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ELEC 4700A

Assignment 4: Circuit Modeling

Part 1:

Using assignment 3, the voltage was swept from 0.1-10V to calculate what the value of R3 is. Using the applied voltage, the current was determined for every voltage step, and figure 1 below was plotted.

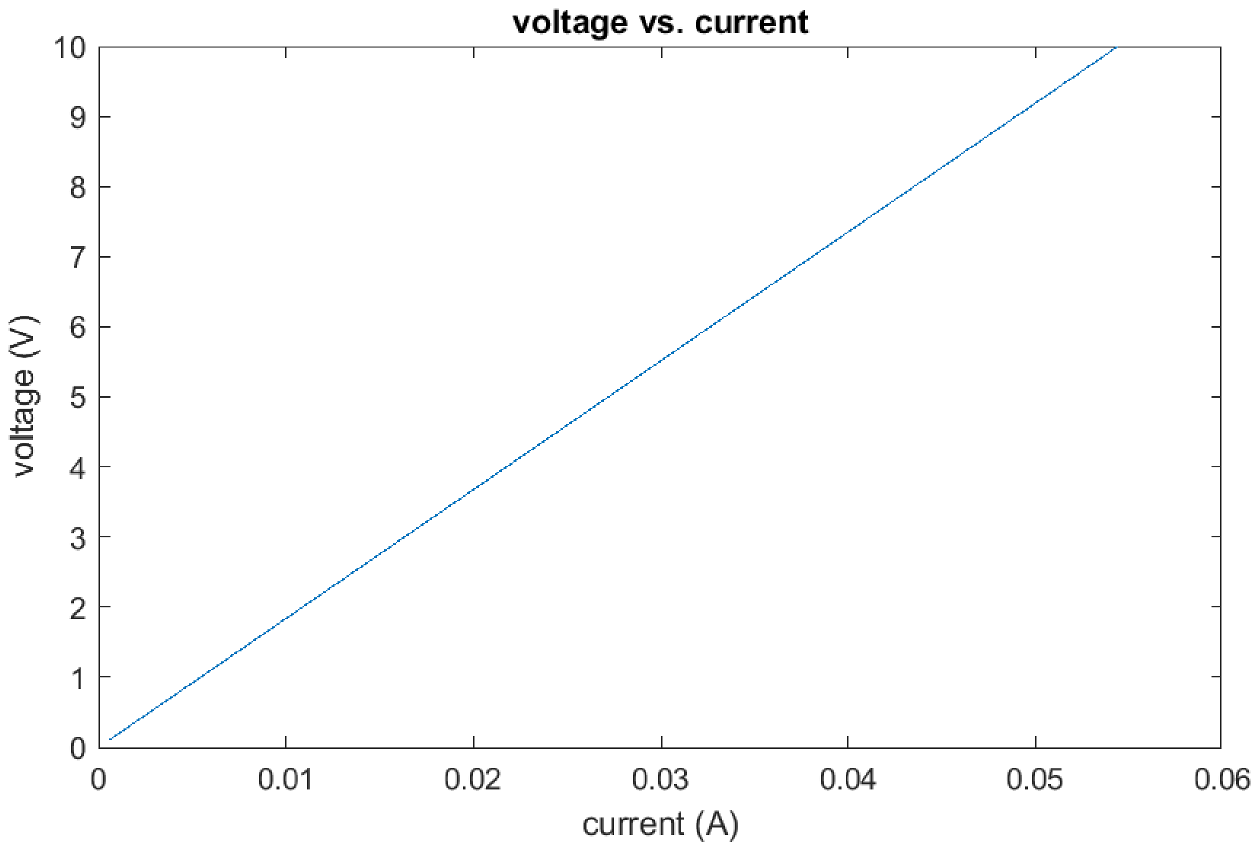

Figure 1: current-voltage characteristics plot

Figure 1 displays the current-voltage characteristics of the semi-conductor being modelled in assignment 3.

Part 2:

Using a linear fit on the data plotted above in Figure 1, the value of R3 was determined. Two different methods were used to determine the value. The first used to polyfit() function in Matlab. The second method was to find the slope of the linear plot using rise/run. The average value found using both methods after many iterations gave R3 = 184 Ohms.

Part 3:

a)

i) The differential equations representing the network in the time domain using KCL are the following:

```
(1) V1 = Vin
(2) G1*(V1-V2) + C*((d(V1-V2))/dt) + I1 = 0
(3) G1*(V2-V1) + C*((d(V2-V1))/dt) + G2*V2 + IL = 0
(4) V2 - V3 - L*(d(I1)/dt) = 0
(5) G3*V3 - IL = 0
(6) G4*(V5 - V4) + Go*V5
(7) G4*(V4 - V5) + Iv = 0
(8) V4 - alpha*G3*V3 = 0
```

ii) In the frequency domain, the differential equations are the following:

```
(1) V1 = Vin
(2) G1*(V1-V2) + C*(j*omega*(V1-V2)) + I1 = 0
(3) G1*(V2-V1) + C*(j*omega*(V2-V1)) + G2*V2 + IL = 0
(4) V2 - V3 - L*(j*omega*I1) = 0
(5) G3*V3 - IL = 0
(6) G4*(V5 - V4) + Go*V5
(7) G4*(V4 - V5) + Iv = 0
(8) V4 - alpha*G3*V3 = 0
```

iii) Using the stamping code written in ELEC 4506, the circuit was modelled using the following equation: (Note that in this course the vector uses F, but in this report, b represents the source vector)

$$C\frac{dV}{dt} + GV = b$$

From the stamps, C, G, and b were found to be the following:

G =

1	2	0	0	0	0	0	1
-1	1.5	0	0	0	1	0	0
0	0	0.0054	0	0	-1	-0.5435	0
0	0	0	10	-10	0	1	0
0	0	0	-10	10.001	0	0	0
0	1	-1	0	0	0	0	0
0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	0

C =

0.25	-0.25	0	0	0	0	0	0
-0.25	0.25	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	-0.2	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

b =

0
0
0
0
0
0
0
Vin

Where Vin is the initial input voltage that varies throughout this assignment.

b)

i) Using the above values for G and b, the input voltage Vin was swept from -10 to 10V. Since this is a DC sweep, the frequency value is 0, therefore the C matrix does not affect the result. Using Matlab, solution vector was calculated, giving a V vector for each iteration. Using this V vector, the third and fifth locations were taken, as they represent V3 and Vout, and plotted in Figure 2 below against Vin.

