Nonequilibrium phase diagram of an atom-cavity system with three-level atoms

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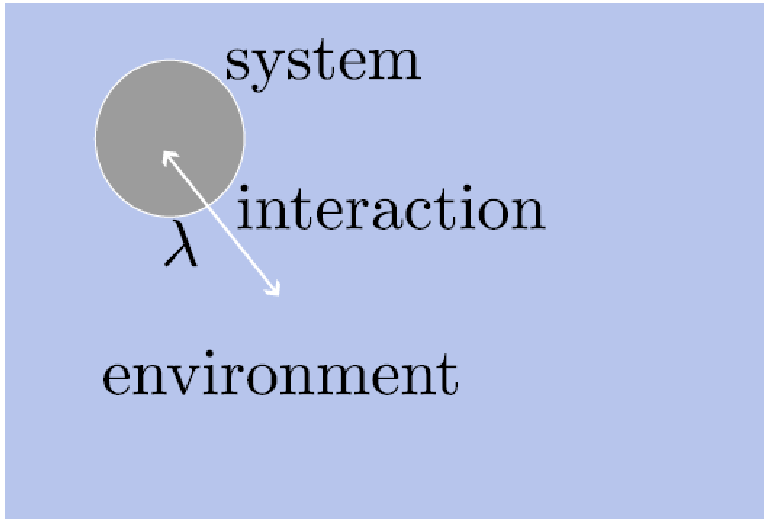
Contents

Theoretical background: Quantum systems

- Open quantum systems interact with their environment.

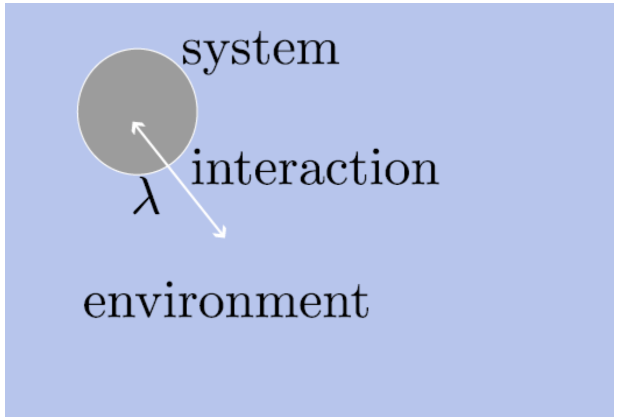


- Their evolution is not purely unitary **[1]**.
- There can be influenced by e.g. driving fields or dissipation.

[1] **H.-P. Breuer and F. Petruccione**, The theory of open quantum systems (2002)

- Quantum systems with described by $|\Psi\rangle$.

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = H|\Psi(t)\rangle$$



With $H = H_{\mathrm{sys}} + H_{\mathrm{env}} + \lambda H_{\mathrm{int}}$.

H_{sys} - represents the „isolated“ system only
H_{env} - is the Hamiltonian of the environment alone
H_{int} - interaction between bath and system
λ - adimensional constant representing interaction strength

Reduced system state is

$$\rho_S(t) = \mathrm{Tr}_{\mathrm{env}}(U\rho_{\mathrm{tot}}(0)U^*).$$

U – time evolution operator

$\rho_{\mathrm{tot}}(0)$ - initial state of the combined system and environment

Challenging or impossible to solve in most cases ! (etwas genauer beschreiben was ich genau meine)

- For ($\lambda \ll 1$) we assume $\rho_{tot}(0) = \rho_S(0) \otimes \rho_{env}(0)$.
- Further we assume Markovian dynamics.

This simplifies the analysis significantly!

$$\frac{d}{dt}\rho_S(t) = \mathcal{L}[\rho_S(t)].\ \mathcal{L}[.]\text{ time independent generator.}$$

- Its action on the density operator is given by

$$\mathcal{L}[\rho_S(t)] = -\frac{i}{\hbar}[H_{sys}, \rho_S] + \sum_k \gamma_k \left(L_k \rho_S L_k^* - \frac{1}{2}(\rho_S L_k^* L_k + L_k^* L_k \rho_S) \right).$$

- The adjoint action is given by

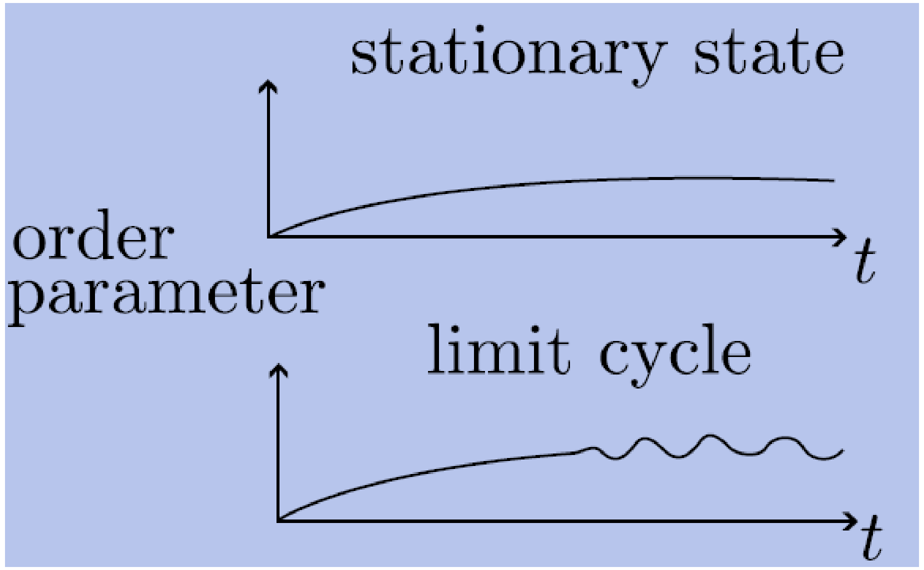
$$\mathcal{L}^*[\mathcal{O}] = \dot{\mathcal{O}} = \frac{i}{\hbar}[H, \mathcal{O}] + \sum_n \gamma_n \left(L_n^* \mathcal{O} L_n - \frac{1}{2}\{L_n^* L_n, \mathcal{O}\} \right).$$

L_k - „jump operators“ ,$\mathcal{O}$ - operator of the system,

γ_k - rates for jump processes.

Theoretical background: Time crystals

- Many systems relax into stationary states.
- Instead of relaxing to a stationary state, some systems exhibit periodic oscillations **[1,2]**.
- They break temporal translation symmetry **[1,2]**.



[1] F. Wilczek, "Quantum time crystals

[2] R. Mattes, I. Lesanovsky, and F. Carollo, "Entangled time-crystal phase in an open quantum light-matter system

Theoretical backgrounds: Dark states

- A dark state is a pure stationary state $\rho_{SS} = |\mathrm{D}\rangle\langle\mathrm{D}|$.

