Week 2: R Tutorial

ResEcon 703: Topics in Advanced Econometrics

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Agenda

Last week

- Structural estimation

This week's topics

- R resources
- Objects in R
- Functions and packages in R
- Math and statistics in R
- Data in R
- R examples

This week's "reading"

- R `swirl` interactive tutorials

R Resources

Hat Tips

This lecture is inspired heavily by notes and slides created by

- Fiona Burlig, University of Chicago
- Grant McDermott, University of Oregon
- Ed Rubin, University of Oregon

Many thanks to them for generously making their course materials available online for all!

Installing R

Installing R is *usually* straightforward

- Download (`cran.r-project.org`) and install R
- Download (`www.rstudio.com/products/rstudio/download`) and install RStudio Desktop (Open Source License)

What is the difference between R and RStudio?

R is like a car's engine. It is the program that powers your data analysis.

RStudio is like a car's dashboard. It is the program you interact with to harness the power of your "engine."

R swirl Interactive Tutorials

swirl is an R package that interactively teaches you how to use R

- Information available here: swirlstats.com

```
## Install swirl package
install.packages('swirl')
## Load swirl package
library(swirl)
## Install swirl tutorials
install_course('R Programming')
install_course('Getting and Cleaning Data')
install_course('Advanced R Programming')
## Start swirl tutorials
swirl()
```

These three swirl tutorials (R Programming, Getting and Cleaning Data, and Advanced R Programming) introduce the main R concepts we will use in this course

More R Resources

These links provide a variety of perspectives and topics related to using R for statistical analysis, all of which may be useful as you learn to use R for structural estimation in this course

- DataCamp's Introduction to R
- R for Data Science book
- Advanced R book
- Ed Rubin's Econometrics lab slides
- Ed Rubin's Econometrics section notes
- Fiona Burlig's Econometrics section notes (warning: puns ahead)
- Grant McDermott's Data Science for Economists lecture slides

Some Complements to R

LaTeX and `knitr`

- LaTeX (`www.latex-project.org`): Typesetting system with great functionality for technical and scientific documents
- `knitr` (`yihui.name/knitr`): R package that integrates R code and output into LaTeX documents (or HTML, Markdown, etc.)

Git, GitHub, and SmartGit

- Git (`git-scm.com`): Version control system
- GitHub (`github.com`): Hosting platform for Git
  - Some alternatives exist: BitBucket, SourceForge, GitLab
- SmartGit (`www.syntevo.com/smartgit`): GUI client for Git
  - Many alternatives exist: GitHub Desktop, GitKraken, SourceTree

Objects in R

Object Basics

Everything is an object, and every object has a name and value

```r
## Assign a value of 1 to an object called a
a <- 1
## Assign a value of 2 to an object called b
b <- 2
## You use these objects in operations and functions
a + b
## [1] 3

## Assign object c to have a value equal to a + b
c <- a + b
c
## [1] 3
```

Classes, Types, and Structures

Every object has a type

- Numeric: `1`, `0.5`, `2/3`, `pi`
- Character: `"Hello"`, `"cruel world"`, `"Metrics is fun!"`
- Logical: `TRUE`, `FALSE`, `T`, `F`

Every object has a structure

- Vector
- Matrix
- List
- Data frame

`class()`, `typeof()`, `str()` give information about an object

Vectors

A vector is a collection of elements of the same type

- c() combines elements into a vector
- seq() and : create sequential vectors of numeric elements

```
## Create a numeric vector
c(1, 1, 2, 3, 5, 8, 13)
## [1]  1  1  2  3  5  8 13

## Create a sequential vector
0:9
##  [1] 0 1 2 3 4 5 6 7 8 9

## Create a character vector
c('Hello', 'world')
## [1] "Hello" "world"
```

If you combine elements of different types, R will convert some

```
## Create a vector with numeric, character, and logical elements
c(1, 'Hello', 3, 'world', TRUE)
## [1] "1"     "Hello" "3"     "world" "TRUE"
```

Matrices

A matrix is a collection of elements of the same type arranged in two dimensions

`matrix()` arranges a vector of data into a matrix

- `data`: Vector of data to create matrix
- `nrow` or `ncol`: Number of rows or columns in the matrix
- `byrow`: Logical indicating how to arrange data

```
## Create a 2 (rows) x 5 (columns) matrix of 1:10 arranged by row
matrix(data = 1:10, nrow = 2, byrow = TRUE)
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    2    3    4    5
## [2,]    6    7    8    9   10
```

Lists

A list is a collection of elements that can have different types and different structures

`list()` combines elements into a list

```
## Create a list with a numeric vector, matrix, and character vector
list(c(2, 4, 6, 8), matrix(1:4, 2), c('a', 'b', 'c'))
## [[1]]
## [1] 2 4 6 8
##
## [[2]]
##      [,1] [,2]
## [1,]    1    3
## [2,]    2    4
##
## [[3]]
## [1] "a" "b" "c"
```

Data Frames

A data frame is a structured table of data arranged in two dimensions

- Each column is a “variable” and each row is an “observation”
- Technically, a data frame is a list of named vectors of the same length
  - Each vector is a “variable”
  - The length of each vector equals the number of “observations”

`data.frame()` combines vectors into a data frame

```
## Create a date frame with 4 variables and 3 observations
data.frame(x = 0:2, y = c(2, 4, 8), z = c(1, 5, 7), w = c('a', 'b', 'c'))
##   x y z w
## 1 0 2 1 a
## 2 1 4 5 b
## 3 2 8 7 c
```

