CS109 Final Exam

This is a timed, open-book exam. Focus, however, also on what you think is the exam. The instructions are the same as for the midterm: Write your answers in the space provided. Show your work. Be clear and concise. Write as much or little as you need to fill the space provided. Assume the current date is the date of the exam. Any calculation is acceptable as long as it is correct and simplified to a single number.

In the case of an honor exam, the expectation is an period of time you dedicate your attention to the exam, without other distractions. Please, do not discuss the exam with anyone else. Please, do not look up information on the internet. Please, do not use other people or services to help you. Please, do not use other people's notes. Please, do not use AI. Please, do not use your own AI. Please, do not use other people's AI. Please, do not use your own AI to help you write your answers to other people's AI questions.

Name: Chris Piech

Name Spelling Check: Chris Piech

Honor Code: I promise to uphold the honor code and agree to the honor code. Lightly or close reading shows that you have read the honor code. Signature: Chris Piech

Family Name (print): Chris Piech

Given Name (print): Chris Piech

1 Random Encounter Redux [10 points]

You walk into a gathering in Stanford and it consists of Stanford students. There is the probability that you encounter a Stanford student at the gathering.

Let E_1 be the number of students you encounter in the gathering. Assume E_1 follows a Poisson distribution with parameter λ. Let E_2 be the number of students you encounter in the gathering. Assume E_2 follows a Poisson distribution with parameter λ. Let E_3 be the number of students you encounter in the gathering. Assume E_3 follows a Poisson distribution with parameter λ.

$E_1 \sim \text{Poisson}(\lambda)$
$E_2 \sim \text{Poisson}(\lambda)$
$E_3 \sim \text{Poisson}(\lambda)$

Let $E = E_1 + E_2 + E_3$. Assume E follows a Poisson distribution with parameter 3λ.

$E \sim \text{Poisson}(3\lambda)$

Let X be the number of students you encounter in the gathering. Assume X follows a Poisson distribution with parameter λ.

$X \sim \text{Poisson}(\lambda)$

Let Y be the number of students you encounter in the gathering. Assume Y follows a Poisson distribution with parameter λ.

$Y \sim \text{Poisson}(\lambda)$

Let Z be the number of students you encounter in the gathering. Assume Z follows a Poisson distribution with parameter λ.

$Z \sim \text{Poisson}(\lambda)$

$P(E = 3\lambda) = \frac{\lambda^{3\lambda} e^{-\lambda}}{(3\lambda)!}$

$P(E = 3\lambda \mid E_1 = \lambda, E_2 = \lambda, E_3 = \lambda) = \frac{\lambda^{3\lambda} e^{-\lambda}}{(3\lambda)!}$

$P(E = 3\lambda \mid E_1 = \lambda) = \frac{\lambda^{3\lambda} e^{-\lambda}}{(3\lambda)!}$

$P(E = 3\lambda \mid X = \lambda) = \frac{\lambda^{3\lambda} e^{-\lambda}}{(3\lambda)!}$

$P(E = 3\lambda \mid Y = \lambda) = \frac{\lambda^{3\lambda} e^{-\lambda}}{(3\lambda)!}$

$P(E = 3\lambda \mid Z = \lambda) = \frac{\lambda^{3\lambda} e^{-\lambda}}{(3\lambda)!}$

$P(E = 3\lambda \mid E_1 = \lambda) = P(E = 3\lambda \mid E_2 = \lambda) = P(E = 3\lambda \mid E_3 = \lambda) = P(E = 3\lambda \mid X = \lambda) = P(E = 3\lambda \mid Y = \lambda) = P(E = 3\lambda \mid Z = \lambda)$

$P(E = 3\lambda \mid E_1 = \lambda) = \frac{\lambda^{3\lambda} e^{-\lambda}}{(3\lambda)!}$

Conditional on $E_1 = \lambda$, E is a sum of three independent Poisson distributions, each with parameter λ. Therefore, E follows a Poisson distribution with parameter 3λ.

$E \mid E_1 = \lambda \sim \text{Poisson}(3\lambda)$