



**Undergraduate Studies** 

## Numerical Analysis

## Homework #1

Preliminaries

DATE: 26th May 2020 DUE: **31th May 2020** 

## 1 INDICATIONS

- 1. You **must** fill this sheet with just the answer for each problem and return it to the professor. However, you have to present the process on a separate exam sheet.
- 2. Answers with no process are not valid.
- 3. Make all calculations with 5 decimal places of precision.

## 2 PRELIMINARIES

1. Let  $\hat{p}$  be an approximation of p, which is given by the sum of the first four terms in the series  $p = \sum_{i=0}^{\infty} cr^i$ .

$$\hat{p} = 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} \tag{2.1}$$

a) Calculate the absolute and relative error between p and  $\hat{p}$ .

 $E = 2.96296296 \times 10^{-6}$   $R = 9.99999999 \times 10^{-7}$ 

b) Determine the number of significant digits  $\boldsymbol{d}$  in the approximation.

$$d = 6$$

c) Suppose that  $\hat{p}$  is stored in a computer of 8 bits of precision: 1 bit for sign, 3 bits for the exponent, and 4 bits for the mantissa. Determine the point float number stored.

$bias = 2^{\circ}(3)$	3-1) - 1=3						
Sign		Exponent			Man	tissa	
0	1	0	0	0	1	1	1

2. Given the Taylor polynomial expansions

$$e^{h} = 1 + h + \frac{h^{2}}{2!} + \frac{h^{3}}{3!} + \frac{h^{4}}{4!} + O(h^{5})$$
 (2.2)

and

$$\sin h = h - \frac{h^3}{3!} + \frac{h^5}{5!} + O(h^7)$$
 (2.3)

Determine the order of approximation of

a)  $e^h + \sin h$ 

Order: O(h^5)

b)  $e^h \times \sin h$ 

Order: O(h^5)

- 3. Convert the following binary numbers to decimal form
  - a) 10011001110<sub>2</sub>

Decimal form: 1230

b)  $100110101.110101101_2$ 

Decimal form: 309.587890625

- 4. Convert the following base  $10\ \mathrm{numbers}\ \mathrm{to}\ \mathrm{binary}\ \mathrm{form}$ 
  - a) 14573<sub>10</sub>

Binary form: 11100011 101101

b) 2135.7314453125<sub>10</sub>

Binary form: 100001010111.1011101101

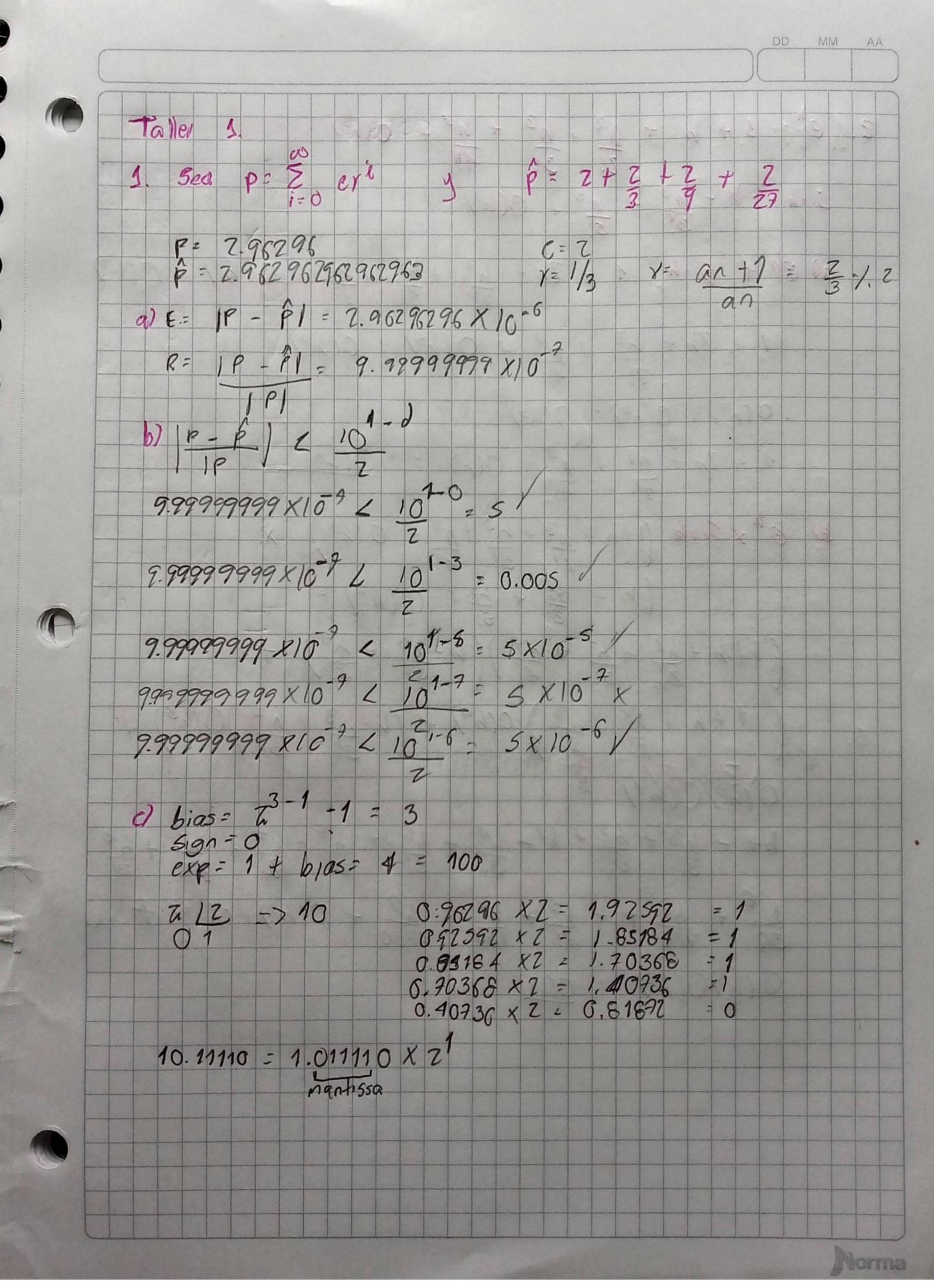
5. Suppose that a computer has 32 bits of precision: 1 bit for sign, 8 bits for the exponent, and 23 bits for the mantissa.