Functions and Packages in R

Functions

A function in R

1. Takes some inputs
2. Performs some internal tasks
3. Returns some output

We have already seen some examples of functions

- `matrix()`
    1. Takes a vector of data, information about the size of the matrix, and information about the arrangement of the matrix
    2. Arranges the data in the way specified by the other inputs
    3. Returns a matrix object

Use `?` (e.g., `?matrix`) to get the help file for a function

Function Inputs

Many functions have default inputs so you do not have to specify all the arguments

- These defaults are shown when you look at the function help file

Use `?matrix` to see the set of default inputs for the `matrix()` function

```
## Matrix function default inputs
matrix(data = NA, nrow = 1, ncol = 1, byrow = FALSE, dimnames = NULL)
```

So the default inputs would create a 1×1 matrix of `NA`

```
## Create matrix with default inputs
matrix()
##      [,1]
## [1,]   NA
```

Inputs can also be highly flexible

- `c()` allows for any number of arguments (as long as you have the memory to create a vector of the specified length)

User-Defined Functions

R makes it easy to define your own functions

Why create your own functions?

- You are performing the same task more than once
- You want to make it easier to parallelize your code
- You want to make your code more readable

How to create your own functions using `function(){}`

1. Specify the inputs in the `()`
2. Write the code for the function tasks in the `{}`
3. Specify the output using `return()` in the `{}`

Function Example

Make a function that calculates the mean sum of squares of three numbers

```
## Define a function that calculates the MSS from three inputs
mean_sum_squares <- function(num1, num2, num3){
  ## Calculate the mean sum of squares
  mss <- (num1^2 + num2^2 + num3^2) / 3
  ## Return the answer
  return(mss)
}
```

Try it out

```
## Calculate the mean sum of squares of 1, 2, and 3
mean_sum_squares(1, 2, 3)
## [1] 4.666667
```

What if we want a default argument?

```
## Make 3 the default input for the third argument
mean_sum_squares <- function(num1, num2, num3 = 3)
```

What if we want a flexible number of inputs?

- That is a little more complicated and context-specific...

Packages

A package is a bundle of code, documentation, and data that has been created and distributed by another R user

- More than 16,000 packages are available on CRAN, the official repository of R packages

What is so great about packages?

- Packages greatly increase the functionality available to you through "canned" routines
- Packages are open source
  - A package can be created by anyone, even you!
  - You can see the source code in any package
- Some packages have vignettes that provide detailed examples for using the package's functionality

Any problems to be aware of?

- A package can be created by anyone, so *caveat utilitor* (user beware)

Using Packages

First download a package from CRAN using `install.packages()`

```r
## Install a few packages we will use in this course
install.packages(c('tidyverse', 'mlogit', 'gmm'))
```

Then load the package into your R session using `library()`

```r
## Load those packages
library(tidyverse)
library(mlogit)
library(gmm)
```

Update packages occasionally using `update.packages()`

Recommended Packages

Packages we will use in this course

- `tidyverse`
  - Collection of packages that improve data analysis and visualization
- `mlogit`
  - Estimating multinomial logit models
- `gmm`
  - Generalized method of moments estimation

Other good packages

- `glue`
  - Character functions
- `lubridate`
  - Date and time functions
- `lfe`
  - Fixed effects models
- `furrr`
  - Parallelization

Math and Statistics in R

Math Operations

```
## Addition
a + b
## [1] 3

## Subtraction
a - b
## [1] -1

## Multiplication
a * b
## [1] 2

## Division
a / b
## [1] 0.5

## Exponents
a^b
## [1] 1
```

Math Functions

```r
## Absolute value
abs(a - b)
## [1] 1

## Exponential
exp(a)
## [1] 2.718282

## Square root
sqrt(b)
## [1] 1.414214

## Natural log
log(b)
## [1] 0.6931472

## Log base 10
log(b, base = 10)
## [1] 0.30103
```

Statistics Functions

```r
## Create a vector 0 to 4
v <- 0:4
v
## [1] 0 1 2 3 4

## Mean
mean(v)
## [1] 2

## Median
median(v)
## [1] 2

## Standard deviation
sd(v)
## [1] 1.581139
```

Sampling Functions

```r
## Set the seed for randomization
set.seed(703)
## Draw from a random normal N(3, 2)
rnorm(n = 5, mean = 3, sd = sqrt(2))
## [1] 1.142567 4.223916 1.236003 3.846436 1.268874

## Draw with replacement from v
sample(v, size = 10, replace = TRUE)
##  [1] 1 0 1 3 1 0 4 1 4 0

# CDF of a standard normal at z = 1.96
pnorm(q = 1.96, mean = 0, sd = 1)
## [1] 0.9750021
```

Vectorization

Many operations and functions are applied to each element of a vector

```
## Addition with each element
v + a
## [1] 1 2 3 4 5

## Multiplication with each element
v * b
## [1] 0 2 4 6 8

## Exponential of each element
exp(v)
## [1]  1.000000  2.718282  7.389056 20.085537 54.598150

## Natural log of each element
log(v)
## [1]      -Inf 0.0000000 0.6931472 1.0986123 1.3862944
```

