Section J
DEALING WITH INFLATION

Preamble

How to deal with inflation is a key question in General Insurance claims reserving. Past inflation of monetary values will affect the shape of the data, and the assumption made as to future rates will significantly affect the final value to be set on the reserve. It is not always necessary, however, to bring inflation explicitly into the calculations — many of the methods so far described will automatically project the level of past inflation into the future. Under stable economic conditions, therefore, the methods can work well on their own. But when inflation is unstable in its rate from year to year, it is necessary to bring it openly into the account.

An important point is that economic inflation is not the only force affecting the average cost per claim. Social influences also play their part, and among these such factors as court awards, attitudes in society towards compensation of accident victims, and legislation, including that of the EU, are of particular importance. Again, technical factors such as a change in the mix of business can produce inflation in the average claim size.

The present section of the Manual concentrates on a number of simple but practical ways of taking inflation into account. The methods are straightforward in use, and have the advantage of allowing the reserver to exercise judgement with regard to the future inflation levels assumed. But all are well based in a prior analysis of the past data which is to hand. This analysis is the homework which needs to be done, to provide the framework and the discipline that lend confidence to the future projections.

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[J1]
INFLATION — GENERAL CONSIDERATIONS

Inflation is a factor that always needs to be taken into account when making reserving estimates. It is, perhaps, the most important source of uncertainty as regards the final liability still to be met on the claims incurred to date. But it is well to ask exactly what is meant by "inflation" in the context of claims reserving. Is one referring to general economic inflation, as measured for example by the decreasing purchasing power of the pound? Does one allow also for some element of what may be called social inflation, as reflected say in the increased level of damages awards by the courts? Or does one mean the inflation of claim amounts themselves, taking into account all possible influences which are operating at the time?

In general, the reserver will be interested in the rate of claims inflation resulting from all causes together. But economic and/or social inflation will be major elements in the package. If they can be assessed in relation to the data, then a more reliable base will be available from which to make projections.

Dealing with inflation for claims reserving purposes normally embraces two aspects:

a) To identify the inflation element implicit in the past data on claim amounts, average costs per claim, etc.

b) To set a suitable inflation assumption for the purpose of future projections of the data.

In many cases, the answer for b) will be to continue with the general level of rates found in a), or to make some simple extrapolation. But other influences can be taken into account, and if expectations are changing rapidly there is no need for future rates to be tied to the past. The forecasts of some economic schools, for example, might be thought just as relevant as the inflation revealed in the office's recent experience.

On the matter of inflation in past data, there are distinct approaches which can be taken. One is to compare the data against some suitable index of inflation. Of such indexes, the Retail Price Index (RPI) is the best known, but will often not be the most apt one for insurance reserving. It is better to seek an index with some more direct connection with the line of business in hand. Thus, motor claims for vehicle damage would be expected to relate more closely to the NAE (index of National Average Earnings), since labour charges are a large element in the cost of repair. An index, if available, on the price of motor spare parts would also be relevant. Similar considerations would apply to health care insurance, where indices on doctors' fees, drug prices and hospital charges will be the appropriate ones to look for.

Given that the relevant index is to hand, the data on claims can then be adjusted to take out the assumed inflationary element. The data should be in a year-by-year rather than a cumulative form. The job is then best done by scaling up payments in past years to make them comparable with payments in the most recent year. The new picture should show how far the variations in the data can be explained by the chosen inflation index.

If the index is satisfactory, the inflation adjusted data can then be projected by any of the normal methods. At this point comes the leading question — should future inflation be taken to reflect the recent values in the chosen index, or should some other assumption be made? There are no hard and fast answers to this question — the reserver must scrutinise the evidence in front of him, and come to his own considered opinion. Once the choice is made, the projected data can be adjusted to take account of the assumed inflation.

A useful step will be to make a number of such projections with different rates of inflation. This will show the sensitivity of the estimates to inflation — the longer the tail of the business, of course, the greater the sensitivity is likely to be.

In the above, it has been assumed that claims inflation is essentially related to the calendar period in which the claim is paid. But it is also possible for the inflation to correspond with the accident period itself. An example might be a business line giving compensation for loss of earnings. Claims paid out will not allow for earnings growth between the accident date and payment date, and hence their relative scale will relate to the accident year only. (In terms of the development triangle, this type of inflation affects the rows only. The normal type of claims inflation works essentially as an effect on the diagonals of the triangle.)

