Laboratorio 2

Taller 3

1. a $\left(P - \frac{a}{V^2}\right)(V-b) = RT$

$$\left(P - \frac{a}{V^2}\right)(V-b) - RT = 0 \Rightarrow \left(5 - \frac{0.245}{V^2}\right)(V - 0.0266) - 28721 = 0$$

2. b) $V_{k+1} = V_k - \frac{F(k)(X_{k-1} - x)}{f(V_k) - f(k-1)}$ $V_0 = 35$ $V_1 = 30$ $|V_k - V_{k-1}| = 5$

$E_K = 29.23766$
$E_K = 21.23766$

$V_2 = V_1 - \frac{F(V_1)(V_1 - V_0)}{F(V_1) - F(V_0)} = 30 - \frac{F(30)(30-35)}{F(30) - f(35)} =$

$E_K = 5.69 \times 10^{-3}$

$V_2 = 30 - \frac{121.15416(30-35)}{121.15416 - 146.15290} = 5.76803$ $|V_k - V_{k-1}| = 21.23196$

$E_K = 0$

$V_3 = 5.76803 - \left[\frac{0.02845(5.76803-30)}{0.02845 - 121.15416}\right] = 5.76234$ $|V_k - V_{k-1}| = 0.00569$

$E_K = 0$

$V_4 = 5.76234 - \left[\frac{0.00003(5.76234 - 5.76803)}{0.00003 - 0.02845}\right] = 5.76234$ $|V_k - V_{k-1}| = 0$

c) $R = 1.618$

$E_{k+1} = A|E_k|^{1.618}$

$A = \frac{E_{k+1}}{E_k} = \frac{29.23766}{21.23766} = 1.20627$

2. a) $F(x) = x^3 - 3x - 2$

$P = P_0 - \frac{F(x)}{F'(x)} = P_0 - \frac{x^3 - 3x - 2}{3x^2 - 3}$

b) $P_0 = 2.1$

$$P_1 = 2.1 - \frac{0.961}{10.23} = 2.00606$$

$$P_2 = 2.00606 - \frac{0.05477}{9.07264} = 2.00002 \qquad |P_K - P_{K-1}| = 0.09394$$

$$P_3 = 2.00002 - \frac{0.00072}{9.00079} = 2 \qquad |P_K - P_{K-1}| = 0.00604$$

$$P_4 = 2 - \frac{0}{8.99999} = 2 \qquad |P_K - P_{K-1}| = 0$$

c) Su orden de convergencia es cuadrático.

3. $N(t) = N_0 e^{\lambda t} + \nu \dfrac{e^{\lambda t} - 1}{\lambda}$; $N_0 = N(t_0)$

$N(t_0) = 1000$; $\nu = 435$ y $N(t_1) = 1564$

a) $$N(\lambda) = 1000e^{\lambda} + 435\left[\frac{e^{\lambda} - 1}{\lambda}\right] - 1564$$

b) $\lambda_0 = 0.5$ y precisión de $|\lambda_k - \lambda_{k-1}| < 10^{-6}$

$$P_1 = P_0 - \frac{F(x)}{F'(x)} = P_0 - \left[\frac{1000e^{\lambda} + 435\left[\frac{e^{\lambda}-1}{\lambda}\right] - 1564}{1000e^{\lambda} + 435\left[\frac{\lambda e^{\lambda} - e^{\lambda} + 1}{\lambda^2}\right]}\right]$$

$$P_2 = 0.5 - \frac{649.10878}{1954.33377} = 0.16786$$

$$P_3 = 0.16786 - \frac{92.41424}{1426.71539} = 0.10306 \qquad |\lambda_k - \lambda_{k-1}| = 0.33214$$

$$P_4 = 0.10306 - \frac{2.77011}{1341.60136} = 0.10100 \qquad |\lambda_k - \lambda_{k-1}| = 0.0648$$

$P_6 = 0.10100 - \frac{0.0027}{1338.99151} = 0.10100$ $|x_k - k_{k-1}| = 0.00266$

1. a) $g(x) = \frac{x^2 - 8x + 25}{3}$, $x \in [1, 7]$

a.1) el teorema dice que:

$y = g(x)$ tiene al menos un punto fijo se se cumple que $y \in [a, b]$ para todos los $x \in [a, b]$

$P = g(P)$

$P = g(P) = \frac{1}{3}(P^2 - 8P + 25)$

$3P = P^2 - 8P + 25$

$P^2 - 11P + 25 = 0$

$P = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$P_1 = 7.79128$

$P_2 = 3.2087$ → Esta raiz se encuentra acotada entre $[a, b]$, por lo tanto se garantiza al menos un punto fijo.

$F(a) > a$ y $f(b) < b$.

$g(1) = \frac{1^2 - 8(1) + 25}{3} = 6$

$g(7) = \frac{49 - 56 + 25}{3} = 6$

a.2) El segundo teorema indica

Si $g'(x)$ existe en (a, b) y existe una constante positiva $k < 1$ con

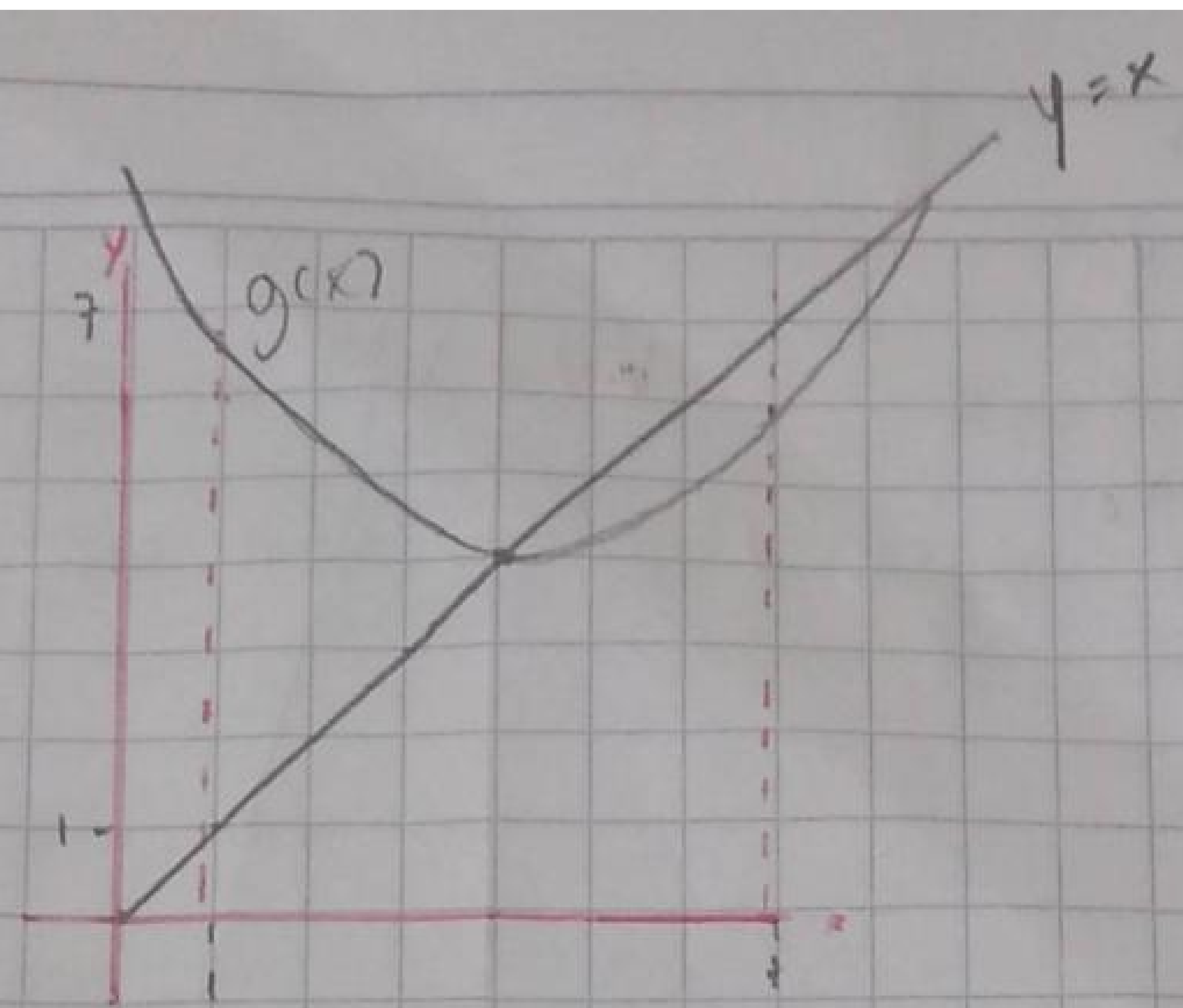
$|g'(x)| \leq k$ para toda $x \in (a, b)$

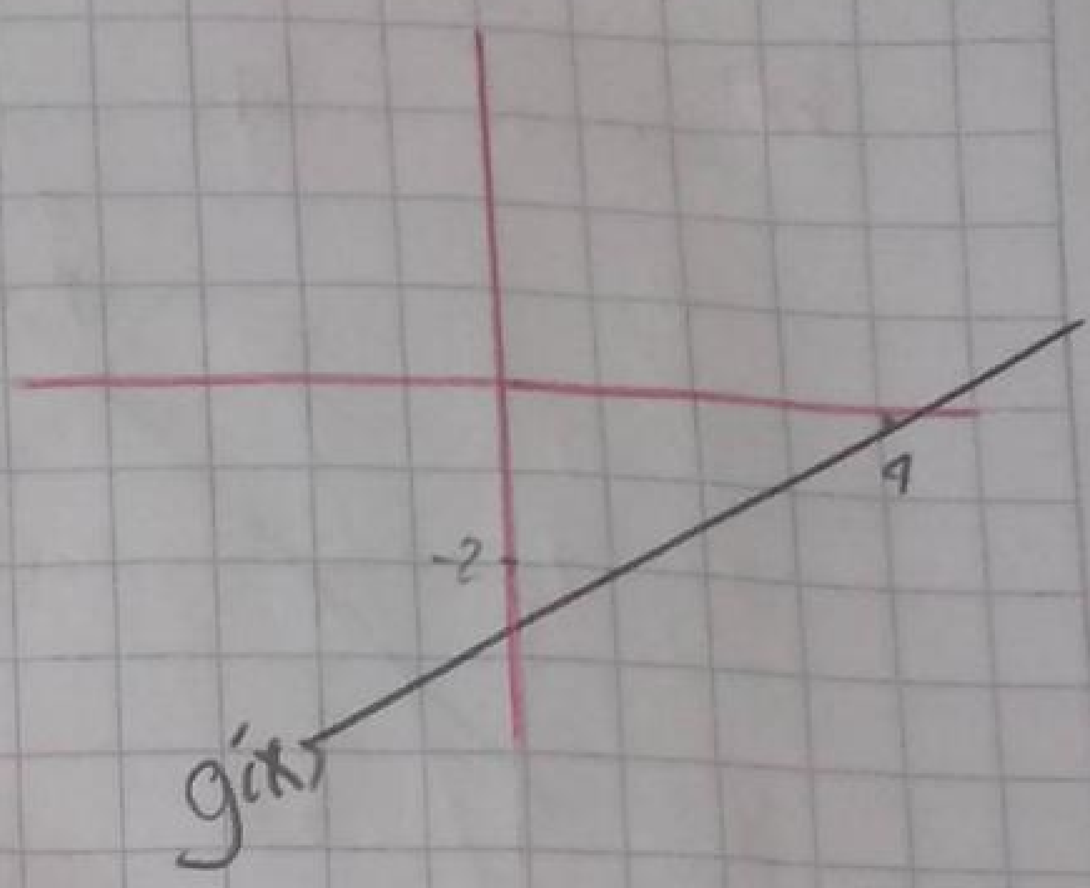
entonces hay exactamente un fijo en $[a, b]$.

$g'(x) = \frac{1}{3}(2P - 8)$

$g(1) = -8$
$g(7) = 2$

$|g'(x)|$ es mayor que k, no se cumple el segundo teorema.

a3 

a4) 

Aplicando Método de punto fijo

$P_0 = 3.15$

$P_1 = F(P_0) = 3.240833333$

$P_2 = F(P_1) = 3.192111343$

$P_3 = F(P_2) = 3.217561361$

$P_4 = 3.204070075$

$P_5 = 3.211168149$

$P_0 = 3.25$

$P_1 = F(P_0) = 3.1875$

$P_2 = 3.220052083$

$P_3 = 3.202772918$

$P_4 = 3.211857007$

$P_5 = 3.207056459$

b. b1) $g(x) = \dfrac{11x^3 - 141x^2 + 556x - 546}{30}$ $x \in [2, 7]$

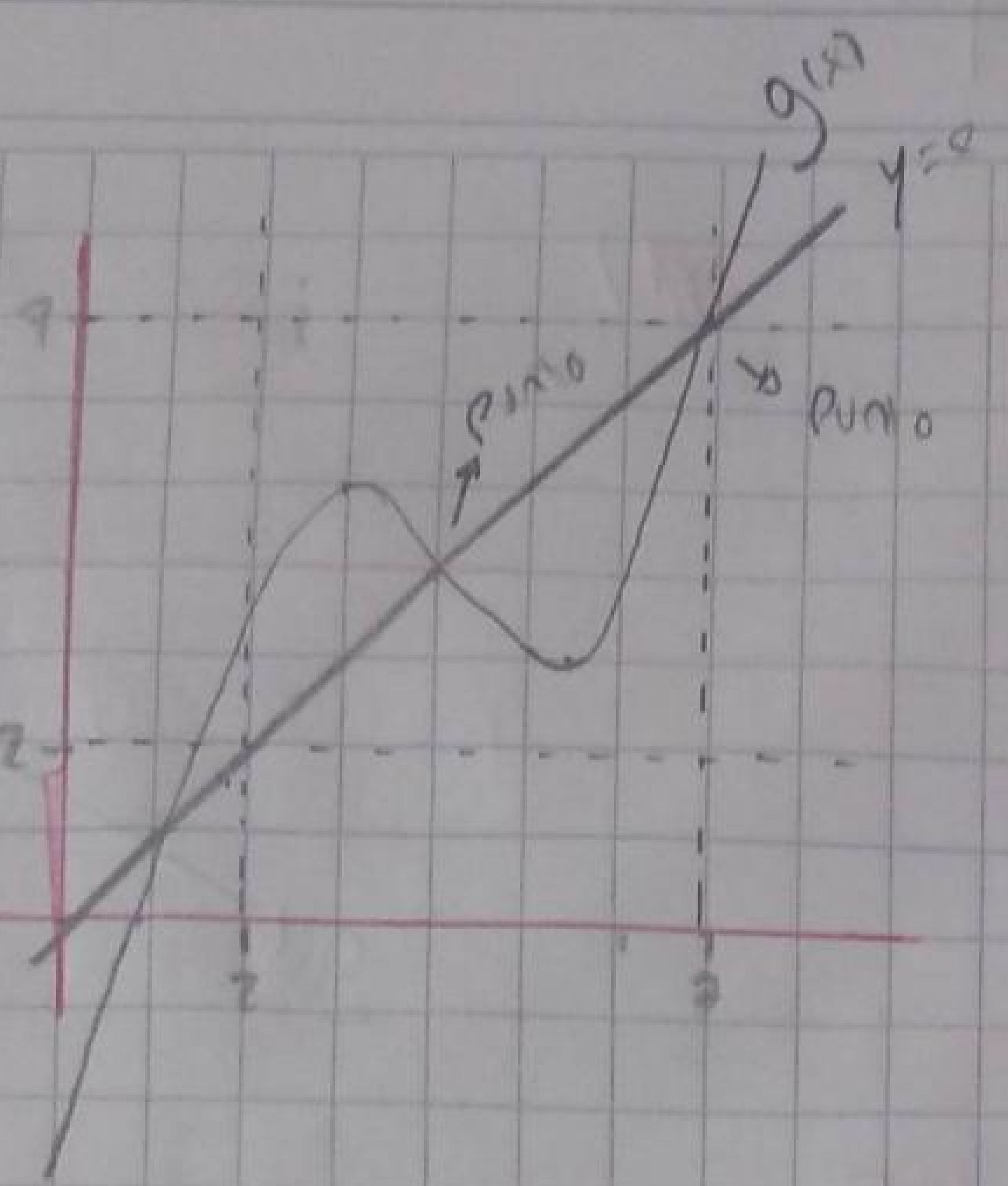
$g(2) = 3$
$g(7) = 7$

$f(a) > a$ $f(b) < b$ → Hay un punto fijo en este extremo
$3 > 2$ $7 < 7$

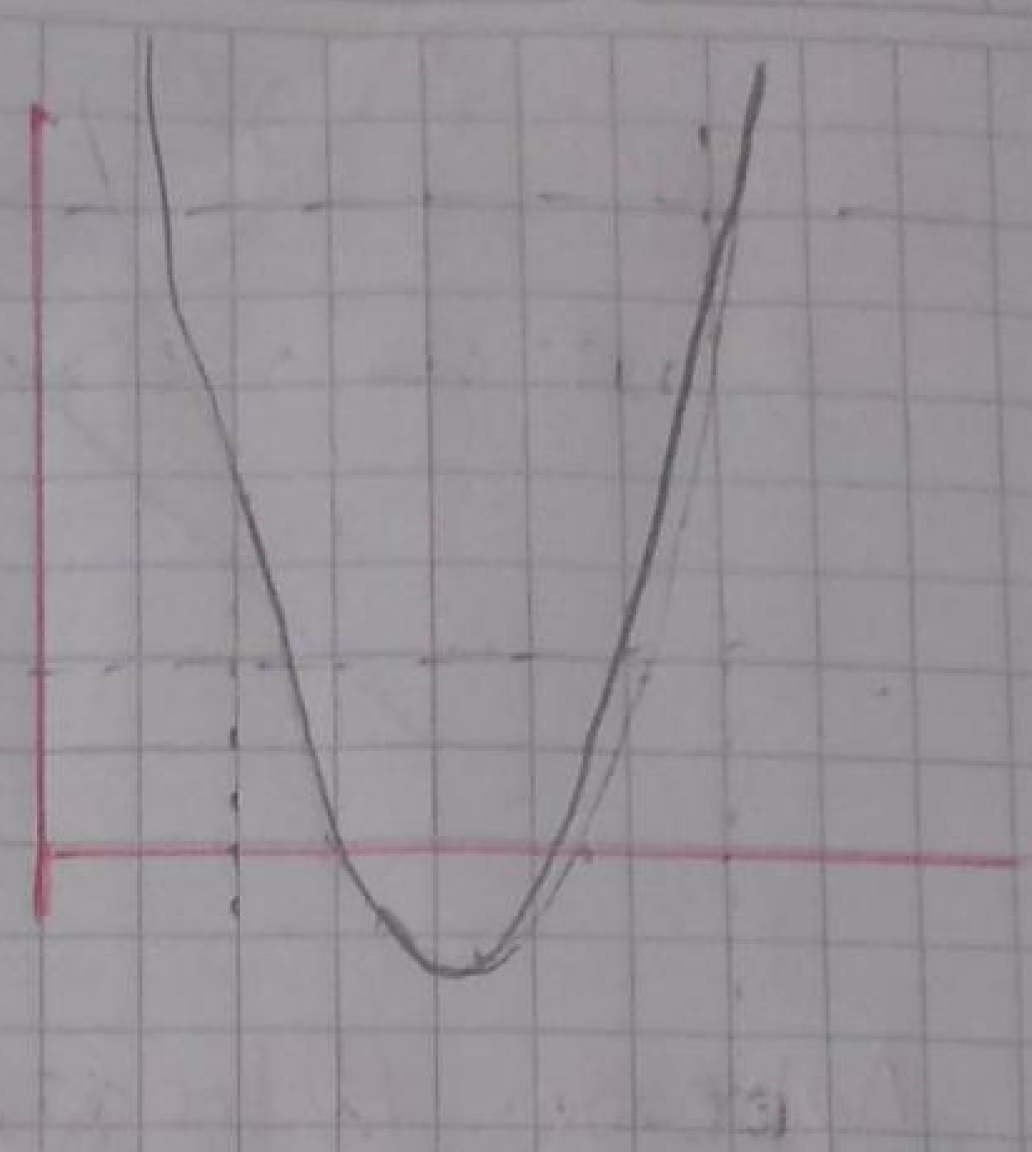
b2) $g'(x) = \dfrac{33x^2 - 282x + 556}{30}$

$\left.\begin{array}{l} g'(2) = 4.133333333 \\ g'(7) = 6.633333333 \end{array}\right] > k$ No se garantiza un punto fijo único.

b3)



b4)



