

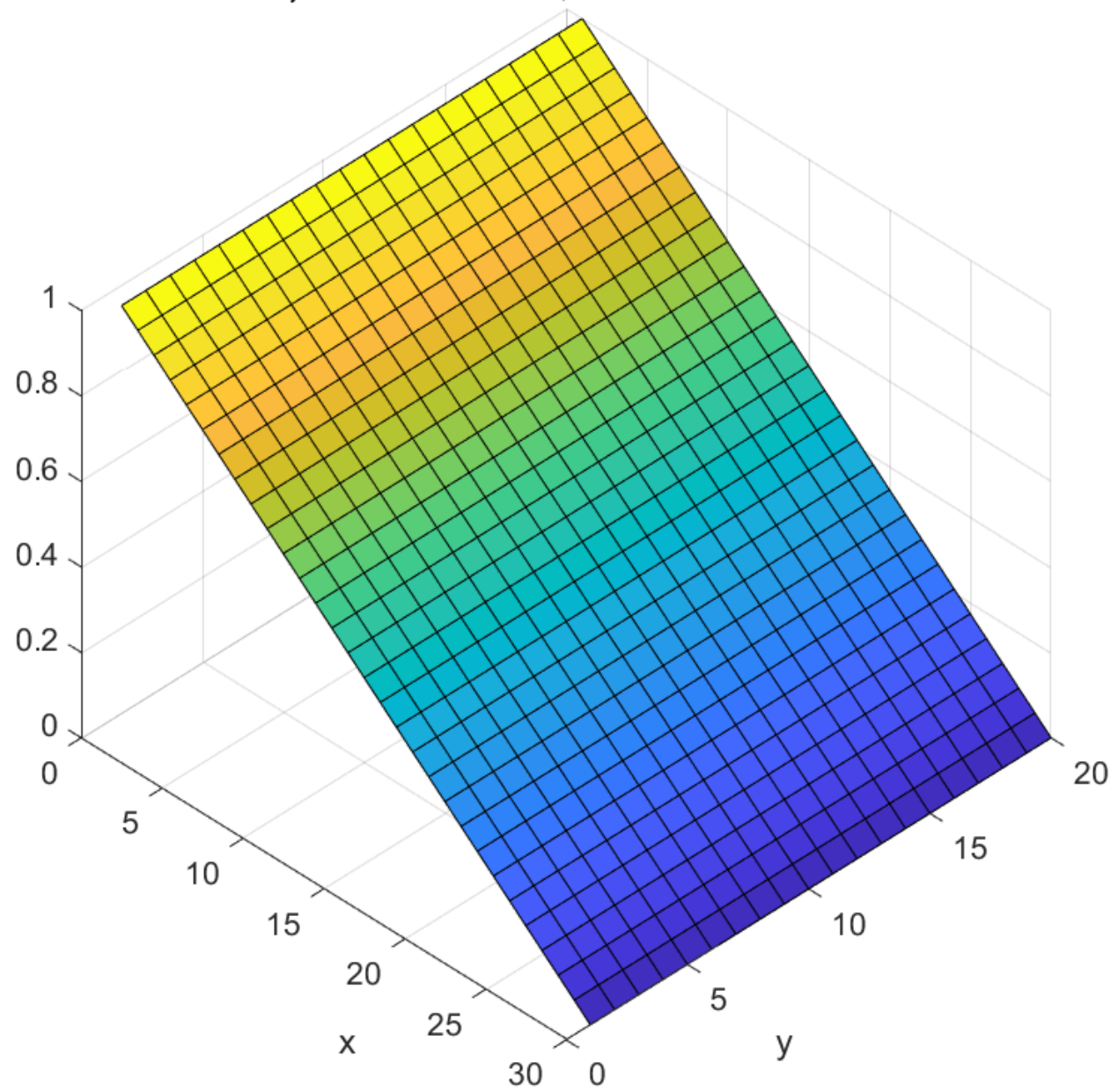
ELEC 4700A

Assignment 2: Finite Difference Method

Part 1:

a)

**1a) F.D.M. Solution, Left Border  $V = V_0$**



*Figure 1: Voltage within a Rectangular Region Using Finite Difference Method (F.D.M.)*

b)

