MUNICH CHAIN LADDER
Closing the gap between paid and incurred IBNR estimates

CIA Seminar for the Appointed Actuary, Toronto, September 23rd 2011

Dr. Gerhard Quarg

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Agenda

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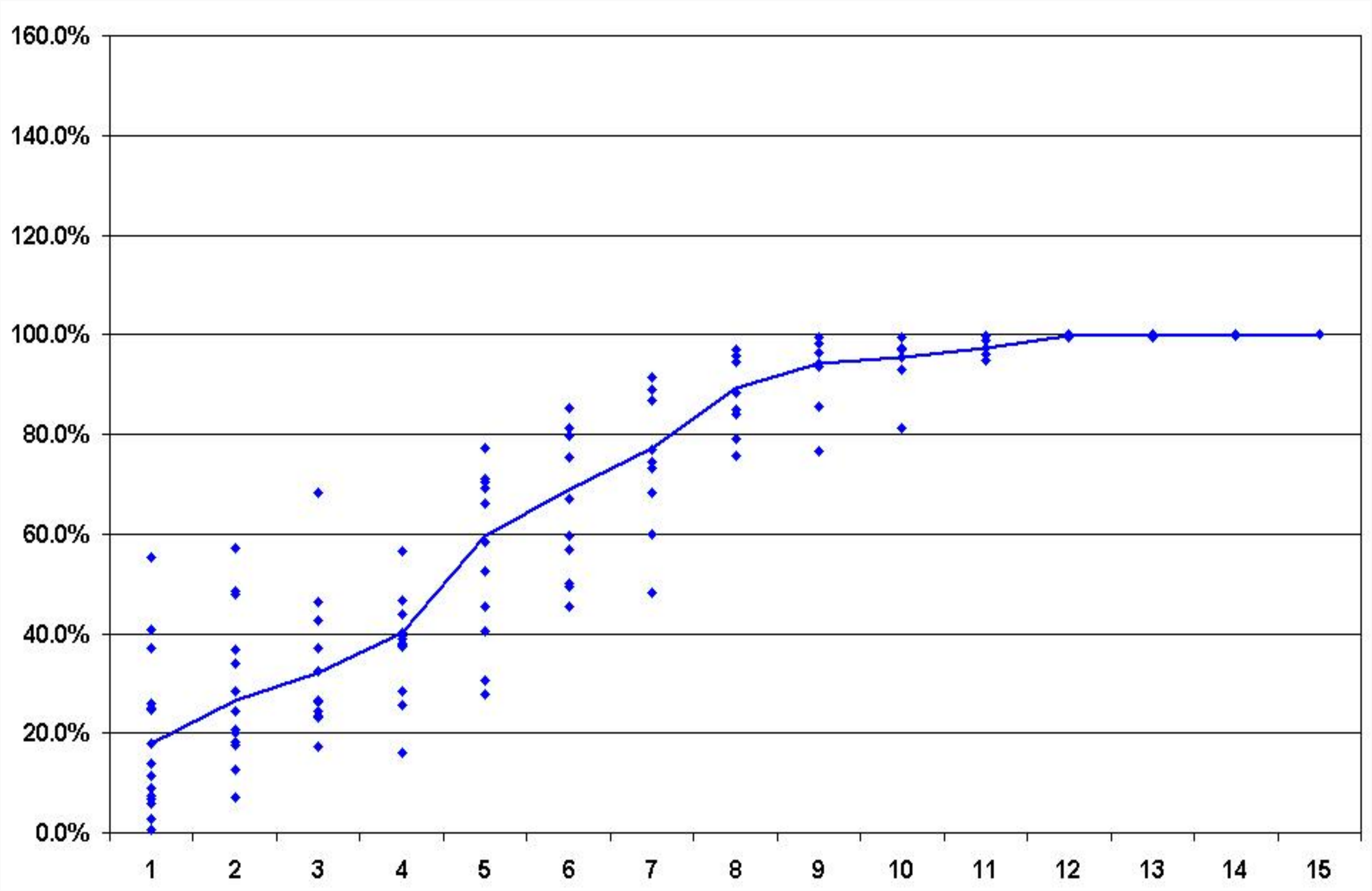
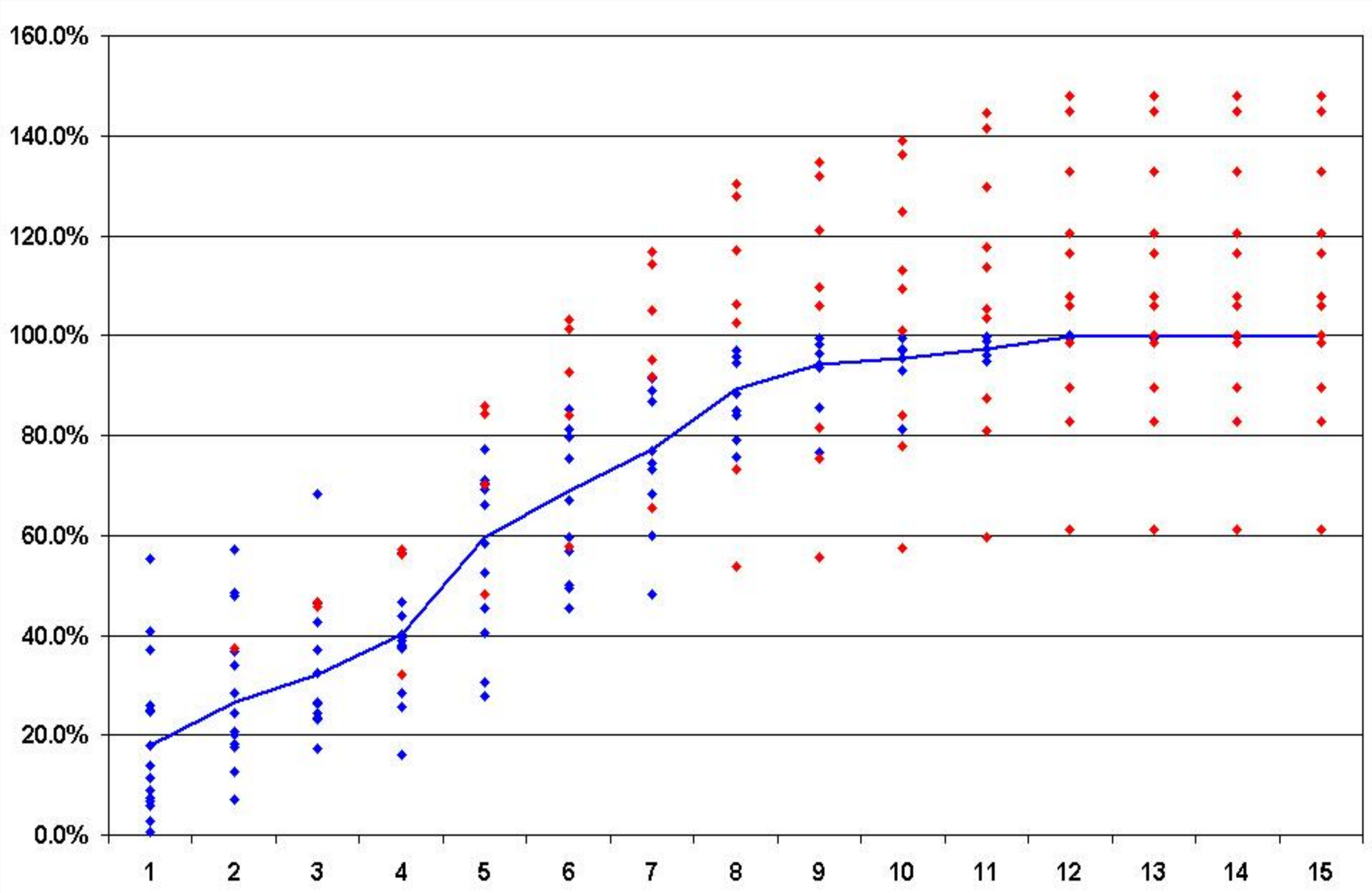
- From Chain Ladder to Munich Chain Ladder
- Performing MCL Calculations
- Backup: On the MCL Prediction Error
- Backup: From the Incremental Loss Ratio method (ILR) to MILR

From Chain Ladder
to Munich Chain Ladder

Triangle of P/I ratios vs. development years



P/I quadrangle (with separate Chain Ladder estimates)



Applying Chain Ladder to paid and incurred separately

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- Interpretation of graphic:

  The development of the projected points is “parallel”.

  A low current P/I ratio yields a low projected ultimate P/I ratio and a high current P/I ratio yields a high projected ultimate P/I ratio.

  The observed P/I ratios converge to 100%, the projected P/I ratios do not.

- This is inherent to separate Chain Ladder calculations.

The P/I problem of separate Chain Ladder calculations

- Thorough mathematical formulation:

$$\frac{(P/I)_{i,k}}{(P/I)_k} = \frac{(P/I)_{i,n-i+1}}{(P/I)_{n-i+1}}$$

where index i denotes the accident year, k the development year, n the size of the triangle and

$$(P/I)_k = \sum_{i=1}^{n} P_{i,k} \Bigg/ \sum_{i=1}^{n} I_{i,k}$$

the average P/I ratio. In words:

  - For each accident year, the quotient of the ultimate P/I ratio and the average ultimate P/I ratio and the quotient of the current P/I ratio and the corresponding average P/I ratio agree.

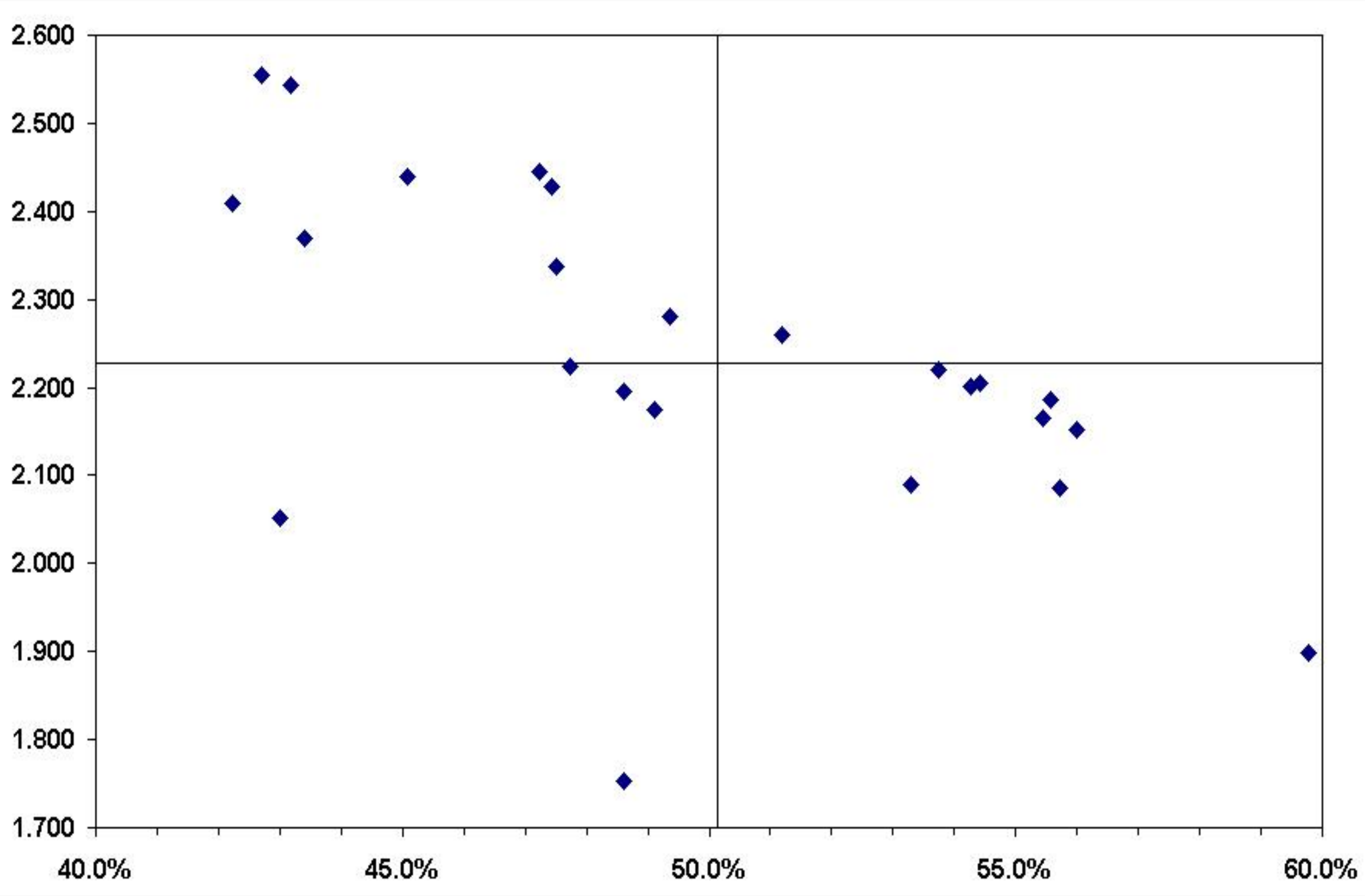
Correlations between paid and incurred data

- For a fixed development year, separate CL calculations use a single projected development factor for all accident years for paid and a single one for incurred.
- This was different in the past:

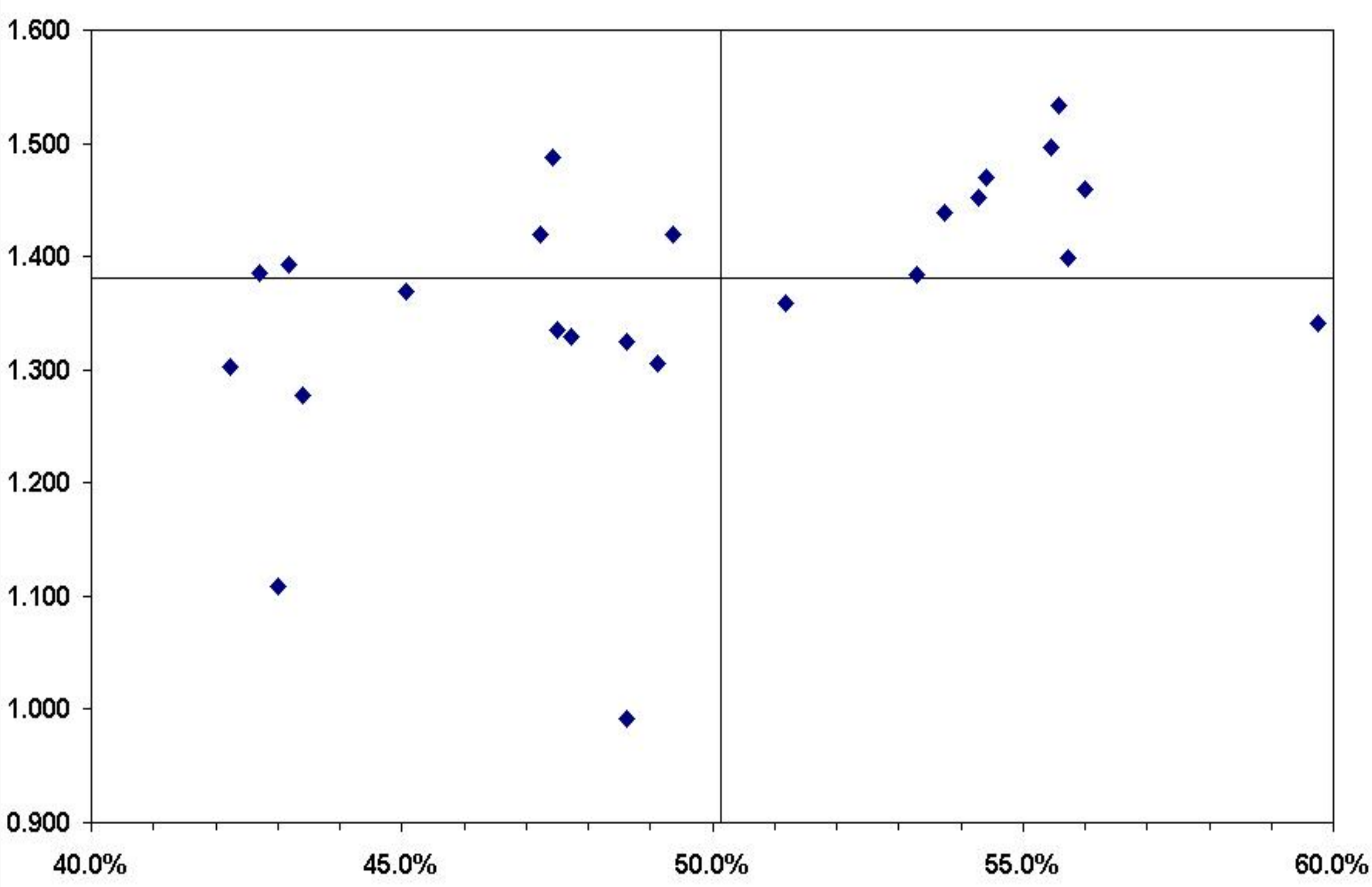
  Below-average P/I ratios were succeeded by relatively high paid and/or relatively low incurred development factors.

  Above-average P/I ratios were succeeded by relatively low paid and/or relatively high incurred development factors.
- This can indeed be seen in the data:

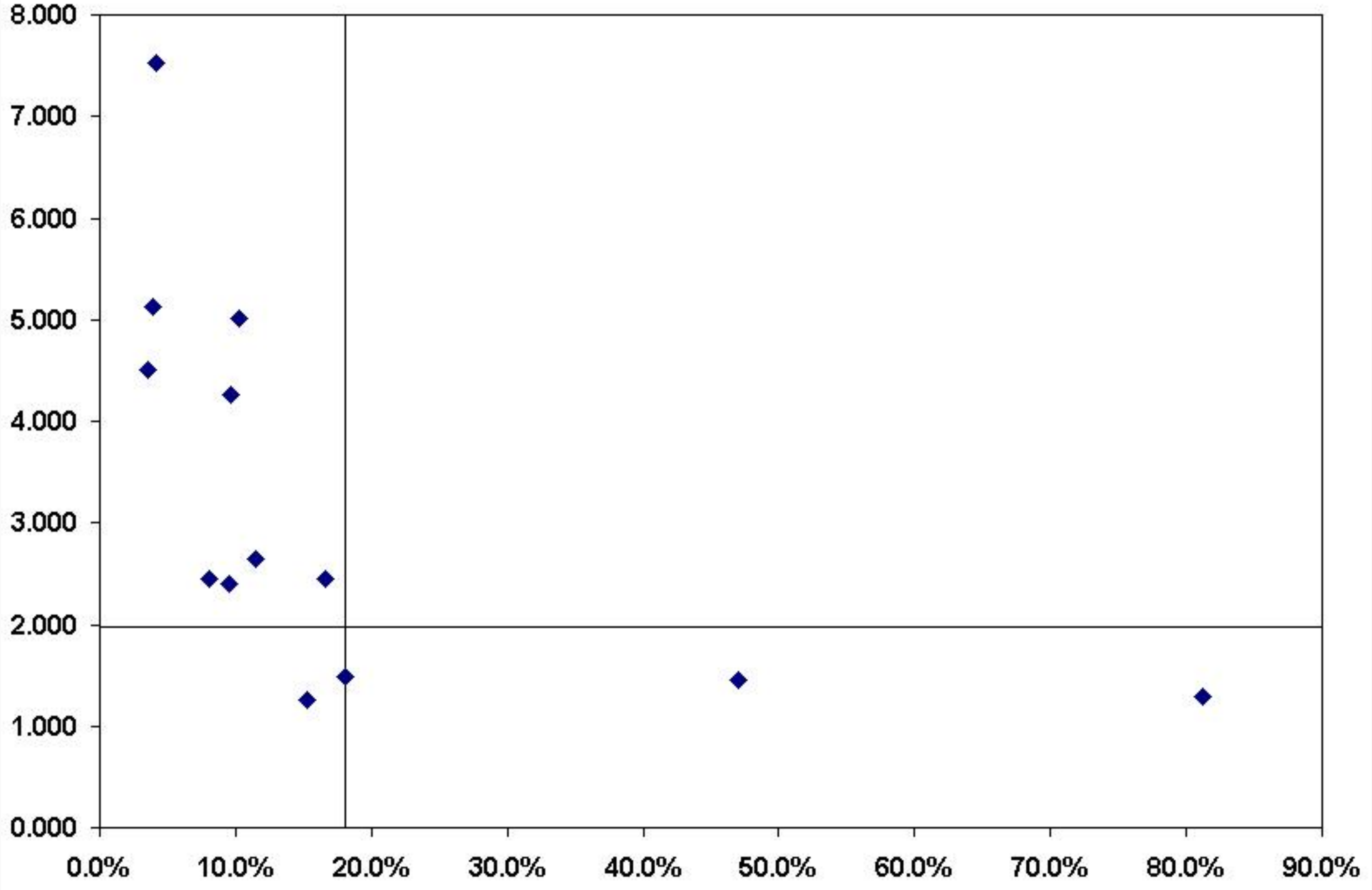
Paid development factors vs. preceding *P/I* ratios



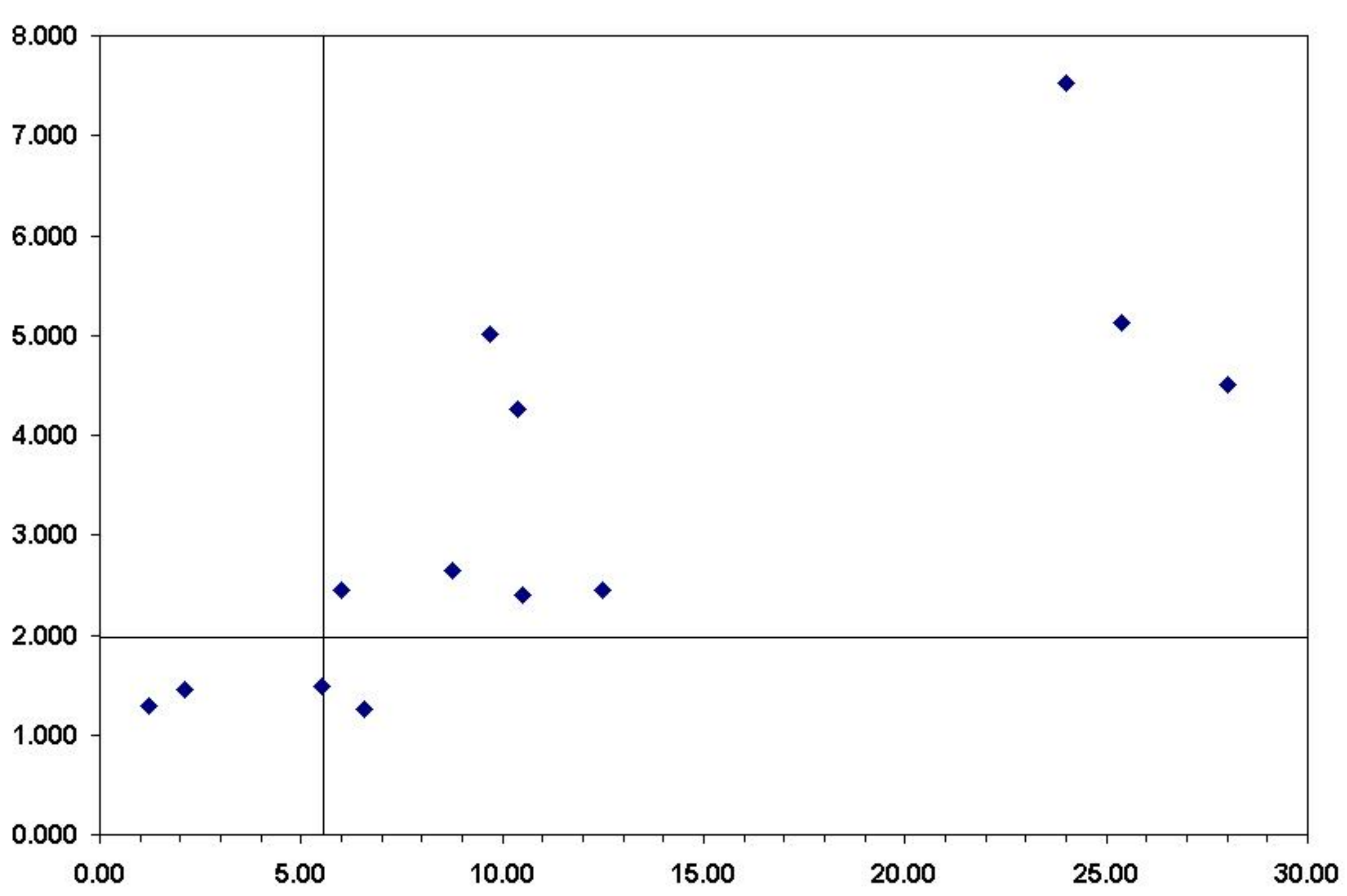
Incurred development factors vs. preceding *P/I* ratios



Problem: paid dev. factors vs. widely scattered P/I ratios



Solution (Th. Mack): paid dev. factors vs. I/P ratios



The residual approach

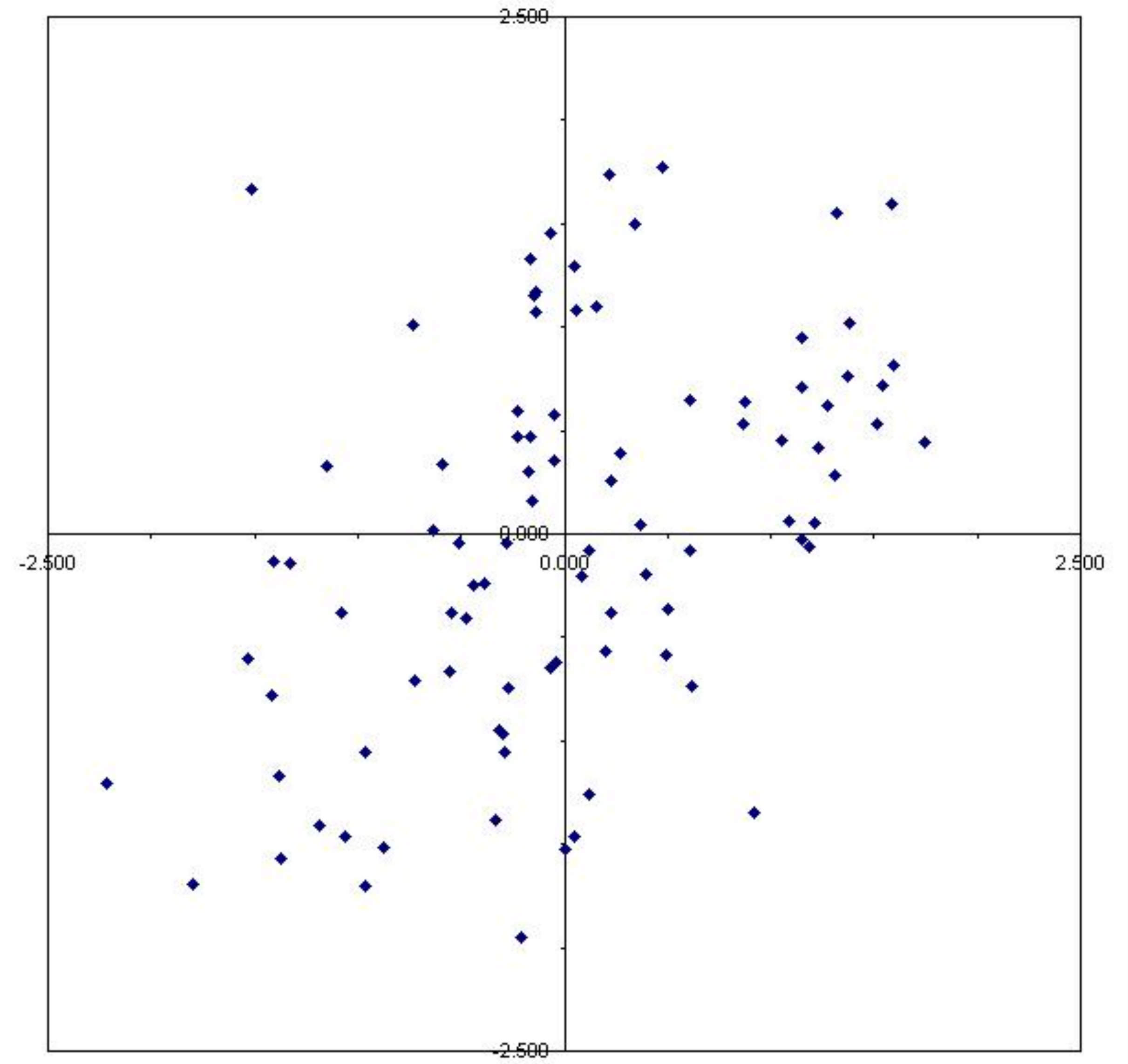
- Problem: high volatility due to not enough data, especially in later development years.

- Solution: consider all development years together.

