Section F
COMPUTERISED ILLUSTRATION OF VOLUME 2 PAPERS

Accompanying this revision of the Claims Reserving Manual is a disk illustrating the application of two of the methods described in Volume 2. The methods included are those described in Sections E5 and E6 of Volume 2, by S Christofides and T Mack respectively.

Both spreadsheet programs on this disk are solely for illustration, and are intended to help the user understand better the mechanics of performing the methods described in the papers. They are designed to replicate exactly the calculations shown in those papers. This will allow the user to follow the intermediate steps, and assist in understanding how the methods can be applied in practice.

Note that the spreadsheets are simply a mechanical reproduction of the particular calculations illustrated in the two papers. As such, they have not been designed to be used as a generalised reserving tool on other data. Readers should not, therefore, attempt to substitute their own data into this software for practical reserving purposes.

[F1]
COMPUTERISED ILLUSTRATION (1) —
REGRESSION MODEL BASED ON LOG-INCREMENTAL PAYMENTS
by S Christofides

The first file on the disk distributed with the Claims Reserving Manual demonstrates the model described in the paper by S Christofides in Section D5 of Volume 2. The filename is **crmsc.xls**, and is written in Excel version 5.

The file illustrates step-by-step the "full parameter" example given in pages D5.16 to D5.33 of the paper. The paper sets out clearly all the steps involved. Further brief instructions are included on the disk as to the operation of the spreadsheet regression analysis and matrix manipulation, so no further instructions are felt necessary here.

The spreadsheet also includes graphs of the various Residual analyses.

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[F2]
COMPUTERISED ILLUSTRATION (2) — MEASURING THE VARIABILITY OF CHAIN LADDER RESERVE ESTIMATES
by T Mack

The Lotus version 3 file, **crmmack.wk3**, on the disk distributed with the Claims Reserving Manual, demonstrates the model described in the paper by T Mack in Section D6 of Volume 2.

The file illustrates the calculation of the standard errors of the reserve estimates, and the use of a variety of diagnostics to test the assumptions made when using the model.

To make the calculation of the standard errors easier to follow, the calculations from the example in Section D6, on pages D6.19 to D6.24, have been broken down into small sections, for ease of reference. This should assist the user in seeing how the techniques can be applied in practice, as the formulae for calculating the standard errors, whilst being quite simple, do look a bit daunting at first sight.

The examples of some of the diagnostic tests are also based on the examples included in section 5 and Appendix H of Mack's paper. The diagnostics involve checking the three assumptions made when using the model. For a summary of the assumptions made, see the précis of this paper given in Section C of Volume 2.

The checks of the three assumptions are briefly described and illustrated below.

Checking Assumption 1

One way of checking assumption 1 is simply to conduct a visual examination of the data, to see if there is a consistent linear relationship between cumulative claims from one period to the next. A further way of checking the assumption is to use regression diagnostics, as explained in the section on checking assumption 3.

The attached tables and graphs are reproduced from the spreadsheet, and illustrate a visual examination of the data and the standardised residuals used to check assumption 1. When checking the residuals, if the model holds good, one expects to see the residuals randomly scattered, without any systematic patterns or distortions.

The diagnostic checks shown here correspond to those in Figure 1 of the paper. They differ slightly in that they plot incremental payments on the Y-axis, and examine the standardised residuals. Both types of diagnostic check are equally valid, and are just two ways of looking at the same thing.

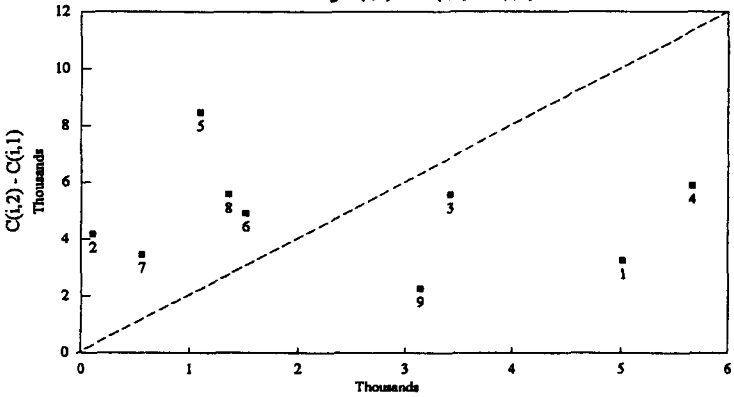
Diagnostic checks of the assumptions of the distribution-free approach