### 1b) F.D.M. Solution, Left+Right Border $V = V_0$

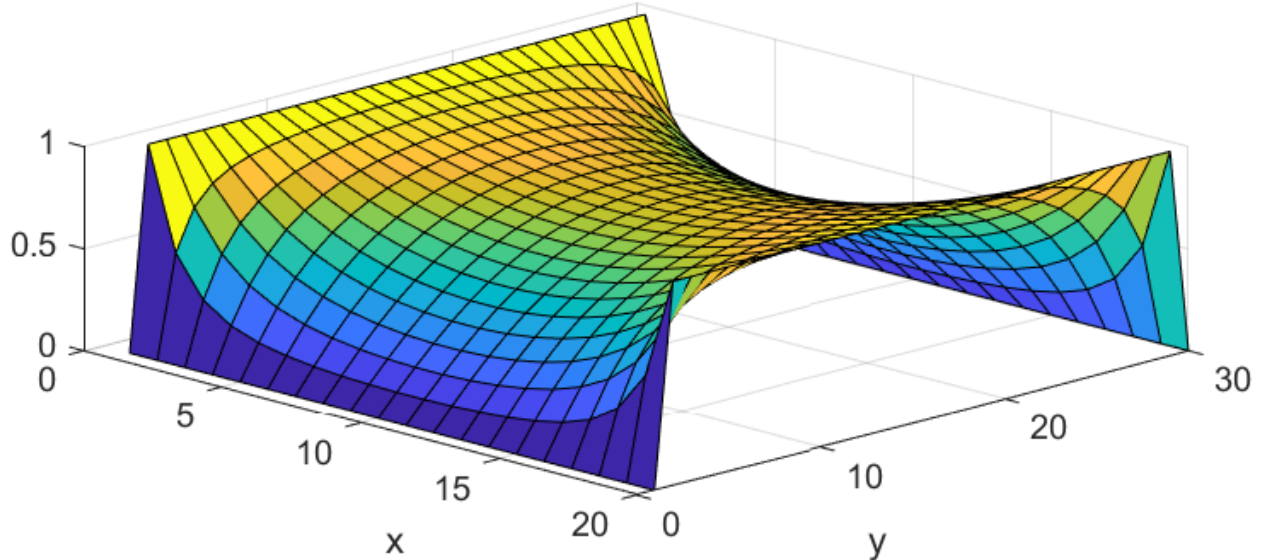


Figure 2: Voltage in Rectangular Region with Ends at Same Potential (1V) and Sides at Same Potential (0V) - F.D.M.

### Analytical Solution, Left+Right Border $V = V_0$

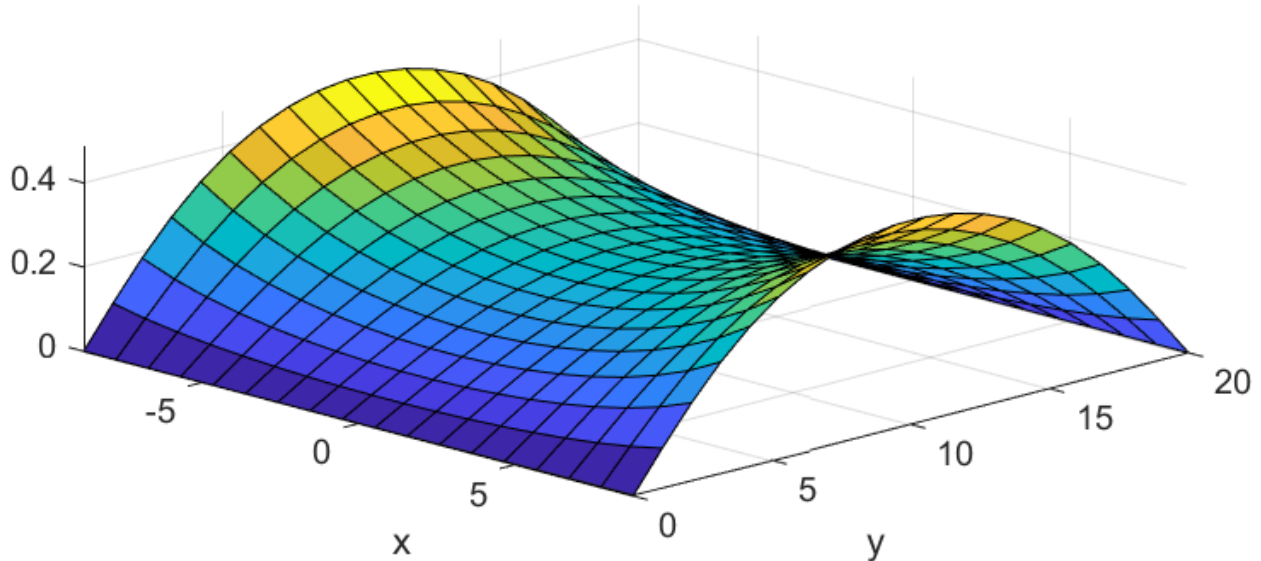


Figure 3: Voltage in Rectangular Region with Ends at Same Potential (1V) and Sides at Same Potential (0V) - Analytical Method

### Discussion:

Figures 1 and 2 above show the use of the finite difference method (FDM) to show the voltage within the rectangular area. Figure 3 above uses an analytical method to show a similar result to the one in figure 2.

