

Exploring the Teapot Effect: Microscale Investigations and Mechanistic Analysis

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Abstract

In this work, we construct a microscopic model to study the fluid dynamics when moving over the teapot surface, which is also called “teapot effect”. This question has been extensively studied via the Navier-Stokes equations and effective models in the past decades. As it is very difficult to solve hydrodynamic equations with a complicated boundary conditions, we developed a new microscopic model, which treat water as many fluid cells. These fluid cells interact with other surrounding fluid cells and also the teapot solid surface due to the adhesive forces. The complex evolution of the fluid is simulated by programming the motion of these droplets. We calculate the viscosity of the fluid via the microscopic model, and also calculate the trajectories of the fluid cells when water moving over the teapot surface.

We conducted a thorough examination of the interaction strength within the water, particularly focusing on the attractive forces between water droplets and the teapot surface. We analyzed how these forces are influenced by the velocity of the water flow and the shape of the teapot. Detailed investigations were carried out to understand the effects of these factors on the behavior of the water on the teapot surface, including its movement on the broken areas of the teapot.

The novel microscopic model developed in this research enables us to delve into the intricate interactions occurring within the fluid system. We were able to explore forces acting between individual water droplets, as well as the intermolecular forces at play. This level of microscopic analysis surpasses the capabilities of traditional macroscopic fluid mechanics. Notably, although our microscale particle collision model was originally designed for the study of incompressible flows, it also exhibits innate potential for simulating compressible fluids. Given the well-known challenges associated with solving problems involving compressible fluids, our microscale particle collision model holds significant promise for a wider range of applications.

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I. INTRODUCTION



FIG. 1: The image illustrates the "Teapot Effect," which is the phenomenon where liquid flows down the walls of a container as it is poured out of the container.

In the realm of science and engineering, the study of fluid dynamics assumes a paramount role. Profound comprehension of fluid dynamics constitutes an indispensable foundation for the advancement of various disciplines, including but not limited to aerodynamics, oceanography, atmospheric science, biology, chemical engineering, and aerospace technology. The simulation and comprehension of fluid flow phenomena prove pivotal within these domains. However, intricate geometries of containers and their internal structures often precipitate distinctive effects on fluid flow. One of these consequential phenomena is known as the "Teapot Effect" [1–3]. The Teapot Effect describes a peculiar behavior observed when fluid emanates from a curved and tilted container, seemingly defying gravity by flowing in a direction counter to the container's tilt. Despite its widespread observation, elucidating the underlying mechanisms governing the Teapot Effect has remained a formidable scientific challenge.

Researchers have proffered various models and theories to expound upon and predict the Teapot Effect. Early attempts involved intricate mathematical models to describe fluid motion. However, these models proved excessively complex and arduous to solve, largely owing to the intricate interplay between fluid and container surfaces. The rapid evolution of computer science and numerical simulation methodologies has catalyzed a shift toward the utilization of Computational Fluid Dynamics (CFD) techniques [4–7] to simulate the Teapot Effect.

The contemporary landscape features a multitude of models and simulation outcomes concerning the Teapot Effect. Some models are grounded in fundamental principles such as the Navier-Stokes equations [8–11], turbulence models [12–14], and boundary conditions accounting for surface interactions[15, 16]. Since the 1950s [17], numerous research groups have conducted exhaustive investigations into the Teapot Effect, treating it as a predominantly hydrodynamic process [18, 19] or scrutinizing it via mass and momentum balance principles, accounting for inertia [20]. This approach has yielded an effective, albeit simplified, semi-quantitative method for exploring the phenomenon. Recent work led by E. Jambon-Pullet, for instance, employed momentum balance techniques to delve into the formation of liquid helixes stemming from the Teapot Effect [21]. Furthermore, research conducted by the Duez Group has highlighted the substantial roles played by capillary effects and surface wettability in the Teapot Effect [22].

These models primarily focus on dissecting the mechanical characteristics of fluid flow and endeavor to simulate the generation and evolution of the Teapot Effect via numerical approaches. Conversely, other investigations adopt an experimental perspective, relying on extensive observations and empirical data to validate model accuracy. These models and experiments collectively furnish invaluable references and foundational knowledge to enhance our comprehension of the Teapot Effect in fluid dynamics.

Nonetheless, notwithstanding the wealth of existing models and experimental research dedicated to the Teapot Effect, numerous enigmas persist. The causative factors and underlying mechanisms of the Teapot Effect remain incompletely elucidated. Additionally, current models and experiments have not comprehensively addressed the microscale interactions occurring between the fluid and the container surface. In the present study, we endeavor to explore the Teapot Effect from an alternate perspective by constructing a simplified model that emphasizes the interplay between fluid layers and their interaction with the container's

surface. This approach aims to unveil the relationship between the Teapot Effect and the strength of interaction between the fluid and the container's surface.

Within this investigation, we probe a range of forces, including adhesive forces between the fluid and the solid surface, internal coupling forces within the fluid, gravitational effects, and the influence of teapot thickness on the dynamics of liquid egress from the container. While our analysis does not purport to offer an exhaustive examination of the Teapot Effect, our goal is to employ fundamental principles derived from basic physics to scrutinize the effects of varying force magnitudes on the fluid's behavior. We explore the fluid's response to differing initial velocities and calculate the conditions under which the Teapot Effect manifests. Additionally, we investigate the occurrence of layer separation within the fluid, where specific layers adhere to the outer wall of the container while others are expelled due to centrifugal forces [23].

We employed the computational fluid mechanics approach to analyze this issue, and the outcome is depicted in the Fig.2, and the simulated streaming video will be placed in the attached file. Nonetheless, resolving the fluid dynamic equation incorporating parameters such

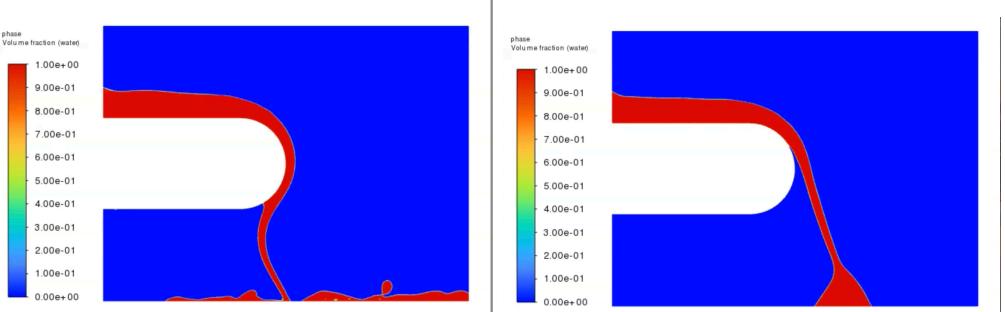


FIG. 2: The left diagram illustrates the flow with adhesion force, where the teapot effect and contact angle are distinctly observable. Conversely, in the right diagram, without adhesion force, the teapot effect is absent.

as viscosity, adhesion force (surface tension), inertia, and gravitational force poses considerable computational challenges. Furthermore, this approach suffers from limited adaptability. Therefore, we have opted to utilize models based on momentum conservation and particle scattering while accounting for surface tension.

II. THEORETICAL METHODS TO ANALYSE THE TEAPOT EFFECT

A. Physical quantity

1. Mean free path

The "certain value" at this time is called the molecular effective diameter d . Define the molecular collision cross section σ , that is, the molecules injected outside the range of this circular cross section will not deviate from the direction of velocity. For this section, the following equations are given:

$$\sigma = \pi d^2 \quad (1)$$

The number of collisions of gas molecules per unit time is called the average collision frequency, which represented by the following equation:

$$\bar{Z} = n\sigma\bar{v}_{rel} \quad (2)$$

We can use the average relative velocity \mathbf{v}_{rel} between particles to drive the mean free path.

$$\mathbf{v}_{rel} = \mathbf{v}_1 - \mathbf{v}_2 \quad (3)$$

$$\bar{\mathbf{v}}_{rel} = \sqrt{\mathbf{v}_{rel} \cdot \mathbf{v}_{rel}} \quad (4)$$

$$\mathbf{v}_{rel} = \sqrt{(\mathbf{v}_1 - \mathbf{v}_2) \cdot (\mathbf{v}_1 - \mathbf{v}_2)} = \sqrt{\mathbf{v}_1 \cdot \mathbf{v}_1 - 2\mathbf{v}_1 \cdot \mathbf{v}_2 + \mathbf{v}_2 \cdot \mathbf{v}_2} \quad (5)$$

the average of the terms leads to

$$\bar{\mathbf{v}}_{rel} = \sqrt{\bar{\mathbf{v}}_1 \cdot \bar{\mathbf{v}}_1 - 2\bar{\mathbf{v}}_1 \cdot \bar{\mathbf{v}}_2 + \bar{\mathbf{v}}_2 \cdot \bar{\mathbf{v}}_2} \quad (6)$$

Since \mathbf{v}_1 and \mathbf{v}_2 are random and uncorrelated,

$$2\bar{\mathbf{v}}_1 \cdot \bar{\mathbf{v}}_2 = 0 \quad (7)$$

Thus, the expression of relative velocity is

$$\overline{\mathbf{v}_{rel}} = \sqrt{\overline{v_1^2} + \overline{v_2^2}} = \sqrt{2\bar{v}} \Rightarrow \overline{v_{rel}} = \sqrt{2\bar{v}} \quad (8)$$

Let the average molecular velocity be \bar{v} , the mean free path is:

$$\bar{\lambda} = \frac{\bar{v}t}{Zt} = \frac{\bar{v}}{n\sigma\overline{v_{rel}}} = \frac{1}{\sqrt{2}n\pi d^2} \quad (9)$$

Where d is the molecular effective diameter, as we mentioned above.

With the formula of the mean free path in the form Eq.(48), one can calculate the “mean-free-path” of the fluid cells, even though this concept was first defined for the case of gas. As we treat the fluid as a “dense gas” with a large cross-section and strong interaction. The mean-free-path will also be used to calculate the viscosity in the next pages.

In the teapot effect, we divide the water into N cells, where \bar{v} is the mean velocity of the fluid cells. n is the density of the fluid cells, while σ is the cross-section between two fluid cells. The value of σ will be estimated according to the formula of Van der Waals interaction in the next section.