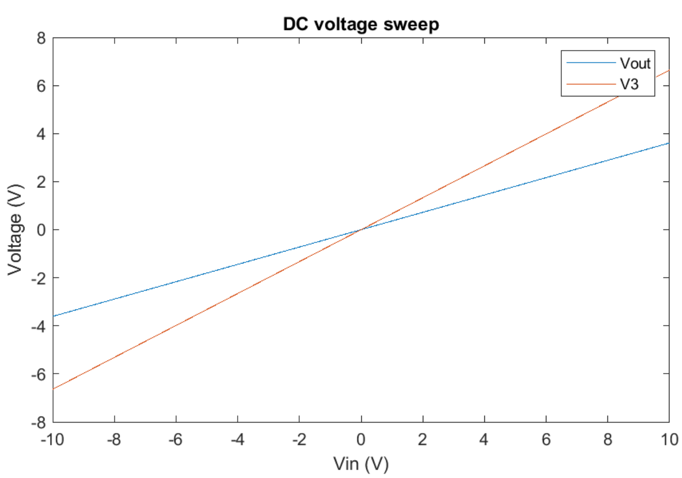

Figure 2: V3 and Vout against Vin swept from -10 to 10V

As it is clearly shown in Figure 2, as Vin approaches roughly zero, V3 becomes greater than Vout.

ii) Next, the AC response was determined by setting Vin to be a constant, then varying the frequency from 1 to 100. The value of ω was determined for each frequency by multiplying it by 2π. The following equation was used to determine the solution vector V:

$$A = G + j * \omega * C$$

The solution vector V was found, and the output voltage at V(5) was tracked to plot against ω in Figure 3 below.

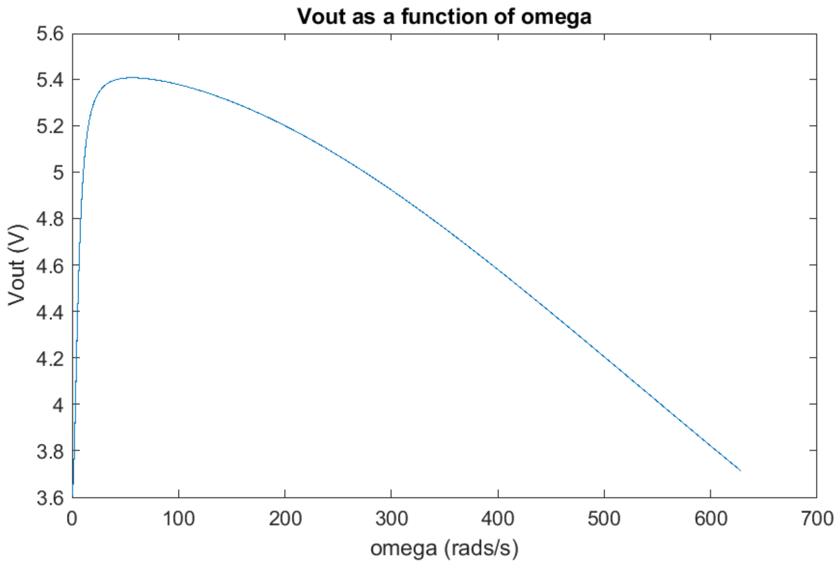

Figure 3: AC plot of Vout against omega

Using the same iterations, the gain was determined using Vout/Vin, then plotted against omega in Figure 4 below.

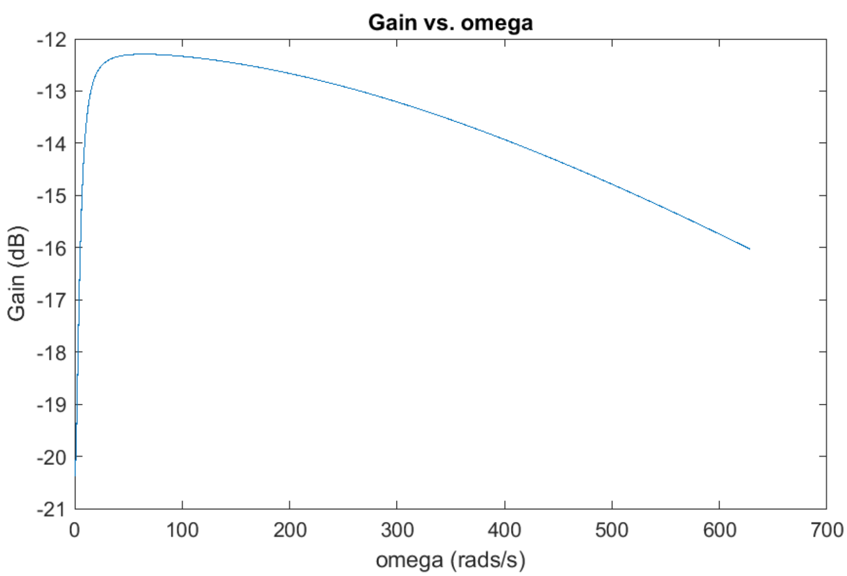

Figure 4: AC sweep Gain vs. omega

iii) Next, the value of C was given random perturbations using a normal distribution with a standard deviation of 0.05. For these iterations, ω was set to equal π. Completing the iterations, the gain was tracked in the histogram below in Figure 5.

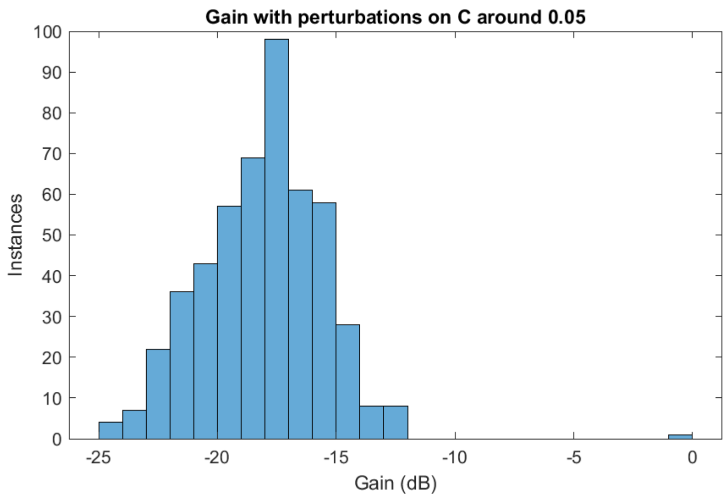

Figure 5: Gain with random perturbations on C

Note that the shape of the histogram resembles a gaussian bell curve. However, there is an outlier which does not affect the distribution results.

