VISUALIZING THE CORRELATION FUNCTIONS OF THE OSCILLATING NEUTRAL MESONS B^o AND K^o

Milo Gerhard
May 2023

CONTENTS

INTRODUCTION

The purpose of this investigation is to explore numerical results in the correlation functions C of the neutral B meson B^o and the neutral kaon K^o. This correlation function describes how the joint probabilities of the oscillating systems evolve over a given Δt, which is useful and necessary in the case of the production of entangled pairs meson-antimeson pairs. Specifically, the aim is to reproduce and examine results from another published paper [1].

The correlation function of the joint probabilities of a system is historically subjected to restraints that probability evolutions of any quantum system are given in order to respect the approach of the classical limit. This is accomplished with the Leggett-Garg inequality, which places the following limitations on three- and four-measurement systems, respectively:

$$-3 \leq C_{12} + C_{23} - C_{13} \leq 1$$
$$C \equiv C_{12} + C_{23} + C_{34} - C_{14} \leq 2$$

Where C_{ij} is the expectation value of a measurement at t_i and t_j (a joint probability), in this case with the possible individual measurement values quantized to ±1 and generally with the same Δt between each consecutive t_i. The implication is that while these inequalities are held, the evolution of the entangled systems respects the classical limit and obeys macroscopic realism.

In practice, both the oscillating B^o system and K^o system exhibit CP violation, and this violation should be reflected in the evaluation of the inequalities. Previous papers have probed this inequality at varying degrees, and have found violations of the inequality on the time scale of the half-lives of the mesons. The following pages will explore in some more detail the properties of these equations, and attempt to reconstruct the aforementioned results.

THE CORRELATION FUNCTION FOR KAONS

The correlation function for kaons is defined in terms of measurement probabilities for the kaon or anti-kaon in the entangled two-kaon system. From this, the Leggett-Garg inequality can be constructed according to the equations in the introduction. The paper [1] uses the four-measurement system and gives the equation for a single C_{12} found ostensibly by the authors of the paper (equation 6). The expected behavior is for a graph of C generated by instances of this function to be identical to the one shown in the paper. Here the focus will be figures 1a and 1b.

Code written for this section can be found in [2], with comments and explanations of the individual methods. Included here will be brief summaries of the code used to produce the graphs and a discussion of the process and results.

First, the C_{12} in the paper was used to construct the overall inequality, both verbatim from the given formulas (equations 6, 1). Values for the parameter ε, the lifetimes, and the mass difference were pulled from the paper. Note that ε is unitless and that the values for the mass difference and lifetimes are in MeV and seconds, respectively. This presents the first hurdle for extracting the same results that the paper found. Δm is given in natural units, and the definition of Γ varies by a factor of ħ depending on whether or not natural units are used.

Effects on the order seen in the paper ($0 < \frac{\Delta t}{\tau_S} < 10$) appeared when using natural units, or $\Gamma = 1/\tau$. However, the graph produced was similar but not identical to the one found in the paper.

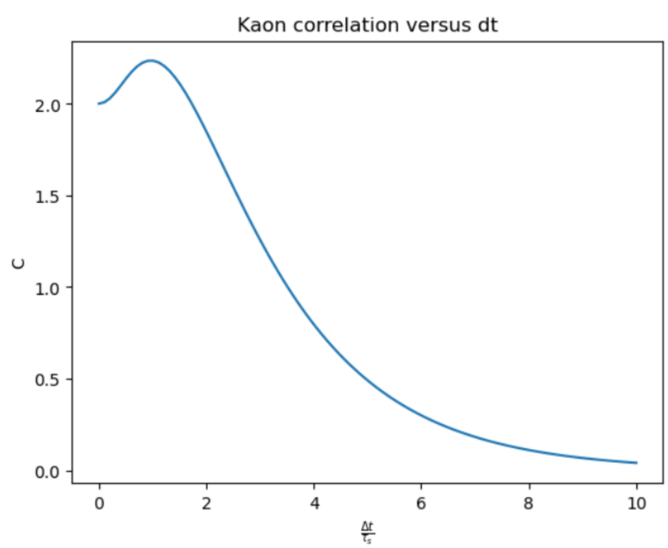

Fig. 1

Compared to figure 1 in the paper, there is no region where the function is negative-valued. The scale of the curve is also different, which is seen more clearly when investigating the impact of the complex parameter on this result.

