Exercice 4

Calculer les transformées de Fourier des signaux suivants :

$$\Delta(X)=\prod(X)\otimes\prod(X)$$

$$\frac{\sin(\pi x)}{\pi x}$$

Calculer :

$$\int\left[\frac{\sin(\pi x)}{\pi x}\right]^n dx \quad avec\ n=2,3,4..$$

Solutions :

Solutions

$$\mathscr{F}(\Delta(x)) = \mathscr{F}(\Pi(x) \otimes \Pi(x)) = \mathscr{F}(\Pi(x)).\mathscr{F}(\Pi(x))$$

$$= \frac{\sin(\pi f)}{\pi f}.\frac{\sin(\pi f)}{\pi f}$$

$$\boxed{= \left(\frac{\sin(\pi f)}{\pi f}\right)^2}$$

$$\mathscr{F}\left(\frac{\sin(\pi x)}{\pi x}\right) = \int \frac{\sin(\pi x)}{\pi x} e^{-2\pi j f x}\, dx = X(f)$$

$$X(-f) = \int \frac{\sin(\pi x)}{\pi x} e^{2\pi j f x}\, dx = \mathscr{F}^{-1}\left[\frac{\sin(\pi x)}{\pi x}\right] = \Pi(-f) = \Pi(f)$$

ensem

$$\int\left[\frac{\sin(\pi x)}{\pi x}\right]^n dx \quad avec\; n=2,3,4..$$

n=2

$$\int\left[\frac{\sin(\pi x)}{\pi x}\right]^2 dx=\int_{-0.5}^{+0.5}(\Pi(f))^2 df=1$$

aide

n=3

$$\int\left[\frac{\sin(\pi x)}{\pi x}\right]^3 dx=\int\left[\frac{\sin(\pi x)}{\pi x}\right]^2 .\left(\frac{\sin(\pi x)}{\pi x}\right)dx=\int\left[\mathscr{F}\left(\frac{\sin(\pi x)}{\pi x}\right)\right]^2 .(\Pi(f))df$$

$$=\int\Pi(f)\otimes\Pi(f).\Pi(f)df=\int\Delta(f)df=3/4$$

n=4

$$\int\left[\frac{\sin(\pi x)}{\pi x}\right]^4 dx=\int\left[\frac{\sin(\pi x)}{\pi x}\right]^2 .\left(\frac{\sin(\pi x)}{\pi x}\right)^2 dx=\int\Delta(f).\Delta(f)df=\int(\Delta(f))^2 df$$

$$=2\int_0^1(-f+1)^2 df=2\left[\frac{-(1-f)^3}{3}\right]_0^1=2/3$$

ensem

C’est une application directe du théorème de Parseval

$$\int \mathrm{f}(x)\overline{\mathrm{g}(x)}dx = \int \widehat{\mathrm{f}}(f)\overline{\widehat{\mathrm{g}}(f)}df$$

$$\int \left|\mathrm{f}(x)\right|^2 = \int \left|\widehat{\mathrm{f}}(f)\right|^2 df$$

Retour