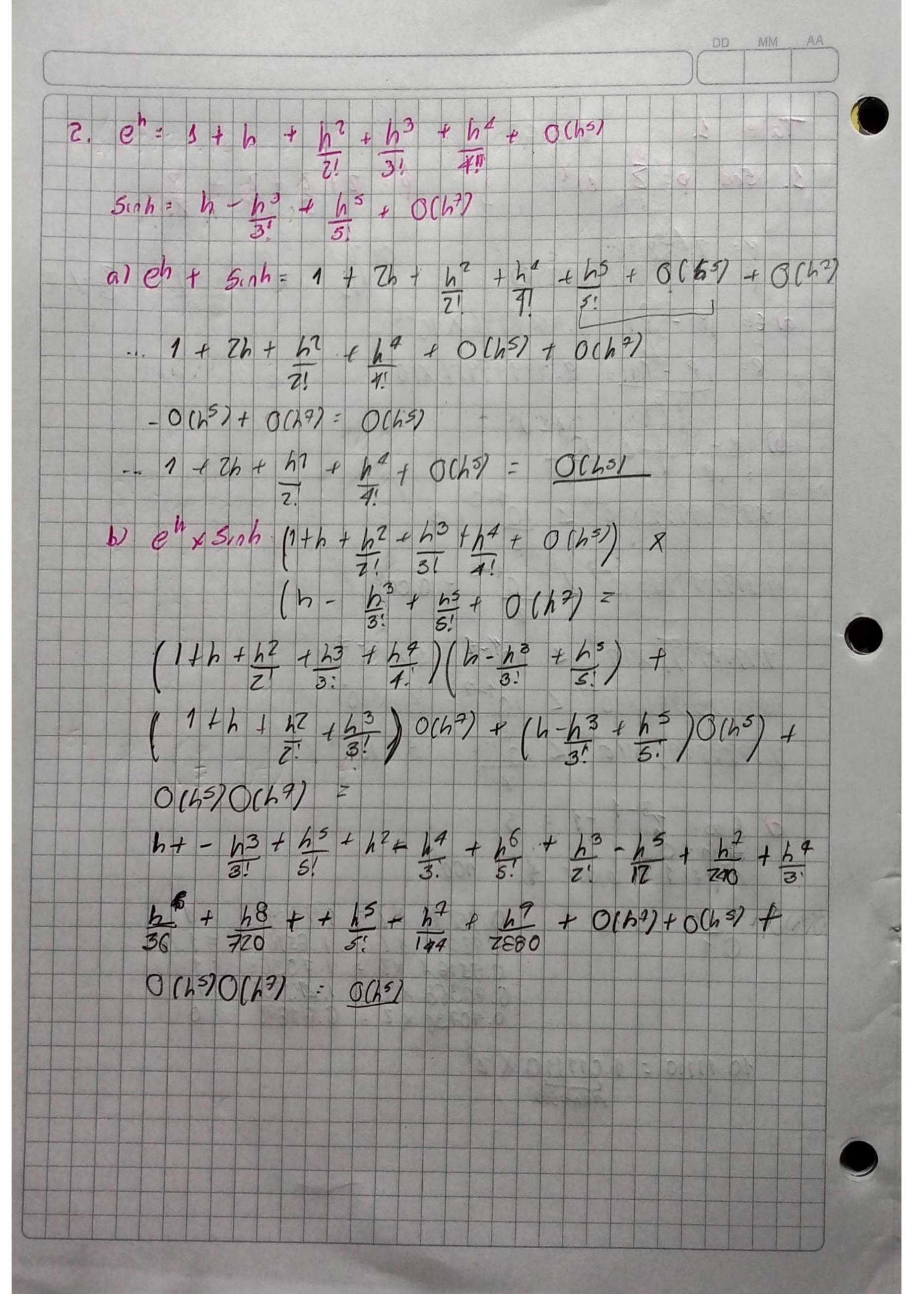
Determine the number  $\hat{p}$  that will be stored for the operation  $p=\left(\frac{9}{5}+\frac{9}{10}\right)+\frac{1}{6}$ . Take into account

