Lecture 24: Endogeneity II

ResEcon 703: Topics in Advanced Econometrics

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Agenda

Last time

- Endogeneity

Today

- Berry, Levinsohn, and Pakes (1995)

Upcoming

- Final exam
  - Final exam is posted, due December 19
- Course surveys
  - Course surveys are open, due December 22
  - Click here (`owl.umass.edu/partners/courseEvalSurvey/uma`)

Endogeneity

In many empirical settings, our variable of interest is endogenous

- The variable of interest is correlated with the error term
- The coefficient on this variable will be biased and cannot be interpreted as causal
  - Sometimes the direction of the bias can be predicted, in which case the estimated coefficient can be interpreted as a bound, but not always

Example: price and unobserved quality

- Products with better unobserved quality are likely to have a higher price, so price is an endogenous attribute of the product
- If we try to estimate a coefficient on price (marginal utility of price or income), our estimate will be biased downward and may even have the wrong sign

Correcting for this endogeneity in a nonlinear structural model is more difficult than in a linear model

Berry, Levinsohn, and Pakes (1995)

Automobile Prices in Market Equilibrium

This paper develops a structural model of demand and supply in the automobile market

- Random coefficient model of demand
- Nash-Bertrand model of firm pricing

Why do we care about automobiles?

- They are important in their own right
  - An automobile is one of the largest purchases a typical household makes
  - Automobiles are an important part of many policy debates, ranging from international trade to environmental regulation
- They are an example of an important kind of industry
  - Differentiated product market with oligopolistic competition
  - This estimation technique can be used to estimate elasticities and cost parameters in other industries

BLP Innovation

What is so innovative about the BLP approach?

- BLP estimate flexible substitution patterns (elasticities) from market-level data using a computationally efficient algorithm that corrects for price endogeneity

Flexible substitution patterns

- BLP model distributions of consumer preferences over attributes
- Alternative: estimate $J(J-1)/2$ elasticity parameters

Market-level data

- Market-level data are more readily available than consumer-level data
- Consumer-level data may not have sufficient power for all elasticities

Computationally efficient algorithm that corrects for price endogeneity

- BLP solve a nonlinear simultaneous equations problem
- BLP develop a "contraction mapping" for efficient estimation

Utility Model

There are T markets ($t = 1, \ldots, T$), each with J_t products ($j = 1, \ldots, J_t$)

- The "outside good" is indexed as $j = 0$

The utility that consumer i in market t obtains from product j is

$$U(x_{jt}, \xi_{jt}, p_{jt}, \tau_i; \theta_1, \theta_2)$$

- x_{jt}: vector of non-price attributes of product j in market t
- ξ_{jt}: utility of unobserved attributes of product j in market t
- p_{jt}: price of product j in market t
- τ_i: characteristics of individual i
- θ_1, θ_2: vectors of unknown parameters

Utility Functional Form

A quasi linear utility function gives the utility expression

$$u_{ijt} = \alpha_i(y_i - p_{jt}) + x_{jt}\beta_i + \xi_{jt} + \varepsilon_{ijt}$$

BLP use a slightly different expression that comes from a Cobb-Douglas utility function

$$u_{ijt} = \alpha_i \ln(y_i - p_{jt}) + x_{jt}\beta_i + \xi_{jt} + \varepsilon_{ijt}$$

but the first expression is easier to follow

Depending on the data and context, we could capture some components of ξ_{jt} through brand and market dummy variables

$$\xi_{jt} = \xi_j + \xi_t + \Delta\xi_{jt}$$

- See Nevo (2000) for details on estimation with these dummy variables

Individual Characteristics and Coefficients

There are two types of individual characteristics

- $D_i \sim \hat{P}^*_D(D)$: "observed" demographic characteristics
- $\nu_i \sim P^*_\nu(\nu)$: additional unobserved characteristics

BLP actually do not observe any individual-specific characteristics, but they do observe the distribution of demographic characteristics in the population, whereas ν_i represents characteristics that are fully unobserved

Individual coefficients are a function of these individual characteristics

$$\begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \Pi D_i + \Sigma \nu_i$$

- Π measures how individual coefficients vary with demographics
- Σ represents variation in individual coefficients for unobserved reasons

BLP have data on the income distribution, but no other demographics

Outside Good

A consumer may choose not to purchase any good, which is denoted the "outside good" and indexed as $j = 0$

The utility that consumer i in market t obtains from the outside good is

$$u_{i0t} = \alpha_i y_i + \xi_{0t} + \pi_0 D_i + \sigma_0 \nu_{i0} + \varepsilon_{i0t}$$

The terms ξ_{0t}, π_0, and σ_0 are set equal to zero

- These terms are not identified without normalizing one ξ_{jt} and components of Π and Σ

Then the utility of the outside good is normalized to be zero (in expectation)

- The utility of income, $\alpha_i y_i$, is common to all products, so it effectively disappears

Full Demand Model

The utility that consumer i in market t obtains from product j is

$$u_{ijt} = \alpha_i y_i + \delta_{jt}(x_{jt}, p_{jt}, \xi_{jt}; \theta_1) + \mu_{ijt}(x_{jt}, p_{jt}, \nu_i, D_i; \theta_2) + \varepsilon_{ijt}$$

where

$$\delta_{jt}(x_{jt}, p_{jt}, \xi_{jt}; \theta_1) = x_{jt}\beta - \alpha p_{jt} + \xi_{jt}$$
$$\mu_{ijt}(x_{jt}, p_{jt}, \nu_i, D_i; \theta_2) = (-p_{jt}, x_{jt})(\Pi D_i + \Sigma \nu_i)$$

