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Undergraduate Studies

Numerical Analysis

Homework #1

Preliminaries

DATE: 26th May 2020 DUE : **31th May 2020**

1 INDICATIONS

1. You **must** fill this sheet with just the answer for each problem and return it to the professor. However, you have to present the process on a separate exam sheet.
2. Answers with no process are **not valid**.
3. Make all calculations with 5 decimal places of precision.

2 PRELIMINARIES

1. Let $\hat{p}$ be an approximation of p, which is given by the sum of the first four terms in the series $p = \sum_{i=0}^{\infty} cr^i$.

$$\hat{p} = 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} \tag{2.1}$$

   a) Calculate the absolute and relative error between p and $\hat{p}$.

$E =$ 2.96296296x10^-6	$R =$ 9.99999999x10^-7

   b) Determine the number of significant digits d in the approximation.

$d =$ 6

   c) Suppose that $\hat{p}$ is stored in a computer of 8 bits of precision: 1 bit for sign, 3 bits for the exponent, and 4 bits for the mantissa. Determine the point float number stored.

$bias =$ 2^(3-1) - 1=3

Sign	Exponent			Mantissa			
0	1	0	0	0	1	1	1

2. Given the Taylor polynomial expansions

$$e^h = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} + \boldsymbol{O}(h^5) \tag{2.2}$$

and

$$\sin h = h - \frac{h^3}{3!} + \frac{h^5}{5!} + \boldsymbol{O}(h^7) \tag{2.3}$$

Determine the order of approximation of

   a) $e^h + \sin h$

Order: O(h^5)

   b) $e^h \times \sin h$

Order: O(h^5)

3. Convert the following binary numbers to decimal form

   a) 10011001110_2

Decimal form: 1230

   b) 100110101.110101101_2

Decimal form: 309.587890625

4. Convert the following base 10 numbers to binary form

   a) 14573_{10}

   Binary form: 11100011101101

   b) 2135.7314453125_{10}

   Binary form: 100001010111.1011101101

5. Suppose that a computer has 32 bits of precision: 1 bit for sign, 8 bits for the exponent, and 23 bits for the mantissa.

   Determine the number $\hat{p}$ that will be stored for the operation $p = \left(\frac{9}{5} + \frac{9}{10}\right) + \frac{1}{6}$. *Take into account the propagation of the error.*

Sign	Exponent								Mantissa																					

6. Convert the following decimal numbers to point float of 32 bits:

   a) 1612.078125_{10}

   b) -981878.78_{10}

   c) 0.897424_{10}

DD MM AA

Taller 1.

1. Sea $p = \sum_{i=0}^{\infty} cr^i$ y $\hat{p} = 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$

$p = 2.962962$
$\hat{p} = 2.962962962962963$

$c = 2$
$r = 1/3$ $r = \frac{a_{n+1}}{a_n} = \frac{2}{3} \wedge .2$

a) $E = |p - \hat{p}| = 2.96296296 \times 10^{-6}$

$R = \frac{|p - \hat{p}|}{|p|} = 9.999999999 \times 10^{-7}$

b) $\left|\frac{p - \hat{p}}{p}\right| < \frac{10^{1-d}}{2}$

$9.999999999 \times 10^{-9} < \frac{10^{1-0}}{2} = 5$ ✓

$9.999999999 \times 10^{-9} < \frac{10^{1-3}}{2} = 0.005$ ✓

$9.999999999 \times 10^{-9} < \frac{10^{1-5}}{2} = 5 \times 10^{-5}$ ✓

$9.999999999 \times 10^{-9} < \frac{10^{1-7}}{2} = 5 \times 10^{-7}$ ✗

$9.999999999 \times 10^{-9} < \frac{10^{1-6}}{2} = 5 \times 10^{-6}$ ✓

c) $bias = 2^{3-1} - 1 = 3$
$sign = 0$
$exp = 1 + bias = 4 = 100$

$2 \lfloor 2$ -> 10
0 1

0.96296 x 2 = 1.92592 = 1
0.92592 x 2 = 1.85184 = 1
0.85184 x 2 = 1.70368 = 1
0.70368 x 2 = 1.40736 = 1
0.40736 x 2 = 0.81472 = 0