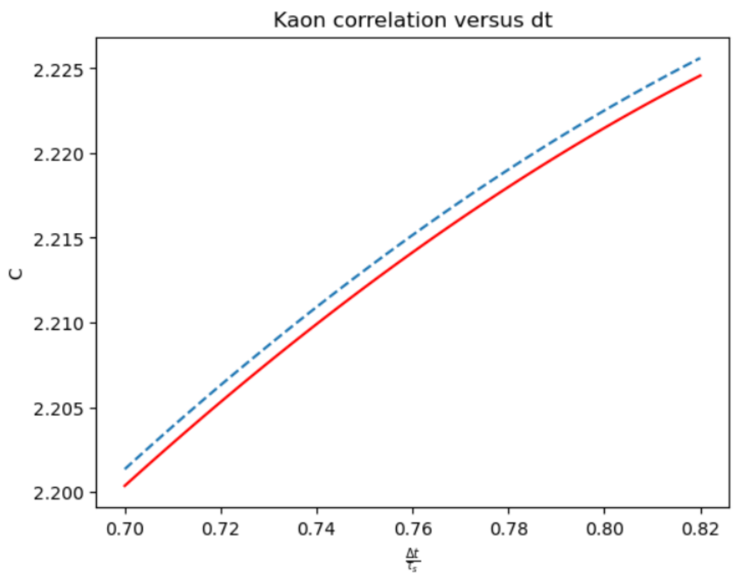

Fig. 2

In Fig. 2, approximately the same region is being displayed as in figure 1(b) in the paper. Rather than containing the peak of the inequality-violating, a part of the ascent is captured. Additionally, the dashed line represents the *non*-CP violating case, which here carries a greater value than the CP violating case. This is the opposite result. The peak here is also lower by approximately 0.13, sitting more comfortably within the upper bound of the limit of QM violation in a Bell-type inequality.

Second, to test the given C_{12} equation, C_{12} was calculated directly from the definition and the joint probabilities themselves.

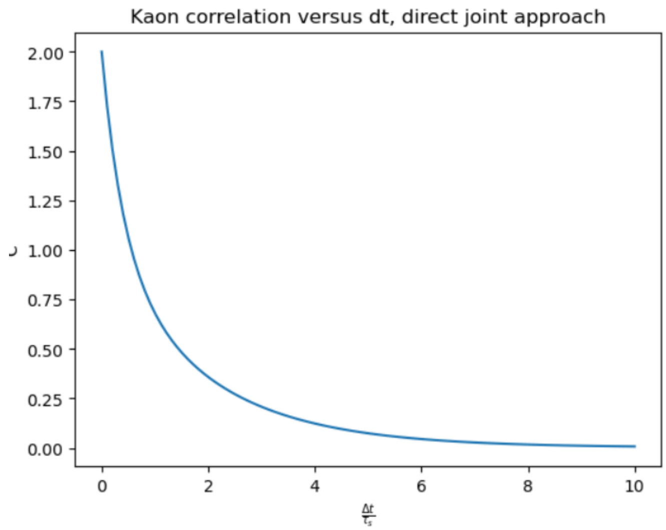

Fig. 3

For further details on the calculation and code, see [2]. In short, the joint probabilities are found by multiplying the distribution for individual measurements at t1 and t2 and then the C_{12} is found by definition.

This result is once again different from that in the paper and is also different from the result in Figure 1. Here, the inequality is not violated at all, and yet again the negative region does not exist.

It is additionally important to note that scaling Γ by $\hbar$ does not affect the contour of the results. Rather, it simply increases the scale in $\Delta t/\tau$ on which the effects appear. Renormalizing the mass difference with c^2 also does not have a significant effect on the outcome.

These results suggest one of three outcomes. The first is that the results in the paper are not correct. This is possible, but would need further review to be confirmed. The second is that there are additional scaling factors hidden in the parameters used in the calculation that are not openly discussed in the paper. Perhaps these are some combination of $\hbar$ factors and other constants not mentioned directly. This is also possible, as the functions are reasonably sensitive to scale changes in certain variables. The final possibility is that there are errors in the code. Human error is generally always a possibility. However, the similarity of the results suggests that the first and second outcomes are more probable, with the second being the most likely due to the nature of the investigation.

THE CORRELATION FUNCTION FOR B MESONS

The correlation function for B mesons follows the methodology used for Kaons, but in most examples the 3-interval inequality is used. That is, the first equation from the introduction, in this case frequently referred to as K_3.

The programmed approach was similar to the calculation of the Kaon correlation function directly from the joint probabilities. See [2] for further detail.

Plots produced are again the value of the inequality versus the dimensionless Δt/τ quantity ranging from zero to ten lifetimes. A noteworthy result is that the use of ħ-scaled quantities has a large effect on the contour of the curve, as shown in the following graphics.

