Section G
METHODS USING LOSS RATIO & LOSS RATIO PROJECTIONS

Preamble

Introduction of the loss ratio into claims reserving methods at first sight seems paradoxical. If one were to know the loss ratio for a class of business with confidence, then the reserving procedure would become almost trivial. But of course the loss ratio is subject to uncertainty, just like other quantities used in claims reserving. Here again the past is no sure guide to the future. But though the reserver cannot have full knowledge of the future for the loss ratio, some familiarity with its past history and the current expectations of underwriters and ratemakers will be of great service.

This familiarity, in fact, should help to provide the reserver with a kind of standard, or benchmark, against which the results of other projections can be assessed. It should help to stabilise results where data are volatile, and provide a first guide to reserves where data are scanty or even non-existent. The loss ratio, and the techniques associated with it, thus form an important part of the reserver's toolkit. The only additional data element required for this work is the premium income (earned or written) for the class of business in question. Being a valid measure of the risk exposures it gives *scale* to the loss data, and hence enables the loss ratio benchmarking to begin.

Contents

[G1]
CONCEPT OF THE LOSS RATIO

The work so far has depended on the use of paid claims and case reserve data. At this stage we introduce a new element — the loss ratio — which brings considerable added scope to the methods.

The loss ratio is a simple concept, but a fundamental one in general insurance. If we take a class or subgroup of business and look at any given cohort, then once the development is complete the loss ratio can be found with certainty. It provides a natural way of summing up the result as a single figure.

$$\text{Loss Ratio} = \text{Ultimate Losses / Related Premium}$$

or

$$\lambda = L\text{-}ult \,/\, P$$

where λ is used as the symbol for loss ratio, and P for the premium earned in relation to the losses.

If we now aggregate the cohorts, we can find the loss ratio for the whole class of business. It will be in such terms that the underwriter thinks when quoting, say, the loss ratio for the motor portfolio. (The one problem with the measure, of course, is that the most recent cohorts will not be fully developed, and so cannot contribute properly to it. The loss ratio is only known with certainty several periods *in arrear*.)

The definition given above requires a little more attention. While the meaning of the ultimate losses *L-ult* should be clear, the premium term P remains in question. Does it denote earned premium or written premium, or perhaps the premium in-force? Is it an office premium including commission and expense, or is it the pure risk premium only? There is no absolute answer, and different forms can be used at different times.

To begin with, if we are using accident year cohorts as the basis of study, then earned premium will be the correct measure. But if policy, or contract, year cohorts are in use (as is common in reinsurance and the London Market), then written premium will be indicated. The point is that the premium definition should correspond to the risk exposure period of the cohort. The rule is:

Accident Year Exposure — *Earned* Premium
Policy Year Exposure — *Written* Premium

The question of including expenses and/or commission in the premium is more tricky. It would be quite possible to work either with the pure or office premiums. However, the picture is a fuller one if commission and expense *are* included, and we shall take that to be the case in the Manual. (This is the usual practice in the

British insurance industry, when quoting a loss ratio — but once again, the practice does differ in the London Market and reinsurance.)

Sources for the Loss Ratio

If we intend to apply loss ratio methods in reserving, the key question that arises is how to select the appropriate ratio for a given class or subgroup of business. There are a number of sources that might be used:

a) Data of past results for the given class.
b) Assumptions being used in the ratemaking process.
c) Opinion of underwriters and claims officials with knowledge of the business.
d) Market statistics for similar types of business, if available.

But whatever the source, it must be recognised that a forecast of some kind is effectively being made. If the class of business has a good stable record of loss ratio in the past 5 years, say, that is encouraging. But there is no guarantee that the level will be adhered to in future years. And more often, the loss ratio will be found to vary appreciably, as the underwriting cycle and other economic influences take their course.

Nevertheless, one has to make the best use of the evidence to hand, and take a rational view of the likely future course for the loss ratio. Then, as the months and years pass, the view must be updated as new influences make their mark, and old ones fade away or return. The importance of setting the loss ratio is that, at least for the time being, it will establish a benchmark against which the emerging loss development can be assessed. And it will give the reserver a standard, albeit a changeable one, to which to refer when other measures fail or cannot be applied.

[G2]
NAIVE LOSS RATIO METHOD

The word "naive" is included in the title as a warning. The estimate of the claims reserve given by this method derives from the simplistic assumption that the loss ratio cannot lie. Unfortunately, the real world does not contain such certainties — but the method still gives a useful reference point against which to view other more sophisticated methods, such as the Bornhuetter-Ferguson in §G3.

To carry out the estimation, some data are needed, and we shall as usual begin from the paid claims figures of the main example. These are repeated here for convenience:

		d					
		0	1	2	3	4	5
	1	1001	1855	2423	2988	3335	3483
	2	1113	2103	2774	3422	3844	
a	3	1265	2433	3233	3977		
	4	1490	2873	3880			
	5	1725	3261				
	6	1889					

Beside these must be set, naturally enough, the accepted loss ratio for the given class of business. We shall take this to be 83%, supposing it to have been settled after a consideration of past data for the class. Finally, we need to know the earned premium for each of the accident years 1 to 6. This is given here, with the figures in £1,000s as usual:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486

(aP is being adopted as the symbol for earned premium. For written premium, applicable to the policy year case, we would write wP.)

To make the actual estimate is simplicity itself. We have only to multiply the earned premium figures by the loss ratio of 83% to obtain the ultimate losses for each accident year. Then deducting the paid claims to date gives the required reserve. The calculations are shown overleaf.

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	83%	83%	83%	83%	83%	83%
$\hat{}L\text{-}ult$	7057	6210	5470	4714	4170	3723
pC^*	1889	3261	3880	3977	3844	3483
$\hat{}V$	5168	2949	1590	737	326	240

Overall Values:	$\sum L\text{-}ult$	31,344
	$\sum pC^*$	20,334
	Reserve	11,010

Not surprisingly, the value now obtained for the claims reserve does not accord with the previous paid and incurred claims projections. But what interpretation is to be placed on the discrepancy? This cannot be answered without some further investigation.

The first need, clearly, will be to re-examine the value taken for the loss ratio. It was based on past data for the business in question. But accident years for which development is complete enough to yield a loss ratio will be relatively old by now. More up-to-date information must be sought. Consultation with underwriters may indicate, say, that the market has softened in the last few years, with a persistent tendency for loss ratios to increase. Further talks with ratemaking staff may show that current rates are being set with an implicit loss ratio closer to 90%. This new evidence suggests that the loss ratio should be *trended*, say by 1% p.a. from 84% in year a=1 to 89% in year a=6. This allows the reserves to be recalculated as follows:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	89%	88%	87%	86%	85%	84%
$\hat{}L\text{-}ult$	7567	6584	5733	4885	4270	3768
pC	1889	3261	3880	3977	3844	3483
$\hat{}V$	5678	3323	1853	908	426	285

Overall Values:	$\sum L\text{-}ult$	32,807
	$\sum PL^*$	20,334
	Reserve	12,473

The result is now in line with the best estimate projections of the paid claims (§E4, E10). This is a satisfactory result, but should not give grounds for complacency. It would not be unusual to find, in later years, that the loss ratio had run ahead rather faster than originally predicted.

The major criticism of the method, however, is that it completely ignores the pattern of claims development to date for the recent accident years. The estimate of overall losses depends *only* on the premium income and the stated loss ratio for the class of business. Important changes shown, or incipient in, the claims development patterns will not be acknowledged or made use of in any way. The method only comes into its own where the claims development data are either scanty, unreliable or missing altogether. The best examples would be in new lines of business, and in the very long-tailed liability classes.

Thus, for the latter, the most recent accident years will not have had time to produce meaningful development figures in the context of the full liability. But the loss ratio approach enables an initial estimate to be made. Of course, it must be modified as time passes and more becomes known about the development. Then in the later years, more reliance can be placed on claims development figures, and the loss ratio estimate gradually phased out. The Bornhuetter-Ferguson method in §G3 in fact gives an automatic means for achieving this end.

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[G3]
BORNHUETTER-FERGUSON METHOD — INTRODUCTION

For any claims projection method based effectively on the use of development factors such as the chain-ladder method, it is often the case that the projected result cannot be relied on with the degree of confidence that one would like. This is particularly likely for more recent underwriting years, where the development factor to project from the current to ultimate claim amount is relatively large and variable, owing to the present lack of claims development.

However, it may be possible to make use of an alternative ultimate figure, usually derived from an assumed loss ratio. This may simply be taken as a fixed rate (such as 100%) as a reasonable first estimate of that experience, or it may be derived from external market views and information.

