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Mobile Robotics

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Path Planning

Lecture Content

- Configuration Space
- Discretized Search Algorithms
- Roadmap Search Algorithms
- Cell Decompositions

Path Planning

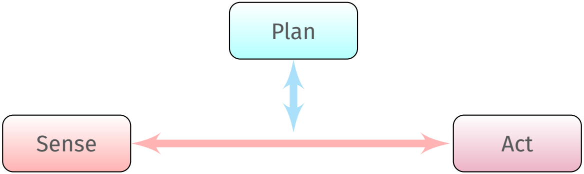
Motion Planning

- Latombe (1991): ”eminently necessary since, by definition, a robot accomplishes tasks by moving int he real world”
- Goals:
  - Collision Free Trajectoery:
  - Robot should reach the goal location as quickly as possible

The challenges..

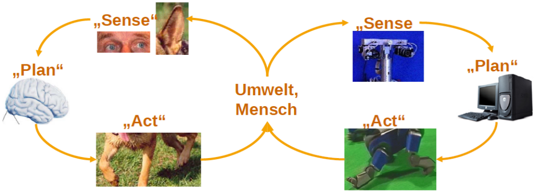
- Calculate the optimal path taking potential uncertainties in the actions into account
- Quickly generate actions in the case of unforeseen objects

Hybrid Deliberative / Reactive Paradigm



Hybrid Paradigm

- Low frequency planning / mapping routines
- High frequency collision avoidance in a closed loop reactive motion control



Path Planning

Configuration Space

- A key concept for motion planning is a configuration:
  – a complete specification of the position of every point in the system
- A simple example: a robot that translates but does not rotate in the plane:
  – what is a sufficient representation of its configuration?
- The space of all possible configurations of the robot (robot arm / etc.) is the configuration space or Cspace.

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Path Planning

Configuration Space

- Let q denote a point in a configuration space Q
- The path planning problem is to find a mapping $c : [0, 1] \rightarrow Q$ so that no configuration along the path intersects an obstacle
- We define a Workspace Obstacle as WO_i
- A configuration space obstacle QO_i is the set of configurations q at which the robot would intersect WO_i, that is

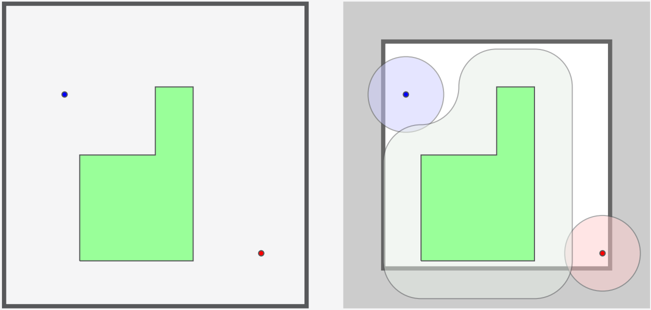
$$QO_i = \{q \in Q | R(q) \cap WO_i \neq \emptyset\} \quad (1)$$

- The free configuration space (or just free space) Q_{free} is

$$Q_{free} = Q - \sum_i QO_i \quad (2)$$

Configuration Space

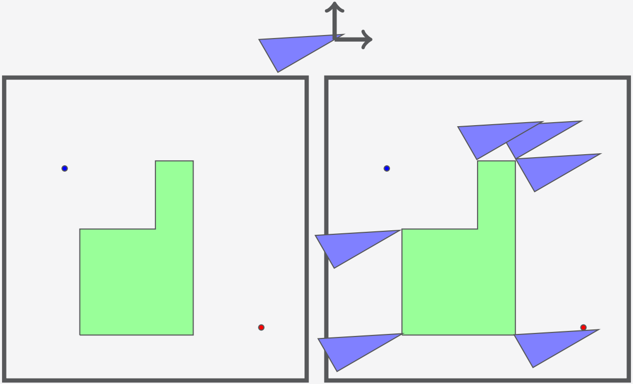
Circular Robot



Path Planning

Configuration Space

Non-circular Symmetry, Translation only

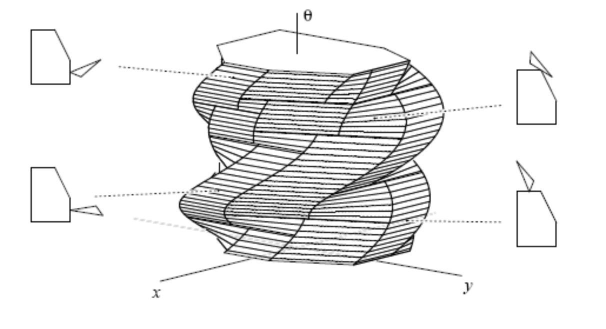


Path Planning

Configuration Space

Non-circular Symmetry, including translation

- Robot planning can be performed with the robot being a point in C-Space.

