Taller 3

1. $P_0 = 0$
$K = 1, 2, 3, 4, 5$

a) $2x + 8y - z = 11$

$5x - 12y + z = 10$

$-x + y + 14z = 3$

Paso 1: Despejar una variable de cada ecuación

$$x = \frac{11 + z - 8y}{2}$$

$$y = -\left[\frac{10 - z - 5x}{12}\right] \Rightarrow y = \frac{5x + z - 10}{12}$$

$$z = \frac{3 + x - y}{14}$$

Paso 2: $x^{(0)}, y^{(0)}, z^{(0)}$

Paso 3 $x^{(1)} = \frac{11 + z^{(0)} - 8y^{(0)}}{2} = \frac{11}{2}$

0

$$y^{(1)} = \frac{5\left(\frac{11}{2}\right) + 0 - 10}{12} = \frac{35}{24}$$

$$z^{(1)} = \frac{3 + x^0 - y^0}{14} = \frac{3 + \frac{11}{2} - \frac{35}{24}}{14} = \frac{169}{336}$$

Paso 4: Repetir paso 3 hasta llegar a la aproximación deseada.

1

$$x^{(2)} = \frac{11 + z^{(1)} - 8y^{(1)}}{2} = \frac{11 + \frac{169}{336} - 8\left(\frac{35}{24}\right)}{2} = \frac{-55}{672}$$

$$y^{(2)} = \frac{5x^{(2)} + z^{(1)} - 10}{12} = \frac{5\left[\frac{-55}{672}\right] + \frac{169}{336} - 10}{12} = \frac{-317}{384}$$

$$z^{(2)} = \frac{3 + x^{(2)} - y^{(2)}}{14} = \frac{3 + \left(\frac{-55}{672}\right) - \left(\frac{-317}{384}\right)}{14} = 0.2674053997$$

2

$$x^{(3)} = \frac{11 + z^{(2)} - 8y^{(2)}}{2} = 8.73579$$

$$y^{(3)} = \frac{5x^{(3)} + z^{(2)} - 10}{12} = 2.91219$$

$$z^{(3)} = \frac{3 + x^{(3)} - y^{(3)}}{14} = 0.64458$$

3

$$x^{(4)} = \frac{11 + z^{(3)} - 8y^{(3)}}{2} = -5.82651$$

$$y^{(4)} = \frac{5x^{(4)} + z^{(3)} - 10}{12} = -3.20733$$

$$z^{(4)} = \frac{3 + x^{(4)} - 8y^{(4)}}{14} = 0.02770$$

4

$$x^{(5)} = \frac{11 + z^{(4)} - 8y^{(4)}}{2} = 18.34293$$

$$y^{(5)} = \frac{5x^{(5)} + z^{(4)} - 10}{12} = 6.81182$$

$$z^{(5)} = \frac{3 + x^{(5)} - 8y^{(5)}}{14} = 1.03794$$

2.

$$x^{(6)} = \frac{11 + z^{(5)} - 8y^{(5)}}{2} = 21.22832$$

$$y^{(6)} = \frac{5x^{(6)} + z^{(5)} - 10}{12} = -9.59197$$

$$z^{(6)} = \frac{3 + 16x^{(6)} - 8y^{(6)}}{2} = -0.61686$$

2. a) $9A + 20B + 5C = 139'500\,000$

$30A + 150B + 30C = 894'000\,000$

$0.01A + 0.002B + 0.3C = 2'372\,800$

b) P→
$$\left[\begin{array}{ccc|c} 9 & 20 & 5 & 139\,500\,000 \\ 30 & 150 & 30 & 894\,000\,000 \\ 0.01 & 0.002 & 0.3 & 2'372\,800 \end{array}\right] \begin{array}{l} \\ (E_2 - \frac{10}{3}E_1) \to \\ (E_3 - \frac{1}{900}E_1) \to \end{array}$$

$$\begin{array}{r} \\ \text{Pivote} \to \\ m_{32} = \frac{-91}{375000} \end{array} \left[\begin{array}{ccc|c} 9 & 20 & 5 & 139\,500\,000 \\ 0 & 250/3 & 40/3 & 429\,000\,000 \\ 0 & -91/4500 & 53/180 & 2217800 \end{array}\right] \begin{array}{l} \\ \\ (E_3 + \frac{91}{375000}E_2) \to \end{array}$$