<u>Part 4:</u>

In this part, the transient circuit is simulated using the equation:

$$C\frac{dV}{dt} + GV = b$$

a) By inspection, this circuit resembles a band-pass structure that would block out lower and higher frequencies, only allowing a specific range of frequencies through. This is evident by the shape in Figure 3 where the initial frequency and the final frequency are low, while the middle frequencies provide a much higher output voltage.

b) The expected frequency response for this circuit would be to have a peak at the ideal frequency, and much lower outputs at any other values.

c) A formulation of this circuit can be found using the finite difference method, providing the following equation:

$$\frac{C}{dt} + GjV(j) = CV(j-1)/dt + b(t(j))$$

d) The circuit was supplied with three different inputs separated into A, B, and C below.

A) The input voltage for this portion was a step that transitions from 0 to 1 at 0.03s

iii) For the duration of one second, the step input was simulated and solved, providing the plot shown below in Figure 6.

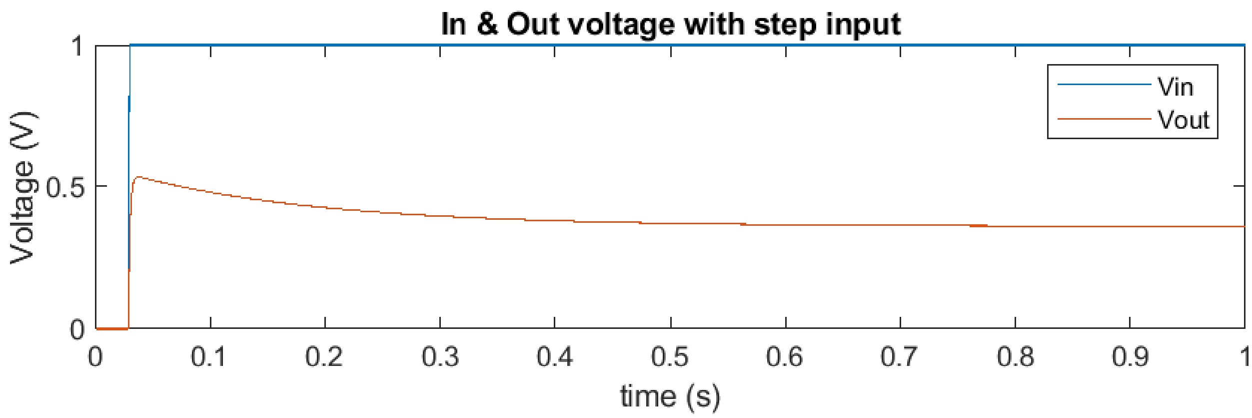

Figure 6: Step input time domain response

iv) Using the fft() and the fftshift() function, the frequency response of the step function was determined for the frequencies from -500 to 500. The response is shown below in Figure 7.

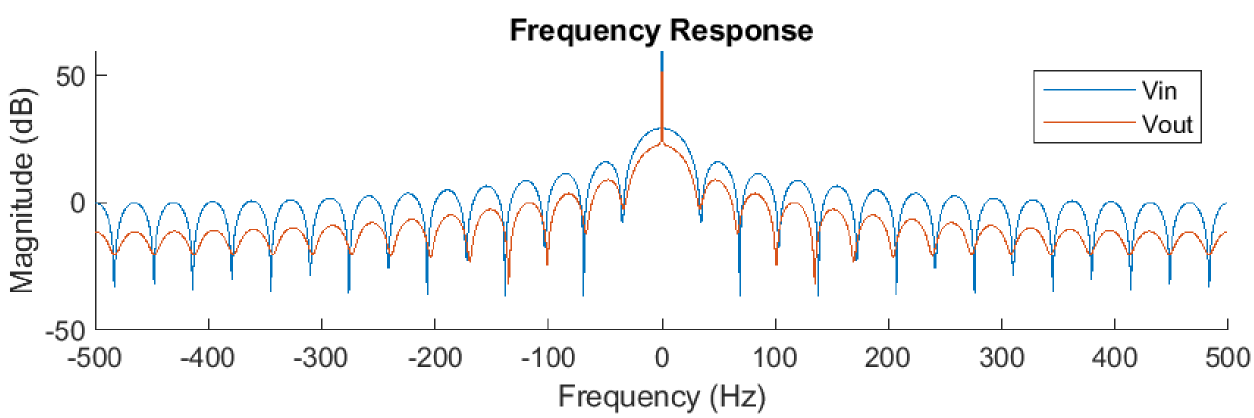

Figure 7: Step input frequency domain response

Note that the band-pass filter only filters through frequencies near 0, and the surrounding responses are far lower.

B) The input voltage for this portion was a sin(2πft) function with f = 1/(0.03) 1/s

iii) For the duration of one second, the sine input was simulated and solved, providing the plot shown below in Figure 8.

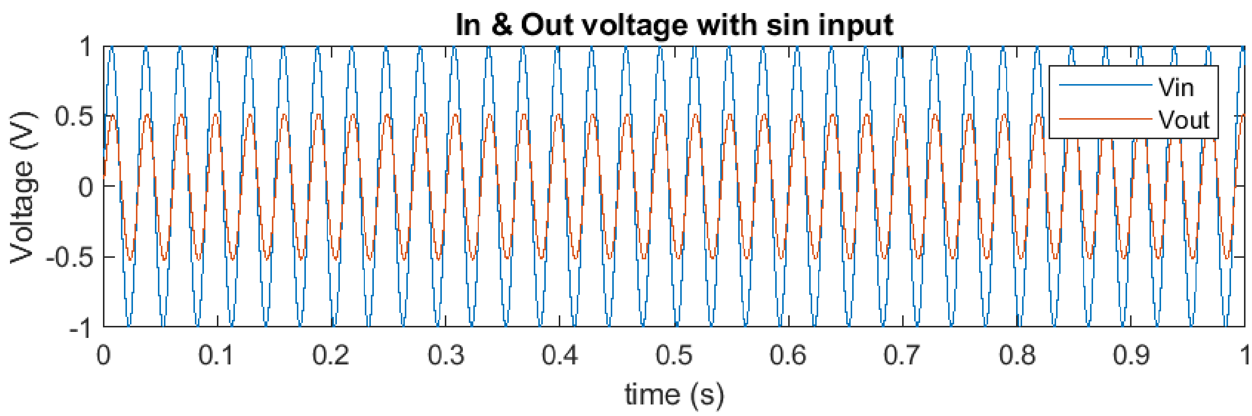

Figure 8: Sine input time domain response

iv) Using the fft() and the fftshift() function, the frequency response of the sine function was determined for the frequencies from -500 to 500. The response is shown below in Figure 9.

