Submission before: 14.12.2015
Discussion on: 15.12.2015
Submission on stud.ip, submission folder for sheet.
Please submit a zip file containing the .m files for Matlab programming tasks.

Exercise 1 *(Understanding Fourier Transform – 6p)*

This exercise aims at getting some intuition of finite, 2d-Fourier transform.

(a) Transform the image `dolly.png` into the frequency space (you may use the Matlab function `fft2`). The result will be a complex matrix. Plot histograms for the amplitude and phase values.

(b) Display the amplitude and phase in separate images. You may again take the logarithm of the amplitude to enhance the contrast. You may also center the base frequency (see Matlab function `fftshift`).

(c) Transform the image back from the frequency space to the image space (again using `fft2`). What do you observe? Explain and repair the result.

(d) Now restore the image, but only keep the amplitude and vary the phase. Try fixed phase values (0, $\pi/2$,...), a random phase matrix, or a noisy version of the original phase values.

(e) Do the same, but now keep the phase while varying the amplitude values. Again try constant, amplitude, randomly distributed amplitudes and noisy versions of the the original values.

Hint: Matlab has some functions to deal with complex numbers: `real` and `imag` provide the real and imaginary parts. `abs` and `angle` provide amplitude and phase. `conj` gives the complex conjugate. To compute a complex value z from amplitude A and phase ϕ you can use the formula $z = A \cdot (\cos(\phi) + i \cdot \sin(\phi))$.

Exercise 2 *(Implementing Fourier Transform – 8p)*

Now implement a finite 2d-Fourier transform:

(a) First implement a one-dimensional discrete version of Fourier transform, i.e. use the formula

$$c_n = \sum_{x=1}^{L} f(x) \cdot e^{\frac{2\pi i}{L} \cdot x} \qquad \text{for } n = 0, \dots, L-1$$

for complex valued coefficients. Plot the graph and the results of your Fourier transform, using the Matlab function `plot(1:L,func)`, for the following discrete functions:

(a) `func1 = sin(2*pi*(1:L)/L);`

(b) `func2 = zeros(1,L); func2(40:60)=1;`

(c) `func3 = gausswin(L,1./0.1)';`

When you plot the complex coefficients, you may use Matlab functions `abs` to get magnitude and `angle` to get the phase angle. Compare your results with the output of the Matlab function `fft`.

(b) Now implement a 2-dimensional version of Fourier transform for images, using the formula from the lecture. Compare your result with the output of `fft2`.

Exercise 3 *(Applying Fourier Transform – 6p)*

(a) Read the image text_deg.jpg, display it and apply Fourier transform. The resulting amplitude should show the angle of the text.

(b) Try to automatically get the rotation angle from the Fourier space. There are different ways to achieve this.

Hints:

- You may apply an erosion operation to strengthen the text sections and thereby get better (i.e. less noisy) amplitude values.
- You may threshold the amplitudes, to only keep "relevant" values. You can then compute the angle of the largest relevant value.
- Alternatively, you may apply methods you know from other lectures to get the main component and compute its angle.

(c) Rotate the image back to its originally intended orientation (*imrotate*).