn (3)} \Rightarrow (Z_1 \angle \theta_1)(Z_4 \angle \theta_4) = (Z_2 \angle \theta_2)(Z_3 \angle \theta_3)$$

→ Two conditions are to be satisfied simultaneously when balancing ac bridge.

→ The first condition is that magnitude of impedances should satisfy

$$Z_1 Z_4 = Z_2 Z_3$$

- The second condition is that the phase angles of impedances satisfy,
- $$\angle \theta_1 + \angle \theta_4 = \angle \theta_2 + \angle \theta_3$$
- Phase angles are positive for an inductive impedance and negative for capacitive impedance.

→ In rectangular co-ordinates,

$$Z_1 = R_1 + jX_1 \quad Z_2 = R_2 + jX_2$$

$$Z_3 = R_3 + jX_3 \quad Z_4 = R_4 + jX_4$$

→ For balance

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + jX_1)(R_4 + jX_4) = (R_2 + jX_2)(R_3 + jX_3)$$

$$R_1 R_4 - X_1 X_4 + j(X_1 R_4 + X_4 R_1) = R_2 R_3 - X_2 X_3 + j(X_2 R_3 + X_3 R_2)$$

$$\boxed{R_1 R_4 - X_1 X_4 = R_2 R_3 - X_2 X_3} \quad (\text{Real parts})$$

$$\boxed{X_1 R_4 + X_4 R_1 = X_2 R_3 + X_3 R_2} \quad (\text{Imaginary parts})$$

Both these conditions should be satisfied for the bridge to be balanced.

Q) The four impedances of an ac bridge shown in fig 1 are ,

$$Z_1 = 400 \Omega \angle 50^\circ \quad Z_2 = 200 \Omega \angle 10^\circ$$

$$Z_3 = 800 \Omega \angle -50^\circ \quad Z_4 = 100 \Omega \angle 20^\circ$$

Find out whether the bridge is balanced under these conditions or not.

For bridge balance , $Z_1 Z_4 = Z_2 Z_3 \neq -1$ —①

$$\angle \theta_1 + \angle \theta_4 = \angle \theta_2 + \angle \theta_3 - ②$$

$$Z_1 Z_4 = 400 \times 400 = 160000$$

$$Z_2 Z_3 = 200 \times 800 = 160000$$

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{satisfied}$$

$$\angle \theta_1 + \angle \theta_4 = 50^\circ + 20^\circ = 70^\circ$$

$$\angle \theta_2 + \angle \theta_3 = 40^\circ - 50^\circ = -10^\circ$$

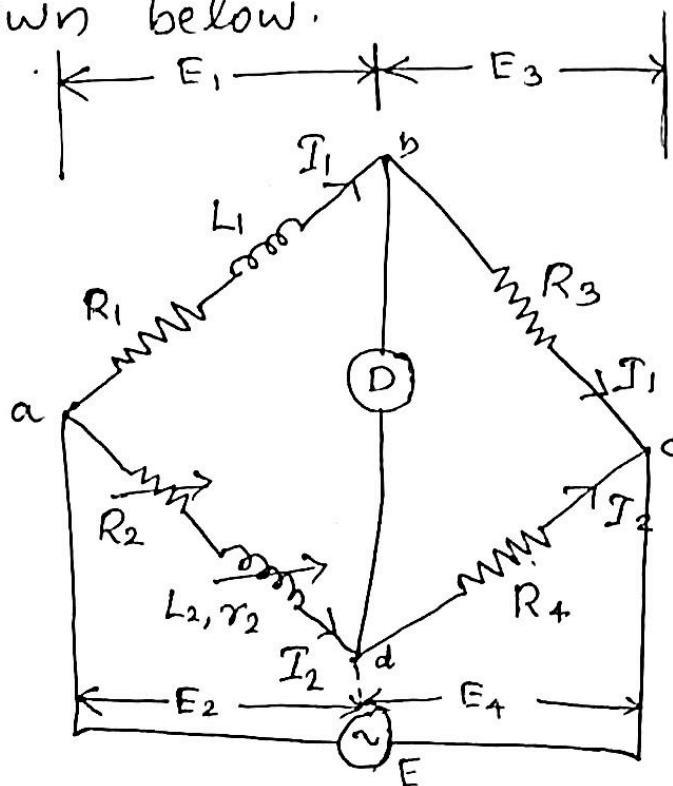
$$\angle \theta_1 + \angle \theta_4 \neq \angle \theta_2 + \angle \theta_3$$

\therefore the bridge is unbalanced

Measurement of self inductance

Maxwell's inductance bridge

→ This bridge circuit measure an inductance by comparing with a variable standard self inductance. The connection diagram is shown below.



Let L_1 = unknown inductance of resistance R_1

L_2 = Variable inductance of fixed resistance r_2

R_2 - Variable resistance connected in series with inductor L_2 .

R_3, R_4 - known non inductive resistances

→ The impedance of the four arms are

$$Z_1 = R_1 + j\omega L_1$$

$$Z_2 = R_2 + r_2 + j\omega L_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4$$

At balance ,

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1) R_4 = (R_2 + r_2 + j\omega L_2) R_3$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + r_2 R_3 + j\omega L_2 R_3$$

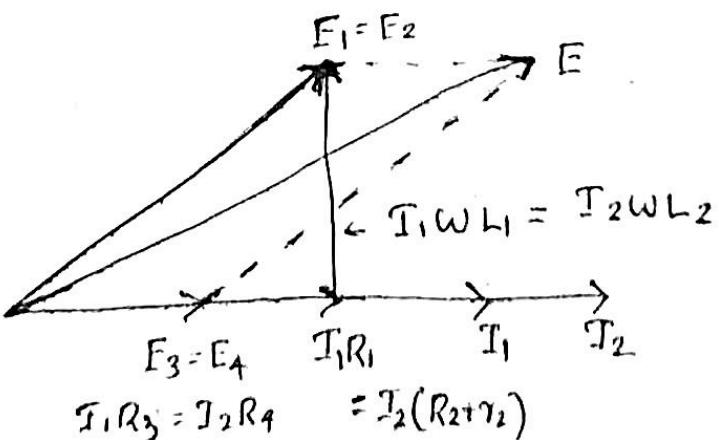
Equating real parts $\Rightarrow R_1 R_4 = R_2 R_3 + r_2 R_3$

