Section H
METHODS BASED ON CLAIM NUMBERS & AVERAGE COST PER CLAIM

Preamble

To this point in the Manual, we have used three main sources of data for the claims projections — paid claims, case reserves and earned or written premiums. These data items, all monetary amounts, lead on naturally to the paid and incurred claim projections and the loss ratio methods. But a further dimension can be provided by a fourth main data item, not itself a monetary unit, which is the *number* of claims. Such data are frequently available in direct insurance work, but seldom in the reinsurance field.

When the claim amounts paid or incurred are divided by the relevant number of claims, an average cost per claim results. This average cost can be projected, just as were the claim amounts themselves. Then, combined with a separate projection for the number of claims, it will yield the new estimate for the ultimate loss. The reserver can also examine the movement of the claim numbers and average costs as the accident years develop, and look for significant trends or discontinuities. A fuller view of the business can thus be obtained, perhaps leading to adjustment of the reserving figures, or showing where further investigation is needed.

One point about average cost per claim methods is that many variations are possible. The reserver should ask, what quantities go into making the average, what is the basis of projection, and what claim numbers are used for the eventual multiplier of the projected average? It is vital to be clear as to exactly what definitions are being used — the term "Average Cost per Claim Method" on its own is rather inadequate. The present section describes some of the average claim methods available, but is far from being exhaustive of the genre.

Contents

[H1]
PAID AVERAGE CLAIMS PROJECTION

Work on claim numbers and average costs per claim methods can begin quite easily from the starting point of paid claim amounts. We will use the data first introduced in §E1.

		d 0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
	2	1113	2103	2774	3422	3844		
a	3	1265	2433	3233	3977			
	4	1490	2873	3880				
	5	1725	3261				[*pC*]	
	6	1889						

The additional data needed are the corresponding claim numbers. Since we are talking about paid claims, the appropriate data are the numbers of claims *settled*, for which we shall use the symbol *nS*. In average cost methods, there are always questions to be answered about the exact definition of claim amounts (for example whether they include partial payments made on claims that are still outstanding) and claim numbers and their correspondence one with another. But we will return to these in §H5, and for the moment press on with describing the projection method itself. Let us suppose that the following data become available for the claim numbers settled:

		d 0	1	2	3	4	5	*ult*
	1	279	379	427	463	482	488	498
	2	303	411	463	500	522		
a	3	328	446	503	544			
	4	343	462	530				
	5	350	469				[*nS*]	
	6	355						

In this table, the numbers refer to the cumulative number of claims settled for each accident year as development time *d* progresses. The final number in the table, 498 in the *ult* column, is not fully objective, since we take it that year $a=1$ has not yet developed beyond $d=5$. The number will have been estimated, presumably, from a study of earlier years' data in which the development to ultimate is complete.

The first step in the working is very simple. The elements in the *nS*-triangle are divided into those of the *pC*-triangle to give the average costs per claim at each stage. The results are as follows, using the symbol *pA* to denote the average cost, which in this case may be called the "paid average" for short. (As with the original data on paid claims, the figures are in £1,000's. They are not supposed to be related to any particular class or grouping of business.)

	d →	0	1	2	3	4	5	*ult*
	1	3.588	4.894	5.674	6.454	6.919	7.137	7.440
	2	3.673	5.117	5.991	6.844	7.364		
a	3	3.857	5.455	6.427	7.311			
	4	4.344	6.219	7.321				
	5	4.929	6.953				[*pA*]	
	6	5.321						

The table of average costs is interesting in itself. There is a marked increase in cost both along the rows and down the columns. The increase results from two major causes. Both rows and columns reflect the general tendency for claim sizes to inflate with the passing of the years. The row increase also reflects the fact that, for any given accident year, the more serious claims tend to take longer to settle — a factor which will be present in non-inflationary conditions.