A final point of some importance refers back to the main methods so far described in the Manual. No mention was made of inflation during their description, which may be surprising in view of the subject's importance. But by dropping inflation until this point, the exposition has been simplified. Also, once the general techniques for dealing with inflation are understood, they can be applied fairly readily to any chosen method.

Lastly, the methods generally do make implicit allowance for inflation — they tend to project forward the inflation already present in the past data. The process only works properly, however, when inflation is fairly stable from year to year. Where it is rapidly varying, as say in the 1970s, the projections will tend to be distorted.

[J2]
INFLATION ADJUSTED CLAIMS PROJECTION

This section presents a general technique for taking inflation explicitly into account in projections. The data are those of the standard paid claims example from earlier sections of the Manual, and the projection is the usual link ratio method. But the technique could equally be applied to many other combinations of data and projection method.

The main steps in the procedure are as follows:

a) If the data are in cumulative form, then the year-by-year values are obtained by subtraction along the rows of the development table.

b) An inflation index relevant for the business class in hand is selected, or a suitable assumption made about claims inflation in recent years.

c) The year-by-year values from previous years are brought into line with the current year by inflating them according to the index.

d) The resulting values are added up along the rows of the table, to produce the adjusted data in the cumulative form.

e) The adjusted data are projected by the reserver's chosen method, in such a way that the development triangle is completed into a rectangle.

f) Year-by-year values are once again obtained, this time relating to the future years' projection (i.e. in the area to the lower right of the development table).

g) An assumption on the level of future inflation is made. This may come from a projection forward of the index in b) above, or be made on independent grounds.

h) The projected year-by-year values are inflated according to the future rate(s) selected in g).

i) Adding the projected inflated year-by-year values along the rows leads to the figure for the estimated reserve on the business.

The procedure is now illustrated by a numerical example. The starting point is the familiar table of paid claims:

		d 0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
	2	1113	2103	2774	3422	3844		
	3	1265	2433	3233	3977			
a	4	1490	2873	3880				
	5	1725	3261				[pC]	
	6	1889						

Step (a)

The year-by-year values are found straight away by subtraction along the rows:

		d 0	1	2	3	4	5	*ult*
	1	1001	854	568	565	347	148	222
	2	1113	990	671	648	422		
	3	1265	1168	800	744			
a	4	1490	1383	1007				
	5	1725	1536				[ΔpC]	
	6	1889						

Step (b)

Suppose that an inflation index relevant to the business class in hand is available. It is based at 100 in year a=4, and the yearly values it shows are:

a	1	2	3	4	5	6
Index	78	82	89	100	111	120

If the index is a monthly one, the value chosen for each year should be that as at 30 June. If the index is annual or quarterly, the 30 June value can be found by simple interpolation. The assumption here is that claims are fairly evenly spread through the year, probably reasonable for most classes of business — to insist on 100% precision in the timing of the index would be out of place. But if there is a bias in the claims, such as might be found with storm damage, then an adjustment may be in order.

From the index, the effective annual inflation in the past can be found. Symbol j will be used for inflation, so that the multiplier for adjustments will be $(1+j)$:

a	1		2		3		4		5		6
Index	78		82		89		100		111		120
$1+j$		1.051		1.085		1.124		1.110		1.081	
$\pi(1+j)$	1.538		1.463		1.348		1.200		1.081		1.000

In the table, 1.051 is 82/78, 1.085 is 89/82 and so on. The annual inflation rates brought out vary from 5.1% to 12.4%, as can be seen. The lowest row is put in to show the cumulative effect — it gives the relation between the earlier years and the current year a=6. In fact, a 53.8% increase has occurred overall since the year a=1.

(Note: the symbol π is used to denote a product of factors. In this case the working runs backwards along the row of annual values $(1+y)$. Thus 1.200 is 1.081×1.110, 1.348 is $1.081 \times 1.110 \times 1.124$, and so on. Alternatively, the values can be calculated as 120/111, 120/100, 120/89, 120/82 and 120/78).

