Classification Model

According to the paper 'Learning with noisy labels', unbiased estimator of the loss from noisy label can be calculated as follows:

$$l'(w,y) = \frac{\left(1-\rho_{-y}\right)l(w,y) - \rho_y l(w,-y)}{1-\rho_{+1}-\rho_{-1}}$$
$$= \frac{1-\rho_{-y}}{1-\rho_{+1}-\rho_{-1}} l(w,y) + \frac{-\rho_y}{1-\rho_{+1}-\rho_{-1}} l(w,-y)$$

where ρ_y denotes the rate of samples with y=1 and ρ_{-y} denotes the rate of samples with y=0, ρ_{+1} and ρ_{-1} denote the noise rate.

Logistical Regression is used to implement the experiment results in the Chapter 5.1.

Log Loss Function for Logistical Regression can be represented as follows:

$$l(w) = -\frac{1}{m}\sum_{i=1}^{m} y_i \log\left(h_w(x_i)\right) + (1-y_i)\log\left(1-h_w(x_i)\right)$$

where h_w is the sigmoid function: $h_w(x_i) = sigmoid(w^T x_i)$, m is the number of all samples.

To calculate the unbiased estimator of the loss from noisy label, we need to calculate the loss with all y=1 samples $l(w,y)$ and the loss with all y=0 samples $l(w,-y)$

For all the samples with y=1,

$$l(w,y) = -\frac{1}{m'}\sum_{i=1}^{m\prime} \log\left(h_w(x_i)\right)$$

where m' is the number of the samples with y=1.
For all the samples with y=0,

$$l(w,-y) = -\frac{1}{m-m'}\sum_{i=1}^{m-m\prime} \log\left(1-h_w(x_i)\right)$$

So the unbiased estimator of the loss can be represented as:

$$l'(w,y) = \frac{1-\rho_{-y}}{1-\rho_{+1}-\rho_{-1}} l(w,y) + \frac{-\rho_y}{1-\rho_{+1}-\rho_{-1}} l(w,-y)$$
$$= \frac{\rho_{-y}-1}{1-\rho_{+1}-\rho_{-1}}\frac{1}{m'}\sum_{\substack{i=1\\y=1}}^{m\prime} \log\left(h_w(x_i)\right) + \frac{\rho_y}{1-\rho_{+1}-\rho_{-1}}\frac{1}{m-m'}\sum_{\substack{i=1\\y=0}}^{m-m\prime} \log\left(1-h_w(x_i)\right)$$

Then the gradient descent is used to minimize the loss function.
For all the samples with y =1, the gradient can be calculated as follows:

$$\frac{\partial l'}{\partial w} = -\frac{1-\rho_{-y}}{1-\rho_{+1}-\rho_{-1}}\frac{1}{m'}\sum_{i=1}^{m'}\frac{1}{h_w}\frac{\partial h}{\partial w}$$
$$= -\frac{1-\rho_{-y}}{1-\rho_{+1}-\rho_{-1}}\frac{1}{m'}\sum_{i=1}^{m'}(1-h_w(x_i))x_i$$

For all the samples with y=0, the gradient can be calculated as follows:

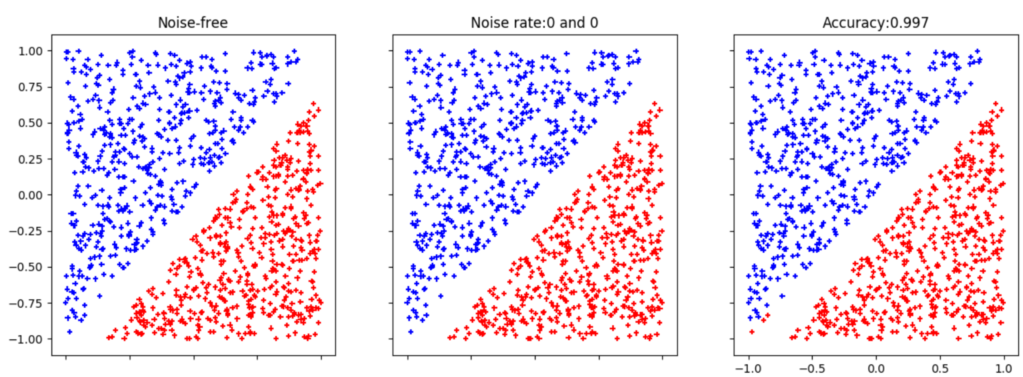
$$\frac{\partial l'}{\partial w} = -\frac{-\rho_{y}}{1-\rho_{+1}-\rho_{-1}}\frac{1}{m-m'}\sum_{i=1}^{m'}\frac{-1}{1-h_w}\frac{\partial h}{\partial w}$$
$$= \frac{\rho_{y}}{1-\rho_{+1}-\rho_{-1}}\frac{1}{m-m'}\sum_{i=1}^{m'}h_w(x_i)x_i$$

