CS 5100 02/10

HW2 (changing P2 → similar to a problem we did in class)

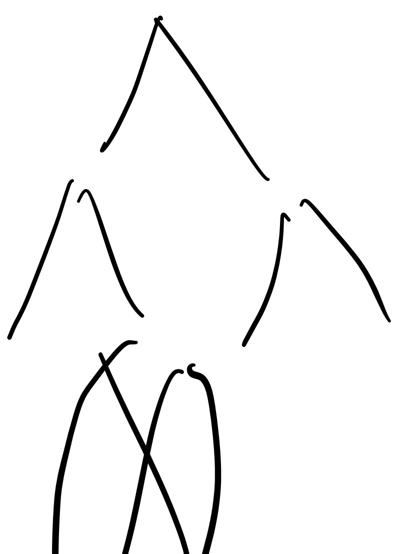
Review

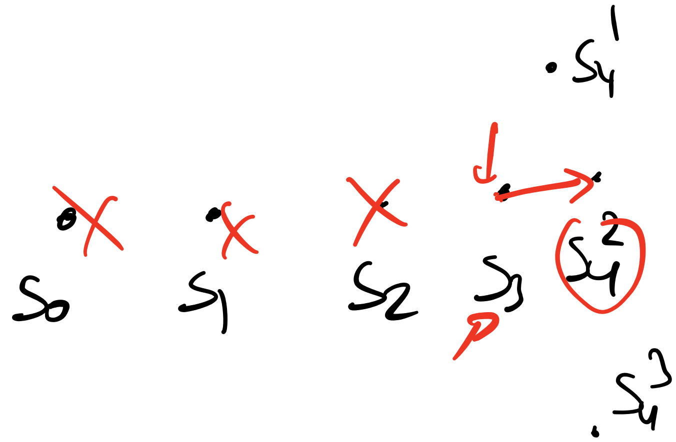
Decision problem

* Transition model (stochastic/random model)

$P(s'|s)$ proba of reaching state s' from state s

assumption on the transition model (Markovian assumption)





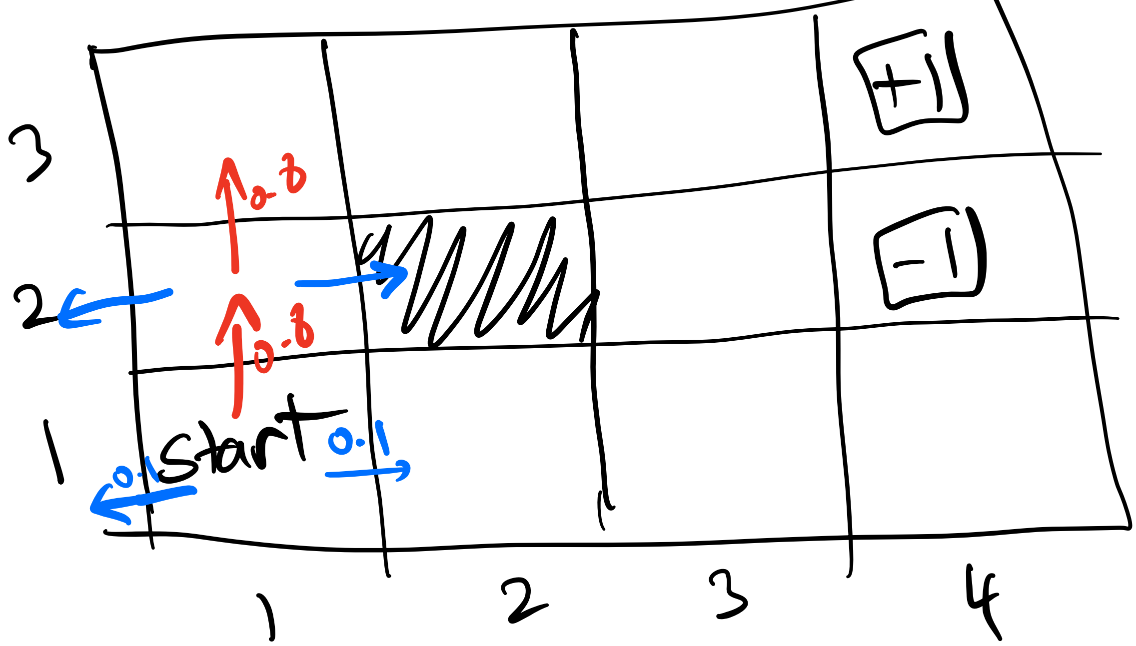
* Reward that we receive at every state (positive or negative)