We now want to project the paid average costs, to estimate the ultimate that will be reached for each accident year. Since we have a triangle of data no different in form from that tackled in earlier chapters, the standard methods can apply. Perhaps the simplest technique is to use a grossing-up procedure, with averaging of factors down the columns. The result is as follows (see ξE3 for full details of method of working):

	d →	0	1	2	3	4	5	*ult*
	1	3.588	4.894	5.674	6.454	6.919	7.137	7.440
		48.2	65.8	76.3	86.7	93.0	<u>95.9%</u>	
	2	3.673	5.117	5.991	6.844	7.364		7.918
		46.4	64.6	75.7	86.4	<u>93.0%</u>		
a	3	3.857	5.455	6.427	7.311			8.442
		45.7	64.6	76.1	<u>86.6%</u>			
	4	4.344	6.219	7.321				9.633
		45.1	64.6	<u>76.0%</u>				
	5	4.929	6.953			[*pA*]		10.713
		46.0	<u>64.9%</u>			[*g*]		
	6	5.321						11.492
		<u>46.3%</u>						

We now have our estimate for the ultimate average claim cost for each accident year. It is only necessary to multiply these figures by the number of expected claims in each accident year to give the final estimate of the loss. The expected claim numbers can, of course, be found by applying a grossing up procedure to the *nS*-table above, and this is done below.

		d 0	1	2	3	4	5	*ult*
	1	279	379	427	463	482	488	498
		56.0	76.1	85.7	93.0	96.8	98.0%	
	2	303	411	463	500	522		539
		56.2	76.3	85.9	92.8	96.8%		
a	3	328	446	503	544			586
		56.0	76.1	85.8	92.9%			
	4	343	462	530				618
		55.5	74.8	85.8%				
	5	350	469			[*nS*]		619
		56.6	75.8%			[*g*]		
	6	355						634
		56.0%						
								3494

The final multiplication of average costs and claim numbers can now be done:

a	^*A-ult*	^*n-ult*	^*L-ult*
1	7.440	498	3705
2	7.918	539	4268
3	8.442	586	4947
4	9.633	618	5953
5	10.713	619	6631
6	11.492	634	7286

Overall Values:	∑^*L-ult*	32,790
	∑*pC**	20,334
	Reserve	12,456

The final value for the reserve is very close indeed to the best estimate from the grossing up of paid claim amounts. (Value £12,461, as given on E4.1.) Since it is much simpler just to use paid claims, is there any real advantage in the more complicated paid average method? In fact, the answer is yes, but not with the version just described. To gain the benefit, a variation must be brought in which concerns the way the claim numbers are handled. This variation arises from looking at the claim settlement pattern, and is the subject of the next section.

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[H2]
NUMBER SETTLED & NUMBER REPORTED

Beginning from paid claim amounts, we were led naturally to consider the numbers of claims settled, and to project the ultimate number of claims from these data. But if we had begun from the incurred claim position, then the corresponding numbers would have been of claims *reported* instead. Of the claims reported at any stage, clearly a subset will be the claims already settled. The remainder will be open claims, i.e. those claims to which (in most classes of business) the data for case reserves will relate.

To continue the example begun in §H1, let us use the symbol *nR* for the number of claims reported, and suppose that the following data have been given:

					d			
		0	1	2	3	4	5	*ult*
	1	414	460	482	488	492	494	494
	2	453	506	526	536	539		
a	3	494	548	572	582			
	4	530	588	615				
	5	545	605				**[*nR*]**	
	6	557						

Again, the numbers refer to the cumulative number of claims reported for each accident year as development time progresses. The final number for year a=1, 494, is derived on the assumption that all claims have been reported by time d=5.

