ENGINEERING PHYSICS

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Engineering Physics

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FOREWORD

It gives me immense pleasure to see the present textbook on "Engineering Physics" which covers almost the entire syllabus taught at undergraduate level at different engineering colleges and institutions throughout India. I complement the authors and appreciate their efforts in bringing out this book written in a very simple language. The text is comprehensive and the explanation of topics is commendable. I understand that this book carries all the elements required for a good presentation.

I have been a student of IIT Kharagpur and later on taught at IIT Delhi. Being a part of the IIT system, I recognise that the rigorous and enriching teaching experience at IITs originating from the interaction with the best engineering students and their strong feedback results in continuous evolution and refinement of the teachers. This spirit is reflected in the comprehensive and in-depth handling of important topics in a very simple manner in this book. I am happy to note that this textbook has been penned down by IITian and hope that it would serve to be a good textbook on the subject. Since this book also covers advanced topics, it will be an important learning resource for the teachers, and those students who wish to develop research skills and pursue higher studies. I hope that the book is well received in the academic world.

Prem Vrat

Professor Prem Vrat
Vice-Chancellor, U.P. Technical University, Lucknow
Founder Director, IIT Roorkee

PREFACE

Physics is a mandatory subject for all engineering students, where almost all the important elements of the subject are covered. Finally, these evolve as different branches of the engineering course. The book entitled Engineering Physics has been written keeping in mind the need of undergraduate students from various engineering and science colleges of all Indian universities. It caters to the complete syllabus for both–Physics-I and Physics-II papers in the first year Engineering Physics course.

The aim of writing this book has been to present the material in a concise and very simple way so that even weak students can grasp the fundamentals. In view of this, every chapter starts with a simple introduction and then related topics are covered with a detailed description along with the help of figures. Particularly the solved problems (compiled from University Question Papers) are at the end of each chapter. These problems are not merely numerical; many of them focus on reasoning and require thoughtful analysis. Finally, the chapters carry unsolved questions based on which the students would be able to test their knowledge as to what they have acquired after going through various chapters. A chapter-end summary and list of important formulae will be helpful to students for a quick review during examinations. The rich pedagogy consists of solved examples (450), objective-type questions (230), short-answer questions (224) and practice problems (617). The manuscript has been formulated in such a way that students shall grasp the subject easily and save their time as well. Since the complete syllabus is covered in a single book, it would be highly convenient to both.

The manuscript contains 22 chapters which have been prepared as per the syllabus taught in various colleges and institutions. In particular, the manuscript discusses optics, lasers, holography, fibre optics, waves, acoustics of buildings, electromagnetism, theory of relativity, nuclear physics, solid state physics, quantum physics, magnetic properties of solids, superconductivity, photoconductivity and photovoltaic, X-rays and nanophysics in a systematic manner. We have discussed advanced topics such as laser cooling, Bose-Einstein condensation, scanning electron microscope (SEM), scanning tunnelling microscope (STM), controlled fusion including plasma, Lawson criterion, inertial confinement fusion (ICF), plasma based accelerators, namely, plasma wake field accelerator, plasma beat wave accelerator, laser wake field accelerator and self-modulated laser wake field accelerator, and nanophysics with special emphasis on properties of nanoparticles, carbon nanotubes, synthesis of nanoparticles and applications of nanotechnology. These will be of interest to the teachers who are involved in teaching postgraduate courses at the universities and the students who opt for higher studies and research as their career. Moreover, a series of review questions and problems at the end of each chapter together with the solved questions would serve as a question bank for the students preparing for various competitive examinations. They will get an opportunity to learn the subject and test their knowledge on the same platform.

The structuring of the book provides in-depth coverage of all topics. **Chapter 1** discusses Interference. **Chapter 2** is on Diffraction. **Chapter 3** is devoted to Polarization. Coherence and Lasers are described in **Chapter 4**. **Chapter 5** discusses Fibre Optics and its Applications, while Electron Optics is dealt with in **Chapter 6**. **Chapter 7** describes Waves and Oscillations. **Chapter 8** is on Sound Waves and Acoustics. **Chapter 9** is on Dielectrics. Electromagnetic Wave Propagation is described in **Chapter 10**. **Chapter 11** discusses the Theory of Relativity.

Chapter 12 is devoted to Nuclear Physics. Crystal Structure is described in **Chapter 13**. **Chapter 14** deals with the Development of Quantum Physics, while **Chapter 15** is on Quantum Mechanics. **Chapter 16** discusses Free Electron Theory. Band Theory of Solids is explained in **Chapter 17**. **Chapter 18** describes the Magnetic Properties of Solids. **Chapter 19** is on Superconductivity. **Chapter 20** explains X-rays in detail while **Chapter 21** is on Photoconductivity and Photovoltaics. Finally, **Chapter 22** discusses Nanophysics in great detail. The manuscript has been organised such that it provides a link between different topics of a chapter. In order to make it simpler, all the necessary mathematical steps have been given and the physical feature of the mathematical expressions is discussed as and when required.

The exhaustive OLC supplements of the book can be accessed at http://www.mhhe.com/malik/ep and contain the following:

For Instructors

- Solution Manual
- Chapter-wise PowerPoint slides with diagrams and notes for effective lecture presentations

For Students

- A sample chapter
- Link to reference material
- Solved Model Question Paper
- Answers to objective type questions given in the book.

We would like to thank the entire team of Tata McGrawHill Education specifically Vibha Mahajan, Shalini Jha, Tina Jajoriya, Dipika Dey, Sohini Mukherji, Priyanka Negi and Baldev Raj for bringing out this book in a very short time span. The reviewers of the book also deserve a special mention for taking out time to review the book. Their names are given below.

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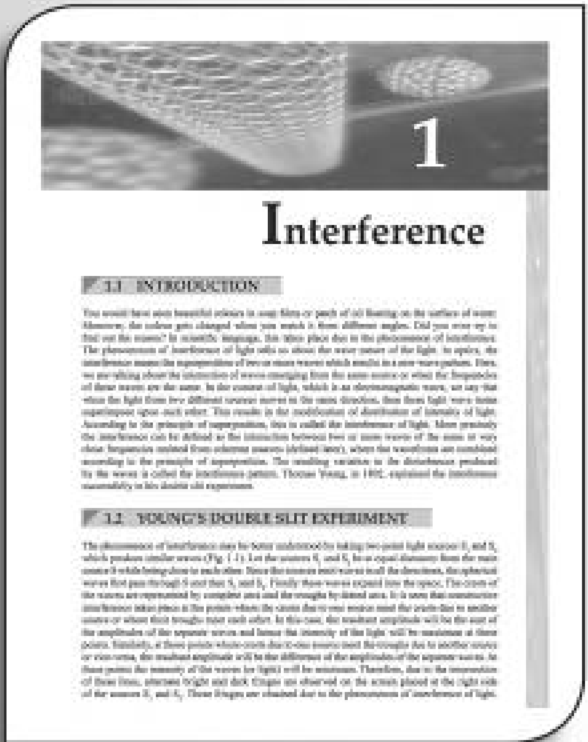
WALKTHROUGH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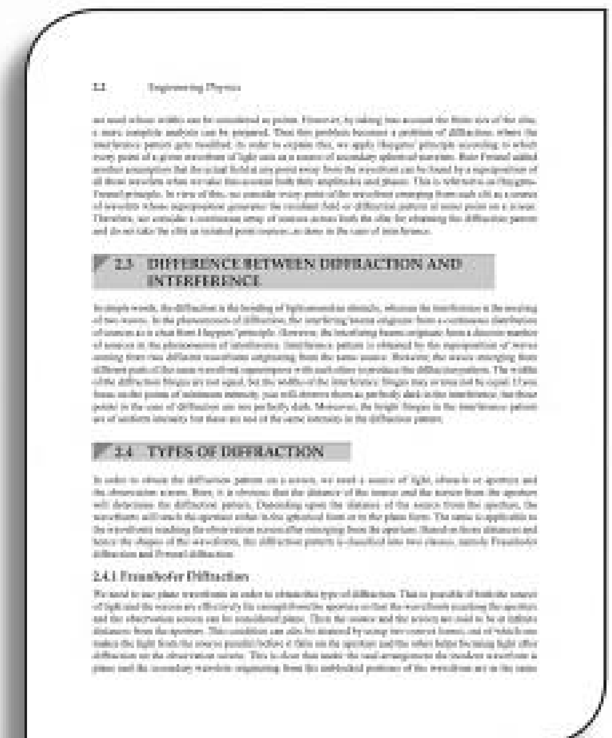
Interference

1.1 INTRODUCTION

1.2 YOUNG'S DOUBLE SLIT EXPERIMENT

Introduction

Each chapter begins with an Introduction that gives a brief summary of the background and contents of the chapter.


2.3 DIFFERENCE BETWEEN DIFFRACTION AND INTERFERENCE

2.4 TYPES OF DIFFRACTION

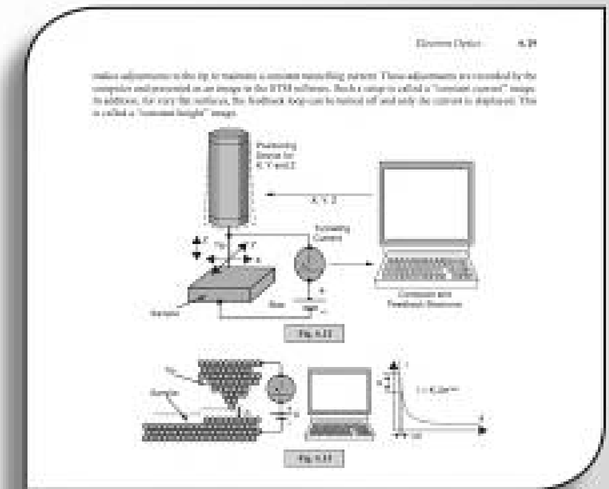
2.4.1 Fraunhofer Diffraction

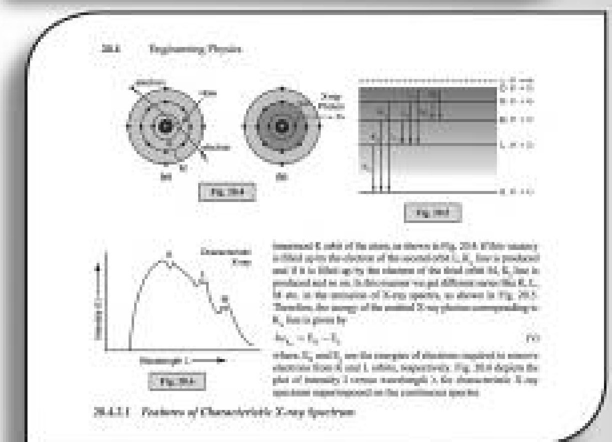
Sections and Sub-sections

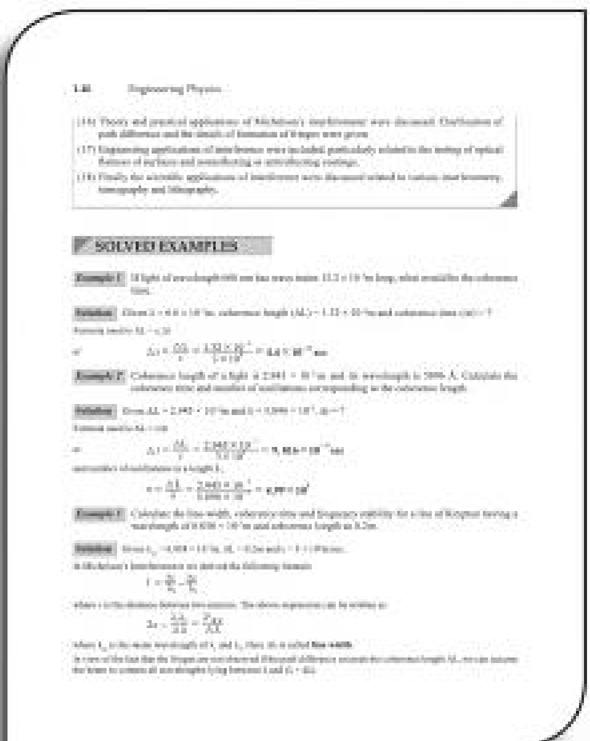
Each chapter has been neatly divided into relevant sections and sub-sections so that the text material is presented in a logical progression of concepts and ideas.

Illustrations

Illustrations are an important tool in the presentation of text material. The reader of the text would come across ample number of diagrams/illustrations provided in each chapter to effectively discuss the concepts of engineering physics.

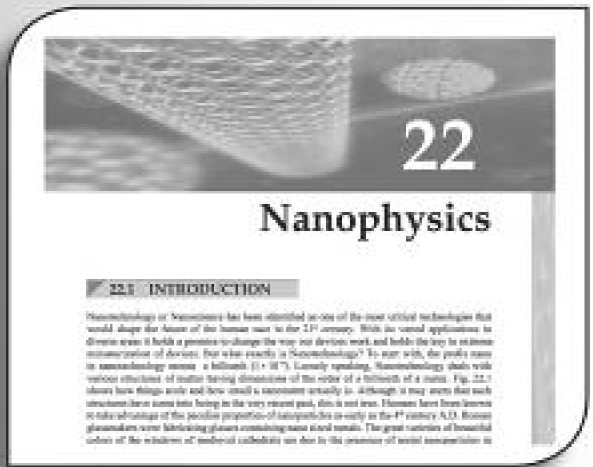






Solved Examples

Solved Examples (450) are provided in sufficient number in each chapter and at appropriate locations, to aid in understanding of the text material.



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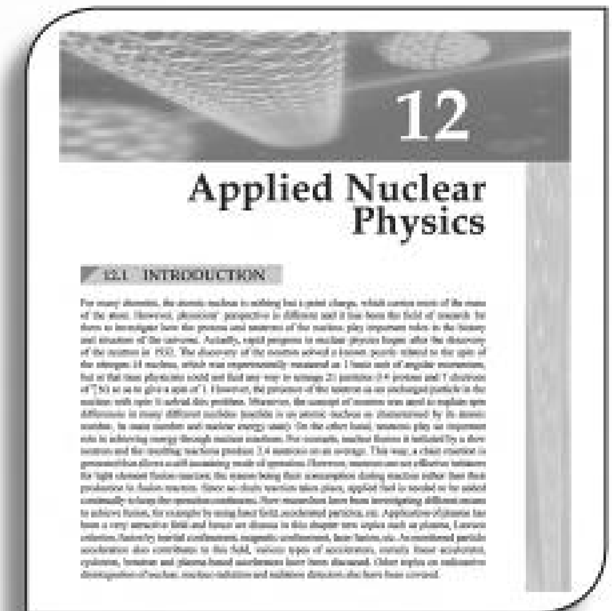
Nanophysics

22.1 INTRODUCTION

Advanced Topics

Advanced Topics like Nanophysics, an essential part of the syllabus, are covered extensively.



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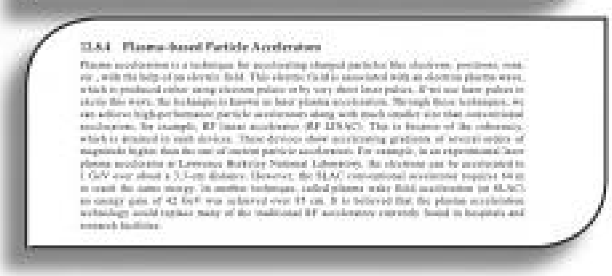
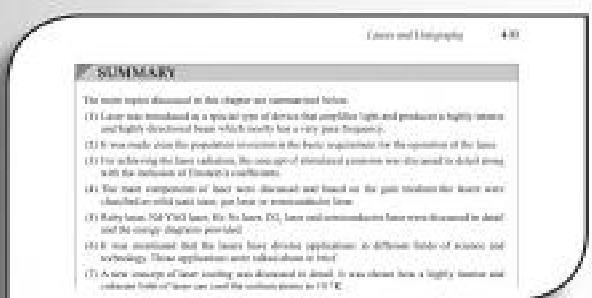
Applied Nuclear Physics

12.1 INTRODUCTION



Applications

Applications like Controlled Fusion Reaction, Particle Accelerators (Basics of Plasma) are explained in detail with relevant topics.



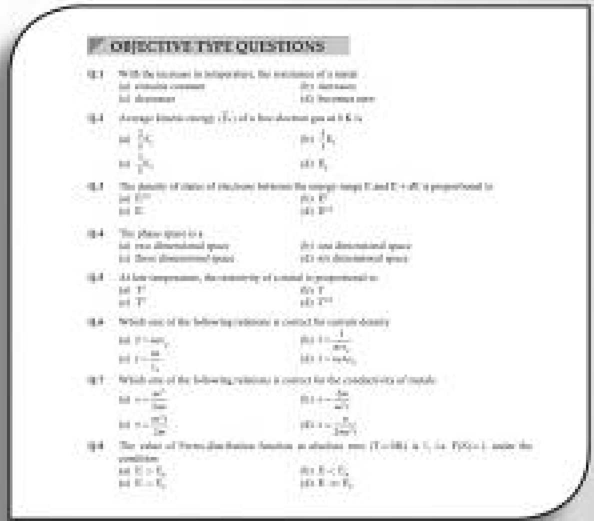
SUMMARY

Summary

A bulleted summary gives the essence of each important topic discussed in the chapter for a quick recap.

Objective-type questions

Objective-type questions enable the user to have a clear comprehension of the subject matter. Answers to all the objective questions are provided in the online learning centre of the book.


OBJECTIVE TYPE QUESTIONS

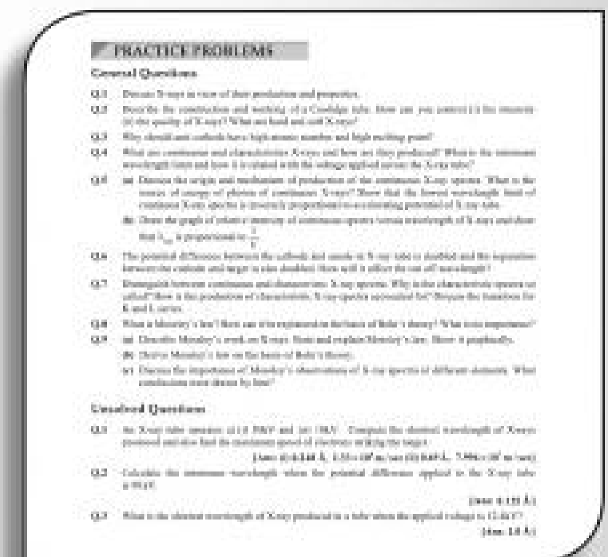

SHORT-ANSWER QUESTIONS

Questions

A set of (850) questions are given as exercise to the students. Further divided into Short-answer Questions and Long-Answer Questions and are very helpful to teachers in setting class work, assignments, quizzes and examinations. In some chapters, numerical problems with answers, under the heading 'Unsolved Questions', are also given. Readers can assess their knowledge by answering the objective-type questions and short-answer questions given at the end of the book.

Practice Problems

Practice problems, in the category of general and unsolved questions provide an opportunity to students to reinforce his or her learning and gain confidence.


PRACTICE PROBLEMS

General Questions

Unsolved Questions

Roadmap to the Model Syllabus

Interference, Diffraction, Polarisation

Chapter 1: Interference
Chapter 2: Diffraction
Chapter 3: Polarisation
Chapter 20: X-Rays

Superconductivity

Chapter 19: Superconductivity

Super Conducting Materials

Chapter 21: Photoconductivity and Photovoltaics

Relativistic Mechanics

Chapter 11: Theory of Relativity

Solid State Physics

Chapter 17: Band theory of Solids
Chapter 18: Magnetic Properties of Solids

Sound Waves

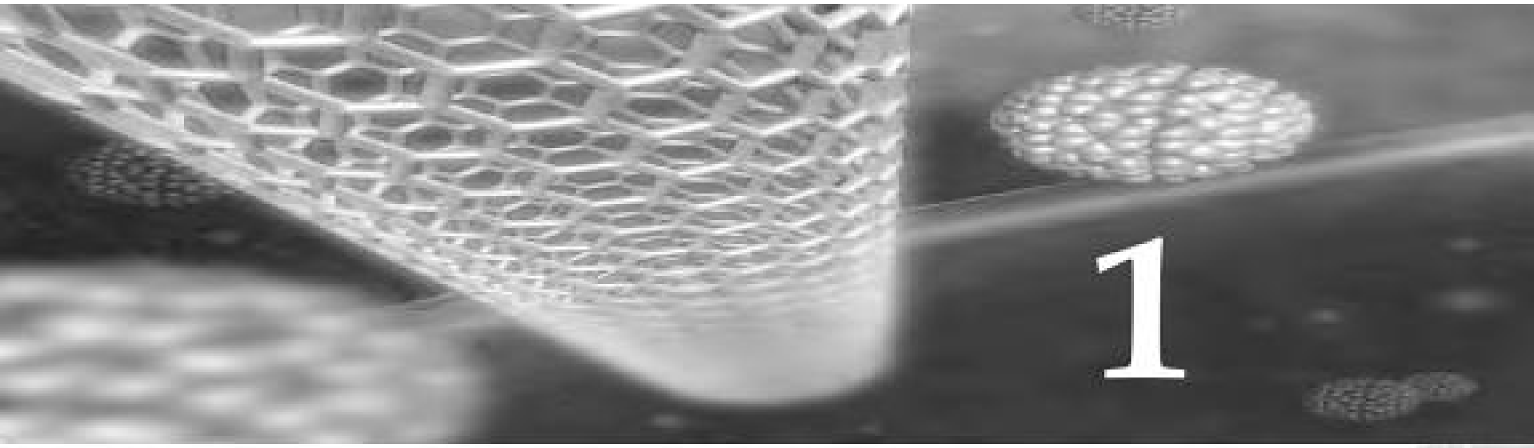
Chapter 7: Waves and Oscillations

Electricity and Magnetism

Chapter 10: Electromagnetic Wave Propagation

Dielectric and Magnetic Properties of Materials

Chapter 9: Dielectrics



1

Interference

1.1 INTRODUCTION

You would have seen beautiful colours in soap films or patch of oil floating on the surface of water. Moreover, the colour gets changed when you watch it from different angles. Did you ever try to find out the reason? In scientific language, this takes place due to the phenomenon of interference. The phenomenon of interference of light tells us about the wave nature of the light. In optics, the interference means the superposition of two or more waves which results in a new wave pattern. Here, we are talking about the interaction of waves emerging from the same source or when the frequencies of these waves are the same. In the context of light, which is an electromagnetic wave, we say that when the light from two different sources moves in the same direction, then these light wave trains superimpose upon each other. This results in the modification of distribution of intensity of light. According to the principle of superposition, this is called the interference of light. More precisely the interference can be defined as the interaction between two or more waves of the same or very close frequencies emitted from coherent sources (defined later), where the wavefronts are combined according to the principle of superposition. The resulting variation in the disturbances produced by the waves is called the interference pattern. Thomas Young, in 1802, explained the interference successfully in his double slit experiment.