* Whenever we were looking at a sequence of states, we were summing up the rewards obtained at every state to get the total reward for the sequence.

reward +1

$\boxed{+1}$ } terminal states
$\boxed{-1}$

reward -1



all other states have a reward of -0.04

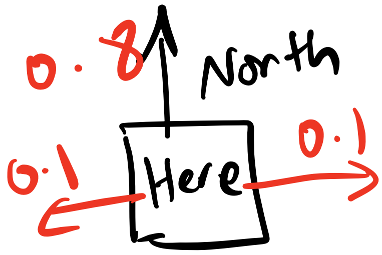
Whenever we selected a direction, we will be able to get there with probability 0.8.

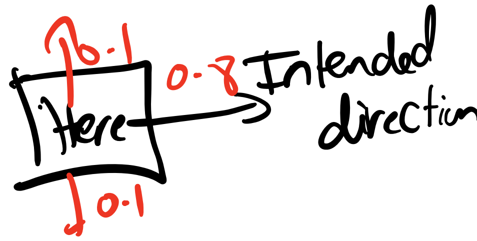
we will end up at

With probability 0.2, any of the states that are perpendicular to the target state.

NN EEE

ex





Go north

$$U(1,1) = -0.04 + \underline{0.8\,U(1,2)} + 0.1\,U(2,1) + 0.1\,U(1,1)$$

At state (1,2) (North)

$$U(1,2) = -0.04 + 0.8\,U(1,3) + 0.1\,U(1,2) + 0.1\,U(1,2)$$

$$= -0.04 + 0.8\,U(1,3) + 0.2\,U(1,2)$$

At state (1,3) (East)

$$U(1,3) = -0.04 + 0.8\,U(2,3) + 0.1\,U(1,3) + 0.1\,U(1,2)$$

At state (2,3) (East)

$$U(2,3) = -0.04 + 0.8\,U(3,3) + 0.1\,U(2,3) + 0.1\,U(2,3)$$

$$= -0.04 + 0.8\,U(3,3) + 0.2\,U(2,3)$$

At state (3,3) (East)

$$U(3,3) = -0.04 + 0.8\,U(4,3) + 0.1\,U(3,3) + 0.1\,U(3,2)$$

At state (4,3)

$$\boxed{U(4,3) = 1}$$

$$\begin{cases} U(1,1) = -0.04 + 0.8\,U(1,2) + 0.1\,U(1,1) + 0.1\,U(2,1) \\ U(1,2) = -0.04 + 0.8\,U(1,3) + 0.2\,U(1,2) \\ U(1,3) = -0.04 + 0.8\,U(2,3) + 0.1\,U(1,3) + 0.1\,U(1,2) \\ U(2,3) = -0.04 + 0.8\,U(3,3) + 0.2\,U(2,3) \\ U(3,3) = -0.04 + 0.8\,U(4,3) + 0.1\,U(3,3) + 0.1\,U(3,2) \\ U(4,3) = 1 \end{cases}$$

$$\begin{cases} x+y=2 \\ 2x-y=4 \end{cases}$$

look at these equations today (Linear algebra)