Vector Math

You can also operate on vectors elementwise

```
## Elementwise addition
v + 1:5
## [1] 1 3 5 7 9

## Elementwise multiplication
v * 1:5
## [1]  0  2  6 12 20
```

But weird things can happen if the vectors are different lengths

```
## Elementwise addition with different lengths
v + 1:4

## Warning in v + 1:4:  longer object length is not a multiple of
shorter object length
## [1] 1 3 5 7 5
```

Indexing Vectors

Access elements within a vector using []

```
## Access the second element of v
v[2]
## [1] 1

## Access the second and fourth elements of v
v[c(2, 4)]
## [1] 1 3

## Access all but the first element of v
v[-1]
## [1] 1 2 3 4

## Replace the first element of v with 5
v[1] <- 5
v
## [1] 5 1 2 3 4
```

Matrices as Vectors

Matrices (usually) work like vectors

```
## Create a matrix
m <- matrix(1:4, nrow = 2)
m
##      [,1] [,2]
## [1,]    1    3
## [2,]    2    4

## Mean
mean(m)
## [1] 2.5

## Natural log of each element
log(m)
##           [,1]     [,2]
## [1,] 0.0000000 1.098612
## [2,] 0.6931472 1.386294
```

Matrix Addition

Matrix addition and subtraction is performed elementwise

```
## Create a second matrix
n <- matrix(c(2, 4, 6, 8), nrow = 2)
n
##      [,1] [,2]
## [1,]    2    6
## [2,]    4    8

## Matrix addition
m + n
##      [,1] [,2]
## [1,]    3    9
## [2,]    6   12
```

Matrix Multiplication

Using ∗ to multiply matrices performs elementwise multiplication

```
## Elementwise matrix multiplication
m * n
##      [,1] [,2]
## [1,]    2   18
## [2,]    8   32
```

You must use %*% to get the matrix product

```
## Matrix product
m %*% n
##      [,1] [,2]
## [1,]   14   30
## [2,]   20   44
```

Matrix Functions

R has many other functions for use with matrices

```
## Transpose
t(m)
##      [,1] [,2]
## [1,]    1    2
## [2,]    3    4

## Inverse
solve(m)
##      [,1] [,2]
## [1,]   -2  1.5
## [2,]    1 -0.5
```

Indexing Matrices

Access elements within a matrix using []

```
## Access the element in the second row and first column of m
m[2, 1]
## [1] 2

## Access the first row of m
m[1, ]
## [1] 1 3

## Access the second column of m
m[, 2]
## [1] 3 4
```

Data in R

Example Data Frame

You will mostly interact with datasets in the form of data frames

- R includes several example data frames

```
## Show an example data frame, mtcars
head(mtcars)
##                    mpg cyl disp  hp drat    wt  qsec vs am gear carb
## Mazda RX4         21.0   6  160 110 3.90 2.620 16.46  0  1    4    4
## Mazda RX4 Wag     21.0   6  160 110 3.90 2.875 17.02  0  1    4    4
## Datsun 710        22.8   4  108  93 3.85 2.320 18.61  1  1    4    1
## Hornet 4 Drive    21.4   6  258 110 3.08 3.215 19.44  1  0    3    1
## Hornet Sportabout 18.7   8  360 175 3.15 3.440 17.02  0  0    3    2
## Valiant           18.1   6  225 105 2.76 3.460 20.22  1  0    3    1
```

Indexing Data Frames

Access elements within a data frame using []

```
## Access the third observation of mtcars
mtcars[3, ]
##             mpg cyl disp hp drat   wt  qsec vs am gear carb
## Datsun 710 22.8   4  108 93 3.85 2.32 18.61  1  1    4    1

## Access the second variable of mtcars
mtcars[, 2]
##  [1] 6 6 4 6 8 6 8 4 4 6 6 8 8 8 8 8 8 4 4 4 4 8 8 8 8 4 4 4 8 6 8 4
```

Access a variable of a data frame using $

```
## Access the cyl variable of mtcars
mtcars$cyl
##  [1] 6 6 4 6 8 6 8 4 4 6 6 8 8 8 8 8 8 4 4 4 4 8 8 8 8 4 4 4 8 6 8 4
```

Adding New Variables

You may want to add new variables to a data frame

```
## Add an id variable to mtcars
mtcars$id <- 1:nrow(mtcars)
## Add a variable that is the power-to-weight ratio (hp / wt)
mtcars$ptw <- mtcars$hp / mtcars$wt
head(mtcars)
##                    mpg cyl disp  hp drat    wt  qsec vs am gear carb
## Mazda RX4         21.0   6  160 110 3.90 2.620 16.46  0  1    4    4
## Mazda RX4 Wag     21.0   6  160 110 3.90 2.875 17.02  0  1    4    4
## Datsun 710        22.8   4  108  93 3.85 2.320 18.61  1  1    4    1
## Hornet 4 Drive    21.4   6  258 110 3.08 3.215 19.44  1  0    3    1
## Hornet Sportabout 18.7   8  360 175 3.15 3.440 17.02  0  0    3    2
## Valiant           18.1   6  225 105 2.76 3.460 20.22  1  0    3    1
##                   id      ptw
## Mazda RX4          1 41.98473
## Mazda RX4 Wag      2 38.26087
## Datsun 710         3 40.08621
## Hornet 4 Drive     4 34.21462
## Hornet Sportabout  5 50.87209
## Valiant            6 30.34682
```

But that can get a little clunky. Is there a better way?