Step (c)

We now bring the year-by-year paid claims into line with the current year by inflating them according to the index. Looking at the development triangle, it is the diagonals which correspond to the years of payment. For example, the cells (a=3, d=0), (a=2, d=1), (a=1, d=2) all represent payments made in year 3. For this year, the inflation factor to the present time is 1.348, from the last row in the above table. It follows that a table of inflation adjustment factors will simply show these $\pi(1+y)$ values arranged on consecutive diagonals of the triangle:

		d						
		0	1	2	3	4	5	*ult*
	1	1.538	1.463	1.348	1.200	1.081	1.000	0.870
	2	1.463	1.348	1.200	1.081	1.000		
	3	1.348	1.200	1.081	1.000			
a	4	1.200	1.081	1.000				
	5	1.081	1.000				**[$\pi(1+j)$]**	
	6	1.000						

There is one slight problem in this table — how to deal with the value under (a=1, d=*ult*). The paid claims value for this cell, 222, is an estimate of a future payment, based on run-off data from earlier years. Exactly what inflation adjustment should be applied to it is therefore a moot point. But if the average time of payment in the run-off comes, say, at 1.5 years on, and 10% is reckoned as the typical rate from the earlier run-offs, then a 15% adjustment will be in order. This leads to the right-hand value in the above table, since 100/115 = .870.

To carry out the adjustment, it remains to multiply the year-by-year claim values by those in the above triangle. The claim values are repeated here for convenience:

		d						
		0	1	2	3	4	5	*ult*
	1	1001	854	568	565	347	148	222
	2	1113	990	671	648	422		
	3	1265	1168	800	744			
a	4	1490	1383	1007				
	5	1725	1536				[ΔpC]	
	6	1889						

The multiplication proceeds on a cell-by-cell basis, and gives the following result:

		d						
		0	1	2	3	4	5	*ult*
	1	1540	1249	766	678	375	148	193
	2	1628	1335	805	700	422		
	3	1705	1402	865	744			
a	4	1788	1495	1007				
	5	1865	1536			[$Adj.\Delta pC$]		
	6	1889						

Step (d)

The next step is the straightforward one of regenerating the cumulative values for the paid claims, now in their adjusted form. This is done by adding the values along the rows of the year-by-year claims triangle:

		d						
		0	1	2	3	4	5	*ult*
	1	1540	2789	3555	4233	4608	4756	4949
	2	1628	2963	3768	4468	4890		
	3	1705	3107	3972	4716			
a	4	1788	3283	4290				
	5	1865	3401				[$Adj.pC$]	
	6	1889						

Step (e)

The adjusted data can now be projected by a standard method. The grossing up procedure, which goes direct to the ultimate values, is not well suited in this case. But the link ratio, which enables the intermediate values for future claim years to be generated, works well. This method is employed here. The ratios at the foot of the table are found as the average of the values in the columns above them.

d

a	0	1	2	3	4	5	*ult*
1	1.811	1.275	1.191	1.089	1.032	1.041	
	1540	2789	3555	4233	4608	4756	4949
2	1.820	1.272	1.186	1.094			
	1628	2963	3768	4468	4890		
3	1.822	1.278	1.187				
	1705	3107	3972	4716			
4	1.836	1.307					
	1788	3283	4290				
5	1.824					[*r*]	
	1865	3401				[*Adj.pC*]	
6	1889						
r	1.823	1.283	1.188	1.092	1.032	1.041	

The projection is now completed by applying the ratios successively to the diagonal elements in the claims triangle. This generates the cumulative claims values for future years, so completing the previously unfilled part of the claims triangle:

d

a	0	1	2	3	4	5	*ult*
1							4949
2		[*Adj.pC*]				5046	5253
3					5150	5315	5533
4				5097	5566	5744	5980
5			4363	5183	5660	5841	6080
6		3444	4419	5250	5733	5916	6159

Along the bottom row, 3444 = 1889 × 1.823, 4419 = 3444 × 1.283, etc. On row a=5, 4363 = 3401 × 1.283, 5183 = 4363 × 1.188, and so on through the triangle.

