Problem 1

Team 2

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Newton's equations:

$$\begin{pmatrix} m\left(v_x' t\right) \\ m\left(v_y' t\right) \end{pmatrix} = \begin{pmatrix} m f(t)\cos\alpha(t) - k\left(v_x t\right) \\ -\left(v_y t\right)k - gm + m f(t)\sin\alpha(t) \end{pmatrix} \tag{1}$$

Here $f(t) = \frac{F(t)}{m}$
Force $f(t)$ acting under angle $\alpha(t)$ Let's solve (1) for v_x, v_y

$$v_x = e^{-\frac{kt}{m}} \int_0^t f(t) e^{\frac{kt_2}{m}} \left(\cos(\alpha(t_2))\right) dt_2 \tag{2}$$

$$v_y = e^{-\frac{kt}{m}} \int_0^t e^{\frac{kt_2}{m}} \left(f(t)(\sin(\alpha(t_2))) - g\right) dt_2 \tag{3}$$

$$\left(a_y + g + \beta v_y\right)^2 + \left(a_x + \beta v_x\right)^2 = f^2(t) \tag{4}$$

From this we can derive $a_x(t)$ and find $v_x(t)$

$$v_x(t) = e^{-\beta t} \int_0^t e^{\beta t_2} \sqrt{f(t_2)^2 - \left(g + v_y'(t_2) + \beta v_y(t_2)\right)^2}\, dt_2 \tag{5}$$

$$L_x = L = \int_0^{t_1} e^{-\beta t} \int_0^t e^{\beta t_2} \sqrt{f(t_2)^2 - \left(g + \beta v_y\left(t_2\right) + v_y'\left(t_2\right)\right)^2}\, dt_2\, dt \tag{6}$$

$$L_y = \int_0^{t_1} \left(\int_0^t v_y'(t_2)\, dt_2 \right) dt = 0 \tag{7}$$

$$F(t, t_2, v_y, v_y') = e^{\beta(t - t_2)} \sqrt{f(t)^2 - (g + \beta v_y + v_y')^2} + \lambda v_y' \tag{8}$$

$$\frac{\partial F}{\partial v_y} - \frac{\partial}{\partial t}\frac{\partial F}{\partial v_y'} = 0 \tag{9}$$

$$f'\left(t\right)\left(g + \beta v_y\left(t\right) + v_y'\left(t\right)\right) = f\left(t\right)\left(\beta v_y'\left(t\right) + v_y''\left(t\right)\right) \tag{10}$$

From (10) we can see that force should act on a constant angle

For the case $F(t) = F_0(1 + \frac{t}{t_0})^2$ and $t_0 >> \frac{m}{k}$

$$v_x(t) = C_2 e^{-\beta t} \left(\beta \mathrm{Ei}((t + t_0)\beta) - \frac{e^{\beta(t + t_0)}}{t + t_0} \right) + C_1 e^{-\beta t} - \frac{g}{\beta} \tag{11}$$

$$v_x(t) = \frac{f_0 \cos\alpha}{\beta \left(\frac{t}{t_0} + 1 \right)^2} \tag{12}$$

$$v_y(t) = \frac{\frac{f_0 \sin\alpha}{\left(\frac{t}{t_0}+1\right)^2} - g}{\beta} \tag{13}$$

$$\frac{g(t_0+t_1) - f_0 t_0 \sin\alpha}{\beta} = 0 \tag{14}$$

$$t_1 = \frac{f_0 t_0 \sin\alpha - g t_0}{g} \tag{15}$$

Finally,

$$L = \frac{\sqrt[3]{f_0} t_0 \left(f_0^{2/3} - g^{2/3}\right) \sqrt{1 - \frac{g^{2/3}}{f_0^{2/3}}}}{\beta} \tag{16}$$