- The state must fulfill the conditions **[1]**:

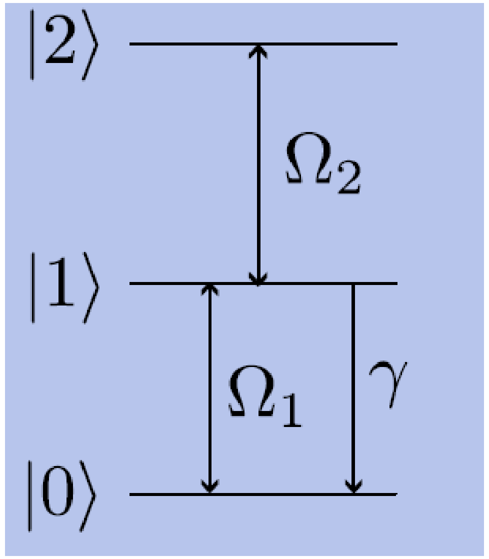
$$H|\mathrm{D}\rangle = E|\mathrm{D}\rangle, \qquad L_k|\mathrm{D}\rangle = 0, \forall k.$$

- A paradigm for such a system would be **[1]**:

$$H = \Omega_2 \left(|1\rangle\langle 2| + |2\rangle\langle 1| \right) + \Omega_1 \left(|0\rangle\langle 1| + |1\rangle\langle 0| \right).$$

- With dark state:

$$|\mathrm{D}\rangle = \frac{1}{\sqrt{\Omega_1^2 + \Omega_2^2}} \left(\Omega_2 |0\rangle - \Omega_1 |2\rangle \right).$$

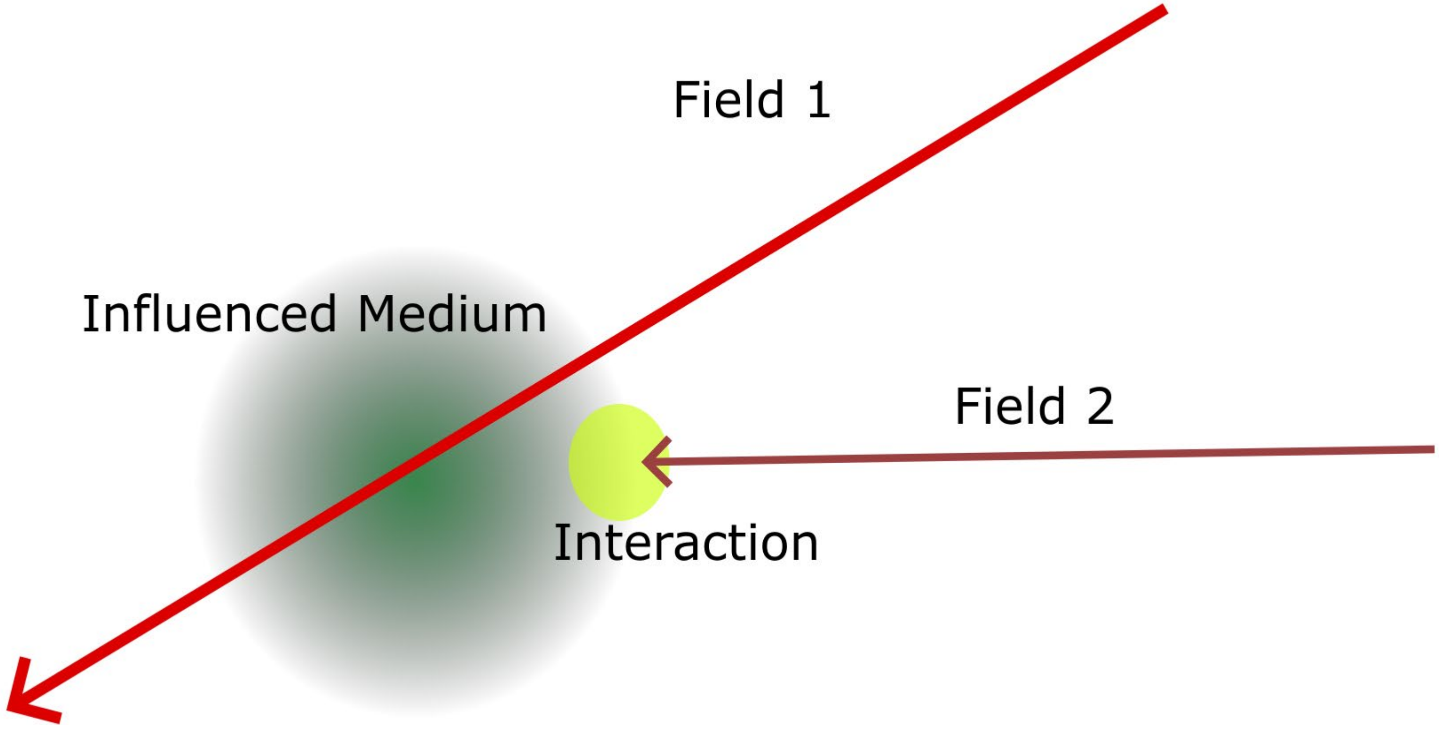


[1] M. O. Scully und M. S. Zubairy, *Quantum Optics*, Cambridge University Press, Cambridge, 1997.

Theoretical backgrounds: EIT (electromagnetically induced transperency)

- Associated with dark states is electromagnetically inducedtransparency (EIT).
- Opaque medium becoming transparent **[1]**.
- A medium needs to be atleast 3-level for EIT **[1]**.

[1] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, "Electromagnetically induced transparency: optics in coherent media