Step (f)

Having projected the cumulative figures with inflation removed, we now wish to put back the future inflation. Before this can be done, the claims must once more be put into their year-by-year form. Again, the procedure is to subtract values along the rows:

		d 0	1	2	3	4	5	*ult*
	1							193
	2	[*Adj.*Δ*pC*]					156	207
	3					434	165	218
a	4				807	469	178	236
	5			962	820	477	181	239
	6		1555	975	831	483	183	243

Step (g)

We come now to the critical question. What rate or rates are to be assumed for future inflation? The evidence from the index used in Step (b) above shows that inflation has been in the range 8.0–12.5% during the last three years, but that it has been declining towards the lower end of the range. In the circumstances, an assumption of future inflation of 10% p.a. seems reasonably cautious. If this is taken up, a further triangle of inflation factors can be constructed:

		d 0	1	2	3	4	5	*ult*
	1							1.150
	2	[$\pi(1+j)$]					1.100	1.265
	3					1.100	1.210	1.392
a	4				1.100	1.210	1.331	1.531
	5			1.100	1.210	1.331	1.464	1.684
	6		1.100	1.210	1.331	1.464	1.610	1.852

The table shows an increase of 10% on the first diagonal, which represents the year following the current one. Then the increase is 21%, 33.1% and so on in sequence, each year having a 10% increase over the one before it, except in the *ult* column where provision is made for one and a half years' increase.

Step (h)

These factors can be applied immediately to the adjusted year-by-year claim values. Again, the multiplication takes place on a cell by cell basis, with the following result:

					d			*row*
		1	2	3	4	5	*ult*	*sum*
	1						222	222
	2		[^Δ*pC*]			172	262	434
	3				477	200	303	980
a	4			888	567	237	361	2053
	5		1058	992	635	265	402	3352
	6	1711	1180	1106	707	295	450	5449

Step (i)

The last step is to add the projected claim values along the rows. This gives the estimated liability for each accident year. Finally, the accident years are totalled to give the required reserve and the estimate of ultimate loss:

Overall Values:	Reserve	12,490
	∑*pC**	20,334
	∑^*L-ult*	32,824

The value obtained is very close to those previously given by the unadjusted data (see §E8.2). The example data are a well behaved set in any case, but the main reason is to do with the rate chosen for future inflation. This was set to be consistent with the values experienced in the past. Whenever this is done, adjusted and unadjusted methods will tend to give similar answers, provided the inflation rates concerned are reasonably stable from year to year.

Sensitivity Calculations

It is worth re-evaluating the above example for different future inflation assumptions. This has been done in the table below, to give an idea of the variation in the result. But the sensitivity will depend very much on the length of the tail of the business under evaluation. A long-tail liability line will show a good deal more variation than the present example.

Inflation	5%	8%	9%	10%	11%	12%	15%
^Reserve	11,228	11,966	12,229	12,490	12,758	13,027	13,885

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[J3]
BENNETT & TAYLOR — METHOD A

Bennett & Taylor's method, entitled "Method A" in their 1979 paper, is essentially a projection of the payments pattern on a per claim basis. It was devised in the context of motor reserving, but can be of quite general application. The authors use data structured on a report year basis, i.e. claims are classified and allowed to develop according to the year in which they are notified to the insurer.

The method begins by taking the year by year figures for the paid claims, and adjusting these to present day values by means of some suitable index. This procedure is identical to Steps 1–3 in the general technique for inflation adjustment described in §J2.

The Crucial Step

The crucial step is then taken, which is to divide the claims figures for each year by the number of claims reported in that year. The result is, or should be, to put the payments data on to a normalised basis. This is because variations in the data from year to year must be attributable to a) average claim size, and b) the number of claims occurring. In principle, the inflation adjustment should deal with much of the type a) variation, so that number of claims is the chief remaining factor.

With the main sources of variation removed, the payments per claim table should show a good degree of regularity in its columns. If the data were perfect then each column would be constant, with a single value repeated down its length. Such a situation will never pertain in practice, but the reserver should soon be able to assess the stability of the data from the calculated table. If all is well, an average can be taken on each column, and used to project the values in the empty cells of the table.

Finally, it is a simple matter to generate the required reserve from the projected payments per claim. There are three main steps:

a) Multiply the figures in each row by the respective number of claims for the report year.

b) Project the inflation index used in the first part of the work, or make some suitable assumption about the level of future inflation.

c) Inflate the future claim payment figures for each report year, and add them to give the estimated reserve.

