

# Assignment 1.

## Problem 1:

$$\begin{aligned} 1. P(z|y, x) &= \frac{P(x, y, z)}{P(x, y)} = \frac{P(y|z, x) P(z, x)}{P(x, y)} \\ &= \frac{P(y|z, x) P(z|x) \cancel{P(x)}}{P(y|z) \cancel{P(x)}} \\ &= \frac{P(y|z, x) P(z|x)}{P(y|z)} \end{aligned}$$

$$2. P(x) = \sum_y P(x, y)$$

$$3. E[X] = \int_{-\infty}^{+\infty} x f(x) dx \quad E[Y] = \int_{-\infty}^{+\infty} y f(y) dy$$

$f(x), f(y)$  are the possibility density function of  $x$  and  $y$ .

$$f(x) = P[X=x], \quad f(y) = P[Y=y]$$

$$E[XY] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} xy f(x, y) dx dy$$

$x, y$  independent

$$= \int_{-\infty}^{+\infty} x f(x) dx \int_{-\infty}^{+\infty} y f(y) dy$$

$$= E[X] \cdot E[Y]$$

4. The posterior probability that the red ball came from box 1 is formed as:

$$P(B=1 | \text{color}=\text{red}) = \frac{P(\text{color}=\text{red} | B=1) P(B=1)}{P(\text{color}=\text{red})}$$

$$= \frac{\frac{7}{10} \times 0.5}{\frac{7+3}{10+12}}$$

$$= 0.77$$

Problem 2:

$$1. P(I' | I^0, d) = \prod_{i,j} P(I'_{i,j} - d_{i,j} | I^0_{i,j}, d_{i,j})$$

Given the corresponding pixels in the first image and the corresponding disparity, the pixels in the second image are conditional independent.

2. The limitations of the Gaussian likelihood function:

Occlusions, shading, shadows and gain control can make the assumption inappropriate. It is very sensitive to outliers.

Alternative: Robust likelihood function, for example Lorentzian.

3. Assumption: Each edge corresponds to a term that models how compatible two neighboring pixels are in terms of their disparity.

In Figure 1  $d_3$  is not in the Markov blanket of  $d_1$ .

The 4 neighboring nodes around  $d_1$  or  $d_3$ .

$$4: f_H(d_{i,j} - d_{i+1,j}) = \frac{1}{Z(T)} \exp \left\{ \frac{1}{T} \delta(d_{i,j} - d_{i+1,j}) \right\}, \delta(a-b) = \begin{cases} 1, a=b \\ 0, a \neq b \end{cases}$$

The compatibility function used in the Potts model is not differentiable.