Universidad Industrial de Santander, Colombia
Numerical Analysis, 2020
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Lab 1. Fixed Point Method

Name: ______________________________

1 Instructions

- Make a **pdf** report including the solution to each point of the practice with name *Lab1_name_lastname.pdf*.
- Send the report and all created files in a rar or zip file with name *Lab1_name_lastname.rar* in the Moodle.
- You are allowed to use internet, notes, and .m files that you have created before.

2 Purposes

- To understand the fixed point method
- To apply the fixed point method.
- To implement the fixed point method in Matlab.
- To interpret problems which can be solved by the fixed point method.
- To propose problems in which the fixed point method can be used.

3 Practice

3.1 Understanding

Answer with your own words the following questions:

- (0.2 points) What is a fixed point?

  Es un punto de una función donde su imagen es el mismo, es decir, si evaluo 0 y 1 en una función f(x), y la imagen da 0 y 1 respectivamente, se dice que es punto fijo.

- (0.2 points) How to calculate a fixed point?

  Como primer paso, debemos llevar la ecuación de f(x)=0 a g(x)=x, luego comprobar si dicha función es continua en un intervalo [a,b], luego debemos comprobar que el dominio de la función esta incluido en su imagen, es decir dentro del intervalo [a,b], luego se procede a tomar el valor p0 como el vaor iniciar, y se comienza a iterar.

- (0.2 points) What applications does the fixed point method have?

  Se utiliza para aproximar soluciones a ecuaciones. Sirve para encontrar ceros en funciones

- (0.2 points) What condition must be satisfied to guarantee that a function has a fixed point?

  Hay un punto fijo si se cumple que g(p)=p

- (0.2 points) What condition must be satisfied to guarantee that the fixed point is unique?

  Si la derivada de g(x) esta definidad dentro de(a,b), y además se cumple
  que k es menor que 1, y a su vez, esta es menos que g'(x), se dice que hay exactamente un punto fijo en [a,b]

3.2 Applying

(0.5 points) Use the fixed point iteration to find a fixed point for $g(x) = 1/x$ on $[0.5, 5.2]$. Do five iterations by hand. Choose the initial point.

Initial point p_o: 1

k	0	1	2	3	4
x	1	1	1	1	1
$g(x)$	1	1	1	1	1

3.3 Implementing

- (0.5 points) Create a Matlab function called *my_fixed_point* to find a fixed point of a function over a given range. The arguments of the function must be: the function to be evaluated $g(x)$ (as an inline function), the initial point of the range a, the final point of the range b, the initial iteration point p_0, and the desired number of iterations. For instance, the sentences to find the fixed point of the exercise in 3.1 should look like:

```
fun = @(x) 1./x;
a=0.5;
b=5.2;
p0=1;
Iter=5;
P=my_fixed_point(fun,a,b,p0,Iter);
```

- (0.5 points) Create a Matlab function called *visual_verification* to visually show if the function has a fixed point over a range, and if that fixed point is unique. The arguments of the function must be: the function to be evaluated $g(x)$ (as an inline function), the initial point of the range a, and the final point of the range b. For instance, the sentences to visually verify if the function of the exercise in 3.1 has an unique fixed point should look like:

```
fun = @(x) 1./x;
a=0.5;
b=5.2;
P=visual_verification(fun,a,b);
```

- (0.5 points) Use the created function *visual_verification* to visually verify if $g(x) = \frac{x^2}{3}$ over $[2.5, 4]$ has an unique fixed point, and use the created function *my_fixed_point* to find the fixed point of $g(x)$ with $p_0 = 3.5$ and 100 iterations.

3.4 Interpreting

The network bandwidth allocation problem is one of the central issues in modern communication networks. When a data rate is allocated to a source participating in a network, it derives a utility, which is modeled as a value of a concave function. The well known utility function of each source s is given by the following function, which satisfies the strict concavity, for all $x \in \mathbb{R}_+$,

$$u_s(x) = \omega_s \log x \tag{1}$$

where $\omega_s > 0$ is the weighted parameter for the source s, and x is the allocated data rate to the network. The share allocated data rate to each source must be regulated with a control mechanism to prevent network congestion. A network congestion is generated when the utility of any source in the network is equal to the allocated data rate in the network.

(1.0) Use the created functions to find the allocated data rate x which could generate a network congestion due to a source with $\omega_s = 4.8$, where $x \in [1, 1.5]$. Choose the initial point p_0.

3.5 Proposing

- (0.5 points) Propose an applicated problem in which the fixed point iteration can be used.
- (0.5 points) Solve the proposed problem using the created functions.