1.2 YOUNG'S DOUBLE SLIT EXPERIMENT

The phenomenon of interference may be better understood by taking two point light sources S_1 and S_2 which produce similar waves (Fig. 1.1). Let the sources S_1 and S_2 be at equal distances from the main source S while being close to each other. Since the sources emit waves in all the directions, the spherical waves first pass through S and then S_1 and S_2. Finally these waves expand into the space. The crests of the waves are represented by complete arcs and the troughs by dotted arcs. It is seen that constructive interference takes place at the points where the crests due to one source meet the crests due to another source or where their troughs meet each other. In this case, the resultant amplitude will be the sum of the amplitudes of the separate waves and hence the intensity of the light will be maximum at these points. Similarly, at those points where crests due to one source meet the troughs due to another source or vice-versa, the resultant amplitude will be the difference of the amplitudes of the separate waves. At these points the intensity of the waves (or light) will be minimum. Therefore, due to the intersection of these lines, alternate bright and dark fringes are observed on the screen placed at the right side of the sources S_1 and S_2. These fringes are obtained due to the phenomenon of interference of light.

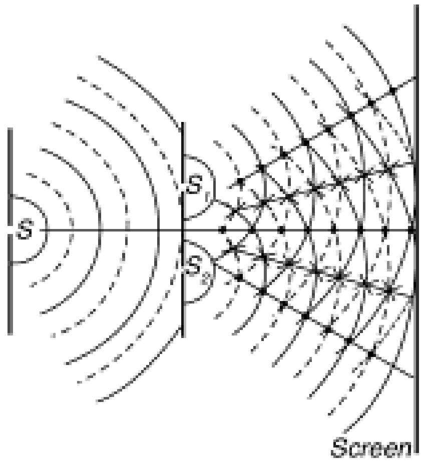


Figure 1.1

1.3 CONCEPT OF WAVES AND HUYGENS' PRINCIPLE

A wave is a disturbance that propagates through space and time, usually with the transference of energy from one point to another without any particle of the medium being permanently displaced. Under this situation, the particles only oscillate about their equilibrium positions. If the oscillations of the particles are in the direction of wave propagation, then the wave is called longitudinal wave. However, if these oscillations take place in perpendicular direction with the direction of wave propagation, the wave is said to be transverse in nature. In electromagnetic waves, such as light waves, it is the changes in electric and magnetic fields which represent the wave disturbance. The progress of the wave is described by the passage of a waveform through the medium with a certain velocity called the phase velocity or wave velocity. However, the energy is transferred at the group velocity of the waves making the waveform.

The wave theory of the light was proposed in 1678 by *Huygens*, a Dutch scientist. On the basis of his wave theory, he explained satisfactorily the phenomena of reflections, refraction etc. In the beginning, Huygens' supposed that these waves are longitudinal waves but later he came to know that these waves are transverse in nature. Huygens' gave a hypothesis for geometrical construction of the position of a common wavefront at any instant when the propagation of waves takes place in a medium. The wavefront is an imaginary surface joining the points of constant phase in a wave propagated through the medium. The way in which the wavefront is propagated further in the medium is given by Huygens' principle. This principle is based on the following assumptions:

(i) Each point on the given wavefront acts as a source of secondary wavelets.

(ii) The secondary wavelets from each point travel through space in all the directions with velocity of light.

(iii) A surface touching the secondary wavelets tangentially in the forward direction at any given time constructs the new wavefront at that instant. This is known as secondary wavefront.

In order to demonstrate the Huygens' principle, we consider the propagation of a spherical wavefront (Fig. 1.2a) or plane wavefront (Fig. 1.2b) in an isotropic (uniform) medium (for example, ether) emerging from a source of light S. At any time, suppose PQ is a section of the primary wavefront drawn from the source S. To find the position of the wavefront after an interval $\Rightarrow t$, we take points 1, 2, 3, … on the primary wavefront

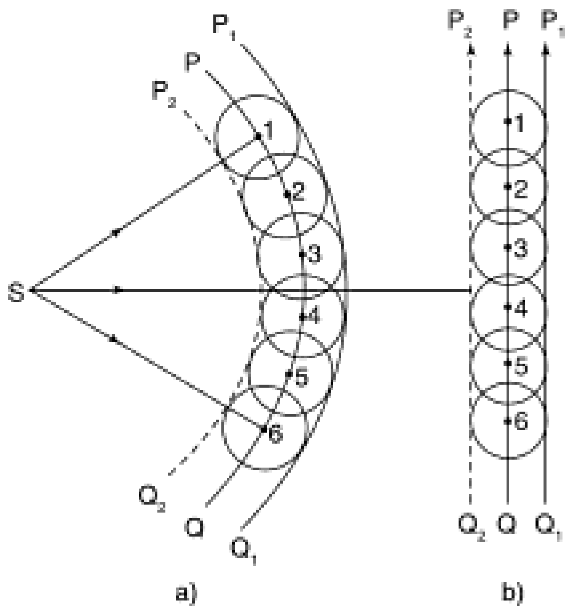


Figure 1.2

PQ. As per Huygens' principle, these points act as the source of secondary wavelets. Taking each point as the centre, we draw spheres of radii ct, where c is the speed of light. These spherical surfaces represent the position of secondary wavelets at time t. Further, we draw a surface P_1Q_1 that touches tangentially all these secondary wavelets in the forward direction. This surface P_1Q_1 is the secondary wavefront. Another surface P_2Q_2 in the backward direction is not called the secondary wavefront as there is no backward flow of the energy during the propagation of the light waves.

1.4 PHASE DIFFERENCE AND PATH DIFFERENCE

As mentioned, the interference pattern is obtained when the two or more waves superimpose each other. In order to understand this pattern it is very important to know about the path and phase differences between the interfering waves.

1.4.1 Phase Difference

Two waves that have the same frequencies and different phases are known to have a phase difference and are said to be out of phase with each other. If the phase difference is 180°, then the two waves are said to be in antiphase and if it is 0°, then they are in phase as shown in Fig. 1.3. If the two interfering waves meet at a point where they are in antiphase, then the destructive interference occurs. However, if these two waves meet at a point where they are in the same phase, then the constructive interference takes place.

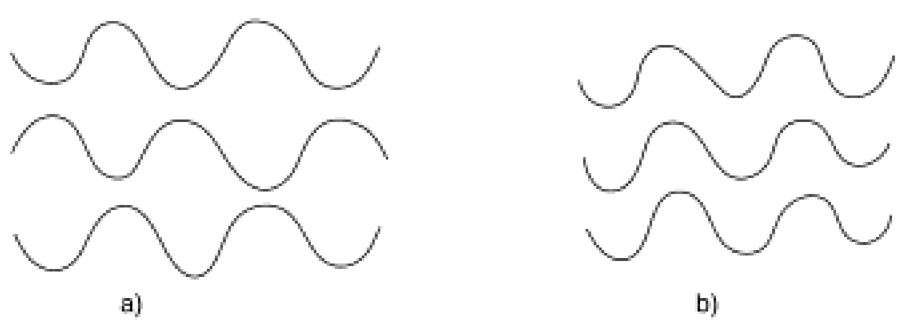


Figure 1.3

1.4.2 Path Difference

In Fig. 1.4, while the two wave crests are traveling a different distance from their sources, they meet at a point P in such a way that a crest meets a crest. For this particular location on the pattern, the difference in distance traveled is known as **path difference**.

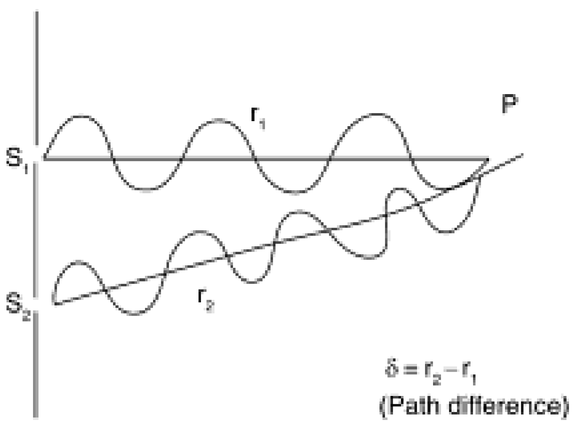


Figure 1.4

1.4.3 Relation between Path Difference and Phase Difference

It is clear from the positions of crests or troughs of the waves that if the path difference between the two waves is equal to the wavelength λ, the corresponding phase difference is 2π (180^0). Suppose for a path difference of δ the corresponding phase difference is ϕ. Then it is clear that

$$\phi = \frac{2\pi}{\lambda}\delta$$

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{Path difference} \qquad \text{(i)}$$

The above can be made clearer with the help of Fig. 1.4, where two sources of waves S_1 and S_2 are shown. The wavelength of these sources is λ and the sources are in phase at S_1 and S_2. The frequencies of both the waves are taken to be the same as f. Therefore, the angular frequency $\omega = 2\pi f$. They travel at the same speed and the propagation constant for them is $k = \frac{2\pi}{\lambda}$. We can write the wave equations for both the waves at point P as

$$y_1 = a\cos(\omega t - kr_1) \text{ for the wave emerging from source } S_1 \text{ and}$$

$$y_2 = a\cos(\omega t - kr_2) \text{ for the wave emerging from source } S_2$$

Here $(\omega t - kr_1)$ is the phase ϕ_1 and $(\omega t - kr_2)$ is the phase ϕ_2. Therefore, the phase difference between them is $\phi_1 - \phi_2$, given by $\phi_1 - \phi_2 = \omega t - kr_1 - \omega t + kr_2 = k(r_2 - r_1)$.

Using Eq.(i) and $k = \frac{2\pi}{\lambda}$, the path difference is obtained as

$$\text{Path difference } \delta = r_2 - r_1.$$

1.5 COHERENCE

Coherence is a property of waves that helps getting stationary interference, i.e. the interference which is temporally and spatially constant. During interference the waves add constructively or subtract destructively, depending on their relative phase. Two waves are said to be coherent if they have a constant relative phase.

that exists between the phases of the wave measured at different points. The coherence of a wave depends on the characteristics of its source.

1.5.1 Temporal Coherence

Temporal coherence is a measure of the correlation between the phases of a wave (light) at different points along the direction of wave propagation. If the phase difference of the wave crossing the two points lying along the direction of wave propagation is independent of time, then the wave is said to have temporal coherence. Temporal coherence is also known as **longitudinal coherence**. This tells us how monochromatic a source is. In Fig. 1.5a, a wave traveling along the positive x-direction is shown, where two points A and B are lying on the x-axis. Let the phases of the wave at these points at any instant t be ϕ_A and ϕ_B, respectively, and at a later time t′ they be ϕ_A' and ϕ_B'. Under this situation, if the phase difference $\phi_B - \phi_A = \phi_B' - \phi_A'$, then the wave is said to have temporal coherence.

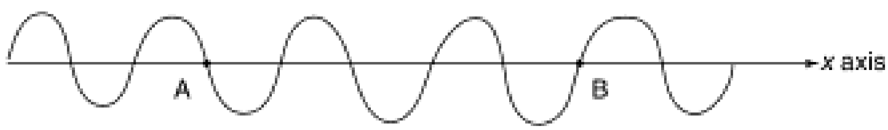


Figure 1.5a

1.5.2 Spatial Coherence

Spatial coherence is a measure of the correlation between the phases of a wave (light) at different points transverse to the direction of propagation. If the phase difference of the waves crossing the two points lying on a plane perpendicular to the direction of wave propagation is independent of time, then the wave is said to have spatial coherence. This tells us how uniform the phase of the wavefront is. In Fig. 1.5b, a wave traveling along the positive x-direction is shown, where PQRS is a transverse plane and A and B are the two points situated on this plane within the waveforms. Let the waves crossing these points at any time t have the same phase ϕ and at a later time t′ the phases of the waves are again the same but equal to ϕ'. Under this situation, the waves are said to have spatial coherence.

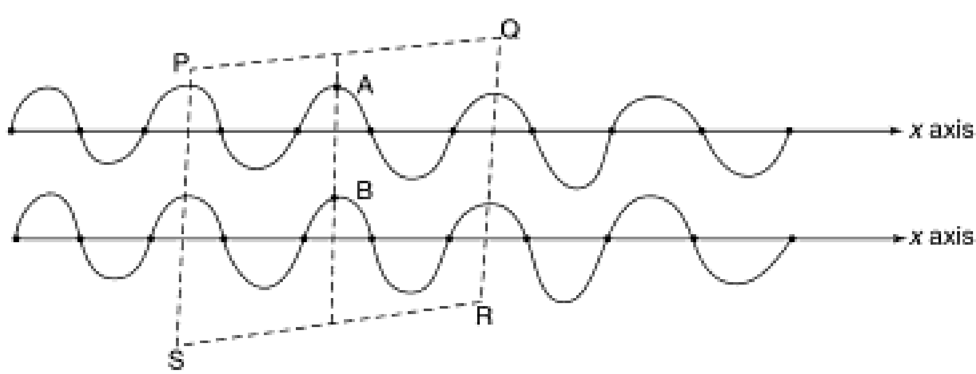


Figure 1.5b

1.5.3 Coherence Time and Coherence Length

A monochromatic source of light emits radiation of a single frequency (or wavelength). In practice, however, even the best source of light emits radiations with a finite range of wavelengths. For a single frequency wave the time interval over which the phase remains constant is called the **coherence time**. The coherence time is generally represented by Δt. In a monochromatic sinusoidal wave the coherence time is infinity because the phase remains constant throughout. However, practially the coherence time exists and the distance traveled by the light pulses during this coherence time is known as coherence length ΔL. The coherence length is also called the **spatial interval**, which is the length over which the phase of the wave remains constant. The coherence length and coherence time are related to each other according to the following formula

$$\Delta L = c\Delta t$$

1.6 COHERENT SOURCES

Two sources of light are said to be coherent, if they emit waves of the same frequency (or wavelength), nearly the same amplitude and maintain a constant phase difference between them. *Laser* is a good example of coherent source. In actual practice it is not possible to have two independent sources which are coherent. This can be explained as follows. A source of light consists of large number of atoms. According to the atomic theory, each atom consists of a central nucleus and the electrons revolve around the nucleus in different orbits. When an atom gets sufficient energy by any means, its electrons jump from lower energy level to higher energy level. This state of an atom is called an **excited state**. The electron lives in this state only for about 10^{-8} seconds. After this interval of time the electrons fall back to the inner orbits. During this process, the atoms radiate energy in the form of light. Out of the large number of atoms some of them emit light at any instant of time and at the next instant other atoms do so and so on. This results in the emission of light waves with different phases. So this is clear that it is difficult to get coherent light from different parts of the same source (Fig. 1.6). Therefore, two independent sources of light can never act as coherent sources.

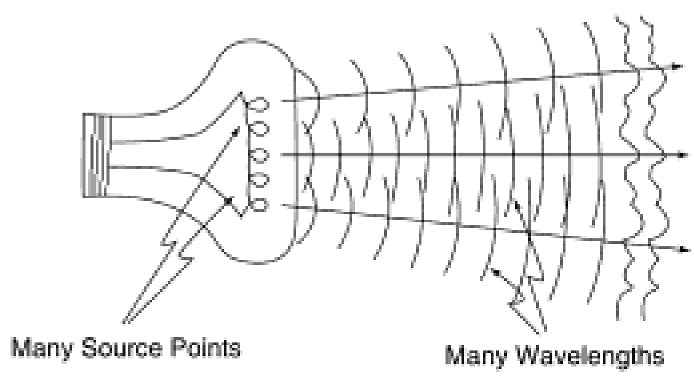


Figure 1.6

1.6.1 Production of Coherent Light from Incoherent Sources

An ordinary light bulb is an example of an incoherent source. We can produce coherent light from such incoherent source, though we will have to throw away a lot of the light. If we spatially filter the light coming from such source, we can increase the spatial coherence (Fig. 1.7). Further, spectrally filtering of the light increases the temporal coherence. This way we can produce the coherent light from the incoherent source.

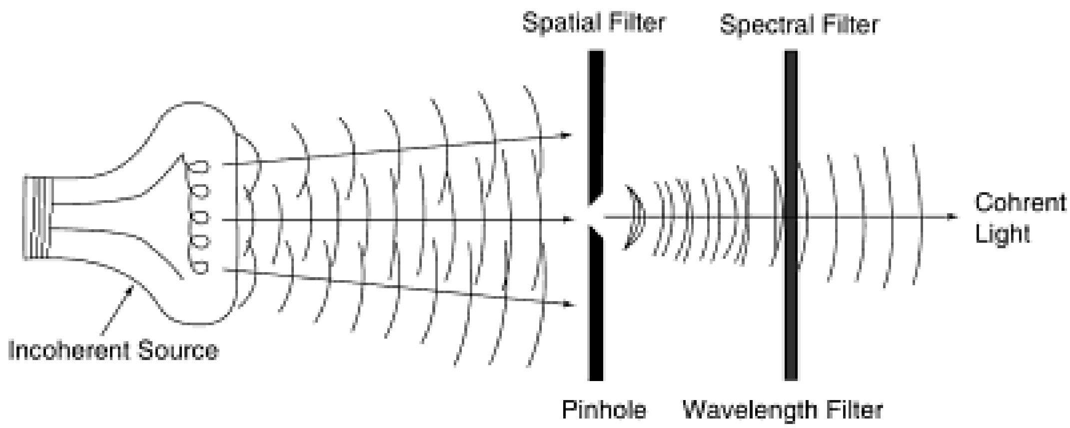


Figure 1.7

1.7 ANALYTICAL TREATMENT OF INTERFERENCE

Let us consider the superposition of two waves of same frequency ω and a constant phase difference ϕ traveling in the same direction. Their amplitudes are taken as a_1 and a_2, respectively. The displacement due to one wave at any instant is given by

$$\psi_1 = a_1 \sin \omega t \tag{i}$$

and the displacement due to another wave at the same instant is given by

$$\psi_2 = a_2 \sin (\omega t + \phi) \tag{ii}$$

According to the principle of superposition, the resultant displacement (ψ_R) is given by

$$\psi_R = \psi_1 + \psi_2 \tag{iii}$$

$$= a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

$$= (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t \tag{iv}$$

Assuming $$a_1 + a_2 \cos \phi = \mathrm{A} \cos \theta \tag{v}$$

$$a_2 \sin \phi = \mathrm{A} \sin \theta \tag{vi}$$

We obtain using Eq. (iv) – (vi)

$$\psi_R = \mathrm{A} \sin (\omega t + \theta) \tag{vii}$$

On squaring and adding Eqs. (v) and (vi), we have

$$\mathrm{A}^2 (\sin^2\theta + \cos^2 \theta)\ a_2^2 \sin^2 \phi + a_1^2 + 2a_1a^2 \cos \phi + a_2^2 \cos^2\phi$$

$$\mathrm{A}^2 = a_1^2 + 2a_1a_2 \cos \phi \tag{viii}$$

The resultant intensity is therefore given by

$$\mathrm{I} = \mathrm{A}^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \phi \tag{ix}$$

The angle θ can be calculated from Eqs. (v) and (vi) as

$$\tan\theta = \frac{a_2 \sin\phi}{a_1 + a_2\cos\phi} \tag{x}$$

1.7.1 Condition for Constructive Interference. It is clear from Eq. (ix) that the intensity, I will be maximum at points where the values of cos ϕ are +1, i.e, phase difference ϕ be $2n\pi$, with $n = 0, 1, 2, 3, \ldots$. Then the maximum intensity is obtained from Eq. (ix) as

$$I_{max} = (a_1 + a_2)^2 \tag{xi}$$

In other words, the intensity will be maximum when the phase difference is an integral multiple of 2π. In this case,

$$I_{max} > a_1^2 + a_2^2$$

Thus, the resultant intensity will be greater than the sum of the individual intensities of the waves.

If $a_1 = a_2 = a$, then

$$I_{max} = 4a^2$$

1.7.2 Condition for Destructive Interference: It is clear from Eq. (ix) that the intensity I will be minimum at points where cos $\phi = -1$. i.e., where phase difference $\phi = (2n + 1)\pi$, with $n = 0, 1, 2, 3, \ldots$. Then Eq. (ix) gives

$$I_{min} = (a_1 - a_2)^2 \tag{xii}$$

Therefore, it is clear that in destructive interference the intensity will be minimum when the phase difference ϕ is an odd multiple of π.

If $a_1 = a_2$, then $I_{min} = 0$

If $a_1 \neq a_2$, then $I_{min} \neq 0$

$$I_{min} < a_1^2 + a_2^2$$

Thus, in the case of destructive interference the resultant intensity will be less than the sum of the individual intensities of the waves.

Fig. 1.8 represents the intensity variation with phase differences ϕ graphically (for $a_1 = a_2 = a$).