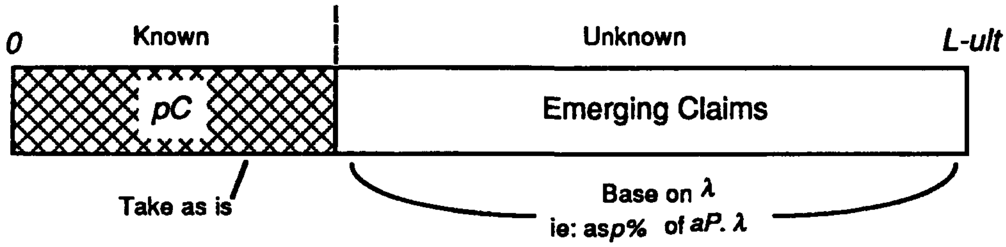
It is then possible to combine the original projected result with this alternative (a priori) value, using a weighted credibility approach. Under this, most weight is initially attached to the a priori value, gradually reducing to zero as the actual claims experience (and hence the projected value) develops towards its ultimate value. The Bornhuetter-Ferguson method adopts this principle.

The Bornhuetter-Ferguson method provides a nicely judged combination of the naive loss ratio method and the earlier paid/incurred claims projections. It is based on the idea of splitting the overall loss for each accident year into its past and future, or emerging, portions. These are then treated separately, according to their merits. The argument goes as follows:

> "As far as the past is concerned, the claims are already well known (paid claims method) or at least well estimated (incurred claims method). But the future is not well known, and the particular claims patterns and case reserves to date of the given accident year do not necessarily provide the right clue to it. It may be better to use a *more general* estimator, based on the overall loss ratio for the class of business in hand. This being done, and the two parts added together, we then have the most reliable estimate we can get for the overall losses, and hence for the required reserves."

The argument has much merit in it. Thus, taking the naive loss ratio method, we have already seen that it pays no attention to the actual claims development in the most recent accident years. Apparently, it flies in the face of reality. On the other hand, the claims development methods rely on the continuation into the future of the patterns for claim reporting and settlement, which seem to be indicated by the particular data in hand. A sudden shift in the pattern for the latest accident year in particular will throw the projections into disarray. The Bornhuetter-Ferguson method steers a safer course in these eventualities — its stability shows through well in the numerical example of §G8.

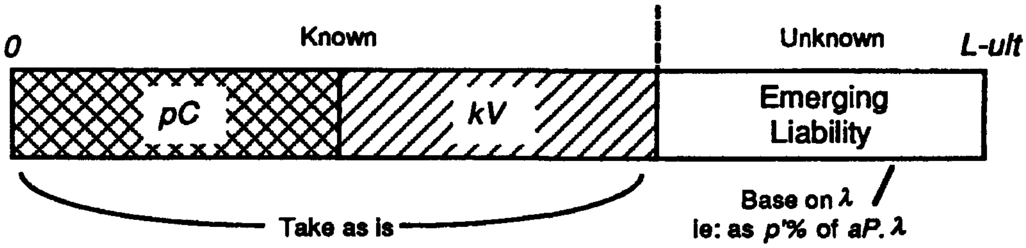
The Bornhuetter-Ferguson principle can equally well be applied using either paid or incurred claims as the base. With paid claims, the picture is as follows:



B/F on pC– Projection

The claim payments are split into 2 parts: those already made, and those which will emerge in the future. The first part has the known value *pC*, while the second is unknown. It is to be estimated as a proportion *p* of the final losses, which in turn are estimated by the simple application of the loss ratio to the earned premium.

With incurred claims, a larger part of the liability is taken as already known, by adding in the case reserves to the paid claims. But the unknown part of the liability is again found as a (smaller) proportion *p*' of the estimated final losses. The picture this time is:



B/F on iC– Projection

The main problem in B/F methods is just to determine the proportion *p* or *p*'. It turns out this can be done via the usual link ratio or grossing up methods applied to the triangle of paid claims or incurred claims data. We shall see that the correspondence is:

$$p = (1 - 1/f) \text{ or } p = (1 - g)$$

where *f* is the final link ratio, and g the grossing up factor, for a given accident year. (Since *f* and *g* can be applied equally for paid and incurred claims, the relations hold just as well whether *p* or *p*' is involved.)

Abbreviations

We are already using B/F as a shorthand for the Bornhuetter-Ferguson method. Further abbreviations to be used are:

$n\lambda$ — Naive Loss Ratio Method
$t\lambda$ — Trended Naive Loss Ratio Method

BF-pC — Bornhuetter-Ferguson Method applied to Paid Claims data
BF-iC — Bornhuetter-Ferguson Method applied to Incurred Claims

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[G4]
BORNHUETTER-FERGUSON ON INCURRED CLAIMS

The section is devoted to a numerical illustration of the B/F method. In the original paper (Bornhuetter & Ferguson 1972), the authors use a static loss ratio, and work with data in the form of *incurred* claims. They also use a link ratio/chain ladder approach in the first part of the exercise. We shall repeat these particular features here. The data will be the adjusted incurred claims triangle (§F7), with the premium figures and loss ratio (83%) from §G2.

The first stage is just to work out the link ratios themselves. It is the final ratios (f-values) that are needed for the method. Although a chain ladder approach is used below, any of the main link ratio variations could be substituted. Again, a grossing up method could be used equally well, with the g-factors substituting for the f-ratios. (Equivalence is: $1/g$ <—> f.) The working runs as follows:

		d 0	1	2	3	4	5	*ult*
	1	2866	3334	3503	3624	3719	3717	3717
	2	3359	3889	4033	4231	4319		
a	3	3848	4503	4779	4946			
	4	4673	5422	5676				
	5	5369	6142					
	6	5818						
$\sum$-*el*		20115	17148	12315	7855	3719		
$\sum$		25933	23290	17991	12801	8038	3717	3717
r		1.158	1.049	1.039	1.023	.999	1.000	
f		1.290	1.114	1.062	1.022	.999	1.000	

This array could be carried through to find the ultimate losses and reserve estimate, just as in the original demonstration of §E8. But these figures are not necessary for the B/F projection. It is the f-ratios which are the crucial output at this stage of the work.

The next step is to invert the f-ratios, and subtract the results from unity. The reason for doing this will soon become apparent.

f	1.290	1.114	1.062	1.022	.999	1.000
$1/f$	.775	.898	.942	.978	1.001	1.000
$1-1/f$	.225	.102	.058	.022	-.001	0

To explain, inverting the *f*-ratio gives the equivalent of a *g*- or grossing up factor. Now a *g*-factor simply shows the proportion that the claims in its column (whether paid or incurred) bears to the ultimate loss, as estimated. Hence $(1-g)$ shows the proportion that the *remaining* claims should bear to the ultimate figure. If we apply these $(1-g)$ factors, or $(1-1/f)$ which comes to the same thing, to the ultimate loss, then we have the remaining claims which should emerge in the future.

Now, at last, we are ready to bring in the loss ratio and premium data. These, of course, give us our benchmark estimates for the final losses, just as in the naive loss ratio method. To recap, the figures are:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	83%	83%	83%	83%	83%	83%
B-ult	7057	6210	5470	4714	4170	3723

These benchmark losses (symbol *B-ult*) are now to be used *as a base* for finding the remaining, or emerging, claims. This point is the crux of the B/F method, and where it distinguishes itself from the usual *pC* and *iC* projection methods. The calculations are straightforward:

a	6	5	4	3	2	1
B-ult	7057	6210	5470	4714	4170	3723
$1-1/f$	.225	.102	.058	.022	-.001	0
$\hat{}eV$	1588	633	317	104	-4	0

The result of multiplying the benchmark losses by the $(1-1/f)$ factors is called here the emerging liability, symbol $\hat{}eV$. (The ^ mark as usual shows that the value is in the nature of an *estimate*.)

$\hat{}eV$ is the liability still to emerge. It is to be contrasted with the liability already established, which is just the case reserves, *kV*. Adding the two parts together will give the whole required reserve:

a	6	5	4	3	2	1
*kV**	3929	2881	1796	969	475	234
$\hat{}eV$	1588	633	317	104	-4	0
CV	5517	3514	2113	1073	471	234

(The *kV*-values are taken from the case reserve data triangle — see §F3. The * indicates use of the main diagonal.)

