CS 5100 02/06

Review

Expectiminimax Algorithm

* Playing against someone
* Uncertainty in the game
* Optimize expected utility

X_1 X_2

$\frac{P(A \cap B)}{P(B)}$

sequence of variables X_1, X_2

A B

$P(X_1 = a, X_2 = b)$ ∩

$P(X_2 = b \mid X_1 = a) = \frac{P(X_2 = b \cap X_1 = a)}{P(X_1 = a)}$

$\Rightarrow$

$P(X_2 = b \cap X_1 = a) = P(X_2 = b \mid X_1 = a) \cdot P(X_1 = a)$

$\parallel$

$P(X_2 = b \mid X_1 = a) \cdot P(X_1 = a)$

Exercise

joint probability

$\boxed{P(X_1 = a, X_2 = b, X_3 = c)}$

$$\underbrace{P(X_1=a)\cdot P(X_2=b\mid X_1=a)}\cdot \boxed{P(X_3=c\mid X_1=a,\,X_2=b)}$$

$$P(A\cap B) = \boxed{P(A\mid B)}\cdot P(B) = P(B\mid A)\cdot P(A)$$

A: $X_3=c$

B: $X_1=a,\,X_2=b$

$$P(X_3=c,\,X_1=a,\,X_2=b) = P(X_3=c\mid X_1=a,X_2=b)\cdot \boxed{P(X_1=a,X_2=b)}$$

$$= P(X_3=c\mid X_1=a,\,X_2=b)\cdot P(X_2=b\mid X_1=a)\cdot P(X_1=a)$$

$$*\,P(\underbrace{X_1=a,\,X_2=b,\,X_3=c}_{B},\,\underbrace{X_4=d}_{A})$$

$$= P(X_4=d\mid X_1=a,X_2=b,\,X_3=c)\cdot \underline{P(X_1=a,X_2=b,X_3=c)}$$

expression we had in the previous example

$$= P(X_4=d\mid X_1=a,\,X_2=b,\,X_3=c)$$
$$P(X_3=c\mid X_1=a,\,X_2=b)\cdot P(X_2=b\mid X_1=a)\cdot P(X_1=a)$$

$P(X_1 = a_1, X_2 = a_2, \ldots, X_n = a_n)$

$= P(X_n = a_n \mid X_1 = a_1, \ldots, X_{n-1} = a_{n-1})$

$\cdot P(X_1 = a_1, \ldots, X_{n-1} = a_{n-1})$

$= P(X_n = a_n \mid X_1 = a_1, \ldots, X_{n-1} = a_{n-1})$

$\cdot P(X_{n-1} = a_{n-1} \mid X_1 = a_1, \ldots, X_{n-2} = a_{n-2})$

$\vdots$

$P(X_2 = a_2 \mid X_1 = a_1) \cdot P(X_1 = a_1)$

$P(X_1 = a_1, \overset{a_2}{\overset{\shortparallel}{X_2}}) = \boxed{P(\overset{a_2}{\overset{\shortparallel}{X_2}} \mid \overset{a_1}{\overset{\shortparallel}{X_1}})} \cdot \boxed{P(\overset{a_1}{\overset{\shortparallel}{X_1}})}$

1 value memorized to represent $P(X_1)$

(1 parameter)

$X_1 \in \{0, 1\}$ 0.4 $\rightarrow$ $\boxed{P(X_1 = 0) \quad P(X_1 = 1)}$

$X_2 \in \{0, 1\}$

$P(X_2 = a_2 \mid X_1 = a_1)$

Fix X_1's value first $(X_1 = 0)$

$P(X_2 = 1 \mid X_1 = 0)$ $\quad$ $P(X_2 = 0 \mid X_1 = 0)$

must sum up to 1

as X_2 could either be 0 or 1

$\Rightarrow$ only need to memorize one of them

Fix $X_1 = 1$

$P(X_2 = 0 \mid X_1 = 1)$ $\quad$ $P(X_2 = 1 \mid X_1 = 1)$

must sum up to 1

$\Rightarrow$ only need to memorize 1 of them

$\Rightarrow$ Total : 3 parameters

$P(X_1 = a_1, X_2 = a_2, X_3 = a_3)$

2^3 $= 8$ outcomes

$\Rightarrow$ need to memorize 7 of the outcomes

$X_1 \in \{0,1\}$

$X_2 \in \{0,1\}$

$X_3 \in \{0,1\}$

$P(X_1 = a_1, X_2 = a_2, \dots, X_n = a_n)$

$X_i \in \{0,1\}$

2^n outcomes

$\Rightarrow 2^n - 1$ outcomes

this can be a huge number of parameter as n increases

We might not be able to store all these parameters to be able to get some information about the stock prices.

