

Section N

GLOSSARY OF NOTATION

Preamble

This section brings together for reference purposes the main symbols used in the system of notation used in describing the methods of Volume 1. Some special symbols which are specific to particular methods, and which fall outside the main structure of the system, have been omitted from the glossary; their use is usually apparent in the context of the method in question.

The notation does not extend to the methods considered in Volume 2 of the Manual. There each method calls for whichever mathematical/statistical notation is appropriate in the particular circumstances.

[N1]
GLOSSARY OF NOTATION

<i>Symbol</i>	<i>Meaning</i>
$\{C\}$	Cohort of claims, which may be identified as
$\{C_a\}$	Accident-year cohort
$\{C_r\}$	Report-year cohort
$\{C_w\}$	Underwriting-year cohort
$\{S\}$	Subgroup of settled claims
$\{U\}$ outstanding claims
$\{O\}$ open claims
$\{R\}$ reported claims
$\{R^{\sim}\}$ IBNR claims
The following relationships hold —	
$\{U\}$	$= \{O R^{\sim}\}$
$\{R\}$	$= \{S O\}$
$\{C\}$	$= \{S O R^{\sim}\}$
	$= \{S U\} \text{ or } \{R R^{\sim}\}$
$\{S^o\}$	Claims settled at nil cost
$\{S^+\}$	Claims settled at some cost
$\{S\}$	$= \{S^o S^+\}$

GLOSSARY OF NOTATION

<i>Symbol</i>	<i>Meaning</i>
nC	Number of claims in cohort, which at a particular review date may be analysed into —
nS	Number of settled claims
nU outstanding claims
nO open claims
nR reported claims
nR^{\sim} IBNR claims
	The following relationships hold —
	$nU = nO + nR^{\sim}$
	$nR = nS + nO$
	$nC = nS + nO + nR^{\sim}$ $= nS + nU \text{ or } nR + nR^{\sim}$
nS°	Number of claims settled at nil cost
nS^{+}	Number of claims settled at some cost
	$nS = nS^{\circ} + nS^{+}$

At the stage of ultimate development of the cohort $nC = nS$ as the other components become zero.

An alternative notation for the ultimate number of claims in the cohort is $n\text{-ult}$. In general -ult is used to denote the ultimate development value of the element involved.

GLOSSARY OF NOTATION

<i>Symbol</i>	<i>Meaning</i>
d	Development time of a cohort. Conventionally $d = 0$ denotes the initial development period (usually year), so that d assumes the values 0, 1, 2, ... <i>ult</i> as the cohort runs off.
Any element, E , of a cohort may be identified by —	
	(i) the year of origin of the cohort, denoted by accident-year a or report-year r or underwriting-year w as the case may be,
	and
	(ii) the development-year d .
$E_a(d)$	The element in development year d in the cohort for accident-year a . The corresponding notation for report-year r and underwriting-year w would be $E_r(d)$ and $E_w(d)$ respectively.
\wedge	Precedes an element to indicate that it is an <u>estimated</u> amount.

GLOSSARY OF NOTATION

Symbol

Meaning

The following are examples of the "cohort development" notation applied to the element of "number of claims".

$nS_a(d)$	Number of claims originating in accident year a which have been settled by the end of development year d .
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$nO_a(d)$	Number of claims originating in accident year a which remain open at end of development year d .
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$nR_a(d)$	Number of claims originating in accident year a which have been reported by the end of development year d $= nS_a(d) + nO_a(d)$
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$\hat{n}R_a \sim (d)$	Estimated number of IBNR claims originating in accident year a , which remain unreported at end of development year d .
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\hat{n}_{a-ult}	Estimated ultimate number of claims attributed to accident year a $= nS_a(d) + nO_a + \hat{n}R_a \sim (d)$
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The same "cohort development" notation may be applied to other cohort elements defined on the following pages.

GLOSSARY OF NOTATION

<i>Symbol</i>	<i>Meaning</i>
pC	Cumulative total claim <u>amounts paid</u> to end of development period.
pC^*	Cumulative claim amounts paid to end of most recent development period. These values lie on the "leading" diagonal of the claims triangle.
pS	Cumulative amounts paid on claims settled to end of development period.
pO	Cumulative amounts paid on claims that are still open at end of development period.
ΔpC	Claim amounts paid in a specified development period (i.e. non-cumulative).