$10.11110 = 1.011110 \times 2^1$

mantissa

DD MM AA

2. $e^h = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} + O(h^5)$

$\sinh = h - \frac{h^3}{3!} + \frac{h^5}{5!} + O(h^7)$

a) $e^h + \sinh = 1 + 2h + \frac{h^2}{2!} + \frac{h^4}{4!} + \frac{h^5}{5!} + O(h^5) + O(h^7)$

$\therefore 1 + 2h + \frac{h^2}{2!} + \frac{h^4}{4!} + O(h^5) + O(h^7)$

$-O(h^5) + O(h^7) = O(h^5)$

$\therefore 1 + 2h + \frac{h^2}{2!} + \frac{h^4}{4!} + O(h^5) = \underline{O(h^5)}$

b) $e^h \times \sinh \left(1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} + O(h^5)\right) \times$

$\left(h - \frac{h^3}{3!} + \frac{h^5}{5!} + O(h^7)\right) =$

$\left(1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!}\right)\left(h - \frac{h^3}{3!} + \frac{h^5}{5!}\right) +$

$\left(1 + h + \frac{h^2}{2!} + \frac{h^3}{3!}\right) O(h^7) + \left(h - \frac{h^3}{3!} + \frac{h^5}{5!}\right) O(h^5) +$

$O(h^5) O(h^7) =$

$h + - \frac{h^3}{3!} + \frac{h^5}{5!} + h^2 + \frac{h^4}{3!} + \frac{h^6}{5!} + \frac{h^3}{2!} - \frac{h^5}{12} + \frac{h^7}{240} + \frac{h^4}{3!}$

$\frac{h^6}{36} + \frac{h^8}{720} + + \frac{h^5}{5!} + \frac{h^7}{144} + \frac{h^9}{2880} + O(h^7) + O(h^5) +$

$O(h^5) O(h^7) = \underline{O(h^5)}$

DD MM AA

3. a) $1001100111 10_2$

$1 \times 2^{10} + 0 \times 2^9 + 0 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0$

$1024 + 128 + 64 + 8 + 4 + 2 = 1230$

b) $100110101.11011011 01_2$

$1 \times 2^8 + 0 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 + 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$256 + 32 + 16 + 4 + 1 = 309$ Parte entera

Para la parte decimal

1 1 0 1 0 1 1 0 1

$2^{-1}\ 2^{-2}\ 2^{-3}\ 2^{-4}\ 2^{-5}\ 2^{-6}\ 2^{-7}\ 2^{-8}\ 2^{-9}$

$\frac{1}{2} + \frac{1}{16} + \frac{1}{64} + \frac{1}{128} + \frac{1}{512} = 0.587890625$

R/ 309.587890625

4. 14573_{10} a Bin

$14573 \div 2 = 1$
$7286 \div 2 = 0$
$3643 \div 2 = 1$
$1821 \div 2 = 1$
$910 \div 2 = 0$
$455 \div 2 = 1$
$227 \div 2 = 1$
$113 \div 2 = 1$
$56 \div 2 = 0$
$28 \div 2 = 0$
$14 \div 2 = 0$
$7 \div 2 = 1$
$3 \div 2 = 1$
$1 \div 2 = 1$

11100011101101_2

DD MM AA

b) 2135.7314453125,10 a B-n

2135 / 2 = 1
1067 / 2 = 1
533 / 2 = 1
266 / 2 = 0
133 / 2 = 1
66 / 2 = 0
33 / 2 = 1
16 / 2 = 0
8 / 2 = 0
4 / 2 = 0
2 / 2 = 0
1 / 2 = 1

10000101011 1 . 10 1110 1101

0.7314453125 x 2 = 1
0.4628906125 x 2 = 0
0.9257812125 x 2 = 1
0.8515625 x 2 = 1
0.703125 x 2 = 1
0.40625 x 2 = 0
0.8125 x 2 = 1
0.625 x 2 = 1
0.25 x 2 = 0
0.5 x 2 = 1

5. $P = \left(\frac{9}{5} + \frac{2}{10}\right) + \frac{1}{6}$

DD MM AA

6.a) 1612.078125$_{10}$

Paso 1.