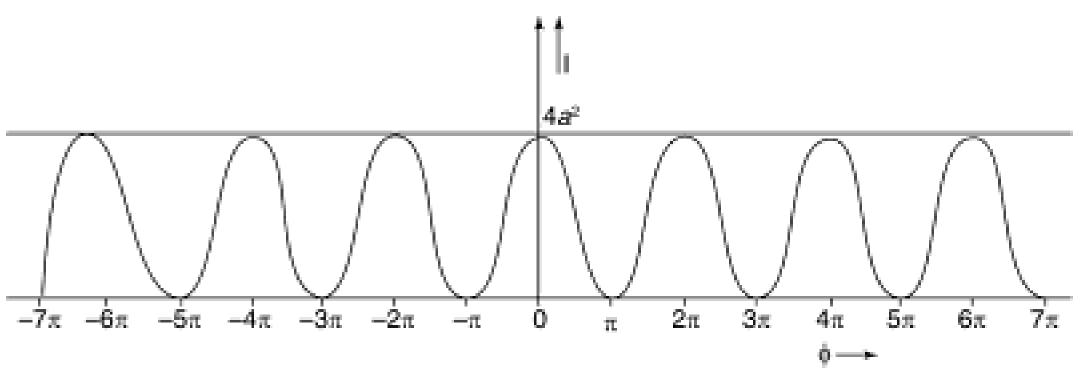


Figure 1.8

1.7.3 Conservation of Energy The resultant intensity due to the interference of two waves $a_1 = \sin \omega t$ and $a_2 = \sin (\omega t + \phi)$ is given by Eq. (ix), reproduced below

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos \phi$$

$$\therefore I_{max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$$

$$\text{and } I_{min} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$$

If $a_1 = a_2 = a$ then

$$I_{max} = 4a^2 \text{ and } I_{min} = 0$$

Therefore, average intensity (I_{av}) will be obtained as

$$I_{av} = 2a^2$$

For unequal amplitudes a_1 and a_2 the average intensity would be $(a_1^2 + a_2^2)$. Thus, in interference only some part of energy is transferred from the position of minima to the position of maxima, and the average intensity or energy remains constant. This shows that the phenomenon of interference is in accordance with the law of conservation of energy.

1.8 CONDITIONS FOR SUSTAINED INTERFERENCE

Sustained interference means a constant interference of light waves. In order to obtain such interference, the following conditions must be satisfied

(i) The two sources should emit waves of the same frequency (wavelength). If it is not so, then the positions of maxima and minima will change with time.

(ii) The waves from the two sources should propagate along the same direction with equal speeds.

(iii) The phase difference between the two interfering waves should be zero or it should remain constant. It means the sources emitting these waves must be coherent.

(iv) The two coherent sources should be very close to each other, otherwise the interference fringes will be very close to each other due to the large path difference between the interfering waves. For the large separation of the sources, the fringes may even overlap and the maxima and minima will not appear distinctly.

(v) A reasonable distance between the sources and screen should be kept, as the maxima and minima appear quite close if this distance is smaller. On the other hand, the large distance of the screen reduces the intensity.

(vi) In order to obtain distinct and clear maxima and minima, the amplitudes of the two interfering waves must be equal or nearly equal.

(vii) If the source is not narrow, it may act as a multi source. This will lead to a number of interference patterns. Therefore, the coherent sources must be narrow.

(viii) In order to obtain the pattern with constant fringe width and good intensity fringes, the sources should be monochromatic and the background should be dark.

1.8.1 Condition of Relative Phase Shift

This is regarding the introduction of additional phase change between the interfering waves when they emerge after reflecting from two different surfaces. In most of the situations, the reflection takes place when the beam

When the reflection occurs with light going from a lower index toward a higher index, the condition is called **internal reflection**. However, when the reflection occurs for light going from a higher index toward a lower index, the condition is referred to as **external reflection**. A relative phase shift of π takes place between the externally and internally reflected beams so that an additional path difference of $\lambda/2$ is introduced between the two beams. If both the interfering beams get either internally or externally reflected, no phase shift takes place between them.

1.9 MULTIPLE BEAM SUPERPOSITION

In Section 1.7, we have given theoretical analysis of the interference due to the superposition of two waves of the same frequency and the constant phase difference. The intensity of the interference pattern showed its dependence on the amplitudes of the interfering waves. However, now we consider a large number of waves of the same frequency and amplitude, which propagate in the same direction. The amount by which each wave train is ahead or lags behind the other is a matter of chance. Based on the amplitude and intensity of the resultant wave, we can examine the interference. We assume n number of wave trains whose individual amplitudes are equal ($= a$, say). The amplitude of the resultant wave can be understood as the amplitude of motion of a particle undergoing n simple harmonic motions (each of amplitude a) at once. In this case, if all these motions are in the same phase, the resultant wave will have an amplitude equal to na and the intensity would be n^2a^2, i.e. n^2 times that of one wave. However, in our case, the phases are distributed purely at random, as shown in Fig. 1.9 as per graphical method of compounding amplitudes. Here, the phases ϕ_1, ϕ_2, ϕ_3, … take arbitrary values between 0 and 2π. The intensity due to the superposition of such waves can be calculated by the square of the resultant amplitude A. In order to find A^2, we should square the sum of the projections of all vectors a along the x-direction and add it to the square of the corresponding sum along the y-direction. The summation of projections along x-direction are given by the expression below

$$a(\cos\phi_1 + \cos\phi_2 + \cos\phi_3 + \ldots + \cos\phi_n).$$

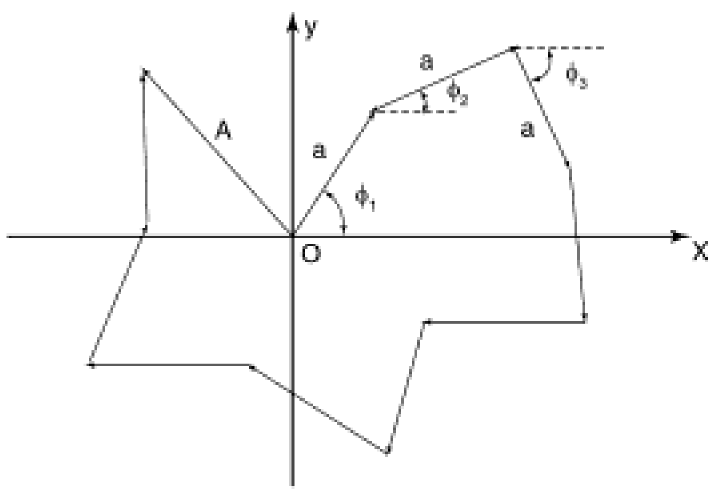


Figure 1.9

The square of quantity in the parentheses gives the terms of the form $\cos^2\phi_1$, $2\cos\phi_1\cos\phi_2$, etc. It is seen that the sum of these cross product terms increases approximately in proportion to number n. So we do not obtain find their average effect in computing the intensity in any physical problem. Under this situation, it is safe to conclude that these cross product terms will average to zero. So we consider only the $\cos^2\phi$ terms. Similarly, for the y projections of the vectors we obtain $\sin^2\phi$ terms. With this we have

$$I \approx A^2 = a^2(\cos^2\phi_1 + \cos^2\phi_2 + \cos^2\phi_3 + \dots + \cos^2\phi_n)$$
$$+ a^2(\sin^2\phi_1 + \sin^2\phi_2 + \sin^2\phi_3 + \dots + \sin^2\phi_n).$$

Using the identity $\sin^2\phi_p + \cos^2\phi_p = 1$, the above expression reduces to $I \approx a^2 \times n$.

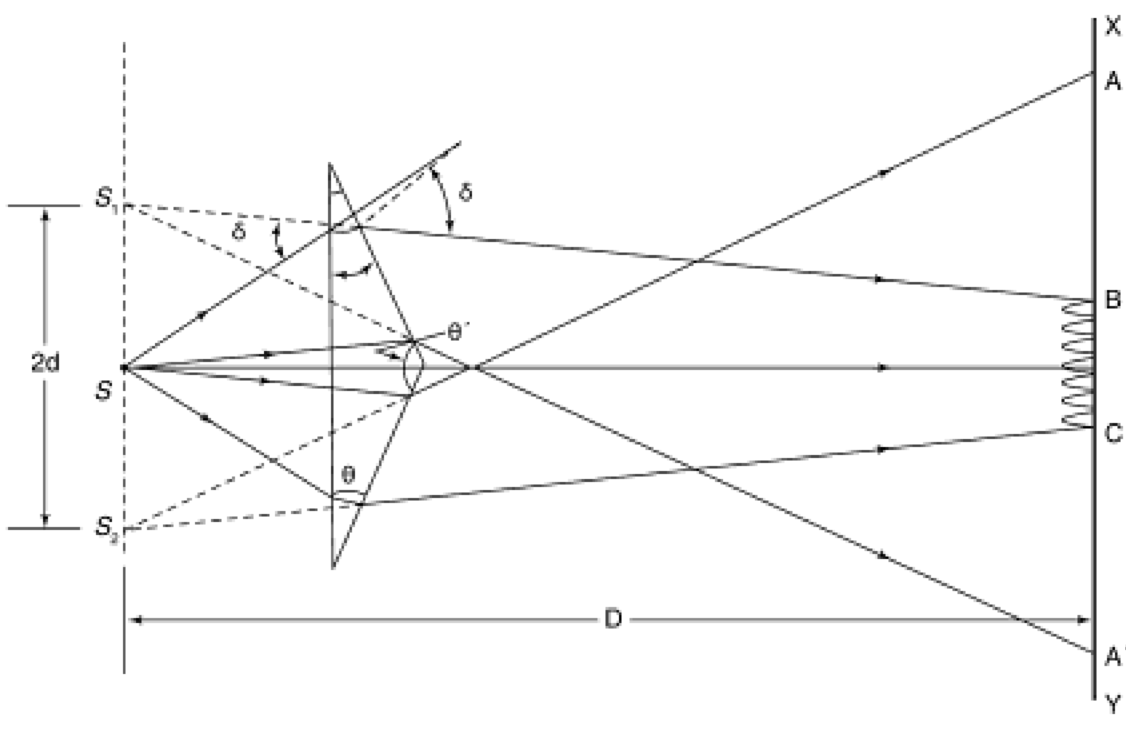
Since a^2 is the intensity due to a single wave, the above relation shows that the average intensity resulting from the superposition of n waves with arbitrary phases is n times of a single wave. It means the resultant amplitude A increases in proportion with $\sqrt{n}$ in length as n gets increased.

1.10 INTERFERENCE BY DIVISION OF WAVEFRONT

This method uses multiple slits, lenses, prisms or mirrors for dividing a single wavefront laterally to form two smaller segments that can interfere with each other. In the division of a wavefront, the interfering beams of radiation that left the source in different directions and some optical means is used to bring the beams back together. This method is useful with small sources. Double slit experiment is an excellent example of interference by division of wavefront. Fresnel's biprism is also used for getting interference pattern based on this method.

1.10.1 Fresnel's Biprism

Fresnel's Biprism is a device by which we can obtain two virtual coherent sources of light to produce sustained interference. It is the combination of two acute angled prisms which are joined with their bases in such a way that one angle becomes obtuse angle θ' of about 179° and remaining two angles are acute angles θ each of about ½°, as shown in Fig. 1.10.



Let monochromatic light from slit S fall on the biprism, placed at a small distance from S. When the light falls on upper part of the biprism, it bends downward and appears to come from source S_1. Similarly, the other part of the light when falls on the lower part of the biprism, bends upward and appears to come from source S_2. Here, the images S_1 and S_2 act as two virtual coherent sources of light (Fig. 1.10). Coherent sources are the ones that have a constant or zero phase difference throughout. In the situation, on placing the screen XY on right side of the biprism, we obtain alternate bright and dark fringes in the overlapping region BC.

1.10.1.1 Theory of Fringes

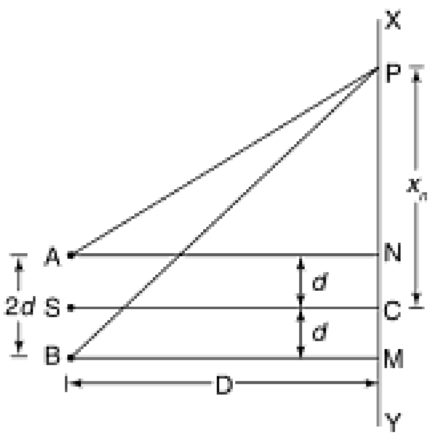


Figure 1.11

Let A and B be two virtual coherent sources of light separated by a distance $2d$. The screen XY, on which the fringes are obtained, is separated by a distance D from the two coherent sources, as shown in Fig. 1.11. The point C on the screen is equidistant from A and B. Therefore, the path difference between the two waves from sources A and B at point C is zero. Thus the point C will be the centre of a bright fringe. On both sides of C, alternately bright and dark fringes are produced.

Draw perpendiculars AN and BM from A and B on the screen. Let the distance of a point P on the screen from the central bright fringe at C be x_n.

From geometry, we have

$$NP = x_n - d; \quad MP = x_n + d$$

In right angled ΔANP,

$$AP^2 = AN^2 + NP^2 \qquad \text{(i)}$$

$$= D^2 + (x_n - d)^2$$

$$D^2\left[1 + \frac{(x_n - d)^2}{D^2}\right]$$

$$AP = D\left[1 + \frac{(x_n - d)^2}{D^2}\right]^{1/2}$$

$$AP = D\left[1 + \frac{1}{2}\frac{(x_n - d)^2}{D^2}\right], \text{[as } (x_n - d) << D]$$

$$AP = D + \frac{1}{2}\frac{(x_n - d)^2}{D} \qquad \text{(ii) [By using Binomial Theorem]}$$

Similarly, in ΔBMP,

$$BP = D + \frac{1}{2}\frac{(x_n + d)^2}{D} \qquad \text{(iii)}$$

Hence, the path difference between the waves reaching via AP and BP paths at the point P on the screen

$$\Delta = BP - AP = \left[D + \frac{1}{2}\frac{(x_n + d)^2}{D}\right] - \left[D + \frac{1}{2}\frac{(x_n - d)^2}{D}\right]$$

$$= \frac{4x_n d}{2D}$$

$$\Delta = \frac{2d}{D} x_n \qquad \text{(iv)}$$

Condition for Bright Fringes: In order to interfere constructively and produce bright fringes, the two rays should arrive at points P in phase. This is possible if the path difference is an integral multiple of λ. Therefore

$$\Delta = n\lambda$$

$$\frac{2d}{D} x_n = n\lambda \text{ where } n = 0, 1, 2 \text{ ………}$$

$$x_n = \frac{n\lambda D}{2d} \qquad \text{(v)}$$

Here it may be recalled that x_n is the distance of the n^{th} order bright fringe from the central maxima.

The distance of the next $(n + 1)^{th}$ maximum from the point C can be calculated by replacing n by $n + 1$ in equation (v). So

$$x_{(n+1)} = (n + 1)\frac{\lambda D}{2d}$$

The separation between two consecutive maxima gives the fringe width β, as follows

$$\beta = x_{n+1} - x_n$$

$$= (n + 1)\frac{\lambda D}{2d} - n\frac{\lambda D}{2d} = \frac{\lambda D}{2d}$$

or fringe width

$$\beta = \frac{\lambda D}{2d} \qquad \text{(vi)}$$

Condition for Dark Fringes: In order to interfere destructively and produce dark fringe at point P, the two rays should arrive at this point in out of phase (phase difference of π). This is possible, if the path difference is an odd multiple of $\frac{\lambda}{2}$. Therefore

$$\Delta = \left(n + \frac{1}{2}\right)\lambda, \text{ where } n = 0, 1, 2, \ldots$$

From Eq. (iv)

$$\Delta = \frac{2d}{D} x_n = (2n + 1)\frac{\lambda}{2} \qquad \text{(vii)}$$

$$x_n = \frac{(2n + 1)\lambda D}{4d} \qquad \text{(viii)}$$

Eq. (viii) gives the distance of n^{th} order dark fringe from the point C. The distance of the next $(n+1)^{th}$ minimum from the point C will be

$$x_{(n+1)} = \frac{[2(n + 1) + 1]\lambda D}{4d}$$

$$= \frac{(2n + 3)\lambda D}{4d} \qquad \text{(ix)}$$

Hence, the fringe width between two consecutive minima would be

$$\beta = x_{(n+1)} - x_n = \frac{(2n+3)\lambda D}{4d} - \frac{(2n+1)\lambda D}{4d}$$

$$\beta = \frac{\lambda D}{2d} \qquad \text{(x)}$$

It is clear from Eqs. (iv) and (x) that the bright and dark fringes are of equal width

1.10.1.2 Experimental Method for Determination of Wavelength of Light

The experimental setup used for the determination of wavelength of light consists of a good quality heavy optical bench of about 1.5 meter length fitted with scale. It has four uprights that carry an adjustable slit S, a biprism, a convex lens and a micrometer eyepiece, respectively. These components are shown in Fig. 1.12. Each upright can be moved along the length of the optical bench and screws are provided to rotate the slit and biprism in their own planes and the eyepiece can also move at right angle to the length of the optical bench.

To obtain well defined and sharp interference fringes, the following adjustments are necessary

(i) Optical bench must be labeled by using sprit level and leveling screws.

(ii) Adjust all uprights to the same height.

(iii) Illuminate the vertical slit by monochromatic source of light. Make the slit narrow

(iv) Now place the biprism on the second upright and try to adjust its edge parallel to the slit until two equally bright virtual sources A and B are observed.

(v) Shift the micrometer eyepiece on the bench away from the slit and also move it at right angle to the length of optical bench until the fringes are observed in the field of view.

(vi) In order to get fine fringes, change the position of the biprism slowly in its own plane such that its edge remains parallel to the slit.

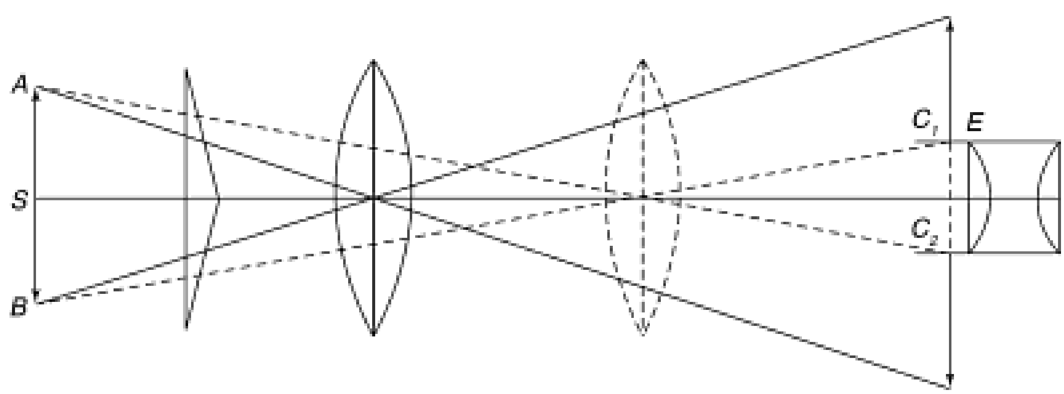
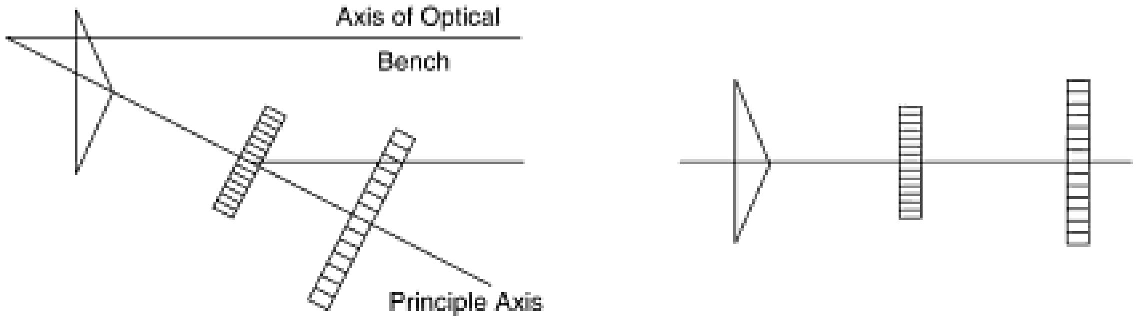


Figure 1.12

Lateral shift and its removal: On moving the micrometer eye piece on the bench towards the biprism, if the fringes appear to shift at right angle to the optical bench then it is known as **lateral shift** (Fig. 1.13(a)). However, if the princple axis and axix of optical bench become parallel, then no lateral shift remains, as shown in Fig. 1.13(b).



(a) *Lateral shift* (b) *No lateral shift of fringes*

Figure 1.13

1.10.1.3 Determination of Distance between Two Virtual Coherent Sources

For measuring $2d$, a convex lens of short focal length is placed between the biprism and the micrometer eye piece. This distance between the biprism and the micrometer eye piece is more than 4 times of the focal length of the convex lens. By moving the lens we **obtain two positions L_1 and L_2 of the convex lens such that two separated images d_1 and d_2** of the two coherent sources respectively can be observed, as shown in Fig. 1.14.

For the first position of lens, L_1, the magnification is given as

$$\frac{v}{u} = \frac{d_1}{2d} \tag{i}$$

and for second position of the lens, the magnification is

$$\frac{u}{v} = \frac{d_2}{2d} \tag{ii}$$

Then from Eqs. (i) and (ii), we get

$$\frac{v}{u} \times \frac{u}{v} = \frac{d_1 d_2}{(2d)^2} \text{ or } (2d)^2 = d_1 d_2$$

or

$$2d = \sqrt{d_1 d_2} \tag{iii}$$

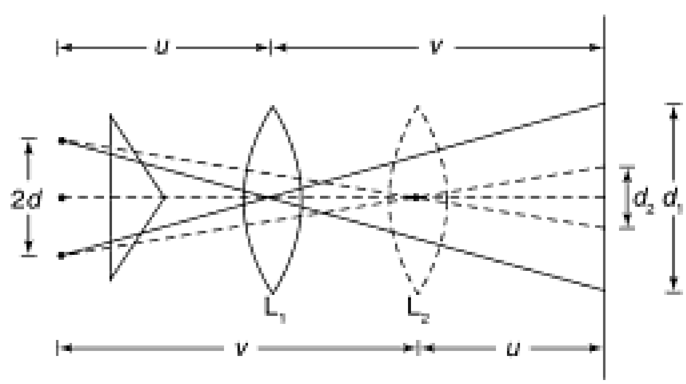


Figure 1.14

Therefore, the measurement of positions of images d_1 and d_2 will determine the distance $2d$ between the sources. The wavelength λ of monochromatic light can be calculated when we substitute the values of β, D

1.10.1.4 Determination of Thickness of Thin Transparent Sheet (Displacement of Fringes)

Let A and B be two virtual coherent sources of light. The point C_0 on the screen is equidistant from both the sources (Fig. 1.15). When a transparent material plate G of thickness t and having refractive index it is placed in the path of one of the light wave, we observe that the fringe which was originally at C_0 shifts to another position P, as shown in Fig. 1.15.

