Computational Applications in International Trade: Problem Set

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1 Hat-algebra assignment

This task asks you to compute a multi-country, multi-sector hat-algebra exercise. The representative agent in country j seeks to maximize:

$$C_j = \Pi_s (C_j^s)^{\beta_j^s} \tag{1}$$

where β_j^s are expenditure share parameters satisfying $\sum_s \beta_j^s = 1$. Sector goods are sourced from different countries following the Armington assumption:

$$(C_j^s)^{\frac{\sigma^s-1}{\sigma^s}} = \sum_i (c_{ij}^s)^{\frac{\sigma^s-1}{\sigma^s}} \tag{2}$$

where $\epsilon^s = \sigma^s - 1$ is the trade elasticity. This implies country j's expenditure on sector s goods from country i:

$$X_{ij}^s = p_{ij}^s c_{ij}^s = \frac{(p_{ij}^s)^{1-\sigma^s}}{\sum_i (p_{ij}^s)^{1-\sigma^s}} X_j^s \tag{3}$$

where X_s^j is country j's total expenditure on sector s goods.
Country j sector s' price index:

$$(P_j^s)^{1-\sigma^s} = \sum_i (p_{ij}^s)^{1-\sigma^s} \tag{4}$$

Country j's consumer price index:

$$P_j = \prod_s (\frac{P_j^s}{\beta_j^s})^{\beta_j^s} \tag{5}$$

Assume countries use domestic labor as the only factor to produce and labor is freely mobile across sectors. Country i is endowed with total labor L_i. The labor market is perfectly competitive. The production function is linear:

$$Q_i^s = A_i^s L_i^s \tag{6}$$

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Further, goods market features perfect competition. This implies:

$$p_{ij}^s = \frac{w_i \tau_{ij}^s (1 + t_{ij}^s)}{A_i^s} \tag{7}$$

where τ_{ij}^s is an iceberg natural trade barrier and t_{ij}^s is tariff rate. To close the model, the goods market clearing condition implies:

$$Y_i^s = \sum_j \frac{1}{1 + t_{ij}^s} X_{ij}^s \tag{8}$$

The labor market clearing condition implies: $L_i = \sum_s L_i^s$.
Total tariff revenue collected by country j in sector s:

$$T_j^s = \sum_i \frac{t_{ij}^s}{1 + t_{ij}^s} X_{ij}^s \tag{9}$$

Budget balance condition:

$$X_j = Y_j + T_j \tag{10}$$

Assume trade is balanced so there is no trade deficit. (One could think about trade deficits as exogenous borrowing and lending among countries.)

1. Change tariffs from $\{t_{ij}^s\}$ to $\{t_{ij}^{s'}\}$. Labor endowment L_i, technologies A_i^s and natural trade barriers τ_{ij}^s are exogenous and held fixed across the two equilibrium. Rewrite the system in "hats" to eliminate L_i, A_i^s and τ_{ij}^s. The reduced system should be equations with only $\hat{w}_i$ as endogenous variables and λ_{ij}^s, Y_i, $(1 + \hat{t_{ij}^s})$, $t_{ij}^{s'}$, σ^s and β_j^s as parameters. λ_{ij}^s is country j's expenditure share on country i for goods s.

2. Take the model to the data. You can find bilateral and sector level trade flows data, Xisj.csv in the attachment. Each row denotes the country of origin and sector, and each column denotes the receiving country. The data is derived from WIOD2008 `http://www.wiod.org/database/wiots13` with negative values, zeros and trade deficits eliminated. It is then collapsed into 34 countries (regions) and 31 sectors in order to reduce the computation for future steps. The concordance for countries and sectors can be found in the attachment. Load the trade elasticity numbers for the 31 sectors. These numbers come from Caliendo and Parro (2015).

   (a) Compute β_n^s, country n's expenditure on sector s, which equals X_n^s / X_n

   (b) Market shares: $\lambda_{nn'}^s = X_{nn'}^s / X_{n'}^s$

3. Assume a trade war breaks out between US and China. The two countries impose 25% reciprocal tariff on imports in all sector from each other. Please compute, starting from the factual equilibrium, the impact of the trade war on welfare of US, China and all other countries/regions in the world. Note a country's welfare is measured with its real income, $\frac{w_i}{P_i}$.

2 Combinatorial choice problem

Implement the Jia (2008) algorithm to solve a combinatorial discrete choice problem with complementarities or the Eckert et al. (2017) algorithm to solve a combinatorial discrete choice problem with substitutes:

(a) Consider your own setting. Incorporate some kind of firm (or agent) heterogeneity, for example in firm-productivity, fixed cost, etc. Write down the profit function to be maximized.

(b) Set the parameters of your problem to values of your choice and specify the distribution(s) from which heterogeneous firm / agent types are drawn.

(c) Code your optimizer. Recall, this will give you bounds in which the optimal solution is contained. If the bounds are not too large (you can evaluate the profits from all feasible choices in between and pick the solution that leads to the largest profits).

(d) Run your optimizer on the problem. Produce a Table similar to Table A.4. in Antras et al. (2017). How wide are your bounds? How does this change if you vary the parameters set in 2)?

References

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