Section M
TOWARDS A FORMALISED APPROACH

Preamble

To this point, the Manual has not gone into a full statistical treatment of claims reserving, and mathematical models have not been explicitly used. But it will be apparent that the subject is a complex one, and that many different concepts and quantities have to be manipulated. The discussion, however, has been on a rather ad hoc basis, without any fuller systematisation. Yet such a systematisation would have its uses, and would help to tie together the many strands that make up general insurance reserving. In particular, it could help to show the relationships which the many methods bear to each other, and so assist in the choice of an appropriate method in particular circumstances.

For these reasons, the present section attempts a more formal approach to the subject, and puts forward some of the elements necessary for a systematised view.

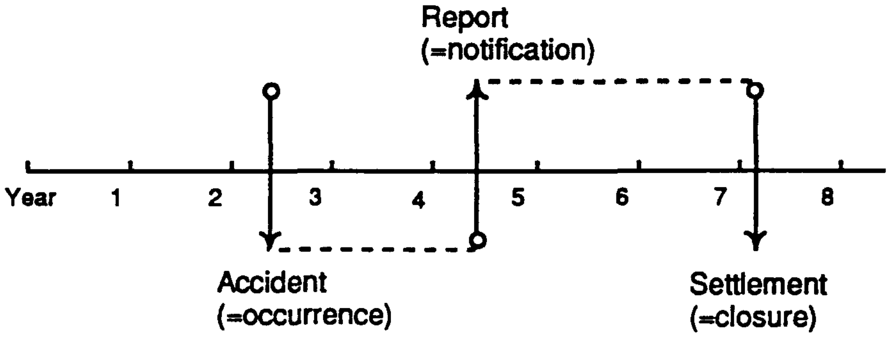
There is a main foundation to the development. It is simply the basic analysis of all claims into three main states: settled, open and IBNR. As time progresses, of course, claims move between these states. But at any accounting or review point, the division can be made. It is of basic relevance to the reserving process — although it is not always possible to make the full division, particularly in reinsurance work. In such cases, substitute measures may be used, whose standing can be assessed through the basic analysis put forward here.

The section also has the function of systematising the notation which is used in Volume 1 of the Manual. The notation is not standard, and could not be, since there is no generally accepted standard notation in General Insurance. But there is a need to be able to express the quantities and ideas which come into reserving in a compact and precise way. The notation should therefore be seen as an attempt towards producing an acceptable algebra for claims reserving.

Contents

[M1]
THE HISTORY OF A CLAIM

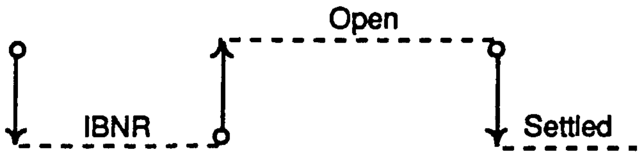
We begin by considering the simple history of an individual claim. This can be shown diagrammatically along a time axis:



In the example shown, the accident or occurrence giving rise to the claim occurs in year 3. The claim is then reported to the insurer in year 5, and settled in year 8. (The alternative terms "notification" and "closure" may also be used for "reporting" and "settlement".) Although the scale is given in years, months or quarters or some other unit could equally be used.

Using a to denote the accident year, r for report year and s for settlement year, the relation: $a \leq r \leq s$ can be seen to hold.

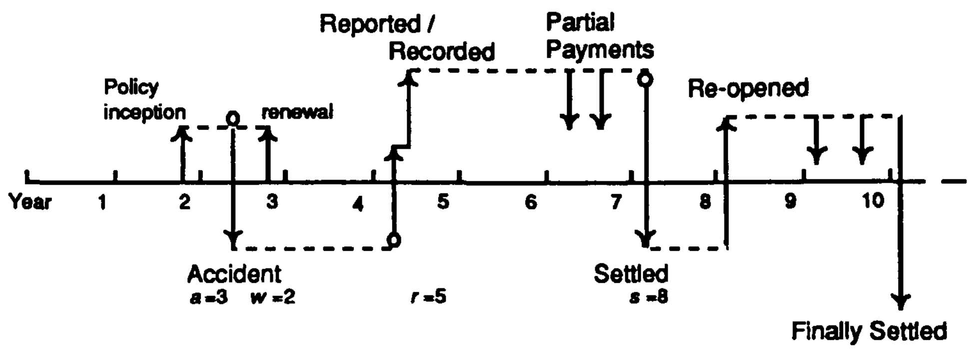
Over the time period in question, the *state* of the claim changes as follows:



Between the accident date and reporting dates, the claim is said to be "incurred but not reported", or IBNR for short. Once reported, the claim becomes an open one in the insurer's records, until it is fully settled with the claimant. In the settled state, the claim file will be kept for some further period, until finally archived or destroyed.

Detailed History

The above simple history gives the three basic states for claims, on which reserving analysis will depend. But there is a fuller story to be told (a good account is given in the first section of Ackman, Green & Young, 1985.) At each stage complications arise, which are shown diagrammatically below:



Accident/Occurrence

Each claim can be classified by underwriting year (i.e. the year in which the risk commences) as an alternative to accident year. If w denotes the underwriting year, then the relation with a) is:

$$w = a \quad \text{or} \quad w = a - 1$$

Reporting/Recording

We can distinguish:

a) Date of report to office by claimant, say r'
b) Date record reaches office's central files, say r''

For reserving purposes, general practice is to use the latter, r''. The term "reported" thus usually means "recorded on file", and is so taken in the Manual.