Other Points

A rider is that since Bennett & Taylor's method uses report year data, it will be necessary to make a separate estimation of the IBNR liability. This must be added in later to give the full reserve.

A second point is to do with the nature and rationale of the method. Bennett & Taylor call it an "average payments" method, which is a fair description. But it should be clearly distinguished from the average cost per claim techniques of §H. The latter allow for claim number development, and do not use a single divisor for each row of the table. Also, they concentrate on whole claims, whether paid, incurred or emerging, at their different stages of development. But Bennett & Taylor's method focuses on the pattern of payment, and in effect distributes the cost for any given claim over all the years of development.

As usual, we illustrate the procedure with a worked example. The data that follow are paid claim information, but is a new set of figures based on a report year definition. It is not intended to be representative of any particular type or class of business. Below are shown the cumulative data, followed by the corresponding year by year claims figures.

		d						
		0	1	2	3	4	5	*ult*
	1	500	737	915	1036	1107	1163	1245
	2	732	1065	1296	1476	1606		
	3	854	1263	1556	1800			
a	4	980	1493	1890				
	5	1101	1688				[*pC*:*r*]	
	6	1189						

		d						
		0	1	2	3	4	5	*ult*
	1	500	237	178	121	71	56	82
	2	732	333	231	180	130		
	3	854	409	293	244			
a	4	980	513	397				
	5	1101	587				[Δ*pC*]	
	6	1189						

An inflation index is needed, or some assumption on the past inflation contained in the claims figures. Here, the following index is taken to be available and relevant to the business in hand:

r	1		2		3		4		5		6
Index	97		100		107		118		126		136
$1+j$		1.031		1.070		1.103		1.068		1.079	
$\pi(1+j)$	1.402		1.360		1.271		1.153		1.079		1.000

The $(1+j)$ line shows inflation varying between 3.1% and 10.3% for the period in question. Overall, there has been a 40% rise in values or costs between the first and last years concerned. The index yields the following full set of factors for inflating the past data:

					d			
		0	1	2	3	4	5	*ult*
	1	1.402	1.360	1.271	1.153	1.079	1.000	0.870
	2	1.360	1.271	1.153	1.079	1.000		
	3	1.271	1.153	1.079	1.000			
a	4	1.153	1.079	1.000				
	5	1.079	1.000				$[\pi(1+j)]$	
	6	1.000						

The adjustment itself follows by straight multiplication of the ΔpC table:

					d			
		0	1	2	3	4	5	*ult*
	1	701	322	226	140	77	56	71
	2	996	423	266	194	130		
	3	1085	472	316	244			
a	4	1130	554	397				
	5	1188	587				$[Adj.\Delta pC]$	
	6	1189						

Now comes the second adjustment, scaling the data according to the number of claims for each of the report years. This produces the pattern of payments per claim, which is the focus of interest. (The single symbol n is used here for number of claims. By definition, the number must be a constant for each of the report years.)

n	r	d 0	1	2	3	4	5	ult
128	1	5.477	2.516	1.766	1.094	.602	.438	.555
167	2	5.964	2.533	1.593	1.162	.778		
190	3	5.711	2.484	1.663	1.284			
203	4	5.567	2.729	1.956				
214	5	5.551	2.743			[$Adj.\Delta pC/n$]		
220	6	5.405						
Col Avge		5.613	2.601	1.745	1.180	.690	.438	.555

The data in this case show good regularity, and it is reasonable to project forward by taking the average of the values in each column. The projection table simply shows the values repeated in the lower right part of the triangle, representing the future years of development:

		d 0	1	2	3	4	5	ult
	1							.555
	2	[$Adj.\Delta pC/n$]					.438	.555
	3					.690	.438	.555
r	4				1.180	.690	.438	.555
	5			1.745	1.180	.690	.438	.555
	6		2.601	1.745	1.180	.690	.438	.555

Now the reverse adjustments are made to the projected data. First, the scaling factor of number of claims is brought back in, by straight multiplication along the rows:

n	r	d 0	1	2	3	4	5	ult
128	1							71
167	2	[$Adj.\Delta pC$]					73	93
190	3					131	83	105
203	4				240	140	89	113
214	5			373	253	148	94	119
220	6		572	384	260	152	96	122

Next, future inflation factors are generated. Here, a uniform rate of 10% is chosen, as an extrapolation of the highest rate shown in the inflation index in the last five years.

