we know: $P[O \cap F] = Pr[O] \times Pr[F]$

2. a) $\bar{O}$ and F are independent

$$Pr[F \cap \bar{O}] = Pr[F - (F \cap O)]$$
$$= Pr[F] - Pr[F \cap O]$$
$$= Pr[F] - Pr[F] Pr[O]$$
$$= Pr[F] (1 - Pr[O])$$
$$= Pr[F] Pr[\bar{O}]$$

b) O and $\bar{F}$ are independent

$$Pr[O \cap \bar{F}] = Pr[O - (O \cap F)]$$
$$= Pr[O] - Pr[F \cap O]$$
$$= Pr[O] \cdot (1 - Pr[F])$$
$$= Pr[O] Pr[\bar{F}]$$

c) $\bar{O}$ and $\bar{F}$ are independent

$$Pr[\bar{O} \cap \bar{F}] = Pr[(\overline{O \cup F})]$$
$$= 1 - Pr(O \cup F)$$
$$= 1 - (Pr(O) + Pr(F) - P(O \cap F))$$
$$= 1 - Pr(O) - Pr(F) + Pr(O) Pr(F)$$
$$= (1 - Pr(O)) - Pr(F)(1 - Pr(O))$$
$$= P(\bar{O}) P(\bar{F})$$

d) O and $F \cap C$ are independent

$$Pr[O \cap (F \cap C)] = Pr[O \cap F \cap C]$$
$$= Pr(O) Pr(F) Pr(C)$$
$$= Pr(O) Pr(F \cap C)$$

e) O and $F \Delta C$ are independent

$$P[O \cap (F \Delta C)] = Pr[O] \cdot Pr[C] \cdot Pr[\bar{F}] + Pr[O] \cdot Pr[F] \cdot Pr[\bar{C}]$$
$$= Pr[O] (Pr[C] Pr[\bar{F}] + Pr[F] Pr[\bar{C}])$$
$$= Pr[O] (P((C - F) \cup (F - C)))$$

a) Throw n balls into n bins w/ depth k-1

$k = 0.1n$

upper bound # ways throw balls to overflow

- select k balls then throw remaining randomly (n-k balls)

$(n C k) \, n^{n-k}$

b) upper bound probability of overflow of first bin

total ways balls can fit in all bin : n^n

total ways balls can fit in n-1 bins (exclude first) = $(n C k)(n^{n-k})$

$$\frac{(n C k)(n^{n-k})}{n^n}$$

c) upper bound probability of overflow of any bin

$$n \cdot \frac{(n C_k)(n^{n-k})}{n^n}$$

a) already helped k students in n total

P[random button => student that hasn't been helped] = $\frac{n-k}{n}$

b) X_i^r = TA hasn't helped student i after pressing random "r" times

$Pr[X_i^r]$ = probability 1 - has helped = $(1 - \frac{1}{n})^r$

bound by $1 - x \leq e^{-x}$

$$(1 - \frac{1}{n})^r < e^{\frac{-r}{n}}$$

c) Tr : TA presses button r times but hasn't helped all n students

union bound - all students of being helped w/ r button clicks

$: T_r = \bigcup_{i=1}^{n} X_i^r$

d) Upper bound for $Pr[Tr]$

$$Pr[Tr] \leq n \cdot Pr[X_i^r] \leq n \cdot e^{-r/n}$$

e) $r = \alpha n \ln n$

$$Pr[X_i^r] \to e^{-\alpha n \ln n / n} \to e^{-\alpha \ln n} = n^{-\alpha}$$

f) upper bound $Pr[T_r] \leq n \cdot Pr[X_i^r] \leq n \cdot n^{-\alpha}$

$$n^{1-\alpha}$$

g) bound tail to $\frac{1}{n^2}$ probability

$$\frac{1}{n^2} = n^{1-\alpha}$$

$\alpha = 3$ $\quad r = 3n \ln n$

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I certify that all solutions are entirely in my words and that I have not looked at another student's solutions. I have credited all external sources in this writeup.

CS 70 HW

1.

a) P[draw straight]

A 2 3 4 5 6 7 8 9 10 J Q K

9 possible starting points for straight

$$\frac{(9C_1)(4C_1)(4C_1)(4C_1)(4C_1)(4C_1)}{52C_5}$$

b) P[one card for each suit]

$$\frac{(1)(13C_1)(13C_1)(13C_1)(13C_1)}{52C_5}$$

c) P[8×8 board 2 square share side]

for each case : $\left(\frac{\#\text{ case instances}}{64}\right)\left(\frac{\#\text{ shared sides}}{64-1}\right)$

square: corner: $\left(\frac{4}{64}\right)\left(\frac{2}{63}\right)$

border: $\left(\frac{24}{64}\right)\left(\frac{3}{63}\right)$

middle: $\left(\frac{36}{64}\right)\left(\frac{4}{63}\right)$

add (+)

d) P[8×8 board, none rooks attacking]

need to place 8 rooks

place 1, place second $\left(\frac{64-\#\text{ in row/column}}{64-1}\right)$

need to keep track that with each rook placed, next rook subtracts 1 from col/rows that prev. rook already has in attack status

$$(1)\left(\frac{49}{63}\right)\left(\frac{36}{62}\right)\left(\frac{25}{61}\right)\left(\frac{16}{60}\right)\left(\frac{9}{59}\right)\left(\frac{4}{58}\right)\left(\frac{1}{57}\right)$$

e) if took Q out

$\frac{2}{3}$ Q — 3/4 replace Q $\boxed{QQP}$ = 2/3 chance Q

— 1/4 replace P $\boxed{QPP}$ = 1/3 chance Q

if took P out

$\frac{1}{3}$ P — 3/4 replace P $\boxed{PQQ}$ = 2/3 chance Q

— 1/4 replace Q $\boxed{QQQ}$ = 100% chance Q

we know we replaced a P

$$\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$$