Taller 4 - Juan Camilo Mairi 6.

1. a) $F(x) = e^{-x^2/2}$ Para $N=4$ y $N=6$ $X_0 = 0$

$F(x) = e^{-x^2/2}$ → 0

$F'(x) = -xe^{-x^2/2}$

$F''(x) = x^2 e^{-x^2/2} - e^{-x^2/2}$ → -1

$F'''(x) = -x^3 e^{-x^2/2} + 3x e^{-x^2/2}$ → 0

$F^4(x) = x^4 e^{-x^2/2} - 6x^2 e^{-x^2/2} + 3e^{-x^2/2}$ → 3

$F^5(x) = -x^5 e^{-x^2/2} + 10x^3 e^{-x^2/2} - 15 x e^{-x^2/2}$ → 0

$F^6(x) = x^6 e^{-x^2/2} - 15x^4 e^{-x^2/2} + 45 x^2 e^{-x^2/2} - 15 e^{-x^2/2}$ → -15

$F^7(x) = -x^7 e^{-x^2/2} + 21 x^5 e^{-x^2/2} - 105 x^3 e^{-x^2/2} + 105 x e^{-x^2/2}$ → 0

$f(x) = F(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a) + \dots$

$\sum_{n=0}^{4} \frac{F^n(a)}{n!}(x-a)^n = 1 + 0 + \frac{(-1)(x)^2}{2!} 0 + \frac{3x^4}{4!} + 0 + \frac{(-15x^6}{6!}$

$P_4(x) = 1 - \frac{x^2}{2!} + \frac{3x^4}{4!} = 1 - \frac{x^2}{4} + \frac{x^4}{8}$

$P_6(x) = 1 - \frac{x^2}{2!} + \frac{3x^4}{4!} - \frac{15x^6}{6!} = 1 - \frac{x^2}{4} + \frac{x^4}{8} - \frac{x^6}{48}$

2.2.b) $f(x) = (3+x)^{1/2}$ Para $N=5$ sobre $X_0 = 3$ $f = 4^{1/2}$

$f'(x) = \frac{1}{2}(3+x)^{-1/2}$

$f''(x) = -\frac{1}{4}(3+x)^{-3/2}$

$f'''(x) = \frac{3}{8}(3+x)^{-5/2}$

$f^4(x) = -\frac{15}{16}(3+x)^{-7/2}$

$f^5(x) = \frac{105}{32}(3+x)^{-9/2}$

105
32

DD MM AA

$$\sum_{n=0}^{3} \frac{f^{(n)}(a)}{n!}(x-a)^n = 6^{1/2} + \frac{(x-3)}{2\times 6^{1/2}} - \frac{(x-3)^2}{8\times 6^{1/2}} + \frac{(x-3)^3}{16\times 6^{1/2}} - \frac{5(x-3)^4}{128\times 6^{1/2}} + \frac{7(x-3)^5}{256\times 6^{1/2}}$$

$$= \sqrt{6} + \frac{(x-3)}{2\sqrt{6}} - \frac{(x-3)^2}{8\sqrt{6}} + \frac{(x-3)^3}{16\sqrt{6}} - \frac{5(x-3)^4}{128\sqrt{6}} + \frac{7(x-3)^5}{256\sqrt{6}}$$

Para $f = 4$

Para $2 = 2.23504$
Para $-2 = 1.05458$

3. a) $f(x) = (x+1)^{1/2}$

K	x_k	$F(x_k)$
0	8	3
1	9	3.16227
2	10	3.31662
3	11	3.46410
4	12	3.600555

Primeras Diferencias Divididas

$$f(x_0,x_1) = \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{3.16227-3}{9-8} = 0.16227$$

$$f(x_1,x_2) = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{3.31662-3.16227}{10-9} = 0.15435$$

$$f(x_2,x_3) = \frac{f(x_3)-f(x_2)}{x_3-x_2} = \frac{3.46410-3.31662}{11-10} = 0.14748$$

$$f(x_3,x_4) = \frac{f(x_4)-f(x_3)}{x_4-x_3} = \frac{3.600555-3.46410}{12-11} = 0.14145$$

Segundas Diferencias Divididas

$$f(x_0,x_1,x_2) = \frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0} = \frac{0.15435-0.16227}{10-8} = -0.00396$$

$$f(x_1,x_2,x_3) = \frac{f(x_2,x_3)-f(x_1,x_2)}{x_3-x_1} = \frac{0.14748-0.15435}{11-9} = -0.00343$$

$$f(x_2,x_3,x_4) = \frac{f(x_3,x_4)-f(x_2,x_3)}{x_4-x_2} = \frac{0.14145-0.14748}{12-10} = -0.00302$$