2. Viscosity Coefficient

As a preliminary theory, assume that the velocities of all molecules are of magnitude \bar{v} (average speed) and are uniformly distributed in the six possible directions. Consider a surface element ΔS on $z = z_0$. Thus, the number of particle that pass through the surface ΔS within time Δt is $1/6$ of the total particle number in volume $n\bar{v}\Delta S\Delta t$ where n is the numerical density of the particle in the space. Thus,

$$N_{pass} = \frac{1}{6}n\bar{v}\Delta S\Delta t \quad (10)$$

Each molecule passing through the plane carries some physical quantity from one side to the other side. Since we are studying viscosity, consider the momentum mu . Since the system

is not uniform along the z direction, the physical quantity Q pass through surface element is

$$\Delta Q = \left(\frac{1}{6} n \bar{v} \Delta S \Delta t Q \right)_A - \left(\frac{1}{6} n \bar{v} \Delta S \Delta t Q \right)_B \quad (11)$$

And the flow should be

$$\mathcal{F} = \frac{\Delta Q}{\Delta t} = \frac{\bar{v}}{6} [(nQ)_A - (nQ)_B] \Delta S \quad (12)$$

Where A and B respectively represent where $z = z_0 \mp \bar{\lambda}$ and $\bar{\lambda} = \bar{v} \Delta t$ is the mean free path of gas. It should be emphasized that the thermal equilibrium distribution is achieved by frequent collisions between molecules. Then, we can make another simplifying assumption that alien molecules are assimilated through a single collision. Thus,

$$\begin{aligned} \mathcal{F} &= \frac{\Delta Q}{\Delta t} = \frac{\bar{v}}{6} [(nQ)_A - (nQ)_B] \Delta S \\ &\sim -\frac{\bar{v}}{6} \left[\frac{d}{dz} (nQ) \right]_{z=z_0} \cdot 2\bar{\lambda} \cdot \Delta S \\ &= -\frac{1}{3} \left[\frac{d}{dz} (nQ) \right]_{z=z_0} \cdot \bar{v} \cdot \bar{\lambda} \cdot \Delta S \end{aligned}$$

Then we make the substitution that $Q = P = mu$; $\mathcal{F} = f$, then

$$f = \frac{\Delta P}{\Delta t} = -\frac{1}{3} \left[\frac{d}{dz} (nm u) \right]_{z=z_0} \bar{v} \bar{\lambda} \Delta S \quad (13)$$

$$= -\frac{1}{3} \rho \bar{v} \bar{\lambda} \left(\frac{du}{dz} \right) \Delta S \quad (14)$$

Notice that the Newton's viscosity law is

$$f = -\mu A \frac{\partial u}{\partial z} \quad (15)$$

Thus, the viscosity coefficient is

$$\mu = \frac{1}{3} \rho \bar{v} \bar{\lambda} \quad (16)$$

where ρ is the mass density and $\bar{\lambda}$ is the mean free path.

B. Model-1: Navier-Stokes equation

1. Continuity Equation

According to the law of conservation of mass, the mass of a fluid remains constant during the flow process. Thus, consider a substance composed of randomly selected mass points in a fluid, with a volume of V and a mass of M , we have that

$$M = \int_V \rho dV \quad (17)$$

Where ρ is the mass density of the fluid. The conservation of mass suggest that

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial t} \int_V \rho dV = - \oint_S \rho \mathbf{v} \cdot d\mathbf{S} \quad (18)$$

Consider the mass derivative of the volume integral,

$$\int_V \left(\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) \right) dV = 0 \Rightarrow \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (19)$$

This is the Continuity Equation, and for incompressible fluid, ρ is a constant. Thus,

$$\nabla \cdot \mathbf{v} = 0 \quad (20)$$

2. Momentum Balance Equation

The mass forces and surface forces acting on the fluid volume V and the surface S , respectively, are as follows:

$$\int_V \rho \mathbf{F} dV \quad (21)$$

$$- \oint_S p d\mathbf{S} = - \int_V \nabla \cdot p dV \quad (22)$$

The viscous force should be expressed by Newton's law of viscosity:

$$\tau = \mu \nabla \mathbf{v} \quad (23)$$

Consider the incompressible fluid, the change of momentum should be

$$\frac{\mathbf{D}\rho\mathbf{v}}{\mathbf{D}t} = \rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) \quad (24)$$

assume the mass force is gravity and there exist a pressure as the active force. We can also take viscosity into consideration. Thus, the momentum balance equation can be written as

$$\int_V \rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) dV = - \oint_S p d\mathbf{S} + \int_V \rho g dV + \oint_S \mu \nabla \mathbf{v} d\mathbf{S} \quad (25)$$

Use Guass' law to eliminate integral

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} + g \quad (26)$$

This is the vector form of the momentum equations in Navier-Stokes Equations, where ρ is the mass density of the fluid, $\nu = \mu/\rho$ is the kinetic viscosity coefficient, \mathbf{v} is the velocity of fluid, g is the gravitational acceleration, p is the pressure.

Consider the ideal fluid without viscosity. The momentum equations in Navier-Stokes Equations will degenerates into Euler's equation:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + g \quad (27)$$

3. Boundary Conditions

In order to find the solution that suitable to teapot effect, we can add some boundary conditions to the equations. The methods of adding boundary conditions is easy. First of all, the fluid cannot enter the solid surface, while considering the interaction between the solid-liquid interface. We can assume that the flow is non-slip, which means

$$v_{S-L} = 0 \quad (28)$$

Similarly, we can limit the maximum value of the flow velocity, that is, the flow velocity at the gas-liquid interface.

$$v_{L-G} = U \quad (29)$$

Simultaneously, we can consider that the flow velocity of the flow in the flat plate part is only related to the viscosity in the y direction and is linearly distributed along the y -axis.

Although finding the numerical solution of Navier-Stokes Equations is a universal methods to solve fluid mechanic problem, it is not easy to apply in teapot effect. That is because the teapot effect is the result of coupling of various factors such as surface tension and viscous force. Surface tension play a significant role in this phenomenon. However, it is hard to consider the affect of surface tension in Navier-Stokes equations, and the fluid mechanics equations is challenging to solve. Thus, the Navier-Stokes Equation is not useful in this problem.

C. Model-2: Momentum balance method

1. Balance State

Take vertically downwards as the positive direction of y -axis. The force balance equation is

$$y - \text{direction: } \pi(p_{up} - p_{down})R^2 \cos \psi + mg + F \cos \psi = 0 \quad (30)$$

$$x - \text{direction: } \pi(p_{up} - p_{down})R^2 \sin \psi + F \sin \psi = 0 \quad (31)$$

Where the first term represent the affect of surface tension, while the second term is the gravitational force. γ_{LG} is the coefficient of surface tension between liquid-gas surface, θ is the contact angle between liquid and solid surface, m is the mass of the liquid and g is the gravitational acceleration. The term F is the normal force and other force that may exist to affect the balance.

2. Unbalance State

The Teapot Effect can be expressed by the 3, where Δn is the fluid thickness, U is the flowing speed, r is the radius of the solid surface, and C is the curvature. To streamline the

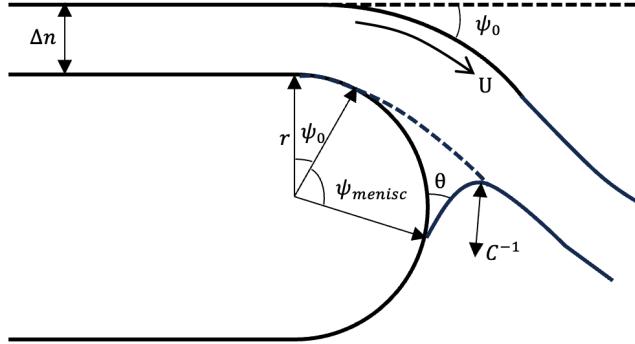


FIG. 3: This is a diagram of the teapot effect

model, we make the simplifying assumption that viscosity can be disregarded. Within this context, let us contemplate the orientation represented by the unit vector \hat{n} for the vertical streamline, arising from the centripetal acceleration.

The centripetal acceleration is given by:

$$a_n = \frac{U^2}{R}$$

Hence, the cumulative force acting on the fluid cell can be expressed as

$$\sum \delta F_n = a_n \delta m = \frac{\rho U^2}{R} \delta V$$

In accordance with Bernoulli's Law, the equation of motion along the vertical streamline direction becomes

$$\left(-\rho g \frac{dh}{dn} - \frac{\partial P}{\partial n} \right) = \frac{\rho U^2}{R} \quad (32)$$

Here, we have defined the relevant parameters: ρ denotes the fluid density, h represents the vertical length, $R = r + \frac{\Delta n}{2}$ stands for the radius of curvature of the streamline, and

P corresponds to the pressure arising from surface tension. Additionally, the volume of the considered flow cell is represented by $V = \delta s \delta n \delta y$.

We can now express this equation in an alternative form as follows:

$$-\rho g \Delta h - \Delta P \sim \frac{\rho U^2 \Delta n}{R} \quad (33)$$

During the computation, it is reasonable to make the assumption that $\cos \psi_0 = \frac{dh}{dn} \approx \frac{\Delta h}{\Delta n}$. The pressure disparity can be attributed to the adhesive force, specifically surface tension. Consequently, in accordance with the Yang-Laplace Equation, we have:

$$\Delta P = \frac{\gamma_{LG}}{C^{-1}} \quad (34)$$

The Weber number is defined as the ratio of inertia to surface tension, expressed as:

$$We = \frac{\rho U^2 \Delta n}{\gamma_{LG}} \quad (35)$$

Here, C represents the curvature of the capillary effect half-moon area. By using Eq.33 and Eq.34, we can derive the expression for curvature as follows:

$$C = \frac{\rho(U^2 \Delta n + g \Delta h R)}{R \gamma_{LG}} \quad (36)$$

Since Eq.32 and Eq. 33 describe the distribution of forces on the surface, and surface tension predominantly affects the wetted area $A_{wet} = r \times \psi_{wet}$, we can express the total force magnitude as:

$$|F_{total}| = \frac{\rho U^2 \Delta n}{R} \times A_{wet} = \frac{\rho U^2 \Delta n}{R} r \psi_{wet} \quad (37)$$

Utilizing geometric relationships, we can derive the expression for the change in momentum flux as:

$$D = \rho U^2 \Delta n (1 - \sin \psi_0) \quad (38)$$

$$= \frac{\gamma_{LG}}{C^{-1}} r (\psi_0 + \psi_{menisc}) + \rho g r \Delta h (\psi_0 + \psi_{menisc}) \quad (39)$$

$$= |F_{total}| \quad (40)$$

Substituting the expression for the Weber number into the above equation, we obtain:

$$\frac{1 - \sin \psi_0}{\psi_0 + \psi_{menisc}} \text{We} = \frac{r}{C^{-1}} \text{We} + 2r \frac{\rho g h}{\gamma} \quad (41)$$

Assuming that the capillary effect half-moon area corresponds to part of a circle, we can determine the expression for ψ_{menisc} .

$$\psi_{menisc} = \left[\frac{2C^{-1}(1 + \cos \theta)}{r} \right]^{1/2} \quad (42)$$

$$= \left[\frac{R}{r} \frac{2\gamma_{LG}(1 + \cos \theta)}{\rho(U^2 \Delta n + g \Delta h R)} \right]^{1/2} \quad (43)$$

These equations show that the teapot effect does not rely on the geometry properties that are represented by the factor r/R in the expression of ψ_{menisc} and the surface wettability of the solid surface which is relied on the contact angle θ . The Weber number is a dimensionless parameter widely employed in fluid mechanics to assess the relative significance of inertial forces in relation to surface tension forces within a fluid medium. Consequently, an increase in the Weber number signifies a heightened role of inertia in fluid dynamics, concomitant with a diminishing influence of surface tension. In situations where the Weber number far exceeds unity ($\text{We} \gg 1$), denoting that $\rho U^2 \Delta n \gg \gamma_{LG}$, the effects of surface tension can be safely neglected, thereby allowing us to assume that adhesion forces exert no discernible impact on the flow.

It is worth noting that the expression for the increment in momentum can only be applied when $U \neq 0$ since the teapot effect is physically non-existent when $U = 0$. Consequently, in a physical context, $\text{We} \neq 0$.