$$R_1 = \frac{R_3}{R_4} (R_2 + r_2)$$

Equating imaginary parts $\Rightarrow L_1 R_4 = L_2 R_3$

$$L_1 = \frac{R_3}{R_4} L_2$$

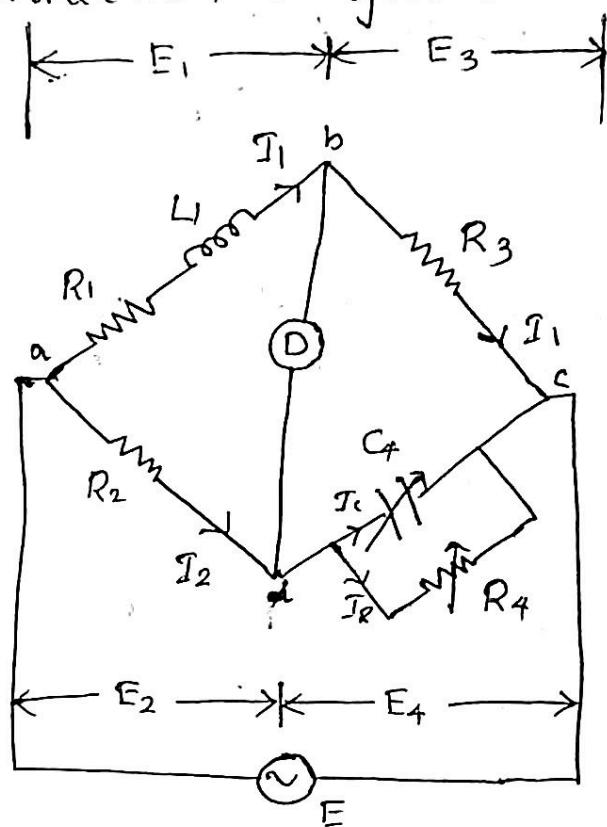
→ The phasor diagram of Maxwell's inductance bridge is shown below



Maxwell's inductance-capacitance bridge

→ In this bridge, an inductance is measured by comparison with a standard variable capacitance.

→ The connection diagram is shown below.



Let L_1 = Unknown inductance

R_1 = Effective resistance of L_1 ,

R_2, R_3, R_4 = known non-inductive resistances

C_4 = variable standard capacitor

→ The impedances are

$$Z_1 = R_1 + j\omega L_1 \quad Z_2 = R_2$$

$$Z_3 = R_3 \quad Z_4 = \frac{R_4}{1 + j\omega C_4 R_4}$$

$$Z_4 = \frac{R_4 \cancel{(1 + j\omega C_4 R_4)}}{\cancel{R_4(1 + j\omega C_4 R_4)}} = \frac{-jR_4}{R_4\omega C_4 - j}$$

$$= \frac{R_4}{1 - R_4\omega C_4 j} = \frac{R_4}{1 + j\omega C_4 R_4}$$

At balance,

$$(R_1 + j\omega L_1) \left(\frac{R_4}{1 + j\omega C_4 R_4} \right) = R_2 R_3$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 (1 + j\omega C_4 R_4)$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega C_4 R_2 R_3 R_4$$

Equate real parts

$$R_1 R_4 = R_2 R_3$$

$$R_1 = \frac{R_3}{R_4} R_2$$

Equate imaginary part

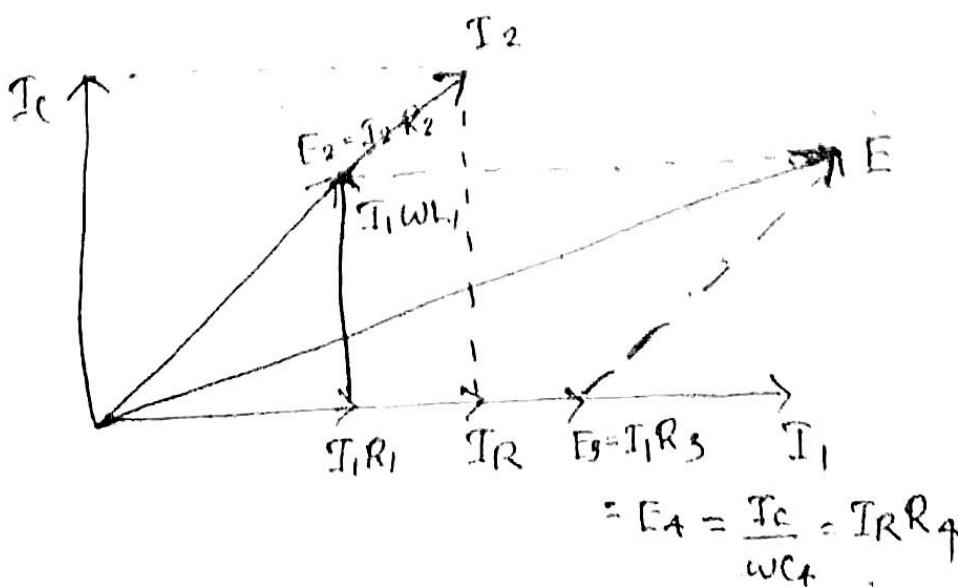
$$L_1 R_4 = C_4 R_2 R_3 R_4$$

$$L_1 = C_4 R_2 R_3$$

$$\text{Q factor} = \frac{\omega L_1 / R_1}{R_3 R_2 / R_4} = \frac{\omega C_4 R_2 R_3}{R_3 R_2 / R_4} = \omega C_4 R_4$$

Admittance

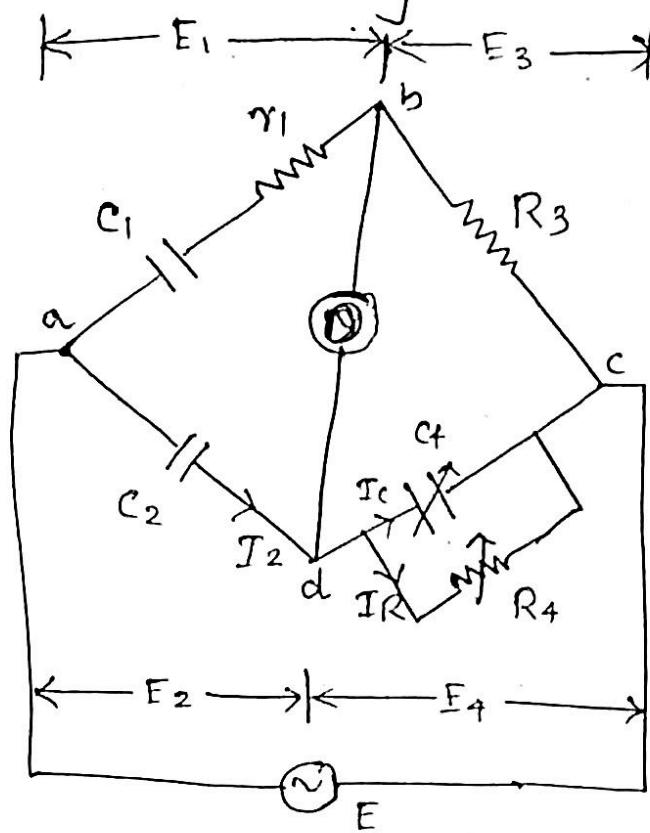
The phasor diagram is shown below.



