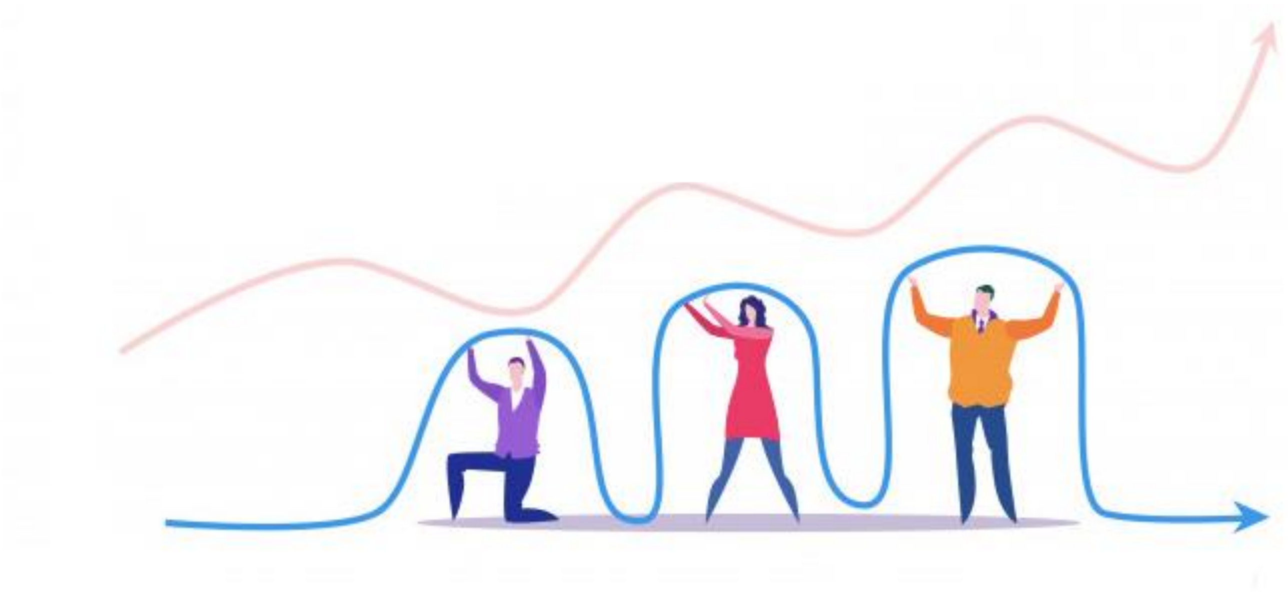
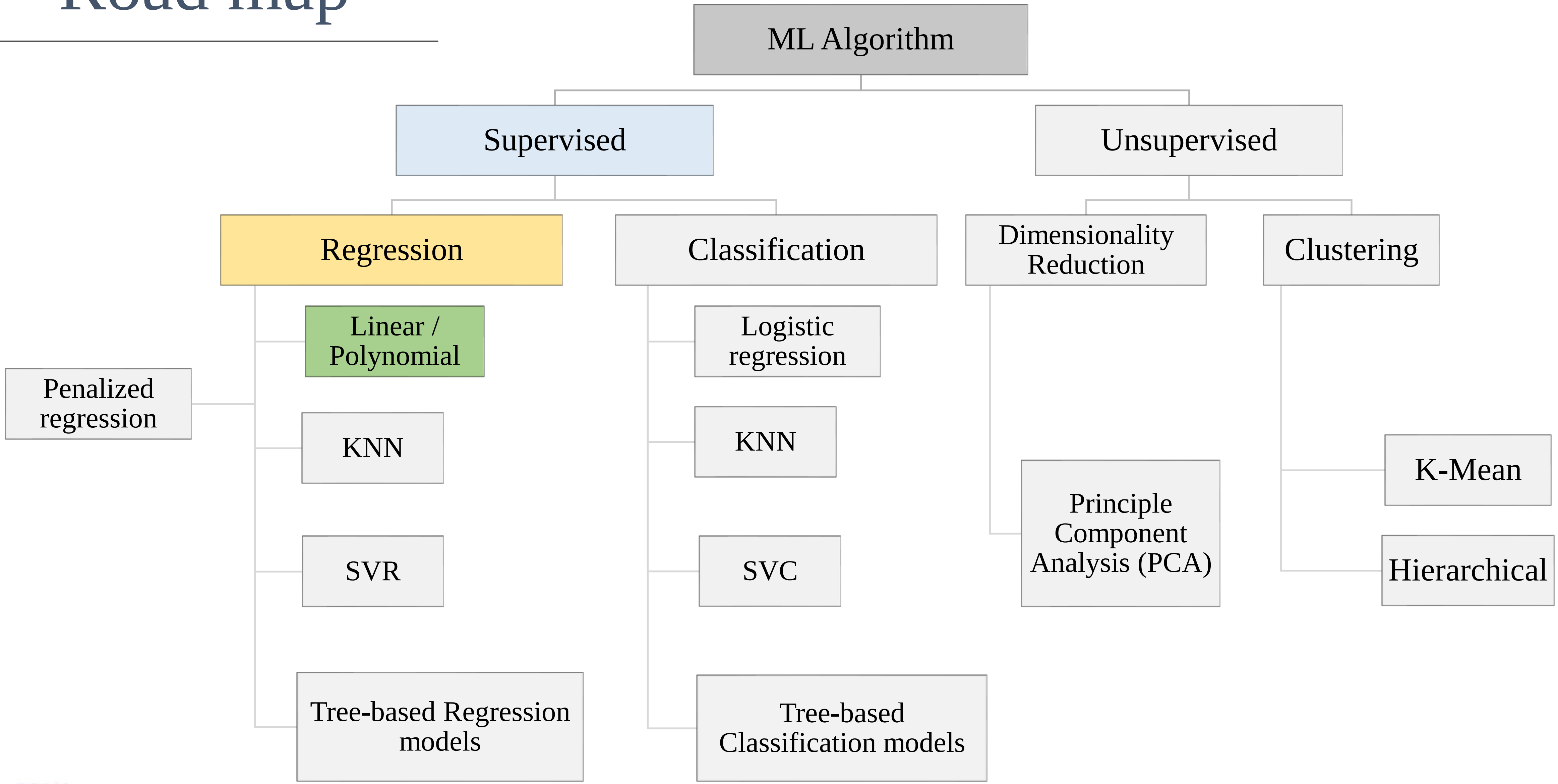
Class 8- Polynomial Regression

