ETH Zürich
D-GESS
Institute for Computational Social Science (COSS)

Seminar Paper

Braess Paradox and Information Supply

Why More Information Is Not Always Better

Controversies in Game Theory Seminar

Kevin Riehl, Cédric Zeiter, Vsevolod Semenov

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Abstract

Braess Paradox shows that providing more options through road infrastructure can result in a worse outcome for everyone, i.e., more congestion. An explanation to this paradox is that a population of rational, selfish individuals who try to maximise their own payoffs leads to a sub-optimal user equilibrium, also known as the Wardrop-equilibrium. Similarly, one can argue that this is not only the case with more options, but also with more provided information which can affect the societal outcome. Today, we are confronted with more options and online information than ever before, but at the same time our consumption behaviour leads to social and ecological problems. This essay aims to examine how the supply of more information affects the consumer decision making processes. In this report, the drivers' route choice behaviour in New York is modeled as a game, and simulated as a case studies in which we control the availability of information. The results exhibit how the drivers' decision behaviour is determined by how much and which type of information is available to them, and how they process it.

1. Introduction

Braess Paradox [1] describes a situation in which the provision with more road infrastructure to choose from, leads to worse outcomes for everyone, i.e., more congestion and longer travel times on average, which might seem counter-intuitive at first. This paradox can be explained by the idea that when a group of rational, selfish individuals all try to maximise their own payoffs through cost minimisation, the result is a sub-optimal user equilibrium, also known as the Wardrop-equilibrium [2]. Similarly, one can argue that not only the supply with more options, but also the supply with more information can affect the societal outcome [3]–[5]. When decisions are based on information, having more information allows individuals to adapt their behaviour better to maximise their own payoff.
Therefore, this study sets out to investigate how supply with more information affects the decision making process for rational individuals. To study the described behaviour, we developed a simulation case study that models drivers' route choices in New York as a game. The drivers are provided with travel time information based on personal experiences and historical reports, both considering varying time horizons.
The outline of this work is structured in the following fashion: Section 2 describes the case study. Section 3 discusses the simulation results and their implications. Section 4 summarises the key points of this work, critically assesses its limitations, and elaborates on future research. The code and the results can be found on this project's GitHub Repository.

2. Case Study: Route Choice Behaviour in New York

This essay investigates the role of information at the example of a congestion game in New York City, where drivers must decide a route to take for their daily commute. The travel times vary, depending on the choices of the entire involved population.

2.1. Background

New York City is the most populous and most densely populated city in the United States of America, with an estimated population of 8.3 million people, and a land area of 1.2 square kilometers. New York is located at the southern tip of New York State, and divided into five boroughs, that are separated by rivers and the sea. Culturally and economically, it is one of the most vibrant cities, being home to financial institutions (New York Stock Exchange), and to headquarters of international corporations and organizations alike (United Nations). Due to its flourishing economy, New York and especially the borough Manhattan, attract many visitors and commuters [6].
The neighbouring state New Jersey is home to a large share of Manhattan's commuting workforce (around 1.23 million people per day). New Jersey and Manhattan are separated by the Hudson river. There are mainly three connections, which drivers can use to cross the river: the Holland Tunnel (Interstate 78), the Lincoln Tunnel (Route 495), and the George Washington Bridge (Interstate 95), as depicted in Fig. 2.1. Together, these three connections handle more than 493,000 vehicles per day [7]. The Holland Tunnel consists of two tubes, has an operating speed of 56 km/h, a length of around 2.5 km, 9 lanes, and is used by around 89,792 vehicles per day [8]. The Lincoln Tunnel

consists of three tubes, has an operating speed of 56 km/h, a length of around 2.4 km, 6 lanes, and is used by around 112,995 vehicles per day [9]. The George Washington Bridge consists of two decks (levels), has an operating speed of 72 km/h, a length of around 1.4 km, 14 lanes, and is used by around 289,827 vehicles per day, making it the world's busiest vehicular bridge [10]. The connections are separated by 4.37 km and 10.87 km respectively [11].

Unfortunately, New York is the city with the worst traffic conditions in America. On a daily average, vehicles spent 154 seconds per kilometer travelled in New York City (average speed 20 km/h) during rush hour, which sums up to 112 wasted hours per year and per vehicle due to congestion [1]. Furthermore, constructions, planned maintenance, scheduled overnight closures, and security incidents cause bottlenecks due to regular closure on these important routes.

Figure 2.1: New Jersey and Manhattan (New York)

2.2. The Decision Problem

For the case study, let us assume, that the Holland Tunnel is irreparably damaged. Therefore, the drivers must decide between taking the Lincoln Tunnel or the George Washington Bridge. A drive from A to B in our case study map (Fig. 2.1) would be around 26.55 kilometers (35 minutes free flow) via the Lincoln Tunnel and 49.89 kilometers (40 minutes free flow) via the George Washington Bridge. Before the closure, it was only around 17.38 kilometers (24 minutes free flow) via the Holland Tunnel. The drivers coming from Interstate 95, which usually take the Holland Tunnel, now try to use the closest alternative passage: the Lincoln Tunnel. As a consequence, the Lincoln Tunnel is facing congestion.

