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Undergraduate Studies

Numerical Analysis

Homework #1

Preliminaries

DATE: 26th May 2020 DUE : **31th May 2020**

1 INDICATIONS

1. You **must** fill this sheet with just the answer for each problem and return it to the professor. However, you have to present the process on a separate exam sheet.
2. Answers with no process are **not valid**.
3. Make all calculations with 5 decimal places of precision.

2 PRELIMINARIES

1. Let $\hat{p}$ be an approximation of p, which is given by the sum of the first four terms in the series $p = \sum_{i=0}^{\infty} cr^i$.

$$\hat{p} = 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} \tag{2.1}$$

   a) Calculate the absolute and relative error between p and $\hat{p}$.

$E =$ 2.96296296x10^-6	$R =$ 9.99999999x10^-7

   b) Determine the number of significant digits d in the approximation.

$d =$ 6

   c) Suppose that $\hat{p}$ is stored in a computer of 8 bits of precision: 1 bit for sign, 3 bits for the exponent, and 4 bits for the mantissa. Determine the point float number stored.

$bias =$ 2^(3-1) - 1=3

Sign	Exponent			Mantissa			
0	1	0	0	0	1	1	1

2. Given the Taylor polynomial expansions

$$e^h = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} + \boldsymbol{O}(h^5) \tag{2.2}$$

and

$$\sin h = h - \frac{h^3}{3!} + \frac{h^5}{5!} + \boldsymbol{O}(h^7) \tag{2.3}$$

Determine the order of approximation of

   a) $e^h + \sin h$

Order: O(h^5)

   b) $e^h \times \sin h$

Order: O(h^5)

3. Convert the following binary numbers to decimal form

   a) 10011001110_2

Decimal form: 1230

   b) 100110101.110101101_2

Decimal form: 309.587890625

4. Convert the following base 10 numbers to binary form

   a) 14573_{10}

   Binary form: 11100011101101

   b) 2135.7314453125_{10}

   Binary form: 100001010111.1011101101

5. Suppose that a computer has 32 bits of precision: 1 bit for sign, 8 bits for the exponent, and 23 bits for the mantissa.

   Determine the number $\hat{p}$ that will be stored for the operation $p = \left(\frac{9}{5} + \frac{9}{10}\right) + \frac{1}{6}$. *Take into account the propagation of the error.*

Sign	Exponent								Mantissa																					

6. Convert the following decimal numbers to point float of 32 bits:

   a) 1612.078125_{10}

   b) -981878.78_{10}

   c) 0.897424_{10}