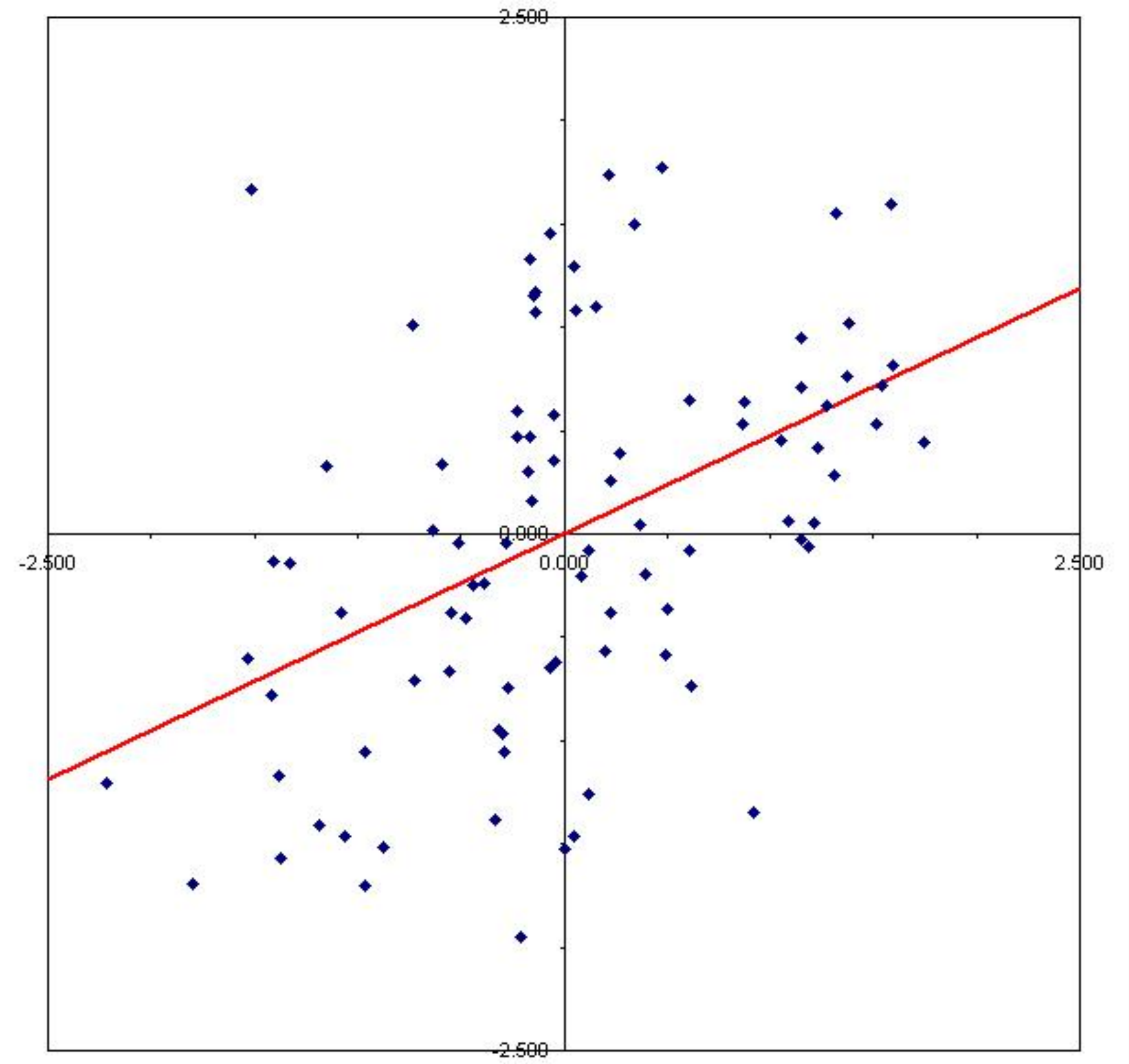
  Use residuals to make different development years comparable.

  Residuals measure deviations from the expected value in multiples of the standard deviation.

Residuals of paid development factors vs. *I/P* residuals



Residuals of paid development factors vs. *I/P* residuals



Residuals of incurred dev. factors vs. P/I residuals

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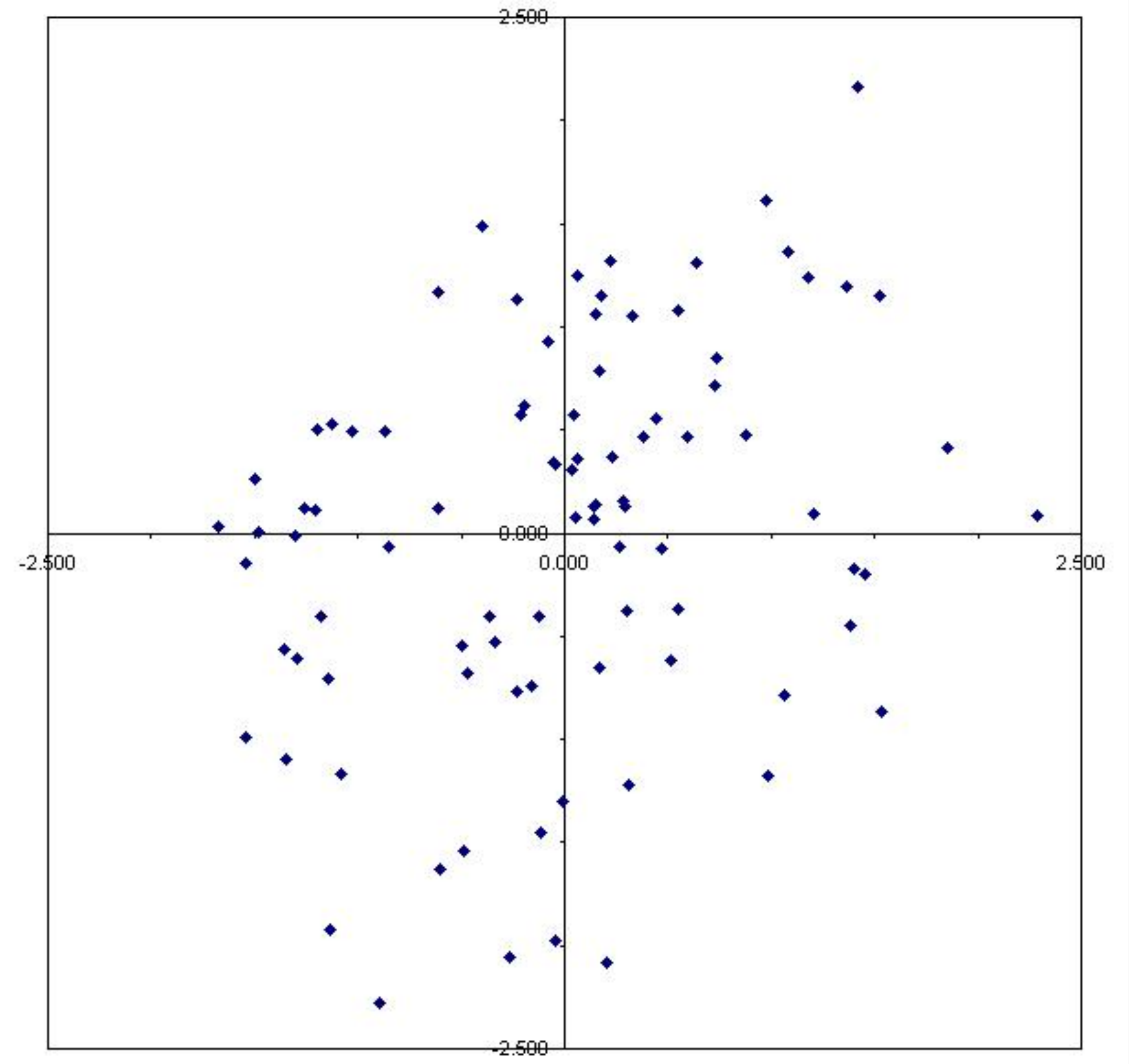

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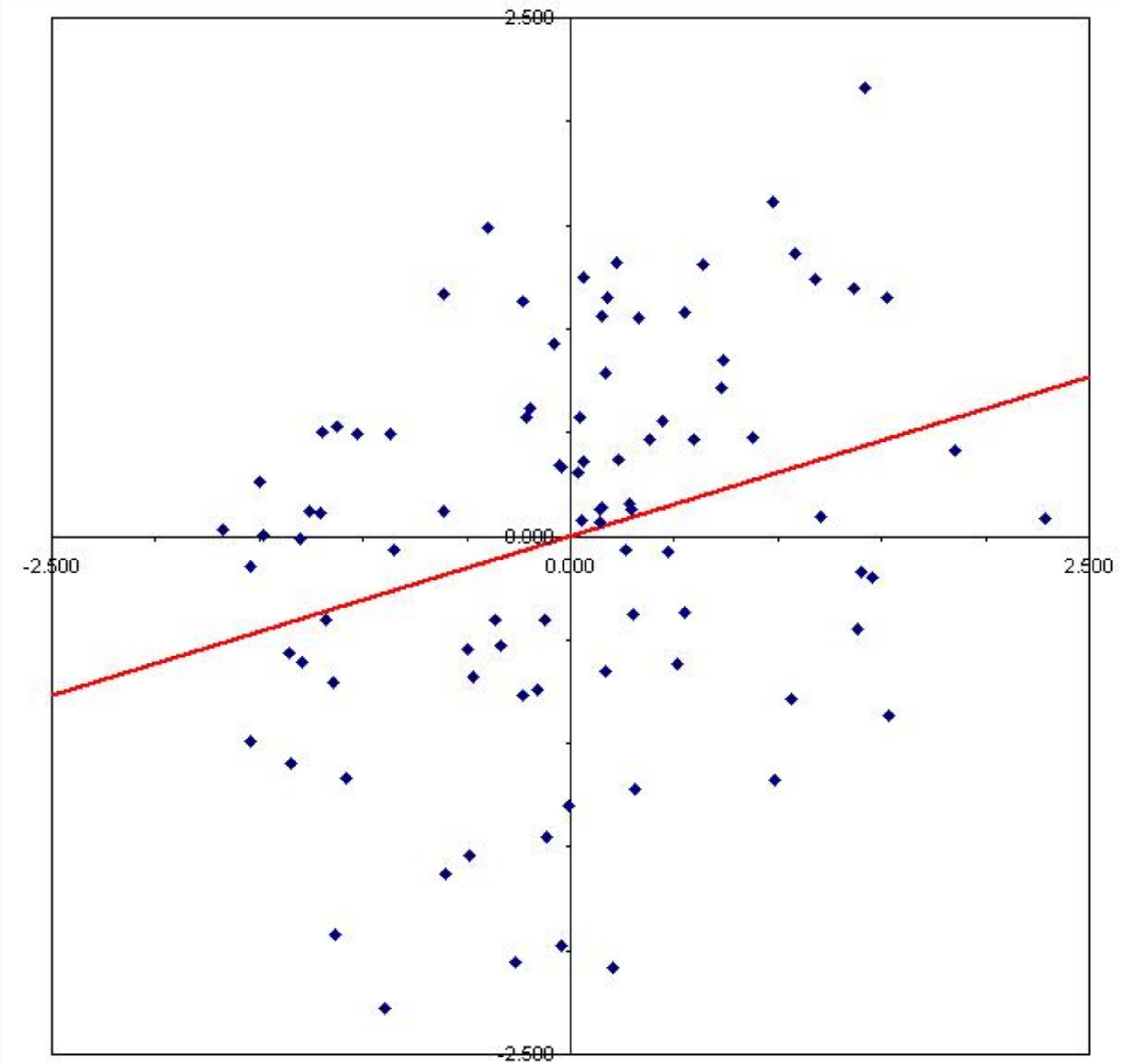
Residuals of incurred dev. factors vs. *P/I* residuals



The standard Chain Ladder model

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- Main assumptions of the standard Chain Ladder model:

$$\mathbf{E}\left(\frac{P_{i,k+1}}{P_{i,k}}|P(k)\right) = f_{k+1}^{P} \qquad \mathbf{E}\left(\frac{I_{i,k+1}}{I_{i,k}}|I(k)\right) = f_{k+1}^{I}$$

where $P(k) = \{P_{i,1}, \ldots, P_{i,k}\}$ and $I(k) = \{I_{i,1}, \ldots, I_{i,k}\}$

- These assumptions are designed for the projection of **one** triangle.
- They ignore systematic correlations between paid losses and incurred losses.

Required model features

➢ In order to combine paid and incurred information we need

$$\mathbf{E}\left(\frac{P_{i,k+1}}{P_{i,k}}|P(k), I(k)\right) = \;\; ?? \qquad \mathbf{E}\left(\frac{I_{i,k+1}}{I_{i,k}}|P(k), I(k)\right) = \;\; ??$$

or equivalently

$$\mathbf{E}\left(\mathbf{Res}\left(\frac{P_{i,k+1}}{P_{i,k}}\right)|P(k), I(k)\right) = \;\; ??$$

$$\mathbf{E}\left(\mathbf{Res}\left(\frac{I_{i,k+1}}{I_{i,k}}\right)|P(k), I(k)\right) = \;\; ??$$

where Res ( ) denotes the conditional residual.

The new model: Munich Chain Ladder

- The Munich Chain Ladder assumptions:

$$\mathbf{E}\left(\mathbf{Res}\left(\frac{P_{i,k+1}}{P_{i,k}}\right)|P(k),I(k)\right)=\lambda^{P}\cdot\mathbf{Res}((I/P)_{i,k})$$

$$\mathbf{E}\left(\mathbf{Res}\left(\frac{I_{i,k+1}}{I_{i,k}}\right)|P(k),I(k)\right)=\lambda^{I}\cdot\mathbf{Res}((P/I)_{i,k})$$

- Lambda is the slope of the regression line through the origin in the respective residual plot.

The new model: Munich Chain Ladder

- Interpretation of lambda as correlation parameter:

$$\mathbf{Corr}\left(\frac{P_{i,k+1}}{P_{i,k}},(I/P)_{i,k}\mid P(k)\right)=\lambda^P$$

$$\mathbf{Corr}\left(\frac{I_{i,k+1}}{I_{i,k}},(P/I)_{i,k}\mid I(k)\right)=\lambda^I$$

- Together, both lambda parameters characterise the interdependency of paid and incurred.

The Munich Chain Ladder recursion formulas

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- The recursion formulas for paid and incurred interact:

$$\widehat{P_{i,k+1}} := \left(\widehat{f^P_{k+1}} + \widehat{\lambda^P} \cdot \frac{\widehat{\sigma^P_{k+1}}}{\widehat{\rho^P_k}} \cdot \left(\frac{\widehat{I_{i,k}}}{\widehat{P_{i,k}}} - \widehat{q_k}^{-1} \right) \right) \cdot \widehat{P_{i,k}}$$

$$\widehat{I_{i,k+1}} := \left(\widehat{f^I_{k+1}} + \widehat{\lambda^I} \cdot \frac{\widehat{\sigma^I_{k+1}}}{\widehat{\rho^I_k}} \cdot \left(\frac{\widehat{P_{i,k}}}{\widehat{I_{i,k}}} - \widehat{q_k} \right) \right) \cdot \widehat{I_{i,k}}$$

- Lambda is the slope of the regression line through the origin in the residual plot, sigma and rho are variance parameters and q is the average P/I ratio.

