Fusion Moves for Correlation Clustering

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Introduction

- Correlation clustering [4], or multicut partitioning [5] , is widely used partitioning an undirected graph with positive and negative edge weights [2, 3, 8, 1].
- NP-hard, exact solvers do not scale and approximative solvers often give bad results.
- Inspired by [6] we define fusion moves for the correlation clustering where we iteratively fuses the current and a proposed partitioning and monotonously improve the partitioning
- Scales well, gives near optimal solutions, has good anytime performance

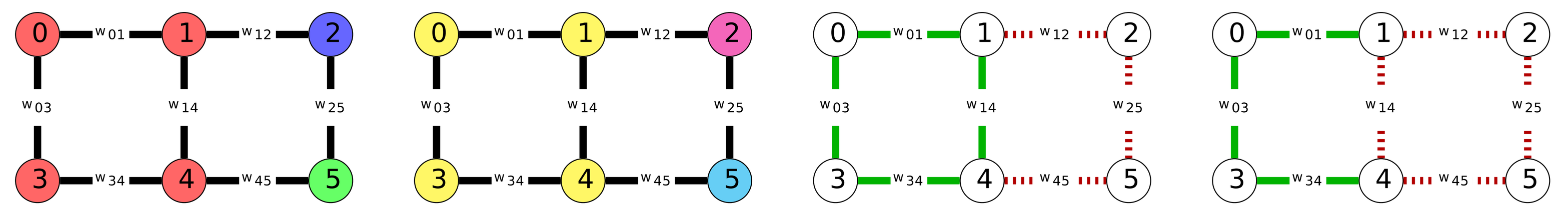
Correlation Clustering / Multicut Objective:

- Given a weighted graph $\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{w})$
- We consider the problem of segmenting $\mathbf{G}$ such that the costs of the edges between distinct segments is minimized.
- Can be formulated in the node domain by assigning each node $\mathbf{i}$ a label $\mathbf{l_i} \in \mathbb{N}$

$$\mathbf{l}^* = \underset{\mathbf{l} \in \mathbb{N}^{|\mathbf{V}|}}{\arg\min} \sum_{(\mathbf{i},\mathbf{j}) \in \mathbf{E}} \mathbf{w_{ij}} \cdot [\mathbf{l_i} \neq \mathbf{l_j}], \tag{1}$$

- Can be formulated in in the edge domain, by labeling each edge $\mathbf{e}$ as cut $\mathbf{y_e} = \mathbf{1}$ or uncut $\mathbf{y_e} = \mathbf{0}$

$$\mathbf{y}^* = \underset{\mathbf{y} \in \mathbf{P}(\mathbf{G})}{\arg\min} \sum_{(\mathbf{i},\mathbf{j}) \in \mathbf{E}} \mathbf{w_{ij}} \cdot \mathbf{y_{ij}}. \tag{2}$$



(a) node labels (b) node labels (c) edge labels (d) dangling edge

Correlation Clustering Fusion Moves

- Given two proposal solutions $\mathbf{y}'$ and $\mathbf{y}''$, $\mathbf{E}_0^{\check{\mathbf{y}}}$ is the set of edges which are uncut in $\mathbf{y}'$ and $\mathbf{y}''$.

$$\check{\mathbf{y}}_{\mathbf{ij}} = \max\{\mathbf{y}'_{\mathbf{ij}}, \mathbf{y}''_{\mathbf{ij}}\} \qquad \forall \mathbf{ij} \in \mathbf{E} \tag{3}$$