google search algorithm

One way

add the two equations

$3x=6 \Rightarrow x=2$

$\Rightarrow$ plug in $x=2$ in the first equation $\Rightarrow y=0$

$y=2-x$

$2x-(2-x)=4 \Rightarrow 3x=6 \Rightarrow x=2$

Many equations and lots of variables

$\Rightarrow$ hard to solve them by hand

Suppose we have this system of 3 equations and 3 unknowns

$$\begin{cases} x + 2y = 3 \\ y - 3z = 4 \\ x + y - 2z = 1 \end{cases}$$

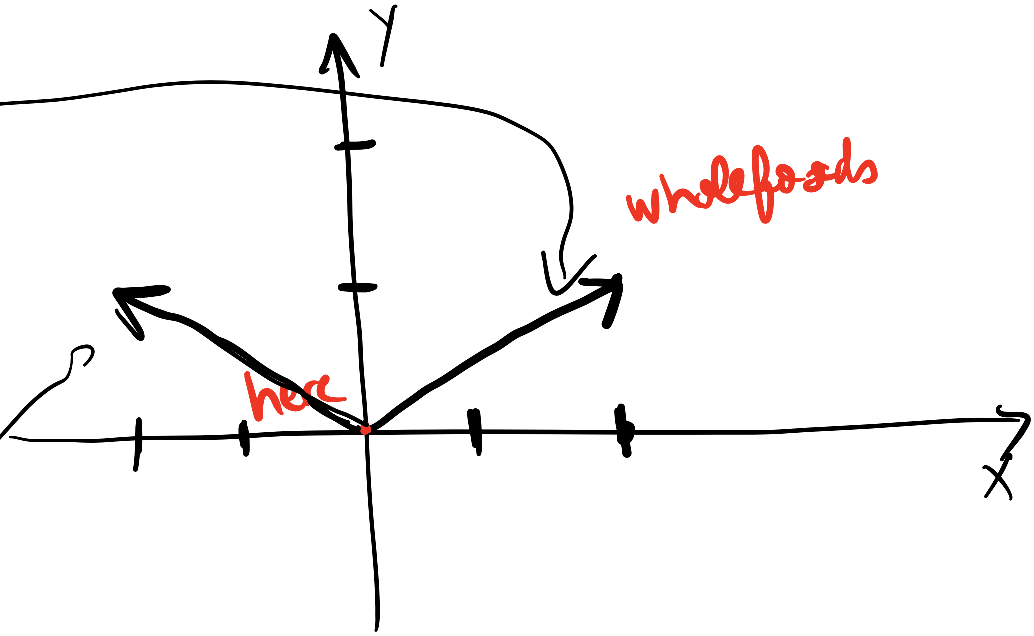
$$\Rightarrow \begin{cases} (1)x + (2)y + (0)z = 3 \\ (0)x + (1)y + (-3)z = 4 \\ (1)x + (1)y + (-2)z = 1 \end{cases}$$

$$\underbrace{\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -3 \\ 1 & 1 & -2 \end{pmatrix}}_{\text{matrix}} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\boxed{\text{vector}}} = \underbrace{\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}}_{\text{vector}}$$

(they're everywhere)

__Vector__ object that has some direction and magnitude

$$\begin{array}{r} x \rightarrow \\ y \rightarrow \end{array} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

__Example where vectors may occur__

Northeastern students are outstanding

search query

Northeastern students

outstanding

are students

outstanding students

	Northeastern	students	are	outstanding
website 1	1	1	0	0
website2	0	0	0	1
website3	0	1	1	0
website4	0	1	0	1