Settlement

We can distinguish:

a) Date claim is first considered settled, say s'
b) Date claim is finally settled, say s''

The problem here is that a settled claim can be re-opened by the claimant. In the Manual, "settled" normally indicates that at least a first-time settlement has taken place, i.e. it uses s' for the definition.

Thus, many potential complications exist. The important point, for claims reserving purposes, is that the fundamental classification of claims is still a 3-fold one:

{ IBNR Claims | Open Claims | Settled Claims }

Each of the groups will have its own requirement for reserves, since settled claims can be re-opened. According to the method chosen, the reserves can be estimated separately, or en bloc, or by some other combination. (See §§ M4, M5.)

◇

[M2]
THE CLAIMS COHORT

For reserving purposes, claims will first be divided according to class and subdivision of business, and geographical territory. Examples would be:

Motor/Non-comp/UK
Liability/Medical Malpractice/USA
Proportional Reinsurance/Aviation/Europe etc.

(The main classes of business are described briefly in §§ A2,A3 above.)

Within each grouping so obtained, it is often necessary and desirable to divide the claims according to their year of origin. The divisions are then known as cohorts, and for these symbol $\{C\}$ will be used:

$\{C\}$ = Cohort of Claims

The origin chosen for cohorts is commonly accident year a or underwriting year w, but report year r can also be used. The resulting cohorts can be distinguished by use of a suffix:

$\{C_a\}$ = Accident Year Cohort
$\{C_r\}$ = Report Year Cohort
$\{C_w\}$ = Underwriting Year Cohort

If report year is used, then the year of occurrence is disregarded in the classification. However, a 2-dimensional classification of report year within accident year can be used especially in analysing IBNR claims. This multiplies the work of reserving, but adds detail.

In the Manual, accident year has been taken as the norm, but there is reference to the other types as well. Report year cohorts in particular give rise to significantly different methods of reserving.

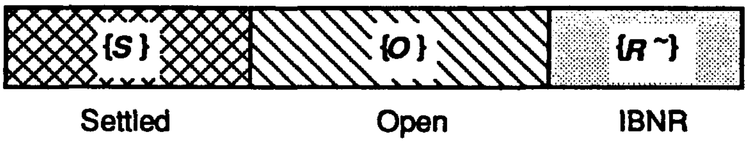
Primary Division of the Cohort

The claims in the cohort develop over time. At any point in time from the end of the year of origin onwards, each claim will be, by the analysis of §M1, either settled, open or IBNR. The claims in the cohort can thus be analysed into three main groups:

$$\{C\} = \{ S \mid O \mid R^{-} \}$$

where: $\{S\}$ denotes the subgroup of settled claims
$\{O\}$ ... ... open claims
$\{R\sim\}$ ... ... IBNR claims

This primary division is not static, and varies according to the moment at which it is made. In most cases, it will be made as at the end of a given accounting period — a year or a quarter. It can also be made at an interim review period, say a quarter or a month. A block diagram can be drawn to represent the analysis:



Development over time of individual claims will, in general, be from $\{R^{\sim}\}$ to $\{O\}$ to $\{S\}$.

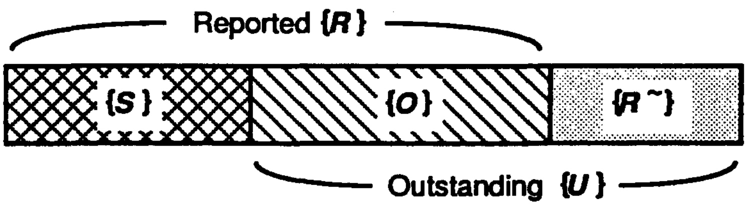
Apart from the primary subgroups of claims, it is also useful to be able to refer to combinations of these:

Reported Claims = Open claims + Settled claims
Outstanding Claims = IBNR claims + Open claims

Symbolically:

$$\{R\} = \{O\} + \{S\}$$
$$\{U\} = \{R^{\sim}\} + \{O\}$$

The full picture in the block diagram becomes:



There is an ambiguity in the term "outstanding claims". As well as being used for (open + IBNR) claims, as here, it is sometimes used as a synonym for open claims per se. This is perhaps a matter of taste. The main point is to be clear as to which set of claims is being referred to at any given time.

The term "IBNR" can also have different definitions. (See §I1.) The usage in this Manual is the most common one, i.e. IBNR claims are those for which liability has been incurred at the reserving date, but which have not *by that time* been reported (i.e. recorded on the insurer's central files).

Claims at Nil Cost

A further division of claims is that of the settled group, $\{S\}$. A claim may be settled at nil cost, i.e. eventually dismissed because it is shown to be valueless, wrongly made, or even fraudulent. Otherwise, it will be settled at some positive cost to the insurer. Symbolically:

$$\{S\} = \{S^{\circ}\} + \{S^{+}\}$$

◇

[M3]
CLAIM NUMBERS & CLAIM AMOUNTS

Given that the cohort $\{C\}$ of claims is defined, we shall be interested in the numbers of claims within the cohort, and the losses that are incurred on their account.

Numbers of Claims

At any review point or accounting date, the numbers of claims in the sets $\{S\}$ and $\{O\}$ can be decided from the office's main files (i.e. leaving aside any technical or data processing problems which may arise in interrogating the database). Symbol n will generally be used for claim numbers:

Number of claims in $\{S\}$	=	nS	"number settled"
Number of claims in $\{O\}$	=	nO	"number open"

In normal circumstances (at least, for a direct writing office), these values can be taken as known. But the number of claims which are IBNR by definition cannot be known and must be estimated:

$$\text{Estimated no. of claims in } \{R^{\sim}\} = \hat{\ } nR^{\sim}$$

If the true number is $nR^{\sim}$, the relation will hold that:

$$nS + nO + nR^{\sim} = nC$$

where nC is the number in the whole cohort $\{C\}$. Ultimately, as time continues, all claims come to fruition, and the exact value of nC becomes known. It can thus be designated as the ultimate number, or *n-ult*.