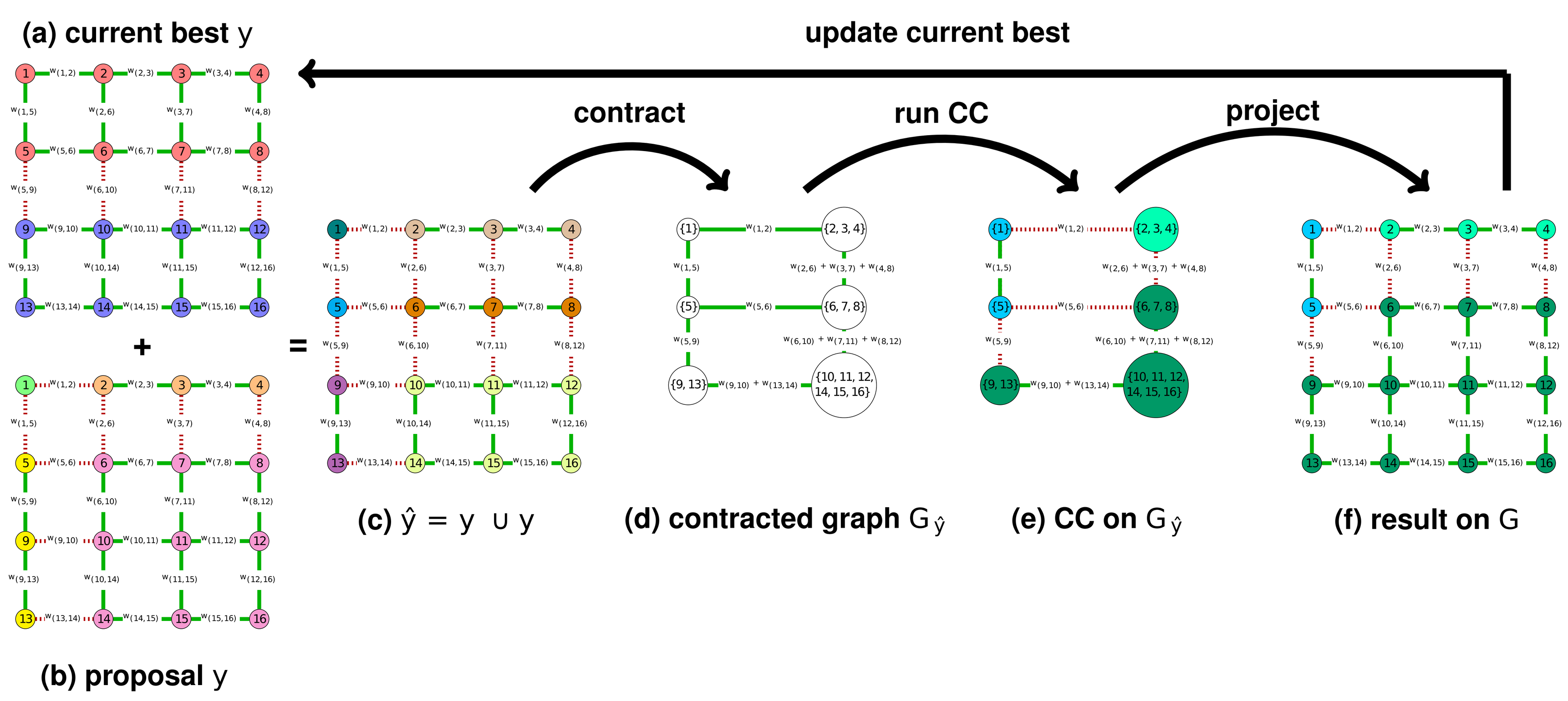
$$\mathbf{E}_0^{\check{\mathbf{y}}} = \{\mathbf{ij} \in \mathbf{E} \mid \check{\mathbf{y}}_{\mathbf{ij}} = \mathbf{0}\} \tag{4}$$

- The fusion move for correlation clustering is solving Eq. 2 with additional *must-link constraints* for all edges in $\mathbf{E}_0^{\check{\mathbf{y}}}$.