What can we do to reduce the number of parameters?

2018	2019	2020	2021	2022	2023
10	20	30	30	30	30

disregarding

<u>Idea</u>: Let's reduce the number of previous years we're looking to predict the new stock price

$P(X_1=a_1, \ldots, X_n=a_n)$

$= P(X_n=a_n \mid X_1=a_1, \ldots, X_{n-1}=a_{n-1}) \cdots\cdots$

$P(X_3 \overset{a_3}{=} \mid X_1=a_1, X_2=a_2) \cdot P(X_2=a_2 \mid X_1=a_1)$

$P(X_1=a_1)$

Tradeoff in the choice of number of years and information stored in the probability functions

One choice : Only look at 1 previous year

$P(X_1=a_1, \ldots, X_n=a_n)$

with the assumption

$= \underbrace{P(X_n=a_n \mid X_{n-1}=a_{n-1})}_{\text{2 parameters}} \cdot \underbrace{P(X_{n-1}=a_{n-1} \mid X_{n-2}=a_{n-2})}_{\text{2 parameters}}$

$\cdots\cdots \underbrace{P(X_3=a_3 \mid X_2=a_2)}_{\text{2 parameters}} \cdot \underbrace{P(X_2=a_2 \mid X_1=a_1)}_{\text{2 param.}} \underbrace{\boxed{P(X_1=a_1)}}_{\text{1 parameter}}$

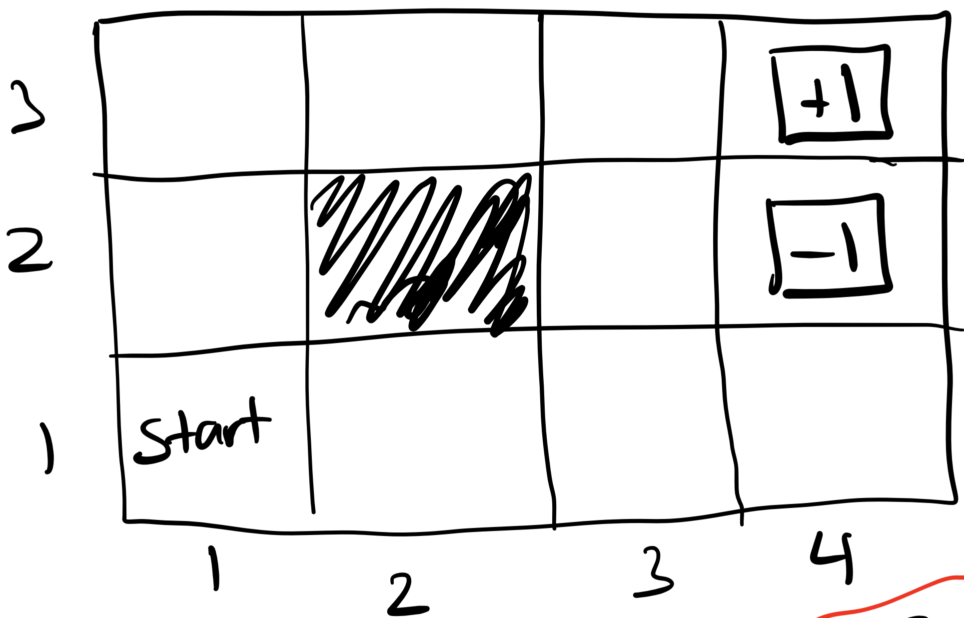
$X_i \in \{0,1\}$

$P(X_2=a_2 \mid X_1=a_1) \longrightarrow$ 2 parameters

Total $= 2(n-1) + 1 = 2n-1$ parameters

Markovian assumption (Looking at one previous year)

Markov



+1 utility

-1 utility

2 terminal states : $\boxed{+1}$ $\boxed{-1}$

all other states : utility -0.04

2 optimal paths $\begin{cases} N, N, E, E, E \rightarrow 0.84 \\ E, E, N, N, E \rightarrow 0.84 \end{cases}$

There's a probability of 0.8 to move to the intended outcome and a probability of 0.2 that is distributed evenly between the other states

N, N, E, E, E

Action : North

$(1,1) \xrightarrow{0.8} (1,2)$

$(1,1) \xrightarrow{0.2} (2,1)$

$$\text{Utility}(1,1) = 0.8 * \boxed{\text{Utility}(1,2)} + 0.2 * \text{Utility}(2,1)$$

(1,2) Action : North

$(1,2) \xrightarrow{0.8} (1,3)$

$(1,2) \xrightarrow{0.2} (1,1)$

$$\text{Utility}(1,2) = 0.8\ \text{Utility}(1,3) + 0.2\ \text{Utility}(1,1)$$