University of Applied Sciences
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Mobile Robotics

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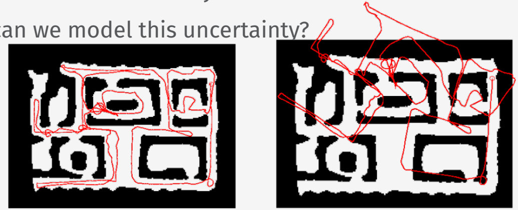
Lecture Content

- Motion and Motion Error
- Coordinate Systems
- Motion Models
  - Odometry Model
  - Velocity Model

Introduction to Mobile Robotics

Motion and Motion Models

- Robot motion is inherently uncertain
- How can we model this uncertainty?



Sources of for Motion Errors

- Uneven or changes in Floor surfaces
- Errors in Robot Construction (poorly aligned axle)
- Slippage by accelleration
- Loose Surfaces

Figure: Source:jebiga.com

Motion Models

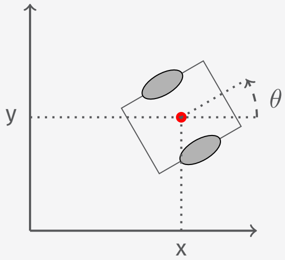
Probabilistic Motion Models

1. To implement the Bayes Filter, we need the transition model $p(x_t | x_{t-1}, u_t)$
2. The term $p(x_t | x_{t-1}, u_t)$ specifies a posterior probability, that action u_t carries the robot from x_{t-1} to x_t.
3. In this section we will discuss, how $p(x_t | x_{t-1}, u_t)$ can be modeled based on the motion equations and the uncertain outcome of the movements.

Motion Models

Coordinate Systems

- The configuration of a typical wheeled robot in 3D can be described by six parameters.
- These are the three-dimensional Cartesian coordinates plus the three Euler angles for roll, pitch, and yaw.
- For simplicity, throughout this section we consider robots operating on a planar surface.
- The state space of such systems is three-dimensional (x, y, θ).



Motion Models

Typical Motion Models

In practice, one often finds two types of motion models:

- Odometry-based
  - Used when systems are equipped with wheel encoders.
- Velocity-based (dead reckoning)
  - Velocity-based models have to be applied when no wheel encoders are given.
  - They calculate the new pose based on the velocities and the time elapsed.

Motion Models

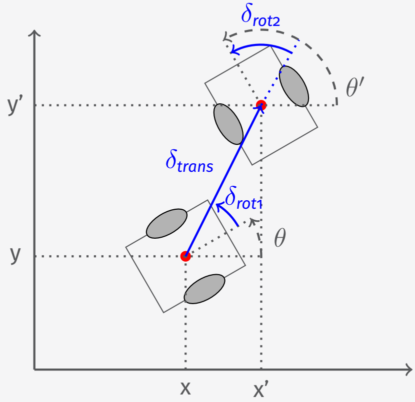
Odometry Model

In time step δt, robot moves from $[x, y, \theta]$ to $[x', y', \theta']$.
Robot Action u, intepreted from Wheel Velocity Commands through kinematic model to translation commands $u = [\delta_{rot1}, \delta_{rot2}, \delta_{trans}]$.

$$\delta_{trans} = \sqrt{(x' - x)^2 + (y' - y)^2} \quad (1)$$

$$\delta_{rot1} = atan2(\frac{y' - y}{x' - x}) - \theta \quad (2)$$

$$\delta_{rot2} = \theta' - \theta - \delta_{rot1} \quad (3)$$



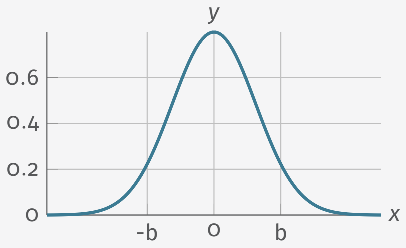
Motion Models

Noise in the Odometry Model

$$\hat{\delta}_{rot1} = \delta_{rot1} + \varepsilon_{\alpha_1|\delta_{rot1}|+\alpha_2|\delta_{trans}|} \tag{4}$$

$$\hat{\delta}_{trans} = \delta_{trans} + \varepsilon_{\alpha_3|\delta_{trans}|+\alpha_4(|\delta_{rot1}|+|\delta_{rot2}|)} \tag{5}$$

$$\hat{\delta}_{rot2} = \delta_{rot2} + \varepsilon_{\alpha_1|\delta_{rot2}|+\alpha_2|\delta_{trans}|} \tag{6}$$