Terceras Diferencias Divididas

$$f(x_0,x_1,x_2,x_3) = \frac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{x_3-x_0} = \frac{-0.00343-(-0.00396)}{11-8} = 0.00018$$

$$f(x_1,x_2,x_3,x_4) = \frac{f(x_2,x_3,x_4)-f(x_1,x_2,x_3)}{x_4-x_1} = \frac{-0.00302-(-0.00343)}{12-9} = 0.00014$$

Cuartas Diferencias Divididas

$$f(x_4,x_3,x_2,x_1,x_0) = \frac{f(x_4,x_3,x_2,x_1) - f(x_3,x_2,x_1,x_0)}{x_4 - x_0}$$

$$f(x_4,x_3,x_2,x_1,x_0) = \frac{0.00041 - 0.00018}{12 - 6} = 1 \times 10^{-5}$$

3	0	0	0	0
3.16227	0.16227	0	0	0
3.31662	0.15435	-0.00396	0	0
3.4641	0.14748	-0.00343	0.00018	0
3.60555	0.14145	-0.00302	0.00014	1×10^{-5}

b) $f(x) = \frac{7.8}{x^2}$

Primeras Diferencias Divididas

$$f(x_0,x_1) = \frac{0.15916 - 0.21666}{7 - 6} = -0.05748$$

k	x_k	$f(x_k)$
0	6	0.21666
1	7	0.15916
2	8	0.12187
3	9	0.09629
4	10	0.07800

$$f(x_1,x_2) = \frac{0.12187 - 0.15916}{8 - 7} = -0.03731$$

$$f(x_2,x_3) = \frac{0.09629 - 0.12187}{9 - 8} = -0.02558$$

$$f(x_3,x_4) = \frac{0.07800 - 0.09629}{10 - 9} = -0.01829$$

Segundas diferencias divididas

$$F(x_0,x_1,x_2) = \frac{-0.03731 - (-0.05748)}{8 - 6} = 0.01009$$

$$F(x_1,x_2,x_3) = \frac{-0.02558 - (-0.03731)}{9 - 7} = 0.00587$$

$$F(x_2,x_3,x_4) = \frac{-0.01829 - (-0.02558)}{10 - 8} = 0.00364$$

Terceras Diferencias Divididas

$$f(x_0,x_1,x_2,x_3) = \frac{0.00587 - 0.01009}{9-6} = -0.00141$$

$$f(x_1,x_2,x_3,x_4) = \frac{0.00364 - 0.00587}{10-7} = -0.00074$$

Cuartas Diferencias Divididas

$$f(x_0,x_1,x_2,x_3,x_4) = \frac{-0.00074 - (-0.00141)}{10-6} = 0.00017$$

$$\begin{bmatrix} 0.21660 & 0 & 0 & 0 & 0 \\ 0.15918 & -0.05742 & 0 & 0 & 0 \\ 0.12187 & -0.03731 & 0.01809 & 0 & 0 \\ 0.09629 & -0.02558 & 0.00587 & -0.00141 & 0 \\ 0.078 & -0.01829 & 0.00364 & -0.00074 & 0.00017 \end{bmatrix}$$

4. a) Calcular Polinomio de Interpolación

$$P(x) = f(x_0) + f(x_0,x_1)(x-x_0) + f(x_0,x_1,x_2)(x-x_0)(x-x_1) + f(x_0,x_1,x_2,x_3)(x-x_0)(x-x_1)(x-x_2) + f(x_0,x_1,x_2,x_3,x_4)(x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

$$P(x) = 3 + 0.16227(x-6) - 0.00396(x-6)(x-9) + 0.00018(x-6)(x-9)(x-10) - 0.00001(x-6)(x-9)(x-10)(x-11)$$

$$P_1(x) = 0.16227x + 1.70184$$

$$P_2(x) = -0.00396(x^2 - 17x + 72) + 0.16227x + 1.70184$$
$$= -0.00396x^2 + 0.06732x - 0.28512 + 0.16227x + 1.70184$$
$$= -0.00396x^2 + 0.22959x + 1.41672$$

$$P_3(x) = 0.00016(x^3 - 27x^2 + 242x - 1080) - 0.00396x^2 + 0.22959x + 1.70184$$

$$P_3(x) = 0.00016(x^3 - 27x^2 + 242x - 720) - 0.00396x^2 + 0.22959x + 1.70184$$
$$P_3(x) = 0.00016x^3 - 0.00432x^2 + 0.03872x - 0.1152 - 0.00396x^2 + 0.22959x + 1.70184$$
$$P_3(x) = 0.00016x^3 - 0.00828x^2 + 0.26831x + 1.58664$$

