ECE-302
PROJECT 3
EXTRA CREDIT

Derek Lee
Professor Keene

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Poisson Distribution

$$f(k) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{1}$$

$$L(\lambda_1, \lambda_2) = \prod_{i=1}^{n} \frac{\lambda_1^{k_i} e^{-\lambda_1}}{k_i!} + \prod_{i=n+1}^{N} \frac{\lambda_2^{k_i} e^{-\lambda_2}}{k_i!} \tag{2}$$

$$= e^{-n\lambda_1} \prod_{i=1}^{n} \frac{\lambda_1^{k_i}}{k_i!} + e^{-(N-n)\lambda_2} \prod_{i=n+1}^{N} \frac{\lambda_2^{k_i}}{k_i!} \tag{3}$$

$$\begin{aligned} LL(\lambda_1, \lambda_2) = -n\lambda_1 + \sum_{i=1}^{n} k_i \ln \lambda_1 - \sum_{i=1}^{n} \ln k_i! \\ -(N-n)\lambda_2 + \sum_{i=n+1}^{N} k_i \ln \lambda_2 - \sum_{i=n+1}^{N} \ln k_i! \end{aligned} \tag{4}$$

Partial Derivatives:

$$LL'(\lambda_1) = -n + \sum_{i=1}^{n} \frac{k_i}{\lambda_1} \tag{5}$$

$$LL'(\lambda_2) = -(N-n) + \sum_{i=n+1}^{N} \frac{k_i}{\lambda_2} \tag{6}$$

Maximizing log likelihood:

$$-n + \sum_{i=1}^{n} \frac{k_i}{\lambda_1} = 0 \tag{7}$$

$$\lambda_1 = \frac{1}{n} \sum_{i=1}^{n} k_i \tag{8}$$

$$-(N-n) + \sum_{i=n+1}^{N} \frac{k_i}{\lambda_2} = 0 \tag{9}$$

$$\lambda_2 = \frac{1}{N-n} \sum_{i=n+1}^{N} k_i \tag{10}$$