Looking at the table, it is clear that the data could be projected just as was done for the numbers settled. Hence we have an alternative route for estimating the ultimate numbers of claims, and a check on the earlier projection. Before carrying this out, it is useful to examine the direct relationship between the numbers settled and reported. For convenience, the data on numbers settled are repeated here:

					d			
		0	1	2	3	4	5	*ult*
	1	279	379	427	463	482	488	498
	2	303	411	463	500	522		
a	3	328	446	503	544			
	4	343	462	530				
	5	350	469				**[*nS*]**	
	6	355						

The obvious route for the comparison is to calculate the proportion which the number settled bears to the number reported at each stage. This is done in the table below:

		d 0	1	2	3	4	5	*ult*
	1	67.4	82.4	88.6	94.9	98.0	98.8	100.8
	2	66.9	81.2	88.0	93.3	96.8		
a	3	66.4	81.4	87.9	93.5			
	4	64.7	78.6	86.2				
	5	64.2	77.5				**[*nS*/*nR*]**	
	6	63.7						

The pattern which emerges is that the number of claims settled has in recent years been a decreasing proportion of the claims reported. If the pattern of the reported claim numbers is a stable one, then we have strong evidence here that the settlement rate for the class of business is tending to slow down. Enquiries should be made to see whether the point can be corroborated by the experience of the claims department staff. If it can, the conclusion will be that the claim number projection on the basis of numbers settled is likely to be at fault.

Indeed, even if the confirmatory evidence is not to hand, the projection of the numbers reported is still likely to be the more reliable. That is simply because, at any point in the development, the numbers reported must of necessity be further advanced towards the ultimate than the numbers settled. It is often found that the numbers reported yield one of the most stable patterns in the claims development scene. In general, numbers of claims tend to be easier to handle and predict than claim amounts or average costs.

Let us now carry out the claim number projection from the given data on claims reported. A grossing up method with averaging of the factors will be used, as for claims settled in §H1.

		d						
		0	1	2	3	4	5	*ult*
	1	414	460	482	488	492	494	494
		83.8	93.1	97.6	98.8	99.6	100.0%	
	2	453	506	526	536	539		541
		83.7	93.5	97.2	99.1	99.6%		
a	3	494	548	572	582			588
		84.0	93.2	97.3	99.0%			
	4	530	588	615				631
		84.0	93.2	97.4%				
	5	545	605			**[*nR*]**		648
		84.1	93.3%			**[*g*]**		
	6	557						664
		83.9%						
								3566

The comparison of the projected claim numbers in the two cases is as follows:

a	6	5	4	3	2	1
^*n-ult* (*nS*-base)	634	619	618	586	539	498
^*n-ult* (*nR*-base)	664	648	631	588	541	494
Δ	+4.7%	+4.7%	+2.1%	+0.3%	+0.4%	-0.8%

The difference is mainly shown in the most recent three accident years.

This shows the importance of choosing the most appropriate claim number projection in an average cost per claim method. Unless there is evidence to the contrary, it will generally be right to prefer the projection of numbers reported, and this principle will be followed throughout the rest of §H.

We can now begin to answer the question posed at the end of §H1, i.e. as to the relevance of the paid average projection. The fact is that such an average cost per claim analysis, if properly applied, can be responsive to certain of the variations in the claim settlement pattern. It will be recalled from §E12 and §G8 above that such variations are a major point of difficulty with the straight projection of paid claim amounts. Indeed, the problem is of such importance that it should never be far from the reserver's mind. The information which comes from the claim numbers, and in particular from the comparison of numbers settled against numbers reported, is very useful in beginning to provide the needed evidence.

The other part of the evidence relates to the claim severities, and in particular to the relative costs of claims settled at different stages of the overall development.