$$\left[\begin{array}{ccc|c} 9 & 20 & 5 & 139\,500\,000 \\ 0 & 250/3 & 40/3 & 429\,000\,000 \\ 0 & 0 & \frac{3721}{12500} & 2321904 \end{array}\right]$$

c) $C = \dfrac{2321904 \times 12500}{3721} = 7800000$

$B = \dfrac{3 * 429000000 - 40/3\,C}{250} = 3900000$

$A = \dfrac{139500000 - 20B - 5C}{9} = 2500000$

3. a) $15x - y + z = 12$

$2x + 8y - z = 11$

$-x + y + 4z = 3$

Paso 1: Despejar una variable de cada ecuación

$$x = \frac{12 + y - z}{15}$$

$$y = \frac{11 + z - 2x}{8}$$

$$z = \frac{3 + x - y}{4}$$

$x^{(0)} = (0,0,0)$

$$x^{(1)} = \frac{12 + y^0 - z^0}{15} = \frac{12}{15}$$

$$y^{(1)} = \frac{11 + z^0 - 2x^0}{8} = \frac{11}{8}$$

$$z^{(1)} = \frac{3 + x^0 - y^0}{4} = \frac{3}{4}$$

$$x^{(2)} = \frac{12 + y^{(1)} - z^{(1)}}{15} = 0.84167$$

$$y^{(2)} = \frac{11 + z^1 - 2x^1}{8} = 1.26875$$

$$z^{(2)} = \frac{3 + x^{(1)} - y^{(1)}}{4} = 0.60625$$

$$x^{(3)} = \frac{12 + y^{(2)} - z^{(2)}}{15} = 0.84419$$

$$y^{(3)} = \frac{11 + z^{(2)} - 2x^{(2)}}{8} = 1.24036$$

$$z^{(3)} = \frac{3 + x^{(2)} - y^{(2)}}{4} = 0.64323$$

$$x^{(4)} = \frac{12 + y^{(3)} - z^{(3)}}{15} = 0.83981$$

$$y^{(4)} = \frac{11 + z^{(3)} - 2x^{(3)}}{8} = 1.24436$$

$$z^{(4)} = \frac{3 + x^{(3)} - y^{(3)}}{4} = 0.65095$$

$$x^{(5)} = \frac{12 + y^{(4)} - z^{(4)}}{15} = 0.83956$$

$$y^{(5)} = \frac{11 + z^{(4)} - 2x^{(4)}}{8} = 1.24872$$

$$z^{(5)} = \frac{3 + x^{(4)} - y^{(4)}}{4} = 0.64886$$

4. a) Demostrar $X \cdot Y = 0$

$$\cos\theta = \frac{\vec{X} \cdot \vec{y}}{|\vec{X}||\vec{y}|}$$

Si $\vec{X} \cdot \vec{y}$ son ortogonales, entonces $\theta = \frac{\pi}{2}$

$$\cos\frac{\pi}{2} = \frac{\vec{X} \cdot \vec{y}}{|\vec{X}||\vec{y}|} \longrightarrow 0 = \frac{\vec{X} \cdot \vec{y}}{|\vec{X}||\vec{y}|} \longrightarrow 0 = \vec{X} \cdot \vec{y}$$

Si $\vec{X} \vec{y} \; 0$

$$\cos\theta = \frac{0}{|\vec{X}||\vec{y}|} \longrightarrow \cos\theta = 0 \qquad (0 \leq \theta \leq 180°)$$

$$\theta = \frac{\pi}{2}$$

b) Buscar dos vectores diferentes, y y z que sean ortogonales a $X = (4, -7, 5, 9)$

- $(4, -7, 5, 9)(a, b, c, d) = 0$

$4a - 7b + 5c + 9d = 0$

Si $a = 1$, $b = 2$ y $c = 3$

$4 - 14 + 15 + 9d = 0$

$9d = -5$

$d = -5/9$

$y = (1, 2, 3, 5/9)$ ✓

$\vec{X} \cdot \vec{y} = 4 \cdot 1 + (-7 \cdot 2) + 5 \cdot 3 + 9 \cdot \frac{5}{9} = 0$

Si $a = 1$, $b = 2$ y $d = 3$

$4 - 14 + 5c + 27 = 0$

$5c = 17$

$c = -\frac{17}{5}$

$z = (1, 2, -\frac{17}{3}, 3)$ ✓

$\vec{X} \cdot \vec{z} = 4 \cdot 1 + (-7 \cdot 2) + 5\left(-\frac{17}{3}\right) + 9 \cdot 3$

