(a) Sonsitivity: 
$$TP = P(D=1 | (R=1) = 99\% \Rightarrow FN = 1\% = P(D=0 | R=1)$$
  
Spentficity:  $TN = P(D=0 | R=0) = 98\% \Rightarrow FP = 2\% = P(D=1 | R=0)$   
(i)  $P(D=1 | H_0) = P(D=1 | R=0) = FP = 2\%$   
(ii)  $P(R=0 | D=1) = P(R=0) = FP = 2\%$   
(iii)  $P(R=0 | D=1) = P(R=0) = P(R=0) = P(D=1 | R=0) =$ 

(iii)

Independence :

$$P(D, \Lambda D_2 \mid R) = P(D, R) P(D_2 \mid R)$$

$$P(D_{2}:1 \mid D_{1}:1) = \frac{P(D_{1}:1 \cap D_{2}:1)}{P(D_{1}:1)}$$

$$= \frac{P(D_{1}:1 \cap D_{2}:1 \mid R:0) + P(D_{1}:1 \cap D_{2}:1 \mid R:4)}{P(D_{1}:1 \cap D_{2}:1 \mid R:4)} P(R:1)$$

$$= \frac{P(D_{1}:1 \cap D_{2}:1 \mid R:0)}{P(D_{1}:1 \cap D_{2}:1 \mid R:0)} P(R:0) + \frac{P(D_{1}:1 \cap D_{2}:1 \mid R:0)}{P(D_{1}:1 \mid R:0)} P(R:0) + \frac{P(D_{1}:1 \mid R:0)}{P(D_{1}:1 \mid R:0)} P(R:0)$$

$$= \frac{P^{2} \cdot \pi + TP^{2} \cdot (+\pi)}{P \cdot \pi + TP \cdot (+\pi)}$$

$$= \frac{(0.02)^{2} \cdot 0.995 + 0.99 \times (0.005)}{0.002 \cdot 0.995 + 0.99 \times 0.005}$$

= 0.213

b)
(i)
$$LR(T) = \frac{f_{1}(T)}{f_{0}(T)} : \frac{2c e^{-2cc}}{c e^{-cc}} = 2 e^{-cc}$$
(ii)
$$2e^{-cc} < \mathcal{I} \qquad T < -\frac{1}{c} \log \left(\frac{\mathcal{I}}{2}\right)$$

$$FPR = P(D=1 | R=0) = P(D=1 | H_{0}) = \alpha$$

$$= P(T < -\frac{1}{c} \log \left(\frac{\mathcal{I}}{2}\right) | H_{0})$$

$$= \int_{-\frac{1}{c}} \log \left(\frac{\mathcal{I}}{2}\right) | H_{0}$$

$$= \int_{0}^{\infty} \left(T < -\frac{1}{c} \log \left(\frac{\mathcal{I}}{2}\right) | H_{0}\right)$$

$$= \int_{0}^{\infty} \left(T < -\frac{1}{c} \log \left(\frac{\mathcal{I}}{2}\right) | H_{0}\right)$$

$$= \int_{0}^{\infty} f_{1}(c) dc = 1 - e^{-2cc} \left(-\frac{1}{c} \log \left(\frac{\mathcal{I}}{2}\right)\right)$$

$$= \int_{0}^{\infty} f_{1}(c) dc = 1 - e^{-2cc}$$

= 1- (1-0)2

# 102 hw1

September 20, 2022

# 1 FA 22 Data 102 Homework 1

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```
[1]: import matplotlib.pyplot as plt
  import numpy as np
  import pandas as pd
  import seaborn as sns
  from datascience import *

from sklearn.model_selection import train_test_split
  from sklearn.preprocessing import LabelEncoder
  from sklearn.linear_model import LogisticRegression
  from scipy.stats import norm
  import timeit
  import hashlib
  %matplotlib inline

