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import numpy as np
import tensorflow as tf
import matplotlib.pyplot as plt
from cycler import cycler

print("TensorFlow version: {}".format(tf.__version__))
print("Eager execution: {}".format(tf.executing_eagerly()))

# Constants
N = 50
sigNoise = 0.1
M = 10 # Number of Gaussians
numEpochs = 200

# Variables
eps = tf.random.normal( [N,1], 0, sigNoise )
x = tf.random.uniform( [N,1], 0, 1 )
y = tf.sin( 2 * np.pi * x ) + eps

# Used for graphing true sinewave without noise
trueX = np.linspace( 0, 1, 500, dtype = 'float32' )
trueX = np.reshape( trueX, (500,1) ) # Reshapes into vector to allow broadcast
ing
trueY = np.sin( 2 * np.pi * trueX )

# Module containing Linear Regression model
class linRegMod( tf.Module ):

    def __init__(self):

        # Trainable Tensorflow variables
        self.w = tf.Variable( tf.random.uniform( [M], -0.5, 0.5 ), name = 'w' )
        self.mu = tf.Variable( tf.linspace( -0.1, 1.1, [M] ), name = 'mu' )
        self.sig = tf.Variable( 0.25 * tf.ones( shape = [M] ), name = 'sig' )
        self.b = tf.Variable( tf.random.uniform( [1], -0.5, 0.5 ), name = 'b' )
```

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# Loss function
@tf.function
def lossFunc( self, x, y ):

    yHat = self.estY( x )    # Applies estY elementwise to x
    MSE = tf.reduce_sum( 0.5 * ( y - yHat )**2 )

    return MSE

# Calculates yHat given x
@tf.function
def estY( self, x ):

    return tf.reduce_sum( self.w * self.gaussian( x ), 1, keepdims = True ) +
self.b    # Sums along rows to get N x 1

# Generates N x M matrix
def gaussian( self, x ):

    return tf.math.exp( -( x - self.mu )**2 / self.sig**2 )

def train( self ):

    # Stochastic Gradient Descent
    # Idea from https://stackoverflow.com/questions/57759563/minimize-
    # multivariate-function-in-tensorflow
    opt = tf.keras.optimizers.SGD()

    # Iterate through epochs
    for _ in range( numEpochs ):

        with tf.GradientTape() as tape:

            loss = self.lossFunc( x, y )
            print(loss)

        grads = tape.gradient( loss, self.variables )
        opt.apply_gradients( zip( grads, self.variables ) )

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# First plot
def plotFit( self, x, y, trueX, trueY ):

    # Calculate manifold from parameters
    yPred = self.estY( trueX ).numpy()

    # Plot
    plt.figure()
    plt.scatter( x, y, color = 'g' )           # Noisy data
    plt.plot( trueX, yPred, color = 'r', linestyle = '--' )
    ' ) # Regression manifold
    plt.plot( trueX, trueY, color = 'b' )       # Noiseless sinewave

    plt.xlabel( 'x' )
    plt.ylabel( 'y', rotation = 0 )
    plt.title( "Fit 1" )
    plt.show()

# Second plot
def plotBases( self, trueX ):

    plt.figure()
    plt.rc( 'axes', prop_cycle = ( cycler( 'color', ['r', 'g', 'b', 'm', 'y', 'c'] ) ) )

    # Plot each Gaussian
    plt.plot( trueX, self.gaussian( trueX ) )

    plt.xlabel( 'x' )
    plt.ylabel( 'y', rotation = 0 )
    plt.title( "Bases for Fit 1" )
    plt.show()

def main():

    model = linRegMod()

    print( "Starting MSE:", model.lossFunc( x, y ).numpy() )
    model.train()
    print( "Final MSE:", model.lossFunc( x, y ).numpy() )

    model.plotFit( x, y, trueX, trueY )
    model.plotBases( trueX )

main()

```

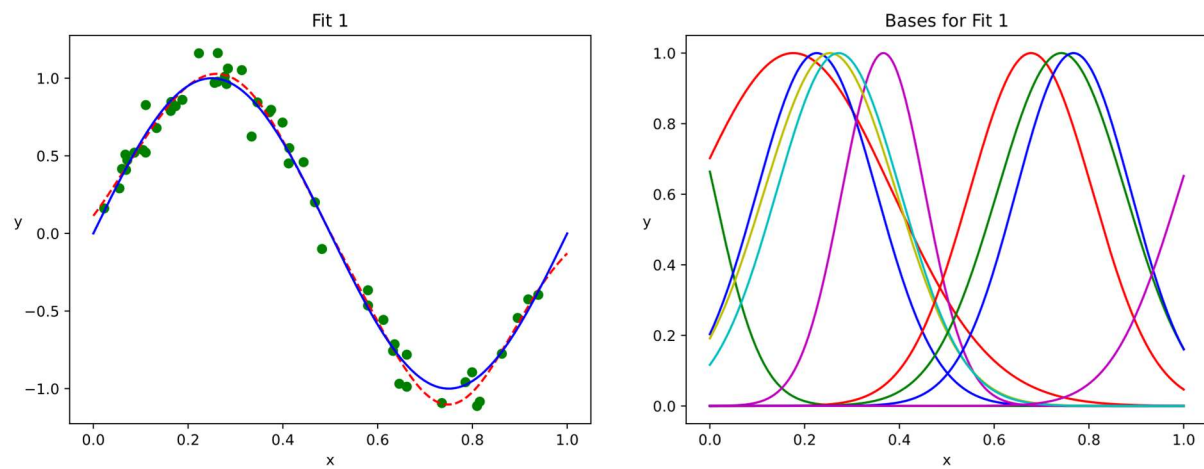


Figure 1: Example plots for linear regression of a noisy sinewave using a set of 10 Gaussian basis functions