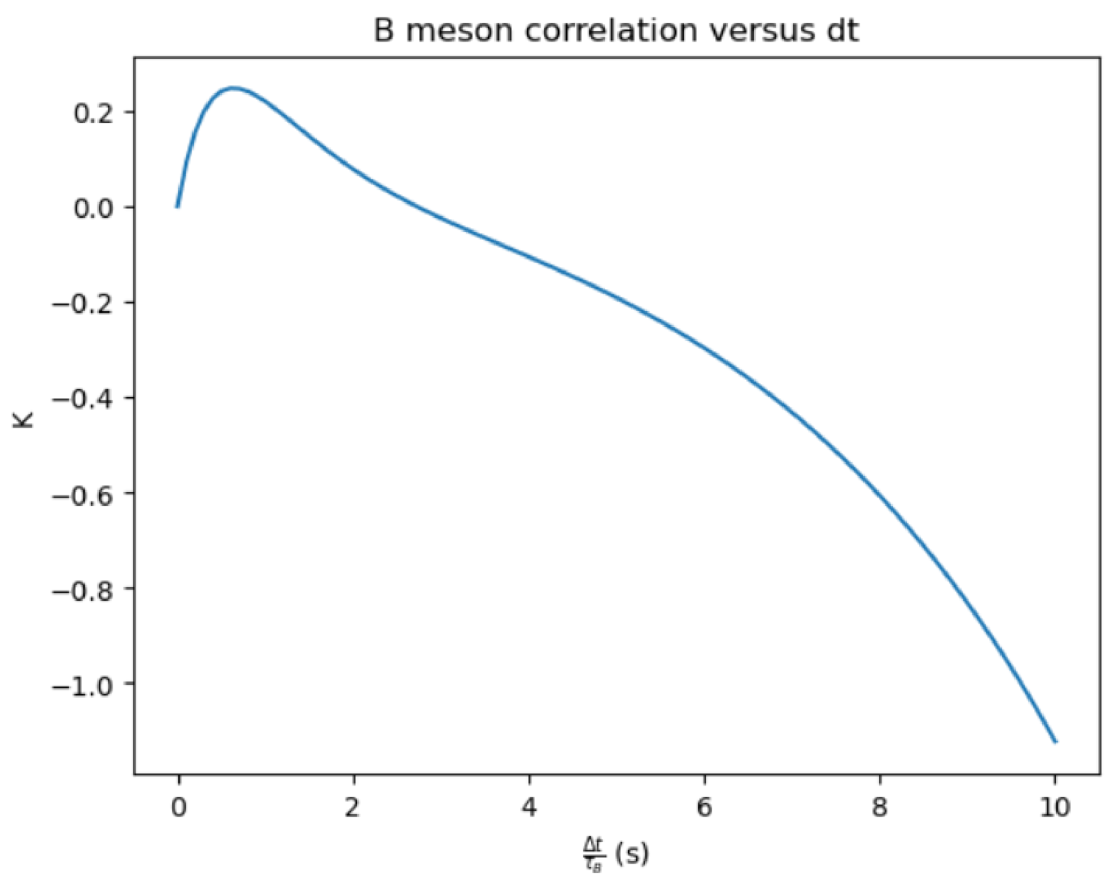

Fig. 4

In Fig. 4, the K_3 without ħ scaling appears using a value of 1 for the complex CP-violating parameter $\frac{q}{p}$. This is the case of no CP violation.

