Introduction

Types of learning

1. Learning with a teacher: supervised learning.
2. Learning with a critic: reinforcement learning.
3. Learning on your own: unsupervised learning.

- How we study learning in neural networks:

Classification

Input vectors $\mathbf{x}$ associated with output vectors $\mathbf{y}$.
Learn mapping: $\mathbf{x} \Rightarrow \mathbf{y}$.
Generalise to data not seen during learning. (“Training set” vs “test set” and also “validation set”).

Approaches to classification

1. Logistic regression (binary outputs). Applied Statistics.
2. Naive Bayes. Machine Learning / probablistic modelling.
3. Multi-layer perceptron. Neural networks part I.
4. Support vector machines. Kernel methods.
5. Decision Trees and Forests.
6. Neural networks part II.

The perceptron (Rosenblatt 1957)

Notation:

- x_i: activity of input unit i (binary or [0,1]).
- w_i synaptic weight from unit i.
- $z = \sum_i w_i x_i$ total input to the output unit.
- y: activity of output unit.
- t: desired output (of use later). (μ superscript denotes training sample.)
- $f(\cdot)$: transfer function; $y = f(z)$.

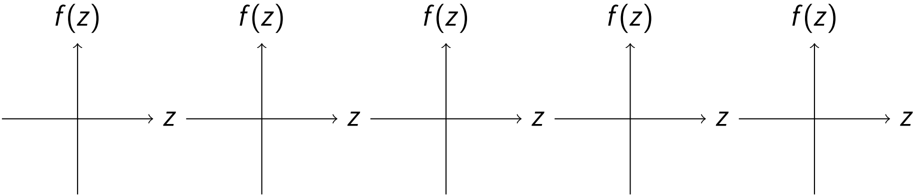
Training set

Sample	x_1	x_2	t
$\mu = 1$	0	0	0
$\mu = 2$	0	1	0
$\mu = 3$	1	0	0
$\mu = 4$	1	1	1

Transfer function

Given total weighted input to neuron, what is its output?

1. identity: $f(z) = z$.
2. threshold: $f(z, \theta) = \begin{cases} 1 & \text{if } z \geq \theta \\ 0 & \text{otherwise} \end{cases}$
3. sigmoidal: $f(z) = \frac{1}{1+\exp(-kz)}$
4. tanh: $f(z) = tanh(z)$
5. rectified linear unit (ReLU): $f(z) = max(0, z)$



How to choose? One key property: differentiable.

Perceptron decisions

Whether a perceptrons output is 0 or 1 depends on whether $\sum_{i=1}^{N} w_i x_i$ is less or greater than θ.

The equation

$$\sum_{i=1}^{N} w_i x_i = \theta$$

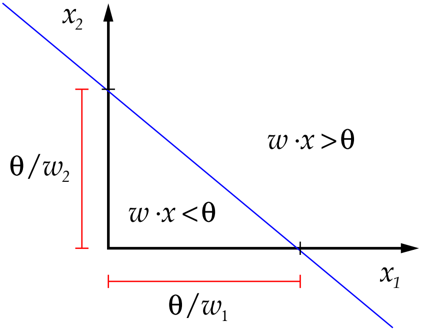
defines a hyperplane in N-dimensional space.

This hyperplane cuts the space in two.

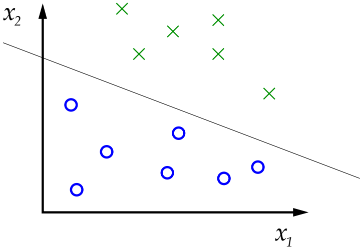
$$w_1 x_1 + w_2 x_2 = \theta$$

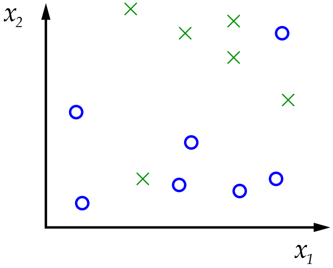
$$x_2 = \left(\frac{-w_1}{w_2}\right) x_1 + \frac{\theta}{w_2}$$

This is an equation for a straight line.



Linearly separable problems and the perceptron

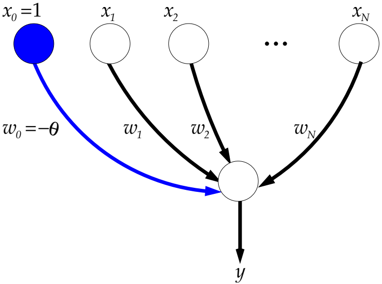




Learning involves adjusting the values of w and θ so that the decision plane can correctly divide the two classes.

Threshold & Bias

The threshold, θ, can be treated as just another weight from a new input unit which always has a value of +1 (or -1). This new input unit is called the **bias** unit.

