Lecture 19: Individual-Specific Parameters I

ResEcon 703: Topics in Advanced Econometrics

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Agenda

Last time

- Simulation-Based Estimation Example in R

Today

- Individual-Specific Parameters
- Applications of Individual-Specific Parameters
- Error in Individual-Specific Parameters

Upcoming

- Reading for next time
  - Optional: Fowlie (2010)
- Problem sets
  - Problem Set 4 is posted, due November 21

Mixed Logit Model

The mixed logit model allows for unobserved variation in coefficients throughout the population

- The distribution of these coefficients in the population is $f(\beta \mid \theta)$
- We estimate the parameters, θ, that define these population distributions
- This population distribution and the parameters that define it tell us nothing about where any individual decision maker falls within that distribution of coefficients

What if we want a better idea of an individual's coefficients?

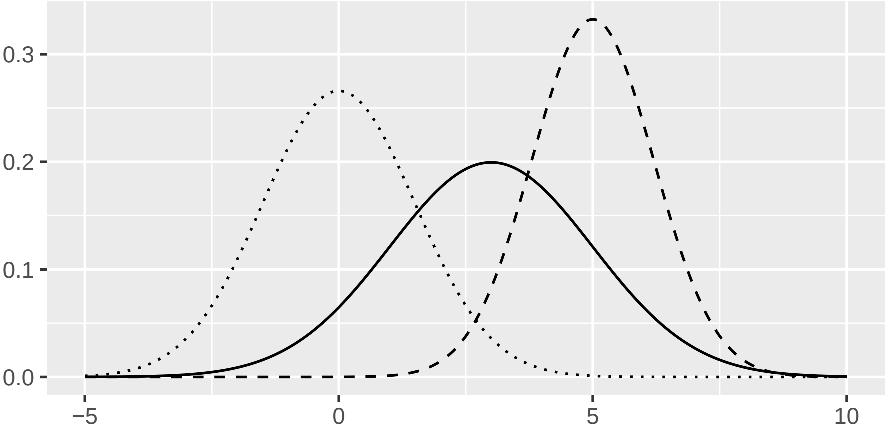
- We can combine the unconditional (or population) distribution of coefficients and the choices made by the individual to define a conditional distribution of coefficients

Individual-Specific Parameters

Example of Individual-Specific Parameters

We are studying how commuters choose their travel mode

- β_0 tells us the relative utility of driving
- We think there is heterogeneity in driving preferences, so we model β_0 as a random coefficient and find $\beta_0 \sim N(3, 4)$
- But what are the conditional distributions for drivers and non-drivers?



Distributions of Coefficients

The unconditional distribution, $f(\beta \mid \theta)$ is the distribution of coefficients throughout the population (of which our sample is representative)

The conditional distribution, $h(\beta \mid y, x, \theta)$ is the distribution of coefficients in the subpopulation of individuals who would make decision(s) y when faced with the choice(s) described by data x

- We cannot recover the actual coefficients for a specific person
- But we can recover the distribution of coefficients for a specific person and others who would make the same decision(s) in the same choice situation(s)

Random Coefficients

The utility that decision maker n obtains from alternative j in time t is

$$U_{njt} = \beta_n' x_{njt} + \varepsilon_{njt}$$

- x_{njt}: data for decision maker n and alternative j in time t
- β_n': individual-specific coefficients $\sim f(\beta \mid \theta)$ in the population
- ε_{njt}: i.i.d. extreme value error term

To simplify notation

- x_n: data collectively defined for all alternatives and times
- y_n: sequence of alternatives chosen by decision maker n

Choice Probabilities with Random Coefficients

If we knew an individual's coefficients, β_n, then the probability of choosing y_n when faced with data x_n would be

$$P(y_n \mid x_n, \beta_n) = \prod_{t=1}^{T} L_{nt}(y_{nt} \mid \beta_n)$$

where

$$L_{nt}(y_{nt} \mid \beta_n) = \frac{e^{\beta_n' x_{ny_{nt}t}}}{\sum_{j=1}^{J} e^{\beta_n' x_{njt}}}$$

But we do not know each individual's coefficients, β_n, so we have to consider the unconditional distribution of parameters in the population and intergrate over this density

$$P(y_n \mid x_n, \theta) = \int P(y_n \mid x_n, \beta) f(\beta \mid \theta) d\beta$$