Measurement of capacitance

Schering Bridge

→ The connection diagram is shown below



Let C_1 = capacitor whose capacitance to be determined

r_1 = a series resistance representing the loss in the capacitor C_1

C_2 = a std capacitor. It is either an air or gas capacitor and hence is loss free.

R_3 = a non inductive resistance

C_4 = a variable capacitor

R_4 = a variable non inductive resistance

The impedances are

$$Z_1 = \gamma_1 + \frac{-j}{\omega C_1} = \gamma_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = \frac{-j}{\omega C_2} = \frac{1}{j\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = \frac{R_4}{1+j\omega C_4 R_4}$$

At balance

$$\boxed{\begin{aligned} Z_4 &= R_4 \times \frac{-j/\omega C_4}{R_4 - j/\omega C_4} \\ &= \frac{-j R_4}{-j(1 - \frac{R_4 \omega C_4}{j})} \\ &= \frac{R_4}{1 + j\omega C_4 R_4} \end{aligned}}$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left(\gamma_1 + \frac{1}{j\omega C_1}\right) \left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = R_3 \times \frac{1}{j\omega C_2}$$

$$\gamma_1 R_4 + \frac{R_4}{j\omega C_1} = R_3 \left(1 + j\omega C_4 R_4\right) / j\omega C_2$$

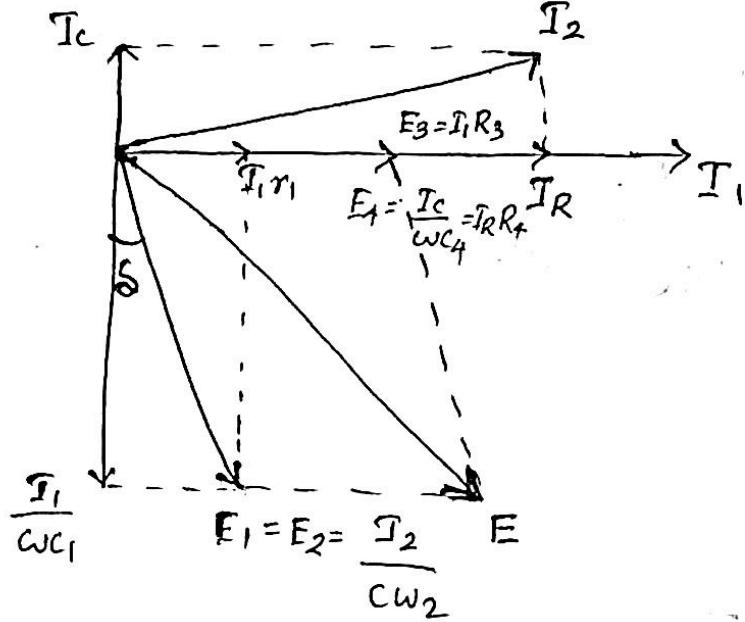
$$\gamma_1 R_4 - \frac{j R_4}{\omega C_1} = \frac{-j R_3}{\omega C_2} + \frac{R_3 C_4 R_4}{C_2}$$

$$\text{Equate real parts} \Rightarrow \gamma_1 R_4 = \frac{R_3 C_4 R_4}{C_2}$$

$$\gamma_1 = \frac{R_3 C_4}{C_2}$$

$$\text{Equate imaginary parts} \Rightarrow C_1 = \frac{R_4 C_2}{R_3}$$

The phasor diagram is shown below



Dissipation factor

$$D = \tan \delta = \frac{I_1 r_1}{\frac{I_1}{wC_1}} = wC_1 \gamma_1$$

$$= w C_2 R_4 / R_3 \times \frac{R_3 C_1}{C_2} = w C_4 R_4$$

$$D = \frac{\text{Ratio of series ESR (effective series resistance)}}{\text{Capacitive reactance}}$$

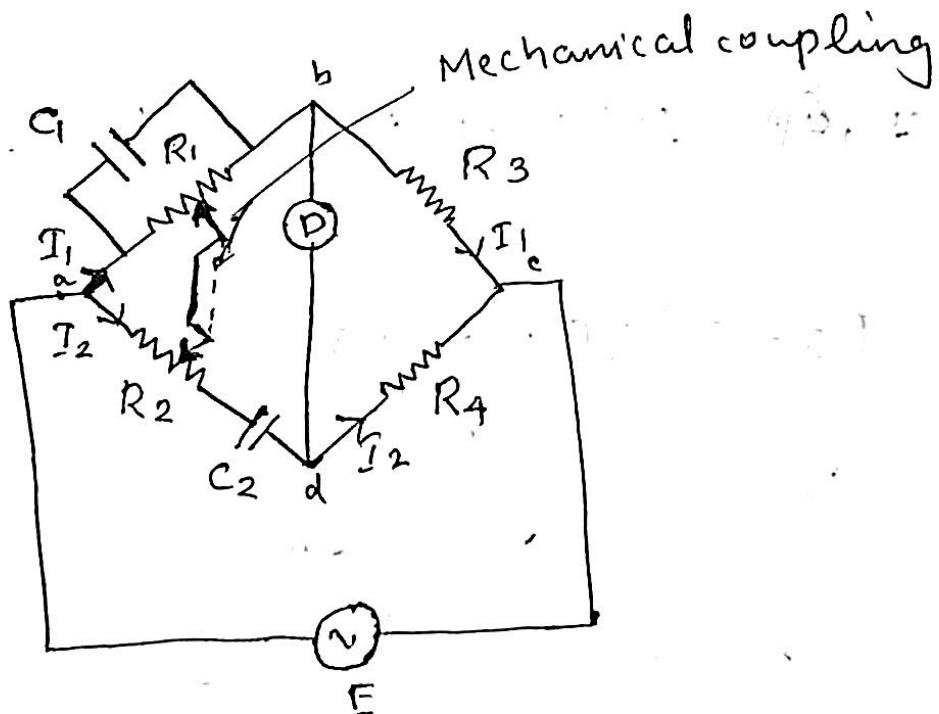
$$= \frac{\gamma_1}{1/C_1 w} = \underline{\underline{w C_1 \gamma_1}} = \underline{\underline{w C_4 R_4}}$$

Measurement of frequency

Wien's Bridge

→ It is primarily known as a frequency determining bridge. But it is also used for other circuits like harmonic distortion analyzers, audio and H.F oscillators etc.