$\vec{X} \cdot \vec{z} = 0$

- $X = (6, 2, -3, -3) \longrightarrow (6, 2, -3, -3) \cdot (a, b, c, d) = 6a + 2b - 3c - 3d$

Si $a = 1$, $b = 2$ y $c = 3$

$6 + 4 - 9 - 3d = 0$

$-3d = -1$

$d = \frac{1}{3}$

$\vec{X} \cdot \vec{y} = 6 \cdot 1 + 2 \cdot 2 + (-3 \cdot 3) + \left(-3 \cdot \frac{1}{3}\right) = 0$

$y = (1, 2, 3, \frac{1}{3})$ ✓

Si $b=1$, $c=2$ y $d=3$ $\quad z=\left(\frac{13}{6}, 1, 2, 3\right)$ ✓

$-6a+2-6-9=0$

$6a=13$

$a=\frac{13}{6}$ $\quad \vec{x}\cdot\vec{z}=\left[6\cdot\frac{13}{6}\right]+2\cdot 1+(-3\cdot 2)+(-3\cdot 3)$

$\vec{x}\cdot\vec{z}=0$

5. a) $T+C+R+A=700$

$T+2C=0.3R \longrightarrow T+2C-0.3R=0$

$0.1T+0.3C+0.84R+0.4A=480$

$C+985=2R+0.35A$

$T+C+R+A=700$

$T+2C-0.3R=0$

$0.1T+0.3C+0.84R+0.4A=480$

$C-2R-0.35A=-985$

$$AX=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 2 & -0.3 & 0\\ 0.1 & 0.3 & 0.84 & 0.4\\ 0 & 1 & -2 & -0.35\end{bmatrix} \quad B=\begin{bmatrix}700\\ 0\\ 480\\ -985\end{bmatrix}$$

b)
$$\begin{bmatrix}l_{11} & 0 & 0 & 0\\ l_{21} & l_{22} & 0 & 0\\ l_{31} & l_{32} & l_{33} & 0\\ l_{41} & l_{42} & l_{43} & l_{44}\end{bmatrix}\begin{bmatrix}U_{11} & U_{12} & U_{13} & U_{14}\\ 0 & U_{22} & U_{23} & U_{24}\\ 0 & 0 & U_{33} & U_{34}\\ 0 & 0 & 0 & U_{44}\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 2 & -0.3 & 0\\ 0.1 & 0.3 & 0.84 & 0.4\\ 0 & 1 & -2 & -0.35\end{bmatrix}$$

$L_{11} U_{11} = 1 \longrightarrow L_{11} = 1$ y $U_{11} = 1$

$L_{11} U_{12} = 1 \longrightarrow U_{12} = 1$

$L_{11} U_{13} = 1 \longrightarrow U_{13} = 1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & U_{22} & U_{23} & U_{24} \\ 0 & 0 & U_{33} & U_{34} \\ 0 & 0 & 0 & U_{44} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -0.3 & 0 \\ 0.1 & 0.33 & 0.84 & 0.4 \\ 0 & 1 & -2 & -0.35 \end{bmatrix}$$

$L_{21} U_{11} = 1$; $L_{31} U_{11} = 0.1$ y $L_{41} U_{11} = 0$

$L_{21} = 1$ $\quad L_{31} = 0.1$ y $L_{41} = 0$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & L_{22} & 0 & 0 \\ 0.1 & L_{32} & L_{33} & 0 \\ 0 & L_{42} & L_{43} & L_{44} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & U_{22} & U_{23} & U_{24} \\ 0 & 0 & U_{33} & U_{34} \\ 0 & 0 & 0 & U_{44} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -0.3 & 0 \\ 0.1 & 0.3 & 0.84 & 0.4 \\ 0 & 1 & -2 & -0.35 \end{bmatrix}$$

