Coursework MAP501 2022

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Contents

You will submit your coursework in the form of a single R notebook (i.e. `.Rmd` file) which can be rendered ("knitted") to an `.pdf` document. Specifically, submit on Learn:

- your R notebook (i.e. the `.Rmd` file),
- the rendered `.pdf` version of your notebook. You might find it easier to knit to html, then print the html file to a pdf.

The coursework will be marked on the basis of correctness of code, interpretation of outputs and commentary as indicated. Therefore, please ensure that all code and outputs are visible in the knit document.

Preamble

```
library(readxl) # not in original coursework doc -> to read csv file.
library(here) # not in original coursework doc -> to get data file.
library(tidyverse) # not in original coursework doc.
library(janitor) # not in original coursework doc -> used to clean column names.
library(lindia)
library(rio)
library(dplyr)
library(tidyr)
library(magrittr)
library(ggplot2)
library(pROC)
library(car)
library(nnet)
```

```
library(caret)
library(lme4)
library(AmesHousing)
```

```
Ames<-make_ames()
```

1. Data Preparation

a. Import the soccer.csv dataset as "footballer_data". (2 points)

```
# Read in data & clean column names.
footballer_data <- read_csv(here("data", "soccer.csv"))
```

b. Ensure all character variables are treated as factors and where variable names have a space, rename the variables without these. (3 points)

```
# Rename variables with spaces.
footballer_data <- footballer_data %>%
  clean_names()

# Convert all character variables to factors.
footballer_data <- footballer_data %>%
  mutate(across(
    c(full_name, birthday_gmt, position, current_club, nationality), as.factor))
footballer_data
```

```
# A tibble: 570 x 45
   full_~1   age birth~2 birth~3 posit~4 curre~5 minut~6 minut~7 minut~8 natio~9
   <fct>   <dbl>   <dbl> <fct>   <fct>   <fct>     <dbl>   <dbl>   <dbl> <fct>
 1 Aaron ~    32  6.30e8 15/12/~ Defend~ West H~    1589     888     701 England
 2 Aaron ~    35  5.46e8 16/04/~ Midfie~ Burnley    1217     487     730 England
 3 Aaron ~    31  6.53e8 15/09/~ Midfie~ Hudder~    2327    1190    1137 Austra~
 4 Aaron ~    31  6.62e8 26/12/~ Midfie~ Arsenal    1327     689     638 Wales
 5 Aaron ~    22  9.68e8 07/09/~ Forward Hudder~      69      14      55 England
 6 Aaron ~    24  8.81e8 26/11/~ Midfie~ Crysta~    3135    1605    1530 England
 7 Abdelh~    25  8.49e8 28/11/~ Midfie~ Hudder~      49       0      49 Morocco
 8 Abdoul~    29  7.26e8 01/01/~ Midfie~ Watford    3062    1566    1496 France
 9 Abouba~    27  7.95e8 07/03/~ Forward Fulham      687     468     219 France
10 Adalbe~    25  8.65e8 31/05/~ Forward Watford       0       0       0 Venezu~
# ... with 560 more rows, 35 more variables: appearances_overall <dbl>,
#   appearances_home <dbl>, appearances_away <dbl>, goals_overall <dbl>,
#   goals_home <dbl>, goals_away <dbl>, assists_overall <dbl>,
#   assists_home <dbl>, assists_away <dbl>, penalty_goals <dbl>,
#   penalty_misses <dbl>, clean_sheets_overall <dbl>, clean_sheets_home <dbl>,
#   clean_sheets_away <dbl>, conceded_overall <dbl>, conceded_home <dbl>,
#   conceded_away <dbl>, yellow_cards_overall <dbl>, ...
```

c. Remove the columns birthday and birthday_GMT. (2 points)

```
# Removes birthday column and birthday_gmt column.
footballer_data <- footballer_data %>%
  select(-c(birthday, birthday_gmt))
footballer_data
```

```
# A tibble: 570 x 43
   full_~1   age posit~2 curre~3 minut~4 minut~5 minut~6 natio~7 appea~8 appea~9
   <fct>   <dbl> <fct>   <fct>     <dbl>   <dbl>   <dbl> <fct>     <dbl>   <dbl>
 1 Aaron ~    32 Defend~ West H~    1589     888     701 England      20      11
 2 Aaron ~    35 Midfie~ Burnley    1217     487     730 England      16       7
 3 Aaron ~    31 Midfie~ Hudder~    2327    1190    1137 Austra~      29      15
 4 Aaron ~    31 Midfie~ Arsenal    1327     689     638 Wales        28      14
 5 Aaron ~    22 Forward Hudder~      69      14      55 England       2       1
 6 Aaron ~    24 Midfie~ Crysta~    3135    1605    1530 England      35      18
 7 Abdelh~    25 Midfie~ Hudder~      49       0      49 Morocco       2       0
 8 Abdoul~    29 Midfie~ Watford    3062    1566    1496 France       35      18
 9 Abouba~    27 Forward Fulham      687     468     219 France       13       8
10 Adalbe~    25 Forward Watford       0       0       0 Venezu~       0       0
# ... with 560 more rows, 33 more variables: appearances_away <dbl>,
#   goals_overall <dbl>, goals_home <dbl>, goals_away <dbl>,
#   assists_overall <dbl>, assists_home <dbl>, assists_away <dbl>,
#   penalty_goals <dbl>, penalty_misses <dbl>, clean_sheets_overall <dbl>,
#   clean_sheets_home <dbl>, clean_sheets_away <dbl>, conceded_overall <dbl>,
#   conceded_home <dbl>, conceded_away <dbl>, yellow_cards_overall <dbl>,
#   red_cards_overall <dbl>, goals_involved_per_90_overall <dbl>, ...
```

d. Remove the cases with age<=15 and age>40. (2 points)

```
footballer_data <- footballer_data %>%
  filter(age > 15 & age <= 40)

# The minimum and maximum age was printed to check range was correct.
footballer_data %>%
  summarise (
    min_age = min(age),
    max_age = max(age)
  )

footballer_data
```

```
# A tibble: 1 x 2
  min_age max_age
    <dbl>   <dbl>
1      20      40
# A tibble: 565 x 43
   full_~1   age posit~2 curre~3 minut~4 minut~5 minut~6 natio~7 appea~8 appea~9
   <fct>   <dbl> <fct>   <fct>     <dbl>   <dbl>   <dbl> <fct>     <dbl>   <dbl>
 1 Aaron ~    32 Defend~ West H~    1589     888     701 England      20      11
 2 Aaron ~    35 Midfie~ Burnley    1217     487     730 England      16       7
 3 Aaron ~    31 Midfie~ Hudder~    2327    1190    1137 Austra~      29      15
 4 Aaron ~    31 Midfie~ Arsenal    1327     689     638 Wales        28      14
 5 Aaron ~    22 Forward Hudder~      69      14      55 England       2       1
```

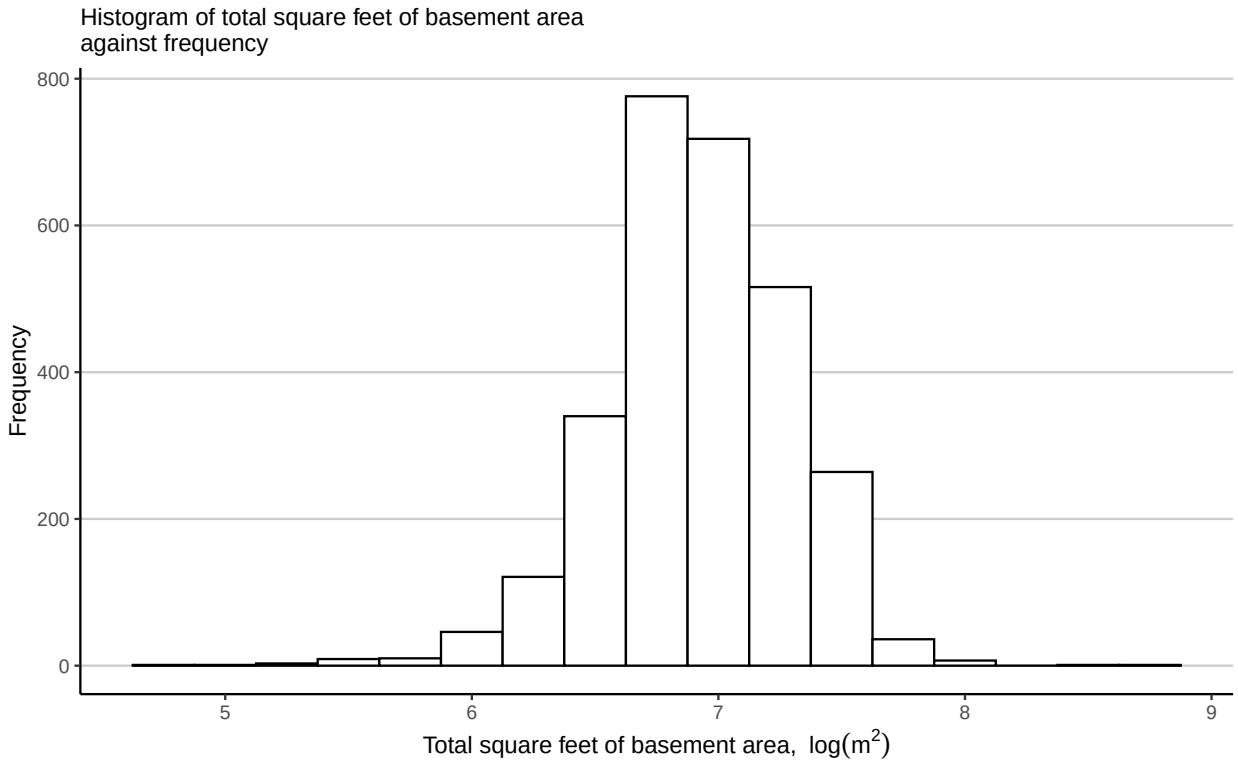
```
 6 Aaron ~    24 Midfie~ Crysta~    3135    1605    1530 England      35      18
 7 Abdelh~    25 Midfie~ Hudder~      49       0      49 Morocco       2       0
 8 Abdoul~    29 Midfie~ Watford    3062    1566    1496 France       35      18
 9 Abouba~    27 Forward Fulham      687     468     219 France       13       8
10 Adalbe~    25 Forward Watford       0       0       0 Venezu~       0       0
# ... with 555 more rows, 33 more variables: appearances_away <dbl>,
#   goals_overall <dbl>, goals_home <dbl>, goals_away <dbl>,
#   assists_overall <dbl>, assists_home <dbl>, assists_away <dbl>,
#   penalty_goals <dbl>, penalty_misses <dbl>, clean_sheets_overall <dbl>,
#   clean_sheets_home <dbl>, clean_sheets_away <dbl>, conceded_overall <dbl>,
#   conceded_home <dbl>, conceded_away <dbl>, yellow_cards_overall <dbl>,
#   red_cards_overall <dbl>, goals_involved_per_90_overall <dbl>, ...
```

2. Linear Regression

In this problem, you are going to investigate the response variable Total_Bsmt_SF in "Ames" dataset through linear regression.

a. By adjusting x axis range and number of bars, create a useful histogram of Total_Bsmt_SF on the full dataset. Ensure that plot titles and axis labels are clear. (4 points)