To conclude the section, we summarise the movement observed in the claim settlement pattern by calculating the numbers settled as a proportion of the estimated ultimate values. (The latter, of course, come from the projection of numbers reported.)

					d			
		0	1	2	3	4	5	*ult*
	1	279	379	427	463	482	488	494
	2	303	411	463	500	522		541
a	3	328	446	503	544			588
	4	343	462	530				631
	5	350	469				**[*nS*]**	648
	6	355						664

	1	56.5	76.7	86.4	93.7	97.6	98.8%
	2	56.0	76.0	85.6	92.4	96.5	
a	3	55.8	75.9	85.5	92.5		
	4	54.4	73.2	84.0			
	5	54.0	72.4				**[*nS/n-ult*]**
	6	53.5					

The ability given to study patterns such as these shows the usefulness to the reserver of the data on claim numbers. The picture that can be built of the development pattern is fuller than can be obtained from using claim amounts alone.

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[H3]
INCURRED AVERAGE CLAIMS PROJECTION

The method already developed for paid claims and the projection of the average cost can also be applied using incurred claims. The mechanics are straightforward, and exactly parallel those set out in §H1. We begin with the original incurred claim data (first given in §F3), and the figures for numbers reported from the previous section:

					d			
		0	1	2	3	4	5	*ult*
	1	2777	3264	3452	3594	3719	3717	3717
	2	3252	3804	3973	4231	4319		
a	3	3725	4404	4779	4946			
	4	4521	5422	5676				
	5	5369	6142				[*iC*]	
	6	5818						
	1	414	460	482	488	492	494	494
	2	453	506	526	536	539		
a	3	494	548	572	582			
	4	530	588	615				
	5	545	605				**[*nR*]**	
	6	557						

Dividing the elements in the *iC* triangle by those of the *nR* triangle gives the average incurred cost per claim at each stage of development. This we shall refer to as the incurred average, symbol *iA*.

					d			
		0	1	2	3	4	5	*ult*
	1	6.708	7.096	7.162	7.365	7.559	7.524	7.524
	2	7.179	7.518	7.553	7.894	8.013		
a	3	7.540	8.036	8.355	8.498			
	4	8.530	9.221	9.229				
	5	9.851	10.152				[*iA*]	
	6	10.445						

The incurred average is projected to ultimate, using some standard method of the grossing up or link ratio type. Here, grossing-up is employed, working backward down the main diagonal and with averaging of factors in the columns:

		d 0	1	2	3	4	5	*ult*
	1	6.708	7.096	7.162	7.365	7.559	7.524	7.524
		89.2	94.3	95.2	97.9	100.5	100.0%	
	2	7.179	7.518	7.553	7.894	8.013		7.973
		90.0	94.3	94.7	99.0	100.5%		
a	3	7.540	8.036	8.355	8.498			8.627
		87.4	93.1	96.8	98.5%			
	4	8.530	9.221	9.229				9.654
		88.4	95.5	95.6%				
	5	9.851	10.152			[*iA*]		10.766
		91.5	94.3%			[*g*]		
	6	10.445						11.697
		89.3%						

It remains to bring in the relevant claim numbers. These are the numbers reported, which have already been projected in §H2 with the result:

a	6	5	4	3	2	1
^*n-ult* (*nR*-base)	644	648	631	588	541	494

Multiplying the projected average claims by the projected numbers then yields the loss estimate in the usual way:

a	^*A-ult*	^*n-ult*	^*L-ult*
1	7.524	494	3717
2	7.973	541	4313
3	8.627	588	5073
4	9.654	631	6092
5	10.766	648	6976
6	11.697	664	7767

Overall Values:	$\sum$^*L-ult*	33,938
	$\sum$*pC**	20,334
	Reserve	13,604

The final figure for the reserve is very close to that obtained by the incurred claims projection itself (§F3.2), which was £13,634. The fact is that the incurred average method will almost always produce such results. For projection purposes, the incurred average cannot be recommended as providing any real advantage over the incurred claims method itself. (It is included in the Manual for purposes of completeness and consistency in the exposition.)