- $\alpha_i y_i$: common to all products and effectively disappears
- δ_{jt}: mean utility of product j in market t that is common to all consumers
- $\mu_{ijt} + \varepsilon_{ijt}$: mean-zero deviation from the common utility that captures the random coefficients and idiosyncratic utility

Market Shares

Consumer i in market t purchases product j if and only if

$$u_{ijt} \geq u_{i\ell t} \; \forall \ell \in J_t$$

Conditional on all product attributes and model parameters, the set of individual characteristics that lead to the choice of product j in market t is

$$A_{jt}(x_t, p_t, \delta_t; \theta_2) = \{(D_i, \nu_i, \varepsilon_{i0t}, \ldots, \varepsilon_{iJ_t t}) \mid u_{ijt} \geq u_{i\ell t} \; \forall \ell \in J_t\}$$

Then the market share of product j in market t is

$$\begin{aligned} s_{jt}(x_t, p_t, \delta_t; \theta_2) &= \int_{A_{jt}} dP^*(D, \nu, \varepsilon) \\ &= \int_{A_{jt}} dP^*_\varepsilon(\varepsilon) dP^*_\nu(\nu) d\hat{P}^*_D(D) \end{aligned}$$

if ε, ν, and D are assumed to be independent

Endogeneity of Price

Product prices are not randomly assigned but set to maximize firm profits

- If firms know how consumers value the unobserved product attributes, ξ_{jt}, then they will consider these values when setting prices
- But then prices are correlated with these unobserved product attributes, so estimates of the price coefficient will be biased
- This problem is analogous to the classic simultaneity problem of demand and supply

BLP propose instrumenting for price

- But they cannot use instruments in a nonlinear model
- BLP use an estimation method that transforms product attributes into a linear function of ξ_{jt}

Cost Function

BLP do not just discuss how profit-maximizing firms lead to price endogeneity, they also explicitly model supply and the pricing decision of firms

There are F firms ($f = 1, \ldots, F$), each of which produces subset $\mathcal{J}_f$ of the products

The marginal cost of producing product j for market t, mc_{jt}, is given by

$$\ln(mc_{jt}) = w_{jt}\gamma + \omega_{jt}$$

- w_{jt}: vector of cost characteristics for product j in market t, which can overlap with product attributes, x_{jt}
- γ: unknown parameters
- ω_{jt}: unobserved component of marginal costs, which can be correlated with ξ_{jt}

Profit-Maximizing Firms

The profits of firm f in market t is given by

$$Profits_{ft} = \sum_{j \in \mathcal{J}_f} (p_{jt} - mc_{jt}) M_t s_{jt}(x_t, p_t, \delta_t; \theta_2)$$

where M_t is the number of consumers in market t

BLP assume a Nash-Bertrand model of pricing, which yields the first-order condition for the price of product j in market t

- Each firm sets prices to maximize its profits conditional on the attributes of all products and the prices of all other firms

$$s_{jt}(x_t, p_t, \delta_t; \theta_2) + \sum_{\ell \in \mathcal{J}_f} (p_{\ell t} - mc_{\ell t}) \frac{\partial s_{\ell t}(x_t, p_t, \delta_t; \theta_2)}{\partial p_{jt}} = 0$$

Markups

The first-order condition can be expressed in vector notation as

$$s_t(x_t, p_t, \delta_t; \theta_2) - \Delta_t(x_t, p_t, \delta_t; \theta_2)(p_t - mc_t) = 0$$

where

$$\Delta_{j\ell} = \begin{cases} \frac{-\partial s_{\ell t}}{\partial p_{jt}} & \text{if } j \text{ and } \ell \text{ are produced by the same firm} \\ 0 & \text{otherwise} \end{cases}$$

Solving this first-order condition for prices gives an optimal markup rule

$$p_t = mc + b_t(x_t, p_t, \delta_t; \theta_2)$$

where b_t is the vector of markups in market t given by

$$b_t(x_t, p_t, \delta_t; \theta_2) = \Delta_t(x_t, p_t, \delta_t; \theta_2)^{-1} s_t(x_t, p_t, \delta_t; \theta_2)$$

Full Supply Model

Putting these pieces together yields an expression for prices in market t as a function of data

$$\ln(p_{jt} - b_{jt}(x_t, p_t, \delta_t; \theta_2)) = w_{jt}\gamma + \omega_{jt}$$

which BLP estimate to construct the marginal cost for every automobile model

As with the demand model, there is an endogeneity problem

- The price vector is a function of unobserved cost characteristics, ω, and the markup, b_t, is a function of prices, so the markup is correlated with unobserved cost characteristics
- The unobserved cost characteristics, ω, are correlated with the utility of unobserved product attributes, ξ, which are correlated with price

Summary of Theoretical Models

BLP develop models of demand and supply

- Random coefficients model of demand for differentiated products
- Nash-Bertrand pricing model with oligopolistic competition

Both of these models suffer from endogeneity

- BLP need instruments to recover unbiased parameter estimates

The endogeneity enters these models nonlinearly

- BLP develop a new estimation method to handle instruments in a nonlinear model

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