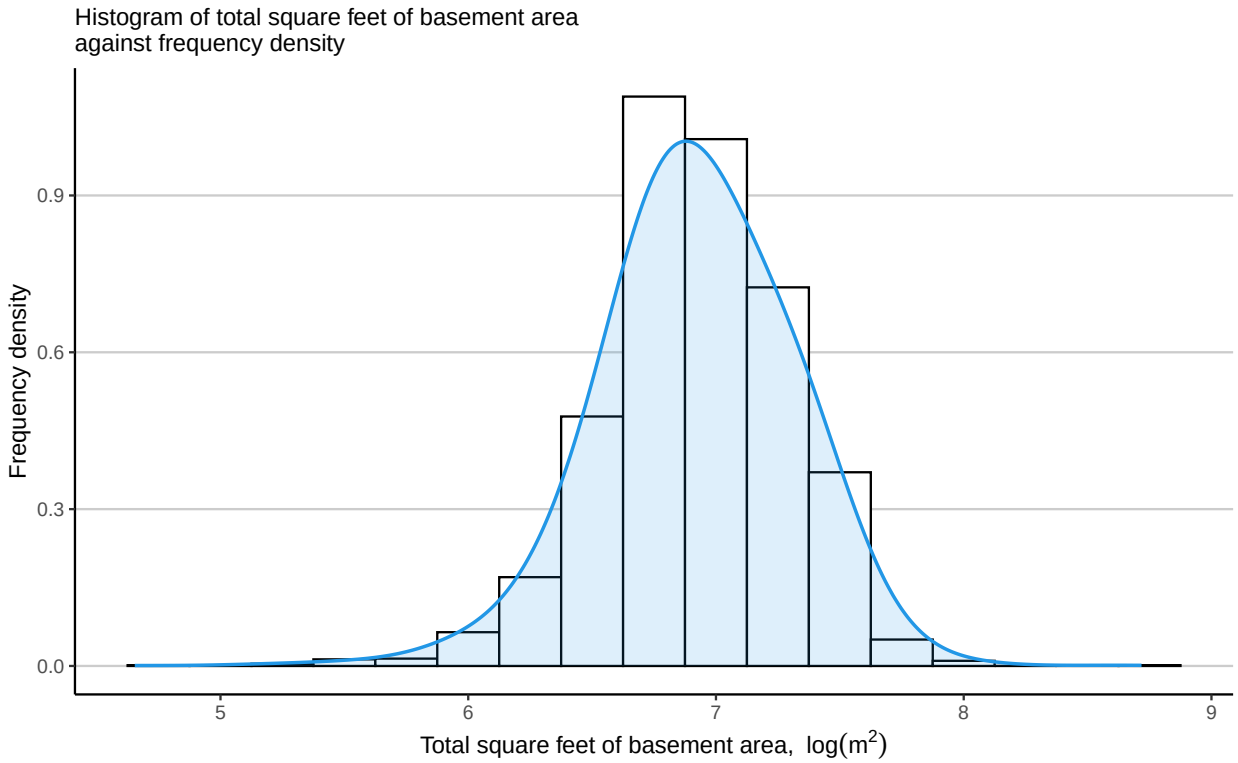
```
Ames
# Where 'Total_Bsmt_SF' is 'Total square feet of basement area'.
# Since it is linear regression we want the histogram to be 'bell-shaped'
# i.e., follow a Gaussian distribution.
# Plotting area as log(area) provided the most useful histogram...
# to linarize the data.
# Bin width was estimated via trial and error.

figure_1 <- Ames %>%
  ggplot(aes(x = log(Total_Bsmt_SF))) +
  geom_histogram(colour = "black", fill = "white", binwidth = 0.25) +
  labs (
    subtitle = "Histogram of total square feet of basement area\nagainst frequency",
    x = "Total square feet of basement area, "~log(m^2),
    y = "Frequency"
  ) +
  theme_classic() +
  theme(
    panel.grid.major.x = element_blank(),
    panel.grid.minor.x = element_blank(),
    panel.grid.major.y = element_line(colour = "grey80"),
    panel.grid.minor.y = element_blank(),
    panel.background   = element_blank()
  )

figure_1
```




```
# A tibble: 2,930 x 81
   MS_Sub~1 MS_Zo~2 Lot_F~3 Lot_A~4 Street Alley Lot_S~5 Land_~6 Utili~7 Lot_C~8
   <fct>    <fct>      <dbl>   <int> <fct>  <fct> <fct>   <fct>   <fct>   <fct>
 1 One_Sto~ Reside~      141   31770 Pave   No_A~ Slight~ Lvl     AllPub  Corner
 2 One_Sto~ Reside~       80   11622 Pave   No_A~ Regular Lvl     AllPub  Inside
 3 One_Sto~ Reside~       81   14267 Pave   No_A~ Slight~ Lvl     AllPub  Corner
 4 One_Sto~ Reside~       93   11160 Pave   No_A~ Regular Lvl     AllPub  Corner
 5 Two_Sto~ Reside~       74   13830 Pave   No_A~ Slight~ Lvl     AllPub  Inside
 6 Two_Sto~ Reside~       78    9978 Pave   No_A~ Slight~ Lvl     AllPub  Inside
 7 One_Sto~ Reside~       41    4920 Pave   No_A~ Regular Lvl     AllPub  Inside
 8 One_Sto~ Reside~       43    5005 Pave   No_A~ Slight~ HLS     AllPub  Inside
 9 One_Sto~ Reside~       39    5389 Pave   No_A~ Slight~ Lvl     AllPub  Inside
10 Two_Sto~ Reside~       60    7500 Pave   No_A~ Regular Lvl     AllPub  Inside
# ... with 2,920 more rows, 71 more variables: Land_Slope <fct>,
#   Neighborhood <fct>, Condition_1 <fct>, Condition_2 <fct>, Bldg_Type <fct>,
#   House_Style <fct>, Overall_Qual <fct>, Overall_Cond <fct>,
#   Year_Built <int>, Year_Remod_Add <int>, Roof_Style <fct>, Roof_Matl <fct>,
#   Exterior_1st <fct>, Exterior_2nd <fct>, Mas_Vnr_Type <fct>,
#   Mas_Vnr_Area <dbl>, Exter_Qual <fct>, Exter_Cond <fct>, Foundation <fct>,
#   Bsmt_Qual <fct>, Bsmt_Cond <fct>, Bsmt_Exposure <fct>, ...
```

```r
# A frequency density plot was created.
# This was overlaid by a distribution curve to validate...
# the normal distribution of 'Total_Bsmt_SF'.

figure_2 <- Ames %>%
  ggplot(aes(x = log(Total_Bsmt_SF))) +
  geom_histogram(aes(y = ..density..), colour = 1, fill = "white", binwidth = 0.25) +
```

```
  geom_density(lwd = 0.75, colour = 4, fill = 4, alpha = 0.15, adjust = 2.75) +
  labs (
    subtitle = "Histogram of total square feet of basement area\nagainst frequency density",
    x = "Total square feet of basement area, "~log(m^2),
    y = "Frequency density"
  ) +
  theme_classic() +
  theme(
    panel.grid.major.x = element_blank(),
    panel.grid.minor.x = element_blank(),
    panel.grid.major.y = element_line(colour = "grey80"),
    panel.grid.minor.y = element_blank(),
    panel.background   = element_blank(),
  )

figure_2
```



b. Using "Ames" dataset to create a new dataset called "Ames2" in which you remove all cases corresponding to:

(i) MS_Zoning categories of A_agr (agricultural), C_all (commercial) and I_all (industrial),
(ii) BsmtFin_Type_1 category of "No_Basement".
(iii) Bldg_Type category of "OneFam"

and drop the unused levels from the dataset "Ames2". (4 points)

```
# Checked levels before removal.
levels(Ames$MS_Zoning)
levels(Ames$BsmtFin_Type_1)
levels(Ames$Bldg_Type)
```

```
[1] "Floating_Village_Residential" "Residential_High_Density"
[3] "Residential_Low_Density"      "Residential_Medium_Density"
[5] "A_agr"                        "C_all"
[7] "I_all"
[1] "ALQ"          "BLQ"          "GLQ"          "LwQ"          "No_Basement"
[6] "Rec"          "Unf"
[1] "OneFam"    "TwoFmCon" "Duplex"   "Twnhs"    "TwnhsE"
```

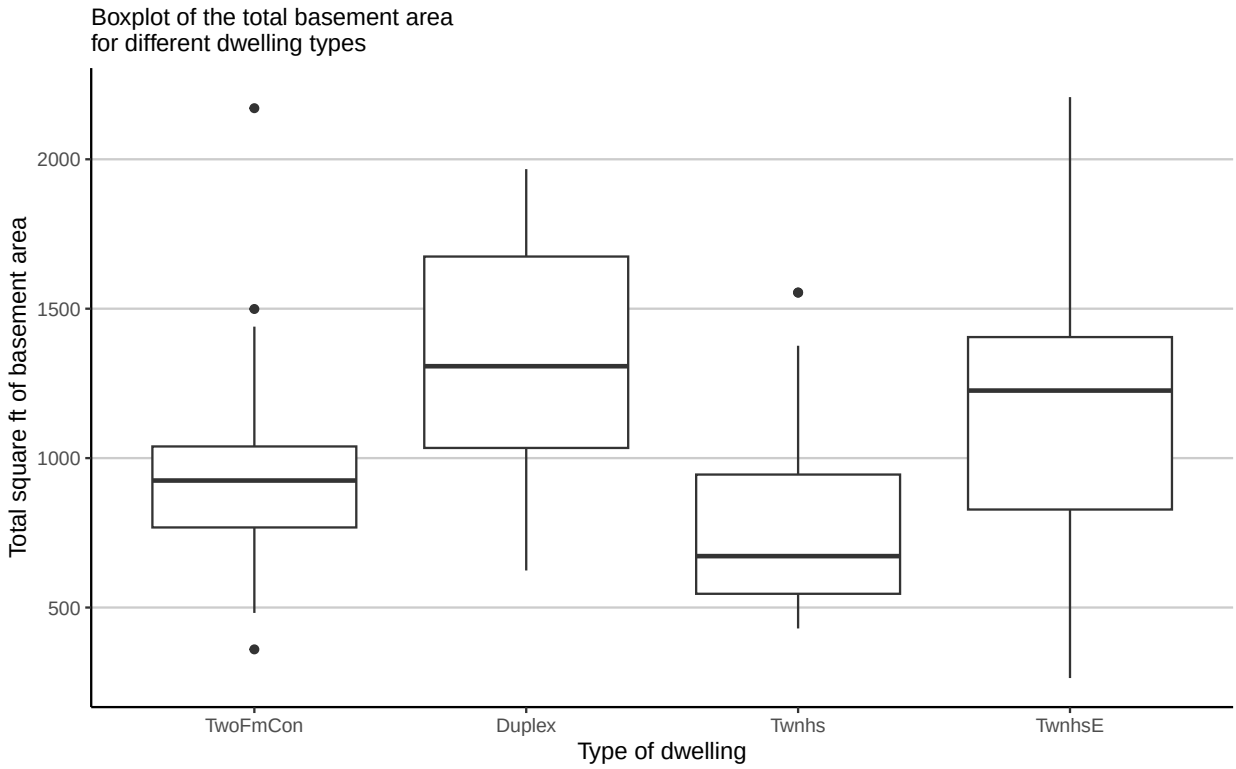
```
# (i), (ii) and (iii)
Ames2 <- Ames %>%
  filter(
      MS_Zoning != "A_agr",
      MS_Zoning != "C_all",
      MS_Zoning != "I_all",
      BsmtFin_Type_1 != "No_Basement",
      Bldg_Type !="OneFam"
      )

Ames2$MS_Zoning <- droplevels(Ames2$MS_Zoning)
Ames2$BsmtFin_Type_1 <- droplevels(Ames2$BsmtFin_Type_1)
Ames2$Bldg_Type <- droplevels(Ames2$Bldg_Type)

# Checked levels after removal.
levels(Ames2$MS_Zoning)
levels(Ames2$BsmtFin_Type_1)
levels(Ames2$Bldg_Type)
```

```
[1] "Floating_Village_Residential" "Residential_High_Density"
[3] "Residential_Low_Density"      "Residential_Medium_Density"
[1] "ALQ" "BLQ" "GLQ" "LwQ" "Rec" "Unf"
[1] "TwoFmCon" "Duplex"   "Twnhs"    "TwnhsE"
```

c. Choose an appropriate plot to investigate the relationship between Bldg_Type and Total_Bsmt_SF in Ames2. (2 points)

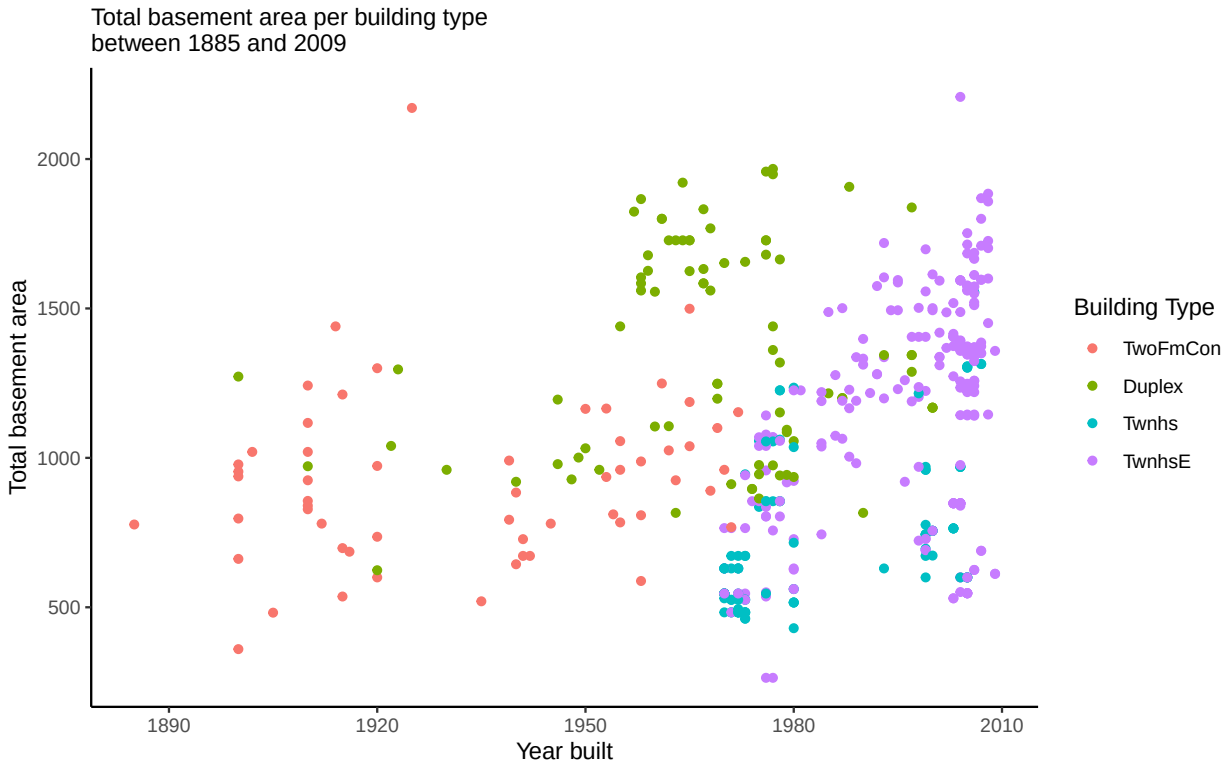
```
# Box plot since the relationship is categorical-numerical.
figure_3 <- Ames2 %>%
  ggplot(aes(x = Bldg_Type, y = Total_Bsmt_SF)) +
  geom_boxplot() +
  labs (
    subtitle = "Boxplot of the total basement area\nfor different dwelling types",
    x = "Type of dwelling",
    y = "Total square ft of basement area"
  ) +
  theme_classic() +
  theme(
    panel.grid.major.x = element_blank(),
```

```
    panel.grid.minor.x = element_blank(),
    panel.grid.major.y = element_line(colour = "grey80"),
    panel.grid.minor.y = element_blank(),
    panel.background   = element_blank(),
  )


figure_3
```