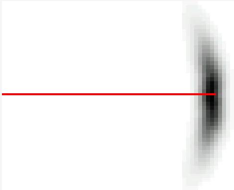
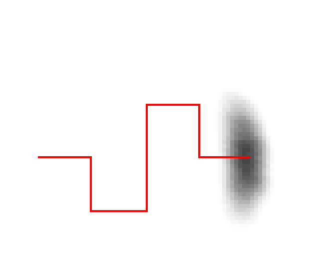


$$\varepsilon_{\sigma^2} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-x^2}{2\sigma^2}} \tag{7}$$

Motion Models

The Calculated Posterior

- Different α for rotation and translation results in a posterior with a typical banana-shaped distributions. Viewed as a 2d-projection of the 3d posterior.
- The following two posteriors show $p(x'|u,x)$ for two different command signals u. x is the begining of the red path, u shows the commanded path.

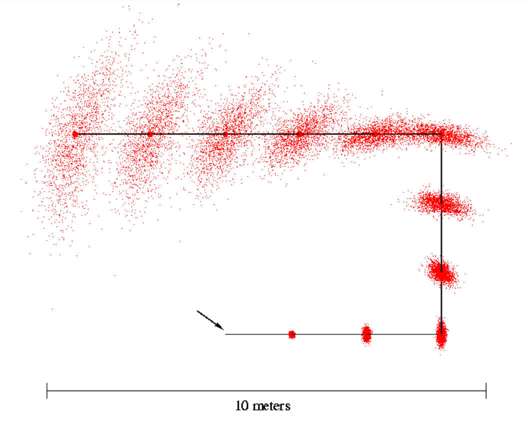



Source: Thrun, Probabilistic Robotics

Motion Models

Resulting Posterior after longer movement

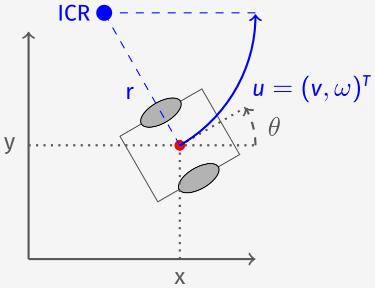
Shown with Sample Based Distribution



Source: Thrun, Probabilistic Robotics

Motion Models

Velocity Based Model



Noise Model

$$\hat{v} = v + \varepsilon_{\alpha_1|v|+\alpha_2|\omega|} \tag{8}$$

$$\hat{\omega} = \omega + \varepsilon_{\alpha_3|v|+\alpha_4|\omega|} \tag{9}$$

- Two degree of Freedom Noise Model, cannot correctly account for noise in a 3DOF state space.
- Additional Term added to account for final rotation.

$$\hat{\gamma} = \varepsilon_{\alpha_5|v|+\alpha_6|\omega|} \tag{10}$$

Motion Models

Motion Model with 3rd Parameter

$$x' = x - \frac{\hat{v}}{\hat{\omega}} \sin\theta + \frac{\hat{v}}{\hat{\omega}} \sin(\theta + \hat{\omega}\delta t) \quad (11)$$

$$y' = x - \frac{\hat{v}}{\hat{\omega}} \cos\theta + \frac{\hat{v}}{\hat{\omega}} \cos(\theta + \hat{\omega}\delta t) \quad (12)$$

$$\theta' = \theta + \hat{\omega}\delta t + \hat{\gamma}\delta t \quad (13)$$

Velocity Motion Model

$$v = \frac{\Delta\theta}{\Delta t} r \quad (14)$$

$$\omega = \frac{\Delta\theta}{\Delta t} \quad (15)$$

r is the radius to ICR. The ICR is the center of a circle that lies on a ray half way between x and x' and is orthogonal to the line between x and x', and orthogonal to the initial heading.

$$r = \sqrt{(x' - x)^2 + (y' - y)^2} \quad (16)$$

$$\Delta\theta = atan\left(\frac{y' - y_{ICR}}{x' - x_{ICR}}\right) - atan\left(\frac{y - y_{ICR}}{x - x_{ICR}}\right) \quad (17)$$

$$\begin{pmatrix} x_{ICR} \\ y_{ICR} \end{pmatrix} = \begin{pmatrix} \frac{x+x'}{2} + \mu(y - y') \\ \frac{y+y'}{2} + \mu(x - x') \end{pmatrix} \qquad \mu = \frac{1}{2} \frac{(x - x')\cos\theta + (y - y')\sin\theta}{(y - y')\cos\theta + (x - x')\sin\theta} \quad (18)$$

Motion Models

Summary

- The requirement for a motion model was discussed with the problems of odometry and sensors.
- Noise Models were proposed for both Odometry based movement and Velocity based movement without Encoders.
- The typical Form of the Posterior was demonstrated.
- In reality, the values of parameters of the models have to be measured.