Ahora, para a_{22}

$L_{21} U_{12} + L_{22} U_{22} = a_{22}$ $\quad$ si $L_{22} = 1$

$1 + 1\, U_{22} = 2 \longrightarrow U_{22} = 1$

$L_{31} U_{12} + L_{32} U_{22} = 0.3$

$0.1 + L_{32} = 0.3 \longrightarrow L_{32} = 0.2$

$L_{21} U_{13} + L_{22} U_{23} = -0.3$

$1 + U_{23} = -0.3$

$U_{23} = -0.3 - 1$

$U_{23} = -1.3$

$L_{21}U_{12} + L_{22}U_{22} = 1$

$1 + L_{42} = 1$

$L_{21}U_{14} + L_{22}U_{24} = 0$

$1 + U_{24} = 0$

$U_{24} = -1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0.1 & 0.2 & L_{33} & 0 \\ 0 & 1 & L_{43} & L_{44} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1.3 & -1 \\ 0 & 0 & U_{33} & U_{34} \\ 0 & 0 & 0 & U_{44} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -0.3 & 0 \\ 0.1 & 0.3 & 0.84 & 0.4 \\ 0 & 1 & -2 & -0.35 \end{bmatrix}$$

$L_{31}U_{13} + L_{32}U_{23} + L_{33}U_{33} = 0.84$ Si $L_{33} = 1$

$0.1 + 0.2(-1.3) + U_{33} = 0.84$

$U_{33} = 0.84 + 0.16 = 1$

$L_{31}U_{14} + L_{32}U_{24} + L_{33}U_{34} = 0.4$

$0.1 + -0.2 \quad U_{34} = 0.4$

$U_{34} = 0.5$

$L_{41}U_{13} + L_{42}U_{23} + L_{43}U = -2$

$0 \quad -1.3 + L_{43} = -2 \rightarrow L_{43} = -2 + 1.3 = -0.7$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0.1 & 0.2 & 1 & 0 \\ 0 & 1 & -0.7 & L_{44} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1.3 & -1 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & U_{44} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -0.3 & 0 \\ 0.1 & 0.3 & 0.84 & 0.4 \\ 0 & 1 & -2 & -0.35 \end{bmatrix}$$

$L_{41}U_{14} + L_{42}U_{24} + L_{43}U_{34} + L_{44}U_{44} = -0.35$ Si $L_{44} = 1$

$0 + -1 \quad -0.35 + L_{44} = -0.35$

$L_{44} = -0.35 + 0.35 + 1$

$L_{44} = 1$

Scanned by TapScanner

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0.1 & 0.2 & 1 & 0 \\ 0 & 1 & -0.7 & 1 \end{bmatrix} \overset{U}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1.3 & -1 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -0.3 & 0 \\ 0.1 & 0.3 & 0.84 & 0.4 \\ 0 & 1 & -2 & -0.35 \end{bmatrix}$$

c) $Ly = B$

$$\begin{aligned} T &= 700 \\ T + C &= 0 \\ 0.1T + 0.2C + R &= 480 \\ C - 0.7R + A &= -985 \end{aligned}$$

$T = 700$
$C = -700$

$0.1(700) + 0.2(-700) + R = 480$
$R = 480 - 70 + 140$
$R = 550$

$-700 - 0.7(550) + A = -985$
$A = -985 + 700 + 385$
$A = 100$

$y = \{700, -700, 550, 100\}$

$Ux = y$

$$\begin{aligned} T + C + R + A &= 700 \\ C - 1.3R - A &= -700 \\ R + 0.5A &= 550 \\ A &= 100 \end{aligned}$$

$A = 100$

$R = 550 - 50$
$R = 500$

$C - 1.3(500) - 100 = -700$
$C = -700 + 100 + 650$
$C = 50$

$T + 50 + 500 + 100 = 700$
$T = 50$

Scanned by TapScanner

6.

$$A = \begin{bmatrix} -5 & 6 & 10 & 12 \\ 15 & 3 & 28 & 1 \\ 10 & -12 & -20 & -24 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & -6 & 3 \\ -6 & 2 & 11 \\ 10 & -12 & -20 \\ 6 & 17 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 8 & 1 \\ 5 & 4 & 1 \\ 2 & 25 & 6 \end{bmatrix}$$

a) Determinar $AB+C \to AB+C$

$AB = 3 \times 3 \qquad A = 3 \times 4 \qquad B = 4 \times 3$

$C_{11} = A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31} + A_{14}B_{41} =$
$C_{11} = -5 \cdot 7 + 6 \cdot (-6) + 10 \cdot 10 + 12 \cdot 6 = 101$

$C_{12} = A_{11}B_{12} + A_{12}B_{22} + A_{13}B_{32} + A_{14}B_{42}$
$C_{12} = -5 \cdot (-6) + 6 \cdot 2 + 10 \cdot (-12) + 12 \cdot 17 = 132$