					d			
		0	1	2	3	4	5	*ult*
	1							1.150
	2		[^(1+*j*)]				1.100	1.265
	3					1.100	1.210	1.392
a	4				1.100	1.210	1.331	1.531
	5			1.100	1.210	1.331	1.464	1.684
	6		1.100	1.210	1.331	1.464	1.610	1.852

A final multiplication yields the future paid claim estimates with inflation allowed for at the assumed rate. The value for the reserve then follows by addition of parts.

					d			*row*
		1	2	3	4	5	*ult*	*sum*
	1						82	82
	2		[^Δ*pC*]			80	118	198
	3				144	100	146	390
a	4			264	169	118	173	724
	5		410	306	197	138	200	1251
	6	629	465	346	223	155	226	2044

The total of the final column is 4,689. Since report year data have been used, this is the estimated reserve for reported claims. To it should be added the IBNR estimate to give the full reserve.

Sensitivity Calculations

As in §J2, the example can be evaluated for different future inflation assumptions, and this is done in the table below.

Inflation	5%	8%	9%	10%	11%	12%	15%
^Reserve	4,196	4,483	4,588	4,689	4,797	4,906	5,244

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[J4]
THE SEPARATION METHOD

The separation method is a mathematical method first devised by Verbeek in 1972. Verbeek applied the model in the reinsurance context to the projection of numbers of claims reported. Later, the method was developed to apply to average payments per claim by the Australian author Taylor. This is the version described here, although the main part of the working is identical with that of Verbeek.

The idea behind the separation method is to distinguish two patterns in the claims data from one another. These are: a) the development pattern for the accident year, and b) calendar year effects, of which inflation is usually the most important. The first pattern is the one which works across the columns of the development table, and which is elicited in the grossing up and link ratio methods. The second pattern is the one operating on the diagonals of the table, and which is the special subject of the present section of the Manual.

The basic assumption behind the method is that the two patterns are independent of one another. The assumption will never completely be satisfied, but in many cases it will serve as a working basis. A test can be applied during the work to test the truth of this assumption.

Using Verbeek's model, the patterns are generated without using any outside information. Thus, unlike the previous methods described in §J, no special inflation indices are required. The data are effectively analysed to reveal their own intrinsic inflation. Once the analysis has been done, it is as if the internal structure of the data was revealed to the light. This structure can then be extended so as to generate data values for future years which are in keeping with the existing data. Estimation of the required reserve then follows straightforwardly.

The style of the earlier parts of the Manual is to present reserving methods by means of arithmetical example. Since the separation method can be applied without going into the full mathematics, the style will be retained here. The method requires, however, that the reserver follow through a detailed set of operations, called here the Separation Algorithm.

Work begins with the standard paid claims data used in the Manual examples:

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
	2	1113	2103	2774	3422	3844		
	3	1265	2433	3233	3977			
a	4	1490	2873	3880				
	5	1725	3261				[*pC*]	
	6	1889						

		d						
		0	1	2	3	4	5	*ult*
	1	1001	854	568	565	347	148	222
	2	1113	990	671	648	422		
	3	1265	1168	800	744			
a	4	1490	1383	1007				
	5	1725	1536				[ΔpC]	
	6	1889						

The first step is to convert the year by year values into the average payment per claim figures. This is done simply by dividing through each row by the related number of claims. With report year data, as in the Bennett & Taylor method, there is no problem since the claim numbers for each year are known with certainty. But with accident year data, claim numbers are not fully developed, and some estimate must be used. There are at least three different possibilities:

a) Use the development to date of claim numbers reported to estimate the ultimate numbers for each year.

b) Estimate the ultimate numbers for each year separately, as at the end of the accident year itself.

c) Substitute for ultimate number of claims the number made in the accident year itself.

Of these, a) is not thought fully satisfactory, since more information is available about the earlier accident years. This means that a bias may be introduced into the working, which should be avoided. As far as b) is concerned, the estimate is likely to depend to a large extent on the number of claims actually made in the accident year. Hence option c) has much to recommend it, apart from being the simplest to use in any case. The fact that claim numbers in c) are not the ultimate ones does not matter, provided the proportionality is constant across the accident years. The claims numbers in c) are being used merely as a standardising factor.