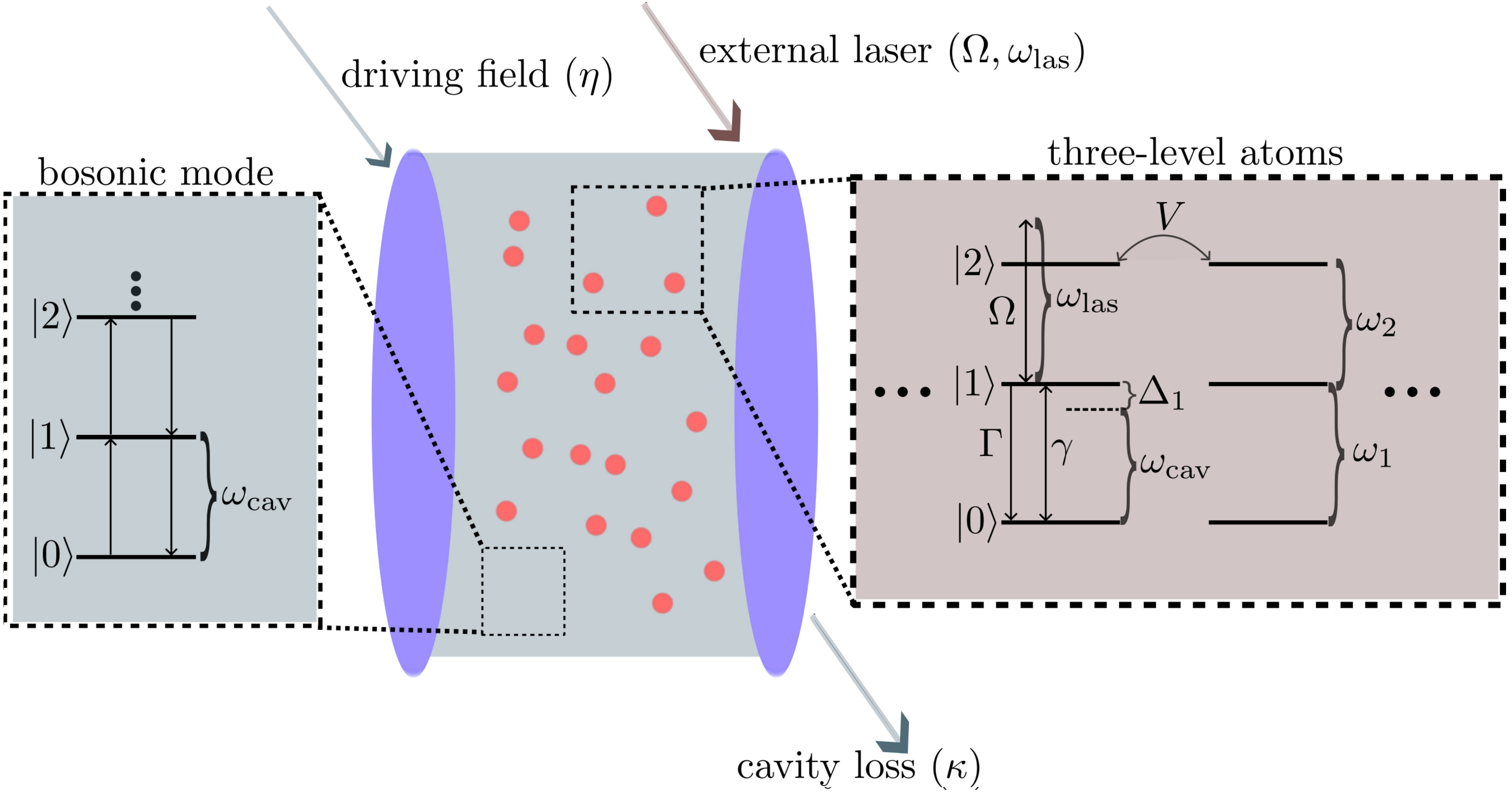
Theoretical backgrounds: EIT

- A stationary state is a dark state if decoupled from a certain interaction (cavity field in our case).
- Transition matrix elements (such as dipole moments) vanish.
- State is immune to certain loss mechanisms e.g. Cavity losses.

Framework

What are we looking at?

- Three-level atoms in optical cavity.



Framework

How do we describe this ?

operators a, a^* to represent the cavity field

and $\sigma_{ab}^{(k)} = |a\rangle^{(k)} \langle b|^{(k)}$ for the atoms.

The Hamiltonian:

$$H = H_{\text{cav}} + H_{\text{at}} + H_{\text{int}} + H_{\text{las}} + H_{\text{at-at}} + H_{\text{pump}},$$

$$= \hbar\omega_{\text{cav}} a^\dagger a + \hbar \sum_k \left(\omega_1 \sigma_{11}^{(k)} + \omega_2 \sigma_{22}^{(k)} \right) + \frac{\hbar \gamma g_0}{\sqrt{N}} \sum_k \left(\sigma_{01}^{(k)} + \sigma_{10}^{(k)} \right) (a + a^\dagger)$$

$$+ \hbar\Omega \sum_k \left(\sigma_{12}^{(k)} + \sigma_{21}^{(k)} \right) \cos(\omega_{\text{las}} t) + \frac{\hbar V}{N} \sum_{k,m} \sigma_{22}^{(k)} \sigma_{22}^{(m)} + i\sqrt{N} \hbar \eta (a^\dagger e^{i\omega_{\text{cav}} t} - a e^{-i\omega_{\text{cav}} t})$$

To simplify, switch to rotating frame.

Hamiltonian time independent after transformation.

We use **[1,2]** $H_{\text{eff}} = ZHZ^* + i\frac{\partial Z}{\partial t}Z^*$, where Z describes the transformation, given by:

$$Z = e^{i\omega_{\text{cav}} a^\dagger a \cdot t} \otimes \prod_k e^{i\omega_{\text{cav}} \sigma_{11}^{(k)} \cdot t} \cdot e^{i(\omega_{\text{cav}} + \omega_{\text{las}})\sigma_{22}^{(k)} \cdot t}.$$

[1] M. O. Scully and M. S. Zubairy, Quantum optics

[2] C. Gerry and P. Knight, Introductory quantum optics

We used a RWA (Rotating wave approximation).
The time independent Hamiltonian in the rotating frame reads

$$H_{\text{eff}} = \hbar \sum_k \left[\left(\Delta_1 \sigma_{11}^{(k)} + \Delta_2 \sigma_{22}^{(k)} \right) + \frac{\gamma g_0}{\sqrt{N}} \left(\sigma_{01}^{(k)} a^* + \sigma_{01}^{(k)} a \right) + \frac{\Omega}{2} \left(\sigma_{12}^{(k)} + \sigma_{21}^{(k)} \right) + i\sqrt{N}\eta(a^* - a) \right] + \hbar \frac{V}{N} \sum_{k,n} \sigma_{22}^{(k)} \sigma_{22}^{(n)}$$

$$\Delta_1 = \omega_1 - \omega_{\text{cav}} \text{ and } \Delta_2 = \omega_2 - \omega_{\text{las}} - \omega_{\text{cav}}.$$

carefull ! Δ_2 is not an actual detuning in the standart sense!

Framework - Equations of motion

- We want to numerically simulate and analyticaly analyze.
- We will use the Heisenberg picture, where we evolve operaotors.
- We do so by applying

$$\mathcal{L}^*[\mathcal{O}] = \dot{\mathcal{O}} = \frac{i}{\hbar}[H, \mathcal{O}] + \sum_n \gamma_n \left(L_n^* \mathcal{O} L_n - \frac{1}{2}\{L_n^* L_n, \mathcal{O}\} \right).$$

- We have two sets of rates and jump operators:

- Dissipation on the atom with rate Γ and operator $L_k = \sigma_{01}^{(k)}$

- cavity dissipation with rate κ and operator $L_a = a$

- Interested in the evolution of collective, macroscopic behaviors.
- Rescaled bosonic operators α^N and average magnetization m_{ij}^N:

$$\alpha^N = \frac{a}{\sqrt{N}}, \qquad m_{ij}^N = \frac{\sum_{k=1}^{N} \sigma_{ij}^{(k)}}{N}.$$

With N being the particle number.