The MCL correction term

- The recursion consists of the usual CL term and a correction term:

$$\widehat{I_{i,k+1}} = \underbrace{\widehat{f^I_{k+1}} \cdot \widehat{I_{i,k}}}_{\text{CL term}} + \underbrace{\widehat{\lambda^I} \cdot \frac{\widehat{\sigma^I_{k+1}}}{\widehat{\rho^I_k}} \cdot \left(\frac{\widehat{P_{i,k}}}{\widehat{I_{i,k}}} - \widehat{q}_k \right) \cdot \widehat{I_{i,k}}}_{\text{correction term}}$$

(Analogously for paid.)

Munich Chain Ladder as regression method (Th. Mack)

➢ Regression formulas

$$\widehat{P_{i,k+1}} = \widehat{a^P_{k+1}} \cdot \widehat{I_{i,k}} + \widehat{b^P_{k+1}} \cdot \widehat{P_{i,k}}$$

$$\widehat{I_{i,k+1}} = \widehat{a^I_{k+1}} \cdot \widehat{P_{i,k}} + \widehat{b^I_{k+1}} \cdot \widehat{I_{i,k}}$$

where

$$\widehat{a^P_{k+1}} := \widehat{\lambda}^P \cdot \frac{\widehat{\sigma^P_{k+1}}}{\widehat{\rho^P_k}} \qquad \widehat{a^I_{k+1}} := \widehat{\lambda}^I \cdot \frac{\widehat{\sigma^I_{k+1}}}{\widehat{\rho^I_k}}$$

$$\widehat{b^P_{k+1}} := \widehat{f^P_{k+1}} - \widehat{a^P_{k+1}} \cdot \widehat{q_k}^{-1} \qquad \widehat{b^I_{k+1}} := \widehat{f^I_{k+1}} - \widehat{a^I_{k+1}} \cdot \widehat{q_k}$$

Munich Chain Ladder as credibility approach (D. Clark)

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- A weighted average of two estimators

$$\widehat{I_{i,k+1}} = \widehat{f^I_{k+1}} \cdot \widehat{I_{i,k}} \cdot \left(1 - \widehat{w^I_{k+1}}\right) + \widehat{f^I_{k+1}} \cdot \frac{\widehat{P_{i,k}}}{\widehat{q_k}} \cdot \widehat{w^I_{k+1}}$$

where the weights are given by

$$\widehat{w^I_{k+1}} := \widehat{\lambda^I} \cdot \frac{\widehat{\sigma^I_{k+1}}}{\widehat{f^I_{k+1}}} \cdot \frac{\widehat{q_k}}{\widehat{\rho^I_k}}$$

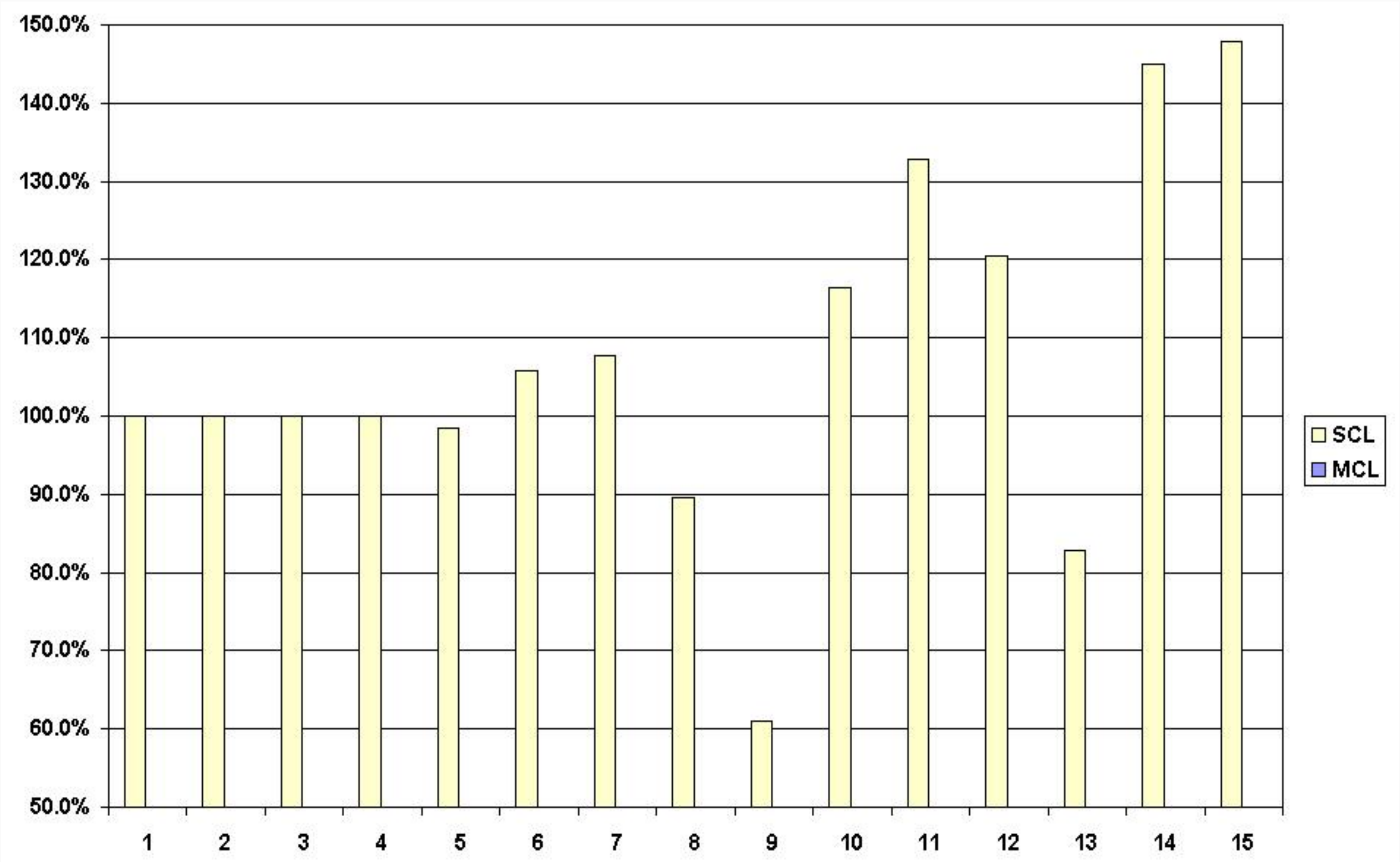
(Analogously for paid.)

Performing MCL Calculations

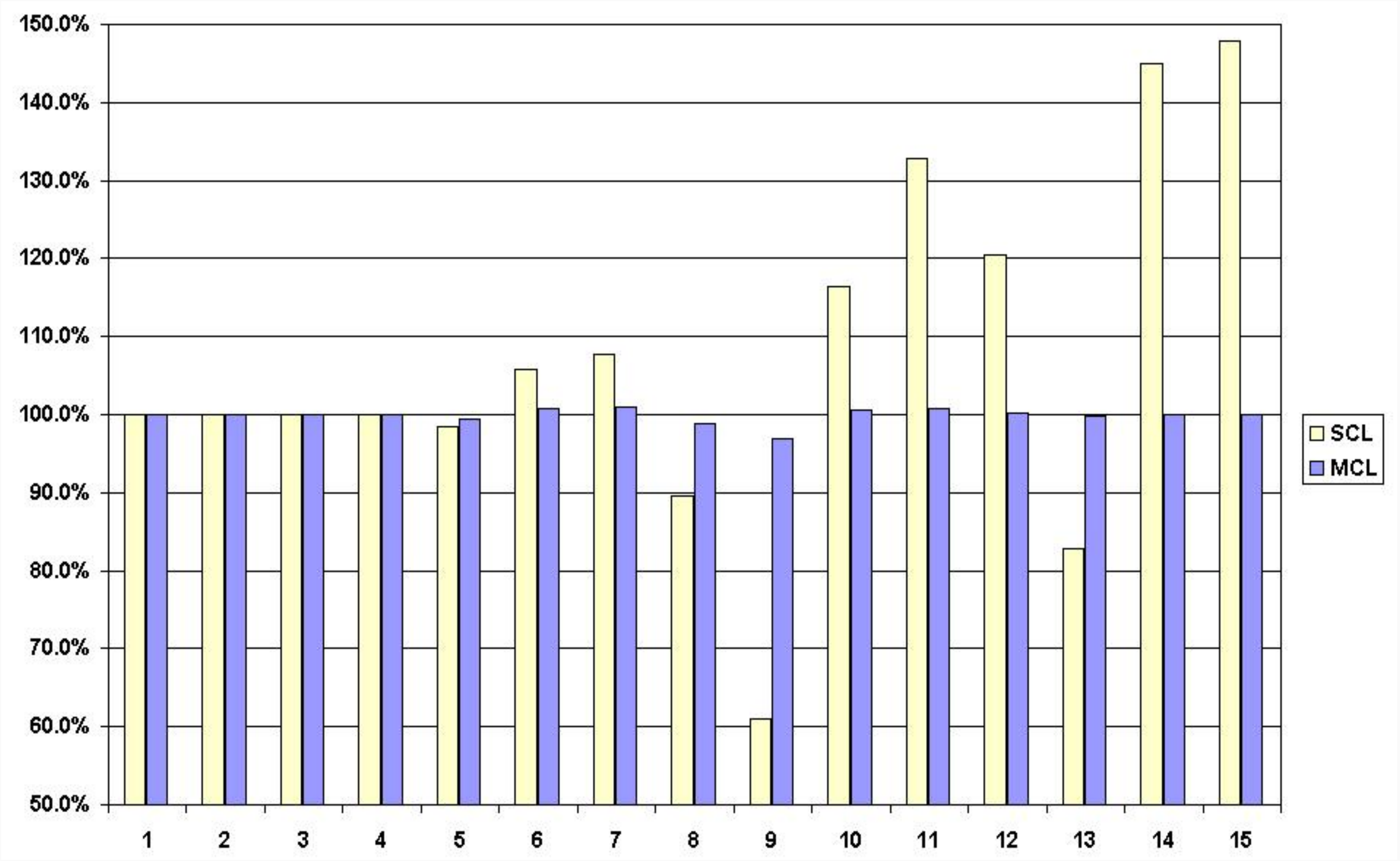
General important facts

- Minimum requirement for MCL: Chain Ladder has to be adequate for the paid and for the incurred triangle individually. The triangles must be large enough.
- Capability and limits: MCL projections for paid and incurred will concur as much as it can be expected based on the data observed so far.
- Back to the introductory example:

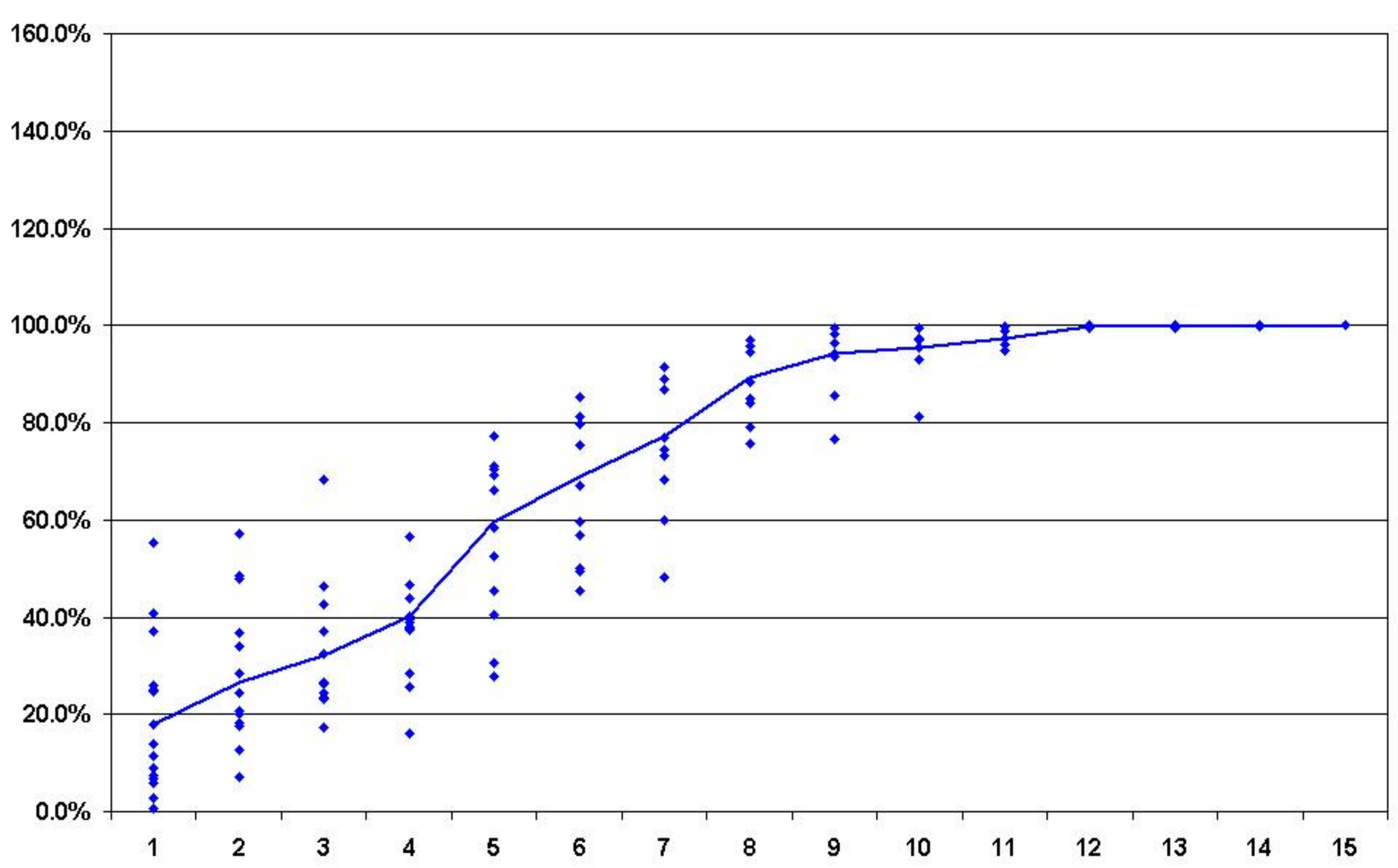
Ultimate *P/I* ratios (Chain Ladder applied separately)



