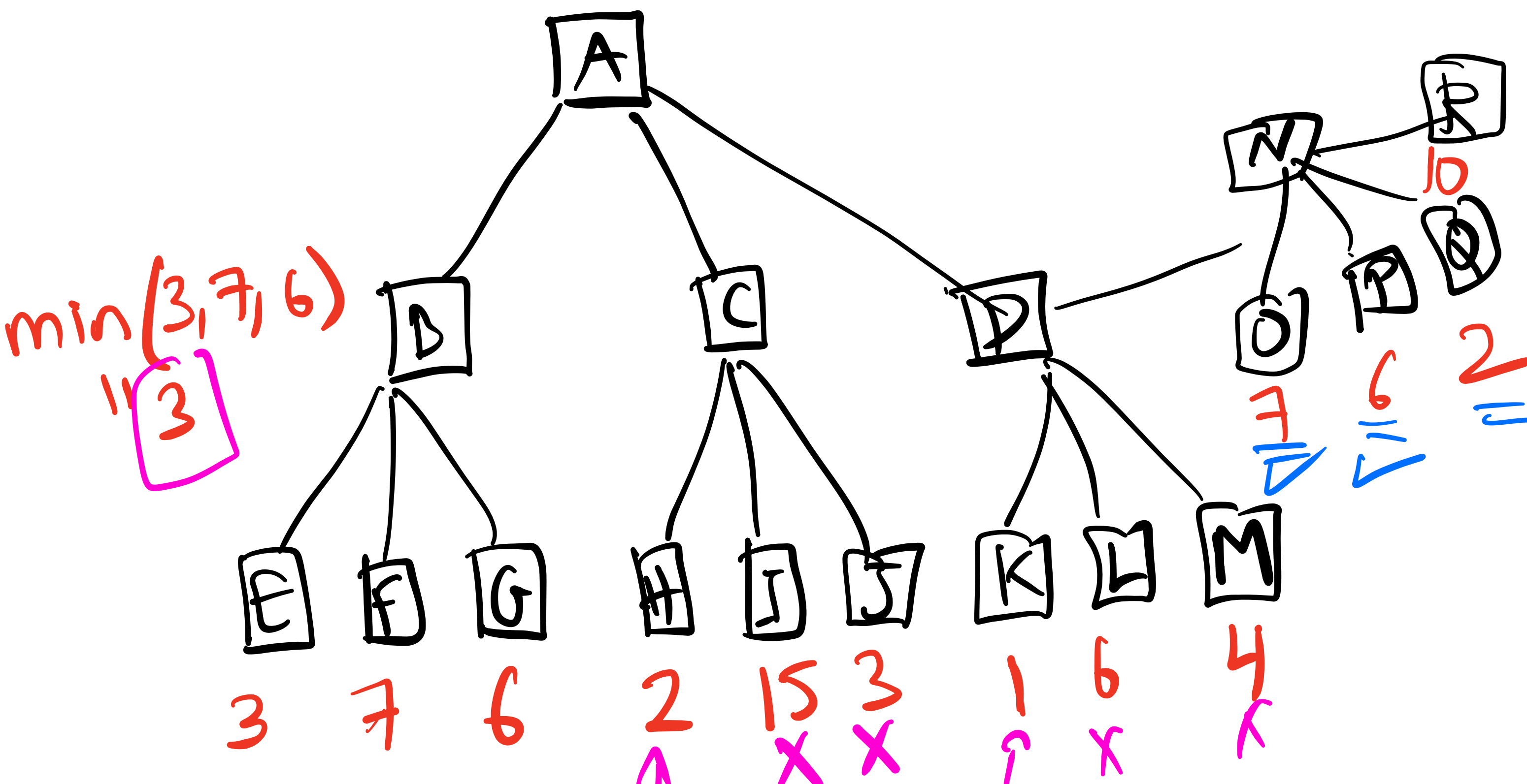
CS 5100 01/30

Adversarial games

starting number

-1 divide by 2

game ends once a player reaches the number 0



min will at most give me 2

pruning (reducing the number of states to look at as there are some states that we don't need to look at)