Prof. Pedram Jahangiry



Road map


ML Algorithm
Supervised
Unsupervised
Regression
Classification
Dimensionality Reduction
Clustering
Linear / Polynomial
Penalized regression
KNN
SVR
Tree-based Regression models
Logistic regression
KNN
SVC
Tree-based Classification models
Principle Component Analysis (PCA)
K-Mean
Hierarchical

Polynomial regression model

The polynomial regression model is a special case of multiple linear regression models!

- Create new variables $X_1 = X,\ X_2 = X^2$, … etc and then treat as multiple linear regression.
- Not really interested in the coefficients; more interested in the fitted function!

$$\hat{f}(X) = f_{w,b}(X) = b + w_1 x + w_2 x^2 + \ldots\ldots w_d x^d$$

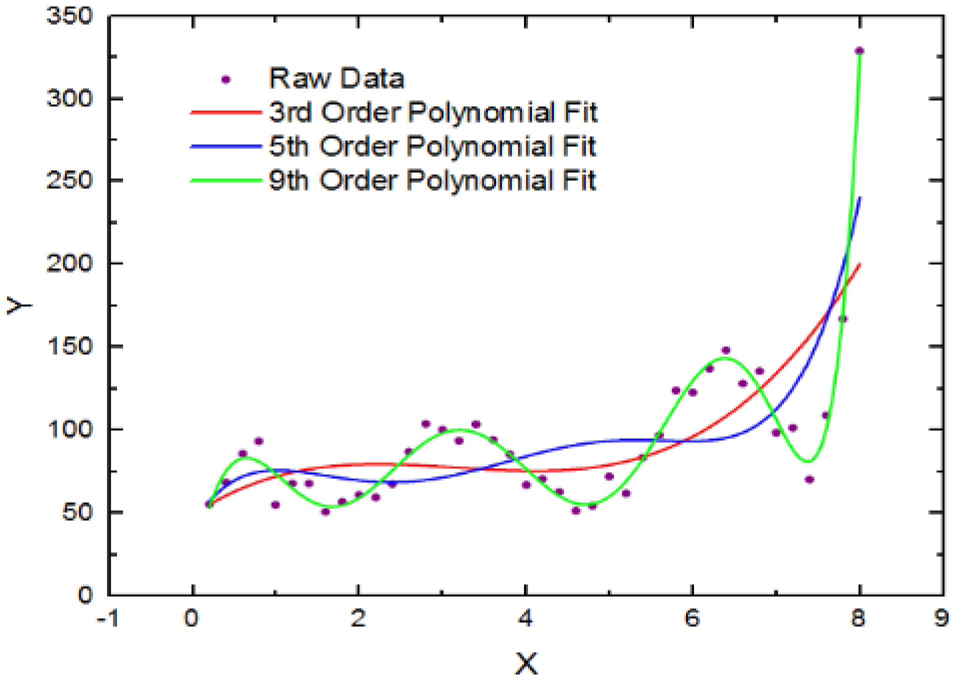
- W is a d-dimensional vector of parameters
- b is a real number
- d is the polynomial degree of the model (we either fix the d at some reasonably low value, else use **cross-validation** to choose d)