→ The connection diagram of Wien's bridge is shown below



$$\rightarrow \text{Here } Z_1 = \frac{R_1}{1+j\omega C_1 R_1} \quad Z_2 = R_2 - \frac{1}{j\omega C_2}$$

$$Z_3 = R_3 \quad Z_4 = R_4$$

Under balanced conditions,

$$\left(\frac{R_1}{1+j\omega C_1 R_1} \right) R_4 = \left(R_2 - \frac{j}{\omega C_2} \right) R_3$$

$$R_1 R_4 = \left(R_2 R_3 - \frac{j R_3}{\omega C_2} \right) (1 + j\omega C_1 R_1)$$

$$R_1 R_4 = R_2 R_3 + j\omega C_1 R_1 R_2 R_3 - \frac{j R_3}{\omega C_2} + \frac{R_1 R_3 C_1}{C_2}$$

Equating imaginary parts,

$$j\omega C_1 R_1 R_2 R_3 - j \frac{R_3}{\omega C_2} = 0$$

$$\text{Since } j\omega^2 C_1 C_2 R_1 R_2 R_3 - j R_3 = 0$$

$$R_3 - \omega^2 C_1 C_2 R_1 R_2 R_3 = 0$$

$$\omega^2 = \frac{R_3}{C_1 C_2 R_1 R_2 R_3}$$

$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad \text{--- (1)}$$

Equating real parts $f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \quad \text{--- (2)}$

$$R_1 R_4 = R_2 R_3 + \frac{R_1 R_3 C_1}{C_2}$$

Equating real parts;

$$R_1 R_4 = R_2 R_3 + \frac{R_1 R_3 C_1}{C_2}$$

$$\therefore R_1 R_3 \Rightarrow \frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2} \quad \text{--- (3)}$$

In most Wien bridges, the components are so chosen that,

$$R_1 = R_2 = R \quad \text{and} \quad C_1 = C_2 = C$$

then (1) $\Rightarrow \omega = \frac{1}{RC}$

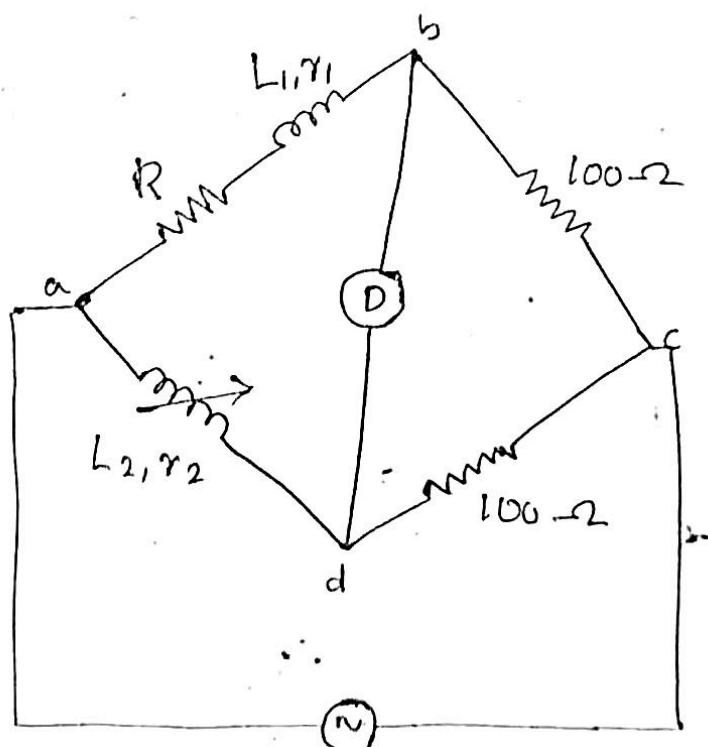
(2) $\Rightarrow f = \frac{1}{2\pi RC}$

(3) $\Rightarrow \frac{R_4}{R_3} = 2$

- Switches for resistors R_1 & R_2 are mechanically linked so as to fulfill the condition $R_1 = R_2$
- C_1 & C_2 are ^{fixed} capacitors equal in value and $R_4 = 2R_3$.

- So the frequency determinations by Wien's bridge can be done by balancing the bridge by a single control.
- Suitable for measurement of Frequencies from 100 Hz to 100 kHz. It is possible to obtain an accuracy of 0.1 to 0.5 %.
- Because of frequency sensitivity, the Wien's bridge may be difficult to balance unless the waveform of applied voltage is sinusoidal.
- Bridge is not balanced for any harmonics present in the applied voltage.
- This difficulty can be overcome by connecting a filter in series with the null detector.

Q1) A Maxwell's inductance comparison bridge is shown in fig below. Arm 'ab' consists of a coil with inductance L_1 and resistance r_1 in series with a non inductive resistance R . Arms 'bc' and 'cd' are each a non-inductive resistance of $100\ \Omega$. Arm 'ad' consists of a standard variable inductor L of resistance $32.7\ \Omega$. Balance is obtained when $L_2 = 47.8\text{ mH}$ and $R = 1.36\ \Omega$. Find the resistance and inductance of the coil in arm ab.



Under balanced condition,

$$(R + r_1 + j\omega L_1) 100 = 100 \times (r_2 + j\omega L_2)$$

~~$$R + r_1 + j\omega L_1 = r_2 + j\omega L_2$$~~

Equal Real parts $\Rightarrow R + r_1 = r_2$
 $r_1 = r_2 - R = 32.7 - 1.36 = 31.34$

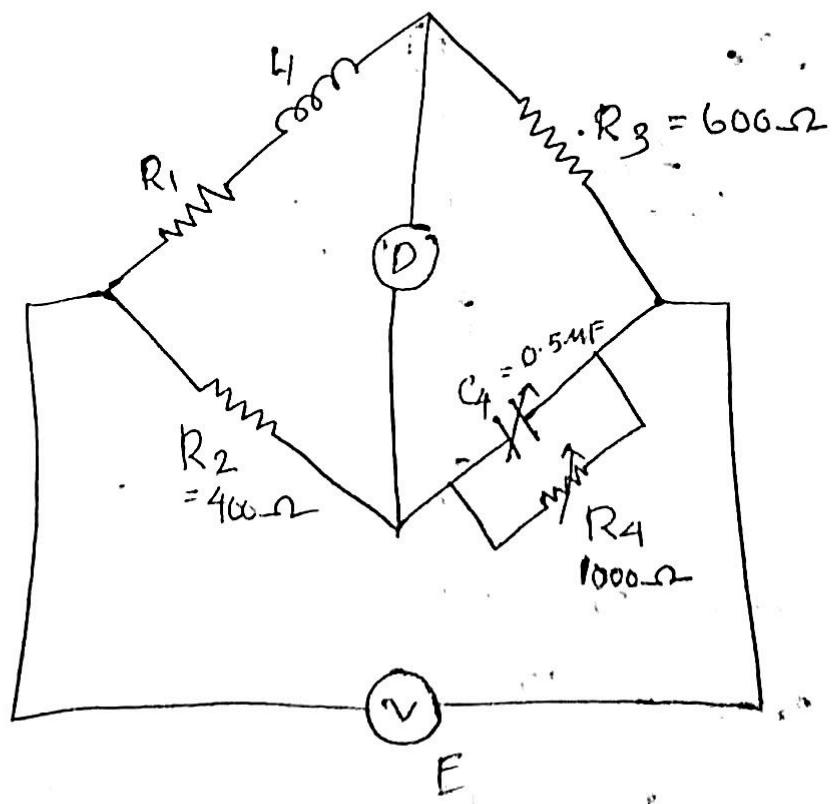
Inductance of coil can be obtained by equating imaginary parts