$$\mathbf{y}^* = \underset{\mathbf{y} \in \mathbf{P}(\mathbf{G})}{\arg\min} \sum_{(\mathbf{i},\mathbf{j}) \in \mathbf{E}} \mathbf{w_{ij}} \cdot \mathbf{y_{ij}}. \tag{5}$$

$$\text{s.t.} \quad \mathbf{y_{ij}} = \mathbf{0} \qquad \forall (\mathbf{i},\mathbf{j}) \in \mathbf{E}_0^{\check{\mathbf{y}}}$$

- We can reformulate 5 into a correlation clustering problem on a coarsened graph, where all nodes which are connected via must-link constraints are merged into single nodes. We call this graph a *contracted graph*.
- Any clustering $\bar{\mathbf{y}}$ of the contracted graph $\mathbf{G_y} = (\mathbf{V_y}, \mathbf{E_y})$ can be *back projected* to a clustering $\tilde{\mathbf{y}}$ of the original graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$



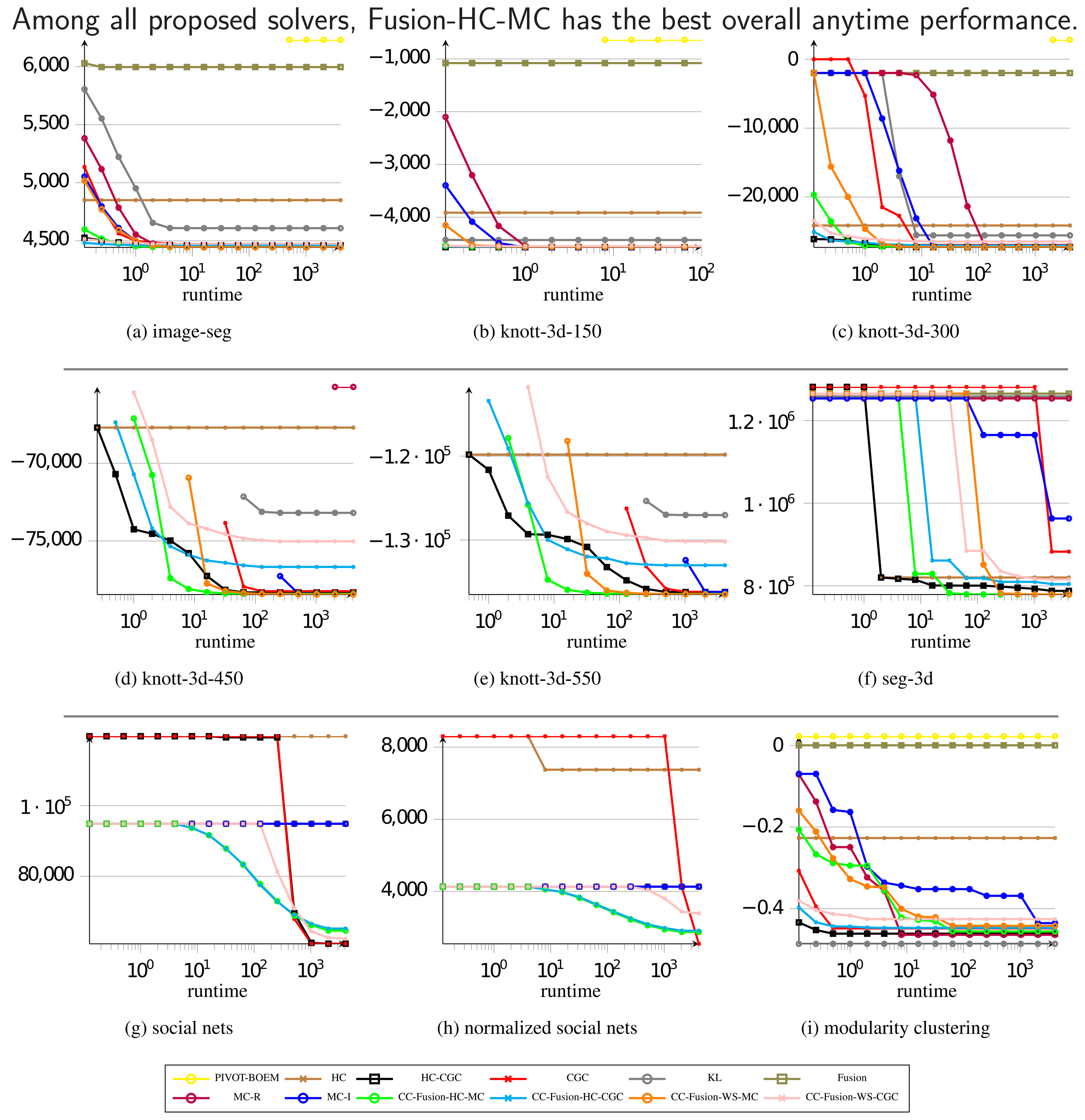
Proposal Generators

As discussed in [6], proposals should have two properties: *high quality* and *large diversity*. For correlation clustering fusion we add a third property: *size*. We use the following two proposal generators (see paper for details):

- **Randomized Hierarchical Clustering (RHC):** we add normally distributed noise $\mathcal{N}(0, \sigma_{ehc})$ to each edge weight. In each step the edge with the highest weight is contracted.
- **Randomized Watersheds (RWS):** The edge weighted watershed algorithm [7] with random seeds can be used to find cheap proposals.

Results

Among all proposed solvers, Fusion-HC-MC has the best overall anytime performance.



