Universidad Industrial de Santander, Colombia
Numerical Analysis, 2020
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Lab 2. Bisection Method

1 Instructions

- Make a **pdf** report including the solution to each point of the practice with name *Lab2_name_lastname.pdf*.
- Send the report and all created files in a rar or zip file with name *Lab2_name_lastname.rar* in the Moodle.
- You are allowed to use internet, notes, and .m files that you have created before.

2 Purposes

- To understand the bisection method.
- To apply the fixed point method.
- To implement the fixed point method in Matlab.
- To interpret problems which can be solved by the fixed point method.

3 Practice

3.1 Understanding

Answer with your own words the following questions:

- (0.2 points) Explain briefly what the bisection method is.

  Primero se eligen dos valores iniciales, que será el intervalo inicial, luego se realiza la primera aproximación a la raiz mediante la formula del punto medio, luego se hace una especie de busqueda binaria para saber en que subintervalo se encuentra la raiz (En que mitas esta o si ya encontramos la raiz), luego se calcula la nueva aproximación de la raiz, y calculamos el error relativo aproximado, según la condición, sabremos si repetir el proceso o no.

- (0.2 points) What condition do the initial points a and b have to satisfy to be used in the bisection method?

  Se debe garantizar que $f(a)f(b)<0$, de tal forma que la función cambie de signo.

- (0.2 points) Describe the decision step at each iteration for the bisection method.

Luego de calcular la primera aproximción de la raiz, se revisa en que intervalo esta la raiz, para ello analizamos si f(a)f(m)<0, entonces la raíz está en el subintervalo [a,m] y b=m, si f(a)f(m)>0, entonces la raíz está en el subintervalo [b, m] y a=m y Si f(a)f(m)=0, entonces aquí se encuentra la raíz.

3.2 Applying

(0.4 points) Use the Bisection method to find the root of the function $f(x) = \tan(x)^2 - x$ on $[1.8, 3]$. Choose two appropriate initial points a and b. Do five iterations by hand.

Initial point a: 1.8 Initial point b: 3

k	a_k	c_k	b_k	$f(c_k)$	$\frac{c_k - c_{k+1}}{c_k}$
0	1.8	2.4	3	-1.5609	0.0625
1	2.1	2.25	2.4	-0.7158	0.0333
2	2.1	2.175	2.25	-0.0766583	0.0172
3	2.1	2.1375	2.175	0.33217	0.0087
4	2.1375	2.1562	2.175	0.11877	0.0043
5	2.1562	2.1656	2.175	0.19236	0.00217

3.3 Implementing

- (0.7 points) Create a Matlab function called *my_finding_interval_name_lastname()* to find two adequate initial points $[a, b]$ given a function $f(x)$. Make a script called *run_2a_name_lastname.m* in which you use the created function with an example (start the search at 0).

- (0.8 points) Create a Matlab function called *my_bisection_function_name_lastname()* to find the root of a function. The arguments of the function must be: the function to be evaluated $f(x)$ (as an inline function), the initial points $[a, b]$, and the stopping criteria (the number of iterations or the relative error). Make a script called *run_2b_name_lastname.m* in which you use the created function to solve any example. For instance,

```
fun = @ XXXXXX;
a=XX;
b=XX;
Iter=X;
root=my_bisection_function_name_lastname(fun,a,b,Iter);
```

- (0.7 points) Given the function $f(x) = (x - 8)(x - 3)^2$ use your script to find each one of the roots. Compare the theoretical number of iterations N with respect to the practical number of iterations when the stopping criteria is established as $\epsilon = 1e^{-2}, 1e^{-4}, 1e^{-6}, 1e^{-8}, 1e^{-10}$. Plot the results where the x-label corresponds to the value of epsilon, and the y-label corresponds to the number of iterations for both cases: theoretical and practical. Conclude about the figure.

- (0.8 points) Create a Matlab function called *my_visual_bisection_function_name_lastname()* to visualize the behaviour of the Bisection method. The arguments of the function must be: the function to be evaluated $f(x)$ (as an inline function), the initial points $[a, b]$, and the number of iterations. Make a script called *run_2d_name_lastname.m* in which you use the created function to visualize the behavior of the bisection method when solving any example and conclude about the convergence of the method. For instance,

```
fun = @ XXXXXX;
a=XX;
b=XX;
Iter= XX
P=my_visual_bisection_function_name_lastname(fun,a,b,Iter);
```

3.4 Interpreting

When the mortality rate is neglected, the world population N can be simulated by a function that grows in proportion to the number of individuals existing at any time t. Also, if λ is the growth rate, φ is a coefficient that simulates the immigration, and N_0 is the population at the beginning of the simulation, the function to determine the quantity of individuals at any time t is given by

$$N(t) = N_0 e^{\lambda t} + \varphi \frac{e^{\lambda t} - 1}{\lambda} \tag{1}$$

Assume that in $t = 0$ the world population has 1500 individuals, also assume that the inmigiriation rate is of 475 individuals per year, and after one year ($t = 1$) the population has amounted to 2264 individuals.

- (0.2 points) Determine a nonlinear equation $f(\lambda) = 0$ to calculate the growth rate λ by finding its root. $f(\lambda) =$____

- (0.8 points) Make a script to find the root of the nonlinear equation $f(\lambda) = 0$ by using the created function in 3.3. Also plot the function between $\lambda = 0.01$ and $\lambda = 1$.