dplyr

dplyr is a package that greatly improves data manipulation in R

- Part of the tidyverse so it is already installed and loaded from earlier code

dplyr is a "grammar of data manipulation"

- Data compose the subjects of your analysis
- dplyr provides the the verbs
  - mutate(): Adds new variables
  - select(): Picks variables
  - filter(): Picks observations
  - arrange(): Changes the order of observations
  - summarize() or summarise(): Summarizes multiple observations

Adding New Variables with `dplyr`

`mutate(.data, ...)`

- `.data`: Existing data frame
- `...`: Names and values of new variables

```
## Add id and power-to-weight ratio variables
mtcars <- mutate(mtcars, id = 1:n(), ptw = hp / wt)
head(mtcars)
##    mpg cyl disp  hp drat    wt  qsec vs am gear carb id      ptw
## 1 21.0   6  160 110 3.90 2.620 16.46  0  1    4    4  1 41.98473
## 2 21.0   6  160 110 3.90 2.875 17.02  0  1    4    4  2 38.26087
## 3 22.8   4  108  93 3.85 2.320 18.61  1  1    4    1  3 40.08621
## 4 21.4   6  258 110 3.08 3.215 19.44  1  0    3    1  4 34.21462
## 5 18.7   8  360 175 3.15 3.440 17.02  0  0    3    2  5 50.87209
## 6 18.1   6  225 105 2.76 3.460 20.22  1  0    3    1  6 30.34682
```

Tibbles

`tidyverse` also introduces a new kind of data frame, the tibble

- Actually, `tibble` is the name of the package that has the code to create and manipulate objects of class `tbl_df`
- But it is easier to say "tibble," so that is what users call both the package and the object
- I will probably use "tibble" and "data frame" interchangeably to mean "tibble"

Why are tibbles better than data frames?

- Data frames sometimes exhibit weird behaviors related to naming variables or trying to convert variable types
- Tibbles are smarter about how much data they show you when you call them
    - You do not have to use `head()` to supress output

Example Tibble

`dplyr` comes with several examples tibbles

```
## Show an example tibble, starwars
starwars
## # A tibble: 87 x 14
##    name   height  mass hair_color skin_color eye_color birth_year sex
##    <chr>   <int> <dbl> <chr>      <chr>      <chr>          <dbl> <chr>
##  1 Luke~     172    77 blond      fair       blue            19   male
##  2 C-3PO     167    75 <NA>       gold       yellow         112   none
##  3 R2-D2      96    32 <NA>       white, bl~ red             33   none
##  4 Dart~     202   136 none       white      yellow          41.9 male
##  5 Leia~     150    49 brown      light      brown           19   fema~
##  6 Owen~     178   120 brown, gr~ light      blue            52   male
##  7 Beru~     165    75 brown      light      blue            47   fema~
##  8 R5-D4      97    32 <NA>       white, red red             NA   none
##  9 Bigg~     183    84 black      light      brown           24   male
## 10 Obi-~     182    77 auburn, w~ fair       blue-gray       57   male
## # ... with 77 more rows, and 6 more variables: gender <chr>,
## #   homeworld <chr>, species <chr>, films <list>, vehicles <list>,
## #   starships <list>
```

Let's play around with the `dplyr` verbs on this tibble

select() Example

```
## Select name, homeworld, and species in starwars
select(starwars, name, homeworld, species)
## # A tibble: 87 x 3
##    name               homeworld species
##    <chr>              <chr>     <chr>
##  1 Luke Skywalker     Tatooine  Human
##  2 C-3PO              Tatooine  Droid
##  3 R2-D2              Naboo     Droid
##  4 Darth Vader        Tatooine  Human
##  5 Leia Organa        Alderaan  Human
##  6 Owen Lars          Tatooine  Human
##  7 Beru Whitesun lars Tatooine  Human
##  8 R5-D4              Tatooine  Droid
##  9 Biggs Darklighter  Tatooine  Human
## 10 Obi-Wan Kenobi     Stewjon   Human
## # ... with 77 more rows
```

filter() Example

```
## Filter to show only droids in starwars
filter(starwars, species == 'Droid')
## # A tibble: 6 x 14
##   name   height  mass hair_color skin_color  eye_color birth_year sex
##   <chr>   <int> <dbl> <chr>      <chr>       <chr>          <dbl> <chr>
## 1 C-3PO     167    75 <NA>       gold        yellow           112 none
## 2 R2-D2      96    32 <NA>       white, bl~  red               33 none
## 3 R5-D4      97    32 <NA>       white, red  red               NA none
## 4 IG-88     200   140 none       metal       red               15 none
## 5 R4-P~      96    NA none       silver, r~  red, blue         NA none
## 6 BB8        NA    NA none       none        black             NA none
## # ... with 6 more variables: gender <chr>, homeworld <chr>,
## #   species <chr>, films <list>, vehicles <list>, starships <list>
```

arrange() Example

```
## Arrange alphabetically by name in starwars
arrange(starwars, name)
## # A tibble: 87 x 14
##    name  height  mass hair_color skin_color eye_color birth_year sex
##    <chr>  <int> <dbl> <chr>      <chr>      <chr>          <dbl> <chr>
##  1 Ackb~    180    83 none       brown mot~ orange          41   male
##  2 Adi ~    184    50 none       dark       blue            NA   fema~
##  3 Anak~    188    84 blond      fair       blue            41.9 male
##  4 Arve~     NA    NA brown      fair       brown           NA   male
##  5 Ayla~    178    55 none       blue       hazel           48   fema~
##  6 Bail~    191    NA black      tan        brown           67   male
##  7 Barr~    166    50 black      yellow     blue            40   fema~
##  8 BB8       NA    NA none       none       black           NA   none
##  9 Ben ~    163    65 none       grey, gre~ orange          NA   male
## 10 Beru~    165    75 brown      light      blue            47   fema~
## # ... with 77 more rows, and 6 more variables: gender <chr>,
## #   homeworld <chr>, species <chr>, films <list>, vehicles <list>,
## #   starships <list>
```