When considering the numerical solution of Eq.41, the resultant graph aligns with our

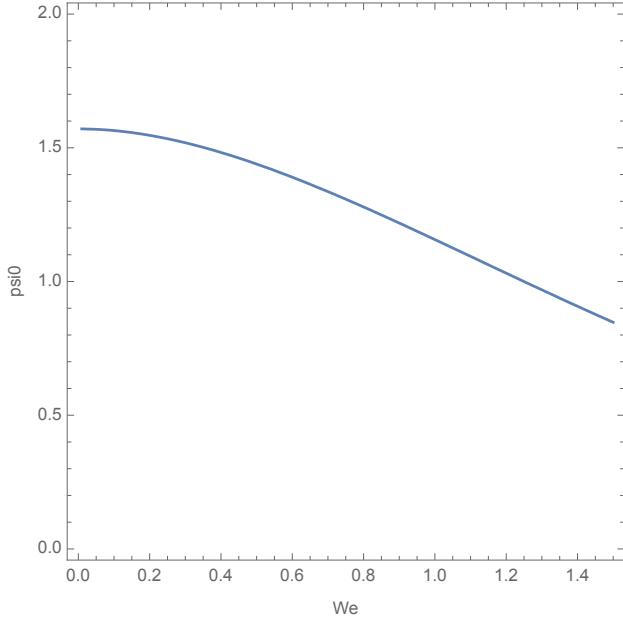


FIG. 4: This is the numerical result of the momentum balance equation Eq.41, which shows that the ejection angle ψ_0 will decrease when the Weber number We increasing.

physical expectations. As the Weber number increases, signifying a reduction in the adhesive effects within the flow, it conforms to our understanding that adhesion forces significantly contribute to strengthening the Teapot Effect. Consequently, with increasing We values, the fluid is more likely to detach from the solid surface, leading to a reduction in ψ_0 as We increases. We employed Mathematica software for the numerical solution of Eq.41. The Mathematica code utilized for solving this equation is provided in the appendix for reference and further detail.

III. NEW APPROACH: MICROSCOPIC MODEL (ORIGINAL WORK)

A. Model construction

Studying fluid dynamics on a solid surface has been one of the difficult physical problems in the past decades. Usually, the general variables to describe the fluid dynamics are the pressure p , density ρ , fluid velocity \vec{u} , viscosity μ , etc. These macroscopic variables satisfy the hydrodynamic equations with specific boundary conditions. For the teapot effect we care about here, the boundary conditions of water on the teapot solid surface is needed, where the adhesive forces between the fluid and the teapot surface must be given as one of conditions. One typical hydrodynamic equation is the famous Navier-Stokes equation. This equation is too complicated for realistic problems due to the complicated boundary conditions. The finite-element method has been applied to study the numerical solutions of Navier-Stokes equation. These have been extensively studied in many previous references.

From statistical physics, we know that these macroscopic variables can be expressed as a summation of small fluid cells. In other words, one can divide the fluid into many small cells. The dynamical evolution of the fluid can be regarded as a **collective motion** of a large number of fluid cells. When the fluid is a strongly coupled system, instead of a weakly coupled medium, then the interaction forces between the fluid cells are large. It means that how the fluid cells move depends on the forces from surrounding fluid cells. The macroscopic variables, like the **density** ρ , **pressure** p , etc can be expressed as an integration of the fluid cells' velocity and momentum. This relation will be given in the later section of this manuscript. Further, the other important physical variables, like **viscosity** μ , **Reynolds number** Re , etc, can also be written as a function of the small fluid cells' positions and momenta. In the following parts, we will treat the fluid cells as a kind of particle. The fluid consists of these particles. Please see the schematic diagram in Fig.5.

As we know, the fluid is strongly coupled together. Therefore, the fluid cells will also interact with the surrounding cells. The interaction force between each cell (or particle) is connected with the coupling strength of the fluid.

We study the dynamic evolutions of the fluid on the solid surface, like the case of water flowing out of the teapot. To simplify the question, we focus on the two-dimensional case,

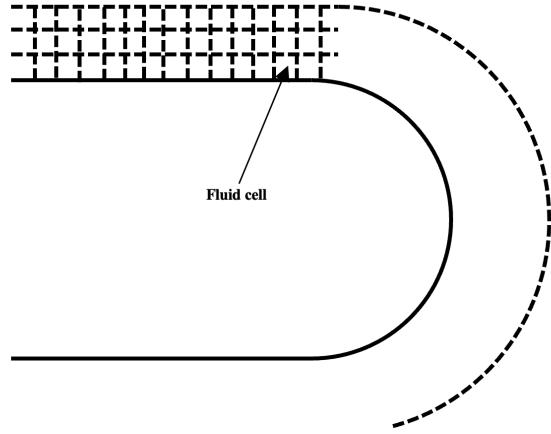


FIG. 5: schematic diagram: the water flows over the solid surface (teapot). 2-dimensional case is considered to simplify the question. The fluid is separated into N parts (consisting of N "particles" or "fluid cells"). The edge of the teapot mouse is simplified as a semicircle. The fluid moves in the right direction with an initial velocity u_0 .

shown as Fig.5. The fluid move above the solid surface, where the edge of the teapot mouse is treated as a semicircle with the radius $r_0 = 2$ cm. The thickness of the water is taken as $e_0 = 3$ cm according to the realistic estimation. How the water flows over the edge of the teapot mouse, i.e., whether the teapot can be observed or not, depends sensitively on the following factors:

- **initial velocity of the fluid**
- **coupling strength between the fluid and teapot**
- **coupling strength inside the fluid**
- **shape of the teapot mouse**
- **other external forces (like gravity)**

In the following sections, I will study how these factors affect the dynamical evolutions of the fluid, i.e., the teapot effect.

B. Parameters and setups

If we study the dynamic evolution of the water on the teapot surface, the boundary conditions like the adhesive force between the water and the teapot, the flow velocity, and the pressure gradient ∇p , must be considered. The Navier-Stokes equation with these complicated boundary conditions is difficult to solve. There has been a comprehensive study on this part. It turns out to be difficult for readers.

In this study, we will develop a new method to study fluid dynamics. The above factors controlling the flow dynamics will be considered via the forces between particles, which consist of the fluid. Furthermore, this method developed to study the non-compressible fluid, can be easily extended to the case of compressible fluid.

After treating the fluid as a large set of particles (fluid cells), the particles' motion is connected with the forces coming from the surrounding object. In Fig.6, the particle suffers the forces including gravity mg with the mass m , the supporting force from the teapot surface, and also the adhesive forces.

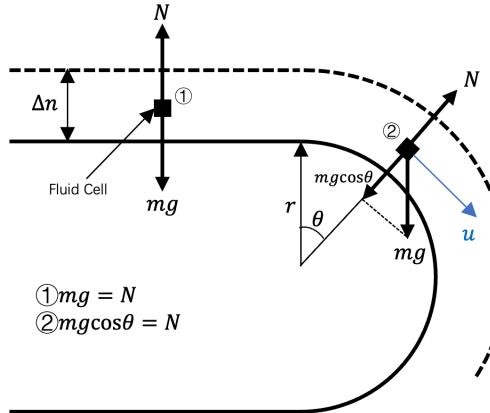


FIG. 6: The forces acting on the fluid cell (particle). For a particle next to the teapot surface, it includes gravity (external force), the (adhesive and supporting) force from the teapot surface, and the interaction force from surrounding particles.

The force between two particles is written as $F_{p-p}(r)$ here, which depends on the distance r between two particles. The force $F_{p-p}(r)$ becomes an attractive force when the distance is large, and the repulsive force when the distance r is small. The repulsive force indicates that the fluid (water) is close to non-compressible. The internal attraction of water, also known as water's cohesion, is caused by intermolecular forces between water molecules. This

attraction is primarily due to hydrogen bonding, which is caused by the electronegativity difference between water molecules. Water molecules are composed of one oxygen atom and two hydrogen atoms, with the oxygen atom being more electronegative than the hydrogen atoms. This results in the oxygen atom carrying a partial negative charge, while the hydrogen atoms carry a partial positive charge. Due to this charge difference, hydrogen bonds can form between water molecules. In water, hydrogen bonds can form a network structure, allowing water molecules to come close to each other and tightly connect. This hydrogen bonding leads to strong attractive forces between water molecules, enabling them to closely aggregate together. The interaction force between different particles is illustrated in Fig.7.

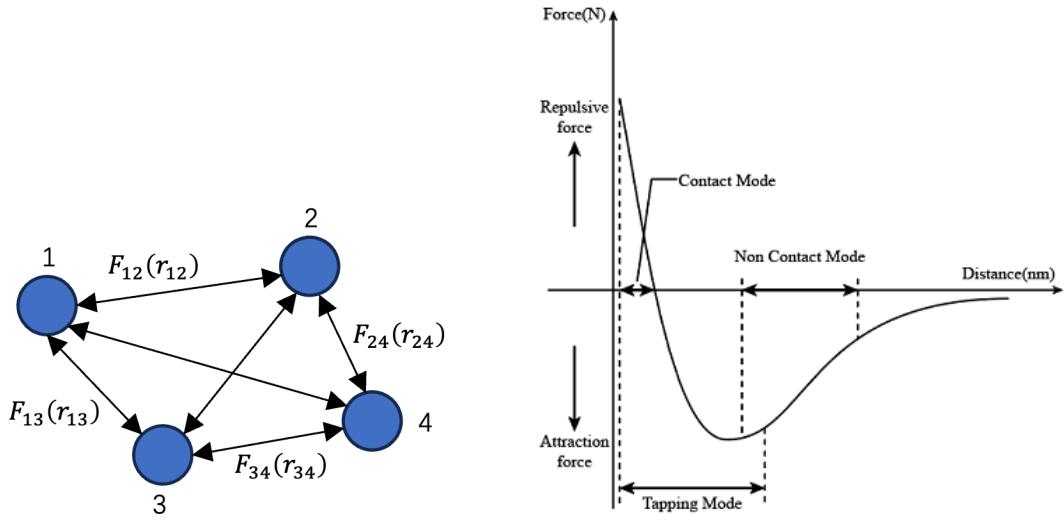


FIG. 7: The interactions between different fluid cells (particles).

Short-range forces: Inspired by the formula of the van der Waals force between water molecules, we approximate the forces between particles to be,

$$f_{p-p}(r) = \begin{cases} -C_1 \times mg, & \text{if } 0 < r \leq L_0 \\ C_2 \times mg, & \text{if } L_0 < r \leq L_1 \\ 0, & \text{if } r > L_1 \end{cases} \quad (44)$$

where m is the mass of the fluid cell (particles), this can be fixed by the fluid density (given latter). L_0 and L_1 is the interaction length.

When the distance is smaller than L_0 , the forces between two particles are repulsive,

represented by the minus before C_1 . When the distance is between $L_0 \sim L_1$, the force becomes an attractive force. f_{p-p} is close to zero when the distance is too large. To get a better estimation about the magnitude of the particle-particle interaction, we use the gravity of the fluid cell mg as a measurement. Therefore, the forces between fluid cells can be regarded as a several times of its gravity. The value of the parameter C_1 and C_2 is closely connected with the coupling strength of the fluid. Different values of the C_1 and C_2 will be employed to calculate different kinds of fluid moving on the teapot surface. The corresponding macroscopic variables like viscosity will be calculated.

There is also adhesive force f_{p-S} between the fluid and also the teapot surface. We also employ a similar formula,

$$f_{p-S}(r) = \begin{cases} -C_3 \times mg, & \text{if } 0 < r \leq L_0 \\ C_4 \times mg, & \text{if } L_0 < r \leq L_1 \\ 0, & \text{if } L_1 < r \end{cases} \quad (45)$$

This equation means the fluid cells can not “move into” the teapot surface due to the repulsive force. Also, there is adhesive force that “pull” the fluid cell toward the teapot surface. This force becomes important for Hydrophilic materials, while becomes zero for Hydrophobic materials.