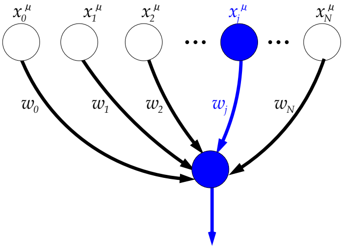


Instead of comparing $\sum_{i=1}^{N} w_i x_i$ with θ, we compare $\sum_{i=0}^{N} w_i x_i$ with 0, or

$$y = \begin{cases} 1 & \text{if } \sum_{i=0}^{N} w_i x_i > 0, \\ 0 & \text{if } \sum_{i=0}^{N} w_i x_i \leqslant 0. \end{cases}$$

Now, learning is about jiggling **weights** only.

Intuitively... (positive valued inputs)



$$y^\mu = \text{step}\left(\sum_{i=0}^{N} w_i x_i^\mu\right)$$

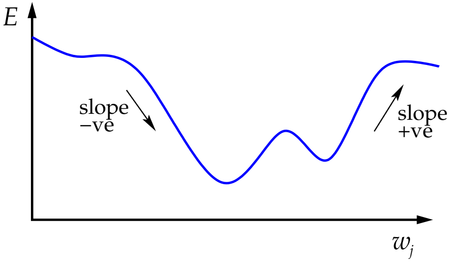
Consider w_j's contribution to $\sum_i w_i x_i^\mu$ in different cases:

1. $y^\mu = t^\mu$ Perceptron has classified input μ correctly – change nothing
2. $x_j^\mu = 0$ Changing w_j will not affect the $\sum_i w_i x_i^\mu$ – change nothing
3. $x_j^\mu \neq 0, y^\mu < t^\mu$ The sum $\sum_i w_i x_i^\mu$ is too low – so increase it
4. $x_j^\mu \neq 0, y^\mu > t^\mu$ The sum $\sum_i w_i x_i^\mu$ is too high – so decrease it

The local rule:

$$\boxed{\Delta w_j \propto (t^\mu - y^\mu)\, x_j^\mu}$$

Perceptron learning rule



$$y = f(\mathbf{w} \cdot \mathbf{x}) \qquad \text{e.g.} \quad f(z) = 1/(1 + \exp(-z)), \quad f(z) = z$$

$$E = \frac{1}{2}(t - y)^2 \qquad t \text{ is target output}$$

$$\Delta w_j = -\epsilon \frac{\partial E}{\partial w_j} = -\epsilon \frac{\partial E}{\partial y} \frac{\partial y}{\partial w_j} =$$

This is the method of gradient descent with learning-rate parameter ϵ.

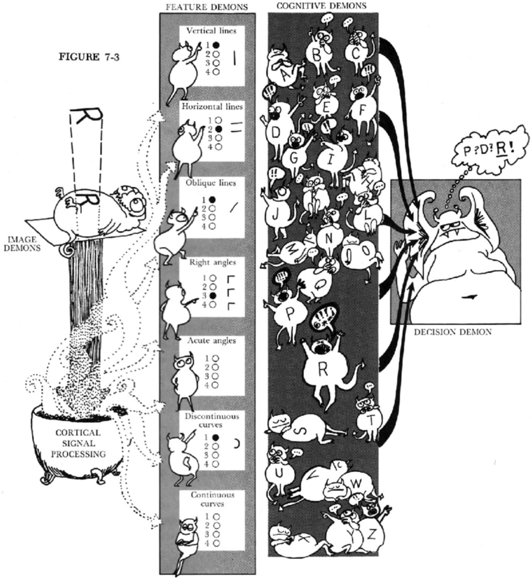
Perceptron pros and cons

- The *perceptron convergence theorem* (Dayan and Abbott, page 327) guarantees that solution will be found iff there is a linearly separable solution.
- Many complex problems are not linearly separable, e.g. XOR problem.

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

Multi-layer perceptrons (MLPs)

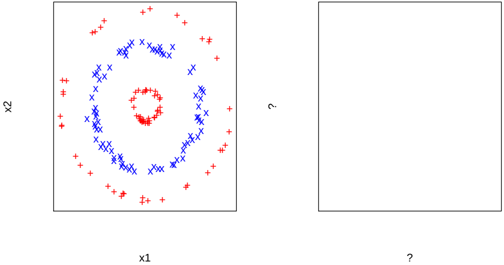
The importance of features / 1



(Lindsay and Norman's view of Selfridge's Pandemonium model, 1959).

The importance of features / 2

- Find the right features to make the task solvable:



- Engineering features by hand is hard.
- Neural networks learn features that they find important.

How many layers of features do you need?

One hidden layer is all you need **in theory** to make a “universal approximator” (Cybenko 1989; Hornik 1991).
With linear transfer functions how many layers do we need?

Neural networks and linear algebra

- Activation of a layer of neurons stored in a **vector**.
- Synapses from one layer to another stored in a **weight matrix**: $\mathbf{W}_{ji}$ is strength of connection from unit i in one layer to unit j in the next layer.
- "The single key fact about vectors and matrices is that each vector represents a point located in space, and a matrix moves that point to a different location. Everything else is just details." (Stone 2019, Appx. C).