1 1 1 1

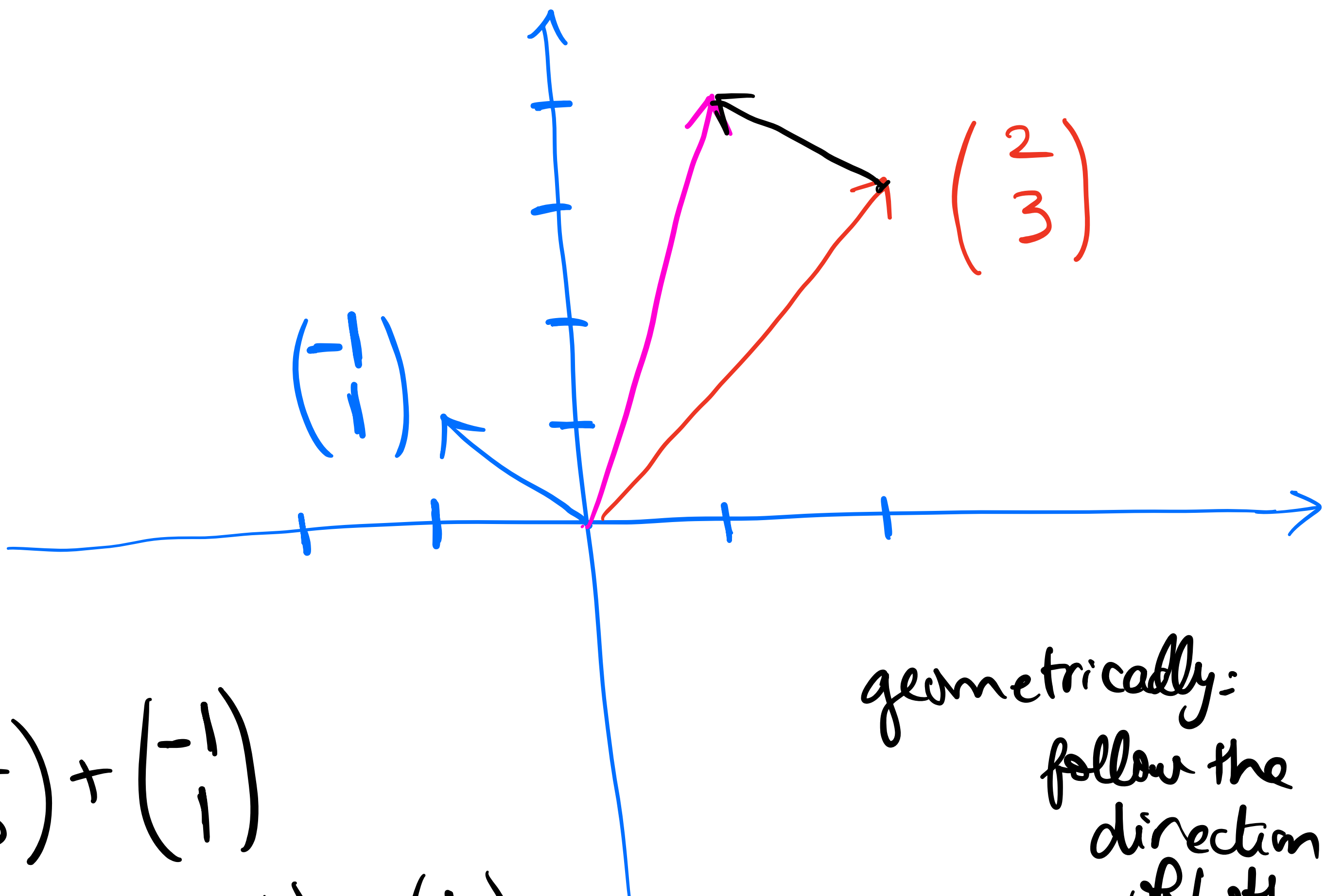
Google will try to figure out which website vectors are the most similar to my search query vector.

Let's answer this question for numbers first

2 and 3 vs 2 and 5

We need to understand how to do operations on vectors to be able to compare vectors

Addition of vectors



$$\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}-1\\1\end{pmatrix}$$

$$=\begin{pmatrix}2+(-1)\\3+1\end{pmatrix}=\begin{pmatrix}1\\4\end{pmatrix}$$ ← algebraically