Overall Values:	$\sum kV^*$	10,284
	$\sum \hat{}eV$	2,638
	Reserve	12,922

An alternative approach at this stage is to add the emerging liability to the incurred claims themselves. This then gives the estimate of the ultimate losses:

a	6	5	4	3	2	1
iC^*	5818	6142	5676	4946	4319	3717
$\hat{}eV$	1588	633	317	104	−4	0
$\hat{}L\text{-}ult$	7406	6775	5993	5050	4315	3717

Overall Values:	$\sum L\text{-}ult$	33,256
	$\sum pC^*$	20,334
	Reserve	12,922

There are one or two slight puzzles here.

a) We now have two new sets of figures for the estimated final losses: i) the benchmark set, ii) the set found immediately above. Which are to be believed? Of course, the B/F method points us towards the second set. The benchmark figures were used along the way, but can now be dropped. If we took them completely to heart, we should just arrive back at the naive loss ratio method of §G2.

b) There is a negative figure in the set of values for $\hat{}eV$, the emerging reserves, at a=2. That is really no problem. It arises because the incurred claims for the earlier accident year a=1 (i.e. at the point d=4) exceeds the ultimate loss for that year. The excess is then projected forward into the figures for year a=2. In this particular case, the value is trivial. Nevertheless, the liability is still reduced — it would be possible to take a cautious view by excluding any such negatives in the projection, just by setting them to zero.

Summary of the Method

To bring the whole procedure together, we now summarise the main steps in the Bornhuetter-Ferguson method using incurred claims:

i) The incurred claims data are set out in the triangular form, and projected using a link ratio or grossing up technique. (The chain ladder variation is used here, but is not obligatory.)

ii) If a link ratio method is used, the final ratios f are inverted (i.e. to produce the equivalent of a g-factor).

iii) The benchmark losses are found by multiplying the earned premium for each accident year by the chosen loss ratio.

iv) The emerging reserves are estimated by applying factor $(1 - 1/f)$, or $(1 - g)$ as the case may be, to the benchmark losses.

v) The emerging reserves are added to the existing incurred claims data to give the estimate of final loss.

vi) The reserve is taken as the estimated final loss minus the paid claims.

Calculations in Full

		d 0	1	2	3	4	5	*ult*
	1	2866	3334	3503	3624	3719	3717	3717
	2	3359	3889	4033	4231	4319		
a	3	3848	4503	4779	4946			
	4	4673	5422	5676				
	5	5369	6142					
	6	5818						
$\sum$-*el*		20115	17148	12315	7855	3719		
$\sum$		25933	23290	17991	12801	8038	3717	3717
r		1.158	1.049	1.039	1.023	.999	1.000	
f		1.290	1.114	1.062	1.022	.999	1.000	
$1/f$		.775	.898	.942	.978	1.001	1.000	
aP		8502	7482	6590	5680	5024	4486	
λ		83%	83%	83%	83%	83%	83%	
B-ult		7057	6210	5470	4714	4170	3723	
$1-1/f$		.225	.102	.058	.022	-.001	0	
^*eV*		1588	633	317	104	-4	0	
*iC**		5818	6142	5676	4946	4319	3717	
^*L-ult*		7406	6775	5993	5050	4315	3717	

Overall Values:	$\sum L\text{-}ult$	33,256
	$\sum pC^*$	20,334
	Reserve	12,922

Key to Symbols

$\sum$	Column sum	r	One-step link ratio
$\sum -el$	Column sum − Last element	f	Final link ratio
aP	Earned premium	λ	Loss ratio
$B\text{-}ult$	Benchmark losses	iC	Incurred claims
eV	Emerging reserves	^	Estimate symbol
$L\text{-}ult$	Ultimate losses (B/F estimate)	pC	Paid claims

(Where a grossing up technique is used in preference to link ratios, g will substitute for $1/f$.)

Formulae

It may help also to put down the main relationships which are used in the calculations. These can be expressed in words and/or symbols as follows:

Benchmark Loss $\quad B\text{-}ult = \lambda .\ aP$
= Loss Ratio × Earned Premium

Emerging Liability $\quad \hat{\ } eV = (1 - 1/f).\ B\text{-}ult$
= B/F Proportion × Benchmark Loss

Estimated Ultimate Loss $\quad \hat{\ } L\text{-}ult = iC + \hat{\ } eV$
= Incurred Claims + Emerging Liability

Reserve Required $\quad CV = \hat{\ } L\text{-}ult - pC$
= Estimated Ultimate Loss − Paid Claims to Date

The first formulation given in the text, using Case Reserves to go direct to the full claims reserve value, is:

Reserve Required $\quad CV = kV + \hat{\ } eV$
= Case Reserves + Emerging Liability

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[G5]
BORNHUETTER-FERGUSON ON PAID CLAIMS

The B/F method is equally applicable to paid claims data as to the incurred. The working is very similar, indeed almost coincident with that used for the incurred claims. As usual, we illustrate by means of numerical example.

The first part of the exercise is to work through the triangle of paid claims by either a link ratio or grossing up method. Having used the former for the incurred claims, we will here choose grossing up by way of contrast. But any of the link ratio variations could equally well be used. The point is to determine apt values for the g-factors, i.e. the equivalent of $1/f$ in the link ratio case. The working is as follows, using the Arabic technique with simple averaging of the factors in each column. The last 2 lines summarise the resulting g and $(1-g)$ values.

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
		27.0	50.1	65.4	80.6	90.0	94.0%	
	2	1113	2103	2774	3422	3844		
		26.1	49.2	64.9	80.1	90.0%		
	3	1265	2433	3233	3977			
a		25.6	49.2	65.4	80.4%			
	4	1490	2873	3880				
		25.0	48.3	65.2%				
	5	1725	3261					
		26.0	49.2%					
	6	1889						
		25.9%						
g		.259	.492	.652	.804	.900	.940	
$1-g$		.741	.508	.348	.196	.100	.060	

The second stage is to bring in the benchmark estimates for the final losses, based of course on the loss ratio and earned premium data. The figures are the same as they were for the BF-iC projection.

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	83%	83%	83%	83%	83%	83%
B-ult	7057	6210	5470	4714	4170	3723

To these benchmark figures, we simply apply the factors (1 -*g*) in order to bring out the remaining, or emerging, claims. The rationale is as before: the *g*-factor shows the proportion that the claims in its column bear to the ultimate loss, as estimated. Hence (1 -*g*) shows the proportion that the *remaining* claims should bear to the ultimate figure. The working is as follows:

a	6	5	4	3	2	1
B-ult	7057	6210	5470	4714	4170	3723
$1-g$	.741	.508	.348	.196	.100	.060
^*eC*	5229	3155	1904	924	417	223

Overall Value: $\sum$ ^*eC* 11,852

The result is here called the emerging claims, symbol ^*eC*. (The ^ denotes an estimate, as usual.) For each accident year, ^*eC* is the claims still to emerge. The summation over the accident years immediately gives the estimate for the full claims reserve.

Adding the emerging claims to the claims already established, i.e. the value *pC*, gives the final estimated losses by accident year:

a	6	5	4	3	2	1
*pC**	1889	3261	3880	3977	3844	3483
^*eC*	5229	3155	1904	924	417	223
^*L-ult*	7118	6416	5784	4901	4261	3706

Overall Values:	$\sum$*L-ult*	32,186
	$\sum$*pC**	20,334
	Reserve	11,852

Summary of the Method

To bring the whole procedure together, we now summarise the main steps in the Bornhuetter-Ferguson method using paid claims:

i) The paid claims data are set out in the triangular form, and projected using a grossing up or link ratio technique. (The Arabic variation is used here, but is not obligatory.)

ii) If a link ratio method is used, the final ratios f are inverted (i.e. to produce the equivalent of a g-factor).

iii) The benchmark losses are found by multiplying the earned premium for each accident year by the chosen loss ratio.

iv) The emerging claims are estimated by applying the factor $(1-g)$, or $(1-1/f)$ as the case may be, to the benchmark losses. Adding these claims together across the accident years gives the required reserve.

v) Finally, the emerging claims are added to the existing paid claims data to give the estimate of final loss.

Calculations in Full

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
		27.0	50.1	65.4	80.6	90.0	94.0%	
	2	1113	2103	2774	3422	3844		
		26.1	49.2	64.9	80.1	90.0%		
	3	1265	2433	3233	3977			
a		25.6	49.2	65.4	80.4%			
	4	1490	2873	3880				
		25.0	48.3	65.2%				
	5	1725	3261					
		26.0	49.2%					
	6	1889						
		25.9%						
g		.259	.492	.652	.804	.900	.940	
aP		8502	7482	6590	5680	5024	4486	
λ		83%	83%	83%	83%	83%	83%	
B-ult		7057	6210	5470	4714	4170	3723	
$1-g$		.741	.508	.348	.196	.100	.060	
^eC		5229	3155	1904	924	417	223	

Overall Reserve: $\sum$ *^eC* 11,852

It is usually worthwhile to complete the calculations to give the final losses aswell as the reserve itself. This is done below.

^eC	5229	3155	1904	924	417	223
*pC**	1889	3261	3880	3977	3844	3483
^L-ult	7118	6416	5784	4901	4261	3706

Overall Losses: $\sum$*L-ult* 32,186

Key to Symbols

g	Grossing up factor		
aP	Earned premium	λ	Loss ratio
B-ult	Benchmark losses	*pC*	Paid claims
eC	Emerging claims	^	Estimate symbol
L-ult	Ultimate losses (B/F estimate)		

(Where a link ratio technique is used in preference to grossing up, $1/f$ will substitute for *g*.)