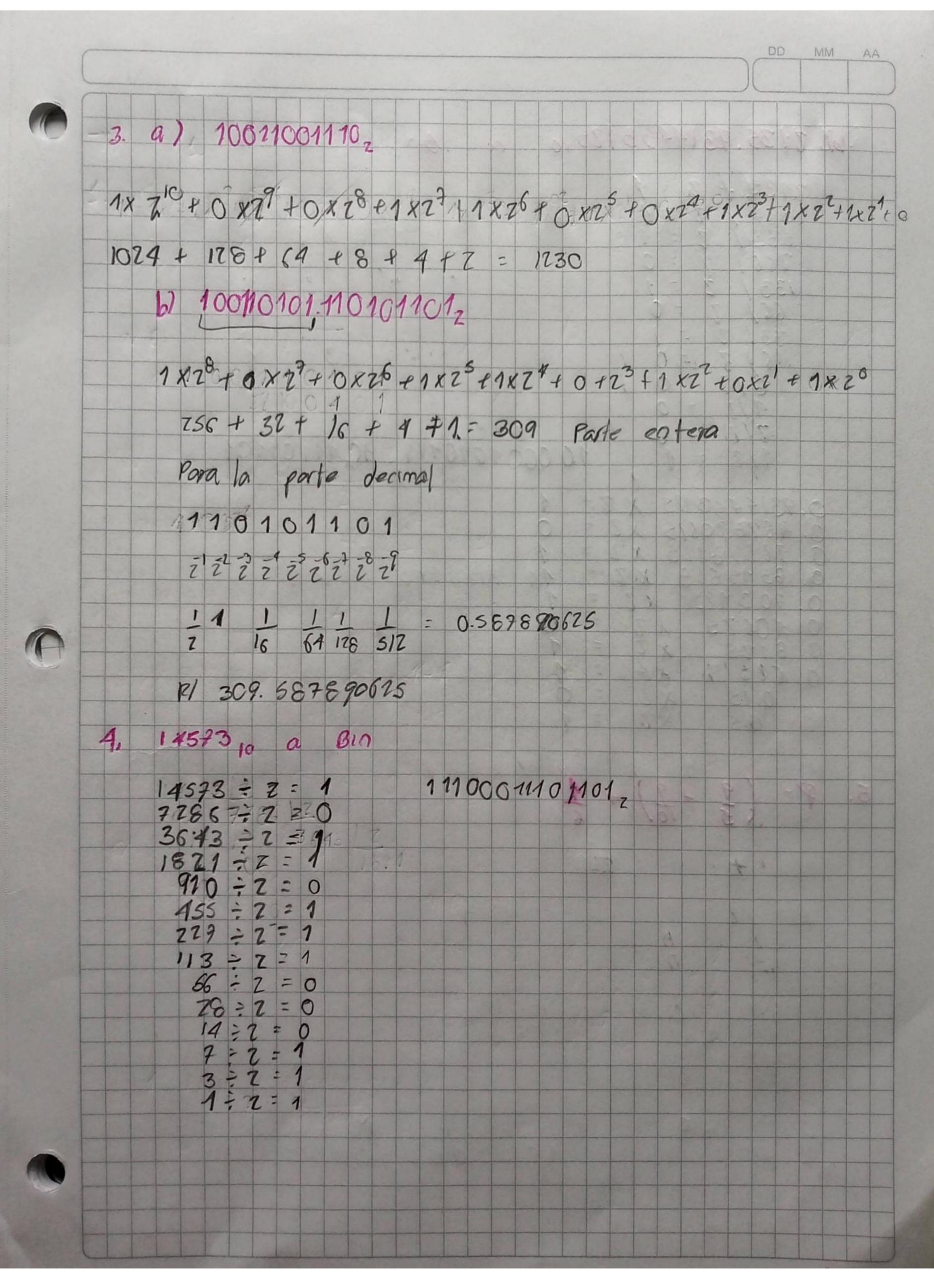
the propagation of the error.