Multiple dplyr Functions

Nest functions inside one another to perform multiple functions

```
## Select, filter, and arrange
arrange(filter(select(starwars, name, homeworld, species), species == 'Droi
## # A tibble: 6 x 3
##   name   homeworld species
##   <chr>  <chr>     <chr>
## 1 BB8    <NA>      Droid
## 2 C-3PO  Tatooine  Droid
## 3 IG-88  <NA>      Droid
## 4 R2-D2  Naboo     Droid
## 5 R4-P17 <NA>      Droid
## 6 R5-D4  Tatooine  Droid
```

```
## Alternative code for those functions
arrange(
  filter(
    select(starwars, name, homeworld, species),
    species == 'Droid'
  ),
  name
)
```

But either option can get very difficult to read and understand

Pipes

Pipes make a sequence of functions or operations much more readable

- Put each new step on its own line rather than all together
- Start with the first step rather than working inside-out

`x %>% f(y)` is the same as `f(x, y)`

```
## Filter with pipes
starwars %>%
  filter(species == 'Droid')
## # A tibble: 6 x 14
##   name   height  mass hair_color skin_color eye_color birth_year sex
##   <chr>   <int> <dbl> <chr>      <chr>      <chr>          <dbl> <chr>
## 1 C-3PO     167    75 <NA>       gold       yellow           112 none
## 2 R2-D2      96    32 <NA>       white, bl~ red               33 none
## 3 R5-D4      97    32 <NA>       white, red red               NA none
## 4 IG-88     200   140 none       metal      red               15 none
## 5 R4-P~      96    NA none       silver, r~ red, blue         NA none
## 6 BB8        NA    NA none       none       black             NA none
## # ... with 6 more variables: gender <chr>, homeworld <chr>,
## #   species <chr>, films <list>, vehicles <list>, starships <list>
```

Multiple dplyr Functions Using Pipes

Let's do the same sequence of three functions but using pipes

```
## Select, filter, and arrange using pipes
starwars %>%
  select(name, homeworld, species) %>%
  filter(species == 'Droid') %>%
  arrange(name)
## # A tibble: 6 x 3
##   name   homeworld species
##   <chr>  <chr>     <chr>
## 1 BB8    <NA>      Droid
## 2 C-3PO  Tatooine  Droid
## 3 IG-88  <NA>      Droid
## 4 R2-D2  Naboo     Droid
## 5 R4-P17 <NA>      Droid
## 6 R5-D4  Tatooine  Droid
```

summarize() Example

summarize() applies a function to a group of observations

- group_by() specifies the grouping to use

```
## Calculate mean height and mass by species
starwars %>%
  group_by(species) %>%
  summarize(mean_height = mean(height), mean_mass = mean(mass))
## # A tibble: 38 x 3
##    species   mean_height mean_mass
##    <chr>           <dbl>     <dbl>
##  1 Aleena            79         15
##  2 Besalisk         198        102
##  3 Cerean           198         82
##  4 Chagrian         196         NA
##  5 Clawdite         168         55
##  6 Droid             NA         NA
##  7 Dug              112         40
##  8 Ewok              88         20
##  9 Geonosian        183         80
## 10 Gungan           209.        NA
## # ... with 28 more rows
```

`NA` and Other Special Values

R has several special values to indicate non-standard objects or elements

- `NA`: Missing value
- `NaN`: Not a number
- `NULL`: "Undefined"
- `Inf` and `-Inf`: ∞ and $-\infty$

Skipping NAs

The argument `na.rm = TRUE` skips missing values

```
## Calculate non-missing mean height and mass by species
starwars %>%
  group_by(species) %>%
  summarize(mean_height = mean(height, na.rm = TRUE),
            mean_mass = mean(mass, na.rm = TRUE))
## # A tibble: 38 x 3
##    species   mean_height mean_mass
##    <chr>           <dbl>     <dbl>
##  1 Aleena             79      15
##  2 Besalisk          198     102
##  3 Cerean            198      82
##  4 Chagrian          196     NaN
##  5 Clawdite          168      55
##  6 Droid             131.     69.8
##  7 Dug               112      40
##  8 Ewok               88      20
##  9 Geonosian         183      80
## 10 Gungan            209.     74
## # ... with 28 more rows
```

R Examples

OLS Regression in R

Using the `mtcars` dataset, regress `mpg` on `hp`

$$\mathtt{mpg}_i = \beta_0 + \beta_1 \mathtt{hp}_i + \varepsilon_i$$

Perform this simple linear OLS regression three ways:

1. "Canned" `lm()` function
2. "Hand-coded" OLS estimators
3. User-defined OLS function

Report parameter estimates, standard errors, t stats, and p values

But before running a regression...