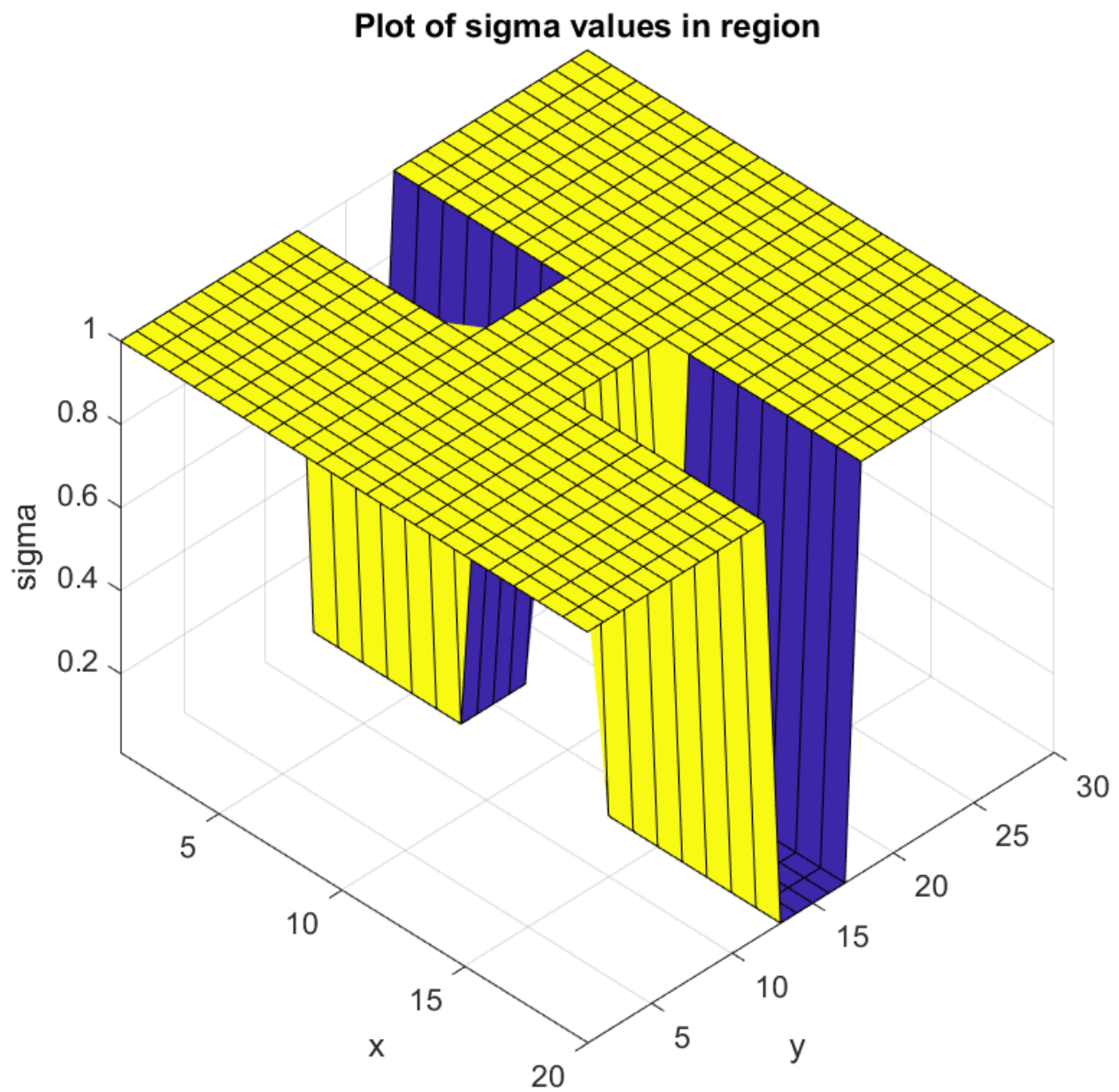
Figure 1 displays an area where one side has a voltage set to  $V_0$  (1V) and the opposite side is set to 0V. The resulting plot is linear as there is nothing to interfere with the inner area.

Figure 2 displays an area where the opposite sides are equal to each other. For the shorter sides, the voltages are set to  $V_0$  (1V) and the longer sides are set to 0V. In figure 2, the opposite is true, which is a coding error I have not been able to figure out - However, the shape of the plot would be the same in both situations. The benefit to using the FDM to determine the voltage is that it can be used to find simple solutions. However, a disadvantage of this numerical method is that for more complex solutions, it requires much more computing power.

Figure 3 displays an analytical solution to a similar area as in figure 2. The benefit of the analytical method is that it is more efficient at finding solutions than the numerical method. A disadvantage is that the analytical method can give incorrect answers at certain amounts of iterations. For this reason, the number of iterations used for figure 3 was limited to 600. Due to the nature of the cosh and sinh terms, which increase as they approach infinity, more iterations would create a larger error.

