Machine Learning 4/M

Maximum Likelihood Laboratory

Aims

- To implement the maximum likelihood estimator of the parameters of a linear model
- To plot predictions and their variance

Tasks

- Download the Olympic data (again)
- Implement the maximum likelihood estimator for the parameters $\mathbf{w}$ and σ^2 of the linear model
- Note that $\mathbf{w}$ should be identical to the value from minimising the loss
- The relevant equations are:

$$\mathbf{w} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{t} \quad \text{(a vector with one value per parameter)}$$

$$\sigma^2 = \frac{1}{N}(\mathbf{t} - \mathbf{X}\mathbf{w})^T(\mathbf{t} - \mathbf{X}\mathbf{w}) \quad \text{(a scalar)}$$

- Plot the training data, the predictive mean (i.e. $\mathbf{X}_{test}\mathbf{w}$, the polynomial function)
- On top of your previous plot add dashed lines to show $\pm\sigma$, i.e. a line at $\mathbf{X}_{test}\mathbf{w} + \sigma$ and one at $\mathbf{X}_{test}\mathbf{w} - \sigma$
- Plot the predictive density for the 2016 Olympics (your x axis will be winning time, t, and your y axis p(t)). I.e. a Gaussian pdf with mean $\mathbf{w}^T\mathbf{x}_{2016}$ and variance σ^2