Look at the mtcars Dataset

You should always double-check the structure of your dataset

```
## Look at the mtcars data
tibble(mtcars)
## # A tibble: 32 x 13
##      mpg   cyl  disp    hp  drat    wt  qsec    vs    am  gear  carb
##    <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
##  1  21       6  160    110  3.9   2.62  16.5     0     1     4     4
##  2  21       6  160    110  3.9   2.88  17.0     0     1     4     4
##  3  22.8     4  108     93  3.85  2.32  18.6     1     1     4     1
##  4  21.4     6  258    110  3.08  3.22  19.4     1     0     3     1
##  5  18.7     8  360    175  3.15  3.44  17.0     0     0     3     2
##  6  18.1     6  225    105  2.76  3.46  20.2     1     0     3     1
##  7  14.3     8  360    245  3.21  3.57  15.8     0     0     3     4
##  8  24.4     4  147.    62  3.69  3.19  20       1     0     4     2
##  9  22.8     4  141.    95  3.92  3.15  22.9     1     0     4     2
## 10  19.2     6  168.   123  3.92  3.44  18.3     1     0     4     4
## # ... with 22 more rows, and 2 more variables: id <int>, ptw <dbl>
```

Summarize the mtcars Dataset

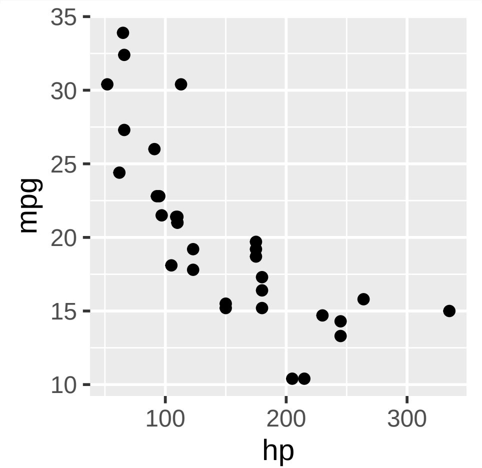
It can be helpful to generate basic summary statistics for your dataset to get a sense for the scale and variation of each variable

```
## Summarize the mtcars dataset
mtcars %>%
  select(mpg, disp, hp, wt, qsec) %>%
  summary()
##       mpg             disp             hp               wt
##  Min.   :10.40   Min.   : 71.1   Min.   : 52.0   Min.   :1.513
##  1st Qu.:15.43   1st Qu.:120.8   1st Qu.: 96.5   1st Qu.:2.581
##  Median :19.20   Median :196.3   Median :123.0   Median :3.325
##  Mean   :20.09   Mean   :230.7   Mean   :146.7   Mean   :3.217
##  3rd Qu.:22.80   3rd Qu.:326.0   3rd Qu.:180.0   3rd Qu.:3.610
##  Max.   :33.90   Max.   :472.0   Max.   :335.0   Max.   :5.424
##       qsec
##  Min.   :14.50
##  1st Qu.:16.89
##  Median :17.71
##  Mean   :17.85
##  3rd Qu.:18.90
##  Max.   :22.90
```

Plot the `mtcars` Dataset

Plotting the data can give an idea of what to expect from your regression

```
## Plot the mtcars dataset
ggplot(data = mtcars, mapping = aes(x = hp, y = mpg)) +
  geom_point()
```



Regression Using `lm()` Function

The `lm()` function fits a linear model to a dataset

- To see how to use the `lm()` function, type `?lm`

```
## See the help file for lm()
?lm
lm(formula, data, subset, weights, na.action,
   method = "qr", model = TRUE, x = FALSE, y = FALSE, qr = TRUE,
   singular.ok = TRUE, contrasts = NULL, offset, ...)
```

The `lm()` function requires a `formula` object

- $\mathtt{y} \sim \mathtt{x1} + \mathtt{x2} + \mathtt{x3}$ regresses variable `y` on variables `x1`, `x2`, and `x3`

Regression Using lm() Function

$$\texttt{mpg}_i = \beta_0 + \beta_1 \texttt{hp}_i + \varepsilon_i$$

```
## Run OLS regression
reg_lm <- lm(formula = mpg ~ hp, data = mtcars)
## Summarize OLS regression results
summary(reg_lm)
##
## Call:
## lm(formula = mpg ~ hp, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.7121 -2.1122 -0.8854  1.5819  8.2360
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 30.09886    1.63392  18.421  < 2e-16 ***
## hp          -0.06823    0.01012  -6.742 1.79e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.863 on 30 degrees of freedom
## Multiple R-squared:  0.6024,Adjusted R-squared:  0.5892
## F-statistic: 45.46 on 1 and 30 DF,  p-value: 1.788e-07
```

Regression Using Hand-Coded Estimators

$$\texttt{mpg}_i = \beta_0 + \beta_1 \texttt{hp}_i + \varepsilon_i$$

How do we estimate the β parameters and their standard errors?

- Reminder: OLS has simple closed-form formulas!