Part 2:

a)



*Figure 4: Plot of Sigma values in Bottleneck*

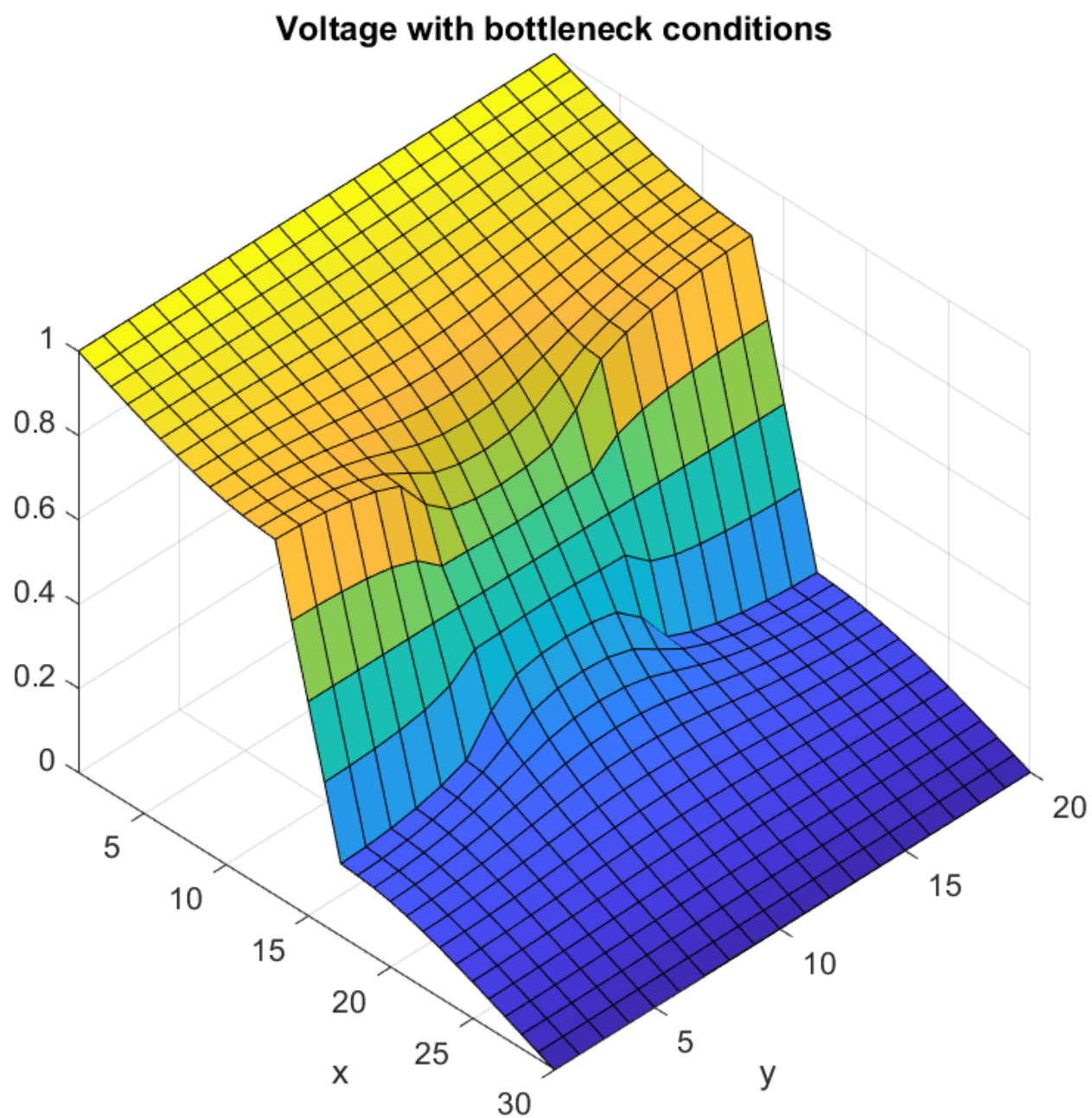


Figure 5: Plot of Voltage in Bottleneck

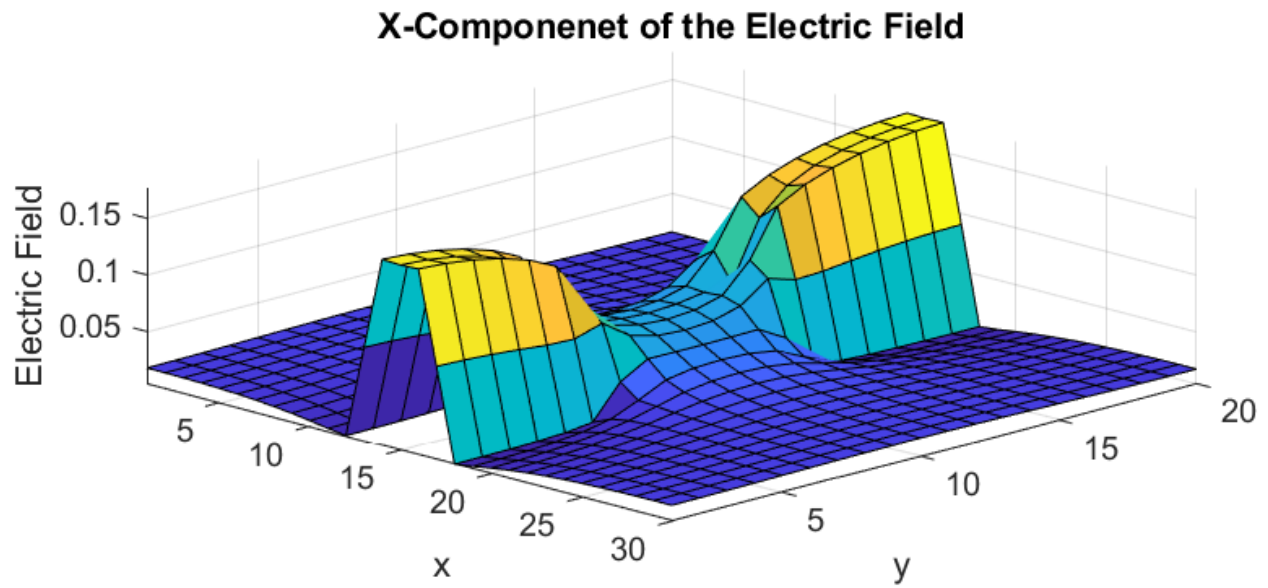