In addition, there is the subsidiary relation for $\{S\}$:

$$nS = nS^{O} + nS^{+}$$

where nS^{O}, nS^{+} denote the numbers of claims settled without payment and with payment respectively.

Claims Amounts

In reserving, a general aim is to estimate the total claim amounts paid from the various claim cohorts, and hence from all cohorts together. For cohort $\{C\}$, we define the total of claims paid out, in past and future together, as the value *L-ult* (the ultimate loss).

At any accounting or review date, $\{C\}$ can be considered as broken into its three main elements:

$$\{C\} = \{S \mid O \mid R^{\sim}\}$$

Hence the value *L-ult* can equally be analysed into losses on settled, open and IBNR claims. Until the end of the development is reached, none of these values is known with absolute certainty (even settled claims can be re-opened). All may therefore have to be estimated. Symbolically:

Incurred claims on $\{S\}$	=	$^{\wedge}iS$
Incurred claims on $\{O\}$	=	$^{\wedge}iO$
Incurred claims on $\{R^{\sim}\}$	=	$^{\wedge}iR^{\sim}$

Symbol i is used to denote the incurred claim values. The above formulae thus read as the amounts "incurred on settled claims", "incurred on open claims", and "incurred on IBNR claims".

True values for these 3 will yield:

$$iS + iO + iR^{\sim} \quad = \quad L\text{-}ult$$

i.e. the full losses on the cohort. These true values will be exactly known only at the end of the development, at which point $iS = L\text{-}ult$

As a first simple approach to the estimates for settled and open claims, it may be possible to use the following:

pC	=	paid losses to date on claims in $\{C\}$
kC	=	total of case estimates on $\{C\}$ at the reserving date

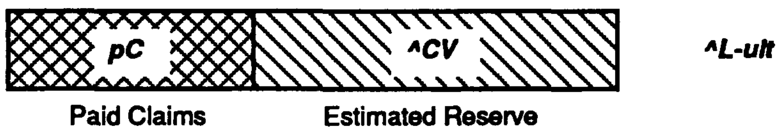
Here, pC will estimate iS, the incurred loss on settled claims, while kC will estimate iO, the loss on open claims. But the estimates will depart from the truth, since:

a) pC does not allow for re-opens on settled claims, but includes part payments on open claims.
b) kC does not allow for future development of the case estimates.

◇

OVERALL LOSS & THE CLAIMS RESERVE

The diagram shows the relationship between the overall loss *L-ult* and the claims reserve itself, *CV*:



Here, the reserve is shown as the estimated one, *^CV*, and the overall loss *^L-ult* is also the estimated value. The true values *CV* and *L-ult* could be substituted in the diagram.

The diagram can represent the overall picture, i.e. for all claims together, or the position for a given cohort $\{C\}$

Algebraically,

$$\hat{}L\text{-}ult = pC + \hat{}CV$$

This equality shows that there can be two distinct approaches to the overall question of the claims reserve:

a) Estimate the overall loss, *L-ult.* Then derive the reserve as:

$$\hat{}CV = \hat{}L\text{-}ult - pC^*$$

where pC^* denotes the paid claims to date.

b) Estimate the required reserve directly, and derive the overall loss from it:

$$\hat{}L\text{-}ult = \hat{}CV + pC^*$$

Either approach can be used, and the choice will often fall out automatically from the particular method used for reserving. It is always good, however, to be clear about the route which is being taken. The choice of routes implies an important distinction between two different approaches to a situation in which the claims paid progress at a higher level than previously anticipated. One approach is to assume that *L-ult* will remain the same and hence that the level of claims paid in the remainder of the cohort will be correspondingly less than expected. The alternative assumption is that the higher level of claims paid implies an increase in *L-ult*.

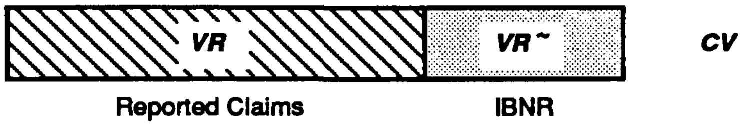
The overall loss/claims reserve identity can be further expanded:

$$
\begin{aligned}
{}^\wedge CV &= {}^\wedge L\text{-}ult - pC^* \\
&= {}^\wedge iS + {}^\wedge iO + {}^\wedge iR^{\sim} - pS^* - pO^*
\end{aligned}
$$

Here, iS, iO, $iR^{\sim}$ are the incurred amounts on the settled, open and IBNR claims. Then pS^* denotes the paid amounts on settled claims, and pO^* the partial payments on the open claims, both to the present date.

[M5]
PRIMARY DIVISION OF THE CLAIMS RESERVE

The primary division of the claims reserve CV is into the reserve for reported claims, and the IBNR reserve:



Algebraically,

$$CV = VR + \mathrm{VR}^{\sim}$$

The equation can apply to claims as a whole, or to particular cohorts of claims. Further to the work of §M4, this analysis again shows that the reserving problem can be tackled from different perspectives:

a) Estimate the overall reserve and the IBNR separately, then find the reported claims as:

$$^\wedge VR = {}^\wedge CV - {}^\wedge ibV$$

b) Estimate the overall reserve and the reported claims separately, then find the IBNR reserve as:

$$^\wedge ibV = {}^\wedge CV - {}^\wedge VR$$

c) Estimate reported claims and IBNR separately, then find the overall reserve as:

$$^\wedge CV = {}^\wedge VR + {}^\wedge ibV$$

Note: In the above, the symbol ibV is used synonymously with $VR^{\sim}$ to denote the IBNR reserve.