Rational drivers always choose a route that minimises their costs. A population of such drivers thus converges to an equilibrium split (flow of vehicles per hour), following Wardrop [2]. Each driver bares two types of costs: Fuel-costs and delay-costs based on their value of time (VOT). We assume an average vehicle consumption of 6.5 l/100km (36 mpg) [2], and a fuel price of 0.96 $/l [3].

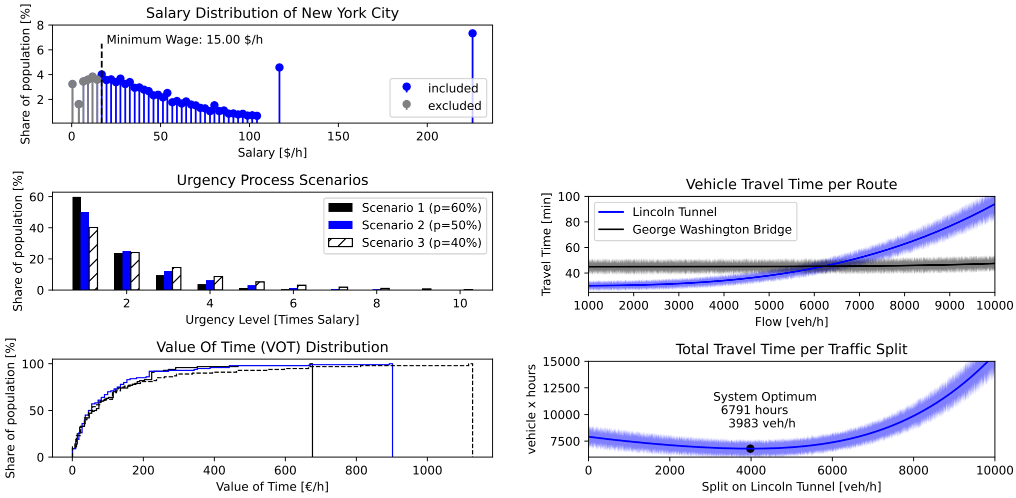
The driver's VOT is calculated as the product of their salary and urgency level, where the k^{th} urgency level represents delay costs of k times the hourly wage [4] (value of time VOT). Fig. 2.2(A) depicts the salary and urgency distribution of the population. We

[1] TomTom Traffic Index Ranking 2023. `https://www.tomtom.com/traffic-index/ranking/?country=US`

[2] Average US fleet fuel consumption in 2021, according to US transportation secretary Pete Buttigieg. `https://edition.cnn.com/2022/04/01/energy/fuel-economy-rules/index.html`

[3] Average US fuel price. `https://tradingeconomics.com/united-states/gasoline-prices`

[4] The salary distribution of New York city according to 2022 US Census data is assumed: `https://en.wikipedia.org/wiki/Household_income_in_the_United_States#Distribution_of_household_income`.



(A) Population Model **(B) Travel Time Model**

Figure 2.2: Case Study: Route Choice Behaviour in New York City

model the population with ten urgency levels (1 to 10), where the urgency levels are assumed to be randomly-geometrically distributed in three different scenarios (p=0.6, p=0.5, p=0.4). Following discussions are restricted to the conservative scenario (p=0.6). Fig. 2.2(B) depicts the travel time model. We assume a total traffic volume of 10,000 veh/h, which is split across the two routes. While the Lincoln Tunnel has a shorter travel time upfront, it gets congested quickly, and after 6,000 veh/h it is much slower compared with the alternative route. The route via George Washington Bridge offers slower travel times, but higher capacity, less congestion and delays, even for larger flows. From a system optimal point of view, a minimum total travel time of 6,791 vehicle hours (40.75 minutes average travel time) can be achieved, if the total flow (10,000 veh/h) splits to 3,983 veh/h on the Lincoln route and 6,017 veh/h on the George Washington route. Unfortunately, Wardrop's user equilibrium lies at a split of 6,169 veh/h on the Lincoln route (45.01 minutes average travel time), which causes 4.26 minutes of additional travel time to every vehicle on average (almost 10 % higher total travel times).

