Problem 4

The change of the angular momentum is related to the change of magnetization,

$$\Delta\mathbf{L} = \frac{1}{g\mu}\Delta\mathbf{M}, \tag{1}$$

where g is g-factor and $\mu = e/2mc$.

Let's consider a toy model to discover some features of the magnetization change during the collision. Let spin Hamiltonian be Heisenberg Hamiltonian. It's spectrum is quadratic for sufficiently small wave vectors and low temperatures,

$$\epsilon_k = \sum_k sJa^2k^2, \tag{2}$$

where J is the exchange integral, a is the length of the primitive vector of the lattice, which is cubic primitive.

The exchange integral is exponential in the distance between atoms, and under compression it should grow. It is equivalent to the decrease of temperature, since properties of the system should depend on the dimensionless parameter J/T. Deformation from the collision appear near the end of the rod, creating region of effectively small temperature. The ground state of a ferromagnet is "all spins in one direction and the state of spins in that region will be close to it. The strain will spread to the other end of the rod, with more and more spins entering the region of low temperature. Every layer of atoms contains the number of spins small relative to the number of spins in the region of effectively small temperature, and barely can change the direction of all spins in the region. Hence, every spin in the rod will soon point to the same direction initial group of spins was pointing.

On account of excitations some group of spins will deviate from the alignment. We are interested in the magnon which will point spins in several atomic layers near the end of the rod to the direction that is different from the ground state direction. The amplitude of deviation will be proportional to the number of such magnons.

$$n = \frac{1}{\mathrm{e}^{\beta\epsilon} - 1} \approx \frac{1}{\mathrm{e}^{\beta sJ} - 1}, \tag{3}$$

and the total deviation will be proportional to $n/2s$, since $2s$ is the maximum number of magnons, corresponding to the full flip of spins. n is small, so that Holstein-Primakoff transformation could be approximated by it's Taylor series.

Thus, the change of the angular momentum in this model is

$$\Delta L \propto \frac{1}{g\mu}\frac{n}{2s}M \propto \frac{M}{2sg\mu(\mathrm{e}^{sJ\beta} - 1)}. \tag{4}$$

It is clear that presented change of the angular momentum is the approximation of the upper bound of possible changes. Less uniformity in the directions of spins near the end of the rod in the moment of collision will result in less change of the magnetization, with the most probable change being equal to zero.

The average deformation in the moment, when the rod has zero velocity, is

$$\epsilon = u\sqrt{\frac{\rho}{E}}. \tag{5}$$

It influences the exchange integral, the efficiency of temperature lowering and the validity of the whole reasoning. However, the exact form of the exchange integral is not known - to me, at least, - so I cannot do anything useful with it.

After the collision, sound waves will propagate in the rod, changing deformation, the exchange integral and magnetization in the same fashion. After the energy of the sound waves dissipates into sound and heat, magnetization (and angular momentum) will not change. (This does not depend on the mechanism of the changing of magnetization; if there is a mechanism that transform a deformation into a change of magnetization, it will work with sound waves, which are, essentially, deformations.)