Computational Applications in International Trade, Lecture 1

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Course Overview

- Three lectures on computational applications in international trade

1. Lecture 1: Gravity Trade model, Solution approach for counterfactuals (hat algebra), Dynamic hat algebra
   - Eaton and Kortum (2002); Dekle, Eaton, and Kortum (2007); Caliendo, Dvorkin, and Parro (forthcoming)
2. Lecture 2: Interdependent Discrete Choice Problems
   - Antras, Fort, and Tintelnot (2017), Jia (2008), Arkolakis and Eckert (2018)
3. Lecture 3: Endogenous Production Networks
   - Tintelnot, Kikkawa, Mogstad, and Dhyne (2018), Lim (2018)

Problem Set

1. Application of hat algebra: Simulate US-China trade conflict
2. Interdependent discrete choice problems: Document performance of Jia (2008) algorithm in your own application

Gravity trade model

- A simple yet powerful framework for quantitative analysis. Widely used in practice.
- We can extend the gravity model to include
  - Multiple sectors
  - Multiple factors of production
  - Intermediate inputs
  - Domestic geography, internal migration, etc

Counterfactuals in Trade using with Hat Algebra

Counterfactuals and estimation

- Often we are interested in predictions "what would happen to Y if X or β changes?"
- Usually this involves estimating various parameters and then re-computing the new equilibrium for changed parameters / covariates
- However, in some contexts we can skip the estimation step and directly move to counterfactuals:
  - Dekle, Eaton, and Kortum (2008)
  - Still need an estimate of trade elasticity
  - You can answer interesting questions simply by solving a system of non-linear equations!

Gravity Trade model (in one slide)

- Recall that the Eaton and Kortum (2002) model implies the following expression for aggregate trade flows from country j to country i:

$$X_{ij} = \frac{T_j (w_j d_{ij})^{-\theta}}{\sum_{i=1}^{N} T_j (w_j d_{ij})^{-\theta}} X_i$$

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- This leads to a system of non-linear equations to solve for wages,

$$w_j L_j = \sum_i \frac{T_j (w_j d_{ij})^{-\theta}}{\sum_k T_k \left(w_k d_{ik}\right)^{-\theta}} w_i L_i.$$

Comparative statics (Dekle, Eaton, and Kortum, 2008)

- Consider a shock to labor endowments, trade costs, or productivity. One could compute the original equilibrium, the new equilibrium and compute the changes in endogenous variables.

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- Consider a shock to labor endowments, trade costs, or productivity. One could compute the original equilibrium, the new equilibrium and compute the changes in endogenous variables.
- But there is a simpler way that uses only information for observables in the initial equilibrium, trade shares and GDP; the trade elasticity, θ; and the exogenous shocks.

Comparative statics (Dekle, Eaton, and Kortum, 2008)

- First solve for changes in wages by solving

$$\hat{w}_j \hat{L}_j Y_j = \sum_i \frac{\pi_{ij} \hat{T}_j \left(\hat{w}_j \hat{d}_{ij} \right)^{-\theta}}{\sum_k \pi_{ik} \hat{T}_k \left(\hat{w}_k \hat{d}_{ik} \right)^{-\theta}} \hat{w}_i \hat{L}_i Y_i$$

and then get changes in trade shares from

$$\hat{\pi}_{ij} = \frac{\hat{T}_j \left(\hat{w}_j \hat{d}_{ij} \right)^{-\theta}}{\sum_k \pi_{ik} \hat{T}_k \left(\hat{w}_k \hat{d}_{ik} \right)^{-\theta}}.$$

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- From here, one can compute welfare changes as described further below.

Comparative statics (Dekle, Eaton, and Kortum, 2008)

- To show this, note that trade shares are

$$\pi_{ij} = \frac{T_j \left(w_j d_{ij}\right)^{-\theta}}{\sum_k T_k \left(w_k d_{ik}\right)^{-\theta}} \text{ and } \pi'_{ij} = \frac{T'_j \left(w'_j d'_{ij}\right)^{-\theta}}{\sum_k T'_k \left(w'_k d'_{ik}\right)^{-\theta}}.$$

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- Letting $\hat{x} \equiv x'/x$, then we have

$$\hat{\pi}_{ij} = \frac{\hat{T}_j \left(\hat{w}_j \hat{d}_{ij}\right)^{-\theta}}{\sum_k T'_k \left(w'_k d'_{ik}\right)^{-\theta} / \sum_\ell T_\ell \left(w_\ell d_{i\ell}\right)^{-\theta}}$$

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$$\begin{aligned}
\hat{\pi}_{ij} &= \frac{\hat{T}_j \left(\hat{w}_j \hat{d}_{ij}\right)^{-\theta}}{\sum_k T'_k \left(w'_k d'_{ik}\right)^{-\theta} / \sum_\ell T_\ell \left(w_\ell d_{i\ell}\right)^{-\theta}} \\
&= \frac{\hat{T}_j \left(\hat{w}_j \hat{d}_{ij}\right)^{-\theta}}{\sum_k \hat{T}_k \left(\hat{w}_k \hat{d}_{ik}\right)^{-\theta} T_k \left(w_k d_{ik}\right)^{-\theta} / \sum_\ell T_\ell \left(w_\ell d_{i\ell}\right)^{-\theta}} \\
&= \frac{\hat{T}_j \left(\hat{w}_j \hat{d}_{ij}\right)^{-\theta}}{\sum_k \pi_{ik} \hat{T}_k \left(\hat{w}_k \hat{d}_{ik}\right)^{-\theta}}.
\end{aligned}$$

Comparative statics (Dekle, Eaton, and Kortum, 2008)

- On the other hand, for equilibrium we have

$$w_j' L_j' = \sum_i \pi_{ij}' w_i' L_i' = \sum_i \hat{\pi}_{ij} \pi_{ij} w_i' L_i'$$

- Letting $Y_i \equiv w_i L_i$ and using the result above for $\hat{\pi}_{ij}$ we get

$$\hat{w}_j \hat{L}_j Y_j = \sum_i \frac{\pi_{ij} \hat{T}_j \left(\hat{w}_j \hat{d}_{ij} \right)^{-\theta}}{\sum_k \pi_{ik} \hat{T}_k \left(\hat{w}_k \hat{d}_{ik} \right)^{-\theta}} \hat{w}_i \hat{L}_i Y_i$$

- This forms a system of N equations in N unknowns, $\hat{w}_j$, from which we can get $\hat{w}_j$ as a function of shocks and initial observables (establishing some numeraire). Here π_{ij} and Y_j are data and we know $\hat{d}_{ij}$, $\hat{T}_j$, $\hat{L}_j$, as well as θ.

