ECE-302
PROJECT 3

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Contents

1 Exponential Distribution

$$f(x) = \lambda \mathrm{e}^{-\lambda x} \tag{1}$$

$$L(\lambda) = \prod_{i=1}^{n} \lambda \mathrm{e}^{-\lambda x_i} = \lambda^n \prod_{i=1}^{n} \mathrm{e}^{-\lambda x_i} = \lambda^n \mathrm{e}^{-\lambda \sum_{i=1}^{n} x_i} \tag{2}$$

$$LL(\lambda) = n \ln \lambda - \lambda \sum_{i=1}^{n} x_i = n \ln \lambda - \lambda n \bar{x} = n(\ln \lambda - \lambda \bar{x}) \tag{3}$$

$$LL'(\lambda) = \frac{n}{\lambda} - n\bar{x} \tag{4}$$

Maximizing log likelihood:

$$\frac{n}{\lambda} - n\bar{x} = 0 \tag{5}$$

$$\frac{n}{\lambda} = n\bar{x} \tag{6}$$

$$\lambda = \frac{1}{\bar{x}} \tag{7}$$

2 Rayleigh Distribution

$$f(x) = \frac{x}{\sigma^2} \mathrm{e}^{-\frac{x^2}{2\sigma^2}} \tag{8}$$

$$L(\sigma) = \prod_{i=1}^{n} \frac{x_i}{\sigma^2} \mathrm{e}^{-\frac{x_i^2}{2\sigma^2}} = \frac{1}{\sigma^{2n}} \prod_{i=1}^{n} x_i \mathrm{e}^{-\frac{x_i^2}{2\sigma^2}} = \frac{1}{\sigma^{2n}} \mathrm{e}^{-\sum_{i=1}^{n} \frac{x_i^2}{2\sigma^2}} \prod_{i=1}^{n} x_i \tag{9}$$

$$LL(\sigma) = -2n \ln \sigma - \sum_{i=1}^{n} \frac{x_i^2}{2\sigma^2} + \sum_{i=1}^{n} \ln x_i \tag{10}$$

$$LL'(\sigma) = -\frac{2n}{\sigma} + \sum_{i=1}^{n} \frac{x_i^2}{\sigma^3} = -\frac{2n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{n} x_i^2 \tag{11}$$

Maximizing log likelihood:

$$-\frac{2n}{\sigma}+\frac{1}{\sigma^3}\sum_{i=1}^{n}x_i^2=0 \tag{12}$$

$$\frac{1}{\sigma^3}\sum_{i=1}^{n}x_i^2=\frac{2n}{\sigma} \tag{13}$$

$$\frac{1}{\sigma^2}\sum_{i=1}^{n}x_i^2=2n \tag{14}$$

$$\sigma^2=\frac{1}{2n}\sum_{i=1}^{n}x_i^2 \tag{15}$$

$$\sigma=\sqrt{\frac{1}{2n}\sum_{i=1}^{n}x_i^2} \tag{16}$$

3 Poisson Distribution

$$f(k)=\frac{\lambda^k e^{-\lambda}}{k!} \tag{17}$$

$$L(\lambda_1,\lambda_2)=\prod_{i=1}^{n}\frac{\lambda_1^{k_i}e^{-\lambda_1}}{k_i!}+\prod_{i=n+1}^{N}\frac{\lambda_2^{k_i}e^{-\lambda_2}}{k_i!} \tag{18}$$

$$=e^{-n\lambda_1}\prod_{i=1}^{n}\frac{\lambda_1^{k_i}}{k_i!}+e^{-n\lambda_2}\prod_{i=n+1}^{N}\frac{\lambda_2^{k_i}}{k_i!} \tag{19}$$

$$\begin{aligned}LL(\lambda_1,\lambda_2)=-n\lambda_1+\sum_{i=1}^{n}k_i\ln\lambda_1-\sum_{i=1}^{n}\ln k_i!\\ -n\lambda_2+\sum_{i=n+1}^{N}k_i\ln\lambda_2-\sum_{i=n+1}^{N}\ln k_i!\end{aligned} \tag{20}$$

Partial Derivatives:

$$LL'(\lambda_1) = -n + \sum_{i=1}^{n} \frac{k_i}{\lambda_1} \tag{21}$$

$$LL'(\lambda_2) = -n + \sum_{i=n+1}^{N} \frac{k_i}{\lambda_2} \tag{22}$$

Maximizing log likelihood:

$$-n + \sum_{i=1}^{n} \frac{k_i}{\lambda_1} = 0 \tag{23}$$

$$\lambda_1 = \frac{1}{n} \sum_{i=1}^{n} k_i \tag{24}$$

$$-n + \sum_{i=n+1}^{N} \frac{k_i}{\lambda_2} = 0 \tag{25}$$

$$\lambda_2 = \frac{1}{n} \sum_{i=n+1}^{N} k_i \tag{26}$$