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Undergraduate Studies

Numerical Analysis

Homework #2

The solution of Nonlinear Equations $f(x) = 0$

DATE: 9th June 2020 DUE : **21th June 2020**

1 INDICATIONS

1. You **must** fill this sheet with just the answer for each problem and return it to the professor. However, you have to present the process on a separate exam sheet.
2. Answers with no process are **not valid**.
3. Make all calculations with 5 decimal places of precision.

2 THE SOLUTION OF NONLINEAR EQUATIONS

1. (1.0 point) The Van der Waals equation relates the density of fluids to the pressure P, volume v, and temperature T conditions. Thus, it is a thermodynamic equation of state given by,

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT, \tag{2.1}$$

where, a, b and R are constasts that depends on the gas.

If $P = 5$, $a = 0.245$, $b = 0.0266$, $R = 0.08206$ and $T = 350$,

a) Determine a nonlinear equation $f(v) = 0$ that allows to calculate the volume v by finding its root.

$f(v) =$

b) Use the Secant method to find the root of the nonlinear equation in literal a). Use $v_0 = 35$, $v_1 = 30$ and iterate until achieving a precision of $|v - v_{k-1}| < 1 \times 10^{-5}$ *Make all calculations with 5 decimal places*.

Iteration rule: $v_{k+1} =$

k	v_k	$f(v_k)$	$\|v_k - v_{k-1}\|$	$E_k = \|5.76234 - v_k\|$
0	35	146.15299	$- - - - -$	29.23766
1	30	121.15416	5	24.23766
2	5.76803	0.02845	24.23197	5.69e-3
3	5.76234	0.00003	0.00569	0
4	5.76234	0.00003	1e-05	0
5	5.76234	0.00003	0.0	0
6				
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c) The rate of convergence for the secant method is given by, $R \approx 1.618$. Thus, the relation between successive error terms is $E_{k+1} = A|E_k|^{1.618}$. Use information found in table of literal b), to calculate the value of A.

$A =$ 1.20629

2. (1.0 point) Let $f(x) = x^3 - 3x - 2$

a) Find the Newton-Rapshon formula $p_k = g(p_{k-1})$

Iteration rule: $p_k =$

b) Start with $p_0 = 2.1$ and find p_1, p_2, p_3, p_4 and p_5.

k	p_k	$f(p_k)$	$f'(p_k)$	$\lvert p_k - p_{k-1} \rvert$
0	2.1	0.96100	10.23000	$-----$
1	2.00606	0.05477	9.07284	0.09394
2	2.00002	0.00022	9.00029	0.00604
3	2.00000	0.00000	8.99999	2e-05
4	2.00000	0.00000	8.99999	0
5	2.00000	0.00000	8.99999	0

c) Is the sequence converging quadratically or linearly?

Answer: Quadratically

3. (1.0 point) The world population N can be simulated by a function that grows in proportion to the number of individuals in a given time t. This is called the *logistic function* and it follows the equation (2.2),

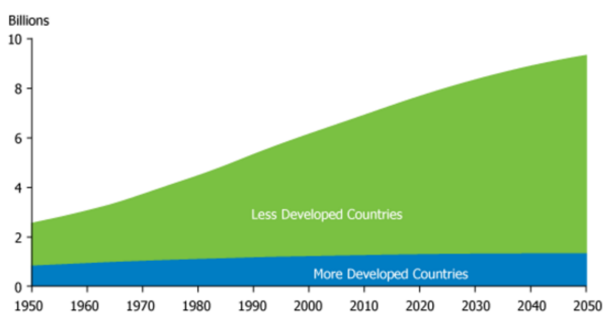

Figure 2.1: World population growth

$$N(t) = N_0 e^{\lambda t} + \mu \frac{e^{\lambda t} - 1}{\lambda}, \tag{2.2}$$

where, $N_0 = N(t_0)$ is the amount of individuals at the beginning of the simulation period, λ is the growth rate and μ simulates the immigration rate.

Suppose that $N(t_0) = 1000$, $\mu = 435$ and $N(t_1) = 1564$.

a) Determine a nonlinear equation $g(\lambda) = 0$ that allows to calculate the growth rate λ by finding its root.

$g(\lambda) =$

b) Use the Newton method to find the root of the nonlinear equation in literal a). Use $\lambda_0 = 0.5$ and iterate until achieving a precision of $|\lambda_k - \lambda_{k-1}| < 1 \times 10^{-6}$ *Make all calculations with 5 decimal places.*

Iteration rule: $\lambda_{k+1} =$

k	λ_k	$g(\lambda_k)$	$g'(\lambda_k)$	$\lvert\lambda_k - \lambda_{k-1}\rvert$
0	0.5	649.10878	1954.33377	— — — — —
1	0.16786	92.41474	1426.21639	0.33214
2	0.10306	2.77011	1341.60135	0.0648
3	0.10100	0.00270	1338.99151	0.00206
4	0.10100	0.00270	1338.99151	0.0
5				
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4. (1.0 point) Determine rigorously if each function has an unique fixed point.

a) $g(x) = \dfrac{x^2 - 8x + 25}{3}$, $x \in [1, 7]$

fixed point existence theorem verification:	unique fixed point theorem verification:
Sketch $g(x)$ and $y = x$	Sketch $g'(x)$

Use the starting value $p_0 = 3.15$ and compute p_1, p_2, p_3, p_4 and p_5.

p_0	p_1	p_2	p_3	p_4	p_5
3.15	3.24083	3.19211	3.21756	3.20407	3.21117
Do the sequence converge? Si.					