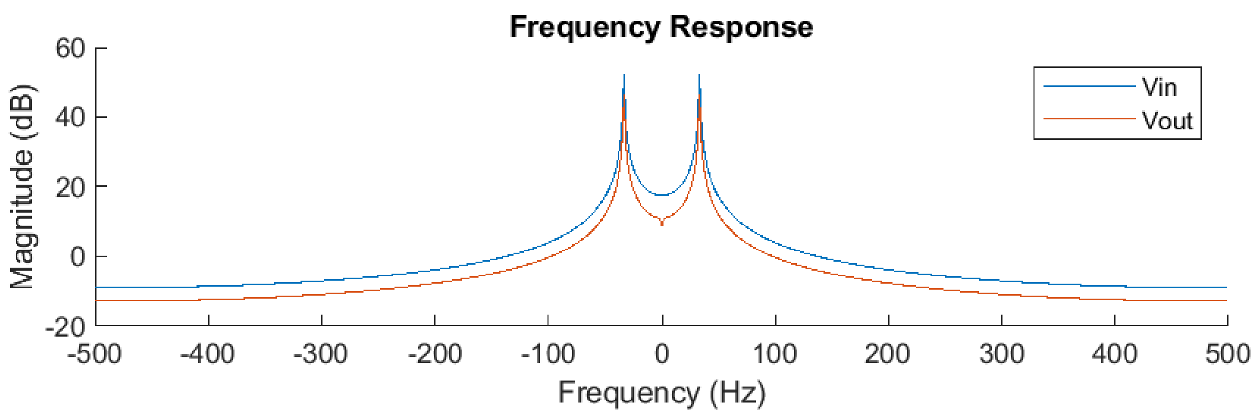

Figure 9: Sine input frequency domain response

C) In this portion, a Gaussian pulse with a magnitude of 1, a standard deviation of 0.03s, and a delay of 0.06s was used as input.

iii) For the duration of one second, the Gaussian input was simulated and solved, providing the plot shown below in Figure 10.

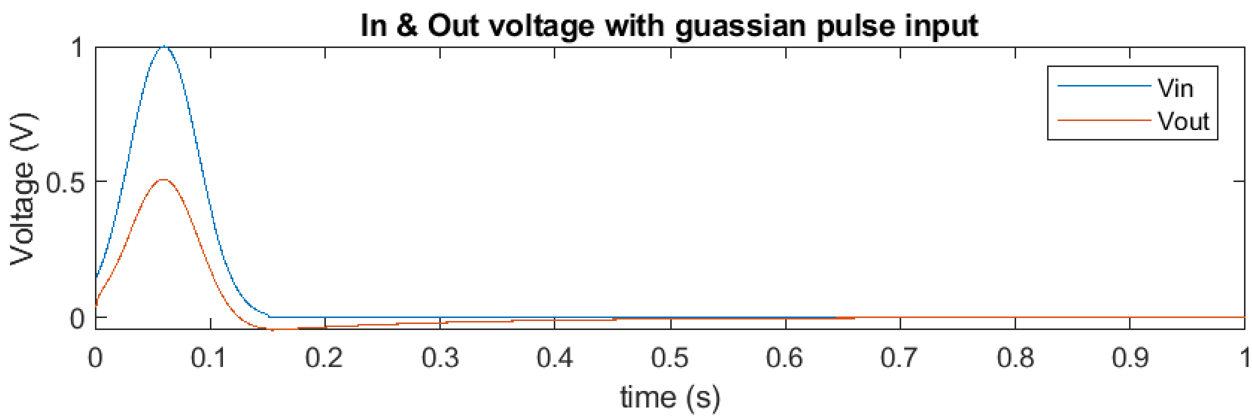

Figure 10: Gaussian input time domain response

iv) Using the fft() and the fftshift() function, the frequency response of the Gaussian function was determined for the frequencies from -500 to 500. The response is shown below in Figure 11.

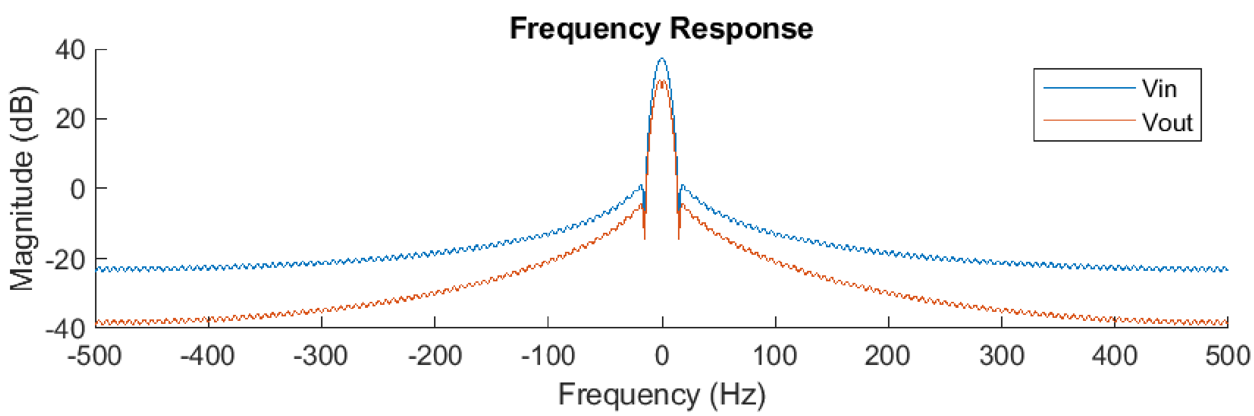

Figure 11: Gaussian input frequency domain response

v) When increasing the time step, the response remained roughly the same with smoother lines for each of the inputs above.

Part 5:

In this portion, noise was introduced into the circuit by placing a current source and a capacitor in parallel with R3. Updating the C matrix with the new additions, the following are the new values.

C =

0.25	-0.25	0	0	0	0	0	0
-0.25	0.25	0	0	0	0	0	0
0	0	Cn	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	-0.2	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Note that because the initial Cn value is 0.00001, Matlab is not precise enough to show it in the matrix. Therefore, since Cn is changed throughout this section, the variable was placed in the correct location.

c) In this portion, five different variations were run to display how the value of Cn and the number of time steps affect the results. For each case, the time domain and frequency domain responses are shown using the same method used in part 4 above. For each of these cases, In is set to 0.001 to ensure the effect it has is visible. The Gaussian input developed above is used for each case.

For this iteration, Cn = 0.01 and there are 1000 time steps. The time domain response is shown in Figure 12, and the frequency domain response is shown in Figure 13.

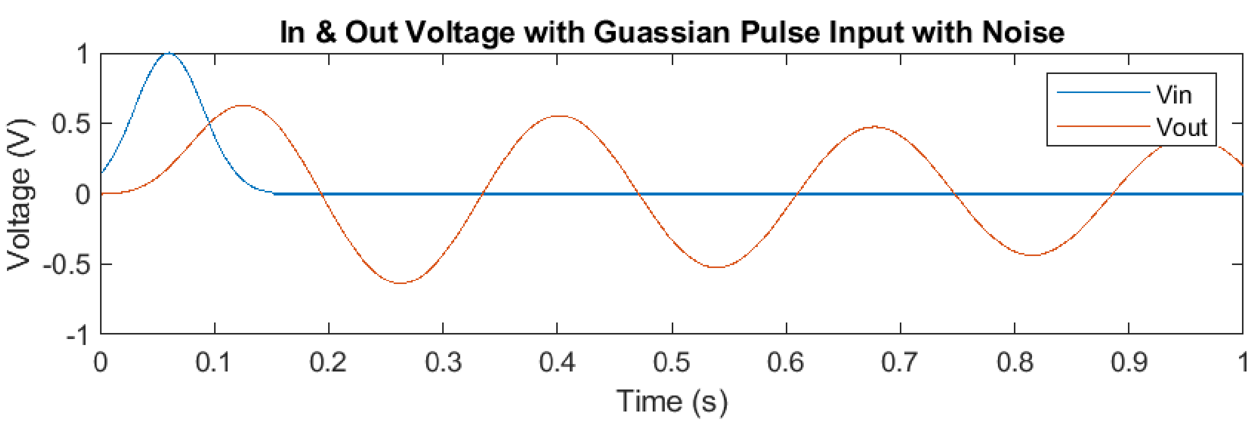

Figure 12: Gaussian input with Cn = 0.01, 1000 time steps time domain response

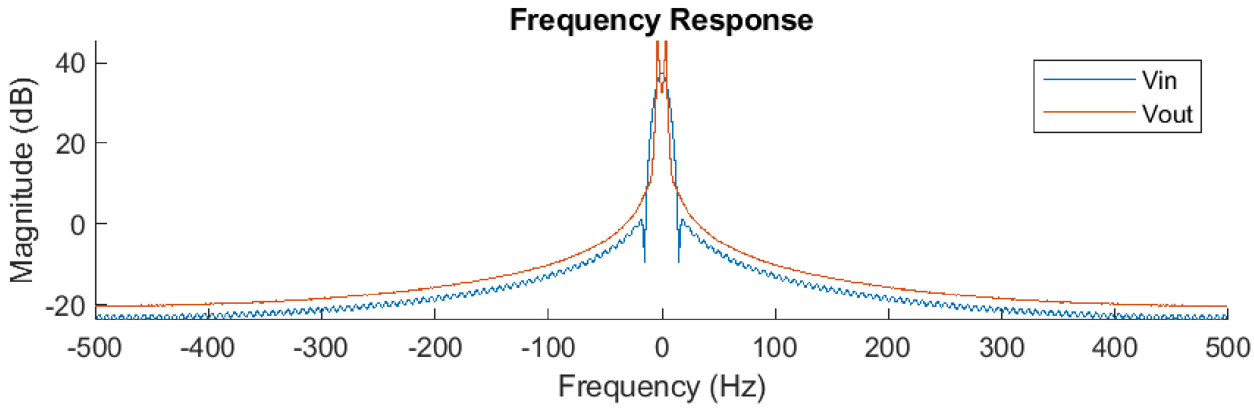

Figure 13: Gaussian input with Cn = 0.01, 1000 time steps frequency domain response

For this iteration, Cn = 0.001 and there are 1000 time steps. The time domain response is shown in Figure 14, and the frequency domain response is shown in Figure 15.

