Data 102 Fall 2022 Lecture 3

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

%matplotlib inline

sns.set()  # This helps make our plots look nicer

# These make our figures bigger
plt.rcParams['figure.figsize'] = (6, 4.5)
plt.rcParams['figure.dpi'] = 100
```

Comparing row-wise and column-wise rates

The FDP is the column-wise error rate for when our decision is 1. It asks: how many of the discoveries we made (i.e., our decision was 1) were false (wrong)?

We can relate this to row-wise rates using Bayes' rule. To simplify notation, we'll use $\pi_0 = P(R = 0)$.

$$\begin{aligned}
FDP &= P(R=0|D=1) \\
\text{(using Bayes' rule)} &= \frac{P(D=1|R=0)P(R=0)}{P(D=1)} \\
\text{(Law of total probability)} &= \frac{P(D=1|R=0)P(R=0)}{P(D=1|R=0)P(R=0)+P(D=1|R=1)P(R=1)} \\
\text{(dividing by the numerator)} &= \frac{1}{1+\frac{P(D=1|R=1)}{P(D=1|R=0)}\frac{P(R=1)}{P(R=0)}} \\
\text{(applying definitions)} &= \frac{1}{1+\frac{TPR}{FPR}\frac{1-\pi_0}{\pi_0}}
\end{aligned}$$

In [2]:
```
def compute_fdp(tpr, fpr, prevalence):
    return 1 / (1 + (tpr/fpr) * (prevalence/(1-prevalence)))
```

Suppose we have an algorithm that predicts whether or not it'll rain. We apply our algorithm in Berkeley (where it almost never rains in the summer) and in Miami, FL (where it rains about half the time in the summer).

In [3]:
```
# Summer in Miami, good test
compute_fdp(tpr=0.99, fpr=0.01, prevalence=0.5)
