$$F_k = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^k\right].$$

F_{1000} = nearest integer to $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{1000}$

◎ Markov Matrices,

∘ State, transition probability.

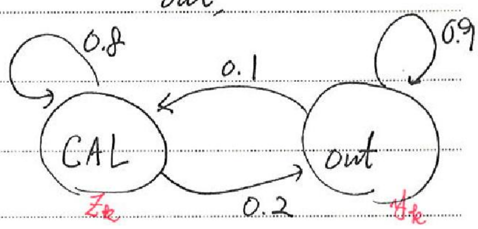
∘ Markov process and Matrix.

1) Each column of Markov matrix adds up to 1.

2) The matrix has no negative entries.

example) $\left(\begin{array}{l}\frac{1}{10} \text{ of the people outside California move in,} \\ \frac{2}{10} \quad \text{〃} \quad \text{inside} \quad \text{〃} \quad \text{out,}\end{array}\right.$

$\left(\begin{array}{l} y_1 : \text{outside people} \\ z_1 : \text{inside} \quad \text{〃} \end{array}\right.$



Difference equation, $\left(\begin{array}{l} y_1 = 0.9\, y_0 + 0.2\, z_0 \\ z_1 = 0.1\, y_0 + 0.8\, z_0 \end{array}\right.$

$$\begin{bmatrix} y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}\begin{bmatrix} y_0 \\ z_0 \end{bmatrix}.$$

$\Rightarrow u_{k+1} = A u_k$

$\det(A-\lambda I) = (0.9-\lambda)(0.8-\lambda) - 0.2\times 0.1 = 0.$

$\lambda^2 - 1.7\lambda + 0.7 = 0 \qquad \lambda_1 = 1, \quad \lambda_2 = 0.7.$

$$A = S\Lambda S^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.7 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = S\Lambda^k S^{-1} \begin{bmatrix} y_0 \\ z_0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.7^k \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} y_0 \\ z_0 \end{bmatrix}}$$

$$= \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.7^k \end{bmatrix} \cdot \begin{bmatrix} y_0 + z_0 \\ y_0 - 2z_0 \end{bmatrix}$$

$$= (y_0 + z_0) \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} \cdot 1^k + (y_0 - 2z_0)\, 0.7^k \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}.$$

Steady-state ; $k \to \infty$, $\begin{bmatrix} y_\infty \\ z_\infty \end{bmatrix} = (y_0 + z_0) \underline{\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}}$

$Au_\infty = u_\infty$.

$$\to \quad A \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}.$$

⇒ The steady state is the eigenvector of A corresponding to $\lambda = 1$

⑨ A Markov Matrix.

1) $a_{ij} \geq 0$. each column adds up to 1.

2) $\lambda_1 = 1$ is an eigenvalue.

3) eigenvector $x_1 \geq 0$, steady state, $Ax_1 = x_1$.

4) The other eigenvalues , $|\lambda_i| \leq 1$.

5) $A^k u_0 \to c x_1 \to u_\infty$; steady state.

6) Markov matrix → transition matrix (for random process)

∘ Stability of $u_{k+1} = Au_k$.

$$\Rightarrow u_k = S\Lambda^k \underbrace{S^{-1}u_0}_{C} = c_1\lambda_1^k x_1 + \cdots + c_n\lambda_n^k x_n$$

↳ stability depends on the eigenvalues,

∘ The difference equation $u_{k+1} = Au_k$ is

stable iff $|\lambda_i| < 1$
neutrally stable if some $|\lambda_i| = 1$ and others $|\lambda_j| < 1$.
unstable if at least one eigenvalue $|\lambda_i| > 1$.

HW 6.3 5, 9, 20, 27

Homogeneous linear Least square

$Ax = 0 \rightarrow \min \|Ax\|^2$

$\Rightarrow x = e_1$ for λ_1

where, $A^TA = R$ $\quad Re_i = \lambda_i e_i$

$0 < \lambda_1 < \lambda_2 \cdots < \lambda_n$.