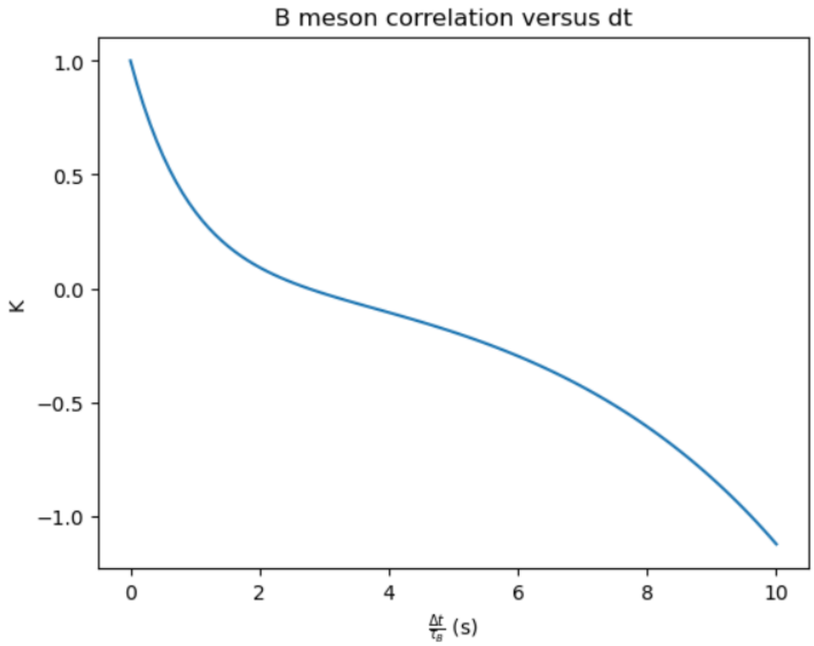

Fig. 5

As an extreme comparison, Fig. 5 shows the same plot but with the CP-violating parameter set to zero (which is not physical). Note the significant contour change during the first two lifetimes, and that increasing the value of the parameter appears to decrease the contributions that might exceed the limits of the inequality.

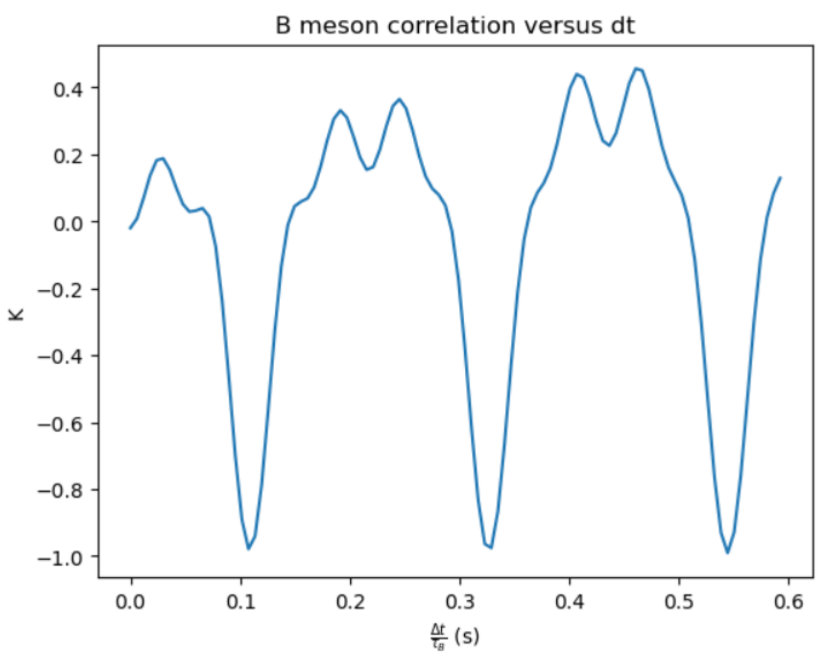

Fig. 6

Fig. 6 shows the change in the plot after introducing ħ scaling, with the Δt/τ scaled up (arbitrarily). This change is quite significant, and the shape of this plot is more similar to what would be expected of the LGI for a B meson system. However, it is still not an exact replica. A reasonable inquiry, then, is to probe the non-scaled function at higher Δt/τ, as the scaling would not be as significant an issue if the contour remained the same.

