

Numerical Analysis

Homework #1

Preliminaries

DATE: 26th May 2020 DUE : 31th May 2020

1 INDICATIONS

- 1. You **must** fill this sheet with just the answer for each problem and return it to the professor. However, you have to present the process on a separate exam sheet.
- 2. Answers with no process are **not valid**.
- 3. Make all calculations with 5 decimal places of precision.

2 PRELIMINARIES

1. Let \hat{p} be an approximation of p , which is given by the sum of the first four terms in the series $p = \sum_{i=0}^{\infty} cr^i$.

$$\hat{p} = 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$$
 (2.1)

- a) Calculate the absolute and relative error between p and \hat{p} .

$E = 2.96296296 \times 10^{-6}$	$R = 9.99999999 \times 10^{-7}$
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- b) Determine the number of significant digits d in the approximation.

$d = 6$

- c) Suppose that \hat{p} is stored in a computer of 8 bits of precision: 1 bit for sign, 3 bits for the exponent, and 4 bits for the mantissa. Determine the point float number stored.

$bias = 2^{(3-1)} - 1 = 3$							
Sign	Exponent			Mantissa			
0	1	0	0	0	1	1	1

2. Given the Taylor polynomial expansions

$$e^h = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} + O(h^5)$$
 (2.2)

and

$$\sin h = h - \frac{h^3}{3!} + \frac{h^5}{5!} + O(h^7)$$
 (2.3)

Determine the order of approximation of

- a) $e^h + \sin h$

Order: $O(h^5)$

- b) $e^h \times \sin h$

Order: $O(h^5)$

3. Convert the following binary numbers to decimal form

- a) 10011001110_2

Decimal form: 1230

- b) 100110101.110101101_2

Decimal form: 309.587890625

4. Convert the following base 10 numbers to binary form

a) 14573_{10}

Binary form: 11100011101101

b) 2135.7314453125_{10}

Binary form: 100001010111.1011101101

5. Suppose that a computer has 32 bits of precision: 1 bit for sign, 8 bits for the exponent, and 23 bits for the mantissa.

Determine the number \hat{p} that will be stored for the operation $p = \left(\frac{9}{5} + \frac{9}{10}\right) + \frac{1}{6}$. Take into account the propagation of the error.

Sign	Exponent								Mantissa																						

6. Convert the following decimal numbers to point float of 32 bits:

a) 1612.078125_{10}

b) -981878.78_{10}

c) 0.897424_{10}

Taller 1.

1. Sea $p = \sum_{i=0}^{\infty} 2^{-i}$

$$y \quad \hat{p} = 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$$

$$P = 2.95296$$

$$\hat{P} = 2.952962962962963$$

$$C = 2$$

$$r = 1/3$$

$$r = \frac{an+1}{an} = \frac{2}{3} \cdot 2$$

a) $E = |P - \hat{P}| = 2.95296296 \times 10^{-6}$

$$R = \frac{|P - \hat{P}|}{|P|} = 9.99999999 \times 10^{-7}$$

b) $\left| \frac{P - \hat{P}}{P} \right| < \frac{10^{1-d}}{2}$

$$9.99999999 \times 10^{-7} < \frac{10^{1-0}}{2} = 5 \quad \checkmark$$

$$9.99999999 \times 10^{-7} < \frac{10^{1-3}}{2} = 0.005 \quad \checkmark$$

$$9.99999999 \times 10^{-7} < \frac{10^{1-6}}{2} = 5 \times 10^{-5} \quad \checkmark$$

$$9.99999999 \times 10^{-7} < \frac{10^{1-7}}{2} = 5 \times 10^{-7} \quad \checkmark$$

$$9.99999999 \times 10^{-7} < \frac{10^{1-8}}{2} = 5 \times 10^{-8} \quad \checkmark$$

