Let us introduce the coordinate system such that the dielectric occupies the semi-space $z < 0$. We then pose a charge e in the point $(0, 0, a)$, where $a > 0$. Then we are going to construct the permittivity tensor, such that the fields E, D were described by the following expressions:

$$\mathbf{E}(\mathbf{x},\mathbf{y},\mathbf{z}) = \mathbf{D}(\mathbf{x},\mathbf{y},\mathbf{z}) = \frac{e(\mathbf{r}-\mathbf{r_a})}{|\mathbf{r}-\mathbf{r_a}|^3}, z > 0 \tag{1}$$

$$\mathbf{E}(\mathbf{x},\mathbf{y},\mathbf{z}) = \frac{e(\mathbf{r}+\mathbf{r_a})}{|\mathbf{r}+\mathbf{r_a}|^3}, z < 0 \tag{2}$$

$$\mathbf{D}(\mathbf{x},\mathbf{y},\mathbf{z}) = \frac{e(\mathbf{r}-\mathbf{r_a})}{|\mathbf{r}-\mathbf{r_a}|^3}, z < 0 \tag{3}$$

where $\mathbf{r}$ and $\mathbf{r_a}$ stand for radius-vectors of the points (x, y, z) and $(0, 0, a)$ respectively. One easily can see that such fields satisfy the boundary conditions

$$E_x(x, y, +0) = E_x(x, y, -0) \tag{4}$$

$$E_y(x, y, +0) = E_y(x, y, -0) \tag{5}$$

$$D_z(x, y, +0) = D_z(x, y, -0) \tag{6}$$

and the Maxwell equation

$$\nabla \cdot \mathbf{D}(\mathbf{x},\mathbf{y},\mathbf{z}) = 4\pi\rho = e\delta(\mathbf{r}-\mathbf{r_a}) \tag{7}$$

Then,

$$\nabla \cdot \mathbf{E}(\mathbf{x},\mathbf{y},\mathbf{z}) = 4\pi(\rho + \rho_{ind}) = e\delta(\mathbf{r}-\mathbf{r_a}) + e\delta(\mathbf{r}+\mathbf{r_a}) \tag{8}$$

and a point charge is induced in the medium. We will now write the tensor in the cylindrical coordinates $r = \sqrt{(x^2 + y^2)}, \phi = \tan^{-1} y/x, z = z$. The fields in the dielectric can be rewritten as:

$$E_r(r, \phi, z) = \frac{er}{(r^2 + (z + a)^2)^{3/2}} \tag{9}$$

$$E_z(r, \phi, z) = \frac{e(z + a)}{(r^2 + (z + a)^2)^{3/2}} \tag{10}$$

$$D_r(r, \phi, z) = \frac{er}{(r^2 + (z - a)^2)^{3/2}} \tag{11}$$

$$D_z(r, \phi, z) = \frac{e(z - a)}{(r^2 + (z - a)^2)^{3/2}} \tag{12}$$

$$\tag{13}$$

while E_ϕ and D_ϕ are both zero. We search the answer in the set of tensors written in the basis $\mathbf{e_r}, \mathbf{e_z}, \mathbf{e_\phi}$ of the following form:

$$\varepsilon = \begin{pmatrix} \frac{D_r(r,z)}{E_r(r,z)} & 0 & 0 \\ 0 & \frac{D_z(r,z)}{E_z(r,z)} & 0 \\ 0 & 0 & v(x,z) \end{pmatrix} + w(r,z) \begin{pmatrix} E_z^2(r,z) & -E_rE_z(r,z) & 0 \\ -E_rE_z(r,z) & E_r^2(r,z) & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{14}$$

Any of such tensors satisfy the equality $\mathbf{D} = \varepsilon\mathbf{E}, z < 0$. Our aim to find any functions $v(x,z), w(r,z)$, such that the result was a continuous function, except, maybe, the point $(0,0,-a)$. One of the possible solutions is:

$$\varepsilon = \begin{pmatrix} \dfrac{D_r(r,z)}{E_r(r,z)} & 0 & 0 \\ 0 & \dfrac{D_z(r,z)}{E_z(r,z)} & 0 \\ 0 & 0 & \dfrac{D_r(r,z)}{E_r(r,z)} \end{pmatrix} - \frac{1}{E_r^2(r,z)+E_z^2(r,z)} \left[\frac{D_z(r,z)}{E_z(r,z)}\right] \begin{pmatrix} E_z^2(r,z) & -E_rE_z(r,z) & 0 \\ -E_rE_z(r,z) & E_r^2(r,z) & 0 \\ 0 & 0 & E_z^2(r,z) \end{pmatrix} \tag{15}$$

The same tensor, rewritten explicitly as a function of r, z:

$$\varepsilon = \frac{(r^2+(z+a)^2)^{1/2}}{(r^2+(z-a)^2)^{3/2}} \begin{pmatrix} r^2+2a(z+a) & -r(z-a) & 0 \\ -r(z-a) & (z+a)(z-a) & 0 \\ 0 & 0 & r^2+2a(z+a) \end{pmatrix} \tag{16}$$

Its continuity is obvious after rewriting it in the basis $\mathbf{e_x}, \mathbf{e_y}, \mathbf{e_z}$:

$$\varepsilon = \frac{(x^2+y^2+(z+a)^2)^{1/2}}{(x^2+y^2+(z-a)^2)^{3/2}} \begin{pmatrix} x^2+y^2+2a(z+a) & 0 & -x(z-a) \\ 0 & x^2+y^2+2a(z+a) & -y(z-a) \\ -x(z-a) & -y(z-a) & (z+a)(z-a) \end{pmatrix} \tag{17}$$