α, β pruning

α: value of the best choice we have found so far for Max (highest)

β: value of the choice we have found so far along the path for Min (lowest)

Some games involve uncertainty (for example in backgammon, we have to roll a pair of dice to determine the next action)

$\Rightarrow$ quantify uncertainty

$\Rightarrow$ probabilities

Review : Probability

probability of an event occurring is the fraction of how many times the event was successful when conducting an infinite number of samples.

toss a coin

$P(H) = \frac{1}{2}$

H T

$P(T) = \frac{1}{2}$

sample space S consists of the set of all outcomes

Axioms (given assumptions)

$\boxed{xH \quad xT}$

* $0 \leq P(A) \leq 1$
* $P(S) = 1$
* $P(H \text{ or } T) = P(H) + P(T)$

as H and T are disjoint (they can never occur at the same time)

$P(A \text{ or } B) = P(A) + P(B)$ if A and B are disjoint

ex

$S = \{1, 2, 3, 4, 5, 6\}$

Roll a die

P(number rolled is even) $\frac{3}{6} = \frac{1}{2}$

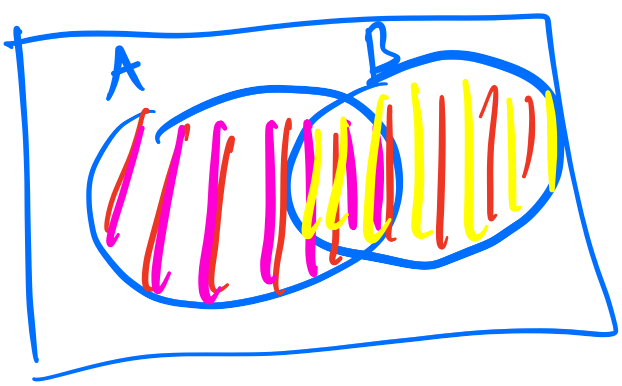
P(number rolled is divisible by 3) $= \frac{2}{6} = \frac{1}{3}$

P(number rolled is larger than 4) $= \frac{2}{6} = \frac{1}{3}$

P(number rolled is even

or number rolled is larger than 4)

{2, 4, 5, 6} $4/6 = 2/3 \neq 1/2 + 1/3$



$P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

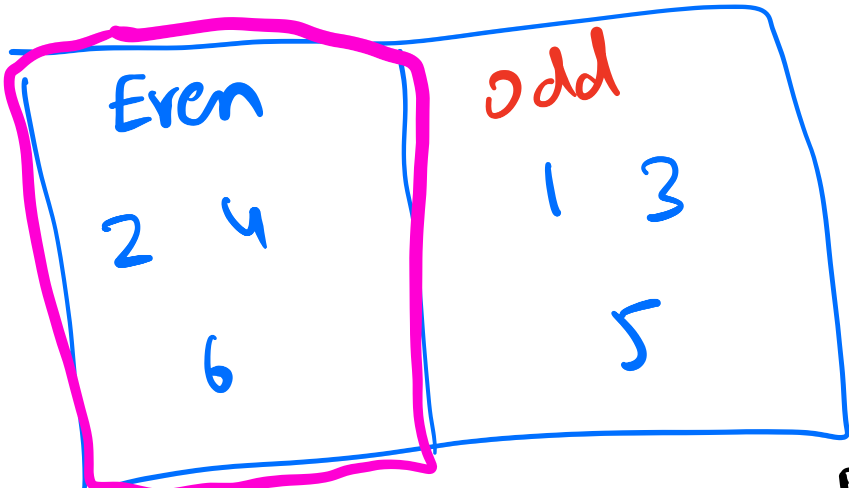
Birthday paradox

(Yay it worked)

*Conditional probabilities

Roll a dice, I know that the number must be even.