Formulae

The main relationships used in the calculations can be expressed in words and/or symbols as follows:

Benchmark Loss $\quad B\text{-}ult = \lambda .\ aP$
= Loss Ratio × Earned Premium

Emerging Claims $\quad \hat{}eC = (1-g).\ B\text{-}ult$
= B/F Proportion × Benchmark Loss

Reserve Required $\quad CV = \sum_a (\hat{}eC)$
= Sum of Emerging Claims by Accident Year

The estimate of the ultimate loss follows from the further relationship:

Estimated Ultimate Loss $\quad \hat{}L\text{-}ult = pC + \hat{}eC$
= Paid Claims + Emerging Claims

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[G6]
COMPARISON OF RESULTS

Having worked through the Bornhuetter-Ferguson method for both paid and incurred claims, it will be worthwhile to make a comparison of the numerical results. Following the order of the main text, let us begin with the BF-*iC* projection. Because this is a hybrid technique, we need to take into account 3 different projections:

a) Incurred Claims Projection (Link Ratio, Best Estimate)
b) B/F on Incurred Claims
c) Naive Loss Ratio Method

Figures for the projected losses and the required reserves are as follows:

		Projected Losses			% Divergence
		iC	BF-iC (from G4.3)	$n\lambda$ (from G2.2)	
Accident Year	6	7505	7406	7057	22.1
	5	6842	6775	6210	10.6
	4	6028	5993	5470	6.3
	3	5055	5050	4714	1.5
	2	4315	4315	4170	0.0
	1	3717	3717	3723	0.0
	$\sum$	33,462	33,256	31,344	9.7

		Required Reserves			% Divergence
		iC	BF-iC (from G4.2)	$n\lambda$ (from G2.2)	
Accident Year	6	5616	5517	5168	22.1
	5	3581	3514	2949	10.6
	4	2148	2113	1590	6.3
	3	1078	1073	737	1.5
	2	471	471	326	0.0
	1	234	234	240	0.0
	$\sum$	13,128	12,922	11,010	9.7

The % divergence is measured as the relative divergence of the BF-*iC* loss (or reserve) value from the incurred claims result towards the naive loss ratio one. Formula:

$$[\hat{}L\{iC\} - \hat{}L\{BF\text{-}iC\}] \,/\, [\hat{}L\{iC\} - \hat{}L\{n\lambda\}]$$

The characteristics of the B/F method begin to emerge well from this comparison. Three points in particular can be made:

a) It yields an overall result intermediate between the incurred claims projection and the naive loss ratio estimate.

b) In the more developed accident years (a=1,2,3), it is very close to or coincident with the iC values.

c) In the less developed years (a=4,5,6), it diverges rather more from the iC projection in the direction of the naive loss ratio estimate.

These characteristics show clearly the general properties of the B/F method which are explored further in §G7. They will be found to be repeated to a large extent with the paid claims projection — but there is a very important difference of emphasis. The figures now follow:

		Projected Losses			% Divergence
		pC (from E3.2)	BF-pC (from G5.2)	$n\lambda$ (from G2.2)	
	6	7293	7118	7057	74.2
	5	6628	6416	6210	50.7
Accident	4	5951	5784	5470	34.7
Year	3	4947	4901	4714	19.7
	2	4271	4261	4170	9.9
	1	3705	3706	3723	5.6
	Σ	32,795	32,186	31,344	42.0

		Required Reserves			% Divergence
		pC (from E4.1)	BF-pC (from G5.2)	$n\lambda$ (from G2.2)	
	6	5404	5229	5168	74.2
	5	3367	3155	2949	50.7
Accident	4	2071	1904	1590	34.7
Year	3	970	924	737	19.7
	2	427	417	326	9.9
	1	222	223	240	5.6
	Σ	12,461	11,852	11,010	42.0

The % divergence is measured as the relative divergence of the BF-*pC* loss (or reserve) value from the paid claims result towards the naive loss ratio one. Formula:

$$[\hat{L}\{pC\} - \hat{L}\{\mathrm{BF}\text{-}pC\}] \,/\, [\hat{L}\{pC\} - \hat{L}\{n\lambda\}]$$

It will be seen that the general pattern whereby the B/F method yields an intermediate result remains. But this time the divergence from the *pC* values is present in all the accident years. In the most recent years (a=5,6), it becomes far more pronounced in the direction of the naive loss ratio estimate. This difference of emphasis is only to be expected. The reason is that the BF-*pC* method refers all claims not actually paid by the reserving date to the benchmark calculation. But in BF-*iC*, only those additional reserves for claims beyond the case reserves are referred in this way. (The diagram in §G3 should make the point clear.)

The particular figures for the divergence of the B/F results from the *pC* and *iC* projections shown here are, of course, illustrative only. The exact values taken will depend completely on the data in hand, and the variations found in practice can be wide.

There is one last interesting point to be made. That is, while for any given accident year the B/F figures must be of an intermediate nature, it is not true in absolutely all cases of the overall result. This is because crossovers can occur, say, in the naive loss ratio and *pC* figures for projected claims by accident year, and these can sometimes push the B/F figure out of alignment. But it would be unusual to find such a result in practice.

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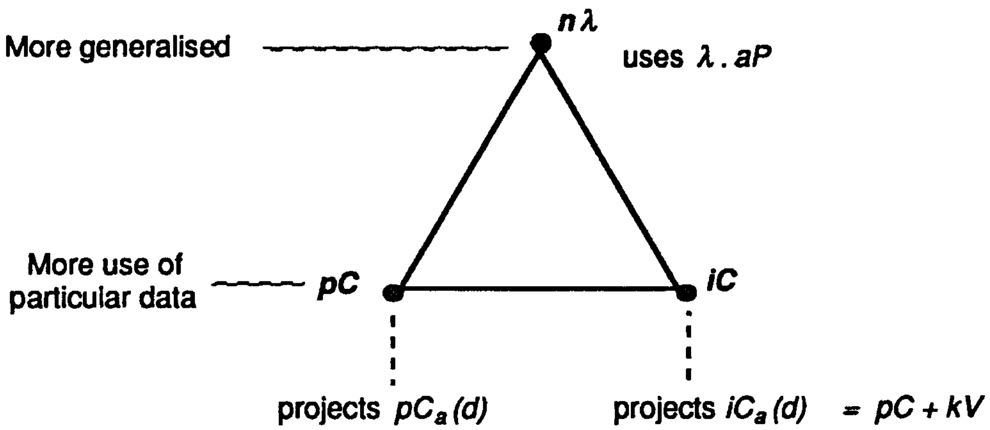
[G7]
TAKING STOCK OF THE METHODS ($pC/iC/n\lambda$/BF)

It is time to take stock of the position. We have by now used three different primary routes to reach the reserve estimate:

a) Paid claims projection
b) Incurred claims projection
c) Naive loss ratio method

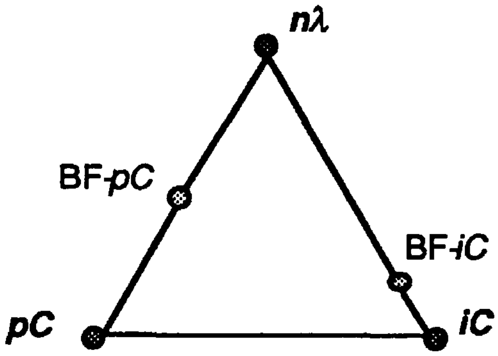
In addition, we have put together route c) with either a) or b) according to the Bornhuetter-Ferguson prescription.

Taking first the three primary routes, the difference between them is not fully characterised by the method of calculation — in fact, the calculations required for the *pC* and *iC* projections are identical in form. Rather, the difference is to be found in the data elements, i.e. the starting point for the method, and in the assumptions which underlie it.



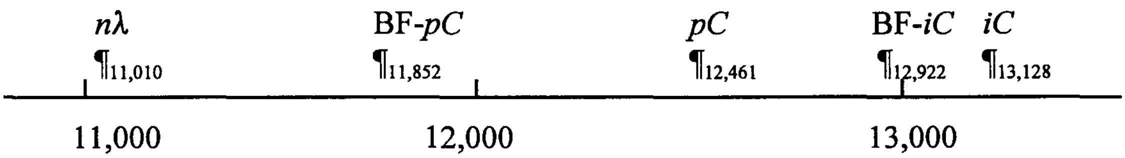
The diagram summarises the position. Note that while the *pC* and *iC* methods make use of developing data, dependent on both accident year and development period, the naive loss ratio method takes no account of these details. In a sense, it is on a loftier, more generalised plane.