- Experimentally accessible (classical variables in the therm limit).
- No emergent correlations between these operators.
- Mean-field equations are exact in the thermodynamic limit **[1]**.

[1] F. Carollo and I. Lesanovsky „ Exactness of Mean-Field Equations for Open Dicke Models with an Application to Pattern Retrieval Dynamics”

Phase diagrm

- Numerically solve equations.
- We use the time averaged order parameter $\bar{m}_{ij}$ and time variance ς_{ij}, defined by

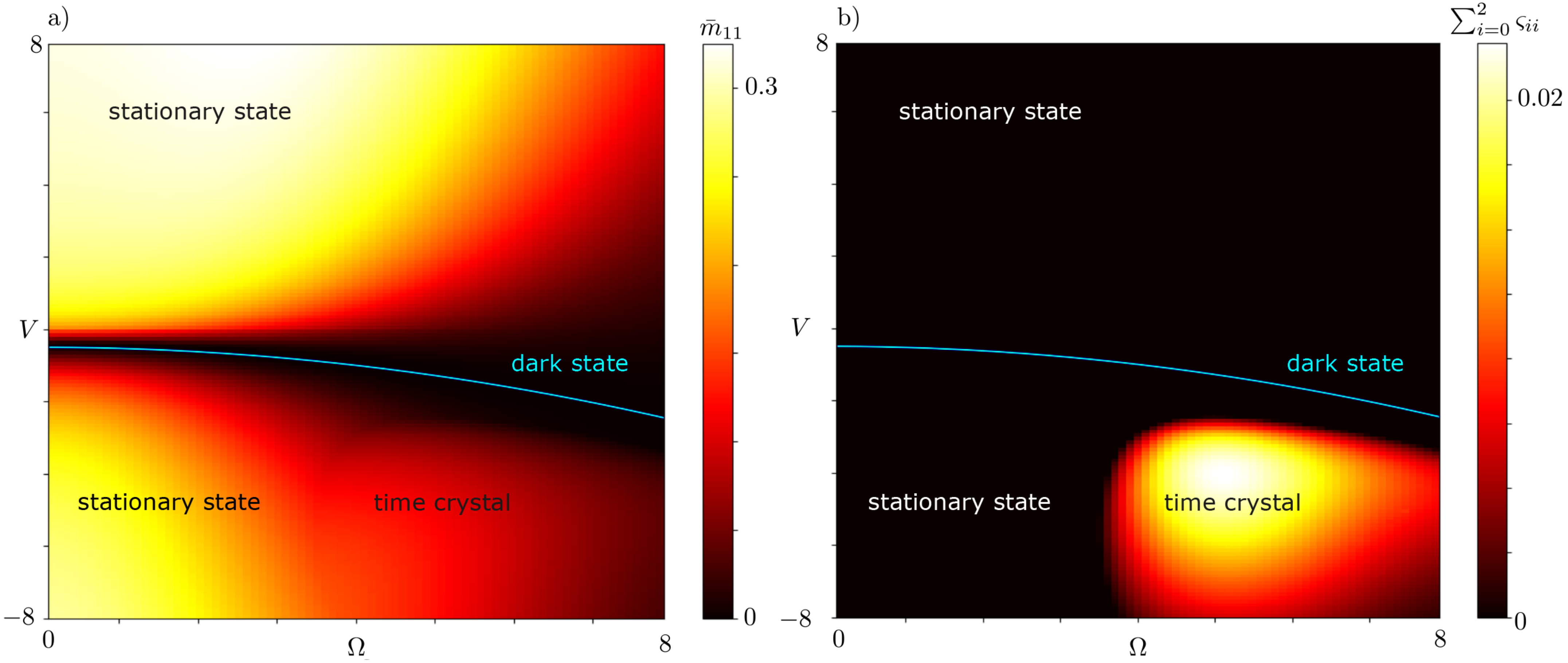
$$\bar{m}_{ij} = \frac{1}{T}\int_{t_{\mathrm{fin}}-T}^{t_{\mathrm{fin}}} m_{ij}(t)dt\,, \qquad \varsigma_{ij} = \frac{1}{T}\int_{t_{\mathrm{fin}}-T}^{t_{\mathrm{fin}}} \left(m_{ij}(t) - \bar{m}_{ij}\right)^2 dt,$$

to classify phases.

t_{fin} - final simulated time step
T - window over which we average

- We use initial state $\mathrm{m}_{00}(0) = 1$, everything else zero.
- Meaning: empty cavity, all atoms equal

Γ=2 κ,
V,Ω in units of κ.
All other parameters are equal to κ



- We analytically cant find stationary, but dark state!
- For finding the dark state we use the ansatz

$$|\mathrm{D}\rangle = c_1\, |0\rangle + c_2\, |2\rangle.$$

- Applying to the condtions

$$H|\mathrm{D}\rangle = E|\mathrm{D}\rangle, \quad L_k|\mathrm{D}\rangle = 0, \forall k.$$

- The condition for a stationary state being dark to be

$$V = V_{\mathrm{D}} = -\frac{\Delta_2}{2}\left(\left(\frac{\Omega\kappa}{4\eta\gamma g_0}\right)^2 + 1\right).$$

Is it possible to interpret this dark state to EIT ?

- Yes.
- No interaction with first excited state!