Option c) is used in the table below. The numbers of claims reported in each accident year, $nR(0)$, are used to divide through the year by year claims figures. The result is a version of the average payments per claim table:

$nR(0)$	a				d		
		0	1	2	3	4	5
414	1	2.418	2.063	1.372	1.365	.838	.357
453	2	2.457	2.185	1.481	1.430	.932	
494	3	2.561	2.364	1.619	1.506		
530	4	2.811	2.609	1.900			
545	5	3.165	2.818				
557	6	3.391					

It will be seen that the cell $a = 1$, $d = ult$ has been dropped from this table. That is because an exact triangle is needed for the main separation calculations — the cell will be brought back into account towards the end of the working. We now begin the separation method proper by calculating the sum of each column and each diagonal in the table:

	diagonal	a			d			
$a+d$	*sums*		0	1	2	3	4	5
		1	2.418	2.063	1.372	1.365	.838	.357
1	2.418	2	2.457	2.185	1.481	1.430	.932	
2	4.520	3	2.561	2.364	1.619	1.506		
3	6.118	4	2.811	2.609	1.900			
4	8.021	5	3.165	2.818			[$\Delta pC/n$]	
5	9.661	6	3.391					
6	10.904							
	col sum		16.803	12.039	6.372	4.301	1.770	.357

With these totals, we can apply the separation algorithm. It is set out in the following table, by example. There are eight rows in the table, and one column for each of the columns in the claim payment table, i.e. six in this case. The work proceeds down the columns, beginning from top left of the table.

	{1}	*diag sum*	10.904	9.661	8.021	6.118	4.520	2.418
D/gen	{2}	{1}/{8}	10.904	9.988	9.088	8.278	7.895	7.757
	{3}	*col sum*	.357	1.770	4.301	6.372	12.039	16.803
	{4}	Σ{2}	10.904	20.892	29.980	38.258	46.153	53.910
C/gen	{5}	{3}/{4}	.0327	.0847	.1435	.1666	.2608	.3117
	{6}	Σ{5}	.0327	.1174	.2609	.4275	.6883	1.0000
	{7}	1−{6}	.9673	.8826	.7391	.5725	.3117	
	{8}	{7} shifted	1	.9673	.8826	.7391	.5725	.3117

To explain in more detail:

Row {1} contains the diagonal sums, in reverse order, starting with the sum of the main diagonal ($a + d = 6$) in the triangle.
Row {3} contains the column sums, in reverse order, from the triangle.
Row {8} starts with the value 1 (the other values being produced progressively by the steps described below).
Row {2} = Row {1} element divided by the corresponding Row {8} element.
Row {4} is the cumulative sum of the Row {2} elements.
Row {5} is Row {3} divided by the Row {4} element.
Row {6} is the cumulative sum of the Row {5} elements.
Row {7} is the value 1 minus the Row {6} element.
This becomes the value of Row {8} in the next column.

The procedure should continue automatically until Row {6} in the final column is reached. The value 1 should be obtained, exactly. This is a good check on the working — if the answer is not unity, then a mistake has occurred somewhere along the line.

In the algorithm table, Row {2} is marked D/gen (= diagonal generator), and Row {5} as C/gen (= column generator). These are the output values from the table, and can be used to remodel the original data. This is done by setting out the Row {5} values in reverse order across the top of a new table, and the Row {2} values down the left hand side, again in reverse order. The element in the table for row a, column d is obtained by multiplying the value of D/gen for $a + d$ by the value of C/gen for column d.

	C/gen		.3117	.2608	.1666	.1435	.0847	.0327
		a				*d*		
a + d	*D/gen*		0	1	2	3	4	5
		1	2.418	2.059	1.379	1.304	.846	.357
1	7.757	2	2.461	2.159	1.514	1.433	.924	
2	7.895	3	2.580	2.370	1.664	1.565		
3	8.278	4	2.833	2.605	1.817			
4	9.088	5	3.113	2.844		**[Gen.Δ*pC/n*]**		
5	9.988	6	3.399					
6	10.904							

For example, on the first row of the table, 2.418 = 7.757 × .3117. Then 2.059 = .2608 × 7.895, and 1.379 = .1666 × 8.278, and so on. On the second row, 2.461 = .3117 × 7.895, and 2.159 = .2608 × 8.278, and so on.