Checking Assumption 1: E(C(i,k+1 | C(i,1),....C(i,k))) = C(i,k) x f(k)

Plot: C(i,k) v. (C(i,k+1) - C(i,k))

	k=1		
i	C(i,1)	C(i,2)	(Ci,2)-C(i,1)
1	5,012	8,269	3,257
2	106	4,285	4,179
3	3,410	8,992	5,582
4	5,655	11,555	5,900
5	1,092	9,565	8,473
6	1,513	6,445	4,932
7	557	4,020	3,463
8	1,351	6,947	5,596
9	3,133	5,395	2,262

Checking Assumption 1
Plotting C(i,1) v. C(i,2) - C(i,1)

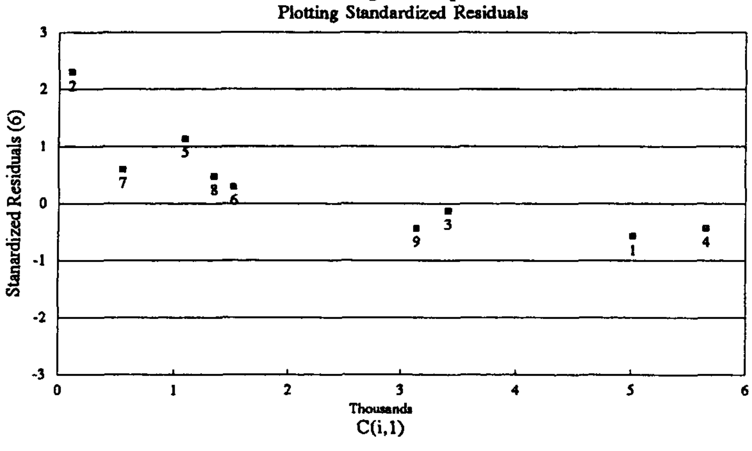


Plot: (C(i,k+1) - C(i,k) x f(k)) / C(i,k)^.5 / s(k) v. C(i,k)

f(1)= 2.9994 s(1)= 166.9835

	k=1					
	C(i,1)	C(i,2)	C(i,1)*f(1)	C(i,2)-C(i,1)*f(1)	C(i,1)^.5	(4)/(5)/(s(1)
i	(1)	(2)	(3)	(4)	(5)	(6)
1	5,012	8,269	15,033	(6,764)	71	-0.5722
2	106	4,285	318	3,967	10	2.3075
3	3,410	8,992	10,228	(1,236)	58	-0.1267
4	5,655	11,555	16,961	(5,406)	75	-0.4305
5	1,092	9,565	3,275	6,290	33	1.1398
6	1,513	6,445	4,538	1,907	39	0.2936
7	557	4,020	1,671	2,349	24	0.5961
8	1,351	6,947	4,052	2,895	37	0.4717
9	3,133	5,395	9,397	(4,002)	56	-0.4282

Checking Assumption 1
Plotting Standardized Residuals



Checking Assumption 2

One possible distortion that may invalidate assumption 2 is the presence of calendar year influences in the data. If there are such calendar year influences (for example, increasing payments in just one calendar year due to a new type of tax), then consecutive sets of development factors will be larger/smaller than expected. It is possible, however, to construct a statistical test to see whether there are diagonals with a preponderance of "Large" or "Small" development factors.

For each development period, k, the development factors are ordered and described as "L" or "S", depending on whether they are larger or smaller than the median. Then, for each of the j different diagonals, the numbers of L or S factors are counted. The actual median development factor, if we are looking at an odd-numbered set of factors, is described as "M", and is excluded from the subsequent construction of the test statistics.