The Accident Year vs. Report Year Comparison

As an example, consider the difference when using report year data as against accident year data.

Accident Year

1) A projection of the paid claims to the ultimate value *L-ult* will give an estimate for the overall loss (i.e. for all claims in the cohort).
2) The deduction of pC^* (paid claims to date) will give the required overall estimate for the claims reserve *CV*.
3) Further deduction of $\hat{}ibV$ (the estimated IBNR) for the cohort will then enable the reserve *VR* to be found.

Report Year

1. Projection of paid claims will give *L-ult* for the cohort. By definition, only claims already reported are members of $\{C_r\}$.
2. Deduction of pC^* will give the estimate of the reserve for reported claims, namely $\hat{}VR$.
3. There is no group of IBNR claims which can be identified with the specific cohort $\{C_r\}$. But there will be IBNR claims for the class of business as a whole. These claims can be estimated separately, after which the overall reserve for the class follows as $\hat{}VR + \hat{}ibV$.

Again, as with the relationship between overall loss and the claims reserve, it is useful to be clear which general approach is being used. Reserving methods will naturally proceed by one route or another to build up the full picture.

IBNR & IBNER

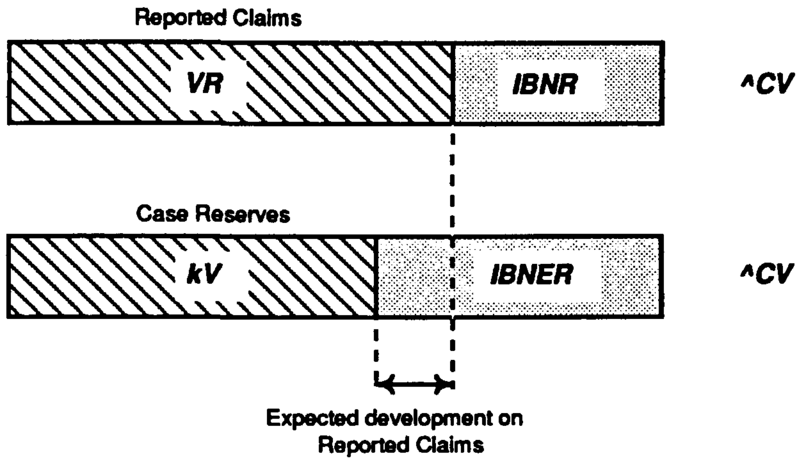
There is a further way of dividing the estimate of the overall claims reserve. This leads to the quantity IBNER, standing for: "Incurred but Not *Enough* Reserved". The term can be confusing, since the initials are so similar to IBNR itself. One derivation of IBNER, as used in the Manual is as follows. In dividing the overall claims reserve, case reserves can be used to estimate reported claims, and then the IBNR reserve found by deduction, i.e.

$$\hat{}VR = kV$$
$$\hat{}ibV = \hat{}CV - kV$$

Strictly speaking, this is satisfactory if case reserves can be shown to be a reliable estimate for reported claims. But usually further development on reported claims would be expected beyond the case value. To recognise this, the IBNR reserve found in the above way is given the name IBNER instead.

Thus:

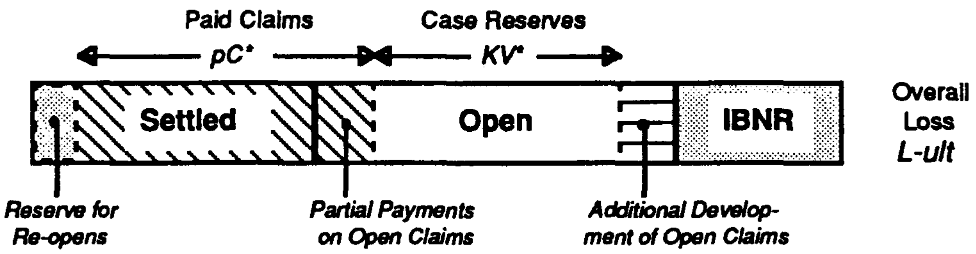
IBNER = IBNR + Expected development on reported claims beyond the current value of case reserves



<>

[M6]
THE FULL ANALYSIS OF LOSS

At this point, we summarise the relationships between all the main quantities which go to make up the overall loss. One new element to be introduced here is the reserve for re-opened claims (i.e. possible re-opens of claims which have been settled already). A diagram may be helpful:



To summarise the analysis:

$$L\text{-}ult = iS + iO + iR^{\sim}$$

or, in words, the overall loss is the sum of incurred amounts on the settled, open and IBNR claims.

At the further level of detail, this leads to:

$$\begin{aligned} iS &= pS + VS \\ iO &= pO + kV + \Delta O \\ iR^{\sim} &= iR^{\sim} \end{aligned}$$

or, in words:

Incurred amount on settled claims	= Amounts paid on such claims + Reserve for possible re-opens (*VS*)
Incurred amount on open claims	= Partial payments on such claims + Case reserves + Additional development
Incurred amount on IBNR claims	= (No further breakdown)

(The symbol ΔO is used here to denote the additional development on open claims, i.e. beyond the current value of the case estimates.)