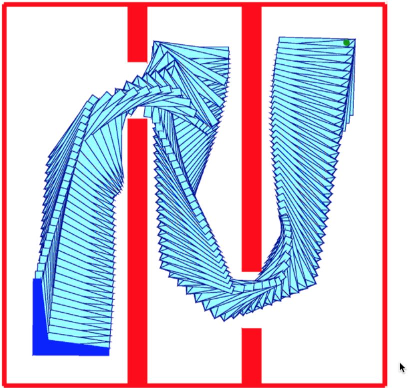


Source: slides from G.D. Hager and Z. Dodds

Path Planning

Configuration Space

Importance of Configuration Space to Path Planning



16-735, Howie Choset with slides from G.D. Hager, Z. Dodds, and Dinesh Mocha

Path Planning

Configuration Space Discretisations

- Continuous Space needs to be discretized for path planning
- There are two general approaches to discretise C-spaces:
  - Combinatorial Planning
    Characterises C_{free} explicitly by capturing the connectivity of C_{free} into a graph and finds solutions using search
  - Sampling-based planning
    Uses collision detection to probe and incrementally search the C-space for a solution

Path Planning

Search

The performance of a search algorithm is measured in four different ways:

- **Completeness**: does the algorithm find a solution when there is one?
- **Optimality**: is the solution the best one of all possible solutions in terms of path cost?
- **Time complexity**: how long does it take to find a solution?
- **Space complexity**: how much memory is needed to perform the search?

Path Planning

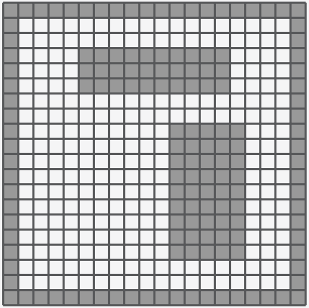
Search

The problem of search: finding a sequence of actions (a path) that leads to desirable states (a goal).

- Uniformed Search: Besides the problem definition, no further information about the domain (”blind search”)
- The only thing one can do is to expand nodes differently
- Example algorithms: breadth first, unform-cost, depth-first, bidirectional, etc.

Path Planning

Discretized Configuration Space



Search

The problem of search: finding a sequence of actions (a path) that leads to desirable states (a goal)

- Informed Search: further information about the domain through heuristics
- Capability to say that a node is ”more promising” than another node
- Example Algorithms: Greedy best-first search, A*, many variants of A*, D* etc.

Path Planning

Uniformed Search

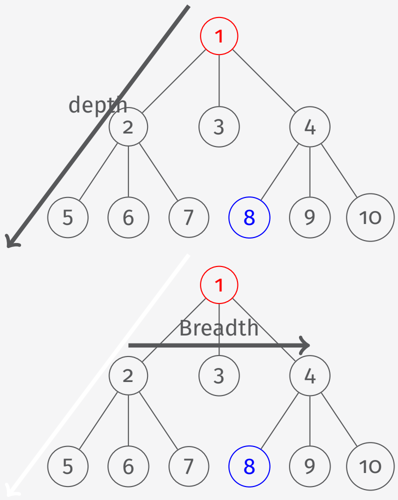
Breadth-first:

- Complete
- Optimal if action costs equal
- Time and Space: $O(b^d)$

Depth-fist:

- Not complete in infinite spaces
- Not optimal
- Time: $O(b^m)$
- Space: $O(b \cdot m)$ (can forget explored subtrees)

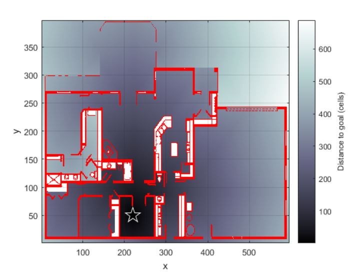
b: Branching factor, d: goal depth, m: max tree depth

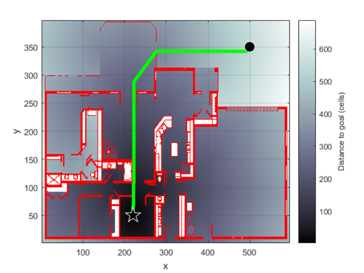


Path Planning

Distance Transform

- Starting with a configuration map, each cell is weighted with its shortest route distance.