Now, the fluid is separated into $N=3000$ parts. Each part is represented with a particle in the figure. As we know the density of the fluid ρ , the length of the fluid over the teapot surface (see fig.5), then the mass of each particle (fluid cell) is,

$$m = \rho \times e_0 l_f / N \quad (46)$$

To get a better numerical simulation of a large set of particles moving on the teapot surface, instead of taking all the initial velocities (u_x^i, u_y^i) of the particles as the same value, we initialize the value of u_x^i with the normal distribution, where the mean value is taken as $\langle u_x \rangle = 5 \text{ cm/s}$, and the width as 0.01 (cm/s)^2 . While for the velocity along the y-direction, u_y^i satisfies the mean value $\langle u_y \rangle = 0$ and the same width. With this setup, a large number of particles move collectively to the right with the velocity $\langle u_x \rangle$, while simultaneously exhibiting a certain degree of random and chaotic motion internally.

Long-range forces: In order to simulate fluid dynamics, introducing short-range forces between fluid cells is not enough. This can be reflected in the numerical simulations between particles. We introduce an additional long-range-force mode, which can give a more realistic simulation of fluid dynamics with the microscopic model. The Long-range force is set to be a weak force, which simulates the collective motion of the fluid. The magnitude of the long-range force inside the particles is set to be on the order of 10^{-1} of the short-range forces between particles (Eq.(44)). With this longe range interaction, the system consisting of particles can better simulate the motion of the fluid.

In future studies, we will employ a more realistic form similar to the Van der Waals force to describe the interactions between adjacent particles (fluid cells). Including the London force (which contributes repulsive force when r is small) and the Column force (which contributes attractive force when r is large), the combined force between two fluid cells is written as,

$$F_{p-p}^{real}(r) = -A/r^6 + B/r^2 \quad (47)$$

This more realistic interaction force will be included in the future work.

IV. RESULTS AND ANALYSIS

A. Calculation of mean-free-path λ , viscosity μ

mean-free-path:

To treat fluid as a large set of fluid cells like “dense gas”, we divide the fluid with the area S into N cells (see the schematic figure above). In the **two-dimensional case** discussed here, the cross-section between fluid cells has been a typical interaction length instead of a area, i.e. $\sigma \approx \times L_1$, where C and L_1 are the parameters given in above sections representing the interaction strength between fluid cells. (Dimension analyse: n is in m^{-2} , C is dimensionless, L_1 is in m^1 , then $\bar{\lambda}$ is in m^1).

$$\bar{\lambda} = \frac{1}{\sqrt{2}n\pi d^2} = \frac{1}{\sqrt{2}\frac{N}{S}(L_1)} \quad (48)$$

In this microscopic simulation of the water following over the teapot surface, we take $N = 3000$, where the “volume”(in 2-dimension, it is “area”) of the water piece is $S = e_0 \times L = 0.03 \times 0.08 (m^2)$. The typical interaction distance is estimated via the relation $L_1 \sim 2 \times \sqrt{S/N}$. **With this system we build, the mean free path is calculated to be $\bar{\lambda} \approx 0.00032 m = 0.32 mm$.** This value is reasonable for the water system. Note that the value of $\bar{\lambda}$ can be adjusted by taking different values of L_1 or N , to study different dynamical systems with different coupling strengths (such as water, oil, etc).

viscosity of the fluid-cell system:

The formula of the viscosity has also been presented in above section. We now calculate the viscosity of our “fluid-cell” system, which use fluid cells to simulalte the dynamical evolution of water.

Based on the formula of viscosity given in Section II, the viscosity in 2-dimensional system is

$$\mu_{2d} = \frac{1}{2}\rho\bar{v}\bar{\lambda} \quad (49)$$

where ρ is the mass density and $\bar{\lambda}$ is the mean free path. \bar{v} is the average speed of the random heat motion of the gas molecules (which are fluid cells in our case). The value of \bar{v}

can be given by the textbook if the distribution of particle velocity satisfying a Boltzmann distribution. In our case, as we already specify the interaction and density of fluid cells, the average speed of the random motion of fluid cells should be calculated via our model. We simulate the random motion of $N=3000$ fluid cells with the above parameters. The time evolution of the average speed is plotted. The initial mean velocity of those fluid cells are set to be $u_0 = 0.05$ m/s. With the cell-cell interactions, the thermal velocity after reaching equilibrium is around $\bar{v} = 0.15$ m/s. With the interactions, the average velocity of fluid cells change with time, plotted in Fig.8.

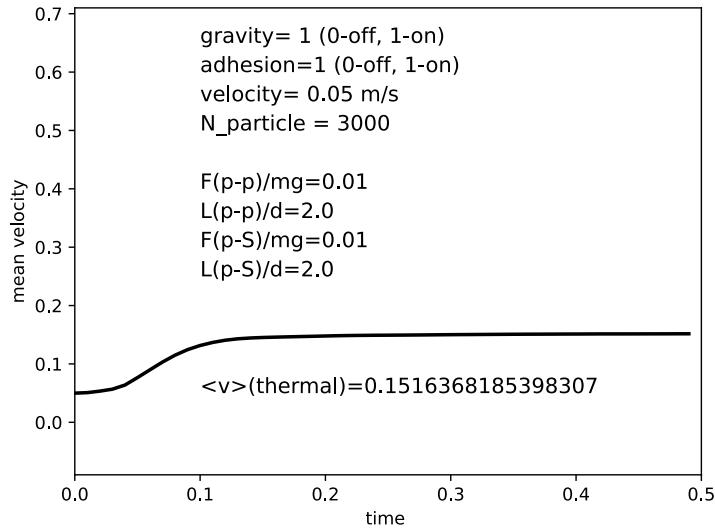


FIG. 8: time evolution of the mean velocity of a large number of fluid cells. This is calculated based on our “Microscopic scattering model”. The interactions between fluid cells are taken as previous sections. The density of the fluid cells is same as the value used in the mean-free-path.

Other relevant parameters in the model are,

$$\rho = 1 \text{ kg/m}^2 \quad (50)$$

$$\bar{\lambda} = 0.32 \text{ mm} \quad (51)$$

One can calculate the dynamical viscosity of the 2-dimensional “fluid” to be $\mu_{2d} = 2.4 \times 10^{-5}$ (kg/s) = 2.4×10^{-2} [(mPa · s) · m]. As we consider the 2-dimensional system, there is additional ‘m’ (length unit) in the μ_{2d} . Note that in the realistic case with 3 dimensions, the dynamical viscosity of the water is around $0.1 \sim 1$ (mPa · s) at different temperatures.

(Dimension units: $1\text{Pa} = 1\text{N/m}^2 = 1(\text{kg} \cdot \text{m/s}^2)/\text{m}^2 = 1\text{kg}/(\text{m} \cdot \text{s}^2)$)

B. Probe microscale force with gravity

How does gravity play the role in fluid dynamics?

When simulating the movement of fluid on the surface of a teapot, the dynamic behavior of the flow is highly sensitive to the coupling strength or the interactions between fluid elements. The key question is: What is the magnitude of the microscopic interactions between fluid elements (or water droplets)?

Due to the interaction forces within the fluid (equations (44)-(45)), controlling the evolution of the fluid, we can gauge the magnitude of microscopic forces within the fluid by using gravity as a reference. To gain a more intuitive sense of these forces, we use gravity as a benchmark for studying their magnitude. For instance, the forces between fluid cells include repulsion and attraction, and their magnitudes are taken as multiples of the gravitational force on a fluid cell, as specified by the parameters in equations (44)-(45): $C_1 = C_2 = C_3 = C_4 = 0.01$. In other words, the attractive and repulsive forces between fluid cells are assumed to be only 10^{-2} times their own gravitational forces.

So, what does the evolution of the fluid look like when the forces between fluid cells, whether attractive or repulsive, are only 10^{-2} times their own gravity? The numerical simulations presented below provide the answer. Furthermore, considering that water is an incompressible fluid, we generally assume the following: 1. When the distance between two fluid cells is less than half of the original distance, there is strong repulsion, indicating incompressibility. 2. When the distance between two fluid cells is greater than twice the originally set distance, there is strong attraction, implying that water is incompressible.

Therefore, in the equations governing the interaction forces, the distance parameters are set as follows: $L_0 = 0.5 \times D$, $L_1 = 2 \times D$. Here, $D = \sqrt{S/N}$ represents the average distance (typical distance) between any two fluid cells when N fluid cells are placed within an area of S .

If the microscopic forces between adjacent fluid cells within the fluid are much smaller

than their own gravitational forces, gravity can be considered an external force that has the potential to significantly influence the collective motion of a large number of fluid cells. Conversely, when the interaction forces between fluid cells are substantial, gravity, acting as a perturbation, may not cause significant changes in the overall motion of a large number of fluid cells. For example, if there is strong attraction and repulsion between a large number of fluid cells, adding gravity may not lead to noticeable alterations in the motion of these fluid cells. This hypothesis is demonstrated in the following model simulations.

The following two sets of figures depict the collective motion of a large number of fluid cells. We define $N=3000$ particles representing water droplets, with their initial positions distributed as shown in the figures. The initial velocity is horizontal to the right, denoted as u_0 . The interaction forces between different particles are described by equations Eq.(44-45). When gravity is not considered, the motion of a large number of particles appears as shown in Fig.9, with four panels representing the distribution of the fluid at times $t=0, 0.5, 1.0,$ and 1.5 seconds.

From Fig.9, it is evident that due to the initial velocity to the right, these red points representing water droplets move to the right. Since the interaction between water droplets and the solid surface accounts for viscosity (Eq.45), you can clearly see that some water droplets become adsorbed on the solid surface, as shown in the 2nd, 3rd, and 4th sub-figures. These red points on the curved surface of the teapot represent water droplets that do not continue moving in a straight line to the right but instead adhere to the vicinity of the curved surface of the solid.

These water droplets near the teapot's solid surface experience forces from both the solid surface and other liquid droplets. The interplay of these forces governs the trajectory of each water droplet, ultimately resulting in the teapot phenomenon.