2.3. Drivers' Time Estimation Models

We assume that each driver $i \in \{1, ..., n\}$ bases their decision on estimates of the travel time $\hat{T}_r$ for route r with two types of information: Previous personal experiences $\mathcal{I}_r^P$, and reported travel times of all agents $\mathcal{I}_r^R$. Each information is a list of the h (information horizon) most recent time-steps. The horizon can be used to control the amount of information, which is available to the drivers. The estimated travel time $\hat{T}_r$ is a weighted sum of two separate travel time estimates $\hat{T}_r^P$ and $\hat{T}_r^R$. Each estimate only depends on the personal history $\mathcal{I}_r^P$ or the reported data $\mathcal{I}_r^R$ with weights w^P and w^R, respectively:

$$\hat{T}_r = \frac{T_r^R \times w^R + T_r^P \times w^P}{w^R + w^P} \tag{2.1}$$

All the models try to process the personal knowledge and publicly reported data of the routes. The publicly reported data could represent government traffic statistics that

are published every day or commercially interpreted data, like the time estimations of navigation systems. Moreover, we assume that all agents decide according to the same model in each experiment.
We model the (human) information processing with four distinct schemes to estimate travel times $\hat{T}_r^P$ and $\hat{T}_r^R$ based on information sets $\mathcal{I}_r^P$ and $\mathcal{I}_r^R$. Each value $v \in \mathcal{I}$ is sorted by time, where v_j represents the index of v in $\mathcal{I}$. One scheme calculates the plain average over previous travel times. Other schemes apply geometric or exponential weights, where the more recent information is weighted higher. In addition, a scheme considering only the maximum travel time from the available information set is also considered. These schemes represent different aspects of human information processing, including weighted average schemes to account for memory loss, and maximum scheme to account for overvaluation-bias of extreme past events. Table 2.1 summarises the estimation schemes.

Scheme	**Formula**
Plain Average	$\hat{T}_r^{\mathcal{I}} = \frac{1}{h}\sum_{v\in\mathcal{I}} v$
Geometric Weights	$\hat{T}_r^{\mathcal{I}} = \frac{1}{h}\sum_{v\in\mathcal{I}}(v \times w_v)$ with $w_v = \frac{1}{v_j}$
Exponential Weights	$\hat{T}_r^{\mathcal{I}} = \frac{1}{h}\sum_{v\in\mathcal{I}}(v \times w_v)$ with $w_v = exp(v_j)$
Maximum Scheme	$\hat{T}_r^{\mathcal{I}} = max_{v\in\mathcal{I}}(v)$

Table 2.1: Different Travel Time Estimation Schemes

2.4. Simulation

Algorithm 1 summarises the daily decision process of drivers. In each iteration (daily commute), every agent of the population decides to take one route, either Lincoln Tunnel or George Washington Bridge. Before doing so, the driver estimates travel times and travel costs consecutively for both routes based on their available information. In most cases, drivers choose the cost-minimizing route (rational behaviour). However, with an exploration rate γ, the driver may choose to deviate from the rationally optimal route.

Algorithm 1 Simulation of Drivers' Daily Route Choice

for each *day* **do**
    for each *driver* **do**
        $\hat{T}_A, \hat{T}_B \leftarrow EstimationScheme(\mathcal{I}_A^P, \mathcal{I}_A^R, \mathcal{I}_B^P, \mathcal{I}_B^R, w^P, w^R)$
        $\hat{C}_A, \hat{C}_B \leftarrow Cost(\hat{T}_A, \hat{T}_B)$
        if $\hat{C}_A < \hat{C}_B$ **then**
            $Choice \leftarrow A$
        else if $\hat{C}_B < \hat{C}_A$ **then**
            $Choice \leftarrow B$
        else
            $Choice \leftarrow random(A, B)$
        end if
        if $random(0, 1) \leq \gamma$ **then**
            if $Choice = A$ **then**
                $Choice \leftarrow B$
            else if $Choice = B$ **then**
                $Choice \leftarrow A$
            end if
        end if
        Driver Takes Route: *Choice*
    end for
    Update Information Sets: $\mathcal{I}_A^P, \mathcal{I}_A^R, \mathcal{I}_B^P, \mathcal{I}_B^R$
end for

3. Results & Discussion

The simulations were conducted for 500 consecutive days, different exploration rates $\gamma \in \{0\ \%, 1\ \%, 2\ \%, 5\ \%\}$ and information horizons $h \in \{5, 10, 15, 50\}$. Moreover, we simulate four pairs of weights (w^R, w^P) representing five distinct information availability situations: only personal information ($w^R = 0$, $w^P = 1$), only reported information ($w^R = 1$, $w^P = 0$), both information ($w^R = 1$, $w^P = 1$). Additionally, the latter was conducted with a focus on personal information ($w^R = 1$, $w^P = 2$) and a focus on reported information ($w^R = 2$, $w^P = 1$).
The availability of information, information horizons, and exploration rates affected the population's speed of convergence towards and stability of the Wardrop-equilibrium, and noise levels. In our model, the different levels of information availability and horizons do not change the position of the user equilibrium. An overview over all simulations is available in the Appendix (see Figures A.1, A.2, A.3 & A.4). In the following, we discuss the most important observations for each proposed decision making scheme.

