Universidad Industrial de Santander, Colombia
Numerical Analysis, 2020-1
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Lab 10. Differential Equations

Name: ______________________________

1 Instructions

- Make a **pdf** report including the solution to each point of the practice with name *Lab10_name_lastname.pdf*.
- Send the report and all created files in a rar or zip file with name *Lab10_name_lastname.rar* in the Moodle.
- You are allowed to use internet, notes, and .m files that you have created before.

2 Purposes

- To understand some numerical methods for solving differential equations.
- To apply some numerical methods for solving differential equations.
- To implement some numerical methods for solving differential equations in Matlab.

3 Practice

3.1 Understanding

Answer with your own words the following questions:

- (0.2 points) How to solve a differential equation with initial value by using Euler's method?

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- (0.2 points) How to solve a differential equation with initial value by using Heun's method?

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- (0.2 points) How to solve a differential equation with initial value by using forth-order Runge-Kutta method?

- (0.2 points) What applications do the differential equations have?

3.2 Applying

- (1.2 points) Solve the differential equation using the forth-order Runge-Kutta method (RK4).

$$y' = e^{-2t} - 2y \tag{1}$$

with $y(0) = \frac{1}{10}, y(t) = \frac{1}{10}e^{-2t} + te^{-2t}$.

  - Take $h = 0.2$ and take two steps calculating the values. Then, take $h = 0.1$ and take four steps calculating the values.
  - Compare the exact solution $y(0.4)$ with the two approximations calculated in the previous point.
  - Does the final global error of the approximations obtained in the previous points behave, as expected when h is divided between two?

- (1.2 points) Let $M(t)$ be the amount of a product that decreases with time t and the rate of decrease is proportional to the amount M.
  - Determine a differential equation that models the phenomena.
  - Solve the differential equation to determine the amount of material at time $t = 1$. Use the Euler method with a step of $h = 0.2$. Consider that $M(0) = 300$.
  - Solve the differential equation to determine the amount of material at time $t = 1$. Use the Heun method with a step of $h = 0.2$. Consider that $M(0) = 300$.

3.3 Implementing

- (0.6 points) Create a Matlab function called *my_euler_function_name_lastname()* using Euler's method to approximate the solution of the initial value $y' = f(t, y)$ with $y(a) = y_0$ over $[a, b]$ for $k = 0, 1, \cdots, M-1$. Make a script called *run_3a_name_lastname.m* in which you use the created function to solve the exercise in 3.2. For instance,

  fun = @ $XXXXXX$;
  $a = XX$;
  $b = XX$;
  $y_0 = XX$;
  $M = X$;
  E=my_euler_function_name_lastname(fun,a,b,y_0,M);

- (0.6 points) Create a Matlab function called *my_heun_function_name_lastname()* using Heun's method to approximate the solution of the initial value $y' = f(t, y)$ with $y(a) = y_0$ over $[a, b]$ for $k = 0, 1, \cdots, M-1$. Make a script called *run_3b_name_lastname.m* in which you use the created function to solve the exercise in 3.2. For instance,

  fun = @ $XXXXXX$;
  $a = XX$;
  $b = XX$;
  $y_0 = XX$;
  $M = X$;
  H=my_heun_function_name_lastname(fun,a,b,y_0,M);

- (0.6 points) Create a Matlab function called *my_RK4_function_name_lastname()* using RK4 method to approximate the solution of the initial value $y' = f(t, y)$ with $y(a) = y_0$ over $[a, b]$ by using the formula $y_{k+1} = y_k + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ for $k = 0, 1, \cdots, M-1$. Make a script called *run_3c_name_lastname.m* in which you use the created function to solve the exercise in 3.2. For instance,

  fun = @ $XXXXXX$;
  $a = XX$;
  $b = XX$;
  $y_0 = XX$;
  $M = X$;
  R=my_Rk4_function_name_lastname(fun,a,b,y_0,M);