sns.set(style="dark")
  plt.style.use("ggplot")
```

## 1.1 Question 2

```
[2]: df = pd.read_csv("policez.csv", index_col = 0)
df
```

```
[2]: x

1 2.411365
2 0.160788
3 -0.852171
4 0.151016
5 1.836084
... ...
2745 2.008162
2746 0.963842
2747 -2.735369
2748 1.074744
```

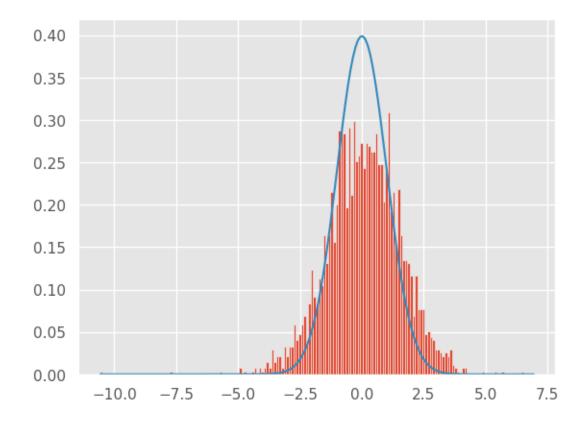
```
2749 -1.978600
```

[2749 rows x 1 columns]

## 1.1.1 a) Normalized Histogram

In one plot, make a normalized histogram of the z-scores and a line plot of the pdf of the theoretical null N(0, 1). Describe how the fit looks.

#### [3]: [<matplotlib.lines.Line2D at 0x7fd5cd23f820>]



The normalized histogram of the z-scores does not fit the line plot of the pdf of the theoretical null N (0, 1) well. There are fewer distributions around the mean(0), and more at around -1.5 and +1.5

#### 1.1.2 b) Compute p-values

Compute p-values  $P_i = \phi(-z_i)$  (where  $\phi$  is the standard normal CDF) and then apply the BH procedure with  $\alpha = 0.2$ . Plot the sorted p-values as well as the decision boundary. How many discoveries did you make?

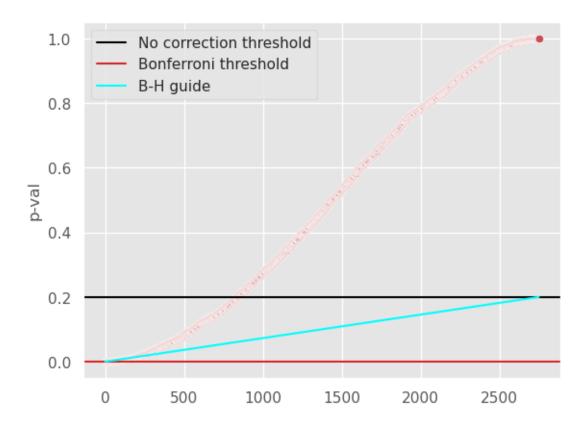
```
[4]: neg_z = -df['x']
     neg_z
[4]: 1
            -2.411365
            -0.160788
     2
     3
             0.852171
     4
            -0.151016
     5
            -1.836084
     2745
            -2.008162
     2746
            -0.963842
     2747
             2.735369
            -1.074744
     2748
     2749
             1.978600
     Name: x, Length: 2749, dtype: float64
[5]: p_val = norm.cdf(neg_z, loc = 0, scale = 1)
     p_val
[5]: array([0.00794646, 0.43613015, 0.80294036, ..., 0.99688448, 0.1412447,
            0.97606948])
[6]: df['p-val'] = p_val
     df
[6]:
                        p-val
                  Х
     1
           2.411365
                     0.007946
     2
           0.160788
                     0.436130
     3
          -0.852171
                     0.802940
     4
           0.151016
                     0.439982
     5
           1.836084
                     0.033173
     2745
           2.008162
                     0.022313
     2746
           0.963842
                     0.167562
     2747 -2.735369
                     0.996884
     2748 1.074744
                     0.141245
     2749 -1.978600
                     0.976069
     [2749 rows x 2 columns]
[7]: p_sorted = df.sort_values(by = ['p-val']).reset_index(drop = True)
     m = len(p_sorted)
```

```
k = np.arange(1, m+1) # index of each test in sorted order
p_sorted['k'] = k
alpha = .2
p_sorted
```

