COMP 1805 - Bonus 1

Derek Yu

Propositional Logic [1-4]

1. Which proposition is equivalent to the following: $a \wedge (\neg b \vee (c \rightarrow d))$
   (a) $\neg(\neg a \vee (b \wedge c \wedge \neg d))$
   (b) $\neg(\neg a \wedge (b \vee c \wedge d))$
   (c) $\neg(\neg a \rightarrow (b \wedge c) \vee \neg d)$
   (d) $\neg(a \vee (b \wedge c \vee d))$

2. If $b =$"I will ball", $t =$"I will take the table", $p =$"I will take the piano", convert the following into a logical expression: I will ball if I take the table or piano.
   (a) $(t \wedge p) \rightarrow b$
   (b) $(t \vee p) \leftrightarrow b$
   (c) $(t \vee p) \rightarrow b$
   (d) $b \rightarrow (t \wedge p)$

3. Find the logically equivalent statement for the following: I won a prize and didn't pay my fee.
   (a) It is false that I didn't win a prize or I paid my fee.
   (b) It is true that I won a prize, it is true that I paid my fee.
   (c) It is false that I either didn't win a prize or paid my fee.
   (d) It is true that I didn't win a prize and I paid my fee.

4. Which statement is not a proposition?
   (a) The apple in bed is void.
   (b) Creating a bus driver removes levels of conundrum.
   (c) Take a deep breath to appreciate that the sky is green.
   (d) Huge wins are for those who are unemployed.

Predicate Logic [5-8]

5. $U =$"All Humans"
$A(x) =$"x has an apple"
$B(x) =$"x beat Michael Jordan"
$C(x) =$"x owns an apartment"

Convert the following into logic:
A person who beat Michael Jordan must have an apple and can't own an apartment.

(a) $\forall x(B(x) \land A(x) \land \neg C(x))$
(b) $\forall x(B(x) \leftrightarrow (A(x) \land \neg C(x)))$
(c) $\forall x(B(x) \rightarrow (A(x) \land \neg C(x)))$
(d) $\forall x(B(x) \lor \neg B(x) \land (A(x) \land \neg C(x)))$

6. Find the correct negation of the following expression: $\forall x(A(x) \leftrightarrow \neg B(x))$

(a) $\exists x((A(x) \land B(x)) \lor (\neg A(x) \land \neg B(x)))$
(b) $\exists x\neg((A(x) \lor B(x)) \land (\neg A(x) \lor B(x)))$
(c) $\forall x((B(x) \land \neg A(x)) \lor (A(x) \land B(x)))$
(d) $\forall x((A(x) \land B(x)) \lor (\neg A(x) \land \neg B(x)))$

7. $U =$"All Humans"
$T(x) =$"x takes the train"
$N(x) =$"x negates the noodle house"
$W(x) =$"x walks the dog"

Convert the following into English: $\exists x(W(x) \land (N(x) \lor T(x)))$

(a) Everyone who walks the dog but either negates the noodle house or takes the train.
(b) There exists someone who walks the dog and negates the noodle house or takes the train.
(c) Everyone walks the dog if they negate the noodle house or takes the train.
(d) There exists someone who walks the dog and either negates the noodle house or takes the train.

8. For what domain is the following expression True: $\forall x((3 < x \leq 18) \land (\frac{x}{3} = 0))$

(a) $U = \{3, 6, 9, 12, 15, 18\}$
(b) $U = \{6, 9, 12, 15, 18\}$
(c) $U = \{4, 6, 8, 10, 12, 14, 16, 18\}$
(d) $U = \{6, 9, 12, 15\}$
(e) $U = \{3, 18\}$
(f) $U = \{6, 9, 12, 15, 18\}$

Validity of Logical Arguments [9-11]

9. (1) $a \to b$

   (2) $c \leftrightarrow b$

   (3) $\neg c \land d$

Given the premises above, we can conclude:
$\therefore$? ☐

(a) $d = False$

(b) $a = False$

(c) $e = True$

(d) $b = True$

10. Select the invalid argument.

(a) $(a \land b), (c \to \neg b) \quad \therefore c$

(b) $(a \lor b \lor c), (\neg b \to \neg c), c \quad \therefore b$

(c) $(a \land (b \lor c)), (a \lor \neg b) \quad \therefore c \lor \neg c$

(d) $(a \lor b \lor c), (\neg b \to \neg c), \neg(a \lor c) \quad \therefore b$

11. Select the correct statement about the argument below:

(1) $b \lor a$

(2) $d \land b$

(3) $b \leftrightarrow c$

(4) $e \lor d$

(5) $\neg c \to b$

Options:

(a) This argument is invalid.

(b) This argument is a tautology.

(c) This argument is a contingency.