Alternative Breakdown

The full analysis in its above form is not often used in claims reserving. It is more common to concentrate on paid claims, incurred claims and the remainder (i.e. IBNER). These quantities can, however, be related to the full analysis:

$$\begin{aligned} pC &= pS + pO \\ iC &= pC + kV \\ IBNER &= iR^{\sim} + \Delta O + VS \end{aligned}$$

or, in words:

Paid claims = Amounts paid on settled claims + Partial payments on open claims
Incurred claims = Paid claims + Current value of case reserves
IBNER = IBNR + Additional development on open claims + Reserve for re-opens

A point to bear in mind through the above analysis is that we are considering the claims quantities as they refer to a given year of origin (accident, underwriting or report year). Where an accounting year is in question in the sense used by companies (as opposed to a Lloyd's "year of account") the analysis does not apply in the same way. For example, the incurred claims on the year are equal to paid claims plus the *increase* in total case reserves over the year.

(Refer to §F2 and §I1 for further discussion of the incurred claims function and the variations within IBNR.)

◇

[M7]
AVERAGE COST PER CLAIM

Data may not always be available on the number of claims, particularly in reinsurance work. But if they are, values for average cost per claim can be developed. These add another dimension to claims estimating, i.e. beyond the use of claim amounts alone, and can improve the picture which is obtained a great deal.

In general, at the end of the development, the average cost per claim will be:

$$A\text{-}ult = L\text{-}ult \,/\, n\text{-}ult$$

This might apply to a whole class of business, or to a cohort. But average claims can also be defined for the subgroups within the cohort. Thus:

Settled claims:	$AS = iS/nS$
Open claims:	$AO = iO/nO$
IBNR claims:	$AR^{\sim} = iR^{\sim}/nR^{\sim}$

The above averages should be considered as holding for the subgroups of the cohort as at a particular review date. The question of development in time will be considered later. (See §§M9,M10.)

If average claims can be estimated in any of the three subgroups of claims, then losses as a whole for that group can be estimated:

Settled	$\hat{}iS = nS \times \hat{}AS$
Open	$\hat{}iO = nO \times \hat{}AO$
IBNR	$\hat{}iR^{\sim} = \hat{}nR^{\sim} \times AR^{\sim}$

(Note that nS, nO should be known, but $nR^{\sim}$ must be estimated.)

Reported and Outstanding Averages

Averages can also be developed for reported claims as a whole, and for outstanding claims:

$$\text{Reported claims} \quad AR = iR \,/\, nR = (iS + iO) \,/\, (nS + nO)$$

$$\text{Outstanding claims} \quad AU = iU \,/\, nU = (iO + iR^{\sim}) \,/\, (nO + nR^{\sim})$$

Thus, if means can be found for estimating AR or AU, their values can be used towards the loss estimates ^iR and ^iU.

Paid and Incurred Averages

Two important averages are the paid and incurred averages. These will be defined as follows:

Paid claims	pA	=	pC / nS
Incurred claims	iA	=	iC / nR

These are the most practical measures, for which data will frequently be available. The problem is that they are not pure measures. The denominators show their relationship to the settled and reported groups of claims respectively. Pure measures would therefore be obtained (by reference to diagram in §M6)
as:

Paid claims	pA	=	$(pS + VS) / nS$
Incurred claims	iA	=	$(iC + \Delta O + VS) / nR$

In practice, the reserve for re-opens (VS) will often be ignored, or taken as part of the IBNR liability. The development on open claims (ΔO) will be included in the incurred claims, if adjusted case reserves are used. Alternatively, it may be left to come out in the IBNR term. Perhaps the more serious objection is the use of the ratio pC/nS in the first paid average definition above. Since:

$$pC = pS + pO$$

the partial payments on open claims (pO) will distort the average. Where these partial payments are small, however, the definition will serve as a reasonable approximation to the true average.

Caveat

As seen above, many different "average claims" can be defined and used in reserving. Indeed, in the literature, quite different methods for reserving can occur under the generic title of average claim method. Hence when speaking of the "average cost per claim" (or "average claim" for short), it is important to be clear to which set of claims precisely the average refers.

◇

[M8]
EXPOSURE MEASURES & LOSS RATIO

Referring to the claims cohort, there is a need for some base measure of it, or rather of the business from which it results. Actuaries customarily employ the expression "exposed-to-risk" to describe the base measure to which claims are related. The most obvious measure is the premium income. Others would be the total of sums assured on the written business, or the number of policy units.

The term "policy unit" may need further definition. It means a policy in force for a year, whether the policy year itself, the accident year or some other 12-month period. Some policies contributing to the claims cohort will have exposure only for a part year, and therefore be counted as a part unit only. The policy unit can also be replaced more graphically by a risk exposure such as the number of motor cars, or dwellings, or individuals covered under the business in question.

Whatever the choice, it will give the base measure X for the cohort. If, say, X is premium income (special symbol P) this will relate to the cohort definition itself:

$\{C_a\}$	Accident year cohort	X_a will be earned premium, aP
$\{C_w\}$	Underwriting year cohort	X_w will be written premium, wP
$\{C_r\}$	Report year cohort	No definition available

The use of report year data has the disadvantage of not allowing any meaningful exposure measure to be developed.

Loss Ratio

If the ultimate losses for a class of business, or a cohort, are known, then its loss ratio can be calculated:

$$\lambda = L\text{-}ult \,/\, P$$

If it can be shown that λ is stable for a given class of business over the years, then it can be used for estimating losses on that class for the current set of claims:

$$\hat{}L\text{-}ult = \lambda \times P$$

where λ is the loss ratio for the given set of claims.