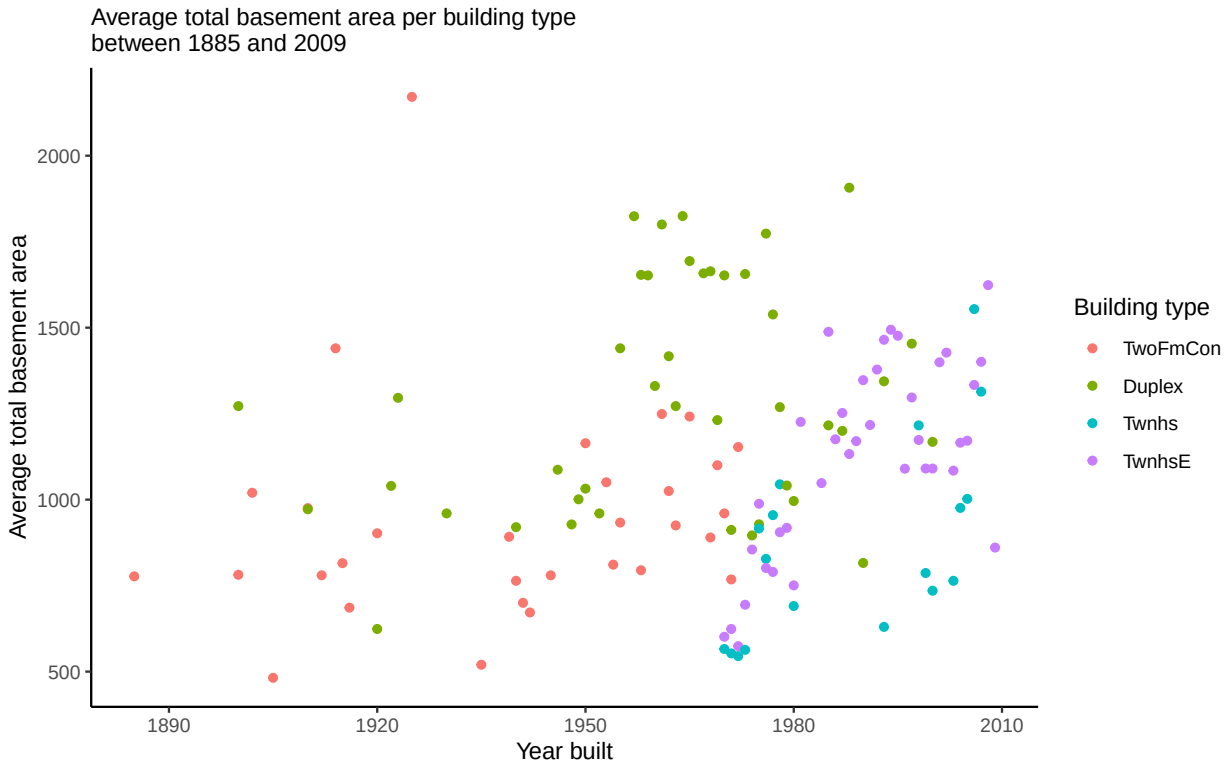


d. Choose an appropriate plot to investigate the relationship between Year_Built and Total_Bsmt_SF in Ames2. Color points according to the factor Bldg_Type. Ensure your plot has a clear title, axis labels and legend. What do you notice about how Basement size has changed over time? Were there any slowdowns in construction over this period? When? Can you think why? (4 points)

```
Ames2 %>%
  ggplot(aes(x = Year_Built, y = Total_Bsmt_SF, colour = Bldg_Type)) +
  geom_point() +
  labs(
    subtitle = "Total basement area per building type\nbetween 1885 and 2009",
    x = "Year built",
    y = "Total basement area",
    colour = "Building Type"
  ) +
  theme_classic()
```




```
# As one can see it is difficult to detect a trend in the relationship...
# between Year_Built and Total_Bsmt_SF from a raw scatter plot.
# Since there are vertically overlapping points, we can compare to average...
# Total_Bsmt_SF for each Bldg_Type built in a given year.

ave_by_year_built <- Ames2 %>%
  group_by(Year_Built, Bldg_Type) %>%
  summarise(mean(Total_Bsmt_SF))

figure_4 <- ave_by_year_built %>%
  ggplot(aes(x = Year_Built, y = `mean(Total_Bsmt_SF)`, colour = Bldg_Type)) +
  geom_point() +
  labs(
    subtitle = "Average total basement area per building type\nbetween 1885 and 2009",
    x = "Year built",
    y = "Average total basement area",
    colour = "Building type"
  ) +
  theme_classic()

figure_4
```




```
# The plot shows the basement size for all building types over time...
# in Ames Iowa, has gradually increased.
# Between 1930 and 1935 the rate of basement construction appears...
# to have slown down i.e., the number of basements constructed...
# is at a lower frequency than before and after this period.
# Reason: This period also included the Great Depression from 1929 to 1933...
# so most businesses and people did not have the means for land acquisition or
# construction.
# The same occurs between 2007 and 2010 which coincides with...
# the 2007-2008 global financial crisis.

# (Link to sources to support these inferences are below.)
```

Sources:

1. Financial Crisis
2. List of Recessions in the United States

e. Why do we make these plots? Comment on your findings from these plots (1 sentence is fine). (2 points)

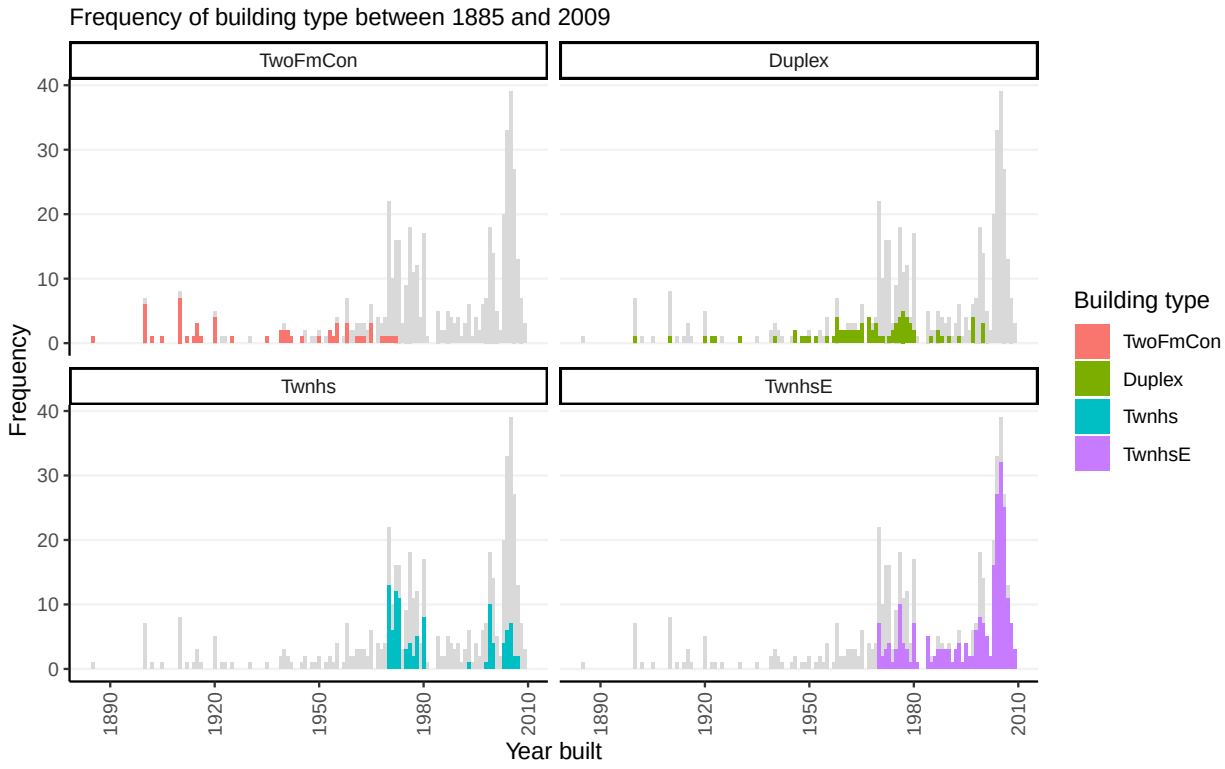
```
# The box plots are to compare the distribution of numeric values...
# in each category. The scatter plot was to determine a trend...
# between the two variables and decide which model may be best fit...
# for prediction i.e., if there is a linear trend this may indicate...
# a linear regression model would best fit the data set.
```

```
# Findings:
# - Generally between 1885 and 2009 the basement size has increased.
# - The number of TwoFmCon constructed completely stopped around 1975 and...
#   around the time Twnhs and TwnhsE basements began to be constructed.
# - Twnhs has the lowest median total basement area with 50% of its areas...
#   ranging between 550 to 850 ft^2.
# - Duplex has the highest median total basement area with 50% of its... areas
#   areas ranging between 1100 to 1700 ft^2. This is the largest spread...
#   in the data set despite TwnshE having more outliers.
```

f. Now choose an appropriate plot to investigate the relationship between Bldg_Type and Year_Built in Ames2. Why should we consider this? What do you notice? (3 points)

```
# This must be considered to compare frequency of each category...
# at different periods of time.

# We should consider this to avoid over generalizing which...
# could increase the residual error of predictions due to not taking...
# the different frequencies (and therefore sample sizes)...
# of each category into account.

# For example if we wanted to investigate Total_Bsmt_SF...
# based on Year_Built and Blg_Type between 1980 and 2008, we now know...
# we should drop the 'TwoFmCon' level because there were zero TwoFmCon...
# buildings constructed during that time period.
# Duplex could also be dropped if the frequency...
# is found to be too small i.e., <30 or small relative to the...
# frequencies of other categories.

# We notice that the 'TwoFmCon' building was only constructed between...
# 1885 to 1965. This is around the same time, Twnhs and TwnhsE building...
# types began construction and at a noticeably higher frequency.
# Duplex building were dispersed more consistently between ...
# 1900 and 1990.

figure_5 <- Ames2 %>%
  ggplot(aes(x = Year_Built, fill = Bldg_Type)) +
  geom_bar(data = select(Ames2, !Bldg_Type), fill = "grey85") +
  geom_bar() +
  facet_wrap(facets = vars(Bldg_Type)) +
  labs(
    subtitle = "Frequency of building type between 1885 and 2009",
    x = "Year built",
    y = "Frequency",
    fill = "Building type"
  ) +
  theme_classic() +
  theme(
    panel.grid.major.x = element_blank(),
    panel.grid.minor.x = element_blank(),
    panel.grid.major.y = element_line(colour = "grey95"),
    panel.grid.minor.y = element_blank(),
    panel.background   = element_blank(),
```

```
    axis.text.x = element_text(angle = 90, vjust = 0.5, hjust = 1)
  )

figure_5
```



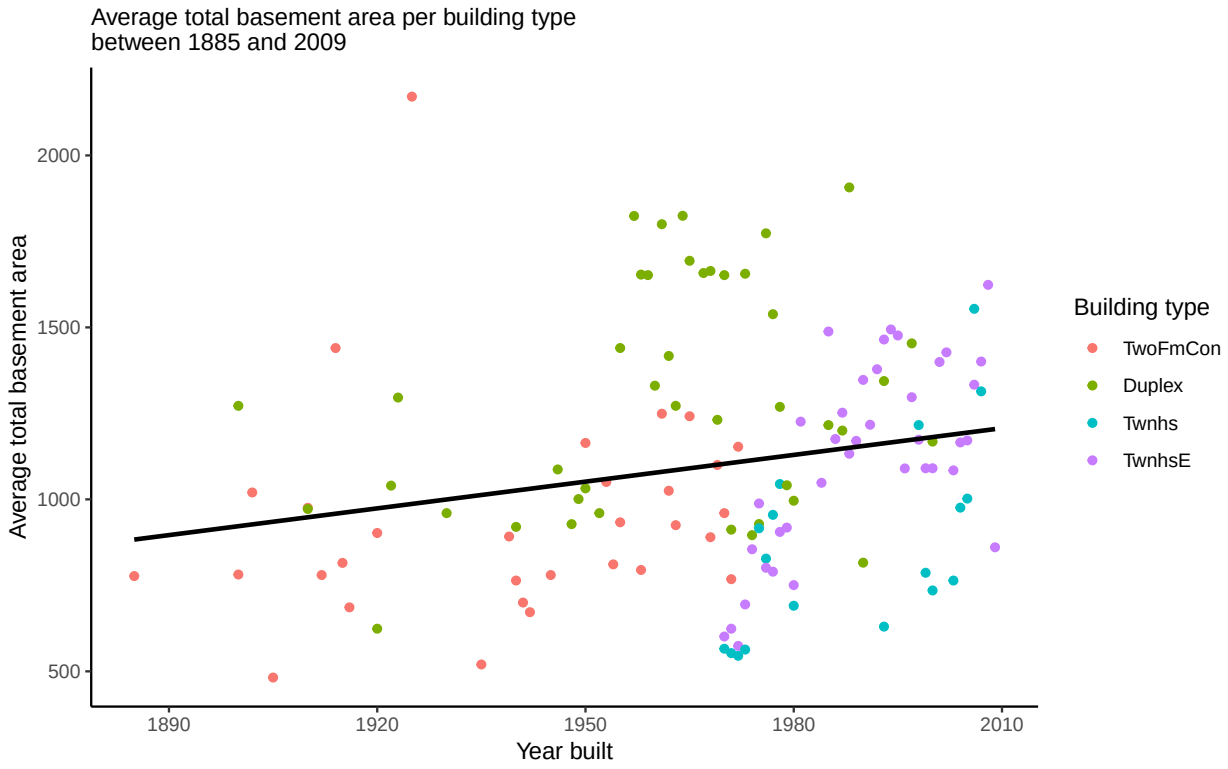
g. Use the lm command to build a linear model, linmod1, of Total_Bsmt_SF as a function of the predictors Bldg_Type and Year_Built for the "Ames2" dataset. (2 points)

```
linmod1 <- lm(formula = Total_Bsmt_SF ~ Bldg_Type + Year_Built,
              data = Ames2)
```