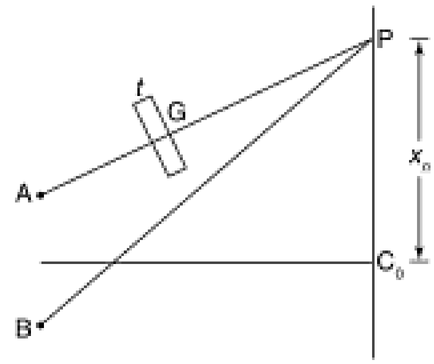


Figure 1.15

The time taken by the light wave from A to P partly through air and partly through the plate is the same as the time taken by the other light wave from B to P in air. If c and v be the velocity of light in air and in the plate, respectively, then

$$\frac{BP}{c} = \frac{AP - t}{c} + \frac{t}{v}$$

or $$\frac{BP}{c} = \frac{AP - t}{c} + \frac{\mu t}{c} \qquad \left[\because \mu = \frac{c}{v}\right]$$

or $$BP = (AP - t) + \mu t$$

or $$BP - AP = (\mu - 1)t \qquad \text{(i)}$$

Here BP – AP is the path difference between the two interfering waves.

If the point P is originally occupied by the n^{th} order bright fringe, then the path difference between the two interfering waves will be

$$BP - AP = n\lambda,$$

$$(\mu - 1)\, t = n\lambda \qquad \text{(ii)}$$

The distance x_n through which the fringe is shifted to point P from the central maximum C_0 is given by

$$x_n = \frac{n\lambda D}{2d} \qquad \text{(iii)}$$

where, $\frac{\lambda D}{2d} = \beta$ = fringe width.

From Eq. (iii), we get

$$\frac{x_n \cdot 2d}{D} = n\lambda \qquad \text{(iv)}$$

From Eqs. (ii) and (iv), we get

$$(\mu - 1)t = \frac{x_n \cdot 2d}{D}$$

or $$t = \frac{x_n \cdot 2d}{(\mu - 1)D} \tag{v}$$

Therefore, by knowing x_n, $2d$, D and μ, we can calculate thickness t of the glass plate by using Eq. (v).

1.11 INTERFERENCE BY DIVISION OF AMPLITUDE

The method, which is used to produce two coherent sources from a common source, is called **division of amplitude** that maintains the same width but reduced amplitude. After following different paths the two waves of reduced amplitudes are combined to produce an interference pattern. In this method, the interfering beams consist of radiation that has left the source in the same direction. This radiation is divided after leaving the source and later combined to produce interference. This method can be used with extended sources. Michelson interferometer is an example of interference by division of amplitude. Thin films are also used for getting interference pattern based on this method.

1.11.1 Interference due to Thin Films

This is clear that the interference takes place when the two waves superimpose each other after traveling some distance, i.e. when there is a path difference between them. Since the thin film has its two surfaces, the waves reflected from these surfaces can attain a path difference and can interfere. The same may be applied on the waves that transmit through the film.

1.11.1.1 Thin Film of Uniform Thickness

Consider a uniform transparent film having thickness t and a refractive index μ. A ray of light AB incident at an angle i on the upper surface of the film is partly reflected along BC and partly refracted along BF at an angle r. At point F the way BF is again partly reflected from the second surface along FD and partly emerges out along FK and so on. In this situation, the interference occurs between reflected waves BC and DE and also between the transmitted waves FK and GL (Fig. 1.16).

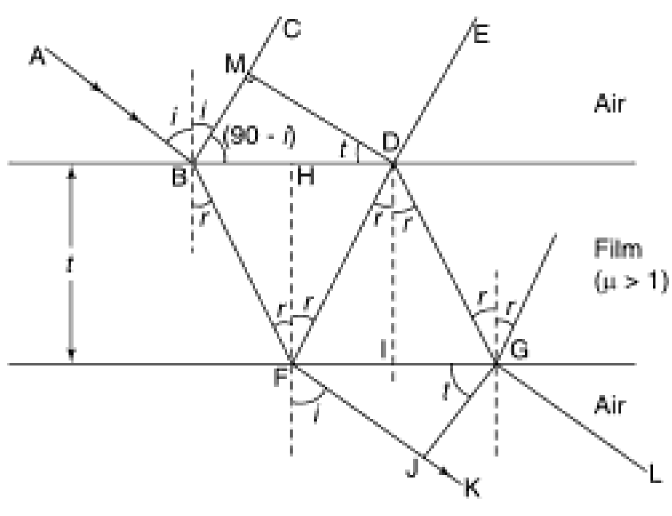


Figure 1.16

The path difference between the reflected rays

$$\Delta = \mu\,(BF + FD) - BM$$

$$\because BF = FD$$

$$\therefore \Delta = 2\mu BF - BM \qquad \text{(i)}$$

In the right angled ΔBFH,

$$\cos r\,\frac{t}{BF} \text{ or } BF = \frac{t}{\cos r} \qquad \text{(ii)}$$

and
$$\tan r\,\frac{BH}{t} \text{ or } BH = t \tan r$$

$$BD = 2 \times BH$$

$$\therefore BD = 2t \tan r \qquad \text{(iii)}$$

In the Δ BMD,

$$\sin i = \frac{BM}{BD} \text{ or } BM = BD \sin i$$

$$\therefore BM = 2t \tan r \sin i \qquad \text{(iv)}$$

From Eqs. (i), (ii) and (iv), we get

$$\Delta = 2\mu \frac{t}{\cos r} - 2t \tan r \sin i \qquad \text{(v)}$$

$$\because \mu = \frac{\sin i}{\sin r} \text{ or } \sin i = \mu \sin r \qquad \text{(vi)}$$

$$\therefore \Delta = \frac{2\mu t}{\cos r} - 2t\,\frac{\sin r}{\cos r}\mu \sin r = \frac{2\mu t}{\cos r}[1 - \sin^2 r]$$

$$\Delta = 2\mu t \cos \qquad \text{(vii)}$$

Equation (vii) represents only the apparent path difference and does not represent the effective total path difference. When the light is reflected from the surface of an optically denser medium in case of BC a phase change of a equivalent to path difference of $\lambda/2$ is introduced. Therefore, the total path difference between BC and DE will be

$$\Delta = 2\mu t \cos r + \lambda/2 \qquad \text{(viii)}$$

Condition for Maxima: To have a maximum at a particular point, the two rays should arrive there in phase. So the path difference must contain a whole number of wavelength, i.e.,

$$\Delta = n\lambda,\ n = 0, 1, 2\ldots, \qquad \text{(ix)}$$

From Eq. (viii) and (ix), we get

$$2\mu t \cos r + \lambda/2 = n\lambda$$

$$2\mu r \cos r + n\lambda - \lambda/2$$

$$2\mu t \cos r = (2n - 1)\lambda/2 \qquad \text{(x)}$$

Condition for Minima: To have a minimum at a particular point, the two rays should arrive there in out of phase (odd multiple of π) for which the path difference must contain a half odd integral number of wavelength, i.e,

$$\Delta = \left(n + \frac{1}{2}\right)\lambda \qquad \text{(xi)}$$

Using Eq. (viii), we obtain

It should be noted that the interference pattern will not be perfect because the intensities of the rays BC and DE are not the same and their amplitudes are different.

In order to obtain the interference between the transmitted waves, we calculate the path difference between the waves, FK and GL as under

$$\Delta = (FD + DG)_{\text{in film}} - (FJ)_{\text{in air}}$$

$$\Delta = \mu[FD + DG] - FJ$$

$$\because FD = DG$$

$$\therefore \Delta = 2\mu FD - FJ \qquad \text{(xiii)}$$

In ΔFDI, $\cos r = \frac{DI}{FD} = \frac{t}{FD}$ or $FD = \frac{t}{\cos r}$ (xiv)

and $\tan r = \frac{FI}{DI} = \frac{FI}{t}$ or $FI = t \tan r$

$$FG = 2t \tan r \qquad \text{(xv)}$$

In right angled ΔFJG,

$$\sin i = \frac{FJ}{FG} \text{ or } FJ = FG \sin i$$

$$\therefore FJ = 2t \tan r \sin i \qquad \text{(xvi)}$$

From Eq. (xiii), (xiv) and (xvi), we get

$$\Delta = \frac{2\mu t}{\cos r} - 2t \tan r \sin i$$

$$= \frac{2\mu t}{\cos r} - 2t\frac{\sin r}{\cos r}\mu \sin r \qquad \left[\mu = \frac{\sin i}{\sin r}\right]$$

$$= \frac{2\mu t}{\cos r}[1 - \sin^2 r] = 2\mu t \cos r$$

Since these two waves are emerging from the same medium, the additional phase difference will not be introduced. Therefore, the total path difference

$$\Delta = 2\mu t \cos r \qquad \text{(xvii)}$$

Condition for Maxima: As discussed, it is possible when

$$\Delta = n\lambda \qquad \text{(xviii)}$$

From Eqs. (xvii) and (xviii), we get

$$2\mu t \cos r = n\lambda \text{ where, } n = 0, 1,2,3,\ldots \qquad \text{(xix)}$$

Condition for Minima: For obtaining minimum intensity, we should have

$$\Delta = \left(n + \frac{1}{2}\right)\lambda,$$

which gives $2\mu t \cos r = \left(n + \frac{1}{2}\right)\lambda$ where, n = 0, 1, 2, 3,.... (xx)

Thus, the conditions for interference with transmitted light are obviously opposite to those obtained with reflected light. Hence, if the film appears dark in the reflected light, it will appear bright in the transmitted light and vice-versa. This shows that the interference pattern in the reflected and transmitted lights are

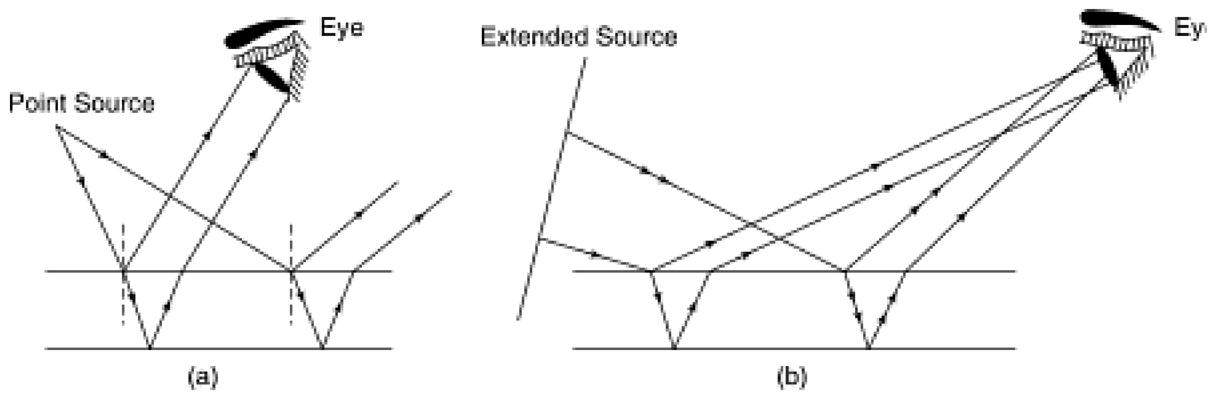


Figure 1.17

(i) Interference using White Light

When a thin transparent film is exposed to white light and seen in the reflected light, different colours are seen in the film. These colours arise due to the interference of the light waves reflected from the top and bottom surfaces of the film. The path difference between the reflected rays depends upon the thickness t, refractive index μ of the film and the angle θ of inclination of the incident rays. The light which comes from any point from the surface of the film will include the colour whose wavelength satisfies the equation $2\mu t \cos r = (2n - 1)\lambda/2$ and only this colour will be present with the maximum intensity in the reflected light.

When the transparent film of a large thickness as compared to the wavelength of the light, is illuminated by white light, the path difference at any point of the film will be zero. In the case of such a thick film, at a given point, the condition of constructive interference is satisfied by a large number of wavelengths, as << t. The condition of destructive interference is also satisfied at the same point for the large number of wavelengths. Therefore, consequently that point receives an average intensity due to the light of all wavelengths and no colours are observed.

In the context of realization of above phenomena it is always needed to use a broad power of light that will enable the eye to see whole of the film simultaneously.

If we use a point source, then we observe that different parts of reflected light cannot reach the eye due to small size of the pupil, as shown in Fig. 1.17(a). The reflected rays only from a small portion of the film can enter the eye. Hence, the whole of the film cannot be seen by the eye placed in a fixed position. However, if a broad source of light is used to illuminate a thin film, the light reflected from each part of the film reaches the eye placed in a fixed position, as shown in Fig. 1.17(b). Hence, one can seen the entire film simultaneously by employing an extended source of light.

1.11.1.2 Non-uniform Thickness Film (Wedge Shaped Film)

Consider two plane surfaces OM and OM' inclined at an angle θ enclosing a wedge shaped air film of increasing thickness, as shown in Fig 1.18. A beam of monochromatic light is incident on the upper surface of the film and the interference occurs between the rays reflected at its upper and lower surfaces. The interference occurs between the reflected rays BK and DL, both of which are obtained from the same incident ray of light AB.

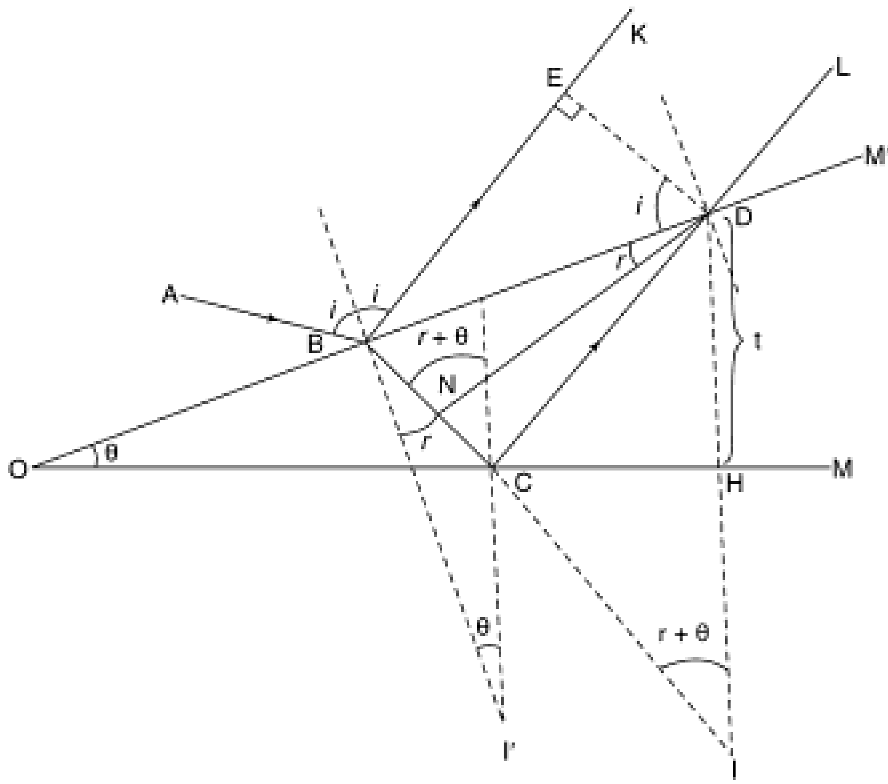


Figure 1.18

The path difference between the two reflected rays

$$\Delta = [BC + CD]_{\text{in film}} - [BE]_{\text{in air}}$$

$$\Delta = \mu(BC + CD) - BE$$

$$\because CD = CI$$

$$\Delta = \mu(BC + CI) - BE$$

$$= \mu BI - BE$$

$$= \mu\,(BN + NI) - BE \qquad \text{(i)}$$

In right angled ΔBED,

$$\sin i = \frac{BE}{BD} \qquad \text{(ii)}$$

In right angled ΔBND,

$$\sin r = \frac{BN}{BD} \qquad \text{(iii)}$$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{\sin i}{\sin r} = \frac{BE}{BN} \text{ or } \frac{\sin i}{\sin r} = \mu = \frac{BE}{BN}$$

$$\text{or } BE = \mu BN \qquad \text{(iv)}$$

From Eqs. (i) and (iv), we get

$$\Delta = \mu\,(BN + NI) - \mu BN$$

or $$\Delta = \mu NI \qquad \text{(v)}$$

In right angled ΔDNI,

$$\cos(r+\theta) = \frac{NI}{DI}$$

$$\because DI = DH + HI = t + t = 2t$$

$$\cos(r+\theta) = \frac{NI}{2t} \text{ or } NI = 2t\cos(r+\theta) \qquad \text{(vi)}$$

From Eqs. (v) and (vi), we get

$$\Delta = 2\mu t\cos(r+\theta) \qquad \text{(vii)}$$

Eq. (vii), in the case of reflected light, does not represent the effective total path difference, as a phase difference of π (Stoke's phase change) has been introduced through the reflection of wave BK. Therefore, the total path difference between the reflected rays,

$$\Delta = 2\mu t\cos(r+\theta) + \lambda/2 \qquad \text{(viii)}$$

This equation shows that the path difference Δ depends on the thickness t. However, t is not uniform and it is different at different positions.

At $t = 0$, Eq. (viii), reads

$$\Delta = \lambda/2$$

which is the condition for darkness. Therefore, the edge of the film appears to be dark. This is called zero order band.

For normal incidence, $i = 0$ and $r = 0$. Then the path difference

$$\Delta = 2\mu t\cos\theta + \lambda/2 \qquad \text{(ix)}$$

Condition for Maxima: As explained earlier, the constructive interference takes place when

$$\Delta = n\lambda \qquad \text{(x)}$$

From Eq. (ix) and (x), we get

$$2\mu t\cos\theta + \lambda/2 = n\lambda$$

$$2\mu t\cos\theta = (2n-1)\,\lambda/2 \text{ where, } n = 0, 1, 2, 3, \ldots \qquad \text{(xi)}$$

Condition for Minima: In order to get destructive interference, the path difference

$$\Delta = \left(n + \frac{1}{2}\right)\lambda \qquad \text{(xii)}$$

or $$2\mu t\cos\theta + \lambda/2 = \left(n + \frac{1}{2}\right)\lambda$$

$$2\mu t\cos\theta = n\lambda \text{ where, } n = 0, 1, 2, 3, \ldots \qquad \text{(xiii)}$$

(i) Nature of Fringes

For normal incidence of the incident waves or a parallel incident beam, the incident angle remains constant and hence the angle of refraction. If the light is monochromatic, then λ is also fixed. Therefore, the change in path difference will take place due to μt or thickness t of the film only. As we move outwards from the point of

t has one and only one value. Since the locii of the points of constant thickness of the film are straight lines parallel to the edge, straight bright and dark fringes parallel to the edge will be obtained in the reflected light. If we use the white light in place of monochromatic light, colored fringes will be observed

(ii) Derivation for Fringe Width

For a wedge shaped film the conditions of maxima and minima are reproduced below.

$$2\mu t \cos(r + \theta) = (2n - 1)\lambda/2$$

$$2\mu t \cos(r + \theta) = n\lambda$$

For normal incidence and small values of θ the above conditions read

$$2\mu t = (2n - 1)\lambda/2 \qquad \text{(xiv)}$$

$$\text{and } 2\mu t = n\lambda \qquad \text{(xv)}$$

If points A and C (Fig. 1.19) represent positions of two consecutive dark fringes corresponding to film thicknesses AB = t_1 and CD = t_2 respectively, then the fringe width (w) will be equal to BE. Now from Eq. (xv), we get the following condition corresponding to the points A and C.

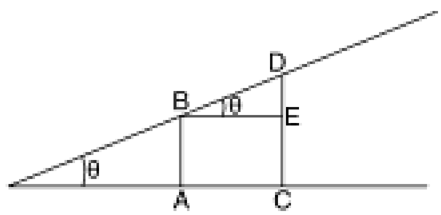


Figure 1.19

$$2\mu t_1 = n\lambda \text{ and } 2\mu t_2 = (n + 1)\lambda$$

or $\quad 2\mu(t_2 - t_1) = \lambda$

or $\quad 2\mu\,(CD - AB) = \lambda$

or $\quad 2\mu(DE) = \lambda \qquad$ (xvi)

But $\quad \tan\theta = DE/BE$ or $DE = BE\tan\theta \qquad$ (xvii)

From Eqs. (xvi) and (xvii), we get

$$2\mu(BE\tan\theta) = \lambda$$

or $\quad BE = \dfrac{\lambda}{2\mu\tan\theta} = w$

For smaller values of θ, $\tan\theta \simeq \theta$ and we get

$$w = \frac{\lambda}{2\mu\theta} \qquad \text{(xviii)}$$

It is clear from (xviii) that the fringe width w is independent of thickness t for smaller angle θ. Therefore the fringes are equally spaced and of same width for fixed λ, μ and θ.

1.11.2 Newton's Rings

If a plano-convex lens is placed such that its curved surface lies on a glass plate, then an air film of gradually increasing thickness is formed between the two surfaces. When a beam of monochromatic (single wavelength) circular fringes are observed. These circular fringes are formed because of the interference between the reflected waves from the top and the bottom surfaces of the air film. These fringes are circular since the air film has a circular symmetry and the thickness of the film corresponding to each fringe is same throughout the circle. The interference fringes so formed were first investigated by Newton and hence known as **Newton's rings**.