Use the starting value $p_0 = 3.25$ and compute p_1, p_2, p_3, p_4 and p_5.

p_0	p_1	p_2	p_3	p_4	p_5
3.25	3.18750	3.22005	3.20277	3.20277	3.20706
Do the sequence converge? Si.					

b) $g(x) = \dfrac{11x^3 - 141x^2 + 556x - 546}{30}$, $x \in [2, 7]$

fixed point existence theorem verification:	unique fixed point theorem verification:
Sketch $g(x)$ and $y = x$	Sketch $g'(x)$

Use the starting value $p_0 = 4.1$ and compute p_1, p_2, p_3, p_4 and p_5.

p_0	p_1	p_2	p_3	p_4	p_5
4.1	4.0507	4.12492	4.01282	4.18115	3.92266
Do the sequence converge? Si.					

Use the starting value $p_0 = 6.95$ and compute p_1, p_2, p_3, p_4 and p_5.

p_0	p_1	p_2	p_3	p_4	p_5
6.95	6.67579	5.12224	2.67727	4.76076	3.07028
Do the sequence converge? No.					

5. (1.0 point) Archimedes' principle indicates that the upward buoyant force that is exerted on a body immersed in a fluid, is equal to the weight of the fluid that the body displaces, and it acts in the upward direction at the centre of mass of the displaced fluid. Suppose that a sphere of radius $r = 15$ constructed with a material of density $\rho = 0.638$ is submerged in water to a depth d as shown in Fig. 2.2. According to Archimedes' principle, the mass of water displaced M_w is equal to the mass of the ball M_b, thus,

$$M_w = M_b \text{ where,} \tag{2.3}$$

$$M_w = \frac{\pi d^2 (3r - d)}{3}, \tag{2.4}$$

$$M_b = \frac{4\pi r^3 \rho}{3}. \tag{2.5}$$

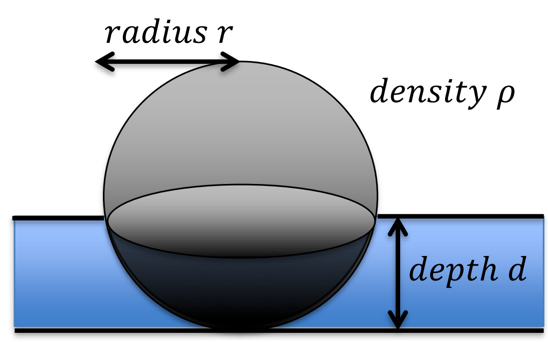

Figure 2.2: Sphere submerged in water

a) Use equations (2.3), (2.4) and (2.5) to find a nonlinear equation of the form $f(d) = 0$, that allows to determine the depth d.

$f(d) =$

b) Use the bisection method of Bolzano to calculate the roots of the nonlinear equation in literal a). Use $a_0 = 17.6$ and $b_0 = 18$ and iterate until achieving a precision of $|c_k - c_{k-1}| < 1 \times 10^{-3}$. *Make all calculations with 5 decimal places.*

k	a_k	b_k	c_k	$f(a_k)$	$f(b_k)$	$f(c_k)$	$\|c_k - c_{k-1}\|$
0	17.6	18	17.8	-175.576	85	-44.952	-------
1	17.8	18	17.9	-44.952	85	20.111	0.1
2	17.8	17.9	17.85	-44.952	20.111	-12.39912	0.05
3	17.85	17.9	17.875	-12.39912	20.111	3.86133	0.025
4	17.85	17.875	17.8625	-12.39912	3.86133	-4.26756	0.0125
5	17.8625	17.875	17.86875	-4.26756	3.86133	-0.20278	6.25e-3
6	17.86875	17.875	17.87187	-0.20278	3.86133	1.82936	3.12e-3
7	17.86875	17.871875	17.87031	-0.20278	1.82936	0.81331	1.56e-3
8	17.86875	17.87031	17.86953	-0.20278	0.81331	0.30527	7.8e-4
9							
10							

c) **Without doing iterations**, determine the required number of iterations N to guarantee that the midpoint c_N is an aproximation to a zero of the nonlinear equation in literal a) with an error less than $\delta = 1 \times 10^{-15}$, where $a_0 = 17$ and $b_0 = 18$.

$N =$ 48.82892