In Fig.9, during the later stages of fluid evolution, such as at $t=1.0$ and 1.5 seconds in the two sub-figures, we can observe that water droplets disperse into a larger space. This dispersion is a result of the interaction forces specified by Eq.44. In real life, when water flows from a height, it can exhibit a phenomenon known as branching diffusion, as seen in waterfalls. Without gravity, different water droplets are not pulled downward and are more prone to dispersion. If we were to employ more realistic interactions between water droplets instead of the form in Eq.44, we should be able to observe a more realistic evolution of the fluid. However, this would introduce new challenges, so for now, we will continue to study

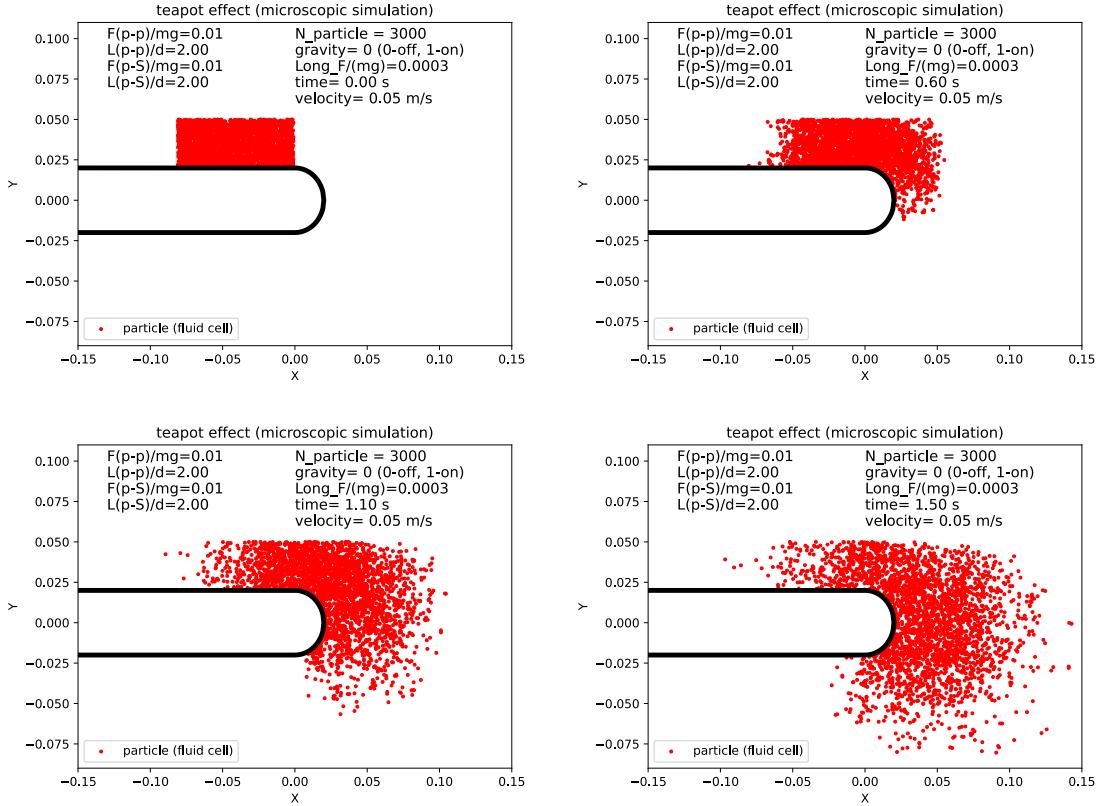


FIG. 9: Using the microscopic model we built to simulate the dynamical evolution of fluids over the teapot solid surface. There are repulsive and attractive forces between fluid cells. Besides, there are adhesive forces between the fluid and the solid surface (plotted with black solid lines). These microscopic forces between water droplets are taken as 10^{-2} of their gravity i.e. $F/(mg) = 10^{-2}$. For figures represent the fluid distributions at different times. The water droplets are plotted with red points, which move in the right direction with an initial velocity $u_0 = 0.05$ m/s. Gravity is not included yet.

the teapot effect based on the current model.

After considering the contribution of gravity, each fluid cell is influenced by gravitational acceleration, causing it to move to the lower-right direction. The trajectories of a large number of fluid cells at different time points are shown in Fig.10. This figure presents the fluid's behavior at $t=0.2$, 0.3 , 0.5 , and 1.0 seconds. It is evident that gravity, acting as an external force, causes all particles to move in the lower-right direction. This significantly alters the particle motion, making Fig.10 noticeably different from Fig.9. This underscores the substantial impact of gravity.

However, how can we roughly estimate the magnitude of the interaction forces between water droplets using gravity? When we assume that the coupling between fluid elements

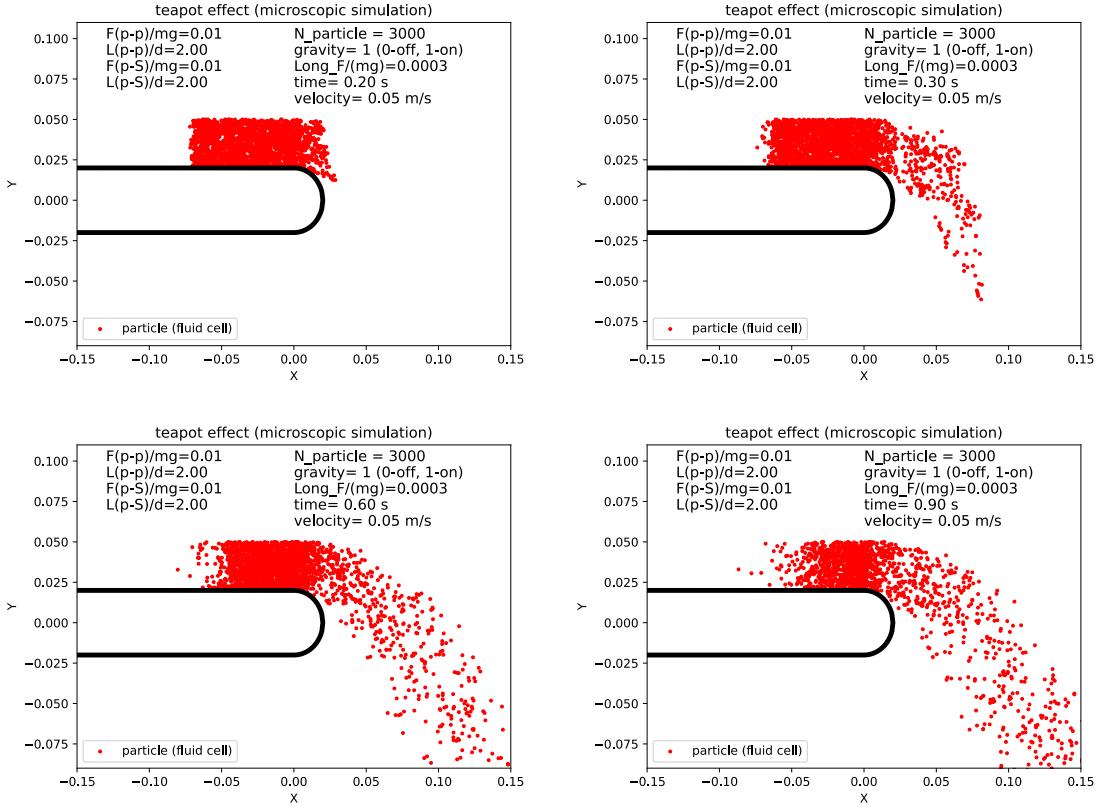


FIG. 10: Water dynamics at different times. Red points represent water droplets (fluid cells). All the parameters are the same with Fig.9, but considering gravity.

is exceptionally strong, essentially manually increasing the interaction forces between water droplets (Eq.44-45) by a factor of 10, satisfying $F/(mg) = 10^{-1}$, where mg represents the gravity acting on a water droplet (or fluid cell). It can be observed that when the repulsive and attractive forces within the fluid are exceedingly large, gravity behaves like a small perturbation on the motion of fluid elements. In this scenario, gravity does not exert a significant influence on the motion of fluid elements, as depicted in Fig.11.

Due to the consideration of gravity in both Fig.10 and Fig.11, the only difference lies in the form of fluid dispersion during flow. It can be observed that if gravity is relatively significant and cannot be ignored compared to the internal interactions within the fluid, then gravity can noticeably alter the fluid's motion pattern. However, conversely, if the microscopic interaction forces (including those between fluid elements and between the fluid and the teapot's solid surface) are very strong, the fluid's motion pattern remains relatively unaffected by gravity.

This intriguing phenomenon can actually be used as a rough estimate of the approximate

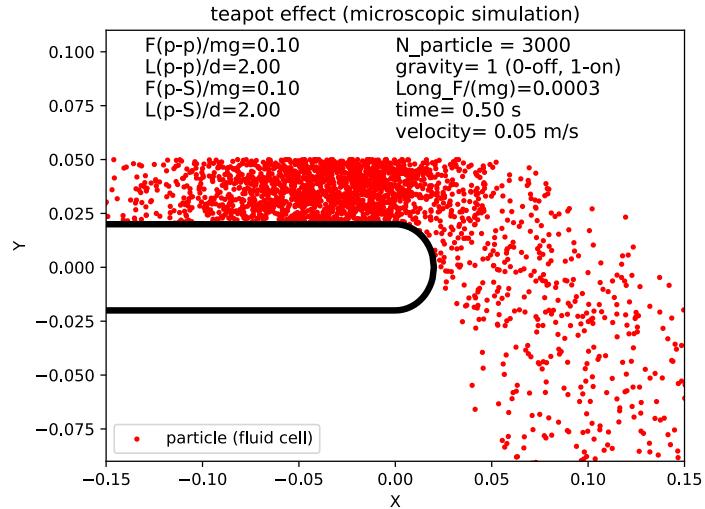


FIG. 11: The dynamical evolution of fluids over the teapot surface. A stronger coupling strength in the fluid is considered, where the interaction forces F_{c-c} , F_{c-s} are taken as 10^{-1} of the fluid gravity, i.e., $F/(mg) = 10^{-1}$, see Eq.(44-45).

magnitude of microscopic interaction forces. From our model calculations, it can be deduced that the interaction forces between water droplets within the fluid are approximately on the order of 10^{-2} times their own gravity, rather than being of the same magnitude. This has implications for understanding fluid mechanics at the microscopic level.

C. Ejection angle in the teapot effect

When a large number of particles move from the curved surface in a manner resembling the flow of water, the average positions of these water droplets can be calculated as follows. As shown in Fig.??, in each region, the position of the fluid can be considered as the average position of the particles in that region. For example, in a sector region $\theta_j \sim \theta_j + \Delta\theta$, where there are a total of N_j particles, and the position of each particle is denoted as $\vec{r}_i(\theta_j)$, the distance of the fluid from the center (the origin of coordinates) can be calculated as follows:

$$\langle r(\theta_j) \rangle = \frac{1}{N_j(\theta_j)} \sum_i |\vec{r}_i(\theta_j)| \quad (52)$$

In this way, we can calculate the "average position" in each region. When there is no attractive force from the teapot's solid surface, we can compute the average positions of the water flow, which represents the line connecting all these $\langle r(\theta_j) \rangle$ points. A schematic representation of these points is shown in Fig. 12.

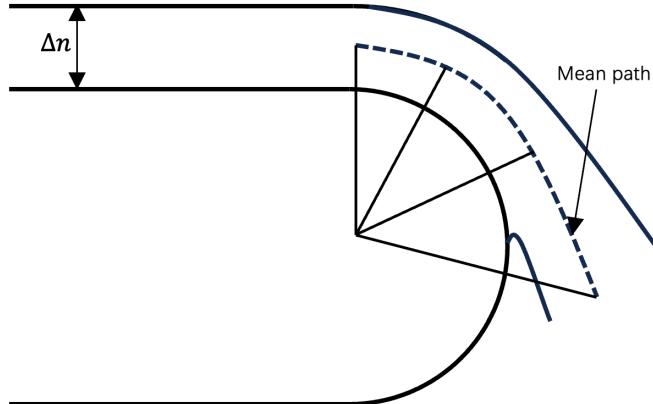


FIG. 12: When the fluid moves along the teapot surface, the fluid flows over the curved surface. We separate the fluid over the curved surface into many pieces, where the center of each piece can be calculated, shown as in the figure.