The path difference between the two reflected rays, can be obtained as done in the case of wedge shaped film. It is reproduced below as

$$\Delta = 2\mu t \cos(r + \theta) + \lambda/2 \qquad \text{(i)}$$

Where ($\lambda/2$) is due to **Stoke's** phase change.

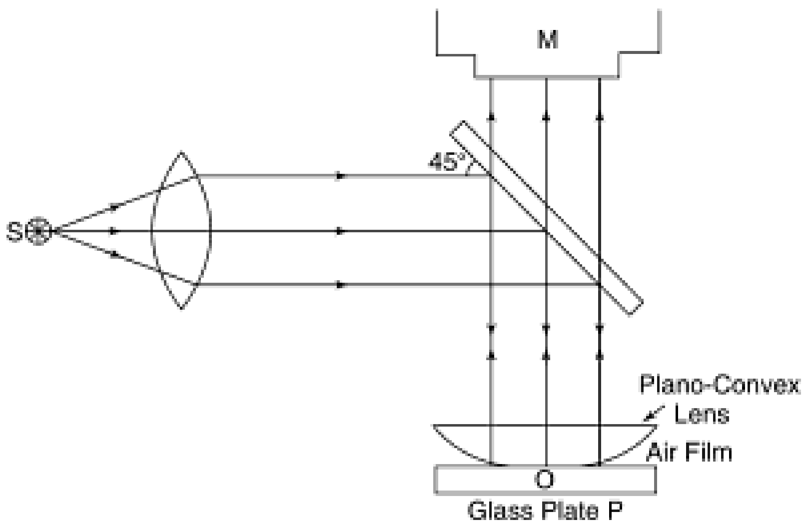


Figure 1.20

For normal incidence and an air film, $i = 0$, $r = 0$, $\mu = 1$. In addition, if θ is also very small, then $\cos\theta = 1$. Under this situation, the path differences becomes

$$\Delta = 2t + \frac{\lambda}{2} \qquad \text{(ii)}$$

Here t is the thickness of the air film at a particular point.

At the point of contact, $t = 0$

$$\Delta = \frac{\lambda}{2}$$

which is the condition of minimum intensity and hence, the central spot of the ring will be dark.

Condition for Maxima: For constructive interference

$$\Delta = n\lambda \qquad \text{(iii)}$$

or
$$2t + \frac{\lambda}{2} = n\lambda$$

or
$$2t = (2n - 1)\frac{\lambda}{2} \text{ where } n = 0, 1, 2, 3\ldots \qquad \text{(iv)}$$

Condition for Minima: For destructive interference

$$\Delta = \left(n + \frac{1}{2}\right)\lambda$$

or $$2t + \frac{\lambda}{2} = n\lambda + \frac{\lambda}{2}$$

or $$2t = n\lambda \text{ where } n = 0, 1, 2, 3.. \quad \text{(v)}$$

Diameter of Dark and Bright Rings: Let us consider the thickness of the air film at point Q as t and r_n as the radius of the fringe at that point together with R as the radius of curvature of the lens (Fig. 1.21)

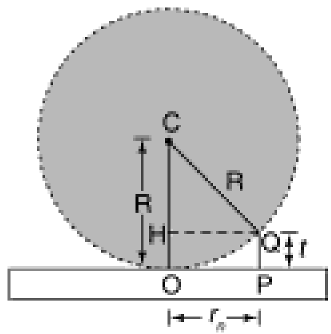


Figure 1.21

Hence, $$OC = CQ = R,\ HQ = r_n$$

$$HC = R - t$$

In right angled ΔCHQ

$$CQ^2 = CH^2 + HQ^2$$

$$R^2 = (R - t)^2 + r_n^2$$

or $$r_n^2 = 2Rt - t^2$$

In actual practice, R is quite large and t is very small. Therefore, t^2 may be neglected in comparison with 2Rt

$$\therefore r_n^2 = 2Rt$$

$$\text{or } r_n^2 = R \times 2t \quad \text{(vi)}$$

For Bright Rings:

From Eq. (iii), we get

$$2t = (2n - 1)\frac{\lambda}{2}$$

When we put this value of $2t$ in Eq. (vi), we get

$$r_n^2 = R \times (2n - 1)\frac{\lambda}{2}$$

$$\left(\frac{D_n}{2}\right)^2 = R \times (2n - 1)\frac{\lambda}{2}$$

or $$D_n^2 = 2\lambda R(2n-1)\frac{\lambda}{2} \qquad \text{(vii)}$$

The above equation gives the diameter D_n of n^{th} order bright fringe as

$$D_n = \sqrt{2\lambda R(2n-1)}$$

$$\therefore D_n \propto \sqrt{(2n-1)} \qquad \text{(viii)}$$

Thus the diameter of the bright circular fringe(s) is proportional to the square root(s) of the odd natural numbers.

For Dark Rings: Applying the condition $2t = n\lambda$ for the dark rings, Eq. (vi) reads

$$r_n^2 = n\lambda R.$$

or $$D_n^2 = 4n\lambda R$$

$$\therefore D_n \propto \sqrt{n} \qquad \text{(ix)}$$

Thus the diameter D_n of dark circular fringe(s) is proportional to the square root(s) of the natural numbers.

1.11.2.1 Determination of Wavelength of Light

We have seen that the diameter of n^{th} order dark fringe in Newton's rings method is

$$D_n^2 = 4n\lambda R \qquad \text{(x)}$$

From the above relation, the diameter of $(n + p)^{th}$ order dark fringe can be written as

$$D_{(n+p)}^2 = 4(n+p)\lambda R \qquad \text{(xi)}$$

Subtracting Eq. (x) from equation (xi), we get

$$D_{(n+p)}^2 - D_n^2 = 4p\lambda R$$

or $$\lambda = \frac{D_{(n+p)}^2 - D_n^2}{4pR}$$

Therefore, the measurement of diameters of the n^{th} and $(n + p)^{th}$ dark fringes together with the radius of curvature of the lens gives us the wavelength of sodium light with the help of above formula.

1.11.2.2 Determination of Radius of Curvature of Plano Convex Lens

This is clear from the theory of Newton's rings that the measurement of diameters of n^{th} order and $(n + p)^{th}$ order dark fringes play an important role in the determination of wavelength of monochromatic light. For this purpose, the following relation is used

$$\lambda = \frac{D_{(n+p)}^2 - D_n^2}{4pR}$$

Therefore, if we use the monochromatic source of light of known wavelength, it would be possible to determine the radius of curvature of the plano convex lens with the help of following formula

$$R = \frac{D_{(n+p)}^2 - D_n^2}{4p\lambda}$$

1.11.2.3 Determination of Refractive Index of a Liquid

The liquid whose refractive index is to be determined is placed between the lens and the glass plate and then we evaluate the diameters of the dark fringes.

The diameter of n^{th} order dark fringe air film is given by

$$D_n^2 = 4n\lambda R$$

Similarly, the diameter of n^{th} order dark fringe in liquid film would be

$$[D_n^2]_{liquid} = \frac{4n\lambda R}{\mu}$$

where μ is the refractive index of the liquid and $D_{liquid} < D_{air}$

Therefore, the refractive index of the liquid can be calculated from the following formula once we are able to determine the diameters of dark fringes.

$$\mu = \frac{[D_n^2]_{air}}{[D_n^2]_{liquid}}$$

1.11.2.4 Newton's Rings in Transmitted Light

Newton's rings can be observed in reflected as well as in transmitted light. Figure 1. 22 shows that the rays QA and HRB are the transmitted rays, which interfere. From the figure it is also clear that the ray QA suffers no reflection at a medium of higher index, so its phase does not change. However, the ray HRB encounters two reflections at the denser medium at Q and H. Since a phase change of π occurs at each reflection, the total phase change due to both reflections would be 2π. Therefore, there will not be any phase shift. In view of this, the path difference between the two transmitted rays QA and HRB would be

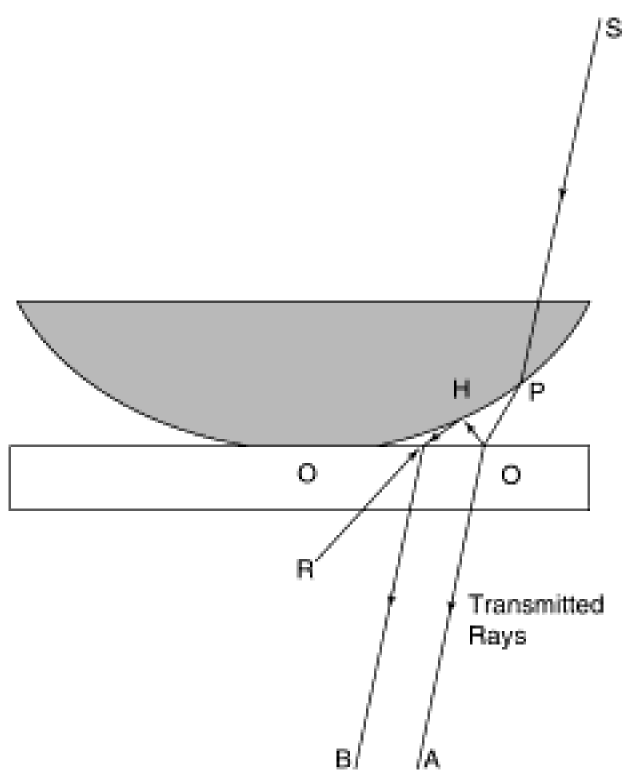


Figure 1.22

$$\Delta = 2\mu t \cos(r + \theta) \quad \text{(i)}$$

For air ($\mu = 1$), normal incidence ($r = 0$) and smaller angle θ ($\cos\theta = 1$), the path difference becomes

$$\Delta = 2t \quad \text{(ii)}$$

The above equations shows that at $t = 0$, the path difference between the two transmitted rays $\Delta = 0$. Therefore, at the centre, the bright fringe will appear.

From Eq. (ii), the conditions for maxima and minima can respectively be obtained as below

$$2t = n\lambda,\ n = 0, 1, 2, \ldots \quad \text{(iii)}$$

$$2t = (n + 1/2)\lambda,\ n = 0, 1, 2, \ldots \quad \text{(iv)}$$

Because of the same reason as discussed earlier, the fringes in the transmitted light will also be circular. The diameter of bright circular fringes can be obtained as

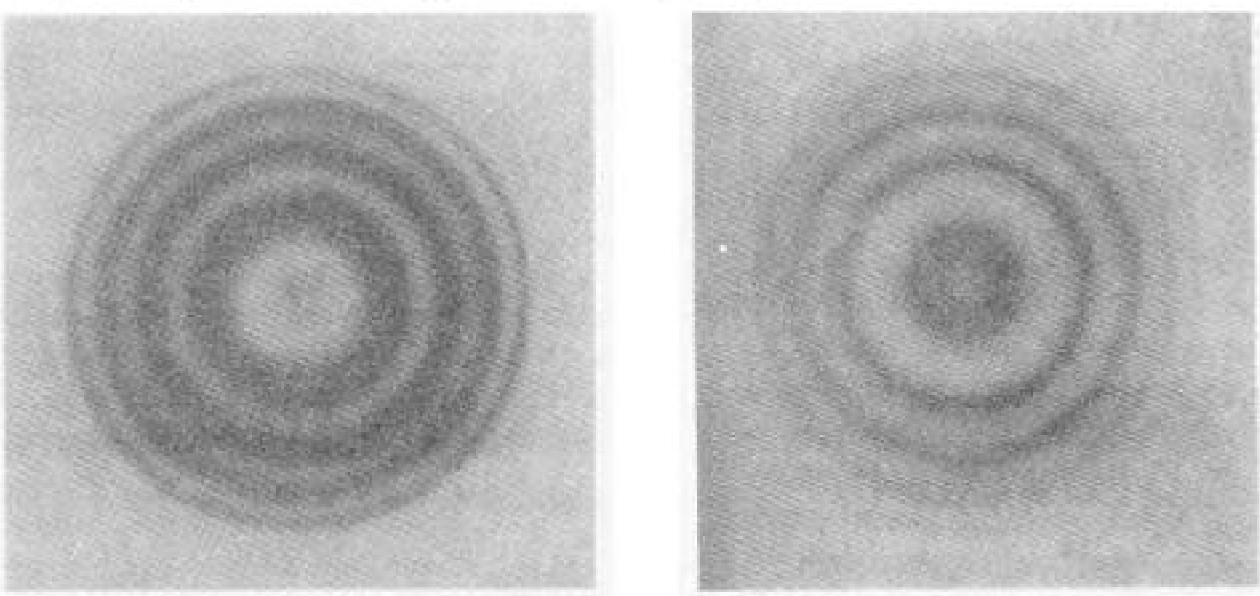
$$D_n = 2\sqrt{n\lambda R}$$

Thus the diameter of the bright fringes is proportional to the square root of natural numbers. When we calculate the diameter of dark circular fringes, it comes out to be

$$D_n = \sqrt{2(n+1)\lambda R}$$

This relation shows that the diameter of the dark fringes is proportional to the square root of odd natural numbers.

From the above relation, it is clear that the fringes observed in the transmitted light are exactly complementary to that of the reflected light. These fringes are much poorer in contrast as the transmitted rays emerge with lower intensity in comparison with the reflected rays. The Newton's rings obtained in the reflected as well as in the transmitted light are shown in Fig. 1.23a and b, respectively.



(a) (b)

Figure 1.23

1.11.2.5 Newton's Rings formed between Two Curved Surfaces

Let us consider two curved surfaces of radii of curvature R_1 and R_2 in contact at the point O. A thin air film is enclosed between the two surfaces (Fig. 1.24). In this arrangement also, dark and bright rings are formed and can be seen with a traveling microscope.

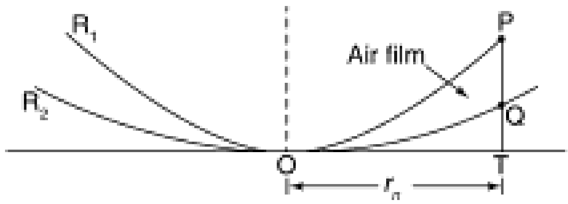


Figure 1.24

The thickness of the air film at P is

$$PQ = PT - QT$$

If the radius of n^{th} dark ring be r_n, then from geometry,

$$PT = \frac{r_n^2}{2R_1} \text{ and QT } \frac{r_n^2}{2R_2}$$

$$\therefore PQ = \frac{r_n^2}{2R_1} - \frac{r_n^2}{2R_2}$$

If we assume the thickness of the film as t, then

$$t = \frac{r_n^2}{2R_1} - \frac{r_n^2}{2R_2}.$$

Now this is clear from Fig. 1.24 that this type of film is similar to the wedge shaped film. Therefore, the path difference between the wave reflected from the upper and lower surfaces of the film would be

$$2\mu t \cos(r + \theta) + \frac{\lambda}{2}$$

For air ($\mu = 1$), normal incidence ($r = 0$) and the smaller angle θ, the path difference takes the form

$$2t + \frac{\lambda}{2}$$

Therefore, in case of reflected light, for n^{th} dark fringes

$$2t + \frac{\lambda}{2} = \left(n + \frac{1}{2}\right)\lambda$$

or $$2t = n\lambda$$

or $$2\left[\frac{r_n^2}{2R_1} - \frac{r_n^2}{2R_2}\right] = n\lambda$$

$$r_n^2\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = n\lambda \text{ where } n = 0, 1, 2, 3\ldots \qquad \text{(i)}$$

Similarly, for n^{th} bright fringe the path difference should satisfy the following condition

$$2t + \frac{\lambda}{2} = n\lambda$$

or $$2t = \left(n - \frac{1}{2}\right)\lambda$$

$$r_n^2\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{(2n-1)\lambda}{2} \text{ where } n = 0, 1, 2, 3\ldots \quad \text{(ii)}$$

Thus, bright and dark fringes are obtained according to Eqs. (i) and (ii). The diameter of the fringes can also be calculated.

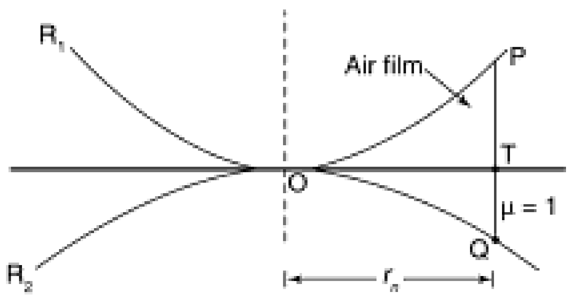


Figure 1.25

Now we invert the lower surface of the film. Under this situation, the film would appear thicker than the previous case (Fig. 1.25). The film thickness PQ in this case would be

$$PQ = PT + QT$$

$$t = \frac{r_n^2}{2R_1} + \frac{r_n^2}{2R_2}$$

For the reasons explained in wedge shape film, the following condition should be satisfied in order to obtain n^{th} order dark fringe of radius r_n

$$2t = n\lambda \text{ (for air)}$$

or $$2r_n^2\left[\frac{1}{2R_1} + \frac{1}{2R_2}\right] = n\lambda$$

$$r_n^2\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = n\lambda \text{ where } n = 0, 1, 2, 3\ldots \quad \text{(iii)}$$

For n^{th} bright fringe

$$2t = (2n-1)\frac{\lambda}{2}$$

or $$2r_n^2\left[\frac{1}{2R_1} + \frac{1}{2R_2}\right] = (2n-1)\frac{\lambda}{2}$$

$$r_n^2\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = (2n-1)\frac{\lambda}{2} \text{ where } n = 0, 1, 2, \ldots\text{etc.} \quad \text{(iv)}$$

A comparison of Eq. (i) with Eq. (iii) reveals that the diameter of dark fringes in the second case, where below curved surface looks like convex when viewed from above, would be smaller than the one in first case. This effect is similar to the situation as if we increase the width or thickness of the film. The same is the case for bright fringes.

1.11.3 Michelson's Interferometer

It consists of two highly polished mirrors M_1 and M_2 and two plane glass plates P and Q parallel to each other, as shown in Fig. 1.26. The glass plate P is half-silvered on its back surface and inclined at an angle of 45° to the beam of incident light. Another glass plate Q is such that P and Q are of equal thickness and of the same material. Two plane mirrors M_1 and M_2 are silvered on their front surfaces and mounted on two arms at right angle to each other. The position of the mirror M_1 can be changed with the help of a fine screw.

Light from a monochromatic source S, rendered parallel by a lens L, falls on the glass plate P. The semi-silvered plate P divides the incident light beam into two parts of nearly equal intensities, namely reflected and transmitted beams. The reflected beam moves towards mirror M_1 and falls normally on it and hence it is reflected back to P and enters the telescope T. The transmitted beam moves towards mirror M_2 and falls normally on it after passing through the plate Q. Therefore, it is reflected by the mirror M_2 and follows the same path. At P it is reflected to enter the telescope T. Since the beams entering the telescope have been derived from the same incident beam, these two rays are capable of giving the phenomenon of interference; thereby producing interference fringes.

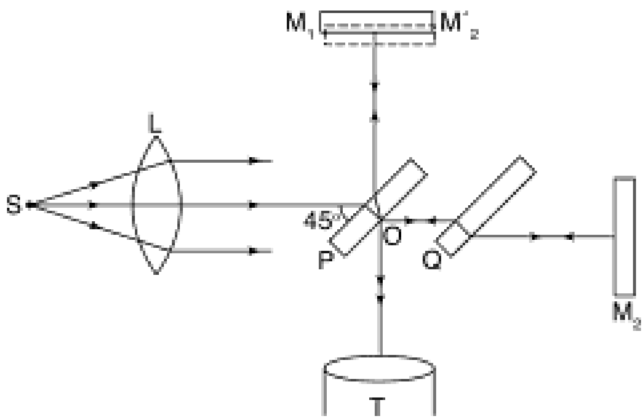


Figure 1.26

Function of Plate Q: The beam going towards the mirror M_1 and reflected back, crosses the plate P twice, while the other beam in the absence of Q would travel wholly in air. Therefore, to compensate the additional path, the plate Q is used between the mirror M_2 and plate P. The light beam going towards the mirror M_2 and reflected back towards P also passes twice through the compensation plate Q. Therefore, the optical paths of the two rays in glass are the same.

Types of Fringes: The fringes in Michelson interferometer depend upon the inclination of M_1 and M_2. Let M'_2 be the image of M_2 formed by the reflection at the half-silvered surface of the plate P so that $OM_2 = OM'_2$. The interference fringes may be regarded as formed by the light reflected from the surfaces of M_1 and M'_2. Thus, the arrangement is equivalent to an air-film enclosed between the reflecting surfaces M_1 and M'_2.

It is obvious that the path difference between the two beams produced by the reflecting surfaces M_1 and M'_2 is equal to the twice of the thickness of the film $M_1M'_2$. This path difference can be varied by moving M_1 backwards or forward parallel to itself. If we use monochromatic light, the pattern of bright and dark fringes will be formed. Here the shape of the fringes will depend upon the inclination of M_1 and M_2.

If M_1 and M_2 are exactly at right angles to each other, the reflecting surfaces M_1 and M'_2 are parallel and hence

These fringes are called as **Haidinger's fringes** that can be seen in the field view of a telescope. When the distance between the mirrors M_1 and M_2 or between M_1 and M'_2 is decreased, the circular fringes shrink and vanish at the centre. A ring disappears each time when the path $2t$ decreases by λ.

