Lab 4

Derek Situ (62222807)

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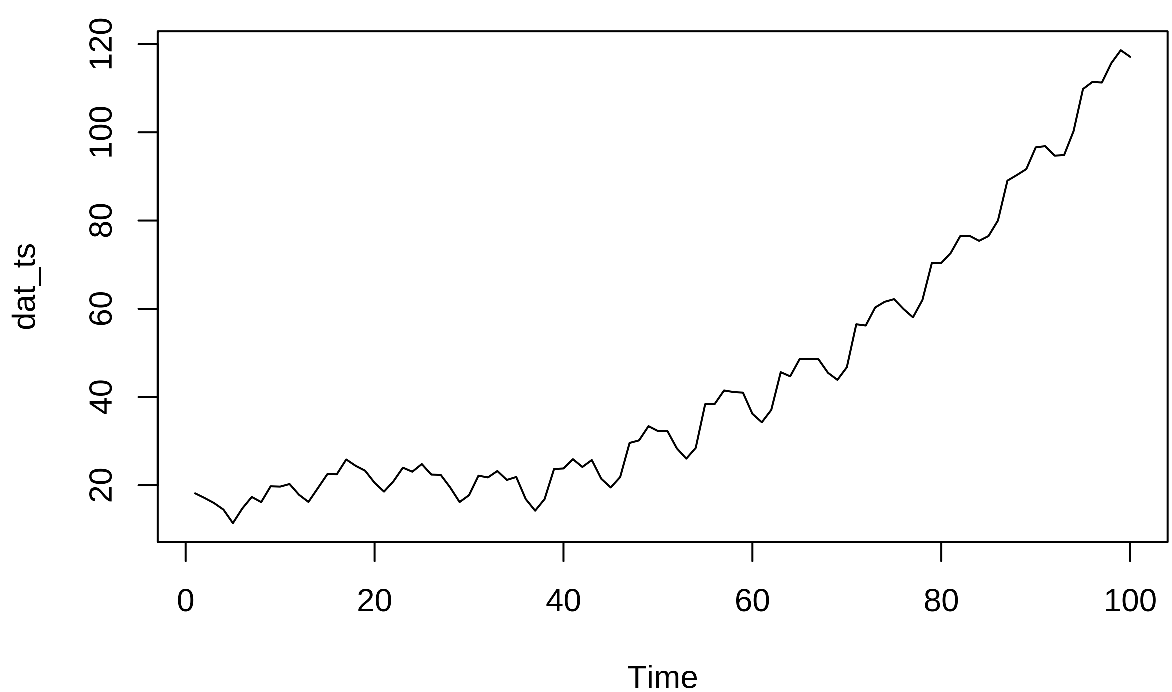
Question 1

A time series is stationary if its mean is constant, its variance is finite, and its autocovariance function only depends on the lag.

