12A: Problem Set 11

Due date: **November 10, 10am**. Deliver completed problem set to the 12A dropbox in room 301, Moses Hall. Reminder: late problem sets do not receive credit.

Also: please put your **section leader's name** and **section meeting time** on the upper right of the first page. This will help us return your problem set promptly.

1. Symbolize into FOL.
   (a) A salesman who greeted every customer was friendly.
   (b) Every farmer who owns a donkey wearing a saddle pets it.
2. Give an FOL semantic derivation for each of these, using the turnstile notation.
   (a) Fa
   (b) Gbx
   (c) $(Fc \wedge P)$
   (d) $\exists x Fx$
   (e) $\exists x Fb$
   (f) $\exists x \exists x Fx$
   (g) $\forall x \exists y (x = y)$
   (h) $\exists y \forall x (x = y)$
3. In FOL the notion of **truth at a model** is defined only for closed formulas. But suppose we removed this restriction, and allowed open formulas to be capable of truth and falsity at a model, too. That is, suppose we worked with this definition instead:

   > **Def.** For all ~~closed~~ wffs ϕ, models $\mathcal{M}$, ϕ is **true relative to** $\mathcal{M}$ (we write this as: '$\mathcal{M} \vDash \phi$') iff for all variable assignments g for $\mathcal{M}$, $\mathcal{M} \vDash_g \phi$. Otherwise ϕ is false relative to $\mathcal{M}$.

   And suppose we give a matching definition of **consistency**:

   > **Def.** A set Γ of ~~closed~~ wffs is **consistent** iff there exists some model $\mathcal{M}$ such that for all $\phi \in \Gamma$, $\mathcal{M} \vDash \phi$. Otherwise Γ is **inconsistent**.

   And suppose our definition **validity** was understood as defined in terms of this more "inclusive" notion of truth at a model:

   > **Def.** $\Gamma \vDash \phi$ iff for any model $\mathcal{M}$, if $\mathcal{M} \vDash \psi$ for all ψ in Γ, then $\mathcal{M} \vDash \phi$.

   (a) Consider the set $\{Gx, \neg Gy\}$. Given the above definitions, is this set consistent? Why or why not? How does this compare to what our official definition of consistency predicts?
   (b) Given these definitions, does Fx entail $(Fa \wedge Fb)$? Why or why not?