Welfare (Dekle, Eaton and Kortum, 2008)

- Recall that $p_i = \gamma \Phi_i^{-1/\theta}$ and $\pi_{ii} = \frac{T_i w_i^{-\theta}}{\Phi_i}$, so

$$\omega_i \equiv w_i / p_i = \gamma^{-1} T_i^{1/\theta} \pi_{ii}^{-1/\theta}.$$

- Hence,

$$\hat{\omega}_i = \left(\hat{T}_i \right)^{1/\theta} \hat{\pi}_{ii}^{-1/\theta}$$

- To compute the implications for welfare of a foreign shock, simply impose that $\hat{L}_i = \hat{T}_i = 1$, solve the system above to get $\hat{w}_j$ and get the implied $\hat{\pi}_{ii}$ through

$$\hat{\pi}_{ij} = \frac{\hat{T}_j \left(\hat{w}_j \hat{d}_{ij} \right)^{-\theta}}{\sum_k \pi_{ik} \hat{T}_k \left(\hat{w}_k \hat{d}_{ik} \right)^{-\theta}}.$$

- Remember, we only needed data for GDP, trade shares, and the knowledge of the trade elasiticity parameter for this.

Caliendo, Dvorkin, and Parro (forthcoming)

Outline

- Dynamic model with migration and trade. Tractable way to conduct counterfactuals
- Recent application: Balboni (2018)
- No capital accumulation. Assume perfect foresight.
- We will go over the paper in various steps:
  1. Dynamic model of migration only
  2. Adding production and trade
  3. Dynamic hat algebra
  4. Full model: multiple sectors, non-employment option
  5. Application: Effect of the rise of China on local labor markets in the US

A simple model of migration

- Start with a simple model of migration dynamics. Take wages as given.
- Dynamic discrete choice problem
  - In response to shocks, worker choose whether to remain where she is or to move to another location
  - If the worker moves, she will pay a cost, which has two components:
    - A portion that is the same for all workers making the same move (moving costs, learning costs, etc.)
    - A time-varying idiosyncratic cost or preference (personal situation)

Idiosyncratic shocks

- No capital accumulation - dynamics arise from idiosyncratic shocks
- Idiosyncratic shocks rationalize some observed labor-market behavior:
  - First, gross flows are an order of magnitude larger than net flows, implying large numbers of workers moving in opposite directions at the same time
  - Evidence shows that a significant fraction of workers who change jobs voluntarily move to jobs which pay less than the job the worker left behind
  - These idiosyncratic costs will generate dynamics

Simple model

- N locations indexed by i and n
- The value of a household in location n at time t given by

$$\mathsf{v}_t^n = U(C_t^n) + \max_{\{i\}_{i=1}^N} \left\{ \beta E \left[\mathsf{v}_{t+1}^i \right] - \tau^{n,i} + \nu \epsilon_t^i \right\},$$

$$s.t.\ U(C_t^n) \equiv \log(w_t^n)$$

  - $\beta \in (0,1)$ discount factor
  - $\tau^{n,i}$ additive, *time invariant* migration costs to i from n
  - ϵ_t^i are stochastic *i.i.d idiosyncratic* taste shocks
    - $\epsilon \sim$ Type-I Extreme Value distribution with zero mean
    - $\nu > 0$ is the dispersion of taste shocks
- *Employed* households supply a unit of labor inelastically
  - Receive the competitive market wage w_t^n

Households' problem - Dynamic discrete control

- Denote by $V_t^n \equiv E[\mathsf{v}_t^n]$ to the expected (expectation over ϵ) lifetime utility of a worker in n

- The value of a household in location n at time t given by

$$E\left[\mathsf{v}_t^n\right] = E\left[U(C_t^n) + \max_{\{i\}_{i=1}^N}\left\{\beta E\left[\mathsf{v}_{t+1}^i\right] - \tau^{n,i} + \nu\epsilon_t^i\right\}\right],$$

- We seek to solve for

$$\Phi_t^n = E\left[\max_{\{i\}_{i=1}^N}\left\{\beta E\left[\mathsf{v}_{t+1}^i\right] - \tau^{n,i} + \nu\,\epsilon_t^i\right\}\right]$$

Households' problem - Dynamic discrete control

- Assumption, Type-I Extreme Value

$$F(\epsilon) = \exp\left(-\exp\left(-\epsilon - \bar{\gamma}\right)\right)$$

- Then

$$\Phi_t^n = \nu \log\left[\sum_{i=1}^{N} \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}\right]$$

Standard type 1 extreme value distribution result (see book by Train, 2009)

Households' problem - Dynamic discrete choice

- Define $\mu_t^{n,i}$ as the fraction of workers that reallocate from location n to i
- This fraction is equal to the probability that a given worker moves from n to i at time t. Formally,

$$\mu_t^{n,i} = \Pr\left(\frac{\beta V_{t+1}^i - \tau^{n,i}}{\nu} + \epsilon_t^i \geq \max_h\{\frac{\beta V_{t+1}^h - \tau^{n,h}}{\nu} + \epsilon_t^h\}\right).$$

- Fraction of workers that reallocate from location n to i

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^N \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

Households' problem - Dynamic discrete control

Equilibrium conditions:

- The expected (expectation over ϵ) lifetime utility of a worker at n

$$V_t^n = U(C_t^n) + \nu \log \left[\sum_{i=1}^N \exp \left(\beta V_{t+1}^i - \tau^{n,i} \right)^{1/\nu} \right]$$

- Fraction of workers that reallocate from market n to i

$$\mu_t^{n,i} = \frac{\exp \left(\beta V_{t+1}^i - \tau^{n,i} \right)^{1/\nu}}{\sum_{h=1}^N \exp \left(\beta V_{t+1}^h - \tau^{n,h} \right)^{1/\nu}}$$

- Finally, evolution of the distribution of labor across markets

$$L_{t+1}^n = \sum_{i=1}^N \mu_t^{i,n} L_t^i$$

- Wages, taken as given $\{w_t^n\}_{t=0}^{\infty}$

Welfare

- Seek to obtain a simple expression to evaluate the welfare gains from migration
- Re-writing the value of being in a particular n is given by

$$v_t^n = \underbrace{\log C_t^n}_{\text{current period utility}} + \underbrace{\beta E\left[\mathsf{v}_{t+1}^n\right]}_{\text{value of staying}} + \underbrace{\max_{\{i\}_{i=1}^N} \left\{\beta E\left[\mathsf{v}_{t+1}^i - \mathsf{v}_{t+1}^n\right] - \tau^{n,i} + \nu\, \epsilon_t^i\right\}}_{\text{option value of migration}}$$

- As before, taking the expected value of this equation, we can write the expected lifetime utility of being in n at time t as

$$V_t^n = \log C_t^n + \beta V_{t+1}^n + \nu \log\left[\sum_{i=1}^N \exp\left(\beta\left(V_{t+1}^i - V_{t+1}^n\right) - \tau^{n,i}\right)^{1/\nu}\right]$$

- Use

$$\mu_t^{n,n} = \frac{\exp\left(\beta V_{t+1}^n\right)^{1/\nu}}{\sum_{h=1}^N \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}},$$

divide numerator and denominator by $\left(\beta V_{t+1}^n\right)^{1/\nu}$, take logs, and rearrange to obtain:

$$\nu \log \sum_{h=1}^N \exp\left(\beta\left(V_{t+1}^h - V_{t+1}^n\right) - \tau^{n,h}\right)^{1/\nu} = -\nu \log \mu_t^{n,n}.$$