$$L_1 = L_2 = \underline{47.8 \text{ mH}}$$

- (Q) A Maxwell's capacitance bridge is used to measure an unknown inductance in comparison with a capacitor. The various values at balance are,

$$R_2 = 400\Omega, R_3 = 600\Omega, R_4 = 1000\Omega; C_4 = 0.5 \mu F$$

Calculate the values of R_1 and L_1 . Also calculate the value of storage (Q) factor of coil if frequency is 1000 Hz



$$(R_1 + j\omega L_1) \left(\frac{R_4}{1 + j\omega C_4 R_4} \right) = R_2 R_3$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega C_4 R_4 R_2 R_3$$

Equate real parts

$$\Rightarrow R_1 R_4 = R_2 R_3$$

$$R_1 = \frac{R_2 R_3}{R_4} = \frac{400 \times 600}{1000}$$

$$= \underline{\underline{240 \Omega}}$$

Equate imaginary parts

$$L_1 R_4 = C_4 R_4 R_2 R_3$$

$$L_1 = R_2 R_3 C_4 = 400 \times 600 \times 0.5 \times 10^{-6}$$

$$= \underline{\underline{0.12 H}}$$

$$Q = \frac{\omega L_1}{R_1} = \frac{2\pi \times 1000 \times 0.12}{240} = \underline{\underline{3.14}}$$

POTENTIOMETERS

- It is an instrument designed to measure unknown voltage by comparing it with a known voltage.
- The known voltage may be supplied by a standard cell or any other known voltage reference source.
- Measurements using comparison methods are capable of a high degree of accuracy because the result obtained does not depend upon on the actual deflection of a pointer, as is the case in deflectional methods, but only upon the accuracy with which the voltage of the reference source is known.

POTENTIOMETERS

- Another advantage of the potentiometers is that since a potentiometer makes use of a balance or null condition, no current flows and hence no power is consumed in the circuit containing the unknown emf when the instrument is balance
- The potentiometer is extensively used for a calibration of voltmeters and ammeters.

BASIC POTENTIOMETER CIRCUIT

- The principle of operation of all potentiometers is based on the circuit shown below which is the schematic diagram of the basic slide wire potentiometer.
- With switch 'S' in the "operate" position and the galvanometer key *K* open, the battery supplies the "working current" through the rheostat *R* and the slide wire.

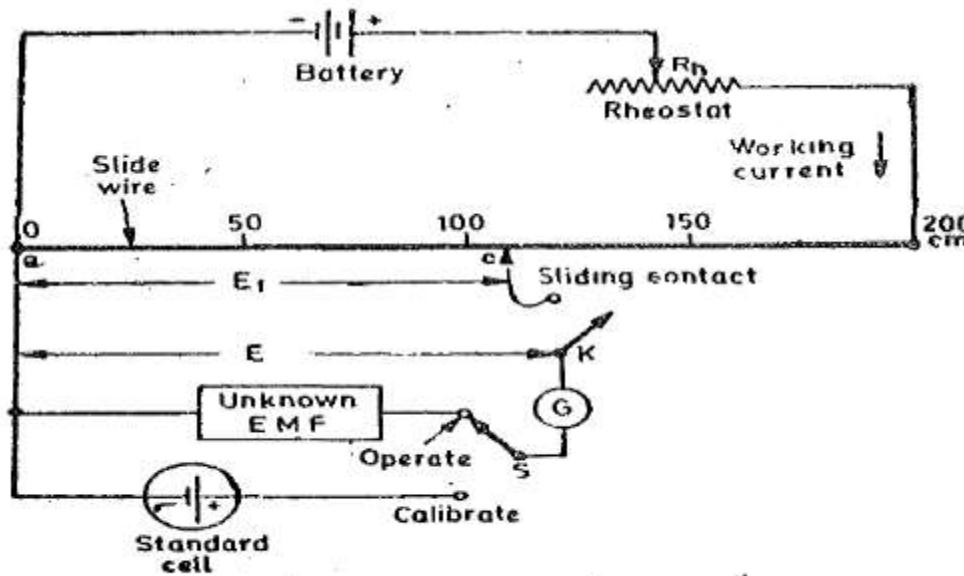


Fig. 14.1. Circuit diagram of a basic slide wire potentiometer.

BASIC POTENTIOMETER CIRCUIT

- The working current through the slide wire may be varied by changing the rheostat setting.
- The method of measuring the unknown voltage, E , depends upon finding a position for the sliding contact such the galvanometer shows zero deflection, i.e., indicates null condition.
- when the galvanometer key K , is closed zero galvanometer deflection or a null means that the unknown voltage, E, is equal to the voltage drop E_1 across portion ac of the slide wire.
- Thus determination of the value of unknown voltage now becomes a matter of evaluating the voltage drop E_1 along the portion ac of the slide wire.

BASIC POTENTIOMETER CIRCUIT

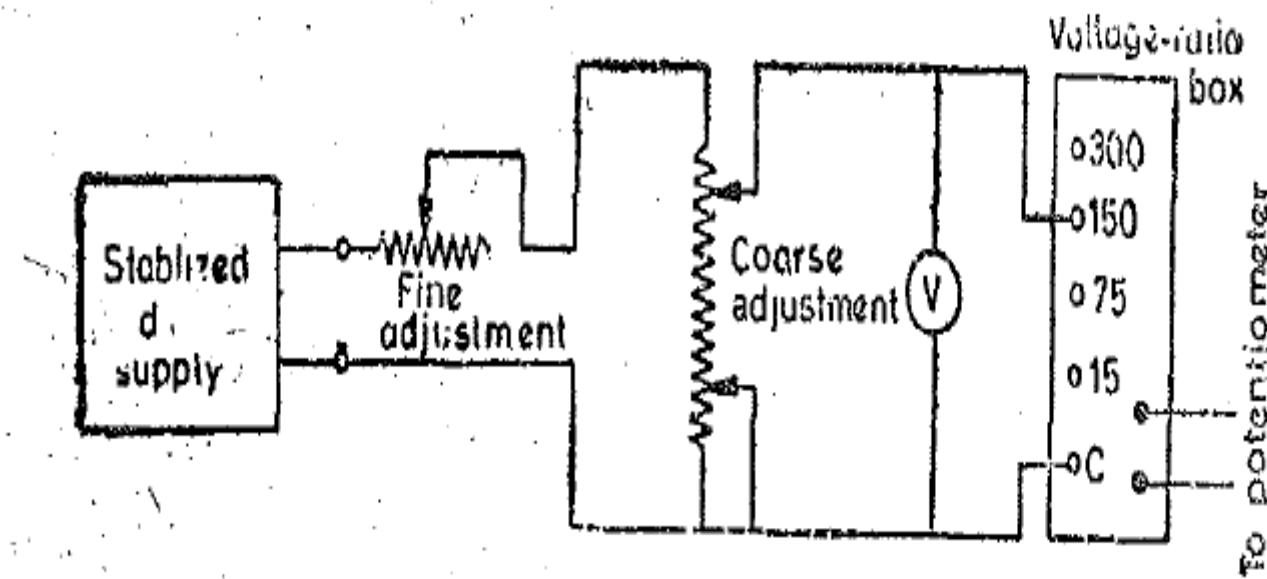
- The slide wire has a uniform cross-section and hence uniform resistance along its entire length.
- A calibrated scale in cm and fractions of cm, is placed along the slide wire so that the sliding contact can be placed accurately at any desired position along the slide wire.
- Since the resistance of slide wire is known accurately, the voltage drop along the slide wire can be controlled by adjusting the value of working current.
- The process of adjusting the working current so as to match the voltage drop across a portion of sliding wire against a standard reference source is known as "*standardisation*".