What's the probability that I will roll the number 6?



sample space changed because of the info about the number being even

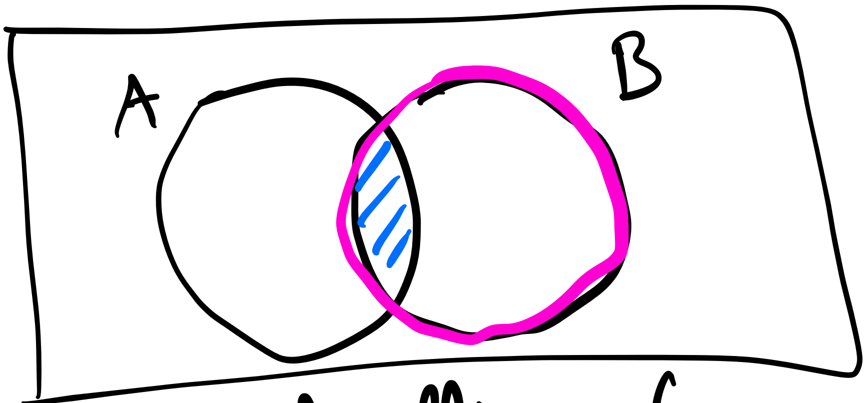
P(A)

$\rightarrow \frac{1}{3}$

$$P(A) = \frac{P(A)}{P(S)}$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

given that



A: proba of rolling a 6
B: proba of rolling a even number

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{1/2} = \frac{2}{6} = \frac{1}{3}$$

ex

Person either likes chocolate or doesn't like chocolate

Two people

Given that one of the two people likes chocolate, what is the probability that the other person likes chocolate?

0 , $\frac{1}{2}$, $\frac{1}{4}$

X X X

$\boxed{C\,C \quad C\,\bar{C} \quad \bar{C}\,C}$ $\cancel{\bar{C}\,\bar{C}}$

1/3

$P($second person likes chocolate $|$ first person likes chocolate$)$

$= P($second person likes chocolate$)$

$P(A|B) = P(A)$

A and B are independent if

$P(A|B) = P(A)$

Die

1, 2, 3, 4, 5, 6

$X=1$ $\quad P(X=1)=1/6$

$X=2$ $\quad P(X=2)=1/6$

$X=3$ $\quad \vdots$

$X=4$

$X=5$

$X=6$ $\quad P(X=6)=1/6$

$P(X=7)=0$

Toss a coin

H, T

1 0

$P(X=1)=1/2$

$P(X=0)=1/2$

$P(X=73)=0$

X: random variable

Roll two dice and we're interested in the probabilities of the sum of the numbers on the

Let X be the sum of the numbers on the dice.

