Economics, Problem Set #1, Dynamic Programming

OSE Lab, Felix Kubler

Due Monday, July 8 at 11:00pm

Instructions:

- Upload your code and PDF of results (preferably using LaTeX).
- Please work in groups

1. **Asset market equilibrium.** Suppose there are two periods and S states of the world in the second period. There is a single perishable consumption good. There are H agents with endowments $e^h = (e_0^h, \tilde{e}^h) \in I\!R_+^{S+1}$ and identical von Neumann-Morgenstern utility

$$U^h(c) = v(c_0) + \frac{1}{S}\sum_{s=1}^{S} v(c_s), \quad \text{with} \quad v(c) \equiv \frac{c^{1-\gamma}}{1-\gamma}, \quad \text{for} \quad \gamma > 1.$$

Suppose $H = 2$, $S = 4$, $e^1 = (1, 1, 2, 1, 2)$, $e^2 = (1, 3, 1, 3, 1)$, $J = 2$, with $A^1 = (1, 1, 1, 1,)$, and $A^2 = (1, 1, 1.5, 1.5)$.

   (a) Define a financial markets equilibrium and write down a system of equations (first order conditions and market clearing) that characterize this equilibrium.

   (b) Use Python to compute the equilibrium prices and allocations for $\gamma = 2$, $\gamma = 4$, $\gamma = 8$, and $\gamma = 166$.

2. **Dynamic programming.** Use value function iteration to write a computer program that cannot lose in Tic-Tac-Toe.

3. **Ramsey I.** Suppose there is a single agent with $\beta = 0.9$, $v(c) = \log(c)$, and that there are two states which are i.i.d. with equal probabilities $\pi_1 = \pi_2 = 0.5$, and the firm's production function is

$$f(k, 1) = 0.9k^{0.3} + 0.3k$$
$$f(k, 2) = 1.1k^{0.3} + 0.9k.$$

Discretize the possible capital values to 50 points. Use value function iteration to compute the policy functions (one for each shock) for consumption and investment. Plot these functions. Now use 500 points for admissible capital levels and redo the exercise.

4. **Stochastic Ramsey model, time iteration collocation.** Suppose there is a single agent with $\beta = 0.9$, $v(c) = \log(c)$.

(a) (Discrete shocks) Assume there are two states which are i.i.d. with equal probabilities $\pi_1 = \pi_2 = 0.5$, and the firm's production function is

$$f(k,1) = 0.9k^{0.3} + 0.3k$$
$$f(k,2) = 1.1k^{0.3} + 0.9k.$$

Approximate the investment policy (as a function of capital and the shock) by a piecewise linear function, take 30 pieces (for each value of the shock) Write down the Euler equations and solve for the optimal investment policy by time iteration. Redo the same exercise with $\beta = 0.999$ and with $v(c) = -c^{-4}$. Try to do the same by approximating the policy function by a polynomial of degree 5 and by cubic splines.

(b) (AR(1) shocks) Now suppose that $f(k,1) = \exp(A_t)k^{0.3} + 0.5k$ and that

$$A_t = 0.9A_{t-1} + \epsilon_t,$$

where ϵ_t is i.i.d. normal with standard deviation 0.1. Redo part(a) of this exercise with this specification (utility is the same). Take the state space to be two-dimensional (A, k) and use tensor products.