Chapter 2. Vector Spaces.

2.1 Vector spaces and Subspaces.

◎ A real vector space R^n (n-dimensional)

1. $x + y = y + x$
2. $x + (y + z) = (x + y) + z$.
3. There is a unique "zero vector" such that $x + 0 = x$. $\forall x$.
4. For each x, there is a unique vector $-x$, such that $-x + x = 0$
5. $1 \cdot x = x$
6. $(c_1 c_2)x = c_1(c_2 x)$
7. $c(x + y) = cx + cy$
8. $(c_1 + c_2)x = c_1 x + c_2 x$.

$\begin{cases} x, y \rightarrow \text{vector.} \\ c, c_1, c_2 \rightarrow \text{scalar.} \end{cases}$

○ x and y should be colsed under addition and scalar multiplication $\Rightarrow$ "space"

ex). 1) R^∞ space

2) $m \times n$ matrix. $\rightarrow R^{mn}$

3) function $f(x) \rightarrow R^\infty$

◎ Subspace. → non empty subset that satisfies the requirements for a vector space.

(i) $x+y \in$ subspace, for $\forall x, y \in$ Subspace

(ii) $cx \in$ subspace, for $\forall x \in$ ~~and~~ subspace, $\forall c$.

⇒ x, and y should be closed under vector addition and scalar multiplications in the subspace.

∘ different from subset

ex) (i) ~~p~~ $\mathbb{R}^3$ vectors on a plane which passes through the origin.

(ii) the origin → smallest subspace.

(iii) $\mathbb{R}^2$ vectors on a line which passes through the origin.

(iv) lower triangular matrices.

◎ The Column Space of A.

∘ Column Space. → contains all linear combinations of the columns of matrix A.

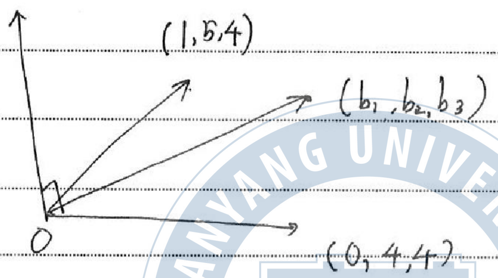
$$\underset{m\times n}{A}x = b \Rightarrow \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$x_1 a_1 + x_2 a_2 + \cdots + x_n a_n = b$$

↳ linear combinations of column vectors equals b.

- $Ax = b$ is solvable iff b can be expressed as a combination of the columns of A. Then b is in the column space.

ex) $u\begin{bmatrix}1\\5\\2\end{bmatrix} + v\begin{bmatrix}0\\4\\4\end{bmatrix} = \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$



→ lie on the plane by (1,5,4) and (0,4,4).

- vector b should be closed to the subspace spanned by column vectors.

- Column Space $\mathbb{C}(A)$
  1) if b and b' ~~line~~ lie in the column Space, → $A(x+x') = b+b'$.
  → closed under addition.
  2) if b lies in the column space → $A(cx) = cb$.
  → closed under scalar multiplication.

ex) 1) for any non-singular 5 x 5 matrix → $\mathbb{R}^5$ (column) space.
↳ 5 pivots in Gauss elimination.

2) zero matrix → 0-dimensional space.