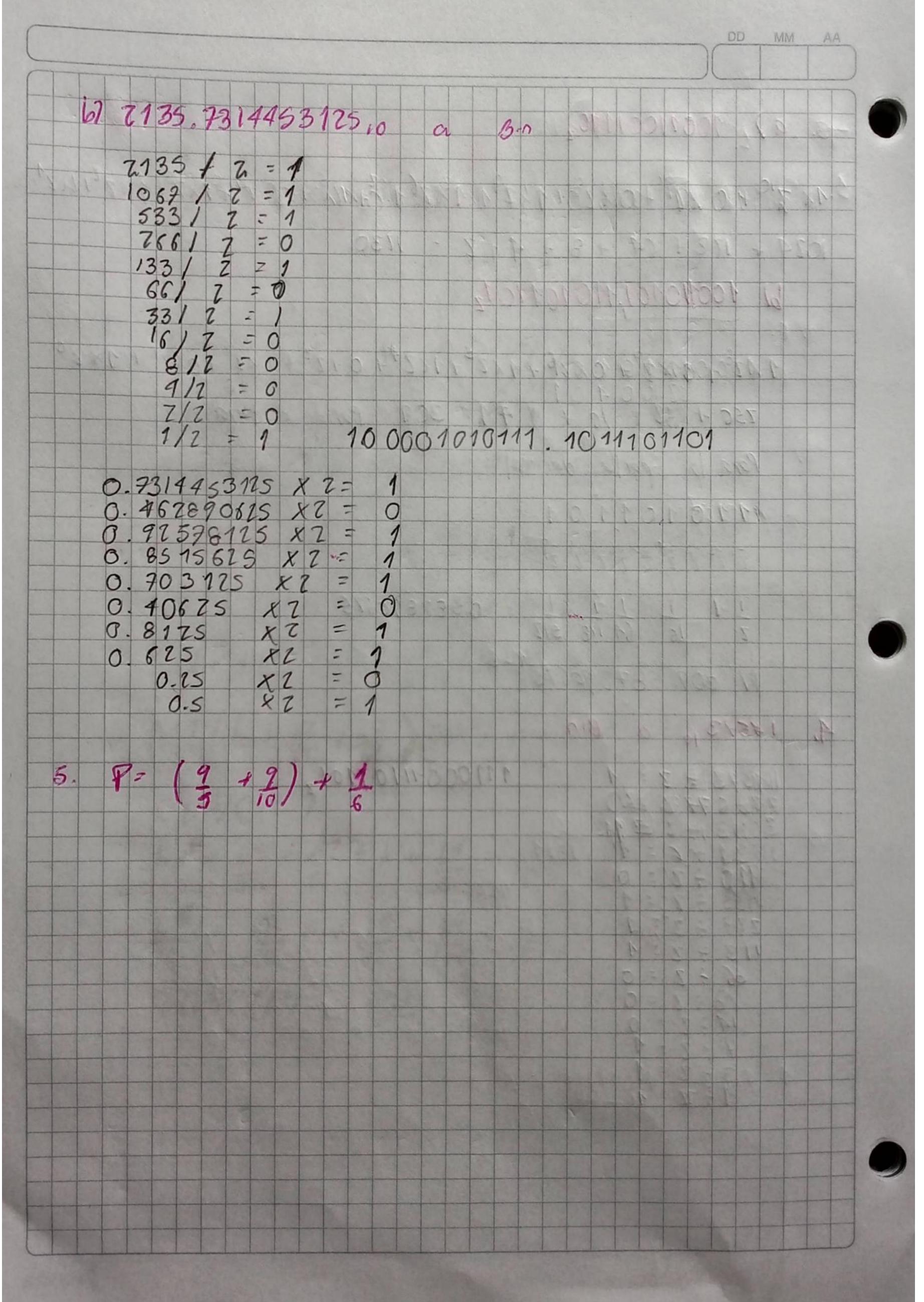
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- 6. Convert the following decimal numbers to point float of 32 bits:
  - a) 1612.078125<sub>10</sub>
  - b)  $-981878.78_{10}$
  - c)  $0.897424_{10}$









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