STAT 443: Course Aims and Objectives

Learning outcomes

The numbered items below each state a learning aim for the course, and the items that follow indicate the learning outcomes (or objectives) through which that aim could be deemed to have been satisfied.

1. Appreciate the important features that describe a time series, and perform simple analyses and computations on series.

   (a) Informally define and explain terminology used to describe time series, including trend, seasonal effects, cyclical effects, outlier and white noise.

   (b) Recognize when curve–fitting may be an appropriate method for modelling a series, identifying linear, quadratic, Gompertz, and Logistic models where appropriate.

   (c) Describe models for seasonal variation, including additive and multiplicative models.

   (d) Apply a filter (that is, a smoother) to a time series, centring if necessary.

   (e) Use a filter to estimate the seasonal indices in a time series that has an additive seasonal component.

   (f) Define and apply the difference operator, including the operator for seasonal differences.

   (g) Recognize the role of transformations for time series, and identify possible transformations to address certain non-stationary features of a series, such as non-constant variance and multiplicative seasonal effects.

   (h) Define the sample autocorrelation function and the correlogram.

   (i) Describe the behaviour of the correlogram for series that alternate, have a trend, or show seasonal fluctuations.

   (j) Use R to perform certain time series analyses including plots, smoothing, computation of the sample autocorrelation function, lagging and differencing.

2. Understand the definitions of the important stochastic processes used in time series modelling, and the properties of those models.

   (a) Define the autocovariance and autocorrelation functions for a time series model.

   (b) Define and explain what it means to say that a process is (weakly) stationary.

   (c) Define what is meant by a *white noise* process.

   (d) Identify a *random walk* model, and derive the basic properties of such a model.

   (e) Identify a *moving average process of order* p, i.e., an MA(q) .

   (f) Derive the mean, variance and autocovariance function of an MA(q) process.

   (g) Define an MA(q) in terms of the backward shift operator B, and hence define when an MA(q) is invertible.

   (h) Recall and test conditions that ensure that an MA(∞) process is stationary.

   (i) Identify an *autoregressive process of order* p, i.e., an AR(p) .

   (j) Derive properties for an AR(1) , including the mean, variance and autocorrelation function.

   (k) Define an AR(p) in terms of the backward shift operator B, and hence define when an AR(p) is stationary.

   (l) Derive the Yule–Walker equations for an AR(p) process.

   (m) Recall the general solution to the Yule–Walker equations, and solve these equations where computationally feasible without the aid of a computer.

   (n) Interpret the solutions to the Yule–Walker equations in the context of fitting an AR model.

   (o) Define an ARMA(p, q) process in terms of the backward shift operator B, and hence identify when an ARMA(p, q) is stationary and/or invertible.

   (p) Express an ARMA(p, q) model as a pure MA process (when $p < 2$) or a pure AR process (when $q < 2$).

(q) Define an ARIMA(p, d, q) process in terms of the backward shift operator B and the difference operator ∇.

(r) Define a SARIMA$(p, d, q) \times (P, D, Q)_s$ process in terms of the backward shift operator B, the difference operator ∇ and the seasonal difference operator ∇_s.

(s) Express a SARIMA$(p, d, q) \times (P, D, Q)_s$ process as an ARMA(p, q) process.

3. Appreciate and apply key concepts of estimation in a time series context.

   (a) Recall the main properties of the sample acvf and acf for a time series.

   (b) Explain issues regarding estimation of the mean of a time series, being able to identify in particular cases how the variance of the sample mean differs from sampling from uncorrelated data.

   (c) Describe the issues related to fitting AR, MA and ARMA models, in particular the choice of the order and general approaches to parameter estimation.

   (d) Fit appropriate ARMA models to time series using the R software, considering diagnostic checks where relevant.

   (e) Explain in broad terms how R fits an ARIMA model to a time series, with reference to the estimation methods such as conditional least squares and maximum likelihood.

   (f) Interpret results of model diagnostic tests based on the residuals of a fitted time series model.

4. Understand and apply key concepts of forecasting in a time series context.

   (a) Describe in general terms the differences and relationship between subjective and model–based approaches to forecasting.