In the absence of any calendar year effects, the number of L's and S's should be about the same. Similarly, the minimum of the number of L's and S's, described as Z_j, should not be significantly different from the average number of L's plus S's. The paper shows how the distribution of Z_j can be calculated and used for a significance test. Where this test indicates the presence of a calendar year effect, it is suggested that the weights of the relevant outlying development factors are reduced.

Alternatively, one can construct a formula for the first two moments of Z_j, which are:

$$E(Z_j) = \frac{n}{2} - \binom{n-1}{m} \times \frac{n}{2^n}$$

$$Var(Z_j) = \frac{n \times (n-1)}{4} - \binom{n-1}{m} \times \frac{n \times (n-1)}{2^n} + E(Z_j) - (E(Z_j))^2$$

Looking at individual diagonals may be misleading, so one considers $Z = Z_2 + ... + Z_{I-1}$. The expected value and variance of Z are just the sums of the individual expectations and variances of the Z_j's respectively (under the initial assumptions, the Z_j's are uncorrelated). Assuming Z is Normal, it can be concluded that there is no significant calendar year affect, at a 95% confidence level, if the actual Z is within two standard errors of the expected Z.

Assumption 2 can also be checked by the use of Residual diagnostics, as described in the section on checking assumption 3. The following examples of some of these tests are from the **crmmack.wk3** spreadsheet.

The statistical test illustrates the alternative approach, based on the first two moments of Z_j. Whilst the examples illustrate the application of these tests, they also show the difficulties in applying statistical tests to the quite small volumes of data one is invariably considering with such reserving exercises.

Diagnostic checks of the assumptions of the distribution-free approach

Checking Assumption 2: {C(i,1)...C(i,I)},{C(j,1,...,C(j,I)}, are independent

Statistical Test Of Independence

i	k	C(i,k)	C(i,k+1)	C(i,k+1)/C(i,k)	S/L/M
(1)	(2)	(3)	(4)	(5)	(7)
1	1	5,012	8,269	1.6498	S
2	1	106	4,285	40.4245	L
3	1	3,410	8,992	2.6370	S
4	1	5,655	11,555	2.0433	S
5	1	1,092	9,565	8.7592	L
6	1	1,513	6,445	4.2597	M
7	1	557	4,020	7.2172	L
8	1	1,351	6,947	5.1421	L
9	1	3,133	5,395	1.7220	S
1	2	8,269	10,907	1.3190	S
2	2	4,285	5,396	1.2593	S
3	2	8,992	13,873	1.5428	S
4	2	11,555	15,766	1.3644	S
5	2	9,565	15,836	1.6556	L
6	2	6,445	11,702	1.8157	L
7	2	4,020	10,946	2.7229	L
8	2	6,947	13,112	1.8874	L
1	3	10,907	11,805	1.0823	S
2	3	5,396	10,666	1.9766	L
3	3	13,873	16,141	1.1635	M
4	3	15,766	21,266	1.3489	L
5	3	15,836	22,169	1.3999	L
6	3	11,702	12,935	1.1054	S
7	3	10,946	12,314	1.1250	S
1	4	11,805	13,539	1.1469	S
2	4	10,666	13,782	1.2921	L
3	4	16,141	18,735	1.1607	S
4	4	21,266	23,425	1.1015	S
5	4	22,169	25,955	1.1708	L
6	4	12,935	15,852	1.2255	L
1	5	13,539	16,181	1.1951	L
2	5	13,782	15,599	1.1318	M
3	5	18,735	22,214	1.1857	L
4	5	23,425	26,083	1.1135	S
5	5	25,955	26,180	1.0087	S
1	6	16,181	18,009	1.1130	L
2	6	15,599	15,496	0.9934	S
3	6	22,214	22,863	1.0292	S
4	6	26,083	27,067	1.0377	L
1	7	18,009	18,608	1.0333	M
2	7	15,496	16,169	1.0434	L
3	7	22,863	23,466	1.0264	S
1	8	18,608	18,662	1.0029	S
2	8	16,169	16,704	1.0331	L
1	9	18,662	18,834	1.0092	M