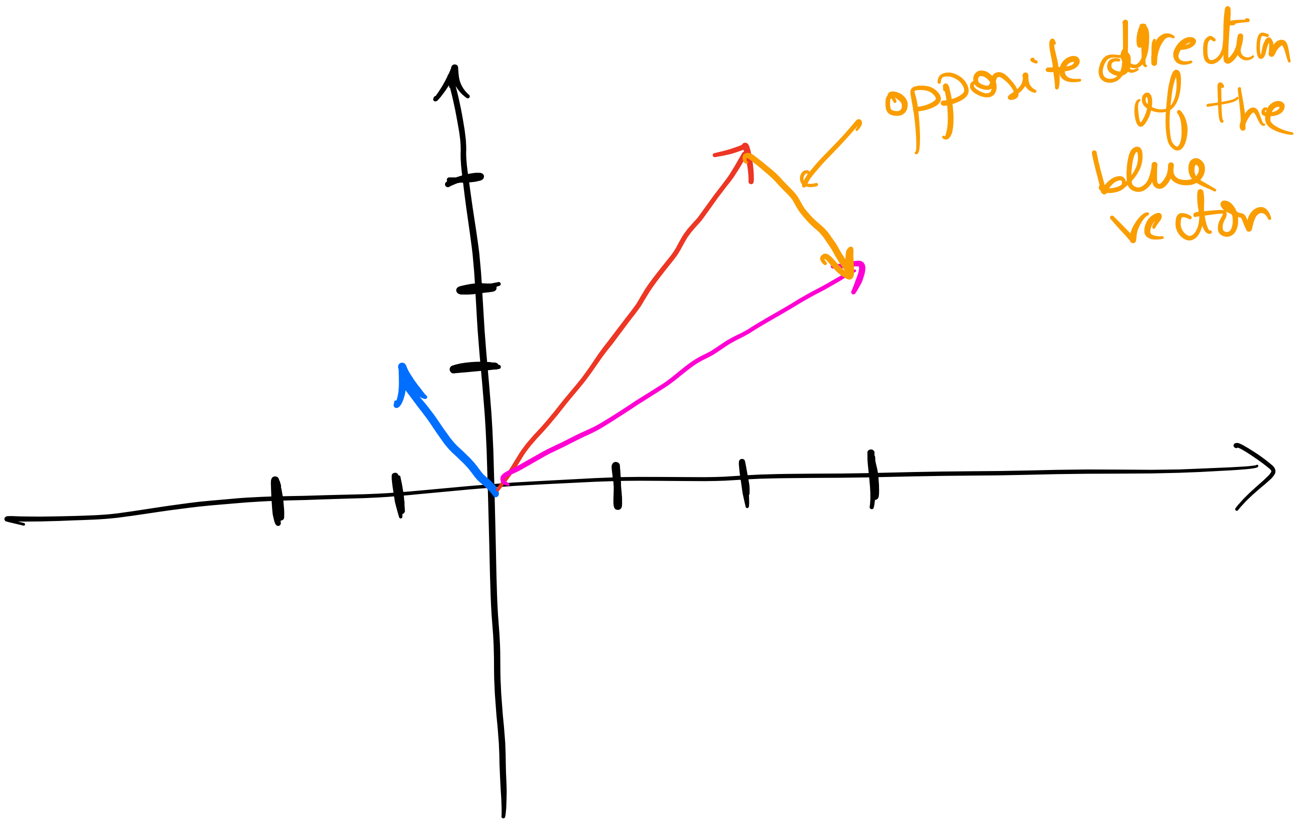
geometrically: follow the direction of both vectors

* Subtraction of vectors

$2-3 = 2+(-3)$ (scalars)

$$\begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix} + \left[-\begin{pmatrix}-1\\1\end{pmatrix}\right]$$

$$= \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix}$$



Addition and subtraction is the same for vectors with more entries

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1+y_1 \\ x_2+y_2 \\ \vdots \\ x_n+y_n \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} - \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1-y_1 \\ x_2-y_2 \\ \vdots \\ x_n-y_n \end{pmatrix}$$

Multiplication of two vectors

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad \vec{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \qquad \vec{x} * \vec{y} = \begin{pmatrix} x_1 \cdot y_1 \\ x_2 \cdot y_2 \\ \vdots \\ x_n \cdot y_n \end{pmatrix}$$