$P_0(x) = y_0 l_0(x) + y_1 l_1(x) =$

$P_1(x) = -382.7016(x-8.3)(x-8.6)(x-8.7) - 731.8766(x-8.1)(x-8.6)(x-8.7)$
$P_1(8.4) = 11.47727$

$P_2(x) = P_1(x) + y_2 l_2(x) = 8.84152 + -1233.676667(x-8.1)(x-8.3)(x-8.7)$
$P_2(8.4) = 22.58237$

$P_3(x) = P_2(x) + y_3 l_3(x) = 22.58237 + 989.2045(x-8.1)(x-8.3)(x-8.4)$
$P_3(9.0) = 17.09971$

b) $F(-1/3) = ?$

k	x	f(x)
0	-0.75	-0.071813
1	-0.5	-0.02975
2	-0.25	0.33494
3	0	1.101

$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$

$l_0(x) = \frac{(x+0.5)(x+0.25)(x-0)}{(-0.75+0.5)(-0.75+0.25)(-0.75+0)}$

$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$

$l_1(x) = \frac{(x+0.75)(x+0.25)(x)}{(-0.5+0.75)(-0.5+0.25)(-0.5+0)}$

$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$

$l_2(x) = \frac{(x+0.75)(x+0.5)(x-0)}{(-0.25+0.75)(-0.25+0.5)(-0.25+0)}$

$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(0+0.75)(0+0.5)(0+0.25)}$

$P_1(x) = y_0 l_0(x) + y_1 l_1(x) = x^3 + x^2 + 0.1875x$

$P_1(x) = 0.76587(x^3 + 0.75x^2 + 0.175x) + -0.792(x^3 + x^2 + 0.1875x)$
$P_1(-1/3) = -0.0562$

$P_2(x) = P_1(x) + -10.718(x^3 + 1.25x^2 + 0.375x)$
$P_2(-1/3) = 0.21216$

$P_3(x) = P_2(x) + 11.744(x+0.75)(x+0.5)(x+0.25)$
$P_3(x) = 0.17452$

c) F(0.25) = ?

k	x	f(x)
0	0.1	0.62045
1	0.2	-0.28397
2	0.3	0.006
3	0.4	0.24842
4		

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$l_0(x) = \frac{(x-0.2)(x-0.3)(x-0.4)}{(0.1-0.2)(0.1-0.3)(0.1-0.4)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x-0.1)(x-0.3)(x-0.4)}{(0.2-0.1)(0.2-0.3)(0.2-0.4)}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x-0.1)(x-0.2)(x-0.4)}{(0.3-0.1)(0.3-0.2)(0.3-0.4)}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x-0.1)(x-0.2)(x-0.3)}{(0.4-0.1)(0.4-0.2)(0.4-0.3)}$$

$$P_1(x) = -103.41667[(x-0.2)(x-0.3)(x-0.4)] + 141.985[(x-0.1)(x-0.3)(x-0.4)]$$
$$P_1(x) = -0.19851$$

$$P_2(x) = P_1(x) + -3(x-0.1)(x-0.2)(x-0.4)$$
$$P_2(x) = -0.19513$$

$$P_3(x) = P_2(x) + 41.40333(x-0.1)(x-0.2)(x-0.3)$$
$$P_3(x) = -0.21067$$

d) F(0.9) = ?

k	x	f(x)
0	0.6	-0.17694
1	0.7	0.01375
2	0.8	0.22363
3	1	0.65809

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$l_0(x) = \frac{(x-0.7)(x-0.8)(x-1)}{(0.6-0.7)(0.6-0.8)(0.6-1)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x-0.6)(x-0.8)(x-1)}{(0.7-0.6)(0.7-0.8)(0.7-1)}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x-0.6)(x-0.7)(x-1)}{(0.8-0.6)(0.8-0.7)(0.8-1)}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x-0.6)(x-0.7)(x-0.8)}{(1-0.6)(1-0.7)(1-0.8)}$$

$$P_1(x) = 22.1175[(x-0.7)(x-0.8)(x-1)] + 1[(x-0.6)(x-0.8)(x-1)]$$
$$P_1(0.75) = -4.3717$$