By way of example, the following relationships hold —

$pC_a(d)$ Cumulative claim amounts paid to end of development year d on claims originating in accident year a :

$$= \Delta pC_a(0) + \Delta pC_a(1) + \dots + \Delta pC_a(d)$$

alternatively

$$= pS_a(d) + pO_a(d)$$

GLOSSARY OF NOTATION

<i>Symbol</i>	<i>Meaning</i>
iC	Cumulative total claim <u>amounts incurred</u> to end of development period. Corresponding <u>amounts incurred</u> on —
iS	Claims settled to end of development period.
iO	Claims still open at end of development period.
iR	Claims reported to end of development period.
$iR\sim$	Claims which are IBNR at end of development period.
iU	Claims which are outstanding at end of development period $= iO + iR\sim$

GLOSSARY OF NOTATION

<i>Symbol</i>	<i>Meaning</i>
$kV_a(d)$	Case reserves on claims originating in accident year a which are open at end of development year d .
$^{\wedge}V_a(d)$	Estimated reserve at end of development year d on claims originating in accident year a .
$hV_a(d)$	Hypothecated reserve on claims outstanding at end of development d originating in accident year a .
$L_a\text{-ult}$	Ultimate liability on claims originating in accident year a .
$^{\wedge}L_a\text{-ult}$	Estimated ultimate liability on claims originating in accident year a .

The following relationships hold —

$$\begin{aligned}
 ^{\wedge}V_a(d) &= ^{\wedge}L_a\text{-ult} - pC_a(d) \\
 hV_a(d) &= ^{\wedge}L_a\text{-ult} - pC_a(d) \\
 iC_a(d) &= pC_a(d) + kV_a(d) \\
 ^{\wedge}L_a\text{-ult} &= iS_a(d) + iO_a(d) + iR_a \sim (d)
 \end{aligned}$$

VR	Reserve for reported claims.
$VR\sim$ (or $ib\ V$)	Reserve for IBNR claims.
VS	Reserve for re-opening of settled claims.

GLOSSARY OF NOTATION

<i>Symbol</i>	<i>Meaning</i>
g	Grossing-up factor = $pC/L\text{-ult}$
$r_a(d)$	Link ratio = $pC_a(d+1)/pC_a(d)$
$f_a(d)$	Final link ratio from the current cumulative claims $pC_a(d)$ to the final ultimate value $L_a\text{-ult}$
$f_a(d)$	$= L_a\text{-ult}/pC_a(d)$ $= r_a(d) \times r_a(d+1) \times \dots \times r_a(u-1)$

GLOSSARY OF NOTATION

<i>Symbol</i>	<i>Meaning</i>
A_{-ult}	<u>Average cost per claim</u> at end of cohort development $= L_{-ult}/n_{-ult}$
Sub-groups of <u>average costs</u> —	
AS	Settled claims iS/nS
AO	Open claims iO/nO
AR_{\sim}	IBNR claims iR_{\sim}/nR_{\sim}
AR	Reported claims iR/nR $= (iS + iO)/(nS + nO)$
AU	Outstanding claims iU/nU $= (iO + iR_{\sim})/(nO + nR_{\sim})$
pA	Paid average cost = pC/nS
iA	Incurred average cost = iC/nR

GLOSSARY OF NOTATION

<i>Symbol</i>	<i>Meaning</i>
X	Base measure of risk exposure (units of exposed-to-risk)
X_a	Unit of exposure for accident-year cohort. (Example: Earned Premium aP)
X_w	Unit of exposure for underwriting-year cohort. (Example: Written Premium wP)
λ	Loss ratio = $L\text{-}ult/P$
$p\lambda_a(d)$	Paid loss ratio at end of development year d $= pC_a(d)/aP$
$i\lambda_a(d)$	Incurred loss ratio at end of development year d $= iC_a(d)/aP$

GLOSSARY OF NOTATION

<i>Symbol</i>	<i>Meaning</i>
$BF-pc$	Bornhuetter-Ferguson Method applied to Paid claims.
$BF-iC$	Bornhuetter-Ferguson Method applied to Incurred claims.
$B-ult$	Benchmark Loss = $\lambda \times aP$
BF Proportion	$1 - 1/f$ where f is the final link ratio, or $1 - g$ where g is grossing-up factor
eV	Emerging Liability $= (1 - 1/f) \times B-ult$
eC	Emerging claims $= (1 - g) \times B-ult$
CV	Required Reserve $= \sum_a (^eC)$ i.e. the sum of Emerging Claims over all accident years.
^L-ult	Estimate Ultimate Loss $= pC + ^eC$