```
[7]:
                           p-val
                                     k
           6.952483 1.794564e-12
    0
                                     1
    1
           6.506210 3.853529e-11
                                     2
    2
           5.703130 5.881356e-09
                                     3
    3
           5.360467 4.150348e-08
                                     4
           4.934229 4.023405e-07
    4
    2744 -4.917371 9.999996e-01 2745
    2745 -4.937951 9.999996e-01 2746
    2746 -5.724645 1.000000e+00 2747
    2747 -7.705945 1.000000e+00 2748
    2748 -10.563937 1.000000e+00 2749
```

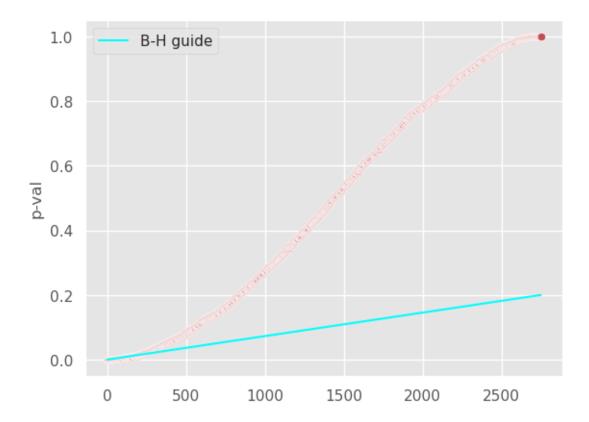
[8]: <matplotlib.legend.Legend at 0x7fc926d71ca0>

[2749 rows x 3 columns]

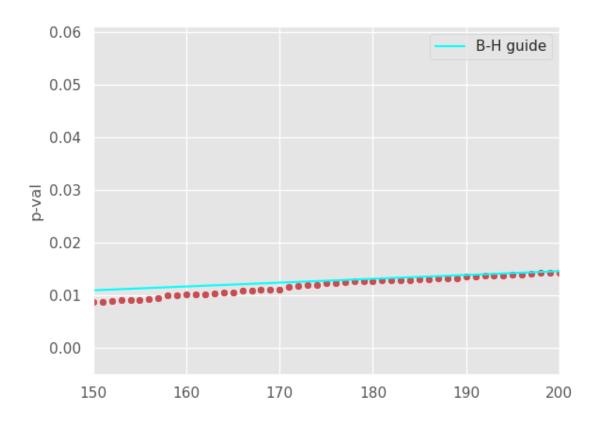


```
[9]: sns.scatterplot(x=k, y=p_sorted['p-val'], color = 'r')
plt.plot(k, k/len(p_sorted)* alpha, label='B-H guide', color='cyan')
plt.legend()
```

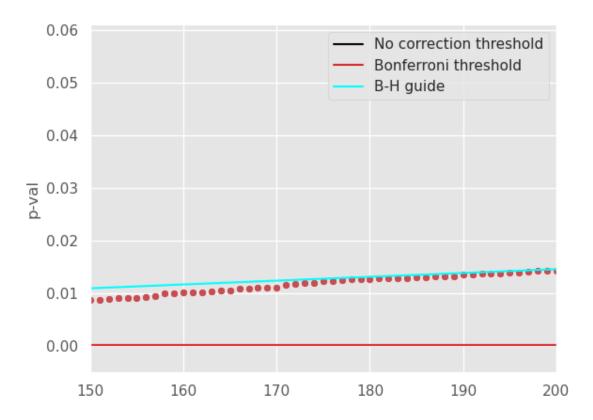
[9]: <matplotlib.legend.Legend at 0x7fc926cf7070>



```
[10]: sns.scatterplot(x=k, y=p_sorted['p-val'], color = 'r')
   plt.plot(k, k/len(p_sorted)* alpha, label='B-H guide', color='cyan')
   plt.legend()
   plt.axis([150, 200, -0.005, .061]);
```



[11]: <matplotlib.legend.Legend at 0x7fc926c4f220>



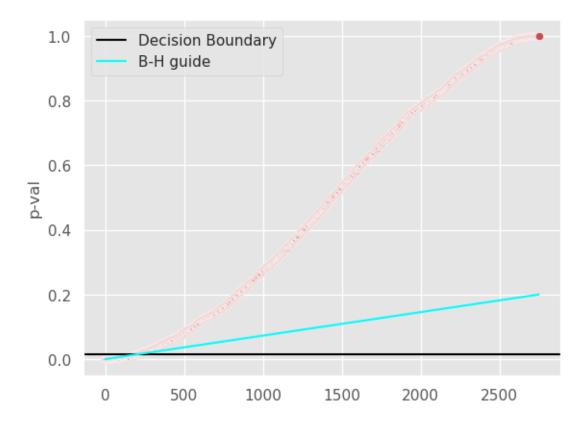
```
[12]: p_sorted['B-H'] = p_sorted['p-val'] - p_sorted['k']/len(p_sorted)* alpha
     p_sorted[p_sorted['B-H']>=-0.00009]
[12]:
                   X
                            p-val
                                      k
                                              B-H
                                      1 -0.000073
     0
            6.952483 1.794564e-12
     201
            2.176679 1.475225e-02
                                     202 0.000056
     202
            2.170774 1.497412e-02
                                     203 0.000205
     203
            2.167356 1.510385e-02
                                     204 0.000262
     204
            2.161422 1.533138e-02
                                     205 0.000417
     2744 -4.917371 9.999996e-01 2745 0.800291
     2745 -4.937951 9.999996e-01
                                   2746 0.800218
     2746 -5.724645 1.000000e+00
                                   2747 0.800146
     2747 -7.705945 1.000000e+00
                                   2748 0.800073
     2748 -10.563937 1.000000e+00 2749 0.800000
     [2549 rows x 4 columns]
[13]: p_sorted[p_sorted['k'] == 201]
[13]:
                       p-val
                               k
                 X
```

200 2.186698 0.014382 201 -0.000241

## **2.2 RESPONSE:** k = 201, $p - val_{B-H} = 0.014382$ , **201 Discoveries were made**

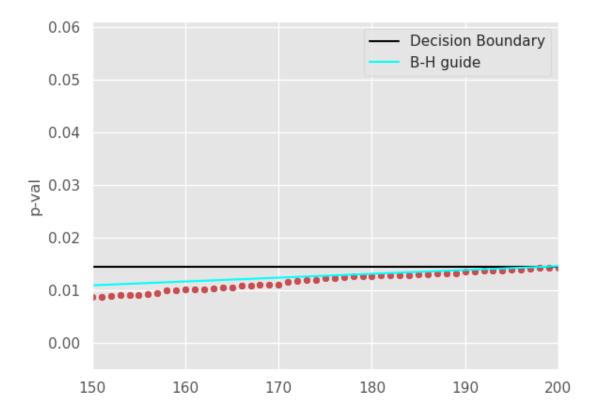
```
[14]: sns.scatterplot(x=k, y=p_sorted['p-val'], color = 'r')
   plt.axhline(0.014382, label='Decision Boundary', color='black')
   plt.plot(k, k/len(p_sorted)* alpha, label='B-H guide', color='cyan')
   plt.legend()
```

#### [14]: <matplotlib.legend.Legend at 0x7fc926c4fac0>