CONSTRUCTIONAL DETAILS OF POTENTIOMETERS

- All the resistors in a potentiometer, with the exception of slide wires are made of manganin. This is because manganin has a high stability, a low temperature co-efficient and has freedom from thermo-electric effects against copper.
- The slide wire is usually made of platinum - silver alloy and the sliding contacts are of a copper-gold-silver alloy. This combination of materials for slide wire and sliding contacts results in a good contact, freedom from thermo-electric emfs and minimum wear of slide wire.
- The current controlling rheostat is usually a combination of a stud dial and a multi turn slide wire.

APPLICATION OF D.C. POTENTIOMETERS

Calibration of Voltmeter



APPLICATION OF D.C. POTENTIOMETERS

Calibration of Voltmeter

- The foremost requirement in this calibration process is a suitable stable d c. voltage supply
- Figure shows a potential divider network, consisting of two rheostats; one for coarse and the other for fine control of calibrating voltage.
- The voltage across the voltmeter is stepped down to a value suitable for application to a potentiometer wjth the help of a volt-ratio box.
- For accuracy of measurements, it is necessary to measure voltages near the maximum range of the potentiometer, as far as possible.

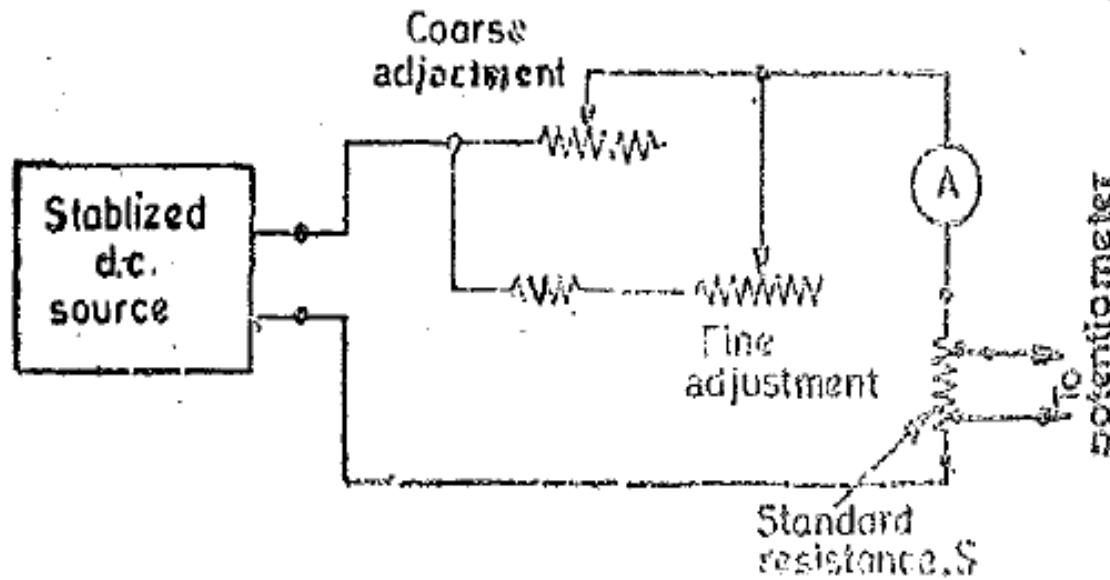
APPLICATION OF D.C. POTENTIOMETERS

Calibration of Voltmeter

- The potentiometer measures the true value of voltage, If the potentiometer reading does not agree with the voltmeter reading, a negative or positive error is indicated.
- This can be corrected by adjusting coarse or fine control.

APPLICATION OF D.C. POTENTIOMETERS

Calibration of ammeter



APPLICATION OF D.C. POTENTIOMETERS

Calibration of ammeter

- A standard resistance of suitable value and sufficient current carrying capacity is placed in series with the ammeter under calibration.
- The voltage across the standard resistor is measured with the help of potentiometer and the current through the standard resistance can be computed.

$$\text{Current } I = \frac{V_s}{S}$$

V_s =voltage across the standard resistor as indicated by the potentiometer

and S = resistance of standard resistor

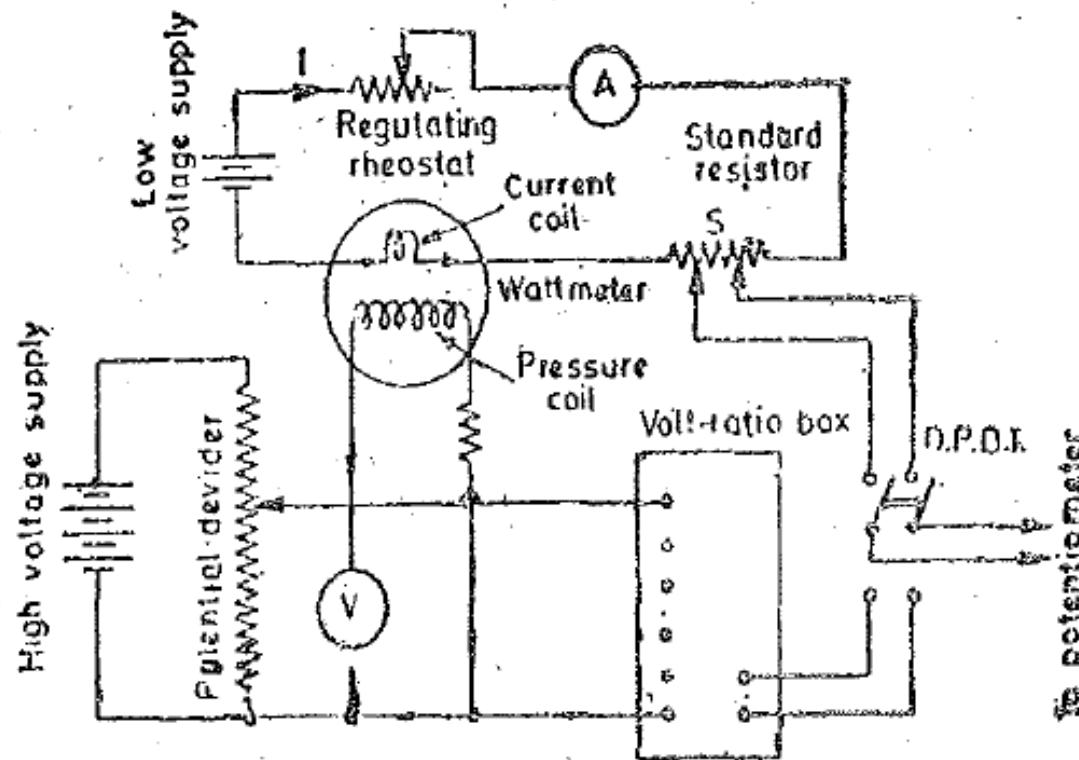
APPLICATION OF D.C. POTENTIOMETERS

Calibration of ammeter

- Since the resistance of the standard resistor is accurately known and the voltage across the standard resistor is measured by a potentiometer, this method of calibrating an ammeter is very accurate.

APPLICATION OF D.C. POTENTIOMETERS

Calibration of wattmeter



APPLICATION OF D.C. POTENTIOMETERS

Calibration of wattmeter

- The current coil of wattmeter is supplied from a low voltage supply and a series rheostat is inserted to adjust the value of current.
- The potential circuit is supplied from a high voltage supply.
- A volt-ratio box is used to step down the voltage for the potentiometer to read.
- The voltage, V and the current, I , are measured in turn with the potentiometer employing a double pole double throw (D.P.D.T.) switch.
- The true power is then VI and the wattmeter reading may be compared with this value.

HIGH VOLTAGE MEASUREMENT

- High voltages can be measured in a variety of ways
 - Direct measurement up to 200 kV
 - by stepping down the voltage using transformers
 - The spark over of sphere gaps
 - Potential dividers and oscilloscopes

ELECTROSTATIC VOLTMETERS

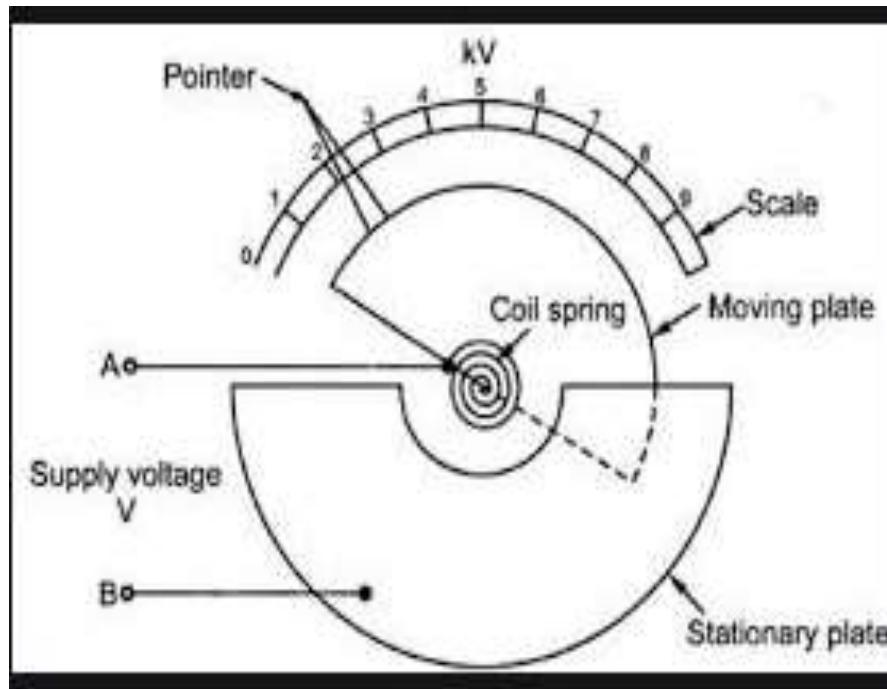
- Use static electrical field to produce the deflecting torque.
- Generally used for the measurement of high voltages .
- The deflecting torque is produced by action of electric field on charged conductors.
- Such instruments are essentially voltmeters, but they may be used with the help of external components, to measure current and power.
- Mainly used in the laboratory is for measurement of high voltages.

TYPES OF ELECTROSTATIC VOLTMETERS

- Quadrant type(10 to 20kV)
- Attracted disc type(above 20kV)

ATTRACTED DISC ELECTROSTATIC VOLTMETER

- The attracted disc type instruments are generally used for the measurement of voltages above 20 kV

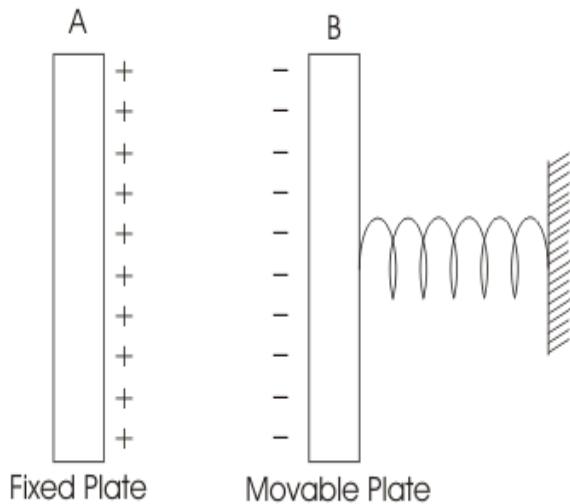


ATTRACTED DISC ELECTROSTATIC VOLTMETER

➤ Construction and working

- The system consists of two plates such that one plate can move freely while other is fixed.
- Both the plates are perfectly insulated from each other.
- The voltage to be measured is applied across the plates as a supply voltage .
- Due to the supply voltage, electrostatic field gets produced which develops a force of attraction between the two plates.
- Due to the force of attraction, the movable plate gets deflected. In this mechanism the controlling torque is provided by a spring .

