Problem Set 2

Topics in Advanced Econometrics (ResEcon 703)
University of Massachusetts Amherst

Due: October 17, 10:00 am ET

Rules

Email a single .pdf file of your problem set writeup, code, and output to `mwoerman@umass.edu` by the date and time above. You may work in groups of up to three, and all group members can submit the same code and output; indicate in your writeup who you worked with. You must submit a unique writeup that answers the problems below. You can discuss answers with your fellow group members, but your writeup must be in your own words. This problem set requires you to code your own estimators, rather than using R's "canned routines."

Data

Download the file `travel_datasets.zip` from the course website (`github.com/woerman/ResEcon703`). This zipped file contains two datasets, `travel_binary.csv` and `travel_multinomial.csv`, that you will use for this problem set. Both datasets contain simulated data on the travel mode choice of 1000 UMass graduate students commuting to campus. The `travel_binary.csv` dataset corresponds to commuting in the middle of winter when only driving a car or taking a bus are feasible options (assume the weather is too severe for even the heartiest graduate students to ride a bike or walk). The `travel_multinomial.csv` dataset corresponds to commuting in spring when riding a bike and walking are feasible alternatives. See the file `travel_descriptions.txt` for descriptions of the variables in each dataset.

Problem 1: Maximum Likelihood Estimation

Use the `travel_multinomial.csv` dataset for this question.

a. Model the travel mode choice to commute to campus during spring as a multinomial logit model. Include the cost and the time of each alternative as explanatory variables with common coefficients; do not include alternative-specific intercepts. That is, the representative utility for alternative j is simply

$$V_{nj} = \beta_1 C_{nj} + \beta_2 T_{nj}$$

where C_{nj} is the cost of alternative j and T_{nj} is the time of alternative j. Estimate the parameters of this model by maximum likelihood estimation. The following steps can provide a rough guide to creating your own maximum likelihood estimator:

I. Create a function that takes a set of parameters and data as inputs: `function(parameters, data)`.

II. Within that function, make the following calculations:

   i. Calculate the representative utility for each alternative and for each decision maker.

   ii. Calculate the choice probability of the chosen alternative for each decision maker.

   iii. Sum the log of these choice probabilities to get the log-likelihood.

   iv. Return the negative of the log-likelihood.

III. Maximize the log-likelihood (or minimize its negative) using `optim()`. Your call of the `optim()` function may look something like:

```
optim(par = your_starting_guesses, fn = your_function, data = your_data,
      method = 'BFGS', hessian = TRUE)
```

Report your parameter estimates, standard errors, z-stats, and p-values. Briefly interpret these results.

b. Again model the travel mode choice to commute to campus during spring as a multinomial logit model, but now add alternative-specific intercepts for all but one alternative. That is, the representative utility for alternative j is

$$V_{nj} = \alpha_j + \beta_1 C_{nj} + \beta_2 T_{nj}$$

where α_j is an alternative-specific intercept, C_{nj} is the cost of alternative j and T_{nj} is the time of alternative j. Estimate the parameters of this model by maximum likelihood estimation. Follow the same set of steps as in (a), but now you have five parameters and the representative utility calculation has an additional component. You can use matrix multiplication in your log-likelihood function to flexibly accommodate different models, or you can create a different function for each model. Report your parameter estimates, standard errors, z-stats, and p-values. Briefly interpret these results.

c. Conduct a likelihood ratio test on the model in part (b) to test the joint significance of the alternative-specific intercepts. That is, test the null hypothesis:

$$H_0\colon\ \alpha_b = \alpha_c = \alpha_w = 0$$

Your null hypothesis may be slightly different, depending on what you consider your "reference alternative." Do you reject this null hypothesis? What is the p-value of the test?

d. Again model the travel mode choice to commute to campus during spring as a multinomial logit model, but now add alternative-specific coefficients on the time variable. That is, the representative utility for alternative j is

$$V_{nj} = \alpha_j + \beta_1 C_{nj} + \beta_j T_{nj}$$

where α_j is an alternative-specific intercept, C_{nj} is the cost of alternative j, T_{nj} is the time of alternative j, and β_j varies for each alternative. Estimate the parameters of this model by maximum likelihood estimation. Follow the same set of steps as in (a), but now you have eight parameters and the representative utility calculation is different. You can use matrix multiplication in your log-likelihood function to flexibly accommodate different models, or you can create a different function for each model. Report your parameter estimates, standard errors, z-stats, and p-values. Briefly interpret these results.

e. Conduct a likelihood ratio test on the model in part (d) to test that the alternative-specific coefficients on time are significantly different from one another. That is, test the null hypothesis:

$$H_0\colon \beta_k = \beta_b = \beta_c = \beta_w$$

Do you reject this null hypothesis? What is the p-value of the test?