3.1. Plain Average Scheme

The first observation is, that higher exploration rates γ lead to faster convergence, but also lead to more noise and stronger oscillations (Fig. 3.1(A)). Contrary to having only personal information, taking only the reported data into account, leads to non-convergence and oscillating behaviour of the population (Fig. 3.1(B)). This observation holds for all investigated schemes.
Combining personal and reported information, with a focus on personal information (2P-1R) leads to a fast convergence and high stability of the user equilibrium(Fig. 3.1(D)). The equal weights (Fig. 3.1(C)) or a focus on reported information (Fig. 3.1(E)) lead to more unstable equilibria and stronger oscillations. These observations are alike for the all tested weighted average schemes (see Figures A.1, A.2 & A.3).

3.2. Weighted Average Scheme

Generally, the weighted average schemes behave similarly to the plain average scheme. Two observed differences are: (i) the weights introduce more oscillations, when compared to the plain average scheme, and (ii) the weighted average schemes react more sensitive to different time horizons (Fig. 3.2).
With the geometric weights a larger horizon provides us with significantly smaller spikes. The amount of these spikes seems to stay the same for different horizons. Exponential weights exhibit higher instability of the user equilibrium.

3.3. Maximum Scheme

The maximum scheme provides substantially different results (Fig. A.4). Shorter time horizons and larger exploration rates lead to faster convergence (Fig. 3.3(A)&(B)). The scheme fails to remain at the highly unstable equilibrium when reported information is taken into account (Fig. 3.3(C) & Fig. A.4).

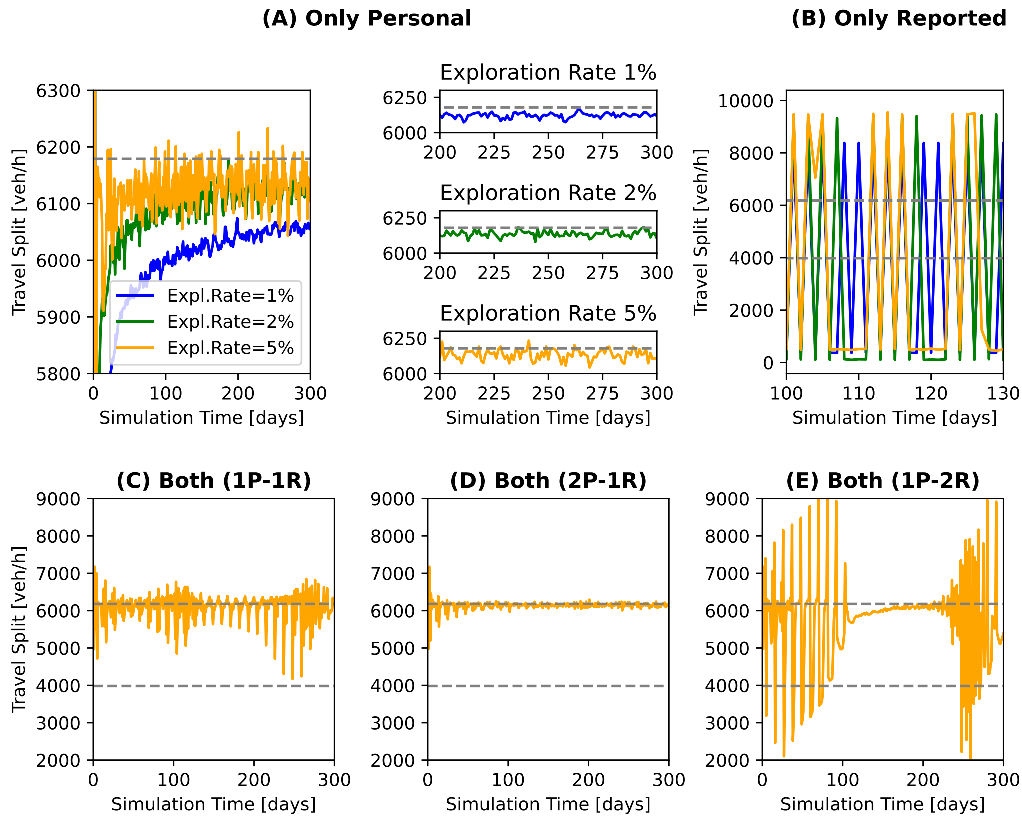


Figure 3.1: Plain Average (Horizon 10)

4. Conclusions

This study investigated how the supply with more information affects the decision making process of rational individuals. For this purpose, we developed a simulation case study that models drivers' route choice behaviour in New York as a game, where drivers are supplied with travel time information based on personal experiences and historical reports of varying time horizons.
The results show that the decision behaviour depends on four factors: (i) how much information is available, (ii) which type of information (personal vs. reported), (iii) how agents estimate travel times, and (iv) deviation from rational behaviour due to the exploration rate.