$C_{13} = A_{11}B_{13} + A_{12}B_{23} + A_{13}B_{33} + A_{14}B_{43} =$
$C_{13} = -5 \cdot 3 + 6 \cdot 11 + 10 \cdot (-20) + 12 \cdot 3 = -118$

$C_{21} = A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31} + A_{24}B_{41} =$
$C_{21} = 15 \cdot 7 + 3 \cdot (-6) + 28 \cdot 10 + 1 \cdot 6 = 373$

$C_{22} = A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32} + A_{24}B_{42} =$
$C_{22} = 15(-6) + 3 \cdot 2 + 28 \cdot (-12) + 1 \cdot 17 = -400$

$C_{23} = A_{21}B_{13} + A_{22}B_{23} + A_{23}B_{33} + A_{24}B_{43} =$
$C_{23} = 15 \cdot 3 + 3 \cdot 11 + 28 \cdot (-20) + 1 \cdot 3 = -474$

$C_{31} = A_{31}B_{11} + A_{32}B_{21} + A_{33}B_{31} + A_{34}B_{41} =$
$C_{31} = 10 \cdot 7 \quad (-12) \cdot (-6) \quad (-20) \cdot 10 \quad (-24) \cdot 6 = -202$

$C_{32} = A_{31}B_{12} + A_{32}B_{22} + A_{33}B_{32} + A_{34}B_{42}$
$C_{32} = 10(-6) \quad (-12)(2) \quad (-20)(-12) \quad (-24)\,17 = -264$

$C_{33} = A_{31}B_{13} + A_{32}B_{23} + A_{33}B_{33} + A_{34}B_{43}$
$C_{33} = 10 \cdot 3 + (-12)\,11 + (-20)(-20) + (-24)\,3 = 236$

$C_{33} = B_{31}A_{13} + B_{32}A_{23} + B_{33}A_{33}$
$C_{33} = 10 \cdot 10 + (-12)\,28 + (-20)(-20) = 164$

$C_{34} = B_{31}A_{14} + B_{32}A_{24} + B_{33}A_{34}$
$C_{34} = 10 \cdot 12 + (-12) \cdot 1 + (-20)(-24) = 588$

$C_{41} = B_{41}A_{11} + B_{42}A_{21} + B_{43}A_{31}$
$C_{41} = 6 \cdot (-5) + 17 \cdot 15 + 3 \cdot 10 = 255$

$C_{42} = B_{41}A_{12} + B_{42}A_{22} + B_{43}A_{32}$
$C_{42} = 6 \cdot 6 + 17 \cdot 3 + 3(-12) = 51$

$C_{43} = B_{41}A_{13} + B_{42}A_{23} + B_{43}A_{33}$
$C_{43} = 6 \cdot 10 + 17 \cdot 28 + 3(-20) = 476$

$C_{44} = B_{41}A_{14} + B_{42}A_{24} + B_{43}A_{34}$
$C_{44} = 6 \cdot 12 + 17 \cdot 1 + 3(-24) = 17$

$$BA = \begin{bmatrix} -95 & -12 & -158 & -6 \\ 170 & -162 & -224 & -334 \\ -480 & 264 & 164 & 528 \\ 255 & 51 & 476 & 17 \end{bmatrix}$$

c) $CA = 3 \times 4$ $\quad C = 3 \times 3 \quad = \quad A = 3 \times 4$

$D_{11} = C_{11}A_{11} + C_{12}A_{21} + C_{13}A_{31}$
$D_{11} = 14\,(-5) + 8 \cdot 15 + 1 \cdot 10 = 60$

$D_{12} = C_{11} \cdot A_{12} + C_{12}A_{22} + C_{13}A_{32}$
$D_{12} = 14\,(6) + 8 \cdot 3 + 1\,(-12) = 96$

$D_{13} = C_{11} \cdot A_{13} + C_{12}A_{23} + C_{13}A_{33}$
$D_{13} = 14 \cdot 10 + 8 \cdot 28 + 1\,(-20) = 344$

$D_{14} = C_{11} \cdot A_{14} + C_{12}A_{24} + C_{13}A_{34}$
$D_{14} = 14 \cdot 12 + 8 \cdot 1 + 1\,(-24) = 152$

$D_{21} = C_{21}A_{11} + C_{22}A_{21} + C_{23}A_{31}$
$D_{21} = 5\,(-5) + 4 \cdot 15 + 1 \cdot 10 = 45$

$D_{22} = C_{21}A_{12} + C_{22}A_{22} + C_{23}A_{32}$
$D_{22} = 5 \cdot 6 + 4 \cdot 3 + 1\,(-12) = 30$

$D_{23} = C_{21} A_{13} + C_{22} A_{23} + C_{23} A_{33}$
$D_{23} = 5 \cdot 10 + 4 \cdot 28 + 1 \cdot 10 = 142$

$D_{24} = C_{21} A_{14} + C_{22} A_{24} + C_{23} A_{34}$
$D_{24} = 5 \cdot 12 + 4 \cdot 1 + 1 \cdot 12 = 50$

$D_{31} = C_{31} A_{11} + C_{32} A_{21} + C_{33} A_{31}$
$D_{31} = 2 \cdot -5 + 25 \cdot 15 + 6 \cdot 10 = 425$

$D_{32} = C_{31} A_{12} + C_{32} A_{22} + C_{33} A_{32}$
$D_{32} = 2 \cdot 6 + 23 \cdot 3 + 6 \cdot -12 = 15$

$D_{33} = C_{31} A_{13} + C_{32} A_{23} + C_{33} A_{33}$
$D_{33} = 2 \cdot 10 + 25 \cdot 28 + 6 \cdot -20 = 600$

$D_{34} = C_{31} A_{14} + C_{32} A_{24} + C_{33} A_{34}$
$D_{34} = 2 \cdot 12 + 25 \cdot 1 + 6 \cdot -24 = -95$

$$CA = \begin{bmatrix} 60 & 96 & 344 & 152 \\ 15 & 30 & 142 & 40 \\ 425 & 15 & 600 & -95 \end{bmatrix}$$

c) Tanto A como B no cumplen que m y n sean iguales por lo tanto no es posible hallar determinante.

Determinante de C

$$C = \begin{bmatrix} 14 & 8 & 1 \\ 5 & 4 & 1 \\ 2 & 25 & 6 \end{bmatrix}$$

$$14 \det \begin{bmatrix} 4 & 1 \\ 25 & 6 \end{bmatrix} - 8 \det \begin{bmatrix} 5 & 1 \\ 2 & 6 \end{bmatrix} + 1 \det \begin{bmatrix} 5 & 4 \\ 2 & 25 \end{bmatrix}$$

$$14(4 \cdot 6 - 25 \cdot 1) - 8(5 \cdot 6 - 2) + (5 \cdot 25 - 2 \cdot 4) = -121$$

Scanned by TapScanner

Universidad Industrial de Santander
Numerical Analysis
Department of Computer Science
PhD. Henry Arguello Fuentes

Facultad de Ingenierías Físico Mecánicas | Universidad Industrial de Santander

Undergraduate Students

Homework #3

Solution of Linear Systems $AX = B$

DATE: 16th June 2020 DUE : **5th July 2020**

Name: Juan Camilo Marin Garcia **Scholar ID**: 2172969

1 INDICATIONS

- Write down the solution process for all problems in this assignment sheet.
- Answers with no process are **not valid**.
- Make all calculations with 5 decimal places of precision.

2 SOLUTION OF LINEAR SYSTEMS $AX = B$

1. (0.8 points) For the following linear system, start with $\mathbf{P_0} = \mathbf{0}$ and use Gauss-Seidel iteration to find $\mathbf{P}_k$ for $k = 1, 2, 3, 4, 5$. Will Gauss-Seidel iteration converge to the solution?

a)

$$\begin{array}{rcrcrcr} 2x & + & 8y & - & z & = & 11 \\ 5x & - & 12y & + & z & = & 10 \\ -x & + & y & + & 14z & = & 3 \end{array}$$

Process:

$\mathbf{P_0} =$	$\mathbf{P_1} =$	$\mathbf{P_2} =$	$\mathbf{P_3} =$	$\mathbf{P_4} =$	$\mathbf{P_5} =$

2. (0.8 points) Suppose that three computers, A, B and C, are working in parallel in three different tasks, T_1, T_2 and T_3. Table 1 shows the consumed time per task per computer and the total of instructions required per task.

Table 1. Time consuming per task per computer

Task	A	B	C	**Instructions**
T_1	$9[s]$	$20[s]$	$5[s]$	139'500,000
T_2	$30[s]$	$150[s]$	$30[s]$	894'000,000
T_3	$0.01[s]$	$0.002[s]$	$0.3[s]$	2'372,800

Distribution of instructions per task per computer. Example: the $139'500,000$ instructions required by T_1 were distributed such that, computer A spend $9[s]$, computer B spend $20[s]$, and, computer C spend $5[s]$ processing the asigned instructions for T_1.

a) Determine a linear system of equations $\mathbf{AX} = \mathbf{B}$, such that, it allows to find the processor speed, V_A, V_B and V_C, of computers A, B and C respectively.

Process and Answer:

b) Determine an equivalent upper-triangular system $\mathbf{UX} = \mathbf{Y}$ for the linear system of equations $\mathbf{AX} = \mathbf{B}$ found in literal a).

Process and Answer:

c) Use backsubstitution method over the upper-triangular system $\mathbf{UX} = \mathbf{Y}$ found in literal b) to determine V_A, V_B and V_C.

Process:

Answer:		
$V_A =$	$V_B =$	$V_C =$

3. (0.8 points) For the following linear system, start with $\mathbf{P_0} = \mathbf{0}$ and use Jacobi iteration to find $\mathbf{P}_k$ for $k = 1, 2, 3, 4, 5$. Will Jacobi iteration converge to the solution?

a)

$$\begin{array}{rcrcrcr} 15x & - & y & + & z & = & 12 \\ 2x & + & 8y & - & z & = & 11 \\ -x & + & y & + & 4z & = & 3 \end{array}$$

Process:

$\mathbf{P_0} =$	$\mathbf{P_1} =$	$\mathbf{P_2} =$	$\mathbf{P_3} =$	$\mathbf{P_4} =$	$\mathbf{P_5} =$

4. (0.8 points) The vectors $\mathbf{X} \in \mathbb{R}^n$ and $\mathbf{Y} \in \mathbb{R}^n$ are said to be orthogonal if the angle between them is $\frac{\pi}{2}$.

a) Prove that $\mathbf{X}$ and $\mathbf{Y}$ are orthogonal **if and only if** $\mathbf{X} \cdot \mathbf{Y} = 0$.

Process and Answer:

b) Find two different vectors, $\mathbf{Y}$ and $\mathbf{Z}$, that are orthogonal to

i. $\mathbf{X} = (4, -7, 5, 9)$

Process:	**Process:**
Answer: $\mathbf{Y} =$	**Answer:** $\mathbf{Z} =$

ii. $\mathbf{X} = (6, 2, -3, -3)$

Process:	**Process:**
Answer: $\mathbf{Y} =$	**Answer:** $\mathbf{Z} =$

5. (0.8 points) In a movie store there are movies in 4 different categories: Thriller, Comedy, Romance and Action. The total amount of movies in the store is 700. Besides, the sum betweeen the amount of Thriller movies and the double of the amount of Comedy movies is equal to the 30% of Romance movies. Also you know that, the 10% of Thriller movies plus the 30% of Comedy movies, plus the 84% of Romance movies, plus the 40% of Action movies is equal to 480. Finally you know that the amount of Comedy movies plus 985 is equal to the double amount of Romance movies plus the 35% of Action movies.

a) Determine a linear system of equations $\mathbf{AX} = \mathbf{B}$ with the given information.

Process and Answer:

b) Determine the $\mathbf{LU}$ factorization for the matrix $\mathbf{A}$ found in literal a).

Process and Answer:

c) Find the solution $\mathbf{X}$ of the linear system of equations $\mathbf{AX} = \mathbf{B}$ by using the $\mathbf{LU}$ factorization found in literal b).

Process and Answer:

6. (1.0 point) Let

$$\mathbf{A} = \begin{bmatrix} -5 & 6 & 10 & 12 \\ 15 & 3 & 28 & 1 \\ 10 & -12 & -20 & -24 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 7 & -6 & 3 \\ -6 & 2 & 11 \\ 10 & -12 & -20 \\ 6 & 17 & 3 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 14 & 8 & 1 \\ 5 & 4 & 1 \\ 2 & 25 & 6 \end{bmatrix}$$

a) Determine $\mathbf{AB} + \mathbf{C}$

Process and Answer:

b) Determine $\mathbf{BA}$

Process and Answer:

c) Determine $\mathbf{CA}$

Process and Answer:

d) Find the determinant of $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ if it exists.

Process and Answer: