Lecture 1: Course Intro, Representative Agent RBC Model, and Misallocation

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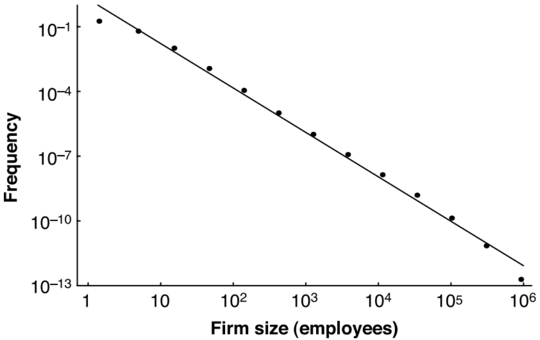
July 22nd, 2019

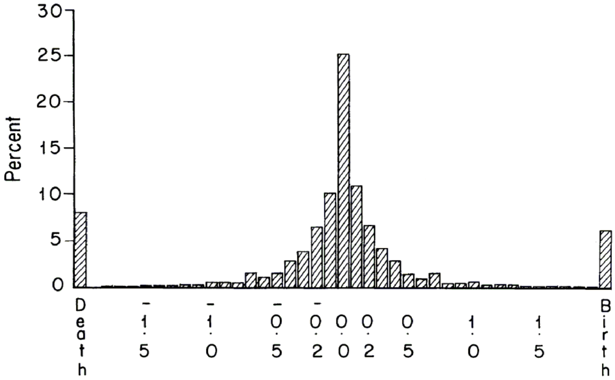
Course Intro

Firm Size Distribution Has Fat Tails



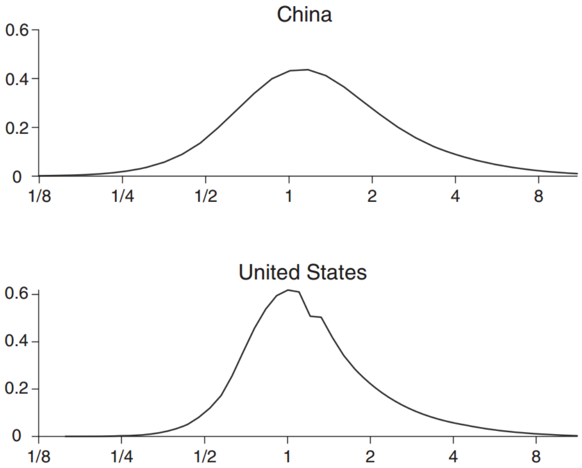
Source: Axtell (2001)

Huge Amount of Churning Among Firms



Source: Davis and Haltiwanger (1992)

Firms Have Very Different Productivity



Source: Hsieh and Klenow (2009)

These Lectures

- **How does firm heterogeneity matter for aggregate outcomes?**
  - Implicit: relative to representative firm models
  - Focus on business cycles

- Two main answers to this question:
  1. Distribution of heterogeneous firms matters for aggregates
  2. Micro data provides information to discipline models

- Emphasize the interaction between
  1. Empirical work: documenting key features of firm behavior
  2. Models: draw implications for aggregate dynamics

- Strong focus on **solving heterogeneous agent models**

Logistics

Three lectures

1. Today: representative agent RBC model + “misallocation”
2. Wednesday: benchmark model of heterogeneous firms + overview of solution methods
3. Friday: details of my solution method + heterogeneous-firm New Keynesian model

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Two homeworks

1. Short homework on using `Dynare` to study representative agent models
2. Main homework on solving heterogeneous firm model with entry and exit

Learning Goals

Economics

1. Basics of representative agent business cycle analysis
2. Conditions for aggregation to representative firm do not hold
3. How firm heterogeneity and capital adjustment costs matter for business cycle
4. Role of financial frictions in monetary transmission
5. (Homework): entry, exit, and the firm lifecycle

Computation

1. Overview of solution methods for heterogeneous agent models with aggregate shocks
2. Details of "Reiter methods" (Winberry 2018)
3. Next week with Tony: details of Krusell-Smith approach

Representative Agent RBC Model

Environment

Preferences

- Representative household with preferences over consumption C_t and labor supply N_t

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\gamma}-1}{1-\gamma}-\chi\frac{N_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)\right]$$

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Technology

- Aggregate production function $Y_t = e^{Z_t}K_t^{\alpha}N_t^{1-\alpha}$
- Output used for consumption or investment $C_t + I_t = Y_t$
- Capital accumulation follows $K_{t+1} = (1-\delta)K_t + I_t$
- Aggregate TFP follows $Z_{t+1} = \rho Z_t + \varepsilon_{t+1}$, where $\varepsilon_{t+1} \sim N(0, \sigma^2)$

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Endowments

- Household endowed with one unit of time each period: $N_t \in [0, 1]$
- Household endowed with K_0 units of capital in $t = 0$

Equilibrium

Definition: Given K_0 and z_0, a sequential markets competitive equilibrium is a list of stochastic processes for C_t, K_{t+1}, N_t, w_t, r_t, and Z_t such that

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1. Household optimization: Taking the processes for w_t and r_t as given, the household solves
$$\max_{C_t, N_t, K_{t+1}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}-1}{1-\gamma} - \chi \frac{N_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)\right]$$
such that $C_t + (K_{t+1} - (1-\delta)K_t) = w_t N_t + r_t K_t$ for all t

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2. Firm optimization: Taking the processes for w_t, r_t, and Z_t as given, the firm solves
$$\max_{K_t, N_t} e^{Z_t} K_t^{\alpha} N_t^{1-\alpha} - r_t K_t - w_t N_t$$

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3. Market clearing + consistency: For all t, $Z_{t+1} = \rho Z_t + \varepsilon_{t+1}$

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$$
\begin{aligned}
w_t C_t^{-\gamma} &= \chi N_t^{\frac{1}{\eta}} \\
C_t^{-\gamma} &= \beta \mathbb{E}_t [C_{t+1}^{-\gamma}(1 - \delta + r_{t+1})]
\end{aligned}
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3. Market clearing + consistency: $C_t + K_{t+1} - (1-\delta)K_t = e^{Z_t} K_t^{\alpha} N_t^{1-\alpha}$ and $Z_{t+1} = \rho Z_t + \varepsilon_{t+1}$

Steady State

Definition: A non-stochastic steady state sequential markets competitive equilibrium is a list C^*, K^*, N^*, w^* and r^* such that if $\sigma = 0$ and $K_0 = K^*$, then $C_t = C^*, K_{t+1} = K^*, N_t = N^*, w_t = w^*$, and $r_t = r^*$ for all t is a sequential markets competitive equilibrium.

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1. Useful in calibrating parameters of the model (steady state $\approx$ long run average)

2. Useful in solving the model using perturbation methods
   - Approximates solution using Taylor expansion around steady state
   - Short homework shows how to use `Dynare` code to this model (you should know how to do this!!!)

Calibration

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   - Choose δ to match $\mathbb{E}[\frac{I_t}{K_t}] = 10\%$ annual
   - Choose α to match $\mathbb{E}[\frac{w_t N_t}{Y_t}] = \frac{2}{3}$
   - Choose β to match $\mathbb{E}[r_t - \delta] = 4\%$ annual

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2. Choose other parameters to match a priori evidence
   - Choose γ to set EIS = 1
   - Choose η to set Frisch = $\frac{1}{2}$ (more on this next slide)

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2. Choose other parameters to match a priori evidence
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   - Choose η to set Frisch = $\frac{1}{2}$ (more on this next slide)
3. Estimate process for TFP from measured Solow residuals

$$Z_t = \log(Y_t) - \alpha \log(K_t) - (1-\alpha)\log(N_t)$$

Indivisible Labor and the Frisch Elasticity

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- Rogerson (1988) showed that micro-level $\neq$ macro-level

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- Even if micro-level $\eta \to 0$, macro-level $\eta \to \infty$!

Impulse Response Analysis

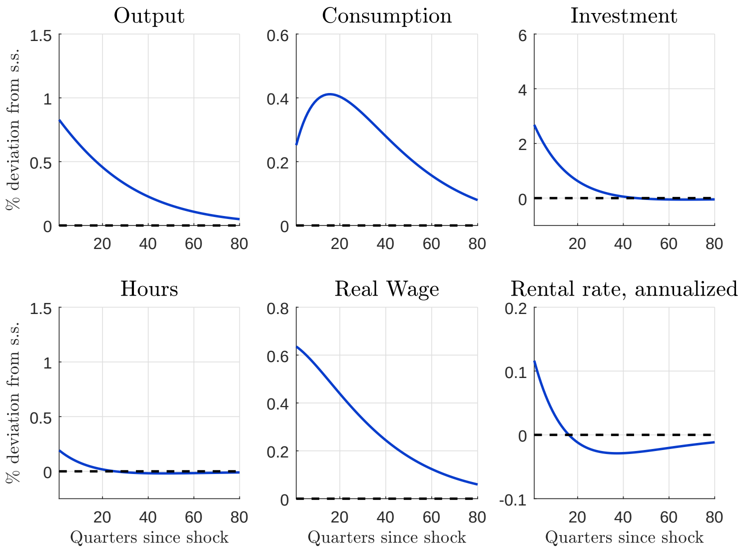
- An **impulse response function** traces out how a one-time shock affects dynamics of the economy

$$\mathbb{E}[Y_{t+s}|\varepsilon_t = \sigma, K_t, z_t] - \mathbb{E}[Y_{t+s}|\varepsilon_t = 0, K_t, z_t]$$

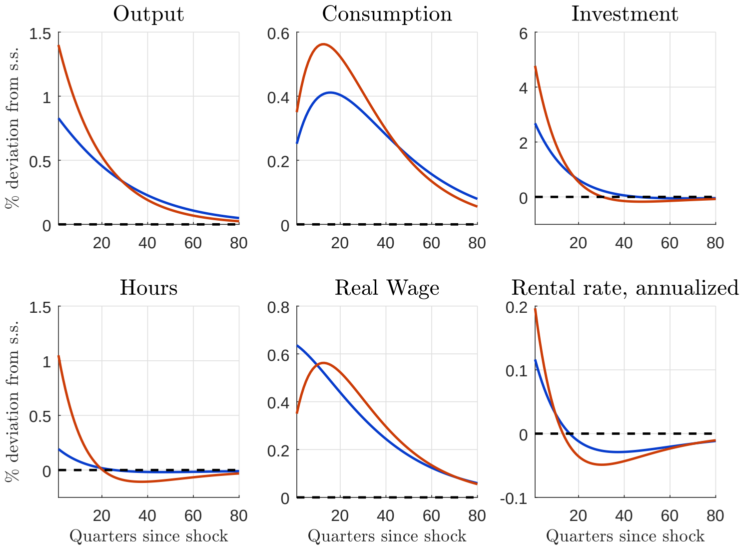
  - In principle, depends on K_t, z_t, and size of the shock
  - But in linear models, does not

- Clear and simple way to understand mechanisms in model

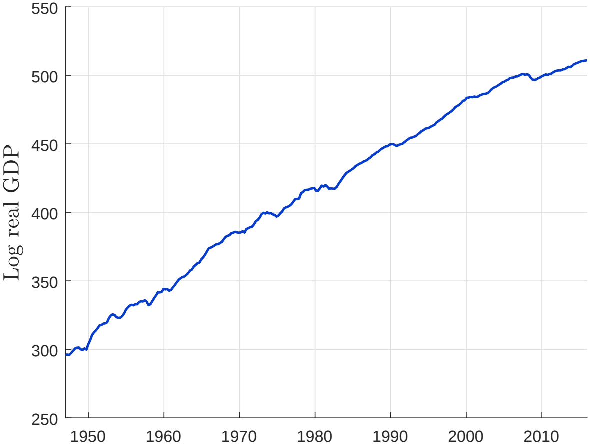
Impulse Response to TFP Shock, $\eta = \frac{1}{2}$



Impulse Response to TFP Shock, $\eta \to \infty$



Cyclical Fluctuations with Hodrick-Prescott Filter



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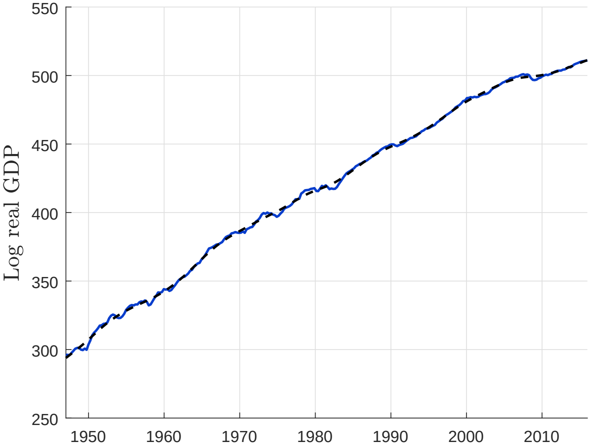

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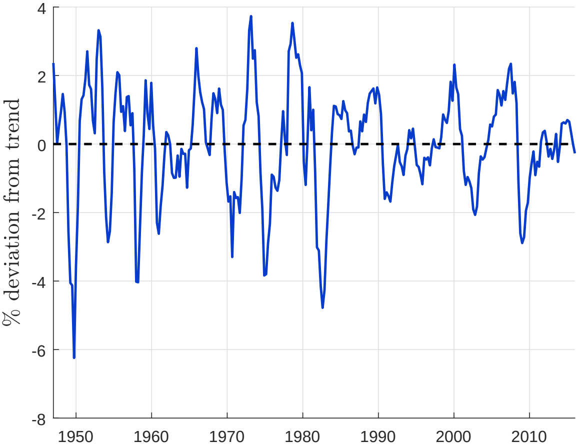

Business Cycle Statistics in the Data

	Volatilities (rel. to $\sigma(y_t)$)				
	$\sigma(y_t)$	$\sigma(c_t)$	$\sigma(i_t)$	$\sigma(n_t)$	$\sigma(r_t)$
Data	(1.62%)	0.53	2.87	1.17	(2.18%)
		Correlations w/ output			
		$\rho(c_t, y_t)$	$\rho(i_t, y_t)$	$\rho(n_t, y_t)$	$\rho(r_t, y_t)$
Data		0.79	0.77	0.87	-0.17

Business Cycle Statistics, $\eta = \frac{1}{2}$

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Model	(1.08%)	0.35	3.24	0.24	(0.15%)

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Business Cycle Statistics, $\eta \to \infty$

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Model	(1.08%)	0.35	3.24	0.24	(0.15%)
Model	(1.82%)	0.30	3.41	0.75	(0.26%)
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Takeaways From RBC Analysis

- Benchmark approach to studying aggregate fluctuations
  - Methodology: model specification, equilibrium, calibration, impulse response analysis, business cycle statistics
  - Economic forces: consumption smoothing, labor supply

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- Benchmark approach to studying aggregate fluctuations
  - Methodology: model specification, equilibrium, calibration, impulse response analysis, business cycle statistics
  - Economic forces: consumption smoothing, labor supply
- Micro data cannot be used to calibrate representative agent
  - Macro "agent" may look very different from micro agents
- Need to build models with explicit micro heterogeneity to use micro data
  - Micro data is the only data we have on how individuals actually make decisions!

Misallocation

Plan for this Discussion

1. Document large and persistent dispersion of firms' productivity

2. Show benchmark irrelevance result: without frictions to inputs, economy still has representative firm

3. Measure input frictions using reduced form "misallocation" measures

    - Substantial frictions at micro-level
    - Implies large aggregate effects

Definitions of Productivity

- Productivity is the amount of **output produced per unit of inputs**
- Depends on unit of analysis:
    1. Establishment: A business or production unit at a single location
    2. Firm: A collection of establishments under common legal control
- Depends on input:
    1. Labor productivity: output per labor input $\frac{y_{it}}{n_{it}}$
    2. Capital productivity: output per capital input $\frac{y_{it}}{k_{it}}$
    3. Total factor productivity: output per composite of inputs $\frac{y_{it}}{k_{it}^{\alpha} n_{it}^{1-\alpha}}$

What Is Productivity?

- Productivity is **anything that influences output other than measured inputs**
    - A useful measure of our ignorance

- What could it be?
    1. Technology
    2. Efficiency
    3. Managerial skill
    4. Market conditions
    5. Regulation
    6. Utilization

Measuring Productivity in Practice

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   - Production function estimation: have to deal with endogeneity problem

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1. **Estimate output elasticity α**
    - Factor shares method: with Cobb-Douglas and perfect competition, $1-\alpha =$ labor share
    - Production function estimation: have to deal with endogeneity problem

2. **Construct measures of y_{it}, k_{it}, and n_{it}**
    - y_{it}: usually gross output (sales) or value added (sales - materials)
    - k_{it}: book value, replacement value, perpetual inventory
    - n_{it}: number of workers, hours worked, wage bill

Stylized Facts About Productivity (Syverson 2011)

1. **Enormous dispersion across establishments, even within narrowly-defined sector**
   - Within average sector, 90th percentile firm is 2 times as productive as 10th
   - SD of this range across sectors is 0.17

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    - Annual autocorrelation 0.6 - 0.8

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2. **Productivity is persistent**
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3. **Productivity matters**
    - Correlated with outcomes like hiring, investment, survival

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3. Measure input frictions using reduced form "misallocation" measures

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Aggregation with Productivity Dispersion

Consider production side of economy in time t with:

- Heterogeneous firms $i \in [0, 1]$ with production function

$$y_{it} = e^{z_{it}} k_{it}^{\alpha_k} n_{it}^{\alpha_n}, \alpha_k + \alpha_n < 1$$

- Perfect competition in factor markets
    - Rent capital at rate r_t
    - Hire labor at rate w_t

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Can we represent this structure with an aggregate production function?

$$Y_t = e^{Z_t} F(K_t, N_t) \text{ where } K_t = \int k_{it} di, N_t = \int n_{it} di, \text{ and } Y_t = \int y_{it} di$$

Aggregation with Productivity Dispersion

Claim: aggregates Y_t, K_t, and N_t are same with representative firm

$$Y_t = e^{Z_t} K_t^{\alpha_k} N_t^{\alpha_n} \text{ with } Z_t = \log\left(\int (e^{Z_{it}})^{\frac{1}{1-\alpha_k-\alpha_n}}\right)$$

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- First order conditions for profit maximization of firm i:

$$\alpha_k e^{Z_{it}} k_{it}^{\alpha_k - 1} n_{it}^{\alpha_n} = r_t$$
$$\alpha_n e^{Z_{it}} k_{it}^{\alpha_k} n_{it}^{\alpha_n - 1} = w_t$$

$\rightarrow$ Firms equalize their marginal products

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$$Y_t = e^{Z_t} K_t^{\alpha_k} N_t^{\alpha_n} \text{ with } Z_t = \log\left(\int (e^{Z_{it}})^{\frac{1}{1-\alpha_k-\alpha_n}}\right)$$

- Manipulate the FOCs to get

$$k_{it} = (e^{Z_{it}})^{\frac{1}{1-\alpha_k-\alpha_n}} \left(\frac{\alpha_k}{r_t}\right)^{\alpha_k} \left(\frac{\alpha_n}{w_t}\right)^{1-\alpha_k}$$

$$n_{it} = (e^{Z_{it}})^{\frac{1}{1-\alpha_k-\alpha_n}} \left(\frac{\alpha_k}{r_t}\right)^{1-\alpha_n} \left(\frac{\alpha_n}{w_t}\right)^{\alpha_n}$$

$$y_{it} = (e^{Z_{it}})^{\frac{1}{1-\alpha_k-\alpha_n}} \left(\frac{\alpha_k}{r_t}\right)^{\frac{\alpha_k}{1-\alpha_k-\alpha_n}} \left(\frac{\alpha_n}{w_t}\right)^{\frac{\alpha_n}{1-\alpha_k-\alpha_n}}$$

Aggregation with Productivity Dispersion

Claim: aggregates Y_t, K_t, and N_t are same with representative firm

$$Y_t = e^{Z_t} K_t^{\alpha_k} N_t^{\alpha_n} \text{ with } Z_t = \log\left(\int (e^{Z_{it}})^{\frac{1}{1-\alpha_k-\alpha_n}}\right)$$

- Aggregate to get

$$K_t = \int k_{it} di = e^{Z_t}\left(\frac{\alpha_k}{r_t}\right)^{\alpha_k}\left(\frac{\alpha_n}{w_t}\right)^{1-\alpha_k}$$

$$N_t = \int n_{it} di = e^{Z_t}\left(\frac{\alpha_k}{r_t}\right)^{1-\alpha_n}\left(\frac{\alpha_n}{w_t}\right)^{\alpha_n}$$

$$Y_t = \int y_{it} di = e^{Z_t}\left(\frac{\alpha_k}{r_t}\right)^{\frac{\alpha_k}{1-\alpha_k-\alpha_n}}\left(\frac{\alpha_n}{w_t}\right)^{\frac{\alpha_n}{1-\alpha_k-\alpha_n}}$$

$\rightarrow$ Same choices as the representative firm!

Plan for this Discussion

1. Document large and persistent dispersion of firms' productivity

2. Show benchmark irrelevance result: without frictions to inputs, economy still has representative firm

3. **Measure input frictions using reduced form "misallocation" measures**
    - Substantial frictions at micro-level
    - Implies large aggregate effects

Simple Version of Hsieh and Klenow (2009) Model

Consider production side of economy in time *t* with:

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- Representative final good producer $Y_t = \left(\int y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$
  - Firm i monopolistic competitor with CES demand curve $\left(\frac{p_{it}}{P_t}\right)^{-\sigma} Y_t$
  - Alternative way to generate curvature in revenue function

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  - Alternative way to generate curvature in revenue function
- Idiosyncratic distortions to factor prices: $(1 + \tau_{it}^{n}) w_t$ and $(1 + \tau_{it}^{k}) r_t$
  - τ_{it}^{n}: hiring costs, regulations, search frictions, ...
  - τ_{it}^{k}: adjustment costs, financial constraints, ...

Firm Behavior Given Wedges

- Optimal input choices:

$$\underbrace{\alpha \left(\frac{\sigma - 1}{\sigma}\right) \frac{p_{it} y_{it}}{k_{it}}}_{\text{MRPK}_{it}} = (1 + \tau^k_{it}) r_t$$

$$\underbrace{(1 - \alpha) \left(\frac{\sigma - 1}{\sigma}\right) \frac{p_{it} y_{it}}{n_{it}}}_{\text{MRPL}_{it}} = (1 + \tau^n_{it}) w_t$$

$\rightarrow \tau^n_{it}$ and τ^k_{it}: how much firms do not equalize marginal products

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$\rightarrow \tau^n_{it}$ and τ^k_{it}: how much firms do not equalize marginal products

- Output:

$$y_{it} = \left(\left(\frac{\sigma-1}{\sigma}\right)\frac{e^{z_{it}}}{\left(\frac{(1+\tau^k_{it})r_t}{\alpha}\right)^{\alpha}\left(\frac{(1+\tau^n_{it})w_t}{1-\alpha}\right)^{1-\alpha}}\right)^{\sigma}$$

Aggregation

- After a lot of algebra (don't worry about it):

$$Y_t = (T_t^p)^{\frac{\sigma}{\sigma-1}} (T_t^k)^{\alpha} (T_t^n)^{1-\alpha} K_t^{\alpha} N_t^{1-\alpha}, \text{ where}$$

$$T_t^p = \left(\int \left(\frac{(1+\tau_{it}^k)^{\alpha}(1+\tau_{it}^n)^{1-\alpha}}{e^{z_{it}}} \right)^{1-\sigma} di \right)^{-1}$$

$$T_t^n = \left(\int \left(\frac{(1+\tau_{it}^k)^{\alpha}(1+\tau_{it}^n)^{-\alpha}}{e^{z_{it}}} \right)^{1-\sigma} \frac{1}{1+\tau_{it}^n} di \right)^{-1}$$

$$T_t^k = \left(\int \left(\frac{(1+\tau_{it}^k)^{\alpha-1}(1+\tau_{it}^n)^{1-\alpha}}{e^{z_{it}}} \right)^{1-\sigma} \frac{1}{1+\tau_{it}^k} di \right)^{-1}$$

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- Compare distribution of wedges in data vs. no wedges

Measuring Wedges and Productivity in the Data

$$(1+\tau_{it}^{k}) = \frac{\mathrm{MRPK}_{it}}{r_t} = \frac{1}{r_t} \times \alpha \left(\frac{\sigma-1}{\sigma}\right) \frac{p_{it}y_{it}}{k_{it}}$$

$$(1+\tau_{it}^{n}) = \frac{\mathrm{MRPL}_{it}}{w_t} = \frac{1}{w_t} \times (1-\alpha) \left(\frac{\sigma-1}{\sigma}\right) \frac{p_{it}y_{it}}{n_{it}}$$

$$e^{z_{it}} = \frac{y_{it}}{k_{it}^{\alpha} n_{it}^{1-\alpha}} = \frac{(p_{it}y_{it})^{\frac{\sigma-1}{\sigma}}}{k_{it}^{\alpha} n_{it}^{1-\alpha}}$$

Measuring Wedges and Productivity in the Data

$$
\begin{aligned}
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\end{aligned}
$$

Want to infer wedges and productivity from data

Measuring Wedges and Productivity in the Data

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Plant level data from Census of Manufactures

- Revenue $p_{it}y_{it}$ is nominal value added
- Capital k_{it} is book value of capital stock
- Labor n_{it} is wage bill of the plant

Measuring Wedges and Productivity in the Data

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Remaining quantities are calibrated

- Rental rate on capital $r_t = 10\%$
- Elasticity of substitution $\sigma = 3$
- Capital share α as 1 - labor share
- NB: actual implementation in paper complicated by sectoral heterogeneity

Dispersion in TFPQ in Line with Literature

TABLE I
DISPERSION OF TFPQ

China	1998	2001	2005
S.D.	1.06	0.99	0.95
75 – 25	1.41	1.34	1.28
90 – 10	2.72	2.54	2.44
N	95,980	108,702	211,304
India	**1987**	**1991**	**1994**
S.D.	1.16	1.17	1.23
75 – 25	1.55	1.53	1.60
90 – 10	2.97	3.01	3.11
N	31,602	37,520	41,006
United States	**1977**	**1987**	**1997**
S.D.	0.85	0.79	0.84
75 – 25	1.22	1.09	1.17
90 – 10	2.22	2.05	2.18
N	164,971	173,651	194,669

Marginal Products Very Disperse

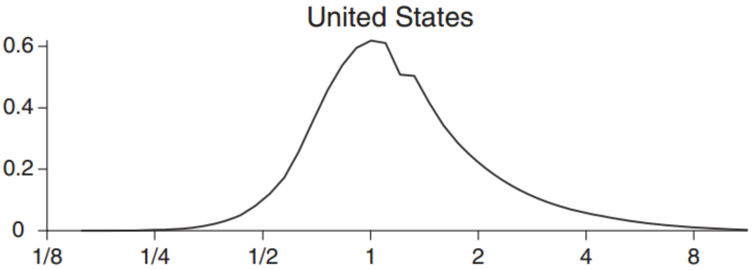


FIGURE II
Distribution of TFPR

$$\text{TFPR}_{it} = \frac{p_{it} y_{it}}{k_{it}^{\alpha} n_{it}^{1-\alpha}} = (\text{MPRK}_{it})^{\alpha} (\text{MRPL}_{it})^{1-\alpha}$$

Marginal Products More Disperse in India and China

TABLE II
DISPERSION OF TFPR

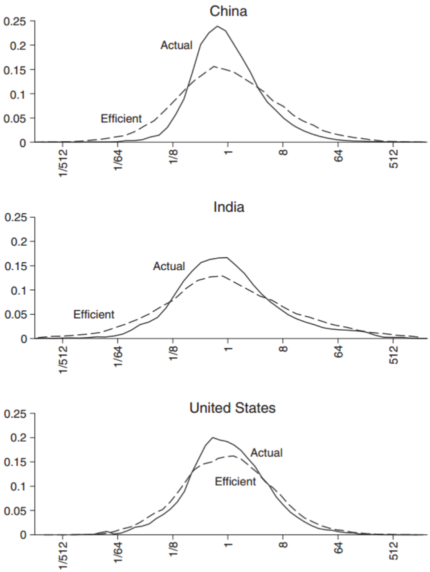
China	1998	2001	2005
S.D.	0.74	0.68	0.63
75 – 25	0.97	0.88	0.82
90 – 10	1.87	1.71	1.59
India	**1987**	**1991**	**1994**
S.D.	0.69	0.67	0.67
75 – 25	0.79	0.81	0.81
90 – 10	1.73	1.64	1.60
United States	**1977**	**1987**	**1997**
S.D.	0.45	0.41	0.49
75 – 25	0.46	0.41	0.53
90 – 10	1.04	1.01	1.19

Large Gains From Equalizing Marginal Products

TABLE IV
TFP GAINS FROM EQUALIZING TFPR WITHIN INDUSTRIES

China	1998	2001	2005
%	115.1	95.8	86.6
India	1987	1991	1994
%	100.4	102.1	127.5
United States	1977	1987	1997
%	36.1	30.7	42.9

Efficient vs. Actual Size Distribution



Takeaways from Misallocation Discussion

- Large dispersion in productivity across firms
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$\Longrightarrow$ **The rest of my lectures is figuring out what these wedges are**