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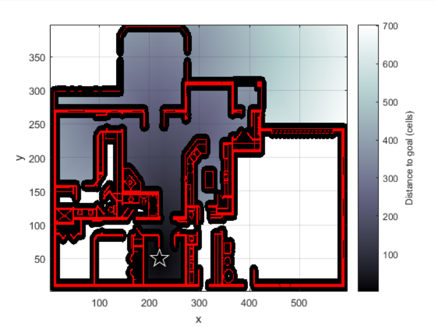
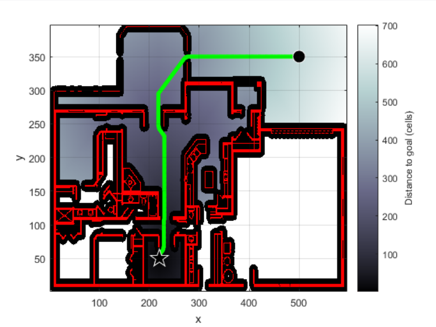
Distance Transform with Configuration Space Considerations

- Discretized Configuration Space used for Path Planning.
- Some possible goals are no longer possible





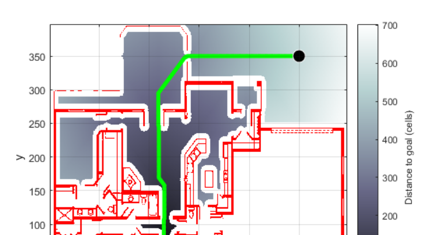
Path Planning

A*

- A* is an informed search algorithm, or a best-first search
- Searches among possible paths for the solution (goal) that incurs the smallest cost,
- and among these paths it first considers the ones that appear to lead most quickly to the solution.
- Weighting is calculated from:

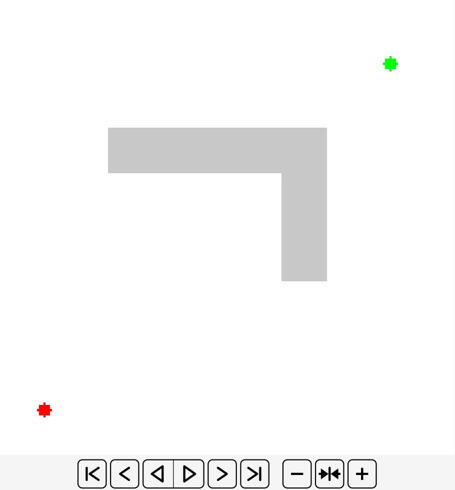
$$f(n) = g(n) + h(n) \tag{3}$$

- g(n) represents the exact cost of the path from the starting point to any vertex n
- h(n) represents the heuristic estimated cost from vertex n to the goal.