Welfare

- Plugging this equation into the value function, we get

$$V_t^n = \log C_t^n + \beta V_{t+1}^n - \nu \log \mu_t^{n,n}$$

- Finally, iterating this equation forward we obtain

$$V_0^n = \sum_{t=0}^{\infty} \beta^t \log \frac{w_t^n}{(\mu_t^{n,n})^{\nu}}$$

  - $\mu_t^{n,n}$ summarizes the option value of migration

Trade, migration, and labor market dynamics

Trade and labor market dynamics

- Next: introduce international trade into the model
- Expand description of the production structure
  - Determine wages such that each labor markets clears
  - Given real wages, labor supply determined as before
  - Production structure and international trade will determine labor demand
  - Prices endogenously determined

Households' - Dynamic problem

- As before, equilibrium conditions:
- The expected (expectation over ϵ) lifetime utility of a worker at n

$$V_t^n = U(C_t^n) + \nu \log \left[\sum_{i=1}^N \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu} \right]$$

but now $\quad U(C_t^n) \equiv \log(w_t^n / P_t^n)$

- Fraction of workers that reallocate from market n to i

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^N \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

- Finally, evolution of the distribution of labor across markets

$$L_{t+1}^n = \sum_{i=1}^N \mu_t^{i,n} L_t^i$$

Production - Static sub-problem

- At each time period, t, simple gravity trade structure
  - Let X_t^n denote the total expenditure on final goods in n
  - Goods market clearing condition is given by

$$X_t^n = w_t^n L_t^n$$

  - The share of total expenditure in market n on goods from i is given by

$$\pi_t^{n,i} = \frac{A_t^i [w_t^i \kappa_t^{n,i}]^{-\theta}}{\sum_{h=1}^N A_t^h [w_t^h \kappa_t^{n,h}]^{-\theta}}$$

- Labor market clearing in n is

$$w_t^n L_t^n = \sum_{i=1}^N \pi_t^{i,n} X_t^i,$$

- Assume balanced trade (for now).

Production - Static sub-problem

- Price index:

$$P_t^n = \bar{\gamma}\left[\sum_{i=1}^{N} A_t^i \left[w_t^i \kappa_t^{n,i}\right]^{-\theta}\right]^{-1/\theta}$$

- Real wages:

$$\frac{w_t^n}{P_t^n} = \left(\pi_t^{n,n} / T_t^{n,n}\right)^{-1/\theta},$$

where $T_t^{n,i} \equiv \bar{\gamma}^{-\theta} A_t^i \left(\kappa_t^{n,i}\right)^{-\theta}$

Welfare

- Now $\log C_t^n = \log w_t^n / P_t^n$, therefore

$$V_0^n = \sum_{t=0}^{\infty} \beta^t \log \frac{\left(\pi_t^{n,n} / T_t^{n,n}\right)^{-\frac{1}{\theta}}}{\left(\mu_t^{n,n}\right)^{\nu}} = \frac{\text{gains from trade}}{\text{gains from migration}}$$

- Sufficient statistic to measure welfare gains from trade and migration relative to autarky $\pi_t^{n,n} = 1$ and no migration $\mu_t^{n,n} = 1$

Sequential and temporary equilibrium

- Given real wages, HH dynamic problem solve for the path of labor supply
- Given labor supply at each time t firms decide production and labor demand. Wages clear markets
- General equilibrium: path of employment and path of wages have to be consistent with both the HH dynamic problem and the static sub-problem

Equilibrium conditions

- Let $\tilde{\tau}^{n,i} \equiv e^{\tau^{n,i}}$, $u_t^n \equiv e^{V_t^n}$, then

$$V_t^n = \log(w_t^n/P_t^n) + \nu \log\left[\sum_{i=1}^{N} \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}\right]$$

- Can be written as

$$\exp\left(V_t^n\right) = \left(w_t^n/P_t^n\right)\left[\sum_{i=1}^{N} \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}\right]^{\nu}$$

- Using $w_t^n/P_t^n = (\pi_t^{n,n}/T_t^{n,n})^{-1/\theta}$

$$u_t^n = \left[\sum_{i=1}^{N} \left(\pi_t^{n,n}/T_t^{n,n}\right)^{-1/\theta\nu}\left(u_{t+1}^i\right)^{\beta/\nu}\left(\tilde{\tau}^{n,i}\right)^{-1/\nu}\right]^{\nu}$$

Equilibrium conditions - simple system of equations

- Temporary equilibrium, trade

$$\pi_t^{n,i} = \frac{(w_t^i)^{-\theta} T_t^{n,i}}{\sum_{h=1}^N (w_t^h)^{-\theta} T_t^{n,h}}$$

$$w_t^n L_t^n = \sum_{i=1}^N \pi_t^{i,n} \, w_t^i L_t^i$$

- Dynamics, migration

$$u_t^n = \left[\sum_{i=1}^N \left(\pi_t^{n,n} / T_t^{n,n}\right)^{-1/\theta\nu} \left(u_{t+1}^i\right)^{\beta/\nu} \left(\tilde{\tau}^{n,i}\right)^{-1/\nu} \right]^\nu$$

$$\mu_t^{n,i} = \frac{\left(u_{t+1}^i\right)^{\beta/\nu} \left(\tilde{\tau}^{n,i}\right)^{-1/\nu}}{\sum_{h=1}^N \left(u_{t+1}^h\right)^{\beta/\nu} \left(\tilde{\tau}^{n,h}\right)^{-1/\nu}}$$

$$L_{t+1}^n = \sum_{i=1}^N \mu_t^{i,n} \, L_t^i$$

Sequential and temporary equilibrium

- State of the economy = distribution of labor $L_t = \{L_t^n\}_{n=1}^N$
  - Exogenous: $\Theta_t \equiv \left(\{A_t^n\}, \{\kappa_t^{n,i}\}, \{\tau^{n,i}\}\right)_{n=1,i=1}^{N,N}$

Definition 1

Given (L_t, Θ_t), a **temporary equilibrium** is a vector of $w_t\,(L_t, \Theta_t)$ that satisfies the equilibrium conditions of the static sub-problem

Definition 2

Given $(L_0, \{\Theta_t\}_{t=0}^\infty)$, a **sequential competitive equilibrium** is a sequence of $\{L_t,\, \mu_t,\, V_t,\, w_t\}_{t=0}^\infty$ that solves HH dynamic problem and the temporary equilibrium at each t

Steady state

Definition 3

A **stationary equilibrium** of the model is a sequential competitive equilibrium such that $\{L_t, \mu_t, V_t, w_t\}_{t=0}^{\infty} = \{\bar{L}, \bar{\mu}, \bar{V}, \bar{w}\}$ are constant for all t.