Since the vertical ray first gets reflected from the inner surface of P (internal reflection), and then from the front surface of the mirror M_1 (external reflection) a phase change of π takes place. The horizontal ray first gets reflected from the front surfaces of M_2 (external reflection) and then from the inner surface of glass plate P (external reflection), so there is no phase change. Therefore, the total path difference for normal incidence would be

$$\Delta = 2t\cos\theta + \frac{\lambda}{2}$$

For bright fringes, the following condition should be satisfied

$$\Delta\ 2t\cos\theta = \left(n - \frac{1}{2}\right)\lambda \qquad [\because\ \Delta = n\lambda] \quad \text{(i)}$$

For dark fringes, the condition reads

$$\Delta\ 2t\cos\theta = n\lambda \qquad [\because\ \Delta = \left(n + \frac{1}{2}\right)\lambda\ ] \quad \text{(ii)}$$

When t is further decreased, a limit is attained where M_1 and M'_2 coincide and the path difference between the two rays becomes zero. Now the field of view is perfectly dark. When M_1 is further moved, the fringes appear again.

If M_1 and M_2 are not perfectly perpendicular, a wedge shaped film will be formed between M_1 and M'_2 then we get almost straight line fringes of equal thickness in the field of view of telescope, as the radius of fringes is very large.

All the above discussed films are shown below in Fig. 1.27

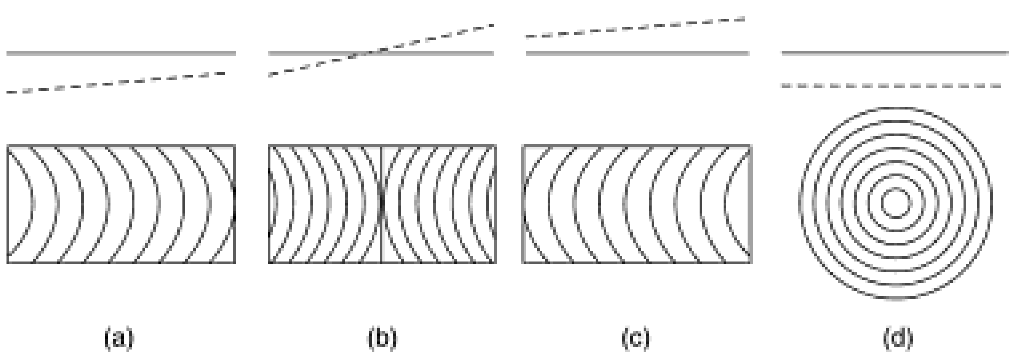


Figure 1.27

1.11.3.1 Applications

Michelson's interferometer uses the concept of interference that takes place with the help of two mirrors. The distance between one mirror and the image of another plays an important role in the formation of fringes. Michelson's interferometer has diverse applications, some of which are listed below.

(i) Determination of Wavelength of Light

First of all the Michelson's interferometer is set for circular fringes with central bright spot, which is possible

(M_1 and M'_2) and n be the order of the spot obtained, then for normal incidence $\cos r = 1$, we have

$$2t + \frac{\lambda}{2} = n\lambda$$

or $$2t = \left(n - \frac{1}{2}\right)\lambda$$

If M_1 is moved $\frac{\lambda}{2}$ away from M'_2, then an additional path difference of λ will be introduced and hence $(n+1)^{th}$ bright spot appears at the centre of the field. Thus each time M_1 moves through a distance $\frac{\lambda}{2}$, a new bright fringe appears. Therefore, if M_1 moves by a distance x (x_1 to x_2) and N new fringes appear at the centre of the field, then we have

$$x = x_2 - x_1 = N\frac{\lambda}{2}$$

or $$\lambda = \frac{2(x_2 - x_1)}{N} = \frac{2x}{N} \qquad\qquad \lambda = \frac{2x}{N}$$

The difference $(x_2 - x_1)$ is measured with the help of micrometer screw and N is actually counted. The experiment is repeated for number of times and the mean value of λ is obtained.

(ii) Determination of Difference in Wavelengths

Michelson's interferometer is adjusted in order to obtain the circular fringes. Let the source be not monochromatic and have two wavelengths λ_1 and λ_2 ($\lambda_1 > \lambda_2$) which are very close to each other (as Sodium D lines). The two wavelengths form their separate fringe patterns but as λ_1 and λ_2 are very close to each other and thickness of air film is small, the two patterns practically coincide with each other. As the mirror M_1 is moved slowly, the two patterns separate slowly and when the thickness of air film is such that the dark fringe of λ_1 falls on bright fringe of λ_2, the result is maximum indistinctness. Now the mirror M_1 is further moved, say through a distance x, so that the next indistinct position is reached. In this position, if n fringes of λ_1 appear at the centre, then $(n+1)$ fringes of λ_2 should appear at the centre of the field of view. Hence

$$x = n\frac{\lambda_1}{2} \text{ and } x = (n+1)\frac{\lambda_2}{2}$$

or $$n = \frac{2x}{\lambda_1} \qquad \text{(i)}$$

and $$(n+1) = \frac{2x}{\lambda_2} \qquad \text{(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$(n+1) - n = \frac{2x}{\lambda_2} - \frac{2x}{\lambda_1}$$

or $$1 = \frac{2x(\lambda_1 - \lambda_2)}{\lambda_1\lambda_2}$$

or $$(\lambda_1 - \lambda_2) = \frac{\lambda_1\lambda_2}{2x} = \frac{\lambda_{av}^2}{2x}$$ where $\lambda_1 \lambda_2 = \lambda_{av}^2$ is the square of mean of λ_1 and λ_2.

Thus measuring the distance x moved by mirror M_1 between the two consecutive positions of maximum indistinctness, the difference between two wavelengths of the source can be determined, if λ_{av} is known.

on the central bright fringe. Now insert thin transparent plate in the path of one of the interfering waves. On the inclusion of a plate of thickness t and refractive index μ, the path difference is increased by a factor of $2(\mu - 1)t$. The fringes are therefore shifted. The mirror M_1 is now moved till the central fringe is again brought back to its initial position. The distance x traveled by the mirror M_1 is measured by micrometer. Therefore

$$2x = 2(\mu - 1)t \text{ or } t = \frac{x}{(\mu - 1)} \quad \text{(iii)}$$

From Eq. (iii), we can write

$$\mu = \frac{x}{t} + 1 \quad \text{(iv)}$$

Thus, by knowing the thickness of the transparent sheet and the distance x, we can calculate the refractive index of the sheet with the help of a Michelson's interferometer.

1.12 APPLICATIONS OF INTERFERENCE IN THE FIELD OF ENGINEERING

The phenomenon of interference arises in many situations and the scientists and engineers have taken advantage of interference in designing and developing various instruments.

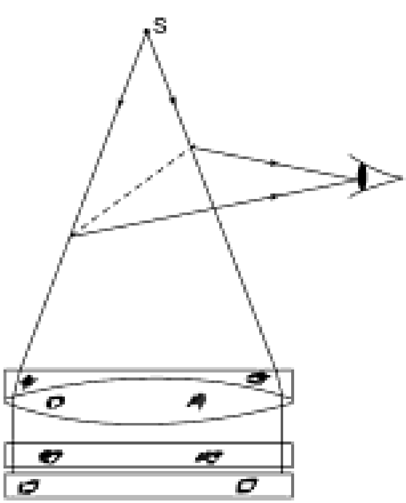
1.12.1 Testing of Optical Flatness of Surfaces

An example of the application of interference method is the testing of optical components for surface quality. The most important example is that of optical flats. However, the methods used for flat surfaces can be adapted simply to test spherical surfaces.

1.12.1.1 Flatness Interferometers

With these interferometers we can compare the flatness of two surfaces by placing them in contact with slight wedge of air between them. This gives a tilt and thus the fringes start originating like that of Newton's ring between the two surfaces. To get half wavelength contours of the space between the surfaces, they should be viewed from infinity. Further, to avoid the risk of scratching, a desirable distance should be there between the two surfaces. Most common examples of flatness interferometers are *Fizeau* and *Twyman interferometers*.

(i) Fizeau Interferometer



In this type of interferometer, the sources and viewing point are kept at infinity (Fig. 1.28). This interferometer generates interference between the surface of a test sample and a reference surface that is brought close to the test sample. The interference images are recorded and analysed by an imaging optic system. However, the contrast and the shape of the interference signals depend on the reflectivity of the test samples.

(ii)Twyman-Green Interferometer

This is an important instrument used to measure defects in optical components such as lenses, prisms, plane parallel windows, laser rods and plane mirrors. Twyman-Green interferometer, shown in Fig. 1.29 resembles Michelson interferometer in the beam splitter and mirror arrangement. However, the difference lies in the way of their a monochromatic point source which is located at the principal focus of a well-corrected lens whereas in Michelson interferometer an extended source is used. If the mirrors M_1 and M_2 are perpendicular to each other and the beam-splitter BS makes an angle of 45° with the normal of each mirror, then the interference is exactly analogous to thin film interference at normal incidence. Therefore, completely constructive interference is obtained when $d = m\lambda/2$, where d is the path difference between the two arms adjusted by translating the mirror M_1. The complete destructive interference is obtained when $d = (m+1/2)\,\lambda/2$. With the help of rotation of mirror M_2 we can see fringes of equal thickness on the screen, as the angle of incidence is constant. This situation is analogous to interference pattern observed with collimated light and a thin film with varying thickness. In order to test the optical components, one of the mirrors is intentionally tilted to create fringes. Then the quality of the component can be determined from the change in the fringe pattern when the component is placed in the interferometer. Lens testing is specifically important for quantifying aberrations and measuring the focal length.

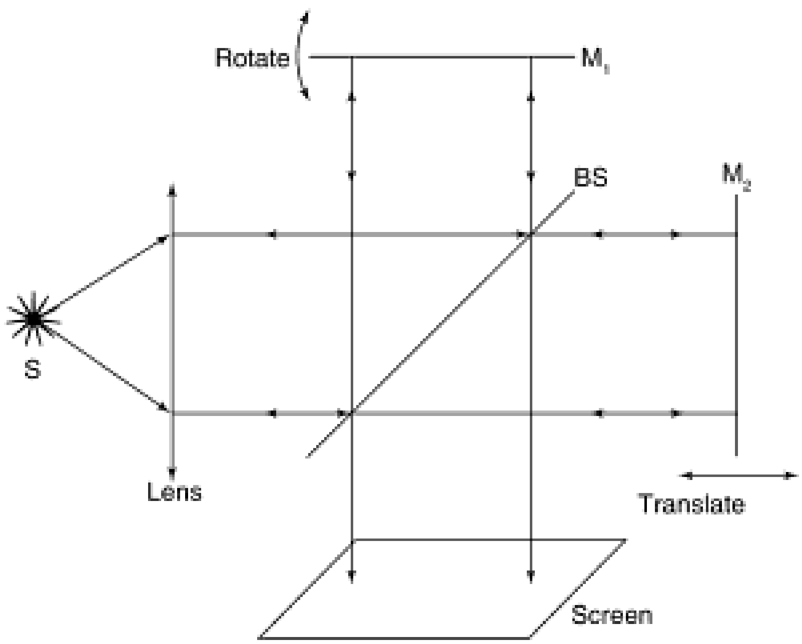


Figure 1.29

1.12.2 Nonreflecting or Antireflecting (AR) Coatings

Interference-based coatings were invented in November 1935 by *Alexander Smakula*, who was working for the *Carl Zeiss optics company*. Antireflecting coatings are a type of optical coatings. These are applied to the surface of lenses and other optical devices for reducing reflection. This way the efficiency of the system gets improved since less light is lost. For example, in a telescope the reduction in reflections improves the contrast of the image by elimination of stray light. In another applications a coating on eyeglass lenses makes the eyes of the wearer more visible. The anti-reflecting coatings can be mainly divided into three groups.

1.12.2.1 Single-layer Interference Coatings

The simplest interference non-reflecting coating consists of a single quarter-wave layer of transparent material. The refractive index of this material is taken to be equal to the square root of the substrate's refractive index. This theoretically gives zero reflectance at the center wavelength and decreased reflectance for wavelengths in a broad band around the center. The use of an intermediate layer to form an antireflection coating can be thought of as analogous to the technique of impedance matching of electrical signals. A similar method is used in fibre optic research where an index matching oil is sometimes used to temporarily defeat total internal reflection so that light may be coupled into or out of a fiber.

The antireflection coatings rely on an intermediate layer not only for its direct reduction of reflection coefficient, but also use the interference effect of a thin layer. If the layer thickness is controlled precisely and it is made exactly one quarter of the light's wavelength ($\lambda/4$), then it is called a quarter-wave coating (Fig. 1.30). In this case, the incident beam I, when reflected from the second interface, will travel exactly half its own wavelength further than the beam reflected from the first surface. The two reflected beams R_1 and R_2 will destructively interfere as they are exactly out of phase and cancel each other if their intensities are equal. Therefore, the reflection from the surface is suppressed and all the energy of the beam is propagated through the transmitted beam T. In the calculation of the reflection from a stack of layers, the transfer-matrix method can be used.

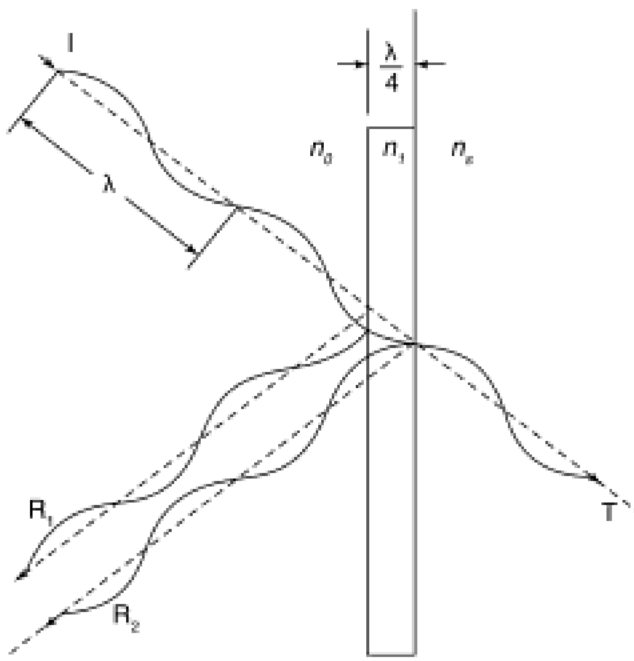


Figure 1.30

1.12.2.2 Multilayer Coatings or Multicoating

Multiple coating layers can also be used for *reflection reduction*. It is possible if we design them such that the reflections from the surfaces undergo maximum destructive interference. This can be done if we add a second quarter-wave thick higher-index layer between the low-index layer (for example, silica) and the substrate. Under this situation, the reflection from all three interfaces produces destructive interference and antireflection. Optical coatings can also be made with near-zero reflectance at multiple wavelengths or optimum performance at angles of incidence other than 0°.

1.12.2.3 Absorbing Antireflecting Coatings

Absorbing antireflecting coatings are an additional category of antireflection coatings. These coatings are useful in situations where *low reflectance* is required and high transmission through a surface is unimportant or undesirable. They can produce very low reflectance with few layers. They can often be produced more cheaply or at greater scale than standard non-absorbing anti-reflecting coatings. In sputter deposition system for such films, titanium nitride and niobium nitride are frequently used.

1.12.2.4 Practical Problems with AR Coatings

Real coatings do not reach perfect performance, though they are capable of reducing a surface's reflection coefficient to less than 0.1%. Practical details include correct calculation of the layer thickness. This is because the wavelength of the light is reduced inside a medium and this thickness will be $\lambda_0/4n_1$, where λ_0 is the vacuum wavelength and n_1 is the refractive index of the film. Finding suitable materials for use on ordinary glass is also another difficulty, since few useful substances have the required refractive index ($n \approx 1.23$) which will make both reflected rays exactly equal in intensity. Since magnesium fluoride (MgF_2) is hard-wearing and can be easily applied to substrates using *physical vapour deposition*, it is often used for this purpose even though its index is higher than desirable ($n = 1.38$).

1.13 SCIENTIFIC APPLICATIONS OF INTERFERENCE

In interferometry, we use the principle of superposition to combine different waves in a way that will cause the result of their combination to have some meaningful property, that is indicative of the original state of the waves. The phenomenon of interference is employed under various situations for its scientific applications. For a better understanding of the applications, we first need to know about the homodyne and heterodyne detections.

1.14 HOMODYNE AND HETERODYNE DETECTION

In standard interferometry, the interference occurs between the two beams at the same wavelength (or *carrier frequency*). The phase difference between the two beams results in a change in the intensity of the light on the detector. Measuring the resulting intensity of the light after the mixing of these two light beams is known as *homodyne detection*. In heterodyne detection, we modulate one of the two beams prior to detection, usually by a frequency shift. A special case of heterodyne detection is optical heterodyne detection, which detects the interference at the beat frequency.

1.14.1 Imaging Interferometry

In this interferometry, the pattern of radiation across a region can be represented as a function of position $i(x,y)$, i.e. an image and the pattern of incoming radiation $i(x,y)$ can be transformed into the Fourier domain $f(u,v)$. A single detector measures information from a single point in (x,y) space. An interferometer measures the difference in phase between two points in the (x,y) domain. This corresponds to a single point in the (u,v) domain. An interferometer builds up a full picture by measuring multiple points in (u,v) space. The image $i(x,y)$ can then be restored by performing an inverse Fourier transform on the measured $f(u,v)$ data.

1.14.2 Holographic Interferometry (HI)

Holographic interferometry (HI) is a technique that enables static and dynamic displacements of objects with optically rough surfaces to be measured to optical interferometric precision, i.e. to fractions of a wavelength of light. These measurements can be applied to stress, strain and vibration analysis, as well as to nondestructive testing. It can also be used to detect optical path length variations in transparent media, which enables, for example, fluid flow to be visualised and analysed. It can also be used to generate contours representing the form of the surface. Holography interferometry is of two types.

(i) Live Holography Interferometry

Holography enables the light field scattered from an object to be recorded and replayed. If this recorded field is superimposed on the "live field" scattered from the object, then the two fields will be identical. However, if a small deformation is applied to the object, the relative phases of the two light fields will alter and it is possible to observe interference. This technique is known as live holographic interferometry.

(ii) Frozen-Fringe Holography

In this holography, it is also possible to obtain fringes by making two recordings of the light field scattered from the object on the same recording medium. The reconstructed light fields may then interfere to give fringes, which map out the displacement of the surface.

1.14.3 Electronic Speckle Pattern Interferometry

Electronic Speckle Pattern Interferometry (ESPI), also known as TV Holography, is a technique that uses laser light together with video detection, recording and processing to visualize static and dynamic displacements of components with optically rough surfaces. The visualisation is in the form of fringes on the image where each fringe normally represents a displacement of half a wavelength of the light used, i.e. quarter of a micrometre or so.

1.14.4 Angle Resolved Low Coherence Interferometry

Angle resolved low coherence interferometry is an emerging biomedical imaging technology that uses the properties of scattered light to measure the average size of cell structures, including the cell nuclei. The technology shows promise as a clinical tool for *in situ* detection of dysplatic or precancerous tissue.

1.14.5 Optical Coherence Tomography

This is a medical imaging technique based on low-coherence interferometry, where subsurface light reflections are resolved to give tomographic visualisation. Recent advances have struggled to combine the nanometre phase retrieval with the ranging capability of low-coherence interferometry.

1.14.6 Geodetic Standard Baseline Measurements

A famous use of white light interferometry is the precise measurement of geodetic standard baselines. Here the light path is split in two, and one leg is folded between a mirror pair 1 m apart. The other leg bounces once off a mirror 6 m away. The fringes will be seen only if the second path is precisely 6 times the first. Starting from a standard quartz gauge of 1 m length, it is possible to measure distances up to 864 m by repeated multiplication. Baselines thus established are used to calibrate geodetic distance measurement equipments. This leads to a metrologically traceable scale for geodetic networks measured by these instruments. More modern geodetic applications of laser interferometry are in calibrating the divisions on levelling staffs and in monitoring the free fall of a reflective prism within a ballistic or absolute gravimeter. This allows determination of gravity, i.e., the acceleration of free fall, directly from the physical definition at a few parts in a billion accuracy.

1.14.7 Interference Lithography

This is a technique for patterning regular arrays of fine features, without the use of complex optical systems or more coherent light waves is set up and recorded in a recording layer. This interference pattern consists of a periodic series of fringes of representing intensity maxima and minima. The benefit of using interference lithography is the quick generation of dense features over a wide area without loss of focus.

1.15 SUMMARY

We summarise the main outcome of the chapter as below.

(1) We first discussed the phenomenon of interference and then explained it based on Young's double slits experiment.

(2) Concepts of wavefront and secondary wavelets were discussed based on Huygens' principle. Then secondary wavefront was introduced as the surface touching the secondary wavelets tangentially in the forward direction at any given time.

(3) Phase difference and path difference between the two waves play a key role for obtaining constructive or destructive interference. Therefore, phase and path differences were explained in detail together with their relation.

(4) For obtaining sustained interference pattern, the two sources should be coherent. So the concept of coherence, both temporal and spatial, was introduced and coherence time and coherence length were talked about.

(5) A short description of a technique for producing coherent light from incoherent sources was given.

(6) Analytical treatment of the interference was discussed where conditions were obtained for the constructive and destructive interferences.

(7) When a light wave gets reflected from a surface, a phase change may take place. Therefore, condition of relative phase shift was explained.

(8) Superposition was extended for n number of waves and it was observed that the resultant amplitude increases in proportion with $\sqrt{n}$ in length as n gets increased.

(9) Interfering waves can be produced either by division of wavefront or by division of amplitude. Therefore, the details of interference were discussed based on these two methods.

(10) Fresnel's biprism, which is used to create two virtual coherent sources, was discussed in detail for obtaining interference pattern and the related conditions for dark and bright fringes.

(11) Application of biprism for the determination of wavelength of light, distance between two virtual coherent sources and thickness of transparent sheet were discussed. The displacement of fringes by the introduction of thin transparent sheet in the path of one light wave was also explored.

(12) Thin films are used for the division of amplitude of light waves which superimpose each other. The interference pattern obtained by thin films of uniform and non-uniform thicknesses was investigated.

(13) When the air film is created between the curved surface of a plano-convex lens and the flat surface of a mirror, the interference takes place between the reflected as well as the transmitted light. Here the fringes are obtained in the form of rings known as Newton's rings.

(14) Newton's rings method was used for determination of the wavelength of light, radius of curvature of a plano-convex lens and the refractive index of liquid.