Conditional Distribution of Random Coefficients

Using Bayes' Rule, we can express the joint density of β and y_n as either side of the expression

$$h(\beta \mid y_n, x_n, \theta) \times P(y_n \mid x_n, \theta) = P(y_n \mid x_n, \beta) \times f(\beta \mid \theta)$$

Rearranging terms gives us an expression for the conditional distribution

$$h(\beta \mid y_n, x_n, \theta) = \frac{P(y_n \mid x_n, \beta) \times f(\beta \mid \theta)}{P(y_n \mid x_n, \theta)}$$

$h(\beta \mid y_n, x_n, \theta)$, or the density of β among the subpopulation who would choose y_n when facing choice(s) defined by x_n, is proportional to the product of

- The density of β in the population; and
- The probability that y_n would be chosen if an individual's coefficients were β (when facing choice(s) defined by x_n)

Applications of Individual-Specific Parameters

Conditional Mean Coefficients

Using the previous formula for the conditional distribution, we can calculate the mean coefficient for an individual (and similar individuals)

$$\begin{aligned}\bar{\beta}_n &= \int \beta h(\beta \mid y_n, x_n, \theta) d\beta \\ &= \frac{\int \beta P(y_n \mid x_n, \beta) f(\beta \mid \theta) d\beta}{\int P(y_n \mid x_n, \beta) f(\beta \mid \theta) d\beta}\end{aligned}$$

These integrals have to be simulated, yielding the simulated conditional mean coefficient

$$\check{\beta}_n = \sum_{r=1}^{R} w^r \beta^r$$

where

$$w^r = \frac{P(y_n \mid x_n, \beta^r)}{\sum_{r=1}^{R} P(y_n \mid x_n, \beta^r)}$$

Future Choices

If we observe a decision maker's past choice, we can refine future choice probabilities by conditioning on those past choices

$$P(i \mid x_{nT+1}, y_n, x_n, \theta) = \int L_{nT+1}(i \mid \beta) h(\beta \mid y_n, x_n, \theta) d\beta$$

where

$$L_{nT+1}(i \mid \beta) = \frac{e^{\beta' x_{niT+1}}}{\sum_{j=1}^{J} e^{\beta' x_{njT+1}}}$$

We can simulate this probability analogously to the previous slide

$$\check{P}_{niT+1}(y_n, x_n, \theta) = \sum_{r=1}^{R} L_{nT+1}(i \mid \beta^r)$$

where

$$w^r = \frac{P(y_n \mid x_n, \beta^r)}{\sum_{r=1}^{R} P(y_n \mid x_n, \beta^r)}$$

Error in Individual-Specific Parameters

Error in Individual-Specific Parameters

Individual-specific parameters (and their applications) depend on the population parameters, θ

- But we do not know the true values of θ
- Instead, we have an estimate of θ, given by $\hat{\theta}$, which has variance-covariance matrix $\hat{W}$

Our estimator, $\hat{\theta}$, is a consistent estimate of the true θ, so we can use our estimated population parameters to get consistent estimates of individual-specific parameters, conditional distribution, mean conditional coefficients, etc.

When using individual-specific parameters (or condition distributions), you may want to represent the underlying uncertainty in the population parameters (or conditional distribution)

- Bootstrap standard errors using the variance-covariance of your estimator (see the Train textbook for more)

Number of Observed Choices

As the number of observed choices (T) increases, the conditional mean coefficient for an individual (and similar individuals), $\bar{\beta}_n$, converges to the individual-specific coefficient, β_n

- $\bar{\beta}_n$ is a consistent estimate of β_n

$$\bar{\beta}_n \xrightarrow{p} \beta_n$$

You must observe (and model) many choices for this convergence to become close

- Train conducts a Monte Carlo simulation exercise to find that even $T = 50$ yields a substantial difference between $\bar{\beta}_n$ and β_n
- See the Train textbook for more details on this point and the Monte Carlo simulation

Announcements

Reading for next time

- Optional: Fowlie (2010)

Office hours

- Reminder: 2:00–3:00 on Tuesdays in 218 Stockbridge

Upcoming

- Problem Set 4 is posted, due November 21