c) $\text{bias} = 2^3 - 1 - 1 = 3$

$$\text{sign} = 0$$

$$\text{exp} = 1 + \text{bias} = 4 = 100$$

$$2 \frac{12}{01} \Rightarrow 10$$

$$0.95296 \times 2 = 1.92592 = 1$$

$$0.92592 \times 2 = 1.85184 = 1$$

$$0.85184 \times 2 = 1.70368 = 1$$

$$0.70368 \times 2 = 1.40736 = 1$$

$$0.40736 \times 2 = 0.81472 = 0$$

$$10.11110 = \underbrace{1.011110}_{\text{mantissa}} \times 2^1$$

$$2. e^h = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} + O(h^5)$$

$$\sinh h = h - \frac{h^3}{3!} + \frac{h^5}{5!} + O(h^7)$$

$$a) e^h + \sinh h = 1 + 2h + \frac{h^2}{2!} + \frac{h^4}{4!} + \frac{h^5}{5!} + O(h^5) + O(h^7)$$

$$\dots 1 + 2h + \frac{h^2}{2!} + \frac{h^4}{4!} + O(h^5) + O(h^7)$$

$$- O(h^5) + O(h^7) = O(h^5)$$

$$\dots 1 + 2h + \frac{h^2}{2!} + \frac{h^4}{4!} + O(h^5) = \underline{O(h^5)}$$

$$b) e^h \times \sinh h = \left(1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} + O(h^5) \right) \times$$

$$\left(h - \frac{h^3}{3!} + \frac{h^5}{5!} + O(h^7) \right) =$$

$$\left(1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \frac{h^4}{4!} \right) \left(h - \frac{h^3}{3!} + \frac{h^5}{5!} \right) +$$

$$\left(1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} \right) O(h^7) + \left(h - \frac{h^3}{3!} + \frac{h^5}{5!} \right) O(h^5) +$$

$$O(h^5) O(h^7) =$$

$$h + -\frac{h^3}{3!} + \frac{h^5}{5!} + h^2 + \frac{h^4}{3!} + \frac{h^6}{5!} + \frac{h^3}{2!} - \frac{h^5}{12} + \frac{h^7}{240} + \frac{h^4}{3!}$$

$$\frac{h^6}{36} + \frac{h^8}{720} + + \frac{h^5}{5!} + \frac{h^7}{144} + \frac{h^9}{2880} + O(h^7) + O(h^5) +$$

$$O(h^5) O(h^7) = \underline{O(h^5)}$$

3. a) 10011001110_2

$$1 \times 2^{10} + 0 \times 2^9 + 0 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0$$

$$1024 + 128 + 64 + 8 + 4 + 2 = 1230$$

b) 100110101.110101101_2

$$1 \times 2^8 + 0 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$256 + 32 + 16 + 4 + 1 = 309 \quad \text{Parte entera}$$

Para la parte decimal

1110101101

 $2^{-1} 2^{-2} 2^{-3} 2^{-4} 2^{-5} 2^{-6} 2^{-7} 2^{-8} 2^{-9}$

$$\frac{1}{2} + \frac{1}{16} + \frac{1}{64} + \frac{1}{128} + \frac{1}{512} = 0.567890625$$

R/ 309.567890625

4. 14573_{10} a Bin

$$14573 \div 2 = 7286$$

$$7286 \div 2 = 3643$$

$$3643 \div 2 = 1821$$

$$1821 \div 2 = 910$$

$$910 \div 2 = 455$$

$$455 \div 2 = 227$$

$$227 \div 2 = 113$$

$$113 \div 2 = 56$$

$$56 \div 2 = 28$$

$$28 \div 2 = 14$$

$$14 \div 2 = 7$$

$$7 \div 2 = 3$$

$$3 \div 2 = 1$$

$$1 \div 2 = 0$$

$$11100011101101_2$$

b) 2135.7314453125₁₀

a) B-n

$$2135 / 2 = 1$$

$$1067 / 2 = 1$$

$$533 / 2 = 1$$

$$266 / 2 = 0$$

$$133 / 2 = 1$$

$$66 / 2 = 0$$

$$33 / 2 = 1$$

$$16 / 2 = 0$$

$$8 / 2 = 0$$

$$4 / 2 = 0$$

$$2 / 2 = 0$$

$$1 / 2 = 1$$

10 0001010111 . 1011101101

$$0.7314453125 \times 2 = 1$$

$$0.462890625 \times 2 = 0$$

$$0.92578125 \times 2 = 1$$

$$0.8515625 \times 2 = 1$$

$$0.703125 \times 2 = 1$$

$$0.40625 \times 2 = 0$$

$$0.8125 \times 2 = 1$$

$$0.625 \times 2 = 1$$

$$0.25 \times 2 = 0$$

$$0.5 \times 2 = 1$$

$$5. \quad P = \left(\frac{9}{5} + \frac{2}{10} \right) + \frac{1}{6}$$

6.25612.078125₁₀

Paso 1.

$$\begin{array}{rcl} 1512 & / & 2 = 0 \\ 806 & / & 2 = 0 \\ 403 & / & 2 = 1 \\ 201 & / & 2 = 1 \\ 100 & / & 2 = 0 \\ 50 & / & 2 = 0 \\ 25 & / & 2 = 1 \\ 12 & / & 2 = 0 \\ 6 & / & 2 = 0 \\ 3 & / & 2 = 1 \\ 1 & / & 2 = 1 \end{array}$$

$$\begin{array}{rcl} 0.078125 & \times & 2 = 0 \\ 0.15625 & \times & 2 = 0 \\ 0.3125 & \times & 2 = 0 \\ 0.625 & \times & 2 = 1 \\ 0.25 & \times & 2 = 0 \\ 0.5 & \times & 2 = 1 \end{array}$$

11001001100.000101

Paso 2.

$$11001001100.000101 = 1.1001001100000101 \times 2^{10}$$

Paso 3.

$$\text{bias} = 2^{E-1} - 1 = 127$$

Paso 4

Mantissa = 1001001100000101

Paso 5.

$$\text{exp} = 10 + \text{bias} = 137$$

$$\begin{array}{rcl} 137 & / & 2 = 1 \\ 68 & / & 2 = 0 \\ 34 & / & 2 = 0 \\ 17 & / & 2 = 1 \\ 8 & / & 2 = 0 \\ 4 & / & 2 = 0 \\ 2 & / & 2 = 0 \\ 1 & / & 2 = 1 \end{array}$$

$$\text{exp} = 10001001$$

S E M

0 10001001 1001001100000101

c) 0.897424₁₀

Paso 1

$0.897424 \times 2 = 1$

$0.79485 \times 2 = 1$

$0.5897 \times 2 = 1$

$0.17939 \times 2 = 0$

$0.35878 \times 2 = 0$

$0.71757 \times 2 = 1$

$0.43514 \times 2 = 0$

$0.87027 \times 2 = 1$

$0.74054 \times 2 = 1$

$0.48109 \times 2 = 0$

$0.96218 \times 2 = 1$

$0.92435 \times 2 = 1$

$0.8487 \times 2 = 1$

$0.69741 \times 2 = 1$

$0.39482 \times 2 = 0$

$0.78963 \times 2 = 1$

$0.57928 \times 2 = 1$

$0.15853 \times 2 = 0$

$0.31706 \times 2 = 0$

$0.63411 \times 2 = 1$

$0.26822 \times 2 = 0$

$0.53645 \times 2 = 1$

$0.072896 \times 2 = 0$

$0.14579 \times 2 = 0$

$0.29158 \times 2 = 0$

$0.58318 \times 2 = 1$

$0.16634 \times 2 = 0$

$0.33267 \times 2 = 0$

$0.66534 \times 2 = 1$

$0.33067 \times 2 = 0$

$0.66138 \times 2 = 1$

$0.32275 \times 2 = 0$

Paso 2.

$1.110010110111011001010001001010 \times 2^1$

Paso 3.

$\text{bias} = 127$

Paso 4.

$\text{Mantissa: } 110010110111011001010001001010$

Paso 5.

$\text{Exp} = 1 + \text{bias} = 128 = 10000000$

S

E

M

0

$10000000 \ 110010110111011001010001001010$