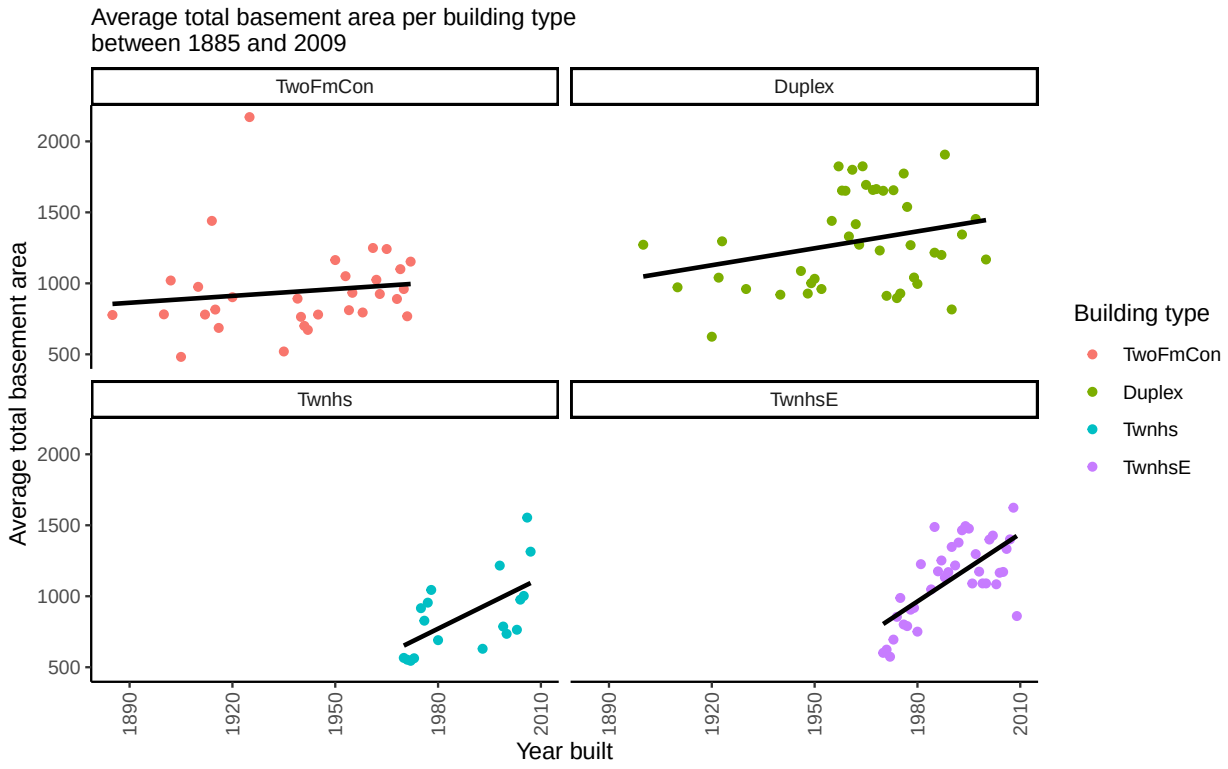
h. State and evaluate the assumptions of the model. (6 points)

```
# Scatter plot of Year Built vs. Total_Bsmt_SF with linear trend line.
# The first one shows the overall trend line of all categories.
# The second one shows the separate trend line of each category.

figure_6a <- ave_by_year_built %>%
  ggplot(aes(x = Year_Built, y = `mean(Total_Bsmt_SF)`, colour = Bldg_Type)) +
  geom_point() +
  labs(
    subtitle = "Average total basement area per building type\nbetween 1885 and 2009",
    x = "Year built",
    y = "Average total basement area",
    colour = "Building type"
  ) +
```

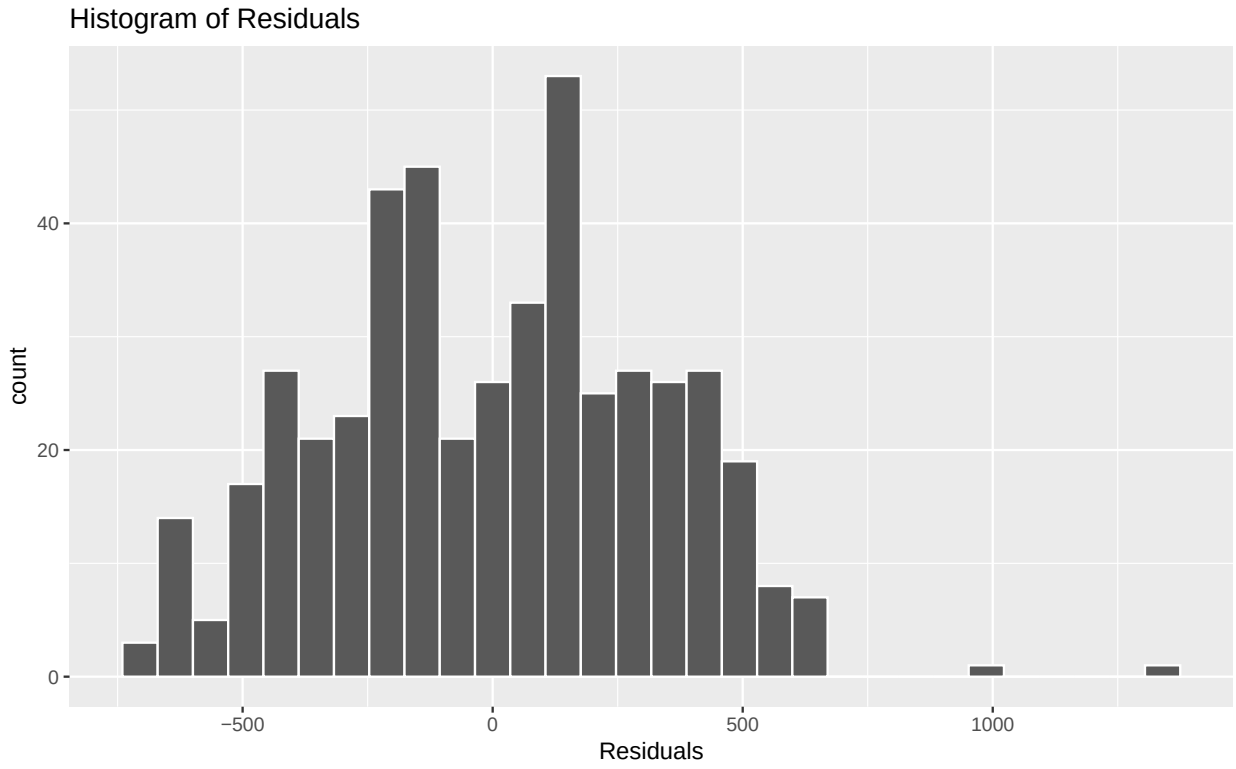
```
  geom_smooth(method = "lm", se = FALSE, colour = "black") +
  theme_classic()

figure_6a
```

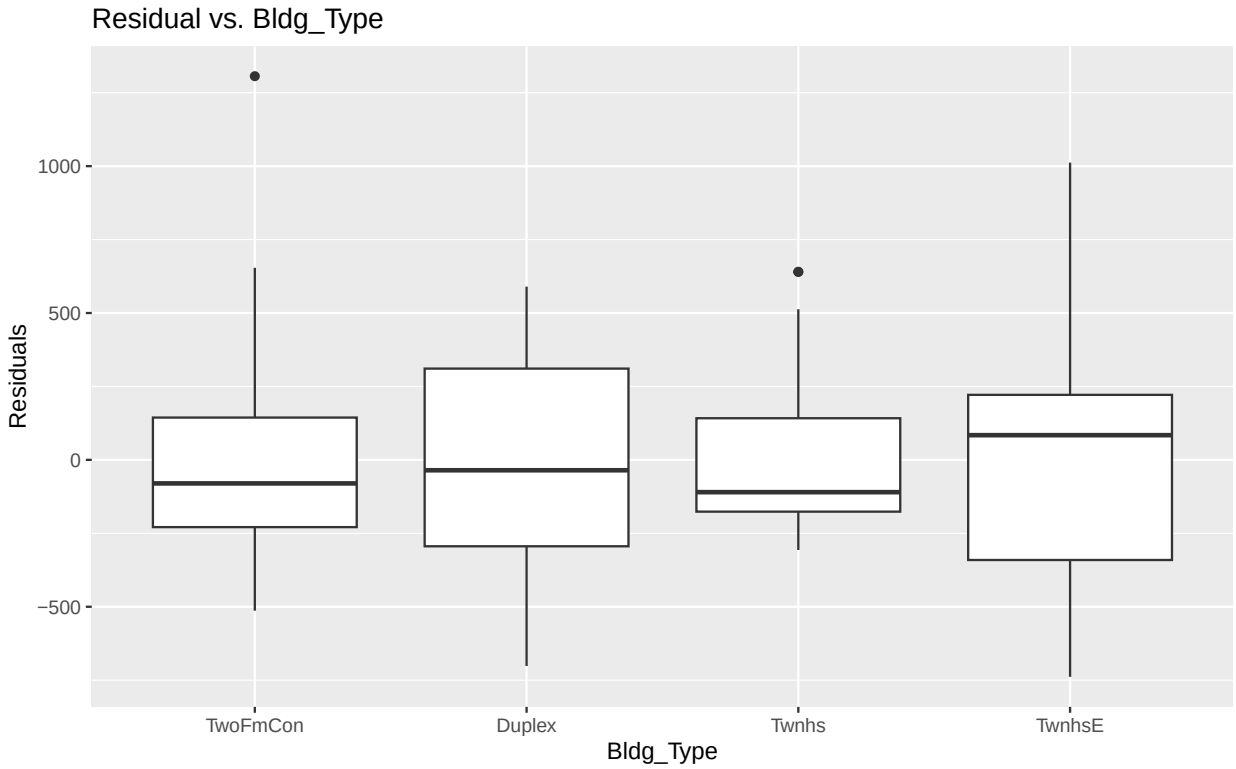


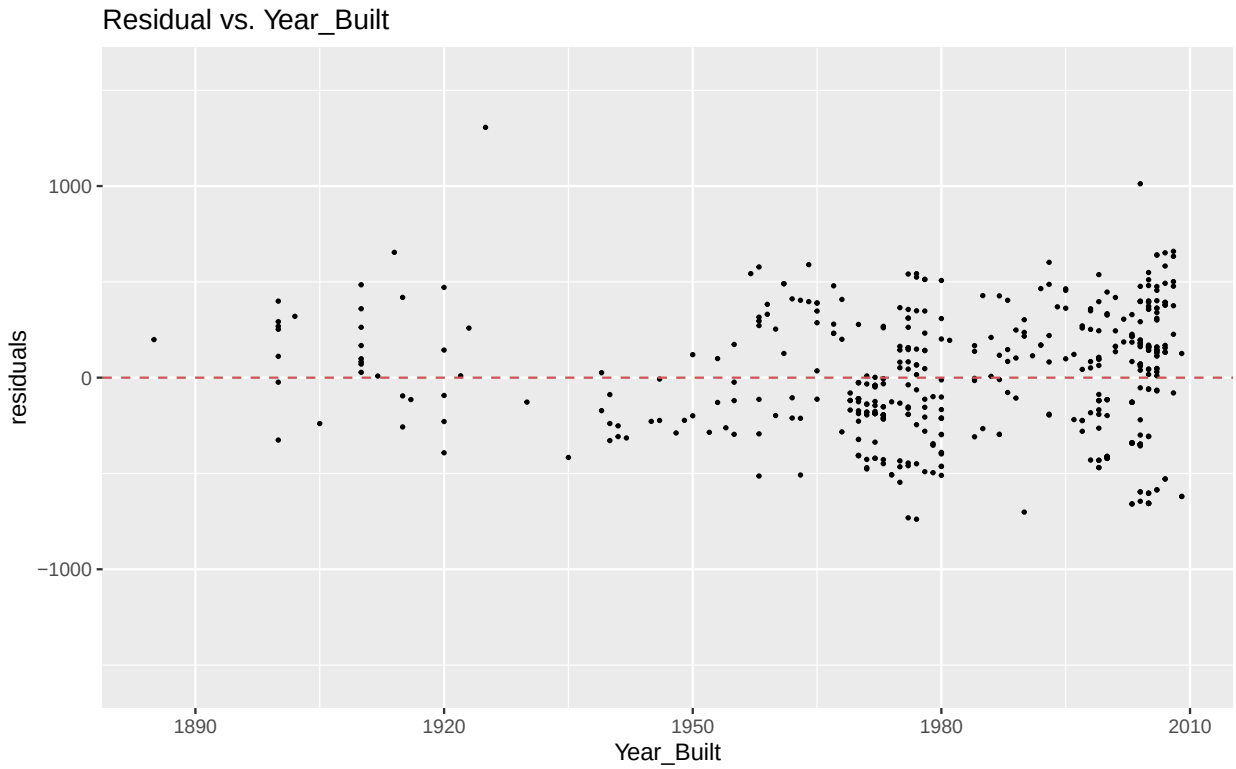

```
figure_6b <- ave_by_year_built %>%
  ggplot(aes(x = Year_Built, y = `mean(Total_Bsmt_SF)`, colour = Bldg_Type)) +
  geom_point() +
   facet_wrap(facets = vars(Bldg_Type)) +
  labs(
    subtitle = "Average total basement area per building type\nbetween 1885 and 2009",
    x = "Year built",
    y = "Average total basement area",
    colour = "Building type"
  ) +
  geom_smooth(method = "lm", se = FALSE, colour = "black") +
  theme_classic() +
  theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust = 1))

figure_6b
```