[H4]
RISK EXPOSURE & CLAIM FREQUENCY

Although the incurred average claims method has few advantages, the numbers reported themselves can be of further use. They can be made to bring out evidence on the claim frequency in the given class of business. What we need in addition are data on the risk exposure for the accident years in question. This exposure can be measured in a number of ways, but a common means will be via a standard exposure unit, which can be on an earned or written basis. The exact definition of the unit will vary with the class of business — it can be a vehicle-year in Motor, or a dwelling-year in domestic Fire, and so on. Let us suppose this information becomes available in the present case as follows (no particular specification of the unit-type is intended):

a	6	5	4	3	2	1
aX	20.59	19.82	19.21	18.94	18.44	18.03

Here, the figures give the 1,000s of exposure units for the years in question. aX is taken as the symbol for exposure X measured on the accident year, i.e. it is the *earned* exposure. (For the *written* exposure, corresponding to the underwriting year, we would write wX.)

The claim frequencies can now be calculated as numbers reported divided by the earned exposure for each accident year. (The symbol used here is Cq, where $Cq = nR/aX$.)

aX	a				d		
		0	1	2	3	4	5
18.03	1	414 23.0	460 25.5	482 26.7	488 27.1	492 27.3	494 27.4
18.44	2	453 24.6	506 27.4	526 28.5	536 29.1	539 29.2	
18.94	3	494 26.1	548 28.9	572 30.2	582 30.7		
19.21	4	530 27.6	588 30.6	615 32.0			
19.82	5	545 27.5	605 30.5			**[*nR*]** **[*Cq*]**	
20.59	6	557 27.1					

The picture shown here is that claim frequencies have been increasing over accident years a=1 to 4, but appear now to be stabilising. But the evidence for the latter point is very far from complete, and it will need further confirmation as the development proceeds. For a clearer view, it is worth setting out the frequencies on their own:

					d		
		0	1	2	3	4	5
	1	23.0	25.5	26.7	27.1	27.3	27.4
	2	24.6	27.4	28.5	29.1	29.2	
a	3	26.1	28.9	30.2	30.7		
	4	27.6	30.6	32.0			
	5	27.5	30.5			**[*Cq*]**	
	6	27.1					

A further useful item that can be discovered from this analysis (given that the exposure data are available) is the premium paid per unit exposure. Repeating the premium data from §G2 yields the figures:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
aX	20.59	19.82	19.21	18.94	18.44	18.03
aP/aX	412.9	377.5	343.1	299.9	272.5	248.8
Pj	1.0938	1.1003	1.1440	1.1006	1.0953	

The premium per unit exposure aP/aX has been increasing steadily during the period in question, and its inflation factors Pj are given in the bottom row of the table. (1.0938 is 412.9÷377.5, and so on.) Premium and claim inflation will not be coincident, of course, but a knowledge of their relationship is very important in the overall control of the business.

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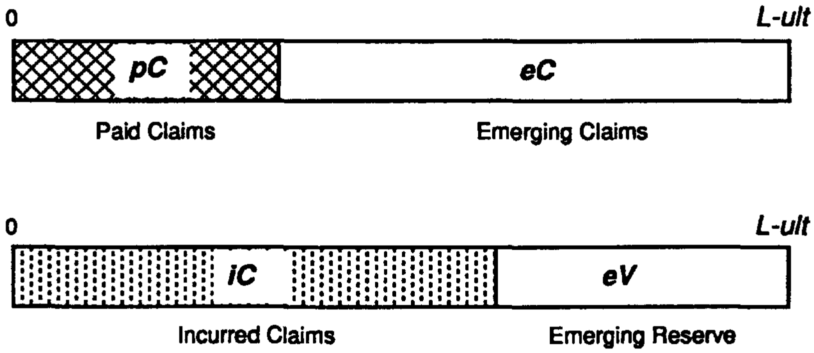
[H5]
CORRESPONDENCE OF CLAIM NUMBERS & CLAIM AMOUNTS