$$\dot{\alpha} = \frac{-\kappa\alpha}{2} + \eta,$$

$$\dot{m}_{00} = 0,$$

$$\dot{m}_{11} = 0,$$

$$\dot{m}_{22} = 0,$$

$$\dot{m}_{20} = i(\Delta_2 m_{20} + 2V m_{20} m_{22}),$$

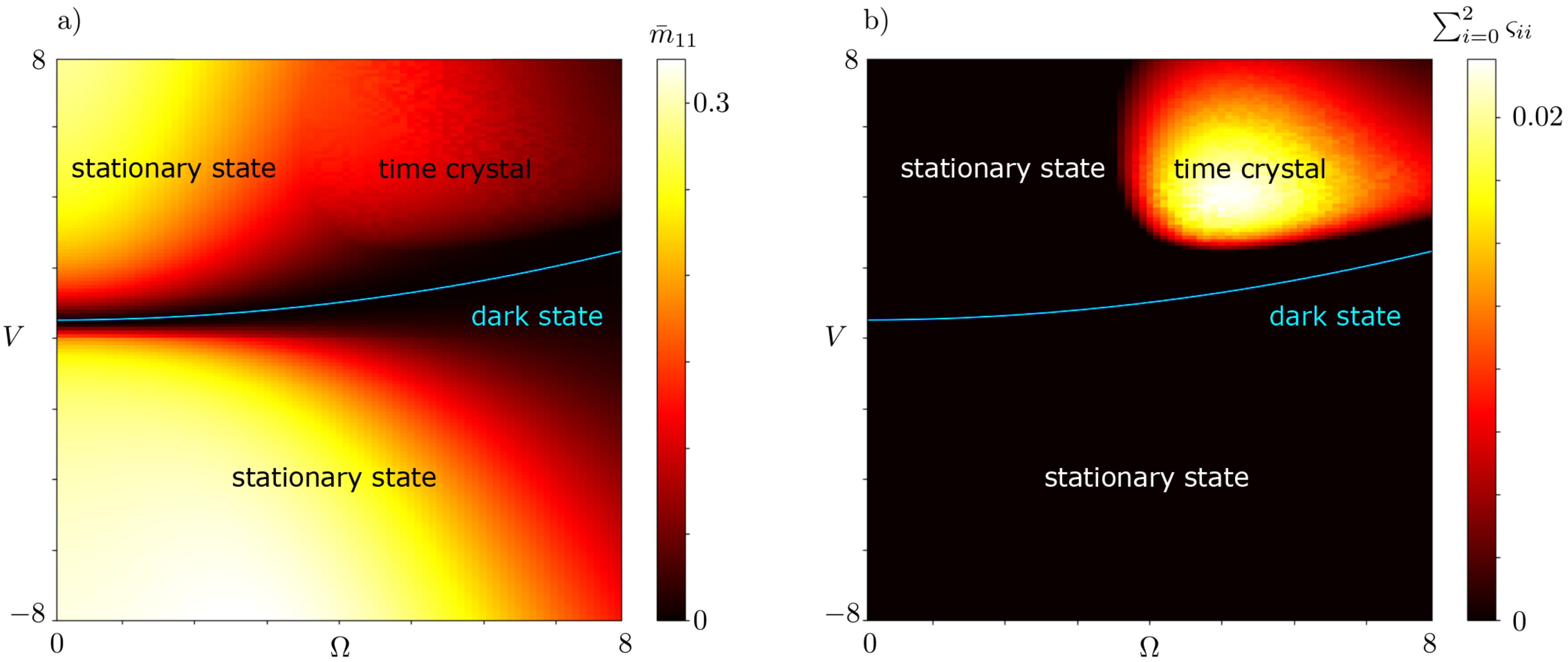
$$\dot{m}_{21} = i\left(-\gamma g_0 m_{20}\alpha + \frac{\Omega}{2} m_{11}\right).$$

Bosonic and atomic systems are decoupled!

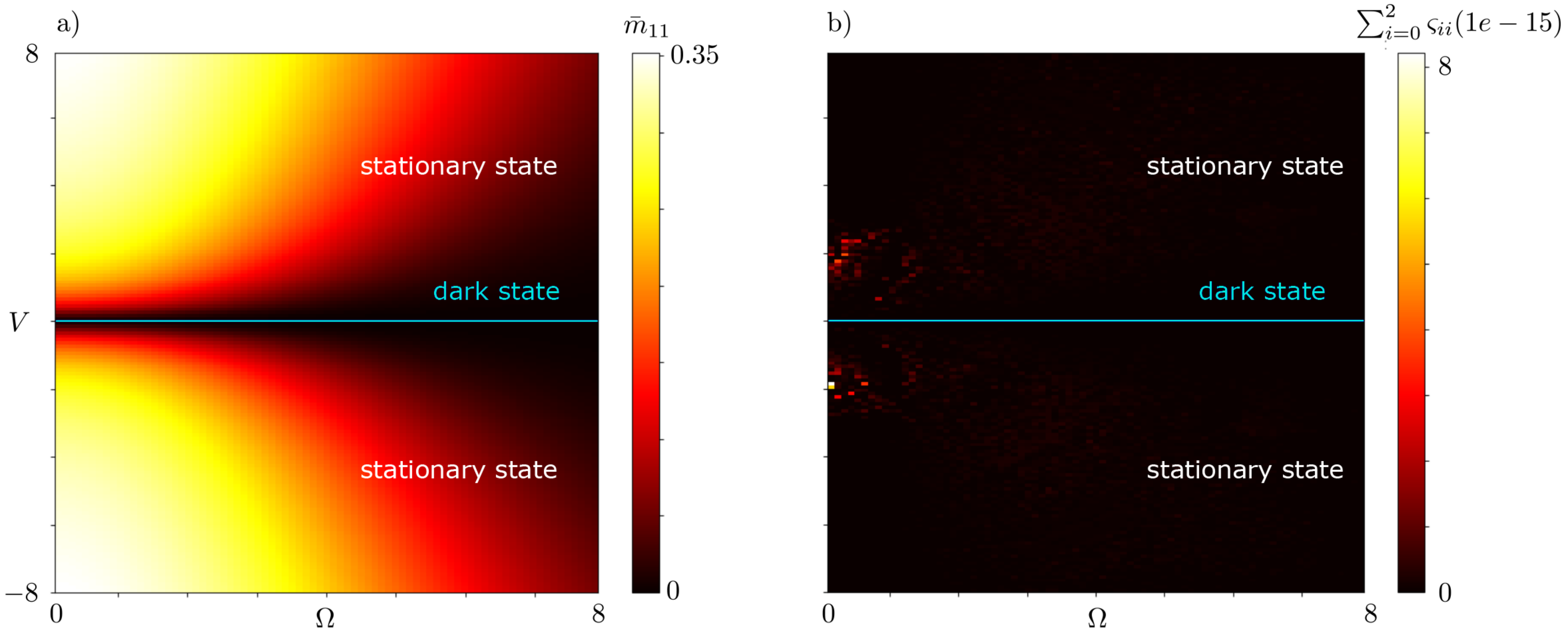
This state is emergent -> only exact in thermodynamic limit.

No dynamics appear in this state.