For the general regression equation

$$\boldsymbol{y} = \boldsymbol{X}\beta + \boldsymbol{\varepsilon}$$

we can estimate $\widehat{\beta}$ and $\widehat{\text{Cov}}(\widehat{\beta})$ using

$$\widehat{\beta} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$$

$$\widehat{\text{Cov}}(\widehat{\beta}) = s^2(\boldsymbol{X}'\boldsymbol{X})^{-1}$$

where

$$s^2 = \frac{\boldsymbol{e}'\boldsymbol{e}}{n-k}$$

$$\boldsymbol{e} = \boldsymbol{y} - \widehat{\boldsymbol{y}}$$

Regression Using Hand-Coded Estimators

$$\widehat{\beta} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$$
$$\widehat{\text{Cov}}(\widehat{\beta}) = s^2(\boldsymbol{X}'\boldsymbol{X})^{-1}$$

Steps to code these estimators

1. Construct matrices $\boldsymbol{X}$ and $\boldsymbol{y}$
2. Estimate parameters $\widehat{\beta}$ using above equation
3. Calculate fitted values of $\boldsymbol{y}$, $\widehat{\boldsymbol{y}}$
4. Calculate residuals, $\boldsymbol{e}$
5. Estimate the variance of error terms, s^2
6. Estimate variance-covariance matrix $\widehat{\text{Cov}}(\widehat{\beta})$ using above equation
7. Calculate standard errors
8. Calculate t stats
9. Calculate p values
10. Organize results table

Regression Using Hand-Coded Estimators

Step 1: Construct matrices $\boldsymbol{X}$ and $\boldsymbol{y}$

```
## Add column of ones for the constant term
reg_data <- mtcars %>%
  mutate(constant = 1)
## Select data for X and convert to a matrix
X <- reg_data %>%
  select(constant, hp) %>%
  as.matrix()
## Select data for y and convert to a matrix
y <- reg_data %>%
  select(mpg) %>%
  as.matrix()
```

Regression Using Hand-Coded Estimators

Step 1b: Make sure matrices look correct

```
## Make sure matrices look correct
head(X)
##      constant  hp
## [1,]        1 110
## [2,]        1 110
## [3,]        1  93
## [4,]        1 110
## [5,]        1 175
## [6,]        1 105

head(y)
##       mpg
## [1,] 21.0
## [2,] 21.0
## [3,] 22.8
## [4,] 21.4
## [5,] 18.7
## [6,] 18.1
```

Regression Using Hand-Coded Estimators

Step 2: Estimate parameters $\widehat{\beta}$ using

$$\widehat{\beta} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$$

```
## Estimate beta parameters
beta_hat <- solve(t(X) %*% X) %*% t(X) %*% y
beta_hat
##                  mpg
## constant 30.09886054
## hp       -0.06822828
```

Regression Using Hand-Coded Estimators

Step 3: Calculate fitted values of $\boldsymbol{y}$, $\widehat{\boldsymbol{y}}$, using

$$\widehat{\boldsymbol{y}} = \boldsymbol{X}\widehat{\beta}$$

```
## Calculate fitted y values
y_hat <- X %*% beta_hat
head(y_hat)
##           mpg
## [1,] 22.59375
## [2,] 22.59375
## [3,] 23.75363
## [4,] 22.59375
## [5,] 18.15891
## [6,] 22.93489
```

Regression Using Hand-Coded Estimators

Step 4: Calculate residuals, $\boldsymbol{e}$, using

$$\boldsymbol{e} = \boldsymbol{y} - \widehat{\boldsymbol{y}}$$

```
## Calculate residuals
resid <- y - y_hat
head(resid)
##             mpg
## [1,] -1.5937500
## [2,] -1.5937500
## [3,] -0.9536307
## [4,] -1.1937500
## [5,]  0.5410881
## [6,] -4.8348913
```

Regression Using Hand-Coded Estimators

Step 5: Estimate the variance of error terms, s^2, using

$$s^2 = \frac{\boldsymbol{e}'\boldsymbol{e}}{n - k}$$

```
## Estimate variance of error term
sigma2_hat <- t(resid) %*% resid / (nrow(X) - ncol(X))
sigma2_hat
##          mpg
## mpg 14.92248
```

Regression Using Hand-Coded Estimators

Step 6: Estimate variance-covariance matrix $\widehat{\text{Cov}}(\widehat{\beta})$ using

$$\widehat{\text{Cov}}(\widehat{\beta}) = s^2(\boldsymbol{X}'\boldsymbol{X})^{-1}$$

```
## Estimate variance-covariance matrix of beta estimates
vcov_hat <- c(sigma2_hat) * solve(t(X) %*% X)
vcov_hat
##               constant            hp
## constant  2.66969767 -0.0150208454
## hp       -0.01502085  0.0001024003
```

Regression Using Hand-Coded Estimators

Steps 7–9: Calculate standard errors, t stats, and p values

```
## Calculate standard errors of beta estimates
std_err <- sqrt(diag(vcov_hat))
std_err
##  constant        hp
## 1.6339210 0.0101193

## Calculate t stats of beta estimates
t_stat <- beta_hat / std_err
t_stat
##                mpg
## constant 18.421246
## hp       -6.742389

## Calculate p values of beta estimates
p_value <- 2 * pt(q = -abs(t_stat), df = nrow(X) - ncol(X))
p_value
##                   mpg
## constant 6.642736e-18
## hp       1.787835e-07
```

Regression Using Hand-Coded Estimators

Step 10: Organize results table

```
## Organize regression results into matrix
results <- cbind(beta_hat, std_err, t_stat, p_value)
results
##                  mpg   std_err       mpg          mpg
## constant 30.09886054 1.6339210 18.421246 6.642736e-18
## hp       -0.06822828 0.0101193 -6.742389 1.787835e-07

## Name columns of results matrix
colnames(results) <- c('Estimate', 'Std. Error', 't stat', 'p value')
results
##             Estimate Std. Error    t stat      p value
## constant 30.09886054  1.6339210 18.421246 6.642736e-18
## hp       -0.06822828  0.0101193 -6.742389 1.787835e-07
```