```
[15]: plt.axis([150, 200, -0.005, .061]);
sns.scatterplot(x=k, y=p_sorted['p-val'], color = 'r')
plt.axhline(0.014382, label='Decision Boundary', color='black')
plt.plot(k, k/len(p_sorted)* alpha, label='B-H guide', color='cyan')
plt.legend()
```

[15]: <matplotlib.legend.Legend at 0x7fc926ce2640>



# 1.1.3 C) Empirical Null

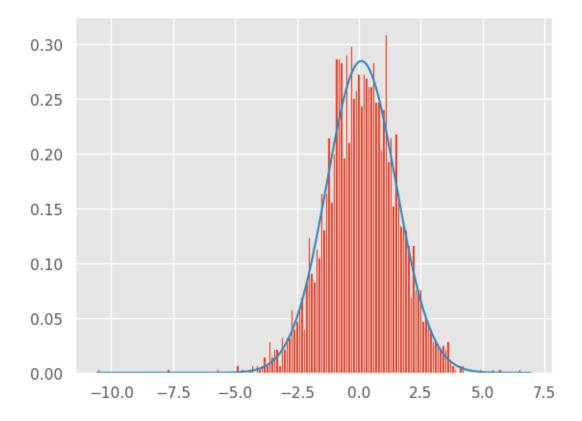
Looking at the data, we can get a better fit to the distribution of z-scores if we use  $N(0.10, 1.40^2)$ , called the empirical null (instead of the theoretical null from part (a)). Repeat steps (a) and (b), treating the empirical null as the null distribution.

```
[16]: df = pd.read_csv("policez.csv", index_col = 0)
df
```

```
[16]:
      1
            2.411365
      2
            0.160788
      3
           -0.852171
      4
            0.151016
      5
            1.836084
            2.008162
      2745
      2746
            0.963842
      2747 -2.735369
      2748
            1.074744
      2749 -1.978600
```

# [2749 rows x 1 columns]

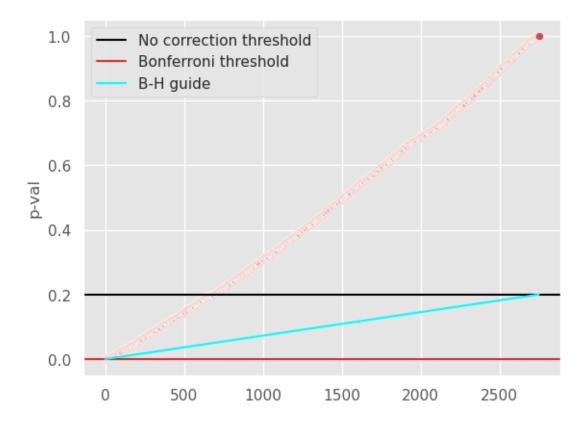
# [17]: [<matplotlib.lines.Line2D at 0x7fc926cdef40>]