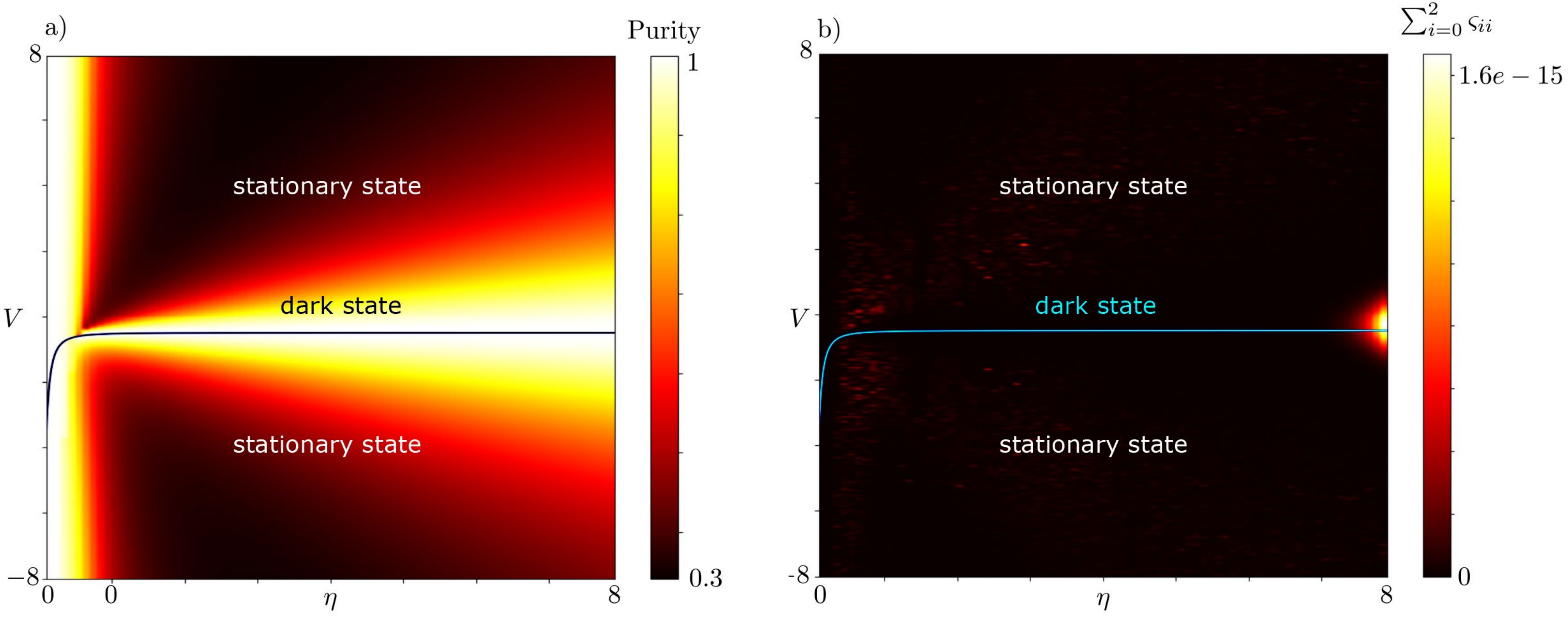
Here $\Delta_1 = \Delta_2 = -\kappa$



Here $\Delta_1 = \Delta_2 = 0$. No time crystal can be found!



$$\Delta_1 = \Delta_2 = 0, \Omega = \kappa.$$



Stability

- Dark state and mixed stationary state stability:
- Small perturbations and linearistation were used.
- Found to be stable for investigated parameters .
- Time crystal stability using Floquet analysis
- Stable for the investigated regions.
- No coexistence could be found or indicated. (Using method of **[1]**)

[1] R. Mattes, I. Lesanovsky, and F. Carollo, “Entangled
time-crystal phase in an open quantum
light-matter system

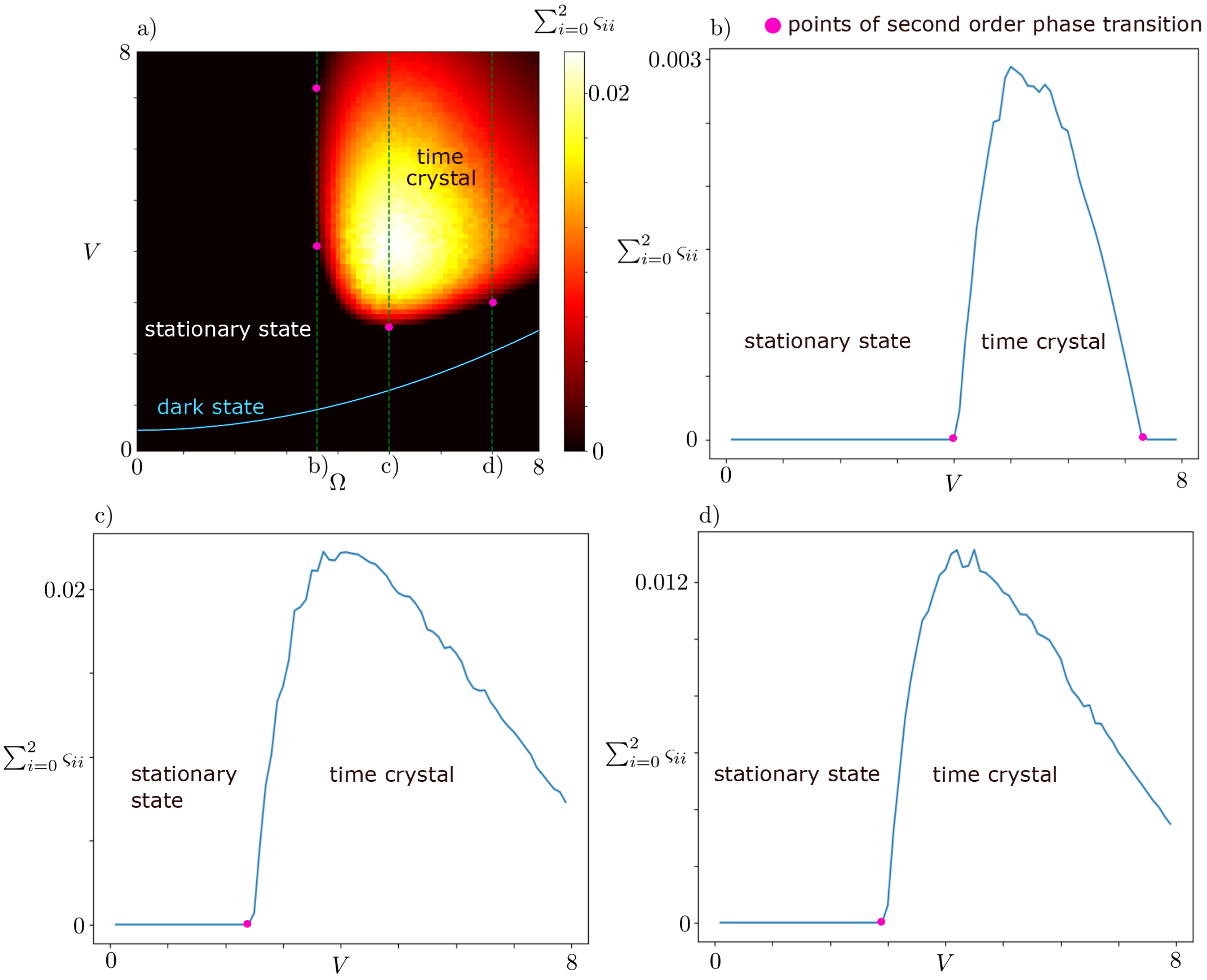
Outlook and conclusion

- Investigate quantum fluctuations.
- Compare with real experimental data of similar systems.
- Explore more parameter regimes for η and Ω, for different couplings and dissipations.