- The use of personal-experience information usually leads to stability of user equilibria, while reported, historical travel time information usually causes oscillations. A more realistic scenario, where drivers consider the combination of both information sources, provided mixed population behaviours.
- Higher exploration rates resulted in faster convergence to the Nash-equilibrium, at the cost of higher noise and oscillations.
- The way drivers employ the available information to estimate travel times and costs, has a strong influence on the population's behaviour. For the plain and weighted average schemes, longer horizons were stabilising the equilibria and

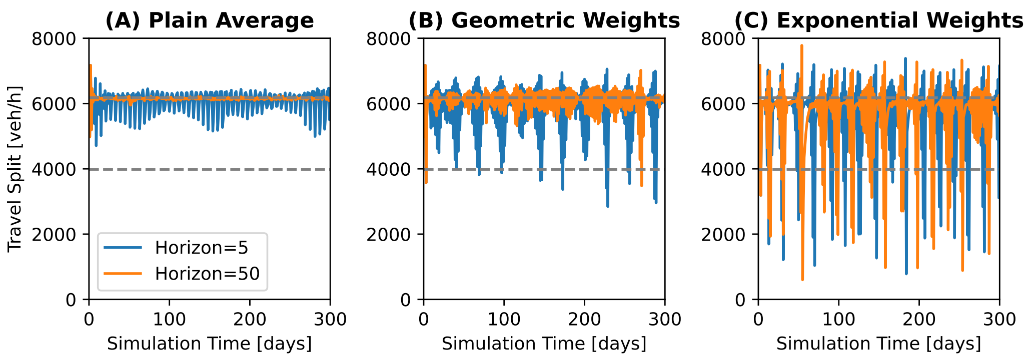


Figure 3.2: Different Weighting Schemes Compared (2P-1R, 5 % Exploration Rate)

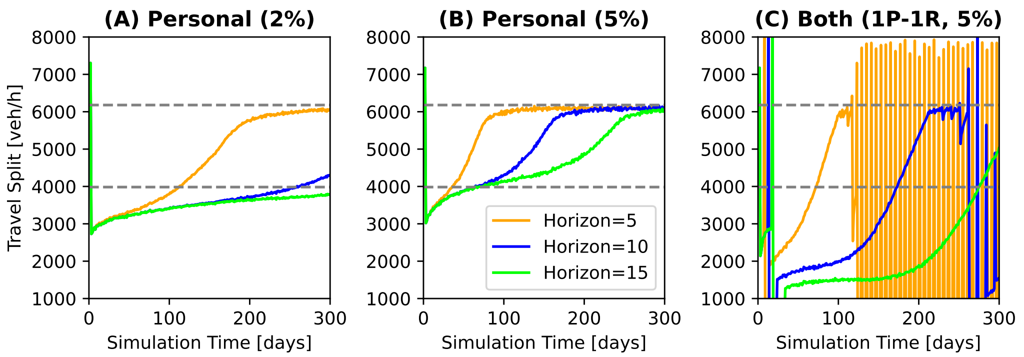


Figure 3.3: Maximum Scheme

accelerating convergence. For the maximum scheme, shorter horizons led to faster convergence.

- Available reported information does not necessarily support the population to converge to its equilibrium state. More information (in the sense of longer horizons) affects the convergence speed and stability of equilibria (depending on the scheme).

While providing first insights, this study is limited to the assumed decision making model and estimation schemes. Moreover, a homogeneous population of drivers with identical information processing is assumed.
Future works could delve into heterogeneous populations with different information availability and information processing. Moreover, more complex scenarios with multiple routes could be taken into account. Finally, one could investigate how systematic misinformation of reported data could lead to a transition of the user equilibrium towards the system optimum.