Problem 2: Nonlinear Least Squares

Use the `travel_binary.csv` dataset for this question.

a. Model the choice to drive to campus during winter as a binary logit model. Include the cost and the time of each alternative as explanatory variables with common coefficients; do not include an intercept. That is, the representative utility for alternative j is simply

$$V_{nj} = \beta_1 C_{nj} + \beta_2 T_{nj}$$

where C_j is the cost of alternative j and T_j is the time of alternative j. Estimate the parameters of this model by nonlinear least squares. The following steps can provide a rough guide to creating your own nonlinear least squares estimator:

   I. Create a function that takes a set of parameters and data as inputs: `function(parameters, data)`.
   II. Within that function, make the following calculations:
      i. Calculate the representative utility for each alternative and for each decision maker.
      ii. Calculate the choice probability of driving for each decision maker.
      iii. Calculate the econometric residual, or the difference between the outcome and the probability, for each decision maker.
      iv. Sum the square of these residuals.
      v. Return the sum of squares.
   III. Minimize the sum of squares using `optim()`. Your call of the `optim()` function may look something like:

```
optim(par = your_starting_guesses, fn = your_function, data = your_data,
      method = 'BFGS')
```

Report your parameter estimates and briefly interpret them. You do not have to estimate standard errors yet.

Hint: For a binary logit model, characterizing one choice probability is sufficient because the two probabilities must sum to 100%. The probability of driving is

$$\begin{aligned} P_{nc} &= \frac{e^{V_{nc}}}{e^{V_{nc}} + e^{V_{nb}}} \\ &= \frac{1}{1 + e^{-V_{nc} + V_{nb}}} \end{aligned}$$

Both of these expressions for the probability of driving may be useful as you solve this problem.

b. Again model the choice to drive to campus during winter as a binary logit model, but now add an intercept term and alternative-specific coefficients. That is, the representative utilities for the alternatives are

$$V_{nc} = \beta_0 + \beta_1 C_{nc} + \beta_2 T_{nc}$$
$$V_{nb} = \beta_3 C_{nb} + \beta_4 T_{nb}$$

where C_{nj} is the cost of alternative j and T_{nj} is the time of alternative j. Estimate the parameters of this model by nonlinear least squares. Follow the same set of steps as in (a), but now you have five parameters and the representative utility calculation is different. You can use matrix multiplication in your sum of squares function to flexibly accommodate different models, or you can create a different function for each model. Report your parameter estimates and briefly interpret them. You do not have to estimate standard errors yet.

c. Conduct a Wald test on the model in part (b) to test that the alternative-specific coefficients on cost are significantly different from one another. That is, test the null hypothesis:

$$H_0\colon\ \beta_1 = \beta_3$$

To conduct a Wald test, you need the variance-covariance matrix of your parameters estimates. For now, use the following parameter variances:

$$Var(\beta_0) = 1.61$$
$$Var(\beta_1) = 0.86$$
$$Var(\beta_2) = 0.09$$
$$Var(\beta_3) = 0.31$$
$$Var(\beta_4) = 0.03$$

and assume no covariances between parameters. The following steps can provide a rough guide to performing a Wald test:

I. Create a vector of J parameter restrictions, $r(\theta) = q$.

II. Calculate the $J \times K$ Jacobian matrix by differentiating each restriction with respect to each of the K parameters.

III. Calculate the Wald test statistic, which is a function of the vector of restrictions, the Jacobian matrix, and the variance-covariance matrix.

IV. Conduct the Wald test using this test statistic, which is distributed χ^2.

Do you reject the null hypothesis? What is the p-value of the test?

d. Estimate the variance-covariance matrix for the model in part (a). The following steps can provide a rough guide to estimating the variance-covariance matrix for nonlinear least squares estimators:

I. Write down the derivative of the nonlinear regression model (in this case, the logit choice probability for driving) with respect to each of the K parameters.

II. Calculate this $K \times 1$ vector of derivatives, at the estimated parameters, for each decision maker.

III. Calculate the $K \times K$ matrix that is the product of the above vector and its transpose for each decision maker.

IV. Sum these matrices for all decision makers.

V. Estimate the variance of the econometric error as the mean sum of squares at the estimated parameters.

VI. Calculate the variance-covariance matrix, which is a function of the above $K \times K$ matrix and the estimated error variance.

Report your parameter estimates (from (a)), standard errors, t-stats, and p-values. Briefly interpret these results.

e. Estimate the variance-covariance matrix for the model in part (b). Follow the same set of steps as in (d), but now you have five parameters to consider. Report your parameter estimates (from (b)), standard errors, t-stats, and p-values. Briefly interpret these results. Additionally, perform the Wald test from part (c) using your estimated variance-covariance matrix. Do you reject the null hypothesis? What is the p-value of the test?