1612 / 2 = 0
806 / 2 = 0
403 / 2 = 1
201 / 2 = 1
100 / 2 = 0
50 / 2 = 0
25 / 2 = 1
12 / 2 = 0
6 / 2 = 0
3 / 2 = 1
1 / 2 = 1

0.078125 x 2 = 0
0.15625 x 2 = 0
0.3125 x 2 = 0
0.625 x 2 = 1
0.25 x 2 = 0
0.5 x 2 = 1

$1100\,1001100.000101_2$

Paso 2.

$1100\,1001100.000101 = 1,1001001100000101 \times 2^{10}$

Paso 3.

$bias = 2^{\varepsilon-1} - 1 = 127$

Paso 4

Mantissa = 10010011000000101

Paso 5

exp = 10 + bias = 137

137 / 2 = 1
68 / 2 = 0
34 / 2 = 0
17 / 2 = 1
8 / 2 = 0
4 / 2 = 0
2 / 2 = 0
1 / 2 = 1

exp = 10001001

S	E	M
0	10001001	10010011000000101

DD MM AA

b) -981878.78_{10}

Paso 1

981878 / 2 = 0	0.78 x 2 = 1
490939 / 2 = 1	0.56 x 2 = 1
245469 / 2 = 1	0.12 x 2 = 0
122734 / 2 = 0	0.24 x 2 = 0
61367 / 2 = 1	0.48 x 2 = 0
30683 / 2 = 1	0.96 x 2 = 1
15341 / 2 = 1	0.92 x 2 = 1
7670 / 2 = 0	0.84 x 2 = 1
3835 / 2 = 1	0.68 x 2 = 1
1917 / 2 = 1	0.36 x 2 = 0
958 / 2 = 0	0.72 x 2 = 1
479 / 2 = 1	0.44 x 2 = 0
239 / 2 = 1	0.88 x 2 = 1
119 / 2 = 1	0.76 x 2 = 1
59 / 2 = 1	0.52 x 2 = 1
29 / 2 = 1	0.04 x 2 = 0
14 / 2 = 0	0.08 x 2 = 0
7 / 2 = 1	0.16 x 2 = 0
3 / 2 = 1	0.32 x 2 = 0
1 / 2 = 1	0.64 x 2 = 1
	0.28 x 2 = 0
	0.56 x 2 = 1
	0.12 x 2 = 0

$11101111101101110110.11000111101011100010$

Paso 2

$1.11011111011011101100011110101110001 0 \times 2^{19}$

Paso 3.

$bias = 2^{8-1} + 1 = 127$

Paso 4.

Mantissa: 11011111011011101100011110101110001 0

Paso 5:

$exp = 19 + bias = 146 = 10010010$

S	E	M
1	10010010	11011111011011101100011110101110001 0

DD MM AA

c) 0.897424_{10}

Paso 1

$0.897424 \times 2 = 1$
$0.794848 \times 2 = 1$
$0.589696 \times 2 = 1$
$0.179392 \times 2 = 0$
$0.358784 \times 2 = 0$
$0.717568 \times 2 = 1$
$0.435136 \times 2 = 0$
$0.870272 \times 2 = 1$
$0.740544 \times 2 = 1$
$0.481088 \times 2 = 0$
$0.962176 \times 2 = 1$
$0.924352 \times 2 = 1$
$0.848704 \times 2 = 1$
$0.697408 \times 2 = 1$
$0.394816 \times 2 = 0$
$0.789632 \times 2 = 1$
$0.579264 \times 2 = 1$
$0.158528 \times 2 = 0$
$0.317056 \times 2 = 0$
$0.634112 \times 2 = 1$
$0.268224 \times 2 = 0$

$0.536448 \times 2 = 1$
$0.072896 \times 2 = 0$
$0.145792 \times 2 = 0$
$0.291584 \times 2 = 0$
$0.583168 \times 2 = 1$
$0.166336 \times 2 = 0$
$0.332672 \times 2 = 0$
$0.665344 \times 2 = 1$
$0.330688 \times 2 = 0$
$0.661376 \times 2 = 1$
$0.322752 \times 2 = 0$

Paso 2.

$1.11001011011110100100010 \times 2^{1}$

Paso 3.

bias = 127

Paso 4.

Mantissa: 11001011011110100100010

Paso 5.

Exp = 1 + bias = 128 = 10000000

S	E	M
0	10000000	11001011011110100100010