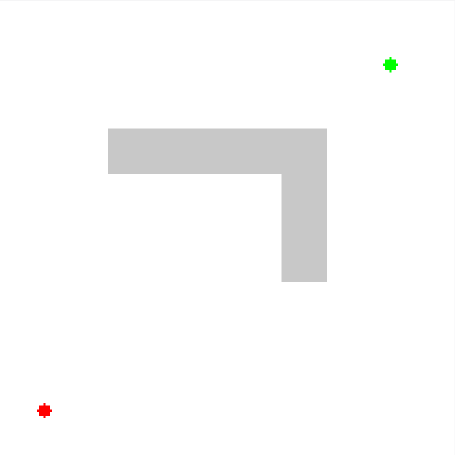
Path Planning - Dijkstra

Dijkstra



Path Planning - A*

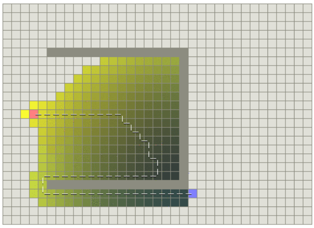
A*



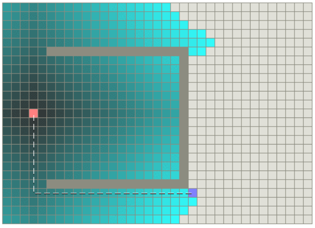
Path Planning

Comparison between Algorithms, Computational Efficiency vs Path Optimality

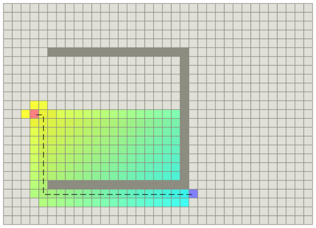
Greedy



Dijkstra



A*



Path Planning

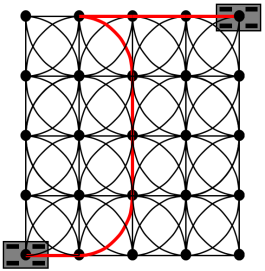
Implementation

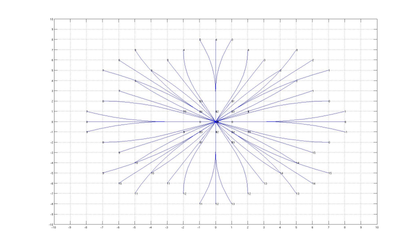
http://theory.stanford.edu/ amitp/GameProgramming/Implementation-Notes.html

Path Planning

>2D Planning

- Lattice Planner





Source: http://www.frc.ri.cmu.edu/ alonzo/video/Lattice.PNG
http://sbpl.net/shared/Images/MotionPrimitives.jpg

Path Planning

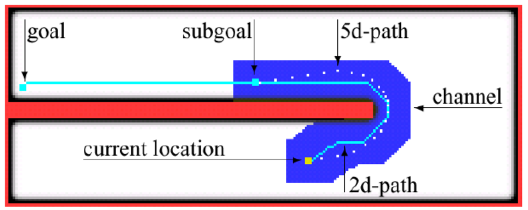
5D Planning

- Planning in 2D does imply compliance with kinematic movement
- Planing in the full $< x, y, \theta, v, \omega >$ configuration space using A*
- Generates a sequence of steering commands to reach the goal location
- Maximises trade-off between driving time an distance to obstacles
- Possible solutions in discrete space can involve a change between maximum steering angles in consecutive steps.
- The search space is too large to be explored within normal planning time constraints

Path Planning

5D Planning

- Alternate solution, use A* to search for the optimal path in the 2D-grid map
- With an assumption, the 5D-plan lies close to the 2D plan.
- In areas with sharp corners, establish sub-goals and a channel where the C-Space allows
- Find solution for 5D movement in Channel with smaller search space



Source: Wolfram Burgard, Uni Freiburg

Path Planning

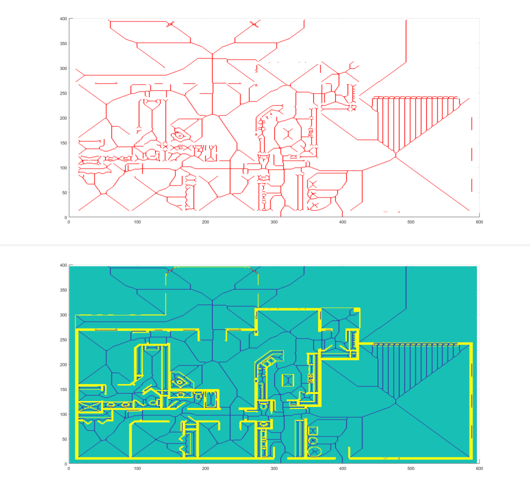
Roadmap Methods

- In robotic planning, analysis of the map is refered to as the planning phase
- the Query phase, uses the result of the planning phase to determine the route from A to B
- A* and other discretised search algorithms require significant amount of computation for the planning phase, but the query phase is cheap.
- The plan depends on the goal, and if the goal changes, the expensive planning phase must be repeated
- With Roadmap planning, similar to a train map, the planning phase is irrelevant of the goal