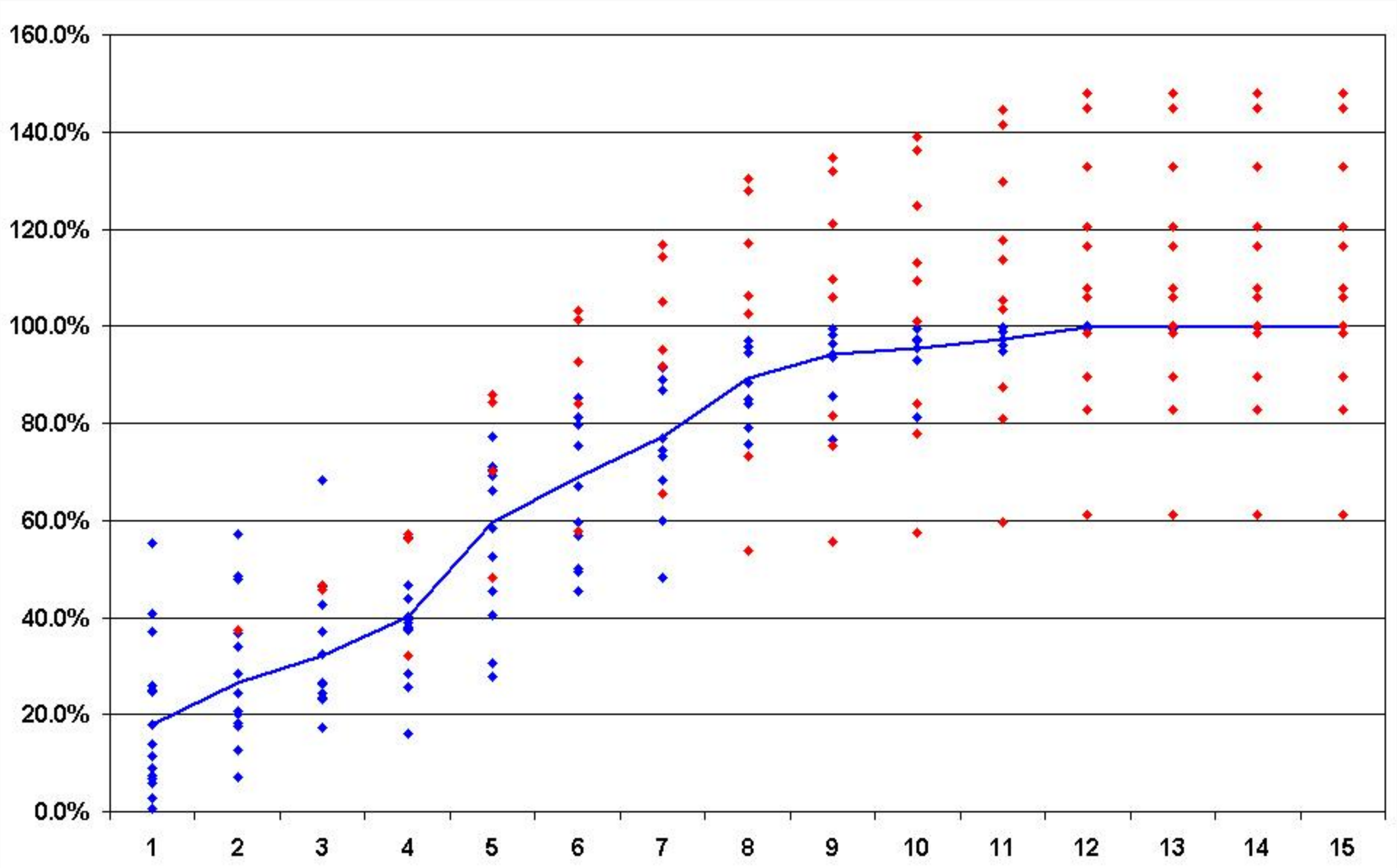
Ultimate P/I ratios (Munich Chain Ladder)



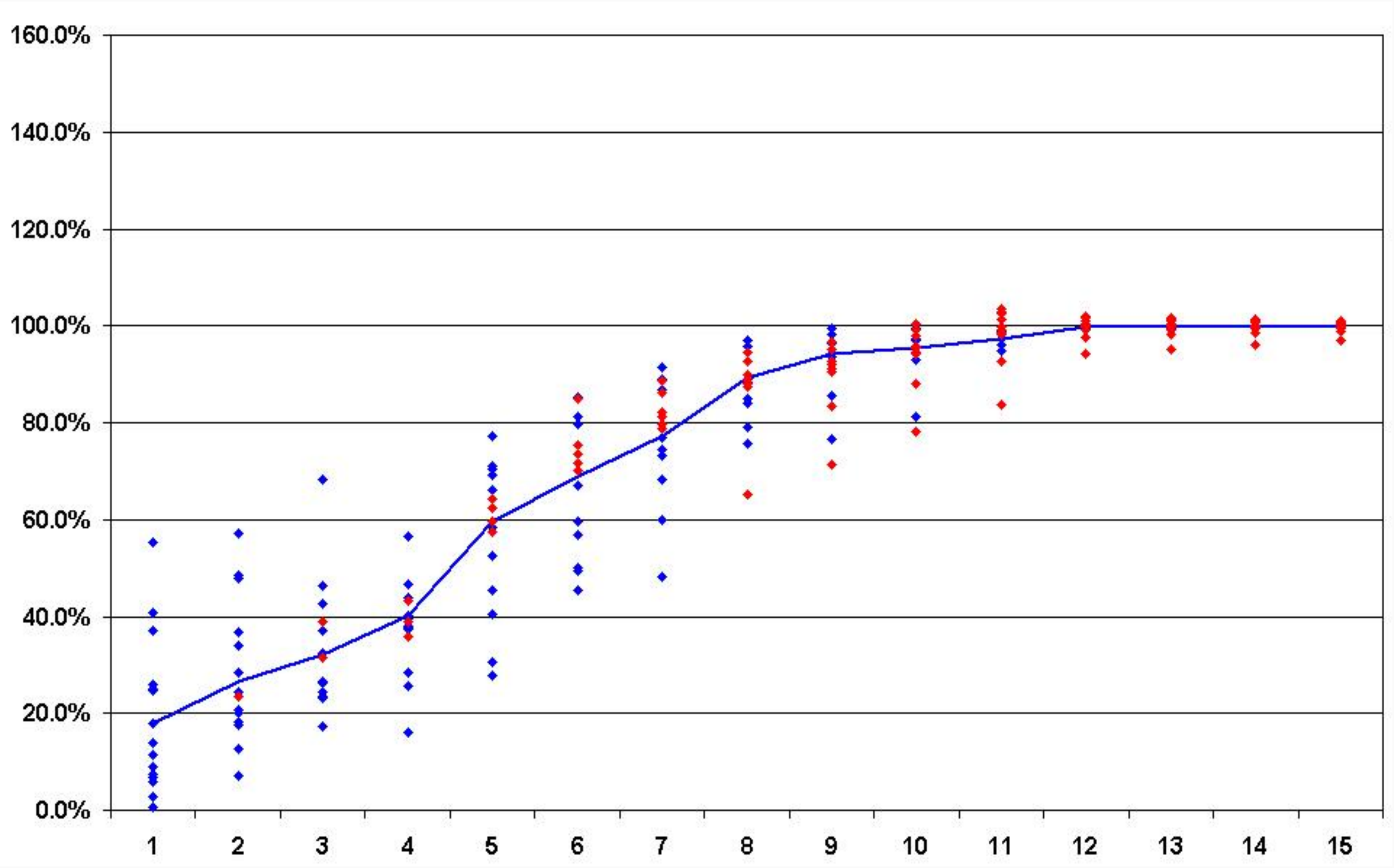
Triangle of P/I ratios vs. development years



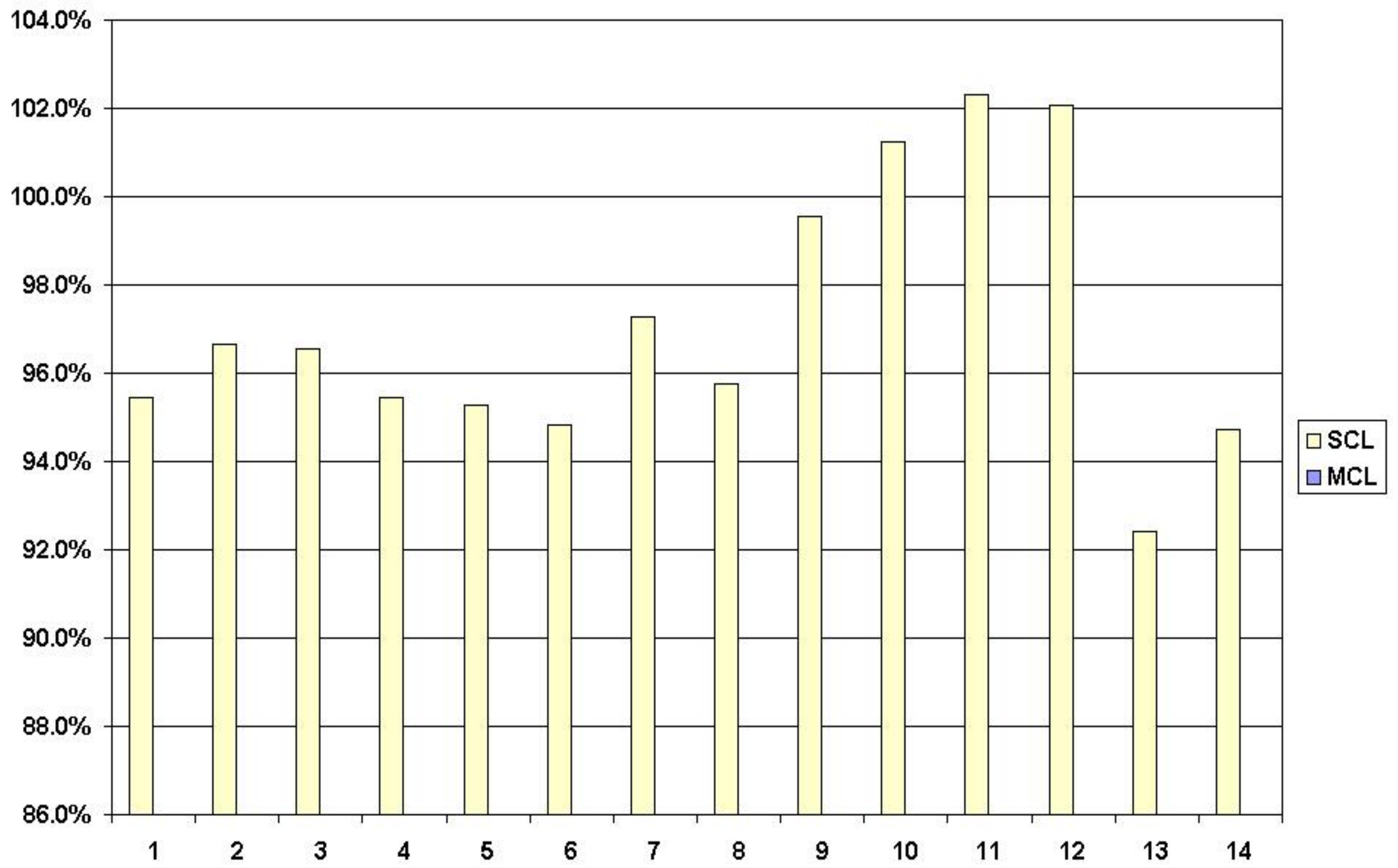
P/I quadrangle (with separate Chain Ladder estimates)



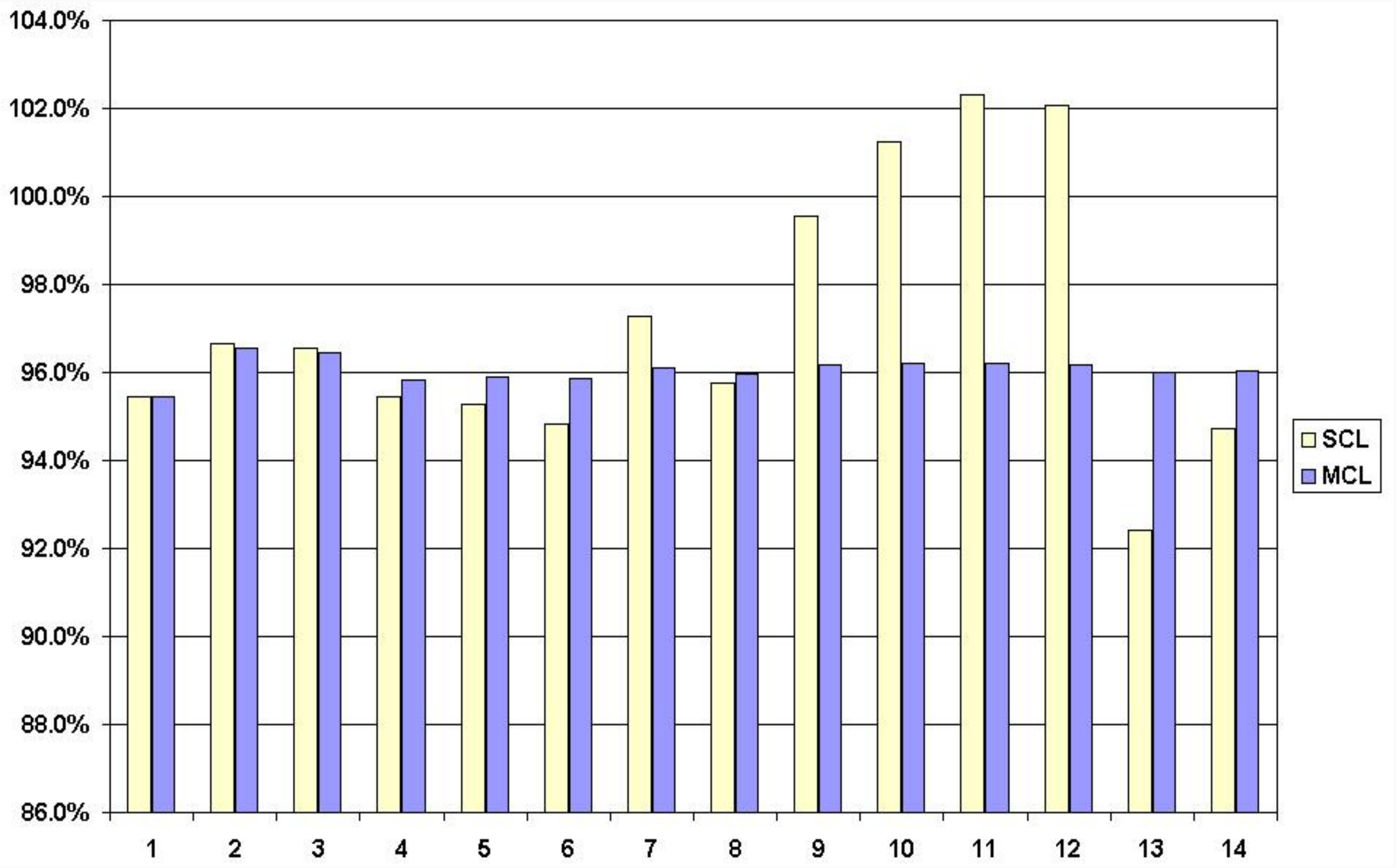
P/I quadrangle (with Munich Chain Ladder)