Next we can place the B/F projections on the diagram. They try to make the best of both worlds, the generalised and the detailed, by combining the naive loss ratio method with either *pC* or *iC*. (The principle behind the combination is given in full in §G3.)



Note that the BF-*iC* point will be relatively closer to *iC* than will BF-*pC* be to *pC*. The exact positioning of the B/F points along the two lines will depend very much on the particular data distributions given, as already noted in the previous section.

Another useful way to illustrate the B/F divergence towards *nλ* from either *pC* or *iC* is to plot the reserve values obtained along an axis:



In this particular case, the BF-*pC* point divides the (*nλ pC*) range in the ratio 58:42. For BF-*iC* and the (*nλ iC*) range, the ratio is 90:10.

Choice of Estimate

Let us return to the central problem — given the widening range of estimates, how is the final choice to be made? As usual, hard and fast rules cannot be laid down. To begin with, the right course is to seek ways of reconciling the different estimates, to look for systematic reasons for the divergences observed. In the present case, the estimate most out of kilter is the naive loss ratio one.

An explanation for this, in fact, is already to hand. We have been working with *a fixed* loss ratio of 83%, which was shown to be a poor assumption in §G2. The 83% value was used, essentially, because the original B/F paper employs such a fixed ratio in its description. But there is nothing to prevent the reserver from applying B/F with a trended ratio, or a ratio varying in some other way — say, cyclically, to match the fluctuations of the underwriting cycle.

To illustrate this, let us bring in the trended ratio, moving from 84% in year *a*=1 to 89% in year *a=6.* This brings out the losses as follows (as in §G2):

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	89%	88%	87%	86%	85%	84%
L-ult	7567	6584	5733	4885	4270	3768

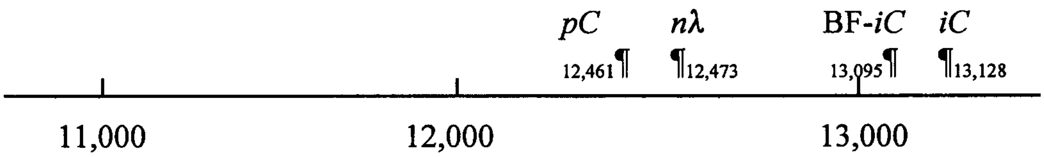
The resulting estimate for the full reserve is £12,473.

We can now bring these revised figures into the main B/F calculations as the benchmark losses. There is no need to rework the basic claims triangles, nor the calculation of the (1−1/*f*) factors, but the subsequent figures must be re-calculated. The results for the BF-*iC* projection are set out here:

a	6	5	4	3	2	1	
B-ult	7567	6584	5733	4885	4270	3768	
$1-1/f$	.225	.102	.058	.022	-.001	0	
							(from G4.2)
^*eV*	1703	672	333	107	-4	0	
*iC**	5818	6142	5676	4946	4319	3717	
							(from G4.3)
^*L-ult*	7521	6814	6009	5053	4315	3717	

Overall Values:	∑*L-ult*	33,429
	∑*pC**	20,334
	Reserve	13,095

It is hardly surprising to find that the BF-*iC* result is now even closer to the original *iC* value of £13,128. (This is because the loss ratio estimate itself has been brought much nearer to the *pC* and *iC* projections.) The values can again be plotted along an axis:



It would be possible to repeat the B/F calculations for the *pC's,* using the new benchmark losses. But since the loss ratio and *pC* estimates are already so close, this is hardly worthwhile. In the present example, we have reached the point where loss ratio and B/F calculations tell us little more than the original *pC* and *iC* projections. We are back to the range of approximately £12,500 to £13,000 for the best estimate of the required reserves.

Is this a typical result, throwing doubt on the usefulness of doing loss ratio and B/F calculations at all? The answer is emphatically *no*. Loss ratio methods, including particularly the B/F variation, are an important tool in the armoury. They come particularly into their own for the very long-tail classes of business, where the build up of paid and incurred claims in the early years is very slow. But there are more general reasons for their importance, which will come out in the remaining sections of this part of the Manual.

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[G8]
SENSITIVITY TESTING & CHOICE OF ESTIMATE

There is one feature of the projections which has been mentioned briefly before (§E1), but not brought out sufficiently. This is the fact that, when the overall claims reserve is analysed, it is the most recent accident years which make the dominant contribution. For example, the *pC* projection we have been using gives the following breakdown:

Total Reserve = 12,461

a	6	5	4	3	2	1
$\hat{V}$	5404	3367	2071	970	427	222
%	43.4	27.0	16.6	7.8	3.4	1.8

Here, 87% of the liability is concentrated in the most recent three accident years. A similar, but not quite coincident, pattern results if we look at the trended loss ratio method's results (from §G2):

Total Reserve = 12,473

a	6	5	4	3	2	1
$\hat{V}$	5678	3323	1853	908	426	285
%	45.5	26.6	14.9	7.3	3.4	2.3

Here again, 87% of the liability comes from the three latter accident years, though this time the emphasis on the last of all (year 6) is even greater.

The corollary which must be noted is that the projections will tend to be particularly sensitive to the actual claims data for the very recent years. They will be most sensitive of all to the figure in the *bottom left hand corner* of the data triangle. But there is a notable exception to this general rule. The B/F projections, in fact, are not sensitive at all to the lower left hand figure. This feature is worth demonstrating in detail.

Consider a projection in which, for the latest accident year, the figures are as follows:

	£		
Paid claims	30	Grossing up factor	30%
Earned premium	125	Loss ratio	80%

The *pC* method gives a projected ultimate loss for the accident year of:

$$pC\,/g = 30\,/\,.3 \;=\; 100$$

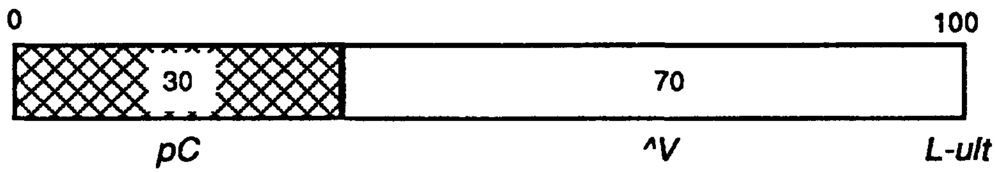
which agrees with the loss ratio method's value of:

$$\lambda\,.\,aP = .8 \times 125 = 100$$

The required reserve in both cases is:

$$(100\;-\;30) = 70$$

The position can be shown in a diagram:



Since *pC* and $n\lambda$ give the same answer, so also will BF-*pC*. The calculation is:

$$\hat{}V = \;(1\;-g)\,.\,B\text{-}ult = (1\;-.3) \times 100 = \;70$$

Now consider what happens to the three estimates if, for some unexplained reason, the paid claims for the accident year come through as 33 instead of 30. It is assumed that every other quantity in the problem retains its former value. In particular, earned premium is still 125 and the *g*-factor is still 30%.

a) *pC Projection*

$$\begin{aligned} \hat{}L\text{-}ult &= pC\,/g &&= 33\,/\,.3 &&= 110 \\ \hat{}V &= \hat{}L\text{-}ult - pC &&= 110\;-\;33 &&= 77 \end{aligned}$$

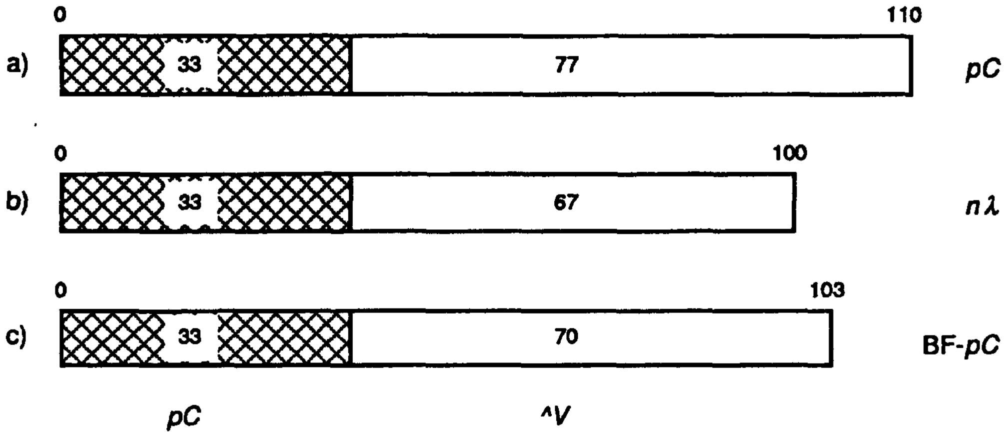
b) *$n\lambda$ Estimate*

$$\begin{aligned} \hat{}L\text{-}ult &= \lambda\,.\,aP &&= .8 \times 125 &&= 100 \\ \hat{}V &= \hat{}L\text{-}ult - pC &&= 100\;-\;33 &&= 67 \end{aligned}$$

c) *BF-pC Projection*

$$\begin{aligned} \hat{}V &= (1\;-g)\,.\,B\text{-}ult &&= .7 \times 100 &&= 70 \\ \hat{}L\text{-}ult &= \hat{}V + pC &&= 70 + 33 &&= 103 \end{aligned}$$

We now have three quite distinct answers! With an increase of 10% in the figure for paid claims, the *pC* method has increased the reserve for the year by the same proportion. The naive loss ratio method has *reduced* the reserve by a lesser amount (about 4%), while the BF-*pC* method has left the reserve exactly where it was before. The position can again be shown most clearly by a diagram:



This demonstrates the stability of the B/F method when data for the recent accident years are in a volatile state, or where there is doubt as to their true values. (In an accounting sense *pC* is a "hard" figure but in fact it is a random variable.) It is an extremely useful technique to apply in such circumstances.