(16) Theory and practical applications of Michelson's interferometer were discussed. Clarification of path difference and the details of formation of fringes were given.

(17) Engineering applications of interference were included, particularly related to the testing of optical flatness of surfaces and nonreflecting or antireflecting coatings.

(18) Finally the scientific applications of interference were discussed related to various interferometry, tomography and lithography.

SOLVED EXAMPLES

Example 1 If light of wavelength 660 nm has wave trains 13.2×10^{-6}m long, what would be the coherence time.

Solution Given $\lambda = 6.6 \times 10^{-7}$m, coherence length ($\Delta L$) = 1.32×10^{-5}m and coherence time (Δt) = ?

Formula used is $\Delta L = c.\Delta t$

or $$\Delta t = \frac{\Delta L}{c} = \frac{1.32 \times 10^{-5}}{3 \times 10^{8}} = \mathbf{4.4 \times 10^{-14} sec}$$

Example 2 Coherence length of a light is 2.945×10^{-2} m and its wavelength is 5896 Å. Calculate the coherence time and number of oscillations corresponding to the coherence length.

Solution Given $\Delta L = 2.945 \times 10^{-2}$m and $\lambda = 5.896 \times 10^{-7}$. Δt = ?

Formula used is $\Delta L = c\Delta t$

or $$\Delta t = \frac{\Delta L}{c} = \frac{2.945 \times 10^{-2}}{3 \times 10^{8}} = \mathbf{9.816 \times 10^{-11} sec}$$

and number of oscillations in a length L,

$$n = \frac{\Delta L}{\lambda} = \frac{2.945 \times 10^{-2}}{5.896 \times 10^{-7}} = \mathbf{4.99 \times 10^{4}}$$

Example 3 Calculate the line-width, coherence time and frequency stability for a line of Krypton having a wavelength of 6.058×10^{-7}m and coherence length as 0.2m.

Solution Given $\lambda_{av} = 6.058 \times 10^{-7}$m, $\Delta L = 0.2$m and $c = 3 \times 10^{8}$m/sec.

In Michelson's Interferometer we derived the following formula

$$1 = \frac{2x}{\lambda_2} - \frac{2x}{\lambda_1}$$

where x is the distance between two mirrors. The above expression can be written as

$$2x = \frac{\lambda_1\lambda_2}{\Delta\lambda} = \frac{\lambda^2{av}}{\Delta\lambda}$$

where λ_{av} is the mean wavelength of λ_1 and λ_2. Here $\Delta\lambda$ is called **line width**.

In view of the fact that the fringes are not observed if the path difference exceeds the coherence length ΔL, we can assume the beam to contain all wavelengths lying between λ and $(\lambda + d\lambda)$.

Therefore, $2x = \Delta L = \dfrac{\lambda^2 av}{\Delta\lambda}$

or $\Delta\lambda = \dfrac{\lambda^2 av}{\Delta L}$

$\because$ frequency, $\nu = \dfrac{c}{\lambda}$

$\therefore$ $\left[\dfrac{\Delta\nu}{\Delta\lambda}\right] = \dfrac{c}{\lambda^2 av}$

or $\Delta\nu = \dfrac{c}{\lambda^2 av}\Delta\lambda = \dfrac{c}{\Delta L}$

Here, Δv is called frequency spread of the line, which can be written in terms of Δt as follows.

$$\Delta\nu = \frac{1}{\Delta t} \text{ or } \Delta t = \frac{1}{\Delta\nu}$$

In addition, frequency stability is defined as the ratio of frequency spread and frequency of any spectral line, i.e.,

$$\text{frequency stability} = \frac{\Delta\nu}{\nu}$$

Line width $\Delta\lambda = \dfrac{\lambda^2 av}{\Delta L} = \dfrac{(6.058 \times 10^{-7})^2}{0.2} = \mathbf{1.834 \times 10^{-12}\ m}$

Frequency spread

$$\Delta\nu = \frac{c}{\Delta L} = \frac{3 \times 10^8}{0.2} = 1.5 \times 10^9 \text{ Hz}$$

Frequency

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{6.058 \times 10^{-7}}$$

$$= \mathbf{4.952 \times 10^{14}\ Hz}$$

and Frequency stability

$$= \frac{\Delta\nu}{\nu} = \frac{1.5 \times 10^9}{4.952 \times 10^{14}} = \mathbf{3.0 \times 10^{-7}}$$

Example 4 The Doppler width for an orange line of Krypton is 550×10^{-15}m. If the wavelength of light is 605.8nm, calculate the coherent length.

Solution Given Doppler line width ($\Delta\lambda$) = 5.5×10^{-13}m.

$\lambda = \lambda_{av} = 6.058 \times 10^{-7}$m, $\Delta L = ?$

Formula used is $\Delta L = \dfrac{\lambda^2 av}{\Delta\lambda} = \dfrac{(6.058 \times 10^{-7})^2}{5.5 \times 10^{-13}}$

$= \mathbf{0.6673\ m}$

Example 5 A mercury vapour lamp emits a light of wavelength 5461 Å with a band width of 6×10^8 Hz. Calculate the ratio of its coherence length with the coherence length of a He−Ne laser operating at a wavelength 6328 Å with a band width of 10^6 Hz.

Solution For mercury vapour lamp, $\lambda_{av} = 5.461 \times 10^{-7}$m, $\Delta\nu = 6 \times 10^8$ Hz

Formula used is $\left[\frac{\Delta\nu}{\Delta\lambda}\right] = \frac{c}{\lambda^2_{av}}$ or $\Delta\lambda = \frac{\lambda^2_{av}\ \Delta\nu}{c}$

or $$\Delta\lambda = \frac{(5.461\times10^{-7})^2\times6\times10^8}{3\times10^8}$$

$$= 5.964 \text{ x } 10^{-13}\text{m}$$

The coherence length is given by

$$\Delta L_1 = \frac{\lambda^2_{av}}{\Delta\lambda} = \frac{(5.641\times10^{-7})^2}{5.964\times10^{-13}} = 0.534 \text{ m}$$

For He – Ne laser,

Given $\lambda_{av} = 6.328 \text{ x } 10^{-7}$m, $\Delta\nu = 10^6$ Hz

$$\Delta\lambda = \frac{\lambda^2_{av}\ \Delta\nu}{c} = \frac{(6.328\times10^{-7})^2\ 10^6}{3\times10^8}$$

$$\Delta\lambda = 1.335\times10^{-15}\text{m}$$

Coherence length (for laser)

$$\Delta L_2 = \frac{\lambda^2_{av}}{\Delta\lambda} = \frac{(6.328\times10^{-7})^2}{1.335\times10^{-15}} = 299.952\text{m}.$$

$$\therefore \quad \frac{\Delta L_1}{\Delta L_2} = \frac{0.534}{299.952} = \frac{1}{562} = \mathbf{1:562}$$

Example 6 Find the coherence length of a laser beam for which the band width is 3000 Hz.

Solution Given $\Delta\nu = 3000$ Hz.

Coherence length $(\Delta L) = c\Delta t$ and coherence time $(\Delta t) = \frac{1}{\Delta\nu}$

So $$\Delta t = \frac{1}{3000} = 3.333\times10^{-4}\text{ sec}$$

$$\Delta L = c\times\Delta t = 3\times10^8\times3.333\times10^{-4} = \mathbf{1.0\times10^5 m}$$

Example 7 Calculate the resultant line-width, band width and coherence length assuming that we chop a continuous perfectly monochromatic beam of wavelength 6328 Å in 10^{-10} seconds using some sort of shutter.

Solution Given $\lambda_{av} = 6.328\times10^{-7}$m and $\Delta t = 10^{-10}$ sec

Coherence length, $\Delta L = c\Delta t = 3\times10^8\times10^{-10} = 3\times10^{-2}$ m

$$\text{Band - width, } \Delta\nu = \frac{1}{\Delta t} = \frac{1}{10^{-10}} = 10^{10}\text{Hz}$$

$$\text{Line - width, } \Delta\lambda = \frac{\lambda^2_{av}}{c}\Delta\nu = \frac{(6.328\times10^{-7})^2\times10^{10}}{3\times10^8}$$

$$= 1.335\times10^{-11}\text{m}$$

$$= \mathbf{0.1335\ Å}$$

Example 8 For a red cadmium line of wavelength 6438 Å and coherence length 38cm deduce the order of

Solution Given coherence length, $\Delta L = 0.38$ m and $\lambda_{av} = 6.438 \times 10^{-7}$m

Coherence time Δt = ?

spectral line width $\Delta\lambda$ = ?

$$\Delta L = c\,\Delta t \text{ or } \Delta t = \frac{\Delta L}{c}$$

or
$$\Delta t = \frac{0.38}{3\times 10^{8}} = 1.266\times 10^{-9} \text{ sec}$$

$$\Delta L = \frac{\lambda^2_{av}}{\Delta\lambda} \text{ or } \Delta\lambda = \frac{\lambda^2_{av}}{\Delta L} = \frac{(6.438\times 10^{-7})^2}{0.38}$$

$$\Delta\lambda = \mathbf{1.09\times 10^{-12}\,m}$$

Example 9 The ratio of intensities of two waves that produce interference pattern is 16:1. Deduce the ratio of maximum to minimum intensities in fringe system.

Solution Given $I_1 : I_2 = 16 : 1$

The intensity, $I \propto a^2$

$$\therefore a^2_1 : a^2_2 = 16 : 1 \quad \text{or} \quad a_1 : a_2 = 4 : 1$$

or
$$a_1 = 4a_2$$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(4a_2 + a_2)^2}{(4a_2 - a_2)^2} = \frac{25}{9}$$

i.e.
$$I_{max} : I_{min} = 25 : 9$$

Example 10 Distance between two slits is 0.1mm and the width of the fringes formed on the screen is 5mm. If the distance between the screen and the slit is one meter, what would be the wavelength of light used?

Solution Given $\beta = 5.0 \times 10^{-3}$m, $2d = 1.0 \times 10^{-4}$m and $D = 1.0$m

Formula used is
$$\lambda = \frac{\beta\, 2d}{D} = \frac{5\times 10^{-3}\times 1.0\times 10^{-4}}{1.0} = 5000\text{Å}$$

$$\boldsymbol{\lambda = 5000\ \text{Å}}$$

Example 11 A biprism of angle 1°and refractive index 1.5 is at a distance of 40 cm from the slit. Find the fringe width at 60cm from the biprism for sodium light of wavelength 5893 Å.

Solution Given $a = 0.4$ cm, $\mu = 1.5$, $\lambda = 5.893 \times 10^{-7}$m and $D = 1.0$m

From the Fig. 1.31
$$\delta = \frac{d}{a} \text{ or } d = a\delta$$

or
$$2d = 2a\delta$$

The deviation produced in the incident light is given by

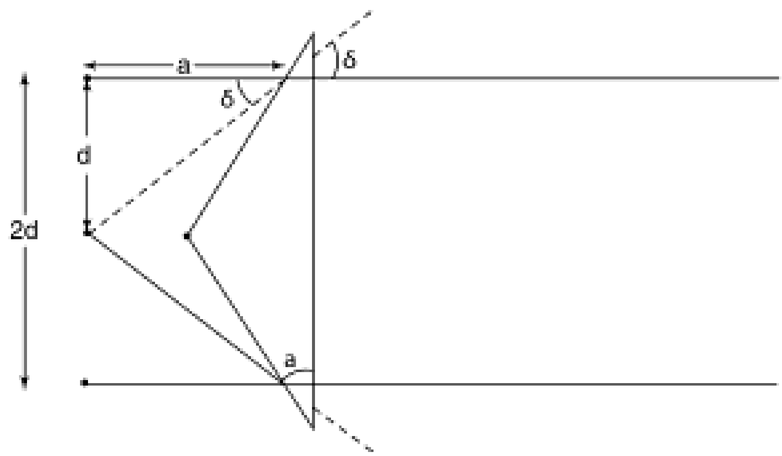
$$\delta = (\mu - 1)\,\alpha$$

$$2d = 2a\,(\mu - 1)\alpha$$

$$1^\circ = \frac{\pi}{180}\ \text{rad}$$

Fringe width $\beta = \dfrac{\lambda D}{2d} = \dfrac{5.893 \times 10^{-7} \times 1.0}{2 \times 0.4(1.5-1)\pi/180} = \dfrac{5.893 \times 10^{-7} \times 180}{2 \times 0.4 \times 0.5 \times 3.14}$

$= \mathbf{0.0844 \times 10^{-3} m}$



Example 12 Interference fringes are produced by Fresnel's bi-prism on the focal plane of a reading microscope which is 1.0m far from the slit. A lens interposed between the biprism and the microscope gives two images of the slit in two positions. If the images of the slits are 4.05mm apart in one position and 2.90mm apart in the other position and the wavelength of the sodium light is 5893 Å, find the distance between the consecutive interference bands?

Solution Given $\lambda = 5.893 \times 10^{-7}$m, D = 1.0m, $d_1 = 4.05 \times 10^{-3}$m and $d_2 = 2.90 \times 10^{-3}$m

Formula used is $2d = \sqrt{d_1 d_2} = \sqrt{4.05 \times 2.90 \times 10^{-6}}$

or $2d = 3.427 \times 10^{-3}$m

Now $\beta = \dfrac{\lambda D}{2d} = \dfrac{5.893 \times 10^{-7} \times 1.0}{3.427 \times 10^{-3}}$

$\beta =$ **0.172mm**

Example 13 In a biprism experiment fringes were first obtained with the sodium light of wavelength 5890 Å and fringe width was measured to be 0.342 mm. Sodium light was then replaced with white light and central fringe was located. On introducing a thin glass sheet in half of the beam, the central fringe was shifted by 2.143mm. Calculate the thickness of the glass sheet if the refractive index of glass is 1.542.

Solution Given $\lambda = 5890 \times 10^{-10}$m, $\mu = 1.542$, $x_n = 2.143 \times 10^{-3}$m

$\beta = 3.42 \times 10^{-4}$m

Formula used is $x_n = n\beta$

or $n = \dfrac{x_n}{\beta} = \dfrac{2.143 \times 10^{-3}}{3.42 \times 10^{-4}} = 6.266$

$\because \quad (\mu - 1)t = n\lambda$

$$\text{or } t = \frac{n\lambda}{(\mu-1)} = \frac{6 \times 5890 \times 10^{-10}}{0.542}$$

$t = \mathbf{6.52 \times 10^{-6} m.}$

Example 14 Biprism is kept 10cm away from the slit illuminated by monochromic light of $\lambda = 5896$ Å. The width of the fringes obtained on a screen placed at a distance of 90cm from the biprism is 9.0×10^{-4} m. What is the distance between two coherent sources?

Solution Given a = 0.10m, b = 0.90m and D = a + b = 1.0 m

$\lambda = 5.896 \times 10^{-7}$m, $\beta = 9.0 \times 10^{-4}$ m

Formula used is $$2d = \frac{\lambda D}{\beta} = \frac{5.896 \times 10^{-7} \times 1.0}{9 \times 10^{-4}}$$

or $2d = \mathbf{6.55 \times 10^{-4} m}$

Example 15 The distance between the slit and biprism and between biprism and screen are 50cm each, Angle of biprism and refractive index are 179° and 1.5, respectively. Calculate the wavelength of light used if the distance between two successive fringes is 0.0135 m.

Solution Given $\beta = 0.0135$m, $\alpha = b = 0.5$m and D = a + b = 1

$$\mu = 1.5,\ A = 179^\circ,\ \alpha = \frac{180 - A}{2} = \left(\frac{1}{2}\right)^\circ \times \frac{\pi}{180} = \frac{\pi}{360} \text{ rad}$$

Formula used is $$\lambda = \frac{2a(\mu-1)\alpha}{D}\beta = \frac{2 \times 0.50 \times (1.5-1)}{1.0} \times \frac{\pi}{360} \times 0.013$$

$\lambda = \mathbf{5893}$ **Å**

Example 16 The distance between the slit and biprism and between biprism and eyepiece are 45 cm each. The obtuse angle of biprism is 178° and its refractive index is 1.5. If the fringe width is 15.6×10^{-3}cm, find the wavelength of light used.

Solution $$\beta = \frac{\lambda D}{2d} \text{ or } \lambda = \frac{\beta(2d)}{D}$$

Given $a = 45$ cm, D = 90cm, $\mu = 1.5$, $\alpha = 1° = \pi/180$ rad

$\beta = 15.6 \times 10^{-3}$cm

$2d$ can be calculated by the relation

$$2d = 2a\,(\mu - 1)\alpha$$

$$= 2 \times 45 \times 0.5 \times (22/7) \times (1/180) = 0.786$$

$$\lambda = \frac{\beta(2d)}{D} = \frac{15.6 \times 10^{-3} \times 0.786}{90}$$

$= 13624 \times 10^{-8}$ cm

$= \mathbf{13624}$ **Å**

Example 17 In a biprism experiment, the eye piece was placed at a distance of 120 cm from the source.

Solution Given $x_n = 1.9$ cm, $n = 20$, D = 120 cm and $2d = 0.06$ cm.

Formula used is $\beta = \dfrac{x_n}{n} = \dfrac{1.9}{20} = 0.095$ cm and $\beta = \dfrac{\lambda D}{2d}$

or $\lambda = \dfrac{\beta\, 2d}{D} = \dfrac{0.095 \times 0.06}{120}$

$\lambda =$ **4750 Å**

Example 18 In a biprism experiment using light of wavelength 5890 Å, 40 fringes are observed in the field of view. If this light is replaced by light of wavelength 4358Å. Calculate how many fringes are observed in the field of view.

Solution Given $\lambda_1 = 5890$ Å, $N_1 = 40$ and $\lambda_2 = 4358$ Å, $N_2 = ?$

$$x = N_1\beta_1 = N_1\frac{\lambda_1 D}{2d} = N_2\frac{\lambda_2 D}{2d}$$

$\therefore$ $N_1\lambda_1 = N_2\lambda_2$

$40 \times 5890 \times 10^{-10} = N_2 \times 4358 \times 10^{-10}$

$N_2 =$ **54**

Example 19 Light of wavelength 5893Å is reflected at normal incidence from a soap film of refractive index 1.42. What is the least thickness of the film that will appear (a) bright and (b) dark?

Solution Given $\lambda = 5.893 \times 10^{-7}$m, $i = r = 0$, $\mu = 1.42$ and for smallest thickness $n = 1$

Condition for thin film to appear bright in reflected light is

$$2\mu t \cos r = (2n - 1)\,\lambda/2$$

or $t = \dfrac{(2n-1)\lambda/2}{2\mu\cos r} = \dfrac{(2-1)\times 5.893\times 10^{-7}}{2\times 1.42\times 2\times 1}$

= **1.038×10^{-4} mm**

Similarly condition for thin film to appear to dark in reflected light is

$$2\mu t \cos r = n\lambda$$

or $t = \dfrac{n\lambda}{2\mu\cos r} = \dfrac{1\times 5.893\times 10^{-7}}{2\times 1.42\times 1}$

= **2.075×10^{-4} mm**

Example 20 A parallel beam of light strikes an oil film ($\mu = 1.4$), floating on a surface of water ($\mu = 1.33$). When viewed at an angle of 30° from the normal 6[th] dark fringe is seen. Find the thickness of the film. (Given wavelength of light = 589 nm).

Solution Given $\lambda = 5.89 \times 10^{-7}$m, $\mu_{oil} = 1.4$, $i = 30°$ and $n = 6$

$$\mu_{oil} = \frac{\sin i}{\sin r} \text{ or } 1.4 = \frac{\sin 30^\circ}{\sin r} \text{ or } \sin r = \frac{0.5}{1.4}$$

or $\sin r =$ **0.3571**

$\therefore$ $\sin^{-1}(0.3571) = 20.92°$

For n^{th} order dark fringe in reflected light, the condition is

$$2\mu t \cos r = n\lambda \text{ or } t = \frac{n\lambda}{2\mu \cos r}$$

$$t = \frac{6 \times 5.89 \times 10^{-7}}{2 \times 1.4 \times 0.934}$$

$$= \mathbf{1.351 \times 10^{-3} mm.}$$

Example 21 Calculate the thickness of a soap film (μ = 1.463) that will result in constructive interference in the reflected light, if the film is illuminated normally with light whose wavelength in free space is 6000Å.

Solution Given $\lambda = 6.0 \times 10^{-7}$ m, μ = 1.463, for normal incidence $i = r = 0°$ and for smallest thickness n = 1.

For constructive interference $2\mu t \cos r = (2n - 1)\lambda/2$

$$t = \frac{(2n-1)\lambda}{2 \times 2 \times 1.463 \times 1} = \frac{(2-1) \times 6.0 \times 10^{-7}}{4 \times 1.463}$$

$$= \mathbf{1.025 \times 10^{-4} mm.}$$

Example 22 A parallel beam of sodium light (λ = 5890Å) strikes a film of oil floating on water. When viewed at an angle of 30° from the normal, θ^{th} dark band is seen. Determine the thickness of the film. (Refractive index of oil = 1.46).

Solution Given $\lambda = 5.89 \times 10^{-7}$m, i = 30°, μ = 1.46 and n = 8

Condition for obtaining dark band is $2\mu t \cos r = n\lambda$ (i)

or $$t = \frac{n\lambda}{2\mu t \cos r} \quad \text{(ii)}$$

As we know, $$\mu = \frac{\sin i}{\sin r} \quad \text{(iii)}$$

or $$r = \frac{\sin i}{\mu}$$

or $$\sin r = \frac{\sin 30°}{1.46} = \frac{1}{2.92}$$

or $$\cos r = \sqrt{1 - \sin^2 r}$$

$$= \sqrt{1 - \left(\frac{1}{2.92}\right)^2}$$

$$= 0.94$$

By using Eq. (ii), we get

$$t = \frac{8 \times 5.89 \times 10^{-7}}{2 \times 1.46 \times 0.94}$$

$$= \mathbf{1.72 \times 10^{-3} mm}$$

Example 23 White light is reflected from an oil film of thickness 0.01 mm and refractive index 1.4 at an angle of 45° to the vertical. If the reflected light falls on the slit of a spectrometer, calculate the number of dark bands seen between wavelengths 4000 and 5000Å.