(a) image-seg (b) knott-3d-150 (c) knott-3d-300 (d) knott-3d-450 (e) knott-3d-550 (f) seg-3d (g) social nets (h) normalized social nets (i) modularity clustering

Evaluation by Segmentation Metrics

Evaluation by Variation of Information (VOI) and Rand Index (RI) for datasets with available ground truth:

VOI	image-seg	knott-3d-150	knott-3d-300	knott-3d-450	3d-seg
PIVOT-BOEM	4.9633	2.9936	4.4986	–	–
HC	2.5967	1.5477	2.3513	2.9155	2.8395
HC-CGC	2.5164	0.9052	1.7636	2.2256	1.7603
CGC	2.5247	0.9267	1.8822	2.3104	6.8908
KL	2.6432	2.0648	4.1318	4.9270	7.1057
FUSION	2.1406	2.8787	4.0744	4.6616	6.5366
MC-R	2.5471	0.9178	1.6369	2.8710	6.5058
MC-I	2.5367	0.9063	1.6352	2.0037	4.3319
CC-Fusion-HC-MC	2.5319	0.9629	1.6516	2.0801	1.3347
CC-Fusion-HC-CGC	2.4961	0.9679	1.7673	2.3809	2.1347
CC-Fusion-WS-MC	2.5340	0.9629	1.6742	2.0739	1.3334
CC-Fusion-WS-CGC	2.5192	1.0585	2.1344	2.7487	3.3514

RI	image-seg	knott-3d-150	knott-3d-300	knott-3d-450	3d-seg
PIVOT-BOEM	0.7438	0.7851	0.8792	–	–
HC	0.7560	0.8139	0.8084	0.7610	0.9651
HC-CGC	0.7724	0.9226	0.8713	0.8433	0.9861
CGC	0.7590	0.9206	0.8666	0.8341	0.6024
KL	0.6400	0.8085	0.6858	0.6409	0.5849
FUSION	0.5480	0.2849	0.1420	0.0998	0.0345
MC-R	0.7822	0.9232	0.8849	0.6713	0.0432
MC-I	0.7821	0.9236	0.8849	0.8670	0.5461
CC-Fusion-HC-MC	0.7801	0.9042	0.8824	0.8573	0.9906
CC-Fusion-HC-CGC	0.7780	0.9031	0.8763	0.8470	0.9775
CC-Fusion-WS-MC	0.7825	0.9042	0.8802	0.8582	0.8895
CC-Fusion-WS-CGC	0.7750	0.8951	0.8596	0.8394	0.9906

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