The optimization problem

The optimization problem is defined as:

$$Min_{w,b}\ MSE = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2 = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - f_{w,b}(X_i)\right)^2$$

- We use the same **loss function** as in linear regression!
- The solution to this optimization problem is w^* and b^*
- Now we can make predictions!



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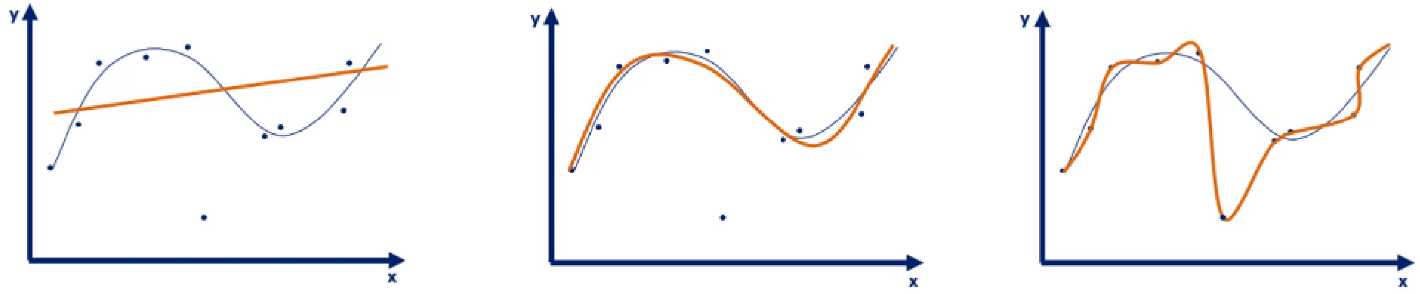
Tuning the hyperparameter d!