Question 2

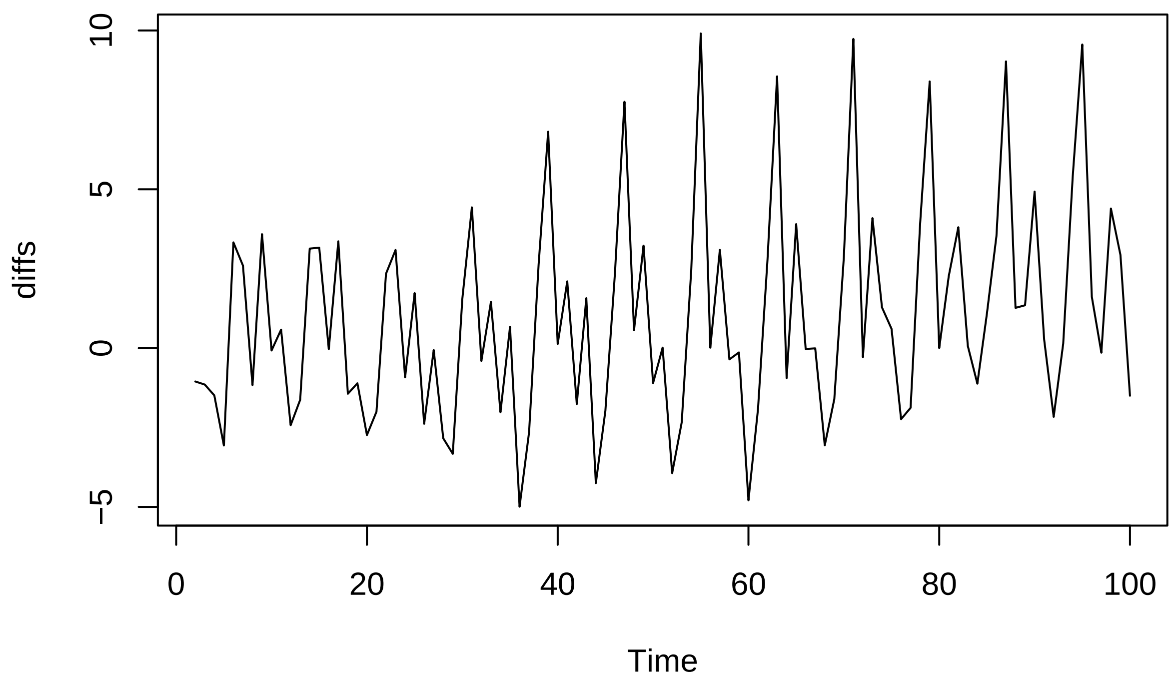
```r
dat <- read.csv("lab4data.csv", header = TRUE)
dat_ts <- ts(dat$x, start = 1, end = 100)
plot(dat_ts)
```



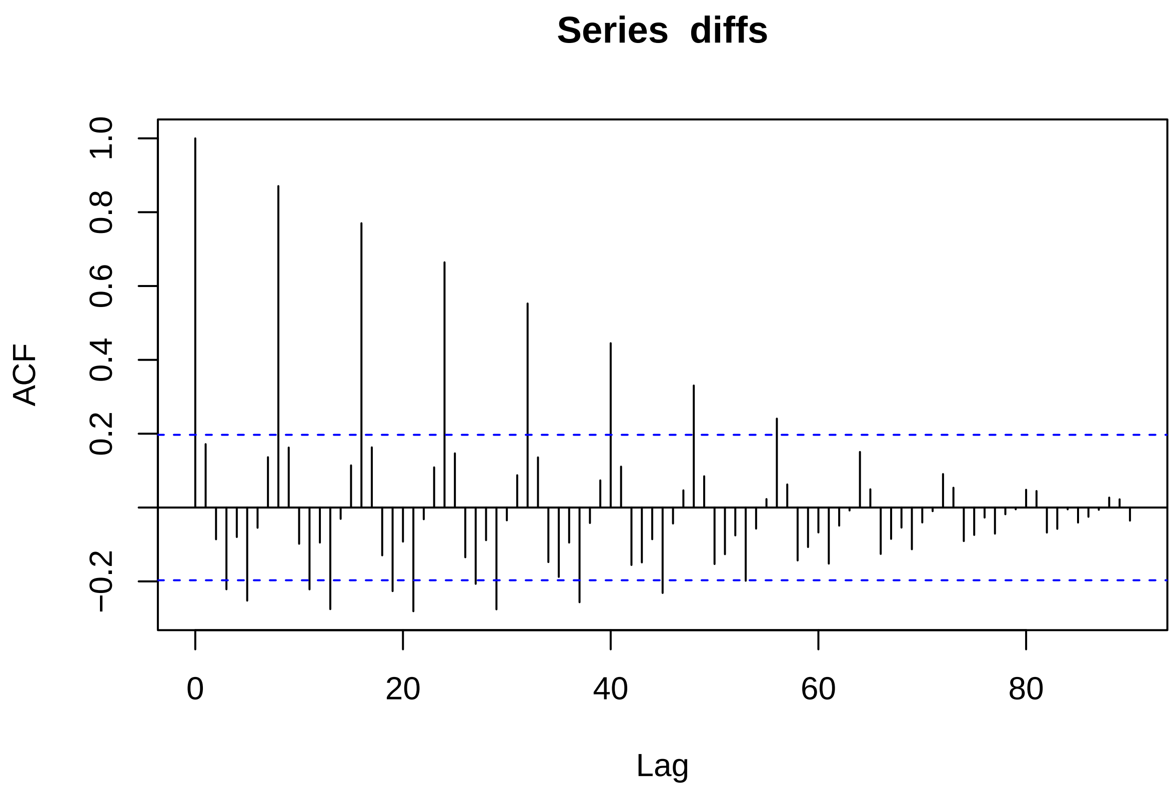
We see that this time series does not appear to satisfy the requirements of stationarity since the mean does not appear to be constant. We can also guess that the autocovariance also depends on the time, i.e. not just on the lag.

