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Reg No.:________________ Name:________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Second Semester B.Tech Degree Examination July 2021 (2019 scheme)

Course Code: MAT102

Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS

(2019 Scheme)

FN Session

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

1 Find the unit tangent vector at a point $t_0 = \frac{\pi}{6}$ to the curve $\bar{r}(t) = \cos 3t\,\hat{\imath} + sin3t\,\hat{\jmath} + 3t\,\hat{k}$. (3)

2 Find the directional derivative of the function $\varphi = 3x^2y - y^3z^2$ at $(-1,-2,-1)$in the direction of negative z axis. (3)

3 Using Green's Theorem evaluate $\int_C 2xy\,dx + (x^2 + x)dy$ where C is the unit circle in the positive direction. (3)

4 Determine whether the vector field $\vec{F}(x,y,z) = (y+z)\hat{\imath} - (xz^3)\hat{\jmath} + (x^2 siny)\hat{k}$ is free of sources and sinks. (3)

5 Solve the initial value problem $y'' + y = 0$: $y(0) = 3, y'(0) = 1$. (3)

6 Find the Wronskian corresponding to the solution of $y'' - 3y' + 2y = 0$. (3)

7 Find the Laplace Transform of $sin3t\ cos2t$. (3)

8 Evaluate $L^{-1}\left[\frac{2}{(s+4)^3}\right]$. (3)

9 Find the Fourier cosine transform of the function $f(x) = \begin{cases} k, & 0 < x < a \\ 0, & x > a \end{cases}$ (3)

10 Express $f(x) = \begin{cases} \frac{1}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ as a Fourier sine integral. (3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11 a) Use a line integral to evaluate the work done by the force field $\vec{F} = xy^2\hat{\imath} + xy\,\hat{\jmath}$ along the triangle with vertices (0,0), (2,1) and (0,1) in the positive direction. (7)

b) Use the given information to find the position and velocity vectors of the particle with acceleration $\vec{a}(t) = -\cos t\,\hat{\imath} - \sin t\,\hat{\jmath}$, $\vec{v}(0) = \hat{\imath}$, $\vec{r}(0) = \hat{\jmath}$ (7)

12 a) Evaluate $\int_C (xyz)dx - \cos(yz)\,dy$ where C is the straight line segment from $(1,1,1)$ to $(-2,1,3)$. (7)

b) Show that the vector field $\vec{F}(x,y) = \cos y\,\hat{\imath} - x\sin y\,\hat{\jmath}$ is conservative and find φ such that $\vec{F} = \nabla\varphi$. Hence evaluate $\int_{(0,1)}^{(\pi,0)} \cos y\,dx - x\sin y\,dy$ (7)

Module-II

13 a) Using divergence theorem, evaluate $\iint_\sigma \vec{F}.\hat{n}\,dS$, where $\vec{F}(x,y,z) = (x^2+y)\,\hat{\imath} + z^2\hat{\jmath} + (e^y - z)\hat{k}$ and σ is the surface of rectangular cube bounded by the coordinate planes and the plane $x = 3, y = 1, z = 3$. (7)

b) Use Stoke's Theorem to evaluate the work done by the force field $\vec{F}(x,y,z) = 3z\,\hat{\imath} + 4x\,\hat{\jmath} + 2y\hat{k}$ over the boundary of the paraboloid $z = 4 - x^2 - y^2, z \geq 0$ with upward orientation. (7)

14 a) Let σ be the portion of the surface $z = 1 - x^2 - y^2$ that lies above the xy −plane and σ is the oriented upwards. Find the flux of the vector field $\vec{F}(x,y,z) = x\,\hat{\imath} + y\,\hat{\jmath} + z\hat{k}$ across σ. (7)

b) Find the mass of the lamina that is the portion of the plane $x + y + z = 2$ lying in the first octant where the density function of the surface is $\sigma = xz$. (7)

Module-III

15 a) Solve using the method of undetermined coefficients: $y'' - 4y' + 4y = 4\sin^2 x$. (7)

b) Solve using the method of variation of parameters: $y'' + 4y = \sec 2x$. (7)

16 a) Solve using the method of undetermined coefficients: $y''' + 2y'' - y' - 2y = e^x$. (7)

b) Solve the initial value problem $x^2y'' - 3xy' + 3y = 0, y(1) = 0, y'(1) = 1$. (7)

Module-IV

17 a) Using Laplace Transform solve $y'' + 4y' + 3y = e^{-t}, y(0) = 1, y'(0) = 1$. (7)

b) Using convolution theorem, find the inverse Laplace Transform of $\frac{18s}{(s^2+36)^2}$ (7)

18 a) Use Laplace Transform to solve $y'' + 3y' + 2y = u(t-1)$, $y(0) = 0, y'(0) = 0$ (7)

b) Evaluate $L^{-1}\left[\frac{2s+1}{s^2+2s+5}\right]$ (7)

Module-V

19 a) Find the Fourier integral representation of the function $f(x) = \begin{cases} 1, & if\ |x| < 1 \\ 0, & if\ |x| > 1 \end{cases}$. (7)

b) Find the Fourier transform of $f(x) = e^{-|x|}, -\infty < x < \infty$ (7)

20 a) Represent $f(x) = \begin{cases} \text{Sin}\, x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ as Fourier Cosine Integral. (7)

b) Find the Fourier sine transform of the function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ -1, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$ (7)

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