$$MSE = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y})^2$$

$$RMSE = \sqrt{MSE}$$

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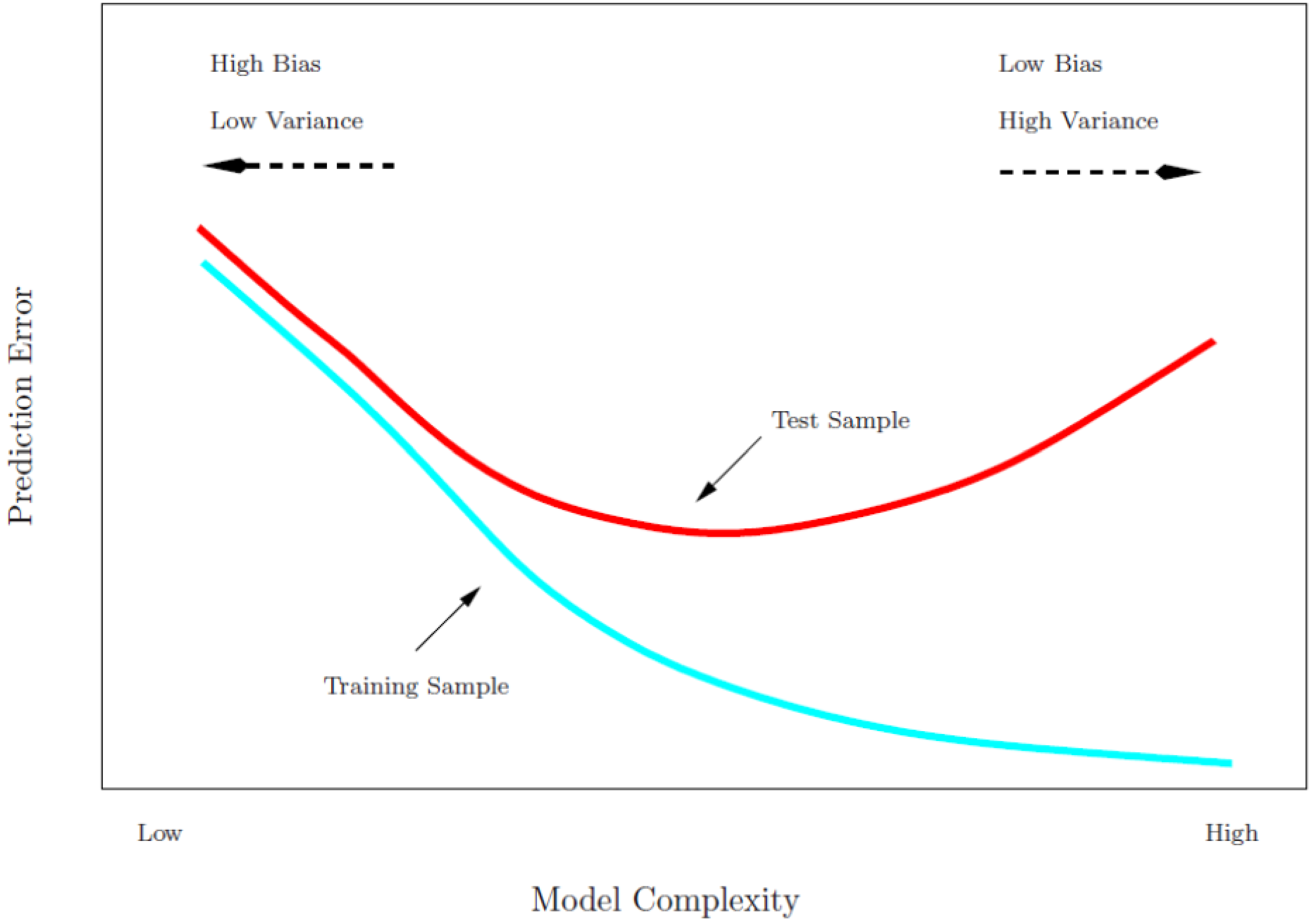
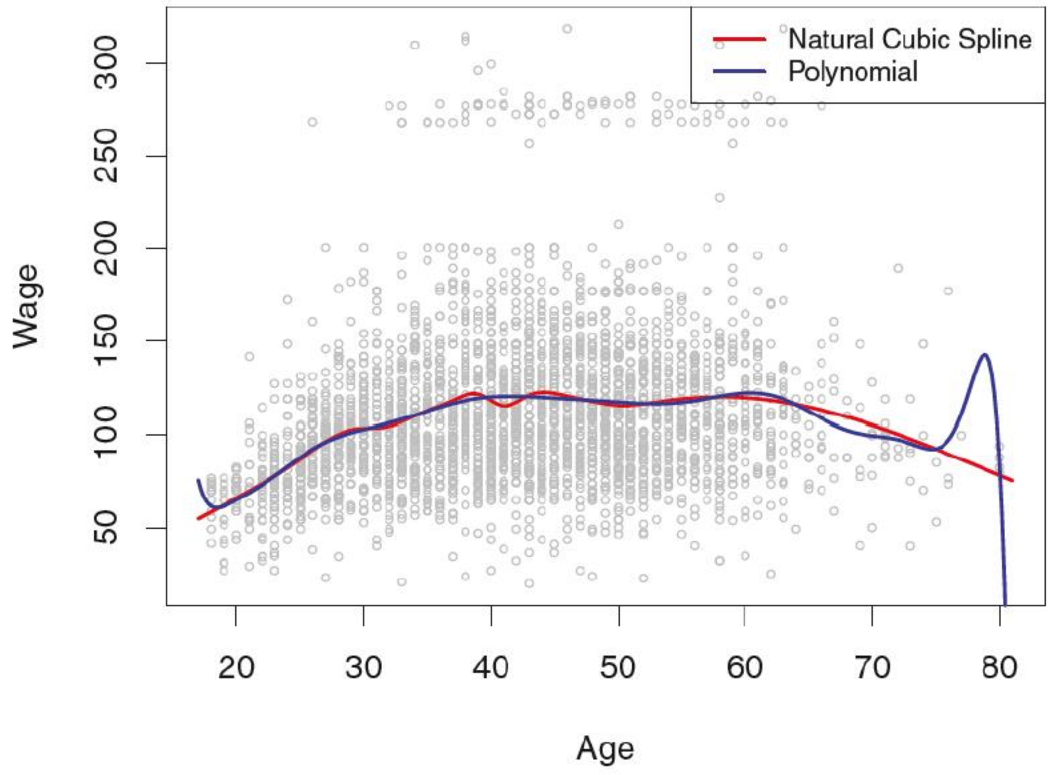
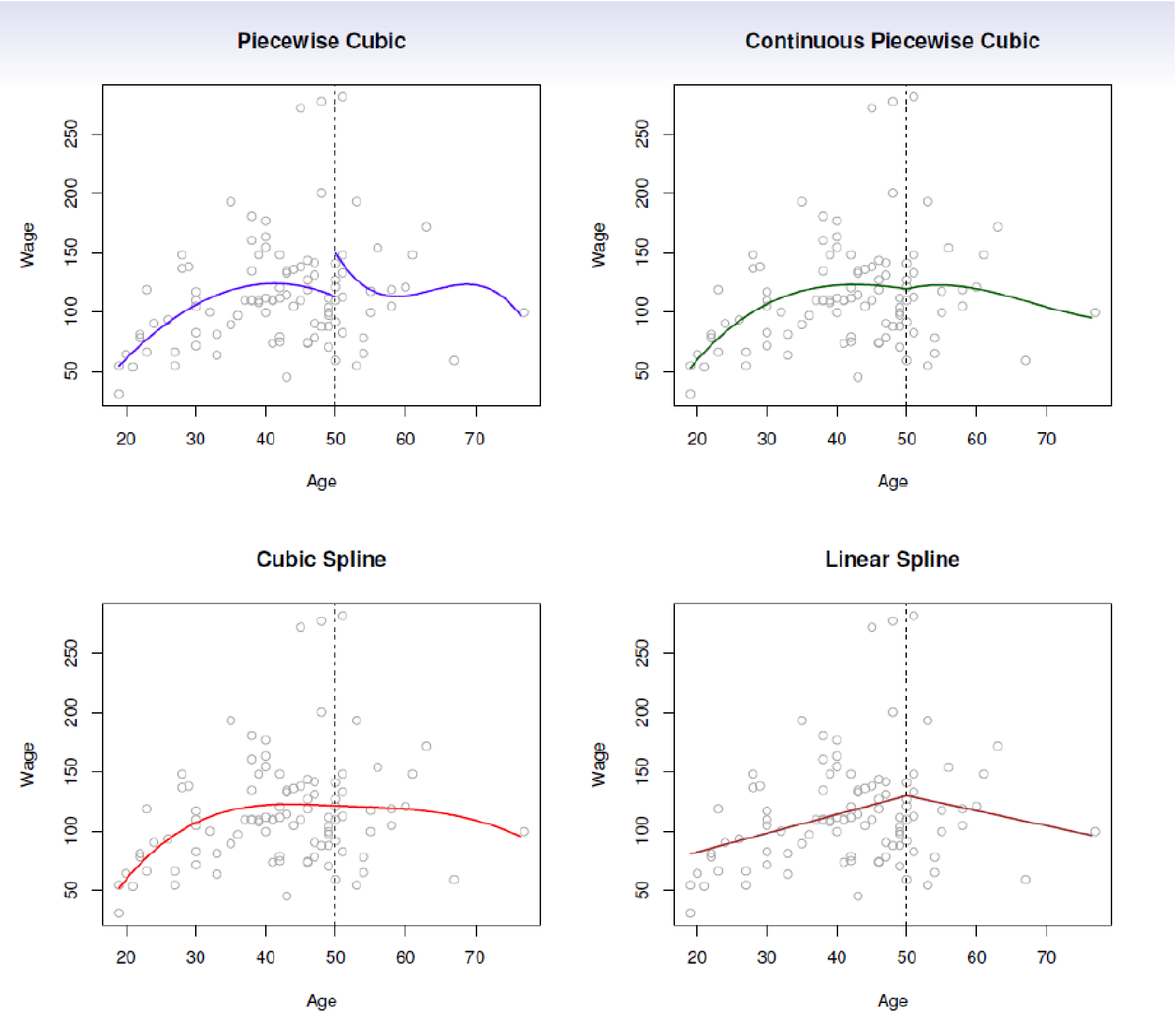


Caveats!

- In general quadratic loss functions are sensitive to outliers
- Polynomials have notorious tail behavior
- Polynomials are global fit!
- Solution: piecewise polynomial, splines and local regressions.



Piecewise polynomials and splines!




thank you all
CLASS DISMISSED