   (b) Identify when curve–fitting and extrapolation would be a viable approach to forecasting future values of a time series.

   (c) Explain the principles underlying exponential smoothing as a forecasting method.

(d) Apply exponential smoothing to forecast future values of a time series, given necessary parameter estimates.

(e) Describe the role of the parameter α in exponential smoothing, and the criteria for how this parameter can be chosen.

(f) Explain the principles underlying Holt's method (double exponential smoothing) as a forecasting method.

(g) Describe the role of the parameters α and β in Holt's method, and the criteria for how these parameter can be chosen.

(h) Apply Holt's method to forecast future values of a time series, given necessary parameter estimates.

(i) Explain the principles underlying Holt–Winters forecasting method for series with additive or multiplicative seasonal components.

(j) Describe the role of the parameters α, β and γ in Holt–Winters forecasting method, and the criteria for how these parameters can be chosen.

(k) Apply Holt–Winters method to forecast future values of a time series, given necessary parameter estimates.

(l) Describe and implement the Box–Jenkins approach to forecasting.

(m) Explain the sense in which Box–Jenkins forecasting is optimal.

(n) Apply Box–Jenkins method to forecast future values of a time series, given necessary parameter estimates.

(o) Compute prediction intervals for a Box–Jenkins forecast.

(p) Describe the behaviour of Box–Jenkins forecasts as the lead time increases.

(q) Implement exponential smoothing, Holt's method, Holt–Winters method, and Box–Jenkins forecasting using the R software.

5. Understand the key concepts related to the stationary processes in the frequency domain.

   (a) Define the terms amplitude, phase, frequency and wavelength in the context of modelling time series using sinusoidal models.

(b) Explain why when modelling an integer–time series in the frequency domain, only frequencies in the range 0 to π need be considered.

(c) Define the Fourier transform of a function and identify when that transform exists.

(d) Define the inverse Fourier transform.

(e) Recall the properties of the Fourier transform of functions that (i) are defined only on the integers, (ii) are even and (iii) functions that satisfy both (i) and (ii).

(f) Define and interpret the spectral density and spectral distribution functions for a time series.

(g) Recall key properties of the spectral density and spectral distribution functions.

(h) Where mathematically tractable, compute the spectral density and spectral distribution functions of a time series model.

(i) Where mathematically tractable, derive the acvf of a time series model from the spectral density function.

6. Understand and apply the theory and methodology of the analysis of time series in the frequency domain.

   (a) Compute the Fourier series representation of an integer–time series.

   (b) Describe the role of the Fourier series coefficients in the fitting of a sinusoidal model to a time series by least squares.

   (c) Recall and explain Parseval's theorem as applied to the harmonic analysis of a time series.

   (d) Define the periodogram for a time series.

   (e) Describe the relationship between the periodogram and sample acvf.

   (f) Explain why the (raw) periodogram is not a consistent estimator of the spectrum of a series.

   (g) Apply asymptotic properties of the periodogram.

(h) Describe and explain methods for modifying the periodogram, in particular approaches that (i) transform and truncate and (ii) smooth the periodogram.

(i) Construct confidence intervals for a spectrum based on a consistent estimator following a modification of the periodogram.

(j) Compare and contrast methods for modifying a periodogram using the spectral window.

(k) Perform a test to determine whether a given time series appears to be a realization of a white noise process.

(l) Analyze time series in the frequency domain using the R software, interpreting the output as appropriate.

7. Understand situations where models with changing variance are appropriate, be able to assess properties of such models and appreciate inference ideas.

   (a) Be familiar with empirical facts about financial time series.

   (b) Define an ARCH(p) process, identify conditions for weak stationarity and verify heavy-tailed behaviour.

   (c) Define a GARCH(p,q) process, identify conditions for weak stationarity and verify heavy-tailed behaviour.

   (d) Define an extension to an ARMA model with GARCH errors; give interpretation of quantities μ_t and σ_t^2.

   (e) Be familiar with the idea of how to fit a GARCH model to data.

   (f) Fit a GARCH model to data using suitable packages in `R`.

   (g) Derive expressions for (multi-step) volatility forecasts.