The Green function of a massive scalar field is

$$G_3(r) = \frac{e^{-\kappa a}}{|r|} \tag{1}$$

in 3D and

$$G_2(r) = K_0(\kappa |r|) \tag{2}$$

in 2D. Assuming that $\kappa\varepsilon << 1$, one can see that a point charge Q induces a charge

$$Q_2(r) = -\frac{K_0(\kappa L)}{\ln \dfrac{2}{\gamma \kappa \varepsilon}} \tag{3}$$

$$Q_3(r) = -\frac{e^{-\kappa L}\varepsilon}{|L|} \tag{4}$$

on a grounded sphere located at distance L from the charge in 2D and 3D respectively.

Consider the 3D case. Then, at big distances, the lattice may be considered as a continuous medium. Each grounded sphere carries a charge $\phi\varepsilon$, where ϕ stands for the potential of all the other spheres. At first, we assume that ϕ is equal to the mean field, and obtain the equation of the mean field. Since

$$-\triangle\phi + \kappa^2\phi = 4\pi\rho = -4\pi\phi\frac{\varepsilon}{a^3} \tag{5}$$

then the propagation of the field in such medium is equivalent to the propagation of the field with effective mass κ_{eff}:

$$\kappa_{eff}^2 = \kappa^2 + 4\pi\frac{\varepsilon}{a^3} \tag{6}$$

Now we are going to take the difference between mean field and field interacting with the sphere into account. To obtain the difference, we assume that the field in the medium is propagating according to the equation (5), however, now the mass κ_{eff} is unknown. Then, we pick a sphere, and replace other spheres with a continuous medium with a spherical cavity around our sphere. The volume of the cavity is equal to the a^3; let r_c be the radius of the cavity. The cavity then can be considered as a medium and an external charge that 'cancels' the induced charge of the medium. Assuming that the relation between the potential and the density of the induced charge is

$$\rho = -\alpha\phi \tag{7}$$

then the cancelling charge is

$$\rho(\mathbf{r}) = \alpha\frac{\exp\left(-\kappa_{eff} r\right)}{r} \approx \alpha\frac{\exp\left(-\kappa_{eff}\mathbf{rn}\right)}{R} \tag{8}$$

where $\mathbf{R} = R\mathbf{n}$ is a radius-vector of our sphere. The potential created by the charge in the center of the cavity is

$$\begin{aligned}
\phi_c &= \int_0^{r_c} \oint r^2 dr d\cos\theta d\phi \, \alpha \frac{\exp\left(-\kappa_{eff}(r\cos\theta + R)\right)}{R} \frac{\exp\left(-\kappa r\right)}{r} \\
&= \frac{2\pi\alpha \exp\left(-\kappa_{eff} R\right)}{R} \int_0^{r_c} dr \, \exp\left(-\kappa r\right) \frac{\exp\left(\kappa_{eff} r\right) - \exp\left(-\kappa_{eff} r\right)}{\kappa_{eff}} \\
&= \frac{2\pi\alpha \exp\left(-\kappa_{eff} R\right)}{\kappa_{eff} R} \left[\frac{1 - \exp\left(-(\kappa - \kappa_{eff}) r_c\right)}{(\kappa - \kappa_{eff})} - \frac{1 - \exp\left(-(\kappa_{eff} + \kappa) r_c\right)}{(\kappa_{eff} + \kappa)} \right]
\end{aligned} \tag{9}$$

The induced charge is

$$\begin{aligned}
Q_i &= \varepsilon \left[\frac{\exp\left(-\kappa_{eff} R\right)}{R} + \frac{2\pi\alpha \exp\left(-\kappa_{eff} R\right)}{\kappa_{eff} R} \left[\frac{1 - \exp\left(-(\kappa - \kappa_{eff}) r_c\right)}{(\kappa - \kappa_{eff})} - \frac{1 - \exp\left(-(\kappa_{eff} + \kappa) r_c\right)}{(\kappa_{eff} + \kappa)} \right] \right] \\
&= a^3 \alpha \frac{\exp\left(-\kappa_{eff} R\right)}{R}
\end{aligned} \tag{10}$$

