The method described below is based on the Lense-Thirring effect - a rotating star causes the precession of the spin vector of a particle/gyroscope. From the quantum point of view, the precession of the particle spin is a consequence of difference in energies that different eigenstates of the particle have. Assume the spin is precessing around z-axis. Then

$$<\hat{S}_+>=<\hat{S}_x> +i <\hat{S}_y>\propto e^{i\omega t} \tag{1}$$

On the other hand, introducing the evolution operator $\hat{U} = \exp\left(-\frac{i\hat{H}t}{\hbar}\right)$, we obtain:

$$\begin{aligned}
<S_+> &=< \psi(t)|\hat{S}_+|\psi(t) >= \sum_{m=-S}^{S-1} < \psi(t)|m+1 >< m+1|\hat{S}_+|m >< m|\psi(t) >= \\
&= \sum_{m=-S}^{S-1} < m+1|\hat{S}_+|m >< \psi(0)|U^+(t)|m+1 >< m|U(t)|\psi(0) > \\
&= \sum_{m=-S}^{S-1} < \psi(0)|m+1 >< m+1|\hat{S}_+|m >< m|\psi(0) > \exp\left(\frac{i(E_{m+1}-E_m)t}{\hbar}\right) \\
&= \sum_{m=-S}^{S-1} < \psi(0)|\hat{S}_+|\psi(0) > \exp\left(\frac{i(E_{m+1}-E_m)t}{\hbar}\right)
\end{aligned} \tag{2}$$

Hence, $E_{m+1} - E_m = \hbar\omega$. The energy for flipping N spins from $m = S$ to $m = -S$ is $E = 2SN\hbar\omega$. Now we a ready to describe our experiment. We measure the spin as follows:

1. We take a rotating star (or a gravitational field of such star).
2. We put a gas of N particles with unknown spin S on the circular orbit along the star, and put a gyroscope on the same orbit.
3. We measure the angular speed ω of the gyroscope precession.
4. We wait before the gas of particles cools to the temperature $kT << \hbar\omega$ by radiating the gravitational waves.
5. We rapidly flip the direction of the star rotation (or alter the gravitational field). Now the direction of the Lense-Thirring precession is opposite, and the gas has the energy $2SN\hbar\omega$ above the ground state.
6. We again cool the gas to the same temperature as in the step 3, now measuring the gravitationally-radiated energy ΔE.

The spin can be obtained from the formula

$$S = \frac{\Delta E}{2N\hbar\omega} \tag{3}$$