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A. APPENDIX

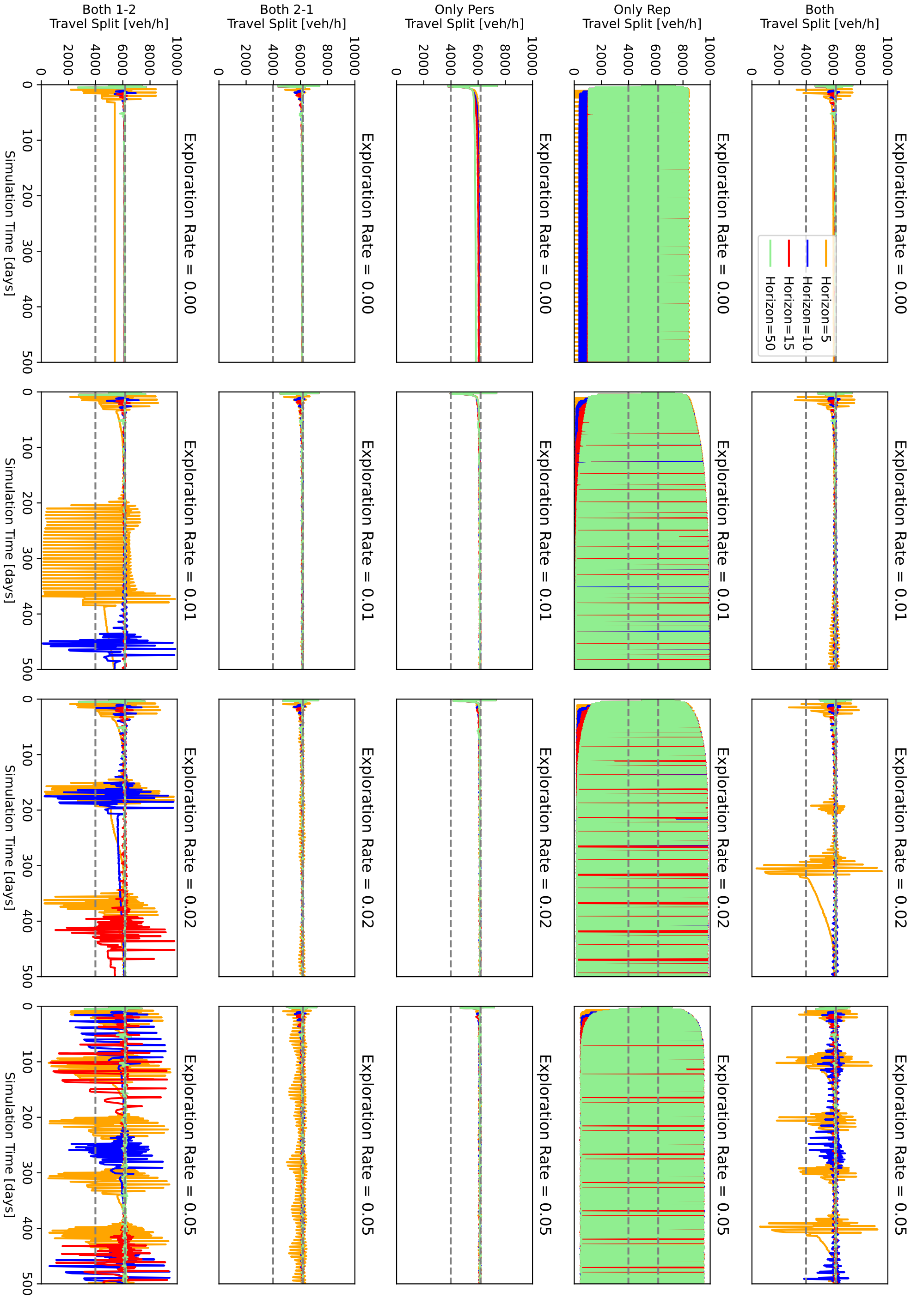

Figure A.1: Time Estimation Method 1: Plain Average

