Non-conservative form of shallow water equation

System of equation
$$\begin{cases} \dfrac{\partial h}{\partial t} + H\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right) = 0 \\ \dfrac{\partial u}{\partial t} - fv = -g\dfrac{\partial h}{\partial x} - bu \\ \dfrac{\partial v}{\partial t} + fu = -g\dfrac{\partial h}{\partial y} - bv \end{cases}$$

$$\frac{\partial h}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$\frac{\partial u}{\partial t} + g\frac{\partial h}{\partial x} + bu - fv = 0$$
$$\frac{\partial v}{\partial t} + g\frac{\partial h}{\partial y} + bv + fu = 0$$

$$\underbrace{\frac{\partial}{\partial t}\begin{bmatrix} h \\ u \\ v \end{bmatrix}}_{h} + \underbrace{\frac{\partial}{\partial x}\begin{bmatrix} u \\ gh \\ 0 \end{bmatrix}}_{U} + \underbrace{\frac{\partial}{\partial y}\begin{bmatrix} v \\ 0 \\ gh \end{bmatrix}}_{V} + \underbrace{\begin{bmatrix} 0 \\ bu-fv \\ bv+fu \end{bmatrix}}_{b} = 0$$

$$= \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \qquad = \begin{bmatrix} h_2 \\ gh_1 \\ 0 \end{bmatrix} \qquad = \begin{bmatrix} h_3 \\ 0 \\ gh_1 \end{bmatrix} \qquad = \begin{bmatrix} 0 \\ bh_2 - fh_3 \\ bh_3 + fh_2 \end{bmatrix}$$

$$\frac{h_{i,j}^{n+1} - h_{i,j}^{n}}{\Delta t} = -\frac{u_{i+1,j}^{n} - u_{i-1,j}^{n}}{2\Delta x} - \frac{v_{i,j+1}^{n} - v_{i,j-1}^{n}}{2\Delta y} - b$$

$$\Rightarrow h_{i,j}^{n+1} = h_{i,j}^{n} - \Delta t\left(\frac{u_{i+1,j}^{n} - u_{i-1,j}^{n}}{2\Delta x} + \frac{v_{i,j+1}^{n} - v_{i,j-1}^{n}}{2\Delta y}\right) - b$$