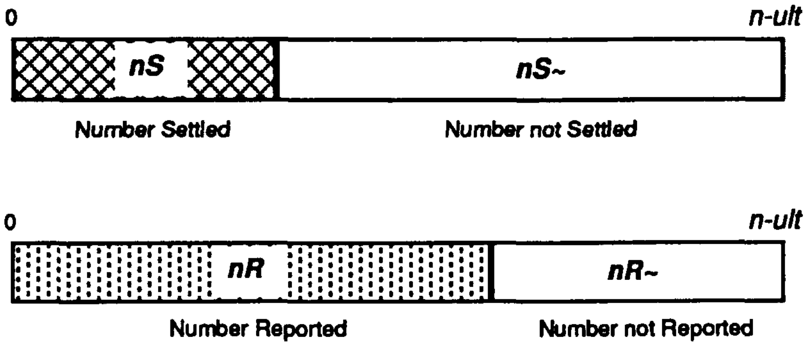
The work of this section of the Manual brings out a basic correspondence in the reserving data on claim amounts and claim numbers. The correspondence is a simple one, between paid claims and numbers settled on the one hand, and incurred claims and numbers reported on the other. It has its uses in establishing a theoretical framework for claims reserving, as will be shown more fully in §M. But its simplicity tends to disguise some very real difficulties which should not be neglected by the reserver, and one or two will be brought out in this section.

First, though, some diagrams to characterize the correspondence may be helpful. These depend on the idea of *completion.* If we are thinking about paid claims then the question is, what further claims are expected to emerge before the ultimate loss is reached? If we are on numbers reported the point is, how many further claims will come in before arriving at the final position? Diagrams can soon be drawn to reflect this idea, and appear as follows (cf those already given in §G3):

Claim Amounts



Claim Numbers



The diagrams are useful conceptually, and help to show the relationships of the various quantities. They lead naturally to the paid and incurred average claim definitions as used earlier in this section of the Manual.

Paid Average	=	Paid Claims/Number Settled
Incurred Average	=	Incurred Claims/Number Reported

But the diagrams conceal the fact that the definitions of the claim amounts and claim numbers are not always properly reconciled. The groups of claims concerned can be subtly different, thus leading to possible bias in the projections. The main point is to know exactly what definitions apply to the data in hand, so that any necessary adjustments can be made. Particular examples of this relate to partially paid claims and claims settled at zero, which can affect the paid average as outlined below.

Partially Paid Claims

Considering the paid claim data, the main element is the full payments on settled claims. If that were all, then numerator and denominator in the paid average would be in harmony with each other. But to the full payments will be added any partial payments made on claims still open at the reserving date. (Such payments arise typically where liability is admitted by the insurer, but where a long period is needed for the liability to be fully assessed.) Hence a discrepancy arises in the calculation of the paid average, which needs to be tackled. Three solutions seem possible.

a) If the partial payments are a small proportion of the whole, the distortion introduced by using pC/nS for pA can perhaps be ignored.

b) Provided the data are available, the paid amounts on settled claims alone can be used in the numerator. Alternatively, the partial payments can be subtracted from payments as a whole. The average is then $pA = pS/nS$.

c) If the general proportion of claim, say k, which is redeemed by a partial payment can be estimated, then the denominator can be adjusted. The addition to nS needed is just k times the number of partially paid claims.

Claims Settled at Zero

This is another point which relates to the paid average. Of the claims settled, a number will be at zero, perhaps through being disproved, or retracted by the insured. The question arises as to whether such claims should be included in the number settled — clearly they will make no difference to the paid amount itself. Often there will be no problem, and the number settled can be taken with or without the zero claims, as most convenient. The choice may, of course, be forced by the availability of the data. But where a choice exists, the main point is to ensure that a consistent definition is used throughout the working.

The problem arises where the proportion of claims settled at zero changes, perhaps as a result of a revised claims handling policy by the insurer, or an attempt to reduce the backlog of waiting claims. The general effect on projections of such changes is not easy to assess, however. It may be best to work with both bases, and assess the two results for their relative dependability.