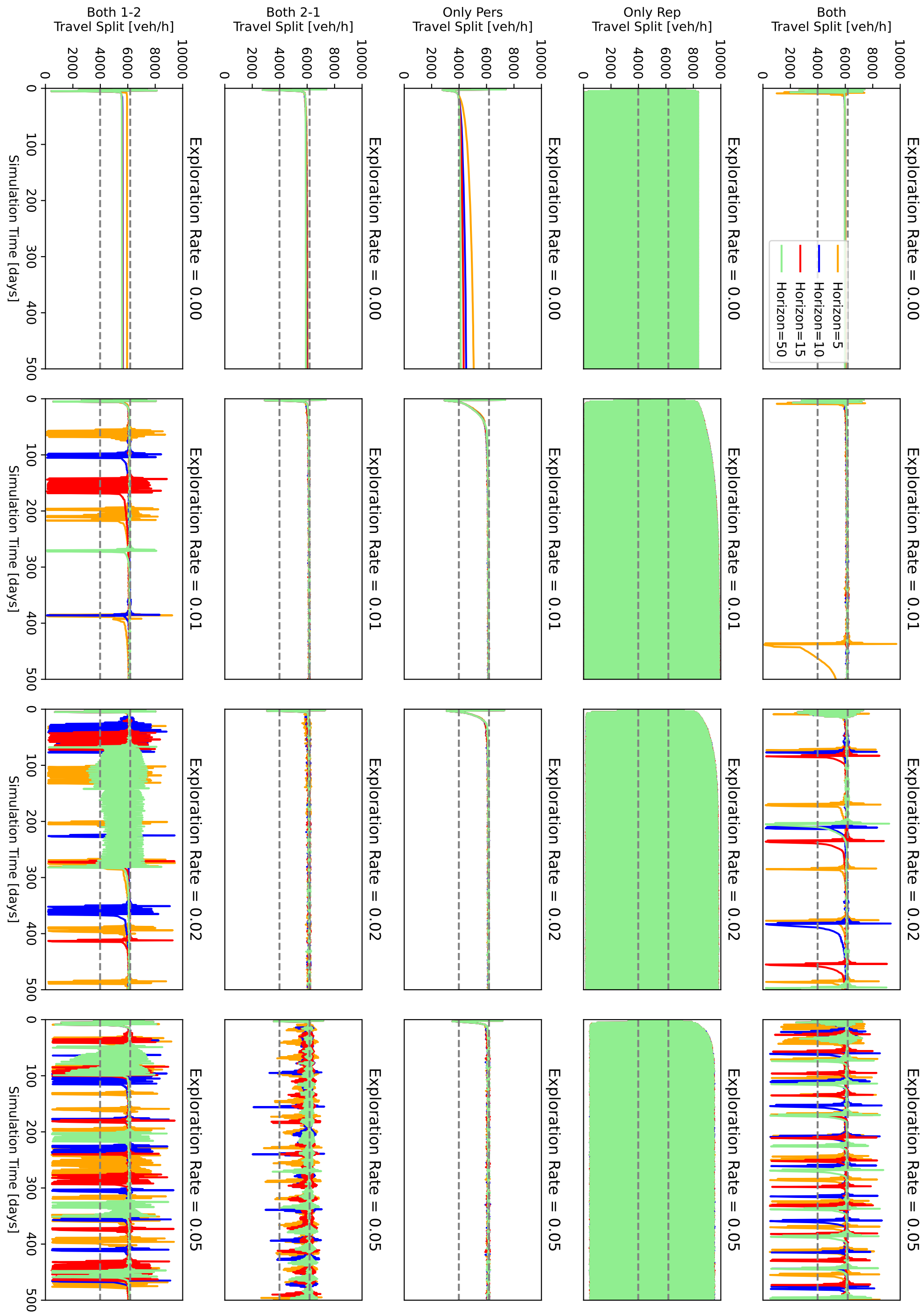

Figure A.2: Time Estimation Method 2: Geometrically Weighted Mean

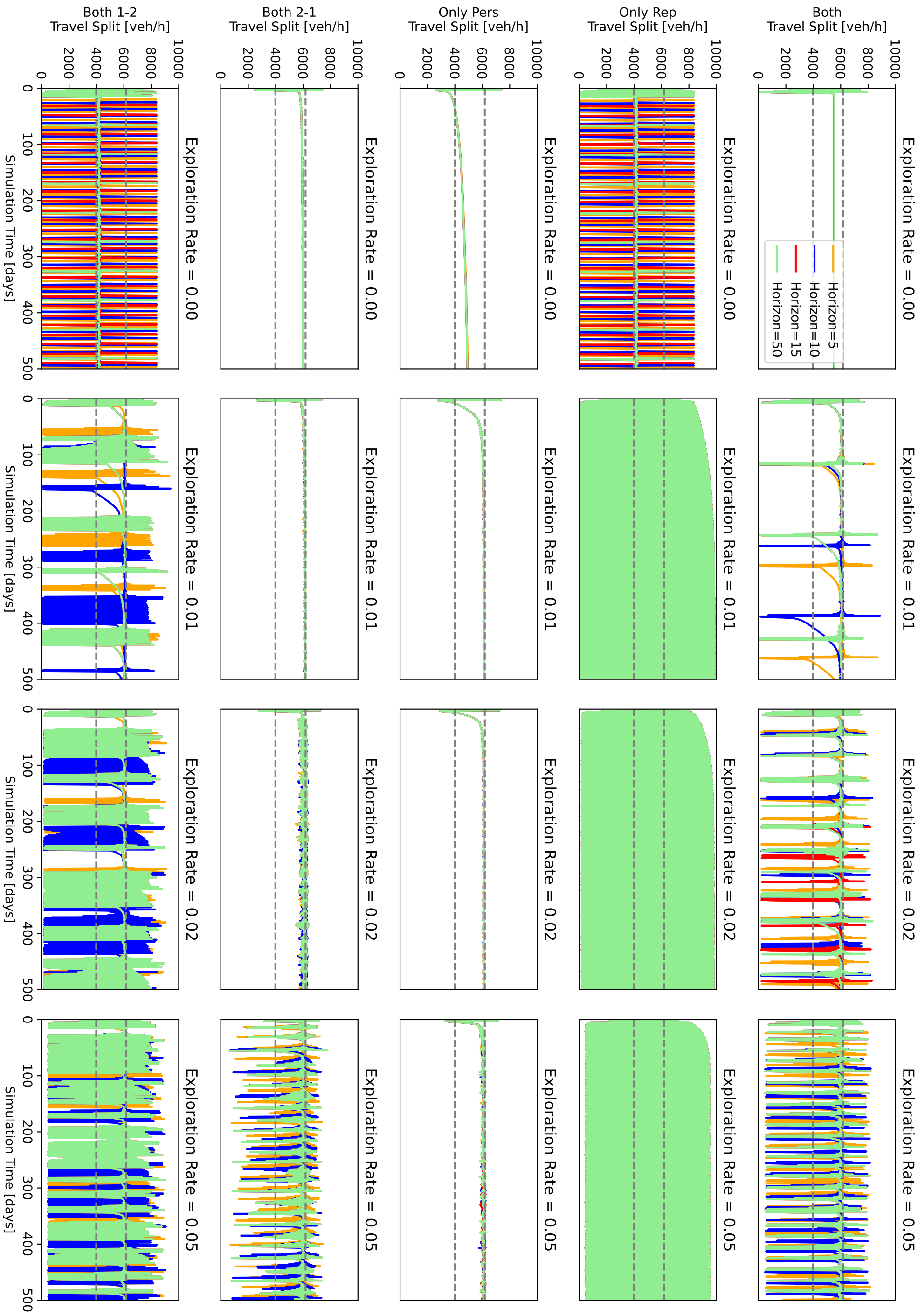