Then

$$\varepsilon \left[1 + \frac{2\pi\alpha}{\kappa_{eff}} \left[\frac{1 - \exp\left(-(\kappa - \kappa_{eff}) r_c\right)}{(\kappa - \kappa_{eff})} - \frac{1 - \exp\left(-(\kappa_{eff} + \kappa) r_c\right)}{(\kappa_{eff} + \kappa)} \right] \right] = a^3 \alpha \tag{11}$$

where

$$\kappa_{eff}^2 = \kappa^2 + 4\pi\alpha \tag{12}$$

This equation cannot be solved analytically. Consider the limit $\kappa a << 1$. Then, $\kappa_{eff} a << 1$. Expanding exponents in (11) into Taylor series up to the second order, we obtain

$$\varepsilon \left[4\pi + 2\pi(\kappa_{eff}^2 - \kappa^2) r_c^2 \right] = a^3 (\kappa_{eff}^2 - \kappa^2) = \frac{4\pi}{3} r_c^3 (\kappa_{eff}^2 - \kappa^2) \tag{13}$$

$$\varepsilon \left[6 + 3(\kappa_{eff}^2 - \kappa^2) r_c^2 \right] = 2 r_c^3 (\kappa_{eff}^2 - \kappa^2) \tag{14}$$

$$6\varepsilon + [2r_c^3 - 3r_c^2\varepsilon]\kappa^2 = [2r_c^3 - 3r_c^2\varepsilon]\kappa_{eff}^2 \tag{15}$$

$$\kappa_{eff}^2 = \kappa^2 + \frac{6\varepsilon}{2r_c^3 - 3r_c^2\varepsilon} \approx \kappa^2 + \frac{4\pi\varepsilon}{a^3} \tag{16}$$

The formula (16) is applicable in the case of electrostatic field too. Hence, the field acting on the sphere is approximately equal to the mean field. Finally, we can write the answer, assuming that the central sphere is placed into a cavity of radius r_c in the medium:

$$\phi(r) = \frac{\exp\left(-\kappa r_c\right) \exp\left(-\sqrt{\kappa^2 + 4\pi\varepsilon/a^3}(r - r_c)\right)}{r}, r_c = \sqrt[3]{\frac{3}{4\pi}} a \tag{17}$$

Now let us move to the 2D case. Now we assume that the field acting on the sphere is approximately equal to the mean field similarly to the 3D case. Now the coefficient α is

$$\alpha = -\frac{\rho}{\phi} = -\frac{1}{a^2 \ln \dfrac{2}{\gamma\kappa\varepsilon}} \tag{18}$$

$$\kappa_{eff}^2 = \kappa^2 + \frac{2\pi}{a^2 \ln \dfrac{2}{\gamma\kappa\varepsilon}} \tag{19}$$

and

$$\phi(r) = \frac{K_0(\kappa r_c) K_0(\kappa_{eff} r)}{K_0(\kappa_{eff} r_c)}, r_c = \sqrt{\frac{1}{\pi}} a \tag{20}$$

The asymptotic behavior is

$$\phi(r) = \sqrt{\frac{\pi}{2}} \frac{K_0(\kappa r_c)}{K_0(\kappa_{eff} r_c)} \frac{\exp(-\kappa_{eff} r)}{\sqrt{r}} \tag{21}$$

In the case of $\kappa a << 1$ we also obtain $\kappa_{eff} a << 1$. This results into

$$\phi(r) = \sqrt{\frac{\pi}{2}} \frac{\ln \dfrac{2}{\gamma\kappa r_c}}{\ln \dfrac{2}{\gamma\kappa_{eff} r_c}} \frac{\exp(-\kappa_{eff} r)}{\sqrt{r}} \tag{22}$$

In the case of $-1/(\ln \kappa\varepsilon) >> \kappa^2 a^2$ we have

$$\kappa_{eff} = \sqrt{\frac{2\pi}{a^2 \ln \dfrac{2}{\gamma\kappa\varepsilon}}} \left[1 + \frac{k^2 a^2 \ln \dfrac{2}{\gamma\kappa\varepsilon}}{2\pi} \right] \tag{23}$$

This shows that the massless limit cannot be obtained by simply letting $\kappa \to 0$. Moreover, since $\phi \propto \ln r$, the induced charge turns out to depend on the capacity of the ground; hence the problem is ill-posed (?!?).

Finally, we answer the last questions. If the spheres were not grounded, then the main contribution to the field would be defined by the dipole momentum. This would result into well-known behavior of the dielectrics - there would be no change in the effective mass of the field, but a change of the field strength. In 3D case, there is an infinite limit: both ε and a should approach zero provided that the ratio ε/a^3 is saved (the relation $\varepsilon = const\, a^3 << a$ will hold for small a). The same is true for 2D, however, the ratio $a^2 \ln \dfrac{2}{\gamma\kappa\varepsilon}$ should hold instead.