Path Planning

Roadmap Methods

- Voronoi Diagram



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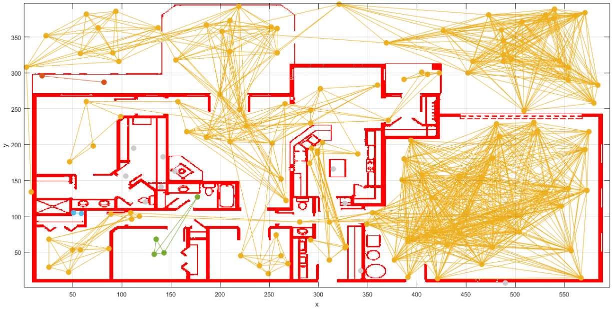
Roadmap Methods

- Probabilistic Road Map (PRM)



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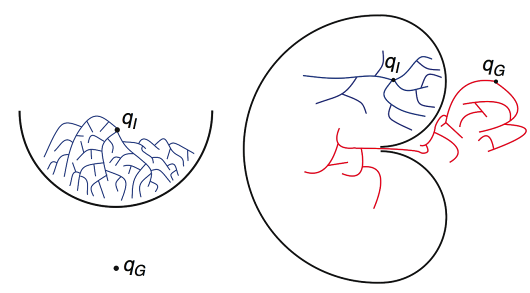
Probabilistic Road Map

- Pros:
  - Can operate in high dimensional C-Space
  - Efficient calculation in planning and query phase
- Cons:
  - Problems with long corridors
  - Not optimal Solution

Path Planning

Roadmap Methods

- Rapidly Exploring Random Tree (RRT)



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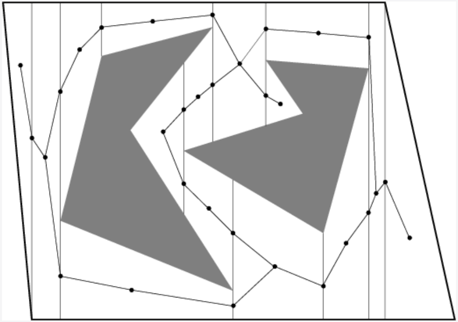
Roadmap Methods

- Cell Decompositions
  - Cell: simple region



- Planning in two steps:
  - Planner determines cells that contain the start and goal
  - Planner searches for a path within adjacency graph

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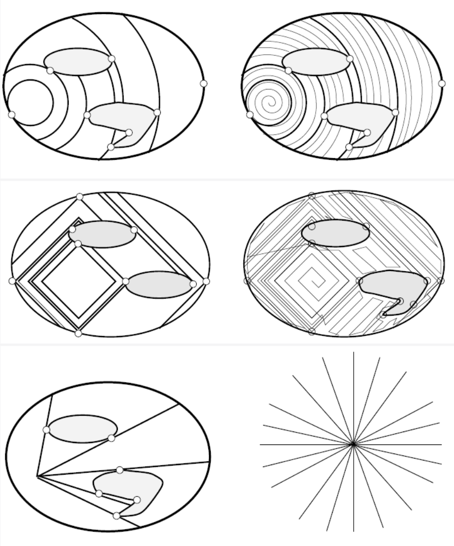
Morse Decomposition



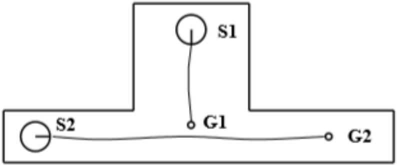
Path Planning

Optimisations ...

- Identify in advance concave areas, and remove from planning
- Large areas can be discretized with a larger cell size
- ...

In a Dynamic Environment ...

- Path Planning with multiple robots requires an additional level of optimisation for the complete solution.



Path Planning

Summary

- Motion Planning commences with the analysis of configuration space
- Discretised Configuration Space is a first step
- Higher dimensions are required for fulfilling kinematic requirements
- Higher dimensions are computationally intense
- Two phases are the planning and the query
- A* and other discretised search algorithms require significant amount of computation for the planning phase, but the query phase is cheap.
- The plan depends on the goal, and if the goal changes, the expensive planning phase must be repeated
- With Roadmap planning, similar to a train map, the planning phase is irrelevant of the goal