

Lab 8. Numerical Differentiation

1 Instructions

- Make a **pdf** report including the solution to each point of the practice with name *Lab8_name_lastname.pdf*.
- Send the report and all created files in a rar or zip file with name *Lab8_name_lastname.rar* in the Moodle.
- You are allowed to use internet, notes, and .m files that you have created before.

2 Purposes

- To apply numerical differentiation.
- To implement numerical differentiation.

3 Practice

3.1 Applying

(2.5 points) Let $f(x) = \cos(x)$ and carry eight or nine decimal places.

- Use the equation (6) with $h = 0.05$ to approximate $f''(1)$
- Use the equation (6) with $h = 0.01$ to approximate $f''(1)$
- Use the equation (12) with $h = 0.1$ to approximate $f''(1)$
- What answer, (a), (b) or (c), is most accurate?

3.2 Implementing

(2.5 points) Use the Program 1 to approximate the derivatives of each of the following functions at the given value of x . Approximations should be accurate to 13 decimal places.

Note. It may be necessary to change the values of `max1` and the initial value of `h` in the program.

- $f(x) = 60x^{45} - 32x^{33} + 233x^5 - 47x^2 - 77; x = 1/\sqrt{3}$
- $f(x) = \tan\left(\cos\left(\frac{\sqrt{5} + \sin(x)}{1 + x^2}\right)\right); x = \frac{1 + \sqrt{5}}{3}$

4 Appendices

- Equation (6)

$$f''(x_0) = \frac{f_1 - 2f_0 + f_{-1}}{h^2} + E(f, h)$$

$$E(f, h) = \frac{4\varepsilon}{h^2} + \frac{h^2 f^{(4)}(c)}{12}$$

- Equation (12)

$$f''(x_0) = \frac{-f_2 + 16f_1 - 30f_0 + 16f_{f-1} - f_{-2}}{12h^2} + E(f, h)$$

$$E(f, h) = \frac{16\varepsilon}{3h^2} + \frac{h^4 f^{(6)}(c)}{90}$$

Listing 1: Program 1 (Differentiation using Limits)

```

function [L,n]=difflim(f,x,toler)

%Input - f is the function input as a string 'f'
%       - x is the differentiation point
%       - toler is the desired tolerance
%Output - L=[H' D' E']: H is the vector of step sizes
%         D is the vector of approximate derivatives
%         E is the vector of error bounds
%       - n is the coordinate of the "best approximation"

% NUMERICAL METHODS: MATLAB Programs
%(c) 1999 by John H. Mathews and Kurtis D. Fink
%To accompany the textbook:
%NUMERICAL METHODS Using MATLAB,
%by John H. Mathews and Kurtis D. Fink
%ISBN 0-13-270042-5, (c) 1999
%PRENTICE HALL, INC.
%Upper Saddle River, NJ 07458

max1=15;
h=1/10;
H(1)=h;
D(1)=(feval(f,x+h)-feval(f,x-h))/(2*h);
E(1)=0;
R(1)=0;

for n=1:2
    h=h/10;
    H(n+1)=h;
    D(n+1)=(feval(f,x+h)-feval(f,x-h))/(2*h);
    E(n+1)=abs(D(n+1)-D(n));
    R(n+1)=2*E(n+1)*(abs(D(n+1))+abs(D(n))+eps);
end

n=2;

while ((E(n)>E(n+1))&(R(n)>toler))&n<max1
    h=h/10;
    H(n+2)=h;
    D(n+2)=(feval(f,x+h)-feval(f,x-h))/(2*h);
    E(n+2)=abs(D(n+2)-D(n+1));
    R(n+2)=2*E(n+2)*(abs(D(n+2))+abs(D(n+1))+eps);
    n=n+1;
end

n=length(D)-1;
L=[H' D' E'];

```