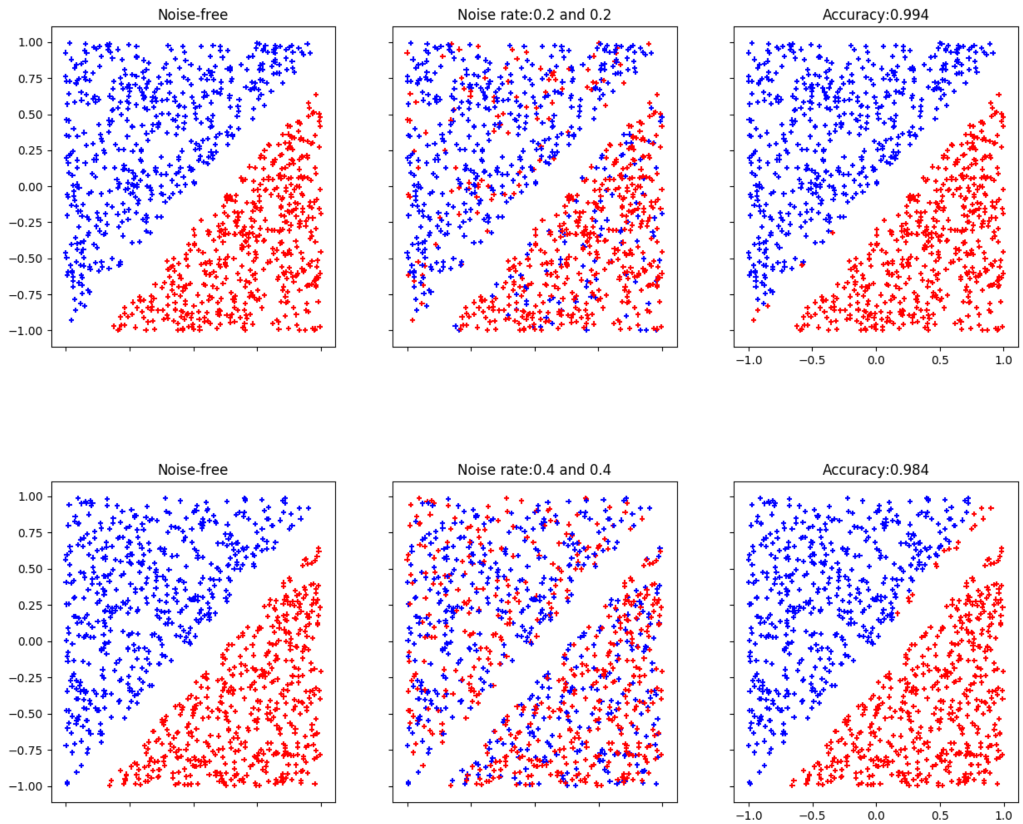
Results

Synthetic 2D linearly separable dataset is shown in figures below.

For the Logistical Regression, the learning rate is 0.1 and the number of iterations is 1000.

Noise rate ρ_{+1}	Noise rate ρ_{-1}	Accuracy
0	0	0.997
0.2	0.2	0.994
0.4	0.4	0.984




Noise-free
Noise rate:0.2 and 0.2
Accuracy:0.994
Noise-free
Noise rate:0.4 and 0.4
Accuracy:0.984
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.0
−0.5
0.0
0.5
1.0