If the loss ratio is not fully stable, but thought to fluctuate about a mean value with a moderate variance (or to be subject to an identifiable trend), then the product of the mean with the premium income can still be used as a first estimate of losses. This will give a target value, from which deviations can be checked as the years of development pass by. ◇

[M9]
TIME AXES

The analysis so far has been relatively static i.e. as at a particular review date. The next step is to bring in the time dimension explicitly.

For a given cohort, there is a fixed reference year — whether accident, underwriting or report year. The chief time axis will then be the development time or period which has passed since the reference year itself. This is to be measured as:

$$d = 0, 1, 2, 3, \ldots\ldots \textit{ult}$$

Here, $d = 1, 2, 3 \ldots$ will be taken to count the development years *subsequent* to the reference year. $d = 0$ then conveniently denotes the reference year itself. $d = u$ or *ult* gives the ultimate development period, i.e. that time after which it is known (or can be taken for practical purposes) that no further development of the cohort of claims will occur. True values for *L-ult, n-ult* and *A-ult* can be established at this time, but not with absolute certainty beforehand.

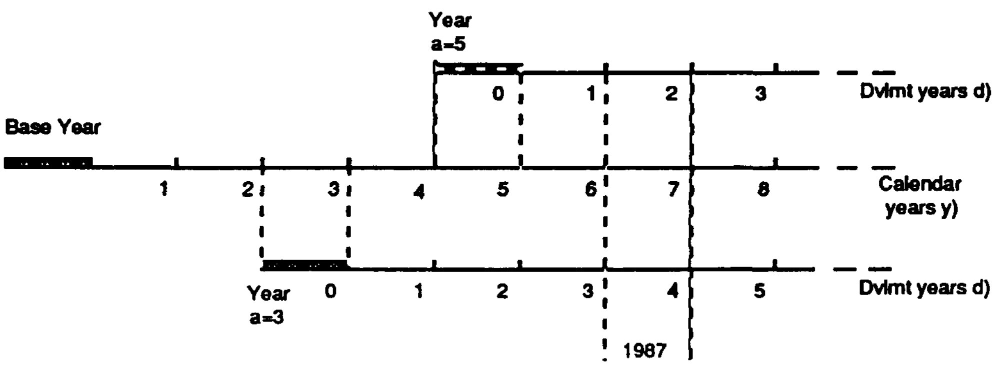
The whole period from inception of the reference year to end of the ultimate year has length $(u + 1)$.

Other Time Axes

Apart from development time, two other time axes are relevant:

a) Accident year (or other origin year for cohort)
b) Calendar year

Using the symbols a and y respectively, and measuring from a base year, the diagram shows the relationship:



In all cases:

$$a + d = y$$

or:	Accident Year	+	Development Year	=	Calendar Year
e.g.	$a = 5$		$d = 2$		$y = 7$
	$a = 3$		$d = 4$		$y = 7$

This is a useful relationship. It is gained by using the convention of starting the development year count at $d = 0$ rather than $d = 1$.

◇

[M10]
DEVELOPMENT OF CLAIMS

We now introduce a fundamental idea in the work. Very many of the claim reserving methods are based upon it. It is that:

> "There exists some recognisable pattern to claim amounts, claim numbers and/or average claims as development time progresses."

If the idea has some truth in it, then it will be worth examining the quantities C, *n*, *A*, considered essentially as functions of *d*.

$$C = C(d)n = n(d)A = A(d)$$

The pattern which is found (or perhaps disallowed) will depend on which subgroup of the cohort $\{C\}$ is examined, i.e.:

Settled claims $\{S\}$ Open claims $\{O\}$ Ibnr claims $\{R^{\sim}\}$
Reported claims $\{R\}$ Outstanding claims $\{U\}$

However, the actual functions chosen for analysis must depend on what data are to hand for reserving purposes.

Paid Claims Development

The most straightforward development to consider is that of paid claims:

$$pC = pC_a(d)$$

Paid claims is just the amount actually paid out to date on the claims overall in the cohort $\{C_a\}$. In effect, it is:

$$pC_a(d) = pS_a(d) + pO_a(d)$$

i.e. the sum of the amounts paid out on settled claims, plus the partial payments on open claims. There could be a case for using $pS_a(d)$ alone, but the extra effort involved may be more trouble than it is worth, especially if $pO_a(d)$ is relatively small in the sum. In any case, it is likely that the amount $pC_a(d)$ will relate mainly to the group $\{S\}$.

Incurred Claims Development

Again, this is a commonly used function. Different definitions are possible, but the usual one is:

$$iC_a(d) = pC_a(d) + kV_a(d)$$

Incurred Claims = Paid Claims + Case Reserves

Here, $kV_a(d)$ is used to signify the total of the individual case reserves at time d.

Incurred claims defined in this way gives a first estimate for the losses on the reported claims $\{R\}$, i.e. groups $\{S\} + \{O\}$. But it omits allowance for any development on the reported claims, and also for any losses to follow on the IBNR claims. (But in some definitions, amounts for reported claim development and/or IBNR can be brought into the iC function.)

A final and important point is that IBNR claims which are late reported will *at that later time* come into the iC value.

Both paid claims and incurred claims are functions that will develop over time. They have the useful property that both must home in to the ultimate loss for the cohort concerned:

$$\lim_{d \text{ becomes large}} pC_a(d) = \lim_{d \text{ becomes large}} iC_a(d) = L_a\text{-}ult$$

or

$$pC_a(u) = iC_a(u) = L_a\text{-}ult$$

For the report year cohort, $pC_r(d)$ and $iC_r(d)$ will also develop, although IBNR claims which are later reported will no longer enter as an element in iC. The development of iC_r will, in fact, directly show the correction that has to be made over time in the case reserves themselves.