In my model, the motion of a large number of water droplets can be described. At a specific moment, we can obtain the positions of these water droplets, as shown in Fig.13. At each azimuthal angle on the curved surface, we can calculate the average position of particles in that region. By connecting the average positions at different azimuthal angles θ_j , we obtain the trajectory of the center of the water flow. In the figure, the blue dashed

line represents the trajectory assuming an extreme case where the water flow closely follows the teapot's solid surface. However, in reality, the water flow hesitates due to its initial velocity, leading it to move to the right. This causes water droplets to tend to depart from the solid surface. Still, due to the attractive force exerted by the solid surface, some water droplets may move along it. The average positions of a large number of water droplets are represented by the solid black line in Fig.13. This black line clearly shows a tendency to move away from the solid surface, but there is also a tendency to be drawn back to the solid surface in the lower-right area, which is due to the teapot surface's adhesive force on the water flow. It is evident that without the teapot surface's adhesive force, the water droplets, represented by the red points, would move to the right, away from the solid surface, under the influence of the initial velocity u_0 .

In Fig.13, to simplify the analysis, we temporarily neglect the influence of gravity. Therefore, the motion of water droplets (represented by the red points) is primarily affected by the interactions between water droplets and the interaction between water droplets and the teapot's solid surface. This simplification enhances the clarity of the analysis.

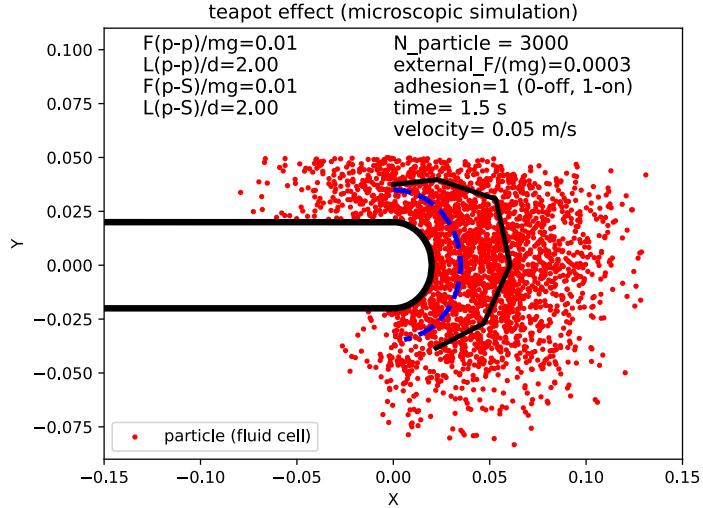


FIG. 13: The mean path of the fluid moving over the teapot surface. The blue dashed line is the mean path when fluid is most strongly coupled with the teapot's solid surface. The black solid line represents the mean path of the red points. As the red points move in the right direction, they also suffer forces from other red points and the solid surface, their position is shown in the figure. They tend to leave the solid surface. Gravity is not included to simplify the analyse.

Through the methods and equation Eq.(52) described above, we can quantitatively determine when water droplets are considered to depart from the solid surface or move along

it. Using this definition, we can introduce a concept known as the "ejection angle," which is the angle at which the fluid departs from the solid surface. In our microscopic model, this ejection angle can be reflected through the calculation of the average positions of particles.

D. Factors Influencing the Teapot Effect

The movement of water flow on the teapot's solid surface depends on several factors, including the coupling strength of water, the viscosity of water with respect to the teapot material. These physical quantities are reflected in the microscopic model as the magnitude and range of interaction forces between particles and between particles and the teapot surface. These factors represent the laws of the physical process. However, there are other factors that can influence fluid motion. One is the initial velocity of the water flow, and the other is the geometric shape of the teapot's spout.

When the initial velocities of the water flow are different, even if the water droplets experience the same forces at the initial time ($t=0$), their trajectories at later times ($t>0$) will differ. As a result, their distances from the teapot surface will also vary at time t . Specifically, when the initial velocity of the water flow is relatively high, the water flow may directly jet out without being significantly influenced by the solid surface's forces.

1. flow velocity:

Studying the influence of these factors on the teapot effect at a microscopic level is one of the main focuses of this research.

We can increase the initial velocities of the particles by a factor of 10, meaning that the average initial velocity of the particles is now $u_0 = 0.5 \text{ m/s}$, horizontally to the right. In our simulation, we provide the spatial distribution of particles at different times. To accurately consider the evolution of particles, we also introduce gravity, as shown in Fig.14. It can be observed that when the particles reach the curved region, their tendency to move to the right is significant due to their high velocity, and the adhesive force from the teapot's surface is not sufficient to significantly alter the fluid's motion. As a result, the red points continue moving to the right. In later times, the influence of gravity causes these particles to fall to the right and downward. Without gravity, these particles might continue moving to the right (close to horizontal motion), rather than flowing along the teapot's surface, as shown in Fig.9.

Therefore, by comparing these figures: Fig.14 and Fig.9-10, it can be observed that the initial velocity of the fluid is crucial for its later evolution. This observation aligns with our

everyday observations. If the initial velocity of the water flow is very low, it may not exhibit the teapot effect as prominently, and the flow behavior will differ.

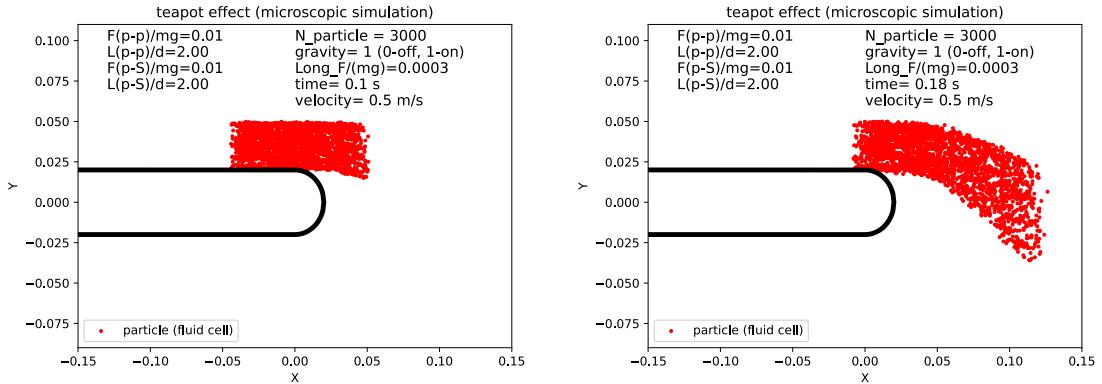


FIG. 14: Fluid with a large initial velocity $u_0 = 0.5$ m/s. Parameters are plotted in the figure. The fluid tend to eject from the teapot surface, without the tendency to flow over the solid surface.

2. Geometry of teapot mouse

In everyday life, when the thickness of the teapot's spout varies, the behavior of water flowing out also differs. This is because the shape of the teapot's spout affects the flow trajectory of the water passing through it. Consequently, the distance between water droplets and the teapot's surface, as well as the adhesive force between them, varies. To study the impact of different teapot spout shapes on fluid evolution, we can change the shape of the teapot spout.

In Fig.15, the radius of the teapot spout is doubled, making the teapot spout thickness $R = 4$ cm. When the teapot spout becomes thicker, some particles that move to the right and downward still remain close to the spout's solid surface, and thus, they experience the adhesive force from the solid surface. This makes it easier for some water droplets to flow along the solid surface. Conversely, when the solid surface is thinner, water droplets can quickly move away from the teapot spout's surface upon departure, and they may not experience the adhesive force of the solid surface. Through our simulation, it can be seen that when the teapot spout thickness is increased, some water molecules are more likely

to be adsorbed to the solid surface. This is evident in Fig.15 (teapot spout with a larger radius).

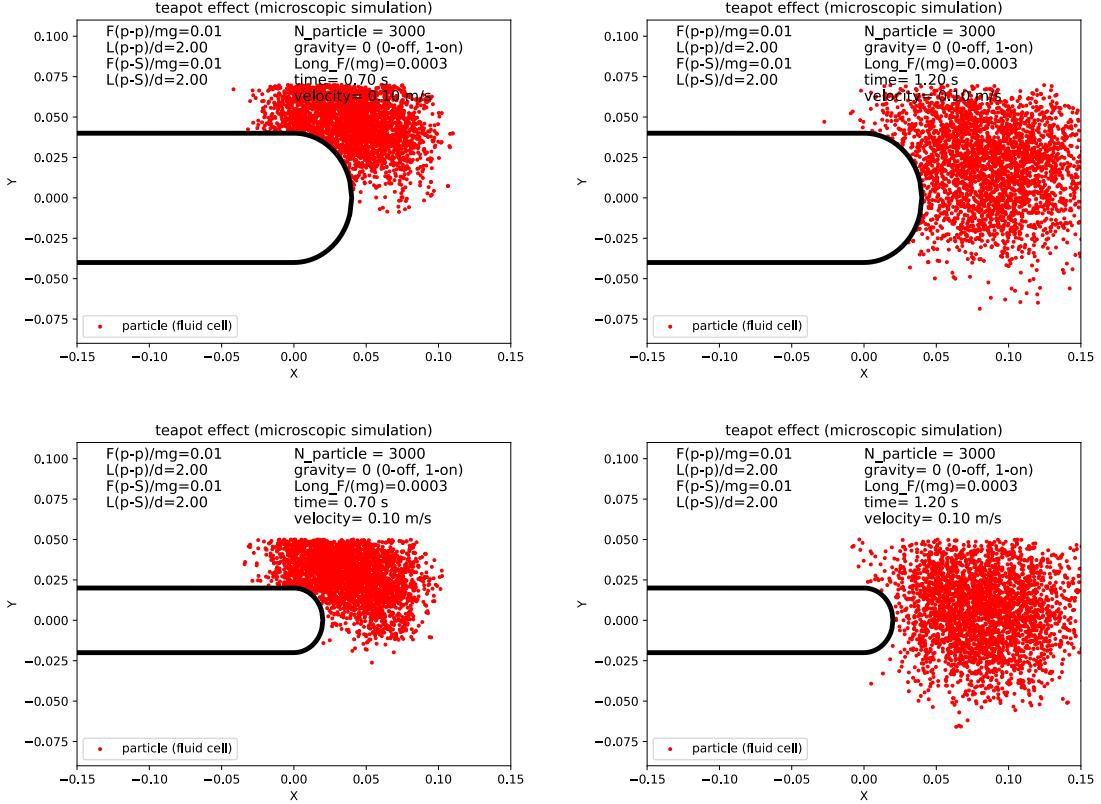


FIG. 15: Fluid with a large initial velocity $u_0 = 0.1 \text{ m/s}$. Parameters are plotted in the figure.

In certain situations, if the teapot spout is not circular but has a notch or rectangular shape, the behavior of the water flow can be different. For example, in the extreme case where the teapot spout has been cut with a knife, forming a rectangular shape, it is evident that in such a situation, the distance between water droplets and the solid surface will change, and the adhesive force experienced by water droplets from the solid surface will also differ.

We use fluids with different velocities to study their flow on the solid surface with this sudden change in teapot spout shape. It can be observed that when the flow velocity is lower, the abrupt change in the teapot spout shape makes it easier for the fluid to move away from the solid surface. Consequently, water droplets are less likely to drip along the solid surface.

3. Broken mouse of the teapot

Based on this model, we also studied the flow behavior when the teapot spout is damaged. When the teapot spout is damaged, the interaction between the water and the teapot surface changes. Due to the inertia of the flowing water, it continues to move forward. If the shape of the teapot spout suddenly changes, altering the distance between the solid surface and the water molecules, the interaction between the solid surface and the water flow can change or even suddenly disappear. This means that the water droplets near the solid surface, when they reach the damaged part of the teapot, will no longer experience adhesive force from the solid surface. This allows water droplets to be attracted by other nearby water droplets and continue moving forward, without leaving a trail along the teapot's solid surface. This implies that the teapot effect is less likely to occur.