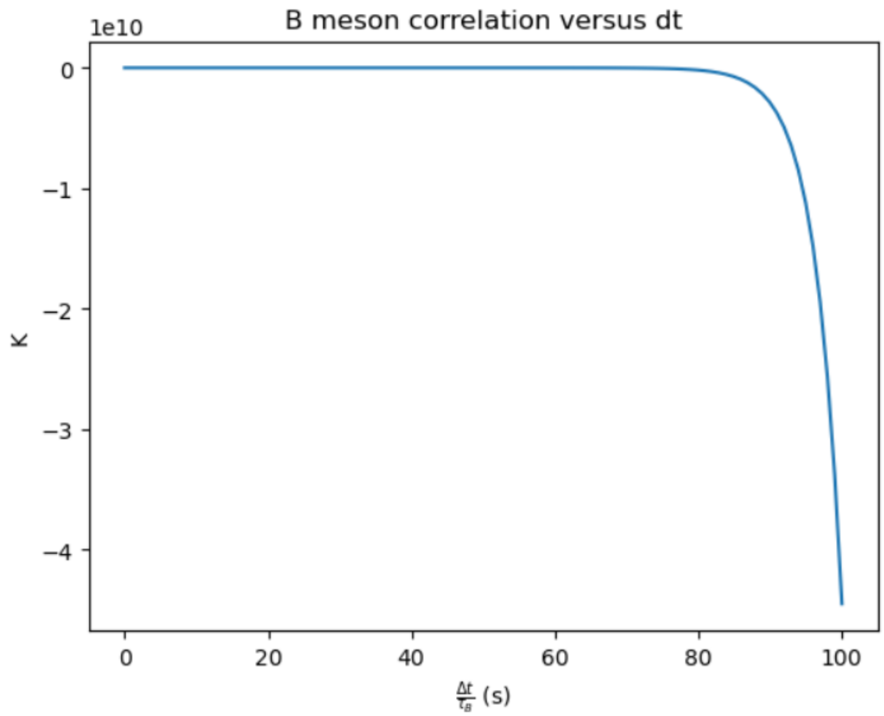

Fig. 7

Fig. 7 shows the unscaled function on a Δt/τ scaled up by a factor of ten. The function does not show the structure of the scaled function and also appears to diverge, exiting the acceptable margin of error for a Bell-type inequality.

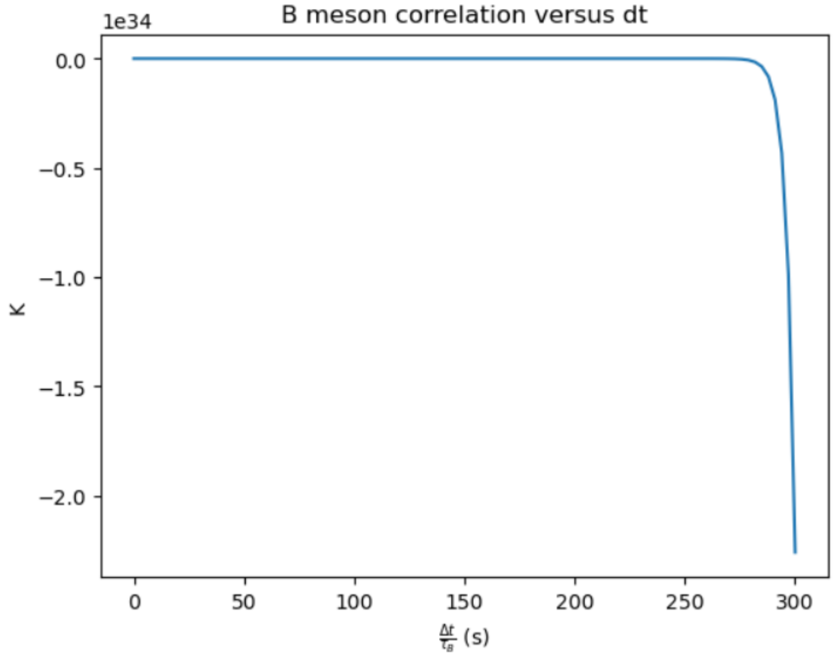

Fig. 8

Fig. 8 shows the unscaled function again, this time with Δt/τ increased by a factor of 30. This plot effectively rules out this version of the function as a tool for probing the B meson, as it conflicts with Fig. 7 due to the disappearance of the divergence at Δt/τ=100. The result also no longer violates the inequality.

These results effectively suggest the same outcomes as those for the kaons. However, due to the second appearance of conflicting results and the new evidence that the scaling factor can significantly change the shape of the curve, evidence for the use of undisclosed scaling conventions in the production of such plots rather than error on the part of the implementation here or in the paper [1] has increased. It therefore appears that the equations and values that were given directly are insufficient to replicate the results in such papers (note that B meson analysis does not appear in the referenced paper, and that this analysis was conducted using available values and formulas for the B meson system).

CONCLUSIONS

This investigation was fundamentally inconclusive. The results from the paper were not directly replicated, and issues with converting the information given into the results generated were abundant throughout. This leaves the investigation with two possible takeaways. The first is that there may be room for increased transparency as to which conventions and shortcuts are being made in the delivery of raw information in such papers. Perhaps this could be provided as a footnote or in an appendix. Perhaps this could appear in the form of code snippets that generated the results in an appendix or referenced in a version control or file sharing service such as GitHub or GitLab. The other possible takeaway is that the methodology here is flawed or incomplete. In this case, the investigation conducted here may be used as a starting off point for further meta-investigations in this area at this level.

Overall, the task attempted was not successfully completed. The remaining outlook is that notational transparency and availability of code could benefit meta-analysis or replication and verification processes.

References

1. arXiv:1304.2761 [quant-ph]

2. https://github.com/DereferenceMyPointer/Meson-Oscillation