Shifts in Payment Pattern & Loss Ratio

We have still not thrown enough light on how to choose the estimate, when given *pC* or *iC*, naive loss ratio and B/F values. But the example just given will be found to yield some clues. To begin with, we can look at the shift in the payment pattern of claims which is implicit in each method. The value to focus on here is the grossing factor *g*, of 30%. This will have been derived from the grossing up or link ratio methods applied to the earlier accident years. Given that the paid claims in the latest accident year now seem to be coming out on the high side, what response do the three methods make? That is, to what extent does each allow for a shift in the underlying payment pattern on the claims?

a) pC Projection

Assumes there is no shift at all. *g* remains static at 30%. This is the way the whole projection works.

b) $n\lambda$ Estimate

Payment pattern is assumed to be speeding up. Effective *g*-value for the accident year changes to: 33/100 = 33%.

c) BF-pC Projection

Again, speed up in payment pattern assumed. But not so pronounced as $n\lambda$ case. The factor is now: 33/103 = 32%.

A second way of looking at the reaction of the three methods is from the point of view of the loss ratio on the business. Taking the paid claims for the most recent accident year as a proportion of the earned premium, this has gone up from: 30/125 = 24% to the higher value of: 33/125 = 26.4%. Should it be assumed from this evidence that the ultimate loss ratio on the year will also increase? Again, the three methods give different answers:

a) pC Projection

Ultimate loss ratio will increase, from 80% to the value: 110/125 = 88%.

b) λA Estimate

In spite of first year increase in paid claims, the loss ratio overall will stay put at 80%.

c) BF-pC Projection

Loss ratio will increase, but not so much as suggested by the *pC* method. Its value will be: 103/125 = 82.4%

Using this analysis, a more general answer can now be given on the choice of estimates as between *pC,* naive loss ratio and BF-*pC*. This runs as follows:

i) Where the data are very stable, there is likely to be little to choose between the different estimates.

ii) Where the data patterns are shifting, the reserver should assess to what extent stability still remains in either: a) the payment patterns, or b) the loss ratio for the succeeding accident years.

iii) Where the evidence is of a stable payment pattern, the *pC* projection will be preferable. Where evidence supports a fairly constant loss ratio, the loss ratio method will be better.

iv) Where data are decidedly volatile, or scanty, particularly in the most recent accident years, a B/F projection will come into its own, and provide the firmest ground.

Very similar considerations apply to the choice between the *iC*, $n\lambda$ and BF-*iC* methods. In this case, however, reporting patterns of claims and stability of case reserving enter the balance in addition to the payment patterns. This may have the merit of bringing in somebody else's judgment on case estimates. It can be seen that the central question to be asked in all cases is: How far are the assumptions underlying the chosen method likely to be satisfied in the data? The weight to be give to each method might depend upon one's judgment on ii), iii), and iv) above.

[G9]
PAID LOSS RATIO — STEP-BY-STEP PROJECTION

There is more that can be learned from the study of the loss ratio in claims reserving. So far, we have only worked with an assumed final, or *ultimate,* loss ratio. This is the proportion of claims to premiums for the business in question when the development is complete. But the progression of the loss ratio towards its final value can also be studied, and it can be done with either the paid or the incurred figures. In the present section, we shall concentrate on the former, leaving the incurred for §G11.

The loss ratio progression can be derived from the usual data. It is only necessary to divide the developing claims for a given accident year by the value of earned premium for that year. In terms of symbols,

$$p\lambda(d) = pC(d)\ / aP$$

where $p\lambda$ is simply called the "paid loss ratio". To look at the numbers, we take the usual data triangle of paid claims and divide along each row in turn by the earned premium for that row. The results are:

aP	*a*	*d* 0	1	2	3	4	5
4486	1	1001	1855	2423	2988	3335	3483
		22.31	41.35	54.01	66.61	74.34	77.64%
5024	2	1113	2103	2774	3422	3844	
		22.15	41.86	55.21	68.11	76.51%	
5680	3	1265	2433	3233	3977		
		22.27	42.83	56.92	70.02%		
6590	4	1490	2873	3880			
		22.61	43.60	58.88%			
7482	5	1725	3261				
		23.06	43.58%				
8502	6	1889					
		22.22%					

(Along the top row, the paid claims 1001, 1855 ... ... 3483 have been divided by the premium value of 4486 to give percentages 22.31, 41.35 ... ... 77.64%. In the second row, the elements have been divided by 5024, and so on.)

The display is informative, but for a clear view it is better to set out the ratios on their own:

		d					
		0	1	2	3	4	5
	1	22.31	41.35	54.01	66.61	74.34	77.64%
	2	22.15	41.86	55.21	68.11	76.51	
a	3	22.27	42.83	56.92	70.02		
	4	22.61	43.60	58.88			
	5	23.06	43.58				
	6	22.22					

It can be seen that ratios increase steadily along each row towards a final value of 80%+. This is in accord with our earlier assumption of an 83% loss ratio. Further, looking down the columns, there is clear evidence of an upward trend in almost all development periods. The exception is the column d=0, which shows a very stable pattern indeed. In period d=l, the upward trend is very approximately +.5%, while in d=2 it rises to +1.5%. In periods d=3,4, the trend is nearly +2%. The pattern is strongly suggestive. If trends so continue, annual increases of 2%+ in the final loss ratio would very much be expected.

How can the triangle be more formally evaluated? For periods d=1,2,3, the trends can certainly be projected, say by the least squares method. For d=4, this becomes more difficult since there are only two data values, and for d=5 no trend at all can be established. Hence direct trending down the columns will not of itself provide any answers for the movement in the ultimate loss ratio.

There is a simple answer to this problem. We move from overall values of the loss ratio to a step by step, or incremental, approach. That is, we calculate by how much the ratios *increase* at each successive development interval. This yields the following triangle:

		d					
		0	1	2	3	4	5
	1	22.31	19.04	12.66	12.60	7.73	3.30%
	2	22.15	19.71	13.35	12.90	8.40	
a	3	22.27	20.56	14.09	13.10		
	4	22.61	20.99	15.28			
	5	23.06	20.52				
	6	22.22					

In detail: for year a=1, $p\lambda$ at the end of period d=0 is 22.31%. Then in period d=1 it goes up to 41.35%, and the increase is 19.04%. Similarly, in d=2 it goes up to 54.01%, a further step of 12.66%. The same process carries on throughout to give the new triangle.

Examining the stepwise data, the columns from d=1 onward all show evidence of the rising trend. We can project them downward by fitting the least squares trendlines, according to the method described in §B8. The required calculations are given in the annex at the end of the section. (They have the same form as those shown in §E9). The results are as follows:

Col	d=1:	one value,	21.44
	d=2:	two values,	16.00, 16.86
	d=3:	three values,	13.37, 13.62, 13.87
	d=4:	trending not appropriate,	use 8.40
	d=5:	no trending possible,	use 3.30

One final matter still has to be settled. That is the value of the step from period d=5 to ultimate. If we take the standard loss ratio of 83% as previously used, this step must be just 83.00 - 77.64% = 5.36%. The full projection can now be put down, converting the previous triangle into a square:

		d: 0	1	2	3	4	*ult*
a	1	22.31	19.04	12.66	12.60	7.73	8.66%
	2	22.15	19.71	13.35	12.90	8.40	***8.66%***
	3	22.27	20.56	14.09	13.10	***8.40***	***8.66%***
	4	22.61	20.99	15.28	***13.37***	***8.40***	***8.66%***
	5	23.06	20.52	***16.00***	***13.62***	***8.40***	***8.66%***
	6	22.22	***21.44***	***16.86***	***13.87***	***8.40***	***8.66%***

The figures in italics are the projected ones. In the final column, 8.66% is just the sum of 3.30 and 5.36%. This column summarises the two steps, d=4 →5 and d=5→*ult*.

