

a) Für  $r$  wählen wir beides mal:  $r = 2 \cdot n - 2 \cdot i - b$

$$b) \mathcal{I} \equiv r = 2n - 2i - b \wedge ((0 \leq i \leq n \wedge b = 0) \vee (0 \leq i < n \wedge b = 1))$$

$$c) \text{WP}[\mathcal{I} \mid r = 2n - 2i - b](\mathcal{I}) \equiv (0 \leq i \leq n \wedge b = 0) \vee (0 \leq i < n \wedge b = 1) \\ \mathcal{L} = r > 2n - 2i - b \wedge ((0 \leq i \leq n \wedge b = 0) \vee (0 \leq i < n \wedge b = 1)) \equiv A$$

Asichert zu, dass  $r$  bei jedem Schleifendurchgang strikt kleiner wird.

$$\text{WP}[\mathcal{I} \mid i = i + 1](A) \equiv r > 2n - 2i - 2 - b \wedge ((0 \leq i + 1 \leq n \wedge b = 0) \vee (0 \leq i + 1 < n \wedge b = 1)) \equiv B$$

$$\text{WP}[\mathcal{I} \mid b = 0](B) \equiv r > 2n - 2i - 2 \wedge (0 \leq i + 1 \leq n) \equiv C$$

$$\text{WP}[\mathcal{I} \mid b = 1](A) \equiv r > 2n - 2i - 1 \wedge (0 \leq i < n) \equiv D$$

$$\text{WP}[\mathcal{I} \mid b = 0](C, D) \equiv (b = 0 \wedge C) \vee (b = 0 \wedge D)$$

$$\equiv (b = 1 \wedge C) \vee (b = 0 \wedge D)$$

$$\equiv (b = 1 \wedge r > 2n - 2i - 1 - b \wedge (0 \leq i + 1 \leq n)) \vee (b = 0 \wedge r > 2n - 2i - 1 - b \wedge (0 \leq i < n))$$

$$\equiv r > 2n - 2i - 1 - b \wedge ((b = 1 \wedge 0 \leq i + 1 \leq n) \vee (b = 0 \wedge 0 \leq i < n))$$

$$\equiv r = 2n - 2i - b \wedge ((b = 1 \wedge 0 \leq i \leq n) \vee (b = 0 \wedge 0 \leq i < n))$$

$$\mathcal{L} = r = 2n - 2i - b \wedge ((b = 1 \wedge 0 \leq i < n) \vee (b = 0 \wedge 0 \leq i < n)) \equiv E$$

$$\Rightarrow r > 0$$

Esichert zu, dass  $r$  bei Betreten der Schleife immer positiv ist.

$$Z \equiv \text{true}$$

$$\text{WP}[\mathcal{I} \mid i \neq n](Z, E) \equiv (i = n) \vee (i \neq n \wedge E)$$

$$\equiv (i = n) \vee E \equiv r = 2n - 2i - b \wedge (i = n \vee (b = 1 \wedge 0 \leq i < n) \vee (b = 0 \wedge 0 \leq i < n))$$

$$\mathcal{L} = r = 2n - 2i - b \wedge ((b = 0 \wedge 0 \leq i \leq n) \vee (b = 1 \wedge 0 \leq i < n)) \equiv \mathcal{I}$$

Die Schleifeninvariante ist also lokal konsistent! Die Schleife terminiert also.

$$\text{WP}[\mathcal{I} \mid r = 2n - 2i - b](\mathcal{I}) \equiv (0 \leq i \leq n \wedge b = 0) \vee (0 \leq i < n \wedge b = 1) \equiv F$$

$$\text{WP}[\mathcal{I} \mid b = 0; i = 0](F) \equiv 0 \leq n \equiv G$$

$$\text{WP}[\mathcal{I} \mid n = -1 * n](G) \equiv 0 \geq n \equiv H$$

$$\text{WP}[\mathcal{I} \mid n < 0](G, H) \equiv 0 \leq n \vee n < 0 \equiv \text{true} \equiv K$$

$$\text{WP}[\mathcal{I} \mid n = \text{read}()](K) \equiv \text{true} \equiv L$$

Das Programm terminiert also immer!

□