$P_2(x) = P_1(x) + -55.9075[(x-0.6)(x-0.4)(x-1)$
$P_2(0.5) = 1.63671$

$P_3(x) = 1.83679 + 27.92091[(x-0.6)(x-0.4)(x-0.3)]$
$P_3(0.5) = -0.53849$

6. $E_N(x) = \left| \frac{[X-X_0)(X-X_1)\ldots(X-X_N)|f^{(N+1)}}{(N+1)!} \right| c$

a) $f(8.4) = \quad c = 8.4$

$E_1(x) = \frac{(8.4-8.1)(8.4-8.3)(8.4-8.6)(8.4-8.7)\,F^{(N+1)}}{2}$

$E_1(x) = \frac{0.0018}{1}(2669.32x - 22720.11533)$

$E_1(8.4) = \frac{(0.0018)|-66.80731|}{2} \cdot 8.4 = 0.50521$

$E_2(x) = \frac{0.0018}{6}(F''')$

$E_2(x) = \frac{0.0018|-4905.74002|}{6} \cdot 8.4 = 11.85920$

$E_3(x) = \frac{0.0018 \cdot F^{4}}{24} = 0$

b) $F(-1/3) = ?$

$P_1'(x) = -0.07809x^2 - 0.43045x - 0.05275$
$P_1''(x) = -0.15618x - 0.43045$

$E_1(x) = \frac{|(-1/3+0.2)(-1/3+0.5)(-1/3+0.23)(-1/3+0)|-0.15618x - 0.43045|}{2} \cdot -1/3$

$E_1(x) = \frac{(0.00192)(-0.36299)-(-1/3)}{2} = 0.00073$

$P_2'(x) = -37.23233x^2 - 27.23025 - 4.07203$
$P_2(x)'' = -64.46466x - 27.23025$
$P_2(x)''' = -64.46466$

$E_2(x) = \frac{(0.00192)(-64.46466)(-1/3)}{6} = 0.00688$

$E_3(x) = 0$

c) $f(0.25) = ?$

$P_1(x)' = -736.2036\,1x^2 + 413.32601x - 53.66548$
$P_1(x)'' = -1472.72002x + 413.32605$

$E_1(x) = \frac{[(0.25-0.1)(0.25-0.3)(0.25-0.5)(0.25-0.1)] \cdot (-1472.72002x + 413.32605 \cdot 0.25}{2}$

$E_1(x) = \frac{0.28153}{2} = 1.59147$

$P_2(x)' = -745.20501x^2 + 413.32601x - 54.88546$
$P_2(x)'' = -1490.41002x + 413.32601$
$P_2(x)''' = -1490.41002$

$E_2(x) = \frac{0.28153(1490.41002)0.25}{6} = 17.48313$

$E_3(x) = 0$

d) $F(0.9) = ?$

$P_1(x)' = 80.10299x^2 - 132.58798x + 54.12871$
$P_1(x)'' = 160.20598x - 132.58798$

$E_1(x) = \frac{(0.9-0.6)(0.9-0.7)(0.6-0.8)(0.6-1)(160.20598x - 132.58798)0.9}{2}$

$E_1(x) = \frac{(-0.0006)(11.597)0.9}{2} = 0.00313$

$P_2(x)' = -87.60001x^2 + 124.58702x - 41.98219$
$P_2(x)'' = -175.21002x + 124.58702$
$P_2(x)''' = -175.21002$

$E_2(x) = \frac{(-0.0006)(-175.21002)\cdot 0.9)}{6}$

$E_2(x) = 0.15772$

6.a) $y = \frac{A}{x}$ g $y = \frac{B}{x^2}$

A

K	X	y
0	0.5	7.1
1	0.8	1.4
2	1.1	3.2
3	1.8	1.9
4	1.0	0.9

K	X^M	X^{2M}
0	2	4
1	1.25	1.5625
2	0.9091	0.8264
3	0.5556	0.3086
4	0.25	0.0625

$$A = \left(\sum_{K=1}^{N} X_K^M y_K\right) \Big/ \left(\sum_{K=1}^{N} X_K^{2M}\right)$$

$$A = \frac{14.2 + 5.32 + 2.709 + 1.0556 + 0.2750}{4 + 1.5625 + 0.8264 + 0.3086 + 0.0625}$$