As shown in Fig.16, even at low water flow speeds, such as 0.05 m/s, water droplets clearly move to the lower right (as indicated by the solid lines showing their movement trends). When the water flow increases to 0.1 m/s, the tendency for the water flow to move to the lower right becomes more pronounced (as shown in the fourth subfigure). The sightlines in the second and fourth subfigures represent the movement trends of the water droplets, while the dashed lines represent the movement trajectory when the water flow completely adheres to the teapot surface.

In the figure, we assume that the upper half of the teapot spout has a regular notch, which is convenient for modeling. When the water droplets reach the notch, the original notch is circular, and the water droplets can come into contact with the solid surface and experience adhesive force from the teapot spout. However, in the presence of the notch, at the notch, the adhesive force on the water droplets suddenly decreases, allowing the water flow to continue moving forward. The key difference between this Fig.16 and the previous figure (Fig.15), where the teapot spout was intact, is that in the former, the water droplets move to the lower right, while in the latter, many water droplets move downward due to the adhesive force from the teapot spout. This is a significant contrast.

In future research, we can even plot the velocity of each region, i.e., $u(x, y, t)$, to clearly demonstrate the movement of water droplets in different regions or study the nature of turbulence.

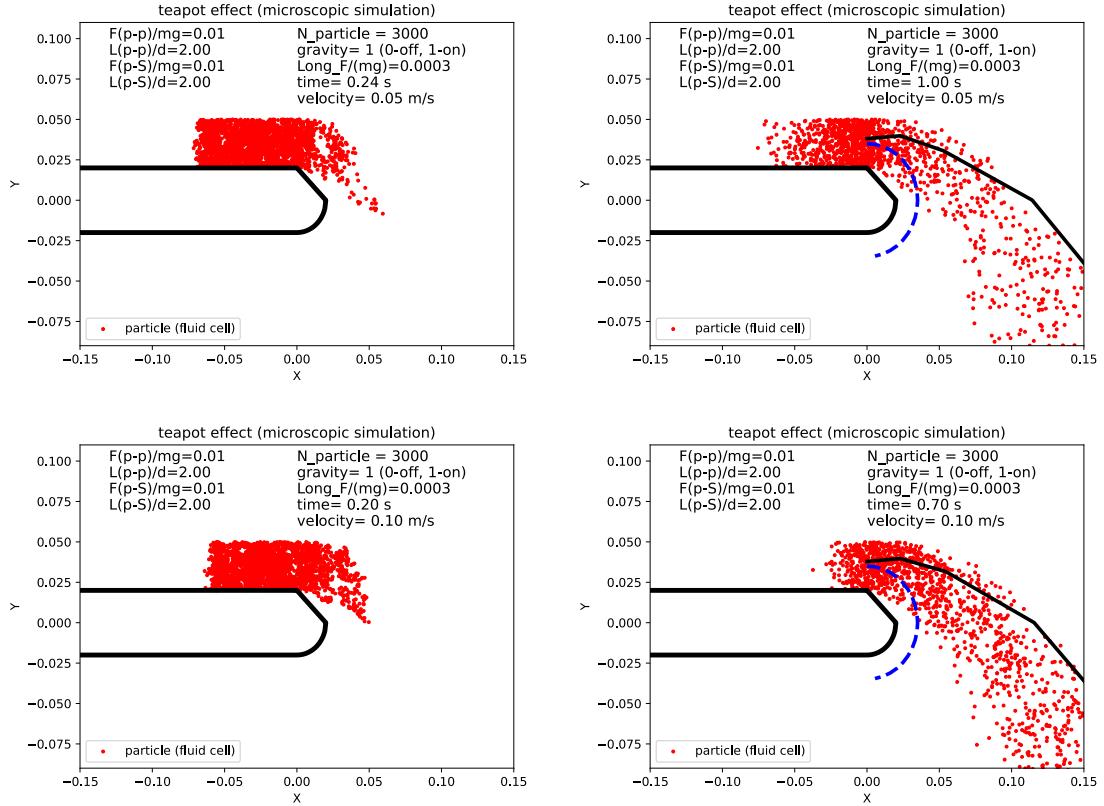


FIG. 16: Water moves above the broken teapot mouse. Assume that the damaged part of the teapot spout is in a regular shape. In the upper sub-figures, fluid moves with a velocity $u = 0.05$ m/s. While in the lower sub-figures, the fluid velocity is $u = 0.1$ m/s. Solid lines represent the moving tendency of the fluid. The dashed line is the moving tendency when the fluid is mostly attached to the solid surface of the teapot mouse.

V. WEAKNESS OF OUR MODEL

(1) The expressions for the interactions between fluid cells and fluid-teapot are rather coarse.

(2) When more fluid is considered, at the far left end of the fluid, those fluid elements will experience collisions from the left, representing pressure from left to right, encouraging the fluid to flow down more quickly. In the simulations described above, 3000 fluid cells were used, allowing the right end of the fluid to flow more realistically. However, to account for two-body interactions between 3000 fluid cells, the number of calculations required at each time step is on the order of $3000^2 \sim 10^7$ two-body interactions. This is time-consuming, and if more fluid cells are to be considered, it would necessitate the use of large-scale computing resources.

Some fluid cells at the far left end of the fluid appear to have a weakened tendency to flow to the right. This is because these fluid cells experience collisions (pressure) from the right due to the presence of fluid cells on the right side, while there is no fluid on the left, preventing the fluid cells on the far left from moving to the right more rapidly.

These issues can be addressed and improved by utilizing greater computational resources.

VI. SUMMARY

In daily life, when water flows out of a teapot, it can either detach from the teapot or flow down along the teapot's spout surface. This phenomenon is commonly referred to as the "teapot effect." Over the past few decades, this phenomenon has been extensively studied, and its underlying principles have been explored through fluid mechanics. However, the teapot effect presents a challenge due to complex boundary conditions and the difficulty of solving fluid dynamics equations, making it a challenging problem.

In this work, we take a different approach by developing a microscopic model inspired by the finite element method, where we divide the fluid into a multitude of discrete water droplets. Each water droplet interacts with its neighboring droplets and interacts with the teapot surface. Consequently, the complex evolution of the fluid can be simulated computationally by tracking the motion of these discrete water droplets. By establishing this mathematical model and implementing it through computer programming, we can effectively study the collective motion of a large number of water droplets. The advantage of this model lies in our ability to investigate interactions at the microscopic level, including the forces between water droplets and even intermolecular forces, which are beyond the reach of macroscopic fluid mechanics.

Furthermore, the microscopic particle collision model we have developed, initially designed for simulating incompressible fluids like water, can naturally be extended to simulate compressible fluids. Considering the significant challenges in solving compressible fluid dynamics equations, our microscopic particle collision model offers a promising avenue for broader applications.

To simplify the problem, we abstract the 3D fluid flow into a 2D scenario. Through the microscopic particle collision model, we investigate the flow of water on the teapot spout surface. We address several aspects:

1. By considering the gravitational force on the water droplets as a reference, we study that the interaction forces between the droplets should be significantly smaller than their own gravity, providing valuable insights into the magnitude of forces at the microscopic level.
2. Based on the model, we examine how the flow of fluid differs on the teapot spout surface at varying fluid velocities, including scenarios where it jets out or flows downward

along the solid surface.

3. Given that the flow of water depends on the interaction forces with the teapot spout's solid surface, we investigate the impact of different solid surface shapes on the fluid's evolution. Complex boundary conditions in fluid mechanics are known to pose significant challenges, but our model allows for the inclusion of various boundary conditions through programming. We examine cases with thin, thick, and rectangular teapot spouts. Additionally, we calculate physical quantities such as the average mean free path and viscosity coefficient for the particle system, which can serve as valuable reference points for comparison with real-world fluid properties, verifying the reliability of our model.

It's important to acknowledge that the current framework employs a simplified version of microscopic interactions between particles, which plays a crucial role in the fluid's evolution. In future research, this work can be extended to consider more realistic forms of intermolecular forces to further explore fluid dynamics.

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VIII. APPENDIX: CODES

A. codes for momentum balance method

```
1 rho = 997;
2 g = 9.8;
3 gamma = 0.078;
4 W = 0.0834;
5 theta = ArcCos[(W - gamma)/gamma];
6 r = 0.001;
7 h = r*Cos[Pi - psi0];
8 n = 0.01;
9 R = r + n/2;
10 c = rho*(gamma*We/rho + g*h*R)/(R*gamma);
11 psimen = Sqrt[2*c^(-1)*(1 + Cos[theta])/r];
12
13 plot = ContourPlot[
14   We*(1 - Sin[psi0])/(psi0 + psimen) == We*r*c + 2*r*rho*g*h/gamma,
15   {We, 0, 1.5}, {psi0, 0, 2*Pi/3},
16   FrameLabel -> {"We", "psi0"}]
```

B. codes for microscopic model

The main codes simulating fluid cell evolution are listed below. (Not all the codes are listed).

```
1
2 % python
3 import os
4 import subprocess
5 import random
6 import math
```

```

7 import numpy as np
8 import matplotlib.pyplot as plt
9
10 import frame_tea
11 import interaction
12 import centerline
13
14 sig_gravity = 1      #1: turn on gravity,  0: turn off gravity
15
16 sig_surface_attraction = 1
17 if sig_surface_attraction ==0:
18     print('turn off surface attraction !!\n')
19
20
21 e0_thick = 0.03  # in: m
22 time_period = 2.20  #10
23 time_step = 0.01  #in s
24
25 radius_teapot = 0.02  #in: m
26 init_mean_vx = 0.05  #in: m/s
27 init_mean_vy = init_mean_vx*0.01 #0.005    #in: m/s      #introduce some
28     random evolution in the y-direction, to avoid non-physical
29     separation between layers.
30
31 it_select = 100
32 list_x_section=[]
33 list_y_section=[]
34 length_fluid = 0.08  # in: m
35 section_range = 0.02  # we divide total area into many sections, and

```

```

    assume only particles within same section can interact, instead of
    calculating interactions with all particles. This simplification
    will significantly save CPU time.

35
36 particle_count = 3000
37 typical_distance = math.sqrt( (length_fluid*e0_thick)/particle_count )
38
39 rho_fluid = 1.0
40 mass = rho_fluid*(length_fluid*e0_thick)/particle_count      #mass of one
        cell (one particle), rho*V/N=mass_eachCell
41
42 N_Jump = 2
43 file_names=[]
44 particles = []
45
46 output_folder = "./figs/"
47
48 print('e0_thick=', e0_thick, 'typical_distance=',typical_distance, '
        init_vx=',init_mean_vx, 'radius_teapot=',radius_teapot)
49 print('time_period=', time_period)
50 print('total particles=', particle_count, 'each particle m*g=', mass
        *10)
51
52
53 with open('./result/parameters.dat', 'w') as f:
54     f.write('particle_num='+str(particle_count)+'\n'+radius='+
              str(radius_teapot)+ '\n' +e0_thick='+
              str(e0_thick)+ '\n' +typical_length='+
              str(typical_distance)+ '\n'+mass_of_particle='+
              str(mass)+ '\n' +init_vx='+
              str(init_mean_vx) )
55

```

```

56 def delete_pdf_files():
57     #delete previous pdf files firstly,
58     pdf_files = [f for f in os.listdir(output_folder) if f.endswith('.pdf')]
59
60     for file in pdf_files:
61         file_path = os.path.abspath(file) #
62
63         if os.path.exists(file_path): #
64             os.remove(file_path) #
65             print(f"File {file} deleted successfully.")
66
67
68
69 #
70 class Particle:
71     def __init__(self, particle_id, position_x, position_y, velocity_x,
72                  velocity_y):
73
74         self.position_x = position_x
75
76         self.position_y = position_y
77
78         self.velocity_x = velocity_x
79
80         self.velocity_y = velocity_y
81
82
83         self.particle_ID= particle_id
84
85         self.particle_section = int(position_x/section_range)      #this
86
87         label will allow particles with the same particle_section
88         can interact with each other. This setup is very helpful
89         and saves computing time.