Another example: ultimate *P/I* ratios (separate CL)



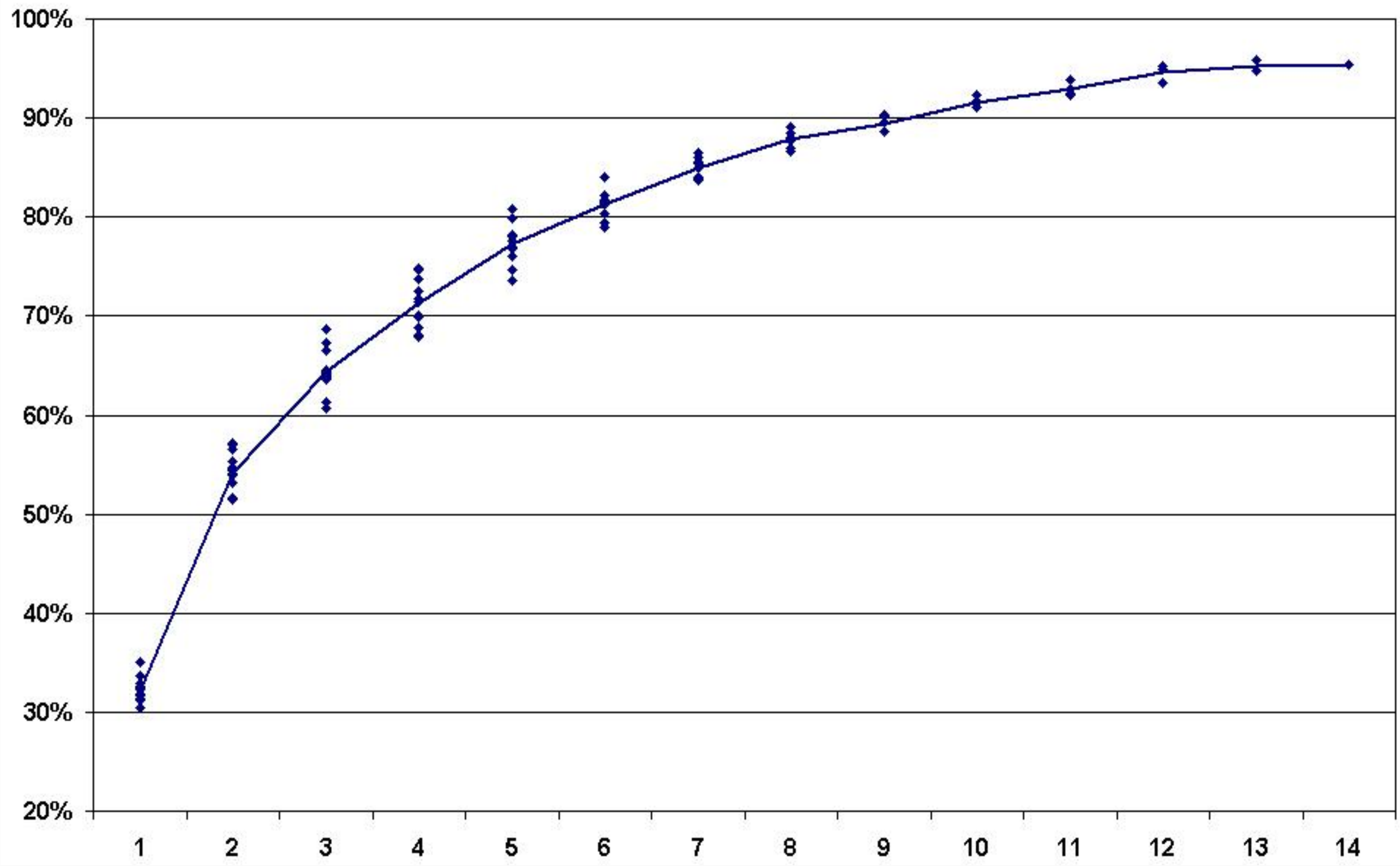
Ultimate P/I ratios (Munich Chain Ladder)



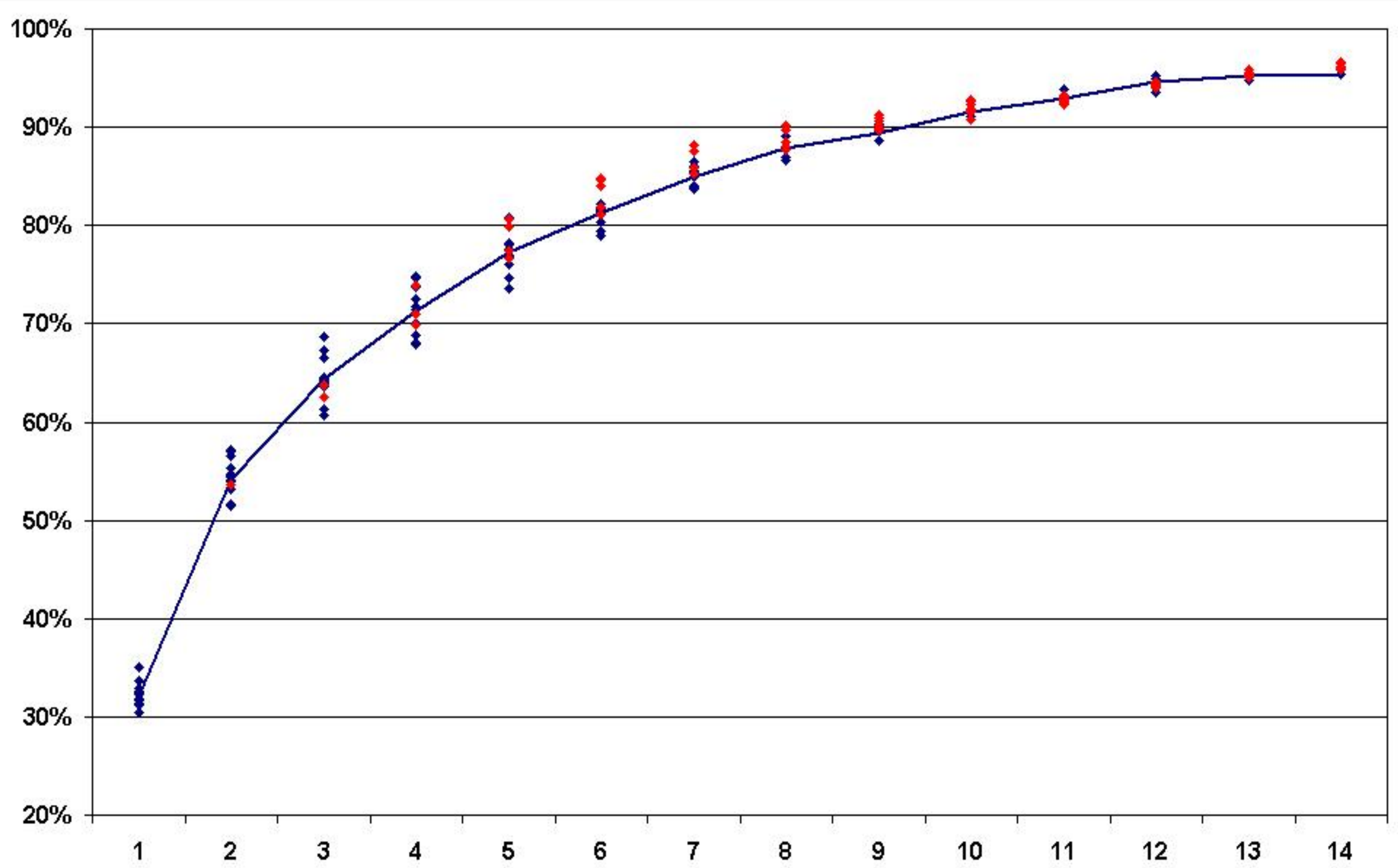
The remaining gap

- Munich Chain Ladder projects ultimate P/I ratios of about “only” 96%.
- There is a remaining gap between paid and incurred IBNR estimates.
- This is not a failure of Munich Chain Ladder. On the contrary, data suggest a remaining gap after 14 years of development:

Triangle of *P/I* ratios vs. development years



P/I quadrangle (with Munich Chain Ladder)



How about an unfinished run-off?

- Estimated parameters of the latest development years are relevant for final result but very volatile.
- Paid and incurred prognosis will/should not concur, if P/I pattern does not reach 100%.
- Smoothing and extrapolating may be appropriate.
- Extrapolation of P/I, sigma and rho pattern is time-consuming, volatile and often unnecessary.
- MCL correction terms should be used as long as necessary to „match“ both projections.

Correlation parameters for each development year

- MCL assumes a single correlation parameter for all development years (one for P and one for I, respectively).
- Check by calculating development year lambdas:

  Usually DY lambdas are widely scattered. This is the very reason for stabilising.

  There should be no clear trend over the DYs.

  Do not look at DY lambdas of last DYs. They are very uncertain and their influence is negligible.

The sign of the MCL correction terms

- The sign of a correction term depends on whether MCL considers the preceding P/I ratio below or above average.
- The sign should not change within one line (accident year) and vary at random within one column (DY).

AY\DY	1	2	3	4	5	6	7	8
1								
2								
3							+	
4							–	
5							–	
6				+	+	+	+	+
7							+	
8							–	

Possible reasons for wrong signs

- Within a line:
  - "Small" changes of sign do not matter.
  - P/I pattern is implausible or does not match paid and incurred development factors.
  - Variance parameters are not reasonable.
- Within a column:
  - P/I pattern is implausible or does not match paid and incurred development factors.
  - New accident years and old accident years have different P/I pattern.

Backup:
On the MCL Prediction Error

General facts

- In order to calculate the prediction error, one needs an assumption on the correlation of paid and incurred (analogously to the correlation assumption of Ch. Braun).
- As with Chain Ladder, the prediction error splits into random error and estimation error.

$$\underbrace{\widehat{\mathbf{Var}}\left(I_{i,n}\right)}_{\text{random error}} + \underbrace{\widehat{\mathbf{Var}}\left(\widehat{I_{i,n}}\right)}_{\text{estimation error}}$$

- Both errors are calculated with recursion formulas that have the same structure. In case of the random error:

Interacting recursion formulas for the random error

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$$\begin{aligned}
\widehat{\mathbf{Var}}\left(P_{i,k+1}\right) &= \widehat{a^P_{k+1}}^{\,2} \cdot \widehat{\mathbf{Var}}\left(I_{i,k}\right) + \widehat{b^P_{k+1}}^{\,2} \cdot \widehat{\mathbf{Var}}\left(P_{i,k}\right) \\
&\quad + 2 \cdot \widehat{a^P_{k+1}} \cdot \widehat{b^P_{k+1}} \cdot \widehat{\mathbf{Cov}}\left(P_{i,k}, I_{i,k}\right) \\
&\quad + \widehat{\mathbf{Var}}^{k}\left(P_{i,k+1}\right)
\end{aligned}$$

$$\begin{aligned}
\widehat{\mathbf{Var}}\left(I_{i,k+1}\right) &= \widehat{a^I_{k+1}}^{\,2} \cdot \widehat{\mathbf{Var}}\left(P_{i,k}\right) + \widehat{b^I_{k+1}}^{\,2} \cdot \widehat{\mathbf{Var}}\left(I_{i,k}\right) \\
&\quad + 2 \cdot \widehat{a^I_{k+1}} \cdot \widehat{b^I_{k+1}} \cdot \widehat{\mathbf{Cov}}\left(P_{i,k}, I_{i,k}\right) \\
&\quad + \widehat{\mathbf{Var}}^{k}\left(I_{i,k+1}\right)
\end{aligned}$$

$$\begin{aligned}
\widehat{\mathbf{Cov}}\left(P_{i,k+1}, I_{i,k+1}\right) &= \widehat{a^P_{k+1}} \cdot \widehat{b^I_{k+1}} \cdot \widehat{\mathbf{Var}}\left(I_{i,k}\right) + \widehat{a^I_{k+1}} \cdot \widehat{b^P_{k+1}} \cdot \widehat{\mathbf{Var}}\left(P_{i,k}\right) \\
&\quad + \left(\widehat{a^P_{k+1}} \cdot \widehat{a^I_{k+1}} + \widehat{b^P_{k+1}} \cdot \widehat{b^I_{k+1}}\right) \cdot \widehat{\mathbf{Cov}}\left(P_{i,k}, I_{i,k}\right) \\
&\quad + \widehat{\mathbf{Cov}}^{k}\left(P_{i,k+1}, I_{i,k+1}\right)
\end{aligned}$$

Questions concerning the MCL prediction error

- Do paid and incurred prediction errors agree?
  - If the eldest accident years are fully developed, they come close. (Due to the interacting recursion formulas!)
- Is the MCL prediction error smaller or larger than the CL prediction error?
  - This is not a valid question. The CL projections and the MCL projections do not agree and are justified under different conditions. Both methods are “not comparable”.
- How about $\widehat{\lambda^I} \approx 0$? In this case CL and MCL agree for the incurred calculation.