Figure 6: Electric Field in Bottleneck (x-component)

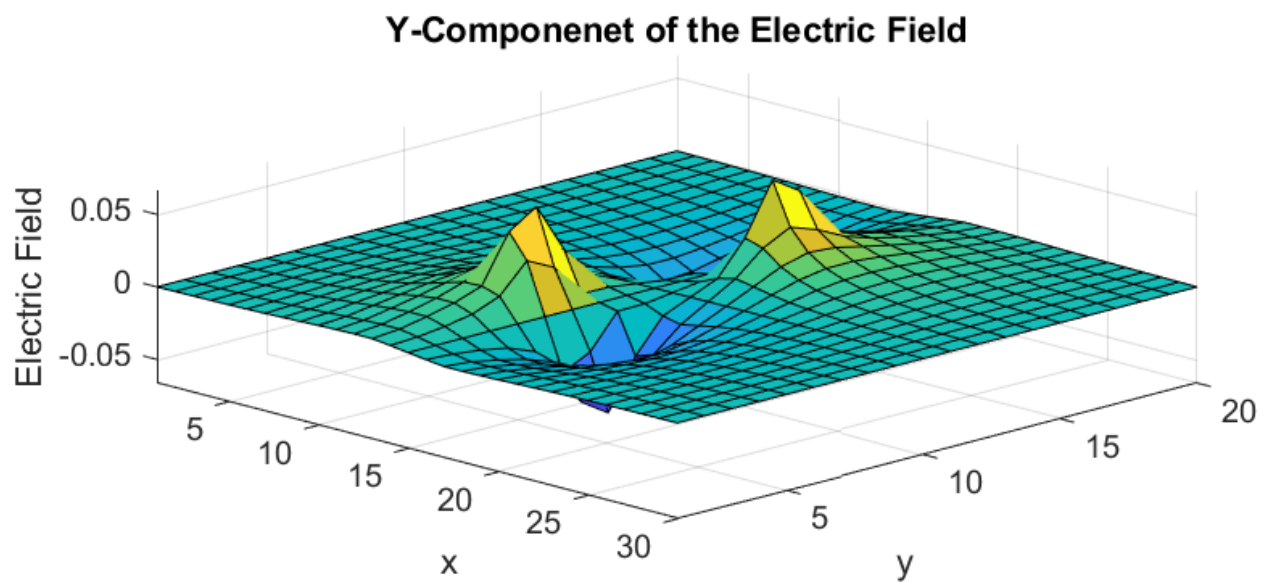


Figure 7: Electric Field in Bottleneck (y-component)



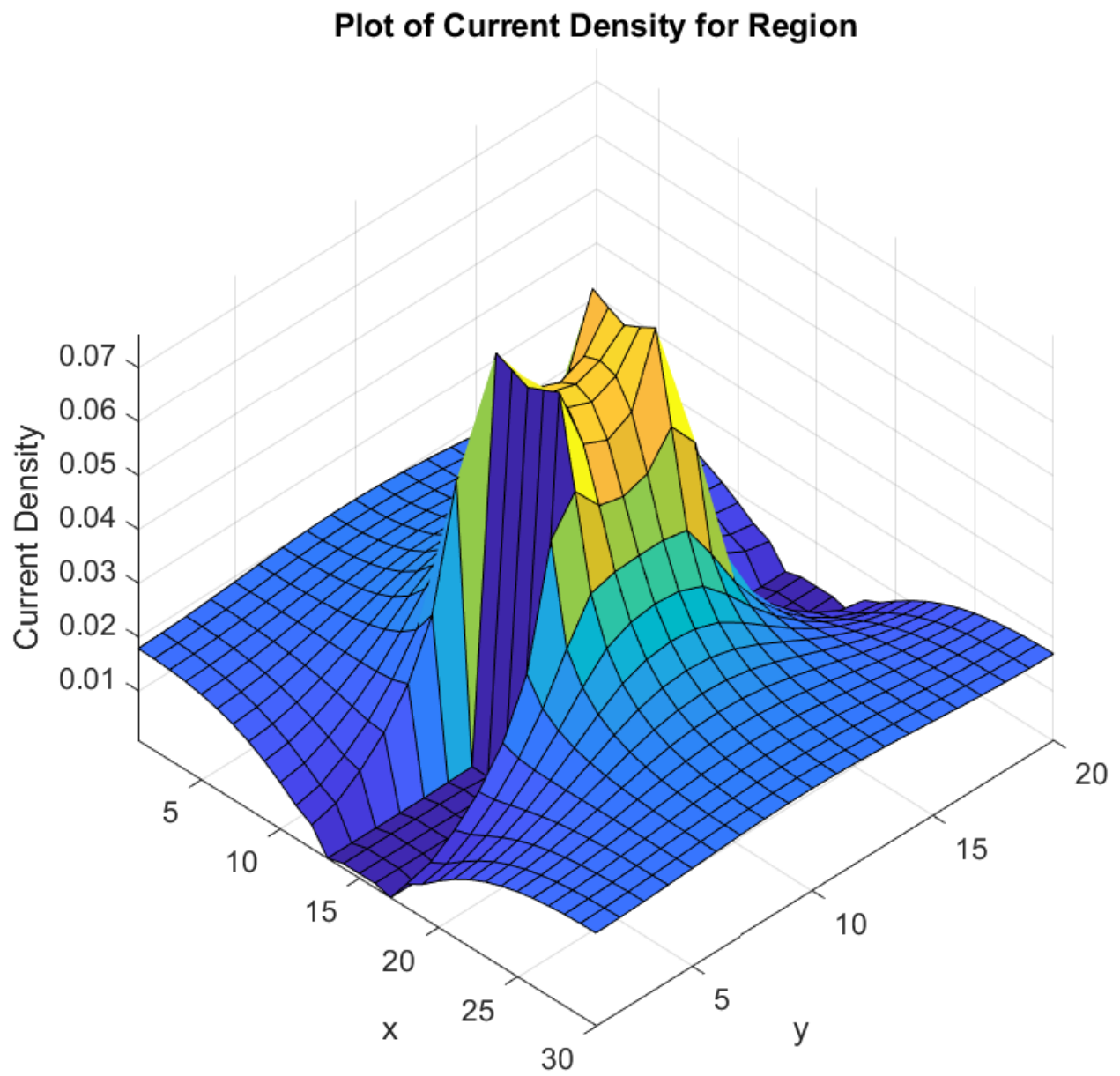


Figure 8: Current Density in Bottleneck



b)

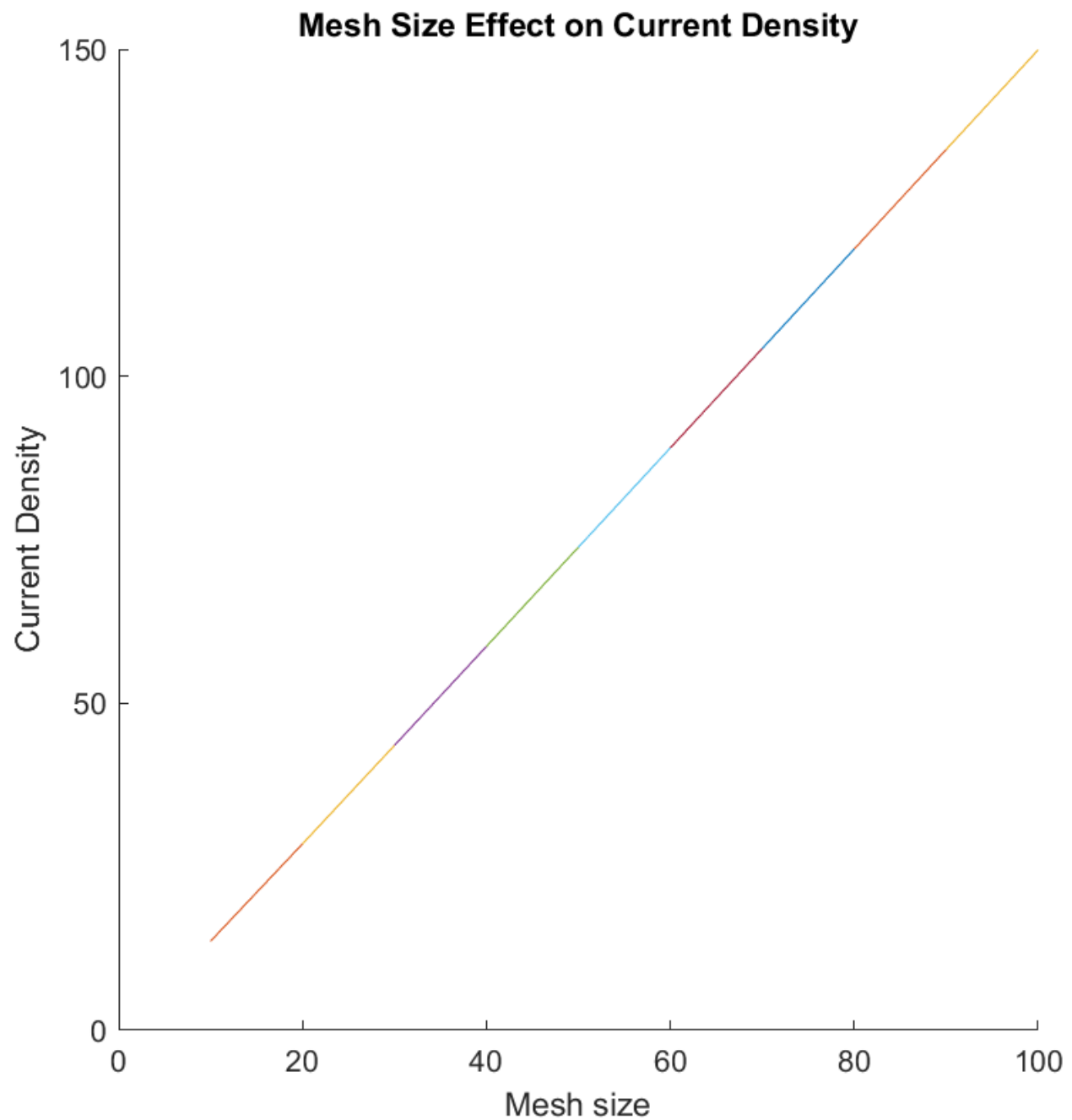


Figure 9: Plot of Mesh Size Effect on Current Density

**Discussion:**

From figure 9 above, it is clear that as mesh size increases, current density increases as well. This relationship is linear. This means that a larger mesh size will have a larger current density.

c)

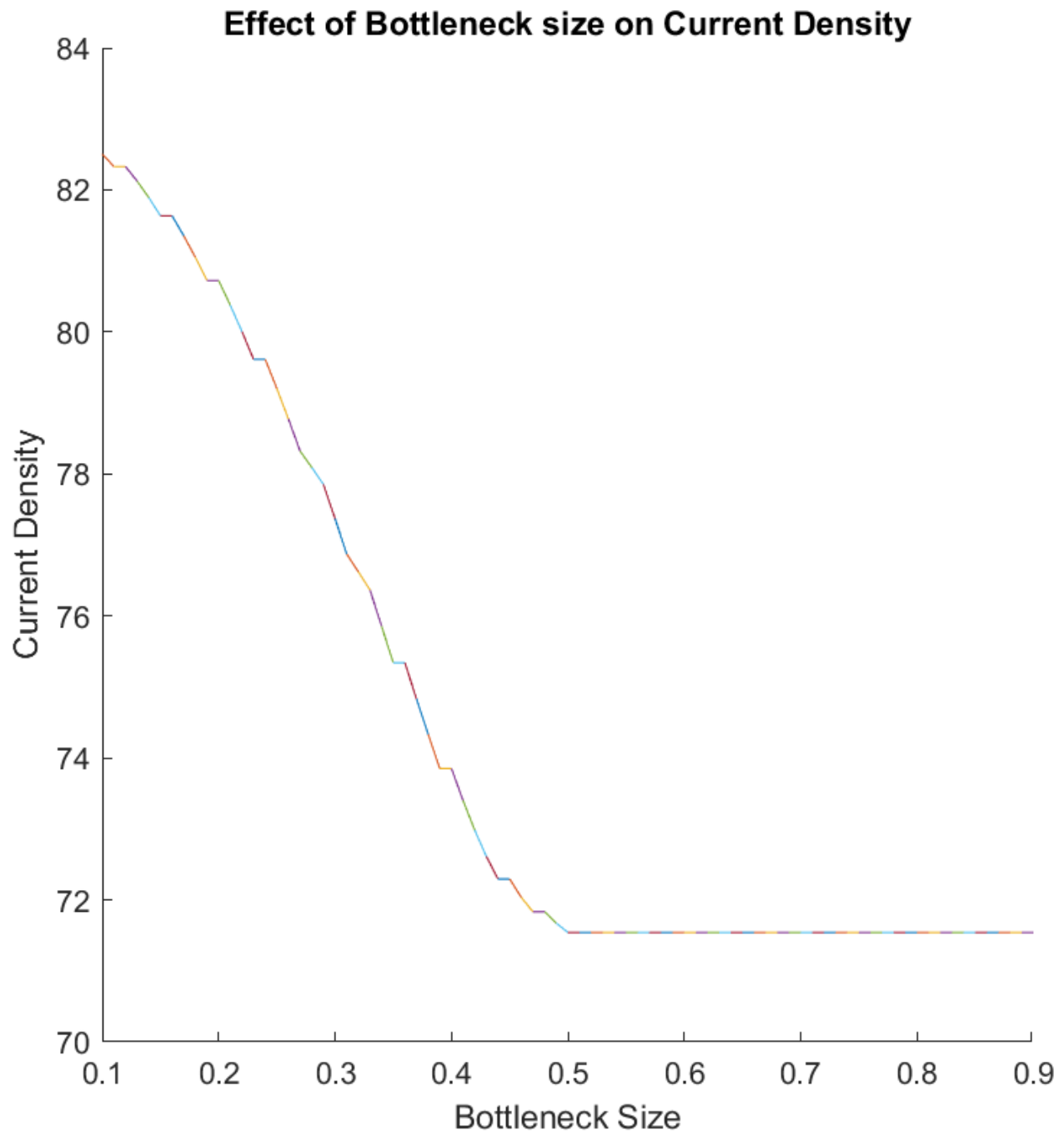


Figure 10: Plot of Bottleneck Size Effect on Current Density

#### Discussion:

From figure 10 above, it is clear that the current density is inversely proportional to the size of the bottleneck. This means that as the bottleneck size gets narrower, the current density gets smaller. It should be noted that as the bottleneck size approaches a specific size (roughly 0.5) the current density reaches its lowest point.

d)

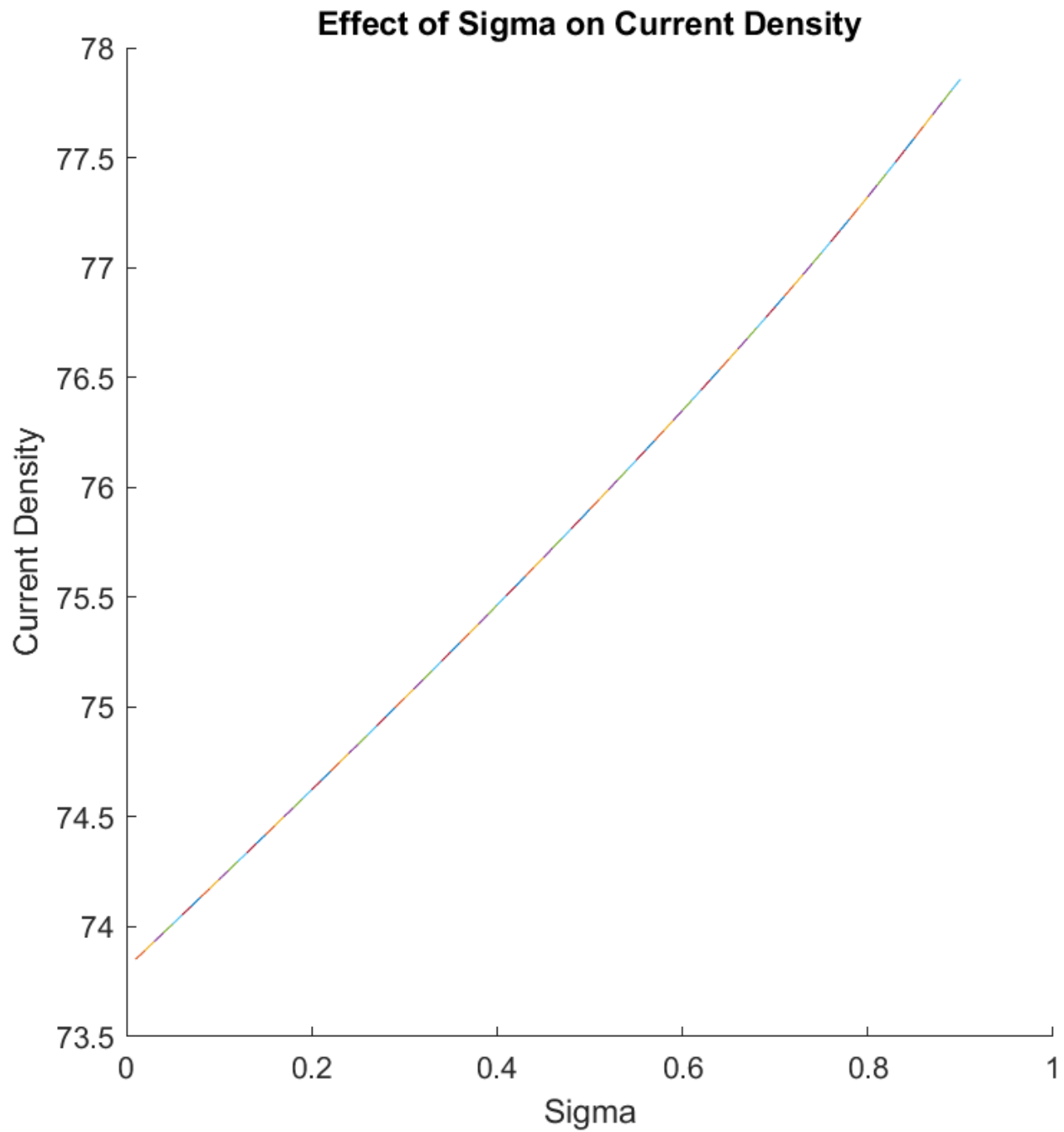


Figure 11: Plot of Sigma Effect on Current Density

Discussion:

From figure 11 above, it is clear the sigma and the current density have a linear relationship. As sigma increases, the current density increases.