	k								
i	1	2	3	4	5	6	7	8	9
1	S	S	S	S	L	L	M	S	M
2	L	S	L	L	M	S	L	L	
3	S	S	M	S	L	S	S		
4	S	S	L	S	S	L			
5	L	L	L	L	S				
6	M	L	S	L					
7	L	L	S						
8	L	L							
9	S								

j	S(j)	L(j)	Z(j)	n	m	E(Z(j))	Var(Z(j))
2	1	1	1	2	1	0.5000	0.2500
3	3	0	0	3	1	0.7500	0.1875
4	3	1	1	4	2	1.2500	0.4375
5	1	3	1	4	2	1.2500	0.4375
6	1	3	1	4	2	1.2500	0.4375
7	2	4	2	6	3	2.0625	0.6211
8	4	4	4	8	4	2.9063	0.8037
9	4	4	4	8	4	2.9063	0.8037
						12.8750	3.97852

Minimum: 8.88576
Maximum: 16.8642

Actual Z: 14

no statistical evidence of CY effects

j= number of diagonal
S(j) = number of Small's
L(j) = number of Large's
Z(j) = Min(L(j),S(j))
n=S(j)+L(j)
m=(n-1)/2
E(Z(j)),Var(E(j)) are our test statistics.

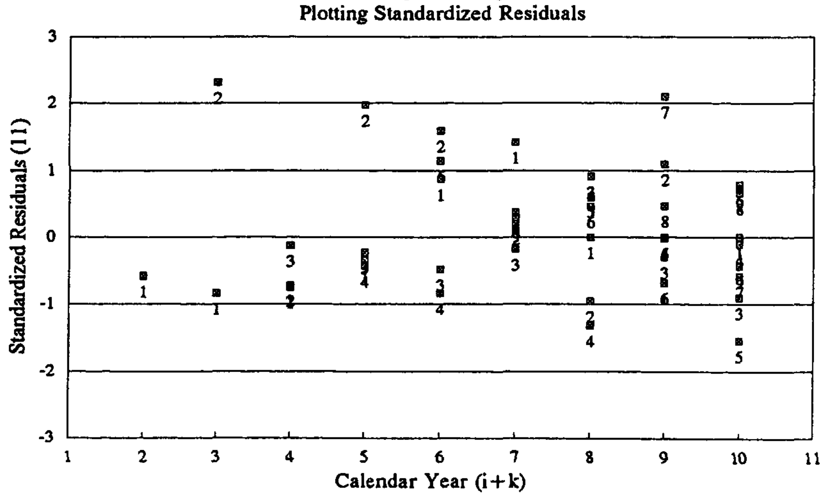
Diagnostic checks of the assumptions of the distribution-free approach

Checking Assumption 2: {C(i,1)...C(i,I)},{C(j,1,...,C(j,I)}, are independent

Plot: (C(i,k+1) - C(i,k) x f(k)) / C(i,k)^.5 / s(k) v. (i+k)