The result is propitious — we now have a complete set of stepwise paid loss ratios for each accident year. Adding along the rows will give the projected final loss ratios. The work is reduced if we replace the upper left triangle of known loss ratios by their cumulative values to date.

		d						
		0	1	2	3	4	*ult*	*row sum*
	1						83.00	83.00
	2					76.51	8.66	85.17
a	3				70.02	8.40	8.66	87.08
	4			58.88	13.37	8.40	8.66	89.31
	5		43.58	16.00	13.62	8.40	8.66	90.26
	6	22.22	21.44	16.86	13.87	8.40	8.66	91.45

The projected final loss ratios are all that is needed to make the full estimate of losses and reserves. They are simply multiplied with the usual earned premium figures:

a	^λ	*aP*	^*L-ult*
1	83.00	4486	3723
2	85.17	5024	4279
3	87.08	5680	4946
4	89.31	6590	5886
5	90.26	7482	6753
6	91.45	8502	7775

Overall Values:	∑*L-ult*	33,362
	∑*pC**	20,334
	Reserve	13,028

The result is interesting for the steady upward progression it yields in the ultimate loss ratio. The trend is of the order of 2% p.a. or a little less. This is satisfying in that it confirms the reaction on first seeing the paid loss ratio figures set out on page G9.2.

Evaluation of the Method

This method is a particularly important one, as may be seen by the following argument. Thus, a strong objection to the usual claim development methods of §E–F is their sensitivity to the actual amount of paid claims for the latest accident year (see §G8). In contrast, the Bornhuetter-Ferguson method is not sensitive in this way. But it is dependent on the particular choice made for the loss ratio — an outdated value can easily spoil the estimates.

The virtue of the present method is that it largely avoids both these criticisms. First, in common with Bornhuetter-Ferguson, it does not depend on the bottom left hand element in the triangle. These latest year paid claims can take any value at all, and the projection will yield the same answer for the final reserve. Second, in common with the claim development methods, it does not depend on some arbitrary value for the loss ratio. Instead, it makes good use of the observed claims patterns of earlier accident years, applying them in a consistent way to forecast the ultimate loss ratios.

Annex: Trendline Calculations

The trendline $y = bx + c$ is to be fitted to the n data points $(x_i\, y_i)$. (See §B8 for theory.)

Formulae: $c = \bar{y}$ $\quad b = \sum x_i y_i \,/\, \sum x_i^2$

where: $\bar{y} = \sum y_i/n$ $\quad$ and x-axis chosen so that: $\bar{x} = \sum x_i \,/\, n = 0$.

d=1:

a	x_i	y_i	x_i^2	$x_i y_i$
1	-2	19.04	4	-38.08
2	-1	19.71	1	-19.71
3	0	20.56	0	0
4	1	20.99	1	20.99
5	2	20.52	4	41.04
$\sum$	0	100.82	10	4.24

Hence: $c = 100.82 / 5 = 20.164$
$b = 4.24 / 10 = .424$

Projection: $20.164 + 3 \times .424 = 21.44$

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d=2:

a	x_i	y_i	x_i^2	$x_i y_i$
1	-1.5	12.66	2.25	-18.99
2	-.5	13.35	.25	-6.68
3	.5	14.09	.25	7.05
4	1.5	15.28	2.25	22.92
$\sum$		55.38	5	4.30

Hence: $c = 55.38 / 4 = 13.845$
$b = 4.30/5 = .860$

Projections: $13.845 + (2.5, 3.5) \times .860 = (16.00, 16.86)$

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d=3:

a	x_i	y_i	x_i^2	$x_i y_i$
1	−1	12.60	1	−12.60
2	0	12.90	0	0
3	1	13.10	1	13.10
$\sum$		38.60	2	.50

Hence:

$$c = 38.60/3 = 12.867$$
$$b = .50/2 = .250$$

Projections: $12.867 + (2, 3, 4) \times .250 = (13.37, 13.62, 13.87)$

◇

[G10]
PAID LOSS RATIO & PAID CLAIMS PROJECTION

In this section, we shall examine the projection of loss ratio as an alternative for projecting the triangle of paid claims. The starting point is the triangle of paid loss ratios, as first derived in the previous section. The figures show the progress of the ratio for each accident year as development time d increases:

		d					
		0	1	2	3	4	5
	1	22.31	41.35	54.01	66.61	74.34	77.64%
	2	22.15	41.86	55.21	68.11	76.51	
a	3	22.27	42.83	56.92	70.02		
	4	22.61	43.60	58.88			
	5	23.06	43.58				
	6	22.22					

The triangle is, of course, closely related to that of the original data on paid claims. It differs in having each row scaled against the earned premium data for the accident years. But it is the same as the paid claims data in that it is a development pattern in triangular form showing the past progress of the business in question. It can, therefore, be worked through by any of the grossing up or link ratio methods of §E. The assumption required, as before, is just that the paid claims run to a stable development pattern over the accident years.

What happens if we take up the suggestion, and do a projection by one of the familiar methods of §E using the paid loss ratio figures? An example is worked through overleaf. It uses the Arabic version of grossing up, with simple averaging of the factors down the columns. The result obtained is very much in accord with the earlier projection of the paid claims itself. Mathematically this is no surprise, since the loss ratio projection as here shown is really a repetition of the paid claims development in a light disguise. *But the difference becomes more important if* **"premiums"** *as well as claims show a development pattern. Such is generally the case with London Market business and reinsurance as opposed to direct business.*

Referring to the calculations themselves, the table is slightly confusing in that *all* the figures in it are proportions of one kind or another. But they are of quite different kinds. In each cell of the table, the upper of the two figures is just the paid loss ratio copied from the data triangle above. The lower figure is then the appropriate grossing up factor, calculated by the usual procedure (see §E3 for full details). The result of the procedure is to generate the ultimate loss ratios in the far right hand column. The topmost of these, however, must be determined by other means — we take it here to be 83%, as previously in §G.

					d			
		0	1	2	3	4	5	*ult*
	1	22.31	41.35	54.01	66.61	74.34	77.64	83.00
		26.88	49.82	65.07	80.25	89.57	93.54%	
	2	22.15	41.86	55.21	68.11	76.51		85.42
		25.93	49.00	64.63	79.74	89.57%		
	3	22.27	42.83	56.92	70.02			87.54
a		25.44	48.93	65.02	79.99%			
	4	22.61	43.60	58.88				90.71
		24.93	48.07	64.91%				
	5	23.06	43.58					89.03
		25.90	48.95%					
	6	22.22						86.06
		25.82%						

Applying the projected loss ratios which appear in the last column to the earned premiums, we can derive the loss and reserve estimates as follows:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	86.06	89.03	90.71	87.54	85.42	83.0%
^L-ult	7317	6661	5978	4972	4292	3723

Overall Values:	$\sum$*L-ult*	32,943
	$\sum$*pC**	20,334
	Reserve	12,609

The comparable result from the series of paid claim projections in §E is that from §E3, Variation 3:

a	6	5	4	3	2	1	
^L-ult	7293	6628	5951	4947	4271	3705	(from E3.2)

Overall Values:	$\sum$*L-ult*	32,795
	$\sum$*pC**	20,334
	Reserve	12,461

The results are so close that it would scarcely seem worthwhile to do both. Nevertheless, the paid loss ratio projection does provide something new — it shows the movement in the estimated final loss ratio down the accident years. The ratio first increases steadily from 83.0% to 90.7% and then falls back quickly to 86.1%. The pattern differs from that brought out by the projection of the previous section, §G9. But if the present forecast is at all sound, a fascinating thought naturally arises. It is that here we could have real evidence of the underwriting cycle at work. We must, therefore, ask carefully whether the observed effect is a genuine one, or a spurious result of some quirk in the data or method.

A first test is to look back at the loss ratios brought out by the original paid claims projection. The calculations are:

a	6	5	4	3	2	1	
^L-ult	7293	6628	5951	4947	4271	3705	
aP	8502	7482	6590	5680	5024	4486	(from E3.2)
λ	85.8	88.6	90.3	87.1	85.0	82.6%	

As expected, the pattern is almost identical, except that it is shifted down by about 0.4%. That is purely the effect of starting from a lower initial loss ratio of 82.6% as opposed to 83% in the paid loss ratio projection. It helps to verify the mathematical equivalence of the two methods when there is a development pattern of premiums.