Regression Using Hand-Coded Estimators

Compare our hand-coded estimates to the canned `lm()` estimates

```
## Compare to lm() results
summary(reg_lm)
##
## Call:
## lm(formula = mpg ~ hp, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.7121 -2.1122 -0.8854  1.5819  8.2360
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 30.09886    1.63392  18.421  < 2e-16 ***
## hp          -0.06823    0.01012  -6.742 1.79e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.863 on 30 degrees of freedom
## Multiple R-squared:  0.6024,Adjusted R-squared:  0.5892
## F-statistic: 45.46 on 1 and 30 DF,  p-value: 1.788e-07

results
##             Estimate Std. Error    t stat      p value
## constant 30.09886054  1.6339210 18.421246 6.642736e-18
## hp       -0.06822828  0.0101193 -6.742389 1.787835e-07
```

Regression Using User-Defined OLS Function

We want to define a new function that does the same 10 steps we just worked through

Why would we want to put these steps inside a function?

- We might want to run more than one regression
- If we define the function to take variable arguments, then we can use the same basic coding framework to run many different OLS regressions

What do we want to be the variable arguments?

- Dataset
- y variable
- x variables
- Anything else?

Regression Using User-Defined OLS Function

```
## Function to perform OLS regression
ols <- function(data, y_var, x_vars){
  ## Add column of ones for the constant term
  reg_data <- data %>%
    mutate(constant = 1)
  ## Select data for X and convert to a matrix
  X <- reg_data %>%
    select(all_of(c('constant', x_vars))) %>%
    as.matrix()
  ## Select data for y and convert to a matrix
  y <- reg_data %>%
    select(all_of(y_var)) %>%
    as.matrix()
  ## Estimate beta parameters
  beta_hat <- solve(t(X) %*% X) %*% t(X) %*% y
  ## Calculate fitted y values
  y_hat <- X %*% beta_hat
  ## Calculate residuals
  resid <- y - y_hat
  ## Estimate variance of error term
  sigma2_hat <- t(resid) %*% resid / (nrow(X) - ncol(X))
  ## Estimate variance-covariance matrix of beta estimates
  vcov_hat <- c(sigma2_hat) * solve(t(X) %*% X)
  ## Calculate standard errors of beta estimates
  std_err <- sqrt(diag(vcov_hat))
  ## Calculate t stats of beta estimates
  t_stat <- beta_hat / std_err
  ## Calculate p values of beta estimates
  p_value <- 2 * pt(q = -abs(t_stat), df = nrow(X) - ncol(X))
  ## Organize regression results into matrix
  results <- cbind(beta_hat, std_err, t_stat, p_value)
  ## Name columns of results matrix
  colnames(results) <- c('Estimate', 'Std. Error', 't stat', 'p value')
  return(results)
```

Regression Using User-Defined OLS Function

$$\texttt{mpg}_i = \beta_0 + \beta_1 \texttt{hp}_i + \varepsilon_i$$

What arguments do we need to specify?

- data, y_var, and x_vars

```
## Regress mpg on hp in mtcars dataset
ols(data = mtcars, y_var = 'mpg', x_vars = 'hp')
##             Estimate Std. Error    t stat      p value
## constant 30.09886054  1.6339210 18.421246 6.642736e-18
## hp       -0.06822828  0.0101193 -6.742389 1.787835e-07
```

We have replicated the results from `lm()` and the earlier hand-coded estimators

Regression Using User-Defined OLS Function

Now use the same function for a different regression

- Regress mpg on hp, disp, wt, qsec

```
## Regress mpg on disp, hp, wt, and qsec in mtcars dataset
ols(data = mtcars,
    y_var = 'mpg',
    x_vars = c('hp', 'disp', 'wt', 'qsec'))
##              Estimate Std. Error     t stat     p value
## constant 27.329637967 8.63903219  3.1635069 0.003833942
## hp       -0.018666202 0.01561305 -1.1955515 0.242266764
## disp      0.002666431 0.01073767  0.2483249 0.805762061
## wt       -4.609122617 1.26585131 -3.6411248 0.001134320
## qsec      0.544160312 0.46649316  1.1664915 0.253616070
```

Regression Using User-Defined OLS Function

Try a different dataset in our OLS function

- R includes a built-in dataset `iris` that includes measurements from 50 iris flowers
- Regress `Petal.Length` on `Petal.Width`, `Sepal.Length`, and `Sepal.Width`

```r
## Regress Petal.Length on Sepal.Length, Sepal.Width, and Petal.Width
## in iris dataset
ols(data = iris,
    y_var = 'Petal.Length',
    x_vars = c('Petal.Width', 'Sepal.Length', 'Sepal.Width'))
##                Estimate Std. Error     t stat      p value
## constant     -0.2627112 0.29740608 -0.8833417 3.785039e-01
## Petal.Width   1.4467934 0.06761125 21.3987078 7.332477e-47
## Sepal.Length  0.7291384 0.05831949 12.5024834 7.656980e-25
## Sepal.Width  -0.6460124 0.06849745 -9.4311891 8.753029e-17
```