Question 3

```
diffs <- diff(dat_ts)
plot(diffs)
```



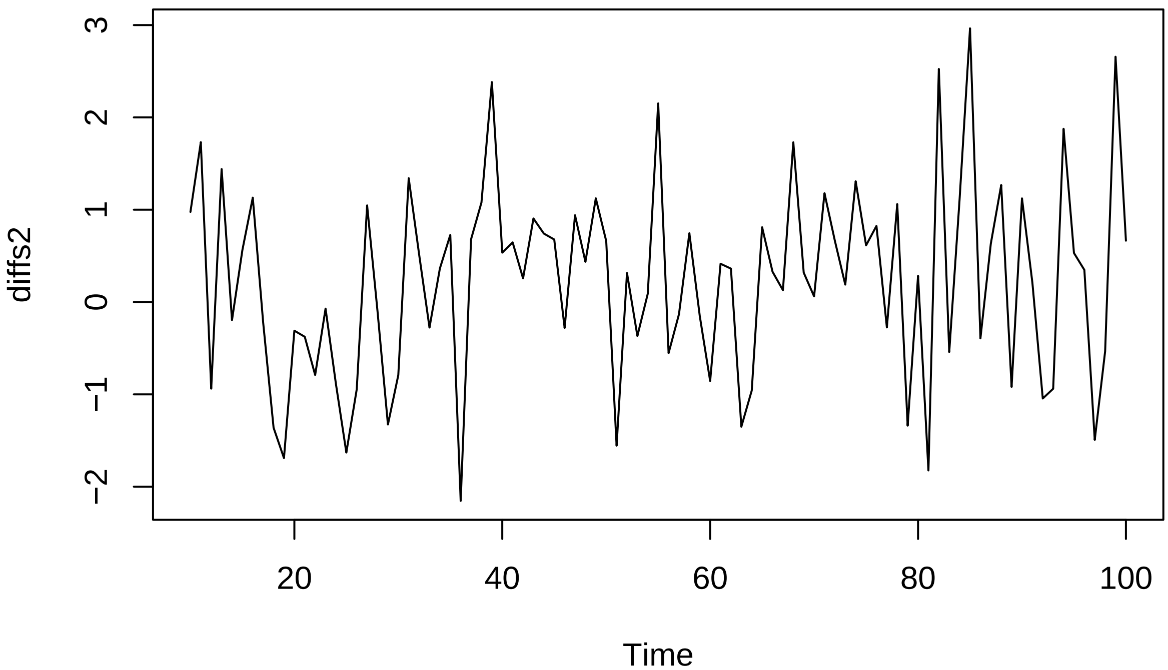

```
acf(diffs, lag.max = 90)
```



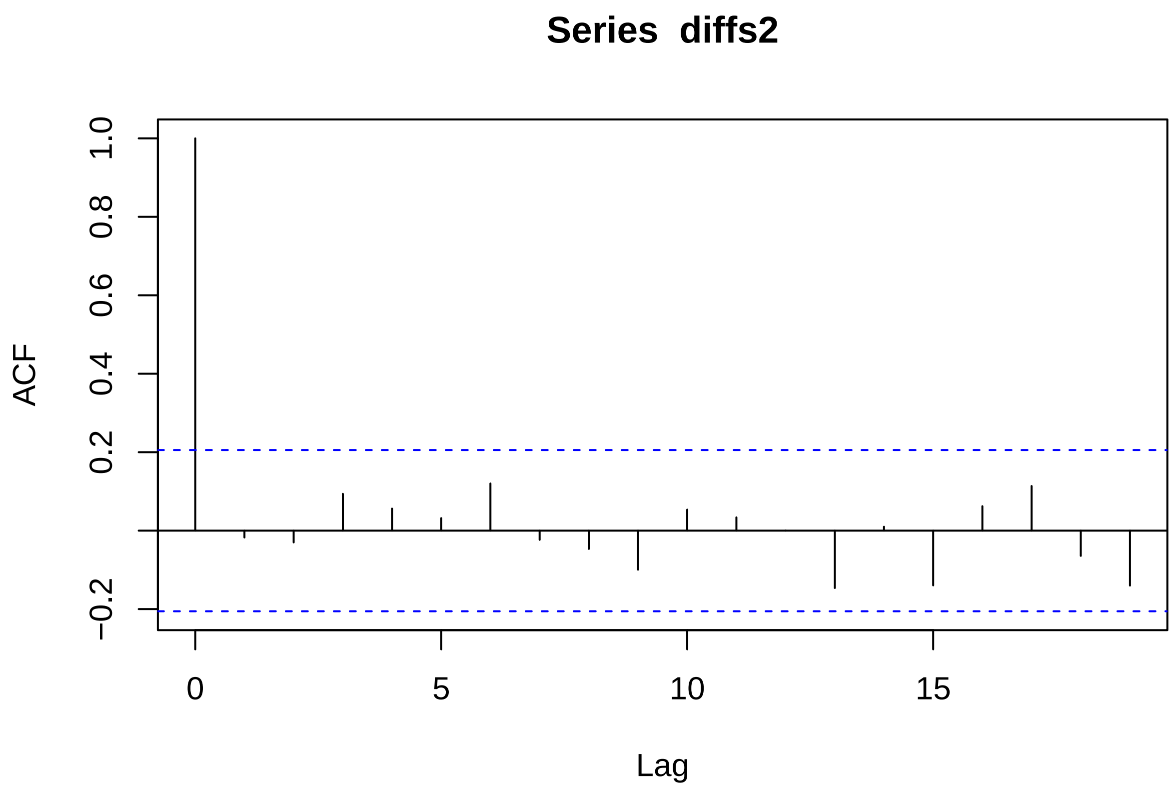
From the plot of the differenced time series we see that the trend is less noticeable and there is seasonality. The acf oscillates with a period of 8.

Question 4

```
diffs2 <- diff(diffs, lag = 8)
plot(diffs2)
```




```
acf(diffs2)
```



This series resembles white noise now that the seasonal effects have been removed.

Question 5

I would use a SARIMA$(0,1,0),(0,1,0)_8$ model since the series has been differenced once to remove trend, and once at lag 8 to remove seasonality, so $1 = d = D$. The resulting series resembles white noise so the original series can be modeled with $0 = p = P = q = Q$.

Question 6

(6a)

Given $Y_t = X_t - X_{t-1}$ and $W_t = Y_t - Y_{t-s}$, then

$$
\begin{aligned}
W_t &= X_t - X_{t-1} - (X_{t-s} - X_{t-s-1}) \\
&= X_t - X_{t_1} - X_{t_s} + X_{t-s-1}
\end{aligned}
$$

(6b)

Recall $Y_t = X_t - X_{t-1}$ and $BX_t = X_{t-1}$. Then,

$$
\begin{aligned}
Y_t &= X_t - BX_t \\
&= (1 - B)X_t
\end{aligned}
$$

(6c)

Given $B^s Y_t = Y_{t-s}$ and $W_t = Y_t - Y_{t_s}$, we have

$$\begin{aligned} W_t &= Y_t - B^s Y_t \\ &= (1 - B^s) Y_t \end{aligned}$$

And since $Y_t = X_t - X_{t-1}$, we see that

$$\begin{aligned} W_t &= (1 - B^s)(X_t - X_{t-1}) \\ &= (1 - B^s)[(1 - B) X_t)] \end{aligned}$$