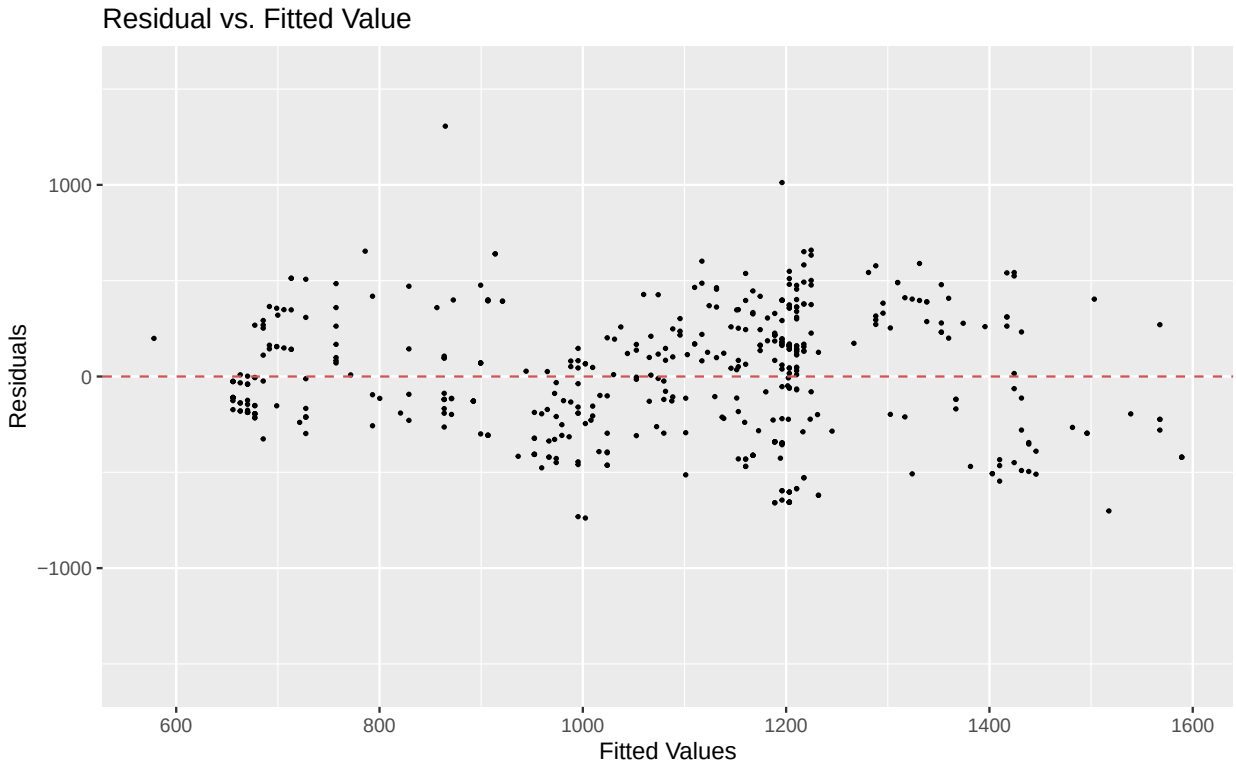


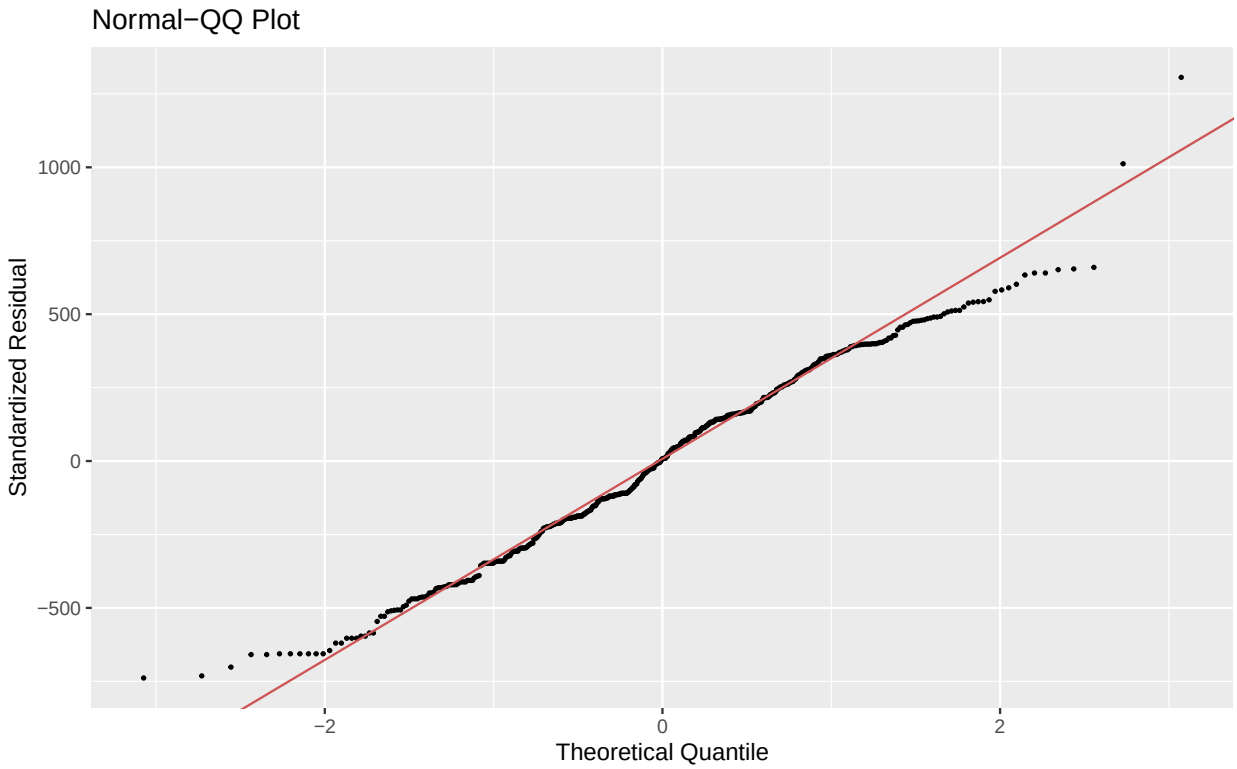

```
figure_7 <- linmod1 %>%
  gg_diagnose(max.per.page = 1)
```

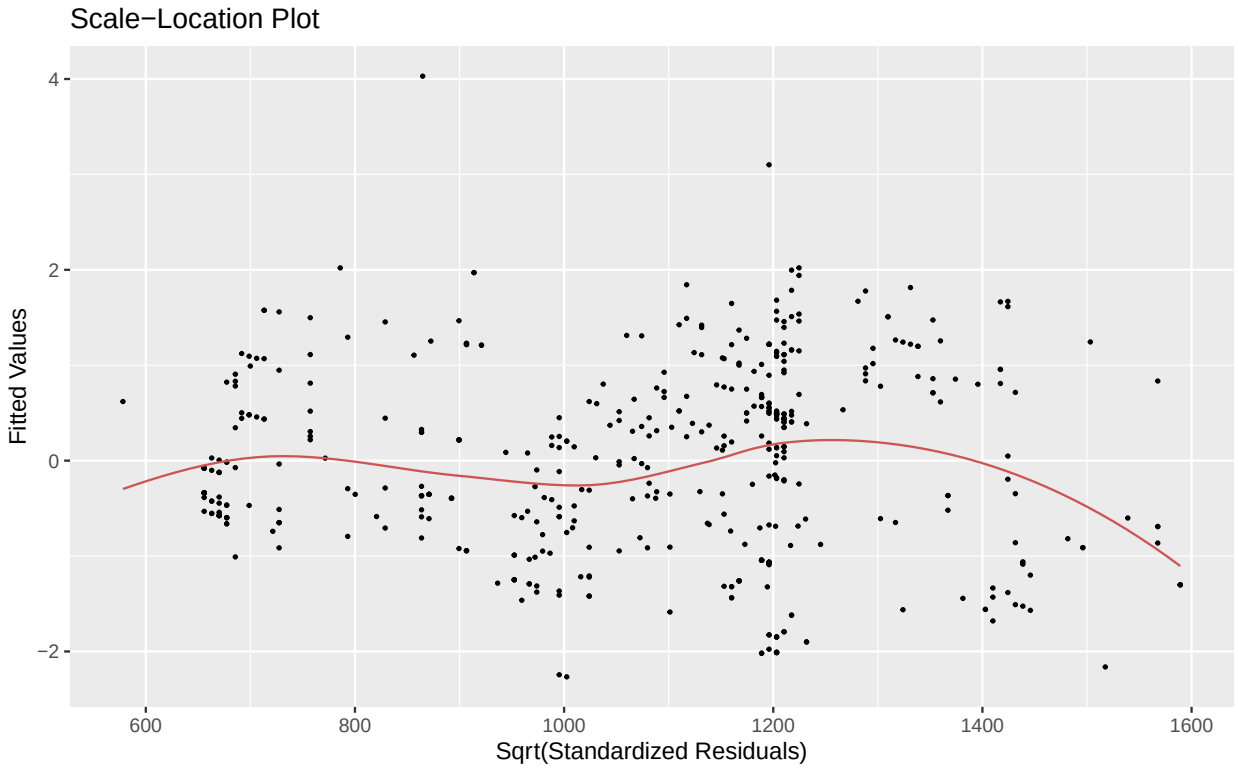


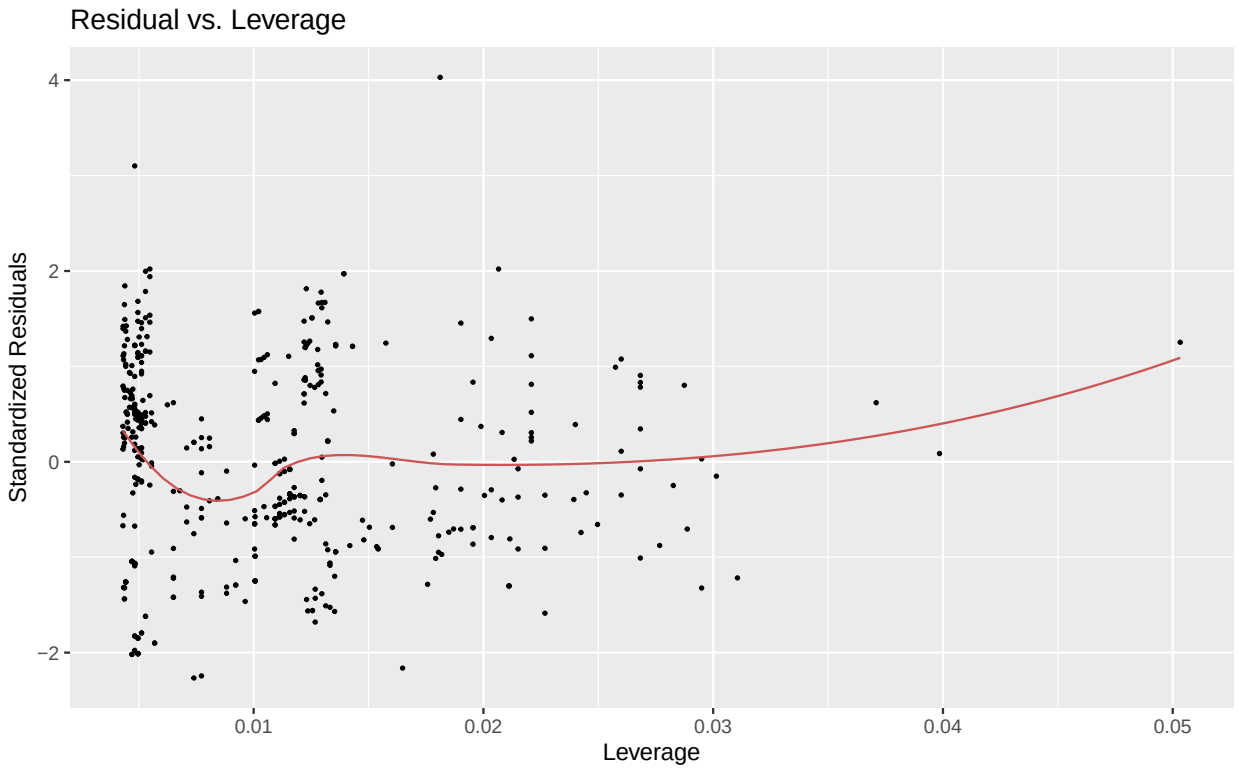

Residual vs. Bldg_Type
Residuals
1000
500
0
−500
TwoFmCon
Duplex
Twnhs
TwnhsE
Bldg_Type

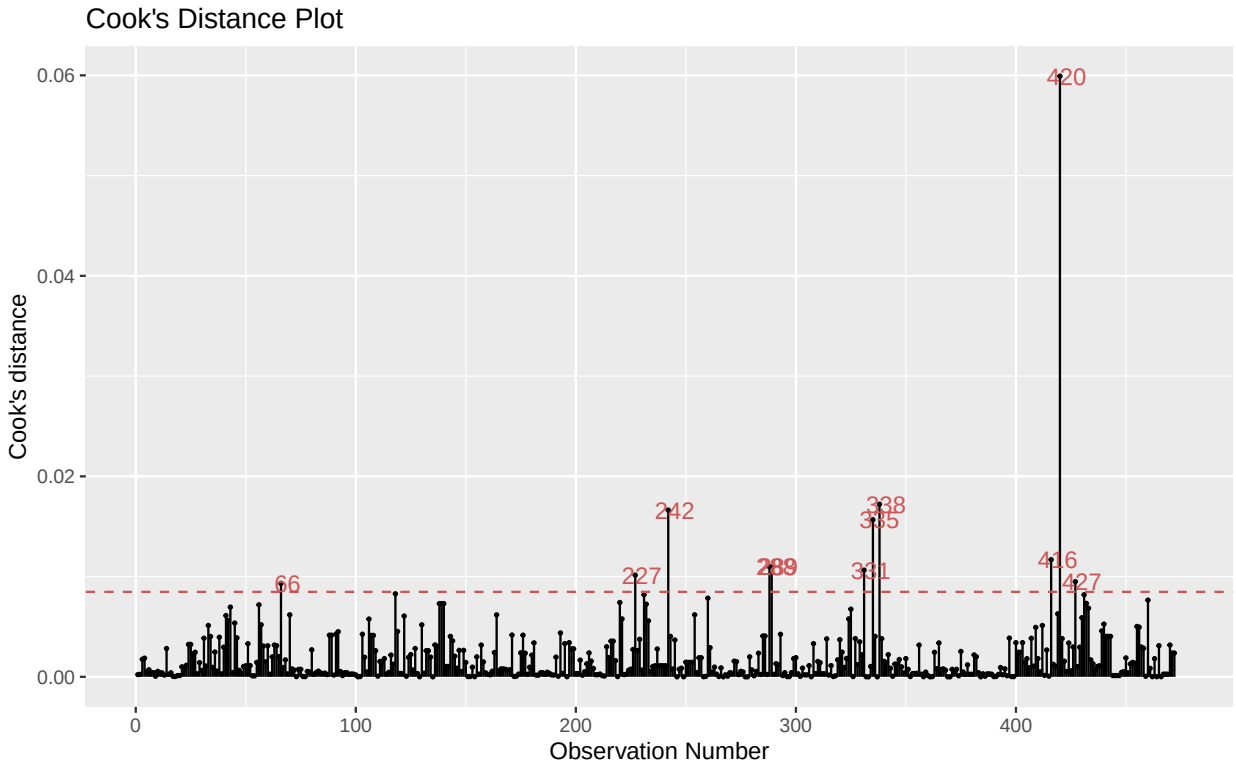

Residual vs. Year_Built
residuals
1000
0
−1000
1890
1920
1950
1980
2010
Year_Built