```

```

81     def update_position(self, time_step, a_x, a_y):
82
83         self.position_x += self.velocity_x * time_step
84
85         self.position_y += self.velocity_y * time_step
86
87
88     def output_setup():
89
90         print('fluid_length=' , fluid_length, '\n')
91
92         print('particle number=' , paerticle_count, '\n')
93
94         print('time_step, and period=' , time_step, time_period, '\n')
95
96
97     def collision_with_surface(j):
98
99         x = particles[j].position_x
100
101        y = particles[j].position_y
102
103        vx = particles[j].velocity_x
104
105        vy = particles[j].velocity_y
106
107
108        if x<0:
109
110            if y<radius_teapot and y>0:
111
112                particles[j].position_y = radius_teapot + np.random.rand()
113
114                *0.1*typical_distance #this np.random.rand() will help
115
116                to avoid putting particles ``on'' the surface of the
117
118                solid, which may results in not-good interactions
119
120                between particles and the solid surface.
121
122                particles[j].velocity_y = abs(particles[j].velocity_y) #
123
124                    particle's velocity vy towards +y direction
125
126            if y<0 and y >-1*radius_teapot:
127
128                particles[j].position_y = -1*radius_teapot - np.random.rand()

```

```

        ()*0.1*typical_distance

106    particles[j].velocity_y = -1*abs(particles[j].velocity_y)

107

108 elif x>0:

109     distance = math.sqrt(particles[j].position_x*particles[j].
110                           position_x + particles[j].position_y*particles[j].position_y)

111     if distance <radius_teapot:

112         particles[j].position_x = (radius_teapot+np.random.rand()
113                               *0.1*typical_distance) * particles[j].position_x/
114                               distance #the term np.random.rand()*0.5*
115                               typical_distance means put particles next to the solid
116                               surface within a small distance, instead of putting ``on
117                               '' the teapot surface. As putting exactly on the surface
118                               will cause difficultly in calculating adhesive force.

119     particles[j].position_y = (radius_teapot +np.random.rand()
120                               *0.1*typical_distance)* particles[j].position_y/distance

121

122     # particles can not jump upward

123     if particles[j].position_y > e0_thick+radius_teapot:
124
125         particles[j].position_y = e0_thick + radius_teapot - np.random.
126                               rand()*0.1*typical_distance
127
128         particles[j].velocity_y = -1.0*abs(particles[j].velocity_y)

129

130 def g_contribution_fun(x, y):
131
132     combined_acc = 0

133

134     if x<=0 and y>0:
135
136         gx = 0
137
138         gy = 0

```

```

126     elif x<=0 and y<0:
127         gx = 0
128         gy = -1.0*interaction.g_gravity
129     elif x>0 and y>0:
130         combined_acc = interaction.g_gravity * x/math.sqrt(x*x + y*y)
131         gx = combined_acc * y/math.sqrt(x*x + y*y)
132         gy = combined_acc * x/math.sqrt(x*x + y*y)* (-1.0)
133     elif x>0 and y<=0:
134         gx = 0.0
135         gy = interaction.g_gravity*(-1.0)
136     else:
137         gx = 0.0
138         gy = 0.0
139
140     return gx, gy
141
142
143 #
144 def simulate_particles(particles, time_steps, filename):
145
146     for i in range(time_steps):
147
148         if i== it_select :          #or i==50 or i==100:
149             global list_x_section, list_y_section
150             list_x_section, list_y_section = centerline.
151                         calculate_center_line(particles, i, sig_gravity)
152
153         if i%N_Jump==0:
154             print('running time step: i=',i,'with total steps=',
155                  time_steps)

```

```

154
155     onestep_file = "./result/"+"step_" + str(i) +"_info.dat"
156     file_names.append(onestep_file)
157
158     with open(onestep_file, 'w') as f:
159
160         f.write("Time,Particle_id,Position_x, position_y, vx, vy\n")
161         )
162
163         for j, particle in enumerate(particles): #particles is a
164             list of the class/object, each element in the ''165
166             particles'' is an object(class) representing one
167             particle.
168
169             a_x, a_y = getting_acceleration(j, particle.position_x,
170                                             particle.position_y)
171
172             gx = 0
173             gy = 0
174
175             if sig_gravity ==1:
176
177                 gx, gy = g_contribution_fun(particle.position_x,
178                                              particle.position_y)
179
180                 #interaction.g_gravity*(-1.0)    #gravity is in -y
181
182                 direction
183
184
185                 #
186
187                 f.write(f"{i+1}\t{particle.particle_ID:d}\t{particle.

```

```

position_x:.6f}\t{particle.position_y:.6f}\t{  

particle.velocity_x:.6f}\t{particle.velocity_y:.6f}\t{  

n")  

177  

178     #  

179     particle.update_position(time_step, a_x, a_y)  

180  

181  

182 def getting_acceleration(particle_id, position_x, position_y):  

183  

184     sum_Fx = 0  

185     sum_Fy = 0  

186     for i in range(len(particles)):  

187  

188         if i == particle_id:  

189             continue  

190         else :  

191             if particles[i].particle_section == particles[particle_id].  

192                 particle_section:  

193                     Fx, Fy = interaction.get_cell_cell_force(particle_id,  

194                         mass, typical_distance, position_x, position_y,  

195                         particles[i].position_x, particles[i].position_y)  

196                     sum_Fx += Fx  

197                     sum_Fy += Fy  

198                     #only when particle-i is located in the same big-cell  

#                     of particle-id, they can interact. This will save  

#                     computing time.  

199  

200         cell_solid_Fx = 0  

201         cell_solid_Fy = 0

```

```

199     if sig_surface_attraction ==1:
200
201         cell_solid_Fx, cell_solid_Fy = interaction.
202             get_cell_teapot_force(position_x, position_y, mass,
203             typical_distance, e0_thick, radius_teapot)
204
205         sum_Fx += cell_solid_Fx
206         sum_Fy += cell_solid_Fy
207
208         #check if collide with surface
209         collision_with_surface(particle_id)
210
211     return a_x, a_y
212
213
214
215 # .      dat
216 def plot_figures(tot_steps):
217
218     for i in range(tot_steps):
219
220         if i%N_Jump ==0:
221
222             data = np.loadtxt(file_names[i], skiprows=1)
223
224             #
225             x = data[:, 2]
226             y = data[:, 3]

```

```

227     plt.figure()
228
229     #
230
231     plt.scatter(x, y, marker='o', color='red', s=5)
232     plt.xlabel('X')
233     plt.ylabel('Y')
234     plt.title('teapot effect (microscopic simulation)')
235
236
237     if i == it_select:
238
239         for kk in range(20):
240
241             list_mean_centerx.append( (0.5*e0_thick+
242                                         radius_teapot)*math.sin(kk*3.14/20) )
243
244             list_mean_centery.append( (0.5*e0_thick+
245                                         radius_teapot)*math.cos(kk*3.14/20) )
246
247             plt.plot(list_x_section, list_y_section, color='black',
248                     linewidth=3)
249
250             plt.plot(list_mean_centerx, list_mean_centery, color='
251                     blue', linestyle='--', linewidth=3)
252
253             #
254
255             time = i*time_step
256
257             plt.legend(['particle (fluid cell)'], loc='lower left')
258             plt.text(0.0, 0.10, f'N_particle = {particle_count}',
259                     fontsize=12)
260             plt.text(0.0, 0.09, f'gravity= {sig_gravity} (0-off, 1-on)'
261                     , fontsize=12)
262             plt.text(0.0, 0.08, f'Long_F/(mg)={interaction.

```

```

    parameter_force.global_mode:.4f}', fontsize=12)

251 #plt.text(0.0, 0.08, f'adhesion={sig_surface_attraction}
252             (0-off, 1-on)}', fontsize=12)

253 plt.text(0.0, 0.07, f'time= {time:.2f} s', fontsize=12)
254 plt.text(0.0, 0.06, f'velocity= {init_mean_vx:.2f} m/s',
255           fontsize=12)

256 plt.text(-0.13, 0.10, f'F(p-p)/mg={interaction.
257             parameter_force.F_cell_cell:.2f}', fontsize=12)
258 plt.text(-0.13, 0.09, f'L(p-p)/d={interaction.
259             parameter_force.L_cell_cell:.2f}', fontsize=12)

260 plt.text(-0.13, 0.08, f'F(p-S)/mg={interaction.
261             parameter_force.F_cell_surf:.2f}', fontsize=12)
262 plt.text(-0.13, 0.07, f'L(p-S)/d={interaction.
263             parameter_force.L_cell_surf:.2f}', fontsize=12)

264 #plot frame of the teapot solid surface
265 plt.plot(frame_tea.datalist_xframe, frame_tea.
266           datalist_yframe, color='black', linewidth=4)

267 #      15
268 plt.xlim(-0.15, 0.15)

269 #      012
270 #plt.ylim(-(e0_thick+radius_teapot), (e0_thick+
271           radius_teapot)*2 )

```

```

272     plt.ylim(-0.09, 0.11 )
273
274     #      PDF
275
276     #plt.savefig('fig_evolve_'+str(i)+'_step.pdf')
277     plt.savefig(os.path.join(output_folder, 'fig_evolve_'+str(i)
278                             +'_step.pdf'))
279
280
281 #
282 def main():
283     frame_tea.generate_data(e0_thick, radius_teapot)
284
285     evolve_steps = int(time_period/time_step)
286
287     #delete_pdf_files()      #delete previous old pdf figures
288     subprocess.run('rm ./figs/*.pdf', shell=True, text=True)
289     subprocess.run('rm ./result/*.dat', shell=True, text=True)
290
291     for i in range(particle_count):
292         position_x = random.uniform(-length_fluid, 0)-(time_step*2*
293
294             init_mean_vx)  #      , randomly distributed in x:-0.5,0 and
295
296             y:
297
298         position_y = random.uniform(radius_teapot, radius_teapot+
299
300             e0_thick)  #
301
302
303         #initial velocity vx=0.2, vy=0
304
305         velocity_x = random.uniform(0.90, 1.1) *init_mean_vx    #

```

```

298     velocity_y = random.uniform(-0.5, 0.5) *init_mean_vy
299
300     particle = Particle(i, position_x, position_y, velocity_x,
301                           velocity_y)
302     particles.append(particle)
303
304     print('starting simulation..\n')
305 #
306 #simulate_particles(particles, evolve_steps, "particle_data.csv")
307 simulate_particles(particles, evolve_steps, "particle_data.dat")
308
309 plot_figures(evolve_steps)
310
311
312 if __name__ == "__main__":
313     main()

```

Listing 1: Python code