i	k	i+k	f(k)	s(k)	C(i,k)	C(i,k+1)	C(i,k)*f(k)	(7)-(8)	C(i,k)^.5	(9)/(10)/(5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	1	2	2.9994	166.9835	5,012	8,269	15,033	(6,764)	71	-0.5722
2	1	3	2.9994	166.9835	106	4,285	318	3,967	10	2.3075
3	1	4	2.9994	166.9835	3,410	8,992	10,228	(1,236)	58	-0.1267
4	1	5	2.9994	166.9835	5,655	11,555	16,961	(5,406)	75	-0.4305
5	1	6	2.9994	166.9835	1,092	9,565	3,275	6,290	33	1.1398
6	1	7	2.9994	166.9835	1,513	6,445	4,538	1,907	39	0.2936
7	1	8	2.9994	166.9835	557	4,020	1,671	2,349	24	0.5961
8	1	9	2.9994	166.9835	1,351	6,947	4,052	2,895	37	0.4717
9	1	10	2.9994	166.9835	3,133	5,395	9,397	(4,002)	56	-0.4282
1	2	3	1.6235	33.2945	8,269	10,907	13,425	(2,518)	91	-0.8317
2	2	4	1.6235	33.2945	4,285	5,396	6,957	(1,561)	65	-0.7161
3	2	5	1.6235	33.2945	8,992	13,873	14,599	(726)	95	-0.2299
4	2	6	1.6235	33.2945	11,555	15,766	18,760	(2,994)	107	-0.8365
5	2	7	1.6235	33.2945	9,565	15,836	15,529	307	98	0.0943
6	2	8	1.6235	33.2945	6,445	11,702	10,464	1,238	80	0.4633
7	2	9	1.6235	33.2945	4,020	10,946	6,527	4,419	63	2.0935
8	2	10	1.6235	33.2945	6,947	13,112	11,279	1,833	83	0.6607
1	3	4	1.2709	26.2953	10,907	11,805	13,862	(2,057)	104	-0.7489
2	3	5	1.2709	26.2953	5,396	10,666	6,858	3,808	73	1.9716
3	3	6	1.2709	26.2953	13,873	16,141	17,631	(1,490)	118	-0.4811
4	3	7	1.2709	26.2953	15,766	21,266	20,037	1,229	126	0.3723
5	3	8	1.2709	26.2953	15,836	22,169	20,126	2,043	126	0.6175
6	3	9	1.2709	26.2953	11,702	12,935	14,872	(1,937)	108	-0.6809
7	3	10	1.2709	26.2953	10,946	12,314	13,911	(1,597)	105	-0.5805
1	4	5	1.1717	7.8250	11,805	13,539	13,832	(293)	109	-0.3442
2	4	6	1.1717	7.8250	10,666	13,782	12,497	1,285	103	1.5900
3	4	7	1.1717	7.8250	16,141	18,735	18,912	(177)	127	-0.1780
4	4	8	1.1717	7.8250	21,266	23,425	24,917	(1,492)	146	-1.3074
5	4	9	1.1717	7.8250	22,169	25,955	25,975	(20)	149	-0.0170
6	4	10	1.1717	7.8250	12,935	15,852	15,156	696	114	0.7825
1	5	6	1.1134	10.9288	13,539	16,181	15,074	1,107	116	0.8704
2	5	7	1.1134	10.9288	13,782	15,599	15,345	254	117	0.1982
3	5	8	1.1134	10.9288	18,735	22,214	20,859	1,355	137	0.9056
4	5	9	1.1134	10.9288	23,425	26,083	26,081	2	153	0.0012
5	5	10	1.1134	10.9288	25,955	26,180	28,898	(2,718)	161	-1.5437
1	6	7	1.0419	6.3890	16,181	18,009	16,860	1,149	127	1.4143
2	6	8	1.0419	6.3890	15,599	15,496	16,253	(757)	125	-0.9488
3	6	9	1.0419	6.3890	22,214	22,863	23,146	(283)	149	-0.2967
4	6	10	1.0419	6.3890	26,083	27,067	27,177	(110)	162	-0.1064
1	7	8	1.0333	1.1591	18,009	18,608	18,608	(0)	134	-0.0003
2	7	9	1.0333	1.1591	15,496	16,169	16,011	158	124	1.0919
3	7	10	1.0333	1.1591	22,863	23,466	23,624	(158)	151	-0.8987
1	8	9	1.0169	2.8077	18,608	18,662	18,923	(261)	136	-0.6819
2	8	10	1.0169	2.8077	16,169	16,704	16,443	261	127	0.7315
1	9	10	1.0092	1.1591	18,662	18,834	18,834	0	137	0.0000