```
[18]: neg_z = -df['x']
      neg_z
[18]: 1
             -2.411365
      2
             -0.160788
      3
              0.852171
      4
             -0.151016
      5
             -1.836084
      2745
             -2.008162
      2746
             -0.963842
      2747
              2.735369
```

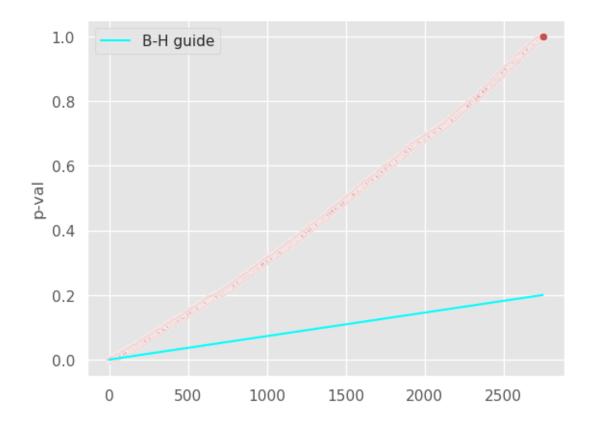
```
2748
            -1.074744
     2749
             1.978600
     Name: x, Length: 2749, dtype: float64
[19]: p_val = norm.cdf(neg_z, loc = 0.10 , scale = 1.40)
     df['p-val'] = p_val
     df
[19]:
                        p-val
           2.411365 0.036420
     1
     2
           0.160788 0.426114
     3
          -0.852171 0.704458
     4
           0.151016 0.428852
     5
           1.836084 0.083345
     2745 2.008162 0.066055
     2746 0.963842 0.223661
     2747 -2.735369 0.970110
     2748 1.074744 0.200706
     2749 -1.978600 0.910179
     [2749 rows x 2 columns]
[20]: p_sorted = df.sort_values(by = ['p-val']).reset_index(drop = True)
     m = len(p_sorted)
     k = np.arange(1, m+1) # index of each test in sorted order
     p_sorted['k'] = k
     alpha = .2
     p_sorted
[20]:
                             p-val
                                       k
                   X
     0
            6.952483 2.358410e-07
                                       1
     1
            6.506210 1.186659e-06
                                       2
     2
            5.703130 1.698380e-05
                                       3
     3
            5.360467 4.803008e-05
                                       4
     4
            4.934229 1.616499e-04
     2744 -4.917371 9.997102e-01 2745
     2745 -4.937951 9.997255e-01 2746
     2746 -5.724645 9.999706e-01 2747
     2747 -7.705945 1.000000e+00 2748
     2748 -10.563937 1.000000e+00 2749
     [2749 rows x 3 columns]
[21]: sns.scatterplot(x=k, y=p_sorted['p-val'], color = 'r')
     plt.axhline(alpha, label='No correction threshold', color='black')
```

[21]: <matplotlib.legend.Legend at 0x7fc92699c160>

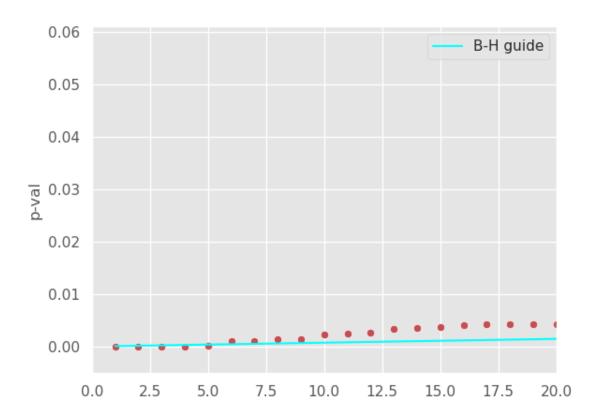