FORCE EQUATION OF ELECTROSTATIC TYPE INSTRUMENT



- Consider two plates as shown in the diagram given below.
- Plate A is positively charged and plate B is negatively charged) we have linear motion between the plates.
- The plate A is fixed and plate B is free to move. Let us assume there exists some force F between the two plates at equilibrium when electrostatic force becomes equal to spring force.
- At this point, the electrostatic energy stored in the plates is

$$\frac{1}{2}CV^2$$

FORCE EQUATION OF ELECTROSTATIC TYPE INSTRUMENT

- Now suppose we increase the applied voltage by an amount dV , due to this the plate B moves towards the plate A by a distance dx .
- The work done against the spring force due to displacement of the plate B be $F.dx$.
- The applied voltage is related to current as

$$i = C \frac{dV}{dt} + V \frac{dC}{dt}$$

- From this value of electric current the input energy can be calculated as

$$Vidt = V^2 dC + CVdV$$

FORCE EQUATION OF ELECTROSTATIC TYPE INSTRUMENT

- From this we can calculate the change in the stored energy and that comes out to be

$$\frac{1}{2}V^2dC + CVdV$$

by neglecting the higher order terms that appears in the expression.

- Now applying the principle of energy conservation we have input energy to the system = increase in the stored energy of the system + mechanical work done by the system. From this we can write,

$$V^2dC + CVdV = \frac{1}{2}V^2dC + CVdV + Fdx$$

FORCE EQUATION OF ELECTROSTATIC TYPE INSTRUMENT

- From the above equation the force can be calculated as

$$F = \frac{1}{2}V^2 \frac{dC}{dx}$$

ADVANTAGES AND DISADVANTAGES OF ELECTROSTATIC TYPE INSTRUMENTS

➤ Advantages

- we can measure both AC and DC voltage.
- Power consumption is quite low in these types of instruments as the current drawn by these instruments is quite low.
- We can measure high value of voltage.

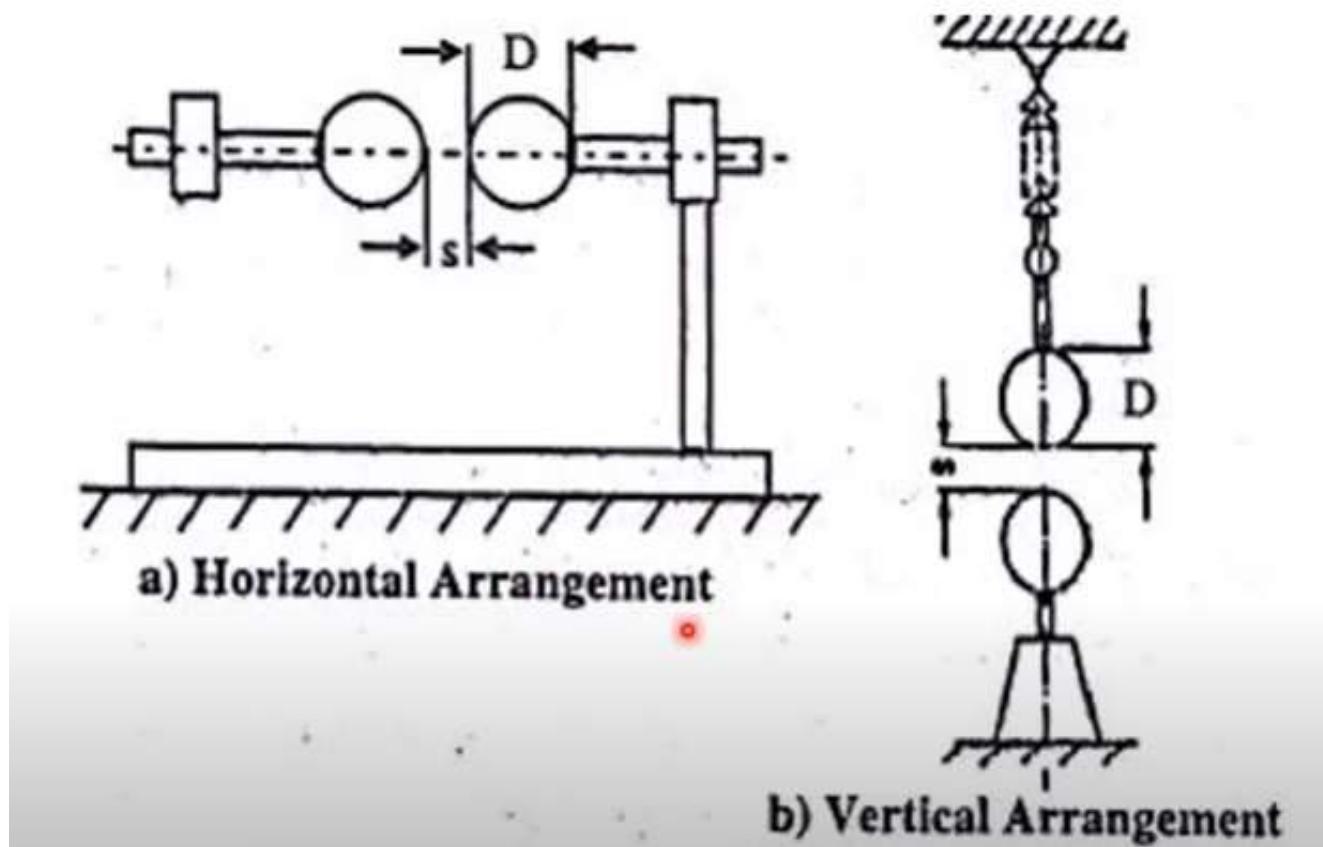
➤ Disadvantages

- These are quite costly as compared to other instruments and also these have large size.
- The scale is not uniform.
- The various operating forces involved are small in magnitude

SPHERE GAPS



SPHERE GAPS



SPHERE GAPS

- A sphere gap consists of an arrangement of two conducting metal spheres separated by a gap usually filled with a gas such as air.
- It is designed to allow an electric spark to pass between the conductors, when the potential difference between the conductors exceeds the breakdown voltage of the gas within the gap.
- Then a spark forms, ionizing the gas and drastically reducing its electrical resistance.
- Sphere gaps are used for voltage measurements of the peak value of either AC, DC or both types of impulse voltages.

SPHERE GAPS

- Working

- A uniform field sphere gap will always have a sparkover voltage within a known tolerance under constant atmospheric conditions.
- Hence a spark gap can be used for measurement of the peak value of the voltage, if the gap distance is known.
- A sparkover voltage of 30 kV (peak) at 1 cm spacing in air at 20°C and 760 ton pressure occurs for a Sphere Gap Measurement or any uniform field gap.
- Sphere Gap Measurement can be arranged either (i) vertically with lower sphere grounded, or (ii) horizontally with both spheres connected to the source voltage or one sphere grounded.

SPHERE GAPS

- In the case of a.c. peak value and d.c. voltage measurements, to know the peak voltage, the gap distance between the sphere gaps are adjusted so that flashover occurs between them.
- From the gap distance, we can understand the peak voltage.
- The flashover voltage for various gap distances and standard diameters of the Sphere Gap Measurement used are given in IS and IEC Standards.

SPHERE GAPS

- Sphere gap chart