Calculo del Punto Fijo

~~P0 = 4.1~~

$P_0 = 4.1$

$P_1 = (4.1) = 4.0507$

$P_2 = (4.0507) = 4.124916638$

$P_3 = f(P_2) = 4.012872374$

$P_4 = 4.18114527$

$P_5 = 3.926655665$

$P_0 = 6.95$

$P_1 = f(P_0) = 6.6757675$

$P_2 = f(P_1) = 5.1522359419$

$P_3 = f(P_2) = 2.6726772679$

$P_4 = f(P_3) = 4.760764107$

$P_5 = f(P_4) = 3.072027873$

5\. a) $M_w = M_b$

$$M_w = \frac{\pi d^2 (3r - d)}{3} \quad ; \quad M_b = \frac{4\pi r^3 \rho}{3}$$

$$M_w - M_b = 0$$

$$\frac{\pi d^2 (3r - d)}{3} = \frac{4\pi r^3 \rho}{3} \cancel{= 0}$$

$$d^2 (3r - d) - 4r^3 \rho = 0$$

$$3rd^2 - d^3 - 4r^3 \rho = 0$$

$$-d^3 + 45 d^2 - 8613 = 0$$

b) $|c_k - c_{k-1}| < 1 \times 10^{3}$ $\quad a_0 = 17.6 \; ; \; b_0 = 18$

$$r = a + \frac{(b-a)}{2}$$

Iteración 0

$a_0 = 17.6$ $\quad c_0 = \frac{a+b}{2} = 17.8$

$b_0 = 18$

$F(x) = -x^3 + 45x^2 - 8613$

$F(a_0) = -175.576$

$F(b_0) = 85$

$F(c_0) = -44.952$

Iteración 1.

$a_1 = 17.8$ $\quad c_1 = \frac{a+b}{2} = 17.9$ $\quad |c_1 - c_0| = 0.1 > 1 \times 10^{-3}$

$b_1 = 18$

$f(a_1) = -44.952$

$f(b_1) = 85$

$F(c_1) = 20.111$

Iteración 2.

$a_2 = 17.8$ $\quad c_2 = \frac{a+b}{2} = 17.85$ $\quad |c_2 - c_1| = 0.05 > 1 \times 10^{-3}$

$b_2 = 17.9$

$F(a_2) = -44.952$

$f(b_2) = 20.111$

$f(c_2) = -12.39917$

Iteración 3

$a_3 = 17.85$ $\quad c_3 = \frac{a+b}{2} = 17.875$ $\quad |c_3 - c_2| = 0.025$

$b_3 = 17.9$

$f(a_3) = -12.39917$

$f(b_3) = 20.111$

$f(c_3) = 3.86133$

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Iteración 4

$a_4 = 17.85$
$b_4 = 17.875$
$F(a_4) = -17.39512$
$F(b_4) = 3.86133$
$F(c_4) = -4.26756$

$c_4 = \frac{a+b}{2} = 17.8625$ $|c_4 - c_3| = 0.025 > 1\times10^{-3}$

Iteración 5

$a_5 = 17.8625$
$b_5 = 17.875$

$F(a_5) = -4.26756$
$F(b_5) = 3.86133$
$f(c_5) = -0.20298$

$c_5 = \frac{a+b}{2} = 17.86875$ $|c_5 - c_4| = 6.25\times10^{-3} > 1\times10^{-3}$

Iteración 6

$a_6 = 17.86875$
$b_6 = 17.875$

$F(a_6) = -0.20278$
$f(b_6) = 3.86133$
$F(c_6) = 1.82936$

$c_6 = \frac{a+b}{2} = 17.87187$ $|c_6 - c_5| = 3.12\times10^{-3} > 1\times10^{-3}$

Iteración 7

$a_7 = 17.86875$
$b_7 = 17.87187$

$F(a_7) = -0.20298$
$f(b_7) = 1.82936$
$f(c_7) = 0.81331$

$c_7 = \frac{a+b}{2} = 17.87031$ $|c_7 - c_6| = 1.56\times10^{-3} > 1\times10^{-3}$

Iteración 8

$a_8 = 17.86875$
$b_8 = 17.87031$

$f(a_8) = -0.20298$
$f(b_8) = 0.81331$
$f(c_8) = 0.30527$

$c_8 = \frac{a+b}{2} = \underline{17.86957}$ $\underline{|c_8 - c_7| = 7.8\times10^{-4} < 1\times10^{-3}}$

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c) Número requerido de iteraciones para garantizar que un punto medio es una aproximación 0.

$\delta = 1 \times 10^{-15}$
$a_0 = 17$
$b_0 = 18$

$$N \geq \frac{Ln(b-a) - Ln(\varepsilon)}{Ln(2)} - 1$$

$$N \geq \frac{L(18-17) - Ln(1 \times 10^{-15})}{Ln(2)} - 1$$

$$N = \underline{48.82892}$$

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