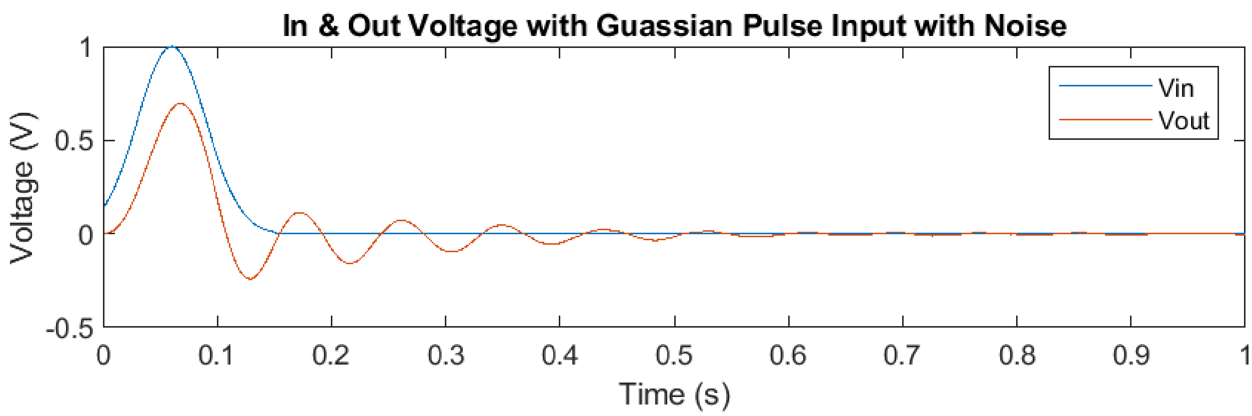

Figure 14: Gaussian input with Cn = 0.001, 1000 time steps time domain response

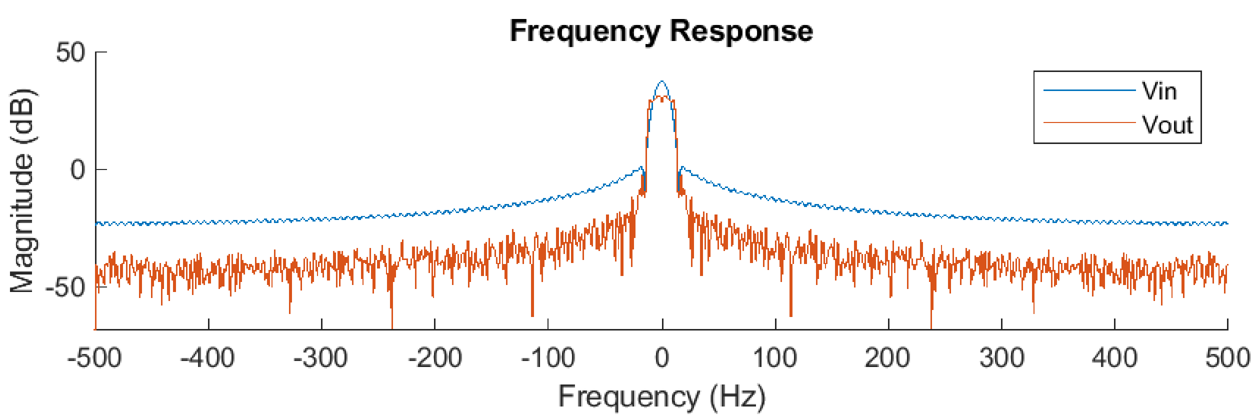

Figure 15: Gaussian input with Cn = 0.001, 1000 time steps frequency domain response

For this iteration, Cn = 0.00001 and there are 1000 time steps. The time domain response is shown in Figure 16, and the frequency domain response is shown in Figure 17.

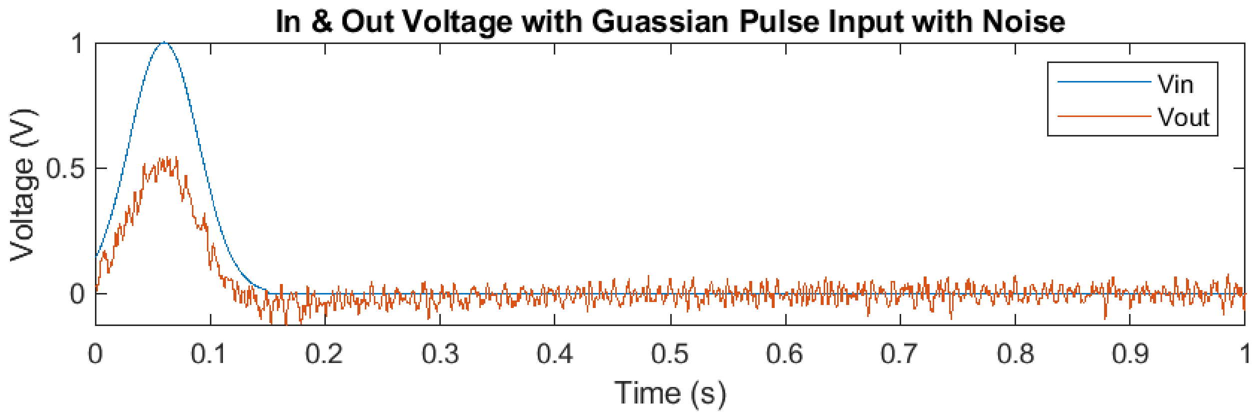

Figure 16: Gaussian input with Cn = 0.00001, 1000 time steps time domain response

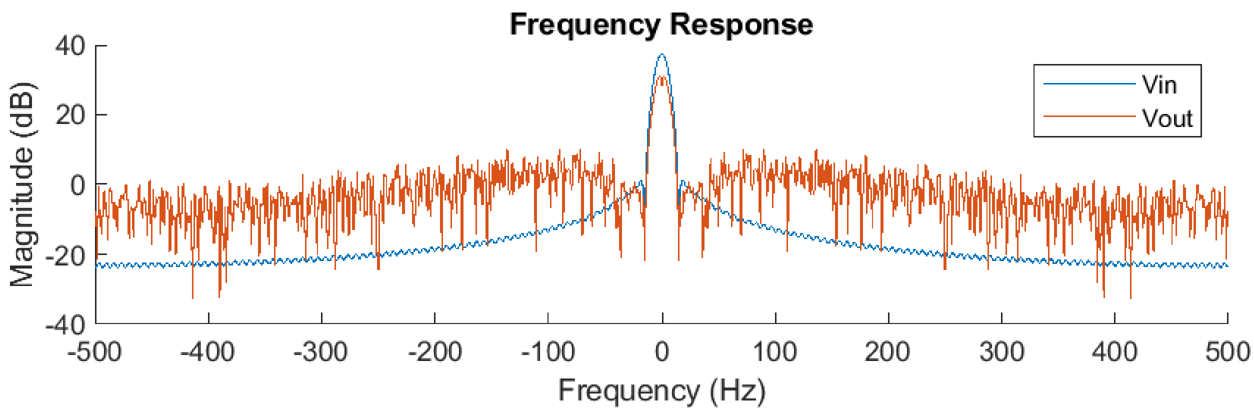

Figure 17: Gaussian input with Cn = 0.00001, 1000 time steps frequency domain response

For this iteration, Cn = 0.00001 and there are 2500 time steps. The time domain response is shown in Figure 18, and the frequency domain response is shown in Figure 19.