The success or otherwise of the remodelling process can be checked by comparing the generated table of average payments with the original version.

This is repeated below for convenience:

		d = 0	1	2	3	4	5
	1	2.418	2.063	1.372	1.365	.838	.357
	2	2.457	2.185	1.481	1.430	.932	
	3	2.561	2.364	1.619	1.506		
a	4	2.811	2.609	1.900			
	5	3.165	2.818			[$\Delta pC/n$]	
	6	3.391					

The agreement between the two sets of figures is very good in this example, with the possible exception of the last element in column $d = 2$ and the first and last in $d = 3$. In practice, one would seldom obtain such a good fit. However, the fit may often be reasonable enough to justify the use of the method.

Further information about the data is afforded by the D/gen values obtained in the analysis. They are a key to the inflation intrinsic in the data on the separation principle. In fact, the D/gen values supply the missing inflation index for the past calendar years, which was brought in externally by the other methods. In the present case, the values are:

a	1	2	3	4	5	6
Index	7.757	7.895	8.278	9.088	9.988	10.904
$1+j$		1.018	1.049	1.098	1.099	1.092
$\pi(1+j)$	1.408	1.383	1.318	1.200	1.092	1.000

The year 2-on-1 inflation looks suspiciously low. However, it should be remembered that the method estimates not only the inflation for past years, but also any other calendar year effects on the data (e.g. the introduction of a new claims administration system). The low index may therefore be explicable by some other special feature of the business in hand.

The last three year's figures are very consistent, and suggest continuing inflation at around the 10% mark. This assumption simplifies the situation, and can be used immediately in projection forward the data.

The projection, then, is to take the latest value in the D/gen set, and uprate it by 10%p.a. compound. This yields:

$a + d$	6	7	8	9	10	11
D/gen projection	10.904	11.994	13.193	14.512	15.963	17.559

We now return to the special table for generation of data. This time, the new values of D/gen are added down the right hand side of the table. Again, the element in the table for row *a*, column *d* is obtained by multiplying the value of D/gen for $a + d$ by the value of C/gen for column *d*.

C/gen		.3117	.2608	.1666	.1435	.0847	.0327		
	a				*d*				*a* + *d*
D/gen		0	1	2	3	4	5	*D/gen*	
	1							11.994	7
7.757	2		[Gen.Δ*pC/n*]				.392	13.193	8
7.895	3					1.016	.431	14.512	9
8.278	4				1.721	1.117	.475	15.963	10
9.088	5			1.998	1.893	1.229	.522	17.559	11
9.988	6		3.128	2.198	2.082	1.352	.574		
10.904									

By way of example, in the bottom row 3.128 = .2608 × 11.994, and 2.198 = .1666 × 13.193, and so on. In the next row up, 1.998 = .1666 × 11.994, and 1.893 = .1435 × 13.193, and so on.

We now have the future pattern of average payments per claim, with inflation included. It remains to multiply through each row by the related number, in order to reinstate the full paid claim amounts. The reader is reminded that the *nR*(0) are being used solely as standardising factors. This is done below:

					d			*row*
nR(0)	*a*	1	2	3	4	5	*ult*	*sum*
414	1						222	222
453	2		[Gen.Δ*pC*]			178	267	445
494	3				502	213	320	1035
530	4			912	592	252	378	2134
545	5		1089	1032	670	284	426	3501
557	6	1742	1224	1160	753	320	480	5679

Overall Values:		
	Reserve	13,016
	$\sum pC^*$	20,334
	$\sum$^*L-ult*	33,350

One final point needs explanation here, relating to the *ult* column in the table. Earlier on, the element for $a = 1$ in the *ult* position had to be dropped, and with it the tail of the claims development from $d = 5$ onwards. Now some means has to be found for bringing the tail back into the reckoning. The simplest way is to take the ratio of the Δ*pC* elements at *d*=5 and *d*=*ult*, and use it as a scaling factor. The elements are 148 and 222 respectively, giving a ratio of 1.500. This ratio is applied in the above table to generate the *ult* column, as 1.5 times the values in the $d = 5$ column alongside. The procedure is not fully satisfactory, but perhaps the best that can be devised in the circumstances.

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