Exercise 8

Deadline: 26.06.2017, 2:15 pm
Regulations: You should hand in the exercises in groups of two or three persons. Please send a *compressed* (!) directory or file containing your solutions including all graphics, descriptions and source code to *thorsten.beier@iwr.uni-heidelberg.de*. The subject line of this email should start with [MLCV17][EX08] followed by the full names of all group members. Please cross-reference your code files in your writeup, such that it is clear which file has to be run for each exercise.

1 Random Walker

Theory[1]:

The random walker minimizes $x^T L x$ for each seed. L is the Laplacian of the weighted graph and x_i is the probability that a seed arrives first a a node i.

The laplacian of the weighted graph is given by:

$$L_{ij} = \begin{cases} d_i & i = j \\ -w_{ij} & i \neq j \end{cases} \tag{1}$$

where d_i is the degree (number of neighbors) of node i.

The laplacian needs to be normalized to have a row sum of zero!!!

The weight $w_i j$ is a decreasing function of the norm of the local gradient. This ensures that diffusion is easier between pixels of similar values.

When the Laplacian is decomposed into blocks of marked and unmarked pixels:

$$L = \begin{bmatrix} M & B^T \\ B & A \end{bmatrix} \tag{2}$$

minimizing $x^T L x$ for a seed is equivalent to:

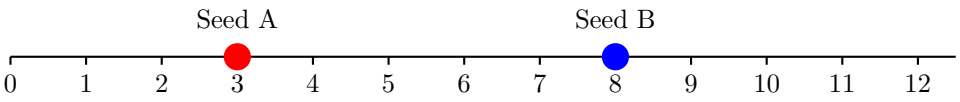
$$Ax = -Bx_m \tag{3}$$

where $x_m = 1$ for the current seed and zero for the other seeds.

For simplicity we will use only two seeds.

1.1 1D Random walker (5P)

Given a 1D image with constant intensity and two seeds as shown below. Show the probabilities for each seeds as function of the position in the 1D image.



[1] based on `https://tinyurl.com/skimage-random-walker`

1.2 2D Random Walker: Implementation (10P)

Implement the random walker algorithm for two seeds. Use $w_{ij} = \exp(-\gamma \cdot ||c_i - c_j||)$ as weights where c_i and c_j are the colors / gray values at pixel i and j.

1.3 Experiments I (1P)

Use an image as image A as input to your algorithm.

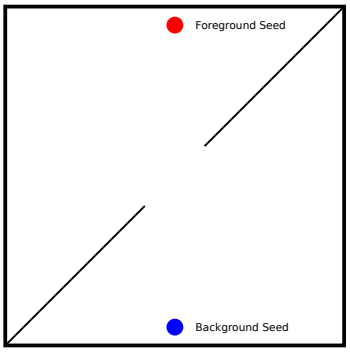


Image A

Show the pixel wise probabilities and the resulting segmentation.

1.4 Experiments II (4P)

What effect has the position of the seeds. Experiment with different positions of the seeds. Also change γ and report what effect different values of γ have.

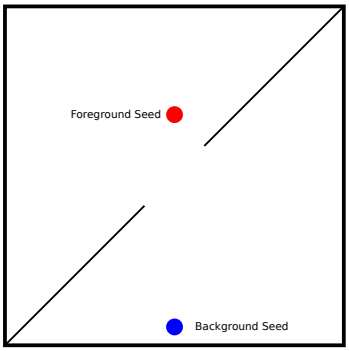


Image B

Show the pixel wise probabilities and the resulting segmentation for different positions of the seeds.

2 Random Walker as Anisotropic Diffusion (Bonus: 10P)

The random walker algorithm solves the diffusion equation at infinite times for sources placed on seeds of each iteration in turn. A pixel is labeled with the seed that has the greatest probability to diffuse first to the pixel. Show the relation between random walks and and the diffusion equation in one dimension.