Residual vs. Fitted Value
Residuals
1000
0
−1000
600
800
1000
1200
1400
1600
Fitted Values


Normal−QQ Plot
Standardized Residual
1000
500
0
−500
−2
0
2
Theoretical Quantile


Scale−Location Plot
Fitted Values
4
2
0
−2
600
800
1000
1200
1400
1600
Sqrt(Standardized Residuals)


Residual vs. Leverage
Standardized Residuals
4
2
0
−2
0.01
0.02
0.03
0.04
0.05
Leverage




```
# The assumption is the model follows the assumptions...
# of a Gaussian distribution that is:

# 1) Linearity -> If there is 1 categorical variable and 1 or more...
#                 continuous variables linearity is assessed by...
#                 a scatter plot with the levels in different colors...
#                 hence figure_6a and figure_6b.
#                 Both figures show the trend is roughly...
#                 linear so this assumption is held.

##### All plots referred to in 2) and 3) are in figure_7. #####

# 2) Normality -> Aside from a few outliers in the histogram of...
#                 of residuals it appears normality assumption is...
#                 is upheld. The normal...
#                 QQ plot also shows this since most of the standardized...
#                 residuals are along the red line.
#
# 3) Homoscedasticity -> The model appears to violate this assumption.
#                        This is because the residuals in the plot...
#                        of Year Built vs Residuals show as the...
#                        Year Built increases the further the residuals...
#                        are from the zero-mean. A similar trend was...
#                        observed in Fitted Values vs Residuals.
#                        The boxplot of Bldg_Type vs Residuals also...
#                        show the IQR are not similar and every category...
#                        aside from 'Duplex' are far from the zero mean.
```

i. Use the lm command to build a second linear model, linmod2, for Total_Bsmt_SF as a function of Bldg_Type, Year_Built and Lot_Area. (2 points)

```
# I assumed the question meant to the model using the...
# Ames2 dataset as well.
linmod2 <- lm(formula = Total_Bsmt_SF ~ Bldg_Type +  Year_Built + Lot_Area,
              data = Ames2)
```

j. Use Anova and Adjusted R-squared to compare these two models, and decide which is a better model. (6 points)

```
summary(linmod1)
summary(linmod2)
anova(linmod1, linmod2)

# Anova -> 8.099e-05 < 0.05 therefore linmod2 is a more statistically...
#          significant model compared to linmod1.
#          In context, this means adding the 'Lot_Area' predictor...
#          had a significant effect on the 'Total Basement Area in...
#          Square Feet'.
# Adjusted R-squared -> linmod1 has an Adjusted R-squared value...
#                       of 0.3282 whereas linmod2 has an...
#                       Adjusted R-squared value of 0.3489.
#                       Therefore linmod2 has a slightly better fit...
#                       than linmod1 as an additional 2.07% of the...
#                       variation in Total_Bsmt_SF can be explained by...
#                       its predictors.
# Therefore the better model is linmod2. That said the Adjusted R-squared...
# value is still low so its predictors are explanatory, there are...
# alot of uncontrolled factors that affect the Total_Bsmt_SF.
```

```
Call:
lm(formula = Total_Bsmt_SF ~ Bldg_Type + Year_Built, data = Ames2)

Residuals:
    Min      1Q  Median      3Q     Max
-738.53 -223.35    7.68  238.36 1306.23

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)     -1.293e+04  1.833e+03  -7.054 6.30e-12 ***
Bldg_TypeDuplex  1.870e+02  6.504e+01   2.875  0.00422 **
Bldg_TypeTwnhs  -5.314e+02  7.252e+01  -7.327 1.04e-12 ***
Bldg_TypeTwnhsE -2.349e+02  7.678e+01  -3.059  0.00235 **
Year_Built       7.166e+00  9.478e-01   7.560 2.15e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 327.1 on 467 degrees of freedom
Multiple R-squared:  0.3339,    Adjusted R-squared:  0.3282
F-statistic: 58.54 on 4 and 467 DF,  p-value: < 2.2e-16
```

```
Call:
lm(formula = Total_Bsmt_SF ~ Bldg_Type + Year_Built + Lot_Area,
    data = Ames2)

Residuals:
    Min      1Q  Median      3Q     Max
-810.32 -212.07   -5.72  233.88 1232.65

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)     -1.176e+04  1.828e+03  -6.435 3.08e-10 ***
Bldg_TypeDuplex  2.378e+02  6.529e+01   3.642 0.000301 ***
Bldg_TypeTwnhs  -4.120e+02  7.745e+01  -5.319 1.62e-07 ***
Bldg_TypeTwnhsE -1.265e+02  8.035e+01  -1.575 0.115942
Year_Built       6.509e+00  9.476e-01   6.868 2.09e-11 ***
Lot_Area         7.793e-03  1.960e-03   3.977 8.10e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 322.1 on 466 degrees of freedom
Multiple R-squared:  0.3558,    Adjusted R-squared:  0.3489
F-statistic: 51.48 on 5 and 466 DF,  p-value: < 2.2e-16

Analysis of Variance Table

Model 1: Total_Bsmt_SF ~ Bldg_Type + Year_Built
Model 2: Total_Bsmt_SF ~ Bldg_Type + Year_Built + Lot_Area
  Res.Df      RSS Df Sum of Sq      F    Pr(>F)
1    467 49980160
2    466 48339705  1   1640455 15.814 8.099e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

k. Construct a confidence interval and a prediction interval for the basement area of a Twnhs built in 1980, with a lot Area of 7300. Explain what these two intervals mean. (6 points)

```r
# Confidence interval
predict(linmod2,
        newdata = data.frame(Bldg_Type = "Twnhs",
                             Year_Built = 1980,
                             Lot_Area = 7300),
                             interval = "confidence")
# Prediction interval
predict(linmod2,
        newdata = data.frame(Bldg_Type = "Twnhs",
                             Year_Built = 1980,
                             Lot_Area = 7300),
                             interval = "prediction")
# Explanation:
# The confidence interval is the uncertainty around the mean at each ...
# total basement area. The expected values of total basement area...
# are expected lie within this interval.
```

```
# Whereas the prediction interval is the range of likely values...
# the expected value of the total basement area could take.
```

```
       fit      lwr      upr
1 768.7589 702.1423 835.3755
       fit      lwr      upr
1 768.7589 132.3605 1405.157
```

l. Now build a linear mixed model, linmod3, for Total_Bsmt_SF as a function of Year_Built, MS_Zoning and Bldg_Type. Use Neighborhood as random effect. What is the critical number to pull out from this, and what does it tell us? (4 points)

```
linmod3 <- lmer(formula = Total_Bsmt_SF ~
                  Year_Built + MS_Zoning + Bldg_Type + (1|Neighborhood),
                data = Ames2)
# Critical number: 187.4
# What it tells us: This is the standard deviation of the effect...
# 'Neighborhood' has on Year_Built and MS_Zoning. In other words...
# Neighbourhood causes 187.4 variance between the Year_Built and...
# MS_Zoning variables.
```

m. Construct 95% confidence intervals around each parameter estimate for linmod3. What does this tell us about the significance of the random effect? (3 points)

```
confint(linmod3)

# .sig01 = Is the random effect.
# Since the range of the confidence interval does not include 0...
# the random effect is significant.
```

```
                                       2.5 %     97.5 %
.sig01                             114.916972  253.19244
.sigma                             244.221572  278.77404
(Intercept)                      -9699.022207 -595.99553
Year_Built                           0.691417    5.33084
MS_ZoningResidential_High_Density  -254.190137  549.38121
MS_ZoningResidential_Low_Density    -91.829020  665.77648
MS_ZoningResidential_Medium_Density -266.136920  487.72182
Bldg_TypeDuplex                     145.073466  377.00462
Bldg_TypeTwnhs                     -249.148897  102.22240
Bldg_TypeTwnhsE                     -68.266514  263.18536
```

n. Write out the full mathematical expression for the model in linmod2 and for the model in linmod3. Round to the nearest integer in all coefficients with modulus > 10 and to three decimal places for coefficients with modulus < 10. (4 points)

```
linmod2
```

```
Call:
```

```
lm(formula = Total_Bsmt_SF ~ Bldg_Type + Year_Built + Lot_Area,
    data = Ames2)

Coefficients:
    (Intercept)  Bldg_TypeDuplex   Bldg_TypeTwnhs  Bldg_TypeTwnhsE
     -1.176e+04        2.378e+02       -4.120e+02       -1.265e+02
     Year_Built         Lot_Area
      6.509e+00        7.793e-03
```

```
linmod3
```

```
Linear mixed model fit by REML ['lmerMod']
Formula:
Total_Bsmt_SF ~ Year_Built + MS_Zoning + Bldg_Type + (1 | Neighborhood)
   Data: Ames2
REML criterion at convergence: 6566.758
Random effects:
 Groups       Name        Std.Dev.
 Neighborhood (Intercept) 187.4
 Residual                 261.8
Number of obs: 472, groups:  Neighborhood, 27
Fixed Effects:
                         (Intercept)                           Year_Built
                           -4890.652                                2.876
  MS_ZoningResidential_High_Density     MS_ZoningResidential_Low_Density
                             148.504                              288.369
MS_ZoningResidential_Medium_Density                        Bldg_TypeDuplex
                             109.234                              264.530
                      Bldg_TypeTwnhs                        Bldg_TypeTwnhsE
                             -63.140                              105.171
```

Mathematical expression for linmod2:

$$E(\text{TotalBsmtSF}) = -11760 + (6.509 \times \text{YearBuilt}) + (0.008 \times \text{LotArea})$$
$$+(238 \times \text{isDuplex}) + (-412 \times \text{isTwnhs}) + (-127 \times \text{isTwnhsE})$$

Mathematical expression for linmod3:

$$E(\text{TotalBsmtSF}) = -4981 + (2.876 \times \text{YearBuilt}) + (149 \times \text{isHighDensity}) + (288 \times \text{isLowDensity})$$
$$+(109 \times \text{isMediumDensity}) + (265 \times \text{isDuplex}) + (-63 \times \text{isTwnhs}) + (105 \times \text{isTwnhsE}) + U$$
$$U \sim N(0, 187)$$
$$\text{TotalBsmtSF} \sim N(E(\text{TotalBsmtSF}), 262)$$

3. Logistic Regression

a. Do the following:

(i) Create a new dataset called "Ames3" that contains all data in "Ames" dataset plus a new variable "excellent_heating" that indicates if the heating quality and condition "Heating_QC" is excellent or not. (2 points)

```
Ames3 <- Ames %>%
  mutate(
    excellent_heating = if_else(Heating_QC == "Excellent", "Yes", "No")
  )

# The code below checks whether this new variable is correct.
Ames3 %>%
  summarise (
    Heating_QC,
    excellent_heating
  )
```

```
# A tibble: 2,930 x 2
   Heating_QC excellent_heating
   <fct>      <chr>
 1 Fair       No
 2 Typical    No
 3 Typical    No
 4 Excellent  Yes
 5 Good       No
 6 Excellent  Yes
 7 Excellent  Yes
 8 Excellent  Yes
 9 Excellent  Yes
10 Good       No
# ... with 2,920 more rows
```

(ii) In "Ames3" dataset, remove all cases "3" and "4" corresponding to the Fireplaces variable. Remove all cases where Lot_Frontage is greater than 130 or smaller than 20. Drop the unused levels from the dataset. (2 points)

```
Ames3 <- Ames3 %>%
  filter(Fireplaces != 3 & Fireplaces != 4) %>%
  filter(Lot_Frontage >= 20 & Lot_Frontage <= 130) %>%
  droplevels()
```

(iii) Save "Fireplaces" as factor in "Ames3" dataset (1 point)