Checking Assumption 2
Plotting Standardized Residuals



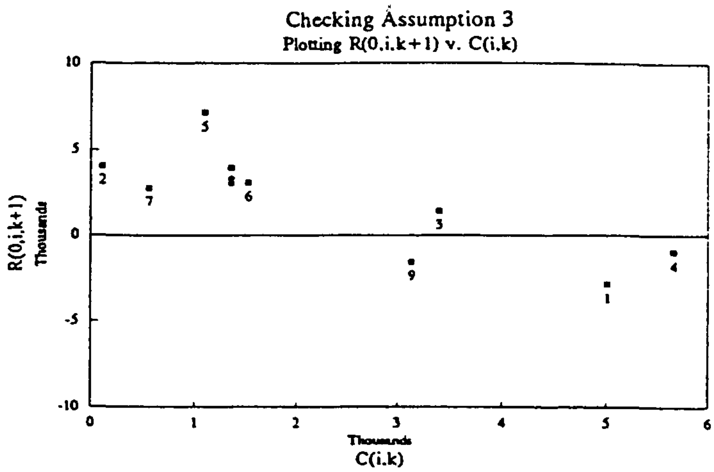
Diagnostic checks of the assumptions of the distribution-free approach

Checking Assumption 3: Var(C(i,k+1|C(i,1),....C(i,k))) = C(i,k) x s(k)^2

Plot: R(0,i,k+1) = (C(i,k+1) - C(i,k) x f(k,0)) v. C(i,k)

f(1,0) = 2.2172

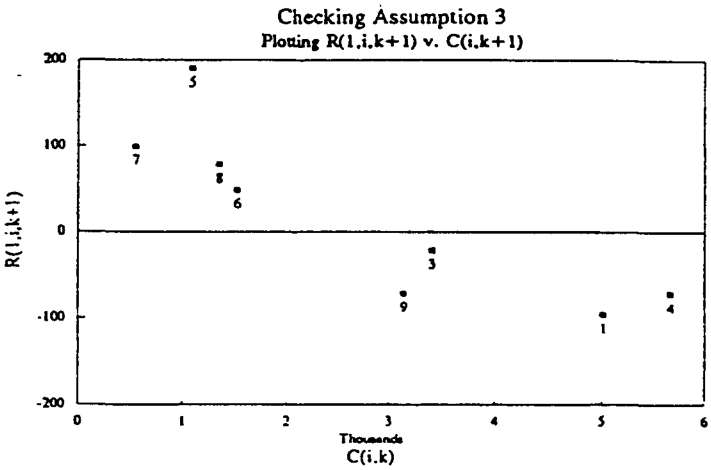
i	k=1 C(i,1)	C(i,2)	R(0,i,k+1)
1	5,012	8,269	(2,844)
2	106	4,285	4,050
3	3,410	8,992	1,431
4	5,655	11,555	(983)
5	1,092	9,565	7,144
6	1,513	6,445	3,090
7	557	4,020	2,785
8	1,351	6,947	3,952
9	3,133	5,395	(1,552)



Plot: R(1,i,k+1) = (C(i,k+1) - C(i,k) x f(k,1)) / C(i,k) ^.5 v. C(i,k)

f(1,1) = 2.9994

i	k=1 C(i,1)	C(i,2)	R(1,i,k+1)
1	5,012	8,269	(96)
2	106	4,285	385
3	3,410	8,992	(21)
4	5,655	11,555	(72)
5	1,092	9,565	190
6	1,513	6,445	49
7	557	4,020	100
8	1,351	6,947	79
9	3,133	5,395	(71)



Plot: R(2,i,k+1) = (C(i,k+1) - C(i,k) x f(k,2)) / C(i,k) v. C(i,k)

f(1,2) = 8.2061

i	k=1 C(i,1)	C(i,2)	R(2,i,k+1)
1	5,012	8,269	-6.5563
2	106	4,285	32.2184
3	3,410	8,992	-5.5691
4	5,655	11,555	-6.1628
5	1,092	9,565	0.5531
6	1,513	6,445	-3.9464
7	557	4,020	-0.9889
8	1,351	6,947	-3.0640
9	3,133	5,395	-6.4841