```
[22]: sns.scatterplot(x=k, y=p_sorted['p-val'], color = 'r')
plt.plot(k, k/len(p_sorted)* alpha, label='B-H guide', color='cyan')
plt.legend()
```

[22]: <matplotlib.legend.Legend at 0x7fc9268f7340>



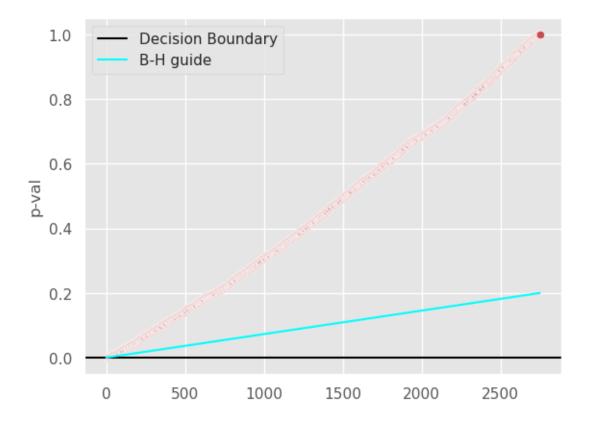
```
[23]: sns.scatterplot(x=k, y=p_sorted['p-val'], color = 'r')
  plt.plot(k, k/len(p_sorted)* alpha, label='B-H guide', color='cyan')
  plt.legend()
  plt.axis([0,20, -0.005, .061]);
```



```
[24]: p_sorted['B-H'] = p_sorted['p-val'] - p_sorted['k']/len(p_sorted)* alpha
     p_sorted[p_sorted['B-H']>=-0.00009]
[24]:
                             p-val
                                              В-Н
                                       k
     0
            6.952483 2.358410e-07
                                       1 -0.000073
     5
            4.224927 1.003367e-03
                                       6 0.000567
            4.214797 1.028083e-03
                                          0.000519
     6
     7
            4.105680 1.332025e-03
                                          0.000750
     8
            4.067040 1.457999e-03
                                         0.000803
     2744 -4.917371 9.997102e-01
                                   2745 0.800001
     2745 -4.937951 9.997255e-01
                                    2746 0.799944
     2746 -5.724645 9.999706e-01
                                    2747
                                          0.800116
     2747 -7.705945 1.000000e+00
                                    2748 0.800073
     2748 -10.563937 1.000000e+00
                                    2749 0.800000
     [2745 rows x 4 columns]
     p_sorted[p_sorted['k'] == 5]
[25]:
[25]:
                     p-val k
     4 4.934229 0.000162 5 -0.000202
```

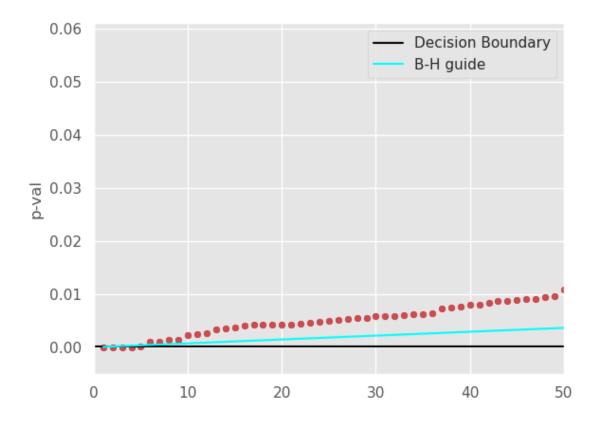
```
[26]: sns.scatterplot(x=k, y=p_sorted['p-val'], color = 'r')
   plt.axhline(0.000162, label='Decision Boundary', color='black')
   plt.plot(k, k/len(p_sorted)* alpha, label='B-H guide', color='cyan')
   plt.legend()
```

[26]: <matplotlib.legend.Legend at 0x7fc926879b80>



```
[27]: plt.axis([0, 50, -0.005, .061]);
sns.scatterplot(x=k, y=p_sorted['p-val'], color = 'r')
plt.axhline(0.000162, label='Decision Boundary', color='black')
plt.plot(k, k/len(p_sorted)* alpha, label='B-H guide', color='cyan')
plt.legend()
```

[27]: <matplotlib.legend.Legend at 0x7fc92679b220>



# 2.3 RESPONSE: k = 5, $p - val_{B-H} = 0.000162$ , 5 Discoveries were made

## 1.2 d) Theoretical Null

Assume the observed data is normally distributed, there is some racial bias existing, and a permutation test will be performed to get the empirical null. Limitation of Theoretical null is that it does not fit the data well, and has the assumption of normally distributed that is hard to reach. There might be covariants, or co-linearity between variables, also Omitted Variable Bias.

I would use the empirical null, because it is a valid method and only have 5 cases discovered, which is much smaller than using the theoretical null. Other limitations include small sample size, ambiguous standard and variation/bias during data collection

## 1.3 Question 3 p-values, FDR and FWER

```
22
      | 58
             | 1
                         | 1
                                          | Female | 40
                                                                      1 0
77
      | 23
             | 1
                         1 0
                                          | Female | 35
                                                                      10
80
      | 27
             1 0
                         1 0
                                          | Male
                                                                      1 0
      | 39
             | 1
                         | 1
32
                                          | Male
                                                    | 40
                                                                      10
59
      1 24
             1 0
                         1 0
                                          | Male
                                                    1 40
                                                                      1 0
                         1 0
                                          | Female | 40
57
      I 49
             | 1
                                                                      1 0
35
      I 35
             1 0
                         I 1
                                          | Male
                                                   I 50
                                                                      1 0
      I 40
                         1 1
                                                                      10
21
             | 1
                                          | Male
                                                    | 40
... (80 rows omitted)
```

## 1.3.1 a) Permutation Test Function

```
[43]: def avg_difference_in_means(dummy, numeral):
    differences = make_array()
    observed = mean_different(df, dummy, numeral)
    repetitions = 25000
    for i in np.arange(repetitions):
        new_difference = one_simulation(dummy, numeral)
        differences = np.append(differences, new_difference)
    empirical_p = np.count_nonzero(differences >= observed) / repetitions
    return empirical_p, differences
```

#### 1.3.2 b) 8 p-values

```
[44]: gender_age, gender_age_diff = avg_difference_in_means("Gender", "Age")
gender_hpw, gender_hpw_diff = avg_difference_in_means("Gender", "Hours per

→Week")
```

```
[45]: gender_age_diff
```

```
[45]: array([ 0.35817308, 2.57572115, 3.0625 , ..., -3.10336538,
             -3.15745192, 5.55048077])
[32]: ps_age = avg_difference_in_means("Post HS?", "Age")
      ps_hpw = avg_difference_in_means("Post HS?", "Hours per Week")
[34]: married_age = avg_difference_in_means("Ever Married?", "Age")
      married_hpw = avg_difference_in_means("Ever Married?", "Hours per Week")
[35]: k_age = avg_difference_in_means("50K?", "Age")
      k_hpw = avg_difference_in_means("50K?", "Hours per Week")
[36]: print("P-values for average difference in Age between Male and Female is:", ___
       ⇔gender_age)
      print("P-values for average difference in Hours Per Week between Male and⊔

→Female is:", gender_hpw)

      print("P-values for average difference in Age between people went to HS and ⊔

→Didn't is:", ps_age)

      print("P-values for average difference in Hours Per Week between people went to...

→HS and Didn't is:", ps_hpw)
      print("P-values for average difference in Age between Married vs not Married is:

→", married_age)

      print("P-values for average difference in Hours Per Week between Married vs not⊔

→Married is:", married_hpw)

      print("P-values for average difference in Age between people with 50K+ Salaries⊔
       →and people without is:", k_age)
      print("P-values for average difference in Hours Per Week between people with⊔
       ⇒50K+ Salaries and people without is:", k_hpw)
     P-values for average difference in Age between Male and Female is: 0.0218
     P-values for average difference in Hours Per Week between Male and Female is:
     0.00588
     P-values for average difference in Age between people went to HS and Didn't is:
     P-values for average difference in Hours Per Week between people went to HS and
     Didn't is: 0.53764
     P-values for average difference in Age between Married vs not Married is: 0.0
     P-values for average difference in Hours Per Week between Married vs not Married
     is: 0.05072
     P-values for average difference in Age between people with 50K+ Salaries and
     people without is: 0.0
     P-values for average difference in Hours Per Week between people with 50K+
     Salaries and people without is: 0.12248
```

#### 1.3.3 c) Naive P-Val Threshold: 0.05

With a Naive P-value Threshold 0.05, following null hypothesis can be rejected: - No age difference between Male and Female - No difference in Working hour per week between Male and Female - No age difference between people married vs not married - No difference in Working hour per week between people married vs not married - No age difference between people making 50k+ salary vs people making less than 50k

## 1.3.4 d) FWER $\leq 0.05$ ,