$$A = \frac{23.6626}{6.5876} = 3.5933$$

$$E_2(A) = \left[\frac{1}{N}\sum_{K=1}^{N} 1\,r(X_K - y_K)^2\right]^{1/2}$$

$$E_2(A) = \left[\frac{7.0609 + 4.4124 + 3.2761 + 1.9633 + 0.8836}{5}\right]^{1/2}$$

$$E_2(A) = 0.043$$

B

K	X^M	X^{2M}
0	28.4	16
1	5.83	2.4414
2	2.646	0.6830
3	0.5864	0.0953
4	0.0583	0.0039

$$B = \frac{28.4 + 5.83 + 2.646 + 0.5864 + 0.0583}{16 + 2.4414 + 0.6830 + 0.0953 + 0.0039}$$

$$B = \frac{38.5620}{19.2197} = 2.0035$$

$$E_2(B) = \left[\frac{8.0240 + 3.1399 + 1.6516 + 0.6191 + 0.1254}{5}\right]^{1/2}$$

$$E_2(B) = 1.1895$$

b)

K	X	y
0	0.7	8.1
1	0.9	4.9
2	1.1	3.3
3	1.6	1.6
4	3	0.5

K	X^M	X^{2M}
0	11.5716	2.0408
1	6.4414	1.2346
2	3	0.8264
3	1	0.3906
4	0.1667	0.1111

$$E_2(X) = \frac{0.5933 + 5.1176 + 9.1830 + 7.6758 + 1.5}{5}$$

$$E_2(X) = 1.08119$$

$$E_2(X) = \frac{8.1137 + 9.7080 + 13.2855 + 1.5529 + 0.}{5}$$

$$E_2(X) = 0.0348.$$

K	X^M	X^{2M}
0	16.5306	4.1649
1	6.0494	1.5247
2	2.7273	0.6530
3	0.6250	0.1526
4	0.0556	0.0123

8.

x^2	XY	X	y	k	Fyk
36	31.8	-6	-5.3	0	-6.02
4	7	-2	-2.5	1	2.9
0	0	0	-1.7	2	-1.34
4	-0.4	2	-0.2	3	0.22
36	24	6	4	4	.334
80	62.9	0	-6.7		0.56721

$$A = \frac{(N * XY - X * y)}{(N * X^2 - X^{\wedge} 2)} = 0.78$$

$$B = \frac{(Y * X^2 - X * XY)}{(N * X^2 - X^{\wedge} 2} = -1.34$$

9. $f(x) = Ax^2 + Bx + C$

k	X_k	Y_k	X_k^2	X_k^3	X_k^4	$X_k y_k$	$X_k^2 y_k$
0	-2	2.8	4	-8	16	-5.6	11.2
1	-1	2.1	1	-1	1	-2.1	2.1
2	0	0.25	0	0	0	0	0
3	1	6	1	1	1	6	6
4	2	11.5	4	8	16	23	46
	0		10	0			

$$\left(\sum_{k=1}^{N} X_k^4\right) A + \left(\sum_{k=1}^{N} X_k^3\right) B + \left(\sum_{k=1}^{N} X^2{}_k\right) C = \sum_{k=1}^{N} y_k x^2{}_k$$

$$\left(\sum_{k=1}^{N} X_k^3\right) A + \left(\sum_{k=1}^{N} X^2{}_k\right) B + \left(\sum_{k=1}^{N} X_k\right) C = \sum_{k=1}^{N} y_k x_k$$

$$\left(\sum_{k=1}^{N} X^2{}_k\right) A + \left(\sum_{k=1}^{N} X_k\right) B + NC = \sum_{k=1}^{N} y_k$$

$34A + 10C = 65.3$

$10B = 21.3$ $\quad\rightarrow\quad C = \frac{21.3}{10} = 2.13$

$10A + 5C = 25.65 \quad (-1)$

$34A + 10C = 65.3$

$-20A + 10C = -51.3$

$14A = 14$

$A = 1$

$C = \frac{25.65 - 10}{5} = 3.13$

$F(x) = x^2 + 2.13x + 3.13$

K	T $F(x)_K - y_K$	T^2
0	2.87	8.2369
1	2	4
2	3.13	9.7969
3	6.26	39.1876
4	11.39	129.7321
		190.9465

0.14765

$\epsilon(k) = \left(\frac{190.9462}{5}\right)^{1/2}$

$\epsilon(k) = 0.14765$