Claim Number Developments

The available claim numbers are (or may be) those on $\{S\}$ and $\{O\}$, the settled and open claims.

$$nS = nS_a(d)$$
$$nO = nO_a(d)$$

Also:

$$nR = nS_a(d) + nO_a(d)$$

Again, as development time increases, the n's home in to a limit:

$$nS_a(u) = nR_a(u) = n_a\text{-}ult$$

and

$$nO_a(u) = 0$$

But for report year cohorts, nR_r is fixed. In fact:

$$nR_r \;=\; n_r\text{-}ult \qquad \text{(by definition)}$$

Average Claim Developments

The idea of development extends naturally to the various types of average cost per claim function. Such functions can be defined for settled, reported or outstanding claims:

Settled claims	$AS_a(d)$	=	$iS_a(d) \,/\, nS_a(d)$
Reported claims	$AR_a(d)$	=	$iR_a(d) \,/\, nR_a(d)$
Outstanding claims	$AU_a(d)$	=	$iU_a(d) \,/\, nU_a(d)$

The settled average might be estimated as: $pC_a(d) \,/\, nS_a(d)$

and the reported average as: $iC_a(d) \,/\, nR_a(d)$

But there is no easy function available for the outstanding average.

Per Claim Payment Patterns

A significant group of methods uses the pattern of payments per claim incurred. The Bennett & Taylor method of §J3 is a good example. In this case, the developing payment values by accident year are divided through by the overall number of claims, or at least by an estimate of this number:

a) $pC_a(d) \,/\, \hat{}n_a\text{-}ult$
b) $pC_a(d) \,/\, \hat{}n_a\text{-}ult(0)$
c) $pC_a(d) \,/\, nR_a(0)$

In a), division is by the current best estimate of *n-ult.* In b), the estimate of *n-ult* is that made at the end of the accident year in question, and not later modified. In c), the number used is the number of claims actually reported in the accident year.

◇

[M11]
THE TRIANGULAR ARRAY

The basic analysis of claim cohorts by accident year (or other year of origin) and by development period leads naturally to a triangular format. The format is virtually identical whatever function is being considered:

		d					
		0	1	2	3	...	*ult*
	1	$el_1(0)$	$el_1(1)$	$el_1(2)$	$el_1(3)$	...	$el_1(u)$
	2	$el_2(0)$	$el_2(1)$	$el_2(2)$	$el_2(3)$		
a	3	$el_3(0)$	$el_3(1)$	$el_3(2)$	$el_3(3)$		
	4	$el_4(0)$	$el_4(1)$	$el_4(2)$		**[*fn*]**	
	...	...	...				
	c	$el_c(0)$					

Years of origin, say accident years *a*, are measured as from some base year, up to and including the most recent (or current) year *c*. Development years *d* are then measured from accident year as base, with the accident year itself appearing as d=0.

Conventionally, the rows of the triangle record the progress for given accident years, while the columns give the state of play for specific development periods. As a result, accident years are listed down the left hand side of the triangle, and development years across the top. Report or underwriting years may replace accident years to name the rows of the triangle.

The elements $el_a(d)$ in the triangle may be any one of a number of claims functions. Common examples are paid claims, incurred claims, number reported, paid average, incurred loss ratio, and so on.

The triangle as drawn above has equal breadth and depth, so that $c = u+1$. But equality of breadth and depth is not essential. In claims reserving many other shapes can be used. Some are shown in M11.3.

In any case, the development may not reach the *ult* value, even in the top row, but may fall short by several periods' length. This final part of the development is called the tail. If it has length l, then the last data value will fall in the cell (a=1, $d= u - l$).

Further points are that the time periods can be other than years, e.g. quarters or even months can be used. Also, there is no absolute need to employ the same time interval on the horizontal and vertical axes. E.g. the accident rows could be by years, and the development columns by quarters. But such triangles are rarely seen.

The Diagonals

In the basic triangle, the diagonals have the useful property of representing the calendar years:

				d			
		0	1	2	3	...	*ult*
	1	$el_1(0)$	$el_1(1)$	$el_1(2)$	$\boldsymbol{el_1(3)}$	...	$el_1(u)$
	2	$el_2(0)$	$el_2(1)$	$\boldsymbol{el_2(2)}$	$el_2(3)$		
a	3	$el_3(0)$	$\boldsymbol{el_3(1)}$	$el_3(2)$	$el_3(3)$		
	4	$\boldsymbol{el_4(0)}$	$el_4(1)$	$el_4(2)$		**[*fn*]**	
	...	...	...				
	c	$el_c(0)$					

The elements in boldface are those relating to calendar year 4. Thus:

$el_4(0)$	is element for	a=4 + d=0	sum = 4
$el_3(1)$	... .	a=3 + d=1	... ...
$el_2(2)$	... .	a=2 + d=2	... ...
$el_1(3)$	... .	a=1 + d=3	... ...

In each case, the sum of accident and development years is 4, i.e. the calendar year value. Thus the general element in the triangle, $el_a(d)$, will in all cases be the value occurring for the calendar year y, where:

$$y = a + d$$

In fact, the element *could* equally be indexed as $el_y(d)$, when a would be deduced as: $a = y - d$. But this convention does not seem to be used, in general.