```
[]: x = 0.05/25000
print("P-val threshold with Bonferroni Correction is:", x)
```

P-value threshold with Bonferroni correction is 0.000002. Under this p-value threshold, following null hypothesis can be rejected: - No age difference between people married vs not married - No age difference between people making 50k+ salary vs people making less than 50k

## 1.3.5 e) FDR <= 0.05

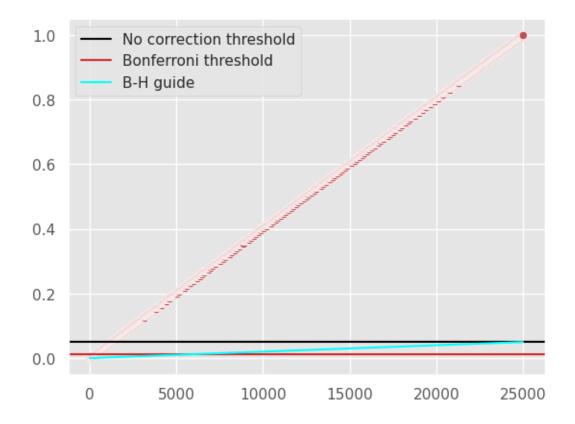
B-H algorithm gives a p-val in between  $\alpha$  (0.05) and Bonferroni Correction. So two null hypothesis rejected under p-val with Bonferroni Correction will be rejected with B-H Algorithm's p-val threshold.

Need to calculate the B-H p-val of the 3 other Null hypothesis, which include: - Age Difference between Male and Female - Working Hour per Week between Male and Female - Working Hour per Week between people married vs not married

```
[87]: def b_h_avg_difference_in_means(dummy, numeral, alpha):
          differences = make array()
          observed = mean_different(df, dummy, numeral)
          repetitions = 25000
          for i in np.arange(repetitions):
              new_difference = one_simulation(dummy, numeral)
              differences = np.append(differences, new_difference)
          p_val = make_array()
          for i in differences:
              p_val = np.append(p_val, (np.count_nonzero(differences >= i) /_
       →repetitions))
          p_val.sort()
          result table = Table().with columns("p-value", p val)
          k = np.arange(1, repetitions+1)
          result_table['k'] = k
          result_table['b_h'] = result_table['k']/len(p_val)* alpha
          result_table['diff'] = result_table['p-value'] - result_table['b_h']
          return result_table
```

```
[88]: gender_age_bh = b_h_avg_difference_in_means("Gender", "Age", 0.05)
[89]: gender_age_bh
[89]: p-value | k
                     | b_h
                               | diff
      4e-05
              1
                     2e-06
                               | 3.8e-05
      8e-05
              1 2
                     l 4e-06
                               1 7.6e-05
      0.00012 | 3
                     | 6e-06
                               0.000114
      0.00016 | 4
                     I 8e-06
                               1 0.000152
      0.0002 | 5
                     l 1e-05
                               0.00019
      0.00024 | 6
                     | 1.2e-05 | 0.000228
      0.00028 | 7
                     | 1.4e-05 | 0.000266
      0.00032 | 8
                     | 1.6e-05 | 0.000304
      0.0004 | 9
                     | 1.8e-05 | 0.000382
      0.0004 | 10
                     l 2e-05
                               1 0.00038
      ... (24990 rows omitted)
[90]: gender hour bh = b h avg difference in means ("Gender", "Hours per Week", 0.05)
[91]: gender_hour_bh.item(100)
     /opt/conda/lib/python3.9/site-packages/datascience/tables.py:222: FutureWarning:
     Implicit column method lookup is deprecated.
       warnings.warn("Implicit column method lookup is deprecated.", FutureWarning)
[91]: p-value | k
                     b_h
                                | diff
      0.0042 | 101 | 0.000202 | 0.003998
[92]: married_hour_bh = b_h_avg_difference_in_means("Ever Married?", "Hours per_
       ⇔Week", 0.05)
[93]: married_hour_bh
[93]: p-value | k
                     | b_h
                               diff
                               | 3.8e-05
      4e-05
              | 1
                     | 2e-06
      8e-05
              | 2
                     l 4e-06
                               | 7.6e-05
                     l 6e-06
      0.00012 | 3
                               0.000114
      0.0002 | 4
                     | 8e-06
                               0.000192
      0.0002 | 5
                     l 1e-05
                               0.00019
      0.00028 | 6
                     | 1.2e-05 | 0.000268
      0.00028 | 7
                     | 1.4e-05 | 0.000266
      0.00032 | 8
                     | 1.6e-05 | 0.000304
      0.00036 | 9
                     | 1.8e-05 | 0.000342
      0.0004 | 10
                     l 2e-05
                               1 0.00038
      ... (24990 rows omitted)
```

[94]: <matplotlib.legend.Legend at 0x7fc92428db20>



t)
FWER is the lowest bound because it is calculated assuming all cases are exclusive, but that's usually not true. And it is about
the probability of at least one FP, which makes this value extremely small. The FWER has a fixed p-val cut off for different
features, but B-H gives different p-val cut offs. As a result, FWER rejects more null hypothesis.

g) Marriage Status can be binarized according to current marriage status instead of ever married. This is because