- At the steady state, $u_t^n = \bar{u}^n$, $\mu_t^{n,i} = \bar{\mu}^{n,i}$, $L_t^n = \bar{L}^n$, $\pi_t^{n,i} = \bar{\pi}^{n,i}$, $w_t^n = \bar{w}^n$, $T_t^{n,i} = \bar{T}^{n,i}$, for all t

Steady state: solution to ....

$$\bar{\pi}^{n,i} = \frac{(\bar{w}^i)^{-\theta} \bar{T}^{n,i}}{\sum_{h=1}^{N} (\bar{w}^h)^{-\theta} \bar{T}^{n,h}}$$

$$\bar{w}^n \bar{L}^n = \sum_{i=1}^{N} \bar{\pi}^{i,n} \, \bar{w}^i \bar{L}^i$$

$$\bar{u}^n = \left(\bar{\pi}^{n,n} / \bar{T}^{n,n}\right)^{-\frac{1}{\theta(1-\beta)}} \left(\bar{\mu}^{n,n}\right)^{-\frac{\nu}{1-\beta}}$$

$$\bar{\mu}^{n,i} = \frac{\left(\bar{u}^i\right)^{\beta/\nu} \left(\tilde{\tau}^{n,i}\right)^{-1/\nu}}{\sum_{h=1}^{N} \left(\bar{u}^h\right)^{\beta/\nu} \left(\tilde{\tau}^{n,h}\right)^{-1/\nu}}$$

$$\bar{L}^n = \sum_{i=1}^{N} \bar{\mu}^{i,n} \, \bar{L}^i$$

Solution Method: Dynamic Hat Algebra

Solution method: Dynamic Hat Algebra

- Solving for an equilibrium of the model requires information on Θ
  - Large $\#$ of unknowns
- As we increase the dimension of the problem—adding countries, regions, or sectors—the number of parameters grows geometrically
- We solve this problem by computing the equilibrium dynamics of the model in time differences
- Why is this progress?
  - Conditioning on observables one can solve the model without knowing the *levels* of Θ
    - Solve for the value function in time differences
- Start description of the *Dynamic Hat Algebra* with *constant fundamentals*
  - Then discuss how to deal with *time varying fundamentals*

Equilibrium conditions

- Expected lifetime utility

$$V_t^n = \log(\frac{w_t^n}{P_t^n}) \; + \nu \log \left[\sum_{i=1}^{N} \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu} \right]$$

- Transition matrix (migration flows)

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

Equilibrium conditions

- *Transition* matrix (migration flows) at $t = -1$, **Data**

$$\mu_{\mathbf{-1}}^{\mathbf{n,i}} = \frac{\exp\left(\beta V_0^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_0^h - \tau^{n,h}\right)^{1/\nu}}$$

- Transition matrix (migration flows) at $t = 0$, Model

$$\mu_0^{n,i} = \frac{\exp\left(\beta V_1^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_1^h - \tau^{n,h}\right)^{1/\nu}}$$

- Take the time difference

$$\frac{\mu_0^{n,i}}{\mu_{\mathbf{-1}}^{\mathbf{n,i}}} = \frac{\frac{\exp\left(\beta V_1^i - \tau^{n,i}\right)^{1/\nu}}{\exp\left(\beta V_0^i - \tau^{n,i}\right)^{1/\nu}}}{\sum_{h=1}^{N} \frac{\exp\left(\beta V_1^h - \tau^{n,h}\right)^{1/\nu}}{\sum_{m=1}^{N} \exp\left(\beta V_0^m - \tau^{n,m}\right)^{1/\nu}}}$$

Equilibrium conditions

- Take the time difference

$$\frac{\mu_0^{n,i}}{\mu_{-1}^{\mathbf{n},\mathbf{i}}} = \frac{\frac{\exp\left(\beta V_1^i - \tau^{n,i}\right)^{1/\nu}}{\exp\left(\beta V_0^i - \tau^{n,i}\right)^{1/\nu}}}{\sum_{h=1}^N \frac{\exp\left(\beta V_1^h - \tau^{n,h}\right)^{1/\nu}}{\sum_{m=1}^N \exp\left(\beta V_0^m - \tau^{n,m}\right)^{1/\nu}}}$$

- Simplify

$$\frac{\mu_0^{n,i}}{\mu_{-1}^{\mathbf{n},\mathbf{i}}} = \frac{\exp\left(V_1^i - V_0^i\right)^{\beta/\nu}}{\sum_{h=1}^N \frac{\exp\left(\beta V_1^h - \tau^{n,h}\right)^{1/\nu}}{\sum_{m=1}^N \exp\left(\beta V_0^m - \tau^{n,m}\right)^{1/\nu}}}$$

- Use $\mu_{-1}^{\mathbf{n},\mathbf{h}}$ once again

$$\mu_0^{n,i} = \frac{\mu_{-1}^{\mathbf{n},\mathbf{i}} \exp\left(V_1^i - V_0^i\right)^{\beta/\nu}}{\sum_{h=1}^N \mu_{-1}^{\mathbf{n},\mathbf{h}} \exp\left(V_1^h - V_0^h\right)^{\beta/\nu}}$$

Equilibrium conditions

- Expected lifetime utility

$$V_t^n = \log(\frac{w_t^n}{P_t^n}) \; + \nu \log \left[\sum_{i=1}^{N} \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu} \right]$$

- Transition matrix

$$\mu_t^{n,i} \quad = \frac{\exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \quad \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

Equilibrium conditions - Time differences

- Expected lifetime utility

$$V_{t+1}^{n} - V_{t}^{n} = \log(\frac{w_{t+1}^{n}/w_{t}^{n}}{P_{t+1}^{n}/P_{t}^{n}}) + \nu \log\left[\sum_{i=1}^{N} \mu_{t}^{n,i} \exp\left(V_{t+2}^{i} - V_{t+1}^{i}\right)^{\beta/\nu}\right]$$

- Transition matrix

$$\frac{\mu_{t+1}^{n,i}}{\mu_{t}^{n,i}} = \frac{\exp\left(V_{t+2}^{i} - V_{t+1}^{i}\right)^{\beta/\nu}}{\sum\limits_{h=1}^{N} \mu_{t}^{n,h} \exp\left(V_{t+2}^{h} - V_{t+1}^{h}\right)^{\beta/\nu}}$$

where $\frac{w_{t+1}^{n}/w_{t}^{n}}{P_{t+1}^{n}/P_{t}^{n}}$ is the solution to the temporary equilibrium in time differences

Temporary equilibrium conditions

- How to solve for the temporary equilibrium in time differences?
  - Trade shares

$$\pi_t^{n,i} = \frac{[w_t^i \kappa^{n,i}]^{-\theta} A^i}{\sum_{h=1}^N [w_t^h \kappa^{n,h}]^{-\theta} A^h},$$

  - Labor market clearing

$$w_t^n L_t^n = \sum_{i=1}^N \pi_t^{i,n} w_t^i L_t^i$$

  - Price index

$$P_t^n = \Gamma^n \left[\sum_{i=1}^N A^i [w_t^i \kappa^{n,i}]^{-\theta}\right]^{-1/\theta},$$