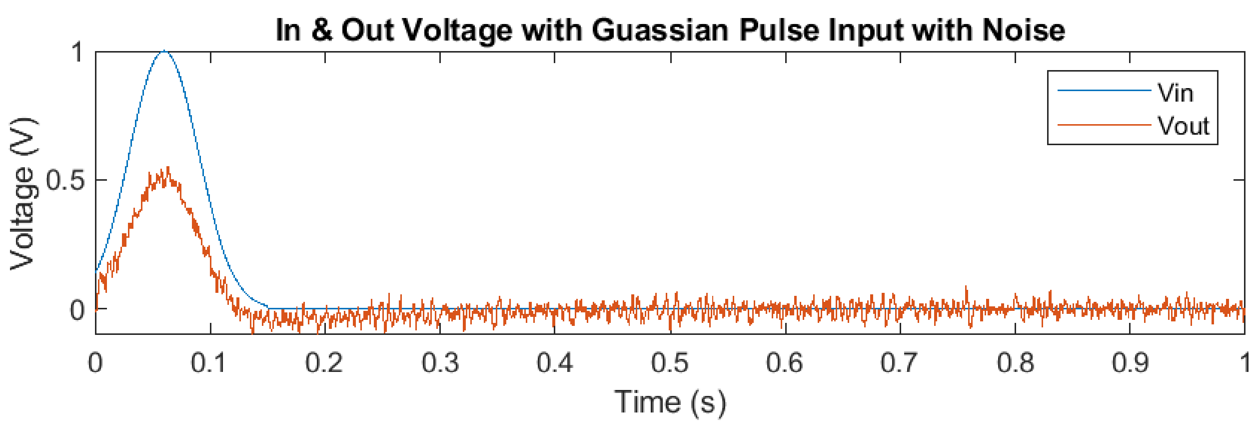

Figure 18: Gaussian input with Cn = 0.00001, 2500 time steps time domain response

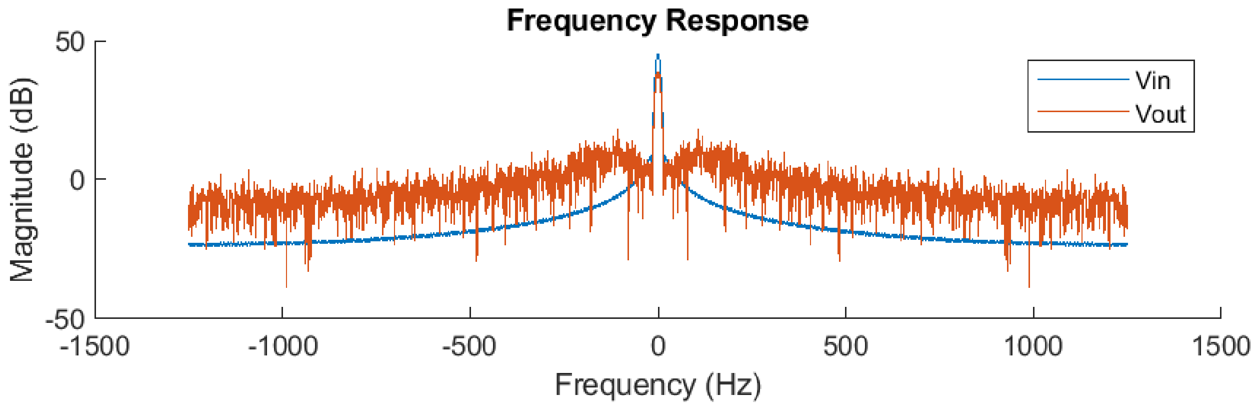

Figure 19: Gaussian input with Cn = 0.00001, 2500 time steps frequency domain response

For this iteration, Cn = 0.00001 and there are 5000 time steps. The time domain response is shown in Figure 20, and the frequency domain response is shown in Figure 21.

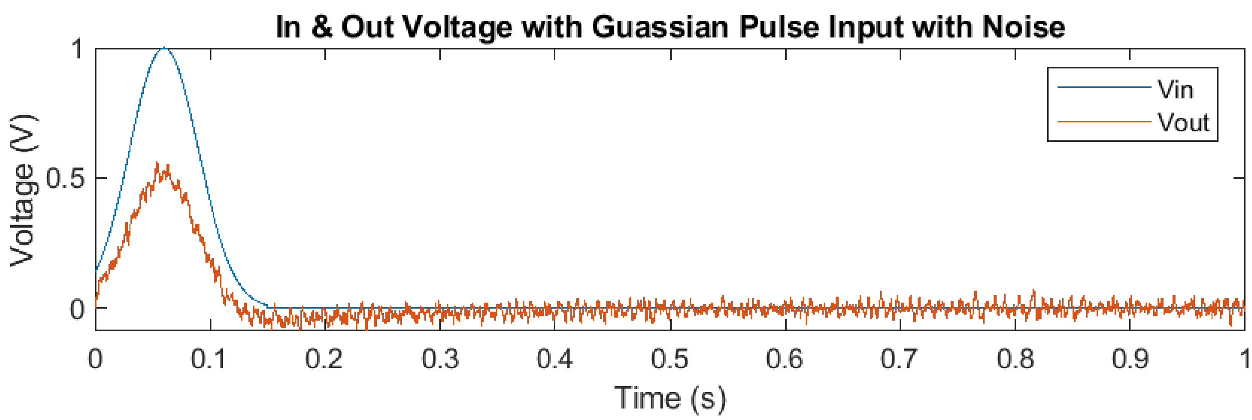

Figure 20: Gaussian input with Cn = 0.00001, 5000 time steps time domain response