(d) This argument is a contradiction.

Validity & Quantifiers [12-14]
$U =$"All Humans"
$T(x) =$"x takes the train"
$N(x) =$"x negates the noodle house"
$W(x) =$"x walks the dog"

12. Find the invalid argument:

    (a) (1) $\forall x(T(x) \to N(x))$
    (2) $\exists x(\neg N(x))$
    $\therefore \exists x(\neg T(x))$

    (b) (1) $\exists x(T(x) \lor N(x))$
    (2) $\exists x(\neg N(x))$
    $\therefore \exists x(T(x) \land \neg N(x))$

    (c) (1) $\forall x(T(x) \land N(x))$
    (2) $\exists x(T(x) \to N(x))$
    $\therefore \exists x(T(x) \lor N(x))$

    (d) (1) $\neg \exists x(T(x) \to N(x))$
    (2) $\exists x(\neg N(x))$
    $\therefore \forall x(T(x) \land \neg N(x))$

13. Select the most appropriate rule of inference to be applied at "?" in the argument below:

    (1) $\forall x(W(x) \lor T(x))$
    (2) ?
    $\therefore \exists x(T(x))$

    Options:

    (a) Universal Instantiation
    (b) Universal Generalization
    (c) Existential Instantiation
    (d) Existential Generalization

14. Select the correct set of elements for the argument below:

    (1) $\exists(x, y)(W(x) \land T(y))$
    (2) $\exists(x, z)(N(z) \land ((z \neq x) \to \neg N(z)))$

    Options:

    (a) $x = John,\ y = Jane,\ z = Jane$
    (b) $x = John,\ y = Jill,\ z = Jane$
    (c) $x = Jane,\ y = Jane,\ z = John$
    (d) $x = John,\ y = Jane,\ z = John$

Proof Techniques [15-17]

15. Find the mistake in the following proof:

    Prove that if integer a is greater than 1, then a is is not equal to 0.
    Indirect proof: Prove that if integer a is equal to 0, then a is smaller than 1.
    Assume a is an integer equal to 0; then $a = 0$.
    $0 < 1 \quad \therefore a < 1$ $\square$

    (a) Circular reasoning
    (b) Incorrect premise
    (c) Lack of evidence in arguments
    (d) Does not prove the original statement

16. What proof technique is being used in the example below:
    Theorem: "If a is an integer greater than integer b, then $\frac{a}{2} < \frac{b}{2}$".
    Let $a = 2, b = 1$. Then $a > b$ since $2 > 1$;
    $(\frac{a}{2} < \frac{b}{2}) = (\frac{2}{2} < \frac{1}{2}) = (1 < 1/2) \quad \therefore$ This statement is False. $\square$

    (a) Contradiction
    (b) Proof by case
    (c) Counterexample
    (d) Direct Proof

17. Which proof technique would be most effective to prove the following theorem:
    "If integer a is negative and integer b is positive, then $|a||b|$ must be positive."

    (a) Direct Proof
    (b) Indirect Proof
    (c) Proof by Contradiction
    (d) Proof by Cases
    (e) Trivial Proof
    (f) Pigeonhole Principle
    (g) Existence Proof

Set Theory [18-20]

Given the following sets:
$A = \{a, b, c, d, e, f\}$
$B = \{b, d, e, g, f\}$
$C = \{d, o, n, m\}$

18. Given:
$F = (A \cap B) \setminus C$
What is the set of F?
    (a) $F = \{a, b, c, r, g, f\}$
    (b) $F = \{a, b, c, d, r, g, f\}$
    (c) F=$\{b, e, f\}$
    (d) $F = \{b, d, e, f\}$
    (e) $F = \{a\}$
    (f) $F = \{b, e, f, g\}$
    (g) $F = \{a, d, g, o, n, m\}$
19. Which statement about the sets is false?
    (a) $\{b, d, e\} \subseteq (A \cap B)$
    (b) $\{d\} \subset (A \cap B \cap C)$
    (c) $(C \setminus B) \subseteq (C)$
    (d) $\{b, e, \emptyset\} \subset (A)$
20. Which is the correct power set of $D = \{1, 2, 3\}$?
    (a) $P(D) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 1\}, \{2, 3\}, \{3, 2\}, \{1, 3\}, \{3, 1\}, \{1, 2, 3\}, \{3, 2, 1\}, \{2, 3, 1\}\}$
    (b) $P(D) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2, 3\}\}$
    (c) $P(D) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$
    (d) $P(D) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 1\}, \{2, 3\}, \{3, 2\}, \{1, 3\}, \{3, 1\}, \{1, 2, 3\}, \{3, 2, 1\}, \{2, 3, 1\}\}$
    (e) $P(D) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$