Suffix Notation

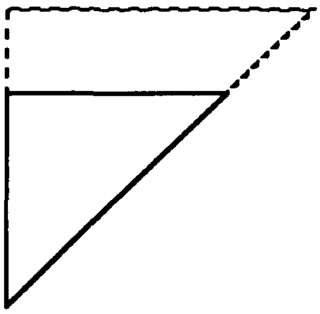
Having introduced the triangle, the general element $el_a(d)$ could equally be written in the double suffix form el_{ad}. This form is frequently used in the literature, though usually as el_{ij}, with i, j in place of a, d. The notation is a derivation from mathematical matrix theory, where i is commonly used as row index, and j as column index. The reason for the different approach of the Manual is that the form $el_a(d)$ emphasises strongly the inherent difference between the accident year classification *a)* and the development period progression *d*).

In General Insurance reserving, we are not dealing with matrices whose rows and columns are, for practical purposes, interchangeable. The mathematical notation tends to suggest a symmetry which is not in reality to be found. Quite apart from this, it is difficult to remember which of i, j refers to accident year or development period. There is no such problem when a, d are used instead.

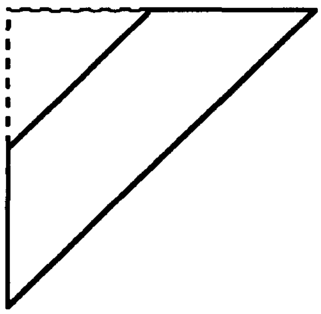
Variations in the Triangle Shape

The following variations can be met with in practice. The precise form will depend on the needs of the analysis, but more upon the data which is available.

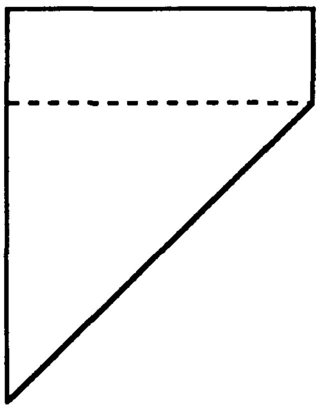
Reduced by Omission of Accident Years



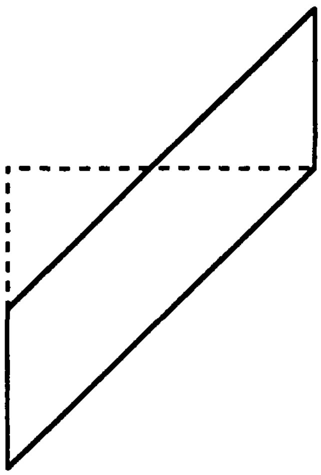
Reduced by Omission of Calendar Years



Extended by Earlier Accident Years



Extended by Accident Year & Reduced by Calendar Year



[M12]
CLAIM DEVELOPMENT & TREND ANALYSIS

Once the development triangle has been set up, it is natural to begin looking at the ratio of one period's claim values to those of the preceding or succeeding period. Given that the triangle contains the general function $el_a(d)$ the development factor can be defined as:

$$r_a(d) = el_a(d+1) / el_a(d)$$

The general term "link ratio" is used in the Manual for such factors. The factor is here defined as a *forward* ratio from the period in question. (The backward ratio $el_a(d) / el_a(d-1)$ could also be used.)

Symbol r is used, since only a single step is being taken, i.e. from one period to the next. But it is also useful to define a ratio forward to the ultimate value, el_a-*ult*. This is the "final development" link ratio, symbol f:

$$f_a(d) = el_a\text{-}ult / el_a(d)$$

An example, using paid claims development, is:

$$r_a(d) = pC_a(d+1) / pC_a(d)$$
$$f_a(d) = L_a\text{-}ult / pC_a(d)$$

A grossing-up factor can also be defined, which, when divided into the current value of el, will yield the final value. In fact, g is just the inverse of f:

$$g_a(d) = el_a(d) / el_a\text{-}ult$$

hence: $el_a(d) / g_a(d) = el_a\text{-}ult$

Multiplying up the one-step link ratios leads to the final value:

$$f_a(d) = r_a(d) \times r_a(d+1) \times \ldots \ldots \times r_a(u-1)$$

Also, the relationship holds that:

$$f_a(d) = r_a(d) \times f_a(d+1)$$

In general, in the upper left-hand part of the triangle, i.e. for: $a + d < c$, the link ratios $r_a(d)$ may be found from the data. In the lower right-hand part of the triangle, i.e. for: $a + d \geq c$, the $r_a(d)$ must be estimated.

Once the one-step links have been estimated for the whole lower right triangle, the final ratios can be calculated by multiplication. The estimate for $L_a\text{-}ult$ can then be made:

$$L_a\text{-}ult \quad = f_a(d) \times pC_a(d)$$

where the work runs through the values $a = 1, 2, \ldots c$, and d is chosen as $(c - a)$. That is, the paid claim values along the main diagonal are picked.

Trend Factors

Another variation using the basic triangular data pattern is to look at the *trend* in values from one accident year to another. The comparison with claim development analysis as just described is instructive:

> Claim development looks at ratios along the *rows* of the triangle, e.g. the relative increase of paid claims over time for given accident years.
>
> Trend analysis looks at ratios down the *columns* of the triangle, e.g. the trend in average cost per claim as experienced at given development periods.

Thus trend analysis constructs vertical rather than horizontal factors in the triangle. Normally, it would not be applied to a function such as paid claims direct, because claims arise in each accident year from a different exposure base. The data would first need to be reduced to an average, or a loss ratio, or a payments pattern. Then trending as applied to such quantities can be useful and valid.

Claim Development:	$r_a(d)$	=	$el_a(d+1) \,/\, el_a(d)$
Trend Analysis:	$h_a(d)$	=	$el_{a+1}(d) \,/\, el_a(d)$

◇