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

$\boxed{11}$ 21 31 … 61

12

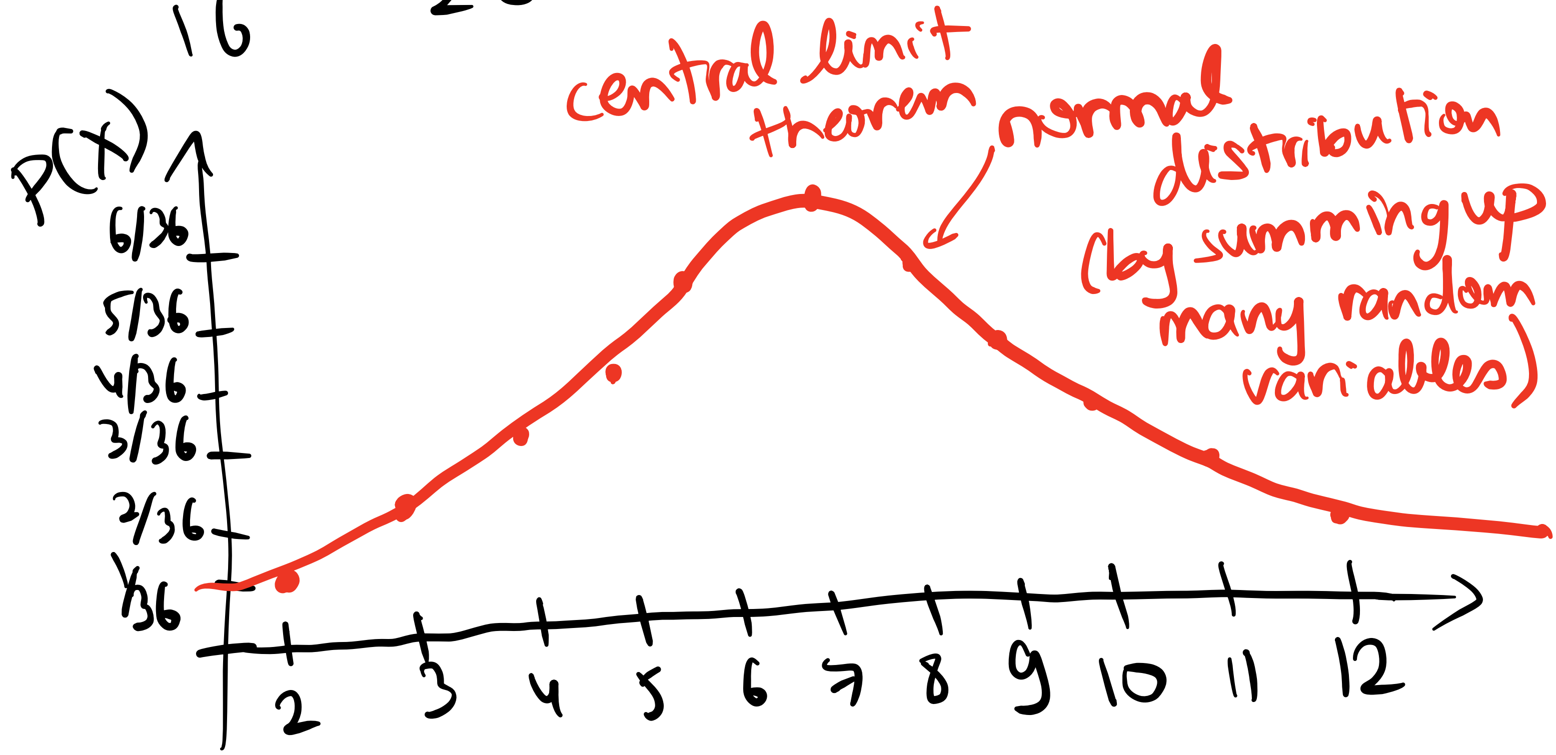
13

14

15

16 26 36 … 66

$\left.\right\}$ 36 outcomes



<u>Random variable</u> X and you know the probabilities

often, we might want to look at the average behavior of a random variable

X	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

$$\frac{2+3+4+5+\cdots+1+12}{11}$$

$$\frac{1}{36}(2)+\frac{2}{36}(3)+\frac{3}{36}(4)+\frac{4}{36}(5)+\frac{5}{36}(6)+\frac{6}{36}(7)$$

$$+\frac{5}{36}(8)+\frac{4}{36}(9)+\frac{3}{36}(10)+\frac{2}{36}(11)+\frac{1}{36}(12)$$

$$=\frac{1\cdot(2)+2(3)+3(4)+\cdots+1(12)}{36}$$

20 50 70

$$=\frac{2+6+12+20+30+42+40+36+30+22+12}{36}$$

$$=\frac{20+50+70+112}{36}=\frac{252}{36}=7$$

average value of a random variable X

= expected value of a random variable X

$= E[X]$

57.5 20 20 90 100

more spread out

57.5 50 50 60 70

more homogeneous

spread of the values is different

$\Rightarrow$ <u>variance</u>