Questions concerning the MCL prediction error

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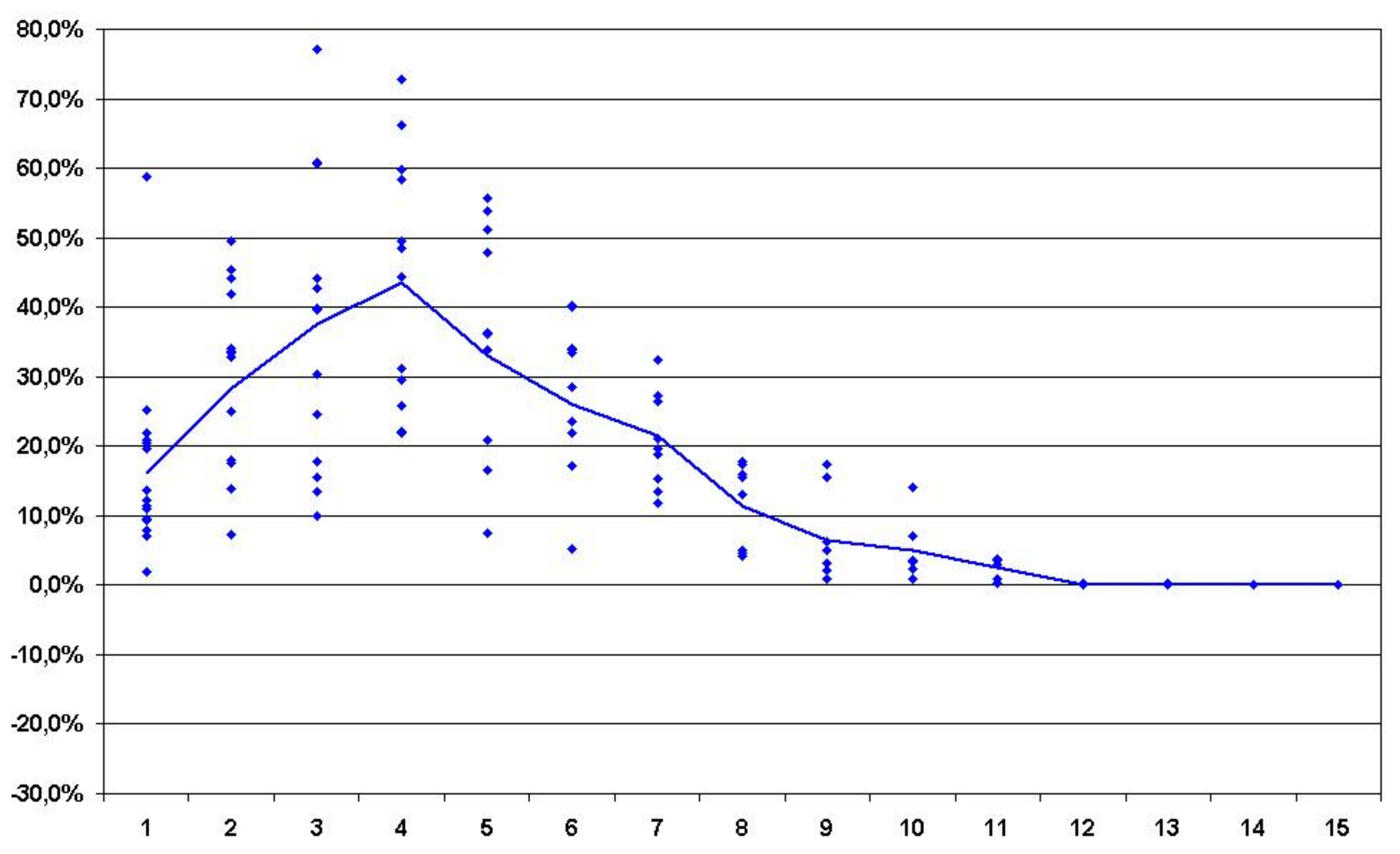
- If $\widehat{\lambda^I} \approx 0$, then the MCL random error is the same as for CL. But the MCL estimation error is larger than the CL estimation error.

- This is reasonable, since CL assumes a vanishing lambda whereas MCL only estimates one. (But the estimation might be wrong!)

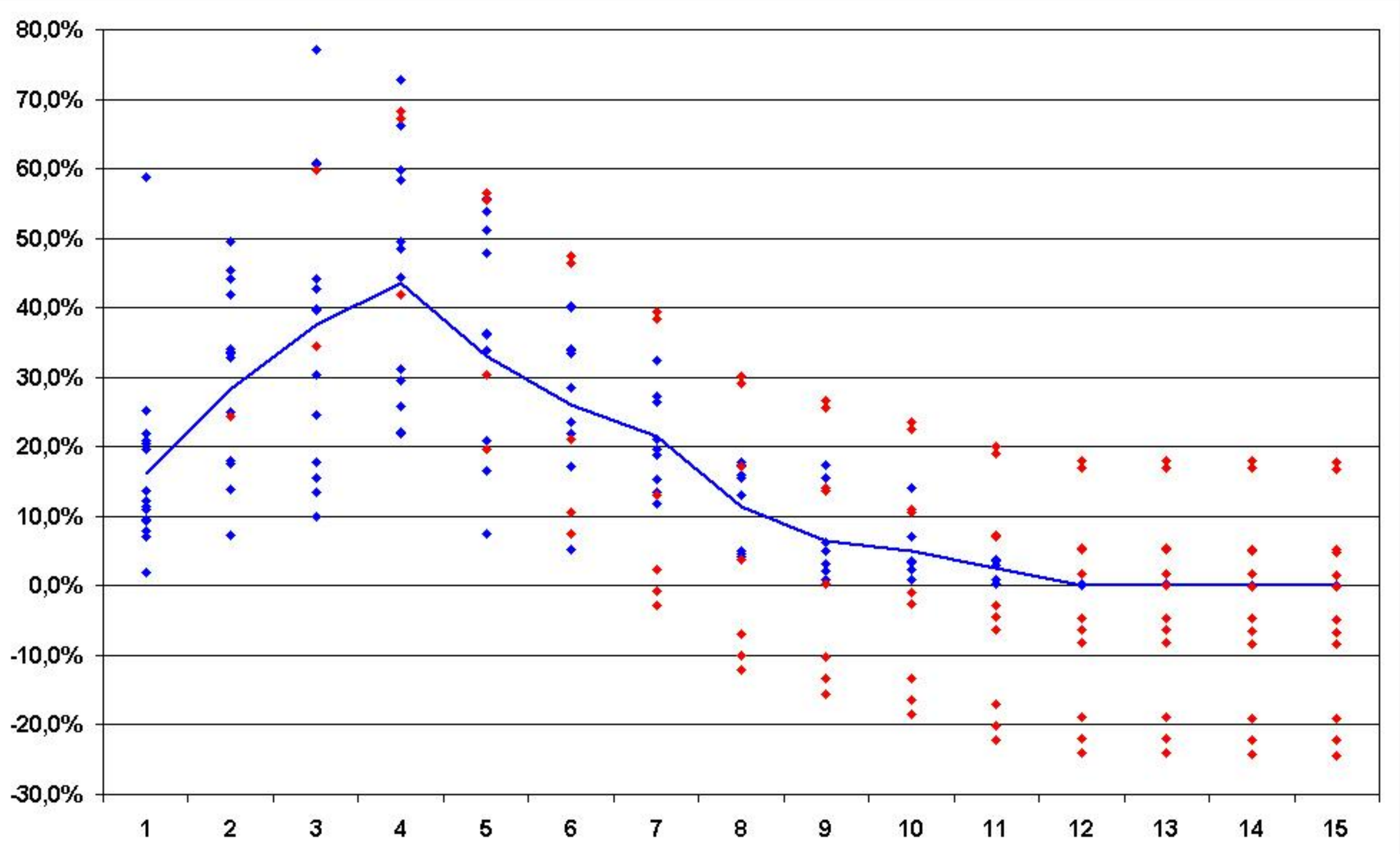
- So CL “hides error” in its model assumptions.

Backup:
From the Incremental Loss Ratio method (ILR) to MILR

Triangle of R/v ratios vs. development years



R/v quadrangle (with separate ILR estimates)



The *R/v* problem of separate ILR calculations

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- The development of the projected points is “parallel” in this plot. The observed *R/v* ratios converge to 0, the projected *R/v* ratios do not.
- This is inherent to separate ILR calculations:

$$(R/v)_{i,k} - (R/v)_k = (R/v)_{i,n-i+1} - (R/v)_{n-i+1}$$

where

$$(R/v)_k := \sum_{i=1}^{n} R_{i,k} \Big/ \sum_{i=1}^{n} v_i$$

is the average *R/v* ratio after *k* development years.

Correlations between paid and incurred data

- For a fixed development year, separate ILR calculations use a single projected incremental loss ratio for all accident years for paid and a single one for incurred.

- This was different in the past:

  Below-average R/v ratios were succeeded by relatively high paid and/or relatively low incurred incremental loss ratios and vice versa.

The residual approach of MILR

- As within the multiplicative setting, there is the problem of high volatility due to not enough data, especially in later development years.

- Analogously one has to consider all development years together, using a residual approach.

- It is convenient to plot (the residuals of) the incurred incremental loss ratios against (the residuals of) the negative preceding ratios $-R/v$. This way any reasonable correlation is positive.

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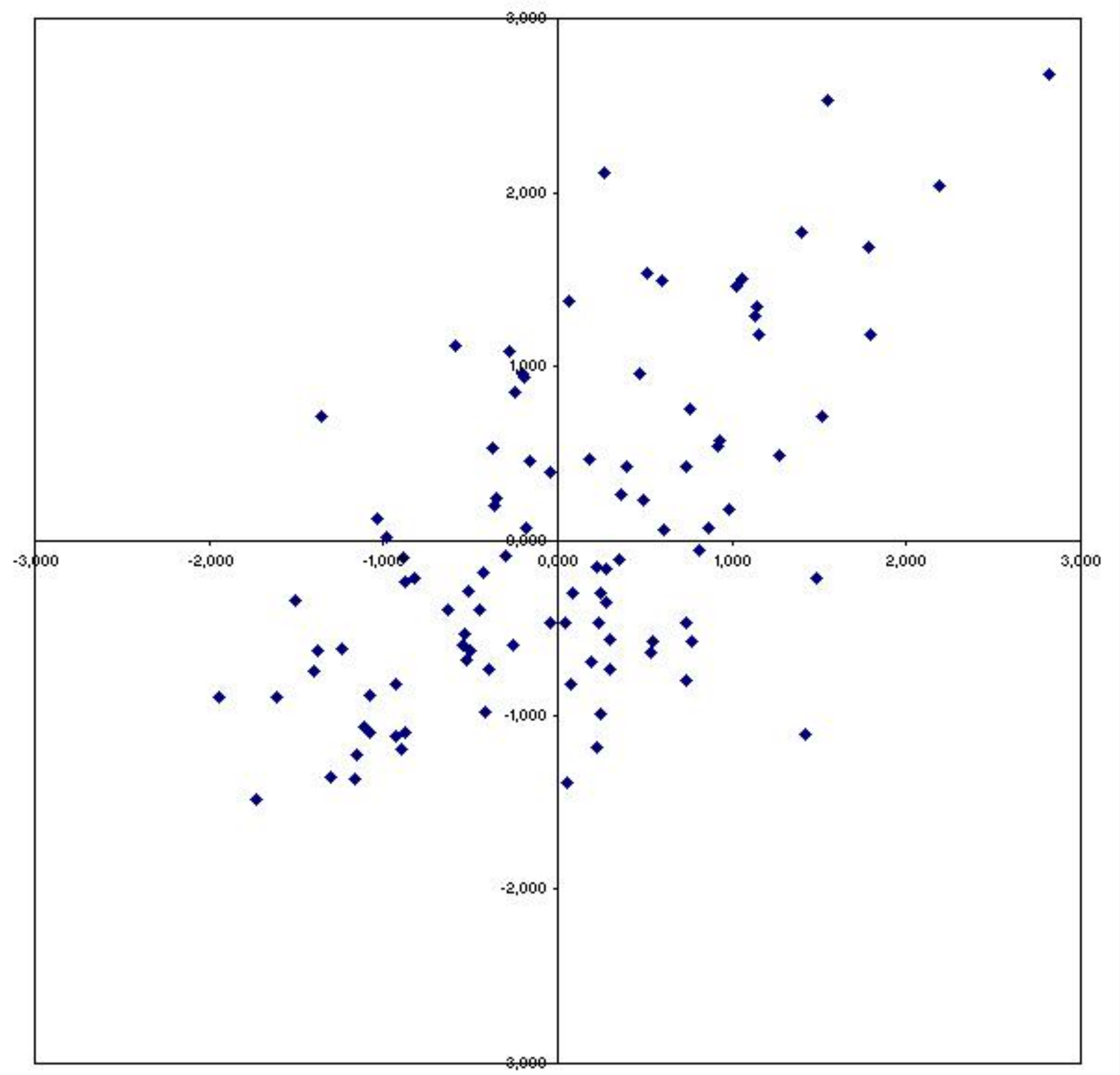
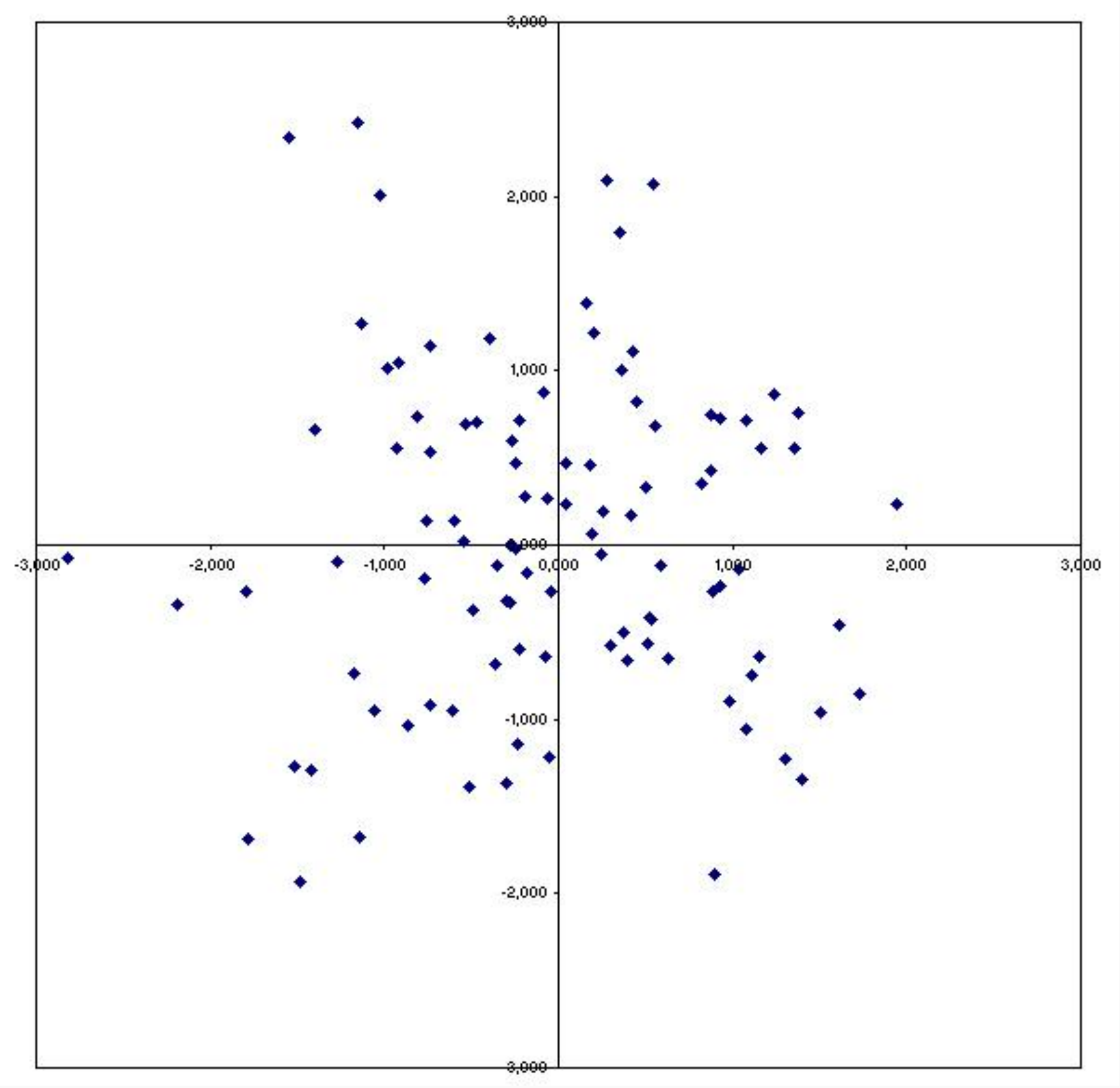
Residuals of paid increm. loss ratios vs. *R/v* residuals