Notation for vector

one way of multiplying vectors

useful for coding

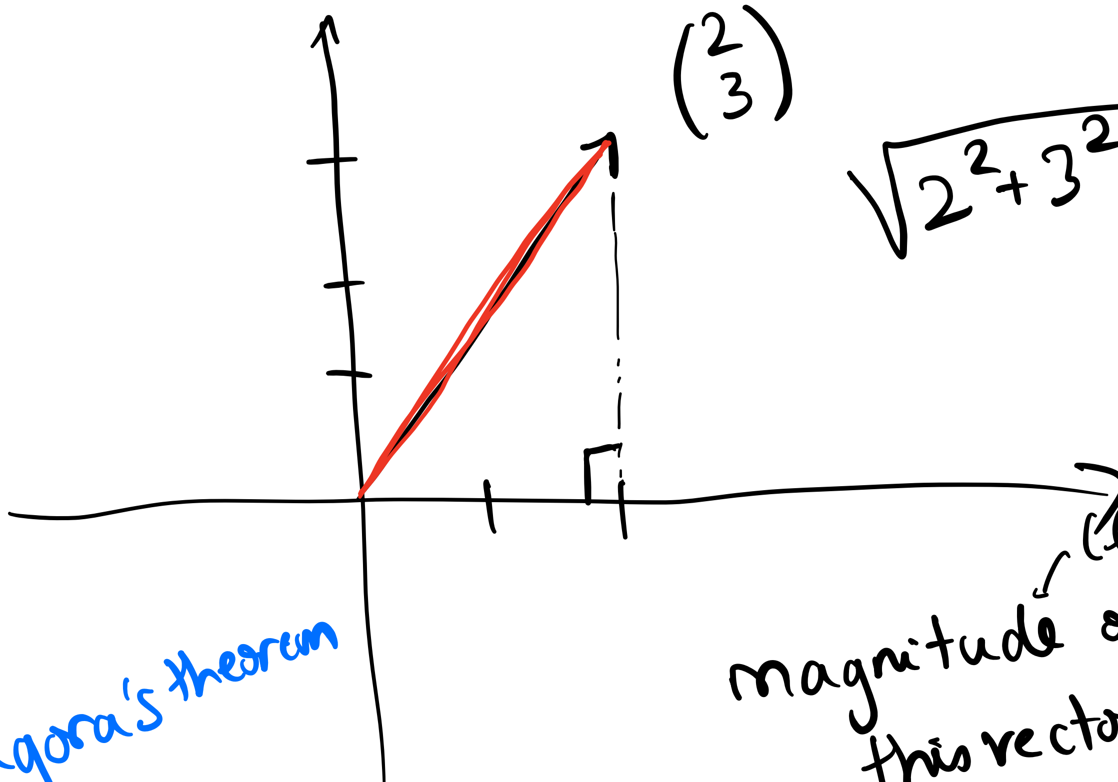
Hadamard product

there is another product that is used more in applications which we call the <u>dot product</u> (inner product)

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \boxed{x_1 y_1 + x_2 y_2 + \cdots + x_n y_n}$$

↓

scalar (not a vector)



$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\sqrt{2^2+3^2}$

Pythagora's theorem

magnitude (length) of this vector?

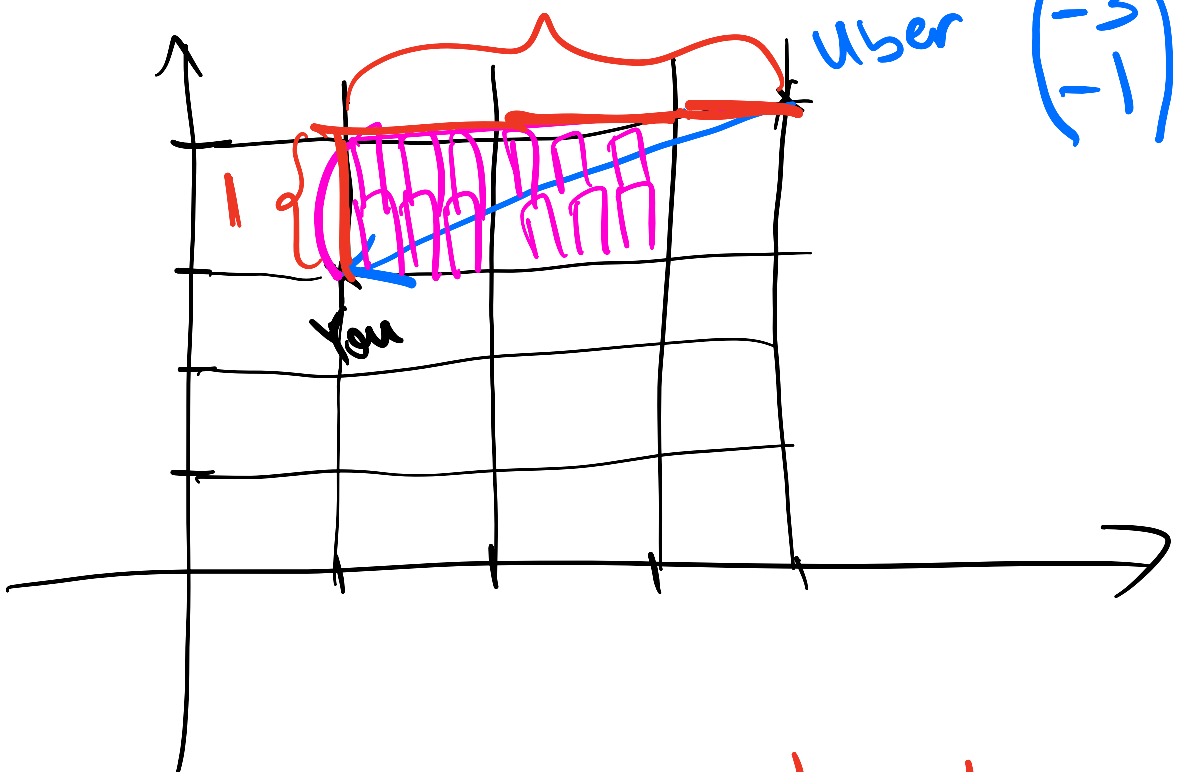
$$\text{magnitude of } \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