Temporary equilibrium - Time differences

- Trade shares

$$\pi_{t+1}^{n,i} = \frac{\pi_t^{n,i} (\dot{w}_{t+1}^i)^{-\theta}}{\sum_{h=1}^N \pi_t^{n,h} (\dot{w}_{t+1}^h)^{-\theta}},$$

- Labor market clearing

$$\dot{w}_{t+1}^n \dot{L}_{t+1}^n w_t^n L_t^n = \sum_{i=1}^N \pi_{t+1}^{i,n} \, \dot{w}_{t+1}^i \dot{L}_{t+1}^i w_t^i L_t^i$$

- Price index

$$\dot{P}_{t+1}^n = \quad \left[\sum_{i=1}^N \pi_t^{n,i} (\dot{w}_{t+1}^i)^{-\theta} \right]^{-1/\theta},$$

- **Notation:** $\dot{P}_{t+1}^n = P_{t+1}^n / P_t^n$, $\dot{w}_{t+1} = \mathbf{w}_{t+1} / \mathbf{w}_t$

- Same "dot trick" applies to all equilibrium conditions

Solving the model with constant fundamentals

Proposition 1.1

Given $(\mathbf{L_0}, \mu_{-1}, \pi_0, \mathbf{X_0})$, (ν, θ, β), *solving the equilibrium in time differences does not require the level of* Θ, *and solves*

$$\dot{u}^n_{t+1} = (\dot{w}^n_{t+1}/\dot{P}^n_{t+1}) \left(\sum_{i=1}^N \mu^{n,i}_t [\dot{u}^i_{t+2}]^{\beta/\nu} \right)^{1/\nu},$$

$$\mu^{n,i}_{t+1} = \frac{\mu^{n,i}_t [\dot{u}^i_{t+2}]^{\beta/\nu}}{\sum_{h=1}^N \mu^{n,h}_t [\dot{u}^h_{t+2}]^{\beta/\nu}},$$

$$\dot{L}^n_{t+1} L^n_t = \sum_{i=1}^N \mu^{i,n}_t L^i_t,$$

where $\dot{u}^i_{t+1} \equiv \exp(V^i_{t+1} - V^i_t)$, *and* $\dot{w}^n_{t+1}/\dot{P}^n_{t+1}$ *solves the temporary equilibrium given* $\dot{L}_{t+1}$.

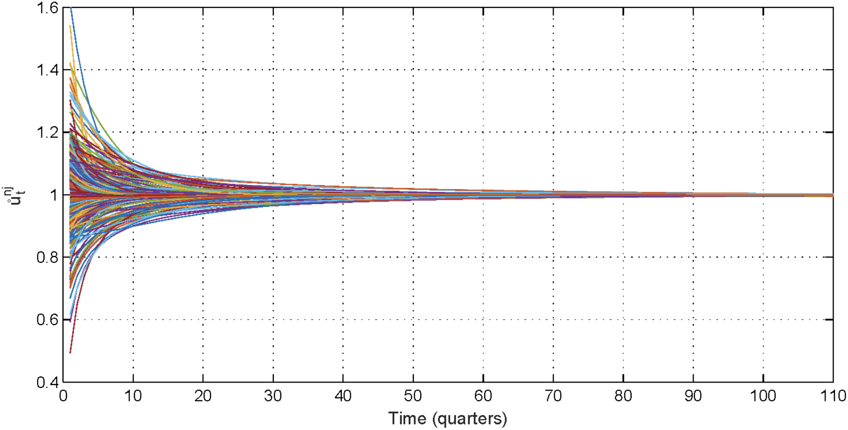
Solution algorithm

1. Initiate guess for a path of $\{\dot{u}_{t+1}^{i(0)}\}_{t=0}^{T}$ for a sufficiently large T.
2. Assume no change in fundamentals after $t = 0$ (could be changed).
3. Path should converge to $\dot{u}_{t+1}^{i(0)}\}_{t=0}^{T+1} = 1$.
4. Solve for $\mu_{t+1}^{n,i}$ and $\dot{L}_{t+1}^{n}$ using the last two equations on the previous slide. Get $\dot{w}_{t+1}^{n}/\dot{P}_{t+1}^{n}$ from temporary equilibrium.
5. Then update $\{\dot{u}_{t+1}^{i(1)}\}_{t=0}^{T}$ solving backward the top equation on the previous slide.

- See Appendix D for details

Solving the model (example)

Figure 1: Equilibrium Value Functions in Time Differences



Solving for counterfactuals

- Want to study the effects of changes in fundamentals Θ'/Θ
  - Recall that $\Theta \equiv \left(\{A^n\}, \{\kappa^{n,i}\}, \{\tau^{n,i}\}\right)_{n=1,i=1}^{N,N}$
  - TFP, trade costs, labor migration costs, endowments of local structures, home production

Solving for counterfactuals

- Suppose we want to study the effects of a change in $A_t'^i / A_t^i$
- **Counterfactual I**
  - Economy with $\dot{\Theta}_t' = \dot{A}_t'^i$ relative to economy with $\dot{\Theta}_t = 1$
    - Pros: requires only data on an initial allocation (for one year)
    - Cons: need to compute the model twice, one under $\dot{\Theta}_t = 1$ "baseline economy", and one under $\dot{\Theta}_t' = \dot{A}_t'^i$
- **Counterfactual II**
  - Economy with actual change in fundamentals $\dot{\Theta}_t$ relative to an economy with all fundamentals changing except $\dot{A}_t^i$
    - Pros: only requires computing the equilibrium once: "baseline economy" is the **data**
    - Cons: larger data requirements, need data for many t, need to deal with $t = T$?

Equilibrium conditions: Time-varying fundamentals

- Transition matrix (migration flows) $\left\{{\mu_t}^{n,i}\right\}_{t=0}^{T}$, **Data**

$$
{\mu_t}^{n,i} = \frac{\exp\left(\beta V_{t+1}^{i} - \tau_t^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_{t+1}^{m} - \tau_t^{n,h}\right)^{1/\nu}}
$$

- Transition matrix at t, from Model given fundamentals τ_t'

$$
\mu_t'^{n,i} = \frac{\exp\left(\beta V_{t+1}'^{i} - \tau_t'^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_{t+1}'^{h} - \tau_t'^{n,h}\right)^{1/\nu}}
$$

- Take the differences at each t, Model relative to Data

$$
\mu_t'^{n,i} = \frac{\mu_t^{n,i} \exp\left(V_{t+1}'^{i} - V_{t+1}^{i}\right)^{\beta/\nu} \exp\left(\tau_t'^{n,i} - \tau_t^{n,i}\right)^{-1/\nu}}{\sum_{h=1}^{N} \mu_t^{n,h} \exp\left(V_{t+1}'^{h} - V_{t+1}^{h}\right)^{\beta/\nu} \exp\left(\tau_t'^{n,h} - \tau_t^{n,h}\right)^{-1/\nu}}
$$

Equilibrium conditions in "hats"

- Denote by

$$\hat{u}_t^n = \dot{u}_t'^n / \dot{u}_t^n,$$

$$\hat{\tau}_t^{n,i} = \exp\left(\tau_t'^{n,i} - \tau_t^{n,i}\right) / \exp\left(\tau_{t-1}'^{n,i} - \tau_{t-1}^{n,i}\right),$$

$$\dot{\mu}_t^{n,i} = \mu_t^{n,i} / \mu_{t-1}^{n,i},$$

and generically

$$\hat{\Theta}_t = \dot{\Theta}_t' / \dot{\Theta}_t$$

- $\hat{}$ counterfactual change; $\dot{}$ time series change
- Take the time difference to obtain

$$\mu_t'^{n,i} = \frac{\mu_{t-1}'^{n,i} \left(\hat{\tau}_t^{n,i}\right)^{-1/\nu} \dot{\mu}_t^{n,i} \left(\hat{u}_{t+1}^i\right)^{\beta/\nu}}{\sum_{h=1}^{N} \mu_{t-1}'^{n,h} \left(\hat{\tau}_t^{n,h}\right)^{-1/\nu} \dot{\mu}_t^{n,h} \left(\hat{u}_{t+1}^h\right)^{\beta/\nu}}$$

Solving the model for counterfactuals

Proposition 1.2

Given $(L_t, \mu_{\mathbf{t-1}}, \pi_{\mathbf{t}}, X_t)\ _{t=0}^{\infty}$, (ν, θ, β), *and* $\{\hat{\Theta}_t\}_{t=1}^{\infty}$, *solving the model with the Dynamic Hat-Algebra does not require* Θ_t, *and solves*

$$\hat{u}_t^n = (\hat{w}_t^n / \hat{P}_t^n) \left(\sum_{i=1}^N \mu_t^{\prime n,i} \, (\hat{\tau}_t^{n,i})^{-1/\nu} \dot{\mu}_t^{n,i} \left(\hat{u}_{t+1}^i \right)^{\beta/\nu} \right)^{\nu} \ ,$$

$$\mu_t^{\prime n,i} = \frac{\mu_{t-1}^{\prime n,i} \left(\hat{\tau}_t^{n,i} \right)^{-1/\nu} \dot{\mu}_t^{n,i} \left(\hat{u}_{t+1}^i \right)^{\beta/\nu}}{\sum_{h=1}^N \mu_{t-1}^{\prime n,h} \left(\hat{\tau}_t^{n,h} \right)^{-1/\nu} \dot{\mu}_t^{n,h} \left(\hat{u}_{t+1}^h \right)^{\beta/\nu}},$$

$$L_{t+1}^{\prime n} = \sum_{i=1}^N \mu_t^{\prime i,n} \, L_t^{\prime i},$$

where $\hat{w}_t^n / \hat{P}_t^n$ *solves the temporary equilibrium.*

Solution algorithm: similar to before, guess path of $\hat{u}_{t+1}^h$ and use the above system to update.

Adding sectors and non-employment: Full model

Households' problem

- N locations (index n and i) and each has J sectors (index j and k)
- The value of a household in market nj at time t given by

$$\mathrm{v}_t^{nj} = u(c_t^{nj}) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{\beta E\left[\mathrm{v}_{t+1}^{ik}\right] - \tau^{nj,ik} + \nu\, \epsilon_t^{ik}\right\},$$

$$s.t.\ u(c_t^{nj}) \equiv \begin{cases} \log(b^n) & if\ \ j=0, \\ \log(w_t^{nj}/P_t^n) & \text{otherwise,} \end{cases}$$

  - $\beta \in (0,1)$ discount factor
  - $\tau^{nj,ik}$ additive, *time invariant* migration costs to ik from nj
  - ϵ_t^{ik} are stochastic *i.i.d idiosyncratic* shocks
    - $\epsilon \sim$ Type-I Extreme Value distribution with zero mean
    - $\nu > 0$ is the dispersion of shocks
- *Non-employed* HH obtain home production b^n
- *Employed* households supply a unit of labor inelastically
  - Receive the competitive market wage w_t^{nj}
  - Consume $c_t^{nj} = \prod_{k=1}^{J} (c_t^{nj,k})^{\alpha^k}$, where P_t^n is the local price index

Households' problem - Dynamic discrete choice

- Using properties of Type-I Extreme Value distributions one obtains:
- The expected (expectation over ϵ) lifetime utility of a worker at nj

$$V_t^{nj} = u(c_t^{nj}) + \nu \log \left[\sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left(\beta V_{t+1}^{ik} - \tau^{nj,ik} \right)^{1/\nu} \right]$$

- Fraction of workers that reallocate from market nj to ik

$$\mu_t^{nj,ik} = \frac{\exp \left(\beta V_{t+1}^{ik} - \tau^{nj,ik} \right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp \left(\beta V_{t+1}^{mh} - \tau^{nj,mh} \right)^{1/\nu}}.$$

- Evolution of the distribution of labor across markets

$$L_{t+1}^{nj} = \sum_{i=1}^{N} \sum_{k=0}^{J} \mu_t^{ik,nj} L_t^{ik}$$

Production - Static sub-problem

- Notice that at each t, labor supply across markets is fully determined
- In each nj there is a continuum of intermediate good producers
  - Perfect competition, CRS technology, *idiosyncratic* productivity z^{nj} ∼Fréchet$(1, \theta^j)$, deterministic sectoral regional TFP A^{nj}

$$q_t^{nj}(z^{nj}) = z^{nj} \left[A^{nj} \, [l_t^{nj}]^{\xi^n} [h_t^{nj}]^{1-\xi^n} \right]^{\gamma^{nj}} \prod_{k=1}^{J} [M_t^{nj,nk}]^{\gamma^{nj,nk}}$$

- Each n, j produces a final good (for final consumption and materials)
  - CES (elasticity η) aggregator of sector j goods from the lowest cost supplier in the world subject to $\kappa^{nj,ij} \geq 1$ "iceberg" bilateral trade cost

Production - Static sub-problem - Equilibrium conditions

- Sectoral price index,

$$P_t^{nj}(\mathbf{w}_t) = \Gamma^{nj} \left[\sum_{i=1}^{N} A^{ij} [x_t^{ij}(\mathbf{w}_t) \kappa^{nj,ij}]^{-\theta^j} \right]^{-1/\theta^j}$$

- Let $X_t^{ij}(\mathbf{w}_t)$ be total expenditure. Expenditure shares given by

$$\pi_t^{nj,ij}(\mathbf{w}_t) = \frac{[x_t^{ij}(\mathbf{w}_t) \kappa^{nj,ij}]^{-\theta^j} A^{ij}}{\sum_{m=1}^{N} [x_t^{mj}(\mathbf{w}_t) \kappa^{nj,mj}]^{-\theta^j} A^{mj}},$$

  where $x_t^{ij}(\mathbf{w}_t)$ is the unit cost of an input bundle

- Labor Market clearing

$$L_t^{nj} = \frac{\gamma^{nj} (1 - \xi^n)}{w_t^{nj}} \sum_{i=1}^{N} \pi_t^{ij,nj} (\mathbf{w}_t) X_t^{ij}(\mathbf{w}_t),$$

  where $\gamma^{nj}(1 - \xi^n)$ labor share

Sequential and temporary equilibrium

- State of the economy = distribution of labor $L_t = \{L_t^{nj}\}_{n=1,j=0}^{N,J}$
  - Let $\Theta \equiv \left(\{A^{nj}\}, \{\kappa^{nj,ij}\}, \{\tau^{nj,ik}\}, \left\{H^{nj}\right\}, \{b^n\}\right)_{n=1,j=0,i=1,k=0}^{N,J,J,N}$

Definition 4

Given (L_t, Θ), a **temporary equilibrium** is a vector of $w\,(L_t, \Theta)$ that satisfies the equilibrium conditions of the static sub-problem

Definition 5

Given (L_0, Θ), a **sequential competitive equilibrium** of the model is a sequence of $\{L_t,\, \mu_t,\, V_t,\, w\,(L_t, \Theta)\}_{t=0}^{\infty}$ that solves HH dynamic problem and the temporary equilibrium at each t

- With $\mu_t = \{\mu_t^{nj,ik}\}_{n=1,j=0,i=1,k=0}^{N,J,J,N}$, and $V_t = \{V_t^{nj}\}_{n=1,j=0}^{N,J}$

Application: The Rise of China

The rise of China

- U.S. imports from China almost doubled from 2000 to 2007
  - At the same time, manufacturing employment fell while employment in other sectors, such as construction and services, grew
- Several studies document that an important part of the employment loss in manufactures was a consequence of China's trade expansion
  - e.g., Pierce and Schott (2012); Autor, Dorn, and Hanson (2013), Acemoglu, Autor, Dorn, and Hanson (2014)
- Use model to quantify the effects of the rise of China's trade expansion, "China shock"
  - Initial period is the year 2000
  - Calculate the sectoral, regional, and aggregate employment and welfare effects of the China shock

Taking the model to the data (quarterly)

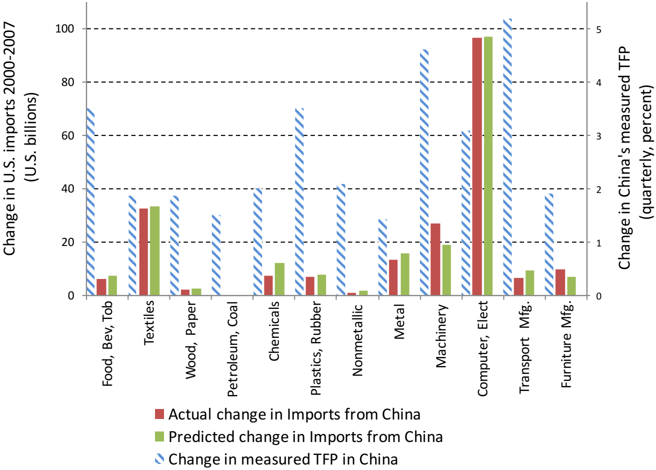
- Model with 50 U.S. states, 22 sectors + non-empl. and 38 countries
  - More than 1000 labor markets
- Need data for $(L_0, \mu_{-1}, \pi_0, VA_0, GO_0)$
  - L_0 : PUMS of the U.S. Census for the year 2000
    - Exclude empl. in farming, mining, utilities, and public sect.
  - μ_{-1} : Use CPS to compute intersectoral mobility and ACS to compute interstate mobility
  - π_0 : CFS and WIOD year 2000
  - VA_0 and GO_0 : BEA VA shares and U.S. IO, WIOD for other countries
- Need values for parameters (ν, θ, β)
  - θ : We use Caliendo and Parro (2015)
  - $\beta = 0.99$ Implies approximately a 4% annual interest rate
  - $\upsilon = 5.34$ (implied elasticity of 0.2) Using ACM's data and specification, adapted to our model
- Trade deficits through owners of housing (rentiers)

Identifying the China "shock"

- Follow Autor, Dorn, and Hanson (2013)
  - Estimate
    $$\Delta M_{USA,j} = a_1 + a_2 \Delta M_{other,j} + u_j,$$
    where j is a NAICS sector, $\Delta M_{USA,j}$ and $\Delta M_{other,j}$ are changes in U.S. and other adv. countries, imports from China from 2000 to 2007
  - Find $a_2 = 1.27$
- Obtain predicted changes in U.S. imports with this specification
- Use the model to solve for the change in China's 12 manufacturing industries TFP $\left\{\hat{A}^{China,j}\right\}_{j=1}^{12}$ such that model's imports match predicted imports from China from 2000 to 2007
  - With model's generated data obtain $a_2 = 1.52$
  - Feed in model $\left\{\hat{A}^{China,j}\right\}_{j=1}^{12}$ by quarter from 2000 to 2007 to study the effects of the shock

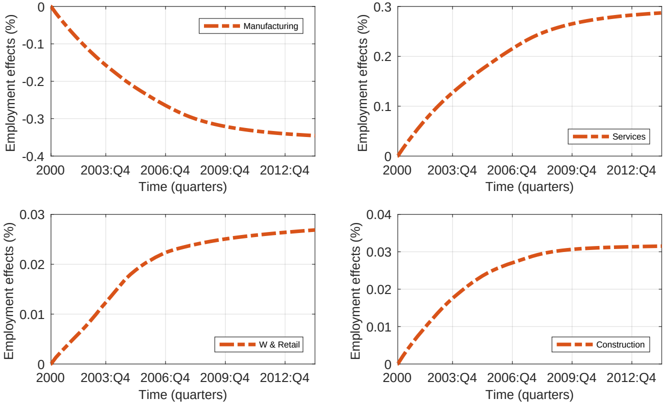
Identifying the China shock

Figure 2: Predicted change in imports vs. model-based Chinese TFP change



Employment effects

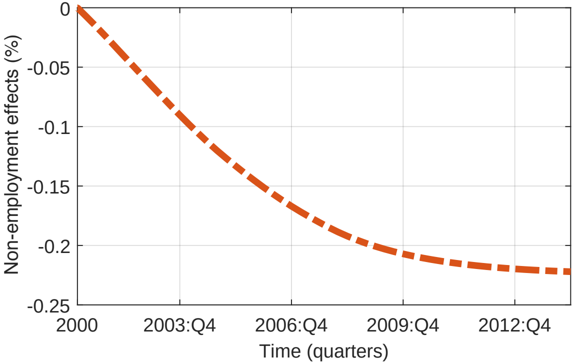
Figure 3: The effect of the China shock on employment shares



- Chinese competition reduced the share of manufacturing employment by 0.36% in the long run, ∼0.55 million employment loss
  - About 36% of the change not explained by a secular trend

Non-employment shares

Figure 4: The effect of the China shock on non-employment shares



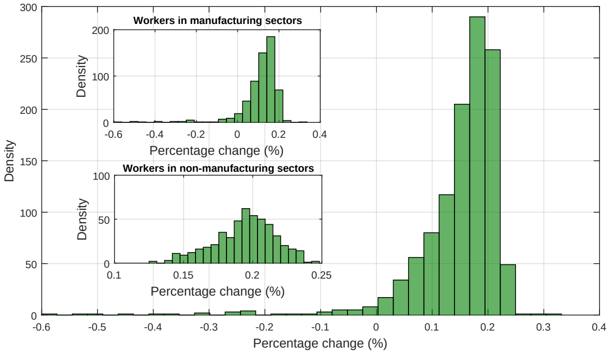
- Fall mainly due to a decline in flows from non-manuf. to non-empl.
- Flow from manuf. to non-empl. increased in states that are concentrated in the manuf. industries
  - Alabama, Arkansas, Mississippi, Michigan, and Ohio, among others

Employment effects: Manufacturing

- Sectors most exposed to Chinese import competition contribute more
  - 1/2 of the decline in manuf. employment originated in the Computer & Electronics and Furniture sectors
  - 1/4 of the total decline comes from the Metal and Textiles sectors
  - Food, Beverage and Tobacco, gained employment
    - Less exposed to China, benefited from cheaper intermediate goods, other sectors, like Services, demanded more of them (I-O linkages)
- Unequal regional effects
  - Regions with a larger concentration of sectors that are more exposed to China lose more jobs
    - California, the region with largest share of employment in Computer & Electronics, contributed to about 12% of the decline

Welfare effects across labor markets

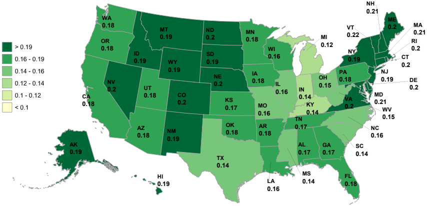
Figure 5: Welfare effects of the China shock across U.S. labor markets



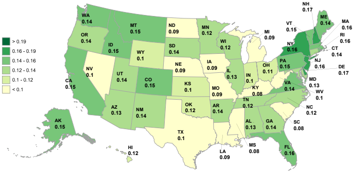
- Very heterogeneous response to the same aggregate shock
  - Most labor markets gain as a consequence of cheaper imports
  - Unequal regional effects

Regional welfare effects

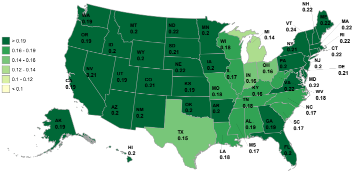
a.1: Regional



a.2: Manufacturing

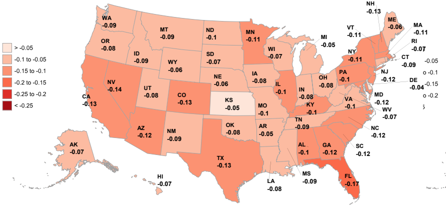


a.3: Non-manufacturing

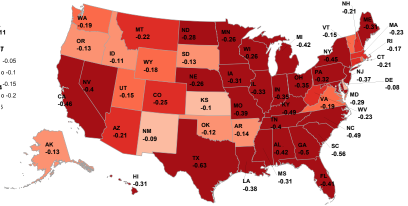


Regional real wage changes in the manuf. sector

a.1: One quarter after 2000

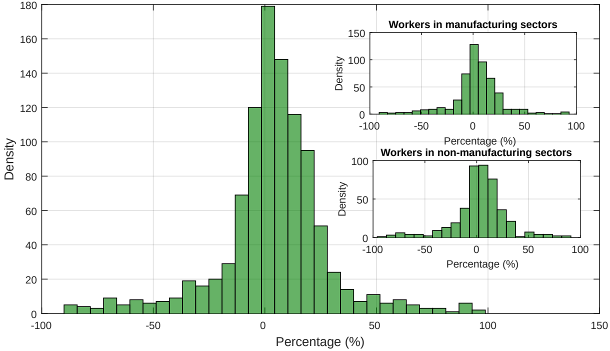


a.2: From 2000 to 2007



Transition cost to the steady state

Figure 6: Adjustment costs



- Adjustment costs reflect the importance of labor market dynamics
  - With free labor mobility AC=0
  - Heterogeneity due to trade/migration frictions and geographic factors

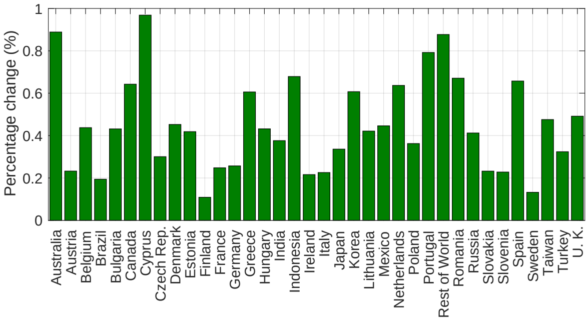
Adjustment costs

- We follow Dix-Carneiro (2014)'s measure of adjustment cost
- The steady-state change in the value function due changes in fundamentals is given by $V_{SS}^{nj}(\hat{\Theta}) - V_{SS}^{nj}$
- Therefore, the transition cost for market nj to the new long-run equilibrium, $AC^{nj}(\hat{\Theta})$, is given by

$$AC^{nj}(\hat{\Theta}) = \log\left(\frac{\frac{1}{1-\beta}\left(V_{SS}^{nj}(\hat{\Theta}) - V_{SS}^{nj}\right)}{\sum_{t=0}^{\infty}\beta^t\left(V_{t+1}^{nj}(\hat{\Theta}) - V_{t+1}^{nj}\right)}\right),$$

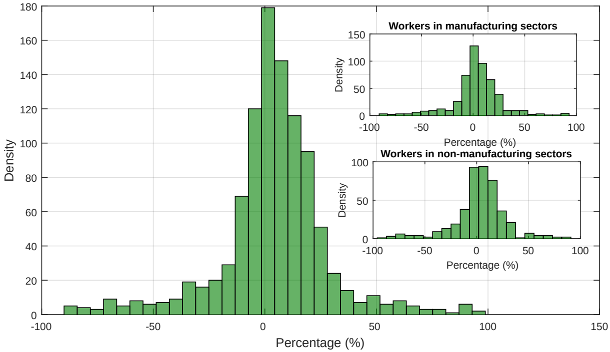
Welfare effects across countries

Figure 7: Welfare effects across countries



Transition cost to the steady state

Figure 8: Adjustment costs



- Adjustment costs reflect the importance of labor market dynamics
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  - Heterogeneity due to trade/migration frictions and geographic factors

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- The steady-state change in the value function due changes in fundamentals is given by $V_{SS}^{nj}(\hat{\Theta}) - V_{SS}^{nj}$
- Therefore, the transition cost for market nj to the new long-run equilibrium, $AC^{nj}(\hat{\Theta})$, is given by

$$AC^{nj}(\hat{\Theta}) = \log \left(\frac{\frac{1}{1-\beta}\left(V_{SS}^{nj}(\hat{\Theta}) - V_{SS}^{nj}\right)}{\sum_{t=0}^{\infty} \beta^t \left(V_{t+1}^{nj}(\hat{\Theta}) - V_{t+1}^{nj}\right)} \right),$$