```

Out[3]: 0.01

In [4]:
```
# Summer in Berkeley, good test
compute_fdp(tpr=0.99, fpr=0.01, prevalence=0.01)
```

Out[4]: 0.5

In [5]:
```python
# Summer in Miami, bad test
compute_fdp(tpr=0.6, fpr=0.4, prevalence=0.5)
```

Out[5]: 0.4

In [6]:
```python
# Summer in Berkeley, bad test
compute_fdp(tpr=0.6, fpr=0.4, prevalence=0.01)
```

Out[6]: 0.9850746268656717

Multiple hypothesis testing and error control

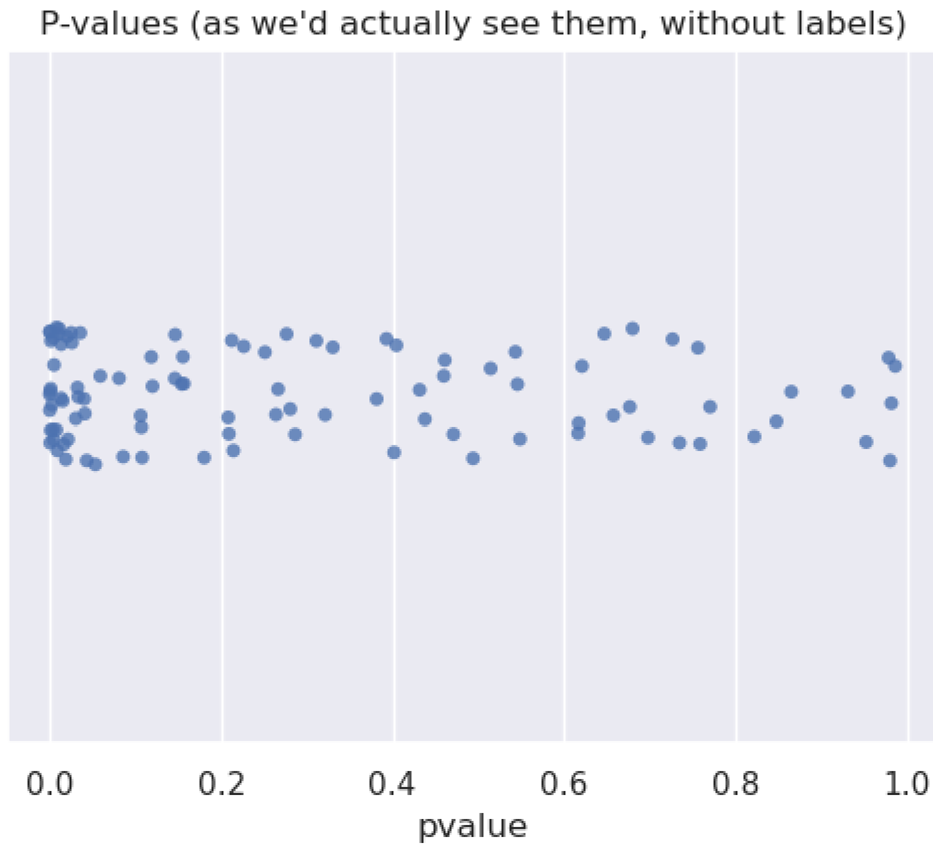
Suppose we have a collection of p-values, with ground truth labels.

In [7]:
```python
p_values = pd.read_csv('pvalues.csv')
p_values.head()
```

Out[7]:

	pvalue	is_alternative
0	0.226410	1
1	0.053205	1
2	0.001128	1
3	0.404488	1
4	0.401786	0

In [8]:
```python
sns.stripplot(
    data=p_values, x='pvalue',
    alpha = 0.8, order = [0, 1], orient = "h",
    #ax=ax
)
plt.title("P-values (as we'd actually see them, without labels)");
```



In [9]:
```
sns.stripplot(
    data=p_values, x='pvalue', y='is_alternative',
    alpha = 0.8, order = [0, 1], orient = "h",
)
plt.title('P-values, with ground truth labels')
```

Out[9]: `Text(0.5, 1.0, 'P-values, with ground truth labels')`

In [10]:
```
p_values
```

Out[10]:

	pvalue	is_alternative
0	0.226410	1
1	0.053205	1
2	0.001128	1
3	0.404488	1
4	0.401786	0
...	...	...
95	0.146346	0
96	0.003678	1
97	0.013303	0
98	0.493987	0
99	0.000219	1

100 rows × 2 columns

In [11]:

```
p_sorted = p_values.sort_values('pvalue')
```

In [19]:

```
m = len(p_sorted)  # number of tests
k = np.arange(1, m+1)  # index of each test in sorted order
p_sorted['k'] = k
p_sorted
```

Out[19]:

	pvalue	is_alternative	k
94	0.000008	1	1
10	0.000109	0	2
86	0.000162	1	3
99	0.000219	1	4
42	0.000436	1	5
...	...	...	...
45	0.952674	0	96
9	0.978843	0	97
22	0.980498	0	98
76	0.982076	0	99
20	0.986542	0	100

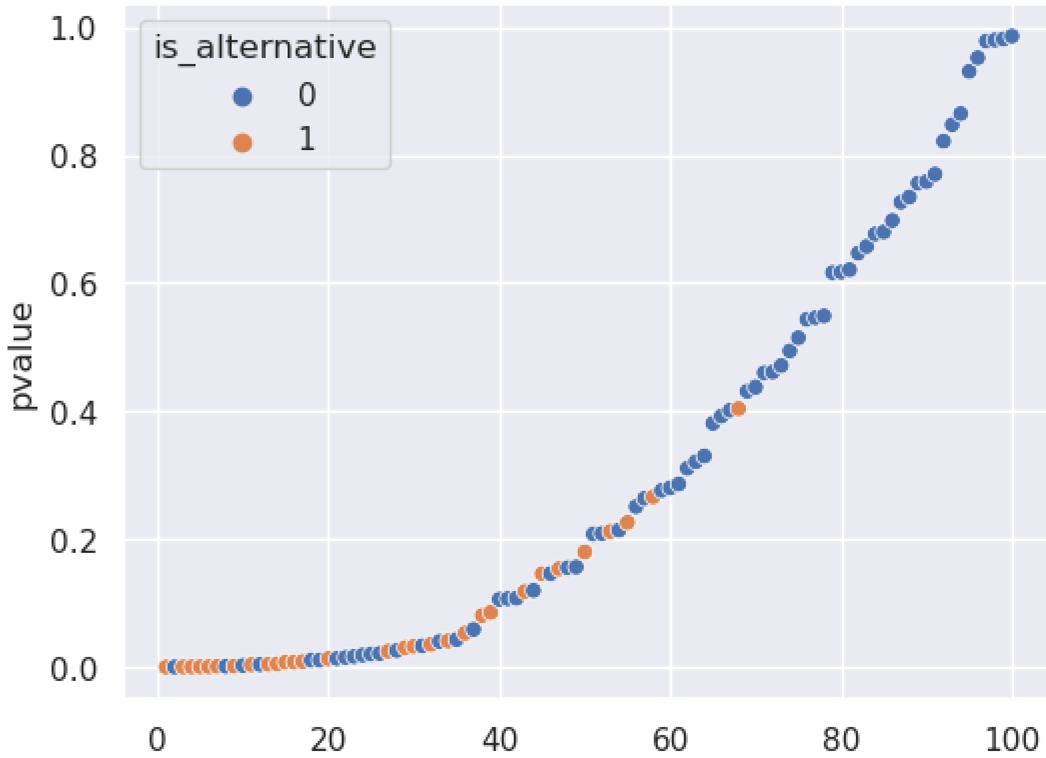
100 rows × 3 columns

In [20]:

```
alpha = .05
```

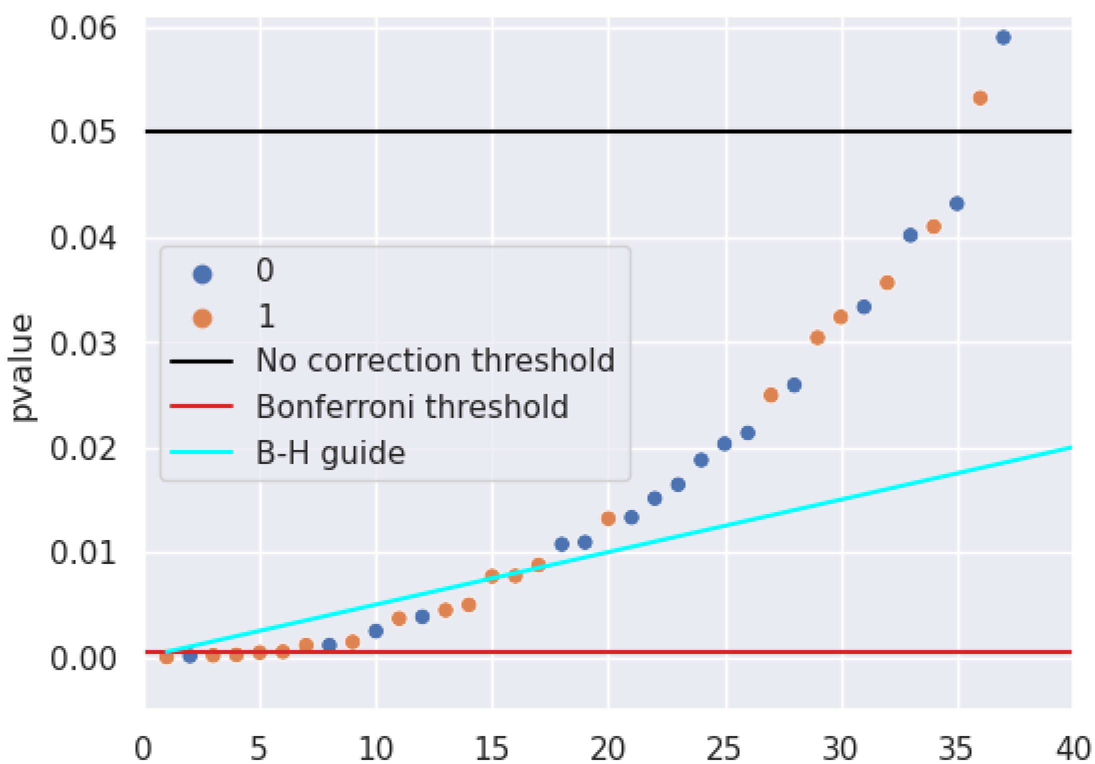
In [21]:

```
sns.scatterplot(x=k, y=p_sorted['pvalue'], hue=p_sorted['is_alternative']);
```

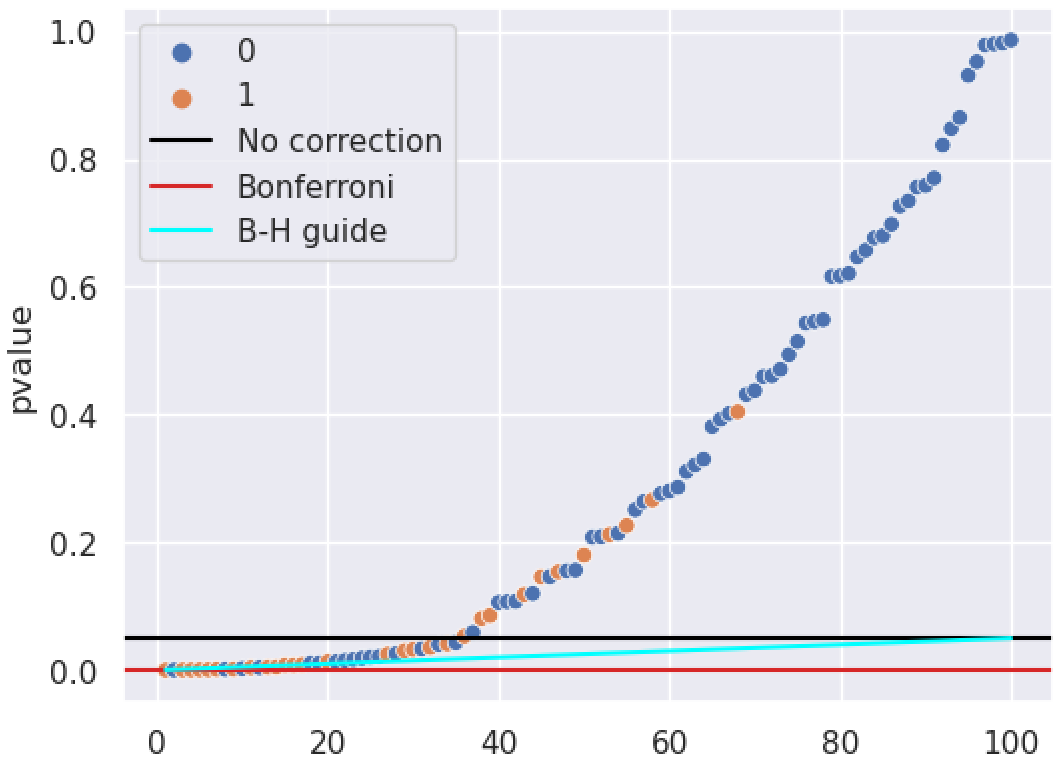


In [22]:
```python
sns.scatterplot(x=k, y=p_sorted['pvalue'], hue=p_sorted['is_alternative'])
plt.axhline(alpha, label='No correction threshold', color='black')
plt.axhline(alpha / len(p_sorted), label='Bonferroni threshold', color='tab:red')
plt.plot(k, k/len(p_sorted)* alpha, label='B-H guide', color='cyan')
plt.legend()