$$= \sqrt{\boxed{x_1 \cdot x_1 + x_2 \cdot x_2 + \cdots + x_n \cdot x_n}}$$

$$= \sqrt{\vec{x} \cdot \vec{x}}$$

one way of computing the magnitude of a vector $\vec{x}$

$\Rightarrow$ dot products are everywhere



$\begin{pmatrix} -3 \\ -1 \end{pmatrix}$

$$|-3| + |-1| = 4$$

$$\|\vec{x}\|_1 = |x_1| + |x_2| + \cdots + |x_n|$$

magnitude
or
norm

1-norm

$$\|\vec{x}\|_2 = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

2-norm

* maximum norm or infinity norm

Testing whether you use some product

perfume

measuring the concentration of some chemical substance in 100 samples of the perfume

If the concentration of any of the substance in the samples exceeds 100, you can't use the perfume.

$$\|\vec{x}\|_\infty = \max(|x_1|, |x_2|, |x_3|, \dots, |x_n|)$$

$$= \max_{1 \le i \le n} |x_i|$$

$$\Rightarrow \begin{cases} (1)x + (2)y + (0)z = 3 \\ (0)x + (1)y + (-3)z = 4 \\ (1)x + (1)y + (-2)z = 1 \end{cases}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -3 \\ 1 & 1 & -2 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$$

matrix

vector

vector

(they're everywhere)

matrix-vector multiplication → extension of a dot product

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -3 \\ 1 & 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1*x + 2*y + 0*z \\ 0*x + 1*y + (-3)*z \\ 1*x + 1*y + (-2)*z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x + 2y \\ y - 3z \\ x + y - 2z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -3 \\ 1 & 1 & -2 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}}_{\vec{b}}$$

$$\boxed{A \quad \vec{x} = \vec{b}}$$

$$\boxed{\text{np.linalg.solve}}(A, \vec{b}) \longrightarrow \vec{x}$$

$$\|A\|_F = \sqrt{1^2+2^2+0^2+0^2+1^2+(-1)^2+1^2+1^2+(-2)^2}$$

Frobenius norm of a matrix

$$A\vec{x} = \vec{b}$$

$$U(1,1) = -0.04 + 0.8\,U(1,2) + 0.1\,U(2,1) + 0.1\,U(1,1)$$

$$U(1,2) = -0.04 + 0.8\,U(1,3) + 0.2\,U(1,2)$$

$$\Rightarrow \quad -0.9\,U(1,1) + 0.8\,U(1,2) + 0.1\,U(2,1) = 0.04$$

$$-0.8\,U(1,2) + 0.8\,U(1,3) = 0.04$$

$$\begin{pmatrix} -0.9 & 0.8 & 0.1 & 0 \\ 0 & -0.8 & 0 & 0.8 \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} U(1,1) \\ U(1,2) \\ U(2,1) \\ U(1,3) \\ \vdots \end{pmatrix} = \begin{pmatrix} 0.04 \\ 0.04 \\ \\ \\ \end{pmatrix}$$