Mobile Robotics

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Introduction to Mobile Robotics - Lecture 3 - Robot Locomotion

Lecture Content

- Kinematics of Mobile Robots
- Drive Systems
- Holonomic and non-Holonomic Systems

Mobile Robots Locomotion

Locomotion

- mechanism used to move
- common approaches:
  - wheels,
  - tracks,
  - legs,
  - wings, …
- considerations:
  - terrain,
  - mechanical complexity,
  - control complexity,
  - energy efficiency,
  - System Requirements

Figure: www.indiamart.com

Mobile Robot Movement

- Workspace: The range of possible poses that the mobile robot can achieve inside it's environment.
- Controllability: The possible paths and trajectories inside it's workspace
- Forward Kinematics:

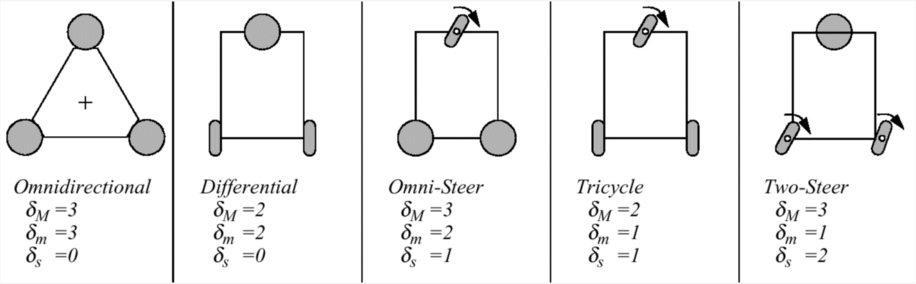
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\phi}_1, \dot{\phi}_r, \cdots) \tag{1}$$

- Mobile Robot Dynamics: provides additional constraints on kinematics and controllability. For example, a high COG limits practical turn rates due to danger of rolling over.

Wheeled Mobile Robot Kinematics

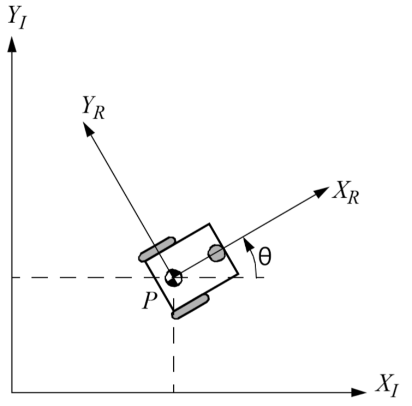
Maneuverability = mobility + steerability

- Instantaneous Center of Curvature
- The mobility available based on the sliding constraints plus additional freedom contributed by the steering



Wheeled Mobile Robot Kinematics

Reference Frame



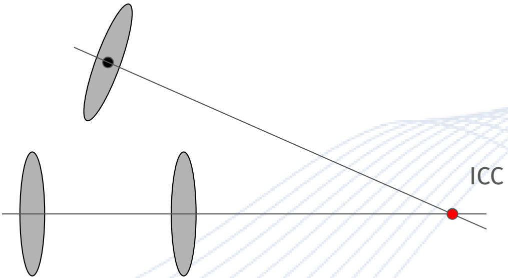
- Robot and it’s local reference frame inside the global reference frame.

Instantaneous Center of Curvature

ICC

Wheels rotate around their long axis.

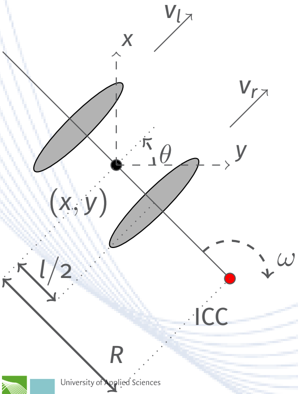
The axes of all wheels intersect at a single point, otherwise slippage will occur.



Differential Drive

Kinematics

While we can vary the velocity of each wheel, for the robot to perform rolling motion, the robot must rotate about a point that lies along their common left and right wheel axis.



Equations

$$\omega(R + l/2) = V_r \quad (2)$$

$$\omega(R - l/2) = V_l \quad (3)$$

$$ICC = [x - R\sin\theta, y - R\cos\theta] \quad (4)$$

$$v = \frac{v_r + v_l}{2} \quad (5)$$

$$\omega = \frac{v_r - v_l}{l} \quad (6)$$

Differential Drive - 2

There are three interesting cases with these kinds of drives.

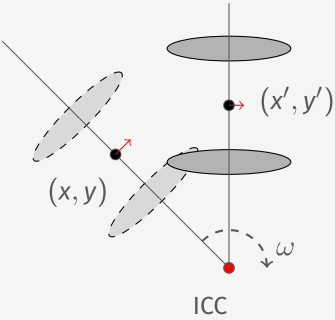
1. If Vl = Vr, then we have forward linear motion in a straight line. R becomes infinite, and there is effectively no rotation - ω is zero.
2. If Vl = -Vr, then R = 0, and we have rotation about the midpoint of the wheel axis - we rotate in place.
3. If Vl = 0, then we have rotation about the left wheel. In this case $R = \frac{l}{2}$. Same is true if we rotate around the right wheel

Note that a differential drive robot cannot move in the direction along the axis - this is a singularity.