A better test is to look back critically at the workings above on the triangle of paid loss ratios. It is particularly instructive to compare the progressions in accident years a=4, 5, 6. (These data are repeated below, for convenience.) Here it can be seen that the paid loss ratios in periods d=0 and d=1 are very stable. But the ultimate loss ratios fall away appreciably, as the result of the influence of values higher in the triangle.

		d						
		0	1	2	3	4	5	*ult*
	4	22.61 24.93	43.60 48.07	58.88 64.91%				90.71
a	5	23.06 25.90	43.58 48.95%					89.03
	6	22.22 25.82%						86.06

From the point of view of projecting the claims for year a=6, this is very unsatisfactory. Following the strong progression of the loss ratio to 90.7% in year 4, there is really no clear evidence at all to support the later fall. The triangle of data, on its own, is insufficient. What the result is pinpointing is not, after all, evidence of the underwriting cycle at work — rather, it is the inherent instability of the projections in the last 2 or 3 accident years.

A final test on the loss ratio progression would be to consult with underwriters for the given class of business. They might be able to confirm that the figures accorded with their own experience of the market in the recent past. Alternatively, they might say there was no evidence as yet to show loss ratios had reached their peak. The upward trend appeared to be continuing, and was not likely to be arrested until such time as a major shakeout in the market occurred. In this event, taking a cautious view, it would be proper to *extend* the loss ratio trend into the accident years 5 and 6. The trend is very close to +2.4% p.a. in years a=1 to 4, so the projection yields:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	95.90	93.50	91.10	87.81	85.42	83.0%
^*L-ult*	8153	6996	6003	4988	4292	3723

Overall Values:	$\sum L\text{-}ult$	34,155
	$\sum pC^*$	20,334
	Reserve	13,821

This is a salutary result. We have introduced very little that is new mathematically, or in computational procedure, but have been led to revise the reserve estimate upward by an appreciable amount. The conclusion is that it is well worthwhile examining the paid loss ratio and its progression for the new clues which the data may provide. Failing this, the results of any paid loss projection should at least be evaluated by means of the loss ratio. Any such projection will implicitly yield a variation of the ultimate loss ratio by accident year, and this pattern should be tested for its reasonableness. ◇

[G11]
INCURRED LOSS RATIO & INCURRED CLAIMS PROJECTION

Having defined the loss ratio for paid claims, and looked into its development, it is a short step to do the same for the incurred claims. This section therefore provides a parallel treatment to that of §G10. The quantity to examine is the incurred loss ratio:

$$i\lambda(d) = iC\,(d)/aP$$

Again, we simply divide the incurred claims for a given accident year by the earned premium, and study the development of the ratio with time. The given data from our main example from F7.1 yield the following:

aP	*a*	*d*					
		0	1	2	3	4	5
4486	1	2866	3334	3503	3624	3719	3717
		63.89	74.32	78.09	80.78	82.90	82.86%
5024	2	3359	3889	4033	4231	4319	
		66.86	77.41	80.27	84.22	85.97%	
5680	3	3848	4503	4779	4946		
		67.75	79.28	84.14	87.08%		
6590	4	4673	5422	5676			
		70.91	82.28	86.13%			
7482	5	5369	6142				
		71.76	82.09%				
8502	6	5818					
		68.43%					

Here, the divisions run along each row. Thus 2866/4486 = 63.89%, 3334/4486 = 74.32% . . 3717/4486 = 82.86% and so on, down to the last row where 5818/8502 = 68.43%.

The patterns of the development can be seen more easily if the loss ratio figures are separated out (as in the following table). Here, as with the paid loss ratios, there are strong increases down the columns. The periods *d*=2, 3, 4 are particularly suggestive, and on the evidence of these one might expect an upward trend of up to 3% p.a. in the ultimate loss ratio.

		d					
		0	1	2	3	4	5
	1	63.89	74.32	78.09	80.78	82.90	82.86%
	2	66.86	77.41	80.27	84.22	85.97	
a	3	67.75	79.28	84.14	87.08		
	4	70.91	82.28	86.13			
	5	71.76	82.09				
	6	68.43					

We shall proceed with a full projection of the triangle, to test out the strength of the loss ratio trend. A grossing up technique was used in §G10 for the paid loss ratios, so we shall here try a link ratio variation for contrast.

		d						
		0	1	2	3	4	5	*ult*
		1.163	1.051	1.034	1.026	.999	1.002	
	1	63.89	74.32	78.09	80.78	82.90	82.86	83.00
							1.002	
		1.158	1.037	1.049	1.021			
	2	66.86	77.41	80.27	84.22	85.97		86.06
						1.001		
		1.170	1.061	1.035				
	3	67.75	79.28	84.14	87.08			89.26
a					*1.025*			
		1.160	1.047					
	4	70.91	82.28	86.13				91.73
				1.065				
		1.144						
	5	71.76	82.09					91.69
			1.117					
	6	68.43						88.62
		1.295						
r		1.159	1.049	1.039	1.024	.999	1.002	
f		1.295	1.117	1.065	1.025	1.001	1.002	
aP		8502	7482	6590	5680	5024	4486	
λ		88.62	91.69	91.73	89.26	86.06	83.00%	
^L-ult		7534	6860	6045	5070	4324	3723	

Overall Values:	$\sum L\text{-}ult$	33,556
	$\sum pC^*$	20,334
	Reserve	13,222

The strong trends in the main data effectively disappear in the sets of link ratios. Hence simple averaging down the columns is used to determine the *r*-factors. But the upward trend reappears clearly in the ultimate loss ratios obtained. The increase is steady from years a=1 to 4, this time at close to 3% p.a., and reaching a high of nearly 92%. The only awkward feature is the falling away to approximately 89% in the year a=6.

Again, it is useful to compare the result with that which would have been obtained from the projection of the incurred claims themselves. The ultimate loss ratios implicit in this earlier projection can be set out as follows:

a	6	5	4	3	2	1
$\hat{}L\text{-}ult$	7505	6842	6028	5055	4315	3717
aP	8502	7482	6590	5680	5024	4486
λ	88.3	91.4	91.5	89.0	85.9	82.9%

The set of values is very similar indeed, helping to confirm the mathematical identity which underlies the two projections. Again, a step-by-step projection, carried out in the manner for paid claims (§G9), will bring out a very similar set of ratios. To settle the estimate in this case, we may perhaps use a set of rounded loss ratios, increasing at 3% p.a. until year a=4, then steadying at the value of 92%. This yields:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	92.0	92.0	92.0	89.0	86.0	83.0%
$\hat{}L\text{-}ult$	7822	6883	6063	5055	4321	3723

Overall Values:	$\sum L\text{-}ult$	33,867
	$\sum pC^*$	20,334
	Reserve	13,533

Finally, for a pessimistic estimate assuming the worst, we can allow the +3% trend to continue right up to the most recent accident year. The figures become:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	98.0	95.0	92.0	89.0	86.0	83.0%
$\hat{}L\text{-}ult$	8332	7108	6063	5055	4321	3723

Overall Values:	$\sum L\text{-}ult$	34,602
	$\sum pC^*$	20,334
	Reserve	14,268

If confirmatory evidence were to be found for the truth of this scenario, clearly it is time that more stringent underwriting criteria and/or financial controls were placed on the class of business in question.

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[G12]
COMPARISON OF RESULTS ($pC/iC/p\lambda/i\lambda$)

It is time to return to the question of our main estimate for the claims reserve. We shall bring together the main results from the paid and incurred loss ratio projections in §G10 and §G11 with the earlier comparison of paid and incurred claims projections themselves (see table in §F7).

		Value of Reserve	
		Best Estimate	Conservative
Paid Claims	pC	12,461	12,931
Paid Loss Ratio	$p\lambda$	13,096	13,821
Incurred Claims	iC	13,151	13,645
Incurred Loss Ratio	$i\lambda$	13,533	14,268

The evidence from the $p\lambda/i\lambda$ projections is undoubtedly strong. Unless we have firm evidence that the ultimate loss ratio is retreating from the high levels forecast for year a=4, we must conclude that the original paid claims projections are giving too low a figure for the reserve. But the incurred claims figures do not seem at all inflated, and are much in agreement with the paid loss ratio evidence. Finally, there are the incurred loss ratio values at the high end of the spectrum. Here, the "conservative" figure is in fact based on a very pessimistic assumption. Unless there is confirmatory evidence, it will be as well to disregard it.

The final determination must rest on the reserver's judgment. In this case, it will be reasonable to set a reserve at say £13,125 for the best estimate, and £13,750 for the conservative value.

Summing Up on Loss Ratios

A brief summing up on the use of loss ratios in reserving may be useful at this stage. The starting point is inauspicious, since the naive loss ratio appears to be of little help and to prejudge the required answers. But when the more subtle methods such as Bornhuetter-Ferguson and Loss Ratio projection are brought into play, it is as if an anchor were provided for the work. The assumptions underlying the claims development methods (both pC and iC) can easily go adrift for the most recent accident years, and B/F or the loss ratio projections can help provide the needed stabilisation.