Solution Given $t = 1.0 \times 10^{-5}$m, $\mu = 1.4$, $i = 45°$, $\lambda_1 = 4.0 \times 10^{-7}$m and $\lambda_2 = 5.0 \times 10^{-7}$m

Condition of dark bands in reflected light is

$$2\mu t \cos r = n\lambda \qquad \text{(i)}$$

$$\mu = \frac{\sin i}{\sin r} \text{ or } \sin r = \frac{\sin i}{\mu}$$

or

$$\sin r = \frac{\sin 45°}{1.4} = \frac{1/\sqrt{2}}{1.4} = \frac{1}{1.4\sqrt{2}}$$

$$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-\frac{1}{2 \times 1.96}}$$

$$= 0.86$$

For wavelength λ_1, i.e, 4.0×10^{-7}m

$$2\mu t \cos r = n_1\lambda_1$$

$$n_1 = \frac{2\mu t \cos r}{\lambda_1} = \frac{2 \times 1.4 \times 1.0 \times 10^{-5} \times 0.86}{4.0 \times 10^{-7}}$$

$$= 60.2$$

$$n_1 \approx 60$$

For wavelength λ_2, i.e, 5.0×10^{-7}m

$$n_2 = \frac{2\mu t \cos r}{\lambda_2} = \frac{2 \times 1.4 \times 1.0 \times 10^{-5} \times 0.86}{5.0 \times 10^{-7}}$$

$$= 48.16 \text{ or } n_2 \approx 48$$

$$n_1 - n_2 = 60 - 46 = \mathbf{12}$$

i.e, **12 dark bands are seen between wavelengths 4000 and 5000Å.**

Example 24 A parallel beam of light of wavelength 5890 Å is incident on a glass plate having refractive index $\mu = 1.5$ such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of glass plate which will appear dark by reflected light.

Solution Given $\lambda = 5.89 \times 10^{-7}$m, $\mu = 1.5$ and $r = 60°$

Condition for the film to appear dark in reflected light is

$$2\mu t \cos r = n\lambda$$

For minimum thickness $n = 1$

$$t = \frac{\lambda}{2\mu \cos r} = \frac{5.89 \times 10^{-7}}{2 \times 1.5 \times 0.5}$$

$$= \mathbf{0.3927 \times 10^{-6}m}$$

Example 25 A soap film of refractive index 1.333 is illuminated by white light incident at an angle of 45°. The light refracted by it is examined by a spectroscope and two consecutive bright bands are focused corresponding to the wavelength 6.1×10^{-5}cm and 6.0×10^{-5}cm. Find the thickness of the film.

$$\mu = \frac{\sin i}{\sin r} \text{ or } \sin r = \frac{\sin 45^\circ}{1.333} = \frac{1/\sqrt{2}}{1.333} = \frac{0.707}{1.333}$$

$$\sin r = 0.53$$

$$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(0.53)^2} = 0.848$$

Condition of bright film to observe in transmitted case is

$$2\mu t \cos r = n\lambda_1 = (n+1)\,\lambda_2$$

$$\text{or } n \times 6.1 \times 10^{-7} = (n+1) \times 6.0 \times 10^{-7}$$

$$n = 60$$

$$\text{and } t = \frac{n\lambda_1}{2\mu \cos r} = \frac{60 \times 6.1 \times 10^{-7}}{2 \times 1.333 \times 0.848}$$

$$= \mathbf{1.62 \times 10^{-5} mm}$$

Example 26 Calculate the thickness of a soap film (μ = 1.463) that will result in constructive interference in the reflected light, if the film is illuminated normally with light whose wavelength in free space is 6000 Å.

Solution Given $\lambda = 6.0 \times 10^{-7}$m, in this case $n = 1$, and for normal incidence $i = 0$ and $r = 0$

Condition for cosntructive interference $2\mu t \cos r = (2n-1)\,\lambda/_2$

$$t = \frac{(2n-1)\lambda}{4\mu \cos r} = \frac{1 \times 6.0 \times 10^{-7}}{4 \times 1.463 \times 1}$$

$$= \mathbf{1.025 \times 10^{-7} m.}$$

Example 27 A thin film is illuminated by white light at an angle of incidence ($i = \sin^{-1}(4/5)$. In reflected light, two dark consecutive overlapping fringes are observed corresponding to wavelengths 6.1×10^{-7}m and 6.0×10^{-7}m. The refractive index of the film is 4/3. Calculate the thickness of the film.

Solution Given $\lambda_1 = 6.1 \times 10^{-7}$m, $\lambda_2 = 6.0 \times 10^{-7}$m and $\mu = 4/3$

$$\mu = \frac{\sin i}{\sin r} \text{ or } \sin r = \frac{\sin i}{\mu} = \frac{4/5}{4/3} = \frac{3}{5}$$

$$\text{and } \cos r = \sqrt{1-\sin^2 r} = \sqrt{1-\frac{9}{25}} = \frac{4}{5} = 0.8$$

Condition for dark fringes is

$$2\mu t \cos r = n\lambda_1 = (n+1)\,\lambda_2$$

$$\text{or } n\lambda_1 = (n+1)\,\lambda_2$$

$$n \times 6.1 \times 10^{-7} = (n+1)\, 6.0 \times 10^{-7}$$

$$n\,(6.1 - 6.0) \times 10^{-7} = 6.0 \times 10^{-7}$$

$$\text{or } n = 60$$

$$\text{and } 2\mu t \cos r = n\lambda_1$$

$$\text{or } t = \frac{60 \times 6.1 \times 10^{-7}}{2 \times \frac{4}{3} \times 0.8}$$

t **= 1.716 × 10^{-2}mm**

Example 28 Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 interference fringes are observed between these edges in sodium light at normal incidence. What is the thickness of wire?

Solution Given λ_{av} = 5893Å, n = 20, $i = r = 0$ and $\mu = 1$

$$w = \frac{\lambda}{2\mu\theta} = \frac{5.893 \times 10^{-7}\text{m}}{2 \times 1 \times \theta}$$

or $$w\theta = \frac{5.893 \times 10^{-7}}{2}$$

From Fig. 1.32

$$\theta = \frac{t}{20\omega}$$

or $$t = 20\omega\theta$$

$$= 20 \times \frac{5.893 \times 10^{-7}}{2}$$

t **= 5.893 × 10^{-3}mm.**

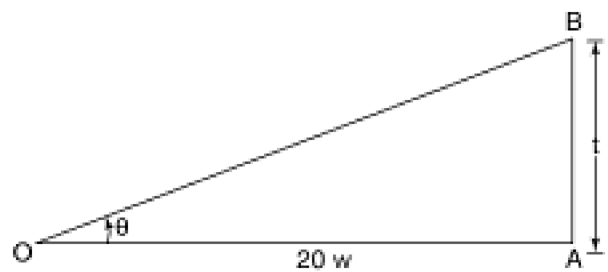


Figure 1.32

Example 29 A wedge air film is enclosed between two glass plates touching at one edge and separated by a wire of 0.06 × 10^{-3}m diameter at a distance of 0.15m from the edge. Calculate the fringe width. The light of wavelength 6.0 × 10^{-7}m from the broad source is allowed to fall normally on the film.

Solution Given $\lambda = 6.0 \times 10^{-7}$m and $\mu = 1$

$$w = \frac{\lambda}{2\mu\theta} \quad \text{(i)}$$

From Fig. 1.33

$$\theta = \frac{6.0 \times 10^{-5}}{0.15} \quad \text{(ii)}$$

$\therefore$ $$w = \frac{6.0 \times 10^{-7} \times 0.15}{2 \times 1 \times 6.0 \times 10^{-5}}$$

w = **0.75 mm.**

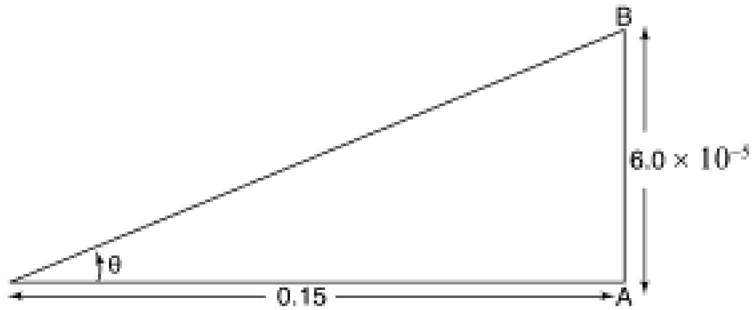


Figure 1.33

Example 30 A wedge shaped film is illuminated by light of wavelength 4650Å. The angle of wedge is 40″. Calculate the fringe separation between two consecutive fringes.

Solution Given $\lambda = 4.56 \times 10^{-7}$m and $\mu = 1$

$$\theta = 40'' = \frac{40}{3600} \times \frac{\pi}{180^\circ} \text{ rad}$$

$$= 1.9 \times 10^{-4} \text{ rad}$$

$$\therefore \quad w = \frac{\lambda}{2\mu\alpha} = \frac{4.65 \times 10^{-7}}{2 \times 1 \times 1.9 \times 10^{-4}}$$

$$\mathbf{w = 1.2\ mm}$$

Example 31 Two glass plates enclose a wedge-shaped air film touching at one edge are separated by a wire of 0.03mm diameter at distance 15cm from the edge. Monochromatic light (λ = 6000Å) from a broad source falls normally on the film. Calculate the fringe-width.

Solution Given $\lambda = 6.0 \times 10^{-7}$m and $\mu = 1$

$$\left[\text{Angle } \theta = \frac{\text{Arc (AB)}}{\text{radius}}\right]$$

$$\theta = \frac{0.03 \times 10^{-3}}{0.15}$$

$$= 2.0 \times 10^{-4} \text{ rad}$$

$$w = \frac{\lambda}{2\mu\theta}$$

$$= \frac{\lambda}{2\theta}$$

$$w = \frac{6.0 \times 10^{-7}}{2 \times 2 \times 10^{-4}} = 1.5 \times 10^{-3}\text{m} = \mathbf{1.5\,mm}$$

Example 32 A glass wedge having angle 0.01 radian is illuminated normally by light of wavelength 5890Å. At what distance from the edge of the wedge, will the 12[th] dark fringe be observed by reflected light ?

Solution Given $\lambda = 5.89 \times 10^{-7}$m, $n = 12$, $\theta = 0.01$ rad and $\mu = 1$

Condition for obtaining dark fringe is

$$2\mu t \cos (r + \theta) = n\lambda \qquad \text{(i)}$$

For normal incidence $i = r = 0$ and when θ is very small

$$\cos \theta \approx 1$$

Eq. (i) reads $2t = n\lambda$ (ii)

Now the angle θ can be written as $\theta = \frac{t}{x}$

where t is the thickness and x is the distance from the edge (Fig. 1.34) then we have $t = \theta.x$ (iii)

By using Eqs. (ii) and (iii), we get

$$2\theta.\, x = n\lambda$$

$$\text{or } x = \frac{n\lambda}{2\theta} = \frac{12 \times 5.89 \times 10^{-7}}{2 \times 0.01} = 3.5 \times 10^{-4}\text{m}$$

$\boldsymbol{x = 0.35}$ **mm.**

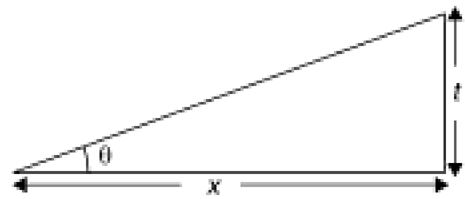


Figure 1.34

Example 33 Interference fringes are produced when monochromatic light is incident normally on a thin wedge-shaped film of refractive index 1.5. If the distance between two consecutive fringes is 0.02 mm. Find the angle of the film, the wavelength of light being 5.5×10^{-5} cm.

Solution Given $\mu = 1.5$, $w = 0.02 \times 10^{-3}$m and $\lambda = 5.5 \times 10^{-7}$m.

$$w = \frac{\lambda}{2\mu\theta} \text{ or } \theta = \frac{\lambda}{2\mu w} = \frac{5.5 \times 10^{-7}}{2 \times 1.5 \times 0.02 \times 10^{-3}}$$

$= 0.009166$ rad $=$ **0.525°**

Example 34 In Newton's rings experiment, the diameter of the 15^{th} ring was found to be 0.59 cm and that of the 5^{th} ring was 0.336 cm. If the radius of the plano convex lens is 100 cm, compute the wavelength of light used.

Solution Given $D_{15} = 5.9 \times 10^{-3}$m, $D_5 = 3.36 \times 10^{-3}$m

$p = 10$ and $r = 1.0$m.

Formula used is $\lambda = \dfrac{D^2(n+p) - D_n^2}{4pR} = \dfrac{[(5.9)^2 - (3.36)^2] \times 10^{-6}}{4 \times 10 \times 1.0}$

$\lambda =$ **5880Å**

Example 35 In a Newton's rings experiment the radius of 10^{th} and 20^{th} rings are 0.2 and 0.3 cm, respectively, and the focal length of the plano-convex lens is 90cm. Calculate the wavelength of light used in nanometers.

Solution Given $f = 0.9$m, $\mu = 1.5$, $D_5 = 0.2$cm and $D_5 = 0.3$ cm. p = 10

Formula used is $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = (1.5 - 1)\left[\dfrac{1}{R_2} - \dfrac{1}{\infty}\right], R = \infty$

$$\frac{1}{0.9} = 0.5\left[\frac{1}{R_1}\right] \text{ or } R_1 = R = 0.45\text{m}$$

and
$$\lambda = \frac{D^2_{(n+p)} - D^2_n}{4pR} = \frac{D^2_{15} - D^2_5}{4 \times 10 \times 0.45} = \frac{[(0.3)^2 - (0.2)^2] \times 10^{-4}}{4 \times 10 \times 0.45}$$

$\lambda =$ **2777 nm**

Example 36 In a Newton's rings arrangement a thin convex lens of focal length 1.0 m. ($\mu = 1.5$) remains in contact with an optical flat and light of wavelength 5896×10^{-10}m is used. Newton's rings are observed normally by reflected light. What is the diameter of 7[th] bright ring?

Solution Given $\mu = 1.5$, $f = 1.0$ m and $\lambda = 5.896 \times 10^{-7}$m

$$R_1 = R \text{ and } R_2 = -R$$

Formula used is
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\text{or } \frac{1}{1.0} = (1.5 - 1)\left[\frac{1}{R} + \frac{1}{R}\right]$$

$$\text{or } \frac{2}{R} = \frac{1}{0.5} \quad \text{or} \quad R = 1.0\text{m}$$

now
$$D^2_n = 4n\lambda R$$

for $n = 7$

$$D_7 = \sqrt{4 \times 7 \times 5.896 \times 10^{-7} \times 1.0}$$

$D_7 = \mathbf{4.063 \times 10^{-3}}$ **m**

Example 37 Light source emitting the light of wavelengths $\lambda_1 = 6.0 \times 10^{-7}$m and $\lambda_2 = 4.8 \times 10^{-7}$m is used to obtain Newton's rings in reflected light. It is found that the n[th] dark ring of λ_1 coincides with $(n+1)^{th}$ dark ring of λ_2. If the radius of curvature of the curved surface of the lens is 0.96 m. Calculate the diameter of $(n+1)^{th}$ dark ring of λ_2.

Solution Given $\lambda_1 = 6.0 \times 10^{-7}$m, $\lambda_2 = 4.8 \times 10^{-7}$m and R = 0.96 n

The diameter of n[th] order dark ring of λ_1 is

$$D^2_n(\lambda_1) = 4n\lambda_1 R$$

Similarly, the diameter of (n + 1)[th] order dark ring of λ_2

$$D^2_{(n+1)}(\lambda_2) = 4(n+1)\lambda_2 R$$

Since,
$$D^2_n(\lambda_1) = D^2_{(n+1)}(\lambda_2)$$

$$4n\lambda_1 = 4(n+1)\lambda_2$$

or
$$\frac{n+1}{n} = \frac{\lambda_1}{\lambda_2}$$

or
$$1 + \frac{1}{n} = \frac{\lambda_1}{\lambda_2} \text{ or } \frac{1}{n} = \frac{\lambda_1 - \lambda_2}{\lambda_2}$$

or
$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{4.8 \times 10^{-7}}{(6.0 - 4.8) \times 10^{-7}} = 4.0$$

Hence, $D^2_{(n+1)}(\lambda_2) = 4(n+1)\lambda_2 R = 4 \times 5 \times 4.8 \times 10^{-7} \times 0.96$

$D_{(n+1)} = 3.0358 \times 10^{-3}$

or $\mathbf{D_{(n+1)} = 3.04 \times 10^{-3}}$ **m.**

Example 38 In Newton's ring arrangement a source is emitting two wavelengths $\lambda_1 = 6.0 \times 10^{-7}$m and $\lambda_2 = 5.9 \times 10^{-7}$m. It is found that n[th] dark ring due to one wavelength coincides with $(n+1)^{th}$ dark ring due to the other. Find the diameter of the n^{th} dark ring if radius of curvature of the lens is 0.9 m.

Solution Given $\lambda_1 = 6.0 \times 10^{-7}$m, $\lambda_2 = 5.9 \times 10^{-7}$m and R = 0.9m.

The diameter of the n^{th} order dark ring of λ_1 is

$$D^2_n(\lambda_1) = 4n\lambda_1 R$$

The diameter of the $(n+1)^{th}$ order dark ring of λ_2 is

$$D^2_{(n+1)}(\lambda_2) = 4_{(n+1)}\lambda_2 R$$

Since two rings coincide

$$4n\lambda_1 R = 4(n+1)\lambda_2 R$$

$$\frac{n+1}{n} = \frac{\lambda_1}{\lambda_2} \quad \text{or} \quad n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$n = \frac{5.9 \times 10^{-7}}{(6.0 - 5.9) \times 10^{-7}} = \mathbf{59}$$

Now $$D_n = \sqrt{4n\lambda_1}\,R = \sqrt{4 \times 59 \times 6.0 \times 10^{-7} \times 0.9}$$

$$= 0.01128 \text{ m}$$

$$\mathbf{D_n = 0.0113\ m}$$

Example 39 Newton's rings are formed using light of wavelength 5896Å in reflected light with a liquid placed between plane and curved surfaces. The diameter of 7[th] bright fringe is 0.4 cm and the radius of curvature is 1.0m. Evaluate the refractive index of liquid.

Solution Given $D_7 = 4.0 \times 10^{-3}$m, $\lambda = 5.896 \times 10^{-7}$m, R = 1.0m and $n = 7$.

$$D'^2_n = \frac{2(2n-1)\lambda R}{\mu} \quad \text{or} \quad \mu = \frac{2(2n-1)\lambda R}{D'^2_n}$$

$$\mu = \frac{2 \times 13 \times 5.896 \times 10^{-7} \times 1.0}{(4 \times 10^{-3})^2}$$

$$\mu = \mathbf{0.96}$$

Example 40 If the diameter of n[th] dark ring in an arrangement giving Newton's ring changes from 0.3 cm and 0.25 cm as liquid is introduced between the lens and the plate, calculate the value of the refractive index of the liquid and also calculate the velocity of light in the liquid. Velocity of light in vacuum is 3×10^8 m/sec.

Solution Given $D_n = 3.0 \times 10^{-3}$m, $D_n = 2.5 \times 10^{-3}$m

Formula used is $D^2 = 4n\lambda R$

$$D^2_n = \frac{4n\lambda R}{\mu}$$

$$\mu = \frac{D^2_n}{D'^2_n} = \left[\frac{3.0\times10^{-3}}{2.5\times10^{-3}}\right]^2 = 1.44$$

$\mu = 1.44$

or $$V_{liq} = \frac{c}{\mu} = \frac{3\times10^8}{1.44}$$

$$\mathbf{V_{liq} = 2.08 \times 10^8\ m/sec}$$

Example 41 The Newton's rings are seen in reflected light of wavelength 5896Å. The radius of curvature of plano-convex lens is 1.0 meter. An air film is replaced by a liquid whose refractive index is to be calculated under the conditions if 16^{th} ring is dark and its diameter is 5.1 mm.

Solution Given $D_{16} = 5.1 \times 10^{-3}$m, $\lambda = 5.896 \times 10^{-7}$m and R = 1.0 m

Formula used is $$D^2_n = \frac{4n\lambda R}{\mu} \text{ or } \mu = \frac{4n\lambda R}{D^2_n}$$

$$\mu = \frac{4\times16\times5.896\times10^{-7}\times1.0}{(5.1\times10^{-3})^2}$$

$\mu = \mathbf{1.45}$

Example 42 The Newton's rings are observed in reflected light of wavelength 6300Å. A thin layer of liquid of refractive index 1.63 is formed between curved surface of plano-convex lens (μ = 1.69) and plane glass plate (μ = 1.03) and the radius of curvature of the curvex lens is 0.9m. Find the radius of smallest dark ring.

Solution Given $\lambda = 6.3 \times 10^{-7}$m, μ = 1.63 and R = 0.9 m

Formula used is $$r'^2_n = \frac{n\lambda R}{\mu} \quad (n = 1 \text{ for a smallest dark ring})$$

$$r'^2_1 = \frac{1\times6.3\times10^{-7}\times0.9}{1.63} = 34.7853 \times 10^{-8} \text{ m}$$

$$\mathbf{r_1{}^2 = 5.9 \times 10^{-4}\ m = 0.59\ mm}$$

Example 43 Newton's rings are observed with two different media between the glass surfaces. The n^{th} rings have diameters as 10:7. Find the ratio of the refractive indices of the two media.

Solution Given Dn' : Dn'' = 10:7

$\because$ $$D^2_n = \frac{4n\lambda R}{\mu} \quad \text{(i)}$$

For the first medium (μ_1)

$$D'^2_n = \frac{4n\lambda R}{\mu_1} \quad \text{(ii)}$$

For the second medium

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