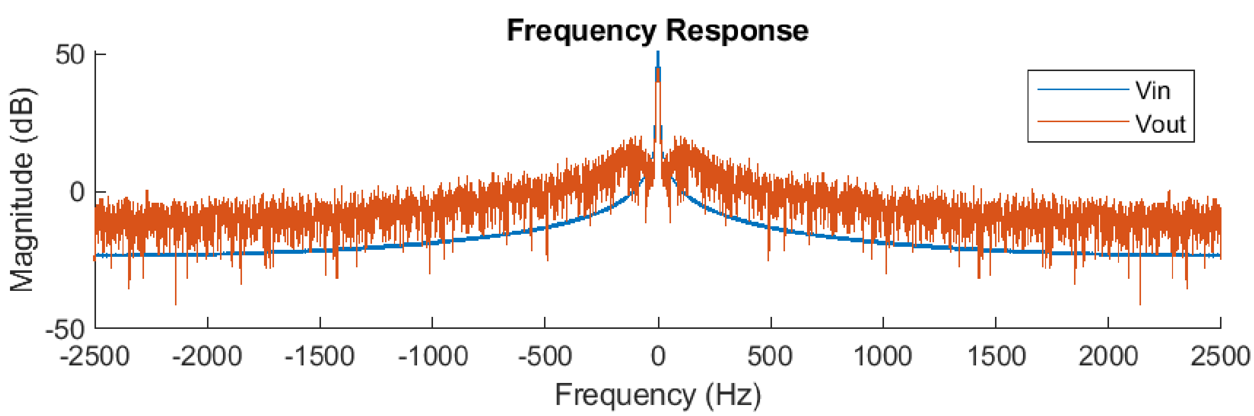

Figure 21: Gaussian input with Cn = 0.00001, 5000 time steps frequency domain response

The randomly selected value of In was tracked and is shown below in Figure 22 in a histogram.

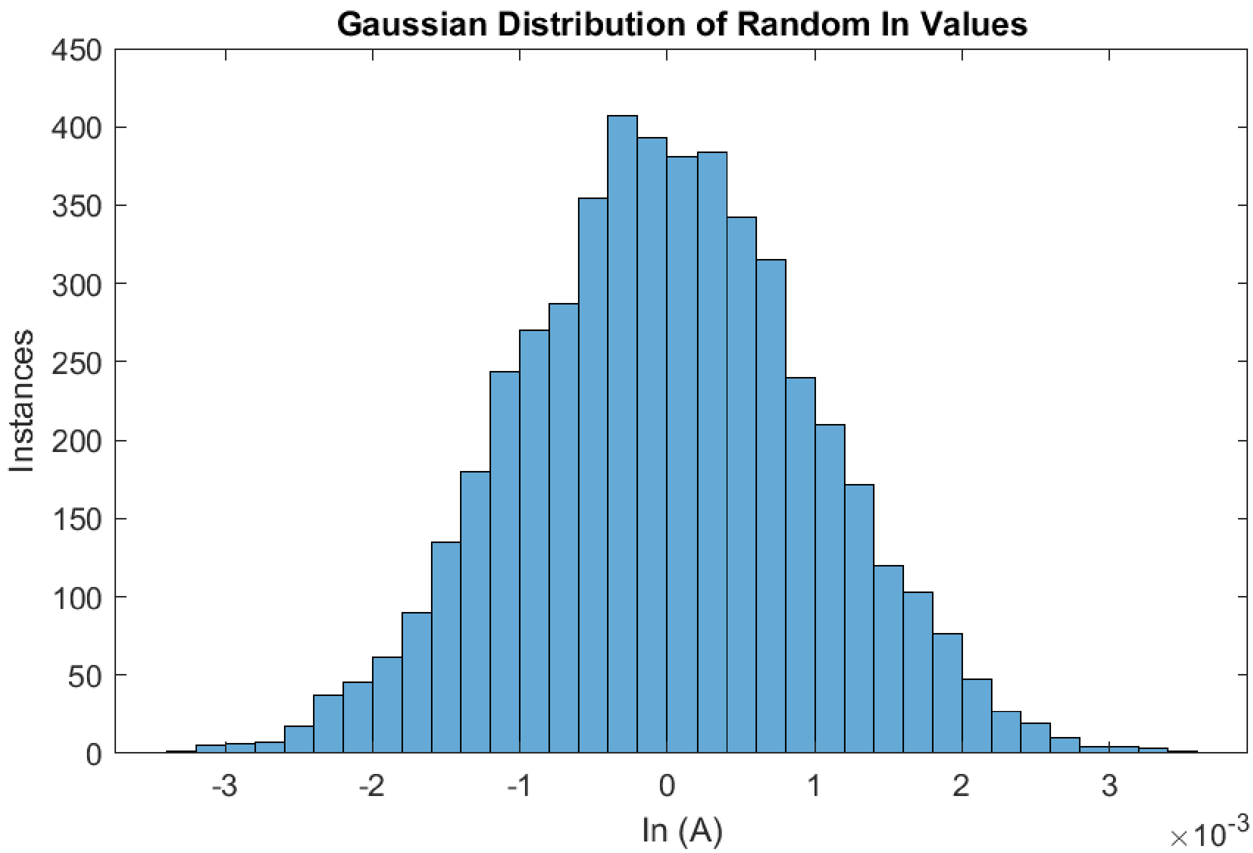

Figure 22: Histogram of In values

<u>Discussion:</u>

As it is clearly shown above, the value of Cn has an effect on the bandwidth of the circuit. The larger capacitance of Cn = 0.1 prevents the output from following the same pulse as the input, instead traveling as a sine wave. The lower the value of Cn, the closer the output follows the input pulse. Therefore, as the value of Cn increases, the bandwidth decreases.

When increasing the timestep above, the frequency domain is affected. As the number of time steps increases, the accuracy of the plot increases as well, as there are more points being plotted. For the three tested time steps, the primary difference is the level of noise visible. This shows that there is a threshold value in which increasing the timestep will not increase the accuracy of the plot. However, if the timestep were to be decreased below the initial 1000, the accuracy would be lowered, and the shape of the output may not be as evident due to the interference from the noise.

<u>Part 6:</u>

a) In this portion, the output voltage source is changed from

$$V = \alpha I_3 \ \ to \ \ V = \alpha I_3 + \beta {I_3}^2 + \gamma {I_3}^3$$

To implement this change into the above code, an additional non-linear vector would need to be added. The non-linear source unknowns would be added into this new vector. In this course, B is used typically, however, in this report, the non-linear vector is represented with F. The new equation to be solved is the following:

$$C\frac{dV}{dt} + GV + F = b$$

b) To add this to the above code, some slight changes need to be made. In the overall for loop that iterates over a defined number of timesteps, an additional nested for loop would need to be added. In this loop, the Jacobian of the above equation would need to be formed. From this, the Newton Raphson equation can be used. This will determine what the non-linear variables are by forcing them to converge to their actual value with a certain degree of error based on the convergence conditions set in the loop. After the non-linear values converge, a new A matrix is formed using both the linear and non-linear converged values, the initial loop continues, and the V vector is solved in the same manor as the linear portion.