

Linearity : $f(x)$ \rightarrow function, operation.

$$\left\{ \begin{array}{l} 1) \text{ superposition} \quad f(x_1 + x_2) = f(x_1) + f(x_2) \\ 2) \text{ homogeneity} \quad f(ax) = a f(x) \end{array} \right.$$

eg. $y = mx + n \quad (\rightarrow n=0, m \neq 0)$

• differentiation

$$\frac{d}{dt} (x_1(t) + x_2(t)) = \frac{d}{dt} x_1(t) + \frac{d}{dt} x_2(t).$$

• integration.

$$\int x_1(t) + x_2(t) dt = \int x_1(t) dt + \int x_2(t) dt.$$

• matrix operation. (multiplication)

$$A(x_1 + x_2) = Ax_1 + Ax_2.$$

vector operation.

② Basic Notation of Matrices.

• vector $v = (a, b, c)$, $w = (a, b, c)$

$$\hookrightarrow \text{columns of a matrix} = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

vector \Rightarrow matrix $\frac{2 \times 3}{3 \times 2}$

• Linear Combinations $\alpha v + \beta w$.

$$\Rightarrow \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} \begin{bmatrix} d \\ \beta \end{bmatrix} = d \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} + \beta \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$$

• central ideas (terms)

- column space : all combinations of the columns
- row space : " " " " rows
- rank : # of independent columns (or rows)
- elimination : to find the rank of a matrix.

~~③ Singular Case \rightarrow not-unique solution \Rightarrow no solution or infinite solu.~~

- ~~row picture $\left\{ \begin{array}{l} \rightarrow \text{parallel lines and planes.} \\ \rightarrow \text{no intersection of 3 planes.} \\ \rightarrow \text{line of intersection.} \end{array} \right.$~~

