Problem 1

Nagaoka bags

January 28-29, 2017

Let's use Caldeira-Leggett equation:

$$\frac{d}{dt}\rho_S(t) = -\frac{i}{\hbar}\left[H_S, \rho_S(t)\right] - \frac{i\Gamma}{2\hbar}\left[x, \{p, \rho_S(t)\}\right] - \Lambda\left[x, [x, \rho_S(t)]\right] \tag{1}$$

Where ρ_S is density matrix, H_S is hamiltonian of the free system, $\{A, B\}$ is anticommutator of operators A and B, $[A, B]$ is commutator of them and

$$\Lambda = \frac{2m\Gamma k_B T}{\hbar^2} \tag{2}$$

First part in right-hand side of equation in (1) stands for free coherent dynamics of the system, second responds to dissipative dynamics, and third responds to fluctuation and leads to decoherence of the system.

$$[x, [x, \rho_S]] = [x, x\rho_S - \rho_S x] = xx\rho_S - x\rho_S x - x\rho_S x + \rho_S xx = x^2\rho_S - 2xx'\rho_S + {x'}^2\rho_S = (x - x')^2\rho_S \tag{3}$$

In a first approximation, we can ignore free evolution and solve eq (1) using just the third part of right-hand side

$$\rho_S(t, x, x') \simeq \exp\left[-\Lambda(x - x')^2 t\right]\rho_S(0, x, x') \tag{4}$$

$$\tau_D = \frac{1}{\Lambda\Delta x^2}, \Delta x = |x - x'| \simeq L \tag{5}$$

Finally,

$$\tau_D = \frac{\hbar^2}{2m\Gamma k_B T L^2} \tag{6}$$