Figure A.3: Time Estimation Method 3: Exponentially Weighted Mean

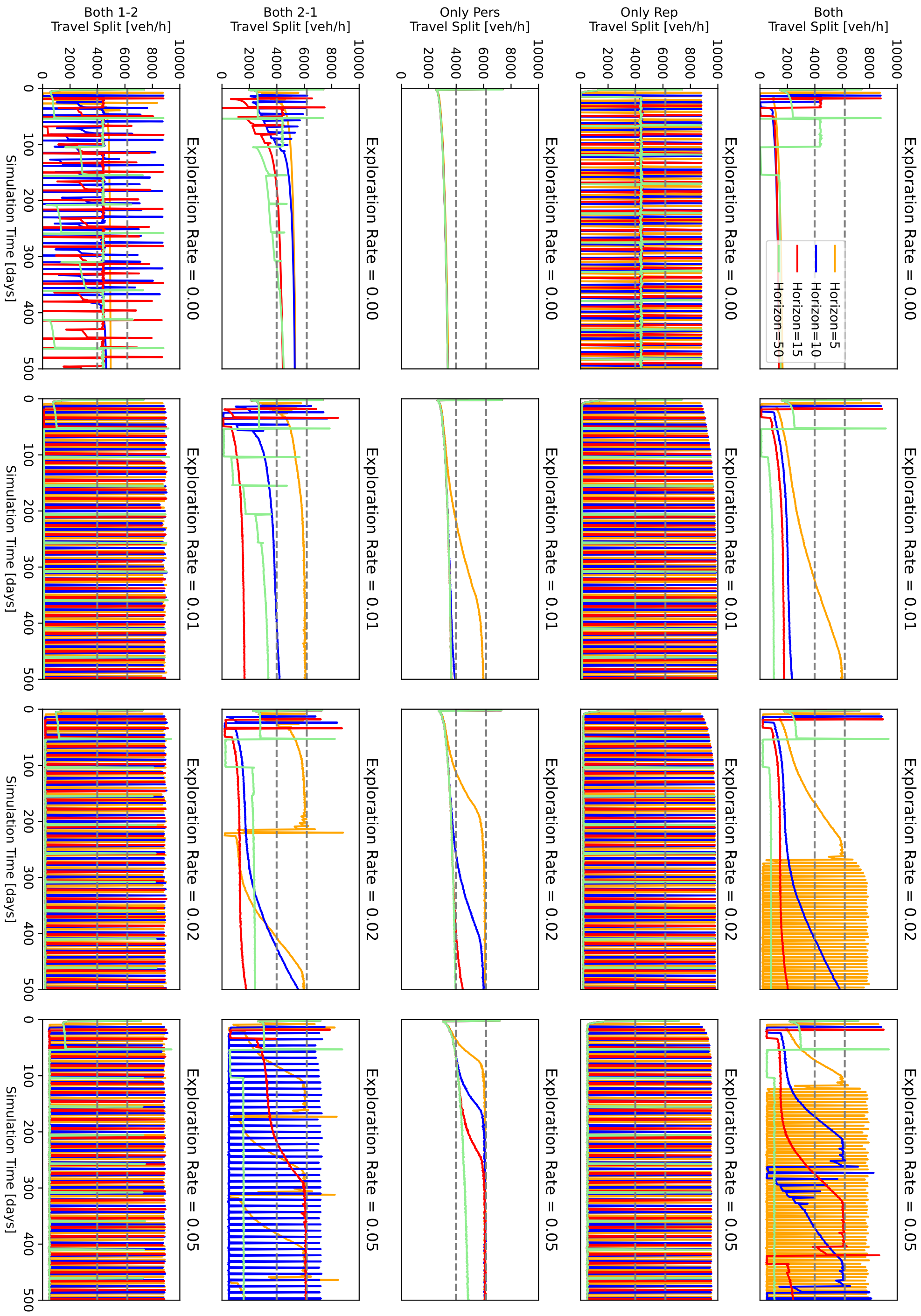

Figure A.4: Time Estimation Method 4: Maximum