```
Ames3 <- Ames3 %>%
  mutate (
    Fireplaces = as.factor(Fireplaces)
  )
```

(iv) Construct a logistic regression model glmod for excellent_heating as a function of Lot_Frontage and Fireplaces for the dataset "Ames3". (2 points)

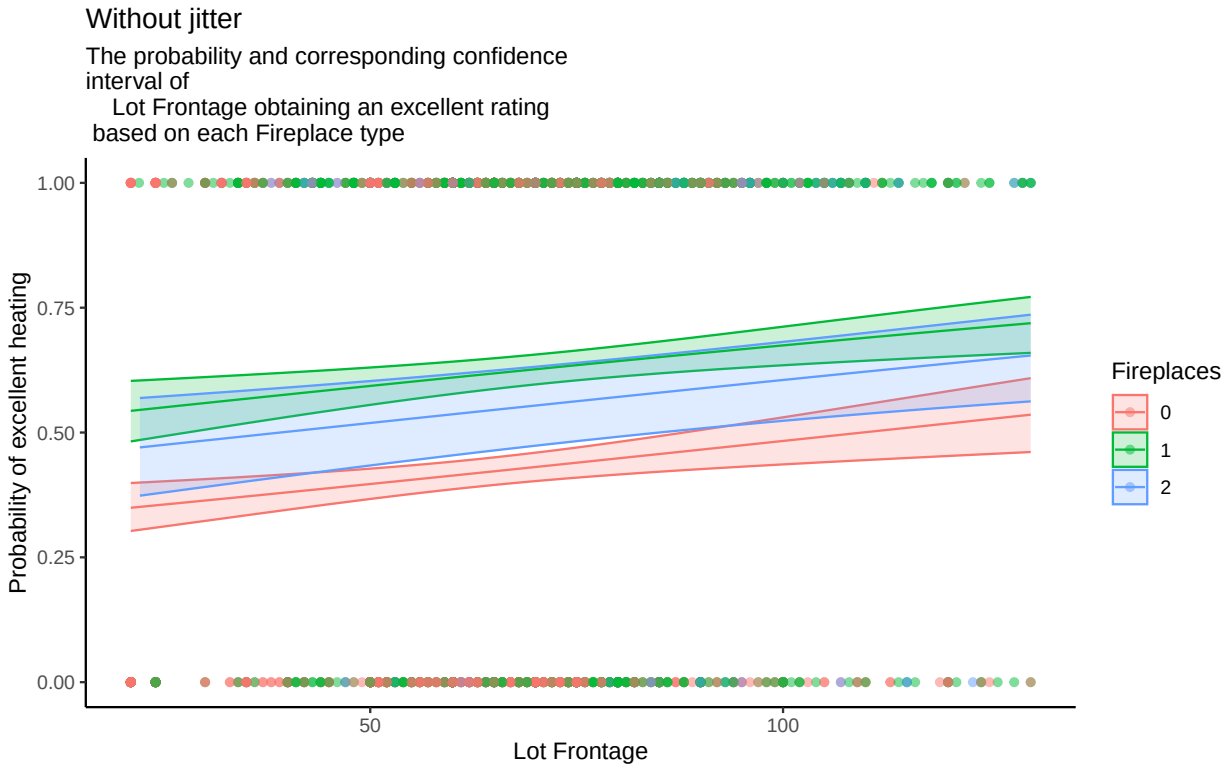
```
glmod <- glm(formula = as.factor(excellent_heating) ~ Lot_Frontage + Fireplaces,
             family = "binomial",
             data = Ames3)
glmod
```

```
Call:  glm(formula = as.factor(excellent_heating) ~ Lot_Frontage + Fireplaces,
    family = "binomial", data = Ames3)

Coefficients:
 (Intercept)  Lot_Frontage   Fireplaces1   Fireplaces2
   -0.769387      0.007018      0.796183      0.494887

Degrees of Freedom: 2399 Total (i.e. Null);  2396 Residual
Null Deviance:      3324
Residual Deviance: 3213     AIC: 3221
```

b. Construct confidence bands for the variable excellent_heating as a function of Lot_Frontage for each number of Fireplaces (hint: create a new data frame for each number of Fireplaces). Colour these with different transparent colours for each number of Fireplaces and plot them together on the same axes. Put the actual data on the plot, coloured to match the bands, and jittered in position to make it possible to see all points. Ensure you have an informative main plot title, axes labels and a legend. (7 points)

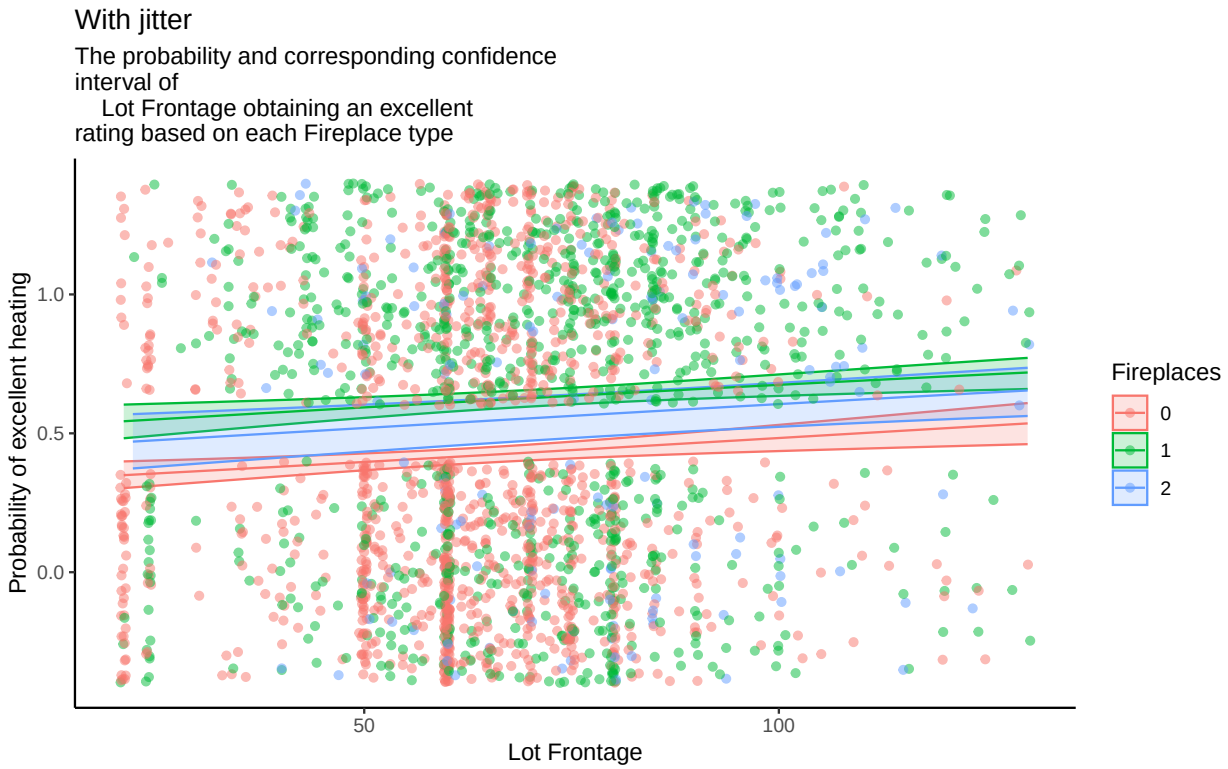
```
ilink <-family(glmod)$linkinv

newf <- with(
  Ames3,
  data.frame(
    Lot_Frontage = seq(min(Ames3$Lot_Frontage),
                       max(Ames3$Lot_Frontage),
                       length = 100),
    Fireplaces))

newf <- cbind(newf,
              predict(
                glmod,
                newf,
                type ="link",
                se.fit=TRUE)[1:2])

newf <-transform(newf,
                 Fitted = ilink(fit),
                 Upper = ilink(fit+(1.96*se.fit)),
                 Lower = ilink(fit-(1.96*se.fit)))

#### Original (actual data without jitter) ####
figure_8a <- ggplot(Ames3,
                    aes(x = Lot_Frontage,
                        y = as.numeric(as.factor(excellent_heating)) - 1,
                        colour = Fireplaces)) + # so the points are coloured.
  geom_ribbon(data = newf,
              aes(
                ymin = Lower,
                ymax = Upper,
                x = Lot_Frontage,
                fill = Fireplaces,
                colour = Fireplaces),
              alpha = 0.2,
```

```
              inherit.aes = FALSE) +
    geom_line(data = newf,
              aes(y = Fitted,
                  x = Lot_Frontage,
                  group = Fireplaces,
                  colour = Fireplaces)) +
  geom_point(alpha = 0.5) + # so all points can be seen.
  labs(
    title = "Without jitter",
    subtitle = "The probability and corresponding confidence\ninterval of
    Lot Frontage obtaining an excellent rating\n based on each Fireplace type",
    x = "Lot Frontage",
    y = "Probability of excellent heating") +
  theme_classic()

figure_8a
```




```
#### Final (actual data with jitter) ####
figure_8b <- ggplot(Ames3,
                    aes(x = Lot_Frontage,
                        y = as.numeric(as.factor(excellent_heating)) - 1,
                        colour = Fireplaces)) + # so the points are coloured.
  geom_ribbon(data = newf,
              aes(
                ymin = Lower,
                ymax = Upper,
                x = Lot_Frontage,
```

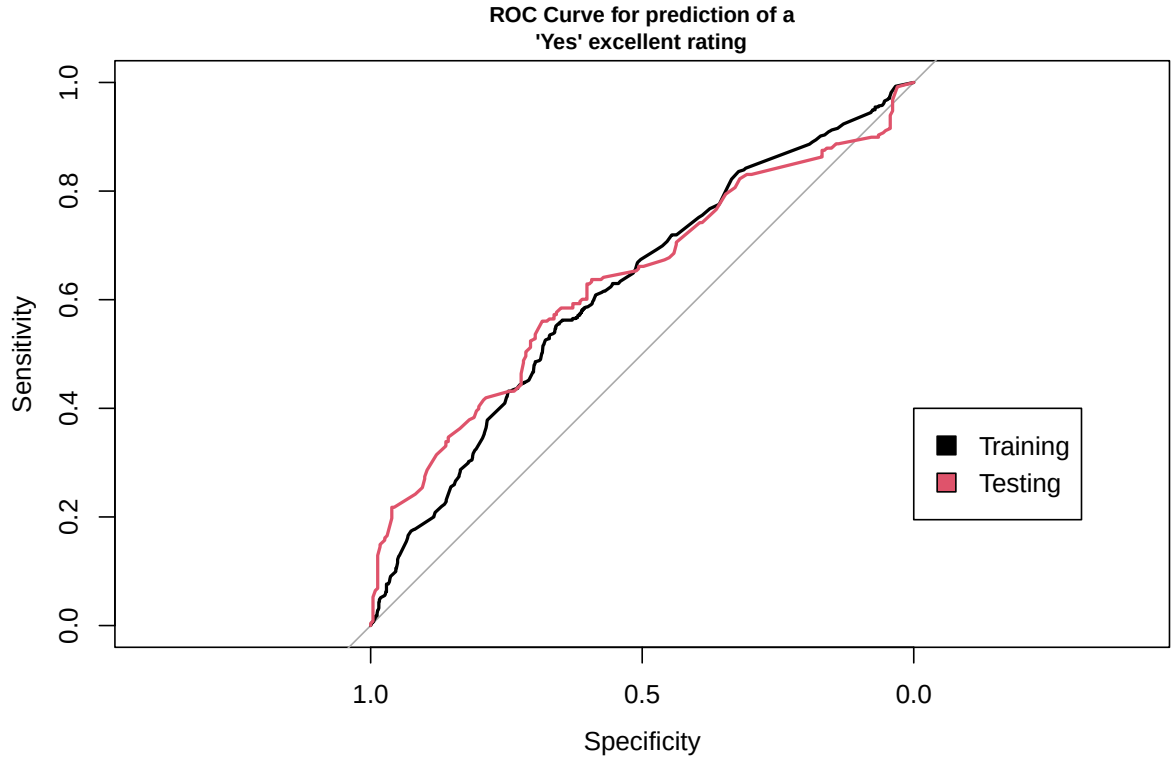
```
                fill = Fireplaces,
                colour = Fireplaces),
              alpha = 0.2,
              inherit.aes = FALSE) +
    geom_line(data = newf,
              aes(y = Fitted,
                  x = Lot_Frontage,
                  group = Fireplaces,
                  colour = Fireplaces)) +
  geom_point(position = "jitter", alpha = 0.5) + # so all points can be seen.
  labs(
    title = "With jitter",
    subtitle = "The probability and corresponding confidence\ninterval of
    Lot Frontage obtaining an excellent\nrating based on each Fireplace type",
    x = "Lot Frontage",
    y = "Probability of excellent heating") +
  theme_classic()

figure_8b
```



c. Split the data using set.seed(120) and rebuild the model on 80% of the data. Cross validate on the remaining 20%. Plot the ROCs for both data and comment on your findings. (6 points)

```
set.seed(120)
training.samples <- c(Ames3$excellent_heating) %>%
  createDataPartition(p = 0.8, list = FALSE)
train.data <- Ames3[training.samples, ]
```

```r
test.data <- Ames3[-training.samples, ]

train.model <- glm(formula = as.factor(excellent_heating) ~ Lot_Frontage + Fireplaces,
             family = "binomial",
             data = train.data)

predtrain <- predict(train.model, type = "response")
predtest <- predict(train.model, newdata = test.data , type = "response")


roctrain <- roc(response = train.data$excellent_heating,
                predictor = predtrain,
                plot = TRUE,
                main = "ROC Curve for prediction of a\n'Yes' excellent rating",
                cex.main = 0.85,
                auc = TRUE)

roc(response = test.data$excellent_heating,
    predictor = predtest,
    plot = TRUE,
    auc = TRUE,
    add = TRUE,
    col = 2)

legend(0, 0.4, legend = c("Training", "Testing"), fill = 1:2)
```



```
# The testing and training ROC curves are similar in shape and closely overlap.
# Therefore this is a good indication the testing data is not...
# over fitted to the training data.
```

4. Multinomial Regression

a. For the dataset "Ames", create a model multregmod to predict BsmtFin_Type_1 from Total_Bsmt_SF and Year_Remod_Add. (3 points)

```
multregmod <- multinom(formula = BsmtFin_Type_1 ~ Total_Bsmt_SF + Year_Remod_Add,
                       data = Ames)
multregmod
```

```
# weights:  28 (18 variable)
initial  value 5701.516737
iter  10 value 4611.614897
iter  20 value 4159.256251
iter  30 value 4153.561922
iter  40 value 4150.324235
iter  50 value 4146.549269
iter  60 value 4144.509436
iter  70 value 4144.474970
final  value 4144.474825
converged
Call:
multinom(formula = BsmtFin_Type_1 ~ Total_Bsmt_SF + Year_Remod_Add,
    data = Ames)