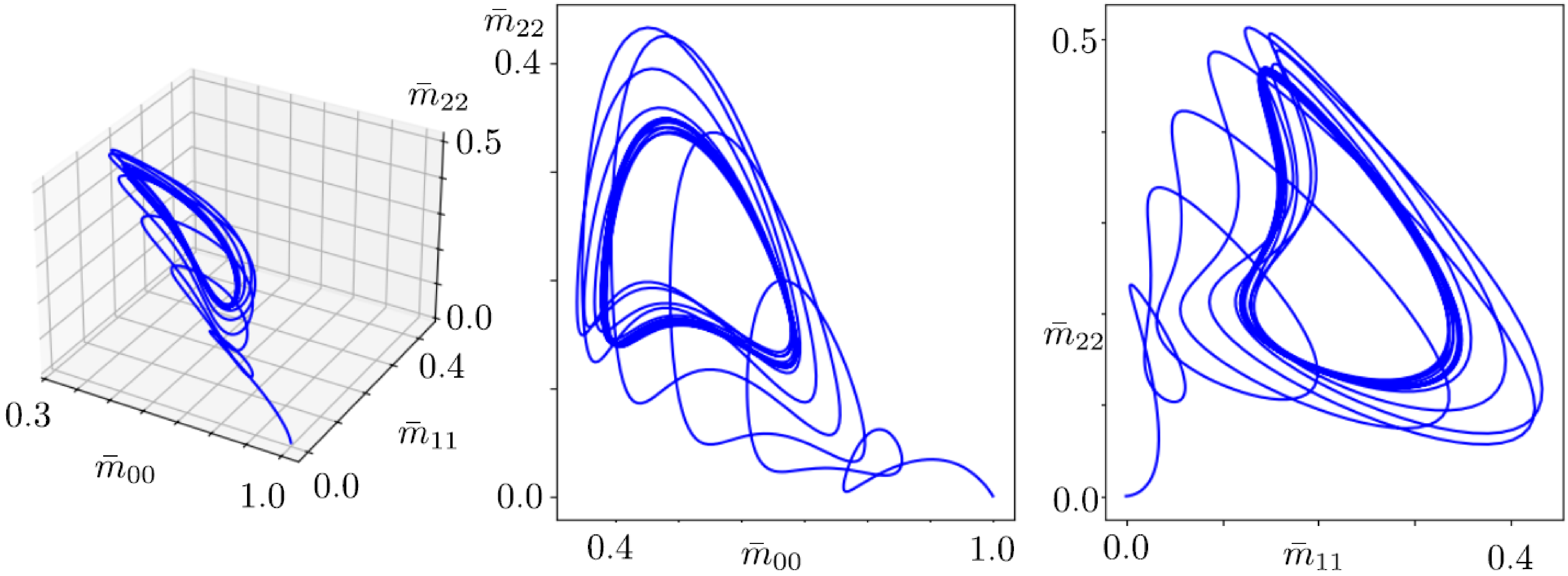
- A general dark state condition for this system was found.
- Dark state was linked to EIT.
- Stability was analyzed.

- Thanks for your attention!

Time crystal extra material



Limit cycle in magnetization space



Stationary- / Dark- state stability

- We define a fixed point and slightly perturb the system.
- We approximate by performing a Taylor expansion, only retaining linear terms.
- The full system equations read

$$A = \begin{bmatrix} -\frac{\kappa}{2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{\kappa}{2} & 0 & 0 & 0 \\ i\gamma g_0 m_{10} & -i\gamma g_0 m_{01} & 0 & \Gamma & 0 \\ -i\gamma g_0 m_{10} & i\gamma g_0 m_{01} & 0 & -\Gamma & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\frac{d}{dt}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix},$$

$$B = \begin{bmatrix} 0 & -i\gamma g_0 & 0 & 0 & 0 & 0 \\ i\gamma g_0 & 0 & 0 & 0 & 0 & 0 \\ i\gamma g_0 \alpha & -i\gamma g_0 \alpha^\dagger & 0 & 0 & 0 & 0 \\ -i\gamma g_0 \alpha & i\gamma g_0 \alpha^\dagger & \frac{i\Omega}{2} & -\frac{i\Omega}{2} & 0 & 0 \\ 0 & 0 & -\frac{i\Omega}{2} & \frac{i\Omega}{2} & 0 & 0 \end{bmatrix},$$

$$\delta x = \begin{bmatrix} \delta\alpha \\ \delta\alpha^\dagger \\ \delta m_{00} \\ \delta m_{11} \\ \delta m_{22} \end{bmatrix}, \qquad \delta y = \begin{bmatrix} \delta m_{10} \\ \delta m_{01} \\ \delta m_{21} \\ \delta m_{12} \\ \delta m_{20} \\ \delta m_{02} \end{bmatrix},$$

