Computer Vision II – Homework Assignment 1
Solution from Group 28

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Problem 1 – Probabilities and Statistics

1. The proof process is as follows:

$$\frac{p(y|z,x)\cdot p(z|x)}{P(y|x)} = \frac{p(y|z,x)\cdot \frac{p(z,x)}{p(x)}}{p(y|x)} = \frac{p(x,y,z)}{p(x,y)} = p(z|y,x)$$

2. The density function of x can be computed by:

$$p(x) = \sum_{y}^{n} p(x,y)$$

3. From the expectation formula, we can get:

$$E[X] = \int_{-\infty}^{+\infty} xf(x)dx \quad , \quad E[Y] = \int_{-\infty}^{+\infty} yf(y)dy$$

Among them, f(x) and f(y) are the possibility density function of x and y. We can therefore derive the following relationship (we assume x and y are independent):

$$\begin{aligned} E[XY] &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xyf(xy)d(xy) \\ &= \int_{-\infty}^{+\infty} xf(x)dx \int_{-\infty}^{+\infty} yf(y)d(y) \\ &= E[X]\cdot E[Y] \end{aligned}$$

4. We can name the event **X**: the chosen ball is red, and the event **Y**: the chosen box is box 1. So the posterior probability that the red ball came from box 1 will be:

$$\begin{aligned} P(Y|X) = \frac{P(X,Y)}{P(X)} &= \frac{P(X|Y)\cdot P(Y)}{P(Y)\cdot P(X|Y) + P(\overline{Y})\cdot P(X|\overline{Y})} \\ &= \frac{\frac{7}{10}\times\frac{1}{2}}{\frac{1}{2}\times\frac{7}{10}+\frac{1}{2}\times\frac{3}{12}} = \frac{\frac{7}{20}}{\frac{19}{40}} \approx 0.7368 \end{aligned}$$

Problem 2 – Modelling

1. The likelihood $p(I^1|I^0,d)$ is the multiplication of the disparity of the corresponding pixels. We can assume that the pixels in the second image are conditional independent of the pixels in the first image. So we can factorise the likelihood into individual terms below:

$$p(I^1|I^0,d) = \prod_{i,j} p(I^1_{i,j-d_{i,j}}|I^0_{i,j}, d_{i,j}) = \prod_{i,j} f(I^0_{i,j} - I^1_{i,j-d_{i,j}})$$

2. **The Limitations:**

(1) The computation cost of Gaussian likelihood can be large.

(2) Some disturbing factors like occlusions, shading, shadows and gain control will make the assumption inappropriate. It means the Gaussian likelihood is very sensitive to outliers.

The Alternatives: The likelihood functions, which are robust. e.g. the Huber's min-max estimator and Lorentzian. [1]

3. This part has three questions:

1) The assumption: Each edge corresponds to a term that models how compatible two neighboring pixel are in terms of their disparity.

2) The node d3 is not in the Markov blanket of node d1.

3) The disparity values of the 4 neighboring nodes around d1 or d3 are required such that d1 and d3 are independent.

4. **The Limitations:**

$$f_H(d_{i,j} - d_{i+1,j}) = \frac{1}{Z(T)} exp(\frac{1}{T}\delta(d_{i,j} - d_{i+1,j})), \quad \delta(A-B) = \begin{cases} 1, a = b \\ 0, a \neq b \end{cases}$$

The compatibility function is not differentiable. And the penalty factor is always same(1 or 0) for different pixel pair. [1]

The Alternatives: A possible solution may be using a different penalty factor for different pixel pair. For example:

$$\delta(d_{i,j} - d_{i+1,j})), \quad \delta(A-B) = \begin{cases} 2, |a-b| = 0 \\ 1, |a-b| > 0.5 \\ 1.5, |a-b| > 0.8 \\ 0, |a-b| > 1 \end{cases}$$

Problem 3 – Stereo Likelihood

1-8. For solutions to programming exercises, please see the A1_problem3.py file.

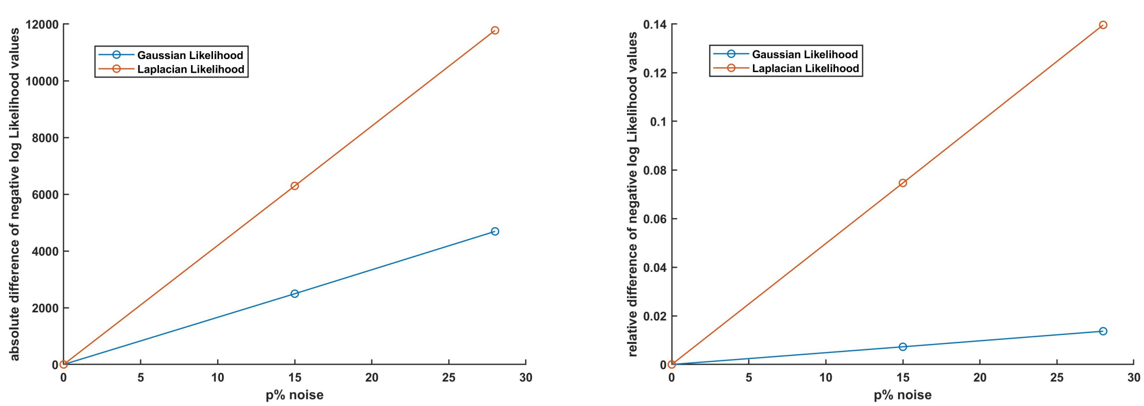

Figure 1: The difference between Gaussian and Laplacian Likelihood involving the noise

9. The results show that, Gaussian Likelihood is more affected by noise than Laplacian Likelihood (meaning smaller difference of negative log Likelihood values) From the figure(Fig. 1) above we can find the difference easily. This result is consistent with what we learned in the lecture, that the Gaussian likelihood is not robust.

References

[1] Stefan Roth. “Computer Vision 2 - Lecture in the sommer semester 2023”. In: *TU Darmstadt* (2023).