Coefficients:
            (Intercept) Total_Bsmt_SF Year_Remod_Add
BLQ           34.465254  6.282504e-05   -0.017706708
GLQ         -105.324418  1.030040e-03    0.052676145
LwQ           39.566891  1.243787e-05   -0.020550529
No_Basement    4.876103 -1.729079e-01    0.004007989
Rec           56.710979  1.596801e-06   -0.028929851
Unf          -29.377212 -6.987213e-04    0.015514051

Residual Deviance: 8288.95
AIC: 8324.95
```

b. Write out the formulas for this model in terms of P(No_Basement), P(Unf) P(Rec),P(BLQ), P(GLQ), P(LwQ),
You may round coefficients to 3 dp. (4 points)

The probabilities in terms of logit are:

$$logit(P(\text{BLQ})) = 34.465 + ((6.283 \times 10^{-5}) \times \text{TotalBmstSF}) + (-0.018 \times \text{YearRemodAdd})$$

$$logit(P(\text{GLQ})) = -105.324 + (0.001 \times \text{TotalBmstSF}) + (0.053 \times \text{YearRemodAdd})$$

$$logit(P(\text{LwQ})) = 39.567 + ((1.244 \times 10^{-5}) \times \text{TotalBmstSF}) + (-0.021 \times \text{YearRemodAdd})$$

$$logit(P(\text{NoBasement})) = 4.876 + (-0.173 \times \text{TotalBmstSF}) + (0.004 \times \text{YearRemodAdd})$$

$$logit(P(\text{Rec})) = 56.711 + ((1.597 \times 10^{-6}) \times \text{TotalBmstSF}) + (-0.029 \times \text{YearRemodAdd})$$

$$logit(P(\text{Unf})) = -29.377 + ((-6.99 \times 10^{-4}) \times \text{TotalBmstSF}) + (0.016 \times \text{YearRemodAdd})$$

Where:

```
# Checked levels to find the base case.
levels(Ames$BsmtFin_Type_1)
```

```
[1] "ALQ"         "BLQ"         "GLQ"         "LwQ"         "No_Basement"
[6] "Rec"         "Unf"
```

$$P(\text{ALQ}) = 1 - P(\text{BLQ}) - P(\text{GLQ}) = P(\text{LwQ}) - P(\text{NoBasement}) - P(\text{Rec}) - P(\text{Unf})$$

c. Evaluate the performance of this model using a confusion matrix and by calculating the sum of sensitivities for the model. Comment on your findings. (4 points)

```
multitable <- table(Ames$BsmtFin_Type_1,
                    predict(multregmod,
                            type ="class"))

names(dimnames(multitable)) <- list("Actual","Predicted")

multitable
```

```
             Predicted
Actual        ALQ BLQ GLQ LwQ No_Basement Rec Unf
  ALQ           1   0 117   0           0  18 293
  BLQ           0   0  50   0           0  30 189
  GLQ           1   0 579   0           0   2 277
  LwQ           1   0  38   0           0  30  85
  No_Basement   0   0   0   0          80   0   0
  Rec           3   0  31   0           0  46 208
  Unf           6   0 291   0           0  76 478
```

Evaluation:

First calculate the sensitivity and specificity for each category:

$$Sensitivity(\text{ALQ}) = \frac{1}{1 + 117 + 18 + 293} = 0.233\%$$

$$Sensitivity(\text{BLQ}) = \frac{0}{50 + 30 + 189} = 0\%$$

$$Sensitivity(\text{GLQ}) = \frac{579}{1 + 579 + 2 + 277} = 67.404\%$$

$$Sensitivity(\text{LwQ}) = \frac{0}{1 + 38 + 30 + 85} = 0\%$$

$$Sensitivity(\text{NoBasement}) = \frac{80}{80} = 100\%$$

$$Sensitivity(\text{Rec}) = \frac{46}{3+31+46+208} = 15.972\%$$

$$Sensitivity(\text{Unf}) = \frac{478}{6+291+76+478} = 56.169\%$$

2. Then calculating the sum of sensitivities for the model:

```
# Calculated manually:
sum_of_sensitivites <- 0.002331002 + 0.6740396 + 1 + 0.1597222 + 0.5616921

sum_of_sensitivites

# Checked by automated sum of sensitivities:
ss_check <- multitable[1,1]/sum(Ames$BsmtFin_Type_1=="ALQ") +
            multitable[2,2]/sum(Ames$BsmtFin_Type_1=="BLQ") +
            multitable[3,3]/sum(Ames$BsmtFin_Type_1=="GLQ") +
            multitable[4,4]/sum(Ames$BsmtFin_Type_1=="LwQ") +
            multitable[5,5]/sum(Ames$BsmtFin_Type_1=="No_Basement") +
            multitable[6,6]/sum(Ames$BsmtFin_Type_1=="Rec") +
            multitable[7,7]/sum(Ames$BsmtFin_Type_1=="Unf")

ss_check
# Since these two values are the same the sum of the sensitivities is correct.

# Comments: Sensitivities per category ->
# Generally the model does a better job at predicting GLQ, No_Basement and Unf.
# Particularly No_Basement which predictions were correct 100% of the time.
# In comparison ALQ and Rec were much less likely to be predicted correctly,
# with sensitivities of 0.002331002 and 0.1597222 respectively.
# The model was worst at predicting BLQ and LwQ with sensitivities of 0.

# Comments: Sum of sensitivities ->
# The sum of the sensitivities is 2.397785.
# This is less than 7 which would be the sum of sensitivities...
# for a perfect model. The model is not very good overall...
# since the sum of sensitivities is well below the...
# perfect model value. Instead only 34% (2.s.f)...
# of all categories are correctly predicted overall.
```

```
[1] 2.397785
[1] 2.397785
```

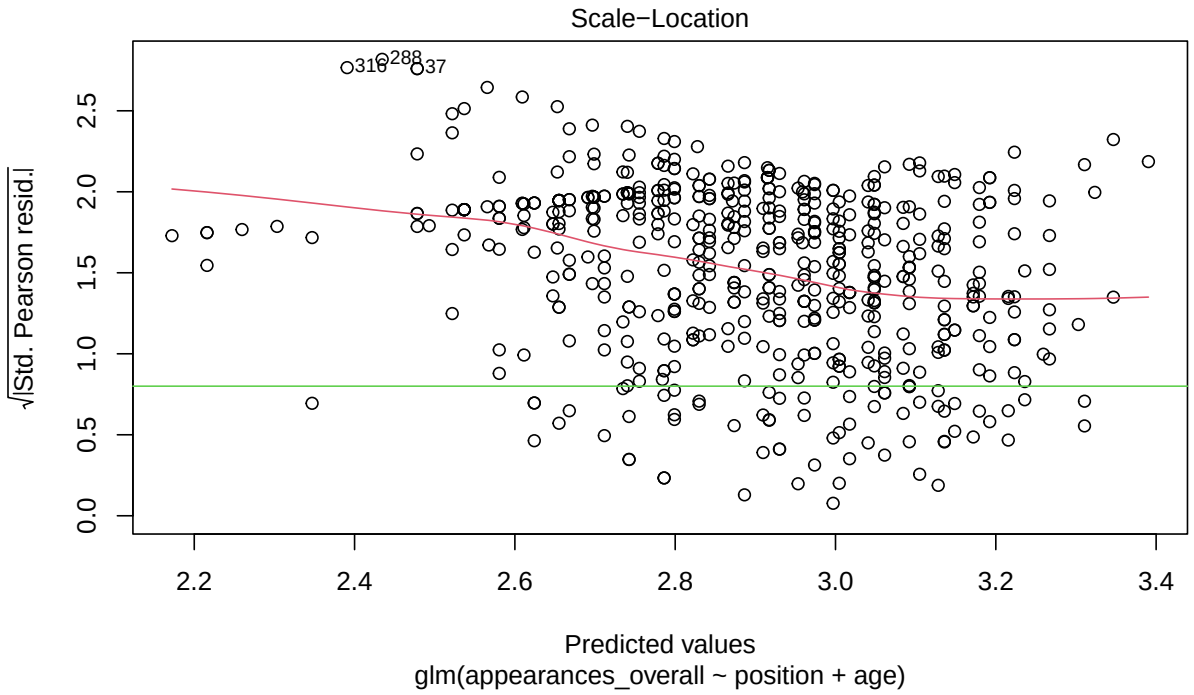
5. Poisson/quasipoisson Regression

a. For the "footballer_data" dataset, create a model appearances_mod to predict the total number of overall appearances a player had based on position and age. (2 points)

```r
appearances_mod <- glm(formula = appearances_overall ~ position + age,
                       data = footballer_data,
                       family = "poisson")
```

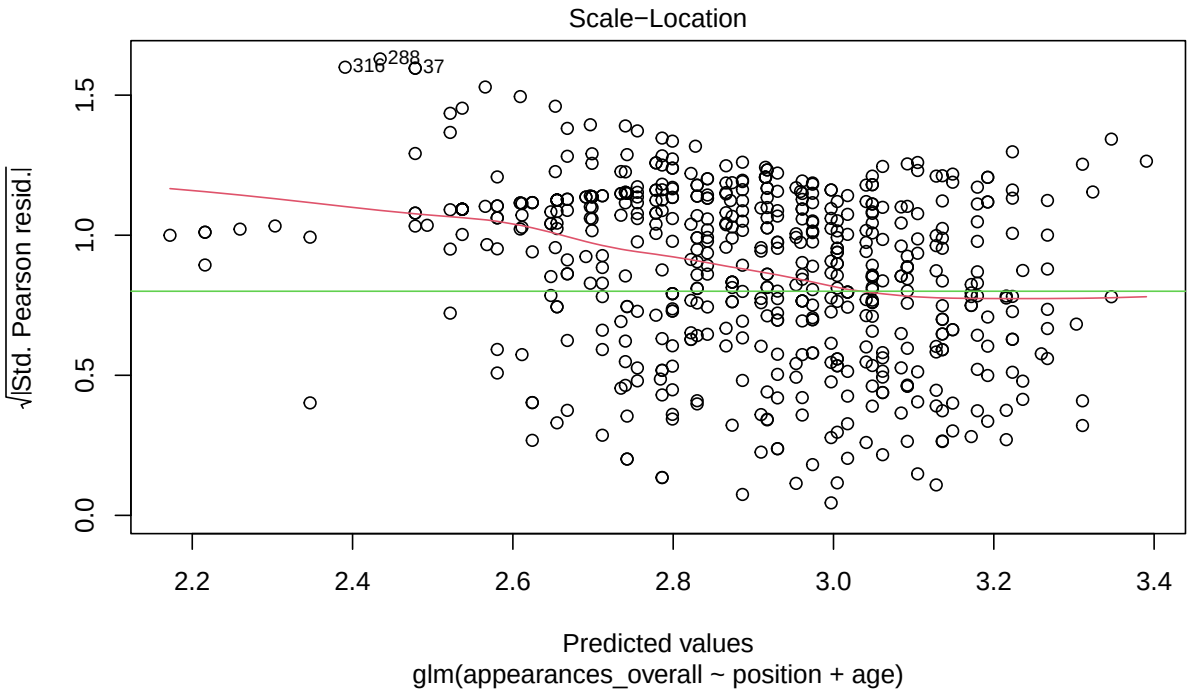
b. Check the assumption of the model using a diagnostic plot and comment on your findings. (3 points)

```r
# The assumption that needs to be checked is whether the variance = mean...
# i.e., the dispersion assumption.

plot(appearances_mod, which = 3)
abline(h = 0.8, col = 3)
```




```r
# Findings:
# The red line is not flat, and it is consistently above 0.8.
# This suggests over dispersion in the data that decreases...
# somewhat linearly as the prediction increases. Therefore...
# the variance is not equal to the mean.

# Since the dispersion appears to roughly be a linear function...
# of the mean, it is possible a quasipoisson model may...
# be a better fit and reduce the magnitude of over dispersion.

# This was tested below:
appearances_mod2 <- glm(formula = appearances_overall ~ position + age,
                        data = footballer_data,
                        family = "quasipoisson")
```

```r
plot(appearances_mod2, which = 3)
abline(h = 0.8, col = 3)
```




```r
# And as expected the over dispersion reduces. This indicates the...
# the quasipoisson model is more suitable than the poisson model.
```

c. What do the coefficients of the model tell us about? which position has the most appearances? How many times more appearances do forwards get on average than goalkeepers? (3 points)

```r
# Used the quasipoission model (since it has less over dispersion):
summary(appearances_mod2)

# The coefficients of the model tells us...
# Age: Every increase of appearances overall by 1...
#      is due to an increase in age by a rate of 0.043704.
# Position: And it tells us the rate of change of each...
#           positions relative to the base case position - the Defender.

# Position with the most appearances: Midfielder

# How many more times forwards appear than goalkeepers: 1.6 times
# Rounded up its 2 times.
forward_goalkeeper_ratio <- exp(0.110606)/exp(-0.364605)
forward_goalkeeper_ratio
```

```
Call:
glm(formula = appearances_overall ~ position + age, family = "quasipoisson",
    data = footballer_data)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-7.5377  -3.5215   0.0351   2.1892   6.1853

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)         1.575316   0.223980   7.033 5.90e-12 ***
positionForward     0.110606   0.082097   1.347  0.17844
positionGoalkeeper -0.364605   0.121975  -2.989  0.00292 **
positionMidfielder  0.118259   0.069717   1.696  0.09039 .
age                 0.043704   0.007153   6.110 1.87e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for quasipoisson family taken to be 8.946343)

    Null deviance: 6539.7  on 564  degrees of freedom
Residual deviance: 6114.4  on 560  degrees of freedom
AIC: NA

Number of Fisher Scoring iterations: 5

[1] 1.608354
```