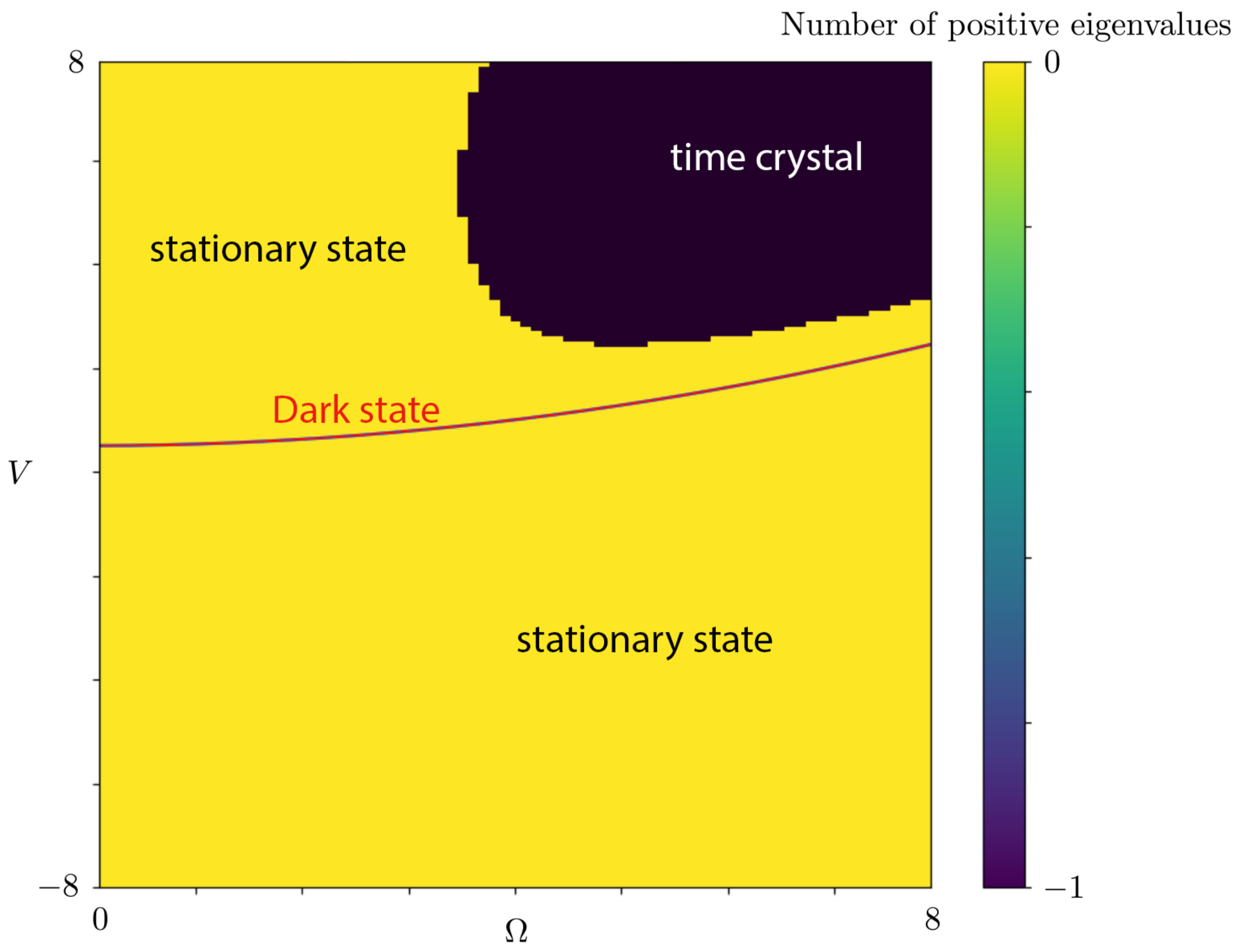
$$C = \begin{bmatrix} 0 & -i\gamma g_0(m_{11} - m_{00}) & i\gamma g_0 \alpha^\dagger & -i\gamma g_0 \alpha^\dagger & 0 \\ i\gamma g_0(m_{11} - m_{00}) & 0 & -i\gamma g_0 \alpha & i\gamma g_0 \alpha & 0 \\ -i\gamma g_0 m_{20} & 0 & 0 & \frac{i\Omega}{2} & 2iVm_{21} - \frac{i\Omega}{2} \\ 0 & i\gamma g_0 m_{02} & 0 & -\frac{i\Omega}{2} & -2iVm_{12} + \frac{i\Omega}{2} \\ 0 & -i\gamma g_0 m_{21} & 0 & 0 & 2iVm_{20} \\ i\gamma g_0 m_{12} & 0 & 0 & 0 & -2iVm_{02} \end{bmatrix},$$

$$\mathrm{D} = \begin{bmatrix} -\frac{\Gamma}{2} + i\Delta_1 & 0 & 0 & 0 & \frac{i\Omega}{2} & 0 \\ 0 & -\frac{\Gamma}{2} - i\Delta_1 & 0 & 0 & 0 & -\frac{i\Omega}{2} \\ 0 & 0 & -\frac{\Gamma}{2} + i(\Delta_2 - \Delta_1 + 2Vm_{22}) & 0 & -i\gamma g_0 \alpha & 0 \\ 0 & 0 & 0 & -\frac{\Gamma}{2} + i(\Delta_1 - \Delta_2 - 2Vm_{22}) & 0 & i\gamma g_0 \alpha^\dagger \\ \frac{i\Omega}{2} & 0 & -i\gamma g_0 \alpha^\dagger & 0 & i(\Delta_2 + 2Vm_{22}) & 0 \\ 0 & -\frac{i\Omega}{2} & 0 & i\gamma g_0 \alpha & 0 & -i(\Delta_2 + 2Vm_{22}) \end{bmatrix}.$$

Our equations of motion read

$$
\begin{aligned}
\dot{\alpha}(t) &= -\frac{\kappa\alpha(t)}{2} - i\gamma g_0 m_{01}(t) + \eta, \\
\dot{m}_{00}(t) &= \Gamma m_{11}(t) + i\gamma g_0\big(m_{10}(t)\alpha(t) - m_{01}(t)\alpha^*(t)\big), \\
\dot{m}_{11}(t) &= -\Gamma m_{11}(t) + i\gamma g_0\big(m_{01}(t)\alpha^*(t) - m_{10}(t)\alpha(t)\big) + \frac{i\Omega}{2}\big(m_{21}(t) - m_{12}(t)\big), \\
\dot{m}_{22}(t) &= i\frac{\Omega}{2}\big(m_{12}(t) - m_{21}(t)\big), \\
\dot{m}_{21}(t) &= -\frac{\Gamma}{2}m_{21}(t) + i\big(\Delta_2 m_{21}(t) - \Delta_1 m_{21}(t) - \gamma g_0 m_{20}(t)\alpha(t)\big)\, i\left(+\frac{\Omega}{2}\big(m_{11}(t) - \right. \\
&\left. m_{22}(t)\big) + 2V m_{21}(t) m_{22}(t)\right), \\
\dot{m}_{01}(t) &= -\frac{\Gamma}{2}m_{01}(t) + i\left(-\Delta_1 m_{01}(t) + \gamma g_0\big(m_{11}(t)\alpha(t) - m_{00}(t)\alpha(t)\big) - \frac{\Omega}{2}m_{02}(t)\right), \\
\dot{m}_{20}(t) &= i\left(\Delta_2 m_{20}(t) + \frac{\Omega}{2}m_{10}(t) + 2V m_{20}(t) m_{22}(t) - \gamma g_0 m_{21}(t)\alpha^*(t)\right).
\end{aligned}
$$

for certain small Γ values, positive eigenvalues appear, indicating instability.



$\Gamma = 2\,\kappa$, $\Delta_1 = \Delta_2 = -\kappa$

V,Ω in units of κ.

All other parameters are equal to κ

ω_{cav} - cavity frequency

ω_{las} - laser frequency (resonant with cavity)

ω_1, ω_2 - cavity transition frequencys

η – driving field amplitude

Γ – atomic dissipation rate

κ – cavity dissipation rate

γ - transition between zero and one

Δ_1- detuning of first excited state

Ω – Rabi frequency of external laser

V – all to all atom interaction potential