Residuals of inc. increm. loss ratios vs. -R/v residuals



The standard ILR model and the new MILR model

- Main assumptions of the standard ILR model:

$$\mathbf{E}\left(\frac{S^P_{i,k+1}}{v_i}|P(k)\right) = m^P_{k+1} \qquad \mathbf{E}\left(\frac{S^I_{i,k+1}}{v_i}|I(k)\right) = m^I_{k+1}$$

- The MILR assumptions:

$$\mathbf{E}\left(\mathbf{Res}\left(\frac{S^P_{i,k+1}}{v_i}\right)|P(k), I(k)\right) = \lambda^P \cdot \mathbf{Res}((R/v)_{i,k})$$

$$\mathbf{E}\left(\mathbf{Res}\left(\frac{S^I_{i,k+1}}{v_i}\right)|P(k), I(k)\right) = \lambda^I \cdot \mathbf{Res}(-(R/v)_{i,k})$$

The MILR recursion formulas

- The recursion formulas for paid and incurred interact:

$$\widehat{P_{i,k+1}} := \widehat{P_{i,k}} + \left(\widehat{m^P_{k+1}} + \widehat{\lambda^P} \cdot \frac{\widehat{\sigma^P_{k+1}}}{\widehat{\rho_k}} \cdot \left(\frac{\widehat{R_{i,k}}}{v_i} - \widehat{r_k} \right) \right) \cdot v_i$$

$$\widehat{I_{i,k+1}} := \widehat{I_{i,k}} + \left(\widehat{m^I_{k+1}} + \widehat{\lambda^I} \cdot \frac{\widehat{\sigma^I_{k+1}}}{\widehat{\rho_k}} \cdot \left(\widehat{r_k} - \frac{\widehat{R_{i,k}}}{v_i} \right) \right) \cdot v_i$$

- Lambda is the slope of the regression line through the origin in the residual plot, sigma and rho are variance parameters and r is the average R/v ratio.

The MILR estimators

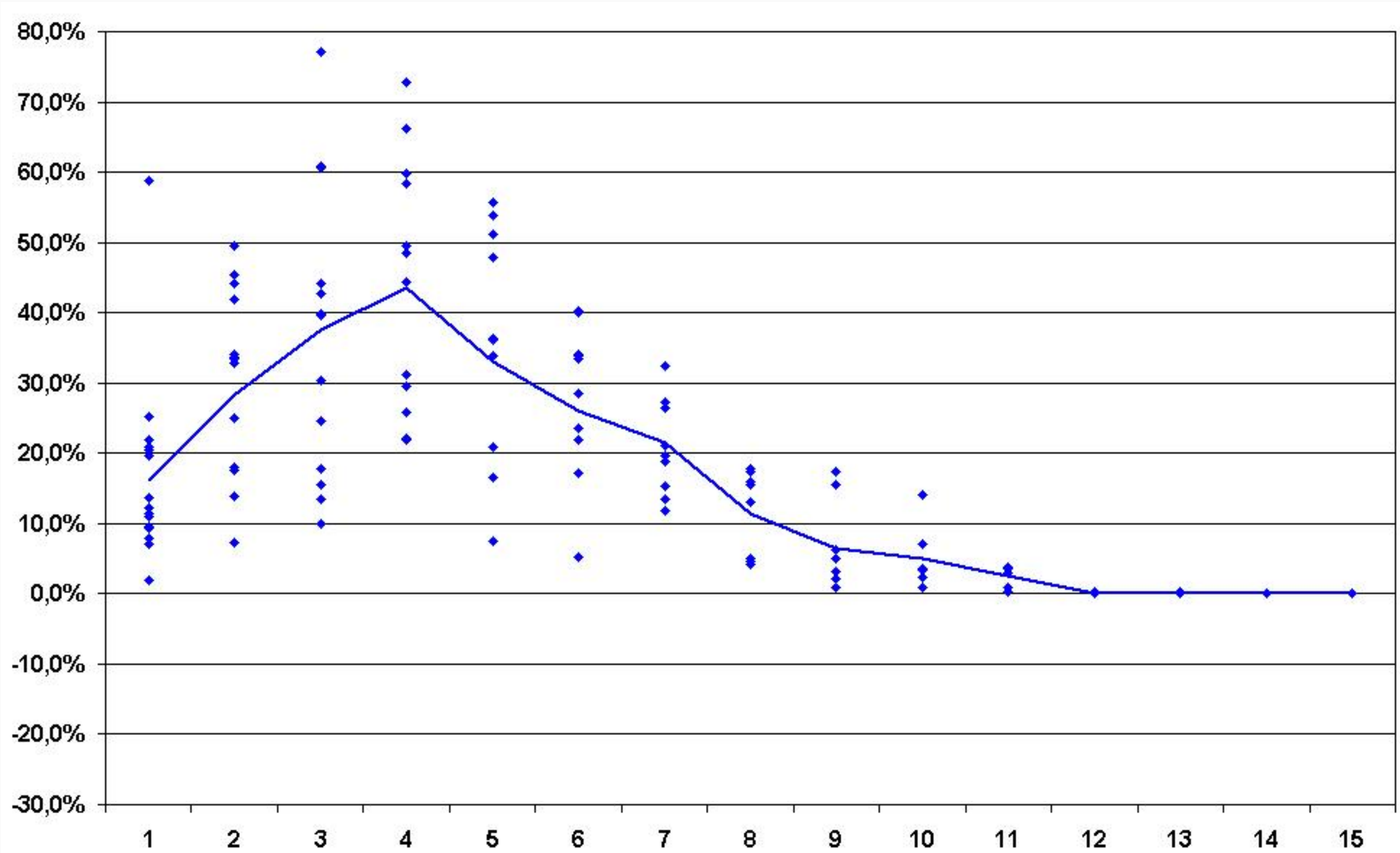
- The average R/v ratio is estimated by

$$\widehat{r_k} := \sum_{i=1}^{n-k+1} R_{i,k} \Bigg/ \sum_{i=1}^{n-k+1} v_i$$

and its variance parameter by

$$\widehat{\rho_k}^2 := \frac{1}{n-k} \cdot \sum_{i=1}^{n-k+1} v_i \cdot \left((R/v)_{i,k} - \widehat{r_k} \right)^2$$

Triangle of *R/v* ratios vs. development years



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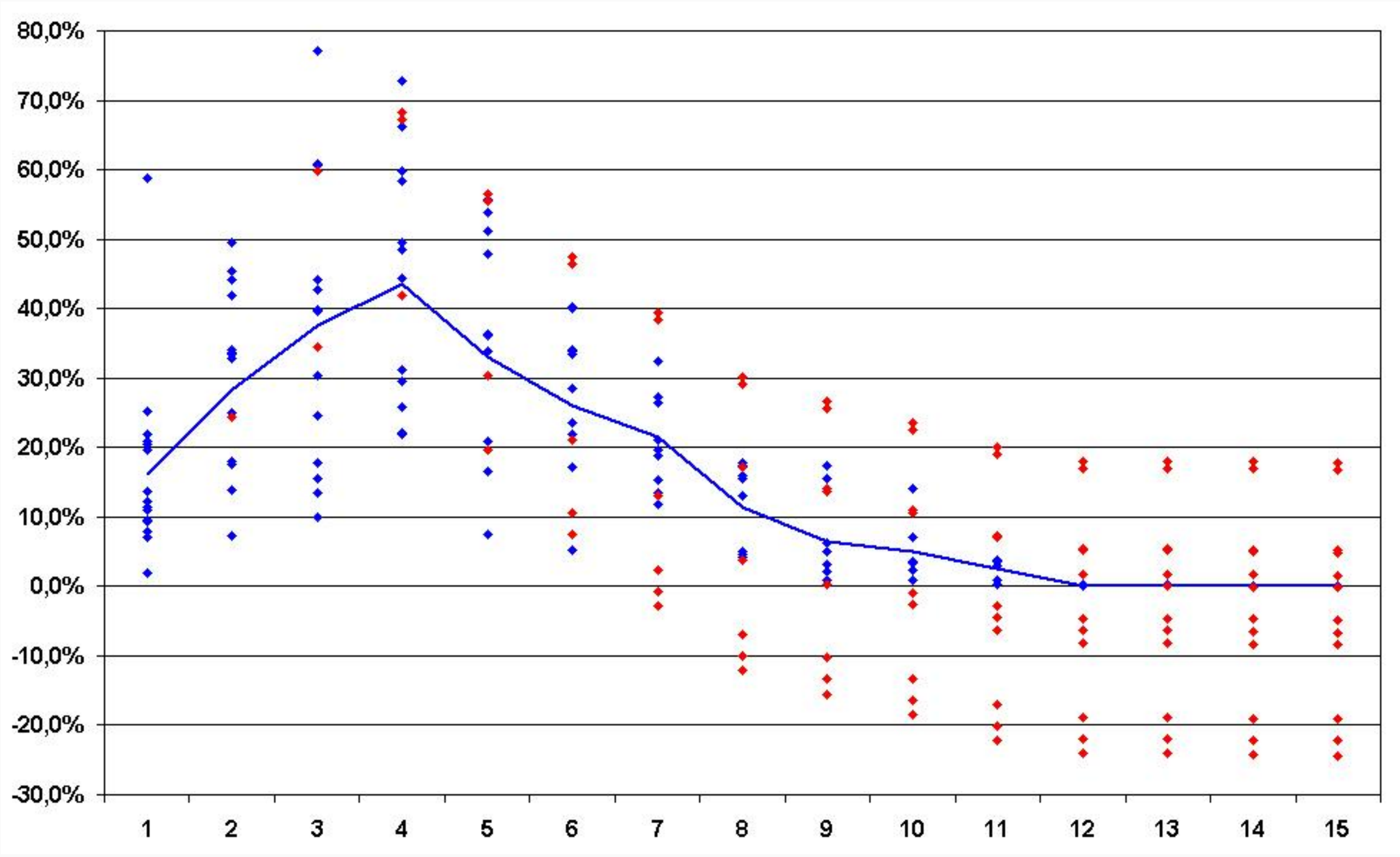
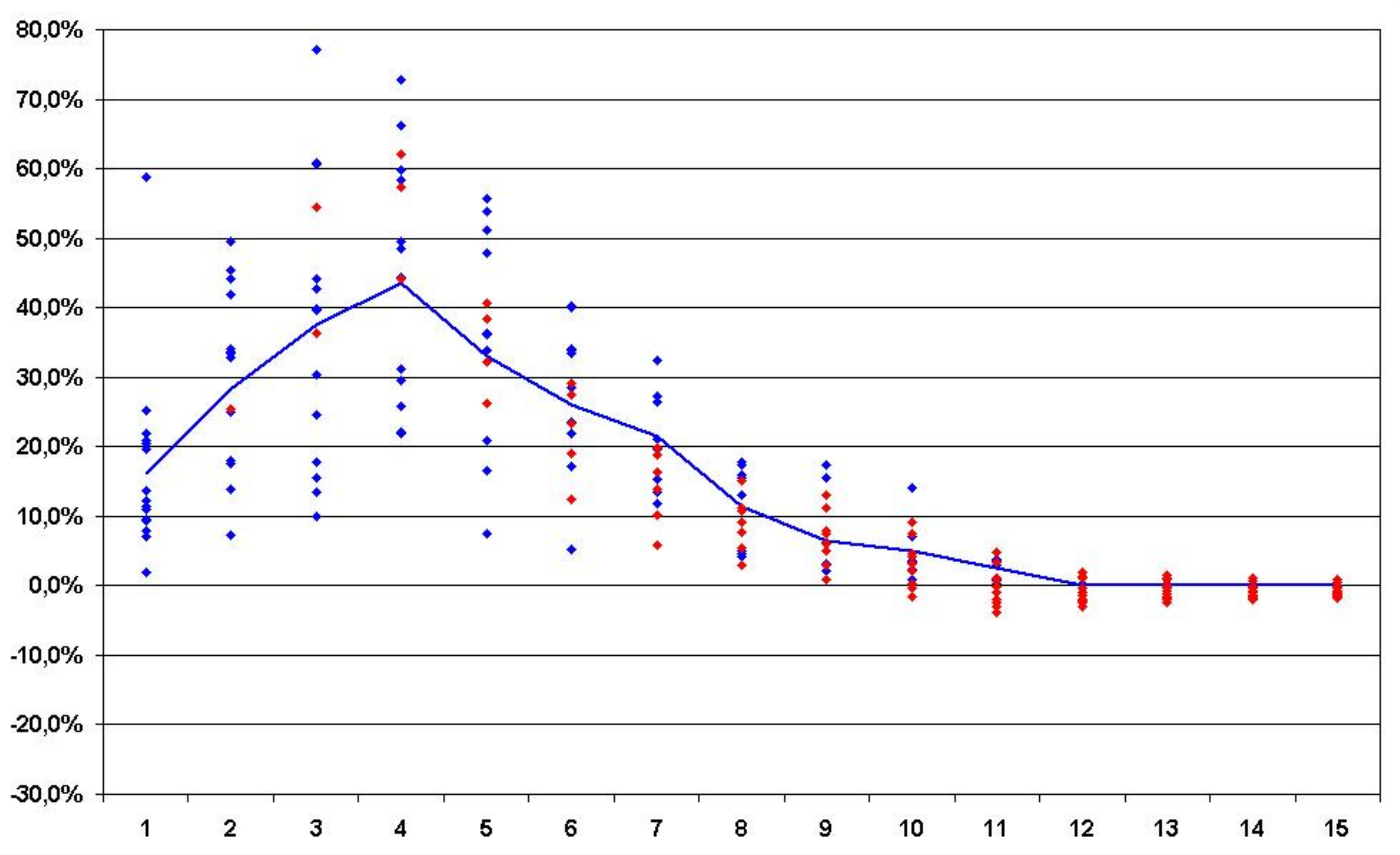
R/v quadrangle (with separate ILR estimates)



R/v quadrangle (with MILR)



THANK YOU VERY MUCH FOR YOUR ATTENTION

Dr. Gerhard Quarg

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