Differential Drive - 3

Forward Kinematics

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega\delta t \end{bmatrix} \tag{7}$$



$$x(t) = \int_0^t v(t')\cos(\theta t')dt' \tag{8}$$

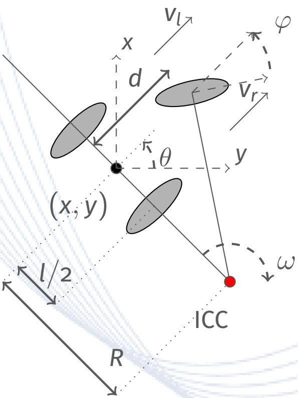
$$y(t) = \int_0^t v(t')\sin(\theta t')dt' \tag{9}$$

$$\theta(t) = \int_0^t \omega(t')dt' \tag{10}$$

Ackermann Drive

Kinematics

One steered wheel. Same Singularity as Differential Drive, ICC must be on axis of rear wheels. When $v = 0$, then $\omega = 0$



Equations

$$R = \frac{d}{\tan \varphi} \tag{11}$$

$$\omega(R + l/2) = V_r \tag{12}$$

$$\omega(R - l/2) = V_l \tag{13}$$

$$ICC = [x - R\sin\theta, y - R\cos\theta] \tag{14}$$

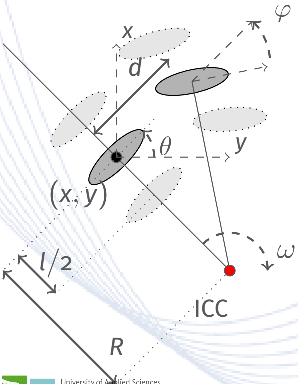
$$v = \frac{v_r + v_l}{2} \tag{15}$$

$$\omega = \frac{v_r - v_l}{l} \tag{16}$$

Car Model

Kinematics of a Car using the Bike Model

The Front and Rear wheels are grouped together to a single steered wheel at the front and a single steered wheel at the back. The ICC is on the axis of the rear wheels. When $v = 0$, then $\omega = 0$.



Equations of Motion

$$R = \frac{d}{\tan\varphi} \tag{17}$$

$$\omega \cdot R = V \tag{18}$$

$$ICC = [x - R\sin\theta, y - R\cos\theta] \tag{19}$$

$$\dot{x} = v\cos(\theta) \tag{20}$$

$$\dot{y} = v\sin(\theta) \tag{21}$$

$$\dot{\theta} = \frac{v}{d}\tan(\varphi) \tag{22}$$

Swedish Wheels

Swedish / Mecanum Wheels

Demo - Kuka youBot

Kinematics

$$v_y = (v_0 + v_1 + v_2 + v_3)/4 \quad (23)$$

$$v_x = (v_0 - v_1 + v_2 - v_3)/4 \quad (24)$$

$$v_\theta = (v_0 + v_1 - v_2 - v_3)/4 \quad (25)$$

Non-Holonomic Systems

Definition

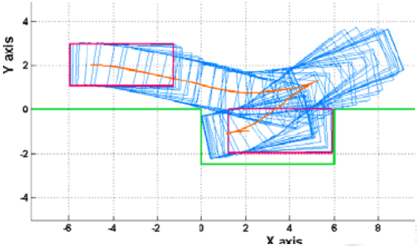
A Holonomic System, is defined as a system where the path integrals in the system depend only upon the initial and final states of the system.
A Non-Holonomic System, is dependant on the path taken to get there.

Constraints

Non-holonomic constraints reduce the control space with respect to the current configuration

- Moving sideways is impossible.
- Holonomic constraints reduce the configuration space.
- E.g., a train on tracks (not all positions and orientations are possible)

Holonomic and Non-Holonomic Systems

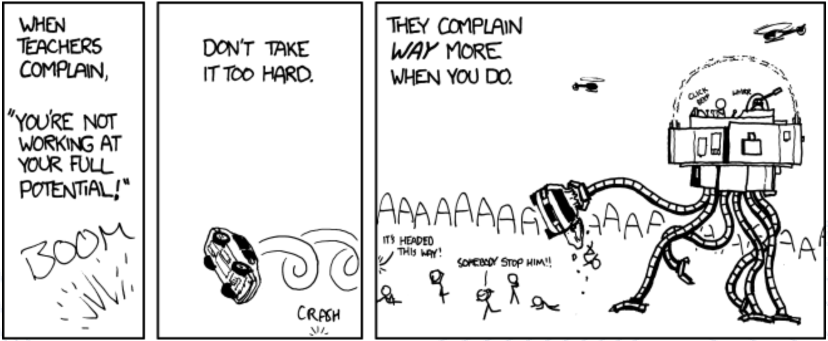


Wheeled Holonomic Systems

- Swedish Wheel
- Synchronous Drive

Wheeled Non-Holonomic Systems

- Differential Drive
- Ackermann Drive
- Car Type Drive
- Skid Steer Drive


WHEN TEACHERS COMPLAIN,
"YOU'RE NOT WORKING AT YOUR FULL POTENTIAL!"
BOOM
DON'T TAKE IT TOO HARD.
CRASH
THEY COMPLAIN WAY MORE WHEN YOU DO.
AAAAAAAA
AAAA
AAA
IT'S HEADED THIS WAY!
SOMEBODY STOP HIM!!

www.xkcd.com