plt.axis([-0.01, 40, -0.005, .061]);
```

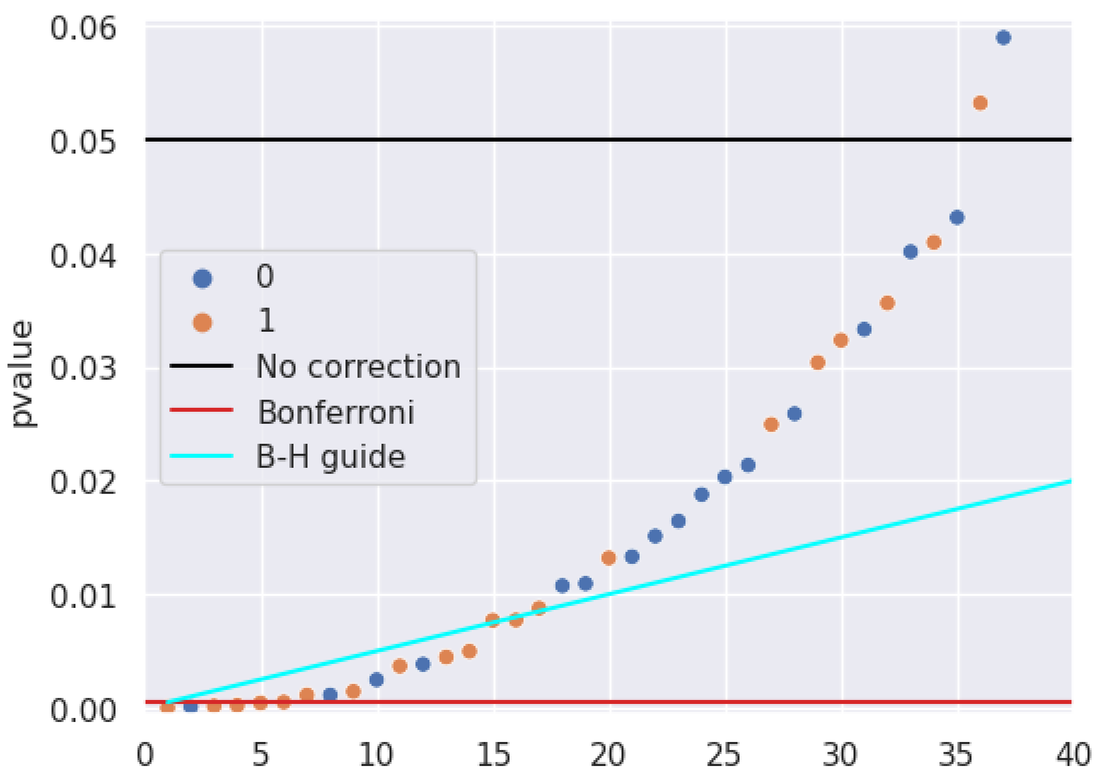


In [23]:
```python
sns.scatterplot(x=k, y=p_sorted['pvalue'], hue=p_sorted['is_alternative']);
plt.axhline(alpha, label='No correction', color='black')
plt.axhline(alpha / m, label='Bonferroni', color='tab:red')
plt.plot(k, k/m * alpha, label='B-H guide', color='cyan')
plt.legend();
```



In [24]:
```
sns.scatterplot(x=k, y=p_sorted['pvalue'], hue=p_sorted['is_alternative']);
plt.axhline(alpha, label='No correction', color='black')
plt.axhline(alpha / m, label='Bonferroni', color='tab:red')
plt.plot(k, k/m * alpha, label='B-H guide', color='cyan')

plt.axis([-0.05, 40, -0.0005, 0.0605])
plt.legend();
```



In [ ]: